Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/1.2.2.2-d-x-^m-a+b-x^2+c-x^4-^p

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [1126]. This is test number [39].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (1126)	% 0.00 (0)
Mathematica	% 100.00 (1126)	% 0.00 (0)
Maple	% 94.32 (1062)	% 5.68 (64)
Maxima	% 61.10 (688)	% 38.90 (438)
Fricas	% 75.13 (846)	% 24.87 (280)
Sympy	% 42.27 (476)	% 57.73 (650)
Giac	% 70.96 (799)	% 29.04 (327)
Mupad	% 61.72 (695)	% 38.28 (431)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

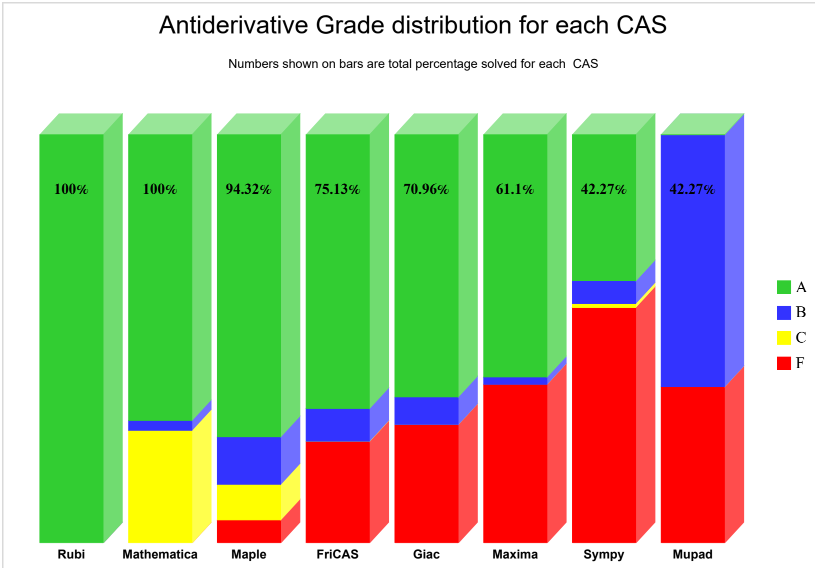
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

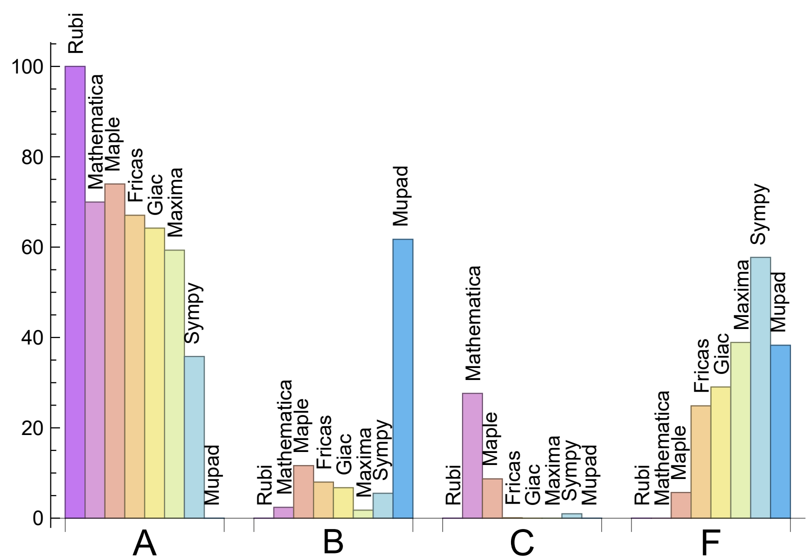
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	69.98	2.40	27.62	0.00
Maple	73.98	11.63	8.70	5.68
Maxima	59.33	1.78	0.00	38.90
Fricas	67.05	7.99	0.09	24.87
Sympy	35.79	5.51	0.98	57.73
Giac	64.21	6.75	0.00	29.04
Mupad	0.00	61.72	0.00	38.28

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	64	100.00 %	0.00 %	0.00 %
Maxima	438	85.39 %	0.00 %	14.61 %
Fricas	280	95.71 %	4.29 %	0.00 %
Sympy	650	80.77 %	19.23 %	0.00 %
Giac	327	89.91 %	0.31 %	9.79 %
Mupad	431	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

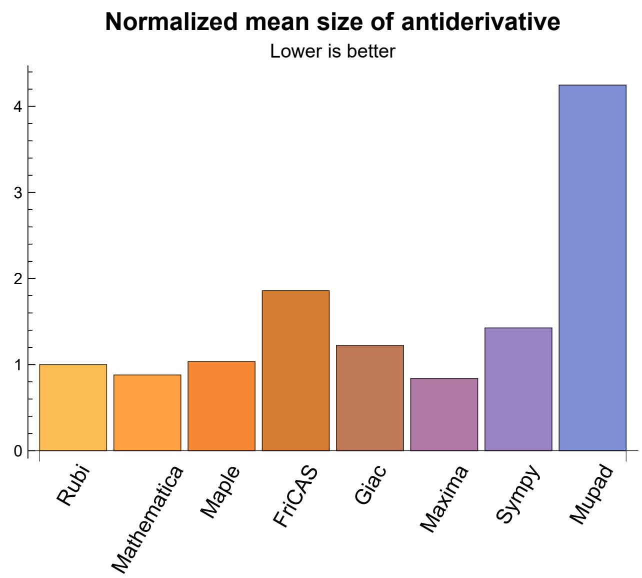
1.3 Performance

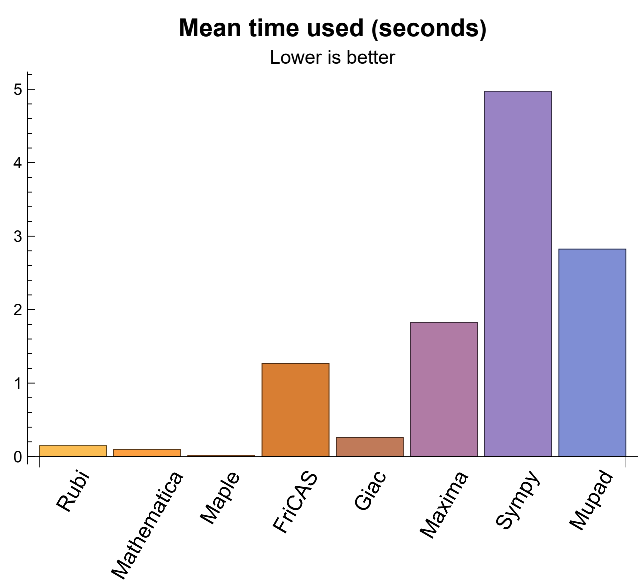
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.15	144.69	1.00	90.00	1.00
Mathematica	0.10	94.62	0.88	61.00	0.89
Maple	0.02	149.27	1.03	78.00	0.87
Maxima	1.82	90.16	0.84	55.00	0.88
Fricas	1.27	342.72	1.86	79.50	0.99
Sympy	4.97	108.84	1.43	52.00	0.98
Giac	0.26	182.51	1.23	69.00	0.87
Mupad	2.82	1459.63	4.25	66.00	0.88

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {14, 17, 18, 19, 23, 24, 25, 27, 28, 30, 31, 32, 36, 37, 38, 40, 41, 43, 44, 45, 46, 49, 50, 51, 53, 54, 56, 57, 58, 62, 63, 64, 78, 79, 81, 124, 125, 126, 128, 129, 130, 132, 133, 918, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command AboluteTiming[] was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command Usage was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's time.time() call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is ValueError. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://`

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

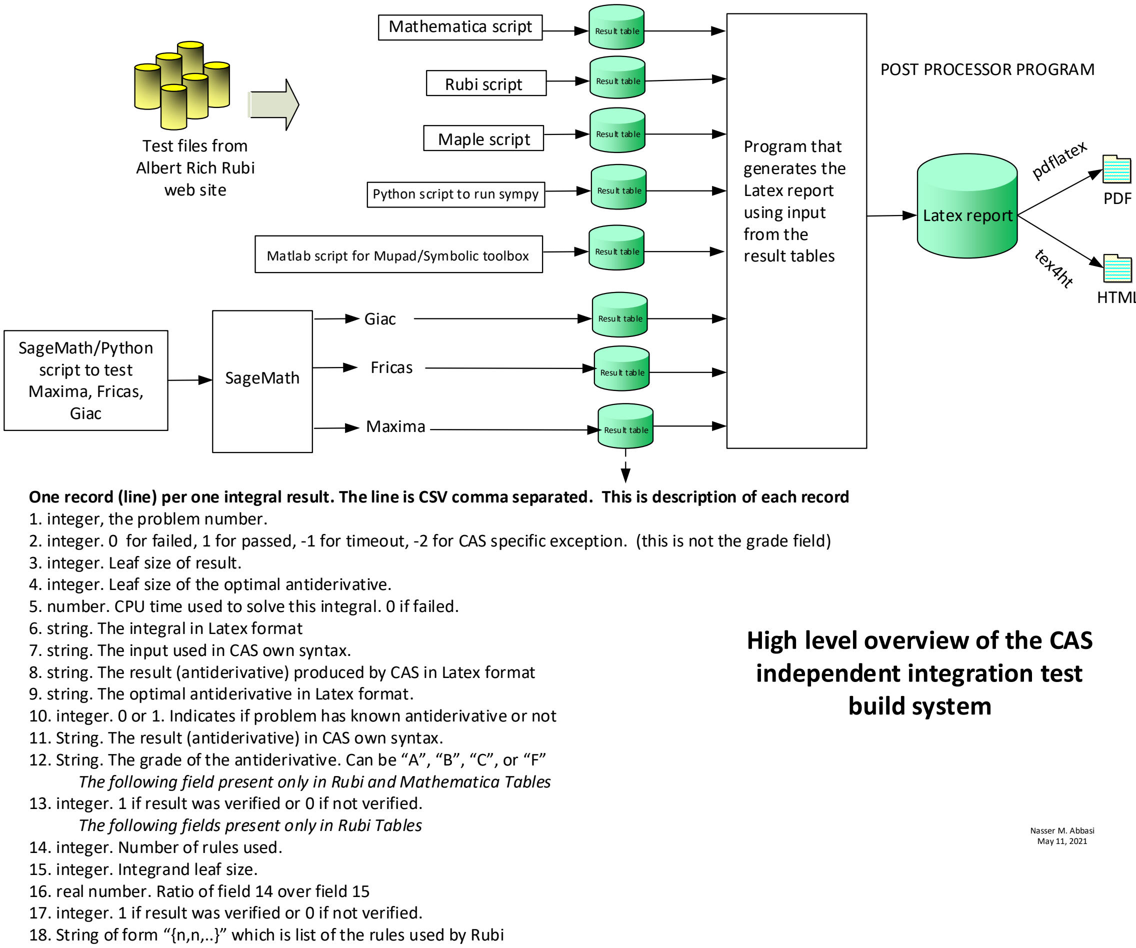
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 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B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 12, 21, 22, 33, 34, 42, 46, 47, 55, 60, 61, 76, 77, 92, 93, 94, 96, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 238, 239, 240, 241, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 256, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 327, 329, 331, 333, 339, 341, 343, 345, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 688, 690, 692, 696, 698, 700, 702, 704, 706, 710, 712, 714, 716, 718, 720, 722, 724, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 757, 759, 761, 763, 765, 769, 771, 773, 775, 777, 779, 781, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 915, 916, 917, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 955, 956, 957, 958, 959, 960, 961, 962, 968, 969, 970, 971, 972, 973, 974, 975, 981, 982, 983, 984, 985, 986, 987, 988, 994, 995, 996, 997, 998, 999, 1000, 1001, 1004, 1006, 1008, 1010, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1097, 1098, 1099, 1105, 1106, 1107, 1111, 1112, 1113, 1114, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126 }

B grade: { 11, 26, 39, 95, 97, 108, 109, 171, 438, 449, 467, 515, 1040, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1100, 1101, 1102, 1103, 1104, 1110 }

C grade: { 8, 10, 13, 14, 15, 16, 17, 18, 19, 20, 23, 24, 25, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 40, 41, 43, 44, 45, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 236, 237, 242, 243, 255, 257, 258, 259, 272, 283, 284, 285, 322, 323, 324, 325, 326, 328, 330, 332, 334, 335, 336, 337, 338, 340, 342, 344, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 659, 660, 661, 662, 663, 664, 687, 689, 691, 693, 694, 695, 697, 699, 701, 703, 705, 707, 708, 709, 711, 713, 715, 717, 719, 721, 723, 725, 726, 727, 754, 755, 756, 758, 760, 762, 764, 766, 767, 768, 770, 772, 774, 776, 778, 780, 782, 783, 784, 909, 910, 911, 912, 913, 914, 918, 919, 930, 931, 932, 933, 934, 935, 947, 948, 949, 950, 951, 952, 953, 954, 963, 964, 965, 966, 967, 976, 977, 978, 979, 980, 989, 990, 991, 992, 993, 1002, 1003, 1005, 1007, 1009, 1011, 1039, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1108, 1109, 1115, 1116, 1117 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 4, 5, 6, 7, 9, 12, 13, 14, 66, 76, 77, 78, 79, 81, 92, 93, 94, 95, 96, 97, 108, 109, 119, 122, 124, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 404, 405, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 592, 593, 594, 595, 596, 597, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 766, 768, 787, 791, 792, 793, 798, 799, 800, 801, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 857, 858, 859, 861, 862, 863, 864, 870, 877, 878, 888, 889, 890, 891, 892, 894, 895, 896, 897, 898, 899, 900, 901, 903, 904, 906, 907, 908, 909, 910, 915, 916, 917, 918, 920, 921, 922, 923, 924, 925, 930, 931, 932, 933, 934, 935, 940, 941, 942, 943, 947, 948, 949, 950, 951, 952, 953, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 999, 1000, 1001, 1002, 1004, 1006, 1008, 1010, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1107 }

B grade: { 8, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 125, 126, 127, 128, 129, 130, 131, 132, 133, 161, 171, 243, 270, 287, 403, 426, 438, 449, 451, 467, 496, 515, 591, 598, 757, 758, 759, 760, 761, 762, 763, 764, 765, 767, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 855, 856, 860, 865, 866, 867, 868, 869, 871, 872, 873, 874, 875, 876, 879, 880, 881, 882, 883, 884, 885, 886, 887, 893, 902, 905, 911, 912, 913, 914, 919, 926, 927, 928, 929, 936, 937, 938, 939, 944, 945, 946, 954, 981, 982, 998, 1039, 1105, 1106 }

C grade: { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 123, 1003, 1005, 1007, 1009, 1011, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088 }

F grade: { 3, 406, 407, 408, 659, 660, 661, 662, 663, 664, 788, 789, 790, 794, 795, 796, 797, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126 }

2.1.4 Maxima

A grade: { 12, 13, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 238, 239, 240, 241, 242, 243, 246, 247, 248, 249, 250, 251, 252, 253, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 403, 404, 405, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 497, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 517, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 761, 762, 763, 764, 773, 774, 775, 776, 777, 778, 779, 780, 785, 786, 787, 791, 792, 793, 798, 799, 800, 801, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 897, 898, 899, 900, 901, 906, 907, 908, 909, 910, 915, 916, 917, 968, 969, 970, 971, 972, 973, 974, 975, 994, 995, 996, 997, 999, 1001, 1004, 1008, 1010, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1105, 1106, 1107 }

B grade: { 11, 161, 171, 210, 244, 245, 426, 438, 449, 451, 467, 496, 498, 515, 516, 518, 519, 598, 647, 1006 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 234, 235, 236, 237, 254, 255, 256, 257, 258, 259, 270, 271, 272, 283, 284, 285, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 406, 407, 408, 659, 660, 661, 662, 663, 664, 757, 758, 759, 760, 765, 766, 767, 768, 769, 770, 771, 772, 781, 782, 783, 784, 788, 789, 790, 794, 795, 796, 797, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 902, 903, 904, 905, 911, 912, 913, 914, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 998, 1000, 1002, 1003, 1005, 1007, 1009, 1011, 1039, 1040, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 12, 13, 15, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 404, 405, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 497, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 517, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 787, 791, 792, 793, 798, 799, 800, 801, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 888, 889, 890, 891, 892, 897, 898, 899, 900, 901, 906, 907, 908, 909, 910, 911, 912, 915, 916, 917, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 955, 956, 957, 958, 959, 960, 961, 962, 968, 969, 970, 971, 972, 973, 974, 975, 981, 982, 983, 984, 985, 987, 988, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1004, 1006, 1008, 1010, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1107 }

B grade: { 8, 10, 11, 14, 161, 171, 210, 403, 426, 438, 449, 451, 467, 496, 498, 499, 500, 515, 516, 518, 519, 520, 521, 522, 598, 647, 785, 786, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 893, 894, 895, 896, 902, 903, 904, 905, 913, 914, 918, 986, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1072, 1073, 1074, 1075, 1076, 1077, 1105, 1106 }

C grade: { 287 }

F grade: { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 406, 407, 408, 659, 660, 661, 662, 663, 664, 788, 789, 790, 794, 795, 796, 797, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 930, 931, 932, 933, 934, 935, 947, 948, 949, 950, 951, 952, 953, 954, 963, 964, 965, 966, 967, 976, 977, 978, 979, 980, 989, 990, 991, 992, 993, 1003, 1005, 1007, 1009, 1011, 1039, 1040, 1071, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126 }

2.1.6 Sympy

A grade: { 8, 10, 12, 13, 14, 21, 34, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 179, 181, 183, 185, 187, 189, 190, 191, 192, 193, 194, 195, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 212, 213, 214, 215, 216, 217, 218, 219, 220, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 318, 319, 320, 321, 322, 323, 324, 325, 403, 404, 405, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 490, 491, 492, 493, 494, 495, 497, 499, 500, 501, 502, 503, 504, 505, 508, 509, 510, 511, 512, 513, 514, 517, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 691, 705, 723, 730, 752, 785, 786, 787, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 855, 856, 857, 858, 859, 860, 868, 869, 870, 893, 894, 895, 896, 899, 902, 903, 904, 905, 911, 912, 913, 914, 915, 916, 917, 918, 919, 1004, 1006, 1008, 1010, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1105, 1106, 1107 }

B grade: { 9, 11, 15, 148, 161, 171, 178, 180, 182, 184, 186, 188, 196, 198, 210, 211, 426, 438, 449, 451, 467, 469, 488, 489, 496, 498, 506, 507, 515, 516, 518, 519, 848, 849, 850, 851, 852, 853, 861, 862, 863, 864, 874, 875, 876, 877, 878, 883, 888, 889, 890, 891, 892, 897, 898, 900, 901, 906, 907, 908, 909, 910 }

C grade: { 47, 60, 71, 87, 102, 114, 1003, 1005, 1007, 1009, 1011 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 316, 317, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 406, 407, 408, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 686, 687, 688, 689, 690, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 724, 725, 726, 727, 728, 729, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 854, 865, 866, 867, 871, 872, 873, 879, 880, 881, 882, 884, 885, 886, 887, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1024, 1039, 1040, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126 }

2.1.7 Giac

A grade: { 1, 3, 4, 8, 9, 12, 13, 14, 15, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 273, 274, 276, 277, 281, 282, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 405, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 592, 593, 594, 595, 596, 597, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 645, 646, 647, 648, 649, 650, 651, 652, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 793, 799, 800, 801, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 861, 862, 863, 864, 865, 866, 873, 874, 875, 876, 877, 878, 879, 880, 888, 889, 890, 891, 892, 897, 898, 899, 900, 901, 906, 907, 908, 909, 910, 911, 912, 913, 915, 916, 917, 918, 919, 920, 921, 922, 923, 925, 955, 956, 957, 958, 959, 960, 968, 969, 970, 971, 972, 973, 981, 982, 983, 984, 985, 986, 987, 988, 995, 996, 997, 998, 999, 1001, 1004, 1006, 1008, 1010, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061 }

B grade: { 11, 161, 171, 226, 227, 228, 244, 245, 246, 247, 248, 403, 404, 426, 438, 449, 451, 467, 515, 567, 591, 598, 785, 786, 787, 791, 792, 798, 855, 856, 857, 858, 859, 860, 867, 868, 869, 870, 871, 872, 881, 882, 883, 884, 885, 886, 887, 893, 894, 895, 896, 902, 903, 904, 905, 914, 926, 927, 928, 929, 936, 937, 938, 939, 942, 943, 944, 945, 946, 961, 962, 974, 975, 1105, 1106, 1107 }

C grade: { }

F grade: { 2, 5, 6, 7, 10, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 268, 271, 272, 275, 278, 279, 280, 283, 284, 285, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 406, 407, 408, 640, 641, 642, 643, 644, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 788, 789, 790, 794, 795, 796, 797, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 924, 930, 931, 932, 933, 934, 935, 940, 941, 947, 948, 949, 950, 951, 952, 953, 954, 963, 964, 965, 966, 967, 976, 977, 978, 979, 980, 989, 990, 991, 992, 993, 994, 1000, 1002, 1003, 1005, 1007, 1009, 1011, 1039, 1040, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126 }

2.1.8 Mupad

A grade: { }

B grade: { 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 21, 34, 47, 60, 71, 87, 102, 114, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 238, 239, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 262, 263, 264, 265, 266, 267, 268, 269, 271, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 403, 404, 405, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 554, 555, 556, 557, 561, 562, 567, 568, 569, 570, 571, 580, 581, 582, 583, 584, 591, 598, 599, 600, 601, 602, 603, 604, 617, 618, 619, 620, 621, 622, 623, 625, 626, 627, 628, 636, 637, 647, 648, 649, 650, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 731, 732, 733, 734, 738, 739, 740, 741, 745, 746, 747, 748, 785, 786, 787, 798, 799, 800, 801, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 938, 939, 957, 958, 959, 960, 970, 971, 972, 973, 983, 984, 985, 994, 995, 996, 997, 999, 1000, 1001, 1004, 1007, 1008, 1009, 1010, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1022, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1105, 1106, 1107 }

C grade: { }

F grade: { 1, 2, 3, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 225, 235, 236, 237, 240, 241, 242, 243, 254, 255, 256, 257, 258, 259, 260, 261, 270, 272, 273, 274, 283, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 406, 407, 408, 550, 551, 552, 553, 558, 559, 560, 563, 564, 565, 566, 572, 573, 574, 575, 576, 577, 578, 579, 585, 586, 587, 588, 589, 590, 592, 593, 594, 595, 596, 597, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 624, 629, 630, 631, 632, 633, 634, 635, 638, 639, 640, 641, 642, 643, 644, 645, 646, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 728, 729, 730, 735, 736, 737, 742, 743, 744, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 961, 962, 963, 964, 965, 966, 967, 968, 969, 974, 975, 976, 977, 978, 979, 980, 981, 982, 986, 987, 988, 989, 990, 991, 992, 993, 998, 1002, 1003, 1005, 1006, 1011, 1012, 1021, 1023, 1024, 1039, 1040, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	97	77	0	177	0	87	-1
normalized size	1	1.00	0.76	0.60	0.00	1.38	0.00	0.68	-0.01
time (sec)	N/A	0.031	0.075	0.031	0.000	0.734	0.000	0.440	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	59	58	0	147	0	0	-1
normalized size	1	1.00	0.65	0.64	0.00	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.020	0.037	0.014	0.000	0.968	0.000	0.000	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	49	0	0	90	0	24	-1
normalized size	1	1.00	0.82	0.00	0.00	1.50	0.00	0.40	-0.02
time (sec)	N/A	0.012	0.015	0.035	0.000	1.126	0.000	0.216	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	25	33	0	34	0	19	34
normalized size	1	1.00	0.74	0.97	0.00	1.00	0.00	0.56	1.00
time (sec)	N/A	0.008	0.011	0.003	0.000	0.861	0.000	0.260	4.139

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	70	40	44	0	58	0	0	45
normalized size	1	1.03	0.59	0.65	0.00	0.85	0.00	0.00	0.66
time (sec)	N/A	0.016	0.012	0.003	0.000	1.139	0.000	0.000	4.205

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	107	51	55	0	80	0	0	56
normalized size	1	1.02	0.49	0.52	0.00	0.76	0.00	0.00	0.53
time (sec)	N/A	0.024	0.015	0.004	0.000	0.564	0.000	0.000	4.210

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	148	62	66	0	102	0	0	141
normalized size	1	1.10	0.46	0.49	0.00	0.76	0.00	0.00	1.04
time (sec)	N/A	0.043	0.021	0.005	0.000	0.822	0.000	0.000	4.129

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	81	1099	0	583	63	75	872
normalized size	1	1.00	0.27	3.68	0.00	1.95	0.21	0.25	2.92
time (sec)	N/A	0.312	0.043	0.133	0.000	0.801	0.802	0.162	4.375

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	43	32	0	269	257	31	85
normalized size	1	1.00	0.91	0.68	0.00	5.72	5.47	0.66	1.81
time (sec)	N/A	0.026	0.023	0.009	0.000	0.812	0.633	0.149	0.102

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	52	1073	0	613	48	0	469
normalized size	1	1.00	0.17	3.59	0.00	2.05	0.16	0.00	1.57
time (sec)	N/A	0.308	0.033	0.106	0.000	0.941	0.581	0.000	4.356

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	37	26	25	25	26	29	11
normalized size	1	1.00	2.18	1.53	1.47	1.47	1.53	1.71	0.65
time (sec)	N/A	0.008	0.007	0.009	1.370	0.850	0.183	0.203	0.036

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	18	17	17	20	17	17
normalized size	1	1.00	1.00	0.75	0.71	0.71	0.83	0.71	0.71
time (sec)	N/A	0.015	0.013	0.006	2.997	0.774	0.161	0.169	4.116

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	91	54	53	53	70	53	83
normalized size	1	1.00	1.36	0.81	0.79	0.79	1.04	0.79	1.24
time (sec)	N/A	0.050	0.071	0.005	3.036	0.792	0.219	0.195	4.147

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	77	57	0	159	63	56	47
normalized size	1	1.00	1.04	0.77	0.00	2.15	0.85	0.76	0.64
time (sec)	N/A	0.049	0.070	0.036	0.000	0.909	0.206	0.189	4.186

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	41	386	0	247	899	143	210
normalized size	1	1.00	0.23	2.19	0.00	1.40	5.11	0.81	1.19
time (sec)	N/A	0.162	0.038	0.109	0.000	0.822	1.133	0.534	4.205

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	65	51	0	0	0	0	-1
normalized size	1	1.00	6.50	5.10	0.00	0.00	0.00	0.00	-0.10
time (sec)	N/A	0.016	0.027	0.039	0.000	0.553	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	49	84	0	0	0	0	-1
normalized size	1	1.00	1.02	1.75	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.134	0.058	0.107	0.000	0.938	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	53	80	0	0	0	0	-1
normalized size	1	1.00	1.10	1.67	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.104	0.061	0.097	0.000	0.621	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	49	84	0	0	0	0	-1
normalized size	1	1.00	1.11	1.91	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.067	0.046	0.097	0.000	0.780	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	63	41	0	0	0	0	-1
normalized size	1	1.00	5.25	3.42	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.012	0.024	0.015	0.000	0.781	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	54	0	0	37	0	16
normalized size	1	1.00	1.00	3.00	0.00	0.00	2.06	0.00	0.89
time (sec)	N/A	0.010	0.022	0.040	0.000	0.875	0.685	0.000	4.200

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	49	0	0	0	0	-1
normalized size	1	1.00	1.00	2.45	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.011	0.026	0.031	0.000	0.811	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	51	84	0	0	0	0	-1
normalized size	1	1.00	1.21	2.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.065	0.044	0.088	0.000	0.821	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	55	80	0	0	0	0	-1
normalized size	1	1.00	1.20	1.74	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.089	0.062	0.094	0.000	0.781	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	49	84	0	0	0	0	-1
normalized size	1	1.00	1.02	1.75	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.094	0.060	0.090	0.000	0.842	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	54	50	0	0	0	0	-1
normalized size	1	1.00	3.00	2.78	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.015	0.023	0.026	0.000	0.785	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	52	84	0	0	0	0	-1
normalized size	1	1.00	1.16	1.87	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.065	0.046	0.107	0.000	0.897	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	84	0	0	0	0	-1
normalized size	1	1.00	0.98	1.91	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.133	0.052	0.107	0.000	0.786	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	65	51	0	0	0	0	-1
normalized size	1	1.00	6.50	5.10	0.00	0.00	0.00	0.00	-0.10
time (sec)	N/A	0.011	0.027	0.027	0.000	0.767	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	51	84	0	0	0	0	-1
normalized size	1	1.00	1.16	1.91	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.077	0.055	0.078	0.000	0.837	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	50	84	0	0	0	0	-1
normalized size	1	1.00	1.11	1.87	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.067	0.051	0.079	0.000	0.821	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	49	84	0	0	0	0	-1
normalized size	1	1.00	1.11	1.91	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.062	0.045	0.079	0.000	0.784	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	47	0	0	0	0	-1
normalized size	1	1.00	1.00	2.35	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.011	0.028	0.027	0.000	0.871	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	54	0	0	37	0	16
normalized size	1	1.00	1.00	3.00	0.00	0.00	2.06	0.00	0.89
time (sec)	N/A	0.006	0.022	0.039	0.000	1.018	0.691	0.000	4.197

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	65	43	0	0	0	0	-1
normalized size	1	1.00	5.42	3.58	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.011	0.023	0.011	0.000	0.665	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	51	84	0	0	0	0	-1
normalized size	1	1.00	1.21	2.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.039	0.076	0.000	0.695	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	52	84	0	0	0	0	-1
normalized size	1	1.00	1.21	1.95	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.044	0.072	0.000	0.773	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	51	84	0	0	0	0	-1
normalized size	1	1.00	1.16	1.91	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.068	0.048	0.076	0.000	0.827	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	54	50	0	0	0	0	-1
normalized size	1	1.00	3.00	2.78	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.012	0.026	0.021	0.000	0.573	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	45	84	0	0	0	0	-1
normalized size	1	1.00	1.07	2.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.096	0.060	0.096	0.000	0.552	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	52	84	0	0	0	0	-1
normalized size	1	1.00	1.16	1.87	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.048	0.046	0.093	0.000	0.635	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	53	0	0	0	0	-1
normalized size	1	1.00	0.81	0.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.008	0.022	0.009	0.000	0.865	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	81	84	0	0	0	0	-1
normalized size	1	1.00	0.57	0.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.053	0.031	0.000	0.858	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	83	84	0	0	0	0	-1
normalized size	1	1.00	0.57	0.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.063	0.035	0.000	0.822	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	83	84	0	0	0	0	-1
normalized size	1	1.00	0.59	0.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.050	0.035	0.000	0.850	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	48	43	0	0	0	0	-1
normalized size	1	1.00	0.76	0.68	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.008	0.028	0.011	0.000	0.879	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	40	56	0	0	34	0	31
normalized size	1	1.00	0.35	0.49	0.00	0.00	0.30	0.00	0.27
time (sec)	N/A	0.016	0.022	0.022	0.000	0.802	0.728	0.000	0.080

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	60	53	0	0	0	0	-1
normalized size	1	1.00	0.92	0.82	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.007	0.023	0.031	0.000	0.801	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	81	84	0	0	0	0	-1
normalized size	1	1.00	0.55	0.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.039	0.035	0.000	0.825	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	81	84	0	0	0	0	-1
normalized size	1	1.00	0.53	0.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.065	0.034	0.000	0.807	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	81	84	0	0	0	0	-1
normalized size	1	1.00	0.55	0.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.065	0.031	0.000	0.689	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	53	0	0	0	0	-1
normalized size	1	1.00	1.03	0.84	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.007	0.025	0.018	0.000	0.835	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	80	84	0	0	0	0	-1
normalized size	1	1.00	0.54	0.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.052	0.033	0.000	0.848	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	77	84	0	0	0	0	-1
normalized size	1	1.00	0.52	0.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.061	0.032	0.000	0.795	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	53	0	0	0	0	-1
normalized size	1	1.00	0.81	0.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.008	0.023	0.006	0.000	0.790	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	83	84	0	0	0	0	-1
normalized size	1	1.00	0.56	0.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.062	0.036	0.000	0.722	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	80	84	0	0	0	0	-1
normalized size	1	1.00	0.55	0.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.068	0.033	0.000	0.952	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	83	84	0	0	0	0	-1
normalized size	1	1.00	0.58	0.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.055	0.034	0.000	0.810	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	51	0	0	0	0	-1
normalized size	1	1.00	1.00	0.81	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.007	0.025	0.030	0.000	0.810	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	40	56	0	0	34	0	31
normalized size	1	1.00	0.36	0.50	0.00	0.00	0.30	0.00	0.28
time (sec)	N/A	0.015	0.024	0.020	0.000	0.893	0.730	0.000	0.082

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	45	0	0	0	0	-1
normalized size	1	1.00	0.78	0.69	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.007	0.024	0.010	0.000	0.854	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	81	84	0	0	0	0	-1
normalized size	1	1.00	0.54	0.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.047	0.032	0.000	0.794	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	78	84	0	0	0	0	-1
normalized size	1	1.00	0.51	0.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.057	0.032	0.000	0.855	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	83	84	0	0	0	0	-1
normalized size	1	1.00	0.54	0.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.064	0.033	0.000	0.791	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	53	0	0	0	0	-1
normalized size	1	1.00	1.03	0.84	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.006	0.025	0.018	0.000	0.804	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	58	44	0	0	0	0	-1
normalized size	1	1.00	1.12	0.85	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.007	0.025	0.017	0.000	0.750	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	-1
normalized size	1	1.00	1.60	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.084	0.158	0.000	0.771	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	0	0	0	-1
normalized size	1	1.00	1.57	0.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.113	0.131	0.000	0.808	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	0	0	0	-1
normalized size	1	1.00	1.57	0.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.083	0.135	0.000	0.909	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	142	85	0	0	0	0	-1
normalized size	1	1.00	1.61	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	0.080	0.133	0.000	0.701	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	25	66	0	0	36	0	16
normalized size	1	1.00	0.35	0.92	0.00	0.00	0.50	0.00	0.22
time (sec)	N/A	0.007	0.026	0.056	0.000	0.828	0.706	0.000	0.089

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	-1
normalized size	1	1.00	1.60	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	0.078	0.115	0.000	0.879	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	-1
normalized size	1	1.00	1.60	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	0.075	0.118	0.000	0.834	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	-1
normalized size	1	1.00	1.60	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.109	0.116	0.000	0.840	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	144	87	0	0	0	0	-1
normalized size	1	1.00	1.64	0.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.012	0.081	0.117	0.000	0.814	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	42	0	0	0	0	-1
normalized size	1	1.00	0.58	0.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	0.024	0.010	0.000	0.872	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	85	82	0	0	0	0	-1
normalized size	1	1.00	0.94	0.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.084	0.085	0.000	0.744	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	97	82	0	0	0	0	-1
normalized size	1	1.00	0.88	0.75	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.071	0.097	0.000	0.768	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	98	82	0	0	0	0	-1
normalized size	1	1.00	0.89	0.75	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.080	0.091	0.000	0.843	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	61	50	0	0	0	0	-1
normalized size	1	1.00	1.02	0.83	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.007	0.026	0.016	0.000	0.767	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	90	82	0	0	0	0	-1
normalized size	1	1.00	0.87	0.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.058	0.084	0.000	0.912	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	58	50	0	0	0	0	-1
normalized size	1	1.00	1.12	0.96	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.006	0.025	0.015	0.000	0.846	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	-1
normalized size	1	1.00	1.60	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.084	0.105	0.000	0.812	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	142	87	0	0	0	0	-1
normalized size	1	1.00	1.54	0.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.101	0.104	0.000	0.826	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	0	0	0	-1
normalized size	1	1.00	1.57	0.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.081	0.102	0.000	0.942	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	140	85	0	0	0	0	-1
normalized size	1	1.00	1.59	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.012	0.071	0.106	0.000	0.929	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	25	66	0	0	36	0	16
normalized size	1	1.00	0.35	0.92	0.00	0.00	0.50	0.00	0.22
time (sec)	N/A	0.007	0.027	0.048	0.000	0.814	1.186	0.000	0.088

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	142	87	0	0	0	0	-1
normalized size	1	1.00	1.58	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.065	0.102	0.000	0.800	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	-1
normalized size	1	1.00	1.60	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.074	0.100	0.000	0.805	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	142	87	0	0	0	0	-1
normalized size	1	1.00	1.58	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.009	0.097	0.103	0.000	0.818	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	144	87	0	0	0	0	-1
normalized size	1	1.00	1.64	0.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.081	0.100	0.000	0.896	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	42	0	0	0	0	-1
normalized size	1	1.00	0.58	0.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.009	0.025	0.013	0.000	0.817	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	81	82	0	0	0	0	-1
normalized size	1	1.00	0.90	0.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.076	0.067	0.000	0.757	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	58	49	0	0	0	0	-1
normalized size	1	1.00	0.63	0.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	0.022	0.011	0.000	0.752	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	58	48	0	0	0	0	-1
normalized size	1	1.00	3.05	2.53	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.012	0.022	0.010	0.000	0.783	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	81	82	0	0	0	0	-1
normalized size	1	1.00	1.84	1.86	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.032	0.024	0.000	0.819	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	53	50	0	0	0	0	-1
normalized size	1	1.00	3.79	3.57	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.012	0.025	0.012	0.000	1.043	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	144	87	0	0	0	0	-1
normalized size	1	1.00	1.64	0.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.054	0.053	0.000	0.932	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	142	87	0	0	0	0	-1
normalized size	1	1.00	1.58	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.093	0.058	0.000	0.974	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	-1
normalized size	1	1.00	1.60	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.071	0.068	0.000	0.889	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	140	85	0	0	0	0	-1
normalized size	1	1.00	1.59	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.009	0.064	0.049	0.000	1.115	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	47	66	0	0	39	0	31
normalized size	1	1.00	0.65	0.92	0.00	0.00	0.54	0.00	0.43
time (sec)	N/A	0.007	0.026	0.022	0.000	0.845	0.713	0.000	4.290

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	142	87	0	0	0	0	-1
normalized size	1	1.00	1.58	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.062	0.047	0.000	0.795	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	0	0	0	-1
normalized size	1	1.00	1.57	0.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.077	0.048	0.000	0.878	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	142	87	0	0	0	0	-1
normalized size	1	1.00	1.54	0.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.097	0.049	0.000	0.878	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	-1
normalized size	1	1.00	1.60	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.009	0.085	0.048	0.000	0.925	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	63	44	0	0	0	0	-1
normalized size	1	1.00	1.19	0.83	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.024	0.013	0.000	0.978	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	85	82	0	0	0	0	-1
normalized size	1	1.00	2.02	1.95	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.077	0.025	0.000	0.870	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	53	42	0	0	0	0	-1
normalized size	1	1.00	8.83	7.00	0.00	0.00	0.00	0.00	-0.17
time (sec)	N/A	0.010	0.025	0.008	0.000	0.911	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	144	87	0	0	0	0	-1
normalized size	1	1.00	1.64	0.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.078	0.048	0.000	0.864	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	-1
normalized size	1	1.00	1.60	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.009	0.108	0.051	0.000	0.700	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	-1
normalized size	1	1.00	1.60	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	0.075	0.063	0.000	0.786	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	142	85	0	0	0	0	-1
normalized size	1	1.00	1.61	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.009	0.077	0.047	0.000	0.743	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	47	66	0	0	39	0	31
normalized size	1	1.00	0.65	0.92	0.00	0.00	0.54	0.00	0.43
time (sec)	N/A	0.008	0.028	0.020	0.000	0.586	0.703	0.000	4.176

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	-1
normalized size	1	1.00	1.60	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.077	0.046	0.000	0.630	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	0	0	0	-1
normalized size	1	1.00	1.57	0.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.079	0.046	0.000	0.819	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	0	0	0	-1
normalized size	1	1.00	1.57	0.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.112	0.046	0.000	0.753	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	-1
normalized size	1	1.00	1.60	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.009	0.081	0.046	0.000	0.922	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	63	50	0	0	0	0	-1
normalized size	1	1.00	1.21	0.96	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.024	0.013	0.000	0.851	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	0	0	0	-1
normalized size	1	1.00	1.57	0.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.112	0.115	0.000	0.848	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	147	87	0	0	0	0	-1
normalized size	1	1.00	1.63	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.085	0.124	0.000	0.736	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	58	44	0	0	0	0	-1
normalized size	1	1.00	1.12	0.85	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.007	0.020	0.000	0.000	0.790	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	48	0	0	0	0	-1
normalized size	1	1.00	1.00	0.83	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.007	0.022	0.014	0.000	0.826	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	103	76	0	0	0	0	-1
normalized size	1	1.00	0.95	0.70	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.082	0.084	0.000	0.840	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	55	80	0	0	0	0	-1
normalized size	1	1.00	1.15	1.67	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.098	0.063	0.099	0.000	0.870	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	52	76	0	0	0	0	-1
normalized size	1	1.00	1.16	1.69	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.062	0.046	0.092	0.000	0.845	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	65	51	0	0	0	0	-1
normalized size	1	1.00	6.50	5.10	0.00	0.00	0.00	0.00	-0.10
time (sec)	N/A	0.012	0.023	0.000	0.000	0.549	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	56	80	0	0	0	0	-1
normalized size	1	1.00	1.14	1.63	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.104	0.061	0.101	0.000	0.571	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	52	80	0	0	0	0	-1
normalized size	1	1.00	1.08	1.67	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.104	0.059	0.108	0.000	0.756	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	56	80	0	0	0	0	-1
normalized size	1	1.00	1.14	1.63	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.078	0.049	0.100	0.000	0.790	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	65	43	0	0	0	0	-1
normalized size	1	1.00	5.42	3.58	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.012	0.026	0.016	0.000	0.820	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	52	76	0	0	0	0	-1
normalized size	1	1.00	1.16	1.69	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.070	0.044	0.097	0.000	0.820	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	56	80	0	0	0	0	-1
normalized size	1	1.00	1.14	1.63	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.079	0.049	0.102	0.000	0.583	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.006	0.002	0.001	1.330	0.581	0.074	0.164	0.023

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.006	0.002	0.001	1.300	0.487	0.065	0.166	0.021

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.003	0.000	0.001	1.363	0.835	0.063	0.153	0.020

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.005	0.001	0.001	1.306	0.528	0.067	0.154	0.020

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
normalized size	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.004	0.000	0.000	1.263	0.691	0.065	0.147	0.017

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	11	10	14	11
normalized size	1	1.00	1.00	0.92	1.08	0.85	0.77	1.08	0.85
time (sec)	N/A	0.005	0.001	0.003	1.347	0.830	0.099	0.147	0.022

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	13	5	10	10
normalized size	1	1.00	1.00	1.10	1.00	1.30	0.50	1.00	1.00
time (sec)	N/A	0.005	0.001	0.005	1.334	0.787	0.103	0.162	0.024

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	17	10	20	11
normalized size	1	1.00	1.00	0.92	1.08	1.31	0.77	1.54	0.85
time (sec)	N/A	0.006	0.002	0.007	1.351	0.766	0.117	0.149	0.039

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	14	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	0.87
time (sec)	N/A	0.006	0.002	0.005	1.296	0.785	0.125	0.181	0.027

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.76
time (sec)	N/A	0.006	0.003	0.004	1.315	0.548	0.131	0.187	0.027

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15
normalized size	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.006	0.002	0.005	1.246	0.821	0.154	0.168	0.028

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.013	0.002	0.001	1.378	0.650	0.072	0.158	0.037

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.025	0.001	0.001	1.350	0.796	0.072	0.195	0.036

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.017	0.001	0.001	1.316	0.867	0.073	0.168	0.035

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	25	24	24	24	24	24
normalized size	1	1.00	1.00	1.56	1.50	1.50	1.50	1.50	1.50
time (sec)	N/A	0.009	0.002	0.001	1.311	0.821	0.097	0.152	0.032

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
normalized size	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.013	0.001	0.001	1.353	0.776	0.075	0.180	0.033

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	24	21	20	24	21
normalized size	1	1.00	1.00	0.96	1.04	0.91	0.87	1.04	0.91
time (sec)	N/A	0.019	0.001	0.003	1.346	0.800	0.113	0.149	0.029

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	25	19	22	22
normalized size	1	1.00	1.00	0.96	0.92	1.04	0.79	0.92	0.92
time (sec)	N/A	0.017	0.001	0.004	1.314	0.777	0.113	0.151	0.035

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	24	27	24	32	23
normalized size	1	1.00	1.00	0.89	0.89	1.00	0.89	1.19	0.85
time (sec)	N/A	0.021	0.001	0.005	1.328	0.762	0.147	0.168	0.032

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	22	26	22	22	24
normalized size	1	1.00	1.00	0.96	0.96	1.13	0.96	0.96	1.04
time (sec)	N/A	0.018	0.001	0.007	1.286	0.731	0.165	0.160	0.027

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	26	28	24	34	24
normalized size	1	1.00	1.00	0.96	1.08	1.17	1.00	1.42	1.00
time (sec)	N/A	0.019	0.001	0.005	1.315	0.850	0.190	0.180	0.045

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	26	26	27	26	25
normalized size	1	1.00	1.00	0.89	0.93	0.93	0.96	0.93	0.89
time (sec)	N/A	0.016	0.001	0.004	1.304	0.736	0.198	0.165	0.036

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	30	25	24	24	26	24	26
normalized size	1	1.00	1.58	1.32	1.26	1.26	1.37	1.26	1.37
time (sec)	N/A	0.010	0.001	0.006	1.227	0.691	0.212	0.174	0.036

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26
normalized size	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87
time (sec)	N/A	0.016	0.001	0.005	1.367	0.841	0.231	0.155	0.037

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81
time (sec)	N/A	0.024	0.002	0.001	1.304	0.649	0.082	0.148	0.044

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	43	36	35	35	37	35	35
normalized size	1	1.00	1.26	1.06	1.03	1.03	1.09	1.03	1.03
time (sec)	N/A	0.039	0.002	0.001	1.322	0.787	0.077	0.177	0.043

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	39	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.021	0.002	0.002	1.344	0.841	0.086	0.166	0.043

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	36	35	35	37	35	35
normalized size	1	1.00	1.00	2.25	2.19	2.19	2.31	2.19	2.19
time (sec)	N/A	0.009	0.002	0.001	1.278	0.782	0.082	0.149	0.042

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	31	32	31	31
normalized size	1	1.00	1.00	0.91	0.89	0.89	0.91	0.89	0.89
time (sec)	N/A	0.017	0.001	0.000	1.318	0.800	0.081	0.162	0.040

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	36	33	37	36	33
normalized size	1	1.00	1.00	0.87	0.92	0.85	0.95	0.92	0.85
time (sec)	N/A	0.026	0.004	0.003	1.327	0.746	0.125	0.153	0.036

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	32	36	29	32	32
normalized size	1	1.00	1.00	0.97	0.94	1.06	0.85	0.94	0.94
time (sec)	N/A	0.019	0.004	0.004	1.378	0.834	0.123	0.151	0.042

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	36	38	37	46	34
normalized size	1	1.00	1.00	0.88	0.90	0.95	0.92	1.15	0.85
time (sec)	N/A	0.029	0.007	0.007	1.266	0.811	0.170	0.158	0.037

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	34	36	36	34	36
normalized size	1	1.00	1.00	0.92	0.92	0.97	0.97	0.92	0.97
time (sec)	N/A	0.019	0.004	0.006	1.286	0.776	0.168	0.167	0.038

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	37	39	37	46	37
normalized size	1	1.00	1.00	0.88	0.92	0.98	0.92	1.15	0.92
time (sec)	N/A	0.026	0.004	0.006	1.343	0.624	0.226	0.150	0.035

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	33	37	34	33	34
normalized size	1	1.00	1.00	0.97	0.97	1.09	1.00	0.97	1.00
time (sec)	N/A	0.020	0.005	0.006	1.303	0.738	0.232	0.166	0.032

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	39	39	37	47	36
normalized size	1	1.00	1.00	0.87	1.00	1.00	0.95	1.21	0.92
time (sec)	N/A	0.026	0.004	0.007	1.319	0.767	0.292	0.156	0.047

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	36	37	37	39	37	35
normalized size	1	1.00	1.00	0.92	0.95	0.95	1.00	0.95	0.90
time (sec)	N/A	0.021	0.004	0.006	1.347	0.702	0.282	0.163	0.029

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	43	36	35	35	37	35	37
normalized size	1	1.00	2.26	1.89	1.84	1.84	1.95	1.84	1.95
time (sec)	N/A	0.010	0.006	0.005	1.338	0.763	0.311	0.152	0.030

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	37	37	39	37	37
normalized size	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.021	0.004	0.006	1.325	0.814	0.304	0.173	0.032

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	43	36	37	37	39	37	37
normalized size	1	1.00	1.08	0.90	0.92	0.92	0.98	0.92	0.92
time (sec)	N/A	0.023	0.004	0.005	1.313	0.704	0.325	0.150	0.033

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	60	60	148	107	65	54
normalized size	1	1.00	1.00	0.88	0.88	2.18	1.57	0.96	0.79
time (sec)	N/A	0.038	0.026	0.008	3.012	0.778	0.225	0.189	0.032

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	46	46	45	44	47	45
normalized size	1	1.00	1.00	0.87	0.87	0.85	0.83	0.89	0.85
time (sec)	N/A	0.045	0.006	0.004	1.329	0.580	0.180	0.159	0.047

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	49	50	126	95	55	43
normalized size	1	1.00	1.00	0.89	0.91	2.29	1.73	1.00	0.78
time (sec)	N/A	0.033	0.027	0.004	2.881	0.826	0.209	0.184	0.052

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	34	33	32	35	33
normalized size	1	1.00	1.00	0.88	0.85	0.82	0.80	0.88	0.82
time (sec)	N/A	0.034	0.006	0.003	1.334	0.844	0.170	0.155	0.046

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	37	99	80	40	32
normalized size	1	1.00	1.00	0.90	0.88	2.36	1.90	0.95	0.76
time (sec)	N/A	0.027	0.019	0.003	2.950	0.837	0.194	0.174	0.047

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	22	20	24	22
normalized size	1	1.00	1.00	0.89	0.85	0.81	0.74	0.89	0.81
time (sec)	N/A	0.027	0.005	0.002	1.313	1.058	0.166	0.157	0.036

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	26	82	56	26	23
normalized size	1	1.00	1.00	0.87	0.84	2.65	1.81	0.84	0.74
time (sec)	N/A	0.018	0.008	0.004	2.968	0.895	0.172	0.169	0.036

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.009	0.002	0.001	1.352	1.157	0.129	0.149	0.029

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	16
normalized size	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67
time (sec)	N/A	0.012	0.004	0.003	2.968	0.704	0.151	0.151	4.198

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	23	18	15	22	18
normalized size	1	1.00	1.00	0.95	1.05	0.82	0.68	1.00	0.82
time (sec)	N/A	0.016	0.005	0.005	1.365	0.844	0.226	0.168	0.064

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	30	29	82	65	29	26
normalized size	1	1.00	1.00	0.88	0.85	2.41	1.91	0.85	0.76
time (sec)	N/A	0.014	0.012	0.005	2.884	0.622	0.190	0.165	4.273

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	33	33	31	43	31
normalized size	1	1.00	1.00	0.91	0.94	0.94	0.89	1.23	0.89
time (sec)	N/A	0.028	0.007	0.006	1.365	0.722	0.280	0.150	0.059

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	39	40	106	87	40	37
normalized size	1	1.00	1.00	0.91	0.93	2.47	2.02	0.93	0.86
time (sec)	N/A	0.023	0.020	0.007	2.961	0.738	0.247	0.168	4.144

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	47	45	42	57	46
normalized size	1	1.00	1.00	0.90	0.96	0.92	0.86	1.16	0.94
time (sec)	N/A	0.035	0.007	0.006	1.317	0.837	0.356	0.172	0.060

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	52	52	132	100	52	48
normalized size	1	1.00	1.00	0.90	0.90	2.28	1.72	0.90	0.83
time (sec)	N/A	0.036	0.026	0.007	2.888	0.496	0.282	0.166	0.052

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	58	58	56	70	58
normalized size	1	1.00	1.00	0.89	0.92	0.92	0.89	1.11	0.92
time (sec)	N/A	0.041	0.007	0.007	1.319	0.528	0.398	0.147	0.068

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	68	71	190	124	73	66
normalized size	1	1.00	0.90	0.86	0.90	2.41	1.57	0.92	0.84
time (sec)	N/A	0.039	0.050	0.010	2.909	0.625	0.357	0.157	0.042

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	52	54	70	53	67	57
normalized size	1	1.00	0.86	0.91	0.95	1.23	0.93	1.18	1.00
time (sec)	N/A	0.051	0.017	0.011	1.319	0.804	0.300	0.180	4.144

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	60	57	59	164	107	61	56
normalized size	1	1.00	0.91	0.86	0.89	2.48	1.62	0.92	0.85
time (sec)	N/A	0.035	0.041	0.009	2.935	0.725	0.331	0.169	0.058

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	41	43	56	39	49	45
normalized size	1	1.00	0.86	0.93	0.98	1.27	0.89	1.11	1.02
time (sec)	N/A	0.037	0.015	0.008	1.324	0.482	0.276	0.174	0.042

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	51	43	45	136	83	42	43
normalized size	1	1.00	0.93	0.78	0.82	2.47	1.51	0.76	0.78
time (sec)	N/A	0.024	0.031	0.010	2.937	0.618	0.282	0.167	4.174

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	27	30	32	35	29	32	29
normalized size	1	1.00	0.82	0.91	0.97	1.06	0.88	0.97	0.88
time (sec)	N/A	0.032	0.008	0.009	1.289	0.626	0.218	0.150	4.178

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	36	120	78	35	33
normalized size	1	1.00	1.00	0.80	0.80	2.67	1.73	0.78	0.73
time (sec)	N/A	0.019	0.020	0.008	3.022	0.637	0.232	0.171	4.149

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	15	15	15	14	14
normalized size	1	1.00	1.00	0.94	0.94	0.94	0.94	0.88	0.88
time (sec)	N/A	0.009	0.002	0.000	1.321	0.476	0.175	0.168	0.024

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	35	120	78	35	33
normalized size	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	0.73
time (sec)	N/A	0.017	0.024	0.005	2.998	0.513	0.251	0.154	0.037

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	35	37	47	34	36	34
normalized size	1	1.00	0.87	0.92	0.97	1.24	0.89	0.95	0.89
time (sec)	N/A	0.035	0.016	0.012	1.296	0.667	0.335	0.155	4.177

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	54	46	49	136	92	47	44
normalized size	1	1.00	0.95	0.81	0.86	2.39	1.61	0.82	0.77
time (sec)	N/A	0.025	0.035	0.010	2.964	0.553	0.317	0.155	0.064

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	46	52	73	51	50	51
normalized size	1	1.00	0.84	0.94	1.06	1.49	1.04	1.02	1.04
time (sec)	N/A	0.041	0.037	0.012	1.342	0.628	0.399	0.175	4.211

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	67	59	64	172	114	59	58
normalized size	1	1.00	0.99	0.87	0.94	2.53	1.68	0.87	0.85
time (sec)	N/A	0.029	0.038	0.014	2.958	0.538	0.392	0.152	4.171

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	57	61	70	90	68	86	67
normalized size	1	1.00	0.86	0.92	1.06	1.36	1.03	1.30	1.02
time (sec)	N/A	0.055	0.052	0.013	1.345	0.651	0.492	0.152	4.170

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	80	70	75	198	126	70	70
normalized size	1	1.00	0.99	0.86	0.93	2.44	1.56	0.86	0.86
time (sec)	N/A	0.045	0.045	0.012	3.027	0.510	0.431	0.176	4.285

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	77	82	230	133	73	77
normalized size	1	1.00	0.91	0.91	0.96	2.71	1.56	0.86	0.91
time (sec)	N/A	0.042	0.047	0.011	2.938	0.774	0.498	0.157	4.211

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	48	58	66	91	68	62	68
normalized size	1	1.00	0.74	0.89	1.02	1.40	1.05	0.95	1.05
time (sec)	N/A	0.054	0.056	0.010	1.358	0.563	0.431	0.195	4.257

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	66	63	68	202	107	54	64
normalized size	1	1.00	0.89	0.85	0.92	2.73	1.45	0.73	0.86
time (sec)	N/A	0.032	0.045	0.011	2.817	0.607	0.458	0.172	4.249

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	46	55	69	53	42	52
normalized size	1	1.00	0.80	0.94	1.12	1.41	1.08	0.86	1.06
time (sec)	N/A	0.045	0.014	0.009	1.351	0.644	0.367	0.179	4.181

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	47	59	188	110	45	56
normalized size	1	1.00	0.86	0.73	0.92	2.94	1.72	0.70	0.88
time (sec)	N/A	0.026	0.042	0.010	2.912	0.942	0.374	0.161	4.226

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	31	36	36	36	22	37
normalized size	1	1.00	1.26	1.63	1.89	1.89	1.89	1.16	1.95
time (sec)	N/A	0.011	0.008	0.008	1.331	0.645	0.307	0.159	4.179

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	49	62	190	110	50	55
normalized size	1	1.00	0.89	0.75	0.95	2.92	1.69	0.77	0.85
time (sec)	N/A	0.025	0.028	0.008	2.859	0.712	0.353	0.159	4.226

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	26	26	27	14	28
normalized size	1	1.00	1.00	0.94	1.62	1.62	1.69	0.88	1.75
time (sec)	N/A	0.009	0.002	0.002	1.272	0.570	0.271	0.154	0.028

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	51	58	188	105	45	55
normalized size	1	1.00	0.89	0.82	0.94	3.03	1.69	0.73	0.89
time (sec)	N/A	0.023	0.033	0.006	2.986	0.750	0.365	0.203	4.207

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	43	49	60	90	56	59	56
normalized size	1	1.00	0.80	0.91	1.11	1.67	1.04	1.09	1.04
time (sec)	N/A	0.044	0.031	0.012	1.378	0.574	0.455	0.155	0.055

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	68	66	71	202	116	57	66
normalized size	1	1.00	0.89	0.87	0.93	2.66	1.53	0.75	0.87
time (sec)	N/A	0.036	0.040	0.013	3.002	0.638	0.452	0.172	4.256

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	59	62	77	119	80	66	75
normalized size	1	1.00	0.88	0.93	1.15	1.78	1.19	0.99	1.12
time (sec)	N/A	0.061	0.056	0.015	1.384	0.484	0.632	0.165	0.064

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	79	79	86	238	138	71	80
normalized size	1	1.00	0.91	0.91	0.99	2.74	1.59	0.82	0.92
time (sec)	N/A	0.042	0.042	0.013	2.934	0.560	0.499	0.176	4.261

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	74	79	92	134	90	79	88
normalized size	1	1.00	0.86	0.92	1.07	1.56	1.05	0.92	1.02
time (sec)	N/A	0.072	0.047	0.014	1.351	0.725	0.569	0.156	4.248

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	90	89	97	264	150	80	92
normalized size	1	1.00	0.90	0.89	0.97	2.64	1.50	0.80	0.92
time (sec)	N/A	0.049	0.053	0.015	2.975	0.633	0.577	0.178	4.235

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	85	90	103	145	104	110	101
normalized size	1	1.00	0.89	0.95	1.08	1.53	1.09	1.16	1.06
time (sec)	N/A	0.085	0.071	0.015	1.363	0.587	0.645	0.154	0.101

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	114	124	121	188	0	101	105
normalized size	1	1.00	0.96	1.04	1.02	1.58	0.00	0.85	0.88
time (sec)	N/A	0.131	0.076	0.016	1.455	0.597	0.000	0.193	4.685

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	103	104	97	167	0	85	77
normalized size	1	1.00	1.13	1.14	1.07	1.84	0.00	0.93	0.85
time (sec)	N/A	0.100	0.061	0.008	1.447	0.491	0.000	0.185	4.364

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	90	84	73	140	0	69	64
normalized size	1	1.00	1.32	1.24	1.07	2.06	0.00	1.01	0.94
time (sec)	N/A	0.064	0.048	0.007	1.428	0.648	0.000	0.189	4.366

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	64	64	49	115	0	52	50
normalized size	1	1.00	1.16	1.16	0.89	2.09	0.00	0.95	0.91
time (sec)	N/A	0.071	0.025	0.005	1.427	0.463	0.000	0.174	4.210

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	60	84	51	115	0	61	-1
normalized size	1	1.00	1.15	1.62	0.98	2.21	0.00	1.17	-0.02
time (sec)	N/A	0.077	0.093	0.007	1.390	1.046	0.000	0.272	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	41	28	0	63	28
normalized size	1	1.00	1.00	1.16	1.64	1.12	0.00	2.52	1.12
time (sec)	N/A	0.039	0.010	0.004	1.438	0.707	0.000	0.225	4.148

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	39	65	42	0	120	41
normalized size	1	1.00	0.67	0.75	1.25	0.81	0.00	2.31	0.79
time (sec)	N/A	0.083	0.011	0.004	1.422	0.979	0.000	0.221	4.263

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	46	50	89	53	0	148	89
normalized size	1	1.00	0.58	0.62	1.11	0.66	0.00	1.85	1.11
time (sec)	N/A	0.120	0.013	0.005	1.426	0.750	0.000	0.238	4.338

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	57	61	113	64	0	178	113
normalized size	1	1.00	0.53	0.56	1.05	0.59	0.00	1.65	1.05
time (sec)	N/A	0.164	0.014	0.007	1.420	0.597	0.000	0.294	4.505

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	68	72	137	75	0	206	137
normalized size	1	1.00	0.50	0.53	1.01	0.55	0.00	1.51	1.01
time (sec)	N/A	0.214	0.015	0.006	1.531	0.651	0.000	0.253	4.619

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	46	50	46	53	0	60	53
normalized size	1	1.00	0.59	0.64	0.59	0.68	0.00	0.77	0.68
time (sec)	N/A	0.094	0.023	0.005	1.406	0.521	0.000	0.159	4.233

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	39	34	41	0	44	41
normalized size	1	1.00	0.67	0.75	0.65	0.79	0.00	0.85	0.79
time (sec)	N/A	0.049	0.018	0.005	1.394	0.741	0.000	0.159	4.135

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	14	28	0	27	29
normalized size	1	1.00	1.00	1.16	0.56	1.12	0.00	1.08	1.16
time (sec)	N/A	0.006	0.005	0.003	1.429	0.539	0.000	0.154	4.136

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	60	65	0	117	0	69	68
normalized size	1	1.00	1.20	1.30	0.00	2.34	0.00	1.38	1.36
time (sec)	N/A	0.049	0.030	0.006	0.000	0.691	0.000	0.180	4.308

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	63	85	0	134	0	50	-1
normalized size	1	1.00	1.12	1.52	0.00	2.39	0.00	0.89	-0.02
time (sec)	N/A	0.052	0.042	0.006	0.000	0.739	0.000	0.200	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	46	106	0	159	0	78	-1
normalized size	1	1.00	0.55	1.26	0.00	1.89	0.00	0.93	-0.01
time (sec)	N/A	0.098	0.014	0.007	0.000	0.555	0.000	0.212	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	46	128	0	185	0	100	-1
normalized size	1	1.00	0.41	1.14	0.00	1.65	0.00	0.89	-0.01
time (sec)	N/A	0.145	0.014	0.010	0.000	0.680	0.000	0.267	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	126	142	142	210	0	115	134
normalized size	1	1.00	1.02	1.15	1.15	1.69	0.00	0.93	1.08
time (sec)	N/A	0.132	0.099	0.010	1.390	0.656	0.000	0.278	4.351

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	115	122	118	189	0	99	99
normalized size	1	1.00	1.14	1.21	1.17	1.87	0.00	0.98	0.98
time (sec)	N/A	0.092	0.096	0.010	1.471	0.718	0.000	0.194	4.445

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	104	102	91	166	0	84	-1
normalized size	1	1.00	1.18	1.16	1.03	1.89	0.00	0.95	-0.01
time (sec)	N/A	0.099	0.075	0.008	1.416	0.648	0.000	0.206	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	84	70	145	0	68	-1
normalized size	1	1.00	0.89	1.05	0.88	1.81	0.00	0.85	-0.01
time (sec)	N/A	0.100	0.103	0.005	1.445	0.753	0.000	0.248	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	54	107	71	139	0	79	-1
normalized size	1	1.00	0.71	1.41	0.93	1.83	0.00	1.04	-0.01
time (sec)	N/A	0.111	0.014	0.007	1.444	0.554	0.000	0.272	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	56	129	89	135	0	122	-1
normalized size	1	1.00	0.75	1.72	1.19	1.80	0.00	1.63	-0.01
time (sec)	N/A	0.104	0.016	0.007	1.505	0.584	0.000	0.437	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	81	39	0	92	30
normalized size	1	1.00	1.00	1.16	3.24	1.56	0.00	3.68	1.20
time (sec)	N/A	0.046	0.014	0.004	1.409	0.551	0.000	0.303	4.380

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	39	105	53	0	178	87
normalized size	1	1.00	0.67	0.75	2.02	1.02	0.00	3.42	1.67
time (sec)	N/A	0.093	0.014	0.005	1.463	0.616	0.000	0.255	4.579

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	46	50	129	64	0	206	111
normalized size	1	1.00	0.58	0.62	1.61	0.80	0.00	2.58	1.39
time (sec)	N/A	0.141	0.015	0.005	1.520	0.475	0.000	0.267	4.722

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	57	61	153	75	0	236	135
normalized size	1	1.00	0.53	0.56	1.42	0.69	0.00	2.19	1.25
time (sec)	N/A	0.184	0.017	0.005	1.492	1.033	0.000	0.276	4.990

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	68	72	177	86	0	264	159
normalized size	1	1.00	0.50	0.53	1.30	0.63	0.00	1.94	1.17
time (sec)	N/A	0.262	0.018	0.007	1.468	0.705	0.000	0.286	5.174

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	75	72	79	86	0	92	73
normalized size	1	1.00	0.56	0.54	0.59	0.64	0.00	0.69	0.54
time (sec)	N/A	0.253	0.036	0.007	1.534	0.784	0.000	0.187	4.478

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	64	61	68	75	0	76	62
normalized size	1	1.00	0.60	0.58	0.64	0.71	0.00	0.72	0.58
time (sec)	N/A	0.196	0.031	0.008	1.503	0.679	0.000	0.165	4.304

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	53	50	57	64	0	60	51
normalized size	1	1.00	0.66	0.62	0.71	0.80	0.00	0.75	0.64
time (sec)	N/A	0.111	0.024	0.006	1.445	0.649	0.000	0.161	4.187

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	42	39	45	52	0	44	40
normalized size	1	1.00	0.81	0.75	0.87	1.00	0.00	0.85	0.77
time (sec)	N/A	0.054	0.020	0.006	1.504	0.760	0.000	0.164	4.160

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	32	39	0	27	30
normalized size	1	1.00	1.00	1.16	1.28	1.56	0.00	1.08	1.20
time (sec)	N/A	0.048	0.008	0.003	1.483	0.650	0.000	0.156	4.149

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	76	78	0	140	0	89	-1
normalized size	1	1.00	1.04	1.07	0.00	1.92	0.00	1.22	-0.01
time (sec)	N/A	0.110	0.052	0.006	0.000	0.866	0.000	0.166	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	44	102	0	147	0	69	-1
normalized size	1	1.00	0.56	1.29	0.00	1.86	0.00	0.87	-0.01
time (sec)	N/A	0.118	0.016	0.006	0.000	0.692	0.000	0.201	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	80	125	0	164	0	76	-1
normalized size	1	1.00	0.99	1.54	0.00	2.02	0.00	0.94	-0.01
time (sec)	N/A	0.115	0.051	0.008	0.000	0.648	0.000	0.240	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	46	145	0	185	0	100	-1
normalized size	1	1.00	0.42	1.33	0.00	1.70	0.00	0.92	-0.01
time (sec)	N/A	0.164	0.018	0.011	0.000	0.896	0.000	0.235	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	46	165	0	207	0	119	-1
normalized size	1	1.00	0.34	1.20	0.00	1.51	0.00	0.87	-0.01
time (sec)	N/A	0.203	0.017	0.016	0.000	0.680	0.000	0.233	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	46	186	0	229	0	138	-1
normalized size	1	1.00	0.28	1.13	0.00	1.39	0.00	0.84	-0.01
time (sec)	N/A	0.255	0.017	0.031	0.000	0.672	0.000	0.379	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	100	105	100	166	0	87	-1
normalized size	1	1.00	0.88	0.92	0.88	1.46	0.00	0.76	-0.01
time (sec)	N/A	0.129	0.047	0.009	1.474	0.522	0.000	0.223	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	89	85	76	145	0	73	-1
normalized size	1	1.00	1.03	0.99	0.88	1.69	0.00	0.85	-0.01
time (sec)	N/A	0.103	0.036	0.009	1.453	0.800	0.000	0.215	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	73	64	52	114	0	59	53
normalized size	1	1.00	1.26	1.10	0.90	1.97	0.00	1.02	0.91
time (sec)	N/A	0.083	0.027	0.007	1.448	0.467	0.000	0.199	4.302

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	52	44	32	74	0	39	33
normalized size	1	1.00	1.68	1.42	1.03	2.39	0.00	1.26	1.06
time (sec)	N/A	0.054	0.013	0.003	1.461	0.566	0.000	0.188	4.360

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	26	21	21	0	25	21
normalized size	1	1.00	1.00	1.13	0.91	0.91	0.00	1.09	0.91
time (sec)	N/A	0.041	0.008	0.005	1.472	0.629	0.000	0.181	4.213

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	37	44	31	0	57	29
normalized size	1	1.00	0.67	0.71	0.85	0.60	0.00	1.10	0.56
time (sec)	N/A	0.083	0.015	0.004	1.429	0.688	0.000	0.186	4.259

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	46	50	68	42	0	90	42
normalized size	1	1.00	0.58	0.62	0.85	0.52	0.00	1.12	0.52
time (sec)	N/A	0.125	0.014	0.005	1.435	0.607	0.000	0.209	4.326

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	57	61	92	53	0	123	92
normalized size	1	1.00	0.53	0.56	0.85	0.49	0.00	1.14	0.85
time (sec)	N/A	0.169	0.015	0.007	1.408	0.639	0.000	0.200	4.273

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	34	37	34	30	0	0	33
normalized size	1	1.00	0.68	0.74	0.68	0.60	0.00	0.00	0.66
time (sec)	N/A	0.079	0.016	0.004	1.505	0.645	0.000	0.000	4.248

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	26	13	20	0	31	20
normalized size	1	1.00	1.00	1.18	0.59	0.91	0.00	1.41	0.91
time (sec)	N/A	0.017	0.005	0.003	1.419	0.632	0.000	0.185	4.258

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	52	50	0	80	0	46	-1
normalized size	1	1.00	1.73	1.67	0.00	2.67	0.00	1.53	-0.03
time (sec)	N/A	0.009	0.009	0.005	0.000	0.821	0.000	0.170	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	68	73	0	133	0	0	76
normalized size	1	1.00	1.15	1.24	0.00	2.25	0.00	0.00	1.29
time (sec)	N/A	0.055	0.060	0.006	0.000	0.664	0.000	0.000	4.472

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	44	94	0	163	0	0	-1
normalized size	1	1.00	0.51	1.08	0.00	1.87	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.012	0.009	0.000	0.817	0.000	0.000	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	88	87	103	209	0	114	-1
normalized size	1	1.00	0.81	0.80	0.94	1.92	0.00	1.05	-0.01
time (sec)	N/A	0.129	0.050	0.010	1.477	0.621	0.000	0.269	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	76	73	77	180	0	99	-1
normalized size	1	1.00	0.94	0.90	0.95	2.22	0.00	1.22	-0.01
time (sec)	N/A	0.109	0.041	0.007	1.506	0.571	0.000	0.240	0.000

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	66	63	54	150	0	0	55
normalized size	1	1.00	1.20	1.15	0.98	2.73	0.00	0.00	1.00
time (sec)	N/A	0.092	0.075	0.007	1.469	0.700	0.000	0.000	4.328

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	28	20	26	0	35	26
normalized size	1	1.00	1.00	1.27	0.91	1.18	0.00	1.59	1.18
time (sec)	N/A	0.055	0.008	0.003	1.453	0.575	0.000	0.179	4.131

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	29	37	41	41	0	28	26
normalized size	1	1.00	1.04	1.32	1.46	1.46	0.00	1.00	0.93
time (sec)	N/A	0.046	0.010	0.005	1.467	0.641	0.000	0.194	4.125

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	48	45	65	54	0	0	51
normalized size	1	1.00	0.65	0.61	0.88	0.73	0.00	0.00	0.69
time (sec)	N/A	0.129	0.010	0.006	1.485	0.538	0.000	0.000	4.236

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	57	59	89	63	0	0	60
normalized size	1	1.00	0.56	0.58	0.87	0.62	0.00	0.00	0.59
time (sec)	N/A	0.185	0.011	0.005	1.511	0.650	0.000	0.000	4.305

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	68	72	113	76	0	0	114
normalized size	1	1.00	0.52	0.55	0.87	0.58	0.00	0.00	0.88
time (sec)	N/A	0.233	0.013	0.006	1.469	0.658	0.000	0.000	4.407

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	29	37	22	39	0	52	38
normalized size	1	1.00	0.62	0.79	0.47	0.83	0.00	1.11	0.81
time (sec)	N/A	0.068	0.014	0.004	1.483	0.630	0.000	0.209	4.225

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	29	14	29	0	17	30
normalized size	1	1.00	1.00	1.38	0.67	1.38	0.00	0.81	1.43
time (sec)	N/A	0.019	0.005	0.003	1.443	0.667	0.000	0.206	4.152

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	38	65	0	162	0	0	-1
normalized size	1	1.00	0.75	1.27	0.00	3.18	0.00	0.00	-0.02
time (sec)	N/A	0.062	0.009	0.007	0.000	0.595	0.000	0.000	0.000

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	40	77	0	199	0	0	42
normalized size	1	1.00	0.49	0.95	0.00	2.46	0.00	0.00	0.52
time (sec)	N/A	0.065	0.008	0.006	0.000	0.630	0.000	0.000	4.341

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	41	94	0	229	0	0	44
normalized size	1	1.00	0.38	0.86	0.00	2.10	0.00	0.00	0.40
time (sec)	N/A	0.153	0.011	0.008	0.000	0.581	0.000	0.000	4.637

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	57	48	26	37	0	26	42
normalized size	1	1.00	1.68	1.41	0.76	1.09	0.00	0.76	1.24
time (sec)	N/A	0.058	0.018	0.008	3.032	0.506	0.000	0.176	4.332

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	52	54	26	59	0	27	41
normalized size	1	1.00	1.53	1.59	0.76	1.74	0.00	0.79	1.21
time (sec)	N/A	0.057	0.018	0.006	2.980	0.645	0.000	0.207	4.363

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	51	48	41	45	0	41	40
normalized size	1	1.00	1.13	1.07	0.91	1.00	0.00	0.91	0.89
time (sec)	N/A	0.056	0.009	0.007	3.047	0.614	0.000	0.171	4.397

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	57	60	41	45	0	42	40
normalized size	1	1.00	1.27	1.33	0.91	1.00	0.00	0.93	0.89
time (sec)	N/A	0.057	0.012	0.007	3.043	0.763	0.000	0.173	4.456

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	73	64	52	114	0	59	53
normalized size	1	1.00	1.26	1.10	0.90	1.97	0.00	1.02	0.91
time (sec)	N/A	0.083	0.033	0.007	1.419	0.572	0.000	0.199	4.707

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	77	67	42	120	0	68	60
normalized size	1	1.00	1.28	1.12	0.70	2.00	0.00	1.13	1.00
time (sec)	N/A	0.082	0.042	0.009	3.026	0.724	0.000	0.201	4.621

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	13	18	19	13	15
normalized size	1	1.00	1.00	0.76	0.62	0.86	0.90	0.62	0.71
time (sec)	N/A	0.005	0.006	0.003	1.302	0.579	11.339	0.147	0.035

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	13	18	19	13	15
normalized size	1	1.00	1.00	0.76	0.62	0.86	0.90	0.62	0.71
time (sec)	N/A	0.005	0.005	0.004	1.358	0.463	5.565	0.154	0.029

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	13	18	19	13	15
normalized size	1	1.00	1.00	0.76	0.62	0.86	0.90	0.62	0.71
time (sec)	N/A	0.005	0.005	0.004	1.344	0.741	2.525	0.161	0.027

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	13	18	19	13	15
normalized size	1	1.00	1.00	0.76	0.62	0.86	0.90	0.62	0.71
time (sec)	N/A	0.005	0.005	0.003	1.327	0.530	1.726	0.212	0.028

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	13	18	19	13	15
normalized size	1	1.00	1.00	0.76	0.62	0.86	0.90	0.62	0.71
time (sec)	N/A	0.005	0.005	0.003	1.333	0.508	0.777	0.150	0.026

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	13	16	19	13	15
normalized size	1	1.00	1.00	0.76	0.62	0.76	0.90	0.62	0.71
time (sec)	N/A	0.005	0.005	0.004	1.304	0.706	0.787	0.200	0.028

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	13	14	17	13	14
normalized size	1	1.00	1.00	0.79	0.68	0.74	0.89	0.68	0.74
time (sec)	N/A	0.005	0.004	0.003	1.310	0.635	1.001	0.153	0.030

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	13	14	17	13	15
normalized size	1	1.00	1.00	0.84	0.68	0.74	0.89	0.68	0.79
time (sec)	N/A	0.005	0.005	0.004	1.343	0.528	1.803	0.163	0.031

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	24	29	34	24	25
normalized size	1	1.00	0.83	0.75	0.67	0.81	0.94	0.67	0.69
time (sec)	N/A	0.016	0.009	0.006	1.278	0.678	35.136	0.147	0.049

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	24	29	34	24	25
normalized size	1	1.00	0.83	0.75	0.67	0.81	0.94	0.67	0.69
time (sec)	N/A	0.018	0.008	0.006	1.353	0.628	20.810	0.151	0.042

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	24	29	34	24	25
normalized size	1	1.00	0.83	0.75	0.67	0.81	0.94	0.67	0.69
time (sec)	N/A	0.015	0.008	0.006	1.231	0.560	11.378	0.159	4.271

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	24	29	34	24	25
normalized size	1	1.00	0.83	0.75	0.67	0.81	0.94	0.67	0.69
time (sec)	N/A	0.014	0.009	0.005	1.358	0.753	2.750	0.145	0.041

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	24	29	34	24	25
normalized size	1	1.00	0.83	0.75	0.67	0.81	0.94	0.67	0.69
time (sec)	N/A	0.014	0.008	0.006	1.251	0.528	4.975	0.151	0.040

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	24	29	34	24	26
normalized size	1	1.00	0.83	0.75	0.67	0.81	0.94	0.67	0.72
time (sec)	N/A	0.016	0.008	0.006	1.330	0.703	5.218	0.152	4.438

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	24	29	34	24	26
normalized size	1	1.00	0.83	0.75	0.67	0.81	0.94	0.67	0.72
time (sec)	N/A	0.015	0.008	0.004	1.349	0.592	5.988	0.149	0.043

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	24	27	34	24	26
normalized size	1	1.00	0.83	0.75	0.67	0.75	0.94	0.67	0.72
time (sec)	N/A	0.016	0.008	0.007	1.325	0.550	8.695	0.147	0.047

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	38	35	40	49	35	35
normalized size	1	1.00	1.00	0.75	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.021	0.013	0.006	1.345	0.682	83.180	0.182	0.050

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	38	35	40	49	35	35
normalized size	1	1.00	1.00	0.75	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.020	0.012	0.005	1.352	0.674	53.376	0.167	0.051

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	38	35	40	49	35	35
normalized size	1	1.00	1.00	0.75	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.020	0.011	0.005	1.309	0.818	33.338	0.154	0.050

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	35	40	49	35	35
normalized size	1	1.00	0.80	0.75	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.019	0.010	0.007	1.327	0.798	4.318	0.151	0.051

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	35	40	49	35	35
normalized size	1	1.00	0.80	0.75	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.020	0.011	0.005	1.311	0.912	17.612	0.151	0.047

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	35	40	49	35	35
normalized size	1	1.00	0.80	0.75	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.020	0.011	0.006	1.306	0.849	19.942	0.152	0.049

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	35	40	49	35	35
normalized size	1	1.00	0.80	0.75	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.019	0.010	0.006	1.352	0.568	22.204	0.151	0.052

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	35	40	49	35	35
normalized size	1	1.00	0.80	0.75	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.022	0.010	0.004	1.370	0.819	26.498	0.178	0.050

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	89	158	198	182	0	197	66
normalized size	1	1.00	0.41	0.73	0.91	0.84	0.00	0.91	0.30
time (sec)	N/A	0.232	0.044	0.014	3.006	0.848	0.000	0.174	0.114

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	203	152	194	170	0	196	67
normalized size	1	1.00	0.94	0.71	0.90	0.79	0.00	0.91	0.31
time (sec)	N/A	0.194	0.064	0.008	2.974	0.558	0.000	0.168	4.494

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	78	143	186	165	180	178	54
normalized size	1	1.00	0.38	0.70	0.91	0.81	0.88	0.87	0.26
time (sec)	N/A	0.186	0.018	0.006	3.124	0.744	166.485	0.177	4.359

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	189	140	185	124	172	178	55
normalized size	1	1.00	0.94	0.69	0.92	0.61	0.85	0.88	0.27
time (sec)	N/A	0.185	0.036	0.007	3.047	0.873	77.501	0.165	4.360

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	54	132	172	126	165	182	38
normalized size	1	1.00	0.28	0.69	0.90	0.66	0.86	0.95	0.20
time (sec)	N/A	0.142	0.024	0.005	3.002	0.599	48.274	0.194	0.079

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	146	132	172	126	160	182	37
normalized size	1	1.00	0.76	0.69	0.90	0.66	0.83	0.95	0.19
time (sec)	N/A	0.139	0.035	0.006	3.024	0.866	27.221	0.161	4.434

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	27	140	186	142	170	190	54
normalized size	1	1.00	0.13	0.69	0.92	0.70	0.84	0.94	0.27
time (sec)	N/A	0.164	0.005	0.008	2.917	1.128	18.320	0.167	4.530

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	29	143	187	167	178	178	53
normalized size	1	1.00	0.14	0.70	0.92	0.82	0.87	0.87	0.26
time (sec)	N/A	0.167	0.006	0.011	2.988	0.605	27.744	0.164	0.099

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	29	152	198	193	190	200	66
normalized size	1	1.00	0.13	0.71	0.92	0.90	0.88	0.93	0.31
time (sec)	N/A	0.188	0.006	0.011	3.131	0.854	48.980	0.167	0.093

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	29	158	201	189	197	192	65
normalized size	1	1.00	0.13	0.73	0.93	0.87	0.91	0.88	0.30
time (sec)	N/A	0.182	0.006	0.011	3.119	0.786	106.893	0.198	4.386

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	29	169	209	204	0	199	77
normalized size	1	1.00	0.13	0.73	0.91	0.89	0.00	0.87	0.33
time (sec)	N/A	0.220	0.007	0.013	3.088	0.858	0.000	0.178	4.466

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	220	172	217	227	0	216	92
normalized size	1	1.00	0.91	0.71	0.89	0.93	0.00	0.89	0.38
time (sec)	N/A	0.210	0.206	0.014	3.016	0.834	0.000	0.195	0.097

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	57	161	207	229	0	196	80
normalized size	1	1.00	0.25	0.70	0.90	1.00	0.00	0.85	0.35
time (sec)	N/A	0.182	0.015	0.011	3.046	0.641	0.000	0.180	0.107

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	221	158	206	192	0	196	80
normalized size	1	1.00	0.96	0.69	0.90	0.83	0.00	0.85	0.35
time (sec)	N/A	0.182	0.104	0.013	3.064	0.562	0.000	0.198	4.320

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	43	149	195	185	0	199	64
normalized size	1	1.00	0.20	0.68	0.89	0.85	0.00	0.91	0.29
time (sec)	N/A	0.166	0.014	0.013	3.005	0.932	0.000	0.182	0.089

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	198	158	195	187	0	199	64
normalized size	1	1.00	0.91	0.72	0.89	0.86	0.00	0.91	0.29
time (sec)	N/A	0.160	0.097	0.012	3.102	0.885	0.000	0.170	4.311

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	29	158	194	182	0	199	64
normalized size	1	1.00	0.13	0.72	0.89	0.83	0.00	0.91	0.29
time (sec)	N/A	0.163	0.005	0.010	3.041	0.862	0.000	0.181	4.340

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	199	149	194	179	0	199	64
normalized size	1	1.00	0.91	0.68	0.89	0.82	0.00	0.91	0.29
time (sec)	N/A	0.169	0.093	0.006	3.047	0.909	0.000	0.187	0.095

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	27	158	208	208	0	210	77
normalized size	1	1.00	0.12	0.69	0.90	0.90	0.00	0.91	0.33
time (sec)	N/A	0.190	0.006	0.017	2.931	0.840	0.000	0.171	0.085

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	29	161	209	228	0	196	77
normalized size	1	1.00	0.13	0.70	0.91	0.99	0.00	0.85	0.33
time (sec)	N/A	0.185	0.007	0.015	3.030	0.883	0.000	0.184	0.107

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	29	172	221	251	0	220	87
normalized size	1	1.00	0.12	0.71	0.91	1.03	0.00	0.91	0.36
time (sec)	N/A	0.217	0.006	0.020	2.969	0.913	0.000	0.194	4.369

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	29	178	224	245	0	212	87
normalized size	1	1.00	0.12	0.73	0.92	1.01	0.00	0.87	0.36
time (sec)	N/A	0.207	0.006	0.018	3.156	0.685	0.000	0.169	0.106

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	29	189	232	262	0	219	99
normalized size	1	1.00	0.11	0.73	0.90	1.02	0.00	0.85	0.38
time (sec)	N/A	0.235	0.007	0.017	3.006	0.903	0.000	0.174	4.369

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	220	178	229	247	0	208	101
normalized size	1	1.00	0.88	0.71	0.91	0.98	0.00	0.83	0.40
time (sec)	N/A	0.211	0.250	0.017	2.950	0.833	0.000	0.184	4.389

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	66	161	218	248	0	209	87
normalized size	1	1.00	0.28	0.67	0.91	1.04	0.00	0.87	0.36
time (sec)	N/A	0.203	0.019	0.017	2.955	0.706	0.000	0.220	4.281

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	242	170	218	254	0	209	87
normalized size	1	1.00	1.01	0.71	0.91	1.06	0.00	0.87	0.36
time (sec)	N/A	0.189	0.104	0.016	2.980	0.828	0.000	0.180	0.097

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	45	169	222	260	0	212	85
normalized size	1	1.00	0.19	0.70	0.92	1.07	0.00	0.88	0.35
time (sec)	N/A	0.188	0.016	0.017	3.086	0.840	0.000	0.185	0.087

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	223	169	221	257	0	211	85
normalized size	1	1.00	0.92	0.70	0.91	1.06	0.00	0.87	0.35
time (sec)	N/A	0.183	0.110	0.015	2.963	0.829	0.000	0.177	0.102

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	29	175	217	250	0	209	86
normalized size	1	1.00	0.12	0.73	0.91	1.05	0.00	0.87	0.36
time (sec)	N/A	0.185	0.005	0.010	3.085	1.113	0.000	0.197	0.089

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	220	166	217	241	0	209	86
normalized size	1	1.00	0.92	0.69	0.91	1.01	0.00	0.87	0.36
time (sec)	N/A	0.186	0.082	0.008	2.966	0.781	0.000	0.207	4.292

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	27	178	230	263	0	220	99
normalized size	1	1.00	0.11	0.71	0.92	1.05	0.00	0.88	0.39
time (sec)	N/A	0.217	0.006	0.019	3.136	0.769	0.000	0.187	4.366

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	29	181	231	283	0	208	99
normalized size	1	1.00	0.12	0.72	0.92	1.13	0.00	0.83	0.39
time (sec)	N/A	0.214	0.007	0.018	3.049	0.954	0.000	0.182	0.128

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	29	192	243	306	0	232	109
normalized size	1	1.00	0.11	0.73	0.92	1.16	0.00	0.88	0.41
time (sec)	N/A	0.233	0.007	0.022	3.144	0.872	0.000	0.269	0.118

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	29	198	246	300	0	224	109
normalized size	1	1.00	0.11	0.75	0.93	1.14	0.00	0.85	0.41
time (sec)	N/A	0.231	0.006	0.022	2.929	0.753	0.000	0.207	4.348

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	29	209	254	317	0	231	121
normalized size	1	1.00	0.10	0.75	0.91	1.14	0.00	0.83	0.43
time (sec)	N/A	0.268	0.007	0.022	3.042	0.806	0.000	0.186	0.139

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	29	209	257	311	0	243	121
normalized size	1	1.00	0.10	0.75	0.92	1.11	0.00	0.87	0.43
time (sec)	N/A	0.260	0.007	0.021	3.063	0.861	0.000	0.171	4.389

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	102	237	0	0	0	0	-1
normalized size	1	1.00	0.32	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.383	0.069	0.062	0.000	0.892	0.000	0.000	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	102	157	0	0	0	0	-1
normalized size	1	1.00	0.58	0.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.249	0.055	0.029	0.000	0.849	0.000	0.000	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	86	226	0	0	0	0	-1
normalized size	1	1.00	0.29	0.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.307	0.036	0.030	0.000	0.746	0.000	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	86	145	0	0	0	0	-1
normalized size	1	1.00	0.59	0.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.034	0.028	0.000	0.830	0.000	0.000	0.000

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	57	213	0	0	0	0	-1
normalized size	1	1.00	0.22	0.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.238	0.015	0.028	0.000	0.908	0.000	0.000	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	55	130	0	0	0	0	-1
normalized size	1	1.00	0.47	1.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.013	0.028	0.000	0.897	0.000	0.000	0.000

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	55	202	0	0	0	0	-1
normalized size	1	1.00	0.22	0.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.238	0.014	0.034	0.000	0.666	0.000	0.000	0.000

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	57	125	0	0	0	0	-1
normalized size	1	1.00	0.48	1.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.013	0.032	0.000	0.760	0.000	0.000	0.000

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	57	224	0	0	0	0	-1
normalized size	1	1.00	0.19	0.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.299	0.014	0.035	0.000	0.756	0.000	0.000	0.000

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	57	142	0	0	0	0	-1
normalized size	1	1.00	0.39	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.013	0.033	0.000	0.845	0.000	0.000	0.000

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	57	239	0	0	0	0	-1
normalized size	1	1.00	0.18	0.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.366	0.015	0.036	0.000	0.850	0.000	0.000	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	57	156	0	0	0	0	-1
normalized size	1	1.00	0.32	0.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.234	0.014	0.035	0.000	0.902	0.000	0.000	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	101	248	0	0	0	0	-1
normalized size	1	1.00	0.29	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.434	0.061	0.030	0.000	0.817	0.000	0.000	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	101	168	0	0	0	0	-1
normalized size	1	1.00	0.50	0.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.289	0.062	0.030	0.000	0.853	0.000	0.000	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	90	237	0	0	0	0	-1
normalized size	1	1.00	0.28	0.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.370	0.045	0.014	0.000	0.818	0.000	0.000	0.000

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	90	157	0	0	0	0	-1
normalized size	1	1.00	0.52	0.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.236	0.037	0.014	0.000	0.885	0.000	0.000	0.000

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	58	226	0	0	0	0	-1
normalized size	1	1.00	0.20	0.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.299	0.017	0.013	0.000	0.776	0.000	0.000	0.000

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	56	145	0	0	0	0	-1
normalized size	1	1.00	0.39	1.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.017	0.015	0.000	0.914	0.000	0.000	0.000

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	56	216	0	0	0	0	-1
normalized size	1	1.00	0.20	0.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.301	0.016	0.019	0.000	0.624	0.000	0.000	0.000

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	58	130	0	0	0	0	-1
normalized size	1	1.00	0.41	0.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.017	0.016	0.000	0.906	0.000	0.000	0.000

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	58	221	0	0	0	0	-1
normalized size	1	1.00	0.20	0.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.305	0.016	0.019	0.000	0.858	0.000	0.000	0.000

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	58	140	0	0	0	0	-1
normalized size	1	1.00	0.41	0.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.017	0.015	0.000	0.882	0.000	0.000	0.000

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	58	239	0	0	0	0	-1
normalized size	1	1.00	0.18	0.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.360	0.018	0.020	0.000	0.785	0.000	0.000	0.000

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	58	156	0	0	0	0	-1
normalized size	1	1.00	0.34	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.237	0.018	0.016	0.000	0.742	0.000	0.000	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	58	250	0	0	0	0	-1
normalized size	1	1.00	0.17	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.426	0.019	0.044	0.000	0.509	0.000	0.000	0.000

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	58	167	0	0	0	0	-1
normalized size	1	1.00	0.29	0.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.296	0.018	0.039	0.000	0.552	0.000	0.000	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	97	148	0	0	0	0	-1
normalized size	1	1.00	0.54	0.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.244	0.036	0.032	0.000	0.935	0.000	0.000	0.000

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	86	217	0	0	0	0	-1
normalized size	1	1.00	0.29	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.298	0.036	0.032	0.000	0.720	0.000	0.000	0.000

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	86	137	0	0	0	0	-1
normalized size	1	1.00	0.58	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.032	0.016	0.000	0.949	0.000	0.000	0.000

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	70	206	0	0	0	0	-1
normalized size	1	1.00	0.26	0.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.237	0.026	0.015	0.000	0.849	0.000	0.000	0.000

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	70	123	0	0	0	0	-1
normalized size	1	1.00	0.58	1.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.026	0.013	0.000	0.615	0.000	0.000	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	57	131	0	0	0	0	-1
normalized size	1	1.00	0.25	0.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.176	0.012	0.013	0.000	0.849	0.000	0.000	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	55	106	0	0	0	0	-1
normalized size	1	1.00	0.61	1.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.013	0.016	0.000	0.775	0.000	0.000	0.000

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	55	195	0	0	0	0	-1
normalized size	1	1.00	0.21	0.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.232	0.015	0.017	0.000	0.843	0.000	0.000	0.000

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	57	119	0	0	0	0	-1
normalized size	1	1.00	0.47	0.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.016	0.015	0.000	0.864	0.000	0.000	0.000

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	57	215	0	0	0	0	-1
normalized size	1	1.00	0.19	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.294	0.014	0.017	0.000	0.866	0.000	0.000	0.000

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	57	134	0	0	0	0	-1
normalized size	1	1.00	0.38	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.014	0.016	0.000	0.843	0.000	0.000	0.000

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	57	230	0	0	0	0	-1
normalized size	1	1.00	0.17	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.359	0.015	0.019	0.000	0.883	0.000	0.000	0.000

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	57	147	0	0	0	0	-1
normalized size	1	1.00	0.32	0.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.233	0.015	0.019	0.000	0.859	0.000	0.000	0.000

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	86	144	0	0	0	0	-1
normalized size	1	1.00	0.49	0.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.242	0.032	0.044	0.000	0.772	0.000	0.000	0.000

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	72	213	0	0	0	0	-1
normalized size	1	1.00	0.25	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.296	0.027	0.039	0.000	0.931	0.000	0.000	0.000

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	73	131	0	0	0	0	-1
normalized size	1	1.00	0.50	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.028	0.017	0.000	0.806	0.000	0.000	0.000

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	61	200	0	0	0	0	-1
normalized size	1	1.00	0.24	0.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.234	0.024	0.018	0.000	0.861	0.000	0.000	0.000

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	60	120	0	0	0	0	-1
normalized size	1	1.00	0.50	1.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.022	0.016	0.000	0.854	0.000	0.000	0.000

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	60	203	0	0	0	0	-1
normalized size	1	1.00	0.23	0.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.237	0.015	0.014	0.000	0.835	0.000	0.000	0.000

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	60	123	0	0	0	0	-1
normalized size	1	1.00	0.51	1.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.019	0.022	0.000	0.872	0.000	0.000	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	58	203	0	0	0	0	-1
normalized size	1	1.00	0.20	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.298	0.017	0.020	0.000	0.810	0.000	0.000	0.000

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	60	127	0	0	0	0	-1
normalized size	1	1.00	0.41	0.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.018	0.023	0.000	0.873	0.000	0.000	0.000

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	60	222	0	0	0	0	-1
normalized size	1	1.00	0.19	0.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.364	0.016	0.021	0.000	0.893	0.000	0.000	0.000

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	60	141	0	0	0	0	-1
normalized size	1	1.00	0.35	0.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.233	0.018	0.020	0.000	0.470	0.000	0.000	0.000

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	60	237	0	0	0	0	-1
normalized size	1	1.00	0.17	0.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.427	0.017	0.022	0.000	0.883	0.000	0.000	0.000

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	59	181	76	161	758	264	171
normalized size	1	1.00	0.81	2.48	1.04	2.21	10.38	3.62	2.34
time (sec)	N/A	0.050	0.039	0.007	1.508	0.586	5.284	0.181	4.290

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	43	96	55	89	352	141	97
normalized size	1	1.00	0.83	1.85	1.06	1.71	6.77	2.71	1.87
time (sec)	N/A	0.039	0.039	0.006	1.448	0.897	2.295	0.175	4.195

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	27	39	34	39	119	56	38
normalized size	1	1.00	0.79	1.15	1.00	1.15	3.50	1.65	1.12
time (sec)	N/A	0.014	0.016	0.003	1.426	0.805	0.757	0.197	4.148

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.011	0.053	0.000	0.681	0.000	0.000	0.000

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	44	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.011	0.047	0.000	0.883	0.000	0.000	0.000

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	44	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.012	0.049	0.000	0.789	0.000	0.000	0.000

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.010	0.002	0.001	1.361	0.659	0.067	0.149	0.038

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.009	0.001	0.000	1.328	0.743	0.068	0.148	0.033

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	16	25	24	24	24	24	24
normalized size	1	1.00	0.53	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.009	0.002	0.001	1.285	0.662	0.067	0.170	0.031

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
normalized size	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.005	0.000	0.001	1.350	0.693	0.070	0.150	0.028

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	24	21	20	24	21
normalized size	1	1.00	1.00	0.96	1.04	0.91	0.87	1.04	0.91
time (sec)	N/A	0.007	0.001	0.004	1.283	0.754	0.104	0.171	4.097

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	25	19	22	22
normalized size	1	1.00	1.00	0.96	0.92	1.04	0.79	0.92	0.92
time (sec)	N/A	0.008	0.001	0.004	1.314	0.775	0.097	0.146	0.034

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	24	27	24	32	23
normalized size	1	1.00	1.00	0.89	0.89	1.00	0.89	1.19	0.85
time (sec)	N/A	0.009	0.001	0.007	1.316	0.840	0.138	0.187	0.032

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	22	26	22	22	24
normalized size	1	1.00	1.00	0.96	0.96	1.13	0.96	0.96	1.04
time (sec)	N/A	0.009	0.001	0.005	1.326	0.756	0.138	0.200	4.106

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	26	28	24	34	24
normalized size	1	1.00	1.00	0.96	1.08	1.17	1.00	1.42	1.00
time (sec)	N/A	0.009	0.001	0.006	1.372	0.754	0.174	0.151	0.044

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	26	26	27	26	25
normalized size	1	1.00	1.00	0.89	0.93	0.93	0.96	0.93	0.89
time (sec)	N/A	0.010	0.001	0.005	1.337	0.723	0.185	0.171	0.036

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	26
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.87
time (sec)	N/A	0.009	0.001	0.005	1.328	0.690	0.196	0.177	0.035

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26
normalized size	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87
time (sec)	N/A	0.009	0.001	0.005	1.348	0.587	0.208	0.168	0.034

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	46	46	53	46	46
normalized size	1	1.00	1.00	0.84	0.82	0.82	0.95	0.82	0.82
time (sec)	N/A	0.028	0.003	0.001	1.328	0.624	0.081	0.147	0.025

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	56	47	46	46	49	46	46
normalized size	1	1.00	1.06	0.89	0.87	0.87	0.92	0.87	0.87
time (sec)	N/A	0.070	0.002	0.000	1.339	0.692	0.081	0.149	0.022

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	46	46	53	46	46
normalized size	1	1.00	1.00	0.84	0.82	0.82	0.95	0.82	0.82
time (sec)	N/A	0.027	0.002	0.001	1.329	0.543	0.082	0.158	0.023

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	56	47	46	46	53	46	46
normalized size	1	1.00	1.65	1.38	1.35	1.35	1.56	1.35	1.35
time (sec)	N/A	0.040	0.002	0.002	1.279	0.674	0.080	0.150	0.022

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	46	46	53	46	46
normalized size	1	1.00	1.00	0.84	0.82	0.82	0.95	0.82	0.82
time (sec)	N/A	0.029	0.002	0.001	1.323	0.673	0.080	0.148	0.022

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	45	44	44	44	44	44
normalized size	1	1.00	1.00	2.81	2.75	2.75	2.75	2.75	2.75
time (sec)	N/A	0.005	0.002	0.001	1.333	0.669	0.079	0.147	0.022

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	44	55	43	49	43	43
normalized size	1	1.00	1.00	0.86	1.08	0.84	0.96	0.84	0.84
time (sec)	N/A	0.024	0.001	0.001	1.282	0.707	0.085	0.143	0.021

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	47	44	49	47	44
normalized size	1	1.00	1.00	0.90	0.94	0.88	0.98	0.94	0.88
time (sec)	N/A	0.034	0.004	0.003	1.341	0.771	0.134	0.148	0.027

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	44	48	44	44	44
normalized size	1	1.00	1.00	0.94	0.92	1.00	0.92	0.92	0.92
time (sec)	N/A	0.026	0.007	0.003	1.338	0.639	0.130	0.148	0.025

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	46	49	46	56	44
normalized size	1	1.00	1.00	0.94	0.96	1.02	0.96	1.17	0.92
time (sec)	N/A	0.037	0.005	0.007	1.340	0.855	0.172	0.156	0.027

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	45	48	49	45	47
normalized size	1	1.00	1.00	0.90	0.90	0.96	0.98	0.90	0.94
time (sec)	N/A	0.026	0.006	0.005	1.445	0.785	0.170	0.172	0.045

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	46	48	49	49	59	48
normalized size	1	1.00	1.00	0.94	0.98	1.00	1.00	1.20	0.98
time (sec)	N/A	0.036	0.005	0.007	1.362	0.808	0.220	0.150	0.038

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	47	48	49	47	47
normalized size	1	1.00	1.00	0.90	0.94	0.96	0.98	0.94	0.94
time (sec)	N/A	0.026	0.009	0.006	1.371	0.753	0.227	0.152	0.045

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	46	48	50	49	57	47
normalized size	1	1.00	1.00	0.94	0.98	1.02	1.00	1.16	0.96
time (sec)	N/A	0.033	0.005	0.007	1.309	0.773	0.304	0.167	0.038

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	44	46	48	48	46	46
normalized size	1	1.00	1.00	0.94	0.98	1.02	1.02	0.98	0.98
time (sec)	N/A	0.025	0.006	0.006	1.449	0.849	0.297	0.156	4.190

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	50	50	49	58	47
normalized size	1	1.00	1.00	0.90	1.00	1.00	0.98	1.16	0.94
time (sec)	N/A	0.033	0.005	0.005	1.376	0.593	0.368	0.152	0.051

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	47	48	48	51	48	47
normalized size	1	1.00	1.00	0.87	0.89	0.89	0.94	0.89	0.87
time (sec)	N/A	0.026	0.008	0.007	1.426	0.753	0.369	0.172	0.034

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	52	47	46	46	49	46	46
normalized size	1	1.00	2.74	2.47	2.42	2.42	2.58	2.42	2.42
time (sec)	N/A	0.006	0.005	0.005	1.346	0.789	0.393	0.184	0.034

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	48	48	51	48	48
normalized size	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.026	0.007	0.006	1.300	0.785	0.401	0.157	4.840

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	56	47	48	48	51	48	48
normalized size	1	1.00	1.40	1.18	1.20	1.20	1.28	1.20	1.20
time (sec)	N/A	0.025	0.004	0.005	1.351	0.571	0.428	0.148	4.218

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	48	48	51	48	48
normalized size	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.027	0.009	0.005	1.463	0.796	0.439	0.214	0.037

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	48	48	51	48	48
normalized size	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.036	0.004	0.007	1.434	0.842	0.445	0.154	4.329

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	48	48	51	48	48
normalized size	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.025	0.007	0.005	1.380	0.842	0.458	0.152	4.351

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	68	68	80	68	68
normalized size	1	1.00	1.00	0.84	0.83	0.83	0.98	0.83	0.83
time (sec)	N/A	0.046	0.003	0.001	1.349	0.717	0.089	0.160	0.033

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	82	69	68	68	78	68	68
normalized size	1	1.00	1.14	0.96	0.94	0.94	1.08	0.94	0.94
time (sec)	N/A	0.117	0.003	0.001	1.431	0.823	0.087	0.152	0.031

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	68	67	67	76	67	67
normalized size	1	1.00	1.00	0.86	0.85	0.85	0.96	0.85	0.85
time (sec)	N/A	0.038	0.003	0.001	1.390	0.763	0.088	0.150	0.031

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	82	69	68	68	80	68	68
normalized size	1	1.00	1.55	1.30	1.28	1.28	1.51	1.28	1.28
time (sec)	N/A	0.085	0.003	0.001	1.362	0.550	0.086	0.153	0.033

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	68	68	80	68	68
normalized size	1	1.00	1.00	0.84	0.83	0.83	0.98	0.83	0.83
time (sec)	N/A	0.040	0.003	0.000	1.294	0.633	0.089	0.165	0.033

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	77	68	67	67	75	67	67
normalized size	1	1.00	2.26	2.00	1.97	1.97	2.21	1.97	1.97
time (sec)	N/A	0.046	0.003	0.000	1.361	0.824	0.087	0.170	0.032

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	68	68	80	68	68
normalized size	1	1.00	1.00	0.84	0.83	0.83	0.98	0.83	0.83
time (sec)	N/A	0.038	0.003	0.002	1.357	0.855	0.086	0.151	0.032

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	69	68	68	78	68	68
normalized size	1	1.00	1.00	4.31	4.25	4.25	4.88	4.25	4.25
time (sec)	N/A	0.005	0.002	0.002	1.379	0.563	0.088	0.155	0.031

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	66	100	65	73	65	65
normalized size	1	1.00	1.00	0.90	1.37	0.89	1.00	0.89	0.89
time (sec)	N/A	0.034	0.001	0.001	1.337	0.909	0.083	0.156	0.030

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	67	69	66	76	69	66
normalized size	1	1.00	1.00	0.88	0.91	0.87	1.00	0.91	0.87
time (sec)	N/A	0.055	0.004	0.003	1.403	0.633	0.166	0.151	0.036

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	67	66	70	70	66	66
normalized size	1	1.00	1.00	0.93	0.92	0.97	0.97	0.92	0.92
time (sec)	N/A	0.039	0.008	0.005	1.349	0.587	0.158	0.156	0.034

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	68	69	72	76	79	67
normalized size	1	1.00	1.00	0.88	0.90	0.94	0.99	1.03	0.87
time (sec)	N/A	0.056	0.008	0.007	1.434	0.982	0.196	0.153	0.039

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	67	67	70	75	67	69
normalized size	1	1.00	1.00	0.91	0.91	0.95	1.01	0.91	0.93
time (sec)	N/A	0.036	0.009	0.005	1.290	0.657	0.207	0.184	0.032

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	67	69	71	73	80	69
normalized size	1	1.00	1.00	0.93	0.96	0.99	1.01	1.11	0.96
time (sec)	N/A	0.052	0.005	0.007	1.366	0.946	0.253	0.159	0.036

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	67	67	70	73	67	69
normalized size	1	1.00	1.00	0.93	0.93	0.97	1.01	0.93	0.96
time (sec)	N/A	0.041	0.007	0.006	1.315	0.874	0.263	0.156	0.033

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	68	70	71	76	81	70
normalized size	1	1.00	1.00	0.86	0.89	0.90	0.96	1.03	0.89
time (sec)	N/A	0.051	0.005	0.006	1.386	0.964	0.325	0.153	4.344

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	67	69	70	73	69	69
normalized size	1	1.00	1.00	0.93	0.96	0.97	1.01	0.96	0.96
time (sec)	N/A	0.037	0.009	0.006	1.381	0.722	0.335	0.151	0.055

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	68	70	72	73	81	69
normalized size	1	1.00	1.00	0.93	0.96	0.99	1.00	1.11	0.95
time (sec)	N/A	0.052	0.008	0.008	1.314	0.750	0.420	0.156	0.047

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	67	69	70	73	69	70
normalized size	1	1.00	1.00	0.91	0.93	0.95	0.99	0.93	0.95
time (sec)	N/A	0.039	0.009	0.007	1.413	0.916	0.435	0.240	0.053

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	68	72	72	75	81	70
normalized size	1	1.00	1.00	0.88	0.94	0.94	0.97	1.05	0.91
time (sec)	N/A	0.051	0.005	0.009	1.381	0.777	0.626	0.151	4.403

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	66	68	70	71	68	68
normalized size	1	1.00	1.00	0.93	0.96	0.99	1.00	0.96	0.96
time (sec)	N/A	0.037	0.006	0.007	1.314	0.908	0.521	0.153	4.300

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	67	72	72	73	80	69
normalized size	1	1.00	1.00	0.88	0.95	0.95	0.96	1.05	0.91
time (sec)	N/A	0.049	0.005	0.007	1.400	0.851	0.618	0.152	0.065

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	69	70	70	75	70	69
normalized size	1	1.00	1.00	0.91	0.92	0.92	0.99	0.92	0.91
time (sec)	N/A	0.040	0.009	0.005	1.340	0.808	0.598	0.177	0.053

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	82	69	68	68	73	68	70
normalized size	1	1.00	4.32	3.63	3.58	3.58	3.84	3.58	3.68
time (sec)	N/A	0.007	0.008	0.006	1.380	0.982	0.628	0.159	4.357

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	70	70	75	70	70
normalized size	1	1.00	1.00	0.84	0.85	0.85	0.91	0.85	0.85
time (sec)	N/A	0.038	0.009	0.005	1.365	0.743	0.617	0.153	0.051

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	78	69	70	70	75	70	69
normalized size	1	1.00	1.95	1.72	1.75	1.75	1.88	1.75	1.72
time (sec)	N/A	0.025	0.005	0.006	1.391	0.971	0.680	0.169	4.366

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	70	70	75	70	70
normalized size	1	1.00	1.00	0.84	0.85	0.85	0.91	0.85	0.85
time (sec)	N/A	0.038	0.007	0.006	1.373	0.540	0.661	0.168	0.047

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	82	69	70	70	75	70	70
normalized size	1	1.00	1.32	1.11	1.13	1.13	1.21	1.13	1.13
time (sec)	N/A	0.039	0.005	0.006	1.336	0.861	0.741	0.166	4.313

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	69	70	70	75	70	69
normalized size	1	1.00	1.00	0.86	0.88	0.88	0.94	0.88	0.86
time (sec)	N/A	0.038	0.009	0.006	1.354	0.775	0.714	0.177	0.053

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	82	69	70	70	75	70	70
normalized size	1	1.00	0.98	0.82	0.83	0.83	0.89	0.83	0.83
time (sec)	N/A	0.056	0.008	0.006	1.416	0.969	0.857	0.154	0.053

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	70	70	75	70	70
normalized size	1	1.00	1.00	0.84	0.85	0.85	0.91	0.85	0.85
time (sec)	N/A	0.040	0.010	0.006	1.338	0.925	0.758	0.172	0.053

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	72	74	77	93	80	92	79
normalized size	1	1.00	0.87	0.89	0.93	1.12	0.96	1.11	0.95
time (sec)	N/A	0.082	0.021	0.009	1.373	1.036	0.323	0.161	4.361

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	60	63	65	81	66	80	68
normalized size	1	1.00	0.86	0.90	0.93	1.16	0.94	1.14	0.97
time (sec)	N/A	0.063	0.022	0.010	1.353	0.811	0.288	0.173	0.044

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	52	54	70	53	67	57
normalized size	1	1.00	0.86	0.91	0.95	1.23	0.93	1.18	1.00
time (sec)	N/A	0.052	0.015	0.008	1.346	0.972	0.273	0.160	0.055

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	41	43	56	39	49	45
normalized size	1	1.00	0.86	0.93	0.98	1.27	0.89	1.11	1.02
time (sec)	N/A	0.037	0.015	0.006	1.296	0.810	0.253	0.169	0.052

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	27	30	32	35	29	30	29
normalized size	1	1.00	0.82	0.91	0.97	1.06	0.88	0.91	0.88
time (sec)	N/A	0.028	0.008	0.006	1.414	0.781	0.209	0.162	0.051

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	15	15	15	14	14
normalized size	1	1.00	1.00	0.94	0.94	0.94	0.94	0.88	0.88
time (sec)	N/A	0.005	0.003	0.003	1.380	0.935	0.165	0.170	4.325

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	35	37	47	34	47	34
normalized size	1	1.00	0.87	0.92	0.97	1.24	0.89	1.24	0.89
time (sec)	N/A	0.038	0.015	0.012	1.359	1.257	0.322	0.152	4.388

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	46	52	73	51	51	51
normalized size	1	1.00	0.84	0.94	1.06	1.49	1.04	1.04	1.04
time (sec)	N/A	0.049	0.036	0.014	1.357	1.151	0.394	0.163	0.077

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	57	61	70	90	68	86	67
normalized size	1	1.00	0.86	0.92	1.06	1.36	1.03	1.30	1.02
time (sec)	N/A	0.054	0.051	0.014	1.415	1.045	0.467	0.173	0.072

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	82	78	82	212	134	84	77
normalized size	1	1.00	0.89	0.85	0.89	2.30	1.46	0.91	0.84
time (sec)	N/A	0.056	0.053	0.013	2.992	1.143	0.348	0.153	0.044

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	68	71	190	124	73	66
normalized size	1	1.00	0.90	0.86	0.90	2.41	1.57	0.92	0.84
time (sec)	N/A	0.045	0.046	0.010	2.946	0.992	0.323	0.151	4.274

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	60	57	59	164	107	61	56
normalized size	1	1.00	0.91	0.86	0.89	2.48	1.62	0.92	0.85
time (sec)	N/A	0.038	0.041	0.009	3.127	0.722	0.303	0.156	0.064

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	51	43	45	136	83	42	43
normalized size	1	1.00	0.93	0.78	0.82	2.47	1.51	0.76	0.78
time (sec)	N/A	0.027	0.032	0.008	3.083	0.877	0.273	0.153	4.286

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	36	120	78	35	33
normalized size	1	1.00	1.00	0.80	0.80	2.67	1.73	0.78	0.73
time (sec)	N/A	0.017	0.021	0.007	2.995	0.973	0.222	0.197	0.046

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	35	120	78	35	33
normalized size	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	0.73
time (sec)	N/A	0.017	0.024	0.004	2.846	0.838	0.220	0.153	0.043

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	54	46	49	136	92	47	44
normalized size	1	1.00	0.95	0.81	0.86	2.39	1.61	0.82	0.77
time (sec)	N/A	0.027	0.036	0.011	2.946	0.909	0.320	0.179	4.489

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	67	59	64	172	114	59	58
normalized size	1	1.00	0.99	0.87	0.94	2.53	1.68	0.87	0.85
time (sec)	N/A	0.036	0.037	0.011	3.017	0.998	0.358	0.152	4.431

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	80	70	75	198	126	70	70
normalized size	1	1.00	0.99	0.86	0.93	2.44	1.56	0.86	0.86
time (sec)	N/A	0.046	0.044	0.013	3.083	1.034	0.428	0.154	4.713

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	78	86	99	137	100	91	98
normalized size	1	1.00	0.86	0.95	1.09	1.51	1.10	1.00	1.08
time (sec)	N/A	0.093	0.031	0.013	1.386	0.828	0.627	0.175	4.483

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	59	74	88	124	90	73	88
normalized size	1	1.00	0.77	0.96	1.14	1.61	1.17	0.95	1.14
time (sec)	N/A	0.073	0.050	0.013	1.405	0.926	0.586	0.156	4.506

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	50	64	77	102	76	53	75
normalized size	1	1.00	0.70	0.90	1.08	1.44	1.07	0.75	1.06
time (sec)	N/A	0.064	0.018	0.010	1.310	0.865	0.472	0.168	4.327

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	35	48	58	58	60	33	60
normalized size	1	1.00	1.84	2.53	3.05	3.05	3.16	1.74	3.16
time (sec)	N/A	0.007	0.013	0.008	1.384	0.911	0.398	0.191	4.287

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	31	47	47	48	22	48
normalized size	1	1.00	0.71	0.91	1.38	1.38	1.41	0.65	1.41
time (sec)	N/A	0.029	0.009	0.006	1.349	1.088	0.366	0.155	4.229

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	37	37	39	14	39
normalized size	1	1.00	1.00	0.94	2.31	2.31	2.44	0.88	2.44
time (sec)	N/A	0.005	0.003	0.004	1.339	0.530	0.333	0.153	4.284

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	54	63	82	134	80	70	78
normalized size	1	1.00	0.77	0.90	1.17	1.91	1.14	1.00	1.11
time (sec)	N/A	0.076	0.041	0.014	1.418	1.207	0.556	0.153	4.467

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	70	77	99	163	102	93	97
normalized size	1	1.00	0.83	0.92	1.18	1.94	1.21	1.11	1.15
time (sec)	N/A	0.086	0.067	0.015	1.396	0.808	0.673	0.160	0.152

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	85	96	114	178	116	108	111
normalized size	1	1.00	0.84	0.95	1.13	1.76	1.15	1.07	1.10
time (sec)	N/A	0.098	0.058	0.015	1.421	1.131	0.718	0.156	4.664

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	99	108	116	322	172	96	109
normalized size	1	1.00	0.85	0.92	0.99	2.75	1.47	0.82	0.93
time (sec)	N/A	0.072	0.056	0.015	2.977	1.088	0.661	0.153	0.062

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	89	97	104	296	156	84	99
normalized size	1	1.00	0.86	0.93	1.00	2.85	1.50	0.81	0.95
time (sec)	N/A	0.059	0.046	0.013	3.010	1.002	0.628	0.163	4.359

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	77	83	90	268	131	65	86
normalized size	1	1.00	0.83	0.89	0.97	2.88	1.41	0.70	0.92
time (sec)	N/A	0.047	0.045	0.012	2.846	0.735	0.572	0.159	0.103

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	66	58	81	254	134	56	78
normalized size	1	1.00	0.80	0.70	0.98	3.06	1.61	0.67	0.94
time (sec)	N/A	0.040	0.038	0.008	3.068	0.908	0.487	0.168	4.388

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	69	58	87	258	143	62	75
normalized size	1	1.00	0.82	0.69	1.04	3.07	1.70	0.74	0.89
time (sec)	N/A	0.041	0.044	0.010	2.928	1.017	0.453	0.156	4.349

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	69	58	87	258	139	62	74
normalized size	1	1.00	0.81	0.68	1.02	3.04	1.64	0.73	0.87
time (sec)	N/A	0.043	0.038	0.011	2.985	0.914	0.437	0.185	4.313

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	66	66	80	254	129	56	77
normalized size	1	1.00	0.84	0.84	1.01	3.22	1.63	0.71	0.97
time (sec)	N/A	0.037	0.035	0.004	3.009	0.923	0.445	0.154	4.356

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	79	86	93	268	139	68	88
normalized size	1	1.00	0.83	0.91	0.98	2.82	1.46	0.72	0.93
time (sec)	N/A	0.056	0.043	0.014	3.028	0.875	0.584	0.160	4.438

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	91	99	108	304	162	82	102
normalized size	1	1.00	0.86	0.93	1.02	2.87	1.53	0.77	0.96
time (sec)	N/A	0.066	0.049	0.016	3.051	0.942	0.633	0.168	4.449

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	101	110	119	330	173	93	114
normalized size	1	1.00	0.85	0.92	1.00	2.77	1.45	0.78	0.96
time (sec)	N/A	0.079	0.052	0.015	3.008	1.052	0.710	0.159	4.465

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	114	120	143	203	150	113	142
normalized size	1	1.00	0.86	0.90	1.08	1.53	1.13	0.85	1.07
time (sec)	N/A	0.143	0.025	0.015	1.415	1.013	0.995	0.157	0.130

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	101	109	132	190	138	95	132
normalized size	1	1.00	0.86	0.92	1.12	1.61	1.17	0.81	1.12
time (sec)	N/A	0.114	0.028	0.014	1.469	0.994	0.974	0.232	4.600

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	72	98	121	168	124	75	119
normalized size	1	1.00	0.66	0.90	1.11	1.54	1.14	0.69	1.09
time (sec)	N/A	0.102	0.024	0.011	1.422	0.787	0.812	0.157	4.370

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	57	81	102	102	107	55	104
normalized size	1	1.00	3.00	4.26	5.37	5.37	5.63	2.89	5.47
time (sec)	N/A	0.007	0.016	0.008	1.369	0.971	0.718	0.197	4.446

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	46	65	91	91	95	44	93
normalized size	1	1.00	1.18	1.67	2.33	2.33	2.44	1.13	2.38
time (sec)	N/A	0.026	0.014	0.008	1.393	1.070	0.668	0.159	0.054

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	48	80	80	83	33	81
normalized size	1	1.00	0.66	0.91	1.51	1.51	1.57	0.62	1.53
time (sec)	N/A	0.045	0.013	0.009	1.347	1.012	0.618	0.192	4.618

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	31	69	69	71	22	70
normalized size	1	1.00	0.71	0.91	2.03	2.03	2.09	0.65	2.06
time (sec)	N/A	0.031	0.008	0.009	1.355	0.843	0.582	0.159	4.478

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	59	59	63	14	61
normalized size	1	1.00	1.00	0.94	3.69	3.69	3.94	0.88	3.81
time (sec)	N/A	0.005	0.003	0.005	1.314	1.018	0.500	0.194	0.058

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	76	91	126	222	128	92	122
normalized size	1	1.00	0.75	0.89	1.24	2.18	1.25	0.90	1.20
time (sec)	N/A	0.105	0.059	0.015	1.450	0.903	0.814	0.152	0.238

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	92	107	143	251	150	115	141
normalized size	1	1.00	0.79	0.92	1.23	2.16	1.29	0.99	1.22
time (sec)	N/A	0.126	0.084	0.018	1.581	1.021	0.901	0.159	4.677

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	107	129	158	266	165	130	155
normalized size	1	1.00	0.76	0.92	1.13	1.90	1.18	0.93	1.11
time (sec)	N/A	0.145	0.061	0.018	1.448	0.927	0.956	0.159	4.912

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	122	148	159	454	218	117	153
normalized size	1	1.00	0.79	0.95	1.03	2.93	1.41	0.75	0.99
time (sec)	N/A	0.106	0.064	0.016	2.885	0.852	1.086	0.169	0.106

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	111	137	148	428	204	106	143
normalized size	1	1.00	0.78	0.96	1.04	3.01	1.44	0.75	1.01
time (sec)	N/A	0.093	0.058	0.017	2.909	0.872	1.032	0.160	4.520

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	100	123	134	400	178	87	130
normalized size	1	1.00	0.76	0.94	1.02	3.05	1.36	0.66	0.99
time (sec)	N/A	0.078	0.052	0.016	2.989	0.865	0.947	0.171	0.158

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	88	80	125	386	182	78	122
normalized size	1	1.00	0.73	0.66	1.03	3.19	1.50	0.64	1.01
time (sec)	N/A	0.069	0.050	0.013	3.070	0.962	0.789	0.185	4.522

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	91	80	131	390	194	84	119
normalized size	1	1.00	0.75	0.66	1.07	3.20	1.59	0.69	0.98
time (sec)	N/A	0.072	0.058	0.012	3.027	0.969	0.757	0.160	4.421

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	91	78	133	390	196	84	117
normalized size	1	1.00	0.74	0.63	1.08	3.17	1.59	0.68	0.95
time (sec)	N/A	0.072	0.060	0.012	2.957	1.034	0.702	0.157	4.504

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	91	78	133	390	196	84	116
normalized size	1	1.00	0.73	0.63	1.07	3.15	1.58	0.68	0.94
time (sec)	N/A	0.073	0.050	0.012	3.039	0.902	0.636	0.164	4.468

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	91	80	131	390	190	84	118
normalized size	1	1.00	0.73	0.64	1.05	3.12	1.52	0.67	0.94
time (sec)	N/A	0.075	0.051	0.012	2.966	0.705	0.624	0.170	4.482

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	89	96	124	386	177	78	121
normalized size	1	1.00	0.79	0.85	1.10	3.42	1.57	0.69	1.07
time (sec)	N/A	0.066	0.045	0.006	3.042	0.702	0.676	0.168	4.707

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	101	126	137	400	187	90	132
normalized size	1	1.00	0.76	0.95	1.03	3.01	1.41	0.68	0.99
time (sec)	N/A	0.089	0.055	0.017	3.121	1.097	0.828	0.158	4.581

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	113	139	152	436	209	104	146
normalized size	1	1.00	0.78	0.97	1.06	3.03	1.45	0.72	1.01
time (sec)	N/A	0.103	0.060	0.019	3.096	1.017	0.888	0.159	4.623

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	123	150	163	462	221	115	158
normalized size	1	1.00	0.78	0.96	1.04	2.94	1.41	0.73	1.01
time (sec)	N/A	0.122	0.063	0.019	3.107	0.939	0.949	0.164	4.648

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	16	15	19	12	15	16
normalized size	1	1.00	0.84	0.84	0.79	1.00	0.63	0.79	0.84
time (sec)	N/A	0.003	0.006	0.005	3.056	0.994	0.110	0.149	0.034

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	9	11
normalized size	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	1.00
time (sec)	N/A	0.002	0.001	0.005	1.347	0.929	0.086	0.149	0.018

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	21	12	15	17
normalized size	1	1.00	1.00	0.84	0.79	1.11	0.63	0.79	0.89
time (sec)	N/A	0.005	0.008	0.006	2.969	0.809	0.105	0.157	0.027

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	18	19	18	23	15	18	18
normalized size	1	1.00	0.82	0.86	0.82	1.05	0.68	0.82	0.82
time (sec)	N/A	0.011	0.005	0.004	1.276	0.909	0.094	0.196	0.035

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	9	9	8	9	11
normalized size	1	1.00	0.85	0.77	0.69	0.69	0.62	0.69	0.85
time (sec)	N/A	0.002	0.002	0.004	1.330	1.014	0.088	0.162	0.046

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	20	19	18	23	14	19	18
normalized size	1	1.00	0.83	0.79	0.75	0.96	0.58	0.79	0.75
time (sec)	N/A	0.015	0.006	0.005	1.319	0.918	0.099	0.151	4.228

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	13	13	12	29	71
normalized size	1	1.00	0.49	0.46	0.16	0.16	0.15	0.37	0.90
time (sec)	N/A	0.064	0.015	0.008	1.353	1.044	0.105	0.157	4.455

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	39	36	13	13	12	23	59
normalized size	1	1.00	0.58	0.54	0.19	0.19	0.18	0.34	0.88
time (sec)	N/A	0.052	0.007	0.004	1.281	0.975	0.103	0.184	4.310

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	14	13	12	22	33
normalized size	1	1.00	1.06	0.97	0.39	0.36	0.33	0.61	0.92
time (sec)	N/A	0.026	0.007	0.010	1.321	0.792	0.098	0.157	4.352

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	37	34	14	11	10	30	109
normalized size	1	1.00	0.49	0.45	0.19	0.15	0.13	0.40	1.45
time (sec)	N/A	0.022	0.012	0.014	1.385	1.097	0.123	0.158	4.394

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	39	38	14	17	10	45	112
normalized size	1	1.00	0.52	0.51	0.19	0.23	0.13	0.60	1.49
time (sec)	N/A	0.022	0.010	0.009	1.402	0.795	0.150	0.235	4.450

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	34	13	13	14	30	33
normalized size	1	1.00	0.95	0.87	0.33	0.33	0.36	0.77	0.85
time (sec)	N/A	0.038	0.008	0.003	1.333	0.732	0.168	0.157	4.212

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	39	36	15	15	15	31	35
normalized size	1	1.00	0.54	0.50	0.21	0.21	0.21	0.43	0.49
time (sec)	N/A	0.016	0.008	0.003	1.303	0.979	0.190	0.155	4.238

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	15	15	15	31	35
normalized size	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44
time (sec)	N/A	0.059	0.008	0.005	1.351	0.796	0.203	0.158	4.242

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	15	15	15	31	35
normalized size	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44
time (sec)	N/A	0.058	0.008	0.004	1.310	0.737	0.216	0.164	4.215

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	13	13	12	29	-1
normalized size	1	1.00	0.49	0.46	0.16	0.16	0.15	0.37	-0.01
time (sec)	N/A	0.023	0.007	0.004	1.348	0.814	0.100	0.166	0.000

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	13	13	12	29	-1
normalized size	1	1.00	0.49	0.46	0.16	0.16	0.15	0.37	-0.01
time (sec)	N/A	0.023	0.007	0.003	1.307	0.881	0.148	0.154	0.000

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	36	33	10	10	8	20	-1
normalized size	1	1.00	0.49	0.45	0.14	0.14	0.11	0.27	-0.01
time (sec)	N/A	0.014	0.007	0.003	1.319	0.800	0.099	0.185	0.000

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	35	34	10	13	5	26	-1
normalized size	1	1.00	0.49	0.47	0.14	0.18	0.07	0.36	-0.01
time (sec)	N/A	0.020	0.008	0.003	1.345	1.009	0.125	0.156	0.000

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	37	34	13	13	14	30	33
normalized size	1	1.00	0.48	0.44	0.17	0.17	0.18	0.39	0.43
time (sec)	N/A	0.023	0.007	0.003	1.383	0.755	0.162	0.173	4.244

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	15	15	15	31	35
normalized size	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44
time (sec)	N/A	0.021	0.008	0.004	1.306	0.772	0.181	0.152	4.211

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	15	15	15	31	35
normalized size	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44
time (sec)	N/A	0.022	0.008	0.002	1.271	0.945	0.197	0.156	4.184

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	15	15	15	31	35
normalized size	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44
time (sec)	N/A	0.022	0.008	0.006	1.294	0.678	0.215	0.161	4.195

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
normalized size	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.114	0.019	0.007	1.315	1.168	0.000	0.156	0.000

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
normalized size	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.114	0.016	0.007	1.317	0.838	0.000	0.158	0.000

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	119	61	58	35	35	0	67	-1
normalized size	1	1.12	0.58	0.55	0.33	0.33	0.00	0.63	-0.01
time (sec)	N/A	0.083	0.016	0.007	1.337	0.818	0.000	0.219	0.000

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	61	58	35	35	0	45	46
normalized size	1	1.00	0.91	0.87	0.52	0.52	0.00	0.67	0.69
time (sec)	N/A	0.051	0.016	0.006	1.326	0.852	0.000	0.154	4.286

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	57	35	35	0	44	36
normalized size	1	1.00	0.75	1.58	0.97	0.97	0.00	1.22	1.00
time (sec)	N/A	0.026	0.012	0.005	1.356	0.798	0.000	0.151	4.253

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	60	57	33	33	0	68	-1
normalized size	1	1.00	0.37	0.35	0.20	0.20	0.00	0.42	-0.01
time (sec)	N/A	0.049	0.021	0.008	1.328	0.838	0.000	0.191	0.000

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	62	59	34	38	0	87	-1
normalized size	1	1.00	0.38	0.36	0.21	0.23	0.00	0.53	-0.01
time (sec)	N/A	0.047	0.021	0.012	1.292	0.803	0.000	0.161	0.000

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	61	60	34	39	0	87	-1
normalized size	1	1.00	0.37	0.37	0.21	0.24	0.00	0.53	-0.01
time (sec)	N/A	0.049	0.016	0.014	1.312	0.826	0.000	0.173	0.000

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	63	60	33	39	0	87	-1
normalized size	1	1.00	0.39	0.37	0.20	0.24	0.00	0.53	-0.01
time (sec)	N/A	0.046	0.021	0.014	1.261	0.949	0.000	0.188	0.000

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	59	56	35	35	0	68	151
normalized size	1	1.00	1.44	1.37	0.85	0.85	0.00	1.66	3.68
time (sec)	N/A	0.039	0.016	0.005	1.358	0.998	0.000	0.174	4.243

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	61	58	35	37	0	69	151
normalized size	1	1.00	0.85	0.81	0.49	0.51	0.00	0.96	2.10
time (sec)	N/A	0.017	0.014	0.008	1.426	0.882	0.000	0.207	4.203

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	37	0	69	151
normalized size	1	1.00	0.37	0.35	0.21	0.22	0.00	0.41	0.90
time (sec)	N/A	0.105	0.014	0.007	1.310	0.789	0.000	0.162	4.207

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	37	0	69	151
normalized size	1	1.00	0.37	0.35	0.21	0.22	0.00	0.41	0.90
time (sec)	N/A	0.106	0.017	0.009	1.133	0.882	0.000	0.156	4.212

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	37	0	69	151
normalized size	1	1.00	0.37	0.35	0.21	0.22	0.00	0.41	0.90
time (sec)	N/A	0.105	0.014	0.008	1.323	0.980	0.000	0.183	4.234

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
normalized size	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.042	0.015	0.009	1.298	0.958	0.000	0.159	0.000

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
normalized size	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.041	0.015	0.008	1.268	0.689	0.000	0.158	0.000

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
normalized size	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.043	0.015	0.007	1.393	0.680	0.000	0.181	0.000

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
normalized size	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.041	0.012	0.007	1.347	0.907	0.000	0.155	0.000

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	59	56	31	31	0	63	-1
normalized size	1	1.00	0.37	0.35	0.19	0.19	0.00	0.40	-0.01
time (sec)	N/A	0.033	0.012	0.004	1.364	0.853	0.000	0.184	0.000

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	60	58	32	36	0	64	-1
normalized size	1	1.00	0.38	0.37	0.20	0.23	0.00	0.41	-0.01
time (sec)	N/A	0.040	0.015	0.007	1.342	0.670	0.000	0.163	0.000

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	59	56	33	36	0	67	-1
normalized size	1	1.00	0.37	0.35	0.20	0.22	0.00	0.42	-0.01
time (sec)	N/A	0.040	0.014	0.007	1.290	0.938	0.000	0.158	0.000

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	59	56	32	37	0	66	-1
normalized size	1	1.00	0.37	0.35	0.20	0.23	0.00	0.42	-0.01
time (sec)	N/A	0.040	0.014	0.006	1.323	0.965	0.000	0.190	0.000

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	61	58	35	37	0	69	151
normalized size	1	1.00	0.37	0.36	0.21	0.23	0.00	0.42	0.93
time (sec)	N/A	0.043	0.014	0.006	1.273	1.094	0.000	0.219	4.255

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	37	0	69	151
normalized size	1	1.00	0.37	0.35	0.21	0.22	0.00	0.41	0.90
time (sec)	N/A	0.039	0.014	0.007	1.338	0.818	0.000	0.190	4.264

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	37	0	69	151
normalized size	1	1.00	0.37	0.35	0.21	0.22	0.00	0.41	0.90
time (sec)	N/A	0.044	0.016	0.007	1.303	0.838	0.000	0.157	4.549

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	37	0	69	151
normalized size	1	1.00	0.37	0.35	0.21	0.22	0.00	0.41	0.90
time (sec)	N/A	0.040	0.014	0.007	1.390	0.731	0.000	0.157	4.630

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	37	0	69	151
normalized size	1	1.00	0.37	0.35	0.21	0.22	0.00	0.41	0.90
time (sec)	N/A	0.041	0.014	0.009	1.294	0.842	0.000	0.206	4.297

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
normalized size	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.162	0.027	0.010	1.319	0.646	0.000	0.157	0.000

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
normalized size	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.159	0.023	0.010	1.374	1.049	0.000	0.154	0.000

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	83	80	57	57	0	105	-1
normalized size	1	1.00	0.41	0.40	0.28	0.28	0.00	0.52	-0.00
time (sec)	N/A	0.132	0.020	0.009	1.433	0.726	0.000	0.160	0.000

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	83	80	57	57	0	105	-1
normalized size	1	1.00	0.52	0.50	0.36	0.36	0.00	0.66	-0.01
time (sec)	N/A	0.118	0.022	0.008	1.400	0.975	0.000	0.156	0.000

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	83	80	56	56	0	104	-1
normalized size	1	1.00	0.70	0.67	0.47	0.47	0.00	0.87	-0.01
time (sec)	N/A	0.097	0.022	0.008	1.336	0.630	0.000	0.193	0.000

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	83	80	56	56	0	67	-1
normalized size	1	1.00	1.24	1.19	0.84	0.84	0.00	1.00	-0.01
time (sec)	N/A	0.052	0.021	0.008	1.249	1.159	0.000	0.202	0.000

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	79	57	57	0	66	36
normalized size	1	1.00	0.75	2.19	1.58	1.58	0.00	1.83	1.00
time (sec)	N/A	0.026	0.014	0.006	1.311	1.044	0.000	0.158	4.404

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	82	79	55	55	0	106	-1
normalized size	1	1.00	0.33	0.31	0.22	0.22	0.00	0.42	-0.00
time (sec)	N/A	0.071	0.025	0.011	1.361	0.957	0.000	0.193	0.000

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	85	82	56	61	0	125	-1
normalized size	1	1.00	0.34	0.33	0.22	0.24	0.00	0.50	-0.00
time (sec)	N/A	0.071	0.026	0.012	1.339	0.689	0.000	0.162	0.000

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	85	82	56	61	0	127	-1
normalized size	1	1.00	0.34	0.33	0.22	0.24	0.00	0.51	-0.00
time (sec)	N/A	0.070	0.026	0.014	1.341	1.053	0.000	0.159	0.000

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	85	82	56	61	0	128	-1
normalized size	1	1.00	0.34	0.33	0.22	0.24	0.00	0.51	-0.00
time (sec)	N/A	0.072	0.024	0.013	1.384	0.877	0.000	0.158	0.000

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	85	82	56	61	0	126	-1
normalized size	1	1.00	0.34	0.33	0.22	0.24	0.00	0.50	-0.00
time (sec)	N/A	0.071	0.020	0.013	1.359	0.678	0.000	0.192	0.000

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	85	82	55	61	0	125	-1
normalized size	1	1.00	0.34	0.33	0.22	0.24	0.00	0.50	-0.00
time (sec)	N/A	0.068	0.027	0.014	1.325	1.095	0.000	0.164	0.000

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	81	78	57	57	0	106	231
normalized size	1	1.00	1.98	1.90	1.39	1.39	0.00	2.59	5.63
time (sec)	N/A	0.039	0.018	0.007	1.396	0.871	0.000	0.158	4.178

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	83	80	57	59	0	107	231
normalized size	1	1.00	1.15	1.11	0.79	0.82	0.00	1.49	3.21
time (sec)	N/A	0.017	0.021	0.009	1.347	0.617	0.000	0.161	4.219

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	83	80	57	59	0	107	231
normalized size	1	1.00	0.65	0.62	0.45	0.46	0.00	0.84	1.80
time (sec)	N/A	0.091	0.018	0.008	1.351	1.205	0.000	0.213	4.235

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	59	0	107	231
normalized size	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91
time (sec)	N/A	0.155	0.017	0.009	1.330	1.173	0.000	0.170	4.266

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	59	0	107	231
normalized size	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91
time (sec)	N/A	0.151	0.018	0.009	1.340	1.134	0.000	0.163	4.225

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	59	0	107	231
normalized size	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91
time (sec)	N/A	0.153	0.018	0.010	1.340	1.001	0.000	0.186	4.233

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	59	0	107	231
normalized size	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91
time (sec)	N/A	0.151	0.022	0.011	1.376	1.162	0.000	0.166	4.223

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
normalized size	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.062	0.022	0.008	1.283	1.151	0.000	0.157	0.000

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
normalized size	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.058	0.020	0.009	1.355	0.637	0.000	0.160	0.000

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
normalized size	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.061	0.020	0.007	1.327	1.141	0.000	0.164	0.000

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
normalized size	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.058	0.020	0.008	1.288	0.652	0.000	0.172	0.000

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
normalized size	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.059	0.020	0.010	1.334	1.108	0.000	0.158	0.000

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	83	80	56	56	0	104	-1
normalized size	1	1.00	0.33	0.32	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.060	0.020	0.008	1.364	0.926	0.000	0.180	0.000

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	81	78	54	54	0	102	-1
normalized size	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.051	0.016	0.004	1.319	0.607	0.000	0.162	0.000

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	83	80	55	59	0	103	-1
normalized size	1	1.00	0.34	0.32	0.22	0.24	0.00	0.42	-0.00
time (sec)	N/A	0.058	0.020	0.007	1.324	1.066	0.000	0.162	0.000

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	83	80	54	59	0	104	-1
normalized size	1	1.00	0.34	0.33	0.22	0.24	0.00	0.42	-0.00
time (sec)	N/A	0.060	0.023	0.007	1.329	0.774	0.000	0.160	0.000

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	83	80	55	59	0	106	-1
normalized size	1	1.00	0.33	0.32	0.22	0.24	0.00	0.43	-0.00
time (sec)	N/A	0.058	0.020	0.007	1.347	0.807	0.000	0.174	0.000

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	83	80	55	59	0	106	-1
normalized size	1	1.00	0.34	0.32	0.22	0.24	0.00	0.43	-0.00
time (sec)	N/A	0.058	0.018	0.007	1.287	0.946	0.000	0.159	0.000

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	83	80	54	59	0	105	-1
normalized size	1	1.00	0.34	0.33	0.22	0.24	0.00	0.43	-0.00
time (sec)	N/A	0.058	0.018	0.006	1.350	1.071	0.000	0.168	0.000

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	83	80	57	59	0	107	231
normalized size	1	1.00	0.33	0.32	0.23	0.24	0.00	0.43	0.92
time (sec)	N/A	0.058	0.017	0.008	1.335	1.057	0.000	0.159	4.221

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	83	80	57	59	0	107	231
normalized size	1	1.00	0.33	0.32	0.23	0.23	0.00	0.42	0.91
time (sec)	N/A	0.060	0.017	0.007	1.335	0.981	0.000	0.178	4.350

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	59	0	107	231
normalized size	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91
time (sec)	N/A	0.058	0.017	0.009	1.352	0.969	0.000	0.160	4.208

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	59	0	107	231
normalized size	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91
time (sec)	N/A	0.059	0.017	0.009	1.355	0.926	0.000	0.168	4.315

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	59	0	107	231
normalized size	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91
time (sec)	N/A	0.058	0.019	0.008	1.345	0.575	0.000	0.199	4.267

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	59	0	107	231
normalized size	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91
time (sec)	N/A	0.058	0.018	0.009	1.337	1.002	0.000	0.161	4.337

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	59	0	107	231
normalized size	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91
time (sec)	N/A	0.058	0.018	0.010	1.376	0.855	0.000	0.160	4.311

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	55	52	34	33	32	59	-1
normalized size	1	1.00	0.43	0.41	0.27	0.26	0.25	0.46	-0.01
time (sec)	N/A	0.102	0.024	0.009	1.389	1.002	0.196	0.181	0.000

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	44	41	23	22	20	33	64
normalized size	1	1.00	0.59	0.55	0.31	0.29	0.27	0.44	0.85
time (sec)	N/A	0.056	0.012	0.007	1.311	0.729	0.183	0.171	4.519

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	35	32	13	13	10	22	33
normalized size	1	1.00	0.80	0.73	0.30	0.30	0.23	0.50	0.75
time (sec)	N/A	0.032	0.008	0.004	1.351	0.762	0.154	0.153	4.415

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	42	39	23	18	15	33	40
normalized size	1	1.00	0.52	0.49	0.29	0.22	0.19	0.41	0.50
time (sec)	N/A	0.034	0.011	0.008	1.281	0.945	0.262	0.152	4.451

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	122	54	51	33	33	31	52	75
normalized size	1	0.98	0.43	0.41	0.26	0.26	0.25	0.42	0.60
time (sec)	N/A	0.050	0.015	0.012	1.285	0.981	0.318	0.156	4.454

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	66	63	37	99	80	64	-1
normalized size	1	1.00	0.51	0.49	0.29	0.77	0.62	0.50	-0.01
time (sec)	N/A	0.046	0.026	0.009	2.931	0.714	0.214	0.160	0.000

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	54	48	26	82	56	42	-1
normalized size	1	1.00	0.61	0.54	0.29	0.92	0.63	0.47	-0.01
time (sec)	N/A	0.032	0.014	0.007	2.914	0.843	0.193	0.231	0.000

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	44	34	15	67	53	23	-1
normalized size	1	1.00	0.83	0.64	0.28	1.26	1.00	0.43	-0.02
time (sec)	N/A	0.015	0.012	0.005	3.030	0.838	0.177	0.177	0.000

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	56	50	29	82	65	37	-1
normalized size	1	1.00	0.61	0.54	0.32	0.89	0.71	0.40	-0.01
time (sec)	N/A	0.033	0.014	0.009	2.941	0.918	0.232	0.153	0.000

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	130	70	69	40	106	87	50	-1
normalized size	1	0.98	0.53	0.52	0.30	0.80	0.65	0.38	-0.01
time (sec)	N/A	0.043	0.025	0.011	3.010	1.191	0.279	0.159	0.000

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	81	103	66	91	0	83	-1
normalized size	1	1.00	0.51	0.65	0.42	0.58	0.00	0.53	-0.01
time (sec)	N/A	0.131	0.031	0.018	1.359	1.169	0.000	0.248	0.000

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	61	81	55	69	0	64	-1
normalized size	1	1.00	0.54	0.72	0.49	0.61	0.00	0.57	-0.01
time (sec)	N/A	0.098	0.022	0.015	1.365	0.957	0.000	0.227	0.000

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	69	39	32	36	36	0	32	42
normalized size	1	1.68	0.95	0.78	0.88	0.88	0.00	0.78	1.02
time (sec)	N/A	0.050	0.012	0.006	1.385	1.161	0.000	0.227	4.244

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	26	26	0	24	34
normalized size	1	1.00	0.71	0.63	0.68	0.68	0.00	0.63	0.89
time (sec)	N/A	0.026	0.008	0.004	1.322	0.896	0.000	0.204	4.343

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	74	107	57	90	0	79	-1
normalized size	1	1.00	0.50	0.73	0.39	0.61	0.00	0.54	-0.01
time (sec)	N/A	0.083	0.027	0.018	1.441	0.840	0.000	0.273	0.000

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	97	133	75	119	0	96	-1
normalized size	1	1.00	0.51	0.70	0.40	0.63	0.00	0.51	-0.01
time (sec)	N/A	0.096	0.037	0.021	1.369	1.002	0.000	0.236	0.000

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	84	97	59	188	0	0	-1
normalized size	1	1.00	0.66	0.76	0.46	1.47	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.032	0.016	3.031	1.010	0.000	0.000	0.000

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	81	97	62	190	0	0	-1
normalized size	1	1.00	0.63	0.75	0.48	1.47	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.028	0.018	2.935	0.936	0.000	0.000	0.000

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	83	97	58	188	0	0	-1
normalized size	1	1.00	0.61	0.72	0.43	1.39	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.024	0.008	2.987	1.073	0.000	0.000	0.000

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	93	119	71	202	0	0	-1
normalized size	1	1.00	0.55	0.70	0.42	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.032	0.018	2.979	0.601	0.000	0.000	0.000

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	105	139	86	238	0	0	-1
normalized size	1	1.00	0.50	0.67	0.41	1.14	0.00	0.00	-0.00
time (sec)	N/A	0.079	0.038	0.021	3.035	0.653	0.000	0.000	0.000

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	103	163	110	157	0	105	-1
normalized size	1	1.00	0.43	0.68	0.46	0.66	0.00	0.44	-0.00
time (sec)	N/A	0.189	0.039	0.021	1.383	1.029	0.000	0.250	0.000

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	83	141	99	135	0	84	-1
normalized size	1	1.00	0.42	0.72	0.51	0.69	0.00	0.43	-0.01
time (sec)	N/A	0.162	0.029	0.017	1.458	1.074	0.000	0.211	0.000

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	61	54	80	80	0	54	144
normalized size	1	1.00	1.49	1.32	1.95	1.95	0.00	1.32	3.51
time (sec)	N/A	0.040	0.017	0.007	1.404	0.922	0.000	0.217	4.291

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	50	43	69	69	0	43	53
normalized size	1	1.00	0.68	0.58	0.93	0.93	0.00	0.58	0.72
time (sec)	N/A	0.016	0.016	0.008	1.788	0.826	0.000	0.213	4.235

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	39	32	58	58	0	32	42
normalized size	1	1.00	0.57	0.46	0.84	0.84	0.00	0.46	0.61
time (sec)	N/A	0.054	0.013	0.008	1.364	0.848	0.000	0.214	4.260

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	48	48	0	24	34
normalized size	1	1.00	0.71	0.63	1.26	1.26	0.00	0.63	0.89
time (sec)	N/A	0.025	0.010	0.007	1.316	0.966	0.000	0.208	4.273

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	96	193	101	178	0	101	-1
normalized size	1	1.00	0.43	0.87	0.45	0.80	0.00	0.45	-0.00
time (sec)	N/A	0.121	0.043	0.019	1.439	0.623	0.000	0.266	0.000

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	119	219	119	207	0	118	-1
normalized size	1	1.00	0.45	0.82	0.45	0.78	0.00	0.44	-0.00
time (sec)	N/A	0.142	0.047	0.023	1.421	0.945	0.000	0.222	0.000

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	105	172	109	324	0	0	-1
normalized size	1	1.00	0.50	0.82	0.52	1.54	0.00	0.00	-0.00
time (sec)	N/A	0.085	0.043	0.018	3.033	0.962	0.000	0.000	0.000

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	105	172	111	324	0	0	-1
normalized size	1	1.00	0.50	0.81	0.52	1.53	0.00	0.00	-0.00
time (sec)	N/A	0.088	0.040	0.018	2.994	1.125	0.000	0.000	0.000

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	105	172	109	324	0	0	-1
normalized size	1	1.00	0.49	0.81	0.51	1.52	0.00	0.00	-0.00
time (sec)	N/A	0.084	0.034	0.017	2.902	1.053	0.000	0.000	0.000

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	105	169	102	320	0	0	-1
normalized size	1	1.00	0.49	0.79	0.48	1.50	0.00	0.00	-0.00
time (sec)	N/A	0.072	0.039	0.008	3.051	1.120	0.000	0.000	0.000

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	115	191	115	334	0	0	-1
normalized size	1	1.00	0.46	0.76	0.46	1.33	0.00	0.00	-0.00
time (sec)	N/A	0.109	0.044	0.022	3.039	0.911	0.000	0.000	0.000

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	127	211	130	370	0	0	-1
normalized size	1	1.00	0.44	0.73	0.45	1.27	0.00	0.00	-0.00
time (sec)	N/A	0.125	0.048	0.024	3.135	1.042	0.000	0.000	0.000

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	64	0	0	0	0	0	-1
normalized size	1	1.00	0.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.229	0.033	0.097	0.000	1.080	0.000	0.000	0.000

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	48	0	0	0	0	0	-1
normalized size	1	1.00	0.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.123	0.011	0.022	0.000	0.842	0.000	0.000	0.000

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	51	0	0	0	0	0	-1
normalized size	1	1.00	0.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.158	0.010	0.070	0.000	0.775	0.000	0.000	0.000

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	618	618	64	0	0	0	0	0	-1
normalized size	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.412	0.033	0.058	0.000	1.011	0.000	0.000	0.000

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	609	609	64	0	0	0	0	0	-1
normalized size	1	1.00	0.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.366	0.020	0.033	0.000	0.969	0.000	0.000	0.000

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	649	649	61	0	0	0	0	0	-1
normalized size	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.444	0.015	0.102	0.000	1.076	0.000	0.000	0.000

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	33	30	41	40	49	48	40
normalized size	1	1.00	0.65	0.59	0.80	0.78	0.96	0.94	0.78
time (sec)	N/A	0.014	0.015	0.008	1.305	0.629	2.671	0.167	0.074

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	33	30	41	34	49	42	41
normalized size	1	1.00	0.65	0.59	0.80	0.67	0.96	0.82	0.80
time (sec)	N/A	0.014	0.012	0.008	1.341	0.866	1.244	0.149	4.224

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	33	30	41	29	49	37	41
normalized size	1	1.00	0.65	0.59	0.80	0.57	0.96	0.73	0.80
time (sec)	N/A	0.013	0.009	0.008	1.364	0.630	0.482	0.150	0.050

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	33	30	41	31	48	41	41
normalized size	1	1.00	0.67	0.61	0.84	0.63	0.98	0.84	0.84
time (sec)	N/A	0.013	0.010	0.009	1.274	0.980	0.628	0.151	0.045

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	33	30	44	34	48	51	31
normalized size	1	1.00	0.67	0.61	0.90	0.69	0.98	1.04	0.63
time (sec)	N/A	0.015	0.012	0.009	1.357	0.735	0.665	0.221	0.052

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	33	30	43	34	48	53	34
normalized size	1	1.00	0.67	0.61	0.88	0.69	0.98	1.08	0.69
time (sec)	N/A	0.014	0.012	0.008	1.403	0.733	0.913	0.154	4.231

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	38	30	47	34	48	48	34
normalized size	1	1.00	0.78	0.61	0.96	0.69	0.98	0.98	0.69
time (sec)	N/A	0.013	0.013	0.006	1.301	0.845	1.969	0.155	0.050

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	55	52	73	68	90	86	71
normalized size	1	1.00	0.60	0.57	0.80	0.75	0.99	0.95	0.78
time (sec)	N/A	0.045	0.022	0.008	1.350	0.848	5.681	0.155	4.199

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	55	52	73	58	90	74	71
normalized size	1	1.00	0.60	0.57	0.80	0.64	0.99	0.81	0.78
time (sec)	N/A	0.044	0.019	0.009	1.350	0.967	2.691	0.171	0.030

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	55	52	73	51	90	69	71
normalized size	1	1.00	0.60	0.57	0.80	0.56	0.99	0.76	0.78
time (sec)	N/A	0.041	0.014	0.008	1.333	1.014	1.275	0.158	0.029

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	55	52	90	53	88	73	71
normalized size	1	1.00	0.62	0.58	1.01	0.60	0.99	0.82	0.80
time (sec)	N/A	0.041	0.016	0.009	1.357	0.819	1.346	0.260	0.031

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	55	52	76	56	88	89	71
normalized size	1	1.00	0.62	0.58	0.85	0.63	0.99	1.00	0.80
time (sec)	N/A	0.042	0.017	0.009	1.211	0.800	1.377	0.177	0.033

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	55	52	76	56	88	92	71
normalized size	1	1.00	0.62	0.58	0.85	0.63	0.99	1.03	0.80
time (sec)	N/A	0.042	0.017	0.008	1.355	0.573	1.737	0.158	0.030

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	60	52	82	56	87	95	75
normalized size	1	1.00	0.69	0.60	0.94	0.64	1.00	1.09	0.86
time (sec)	N/A	0.042	0.020	0.008	1.438	0.806	2.474	0.175	0.058

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	77	74	105	96	129	124	103
normalized size	1	1.00	0.60	0.57	0.81	0.74	1.00	0.96	0.80
time (sec)	N/A	0.066	0.030	0.009	1.401	0.959	10.798	0.161	0.039

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	77	74	105	82	131	106	103
normalized size	1	1.00	0.59	0.56	0.80	0.63	1.00	0.81	0.79
time (sec)	N/A	0.061	0.025	0.010	1.333	1.046	5.338	0.191	0.037

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	77	74	105	73	131	101	103
normalized size	1	1.00	0.59	0.56	0.80	0.56	1.00	0.77	0.79
time (sec)	N/A	0.061	0.021	0.008	1.392	0.888	3.003	0.175	0.038

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	77	74	155	75	129	105	103
normalized size	1	1.00	0.60	0.57	1.20	0.58	1.00	0.81	0.80
time (sec)	N/A	0.060	0.023	0.009	1.361	0.869	2.938	0.180	0.037

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	77	74	108	78	126	127	103
normalized size	1	1.00	0.62	0.59	0.86	0.62	1.01	1.02	0.82
time (sec)	N/A	0.060	0.022	0.010	1.389	1.103	3.006	0.199	0.039

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	77	74	108	78	128	130	103
normalized size	1	1.00	0.61	0.58	0.85	0.61	1.01	1.02	0.81
time (sec)	N/A	0.062	0.022	0.009	1.388	0.692	3.571	0.169	0.038

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	82	74	114	78	128	133	107
normalized size	1	1.00	0.65	0.58	0.90	0.61	1.01	1.05	0.84
time (sec)	N/A	0.064	0.027	0.010	1.346	0.767	4.556	0.174	0.039

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	235	242	300	283	0	297	129
normalized size	1	1.00	0.74	0.77	0.95	0.90	0.00	0.94	0.41
time (sec)	N/A	0.383	0.340	0.023	3.037	1.014	0.000	0.213	4.273

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	63	226	273	283	0	277	112
normalized size	1	1.00	0.21	0.76	0.92	0.95	0.00	0.93	0.38
time (sec)	N/A	0.299	0.019	0.018	3.123	0.863	0.000	0.192	0.124

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	244	223	282	247	0	263	112
normalized size	1	1.00	0.82	0.75	0.95	0.83	0.00	0.88	0.38
time (sec)	N/A	0.295	0.163	0.018	3.029	0.889	0.000	0.189	0.120

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	54	209	256	247	0	277	92
normalized size	1	1.00	0.19	0.74	0.91	0.88	0.00	0.99	0.33
time (sec)	N/A	0.280	0.016	0.016	3.103	0.735	0.000	0.232	4.252

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	210	212	265	234	0	261	92
normalized size	1	1.00	0.75	0.75	0.94	0.83	0.00	0.93	0.33
time (sec)	N/A	0.260	0.170	0.014	3.059	0.846	0.000	0.234	4.355

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	32	210	255	232	78	264	90
normalized size	1	1.00	0.11	0.74	0.90	0.82	0.28	0.93	0.32
time (sec)	N/A	0.271	0.006	0.012	3.055	1.134	6.667	0.190	0.112

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	211	207	261	232	0	269	90
normalized size	1	1.00	0.75	0.73	0.92	0.82	0.00	0.95	0.32
time (sec)	N/A	0.265	0.171	0.011	3.001	1.000	0.000	0.192	0.105

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	30	223	268	276	0	294	102
normalized size	1	1.00	0.10	0.74	0.89	0.92	0.00	0.98	0.34
time (sec)	N/A	0.323	0.010	0.019	3.122	1.034	0.000	0.181	0.122

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	32	226	275	300	0	276	102
normalized size	1	1.00	0.11	0.75	0.92	1.00	0.00	0.92	0.34
time (sec)	N/A	0.299	0.011	0.019	3.077	1.086	0.000	0.195	4.396

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	37	242	290	323	0	307	113
normalized size	1	1.00	0.12	0.76	0.91	1.02	0.00	0.97	0.36
time (sec)	N/A	0.343	0.010	0.021	3.008	1.056	0.000	0.185	4.350

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	347	306	361	399	0	336	188
normalized size	1	1.00	0.94	0.83	0.98	1.08	0.00	0.91	0.51
time (sec)	N/A	0.418	0.227	0.023	3.103	0.752	0.000	0.216	0.129

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	87	290	334	399	0	316	171
normalized size	1	1.00	0.25	0.83	0.95	1.14	0.00	0.90	0.49
time (sec)	N/A	0.393	0.028	0.023	3.083	1.119	0.000	0.235	4.333

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	324	287	343	363	0	302	171
normalized size	1	1.00	0.93	0.82	0.98	1.04	0.00	0.86	0.49
time (sec)	N/A	0.382	0.140	0.024	3.141	0.878	0.000	0.199	4.300

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	83	271	317	370	0	314	153
normalized size	1	1.00	0.25	0.81	0.95	1.11	0.00	0.94	0.46
time (sec)	N/A	0.345	0.027	0.019	3.444	0.801	0.000	0.202	0.110

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	299	280	326	373	0	301	153
normalized size	1	1.00	0.90	0.84	0.98	1.12	0.00	0.90	0.46
time (sec)	N/A	0.345	0.130	0.021	3.131	1.350	0.000	0.213	4.289

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	74	277	323	390	0	317	150
normalized size	1	1.00	0.22	0.82	0.96	1.16	0.00	0.94	0.45
time (sec)	N/A	0.351	0.022	0.021	3.017	0.956	0.000	0.215	4.260

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	279	277	332	389	0	304	150
normalized size	1	1.00	0.83	0.82	0.99	1.16	0.00	0.90	0.45
time (sec)	N/A	0.341	0.166	0.019	2.962	0.939	0.000	0.236	4.264

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	60	277	323	396	0	317	149
normalized size	1	1.00	0.18	0.83	0.96	1.18	0.00	0.95	0.44
time (sec)	N/A	0.348	0.020	0.019	3.019	0.797	0.000	0.218	4.232

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	260	271	332	373	0	302	149
normalized size	1	1.00	0.78	0.81	0.99	1.11	0.00	0.90	0.44
time (sec)	N/A	0.347	0.131	0.021	3.019	1.048	0.000	0.197	4.272

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	32	272	317	359	252	302	150
normalized size	1	1.00	0.10	0.81	0.95	1.07	0.75	0.90	0.45
time (sec)	N/A	0.352	0.006	0.019	2.983	1.007	28.986	0.221	0.099

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	253	269	322	357	0	308	150
normalized size	1	1.00	0.76	0.80	0.96	1.07	0.00	0.92	0.45
time (sec)	N/A	0.348	0.112	0.018	3.091	1.086	0.000	0.268	4.284

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	30	285	328	410	0	327	166
normalized size	1	1.00	0.09	0.81	0.93	1.16	0.00	0.93	0.47
time (sec)	N/A	0.402	0.011	0.025	3.071	1.233	0.000	0.195	0.138

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	32	288	335	434	0	308	166
normalized size	1	1.00	0.09	0.82	0.95	1.23	0.00	0.88	0.47
time (sec)	N/A	0.388	0.013	0.024	3.071	1.115	0.000	0.216	4.253

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	37	304	350	457	0	349	179
normalized size	1	1.00	0.10	0.82	0.95	1.24	0.00	0.94	0.48
time (sec)	N/A	0.447	0.011	0.027	3.173	0.990	0.000	0.218	4.327

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	432	370	421	515	0	374	248
normalized size	1	1.00	1.03	0.88	1.00	1.23	0.00	0.89	0.59
time (sec)	N/A	0.529	0.180	0.029	3.181	0.910	0.000	0.242	4.402

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	109	354	394	515	0	354	231
normalized size	1	1.00	0.27	0.88	0.98	1.28	0.00	0.88	0.57
time (sec)	N/A	0.469	0.034	0.031	3.204	1.129	0.000	0.224	0.236

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	408	351	403	479	0	340	231
normalized size	1	1.00	1.01	0.87	1.00	1.19	0.00	0.85	0.57
time (sec)	N/A	0.486	0.297	0.031	3.226	0.795	0.000	0.248	4.358

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	104	335	377	486	0	352	213
normalized size	1	1.00	0.27	0.87	0.98	1.26	0.00	0.91	0.55
time (sec)	N/A	0.448	0.036	0.025	3.161	1.095	0.000	0.209	0.214

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	381	344	386	489	0	339	213
normalized size	1	1.00	0.99	0.89	1.00	1.27	0.00	0.88	0.55
time (sec)	N/A	0.446	0.195	0.026	3.114	1.175	0.000	0.244	4.272

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	96	341	383	506	0	355	210
normalized size	1	1.00	0.25	0.88	0.99	1.30	0.00	0.91	0.54
time (sec)	N/A	0.449	0.031	0.026	3.043	1.078	0.000	0.210	4.289

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	359	341	392	505	0	342	210
normalized size	1	1.00	0.93	0.88	1.01	1.30	0.00	0.88	0.54
time (sec)	N/A	0.446	0.259	0.025	3.221	1.142	0.000	0.223	0.127

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	85	339	385	518	0	355	208
normalized size	1	1.00	0.22	0.87	0.98	1.32	0.00	0.91	0.53
time (sec)	N/A	0.479	0.031	0.025	3.003	1.007	0.000	0.226	4.322

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	337	339	394	513	0	342	208
normalized size	1	1.00	0.86	0.87	1.01	1.31	0.00	0.87	0.53
time (sec)	N/A	0.473	0.182	0.025	3.162	1.089	0.000	0.213	4.230

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	61	339	385	520	0	355	207
normalized size	1	1.00	0.15	0.86	0.98	1.32	0.00	0.90	0.53
time (sec)	N/A	0.461	0.026	0.028	3.173	0.903	0.000	0.259	0.116

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	317	339	394	513	0	342	207
normalized size	1	1.00	0.80	0.86	1.00	1.30	0.00	0.87	0.53
time (sec)	N/A	0.449	0.170	0.025	3.149	1.046	0.000	0.221	4.265

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	48	341	383	512	0	355	209
normalized size	1	1.00	0.12	0.88	0.98	1.32	0.00	0.91	0.54
time (sec)	N/A	0.455	0.020	0.025	3.156	1.174	0.000	0.217	4.280

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	298	335	392	485	0	340	209
normalized size	1	1.00	0.77	0.86	1.01	1.25	0.00	0.87	0.54
time (sec)	N/A	0.499	0.173	0.025	3.232	1.199	0.000	0.209	0.131

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	32	336	377	469	547	340	210
normalized size	1	1.00	0.08	0.87	0.97	1.21	1.41	0.88	0.54
time (sec)	N/A	0.490	0.006	0.026	3.192	0.986	89.238	0.221	4.247

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	295	333	382	475	0	346	210
normalized size	1	1.00	0.76	0.86	0.99	1.23	0.00	0.89	0.54
time (sec)	N/A	0.496	0.160	0.026	3.106	0.800	0.000	0.192	4.289

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	404	30	349	388	544	0	365	226
normalized size	1	1.00	0.07	0.86	0.96	1.35	0.00	0.90	0.56
time (sec)	N/A	0.529	0.012	0.031	3.229	1.140	0.000	0.203	0.208

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	404	32	352	395	568	0	356	226
normalized size	1	1.00	0.08	0.87	0.98	1.41	0.00	0.88	0.56
time (sec)	N/A	0.509	0.014	0.031	3.234	1.124	0.000	0.201	4.463

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	37	368	410	591	0	362	239
normalized size	1	1.00	0.09	0.87	0.97	1.40	0.00	0.86	0.57
time (sec)	N/A	0.554	0.013	0.041	3.317	1.102	0.000	0.203	0.273

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	44	39	25	26	0	45	-1
normalized size	1	1.00	0.47	0.42	0.27	0.28	0.00	0.48	-0.01
time (sec)	N/A	0.030	0.020	0.004	1.325	0.829	0.000	0.177	0.000

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	44	39	25	22	0	42	-1
normalized size	1	1.00	0.47	0.42	0.27	0.24	0.00	0.45	-0.01
time (sec)	N/A	0.029	0.014	0.002	1.252	0.635	0.000	0.157	0.000

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	44	39	25	18	27	37	-1
normalized size	1	1.00	0.47	0.42	0.27	0.19	0.29	0.40	-0.01
time (sec)	N/A	0.029	0.013	0.004	1.397	0.914	133.053	0.169	0.000

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	43	38	24	19	0	40	47
normalized size	1	1.00	0.47	0.42	0.26	0.21	0.00	0.44	0.52
time (sec)	N/A	0.028	0.012	0.003	1.370	0.728	0.000	0.171	4.358

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	43	39	25	22	0	41	52
normalized size	1	1.00	0.47	0.43	0.27	0.24	0.00	0.45	0.57
time (sec)	N/A	0.028	0.014	0.003	1.355	0.517	0.000	0.158	4.345

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	42	37	24	23	0	42	53
normalized size	1	1.00	0.46	0.41	0.26	0.25	0.00	0.46	0.58
time (sec)	N/A	0.028	0.016	0.003	1.403	0.799	0.000	0.162	4.377

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	42	37	25	21	0	44	56
normalized size	1	1.00	0.46	0.41	0.27	0.23	0.00	0.48	0.62
time (sec)	N/A	0.029	0.016	0.003	1.270	0.770	0.000	0.181	4.319

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	66	61	83	54	0	99	-1
normalized size	1	1.00	0.34	0.31	0.43	0.28	0.00	0.51	-0.01
time (sec)	N/A	0.059	0.030	0.006	1.458	0.852	0.000	0.163	0.000

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	66	61	83	46	0	90	-1
normalized size	1	1.00	0.34	0.31	0.43	0.24	0.00	0.46	-0.01
time (sec)	N/A	0.058	0.024	0.007	1.409	0.685	0.000	0.214	0.000

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	66	61	83	40	0	85	-1
normalized size	1	1.00	0.34	0.31	0.43	0.21	0.00	0.44	-0.01
time (sec)	N/A	0.055	0.021	0.007	1.436	0.961	0.000	0.161	0.000

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	66	61	87	42	0	89	76
normalized size	1	1.00	0.34	0.32	0.45	0.22	0.00	0.46	0.39
time (sec)	N/A	0.054	0.021	0.006	1.459	0.936	0.000	0.167	4.496

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	66	61	87	45	0	102	87
normalized size	1	1.00	0.35	0.32	0.46	0.24	0.00	0.53	0.46
time (sec)	N/A	0.058	0.024	0.006	1.424	1.062	0.000	0.168	4.535

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	66	61	86	45	0	105	88
normalized size	1	1.00	0.34	0.32	0.45	0.23	0.00	0.54	0.46
time (sec)	N/A	0.055	0.026	0.007	1.475	0.783	0.000	0.174	4.490

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	66	61	86	45	0	107	91
normalized size	1	1.00	0.35	0.32	0.45	0.24	0.00	0.56	0.48
time (sec)	N/A	0.055	0.027	0.007	1.452	1.069	0.000	0.220	4.533

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	88	83	147	82	0	153	-1
normalized size	1	1.00	0.30	0.28	0.49	0.28	0.00	0.52	-0.00
time (sec)	N/A	0.082	0.043	0.006	1.471	0.637	0.000	0.171	0.000

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	88	83	147	70	0	138	-1
normalized size	1	1.00	0.30	0.28	0.49	0.24	0.00	0.46	-0.00
time (sec)	N/A	0.077	0.035	0.007	1.416	0.639	0.000	0.202	0.000

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	88	83	147	62	0	133	-1
normalized size	1	1.00	0.30	0.28	0.49	0.21	0.00	0.45	-0.00
time (sec)	N/A	0.081	0.030	0.007	1.526	1.009	0.000	0.202	0.000

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	88	83	151	64	0	137	112
normalized size	1	1.00	0.30	0.28	0.52	0.22	0.00	0.47	0.38
time (sec)	N/A	0.079	0.031	0.006	1.450	0.915	0.000	0.174	4.565

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	88	83	151	67	0	156	116
normalized size	1	1.00	0.30	0.28	0.51	0.23	0.00	0.53	0.39
time (sec)	N/A	0.079	0.034	0.005	1.504	0.819	0.000	0.188	4.536

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	88	83	151	67	0	159	116
normalized size	1	1.00	0.30	0.28	0.52	0.23	0.00	0.54	0.40
time (sec)	N/A	0.082	0.035	0.007	1.537	0.935	0.000	0.187	4.564

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	88	83	150	67	0	162	118
normalized size	1	1.00	0.30	0.28	0.51	0.23	0.00	0.55	0.40
time (sec)	N/A	0.079	0.037	0.007	1.554	1.204	0.000	0.204	4.716

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	238	239	266	223	0	273	-1
normalized size	1	1.00	0.52	0.52	0.58	0.49	0.00	0.60	-0.00
time (sec)	N/A	0.325	0.091	0.011	2.943	1.087	0.000	0.196	0.000

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	110	221	241	219	0	254	-1
normalized size	1	1.00	0.27	0.54	0.58	0.53	0.00	0.62	-0.00
time (sec)	N/A	0.286	0.054	0.009	2.971	1.092	0.000	0.197	0.000

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	221	214	250	170	0	238	-1
normalized size	1	1.00	0.54	0.52	0.61	0.41	0.00	0.58	-0.00
time (sec)	N/A	0.275	0.076	0.008	3.103	0.949	0.000	0.243	0.000

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	85	183	216	173	41	242	-1
normalized size	1	1.00	0.23	0.50	0.59	0.47	0.11	0.66	-0.00
time (sec)	N/A	0.246	0.044	0.008	3.005	0.826	57.265	0.193	0.000

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	178	182	226	165	0	251	-1
normalized size	1	1.00	0.48	0.49	0.61	0.45	0.00	0.68	-0.00
time (sec)	N/A	0.238	0.049	0.007	3.056	0.696	0.000	0.256	0.000

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	50	224	234	198	0	264	-1
normalized size	1	1.00	0.12	0.54	0.57	0.48	0.00	0.64	-0.00
time (sec)	N/A	0.279	0.012	0.010	3.255	0.978	0.000	0.234	0.000

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	52	239	242	227	0	256	-1
normalized size	1	1.00	0.13	0.58	0.58	0.55	0.00	0.62	-0.00
time (sec)	N/A	0.276	0.013	0.010	3.076	0.815	0.000	0.245	0.000

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	52	251	259	253	0	284	-1
normalized size	1	1.00	0.11	0.55	0.56	0.55	0.00	0.62	-0.00
time (sec)	N/A	0.327	0.013	0.013	3.034	1.121	0.000	0.316	0.000

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	551	551	498	737	0	341	0	419	-1
normalized size	1	1.00	0.90	1.34	0.00	0.62	0.00	0.76	-0.00
time (sec)	N/A	0.399	0.165	0.023	0.000	0.946	0.000	0.349	0.000

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	504	504	88	679	0	341	0	399	-1
normalized size	1	1.00	0.17	1.35	0.00	0.68	0.00	0.79	-0.00
time (sec)	N/A	0.369	0.034	0.023	0.000	1.167	0.000	0.410	0.000

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	504	504	484	696	0	305	0	385	-1
normalized size	1	1.00	0.96	1.38	0.00	0.61	0.00	0.76	-0.00
time (sec)	N/A	0.371	0.166	0.022	0.000	1.013	0.000	0.314	0.000

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	458	458	84	612	0	312	0	380	-1
normalized size	1	1.00	0.18	1.34	0.00	0.68	0.00	0.83	-0.00
time (sec)	N/A	0.333	0.032	0.020	0.000	0.963	0.000	0.318	0.000

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	458	458	447	666	279	315	0	367	-1
normalized size	1	1.00	0.98	1.45	0.61	0.69	0.00	0.80	-0.00
time (sec)	N/A	0.322	0.144	0.019	3.230	0.892	0.000	0.307	0.000

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	73	617	272	326	0	383	-1
normalized size	1	1.00	0.16	1.34	0.59	0.71	0.00	0.83	-0.00
time (sec)	N/A	0.333	0.026	0.018	3.186	0.919	0.000	0.319	0.000

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	272	668	281	308	0	367	-1
normalized size	1	1.00	0.59	1.46	0.61	0.67	0.00	0.80	-0.00
time (sec)	N/A	0.327	0.205	0.019	3.220	1.052	0.000	0.337	0.000

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	54	617	265	304	0	368	-1
normalized size	1	1.00	0.12	1.34	0.58	0.66	0.00	0.80	-0.00
time (sec)	N/A	0.333	0.014	0.012	3.205	0.974	0.000	0.336	0.000

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	272	638	0	298	0	374	-1
normalized size	1	1.00	0.59	1.39	0.00	0.65	0.00	0.81	-0.00
time (sec)	N/A	0.336	0.188	0.010	0.000	1.079	0.000	0.291	0.000

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	506	506	52	645	0	343	0	410	-1
normalized size	1	1.00	0.10	1.27	0.00	0.68	0.00	0.81	-0.00
time (sec)	N/A	0.384	0.014	0.021	0.000	0.952	0.000	0.314	0.000

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	506	506	54	707	0	367	0	401	-1
normalized size	1	1.00	0.11	1.40	0.00	0.73	0.00	0.79	-0.00
time (sec)	N/A	0.376	0.014	0.022	0.000	0.817	0.000	0.351	0.000

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	54	687	0	390	0	432	-1
normalized size	1	1.00	0.10	1.24	0.00	0.71	0.00	0.78	-0.00
time (sec)	N/A	0.432	0.015	0.026	0.000	1.192	0.000	0.339	0.000

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	647	647	401	1287	0	457	0	457	-1
normalized size	1	1.00	0.62	1.99	0.00	0.71	0.00	0.71	-0.00
time (sec)	N/A	0.509	0.304	0.027	0.000	1.118	0.000	0.445	0.000

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	600	600	110	1171	0	457	0	437	-1
normalized size	1	1.00	0.18	1.95	0.00	0.76	0.00	0.73	-0.00
time (sec)	N/A	0.468	0.048	0.027	0.000	0.716	0.000	0.419	0.000

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	600	600	384	1202	0	421	0	423	-1
normalized size	1	1.00	0.64	2.00	0.00	0.70	0.00	0.70	-0.00
time (sec)	N/A	0.463	0.283	0.028	0.000	0.844	0.000	0.420	0.000

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	554	554	106	1046	0	428	0	418	-1
normalized size	1	1.00	0.19	1.89	0.00	0.77	0.00	0.75	-0.00
time (sec)	N/A	0.419	0.045	0.025	0.000	1.057	0.000	0.366	0.000

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	554	554	366	1134	583	431	0	405	-1
normalized size	1	1.00	0.66	2.05	1.05	0.78	0.00	0.73	-0.00
time (sec)	N/A	0.422	0.264	0.025	3.762	1.067	0.000	0.348	0.000

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	97	1051	577	448	0	421	-1
normalized size	1	1.00	0.17	1.89	1.04	0.80	0.00	0.76	-0.00
time (sec)	N/A	0.425	0.038	0.024	3.752	1.161	0.000	0.363	0.000

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	352	1136	595	447	0	408	-1
normalized size	1	1.00	0.63	2.04	1.07	0.80	0.00	0.73	-0.00
time (sec)	N/A	0.430	0.288	0.023	3.710	1.048	0.000	0.345	0.000

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	560	560	86	1051	584	462	0	421	-1
normalized size	1	1.00	0.15	1.88	1.04	0.82	0.00	0.75	-0.00
time (sec)	N/A	0.424	0.038	0.026	3.688	0.944	0.000	0.365	0.000

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	560	560	341	1136	597	455	0	408	-1
normalized size	1	1.00	0.61	2.03	1.07	0.81	0.00	0.73	-0.00
time (sec)	N/A	0.438	0.305	0.024	3.811	1.086	0.000	0.352	0.000

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	73	1051	582	454	0	421	-1
normalized size	1	1.00	0.13	1.89	1.04	0.82	0.00	0.76	-0.00
time (sec)	N/A	0.454	0.032	0.024	3.708	1.093	0.000	0.370	0.000

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	324	1136	586	429	0	406	-1
normalized size	1	1.00	0.58	2.04	1.05	0.77	0.00	0.73	-0.00
time (sec)	N/A	0.426	0.296	0.024	3.638	1.144	0.000	0.348	0.000

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	556	556	54	1051	569	414	0	406	-1
normalized size	1	1.00	0.10	1.89	1.02	0.74	0.00	0.73	-0.00
time (sec)	N/A	0.434	0.013	0.023	3.789	1.317	0.000	0.378	0.000

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	556	556	319	1133	0	416	0	412	-1
normalized size	1	1.00	0.57	2.04	0.00	0.75	0.00	0.74	-0.00
time (sec)	N/A	0.429	0.134	0.024	0.000	0.939	0.000	0.334	0.000

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	602	602	52	1081	0	477	0	448	-1
normalized size	1	1.00	0.09	1.80	0.00	0.79	0.00	0.74	-0.00
time (sec)	N/A	0.485	0.014	0.030	0.000	1.106	0.000	0.372	0.000

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	602	602	54	1183	0	501	0	439	-1
normalized size	1	1.00	0.09	1.97	0.00	0.83	0.00	0.73	-0.00
time (sec)	N/A	0.484	0.018	0.029	0.000	1.129	0.000	0.739	0.000

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	649	649	54	1129	0	524	0	470	-1
normalized size	1	1.00	0.08	1.74	0.00	0.81	0.00	0.72	-0.00
time (sec)	N/A	0.534	0.017	0.032	0.000	0.767	0.000	0.365	0.000

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	105	602	144	507	3188	847	540
normalized size	1	1.00	0.70	4.01	0.96	3.38	21.25	5.65	3.60
time (sec)	N/A	0.121	0.069	0.011	1.542	0.910	7.605	0.251	4.579

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	73	292	100	253	1321	415	263
normalized size	1	1.00	0.70	2.81	0.96	2.43	12.70	3.99	2.53
time (sec)	N/A	0.075	0.035	0.009	1.487	1.010	3.200	0.181	4.513

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	41	94	56	87	345	135	95
normalized size	1	1.00	0.71	1.62	0.97	1.50	5.95	2.33	1.64
time (sec)	N/A	0.023	0.033	0.006	1.397	1.021	1.007	0.160	4.270

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.009	0.052	0.000	0.739	0.000	0.000	0.000

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.009	0.032	0.000	1.139	0.000	0.000	0.000

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.009	0.041	0.000	0.944	0.000	0.000	0.000

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	111	453	243	369	0	900	-1
normalized size	1	1.00	0.35	1.45	0.78	1.18	0.00	2.88	-0.00
time (sec)	N/A	0.120	0.092	0.006	1.414	0.859	0.000	0.278	0.000

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	131	199	119	159	0	384	-1
normalized size	1	1.00	0.64	0.97	0.58	0.78	0.00	1.87	-0.00
time (sec)	N/A	0.076	0.070	0.006	1.443	1.130	0.000	0.210	0.000

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	53	56	35	35	0	83	-1
normalized size	1	1.00	0.55	0.58	0.36	0.36	0.00	0.86	-0.01
time (sec)	N/A	0.034	0.024	0.004	1.398	1.076	0.000	0.163	0.000

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	62	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.020	0.041	0.000	0.674	0.000	0.000	0.000

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.022	0.024	0.000	1.037	0.000	0.000	0.000

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.019	0.039	0.000	0.724	0.000	0.000	0.000

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	77	66	0	0	0	0	0	-1
normalized size	1	1.04	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.019	0.135	0.000	1.044	0.000	0.000	0.000

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	110	150	115	163	0	375	206
normalized size	1	1.00	0.63	0.86	0.66	0.94	0.00	2.16	1.18
time (sec)	N/A	0.109	0.059	0.010	1.459	0.955	0.000	0.193	4.403

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	77	96	79	108	0	235	137
normalized size	1	1.00	0.59	0.74	0.61	0.83	0.00	1.81	1.05
time (sec)	N/A	0.082	0.034	0.008	1.437	1.118	0.000	0.186	4.267

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	51	60	54	70	0	132	85
normalized size	1	1.00	0.61	0.71	0.64	0.83	0.00	1.57	1.01
time (sec)	N/A	0.057	0.021	0.007	1.431	1.114	0.000	0.217	4.264

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	29	40	30	37	0	58	46
normalized size	1	1.00	0.71	0.98	0.73	0.90	0.00	1.41	1.12
time (sec)	N/A	0.025	0.004	0.004	1.324	0.915	0.000	0.262	4.669

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	54	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.011	0.027	0.000	1.103	0.000	0.000	0.000

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.012	0.045	0.000	0.935	0.000	0.000	0.000

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.007	0.067	0.000	0.893	0.000	0.000	0.000

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.006	0.044	0.000	0.869	0.000	0.000	0.000

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	46	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.012	0.006	0.015	0.000	0.904	0.000	0.000	0.000

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	49	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.006	0.041	0.000	1.107	0.000	0.000	0.000

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.006	0.055	0.000	1.030	0.000	0.000	0.000

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.009	0.012	0.000	1.064	0.000	0.000	0.000

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.020	0.008	0.012	0.000	0.724	0.000	0.000	0.000

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	54	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.008	0.013	0.000	0.672	0.000	0.000	0.000

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	54	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.009	0.013	0.000	0.757	0.000	0.000	0.000

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.009	0.012	0.000	0.881	0.000	0.000	0.000

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.007	0.002	0.002	1.332	0.757	0.068	0.148	0.030

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.007	0.001	0.000	1.385	0.872	0.067	0.151	0.028

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
normalized size	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.003	0.000	0.001	1.352	0.883	0.064	0.148	0.024

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	20	17	17	20	17
normalized size	1	1.00	1.00	0.86	0.95	0.81	0.81	0.95	0.81
time (sec)	N/A	0.005	0.002	0.003	1.360	0.797	0.098	0.149	0.025

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	20	12	16	16
normalized size	1	1.00	1.00	0.94	0.89	1.11	0.67	0.89	0.89
time (sec)	N/A	0.007	0.002	0.004	1.385	0.733	0.099	0.147	0.029

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	20	22	17	26	17
normalized size	1	1.00	1.00	0.86	0.95	1.05	0.81	1.24	0.81
time (sec)	N/A	0.007	0.002	0.005	1.337	0.982	0.126	0.152	0.029

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	17	21	17	17	18
normalized size	1	1.00	1.00	0.94	0.94	1.17	0.94	0.94	1.00
time (sec)	N/A	0.007	0.004	0.006	1.369	0.627	0.133	0.153	0.023

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	21	23	19	27	20
normalized size	1	1.00	1.00	0.86	1.00	1.10	0.90	1.29	0.95
time (sec)	N/A	0.007	0.003	0.005	1.305	1.009	0.237	0.163	0.043

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	21	21	22	21	20
normalized size	1	1.00	1.00	0.87	0.91	0.91	0.96	0.91	0.87
time (sec)	N/A	0.007	0.002	0.005	1.362	0.910	0.256	0.152	0.031

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	21	21	22	21	21
normalized size	1	1.00	1.00	0.80	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.007	0.002	0.005	1.295	0.672	0.340	0.148	0.031

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	21	21	22	21	21
normalized size	1	1.00	1.00	0.80	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.007	0.002	0.005	1.344	1.397	0.322	0.167	0.033

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	46	51	46	45
normalized size	1	1.00	1.00	0.83	0.81	0.85	0.94	0.85	0.83
time (sec)	N/A	0.030	0.007	0.000	1.313	0.818	0.078	0.148	0.027

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	45	44	46	46	46	45
normalized size	1	1.00	0.89	0.83	0.81	0.85	0.85	0.85	0.83
time (sec)	N/A	0.038	0.008	0.001	1.378	0.751	0.080	0.163	0.021

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	45	43	48	43	42
normalized size	1	1.00	1.00	0.86	0.92	0.88	0.98	0.88	0.86
time (sec)	N/A	0.021	0.005	0.001	1.329	0.781	0.076	0.149	0.020

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	44	44	41	42	46	42
normalized size	1	1.00	1.00	0.94	0.94	0.87	0.89	0.98	0.89
time (sec)	N/A	0.041	0.012	0.001	1.339	0.547	0.143	0.152	0.024

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	42	46	44	44	43
normalized size	1	1.00	1.00	0.94	0.88	0.96	0.92	0.92	0.90
time (sec)	N/A	0.021	0.018	0.004	1.220	0.894	0.136	0.184	0.023

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	46	45	44	47	44	53	43
normalized size	1	1.00	0.90	0.88	0.86	0.92	0.86	1.04	0.84
time (sec)	N/A	0.041	0.016	0.006	1.374	0.721	0.169	0.152	0.026

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	42	42	46	46	42	44
normalized size	1	1.00	1.00	0.89	0.89	0.98	0.98	0.89	0.94
time (sec)	N/A	0.024	0.019	0.006	1.371	0.932	0.180	0.150	0.041

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	43	45	47	44	60	43
normalized size	1	1.00	0.91	0.96	1.00	1.04	0.98	1.33	0.96
time (sec)	N/A	0.038	0.019	0.008	1.336	0.859	0.374	0.177	0.037

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	49	43	45	46	48	47	44
normalized size	1	1.00	1.02	0.90	0.94	0.96	1.00	0.98	0.92
time (sec)	N/A	0.023	0.021	0.006	1.323	0.866	0.428	0.148	0.041

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	46	45	48	48	54	46
normalized size	1	1.00	0.98	0.90	0.88	0.94	0.94	1.06	0.90
time (sec)	N/A	0.036	0.018	0.008	1.338	1.041	0.776	0.156	4.137

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	49	42	44	46	46	46	45
normalized size	1	1.00	1.04	0.89	0.94	0.98	0.98	0.98	0.96
time (sec)	N/A	0.024	0.023	0.006	1.342	0.926	0.759	0.167	4.173

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	50	45	48	48	48	58	45
normalized size	1	1.00	1.04	0.94	1.00	1.00	1.00	1.21	0.94
time (sec)	N/A	0.035	0.026	0.006	1.364	0.883	1.307	0.151	4.182

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	50	45	46	46	49	48	46
normalized size	1	1.00	0.96	0.87	0.88	0.88	0.94	0.92	0.88
time (sec)	N/A	0.025	0.020	0.005	1.330	0.888	1.474	0.149	0.035

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	45	46	46	49	48	47
normalized size	1	1.00	0.98	0.83	0.85	0.85	0.91	0.89	0.87
time (sec)	N/A	0.037	0.015	0.005	1.279	0.915	2.078	0.216	4.119

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	56	45	46	46	49	48	47
normalized size	1	1.00	1.04	0.83	0.85	0.85	0.91	0.89	0.87
time (sec)	N/A	0.025	0.025	0.006	1.380	0.800	1.958	0.195	4.160

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	45	46	46	49	48	47
normalized size	1	1.00	0.93	0.83	0.85	0.85	0.91	0.89	0.87
time (sec)	N/A	0.035	0.016	0.005	1.341	0.887	2.698	0.146	4.158

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	111	81	87	97	87	76
normalized size	1	1.00	1.00	1.25	0.91	0.98	1.09	0.98	0.85
time (sec)	N/A	0.059	0.013	0.000	1.385	0.660	0.094	0.193	0.034

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	79	111	81	87	92	87	76
normalized size	1	1.00	0.89	1.25	0.91	0.98	1.03	0.98	0.85
time (sec)	N/A	0.083	0.016	0.002	1.365	0.968	0.098	0.169	0.030

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	107	85	83	87	83	72
normalized size	1	1.00	1.00	1.32	1.05	1.02	1.07	1.02	0.89
time (sec)	N/A	0.045	0.010	0.001	1.355	0.493	0.094	0.150	0.030

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	85	82	79	92	87	73
normalized size	1	1.00	1.00	1.00	0.96	0.93	1.08	1.02	0.86
time (sec)	N/A	0.074	0.021	0.002	1.387	0.892	0.222	0.160	0.034

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	84	78	83	82	83	73
normalized size	1	1.00	1.00	1.05	0.98	1.04	1.02	1.04	0.91
time (sec)	N/A	0.039	0.024	0.004	1.362	0.991	0.216	0.149	0.033

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	78	87	82	85	92	98	75
normalized size	1	1.00	0.91	1.01	0.95	0.99	1.07	1.14	0.87
time (sec)	N/A	0.078	0.034	0.007	1.366	0.980	0.267	0.164	0.036

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	84	80	83	90	84	77
normalized size	1	1.00	1.00	1.01	0.96	1.00	1.08	1.01	0.93
time (sec)	N/A	0.041	0.025	0.006	1.345	0.883	0.239	0.177	0.031

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	93	142	0	313	391	92	842
normalized size	1	1.00	0.93	1.42	0.00	3.13	3.91	0.92	8.42
time (sec)	N/A	0.117	0.089	0.008	0.000	0.636	2.912	0.564	4.396

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	111	0	254	316	75	655
normalized size	1	1.00	0.96	1.37	0.00	3.14	3.90	0.93	8.09
time (sec)	N/A	0.080	0.044	0.004	0.000	0.870	2.135	0.620	4.750

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	62	60	0	197	223	59	118
normalized size	1	1.00	0.98	0.95	0.00	3.13	3.54	0.94	1.87
time (sec)	N/A	0.055	0.024	0.002	0.000	0.588	1.032	0.573	4.263

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	39	36	0	129	131	35	41
normalized size	1	1.00	1.08	1.00	0.00	3.58	3.64	0.97	1.14
time (sec)	N/A	0.034	0.009	0.002	0.000	1.189	0.586	0.570	4.270

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	113	66	0	223	253	68	1014
normalized size	1	1.00	1.64	0.96	0.00	3.23	3.67	0.99	14.70
time (sec)	N/A	0.070	0.069	0.006	0.000	0.972	4.669	0.575	4.936

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	135	119	0	293	345	94	2033
normalized size	1	1.00	1.52	1.34	0.00	3.29	3.88	1.06	22.84
time (sec)	N/A	0.131	0.125	0.009	0.000	1.126	137.798	0.580	5.892

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	188	159	0	374	0	126	2451
normalized size	1	1.00	1.65	1.39	0.00	3.28	0.00	1.11	21.50
time (sec)	N/A	0.196	0.235	0.011	0.000	0.782	0.000	0.553	6.367

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	250	467	0	1564	194	2457	4127
normalized size	1	1.00	1.23	2.30	0.00	7.70	0.96	12.10	20.33
time (sec)	N/A	0.670	0.150	0.046	0.000	1.185	5.160	1.013	5.014

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	202	343	0	1059	129	2109	3026
normalized size	1	1.00	1.13	1.92	0.00	5.92	0.72	11.78	16.91
time (sec)	N/A	0.269	0.107	0.026	0.000	0.965	5.506	0.974	0.653

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	165	208	0	559	75	503	416
normalized size	1	1.00	1.10	1.39	0.00	3.73	0.50	3.35	2.77
time (sec)	N/A	0.109	0.082	0.018	0.000	0.787	2.620	1.050	4.457

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	129	116	0	613	87	1024	763
normalized size	1	1.00	0.86	0.77	0.00	4.09	0.58	6.83	5.09
time (sec)	N/A	0.086	0.075	0.016	0.000	0.660	2.840	0.575	4.612

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	191	232	0	1116	148	1839	2997
normalized size	1	1.00	1.10	1.33	0.00	6.41	0.85	10.57	17.22
time (sec)	N/A	0.222	0.383	0.022	0.000	1.057	4.919	0.967	4.854

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	216	368	0	1622	211	1640	4160
normalized size	1	1.00	1.10	1.88	0.00	8.28	1.08	8.37	21.22
time (sec)	N/A	0.423	0.138	0.023	0.000	0.918	16.671	1.158	0.788

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	121	222	0	663	745	152	1336
normalized size	1	1.00	0.92	1.68	0.00	5.02	5.64	1.15	10.12
time (sec)	N/A	0.168	0.168	0.016	0.000	1.205	40.654	0.600	5.098

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	93	104	0	407	282	96	187
normalized size	1	1.00	1.19	1.33	0.00	5.22	3.62	1.23	2.40
time (sec)	N/A	0.066	0.085	0.010	0.000	0.789	3.905	0.913	0.177

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	79	77	0	360	269	82	178
normalized size	1	1.00	1.05	1.03	0.00	4.80	3.59	1.09	2.37
time (sec)	N/A	0.061	0.063	0.006	0.000	0.944	1.885	0.619	4.566

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	79	75	0	361	267	82	172
normalized size	1	1.00	1.07	1.01	0.00	4.88	3.61	1.11	2.32
time (sec)	N/A	0.058	0.078	0.006	0.000	0.872	2.777	0.577	4.311

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	207	253	0	813	0	166	5048
normalized size	1	1.00	1.70	2.07	0.00	6.66	0.00	1.36	41.38
time (sec)	N/A	0.198	0.326	0.018	0.000	1.005	0.000	0.556	8.292

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	248	352	0	1007	0	182	5491
normalized size	1	1.00	1.53	2.17	0.00	6.22	0.00	1.12	33.90
time (sec)	N/A	0.251	0.268	0.020	0.000	1.463	0.000	0.589	8.812

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	327	844	0	2856	0	3339	7599
normalized size	1	1.00	0.99	2.55	0.00	8.63	0.00	10.09	22.96
time (sec)	N/A	0.844	0.657	0.038	0.000	1.101	0.000	1.168	1.566

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	282	602	0	2257	379	2736	6293
normalized size	1	1.00	1.04	2.22	0.00	8.33	1.40	10.10	23.22
time (sec)	N/A	0.572	0.504	0.033	0.000	1.231	51.730	1.060	5.999

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	235	452	0	1668	296	2132	4973
normalized size	1	1.00	0.99	1.91	0.00	7.04	1.25	9.00	20.98
time (sec)	N/A	0.411	0.402	0.027	0.000	1.128	9.011	1.055	5.910

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	222	342	0	1680	298	1970	4854
normalized size	1	1.00	1.00	1.55	0.00	7.60	1.35	8.91	21.96
time (sec)	N/A	0.260	0.432	0.083	0.000	0.980	20.750	0.984	1.348

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	243	733	0	2309	0	2682	6404
normalized size	1	1.00	0.96	2.91	0.00	9.16	0.00	10.64	25.41
time (sec)	N/A	0.513	0.419	0.065	0.000	1.184	0.000	0.864	5.996

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	302	712	0	2912	0	3087	7555
normalized size	1	1.00	0.98	2.31	0.00	9.45	0.00	10.02	24.53
time (sec)	N/A	1.443	0.601	0.038	0.000	1.377	0.000	1.339	6.716

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	244	547	0	1631	0	306	2588
normalized size	1	1.00	1.17	2.62	0.00	7.80	0.00	1.46	12.38
time (sec)	N/A	0.401	0.325	0.024	0.000	1.129	0.000	1.841	7.296

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	194	267	0	973	554	212	444
normalized size	1	1.00	1.60	2.21	0.00	8.04	4.58	1.75	3.67
time (sec)	N/A	0.112	0.172	0.017	0.000	0.896	4.641	1.872	4.532

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	137	230	0	892	524	171	423
normalized size	1	1.00	1.15	1.93	0.00	7.50	4.40	1.44	3.55
time (sec)	N/A	0.104	0.193	0.016	0.000	1.107	3.811	1.772	4.444

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	145	270	0	907	580	161	460
normalized size	1	1.00	1.12	2.08	0.00	6.98	4.46	1.24	3.54
time (sec)	N/A	0.129	0.134	0.016	0.000	0.852	5.406	1.813	4.457

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	114	142	0	808	491	143	400
normalized size	1	1.00	1.01	1.26	0.00	7.15	4.35	1.27	3.54
time (sec)	N/A	0.090	0.099	0.009	0.000	1.091	3.041	1.786	4.392

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	106	141	0	809	481	144	386
normalized size	1	1.00	0.94	1.25	0.00	7.16	4.26	1.27	3.42
time (sec)	N/A	0.088	0.099	0.008	0.000	0.816	2.908	1.774	4.337

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	342	822	0	2017	0	323	9339
normalized size	1	1.00	1.71	4.11	0.00	10.08	0.00	1.62	46.70
time (sec)	N/A	0.298	0.522	0.028	0.000	2.393	0.000	1.879	10.945

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	402	1002	0	2312	0	382	10074
normalized size	1	1.00	1.58	3.93	0.00	9.07	0.00	1.50	39.51
time (sec)	N/A	0.391	0.621	0.033	0.000	3.063	0.000	1.801	11.756

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	455	1141	0	4279	0	2430	10912
normalized size	1	1.00	1.14	2.85	0.00	10.70	0.00	6.08	27.28
time (sec)	N/A	1.727	1.166	0.049	0.000	1.651	0.000	3.626	9.036

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	381	953	0	3725	0	4558	9575
normalized size	1	1.00	1.09	2.74	0.00	10.70	0.00	13.10	27.51
time (sec)	N/A	0.886	0.961	0.045	0.000	0.972	0.000	2.457	8.537

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	343	753	0	3128	627	1750	8521
normalized size	1	1.00	1.15	2.53	0.00	10.50	2.10	5.87	28.59
time (sec)	N/A	0.683	0.845	0.038	0.000	1.058	23.392	2.817	8.179

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	285	617	0	3128	0	1861	8397
normalized size	1	1.00	0.99	2.13	0.00	10.82	0.00	6.44	29.06
time (sec)	N/A	0.705	0.710	0.038	0.000	1.086	0.000	2.642	7.590

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	334	2958	0	3777	0	4270	9731
normalized size	1	1.00	1.07	9.51	0.00	12.14	0.00	13.73	31.29
time (sec)	N/A	0.701	0.851	0.161	0.000	1.298	0.000	2.454	8.367

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	372	3360	0	4323	0	2705	10979
normalized size	1	1.00	1.05	9.46	0.00	12.18	0.00	7.62	30.93
time (sec)	N/A	1.838	1.023	0.131	0.000	1.596	0.000	1.433	8.997

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	454	1567	0	4924	0	5273	12130
normalized size	1	1.00	1.07	3.69	0.00	11.59	0.00	12.41	28.54
time (sec)	N/A	0.963	1.758	0.055	0.000	2.197	0.000	2.616	9.370

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	80	116	0	259	311	78	656
normalized size	1	1.00	0.98	1.41	0.00	3.16	3.79	0.95	8.00
time (sec)	N/A	0.092	0.052	0.005	0.000	0.804	2.761	0.533	4.736

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	63	0	206	223	62	120
normalized size	1	1.00	1.02	0.98	0.00	3.22	3.48	0.97	1.88
time (sec)	N/A	0.062	0.023	0.004	0.000	0.815	1.448	0.566	4.398

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	41	38	0	134	131	37	42
normalized size	1	1.00	1.17	1.09	0.00	3.83	3.74	1.06	1.20
time (sec)	N/A	0.043	0.008	0.002	0.000	0.797	0.721	0.573	4.297

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	117	69	0	230	253	71	1015
normalized size	1	1.00	1.67	0.99	0.00	3.29	3.61	1.01	14.50
time (sec)	N/A	0.084	0.074	0.008	0.000	0.977	5.742	0.569	4.892

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	139	123	0	298	350	95	2032
normalized size	1	1.00	1.56	1.38	0.00	3.35	3.93	1.07	22.83
time (sec)	N/A	0.140	0.141	0.008	0.000	0.924	142.971	0.588	5.844

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	208	343	0	1051	129	2153	3000
normalized size	1	1.00	1.16	1.92	0.00	5.87	0.72	12.03	16.76
time (sec)	N/A	0.365	0.123	0.028	0.000	0.995	2.775	0.984	0.673

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	137	208	0	551	75	513	416
normalized size	1	1.00	0.91	1.39	0.00	3.67	0.50	3.42	2.77
time (sec)	N/A	0.109	0.107	0.013	0.000	0.806	1.245	1.063	4.537

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	137	116	0	605	87	1050	763
normalized size	1	1.00	0.91	0.77	0.00	4.03	0.58	7.00	5.09
time (sec)	N/A	0.072	0.079	0.013	0.000	0.607	1.245	0.567	0.486

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	199	232	0	1108	148	1877	2979
normalized size	1	1.00	1.16	1.35	0.00	6.44	0.86	10.91	17.32
time (sec)	N/A	0.203	0.399	0.016	0.000	0.809	3.834	1.014	4.932

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	62	86	74	156	138	60	166
normalized size	1	1.00	0.90	1.25	1.07	2.26	2.00	0.87	2.41
time (sec)	N/A	0.084	0.038	0.005	2.998	0.805	1.785	0.260	0.391

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	51	49	60	134	110	46	153
normalized size	1	1.00	0.91	0.88	1.07	2.39	1.96	0.82	2.73
time (sec)	N/A	0.049	0.019	0.002	2.971	0.836	0.845	0.260	0.170

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	37	91	53	23	31
normalized size	1	1.00	1.00	0.84	1.19	2.94	1.71	0.74	1.00
time (sec)	N/A	0.028	0.008	0.002	3.015	0.858	0.339	0.262	4.341

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	90	71	85	151	184	71	183
normalized size	1	1.00	1.17	0.92	1.10	1.96	2.39	0.92	2.38
time (sec)	N/A	0.072	0.049	0.006	3.125	0.863	5.311	0.328	4.563

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	146	122	123	209	372	126	389
normalized size	1	1.00	1.51	1.26	1.27	2.15	3.84	1.30	4.01
time (sec)	N/A	0.141	0.094	0.010	2.970	0.889	32.928	0.282	4.870

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	144	210	0	603	105	511	1097
normalized size	1	1.00	1.26	1.84	0.00	5.29	0.92	4.48	9.62
time (sec)	N/A	0.165	0.086	0.030	0.000	0.844	2.019	0.357	4.791

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	128	134	0	267	44	199	216
normalized size	1	1.00	1.17	1.23	0.00	2.45	0.40	1.83	1.98
time (sec)	N/A	0.054	0.104	0.011	0.000	0.833	0.596	0.360	0.297

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	105	74	0	553	63	299	322
normalized size	1	1.00	0.96	0.68	0.00	5.07	0.58	2.74	2.95
time (sec)	N/A	0.047	0.063	0.012	0.000	0.963	0.947	0.247	5.779

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	143	180	0	1612	134	698	2774
normalized size	1	1.00	1.18	1.49	0.00	13.32	1.11	5.77	22.93
time (sec)	N/A	0.113	0.146	0.013	0.000	0.792	6.144	0.384	5.116

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	62	84	58	157	144	58	302
normalized size	1	1.00	0.90	1.22	0.84	2.28	2.09	0.84	4.38
time (sec)	N/A	0.076	0.036	0.006	3.028	0.906	1.604	0.248	0.180

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	49	47	42	131	117	42	85
normalized size	1	1.00	0.91	0.87	0.78	2.43	2.17	0.78	1.57
time (sec)	N/A	0.045	0.018	0.003	2.875	0.872	0.600	0.230	0.086

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	21	91	60	21	24
normalized size	1	1.00	1.00	0.84	0.68	2.94	1.94	0.68	0.77
time (sec)	N/A	0.026	0.007	0.003	2.940	0.897	0.459	0.241	0.051

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	105	63	61	147	194	61	71
normalized size	1	1.00	1.52	0.91	0.88	2.13	2.81	0.88	1.03
time (sec)	N/A	0.067	0.057	0.009	3.013	0.950	5.964	0.234	4.641

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	163	110	104	208	386	125	3313
normalized size	1	1.00	1.83	1.24	1.17	2.34	4.34	1.40	37.22
time (sec)	N/A	0.132	0.098	0.010	2.904	0.861	41.752	0.277	7.390

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	432	432	164	1658	0	615	105	533	1147
normalized size	1	1.00	0.38	3.84	0.00	1.42	0.24	1.23	2.66
time (sec)	N/A	0.891	0.097	0.138	0.000	0.840	2.197	0.346	4.650

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	143	724	0	279	44	203	222
normalized size	1	1.00	0.43	2.19	0.00	0.84	0.13	0.61	0.67
time (sec)	N/A	0.256	0.116	0.059	0.000	0.704	0.827	0.343	0.283

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	119	913	0	567	63	307	986
normalized size	1	1.00	0.33	2.54	0.00	1.58	0.18	0.86	2.75
time (sec)	N/A	0.261	0.070	0.079	0.000	0.828	1.235	0.249	5.157

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	174	3318	0	1582	134	742	2848
normalized size	1	1.00	0.40	7.66	0.00	3.65	0.31	1.71	6.58
time (sec)	N/A	0.519	0.150	0.070	0.000	0.778	4.537	0.375	5.274

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	18	26	18	20
normalized size	1	1.00	1.00	0.95	0.90	0.90	1.30	0.90	1.00
time (sec)	N/A	0.020	0.006	0.002	2.837	0.909	0.168	0.151	0.057

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
normalized size	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.016	0.005	0.003	2.920	0.768	0.124	0.586	0.059

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	20	18	18
normalized size	1	1.00	1.00	0.83	0.78	0.78	0.87	0.78	0.78
time (sec)	N/A	0.012	0.014	0.009	2.982	0.786	0.209	0.192	4.368

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	94	57	0	159	63	56	44
normalized size	1	1.00	1.27	0.77	0.00	2.15	0.85	0.76	0.59
time (sec)	N/A	0.050	0.144	0.022	0.000	0.569	0.308	0.172	0.082

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	39	308	0	247	24	147	101
normalized size	1	1.00	0.21	1.64	0.00	1.31	0.13	0.78	0.54
time (sec)	N/A	0.177	0.031	0.105	0.000	0.861	0.830	0.849	4.371

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	164	296	0	367	0	172	315
normalized size	1	1.00	0.96	1.73	0.00	2.15	0.00	1.01	1.84
time (sec)	N/A	0.155	0.147	0.024	0.000	0.943	0.000	0.250	5.313

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	136	247	0	303	0	134	193
normalized size	1	1.00	0.89	1.61	0.00	1.98	0.00	0.88	1.26
time (sec)	N/A	0.129	0.067	0.017	0.000	0.966	0.000	0.232	4.639

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	101	139	0	237	0	98	87
normalized size	1	1.00	0.94	1.29	0.00	2.19	0.00	0.91	0.81
time (sec)	N/A	0.081	0.049	0.014	0.000	0.780	0.000	0.215	4.520

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	101	0	197	0	76	72
normalized size	1	1.00	1.00	1.22	0.00	2.37	0.00	0.92	0.87
time (sec)	N/A	0.055	0.022	0.010	0.000	0.881	0.000	0.204	4.622

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	106	91	0	566	0	0	88
normalized size	1	1.00	0.97	0.83	0.00	5.19	0.00	0.00	0.81
time (sec)	N/A	0.109	0.043	0.012	0.000	0.928	0.000	0.000	4.423

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	112	140	0	601	0	148	91
normalized size	1	1.00	1.00	1.25	0.00	5.37	0.00	1.32	0.81
time (sec)	N/A	0.104	0.049	0.011	0.000	0.822	0.000	0.292	4.553

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	193	0	215	0	241	-1
normalized size	1	1.00	1.00	2.19	0.00	2.44	0.00	2.74	-0.01
time (sec)	N/A	0.071	0.040	0.012	0.000	0.916	0.000	0.224	0.000

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	108	222	0	261	0	359	-1
normalized size	1	1.00	0.93	1.91	0.00	2.25	0.00	3.09	-0.01
time (sec)	N/A	0.096	0.075	0.013	0.000	0.952	0.000	0.301	0.000

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	141	387	0	325	0	617	-1
normalized size	1	1.00	0.88	2.40	0.00	2.02	0.00	3.83	-0.01
time (sec)	N/A	0.150	0.095	0.017	0.000	0.755	0.000	0.281	0.000

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	173	442	0	389	0	842	-1
normalized size	1	1.00	0.87	2.22	0.00	1.95	0.00	4.23	-0.01
time (sec)	N/A	0.229	0.120	0.020	0.000	1.099	0.000	0.357	0.000

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	538	476	0	0	0	0	-1
normalized size	1	1.00	1.36	1.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.251	1.549	0.052	0.000	0.819	0.000	0.000	0.000

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	479	417	0	0	0	0	-1
normalized size	1	1.00	1.40	1.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.141	1.250	0.010	0.000	0.536	0.000	0.000	0.000

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	445	379	0	0	0	0	-1
normalized size	1	1.00	1.44	1.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.101	0.822	0.011	0.000	0.831	0.000	0.000	0.000

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	435	381	0	0	0	0	-1
normalized size	1	1.00	1.44	1.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.084	0.811	0.013	0.000	0.679	0.000	0.000	0.000

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	459	404	0	0	0	0	-1
normalized size	1	1.00	1.35	1.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.155	0.896	0.015	0.000	0.743	0.000	0.000	0.000

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	530	452	0	0	0	0	-1
normalized size	1	1.00	1.34	1.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.261	1.341	0.019	0.000	0.817	0.000	0.000	0.000

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	192	534	0	535	0	669	-1
normalized size	1	1.00	0.86	2.39	0.00	2.40	0.00	3.00	-0.00
time (sec)	N/A	0.212	0.253	0.035	0.000	0.925	0.000	0.405	0.000

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	175	432	0	451	0	535	-1
normalized size	1	1.00	0.86	2.12	0.00	2.21	0.00	2.62	-0.00
time (sec)	N/A	0.182	0.165	0.024	0.000	0.942	0.000	0.397	0.000

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	149	316	0	361	0	414	223
normalized size	1	1.00	0.99	2.11	0.00	2.41	0.00	2.76	1.49
time (sec)	N/A	0.117	0.142	0.020	0.000	0.786	0.000	0.394	4.880

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	126	242	0	297	0	317	115
normalized size	1	1.00	1.02	1.95	0.00	2.40	0.00	2.56	0.93
time (sec)	N/A	0.086	0.086	0.015	0.000	0.779	0.000	0.389	4.965

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	143	192	0	727	0	0	-1
normalized size	1	1.00	0.92	1.24	0.00	4.69	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.145	0.019	0.000	1.062	0.000	0.000	0.000

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	134	170	0	713	0	0	-1
normalized size	1	1.00	0.89	1.13	0.00	4.75	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.118	0.019	0.000	0.843	0.000	0.000	0.000

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	134	174	0	713	0	302	-1
normalized size	1	1.00	0.89	1.15	0.00	4.72	0.00	2.00	-0.01
time (sec)	N/A	0.163	0.170	0.018	0.000	1.063	0.000	0.446	0.000

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	149	202	0	771	0	412	-1
normalized size	1	1.00	0.91	1.24	0.00	4.73	0.00	2.53	-0.01
time (sec)	N/A	0.184	0.216	0.019	0.000	1.163	0.000	0.677	0.000

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	138	260	0	319	0	606	-1
normalized size	1	1.00	1.04	1.95	0.00	2.40	0.00	4.56	-0.01
time (sec)	N/A	0.117	0.167	0.020	0.000	1.055	0.000	0.495	0.000

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	167	337	0	383	0	832	-1
normalized size	1	1.00	1.03	2.08	0.00	2.36	0.00	5.14	-0.01
time (sec)	N/A	0.140	0.140	0.024	0.000	1.057	0.000	0.530	0.000

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	206	457	0	473	0	1235	-1
normalized size	1	1.00	0.95	2.12	0.00	2.19	0.00	5.72	-0.00
time (sec)	N/A	0.217	0.212	0.027	0.000	1.542	0.000	0.701	0.000

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	495	495	657	674	0	0	0	0	-1
normalized size	1	1.00	1.33	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.440	2.208	0.011	0.000	0.893	0.000	0.000	0.000

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	443	443	602	545	0	0	0	0	-1
normalized size	1	1.00	1.36	1.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.285	1.889	0.010	0.000	0.728	0.000	0.000	0.000

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	533	471	0	0	0	0	-1
normalized size	1	1.00	1.40	1.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.251	1.517	0.009	0.000	0.794	0.000	0.000	0.000

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	505	430	0	0	0	0	-1
normalized size	1	1.00	1.40	1.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.206	1.274	0.015	0.000	0.746	0.000	0.000	0.000

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	473	428	0	0	0	0	-1
normalized size	1	1.00	1.34	1.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.153	0.950	0.015	0.000	0.656	0.000	0.000	0.000

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	527	450	0	0	0	0	-1
normalized size	1	1.00	1.32	1.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.251	1.365	0.018	0.000	0.653	0.000	0.000	0.000

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	447	572	495	0	0	0	0	-1
normalized size	1	1.00	1.28	1.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.395	1.562	0.021	0.000	0.648	0.000	0.000	0.000

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	59	114	0	0	0	0	-1
normalized size	1	1.00	1.23	2.38	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.064	0.016	0.000	0.740	0.000	0.000	0.000

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	104	162	0	241	0	103	-1
normalized size	1	1.00	0.86	1.34	0.00	1.99	0.00	0.85	-0.01
time (sec)	N/A	0.114	0.063	0.017	0.000	0.789	0.000	0.211	0.000

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	88	116	0	203	0	82	-1
normalized size	1	1.00	0.85	1.12	0.00	1.95	0.00	0.79	-0.01
time (sec)	N/A	0.099	0.034	0.014	0.000	0.689	0.000	0.240	0.000

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	56	0	161	0	61	55
normalized size	1	1.00	1.00	0.82	0.00	2.37	0.00	0.90	0.81
time (sec)	N/A	0.054	0.014	0.011	0.000	0.696	0.000	0.213	4.428

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	35	0	118	0	40	34
normalized size	1	1.00	1.00	0.81	0.00	2.74	0.00	0.93	0.79
time (sec)	N/A	0.031	0.006	0.009	0.000	0.819	0.000	0.211	4.690

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	0	124	0	38	44
normalized size	1	1.00	1.00	0.89	0.00	2.82	0.00	0.86	1.00
time (sec)	N/A	0.041	0.011	0.011	0.000	0.858	0.000	0.250	4.441

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	63	0	179	0	114	56
normalized size	1	1.00	1.00	0.88	0.00	2.49	0.00	1.58	0.78
time (sec)	N/A	0.061	0.021	0.013	0.000	0.923	0.000	0.434	4.484

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	91	127	0	221	0	221	-1
normalized size	1	1.00	0.84	1.18	0.00	2.05	0.00	2.05	-0.01
time (sec)	N/A	0.105	0.054	0.014	0.000	0.850	0.000	0.298	0.000

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	112	176	0	265	0	335	-1
normalized size	1	1.00	0.77	1.21	0.00	1.83	0.00	2.31	-0.01
time (sec)	N/A	0.166	0.079	0.016	0.000	0.924	0.000	0.265	0.000

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	444	388	0	0	0	0	-1
normalized size	1	1.00	1.42	1.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.113	0.860	0.011	0.000	0.724	0.000	0.000	0.000

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	278	216	0	0	0	0	-1
normalized size	1	1.00	1.04	0.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.069	0.133	0.009	0.000	0.740	0.000	0.000	0.000

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	186	144	0	0	0	0	-1
normalized size	1	1.00	1.63	1.26	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	0.099	0.007	0.000	0.755	0.000	0.000	0.000

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	298	239	0	0	0	0	-1
normalized size	1	1.00	1.01	0.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.116	0.476	0.013	0.000	0.870	0.000	0.000	0.000

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	459	413	0	0	0	0	-1
normalized size	1	1.00	1.33	1.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.156	0.891	0.015	0.000	0.962	0.000	0.000	0.000

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	107	168	153	249	0	112	-1
normalized size	1	1.00	0.86	1.35	1.23	2.01	0.00	0.90	-0.01
time (sec)	N/A	0.113	0.077	0.022	2.430	0.921	0.000	0.259	0.000

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	89	120	105	211	0	91	-1
normalized size	1	1.00	0.83	1.12	0.98	1.97	0.00	0.85	-0.01
time (sec)	N/A	0.093	0.044	0.017	2.420	0.809	0.000	0.212	0.000

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	58	50	169	0	70	62
normalized size	1	1.00	1.00	0.83	0.71	2.41	0.00	1.00	0.89
time (sec)	N/A	0.058	0.016	0.014	2.466	0.833	0.000	0.265	4.593

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	36	28	124	0	45	40
normalized size	1	1.00	1.00	0.82	0.64	2.82	0.00	1.02	0.91
time (sec)	N/A	0.038	0.006	0.010	2.394	0.940	0.000	0.205	4.789

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	46	45	36	129	0	36	52
normalized size	1	1.00	0.98	0.96	0.77	2.74	0.00	0.77	1.11
time (sec)	N/A	0.042	0.014	0.016	2.330	0.819	0.000	0.207	4.520

Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	76	74	62	188	0	111	64
normalized size	1	1.00	0.99	0.96	0.81	2.44	0.00	1.44	0.83
time (sec)	N/A	0.062	0.022	0.012	2.410	0.836	0.000	0.217	4.546

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	95	149	126	230	0	224	-1
normalized size	1	1.00	0.83	1.30	1.10	2.00	0.00	1.95	-0.01
time (sec)	N/A	0.123	0.046	0.014	2.432	0.748	0.000	0.225	0.000

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	116	202	179	272	0	344	-1
normalized size	1	1.00	0.75	1.31	1.16	1.77	0.00	2.23	-0.01
time (sec)	N/A	0.172	0.076	0.018	2.258	0.660	0.000	0.267	0.000

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	459	391	0	0	0	0	-1
normalized size	1	1.00	1.12	0.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.588	0.769	0.052	0.000	0.837	0.000	0.000	0.000

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	271	217	0	0	0	0	-1
normalized size	1	1.00	0.72	0.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.245	0.118	0.011	0.000	0.861	0.000	0.000	0.000

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	177	145	0	0	0	0	-1
normalized size	1	1.00	1.05	0.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.084	0.007	0.000	0.853	0.000	0.000	0.000

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	283	241	0	0	0	0	-1
normalized size	1	1.00	0.69	0.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.332	0.428	0.013	0.000	0.866	0.000	0.000	0.000

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	445	445	472	417	0	0	0	0	-1
normalized size	1	1.00	1.06	0.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.469	0.763	0.015	0.000	0.631	0.000	0.000	0.000

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	181	354	0	591	0	215	-1
normalized size	1	1.00	0.95	1.86	0.00	3.11	0.00	1.13	-0.01
time (sec)	N/A	0.237	0.194	0.020	0.000	1.035	0.000	0.274	0.000

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	137	264	0	459	0	154	-1
normalized size	1	1.00	1.02	1.97	0.00	3.43	0.00	1.15	-0.01
time (sec)	N/A	0.111	0.117	0.015	0.000	0.742	0.000	0.273	0.000

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	107	149	0	387	0	101	84
normalized size	1	1.00	0.93	1.30	0.00	3.37	0.00	0.88	0.73
time (sec)	N/A	0.092	0.098	0.016	0.000	0.988	0.000	0.259	4.765

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	38	0	67	0	44	37
normalized size	1	1.00	1.00	1.06	0.00	1.86	0.00	1.22	1.03
time (sec)	N/A	0.029	0.096	0.006	0.000	0.946	0.000	0.284	4.474

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	37	36	0	67	0	45	35
normalized size	1	1.00	1.03	1.00	0.00	1.86	0.00	1.25	0.97
time (sec)	N/A	0.023	0.020	0.004	0.000	0.896	0.000	0.195	4.362

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	99	0	389	0	110	-1
normalized size	1	1.00	1.00	1.11	0.00	4.37	0.00	1.24	-0.01
time (sec)	N/A	0.081	0.103	0.014	0.000	1.029	0.000	0.203	0.000

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	137	195	0	485	0	200	-1
normalized size	1	1.00	0.99	1.40	0.00	3.49	0.00	1.44	-0.01
time (sec)	N/A	0.126	0.084	0.015	0.000	0.991	0.000	0.276	0.000

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	179	314	0	615	0	350	-1
normalized size	1	1.00	0.92	1.61	0.00	3.15	0.00	1.79	-0.01
time (sec)	N/A	0.211	0.126	0.018	0.000	1.400	0.000	0.377	0.000

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	489	482	0	0	0	0	-1
normalized size	1	1.00	1.20	1.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.210	1.225	0.024	0.000	0.742	0.000	0.000	0.000

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	452	450	0	0	0	0	-1
normalized size	1	1.00	1.32	1.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.130	0.839	0.015	0.000	0.884	0.000	0.000	0.000

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	437	446	0	0	0	0	-1
normalized size	1	1.00	1.28	1.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.125	0.782	0.014	0.000	0.554	0.000	0.000	0.000

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	456	481	0	0	0	0	-1
normalized size	1	1.00	1.29	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.135	0.844	0.012	0.000	0.857	0.000	0.000	0.000

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	428	428	515	536	0	0	0	0	-1
normalized size	1	1.00	1.20	1.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.217	1.329	0.020	0.000	0.859	0.000	0.000	0.000

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	34	37	34	30	0	0	33
normalized size	1	1.00	0.68	0.74	0.68	0.60	0.00	0.00	0.66
time (sec)	N/A	0.068	0.021	0.005	1.264	0.742	0.000	0.000	4.604

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	73	64	52	114	0	59	53
normalized size	1	1.00	1.26	1.10	0.90	1.97	0.00	1.02	0.91
time (sec)	N/A	0.084	0.036	0.007	1.157	0.716	0.000	0.198	4.613

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	26	13	20	0	31	20
normalized size	1	1.00	1.00	1.18	0.59	0.91	0.00	1.41	0.91
time (sec)	N/A	0.017	0.005	0.005	1.219	0.802	0.000	0.183	4.369

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	52	44	32	74	0	39	33
normalized size	1	1.00	1.68	1.42	1.03	2.39	0.00	1.26	1.06
time (sec)	N/A	0.049	0.012	0.003	1.055	0.777	0.000	0.185	4.563

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	52	50	0	80	0	46	-1
normalized size	1	1.00	1.73	1.67	0.00	2.67	0.00	1.53	-0.03
time (sec)	N/A	0.010	0.011	0.005	0.000	0.801	0.000	0.189	0.000

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	26	21	21	0	25	21
normalized size	1	1.00	1.00	1.13	0.91	0.91	0.00	1.09	0.91
time (sec)	N/A	0.040	0.007	0.004	1.022	0.831	0.000	0.180	4.315

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	68	73	0	133	0	0	76
normalized size	1	1.00	1.15	1.24	0.00	2.25	0.00	0.00	1.29
time (sec)	N/A	0.056	0.064	0.008	0.000	0.900	0.000	0.000	4.637

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	37	44	31	0	57	29
normalized size	1	1.00	0.67	0.71	0.85	0.60	0.00	1.10	0.56
time (sec)	N/A	0.084	0.015	0.005	1.086	0.895	0.000	0.185	4.471

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	44	94	0	163	0	0	-1
normalized size	1	1.00	0.51	1.08	0.00	1.87	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.012	0.007	0.000	0.832	0.000	0.000	0.000

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	62	91	0	0	37	0	-1
normalized size	1	1.00	0.57	0.84	0.00	0.00	0.34	0.00	-0.01
time (sec)	N/A	0.025	0.023	0.050	0.000	0.774	1.902	0.000	0.000

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	22	14	14
normalized size	1	1.00	1.00	0.83	0.78	0.78	1.22	0.78	0.78
time (sec)	N/A	0.004	0.004	0.006	0.969	0.618	0.886	0.187	4.663

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	51	97	0	0	37	0	-1
normalized size	1	1.00	0.24	0.46	0.00	0.00	0.18	0.00	-0.00
time (sec)	N/A	0.053	0.009	0.006	0.000	0.843	0.915	0.000	0.000

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	24	45	63	20	25	-1
normalized size	1	1.00	1.00	0.80	1.50	2.10	0.67	0.83	-0.03
time (sec)	N/A	0.016	0.006	0.008	2.426	0.829	1.099	0.164	0.000

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	74	70	0	0	36	0	37
normalized size	1	1.00	0.84	0.80	0.00	0.00	0.41	0.00	0.42
time (sec)	N/A	0.010	0.035	0.005	0.000	0.871	0.855	0.000	4.347

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	29	37	63	22	23	19
normalized size	1	1.00	1.00	1.07	1.37	2.33	0.81	0.85	0.70
time (sec)	N/A	0.020	0.006	0.011	2.294	0.677	1.251	0.152	4.549

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	49	115	0	0	39	0	40
normalized size	1	1.00	0.21	0.50	0.00	0.00	0.17	0.00	0.17
time (sec)	N/A	0.072	0.009	0.011	0.000	0.820	1.109	0.000	4.562

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	20	31	17
normalized size	1	1.00	1.00	0.86	0.81	0.81	0.95	1.48	0.81
time (sec)	N/A	0.005	0.005	0.005	1.081	0.788	0.843	0.169	4.514

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	51	93	0	0	41	0	-1
normalized size	1	1.00	0.46	0.85	0.00	0.00	0.37	0.00	-0.01
time (sec)	N/A	0.022	0.009	0.013	0.000	0.759	1.075	0.000	0.000

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	62	59	51	124	95	54	-1
normalized size	1	1.00	0.85	0.81	0.70	1.70	1.30	0.74	-0.01
time (sec)	N/A	0.022	0.028	0.007	0.971	0.754	4.612	0.293	0.000

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	25	33	23	44	30	24
normalized size	1	1.00	0.75	0.69	0.92	0.64	1.22	0.83	0.67
time (sec)	N/A	0.023	0.014	0.005	1.022	0.760	0.545	0.150	4.599

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	39	31	93	42	40	56
normalized size	1	1.00	1.00	0.80	0.63	1.90	0.86	0.82	1.14
time (sec)	N/A	0.013	0.019	0.006	1.024	0.912	2.911	0.185	4.640

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	20	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	1.33	0.87	0.87
time (sec)	N/A	0.003	0.002	0.003	1.048	0.799	0.400	0.155	4.331

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	13	59	17	23	20
normalized size	1	1.00	1.00	0.84	0.52	2.36	0.68	0.92	0.80
time (sec)	N/A	0.006	0.005	0.003	1.018	0.800	1.197	0.183	0.124

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	17	60	19	22	19
normalized size	1	1.00	1.00	1.16	0.68	2.40	0.76	0.88	0.76
time (sec)	N/A	0.017	0.006	0.005	1.079	0.589	1.203	0.158	4.573

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	19	30	17
normalized size	1	1.00	1.00	0.95	0.89	0.89	1.00	1.58	0.89
time (sec)	N/A	0.004	0.004	0.005	1.005	0.744	0.749	0.181	0.040

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	61	48	36	105	42	51	38
normalized size	1	1.00	1.22	0.96	0.72	2.10	0.84	1.02	0.76
time (sec)	N/A	0.027	0.054	0.006	1.029	0.698	3.473	0.168	4.535

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	29	26	36	27	46	55	25
normalized size	1	1.00	0.66	0.59	0.82	0.61	1.05	1.25	0.57
time (sec)	N/A	0.010	0.006	0.005	1.000	0.802	1.049	0.283	4.552

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	13	15	8	-1
normalized size	1	1.00	1.00	0.81	0.75	0.81	0.94	0.50	-0.06
time (sec)	N/A	0.002	0.002	0.004	0.965	0.832	0.627	0.151	0.000

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	15	8	10
normalized size	1	1.00	1.00	0.81	0.75	0.75	0.94	0.50	0.62
time (sec)	N/A	0.002	0.002	0.003	1.038	0.800	0.527	0.148	4.505

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	14	14	5	-1
normalized size	1	1.00	1.00	0.92	0.85	1.08	1.08	0.38	-0.08
time (sec)	N/A	0.001	0.001	0.003	0.984	0.587	0.482	0.170	0.000

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	16	0	7	-1
normalized size	1	1.00	1.00	0.93	0.87	1.07	0.00	0.47	-0.07
time (sec)	N/A	0.001	0.002	0.003	0.985	0.695	0.000	0.150	0.000

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	15	14	8	13
normalized size	1	1.00	1.00	0.92	0.83	1.25	1.17	0.67	1.08
time (sec)	N/A	0.001	0.001	0.002	0.977	0.811	0.472	0.150	4.300

Problem 1026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	15	15	8	10
normalized size	1	1.00	1.00	0.77	0.69	1.15	1.15	0.62	0.77
time (sec)	N/A	0.001	0.001	0.002	1.026	0.814	0.514	0.159	4.338

Problem 1027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	15	17	8	13
normalized size	1	1.00	1.00	0.81	0.75	0.94	1.06	0.50	0.81
time (sec)	N/A	0.001	0.002	0.002	1.075	0.544	0.556	0.155	4.305

Problem 1028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	17	13	12	15	19	8	13
normalized size	1	1.00	1.06	0.81	0.75	0.94	1.19	0.50	0.81
time (sec)	N/A	0.002	0.004	0.003	1.048	0.482	0.658	0.162	4.272

Problem 1029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	15	13	12	15	19	8	13
normalized size	1	1.00	0.94	0.81	0.75	0.94	1.19	0.50	0.81
time (sec)	N/A	0.002	0.002	0.003	1.064	0.826	0.708	0.153	4.331

Problem 1030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	8	8	8
normalized size	1	1.00	1.00	0.75	0.67	0.67	0.67	0.67	0.67
time (sec)	N/A	0.001	0.001	0.001	1.009	0.832	0.066	0.149	0.016

Problem 1031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	8	8	8
normalized size	1	1.00	1.00	0.75	0.67	0.67	0.67	0.67	0.67
time (sec)	N/A	0.001	0.001	0.000	1.040	0.655	0.063	0.175	0.027

Problem 1032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	8	8	8
normalized size	1	1.00	1.00	0.75	0.67	0.67	0.67	0.67	0.67
time (sec)	N/A	0.001	0.001	0.001	0.949	0.839	0.066	0.150	0.013

Problem 1033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	8	8	8
normalized size	1	1.00	1.00	0.75	0.67	0.67	0.67	0.67	0.67
time (sec)	N/A	0.001	0.001	0.000	1.074	0.819	0.081	0.150	0.016

Problem 1034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	5	5	5
normalized size	1	1.00	1.00	0.86	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.001	0.000	0.000	0.981	0.813	0.135	0.150	0.002

Problem 1035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	7	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.88	0.88	0.75
time (sec)	N/A	0.001	0.000	0.001	1.032	0.622	0.081	0.148	4.241

Problem 1036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.001	0.001	0.000	0.882	0.787	0.077	0.182	0.028

Problem 1037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	12	8	8
normalized size	1	1.00	1.00	0.75	0.67	0.67	1.00	0.67	0.67
time (sec)	N/A	0.002	0.001	0.001	1.038	0.789	0.075	0.148	4.400

Problem 1038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	12	8	8
normalized size	1	1.00	1.00	0.75	0.67	0.67	1.00	0.67	0.67
time (sec)	N/A	0.001	0.001	0.000	1.003	0.832	0.074	0.178	4.329

Problem 1039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	43	0	0	0	0	-1
normalized size	1	1.00	1.50	3.58	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.010	0.020	0.008	0.000	0.831	0.000	0.000	0.000

Problem 1040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	87	82	0	0	0	0	-1
normalized size	1	1.00	2.23	2.10	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.073	0.113	0.083	0.000	0.900	0.000	0.000	0.000

Problem 1041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	22	19	24	29	19	21
normalized size	1	1.00	0.81	0.71	0.61	0.77	0.94	0.61	0.68
time (sec)	N/A	0.006	0.007	0.003	0.979	0.751	6.725	0.164	4.291

Problem 1042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	22	19	24	29	19	21
normalized size	1	1.00	0.81	0.71	0.61	0.77	0.94	0.61	0.68
time (sec)	N/A	0.007	0.009	0.004	1.042	0.804	2.672	0.209	0.036

Problem 1043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	22	19	22	29	19	21
normalized size	1	1.00	0.81	0.71	0.61	0.71	0.94	0.61	0.68
time (sec)	N/A	0.007	0.007	0.004	1.013	0.764	2.102	0.169	0.034

Problem 1044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	22	19	21	27	19	21
normalized size	1	1.00	0.86	0.76	0.66	0.72	0.93	0.66	0.72
time (sec)	N/A	0.006	0.008	0.005	1.028	0.502	0.818	0.152	0.031

Problem 1045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	22	19	21	27	19	21
normalized size	1	1.00	0.86	0.76	0.66	0.72	0.93	0.66	0.72
time (sec)	N/A	0.006	0.009	0.004	1.095	0.753	1.041	0.149	0.039

Problem 1046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	22	19	21	27	19	21
normalized size	1	1.00	0.86	0.76	0.66	0.72	0.93	0.66	0.72
time (sec)	N/A	0.006	0.009	0.004	1.023	0.594	1.282	0.326	0.034

Problem 1047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	22	20	21	27	20	21
normalized size	1	1.00	0.86	0.76	0.69	0.72	0.93	0.69	0.72
time (sec)	N/A	0.006	0.009	0.004	1.081	0.800	1.871	0.209	4.326

Problem 1048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	49	44	49	70	46	45
normalized size	1	1.00	1.00	0.77	0.69	0.77	1.09	0.72	0.70
time (sec)	N/A	0.023	3.696	0.006	1.026	0.864	22.351	0.206	4.415

Problem 1049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	66	49	44	49	70	46	45
normalized size	1	1.00	1.03	0.77	0.69	0.77	1.09	0.72	0.70
time (sec)	N/A	0.022	0.057	0.007	1.160	0.796	12.365	0.172	0.026

Problem 1050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	50	49	44	47	63	46	45
normalized size	1	1.00	0.78	0.77	0.69	0.73	0.98	0.72	0.70
time (sec)	N/A	0.023	3.384	0.006	1.112	0.868	3.454	0.150	0.028

Problem 1051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	63	49	48	46	68	46	45
normalized size	1	1.00	1.02	0.79	0.77	0.74	1.10	0.74	0.73
time (sec)	N/A	0.022	0.042	0.006	1.139	0.837	4.996	0.151	0.025

Problem 1052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	54	49	44	46	68	46	45
normalized size	1	1.00	0.87	0.79	0.71	0.74	1.10	0.74	0.73
time (sec)	N/A	0.022	0.044	0.007	1.113	0.815	5.654	0.154	0.026

Problem 1053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	53	49	44	46	68	46	45
normalized size	1	1.00	0.85	0.79	0.71	0.74	1.10	0.74	0.73
time (sec)	N/A	0.022	0.065	0.007	1.130	0.719	6.892	0.151	0.027

Problem 1054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	50	49	45	46	68	47	48
normalized size	1	1.00	0.81	0.79	0.73	0.74	1.10	0.76	0.77
time (sec)	N/A	0.023	0.049	0.007	1.118	0.775	9.109	0.160	0.046

Problem 1055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	103	90	81	86	129	87	76
normalized size	1	1.00	1.00	0.87	0.79	0.83	1.25	0.84	0.74
time (sec)	N/A	0.048	3.724	0.008	0.999	0.571	60.632	0.157	0.041

Problem 1056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	105	90	81	86	129	87	76
normalized size	1	1.00	1.02	0.87	0.79	0.83	1.25	0.84	0.74
time (sec)	N/A	0.043	0.098	0.007	1.061	0.722	39.321	0.166	0.037

Problem 1057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	103	90	81	84	112	87	76
normalized size	1	1.00	1.00	0.87	0.79	0.82	1.09	0.84	0.74
time (sec)	N/A	0.042	3.379	0.008	1.043	0.607	6.047	0.160	0.035

Problem 1058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	102	90	88	83	128	87	76
normalized size	1	1.00	1.01	0.89	0.87	0.82	1.27	0.86	0.75
time (sec)	N/A	0.044	0.074	0.007	1.043	0.829	23.501	0.171	0.035

Problem 1059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	100	90	81	83	126	87	76
normalized size	1	1.00	1.01	0.91	0.82	0.84	1.27	0.88	0.77
time (sec)	N/A	0.043	0.084	0.006	0.982	0.675	19.835	0.156	0.038

Problem 1060	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	103	90	81	83	128	87	76
normalized size	1	1.00	1.02	0.89	0.80	0.82	1.27	0.86	0.75
time (sec)	N/A	0.044	0.075	0.008	1.063	0.746	25.437	0.198	0.037

Problem 1061	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	100	90	82	83	124	88	79
normalized size	1	1.00	1.01	0.91	0.83	0.84	1.25	0.89	0.80
time (sec)	N/A	0.042	0.077	0.007	1.055	0.752	31.860	0.174	0.036

Problem 1062	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	80	65	0	6649	0	0	12789
normalized size	1	1.00	0.21	0.17	0.00	17.09	0.00	0.00	32.88
time (sec)	N/A	0.860	0.053	0.059	0.000	8.226	0.000	0.000	5.820

Problem 1063	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	80	64	0	5319	0	0	10449
normalized size	1	1.00	0.21	0.17	0.00	13.82	0.00	0.00	27.14
time (sec)	N/A	0.797	0.045	0.010	0.000	3.182	0.000	0.000	6.858

Problem 1064	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	48	45	0	4058	0	0	8093
normalized size	1	1.00	0.15	0.14	0.00	12.26	0.00	0.00	24.45
time (sec)	N/A	0.442	0.029	0.008	0.000	1.663	0.000	0.000	6.512

Problem 1065	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	46	45	0	2482	0	0	8229
normalized size	1	1.00	0.14	0.14	0.00	7.50	0.00	0.00	24.86
time (sec)	N/A	0.403	0.028	0.008	0.000	1.294	0.000	0.000	6.024

Problem 1066	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	47	45	0	2769	0	0	6133
normalized size	1	1.00	0.14	0.14	0.00	8.37	0.00	0.00	18.53
time (sec)	N/A	0.366	0.029	0.007	0.000	1.168	0.000	0.000	5.308

Problem 1067	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	49	42	0	4045	0	0	10401
normalized size	1	1.00	0.15	0.13	0.00	12.22	0.00	0.00	31.42
time (sec)	N/A	0.415	0.032	0.006	0.000	1.412	0.000	0.000	6.258

Problem 1068	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	78	65	0	5384	0	0	10573
normalized size	1	1.00	0.21	0.18	0.00	14.51	0.00	0.00	28.50
time (sec)	N/A	0.567	0.049	0.010	0.000	3.523	0.000	0.000	5.737

Problem 1069	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	82	64	0	6671	0	0	16557
normalized size	1	1.00	0.22	0.17	0.00	17.98	0.00	0.00	44.63
time (sec)	N/A	0.512	0.054	0.011	0.000	6.253	0.000	0.000	8.637

Problem 1070	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	107	82	0	7995	0	0	15149
normalized size	1	1.00	0.26	0.20	0.00	19.41	0.00	0.00	36.77
time (sec)	N/A	0.981	0.073	0.013	0.000	20.361	0.000	0.000	6.479

Problem 1071	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	544	544	144	149	0	0	0	0	28774
normalized size	1	1.00	0.26	0.27	0.00	0.00	0.00	0.00	52.89
time (sec)	N/A	2.577	0.281	0.021	0.000	0.000	0.000	0.000	7.012

Problem 1072	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	520	520	144	146	0	11906	0	0	31964
normalized size	1	1.00	0.28	0.28	0.00	22.90	0.00	0.00	61.47
time (sec)	N/A	1.371	0.266	0.020	0.000	82.621	0.000	0.000	11.849

Problem 1073	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	471	124	120	0	11032	0	0	23808
normalized size	1	1.00	0.26	0.25	0.00	23.42	0.00	0.00	50.55
time (sec)	N/A	0.917	0.213	0.019	0.000	72.302	0.000	0.000	6.445

Problem 1074	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	483	483	127	118	0	9245	0	0	26432
normalized size	1	1.00	0.26	0.24	0.00	19.14	0.00	0.00	54.72
time (sec)	N/A	1.032	0.213	0.020	0.000	8.616	0.000	0.000	10.885

Problem 1075	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	450	109	121	0	9757	0	0	21913
normalized size	1	1.00	0.24	0.27	0.00	21.68	0.00	0.00	48.70
time (sec)	N/A	0.710	0.214	0.019	0.000	22.264	0.000	0.000	6.058

Problem 1076	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	442	442	111	118	0	10570	0	0	28713
normalized size	1	1.00	0.25	0.27	0.00	23.91	0.00	0.00	64.96
time (sec)	N/A	0.705	0.230	0.020	0.000	19.510	0.000	0.000	10.634

Problem 1077	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	489	489	149	146	0	12411	0	0	26373
normalized size	1	1.00	0.30	0.30	0.00	25.38	0.00	0.00	53.93
time (sec)	N/A	0.998	0.243	0.020	0.000	114.215	0.000	0.000	6.560

Problem 1078	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	503	503	153	144	0	0	0	0	35171
normalized size	1	1.00	0.30	0.29	0.00	0.00	0.00	0.00	69.92
time (sec)	N/A	1.276	0.249	0.019	0.000	0.000	0.000	0.000	7.286

Problem 1079	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	573	573	190	245	0	0	0	0	31145
normalized size	1	1.00	0.33	0.43	0.00	0.00	0.00	0.00	54.35
time (sec)	N/A	2.446	0.339	0.027	0.000	0.000	0.000	0.000	11.419

Problem 1080	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	621	621	254	275	0	0	0	0	50970
normalized size	1	1.00	0.41	0.44	0.00	0.00	0.00	0.00	82.08
time (sec)	N/A	1.772	0.442	0.039	0.000	0.000	0.000	0.000	9.350

Problem 1081	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	569	569	216	242	0	0	0	0	39697
normalized size	1	1.00	0.38	0.43	0.00	0.00	0.00	0.00	69.77
time (sec)	N/A	1.908	0.408	0.038	0.000	0.000	0.000	0.000	8.015

Problem 1082	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	569	569	219	241	0	0	0	0	45495
normalized size	1	1.00	0.38	0.42	0.00	0.00	0.00	0.00	79.96
time (sec)	N/A	1.965	0.387	0.036	0.000	0.000	0.000	0.000	8.524

Problem 1083	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	533	533	176	244	0	0	0	0	37678
normalized size	1	1.00	0.33	0.46	0.00	0.00	0.00	0.00	70.69
time (sec)	N/A	1.452	0.380	0.036	0.000	0.000	0.000	0.000	7.659

Problem 1084	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	533	533	177	237	0	0	0	0	47803
normalized size	1	1.00	0.33	0.44	0.00	0.00	0.00	0.00	89.69
time (sec)	N/A	1.363	0.441	0.036	0.000	0.000	0.000	0.000	8.384

Problem 1085	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	594	594	222	277	0	0	0	0	42197
normalized size	1	1.00	0.37	0.47	0.00	0.00	0.00	0.00	71.04
time (sec)	N/A	2.310	0.406	0.037	0.000	0.000	0.000	0.000	8.019

Problem 1086	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	594	594	224	270	0	0	0	0	54027
normalized size	1	1.00	0.38	0.45	0.00	0.00	0.00	0.00	90.95
time (sec)	N/A	2.366	0.412	0.037	0.000	0.000	0.000	0.000	9.347

Problem 1087	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	658	658	254	321	0	0	0	0	46948
normalized size	1	1.00	0.39	0.49	0.00	0.00	0.00	0.00	71.35
time (sec)	N/A	5.485	0.495	0.053	0.000	0.000	0.000	0.000	8.746

Problem 1088	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	658	658	258	316	0	0	0	0	60099
normalized size	1	1.00	0.39	0.48	0.00	0.00	0.00	0.00	91.34
time (sec)	N/A	5.792	0.464	0.038	0.000	0.000	0.000	0.000	9.847

Problem 1089	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	365	0	0	0	0	0	-1
normalized size	1	1.00	2.48	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.207	0.579	0.083	0.000	1.010	0.000	0.000	0.000

Problem 1090	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	342	0	0	0	0	0	-1
normalized size	1	1.00	2.33	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.432	0.057	0.000	1.174	0.000	0.000	0.000

Problem 1091	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	342	0	0	0	0	0	-1
normalized size	1	1.00	2.36	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.380	0.050	0.000	1.019	0.000	0.000	0.000

Problem 1092	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	345	0	0	0	0	0	-1
normalized size	1	1.00	2.38	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.356	0.053	0.000	1.051	0.000	0.000	0.000

Problem 1093	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	459	0	0	0	0	0	-1
normalized size	1	1.00	3.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.879	0.062	0.000	1.025	0.000	0.000	0.000

Problem 1094	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	417	0	0	0	0	0	-1
normalized size	1	1.00	2.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.710	0.061	0.000	1.041	0.000	0.000	0.000

Problem 1095	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	415	0	0	0	0	0	-1
normalized size	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.706	0.057	0.000	0.625	0.000	0.000	0.000

Problem 1096	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	384	0	0	0	0	0	-1
normalized size	1	1.00	2.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.731	0.058	0.000	0.670	0.000	0.000	0.000

Problem 1097	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	173	0	0	0	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.092	0.028	0.000	0.599	0.000	0.000	0.000

Problem 1098	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	173	0	0	0	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.099	0.023	0.000	0.581	0.000	0.000	0.000

Problem 1099	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	171	0	0	0	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.080	0.024	0.000	0.757	0.000	0.000	0.000

Problem 1100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	348	0	0	0	0	0	-1
normalized size	1	1.00	2.40	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.478	0.055	0.000	0.626	0.000	0.000	0.000

Problem 1101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	348	0	0	0	0	0	-1
normalized size	1	1.00	2.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.344	0.026	0.000	0.676	0.000	0.000	0.000

Problem 1102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	367	0	0	0	0	0	-1
normalized size	1	1.00	2.45	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.510	0.027	0.000	0.676	0.000	0.000	0.000

Problem 1103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	366	0	0	0	0	0	-1
normalized size	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.472	0.034	0.000	0.709	0.000	0.000	0.000

Problem 1104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	409	0	0	0	0	0	-1
normalized size	1	1.00	2.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.693	0.123	0.000	0.800	0.000	0.000	0.000

Problem 1105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	111	782	195	594	4451	1132	546
normalized size	1	1.00	0.71	5.01	1.25	3.81	28.53	7.26	3.50
time (sec)	N/A	0.100	0.129	0.009	1.308	0.690	7.268	0.226	4.834

Problem 1106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	70	301	110	241	1486	449	260
normalized size	1	1.00	0.69	2.98	1.09	2.39	14.71	4.45	2.57
time (sec)	N/A	0.053	0.054	0.008	1.107	0.656	2.871	0.181	4.584

Problem 1107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	78	50	71	314	119	89
normalized size	1	1.00	0.67	1.50	0.96	1.37	6.04	2.29	1.71
time (sec)	N/A	0.020	0.029	0.004	1.087	0.505	0.989	0.160	4.395

Problem 1108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	82	0	0	0	0	0	-1
normalized size	1	1.00	0.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.251	0.057	0.035	0.000	0.715	0.000	0.000	0.000

Problem 1109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	78	0	0	0	0	0	-1
normalized size	1	1.00	0.25	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.696	0.080	0.027	0.000	0.645	0.000	0.000	0.000

Problem 1110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	357	0	0	0	0	0	-1
normalized size	1	1.00	2.26	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.281	0.011	0.000	0.642	0.000	0.000	0.000

Problem 1111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	181	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.099	0.010	0.000	0.887	0.000	0.000	0.000

Problem 1112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	181	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.112	0.014	0.000	0.623	0.000	0.000	0.000

Problem 1113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	221	0	0	0	0	0	-1
normalized size	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.172	0.011	0.000	0.652	0.000	0.000	0.000

Problem 1114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	179	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.241	0.074	0.000	0.644	0.000	0.000	0.000

Problem 1115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	162	0	0	0	0	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.373	0.234	0.060	0.000	0.552	0.000	0.000	0.000

Problem 1116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	162	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.211	0.224	0.048	0.000	0.795	0.000	0.000	0.000

Problem 1117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	162	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.198	0.035	0.000	0.581	0.000	0.000	0.000

Problem 1118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	135	0	0	0	0	0	-1
normalized size	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.098	0.023	0.000	0.497	0.000	0.000	0.000

Problem 1119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	152	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.204	0.018	0.000	0.535	0.000	0.000	0.000

Problem 1120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	163	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.203	0.032	0.000	0.768	0.000	0.000	0.000

Problem 1121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	159	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.221	0.042	0.000	0.553	0.000	0.000	0.000

Problem 1122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.170	0.038	0.000	0.503	0.000	0.000	0.000

Problem 1123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.154	0.029	0.000	0.723	0.000	0.000	0.000

Problem 1124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	161	0	0	0	0	0	-1
normalized size	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.156	0.016	0.000	0.731	0.000	0.000	0.000

Problem 1125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	164	0	0	0	0	0	-1
normalized size	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.159	0.030	0.000	0.634	0.000	0.000	0.000

Problem 1126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.169	0.038	0.000	0.685	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [327] had the largest ratio of [.5263]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	22	0.136
2	A	3	3	1.00	22	0.136
3	A	2	2	1.00	22	0.091
4	A	2	2	1.00	22	0.091
5	A	3	3	1.03	22	0.136
6	A	4	3	1.02	22	0.136
7	A	5	3	1.10	22	0.136
8	A	9	5	1.00	16	0.312
9	A	3	3	1.00	16	0.188
10	A	9	5	1.00	16	0.312
11	A	3	2	1.00	12	0.167
12	A	3	2	1.00	12	0.167
13	A	9	5	1.00	12	0.417
14	A	9	5	1.00	12	0.417
15	A	9	5	1.00	12	0.417
16	A	2	2	1.00	16	0.125
17	A	2	2	1.00	16	0.125
18	A	2	2	1.00	16	0.125
19	A	2	2	1.00	16	0.125
20	A	2	2	1.00	14	0.143
21	A	1	1	1.00	11	0.091
22	A	2	2	1.00	16	0.125
23	A	2	2	1.00	16	0.125
24	A	2	2	1.00	16	0.125
25	A	2	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	2	2	1.00	16	0.125
27	A	2	2	1.00	16	0.125
28	A	2	2	1.00	16	0.125
29	A	2	2	1.00	16	0.125
30	A	2	2	1.00	16	0.125
31	A	2	2	1.00	16	0.125
32	A	2	2	1.00	16	0.125
33	A	2	2	1.00	14	0.143
34	A	1	1	1.00	11	0.091
35	A	2	2	1.00	16	0.125
36	A	2	2	1.00	16	0.125
37	A	2	2	1.00	16	0.125
38	A	2	2	1.00	16	0.125
39	A	2	2	1.00	16	0.125
40	A	2	2	1.00	16	0.125
41	A	2	2	1.00	16	0.125
42	A	1	1	1.00	16	0.062
43	A	1	1	1.00	16	0.062
44	A	1	1	1.00	16	0.062
45	A	1	1	1.00	16	0.062
46	A	1	1	1.00	14	0.071
47	A	1	1	1.00	11	0.091
48	A	1	1	1.00	16	0.062
49	A	1	1	1.00	16	0.062
50	A	1	1	1.00	16	0.062
51	A	1	1	1.00	16	0.062

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
52	A	1	1	1.00	16	0.062
53	A	1	1	1.00	16	0.062
54	A	1	1	1.00	16	0.062
55	A	1	1	1.00	16	0.062
56	A	1	1	1.00	16	0.062
57	A	1	1	1.00	16	0.062
58	A	1	1	1.00	16	0.062
59	A	1	1	1.00	14	0.071
60	A	1	1	1.00	11	0.091
61	A	1	1	1.00	16	0.062
62	A	1	1	1.00	16	0.062
63	A	1	1	1.00	16	0.062
64	A	1	1	1.00	16	0.062
65	A	1	1	1.00	16	0.062
66	A	1	1	1.00	16	0.062
67	A	1	1	1.00	16	0.062
68	A	1	1	1.00	16	0.062
69	A	1	1	1.00	16	0.062
70	A	1	1	1.00	14	0.071
71	A	1	1	1.00	11	0.091
72	A	1	1	1.00	16	0.062
73	A	1	1	1.00	16	0.062
74	A	1	1	1.00	16	0.062
75	A	1	1	1.00	16	0.062
76	A	1	1	1.00	16	0.062
77	A	1	1	1.00	16	0.062

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
78	A	1	1	1.00	16	0.062
79	A	1	1	1.00	16	0.062
80	A	1	1	1.00	16	0.062
81	A	1	1	1.00	16	0.062
82	A	1	1	1.00	16	0.062
83	A	1	1	1.00	16	0.062
84	A	1	1	1.00	16	0.062
85	A	1	1	1.00	16	0.062
86	A	1	1	1.00	14	0.071
87	A	1	1	1.00	11	0.091
88	A	1	1	1.00	16	0.062
89	A	1	1	1.00	16	0.062
90	A	1	1	1.00	16	0.062
91	A	1	1	1.00	16	0.062
92	A	1	1	1.00	16	0.062
93	A	1	1	1.00	16	0.062
94	A	1	1	1.00	16	0.062
95	A	2	2	1.00	16	0.125
96	A	2	2	1.00	16	0.125
97	A	2	2	1.00	16	0.125
98	A	1	1	1.00	16	0.062
99	A	1	1	1.00	16	0.062
100	A	1	1	1.00	16	0.062
101	A	1	1	1.00	14	0.071
102	A	1	1	1.00	11	0.091
103	A	1	1	1.00	16	0.062

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
104	A	1	1	1.00	16	0.062
105	A	1	1	1.00	16	0.062
106	A	1	1	1.00	16	0.062
107	A	2	2	1.00	16	0.125
108	A	2	2	1.00	16	0.125
109	A	2	2	1.00	16	0.125
110	A	1	1	1.00	16	0.062
111	A	1	1	1.00	16	0.062
112	A	1	1	1.00	16	0.062
113	A	1	1	1.00	14	0.071
114	A	1	1	1.00	11	0.091
115	A	1	1	1.00	16	0.062
116	A	1	1	1.00	16	0.062
117	A	1	1	1.00	16	0.062
118	A	1	1	1.00	16	0.062
119	A	2	2	1.00	16	0.125
120	A	1	1	1.00	16	0.062
121	A	1	1	1.00	16	0.062
122	A	1	1	1.00	16	0.062
123	A	1	1	1.00	16	0.062
124	A	1	1	1.00	14	0.071
125	A	2	2	1.00	16	0.125
126	A	2	2	1.00	16	0.125
127	A	2	2	1.00	16	0.125
128	A	2	2	1.00	16	0.125
129	A	2	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	2	2	1.00	16	0.125
131	A	2	2	1.00	16	0.125
132	A	2	2	1.00	16	0.125
133	A	2	2	1.00	16	0.125
134	A	2	1	1.00	15	0.067
135	A	2	1	1.00	13	0.077
136	A	1	0	1.00	11	0.000
137	A	2	1	1.00	15	0.067
138	A	2	1	1.00	15	0.067
139	A	2	1	1.00	15	0.067
140	A	2	1	1.00	15	0.067
141	A	2	1	1.00	15	0.067
142	A	2	1	1.00	15	0.067
143	A	2	1	1.00	15	0.067
144	A	2	1	1.00	15	0.067
145	A	3	2	1.00	13	0.154
146	A	4	3	1.00	17	0.176
147	A	3	2	1.00	17	0.118
148	A	2	2	1.00	17	0.118
149	A	3	2	1.00	17	0.118
150	A	4	3	1.00	17	0.176
151	A	3	2	1.00	17	0.118
152	A	4	3	1.00	17	0.176
153	A	3	2	1.00	17	0.118
154	A	4	3	1.00	17	0.176
155	A	3	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
156	A	2	2	1.00	17	0.118
157	A	3	2	1.00	17	0.118
158	A	3	2	1.00	17	0.118
159	A	4	3	1.00	17	0.176
160	A	3	2	1.00	17	0.118
161	A	2	2	1.00	17	0.118
162	A	3	2	1.00	17	0.118
163	A	4	3	1.00	17	0.176
164	A	3	2	1.00	17	0.118
165	A	4	3	1.00	17	0.176
166	A	3	2	1.00	17	0.118
167	A	4	3	1.00	17	0.176
168	A	3	2	1.00	17	0.118
169	A	4	3	1.00	17	0.176
170	A	3	2	1.00	17	0.118
171	A	2	2	1.00	17	0.118
172	A	3	2	1.00	17	0.118
173	A	4	4	1.00	17	0.235
174	A	4	3	1.00	17	0.176
175	A	4	3	1.00	17	0.176
176	A	4	3	1.00	17	0.176
177	A	4	3	1.00	17	0.176
178	A	4	3	1.00	17	0.176
179	A	4	3	1.00	17	0.176
180	A	3	3	1.00	17	0.176
181	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	2	2	1.00	17	0.118
183	A	5	5	1.00	15	0.333
184	A	3	3	1.00	13	0.231
185	A	4	3	1.00	17	0.176
186	A	4	3	1.00	17	0.176
187	A	4	3	1.00	17	0.176
188	A	5	3	1.00	17	0.176
189	A	4	3	1.00	17	0.176
190	A	5	4	1.00	17	0.235
191	A	4	3	1.00	17	0.176
192	A	5	4	1.00	17	0.235
193	A	4	3	1.00	17	0.176
194	A	4	4	1.00	17	0.235
195	A	4	3	1.00	17	0.176
196	A	3	3	1.00	17	0.176
197	A	2	2	1.00	17	0.118
198	A	3	3	1.00	17	0.176
199	A	4	3	1.00	17	0.176
200	A	4	4	1.00	17	0.235
201	A	4	3	1.00	15	0.200
202	A	5	4	1.00	13	0.308
203	A	4	3	1.00	17	0.176
204	A	6	4	1.00	17	0.235
205	A	6	4	1.00	17	0.235
206	A	4	3	1.00	17	0.176
207	A	5	4	1.00	17	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
208	A	4	3	1.00	17	0.176
209	A	4	3	1.00	17	0.176
210	A	2	2	1.00	17	0.118
211	A	4	4	1.00	17	0.235
212	A	2	2	1.00	17	0.118
213	A	4	3	1.00	17	0.176
214	A	4	3	1.00	17	0.176
215	A	5	4	1.00	17	0.235
216	A	4	3	1.00	17	0.176
217	A	6	4	1.00	17	0.235
218	A	4	3	1.00	15	0.200
219	A	7	4	1.00	13	0.308
220	A	4	3	1.00	17	0.176
221	A	6	6	1.00	19	0.316
222	A	5	5	1.00	19	0.263
223	A	4	4	1.00	17	0.235
224	A	4	4	1.00	19	0.210
225	A	4	4	1.00	19	0.210
226	A	1	1	1.00	19	0.053
227	A	2	2	1.00	19	0.105
228	A	3	2	1.00	19	0.105
229	A	4	2	1.00	19	0.105
230	A	5	2	1.00	19	0.105
231	A	3	2	1.00	19	0.105
232	A	2	2	1.00	19	0.105
233	A	1	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
234	A	3	3	1.00	19	0.158
235	A	3	3	1.00	19	0.158
236	A	4	4	1.00	19	0.210
237	A	5	4	1.00	19	0.210
238	A	6	5	1.00	19	0.263
239	A	5	4	1.00	17	0.235
240	A	5	5	1.00	19	0.263
241	A	5	4	1.00	19	0.210
242	A	5	5	1.00	19	0.263
243	A	5	4	1.00	19	0.210
244	A	1	1	1.00	19	0.053
245	A	2	2	1.00	19	0.105
246	A	3	2	1.00	19	0.105
247	A	4	2	1.00	19	0.105
248	A	5	2	1.00	19	0.105
249	A	5	3	1.00	19	0.158
250	A	4	3	1.00	19	0.158
251	A	3	3	1.00	19	0.158
252	A	2	2	1.00	15	0.133
253	A	1	1	1.00	19	0.053
254	A	4	3	1.00	19	0.158
255	A	4	4	1.00	19	0.210
256	A	4	3	1.00	19	0.158
257	A	5	4	1.00	19	0.210
258	A	6	4	1.00	19	0.210
259	A	7	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
260	A	6	5	1.00	19	0.263
261	A	5	5	1.00	19	0.263
262	A	4	4	1.00	19	0.210
263	A	3	3	1.00	17	0.176
264	A	1	1	1.00	19	0.053
265	A	2	2	1.00	19	0.105
266	A	3	2	1.00	19	0.105
267	A	4	2	1.00	19	0.105
268	A	2	2	1.00	19	0.105
269	A	1	1	1.00	19	0.053
270	A	2	2	1.00	15	0.133
271	A	3	3	1.00	19	0.158
272	A	4	3	1.00	19	0.158
273	A	6	6	1.00	19	0.316
274	A	5	5	1.00	19	0.263
275	A	4	4	1.00	19	0.210
276	A	1	1	1.00	19	0.053
277	A	2	2	1.00	17	0.118
278	A	3	3	1.00	19	0.158
279	A	4	3	1.00	19	0.158
280	A	5	3	1.00	19	0.158
281	A	2	2	1.00	19	0.105
282	A	1	1	1.00	19	0.053
283	A	3	3	1.00	19	0.158
284	A	4	4	1.00	15	0.267
285	A	5	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
286	A	4	4	1.00	19	0.210
287	A	4	4	1.00	19	0.210
288	A	4	4	1.00	19	0.210
289	A	4	4	1.00	19	0.210
290	A	4	4	1.00	19	0.210
291	A	4	4	1.00	20	0.200
292	A	2	1	1.00	17	0.059
293	A	2	1	1.00	17	0.059
294	A	2	1	1.00	17	0.059
295	A	2	1	1.00	17	0.059
296	A	2	1	1.00	17	0.059
297	A	2	1	1.00	17	0.059
298	A	2	1	1.00	17	0.059
299	A	2	1	1.00	17	0.059
300	A	3	2	1.00	19	0.105
301	A	3	2	1.00	19	0.105
302	A	3	2	1.00	19	0.105
303	A	3	2	1.00	19	0.105
304	A	3	2	1.00	19	0.105
305	A	3	2	1.00	19	0.105
306	A	3	2	1.00	19	0.105
307	A	3	2	1.00	19	0.105
308	A	3	2	1.00	19	0.105
309	A	3	2	1.00	19	0.105
310	A	3	2	1.00	19	0.105
311	A	3	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
312	A	3	2	1.00	19	0.105
313	A	3	2	1.00	19	0.105
314	A	3	2	1.00	19	0.105
315	A	3	2	1.00	19	0.105
316	A	13	9	1.00	19	0.474
317	A	13	9	1.00	19	0.474
318	A	12	9	1.00	19	0.474
319	A	12	9	1.00	19	0.474
320	A	11	8	1.00	19	0.421
321	A	11	8	1.00	19	0.421
322	A	12	9	1.00	19	0.474
323	A	12	9	1.00	19	0.474
324	A	13	9	1.00	19	0.474
325	A	13	9	1.00	19	0.474
326	A	14	9	1.00	19	0.474
327	A	14	10	1.00	19	0.526
328	A	13	10	1.00	19	0.526
329	A	13	10	1.00	19	0.526
330	A	12	9	1.00	19	0.474
331	A	12	9	1.00	19	0.474
332	A	12	9	1.00	19	0.474
333	A	12	9	1.00	19	0.474
334	A	13	10	1.00	19	0.526
335	A	13	10	1.00	19	0.526
336	A	14	10	1.00	19	0.526
337	A	14	10	1.00	19	0.526

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
338	A	15	10	1.00	19	0.526
339	A	14	10	1.00	19	0.526
340	A	13	9	1.00	19	0.474
341	A	13	9	1.00	19	0.474
342	A	13	10	1.00	19	0.526
343	A	13	10	1.00	19	0.526
344	A	13	9	1.00	19	0.474
345	A	13	9	1.00	19	0.474
346	A	14	10	1.00	19	0.526
347	A	14	10	1.00	19	0.526
348	A	15	10	1.00	19	0.526
349	A	15	10	1.00	19	0.526
350	A	16	10	1.00	19	0.526
351	A	16	10	1.00	19	0.526
352	A	8	7	1.00	21	0.333
353	A	6	5	1.00	21	0.238
354	A	7	7	1.00	21	0.333
355	A	5	5	1.00	21	0.238
356	A	6	6	1.00	21	0.286
357	A	4	4	1.00	21	0.190
358	A	6	6	1.00	21	0.286
359	A	4	4	1.00	21	0.190
360	A	7	7	1.00	21	0.333
361	A	5	5	1.00	21	0.238
362	A	8	7	1.00	21	0.333
363	A	6	5	1.00	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
364	A	9	7	1.00	21	0.333
365	A	7	5	1.00	21	0.238
366	A	8	7	1.00	21	0.333
367	A	6	5	1.00	21	0.238
368	A	7	6	1.00	21	0.286
369	A	5	4	1.00	21	0.190
370	A	7	7	1.00	21	0.333
371	A	5	5	1.00	21	0.238
372	A	7	6	1.00	21	0.286
373	A	5	4	1.00	21	0.190
374	A	8	7	1.00	21	0.333
375	A	6	5	1.00	21	0.238
376	A	9	7	1.00	21	0.333
377	A	7	5	1.00	21	0.238
378	A	6	4	1.00	21	0.190
379	A	7	6	1.00	21	0.286
380	A	5	4	1.00	21	0.190
381	A	6	6	1.00	21	0.286
382	A	4	4	1.00	21	0.190
383	A	5	5	1.00	21	0.238
384	A	3	3	1.00	21	0.143
385	A	6	6	1.00	21	0.286
386	A	4	4	1.00	21	0.190
387	A	7	6	1.00	21	0.286
388	A	5	4	1.00	21	0.190
389	A	8	6	1.00	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
390	A	6	4	1.00	21	0.190
391	A	6	5	1.00	21	0.238
392	A	7	7	1.00	21	0.333
393	A	5	5	1.00	21	0.238
394	A	6	6	1.00	21	0.286
395	A	4	4	1.00	21	0.190
396	A	6	6	1.00	21	0.286
397	A	4	4	1.00	21	0.190
398	A	7	7	1.00	21	0.333
399	A	5	5	1.00	21	0.238
400	A	8	7	1.00	21	0.333
401	A	6	5	1.00	21	0.238
402	A	9	7	1.00	21	0.333
403	A	4	3	1.00	19	0.158
404	A	4	3	1.00	19	0.158
405	A	2	1	1.00	17	0.059
406	A	3	3	1.00	19	0.158
407	A	3	3	1.00	19	0.158
408	A	3	3	1.00	19	0.158
409	A	2	1	1.00	22	0.045
410	A	2	1	1.00	22	0.045
411	A	2	1	1.00	20	0.050
412	A	1	0	1.00	18	0.000
413	A	2	1	1.00	22	0.045
414	A	2	1	1.00	22	0.045
415	A	2	1	1.00	22	0.045

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
416	A	2	1	1.00	22	0.045
417	A	2	1	1.00	22	0.045
418	A	2	1	1.00	22	0.045
419	A	2	1	1.00	22	0.045
420	A	2	1	1.00	22	0.045
421	A	3	2	1.00	24	0.083
422	A	4	3	1.00	24	0.125
423	A	3	2	1.00	24	0.083
424	A	4	3	1.00	24	0.125
425	A	3	2	1.00	24	0.083
426	A	2	2	1.00	22	0.091
427	A	3	2	1.00	20	0.100
428	A	4	3	1.00	24	0.125
429	A	3	2	1.00	24	0.083
430	A	4	3	1.00	24	0.125
431	A	3	2	1.00	24	0.083
432	A	4	3	1.00	24	0.125
433	A	3	2	1.00	24	0.083
434	A	4	3	1.00	24	0.125
435	A	3	2	1.00	24	0.083
436	A	4	3	1.00	24	0.125
437	A	3	2	1.00	24	0.083
438	A	2	2	1.00	24	0.083
439	A	3	2	1.00	24	0.083
440	A	4	4	1.00	24	0.167
441	A	3	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
442	A	4	3	1.00	24	0.125
443	A	3	2	1.00	24	0.083
444	A	3	2	1.00	24	0.083
445	A	4	3	1.00	24	0.125
446	A	3	2	1.00	24	0.083
447	A	4	3	1.00	24	0.125
448	A	3	2	1.00	24	0.083
449	A	4	3	1.00	24	0.125
450	A	3	2	1.00	24	0.083
451	A	2	2	1.00	22	0.091
452	A	3	2	1.00	20	0.100
453	A	4	3	1.00	24	0.125
454	A	3	2	1.00	24	0.083
455	A	4	3	1.00	24	0.125
456	A	3	2	1.00	24	0.083
457	A	4	3	1.00	24	0.125
458	A	3	2	1.00	24	0.083
459	A	4	3	1.00	24	0.125
460	A	3	2	1.00	24	0.083
461	A	4	3	1.00	24	0.125
462	A	3	2	1.00	24	0.083
463	A	4	3	1.00	24	0.125
464	A	3	2	1.00	24	0.083
465	A	4	3	1.00	24	0.125
466	A	3	2	1.00	24	0.083
467	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
468	A	3	2	1.00	24	0.083
469	A	4	4	1.00	24	0.167
470	A	3	2	1.00	24	0.083
471	A	5	4	1.00	24	0.167
472	A	3	2	1.00	24	0.083
473	A	6	4	1.00	24	0.167
474	A	3	2	1.00	24	0.083
475	A	4	3	1.00	24	0.125
476	A	4	3	1.00	24	0.125
477	A	4	3	1.00	24	0.125
478	A	4	3	1.00	24	0.125
479	A	4	3	1.00	24	0.125
480	A	2	2	1.00	22	0.091
481	A	4	3	1.00	24	0.125
482	A	4	3	1.00	24	0.125
483	A	4	3	1.00	24	0.125
484	A	5	4	1.00	24	0.167
485	A	5	4	1.00	24	0.167
486	A	5	4	1.00	24	0.167
487	A	4	4	1.00	24	0.167
488	A	3	3	1.00	24	0.125
489	A	3	3	1.00	20	0.150
490	A	4	4	1.00	24	0.167
491	A	5	4	1.00	24	0.167
492	A	6	4	1.00	24	0.167
493	A	4	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
494	A	4	3	1.00	24	0.125
495	A	4	3	1.00	24	0.125
496	A	2	2	1.00	24	0.083
497	A	4	3	1.00	24	0.125
498	A	2	2	1.00	22	0.091
499	A	4	3	1.00	24	0.125
500	A	4	3	1.00	24	0.125
501	A	4	3	1.00	24	0.125
502	A	7	4	1.00	24	0.167
503	A	7	4	1.00	24	0.167
504	A	6	4	1.00	24	0.167
505	A	5	3	1.00	24	0.125
506	A	5	4	1.00	24	0.167
507	A	5	4	1.00	24	0.167
508	A	5	3	1.00	20	0.150
509	A	6	4	1.00	24	0.167
510	A	7	4	1.00	24	0.167
511	A	8	4	1.00	24	0.167
512	A	4	3	1.00	24	0.125
513	A	4	3	1.00	24	0.125
514	A	4	3	1.00	24	0.125
515	A	2	2	1.00	24	0.083
516	A	4	4	1.00	24	0.167
517	A	4	3	1.00	24	0.125
518	A	4	3	1.00	24	0.125
519	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
520	A	4	3	1.00	24	0.125
521	A	4	3	1.00	24	0.125
522	A	4	3	1.00	24	0.125
523	A	9	4	1.00	24	0.167
524	A	9	4	1.00	24	0.167
525	A	8	4	1.00	24	0.167
526	A	7	3	1.00	24	0.125
527	A	7	4	1.00	24	0.167
528	A	7	4	1.00	24	0.167
529	A	7	4	1.00	24	0.167
530	A	7	4	1.00	24	0.167
531	A	7	3	1.00	20	0.150
532	A	8	4	1.00	24	0.167
533	A	9	4	1.00	24	0.167
534	A	10	4	1.00	24	0.167
535	A	3	3	1.00	12	0.250
536	A	2	2	1.00	14	0.143
537	A	3	3	1.00	16	0.188
538	A	4	3	1.00	16	0.188
539	A	2	2	1.00	14	0.143
540	A	4	3	1.00	16	0.188
541	A	4	3	1.00	26	0.115
542	A	3	3	1.00	26	0.115
543	A	2	2	1.00	24	0.083
544	A	3	2	1.00	26	0.077
545	A	3	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
546	A	3	3	1.00	26	0.115
547	A	1	1	1.00	26	0.038
548	A	4	3	1.00	26	0.115
549	A	4	3	1.00	26	0.115
550	A	3	2	1.00	26	0.077
551	A	3	2	1.00	26	0.077
552	A	2	1	1.00	22	0.045
553	A	3	2	1.00	26	0.077
554	A	3	2	1.00	26	0.077
555	A	3	2	1.00	26	0.077
556	A	3	2	1.00	26	0.077
557	A	3	2	1.00	26	0.077
558	A	4	3	1.00	26	0.115
559	A	4	3	1.00	26	0.115
560	A	3	2	1.12	26	0.077
561	A	3	3	1.00	26	0.115
562	A	2	2	1.00	24	0.083
563	A	4	3	1.00	26	0.115
564	A	4	3	1.00	26	0.115
565	A	4	3	1.00	26	0.115
566	A	4	3	1.00	26	0.115
567	A	3	3	1.00	26	0.115
568	A	1	1	1.00	26	0.038
569	A	4	3	1.00	26	0.115
570	A	4	3	1.00	26	0.115
571	A	4	3	1.00	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
572	A	3	2	1.00	26	0.077
573	A	3	2	1.00	26	0.077
574	A	3	2	1.00	26	0.077
575	A	3	2	1.00	26	0.077
576	A	3	2	1.00	22	0.091
577	A	3	2	1.00	26	0.077
578	A	3	2	1.00	26	0.077
579	A	3	2	1.00	26	0.077
580	A	3	2	1.00	26	0.077
581	A	3	2	1.00	26	0.077
582	A	3	2	1.00	26	0.077
583	A	3	2	1.00	26	0.077
584	A	3	2	1.00	26	0.077
585	A	4	3	1.00	26	0.115
586	A	4	3	1.00	26	0.115
587	A	3	2	1.00	26	0.077
588	A	4	3	1.00	26	0.115
589	A	4	3	1.00	26	0.115
590	A	3	3	1.00	26	0.115
591	A	2	2	1.00	24	0.083
592	A	4	3	1.00	26	0.115
593	A	4	3	1.00	26	0.115
594	A	4	3	1.00	26	0.115
595	A	4	3	1.00	26	0.115
596	A	4	3	1.00	26	0.115
597	A	4	3	1.00	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
598	A	3	3	1.00	26	0.115
599	A	1	1	1.00	26	0.038
600	A	5	4	1.00	26	0.154
601	A	4	3	1.00	26	0.115
602	A	4	3	1.00	26	0.115
603	A	4	3	1.00	26	0.115
604	A	4	3	1.00	26	0.115
605	A	3	2	1.00	26	0.077
606	A	3	2	1.00	26	0.077
607	A	3	2	1.00	26	0.077
608	A	3	2	1.00	26	0.077
609	A	3	2	1.00	26	0.077
610	A	3	2	1.00	26	0.077
611	A	3	2	1.00	22	0.091
612	A	3	2	1.00	26	0.077
613	A	3	2	1.00	26	0.077
614	A	3	2	1.00	26	0.077
615	A	3	2	1.00	26	0.077
616	A	3	2	1.00	26	0.077
617	A	3	2	1.00	26	0.077
618	A	3	2	1.00	26	0.077
619	A	3	2	1.00	26	0.077
620	A	3	2	1.00	26	0.077
621	A	3	2	1.00	26	0.077
622	A	3	2	1.00	26	0.077
623	A	3	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
624	A	4	3	1.00	26	0.115
625	A	4	4	1.00	26	0.154
626	A	3	3	1.00	24	0.125
627	A	5	5	1.00	26	0.192
628	A	4	3	0.98	26	0.115
629	A	4	3	1.00	26	0.115
630	A	3	3	1.00	26	0.115
631	A	2	2	1.00	22	0.091
632	A	3	3	1.00	26	0.115
633	A	4	3	0.98	26	0.115
634	A	4	3	1.00	26	0.115
635	A	4	3	1.00	26	0.115
636	A	3	3	1.68	26	0.115
637	A	2	2	1.00	24	0.083
638	A	4	3	1.00	26	0.115
639	A	4	3	1.00	26	0.115
640	A	4	3	1.00	26	0.115
641	A	4	4	1.00	26	0.154
642	A	4	3	1.00	22	0.136
643	A	5	4	1.00	26	0.154
644	A	6	4	1.00	26	0.154
645	A	4	3	1.00	26	0.115
646	A	4	3	1.00	26	0.115
647	A	3	3	1.00	26	0.115
648	A	1	1	1.00	26	0.038
649	A	3	3	1.00	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
650	A	2	2	1.00	24	0.083
651	A	4	3	1.00	26	0.115
652	A	4	3	1.00	26	0.115
653	A	6	4	1.00	26	0.154
654	A	6	4	1.00	26	0.154
655	A	6	4	1.00	26	0.154
656	A	6	3	1.00	22	0.136
657	A	7	4	1.00	26	0.154
658	A	8	4	1.00	26	0.154
659	A	4	4	1.00	26	0.154
660	A	3	3	1.00	22	0.136
661	A	4	4	1.00	26	0.154
662	A	6	6	1.00	26	0.231
663	A	6	6	1.00	22	0.273
664	A	7	7	1.00	26	0.269
665	A	2	1	1.00	26	0.038
666	A	2	1	1.00	26	0.038
667	A	2	1	1.00	26	0.038
668	A	2	1	1.00	26	0.038
669	A	2	1	1.00	26	0.038
670	A	2	1	1.00	26	0.038
671	A	2	1	1.00	26	0.038
672	A	3	2	1.00	28	0.071
673	A	3	2	1.00	28	0.071
674	A	3	2	1.00	28	0.071
675	A	3	2	1.00	28	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
676	A	3	2	1.00	28	0.071
677	A	3	2	1.00	28	0.071
678	A	3	2	1.00	28	0.071
679	A	3	2	1.00	28	0.071
680	A	3	2	1.00	28	0.071
681	A	3	2	1.00	28	0.071
682	A	3	2	1.00	28	0.071
683	A	3	2	1.00	28	0.071
684	A	3	2	1.00	28	0.071
685	A	3	2	1.00	28	0.071
686	A	14	10	1.00	28	0.357
687	A	13	10	1.00	28	0.357
688	A	13	10	1.00	28	0.357
689	A	12	9	1.00	28	0.321
690	A	12	9	1.00	28	0.321
691	A	12	9	1.00	28	0.321
692	A	12	9	1.00	28	0.321
693	A	13	10	1.00	28	0.357
694	A	13	10	1.00	28	0.357
695	A	14	10	1.00	28	0.357
696	A	16	10	1.00	28	0.357
697	A	15	10	1.00	28	0.357
698	A	15	10	1.00	28	0.357
699	A	14	9	1.00	28	0.321
700	A	14	9	1.00	28	0.321
701	A	14	10	1.00	28	0.357

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
702	A	14	10	1.00	28	0.357
703	A	14	10	1.00	28	0.357
704	A	14	10	1.00	28	0.357
705	A	14	9	1.00	28	0.321
706	A	14	9	1.00	28	0.321
707	A	15	10	1.00	28	0.357
708	A	15	10	1.00	28	0.357
709	A	16	10	1.00	28	0.357
710	A	18	10	1.00	28	0.357
711	A	17	10	1.00	28	0.357
712	A	17	10	1.00	28	0.357
713	A	16	9	1.00	28	0.321
714	A	16	9	1.00	28	0.321
715	A	16	10	1.00	28	0.357
716	A	16	10	1.00	28	0.357
717	A	16	10	1.00	28	0.357
718	A	16	10	1.00	28	0.357
719	A	16	10	1.00	28	0.357
720	A	16	10	1.00	28	0.357
721	A	16	10	1.00	28	0.357
722	A	16	10	1.00	28	0.357
723	A	16	9	1.00	28	0.321
724	A	16	9	1.00	28	0.321
725	A	17	10	1.00	28	0.357
726	A	17	10	1.00	28	0.357
727	A	18	10	1.00	28	0.357

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
728	A	3	2	1.00	30	0.067
729	A	3	2	1.00	30	0.067
730	A	3	2	1.00	30	0.067
731	A	3	2	1.00	30	0.067
732	A	3	2	1.00	30	0.067
733	A	3	2	1.00	30	0.067
734	A	3	2	1.00	30	0.067
735	A	3	2	1.00	30	0.067
736	A	3	2	1.00	30	0.067
737	A	3	2	1.00	30	0.067
738	A	3	2	1.00	30	0.067
739	A	3	2	1.00	30	0.067
740	A	3	2	1.00	30	0.067
741	A	3	2	1.00	30	0.067
742	A	3	2	1.00	30	0.067
743	A	3	2	1.00	30	0.067
744	A	3	2	1.00	30	0.067
745	A	3	2	1.00	30	0.067
746	A	3	2	1.00	30	0.067
747	A	3	2	1.00	30	0.067
748	A	3	2	1.00	30	0.067
749	A	13	9	1.00	30	0.300
750	A	12	9	1.00	30	0.300
751	A	12	9	1.00	30	0.300
752	A	11	8	1.00	30	0.267
753	A	11	8	1.00	30	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
754	A	12	9	1.00	30	0.300
755	A	12	9	1.00	30	0.300
756	A	13	9	1.00	30	0.300
757	A	15	10	1.00	30	0.333
758	A	14	10	1.00	30	0.333
759	A	14	10	1.00	30	0.333
760	A	13	9	1.00	30	0.300
761	A	13	9	1.00	30	0.300
762	A	13	10	1.00	30	0.333
763	A	13	10	1.00	30	0.333
764	A	13	9	1.00	30	0.300
765	A	13	9	1.00	30	0.300
766	A	14	10	1.00	30	0.333
767	A	14	10	1.00	30	0.333
768	A	15	10	1.00	30	0.333
769	A	17	10	1.00	30	0.333
770	A	16	10	1.00	30	0.333
771	A	16	10	1.00	30	0.333
772	A	15	9	1.00	30	0.300
773	A	15	9	1.00	30	0.300
774	A	15	10	1.00	30	0.333
775	A	15	10	1.00	30	0.333
776	A	15	10	1.00	30	0.333
777	A	15	10	1.00	30	0.333
778	A	15	10	1.00	30	0.333
779	A	15	10	1.00	30	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
780	A	15	9	1.00	30	0.300
781	A	15	9	1.00	30	0.300
782	A	16	10	1.00	30	0.333
783	A	16	10	1.00	30	0.333
784	A	17	10	1.00	30	0.333
785	A	3	2	1.00	26	0.077
786	A	3	2	1.00	26	0.077
787	A	2	1	1.00	24	0.042
788	A	2	2	1.00	26	0.077
789	A	2	2	1.00	26	0.077
790	A	2	2	1.00	26	0.077
791	A	3	2	1.00	28	0.071
792	A	3	2	1.00	28	0.071
793	A	3	2	1.00	28	0.071
794	A	2	2	1.00	28	0.071
795	A	2	2	1.00	28	0.071
796	A	2	2	1.00	28	0.071
797	A	2	2	1.04	26	0.077
798	A	4	3	1.00	24	0.125
799	A	4	3	1.00	24	0.125
800	A	4	3	1.00	24	0.125
801	A	2	2	1.00	22	0.091
802	A	3	3	1.00	24	0.125
803	A	3	3	1.00	24	0.125
804	A	2	2	1.00	24	0.083
805	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
806	A	2	2	1.00	20	0.100
807	A	2	2	1.00	24	0.083
808	A	2	2	1.00	24	0.083
809	A	2	2	1.00	28	0.071
810	A	2	2	1.00	28	0.071
811	A	2	2	1.00	28	0.071
812	A	2	2	1.00	28	0.071
813	A	2	2	1.00	28	0.071
814	A	2	1	1.00	16	0.062
815	A	2	1	1.00	14	0.071
816	A	1	0	1.00	12	0.000
817	A	2	1	1.00	16	0.062
818	A	2	1	1.00	16	0.062
819	A	2	1	1.00	16	0.062
820	A	2	1	1.00	16	0.062
821	A	2	1	1.00	16	0.062
822	A	2	1	1.00	16	0.062
823	A	2	1	1.00	16	0.062
824	A	2	1	1.00	16	0.062
825	A	2	1	1.00	18	0.056
826	A	3	2	1.00	16	0.125
827	A	2	1	1.00	14	0.071
828	A	3	2	1.00	18	0.111
829	A	2	1	1.00	18	0.056
830	A	3	2	1.00	18	0.111
831	A	2	1	1.00	18	0.056

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
832	A	3	2	1.00	18	0.111
833	A	2	1	1.00	18	0.056
834	A	3	2	1.00	18	0.111
835	A	2	1	1.00	18	0.056
836	A	3	2	1.00	18	0.111
837	A	2	1	1.00	18	0.056
838	A	3	2	1.00	18	0.111
839	A	2	1	1.00	18	0.056
840	A	3	2	1.00	18	0.111
841	A	2	1	1.00	18	0.056
842	A	3	2	1.00	16	0.125
843	A	2	1	1.00	14	0.071
844	A	3	2	1.00	18	0.111
845	A	2	1	1.00	18	0.056
846	A	3	2	1.00	18	0.111
847	A	2	1	1.00	18	0.056
848	A	7	6	1.00	18	0.333
849	A	6	6	1.00	18	0.333
850	A	5	5	1.00	18	0.278
851	A	3	3	1.00	16	0.188
852	A	7	7	1.00	18	0.389
853	A	8	7	1.00	18	0.389
854	A	8	7	1.00	18	0.389
855	A	5	4	1.00	18	0.222
856	A	4	3	1.00	18	0.167
857	A	3	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
858	A	3	2	1.00	14	0.143
859	A	4	3	1.00	18	0.167
860	A	5	4	1.00	18	0.222
861	A	7	7	1.00	18	0.389
862	A	4	4	1.00	18	0.222
863	A	4	4	1.00	18	0.222
864	A	4	4	1.00	16	0.250
865	A	8	7	1.00	18	0.389
866	A	8	7	1.00	18	0.389
867	A	6	4	1.00	18	0.222
868	A	5	4	1.00	18	0.222
869	A	4	3	1.00	18	0.167
870	A	4	3	1.00	18	0.167
871	A	4	3	1.00	14	0.214
872	A	5	4	1.00	18	0.222
873	A	8	8	1.00	18	0.444
874	A	5	4	1.00	18	0.222
875	A	5	5	1.00	18	0.278
876	A	5	5	1.00	18	0.278
877	A	5	5	1.00	18	0.278
878	A	5	4	1.00	16	0.250
879	A	9	8	1.00	18	0.444
880	A	9	8	1.00	18	0.444
881	A	7	5	1.00	18	0.278
882	A	6	5	1.00	18	0.278
883	A	5	4	1.00	18	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
884	A	5	4	1.00	18	0.222
885	A	5	4	1.00	18	0.222
886	A	5	4	1.00	14	0.286
887	A	6	5	1.00	18	0.278
888	A	6	6	1.00	19	0.316
889	A	5	5	1.00	19	0.263
890	A	3	3	1.00	17	0.176
891	A	7	7	1.00	19	0.368
892	A	8	7	1.00	19	0.368
893	A	4	3	1.00	19	0.158
894	A	3	2	1.00	19	0.105
895	A	3	2	1.00	15	0.133
896	A	4	3	1.00	19	0.158
897	A	6	6	1.00	22	0.273
898	A	5	5	1.00	22	0.227
899	A	3	3	1.00	20	0.150
900	A	7	7	1.00	22	0.318
901	A	8	7	1.00	22	0.318
902	A	4	3	1.00	22	0.136
903	A	3	2	1.00	22	0.091
904	A	3	2	1.00	18	0.111
905	A	4	3	1.00	22	0.136
906	A	6	6	1.00	20	0.300
907	A	5	5	1.00	20	0.250
908	A	3	3	1.00	18	0.167
909	A	7	7	1.00	20	0.350

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
910	A	8	7	1.00	20	0.350
911	A	10	6	1.00	20	0.300
912	A	9	5	1.00	20	0.250
913	A	9	5	1.00	16	0.312
914	A	10	6	1.00	20	0.300
915	A	3	3	1.00	12	0.250
916	A	3	3	1.00	14	0.214
917	A	3	2	1.00	16	0.125
918	A	9	6	1.00	16	0.375
919	A	9	6	1.00	16	0.375
920	A	6	6	1.00	20	0.300
921	A	6	6	1.00	20	0.300
922	A	5	5	1.00	20	0.250
923	A	4	4	1.00	18	0.222
924	A	7	6	1.00	20	0.300
925	A	7	6	1.00	20	0.300
926	A	4	4	1.00	20	0.200
927	A	5	5	1.00	20	0.250
928	A	6	6	1.00	20	0.300
929	A	7	7	1.00	20	0.350
930	A	5	5	1.00	20	0.250
931	A	4	4	1.00	20	0.200
932	A	4	4	1.00	16	0.250
933	A	4	4	1.00	20	0.200
934	A	5	5	1.00	20	0.250
935	A	6	5	1.00	20	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
936	A	7	6	1.00	20	0.300
937	A	7	6	1.00	20	0.300
938	A	6	5	1.00	20	0.250
939	A	5	4	1.00	18	0.222
940	A	8	7	1.00	20	0.350
941	A	8	7	1.00	20	0.350
942	A	8	7	1.00	20	0.350
943	A	8	7	1.00	20	0.350
944	A	5	4	1.00	20	0.200
945	A	6	5	1.00	20	0.250
946	A	7	6	1.00	20	0.300
947	A	6	6	1.00	20	0.300
948	A	5	5	1.00	20	0.250
949	A	5	5	1.00	16	0.312
950	A	5	5	1.00	20	0.250
951	A	5	5	1.00	20	0.250
952	A	6	6	1.00	20	0.300
953	A	7	6	1.00	20	0.300
954	A	5	5	1.00	16	0.312
955	A	5	5	1.00	20	0.250
956	A	5	5	1.00	20	0.250
957	A	4	4	1.00	20	0.200
958	A	3	3	1.00	18	0.167
959	A	3	3	1.00	20	0.150
960	A	4	4	1.00	20	0.200
961	A	5	5	1.00	20	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
962	A	6	6	1.00	20	0.300
963	A	4	4	1.00	20	0.200
964	A	3	3	1.00	20	0.150
965	A	1	1	1.00	16	0.062
966	A	5	5	1.00	20	0.250
967	A	5	5	1.00	20	0.250
968	A	5	5	1.00	21	0.238
969	A	5	5	1.00	21	0.238
970	A	4	4	1.00	21	0.190
971	A	3	3	1.00	19	0.158
972	A	3	3	1.00	22	0.136
973	A	4	4	1.00	22	0.182
974	A	5	5	1.00	22	0.227
975	A	6	6	1.00	22	0.273
976	A	5	5	1.00	21	0.238
977	A	4	4	1.00	21	0.190
978	A	2	2	1.00	17	0.118
979	A	6	6	1.00	21	0.286
980	A	6	6	1.00	21	0.286
981	A	6	6	1.00	20	0.300
982	A	5	5	1.00	20	0.250
983	A	5	5	1.00	20	0.250
984	A	2	2	1.00	20	0.100
985	A	2	2	1.00	18	0.111
986	A	5	5	1.00	20	0.250
987	A	5	5	1.00	20	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
988	A	6	6	1.00	20	0.300
989	A	5	5	1.00	20	0.250
990	A	4	4	1.00	20	0.200
991	A	4	4	1.00	20	0.200
992	A	4	4	1.00	16	0.250
993	A	5	5	1.00	20	0.250
994	A	3	3	1.00	28	0.107
995	A	5	5	1.00	28	0.179
996	A	2	2	1.00	28	0.071
997	A	4	4	1.00	26	0.154
998	A	3	3	1.00	24	0.125
999	A	2	2	1.00	28	0.071
1000	A	4	4	1.00	28	0.143
1001	A	3	3	1.00	28	0.107
1002	A	5	4	1.00	28	0.143
1003	A	3	3	1.00	29	0.103
1004	A	2	2	1.00	29	0.069
1005	A	4	4	1.00	29	0.138
1006	A	4	4	1.00	27	0.148
1007	A	2	2	1.00	25	0.080
1008	A	4	4	1.00	29	0.138
1009	A	5	5	1.00	29	0.172
1010	A	2	2	1.00	29	0.069
1011	A	3	3	1.00	29	0.103
1012	A	5	4	1.00	29	0.138
1013	A	4	3	1.00	29	0.103

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1014	A	4	4	1.00	29	0.138
1015	A	2	2	1.00	27	0.074
1016	A	3	3	1.00	25	0.120
1017	A	4	4	1.00	29	0.138
1018	A	2	2	1.00	29	0.069
1019	A	5	5	1.00	29	0.172
1020	A	3	3	1.00	29	0.103
1021	A	3	3	1.00	23	0.130
1022	A	3	3	1.00	23	0.130
1023	A	3	3	1.00	23	0.130
1024	A	3	3	1.00	21	0.143
1025	A	3	3	1.00	19	0.158
1026	A	3	3	1.00	23	0.130
1027	A	3	3	1.00	23	0.130
1028	A	3	3	1.00	23	0.130
1029	A	3	3	1.00	23	0.130
1030	A	3	3	1.00	24	0.125
1031	A	3	3	1.00	24	0.125
1032	A	3	3	1.00	24	0.125
1033	A	3	3	1.00	22	0.136
1034	A	2	2	1.00	20	0.100
1035	A	3	3	1.00	24	0.125
1036	A	3	3	1.00	24	0.125
1037	A	3	3	1.00	24	0.125
1038	A	3	3	1.00	24	0.125
1039	A	2	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1040	A	2	2	1.00	16	0.125
1041	A	2	1	1.00	18	0.056
1042	A	2	1	1.00	18	0.056
1043	A	2	1	1.00	18	0.056
1044	A	2	1	1.00	18	0.056
1045	A	2	1	1.00	18	0.056
1046	A	2	1	1.00	18	0.056
1047	A	2	1	1.00	18	0.056
1048	A	2	1	1.00	20	0.050
1049	A	2	1	1.00	20	0.050
1050	A	2	1	1.00	20	0.050
1051	A	2	1	1.00	20	0.050
1052	A	2	1	1.00	20	0.050
1053	A	2	1	1.00	20	0.050
1054	A	2	1	1.00	20	0.050
1055	A	2	1	1.00	20	0.050
1056	A	2	1	1.00	20	0.050
1057	A	2	1	1.00	20	0.050
1058	A	2	1	1.00	20	0.050
1059	A	2	1	1.00	20	0.050
1060	A	2	1	1.00	20	0.050
1061	A	2	1	1.00	20	0.050
1062	A	9	6	1.00	20	0.300
1063	A	9	6	1.00	20	0.300
1064	A	8	5	1.00	20	0.250
1065	A	8	5	1.00	20	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1066	A	8	5	1.00	20	0.250
1067	A	8	5	1.00	20	0.250
1068	A	9	6	1.00	20	0.300
1069	A	9	6	1.00	20	0.300
1070	A	10	7	1.00	20	0.350
1071	A	10	7	1.00	20	0.350
1072	A	10	7	1.00	20	0.350
1073	A	9	6	1.00	20	0.300
1074	A	9	6	1.00	20	0.300
1075	A	9	6	1.00	20	0.300
1076	A	9	6	1.00	20	0.300
1077	A	9	6	1.00	20	0.300
1078	A	9	6	1.00	20	0.300
1079	A	10	7	1.00	20	0.350
1080	A	11	8	1.00	20	0.400
1081	A	10	7	1.00	20	0.350
1082	A	10	7	1.00	20	0.350
1083	A	10	7	1.00	20	0.350
1084	A	10	7	1.00	20	0.350
1085	A	10	7	1.00	20	0.350
1086	A	10	7	1.00	20	0.350
1087	A	10	7	1.00	20	0.350
1088	A	10	7	1.00	20	0.350
1089	A	2	2	1.00	24	0.083
1090	A	2	2	1.00	24	0.083
1091	A	2	2	1.00	24	0.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1092	A	2	2	1.00	24	0.083
1093	A	2	2	1.00	24	0.083
1094	A	2	2	1.00	24	0.083
1095	A	2	2	1.00	24	0.083
1096	A	2	2	1.00	24	0.083
1097	A	2	2	1.00	24	0.083
1098	A	2	2	1.00	24	0.083
1099	A	2	2	1.00	24	0.083
1100	A	2	2	1.00	24	0.083
1101	A	2	2	1.00	24	0.083
1102	A	2	2	1.00	24	0.083
1103	A	2	2	1.00	24	0.083
1104	A	2	2	1.00	24	0.083
1105	A	2	1	1.00	20	0.050
1106	A	2	1	1.00	20	0.050
1107	A	2	1	1.00	18	0.056
1108	A	3	2	1.00	20	0.100
1109	A	4	3	1.00	20	0.150
1110	A	2	2	1.00	22	0.091
1111	A	2	2	1.00	22	0.091
1112	A	2	2	1.00	22	0.091
1113	A	2	2	1.00	22	0.091
1114	A	2	2	1.00	20	0.100
1115	A	4	4	1.00	18	0.222
1116	A	4	4	1.00	18	0.222
1117	A	3	3	1.00	18	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1118	A	2	2	1.00	16	0.125
1119	A	3	3	1.00	18	0.167
1120	A	3	3	1.00	18	0.167
1121	A	3	3	1.00	18	0.167
1122	A	2	2	1.00	18	0.111
1123	A	2	2	1.00	18	0.111
1124	A	2	2	1.00	14	0.143
1125	A	2	2	1.00	18	0.111
1126	A	2	2	1.00	18	0.111

Chapter 3

Listing of integrals

3.1 $\int \left(a^2+2abx^2+b^2x^4\right)^{3/4} dx$

Optimal. Leaf size=128

$$\frac{3ax\left(a^2+2abx^2+b^2x^4\right)^{3/4}}{8\left(a+bx^2\right)}+\frac{1}{4}x\left(a^2+2abx^2+b^2x^4\right)^{3/4}+\frac{3\sqrt{a}\left(a^2+2abx^2+b^2x^4\right)^{3/4}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

```
[Out] 1/4*x*(b^2*x^4+2*a*b*x^2+a^2)^(3/4)+3/8*a*x*(b^2*x^4+2*a*b*x^2+a^2)^(3/4)/(
b*x^2+a)+3/8*(b^2*x^4+2*a*b*x^2+a^2)^(3/4)*arcsinh(x*b^(1/2)/a^(1/2))*a^(1/
2)/(1+b*x^2/a)^(3/2)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1089, 195, 215}

$$\frac{3ax\left(a^2+2abx^2+b^2x^4\right)^{3/4}}{8\left(a+bx^2\right)}+\frac{1}{4}x\left(a^2+2abx^2+b^2x^4\right)^{3/4}+\frac{3\sqrt{a}\left(a^2+2abx^2+b^2x^4\right)^{3/4}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4),x]
```

```
[Out] (x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4))/4 + (3*a*x*(a^2 + 2*a*b*x^2 + b^2*x^4
)^(3/4))/(8*(a + b*x^2)) + (3*Sqrt[a]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4)*Arc
Sinh[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[b]*(1 + (b*x^2)/a)^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/4} dx = \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/4}\int \left(1+\frac{bx^2}{a}\right)^{3/2} dx}{\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{1}{4}x\left(a^2+2abx^2+b^2x^4\right)^{3/4} + \frac{\left(3\left(a^2+2abx^2+b^2x^4\right)^{3/4}\right)\int \sqrt{1+\frac{bx^2}{a}}\, dx}{4\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{1}{4}x\left(a^2+2abx^2+b^2x^4\right)^{3/4} + \frac{3ax\left(a^2+2abx^2+b^2x^4\right)^{3/4}}{8\left(a+bx^2\right)} + \frac{\left(3\left(a^2+2abx^2+b^2x^4\right)\right.}{8\left(1+\frac{bx^2}{a}\right.}$$

$$= \frac{1}{4}x\left(a^2+2abx^2+b^2x^4\right)^{3/4} + \frac{3ax\left(a^2+2abx^2+b^2x^4\right)^{3/4}}{8\left(a+bx^2\right)} + \frac{3\sqrt{a}\left(a^2+2abx^2+b^2\right.}{8\sqrt{b}\left(1+\right.}$$

Mathematica [A] time = 0.08, size = 97, normalized size = 0.76

$$\frac{\left(\left(a+bx^2\right)^2\right)^{3/4}\left(3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+\sqrt{b}\,x\left(5a+2bx^2\right)\sqrt{\frac{bx^2}{a}+1}\right)}{8\sqrt{b}\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4),x]
```

```
[Out] (((a + b*x^2)^2)^(3/4)*(Sqrt[b]*x*(5*a + 2*b*x^2)*Sqrt[1 + (b*x^2)/a] + 3*a
^(3/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]]))/(8*Sqrt[b]*(a + b*x^2)*Sqrt[1 + (b*x^
2)/a])
```

fricas [A] time = 0.73, size = 177, normalized size = 1.38

$$\left[\frac{3\,a^2\sqrt{b}\,\log\left(-2\,bx^2-2\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}\sqrt{b}\,x-a\right)+2\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}\left(2\,b^2x^3+5\,abx\right)}{16\,b},-\frac{3\,a^2\sqrt{-b}\,a}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqr
t(b)*x - a) + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*(2*b^2*x^3 + 5*a*b*x))/b,
 -1/8*(3*a^2*sqrt(-b)*arctan((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(-b)*x/(
b*x^2 + a)) - (b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*(2*b^2*x^3 + 5*a*b*x))/b]
```

giac [A] time = 0.44, size = 87, normalized size = 0.68

$$-\frac{\frac{3\,a^3\arctan\left(\frac{\sqrt{-\frac{bx^2+a}{x^2}}}{\sqrt{b}}\right)}{\sqrt{b}}+\frac{\left(5\,a^3\left(b+\frac{a}{x^2}\right)\sqrt{-\frac{bx^2+a}{x^2}}-3\,a^3b\sqrt{-\frac{bx^2+a}{x^2}}\right)x^4}{a^2}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="giac")
```

```
[Out] -1/8*(3*a^3*arctan(sqrt(-(b*x^2 + a)/x^2)/sqrt(b))/sqrt(b) + (5*a^3*(b + a/
x^2)*sqrt(-(b*x^2 + a)/x^2) - 3*a^3*b*sqrt(-(b*x^2 + a)/x^2))*x^4/a^2)/a
```

maple [A] time = 0.03, size = 77, normalized size = 0.60

$$\frac{3\sqrt{b\,x^{2}+a}\;a^{2}\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{8\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{1}{4}}\sqrt{b}}+\frac{\left(2b\,x^{2}+5a\right)\left(b\,x^{2}+a\right)x}{8\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/4),x)
```

```
[Out] 1/8*x*(2*b*x^2+5*a)*(b*x^2+a)/((b*x^2+a)^2)^(1/4)+3/8*a^2*ln(x*b^(1/2)+(b*x
^2+a)^(1/2))/b^(1/2)/((b*x^2+a)^2)^(1/4)*(b*x^2+a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}\right)^{3/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/4),x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(3/4), x)
```

3.2 $\int \sqrt[4]{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=91

$$\frac{1}{2}x\sqrt[4]{a^2+2abx^2+b^2x^4}+\frac{\sqrt{a}\,\sqrt[4]{a^2+2abx^2+b^2x^4}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}\sqrt{\frac{bx^2}{a}+1}}$$

```
[Out] 1/2*x*(b^2*x^4+2*a*b*x^2+a^2)^(1/4)+1/2*(b^2*x^4+2*a*b*x^2+a^2)^(1/4)*arcsi
nh(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(1/2)/(1+b*x^2/a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1089, 195, 215}

$$\frac{1}{2}x\sqrt[4]{a^2+2abx^2+b^2x^4}+\frac{\sqrt{a}\,\sqrt[4]{a^2+2abx^2+b^2x^4}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4),x]
```

```
[Out] (x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4))/2 + (Sqrt[a]*(a^2 + 2*a*b*x^2 + b^2*x
^4)^(1/4)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[b]*Sqrt[1 + (b*x^2)/a])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
```

```
*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \sqrt[4]{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt[4]{a^2+2abx^2+b^2x^4}\int \sqrt{1+\frac{bx^2}{a}}\,dx}{\sqrt{1+\frac{bx^2}{a}}}$$

$$= \frac{1}{2}x\sqrt[4]{a^2+2abx^2+b^2x^4} + \frac{\sqrt[4]{a^2+2abx^2+b^2x^4}\int \frac{1}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{2\sqrt{1+\frac{bx^2}{a}}}$$

$$= \frac{1}{2}x\sqrt[4]{a^2+2abx^2+b^2x^4} + \frac{\sqrt{a}\sqrt[4]{a^2+2abx^2+b^2x^4}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}\sqrt{1+\frac{bx^2}{a}}}$$

Mathematica [A] time = 0.04, size = 59, normalized size = 0.65

$$\frac{1}{2}\sqrt[4]{\left(a+bx^2\right)^2}\left(\frac{a\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{\sqrt{b}\sqrt{a+bx^2}}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4),x]
[Out] (((a + b*x^2)^2)^(1/4)*(x + (a*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(Sqrt[b]
*Sqrt[a + b*x^2])))/2
```

fricas [A] time = 0.97, size = 147, normalized size = 1.62

$$\left[\frac{a\sqrt{b}\log\left(-2\,bx^2-2\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}\sqrt{b}\,x-a\right)+2\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}bx}{4\,b}, -\frac{a\sqrt{-b}\arctan\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right.}{bx^2+a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="fricas")
```

```
[Out] [1/4*(a*sqrt(b)*log(-2*b*x^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(b)*
x - a) + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*b*x)/b, -1/2*(a*sqrt(-b)*arcta
n((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(-b)*x/(b*x^2 + a)) - (b^2*x^4 + 2*
a*b*x^2 + a^2)^(1/4)*b*x)/b]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4), x)
```

maple [A] time = 0.01, size = 58, normalized size = 0.64

$$\frac{\left(\left(b\,x^2+a\right)^2\right)^{\frac{1}{4}} a \ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b\,x^2+a}\;\sqrt{b}}+\frac{\left(\left(b\,x^2+a\right)^2\right)^{\frac{1}{4}} x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(1/4),x)
```

```
[Out] 1/2*x*((b*x^2+a)^2)^(1/4)+1/2*a*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)*((b*x
^2+a)^2)^(1/4)/(b*x^2+a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4),x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[4]{a^2 + 2abx^2 + b^2x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(1/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(1/4), x)
```

3.3 $\int \frac{1}{\sqrt[4]{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=60

$$\frac{\sqrt{a}\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

```
[Out] arcsinh(x*b^(1/2)/a^(1/2))*a^(1/2)*(1+b*x^2/a)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2
)^(1/4)/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1089, 215}

$$\frac{\sqrt{a}\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1/4),x]
```

```
[Out] (Sqrt[a]*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(a^2 +
2*a*b*x^2 + b^2*x^4)^(1/4))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a^2+2abx^2+b^2x^4}}\,dx = \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{1}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

Mathematica [**A**] time = 0.01, size = 49, normalized size = 0.82

$$\frac{\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\,\sqrt[4]{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1/4),x]
```

```
[Out] (Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*((a + b*x^2
)^2)^(1/4))
```

fricas [**A**] time = 1.13, size = 90, normalized size = 1.50

$$\left[\frac{\log\left(-2\,bx^2-2\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}\sqrt{b}\,x-a\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}\sqrt{-b}\,x}{bx^2+a}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*b*x^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(b)*x - a)/sqrt
(b), -sqrt(-b)*arctan((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(-b)*x/(b*x^2 +
 a))/b]
```

giac [**A**] time = 0.22, size = 24, normalized size = 0.40

$$-\frac{\arctan\left(\frac{\sqrt{-\frac{bx^2+a}{x^2}}}{\sqrt{b}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="giac")
[Out] -arctan(sqrt(-(b*x^2 + a)/x^2)/sqrt(b))/sqrt(b)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^4+2ab\,x^2+a^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x)
[Out] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="maxima")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4),x)
[Out] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a^2+2abx^2+b^2x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(1/4),x)
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-1/4), x)
```

3.4 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{3/4}}\,dx$

Optimal. Leaf size=34

$$\frac{x\left(a+bx^2\right)}{a\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

```
[Out] x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(3/4)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1089, 191}

$$\frac{x\left(a+bx^2\right)}{a\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/4),x]
```

```
[Out] (x*(a + b*x^2))/(a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{3/4}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{3/2}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/2}}\,dx}{\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$
$$= \frac{x\left(a+bx^2\right)}{a\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.74

$$\frac{x\left(a+bx^2\right)}{a\left(\left(a+bx^2\right)^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/4),x]
[Out] (x*(a + b*x^2))/(a*((a + b*x^2)^2)^(3/4))
```

fricas [A] time = 0.86, size = 34, normalized size = 1.00

$$\frac{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}x}{abx^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="fricas")
[Out] (b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*x/(a*b*x^2 + a^2)
```

giac [A] time = 0.26, size = 19, normalized size = 0.56

$$-\frac{1}{a\sqrt{-\frac{bx^2+a}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="giac")
[Out] -1/(a*sqrt(-(b*x^2 + a)/x^2))
```

maple [A] time = 0.00, size = 33, normalized size = 0.97

$$\frac{\left(b\,x^{2}+a\right)x}{\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{3}{4}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/4),x)
```

```
[Out] x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(3/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-3/4), x)
```

mupad [B] time = 4.14, size = 34, normalized size = 1.00

$$\frac{x\left(a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}\right)^{1/4}}{a\,\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/4),x)
```

```
[Out] (x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4))/(a*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(3/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-3/4), x)
```

3.5 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{5/4}}\,dx$

Optimal. Leaf size=68

$$\frac{2x}{3a^2\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{x\left(a+bx^2\right)}{3a\left(a^2+2abx^2+b^2x^4\right)^{5/4}}$$

```
[Out] 1/3*x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(5/4)+2/3*x/a^2/(b^2*x^4+2*a*b*x^
2+a^2)^(1/4)
```

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1089, 192, 191}

$$\frac{x}{3a\left(a+bx^2\right)\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{2x}{3a^2\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/4),x]
```

```
[Out] (2*x)/(3*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4)) + x/(3*a*(a + b*x^2)*(a^2 +
 2*a*b*x^2 + b^2*x^4)^(1/4))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{5/4}}\,dx = \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/2}}\,dx}{a^2\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{x}{3a\left(a+bx^2\right)\sqrt[4]{a^2+2abx^2+b^2x^4}} + \frac{\left(2\sqrt{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/2}}\,dx}{3a^2\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2x}{3a^2\sqrt[4]{a^2+2abx^2+b^2x^4}} + \frac{x}{3a\left(a+bx^2\right)\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.59

$$\frac{x\left(3a+2bx^2\right)}{3a^2\left(a+bx^2\right)\sqrt[4]{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/4),x]
[Out] (x*(3*a + 2*b*x^2))/(3*a^2*(a + b*x^2)*((a + b*x^2)^2)^(1/4))
```

fricas [A] time = 1.14, size = 58, normalized size = 0.85

$$\frac{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}\left(2\,bx^3+3\,ax\right)}{3\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/4),x, algorithm="fricas")
[Out] 1/3*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*(2*b*x^3 + 3*a*x)/(a^2*b^2*x^4 + 2*a^
3*b*x^2 + a^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-5/4), x)
```

maple [A] time = 0.00, size = 44, normalized size = 0.65

$$\frac{\left(b\,x^{2}+a\right)\left(2b\,x^{2}+3a\right)x}{3\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{4}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/4),x)
```

```
[Out] 1/3*(b*x^2+a)*x*(2*b*x^2+3*a)/a^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-5/4), x)
```

mupad [B] time = 4.20, size = 45, normalized size = 0.66

$$\frac{x\,\left(2\,b\,x^2+3\,a\right)\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/4}}{3\,a^2\,{\left(b\,x^2+a\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/4),x)
```

```
[Out] (x*(3*a + 2*b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/4))/(3*a^2*(a + b*x^2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(5/4),x)
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-5/4), x)
```

3.6 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{7/4}}\,dx$

Optimal. Leaf size=105

$$\frac{4x}{15a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}}+\frac{x\left(a+bx^2\right)}{5a\left(a^2+2abx^2+b^2x^4\right)^{7/4}}+\frac{8x\left(a+bx^2\right)}{15a^3\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

```
[Out] 1/5*x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(7/4)+4/15*x/a^2/(b^2*x^4+2*a*b*x
^2+a^2)^(3/4)+8/15*x*(b*x^2+a)/a^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/4)
```

Rubi [A] time = 0.02, antiderivative size = 107, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1089, 192, 191}

$$\frac{8x\left(a+bx^2\right)}{15a^3\left(a^2+2abx^2+b^2x^4\right)^{3/4}}+\frac{x}{5a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/4}}+\frac{4x}{15a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-7/4),x]
```

```
[Out] (4*x)/(15*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4)) + x/(5*a*(a + b*x^2)*(a^2
+ 2*a*b*x^2 + b^2*x^4)^(3/4)) + (8*x*(a + b*x^2))/(15*a^3*(a^2 + 2*a*b*x^2
+ b^2*x^4)^(3/4))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
```

```
*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{7/4}}\,dx=\frac{\left(1+\frac{bx^2}{a}\right)^{3/2}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{7/2}}\,dx}{a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

$$=\frac{x}{5a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/4}}+\frac{\left(4\left(1+\frac{bx^2}{a}\right)^{3/2}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/2}}\,dx}{5a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

$$=\frac{4x}{15a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}}+\frac{x}{5a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/4}}+\frac{\left(8\left(1+\frac{bx^2}{a}\right.\right.}{15a^2\left(a^2+\right.}$$

$$=\frac{4x}{15a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}}+\frac{x}{5a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/4}}+\frac{8x\left(\right.}{15a^3\left(a^2+2\right.}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.49

$$\frac{x\left(15a^2+20abx^2+8b^2x^4\right)}{15a^3\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-7/4),x]
```

```
[Out] (x*(15*a^2 + 20*a*b*x^2 + 8*b^2*x^4))/(15*a^3*(a + b*x^2)*((a + b*x^2)^2)^(
3/4))
```

fricas [A] time = 0.56, size = 80, normalized size = 0.76

$$\frac{\left(8\,b^2x^5+20\,abx^3+15\,a^2x\right)\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}}{15\left(a^3b^3x^6+3\,a^4b^2x^4+3\,a^5bx^2+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(7/4),x, algorithm="fricas")
```

```
[Out] 1/15*(8*b^2*x^5 + 20*a*b*x^3 + 15*a^2*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)/
(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-7/4), x)
```

maple [A] time = 0.00, size = 55, normalized size = 0.52

$$\frac{\left(b\,x^2 + a\right)\left(8b^2x^4 + 20ab\,x^2 + 15a^2\right)x}{15\left(b^2x^4 + 2ab\,x^2 + a^2\right)^{\frac{7}{4}}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(7/4),x)
```

```
[Out] 1/15*(b*x^2+a)*x*(8*b^2*x^4+20*a*b*x^2+15*a^2)/a^3/(b^2*x^4+2*a*b*x^2+a^2)^
(7/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-7/4), x)
```

mupad [B] time = 4.21, size = 56, normalized size = 0.53

$$\frac{x\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{1/4}\,\left(15\,a^2 + 20\,a\,b\,x^2 + 8\,b^2\,x^4\right)}{15\,a^3\left(b\,x^2 + a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(7/4),x)
```

```
[Out] (x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4)*(15*a^2 + 8*b^2*x^4 + 20*a*b*x^2))/(15
*a^3*(a + b*x^2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2 + 2abx^2 + b^2x^4\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(7/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-7/4), x)
```

3.7 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{9/4}}\,dx$

Optimal. Leaf size=135

$$\frac{6x}{35a^2\left(a^2+2abx^2+b^2x^4\right)^{5/4}}+\frac{x\left(a+bx^2\right)}{7a\left(a^2+2abx^2+b^2x^4\right)^{9/4}}+\frac{16x}{35a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{8x\left(a+bx^2\right)}{35a^3\left(a^2+2abx^2+b^2x^4\right)^{5/4}}$$

```
[Out] 1/7*x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(9/4)+6/35*x/a^2/(b^2*x^4+2*a*b*x
^2+a^2)^(5/4)+8/35*x*(b*x^2+a)/a^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/4)+16/35*x/a^
4/(b^2*x^4+2*a*b*x^2+a^2)^(1/4)
```

Rubi [A] time = 0.04, antiderivative size = 148, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1089, 192, 191}

$$\frac{8x}{35a^3\left(a+bx^2\right)\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{6x}{35a^2\left(a+bx^2\right)^2\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{x}{7a\left(a+bx^2\right)^3\sqrt[4]{a^2+2abx^2+b^2x^4}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-9/4),x]
```

```
[Out] (16*x)/(35*a^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4)) + x/(7*a*(a + b*x^2)^3*(a
^2 + 2*a*b*x^2 + b^2*x^4)^(1/4)) + (6*x)/(35*a^2*(a + b*x^2)^2*(a^2 + 2*a*b
*x^2 + b^2*x^4)^(1/4)) + (8*x)/(35*a^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x
^4)^(1/4))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
```

```
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] && !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{9/4}}\,dx=\frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{9/2}}\,dx}{a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

$$=\frac{x}{7a\left(a+bx^2\right)^3\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{\left(6\sqrt{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{7/2}}\,dx}{7a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

$$=\frac{x}{7a\left(a+bx^2\right)^3\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{6x}{35a^2\left(a+bx^2\right)^2\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{(24}{35}$$

$$=\frac{x}{7a\left(a+bx^2\right)^3\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{6x}{35a^2\left(a+bx^2\right)^2\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{}{35a}$$

$$=\frac{16x}{35a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{x}{7a\left(a+bx^2\right)^3\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{}{35a^2\left(a+bx^2\right)^2}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.46

$$\frac{x\left(35a^3+70a^2bx^2+56ab^2x^4+16b^3x^6\right)}{35a^4\left(a+bx^2\right)^3\sqrt[4]{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-9/4),x]
```

```
[Out] (x*(35*a^3 + 70*a^2*b*x^2 + 56*a*b^2*x^4 + 16*b^3*x^6))/(35*a^4*(a + b*x^2)
^3*((a + b*x^2)^2)^(1/4))
```

fricas [A] time = 0.82, size = 102, normalized size = 0.76

$$\frac{\left(16\,b^{3}x^{7}+56\,ab^{2}x^{5}+70\,a^{2}bx^{3}+35\,a^{3}x\right)\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{\frac{1}{4}}}{35\left(a^{4}b^{4}x^{8}+4\,a^{5}b^{3}x^{6}+6\,a^{6}b^{2}x^{4}+4\,a^{7}bx^{2}+a^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(9/4),x, algorithm="fricas")
```

```
[Out] 1/35*(16*b^3*x^7 + 56*a*b^2*x^5 + 70*a^2*b*x^3 + 35*a^3*x)*(b^2*x^4 + 2*a*b
*x^2 + a^2)^(1/4)/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^
2 + a^8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-9/4), x)
```

maple [A] time = 0.00, size = 66, normalized size = 0.49

$$\frac{\left(b\,x^{2}+a\right)\left(16b^{3}x^{6}+56b^{2}x^{4}a+70a^{2}b\,x^{2}+35a^{3}\right)x}{35\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{9}{4}}\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(9/4),x)
```

```
[Out] 1/35*(b*x^2+a)*x*(16*b^3*x^6+56*a*b^2*x^4+70*a^2*b*x^2+35*a^3)/a^4/(b^2*x^4
+2*a*b*x^2+a^2)^(9/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-9/4), x)
```

mupad [B] time = 4.13, size = 141, normalized size = 1.04

$$\frac{x\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/4}}{7\,a\left(b\,x^2+a\right)^5}+\frac{6\,x\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/4}}{35\,a^2\left(b\,x^2+a\right)^4}+\frac{8\,x\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/4}}{35\,a^3\left(b\,x^2+a\right)^3}+\frac{16\,x\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right.}{35\,a^4\left(b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(9/4),x)
```

```
[Out] (x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/4))/(7*a*(a + b*x^2)^5) + (6*x*(a^2 + b^2
*x^4 + 2*a*b*x^2)^(3/4))/(35*a^2*(a + b*x^2)^4) + (8*x*(a^2 + b^2*x^4 + 2*a
*b*x^2)^(3/4))/(35*a^3*(a + b*x^2)^3) + (16*x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(
3/4))/(35*a^4*(a + b*x^2)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(9/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-9/4), x)
```

3.8 $\int \frac{1}{a^2+b+2ax^2+x^4}\,dx$

Optimal. Leaf size=299

$$-\frac{\log\left(-\sqrt{2}\,x\sqrt{\sqrt{a^2+b}-a}+\sqrt{a^2+b}+x^2\right)}{4\sqrt{2}\,\sqrt{a^2+b}\,\sqrt{\sqrt{a^2+b}-a}}+\frac{\log\left(\sqrt{2}\,x\sqrt{\sqrt{a^2+b}-a}+\sqrt{a^2+b}+x^2\right)}{4\sqrt{2}\,\sqrt{a^2+b}\,\sqrt{\sqrt{a^2+b}-a}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+b}-a}\cdots}{\sqrt{\sqrt{a^2+b}\cdots}}\right)}{2\sqrt{2}\,\sqrt{a^2+b}\,\sqrt{\sqrt{a^2\cdots}}}$$

```
[Out] -1/8*ln(x^2+(a^2+b)^(1/2)-x*2^(1/2)*(-a+(a^2+b)^(1/2))^(1/2))*2^(1/2)/(a^2+
b)^(1/2)/(-a+(a^2+b)^(1/2))^(1/2)+1/8*ln(x^2+(a^2+b)^(1/2)+x*2^(1/2)*(-a+(a
^2+b)^(1/2))^(1/2))*2^(1/2)/(a^2+b)^(1/2)/(-a+(a^2+b)^(1/2))^(1/2)-1/4*arct
an((-x*2^(1/2)+(-a+(a^2+b)^(1/2))^(1/2))/(a+(a^2+b)^(1/2))^(1/2))*2^(1/2)/(
a^2+b)^(1/2)/(a+(a^2+b)^(1/2))^(1/2)+1/4*arctan((x*2^(1/2)+(-a+(a^2+b)^(1/2
))^(1/2))/(a+(a^2+b)^(1/2))^(1/2))*2^(1/2)/(a^2+b)^(1/2)/(a+(a^2+b)^(1/2))^
(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{\log\left(-\sqrt{2}\,x\sqrt{\sqrt{a^2+b}-a}+\sqrt{a^2+b}+x^2\right)}{4\sqrt{2}\,\sqrt{a^2+b}\,\sqrt{\sqrt{a^2+b}-a}}+\frac{\log\left(\sqrt{2}\,x\sqrt{\sqrt{a^2+b}-a}+\sqrt{a^2+b}+x^2\right)}{4\sqrt{2}\,\sqrt{a^2+b}\,\sqrt{\sqrt{a^2+b}-a}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+b}-a}\cdots}{\sqrt{\sqrt{a^2+b}\cdots}}\right)}{2\sqrt{2}\,\sqrt{a^2+b}\,\sqrt{\sqrt{a^2\cdots}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[-a + Sqrt[a^2 + b]] - Sqrt[2]*x)/Sqrt[a + Sqrt[a^2 + b]]]/(2*
Sqrt[2]*Sqrt[a^2 + b]*Sqrt[a + Sqrt[a^2 + b]]) + ArcTan[(Sqrt[-a + Sqrt[a^2
 + b]] + Sqrt[2]*x)/Sqrt[a + Sqrt[a^2 + b]]]/(2*Sqrt[2]*Sqrt[a^2 + b]*Sqrt[
a + Sqrt[a^2 + b]]) - Log[Sqrt[a^2 + b] - Sqrt[2]*Sqrt[-a + Sqrt[a^2 + b]]*
x + x^2]/(4*Sqrt[2]*Sqrt[a^2 + b]*Sqrt[-a + Sqrt[a^2 + b]]) + Log[Sqrt[a^2
+ b] + Sqrt[2]*Sqrt[-a + Sqrt[a^2 + b]]*x + x^2]/(4*Sqrt[2]*Sqrt[a^2 + b]*S
qrt[-a + Sqrt[a^2 + b]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a^2+b+2ax^2+x^4}\,dx = \frac{\int \frac{\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}-x}{\sqrt{a^2+b}-\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}\,x+x^2}\,dx}{2\sqrt{2}\sqrt{a^2+b}\sqrt{-a+\sqrt{a^2+b}}} + \frac{\int \frac{\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}+x}{\sqrt{a^2+b}+\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}\,x+x^2}\,dx}{2\sqrt{2}\sqrt{a^2+b}\sqrt{-a+\sqrt{a^2+b}}}$$

$$= \frac{\int \frac{1}{\sqrt{a^2+b}-\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}\,x+x^2}\,dx}{4\sqrt{a^2+b}} + \frac{\int \frac{1}{\sqrt{a^2+b}+\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}\,x+x^2}\,dx}{4\sqrt{a^2+b}} - \frac{\int \frac{-\sqrt{2}\sqrt{-a+\sqrt{a^2}\cdots}}{\sqrt{a^2+b}-\sqrt{2}\sqrt{-a+\cdots}}}{4\sqrt{2}\sqrt{a^2+b}\sqrt{-\cdots}}$$

$$= -\frac{\log\left(\sqrt{a^2+b}-\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}\,x+x^2\right)}{4\sqrt{2}\sqrt{a^2+b}\sqrt{-a+\sqrt{a^2+b}}} + \frac{\log\left(\sqrt{a^2+b}+\sqrt{2}\sqrt{-a+\sqrt{a^2+\cdots}}\cdots\right.}{4\sqrt{2}\sqrt{a^2+b}\sqrt{-a+\sqrt{a^2+\cdots}}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-a+\sqrt{a^2+b}}-\sqrt{2}\,x}{\sqrt{a+\sqrt{a^2+b}}}\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{a+\sqrt{a^2+b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{-a+\sqrt{a^2+b}}+\sqrt{2}\,x}{\sqrt{a+\sqrt{a^2+b}}}\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{a+\sqrt{a^2+b}}} - \frac{\log\left(\sqrt{a^2+b}-\sqrt{2}\cdots\right.}{4\sqrt{2}\sqrt{a^2+\cdots}}$$

Mathematica [C] time = 0.04, size = 81, normalized size = 0.27

$$-\frac{i\left(\frac{\tan^{-1}\left(\frac{x}{\sqrt{a-i\sqrt{b}}}\right)}{\sqrt{a-i\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{a+i\sqrt{b}}}\right)}{\sqrt{a+i\sqrt{b}}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] ((-1/2*I)*(ArcTan[x/Sqrt[a - I*Sqrt[b]]]/Sqrt[a - I*Sqrt[b]] - ArcTan[x/Sqr
t[a + I*Sqrt[b]]]/Sqrt[a + I*Sqrt[b]]))/Sqrt[b]
```

fricas [B] time = 0.80, size = 583, normalized size = 1.95

$$\frac{1}{4}\sqrt{\frac{\left(a^2b+b^2\right)\sqrt{-\frac{1}{a^4b+2\,a^2b^2+b^3}}+a}{a^2b+b^2}}\,\log\left(\left(\left(a^3b+ab^2\right)\sqrt{-\frac{1}{a^4b+2\,a^2b^2+b^3}}+b\right)\sqrt{\frac{\left(a^2b+b^2\right)\sqrt{-\frac{1}{a^4b+2\,a^2b^2+b^3}}+\cdots}{a^2b+b^2}}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+b),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + a)/(a^2*b + b^
2))*log(((a^3*b + a*b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + b)*sqrt(((a^2
*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + a)/(a^2*b + b^2)) + x) - 1/4
*sqrt(((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + a)/(a^2*b + b^2))
*log(-((a^3*b + a*b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + b)*sqrt(((a^2*b
 + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + a)/(a^2*b + b^2)) + x) - 1/4*s
qrt(-((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - a)/(a^2*b + b^2))*
log(((a^3*b + a*b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - b)*sqrt(-((a^2*b
+ b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - a)/(a^2*b + b^2)) + x) + 1/4*sq
rt(-((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - a)/(a^2*b + b^2))*l
og(-((a^3*b + a*b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - b)*sqrt(-((a^2*b
+ b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - a)/(a^2*b + b^2)) + x)
```

giac [A] time = 0.16, size = 75, normalized size = 0.25

$$-\frac{\sqrt{a+\sqrt{-b}}\arctan\left(\frac{x}{\sqrt{a+\sqrt{-b}}}\right)}{2\left(a\sqrt{-b}-b\right)}+\frac{\sqrt{a-\sqrt{-b}}\arctan\left(\frac{x}{\sqrt{a-\sqrt{-b}}}\right)}{2\left(a\sqrt{-b}+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+b),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + sqrt(-b))*arctan(x/sqrt(a + sqrt(-b)))/(a*sqrt(-b) - b) + 1/2
*sqrt(a - sqrt(-b))*arctan(x/sqrt(a - sqrt(-b)))/(a*sqrt(-b) + b)
```

maple [B] time = 0.13, size = 1099, normalized size = 3.68

$$\frac{a^4\arctan\left(\frac{2x+\sqrt{-2a+2\sqrt{a^2+b}}}{\sqrt{2a+2\sqrt{a^2+b}}}\right)}{2\left(a^2+b\right)^{\frac{3}{2}}\sqrt{2a+2\sqrt{a^2+b}}\;b}-\frac{a^4\arctan\left(\frac{-2x+\sqrt{-2a+2\sqrt{a^2+b}}}{\sqrt{2a+2\sqrt{a^2+b}}}\right)}{2\left(a^2+b\right)^{\frac{3}{2}}\sqrt{2a+2\sqrt{a^2+b}}\;b}+\frac{\sqrt{-2a+2\sqrt{a^2+b}}\;a^3\ln\left(x^2+\sqrt{-2a+2\sqrt{a^2+}}\right.}{8\left(a^2+b\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2*a*x^2+a^2+b),x)
```

```
[Out] 1/8/b/(a^2+b)*ln(x^2+x*(2*(a^2+b)^(1/2)-2*a)^(1/2)+(a^2+b)^(1/2))*(2*(a^2+b
)^(1/2)-2*a)^(1/2)*a^2+1/8/b/(a^2+b)^(3/2)*ln(x^2+x*(2*(a^2+b)^(1/2)-2*a)^(
1/2)+(a^2+b)^(1/2))*(2*(a^2+b)^(1/2)-2*a)^(1/2)*a^3+1/8/(a^2+b)*ln(x^2+x*(2
*(a^2+b)^(1/2)-2*a)^(1/2)+(a^2+b)^(1/2))*(2*(a^2+b)^(1/2)-2*a)^(1/2)+1/8/(a
^2+b)^(3/2)*ln(x^2+x*(2*(a^2+b)^(1/2)-2*a)^(1/2)+(a^2+b)^(1/2))*(2*(a^2+b)^
(1/2)-2*a)^(1/2)*a-1/2/b/(a^2+b)^(1/2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan((
```

```
2*x+(2*(a^2+b)^(1/2)-2*a)^(1/2))/(2*(a^2+b)^(1/2)+2*a)^(1/2))*a^2+1/2/b/(a^
2+b)^(3/2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+b)^(1/2)-2*a)^(1
/2))/(2*(a^2+b)^(1/2)+2*a)^(1/2))*a^4-1/2/(a^2+b)^(1/2)/(2*(a^2+b)^(1/2)+2*
a)^(1/2)*arctan((2*x+(2*(a^2+b)^(1/2)-2*a)^(1/2))/(2*(a^2+b)^(1/2)+2*a)^(1/
2))+3/2/(a^2+b)^(3/2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+b)^(1
/2)-2*a)^(1/2))/(2*(a^2+b)^(1/2)+2*a)^(1/2))*a^2+b/(a^2+b)^(3/2)/(2*(a^2+b)
^(1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+b)^(1/2)-2*a)^(1/2))/(2*(a^2+b)^(1/2)
+2*a)^(1/2))-1/8/b/(a^2+b)*ln(x*(2*(a^2+b)^(1/2)-2*a)^(1/2)-x^2-(a^2+b)^(1/
2))*(2*(a^2+b)^(1/2)-2*a)^(1/2)*a^2-1/8/b/(a^2+b)^(3/2)*ln(x*(2*(a^2+b)^(1/
2)-2*a)^(1/2)-x^2-(a^2+b)^(1/2))*(2*(a^2+b)^(1/2)-2*a)^(1/2)*a^3-1/8/(a^2+b
)*ln(x*(2*(a^2+b)^(1/2)-2*a)^(1/2)-x^2-(a^2+b)^(1/2))*(2*(a^2+b)^(1/2)-2*a)
^(1/2)-1/8/(a^2+b)^(3/2)*ln(x*(2*(a^2+b)^(1/2)-2*a)^(1/2)-x^2-(a^2+b)^(1/2)
)*(2*(a^2+b)^(1/2)-2*a)^(1/2)*a+1/2/b/(a^2+b)^(1/2)/(2*(a^2+b)^(1/2)+2*a)^(
1/2)*arctan(((2*(a^2+b)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+b)^(1/2)+2*a)^(1/2))*
a^2-1/2/b/(a^2+b)^(3/2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan(((2*(a^2+b)^(1/2
)-2*a)^(1/2)-2*x)/(2*(a^2+b)^(1/2)+2*a)^(1/2))*a^4+1/2/(a^2+b)^(1/2)/(2*(a^
2+b)^(1/2)+2*a)^(1/2)*arctan(((2*(a^2+b)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+b)^(
1/2)+2*a)^(1/2))-3/2/(a^2+b)^(3/2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan(((2*(
a^2+b)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+b)^(1/2)+2*a)^(1/2))*a^2-b/(a^2+b)^(3/
2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan(((2*(a^2+b)^(1/2)-2*a)^(1/2)-2*x)/(2*
(a^2+b)^(1/2)+2*a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 + 2\,ax^2 + a^2 + b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+b),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + 2*a*x^2 + a^2 + b), x)
```

mupad [B] time = 4.38, size = 872, normalized size = 2.92

$$-2\,\mathrm{atanh}\left(\frac{8\,x\,\sqrt{\frac{a\,b}{16\,\left(a^2\,b^2+b^3\right)}-\frac{\sqrt{-b^3}}{16\,\left(a^2\,b^2+b^3\right)}}}{\frac{2\,b\,\sqrt{-b^3}}{a^2\,b^2+b^3}-\frac{2\,a\,b^2}{a^2\,b^2+b^3}}-\frac{8\,a^2\,b^2\,x\,\sqrt{\frac{a\,b}{16\,\left(a^2\,b^2+b^3\right)}-\frac{\sqrt{-b^3}}{16\,\left(a^2\,b^2+b^3\right)}}}{\frac{2\,b^4\,\sqrt{-b^3}}{a^2\,b^2+b^3}-\frac{2\,a^3\,b^4}{a^2\,b^2+b^3}-\frac{2\,a\,b^5}{a^2\,b^2+b^3}+\frac{2\,a^2\,b^3\,\sqrt{-b^3}}{a^2\,b^2+b^3}}+\frac{8\,a\,b\,x\,\sqrt{\frac{a\,b}{16\,\left(a^2\,b^2+b^3\right)}-}}{\frac{2\,b^4\,\sqrt{-b^3}}{a^2\,b^2+b^3}-\frac{2\,a^3\,b^4}{a^2\,b^2+b^3}-\frac{2}{a^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b + 2*a*x^2 + a^2 + x^4),x)
```

```
[Out] - 2*atanh((8*x*((a*b)/(16*(b^3 + a^2*b^2)) - (-b^3)^(1/2)/(16*(b^3 + a^2*b^
2)))^(1/2))/((2*b*(-b^3)^(1/2))/(b^3 + a^2*b^2) - (2*a*b^2)/(b^3 + a^2*b^2)
```

```
) - (8*a^2*b^2*x*((a*b)/(16*(b^3 + a^2*b^2)) - (-b^3)^(1/2)/(16*(b^3 + a^2*
b^2)))^(1/2))/((2*b^4*(-b^3)^(1/2))/(b^3 + a^2*b^2) - (2*a^3*b^4)/(b^3 + a^
2*b^2) - (2*a*b^5)/(b^3 + a^2*b^2) + (2*a^2*b^3*(-b^3)^(1/2))/(b^3 + a^2*b^
2)) + (8*a*b*x*((a*b)/(16*(b^3 + a^2*b^2)) - (-b^3)^(1/2)/(16*(b^3 + a^2*b^
2)))^(1/2)*(-b^3)^(1/2))/((2*b^4*(-b^3)^(1/2))/(b^3 + a^2*b^2) - (2*a^3*b^4
)/(b^3 + a^2*b^2) - (2*a*b^5)/(b^3 + a^2*b^2) + (2*a^2*b^3*(-b^3)^(1/2))/(b
^3 + a^2*b^2)))*((a*b - (-b^3)^(1/2))/(16*(b^3 + a^2*b^2)))^(1/2) - 2*atanh
((8*a^2*b^2*x*((-b^3)^(1/2)/(16*(b^3 + a^2*b^2)) + (a*b)/(16*(b^3 + a^2*b^2
)))^(1/2))/((2*b^4*(-b^3)^(1/2))/(b^3 + a^2*b^2) + (2*a^3*b^4)/(b^3 + a^2*b
^2) + (2*a*b^5)/(b^3 + a^2*b^2) + (2*a^2*b^3*(-b^3)^(1/2))/(b^3 + a^2*b^2))
 - (8*x*((-b^3)^(1/2)/(16*(b^3 + a^2*b^2)) + (a*b)/(16*(b^3 + a^2*b^2)))^(1
/2))/((2*b*(-b^3)^(1/2))/(b^3 + a^2*b^2) + (2*a*b^2)/(b^3 + a^2*b^2)) + (8*
a*b*x*((-b^3)^(1/2)/(16*(b^3 + a^2*b^2)) + (a*b)/(16*(b^3 + a^2*b^2)))^(1/2
)*(-b^3)^(1/2))/((2*b^4*(-b^3)^(1/2))/(b^3 + a^2*b^2) + (2*a^3*b^4)/(b^3 +
a^2*b^2) + (2*a*b^5)/(b^3 + a^2*b^2) + (2*a^2*b^3*(-b^3)^(1/2))/(b^3 + a^2*
b^2)))*((a*b + (-b^3)^(1/2))/(16*(b^3 + a^2*b^2)))^(1/2)
```

sympy [A] time = 0.80, size = 63, normalized size = 0.21

$$\operatorname{RootSum}\left(t^4\left(256a^2b^2+256b^3\right)-32t^2ab+1,\left(t\mapsto t\log\left(64t^3a^3b+64t^3ab^2-4ta^2+4tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+2*a*x**2+a**2+b),x)
```

```
[Out] RootSum(_t**4*(256*a**2*b**2 + 256*b**3) - 32*_t**2*a*b + 1, Lambda(_t, _t*
log(64*_t**3*a**3*b + 64*_t**3*a*b**2 - 4*_t*a**2 + 4*_t*b + x)))
```

3.9 $\int \frac{1}{-1+a^2+2ax^2+x^4}\,dx$

Optimal. Leaf size=47

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt{a+1}}\right)}{2\sqrt{a+1}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1-a}}\right)}{2\sqrt{1-a}}$$

```
[Out] -1/2*arctanh(x/(1-a)^(1/2))/(1-a)^(1/2)-1/2*arctan(x/(1+a)^(1/2))/(1+a)^(1/
2)
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1093, 207, 203}

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt{a+1}}\right)}{2\sqrt{a+1}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1-a}}\right)}{2\sqrt{1-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[x/Sqrt[1 + a]]/(2*Sqrt[1 + a]) - ArcTanh[x/Sqrt[1 - a]]/(2*Sqrt[1 -
 a])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{-1+a^2+2ax^2+x^4}\,dx = \frac{1}{2}\int \frac{1}{-1+a+x^2}\,dx - \frac{1}{2}\int \frac{1}{1+a+x^2}\,dx$$
$$= -\frac{\tan^{-1}\left(\frac{x}{\sqrt{1+a}}\right)}{2\sqrt{1+a}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{1-a}}\right)}{2\sqrt{1-a}}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.91

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{a-1}}\right)}{2\sqrt{a-1}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{a+1}}\right)}{2\sqrt{a+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTan[x/Sqrt[-1 + a]]/(2*Sqrt[-1 + a]) - ArcTan[x/Sqrt[1 + a]]/(2*Sqrt[1 +
 a])
```

fricas [A] time = 0.81, size = 269, normalized size = 5.72

$$\left[-\frac{(a-1)\sqrt{-a-1}\,\log\left(\frac{x^2+2\sqrt{-a-1}\,x-a-1}{x^2+a+1}\right)+(a+1)\sqrt{-a+1}\,\log\left(\frac{x^2-2\sqrt{-a+1}\,x-a+1}{x^2+a-1}\right)}{4\left(a^2-1\right)}, \frac{2\,(a+1)\sqrt{a-1}\,\arctan\left(\frac{x}{\sqrt{a-1}}\right)\ldots}{4\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2-1),x, algorithm="fricas")
```

```
[Out] [-1/4*((a - 1)*sqrt(-a - 1)*log((x^2 + 2*sqrt(-a - 1)*x - a - 1)/(x^2 + a +
 1)) + (a + 1)*sqrt(-a + 1)*log((x^2 - 2*sqrt(-a + 1)*x - a + 1)/(x^2 + a -
 1)))/(a^2 - 1), 1/4*(2*(a + 1)*sqrt(a - 1)*arctan(x/sqrt(a - 1)) - (a - 1)
*sqrt(-a - 1)*log((x^2 + 2*sqrt(-a - 1)*x - a - 1)/(x^2 + a + 1)))/(a^2 - 1
), -1/4*(2*sqrt(a + 1)*(a - 1)*arctan(x/sqrt(a + 1)) + (a + 1)*sqrt(-a + 1)
*log((x^2 - 2*sqrt(-a + 1)*x - a + 1)/(x^2 + a - 1)))/(a^2 - 1), -1/2*(sqrt
(a + 1)*(a - 1)*arctan(x/sqrt(a + 1)) - (a + 1)*sqrt(a - 1)*arctan(x/sqrt(a
 - 1)))/(a^2 - 1)]
```

giac [A] time = 0.15, size = 31, normalized size = 0.66

$$-\frac{\arctan\left(\frac{x}{\sqrt{a+1}}\right)}{2\,\sqrt{a+1}} + \frac{\arctan\left(\frac{x}{\sqrt{a-1}}\right)}{2\,\sqrt{a-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2-1),x, algorithm="giac")
```

```
[Out] -1/2*arctan(x/sqrt(a + 1))/sqrt(a + 1) + 1/2*arctan(x/sqrt(a - 1))/sqrt(a -
 1)
```

maple [A] time = 0.01, size = 32, normalized size = 0.68

$$-\frac{\arctan\left(\frac{x}{\sqrt{a+1}}\right)}{2\sqrt{a+1}}+\frac{\arctan\left(\frac{x}{\sqrt{a-1}}\right)}{2\sqrt{a-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2*a*x^2+a^2-1),x)
```

```
[Out] -1/2*arctan(x/(1+a)^(1/2))/(1+a)^(1/2)+1/2/(a-1)^(1/2)*arctan(x/(a-1)^(1/2)
)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2-1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1.0>0)', see `assume?` for more d
etails)Is a-1.0 positive or negative?
```

mupad [B] time = 0.10, size = 85, normalized size = 1.81

$$\frac{\operatorname{atanh}\left(\frac{2\,x\,\left(\frac{a}{2}-\frac{1}{2}\right)}{\sqrt{1-a}}+\frac{2\,a\,x\,\left(\frac{a}{2}-\frac{1}{2}\right)}{{\left(1-a\right)}^{3/2}}\right)}{2\,\sqrt{1-a}}+\frac{\operatorname{atanh}\left(\frac{2\,x\,\left(\frac{a}{2}+\frac{1}{2}\right)}{\sqrt{-a-1}}+\frac{2\,a\,x\,\left(\frac{a}{2}+\frac{1}{2}\right)}{{\left(-a-1\right)}^{3/2}}\right)}{2\,\sqrt{-a-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a*x^2 + a^2 + x^4 - 1),x)
```

```
[Out] atanh((2*x*(a/2 - 1/2))/(1 - a)^(1/2) + (2*a*x*(a/2 - 1/2))/(1 - a)^(3/2))/
(2*(1 - a)^(1/2)) + atanh((2*x*(a/2 + 1/2))/(- a - 1)^(1/2) + (2*a*x*(a/2 +
 1/2))/(- a - 1)^(3/2))/(2*(- a - 1)^(1/2))
```

sympy [B] time = 0.63, size = 257, normalized size = 5.47

$$\frac{\sqrt{-\frac{1}{a-1}}\log\left(-a^3\left(-\frac{1}{a-1}\right)^{\frac{3}{2}} - a^2\sqrt{-\frac{1}{a-1}} + a\left(-\frac{1}{a-1}\right)^{\frac{3}{2}} + x - \sqrt{-\frac{1}{a-1}}\right)}{4} - \frac{\sqrt{-\frac{1}{a-1}}\log\left(a^3\left(-\frac{1}{a-1}\right)^{\frac{3}{2}} + a^2\sqrt{-\frac{1}{a-1}} - a\left(- \cdots\right.\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+2*a*x**2+a**2-1),x)
```

```
[Out] sqrt(-1/(a - 1))*log(-a**3*(-1/(a - 1))**(3/2) - a**2*sqrt(-1/(a - 1)) + a*
(-1/(a - 1))**(3/2) + x - sqrt(-1/(a - 1)))/4 - sqrt(-1/(a - 1))*log(a**3*(
-1/(a - 1))**(3/2) + a**2*sqrt(-1/(a - 1)) - a*(-1/(a - 1))**(3/2) + x + sq
rt(-1/(a - 1)))/4 + sqrt(-1/(a + 1))*log(-a**3*(-1/(a + 1))**(3/2) - a**2*s
qrt(-1/(a + 1)) + a*(-1/(a + 1))**(3/2) + x - sqrt(-1/(a + 1)))/4 - sqrt(-1
/(a + 1))*log(a**3*(-1/(a + 1))**(3/2) + a**2*sqrt(-1/(a + 1)) - a*(-1/(a +
 1))**(3/2) + x + sqrt(-1/(a + 1)))/4
```

3.10 $\int \frac{1}{1+a^2+2ax^2+x^4}\,dx$

Optimal. Leaf size=299

$$-\frac{\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\,x+\sqrt{a^2+1}+x^2\right)}{4\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}+\frac{\log\left(\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\,x+\sqrt{a^2+1}+x^2\right)}{4\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+1}-a-}}{\sqrt{\sqrt{a^2+1}-}}\right.}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2}}}$$

```
[Out] -1/8*ln(x^2+(a^2+1)^(1/2)-x*2^(1/2)*(-a+(a^2+1)^(1/2))^(1/2))*2^(1/2)/(a^2+
1)^(1/2)/(-a+(a^2+1)^(1/2))^(1/2)+1/8*ln(x^2+(a^2+1)^(1/2)+x*2^(1/2)*(-a+(a
^2+1)^(1/2))^(1/2))*2^(1/2)/(a^2+1)^(1/2)/(-a+(a^2+1)^(1/2))^(1/2)-1/4*arct
an((-x*2^(1/2)+(-a+(a^2+1)^(1/2))^(1/2))/(a+(a^2+1)^(1/2))^(1/2))*2^(1/2)/(
a^2+1)^(1/2)/(a+(a^2+1)^(1/2))^(1/2)+1/4*arctan((x*2^(1/2)+(-a+(a^2+1)^(1/2
))^(1/2))/(a+(a^2+1)^(1/2))^(1/2))*2^(1/2)/(a^2+1)^(1/2)/(a+(a^2+1)^(1/2))^
(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\,x+\sqrt{a^2+1}+x^2\right)}{4\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}+\frac{\log\left(\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\,x+\sqrt{a^2+1}+x^2\right)}{4\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+1}-a-}}{\sqrt{\sqrt{a^2+1}-}}\right.}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[-a + Sqrt[1 + a^2]] - Sqrt[2]*x)/Sqrt[a + Sqrt[1 + a^2]]]/(2*
Sqrt[2]*Sqrt[1 + a^2]*Sqrt[a + Sqrt[1 + a^2]]) + ArcTan[(Sqrt[-a + Sqrt[1 +
 a^2]] + Sqrt[2]*x)/Sqrt[a + Sqrt[1 + a^2]]]/(2*Sqrt[2]*Sqrt[1 + a^2]*Sqrt[
a + Sqrt[1 + a^2]]) - Log[Sqrt[1 + a^2] - Sqrt[2]*Sqrt[-a + Sqrt[1 + a^2]]*
x + x^2]/(4*Sqrt[2]*Sqrt[1 + a^2]*Sqrt[-a + Sqrt[1 + a^2]]) + Log[Sqrt[1 +
a^2] + Sqrt[2]*Sqrt[-a + Sqrt[1 + a^2]]*x + x^2]/(4*Sqrt[2]*Sqrt[1 + a^2]*S
qrt[-a + Sqrt[1 + a^2]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{1+a^2+2ax^2+x^4}\,dx = \frac{\int \frac{\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}-x}{\sqrt{1+a^2}-\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\,x+x^2}\,dx}{2\sqrt{2}\sqrt{1+a^2}\sqrt{-a+\sqrt{1+a^2}}} + \frac{\int \frac{\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}+x}{\sqrt{1+a^2}+\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\,x+x^2}\,dx}{2\sqrt{2}\sqrt{1+a^2}\sqrt{-a+\sqrt{1+a^2}}}$$

$$= \frac{\int \frac{1}{\sqrt{1+a^2}-\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\,x+x^2}\,dx}{4\sqrt{1+a^2}} + \frac{\int \frac{1}{\sqrt{1+a^2}+\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\,x+x^2}\,dx}{4\sqrt{1+a^2}} - \frac{\int \frac{-\sqrt{2}\sqrt{-a+\sqrt{1-}}}{\sqrt{1+a^2}-\sqrt{2}\sqrt{-a+}}}{4\sqrt{2}\sqrt{1+a^2}\sqrt{-}}$$

$$= -\frac{\log\left(\sqrt{1+a^2}-\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\,x+x^2\right)}{4\sqrt{2}\sqrt{1+a^2}\sqrt{-a+\sqrt{1+a^2}}} + \frac{\log\left(\sqrt{1+a^2}+\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\right.}{4\sqrt{2}\sqrt{1+a^2}\sqrt{-a+\sqrt{1+a}}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-a+\sqrt{1+a^2}}-\sqrt{2}\,x}{\sqrt{a+\sqrt{1+a^2}}}\right)}{2\sqrt{2}\sqrt{1+a^2}\sqrt{a+\sqrt{1+a^2}}} + \frac{\tan^{-1}\left(\frac{\sqrt{-a+\sqrt{1+a^2}}+\sqrt{2}\,x}{\sqrt{a+\sqrt{1+a^2}}}\right)}{2\sqrt{2}\sqrt{1+a^2}\sqrt{a+\sqrt{1+a^2}}} - \frac{\log\left(\sqrt{1+a^2}-\sqrt{2}\right.}{4\sqrt{2}\sqrt{1+a}}$$

Mathematica [C] time = 0.03, size = 52, normalized size = 0.17

$$-\frac{1}{2}i\left(\frac{\tan^{-1}\left(\frac{x}{\sqrt{a-i}}\right)}{\sqrt{a-i}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{a+i}}\right)}{\sqrt{a+i}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] (-1/2*I)*(ArcTan[x/Sqrt[-I + a]]/Sqrt[-I + a] - ArcTan[x/Sqrt[I + a]]/Sqrt[
I + a])
```

fricas [B] time = 0.94, size = 613, normalized size = 2.05

$$\frac{\sqrt{2\,a^2-\frac{2\left(a^3+a\right)}{\sqrt{a^2+1}}+2}\left(\frac{a}{\sqrt{a^2+1}}+1\right)\log\left(x^2+\frac{\sqrt{2\,a^2-\frac{2\left(a^3+a\right)}{\sqrt{a^2+1}}+2}\,x}{\left(a^2+1\right)^{\frac{1}{4}}}+\sqrt{a^2+1}\right)}{8\left(a^2+1\right)^{\frac{1}{4}}} - \frac{\sqrt{2\,a^2-\frac{2\left(a^3+a\right)}{\sqrt{a^2+1}}+2}\left(\frac{a}{\sqrt{a^2+1}}+1\right)\log\left(\right.}{8\left(a^2+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+1),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*(a/sqrt(a^2 + 1) + 1)*log(x
^2 + sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*x/(a^2 + 1)^(1/4) + sqrt(a
^2 + 1))/(a^2 + 1)^(1/4) - 1/8*sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*
(a/sqrt(a^2 + 1) + 1)*log(x^2 - sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)
*x/(a^2 + 1)^(1/4) + sqrt(a^2 + 1))/(a^2 + 1)^(1/4) - 1/2*sqrt(2*a^2 - 2*(a
^3 + a)/sqrt(a^2 + 1) + 2)*(a^2 + 1)^(1/4)*arctan(-sqrt(a^4 + 2*a^2 + 1)*sq
rt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*x/(a^2 + 1)^(5/4) + (a^3 + a)/sqr
t(a^4 + 2*a^2 + 1) + sqrt(a^4 + 2*a^2 + 1)*sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^
2 + 1) + 2)*sqrt(x^2 + sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*x/(a^2 +
 1)^(1/4) + sqrt(a^2 + 1))/(a^2 + 1)^(5/4) - sqrt(a^4 + 2*a^2 + 1)/sqrt(a^2
 + 1))/sqrt(a^4 + 2*a^2 + 1) - 1/2*sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) +
 2)*(a^2 + 1)^(1/4)*arctan(-sqrt(a^4 + 2*a^2 + 1)*sqrt(2*a^2 - 2*(a^3 + a)/
sqrt(a^2 + 1) + 2)*x/(a^2 + 1)^(5/4) - (a^3 + a)/sqrt(a^4 + 2*a^2 + 1) + sq
rt(a^4 + 2*a^2 + 1)*sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*sqrt(x^2 -
sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*x/(a^2 + 1)^(1/4) + sqrt(a^2 +
1))/(a^2 + 1)^(5/4) + sqrt(a^4 + 2*a^2 + 1)/sqrt(a^2 + 1))/sqrt(a^4 + 2*a^2
 + 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+1),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.11, size = 1073, normalized size = 3.59

$$\frac{a^{4} \arctan\left(\frac{2x+\sqrt{-2a+2\sqrt{a^{2}+1}}}{\sqrt{2a+2\sqrt{a^{2}+1}}}\right)}{2\left(a^{2}+1\right)^{\frac{3}{2}} \sqrt{2a+2\sqrt{a^{2}+1}}}-\frac{a^{4} \arctan\left(\frac{-2x+\sqrt{-2a+2\sqrt{a^{2}+1}}}{\sqrt{2a+2\sqrt{a^{2}+1}}}\right)}{2\left(a^{2}+1\right)^{\frac{3}{2}} \sqrt{2a+2\sqrt{a^{2}+1}}}+\frac{\sqrt{-2a+2\sqrt{a^{2}+1}}\; a^{3} \ln\left(x^{2}+\sqrt{-2a+2\sqrt{a^{2}+1}}\right.}{8\left(a^{2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2*a*x^2+a^2+1),x)
```

```
[Out] -1/8/(a^2+1)*ln(x*(2*(a^2+1)^(1/2)-2*a)^(1/2)-x^2-(a^2+1)^(1/2))*(2*(a^2+1)
^(1/2)-2*a)^(1/2)*a^2-1/8/(a^2+1)^(3/2)*ln(x*(2*(a^2+1)^(1/2)-2*a)^(1/2)-x^
2-(a^2+1)^(1/2))*(2*(a^2+1)^(1/2)-2*a)^(1/2)*a^3-1/8/(a^2+1)*ln(x*(2*(a^2+1
```

```
)^(1/2)-2*a)^(1/2)-x^2-(a^2+1)^(1/2))*(2*(a^2+1)^(1/2)-2*a)^(1/2)-1/8/(a^2+
1)^(3/2)*ln(x*(2*(a^2+1)^(1/2)-2*a)^(1/2)-x^2-(a^2+1)^(1/2))*(2*(a^2+1)^(1/
2)-2*a)^(1/2)*a+1/2/(a^2+1)^(1/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*arctan(((2*(a
^2+1)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+1)^(1/2)+2*a)^(1/2))*a^2-1/2/(a^2+1)^(3
/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*arctan(((2*(a^2+1)^(1/2)-2*a)^(1/2)-2*x)/(2
*(a^2+1)^(1/2)+2*a)^(1/2))*a^4+1/2/(a^2+1)^(1/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2
)*arctan(((2*(a^2+1)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+1)^(1/2)+2*a)^(1/2))-3/2
/(a^2+1)^(3/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*arctan(((2*(a^2+1)^(1/2)-2*a)^(1
/2)-2*x)/(2*(a^2+1)^(1/2)+2*a)^(1/2))*a^2-1/(a^2+1)^(3/2)/(2*(a^2+1)^(1/2)+
2*a)^(1/2)*arctan(((2*(a^2+1)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+1)^(1/2)+2*a)^(
1/2))+1/8/(a^2+1)*ln(x^2+x*(2*(a^2+1)^(1/2)-2*a)^(1/2)+(a^2+1)^(1/2))*(2*(a
^2+1)^(1/2)-2*a)^(1/2)*a^2+1/8/(a^2+1)^(3/2)*ln(x^2+x*(2*(a^2+1)^(1/2)-2*a)
^(1/2)+(a^2+1)^(1/2))*(2*(a^2+1)^(1/2)-2*a)^(1/2)*a^3+1/8/(a^2+1)*ln(x^2+x*
(2*(a^2+1)^(1/2)-2*a)^(1/2)+(a^2+1)^(1/2))*(2*(a^2+1)^(1/2)-2*a)^(1/2)+1/8/
(a^2+1)^(3/2)*ln(x^2+x*(2*(a^2+1)^(1/2)-2*a)^(1/2)+(a^2+1)^(1/2))*(2*(a^2+1
)^(1/2)-2*a)^(1/2)*a-1/2/(a^2+1)^(1/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*arctan((
2*x+(2*(a^2+1)^(1/2)-2*a)^(1/2))/(2*(a^2+1)^(1/2)+2*a)^(1/2))*a^2+1/2/(a^2+
1)^(3/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+1)^(1/2)-2*a)^(1/2
))/(2*(a^2+1)^(1/2)+2*a)^(1/2))*a^4-1/2/(a^2+1)^(1/2)/(2*(a^2+1)^(1/2)+2*a)
^(1/2)*arctan((2*x+(2*(a^2+1)^(1/2)-2*a)^(1/2))/(2*(a^2+1)^(1/2)+2*a)^(1/2)
)+3/2/(a^2+1)^(3/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+1)^(1/2
)-2*a)^(1/2))/(2*(a^2+1)^(1/2)+2*a)^(1/2))*a^2+1/(a^2+1)^(3/2)/(2*(a^2+1)^(
1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+1)^(1/2)-2*a)^(1/2))/(2*(a^2+1)^(1/2)+2
*a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 + 2\,ax^2 + a^2 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + 2*a*x^2 + a^2 + 1), x)
```

mupad [B] time = 4.36, size = 469, normalized size = 1.57

$$-\frac{\operatorname{atanh}\left(-\frac{2\,x\,\sqrt{\frac{a}{a^2+1}+\frac{1\mathrm{i}}{a^2+1}}}{\frac{2\,a}{a^2+1}+\frac{2\mathrm{i}}{a^2+1}}+\frac{a\,x\,\sqrt{\frac{a}{a^2+1}+\frac{1\mathrm{i}}{a^2+1}}\,2\mathrm{i}}{\frac{2\,a}{a^2+1}+\frac{2\,a^3}{a^2+1}+\frac{2\mathrm{i}}{a^2+1}+\frac{a^2\,2\mathrm{i}}{a^2+1}}+\frac{2\,a^2\,x\,\sqrt{\frac{a}{a^2+1}+\frac{1\mathrm{i}}{a^2+1}}}{\frac{2\,a}{a^2+1}+\frac{2\,a^3}{a^2+1}+\frac{2\mathrm{i}}{a^2+1}+\frac{a^2\,2\mathrm{i}}{a^2+1}}\right)\sqrt{\frac{a+1\mathrm{i}}{a^2+1}}}{2}+2\operatorname{atanh}\left(\frac{8\,x\,\sqrt{\frac{a}{16\,a^2+16}-\frac{1\mathrm{i}}{16\,a^2+16}}}{\frac{32\,a}{16\,a^2+16}-\frac{32\mathrm{i}}{16\,a^2+16}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a*x^2 + a^2 + x^4 + 1),x)
```

```
[Out] 2*atanh((8*x*(a/(16*a^2 + 16) - 1i/(16*a^2 + 16))^(1/2))/((32*a)/(16*a^2 +
16) - 32i/(16*a^2 + 16)) + (a*x*(a/(16*a^2 + 16) - 1i/(16*a^2 + 16))^(1/2)*
128i)/((512*a)/(16*a^2 + 16) - 512i/(16*a^2 + 16) - (a^2*512i)/(16*a^2 + 16
) + (512*a^3)/(16*a^2 + 16)) - (128*a^2*x*(a/(16*a^2 + 16) - 1i/(16*a^2 + 1
6))^(1/2))/((512*a)/(16*a^2 + 16) - 512i/(16*a^2 + 16) - (a^2*512i)/(16*a^2
 + 16) + (512*a^3)/(16*a^2 + 16)))*((a - 1i)/(16*a^2 + 16))^(1/2) - (atanh(
(a*x*(a/(a^2 + 1) + 1i/(a^2 + 1))^(1/2)*2i)/((2*a)/(a^2 + 1) + 2i/(a^2 + 1)
 + (a^2*2i)/(a^2 + 1) + (2*a^3)/(a^2 + 1)) - (2*x*(a/(a^2 + 1) + 1i/(a^2 +
1))^(1/2))/((2*a)/(a^2 + 1) + 2i/(a^2 + 1)) + (2*a^2*x*(a/(a^2 + 1) + 1i/(a
^2 + 1))^(1/2))/((2*a)/(a^2 + 1) + 2i/(a^2 + 1) + (a^2*2i)/(a^2 + 1) + (2*a
^3)/(a^2 + 1)))*((a + 1i)/(a^2 + 1))^(1/2))/2
```

sympy [A] time = 0.58, size = 48, normalized size = 0.16

$$\operatorname{RootSum}\left(t^4\left(256a^2+256\right)-32t^2a+1,\left(t\mapsto t\log\left(64t^3a^3+64t^3a-4ta^2+4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+2*a*x**2+a**2+1),x)
```

```
[Out] RootSum(_t**4*(256*a**2 + 256) - 32*_t**2*a + 1, Lambda(_t, _t*log(64*_t**3
*a**3 + 64*_t**3*a - 4*_t*a**2 + 4*_t + x)))
```

3.11 $\int \frac{1}{4-5x^2+x^4} \, dx$

Optimal. Leaf size=17

$$\frac{1}{3}\tanh^{-1}(x)-\frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right)$$

```
[Out] -1/6*arctanh(1/2*x)+1/3*arctanh(x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1093, 207}

$$\frac{1}{3}\tanh^{-1}(x)-\frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 5*x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTanh[x/2]/6 + ArcTanh[x]/3
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4-5x^2+x^4} \, dx &= \frac{1}{3}\int \frac{1}{-4+x^2} \, dx-\frac{1}{3}\int \frac{1}{-1+x^2} \, dx\\ &= -\frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}\tanh^{-1}(x)\end{aligned}$$

Mathematica [B] time = 0.01, size = 37, normalized size = 2.18

$$-\frac{1}{6}\log(1-x)+\frac{1}{12}\log(2-x)+\frac{1}{6}\log(x+1)-\frac{1}{12}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 5*x^2 + x^4)^(-1),x]
[Out] -1/6*Log[1 - x] + Log[2 - x]/12 + Log[1 + x]/6 - Log[2 + x]/12
```

fricas [B] time = 0.85, size = 25, normalized size = 1.47

$$-\frac{1}{12}\,\log\left(x+2\right)+\frac{1}{6}\,\log\left(x+1\right)-\frac{1}{6}\,\log\left(x-1\right)+\frac{1}{12}\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-5*x^2+4),x, algorithm="fricas")
[Out] -1/12*log(x + 2) + 1/6*log(x + 1) - 1/6*log(x - 1) + 1/12*log(x - 2)
```

giac [B] time = 0.20, size = 29, normalized size = 1.71

$$-\frac{1}{12}\,\log\left(|x+2|\right)+\frac{1}{6}\,\log\left(|x+1|\right)-\frac{1}{6}\,\log\left(|x-1|\right)+\frac{1}{12}\,\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-5*x^2+4),x, algorithm="giac")
[Out] -1/12*log(abs(x + 2)) + 1/6*log(abs(x + 1)) - 1/6*log(abs(x - 1)) + 1/12*lo
g(abs(x - 2))
```

maple [B] time = 0.01, size = 26, normalized size = 1.53

$$\frac{\ln\left(x+1\right)}{6}-\frac{\ln\left(x+2\right)}{12}+\frac{\ln\left(x-2\right)}{12}-\frac{\ln\left(x-1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-5*x^2+4),x)
[Out] -1/12*ln(x+2)+1/6*ln(1+x)+1/12*ln(x-2)-1/6*ln(x-1)
```

maxima [B] time = 1.37, size = 25, normalized size = 1.47

$$-\frac{1}{12}\,\log\left(x+2\right)+\frac{1}{6}\,\log\left(x+1\right)-\frac{1}{6}\,\log\left(x-1\right)+\frac{1}{12}\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-5*x^2+4),x, algorithm="maxima")
[Out] -1/12*log(x + 2) + 1/6*log(x + 1) - 1/6*log(x - 1) + 1/12*log(x - 2)
```

mupad [B] time = 0.04, size = 11, normalized size = 0.65

$$\frac{\operatorname{atanh}(x)}{3} - \frac{\operatorname{atanh}\left(\frac{x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 - 5*x^2 + 4),x)
```

```
[Out] atanh(x)/3 - atanh(x/2)/6
```

sympy [B] time = 0.18, size = 26, normalized size = 1.53

$$\frac{\log(x-2)}{12} - \frac{\log(x-1)}{6} + \frac{\log(x+1)}{6} - \frac{\log(x+2)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-5*x**2+4),x)
```

```
[Out] log(x - 2)/12 - log(x - 1)/6 + log(x + 1)/6 - log(x + 2)/12
```

3.12 $\int \frac{1}{3+4x^2+x^4}\,dx$

Optimal. Leaf size=24

$$\frac{1}{2}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] 1/2*arctan(x)-1/6*arctan(1/3*x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1093, 203}

$$\frac{1}{2}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTan[x]/2 - ArcTan[x/Sqrt[3]]/(2*Sqrt[3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{3+4x^2+x^4}\,dx &= \frac{1}{2}\int \frac{1}{1+x^2}\,dx-\frac{1}{2}\int \frac{1}{3+x^2}\,dx\\ &= \frac{1}{2}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x^2 + x^4)^(-1),x]
[Out] ArcTan[x]/2 - ArcTan[x/Sqrt[3]]/(2*Sqrt[3])
```

fricas [A] time = 0.77, size = 17, normalized size = 0.71

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,x\right)+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^2+3),x, algorithm="fricas")
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/2*arctan(x)
```

giac [A] time = 0.17, size = 17, normalized size = 0.71

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,x\right)+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^2+3),x, algorithm="giac")
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/2*arctan(x)
```

maple [A] time = 0.01, size = 18, normalized size = 0.75

$$\frac{\arctan(x)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+4*x^2+3),x)
[Out] 1/2*arctan(x)-1/6*arctan(1/3*x*3^(1/2))*3^(1/2)
```

maxima [A] time = 3.00, size = 17, normalized size = 0.71

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,x\right)+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^2+3),x, algorithm="maxima")
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/2*arctan(x)
```

mupad [B] time = 4.12, size = 17, normalized size = 0.71

$$\frac{\operatorname{atan}(x)}{2} - \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}\,x}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 + x^4 + 3),x)
[Out] atan(x)/2 - (3^(1/2)*atan((3^(1/2)*x)/3))/6
```

sympy [A] time = 0.16, size = 20, normalized size = 0.83

$$\frac{\operatorname{atan}{\left(x \right)}}{2} - \frac{\sqrt{3}\,\operatorname{atan}{\left(\frac{\sqrt{3}\,x}{3} \right)}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+4*x**2+3),x)
[Out] atan(x)/2 - sqrt(3)*atan(sqrt(3)*x/3)/6
```

3.13 $\int \frac{1}{9+5x^2+x^4} dx$

Optimal. Leaf size=67

$$-\frac{1}{12}\log\left(x^2-x+3\right)+\frac{1}{12}\log\left(x^2+x+3\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{11}}\right)}{6\sqrt{11}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{11}}\right)}{6\sqrt{11}}$$

```
[Out] -1/12*ln(x^2-x+3)+1/12*ln(x^2+x+3)-1/66*arctan(1/11*(1-2*x)*11^(1/2))*11^(1
/2)+1/66*arctan(1/11*(1+2*x)*11^(1/2))*11^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{1}{12}\log\left(x^2-x+3\right)+\frac{1}{12}\log\left(x^2+x+3\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{11}}\right)}{6\sqrt{11}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{11}}\right)}{6\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(9 + 5*x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[11]]/(6*Sqrt[11]) + ArcTan[(1 + 2*x)/Sqrt[11]]/(6*Sq
rt[11]) - Log[3 - x + x^2]/12 + Log[3 + x + x^2]/12
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{9+5x^2+x^4}\,dx &= \frac{1}{6}\int \frac{1-x}{3-x+x^2}\,dx+\frac{1}{6}\int \frac{1+x}{3+x+x^2}\,dx\\ &= \frac{1}{12}\int \frac{1}{3-x+x^2}\,dx-\frac{1}{12}\int \frac{-1+2x}{3-x+x^2}\,dx+\frac{1}{12}\int \frac{1}{3+x+x^2}\,dx+\frac{1}{12}\int \frac{1+2x}{3+x+x^2}\,dx\\ &= -\frac{1}{12}\log\left(3-x+x^2\right)+\frac{1}{12}\log\left(3+x+x^2\right)-\frac{1}{6}\mathrm{Subst}\left(\int \frac{1}{-11-x^2}\,dx,x,-1+2x\right)-\frac{1}{6}\mathrm{Sub}\\ &= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{11}}\right)}{6\sqrt{11}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{11}}\right)}{6\sqrt{11}}-\frac{1}{12}\log\left(3-x+x^2\right)+\frac{1}{12}\log\left(3+x+x^2\right)\end{aligned}$$

Mathematica [C] time = 0.07, size = 91, normalized size = 1.36

$$\frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(5+i\sqrt{11}\right)}}\right)}{\sqrt{\frac{11}{2}\left(5+i\sqrt{11}\right)}}-\frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(5-i\sqrt{11}\right)}}\right)}{\sqrt{\frac{11}{2}\left(5-i\sqrt{11}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 5*x^2 + x^4)^(-1),x]
```

```
[Out] ((-I)*ArcTan[x/Sqrt[(5 - I*Sqrt[11])/2]])/Sqrt[(11*(5 - I*Sqrt[11]))/2] + (
I*ArcTan[x/Sqrt[(5 + I*Sqrt[11])/2]])/Sqrt[(11*(5 + I*Sqrt[11]))/2]
```

fricas [A] time = 0.79, size = 53, normalized size = 0.79

$$\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\,(2x+1)\right)+\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\,(2x-1)\right)+\frac{1}{12}\log\left(x^2+x+3\right)-\frac{1}{12}\log\left(x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+5*x^2+9),x, algorithm="fricas")
```

```
[Out] 1/66*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 1)) + 1/66*sqrt(11)*arctan(1/11*s
qrt(11)*(2*x - 1)) + 1/12*log(x^2 + x + 3) - 1/12*log(x^2 - x + 3)
```

giac [A] time = 0.20, size = 53, normalized size = 0.79

$$\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x+1)\right)+\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x-1)\right)+\frac{1}{12}\log\left(x^2+x+3\right)-\frac{1}{12}\log\left(x^2-x+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+5*x^2+9),x, algorithm="giac")
```

```
[Out] 1/66*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 1)) + 1/66*sqrt(11)*arctan(1/11*s
qrt(11)*(2*x - 1)) + 1/12*log(x^2 + x + 3) - 1/12*log(x^2 - x + 3)
```

maple [A] time = 0.00, size = 54, normalized size = 0.81

$$\frac{\sqrt{11}\,\arctan\left(\frac{(2x+1)\sqrt{11}}{11}\right)}{66}+\frac{\sqrt{11}\,\arctan\left(\frac{(2x-1)\sqrt{11}}{11}\right)}{66}-\frac{\ln\left(x^2-x+3\right)}{12}+\frac{\ln\left(x^2+x+3\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+5*x^2+9),x)
```

```
[Out] -1/12*ln(x^2-x+3)+1/66*11^(1/2)*arctan(1/11*(2*x-1)*11^(1/2))+1/12*ln(x^2+x
+3)+1/66*arctan(1/11*(2*x+1)*11^(1/2))*11^(1/2)
```

maxima [A] time = 3.04, size = 53, normalized size = 0.79

$$\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x+1)\right)+\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x-1)\right)+\frac{1}{12}\log\left(x^2+x+3\right)-\frac{1}{12}\log\left(x^2-x+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+5*x^2+9),x, algorithm="maxima")
```

```
[Out] 1/66*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 1)) + 1/66*sqrt(11)*arctan(1/11*s
qrt(11)*(2*x - 1)) + 1/12*log(x^2 + x + 3) - 1/12*log(x^2 - x + 3)
```

mupad [B] time = 4.15, size = 83, normalized size = 1.24

$$\mathrm{atan}\left(\frac{x\,8\mathrm{i}}{27\left(-\frac{5}{9}+\frac{\sqrt{11}\,1\mathrm{i}}{9}\right)}-\frac{2\sqrt{11}\,x}{27\left(-\frac{5}{9}+\frac{\sqrt{11}\,1\mathrm{i}}{9}\right)}\right)\left(\frac{\sqrt{11}}{66}+\frac{1}{6}\mathrm{i}\right)+\mathrm{atan}\left(\frac{x\,8\mathrm{i}}{27\left(\frac{5}{9}+\frac{\sqrt{11}\,1\mathrm{i}}{9}\right)}+\frac{2\sqrt{11}\,x}{27\left(\frac{5}{9}+\frac{\sqrt{11}\,1\mathrm{i}}{9}\right)}\right)\left(\frac{\sqrt{11}}{66}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 + x^4 + 9),x)
```

```
[Out] atan((x*8i)/(27*((11^(1/2)*1i)/9 - 5/9)) - (2*11^(1/2)*x)/(27*((11^(1/2)*1i
)/9 - 5/9)))*(11^(1/2)/66 + 1i/6) + atan((x*8i)/(27*((11^(1/2)*1i)/9 + 5/9)
) + (2*11^(1/2)*x)/(27*((11^(1/2)*1i)/9 + 5/9)))*(11^(1/2)/66 - 1i/6)
```

sympy [A] time = 0.22, size = 70, normalized size = 1.04

$$-\frac{\log\left(x^2-x+3\right)}{12}+\frac{\log\left(x^2+x+3\right)}{12}+\frac{\sqrt{11}\ \operatorname{atan}\left(\frac{2\sqrt{11}\,x}{11}-\frac{\sqrt{11}}{11}\right)}{66}+\frac{\sqrt{11}\ \operatorname{atan}\left(\frac{2\sqrt{11}\,x}{11}+\frac{\sqrt{11}}{11}\right)}{66}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+5*x**2+9),x)
```

```
[Out] -log(x**2 - x + 3)/12 + log(x**2 + x + 3)/12 + sqrt(11)*atan(2*sqrt(11)*x/1
1 - sqrt(11)/11)/66 + sqrt(11)*atan(2*sqrt(11)*x/11 + sqrt(11)/11)/66
```

3.14 $\int \frac{1}{1-x^2+x^4} \, dx$

Optimal. Leaf size=74

$$-\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{2}\tan^{-1}\left(2x+\sqrt{3}\right)$$

```
[Out] 1/2*arctan(2*x-3^(1/2))+1/2*arctan(2*x+3^(1/2))-1/12*ln(1+x^2-x*3^(1/2))*3^
(1/2)+1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{2}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x]/2 + ArcTan[Sqrt[3] + 2*x]/2 - Log[1 - Sqrt[3]*x + x^
2]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1-x^2+x^4}\,dx &= \frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2}\,dx}{2\sqrt{3}}+\frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2}\,dx}{2\sqrt{3}}\\
&= \frac{1}{4}\int \frac{1}{1-\sqrt{3}x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+\sqrt{3}x+x^2}\,dx-\frac{\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}+\frac{\int \frac{\sqrt{3}+2x}{1+\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}\\
&= -\frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,-\sqrt{3}+2x\right)-\frac{1}{2}\text{Su}\\
&= -\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{2}\tan^{-1}\left(\sqrt{3}+2x\right)-\frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 77, normalized size = 1.04

$$\frac{i\left(\sqrt{-1-i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)-\sqrt{-1+i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)\right)}{\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x^2 + x^4)^(-1),x]
```

```
[Out] (I*(Sqrt[-1 - I*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - Sqrt[-1 + I*Sqrt[3
]]*ArcTan[((1 + I*Sqrt[3])*x)/2]))/Sqrt[6]
```

fricas [B] time = 0.91, size = 159, normalized size = 2.15

$$-\frac{1}{6}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)-\frac{1}{6}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sq
rt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) - 1/6*sqrt(6)*
sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(3)
*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) + 1/24*sqrt(6)*sqrt(2)*log
(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - 1/24*sqrt(6)*sqrt(2)*log(-sqrt(6)*sqrt(2)
*x + 2*x^2 + 2)
```

giac [A] time = 0.19, size = 56, normalized size = 0.76

$$\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}\,x+1\right)-\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}\,x+1\right)+\frac{1}{2}\arctan\left(2\,x+\sqrt{3}\right)+\frac{1}{2}\arctan\left(2\,x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12*sqrt(3)*log(x^2 - sqrt(3)*x +
1) + 1/2*arctan(2*x + sqrt(3)) + 1/2*arctan(2*x - sqrt(3))
```

maple [A] time = 0.04, size = 57, normalized size = 0.77

$$\frac{\arctan\left(2x-\sqrt{3}\right)}{2}+\frac{\arctan\left(2x+\sqrt{3}\right)}{2}-\frac{\sqrt{3}\,\ln\left(x^2-\sqrt{3}\,x+1\right)}{12}+\frac{\sqrt{3}\,\ln\left(x^2+\sqrt{3}\,x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-x^2+1),x)
```

```
[Out] 1/2*arctan(2*x-3^(1/2))+1/2*arctan(2*x+3^(1/2))-1/12*ln(1+x^2-3^(1/2)*x)*3^
(1/2)+1/12*ln(1+x^2+3^(1/2)*x)*3^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 - x^2 + 1), x)
```

mupad [B] time = 4.19, size = 47, normalized size = 0.64

$$\mathrm{atan}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\mathrm{atan}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 - x^2 + 1),x)
```

```
[Out] atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/6 - 1/2) + atan((2*x)/(3^(1/2)*1
i + 1))*((3^(1/2)*1i)/6 + 1/2)
```

sympy [A] time = 0.21, size = 63, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}\,x+1\right)}{12}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}\,x+1\right)}{12}+\frac{\operatorname{atan}\left(2x-\sqrt{3}\right)}{2}+\frac{\operatorname{atan}\left(2x+\sqrt{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-x**2+1),x)
```

```
[Out] -sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2 + atan(2*x - sqrt(3))/2 + atan(2*x + sqrt(3))/2
```

3.15 $\int \frac{1}{2+2x^2+x^4} \, dx$

Optimal. Leaf size=176

$$-\frac{\log \left(x^2-\sqrt{2 \left(\sqrt{2}-1\right)} x+\sqrt{2}\right)}{8 \sqrt{\sqrt{2}-1}}+\frac{\log \left(x^2+\sqrt{2 \left(\sqrt{2}-1\right)} x+\sqrt{2}\right)}{8 \sqrt{\sqrt{2}-1}}-\frac{1}{4} \sqrt{\sqrt{2}-1} \tan ^{-1}\left(\frac{\sqrt{2 \left(\sqrt{2}-1\right)}-2 x}{\sqrt{2 \left(1+\sqrt{2}\right)}}\right)+$$

```
[Out] -1/8*ln(x^2+2^(1/2)-x*(-2+2*2^(1/2))^(1/2))/(2^(1/2)-1)^(1/2)+1/8*ln(x^2+2^
(1/2)+x*(-2+2*2^(1/2))^(1/2))/(2^(1/2)-1)^(1/2)-1/4*arctan((-2*x+(-2+2*2^(1
/2))^(1/2))/(2+2*2^(1/2))^(1/2))*(2^(1/2)-1)^(1/2)+1/4*arctan((2*x+(-2+2*2^
(1/2))^(1/2))/(2+2*2^(1/2))^(1/2))*(2^(1/2)-1)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{\log \left(x^2-\sqrt{2 \left(\sqrt{2}-1\right)} x+\sqrt{2}\right)}{8 \sqrt{\sqrt{2}-1}}+\frac{\log \left(x^2+\sqrt{2 \left(\sqrt{2}-1\right)} x+\sqrt{2}\right)}{8 \sqrt{\sqrt{2}-1}}-\frac{1}{4} \sqrt{\sqrt{2}-1} \tan ^{-1}\left(\frac{\sqrt{2 \left(\sqrt{2}-1\right)}-2 x}{\sqrt{2 \left(1+\sqrt{2}\right)}}\right)+$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x^2 + x^4)^(-1),x]
```

```
[Out] -(Sqrt[-1 + Sqrt[2]]*ArcTan[(Sqrt[2*(-1 + Sqrt[2])] - 2*x)/Sqrt[2*(1 + Sqrt
[2])]])/4 + (Sqrt[-1 + Sqrt[2]]*ArcTan[(Sqrt[2*(-1 + Sqrt[2])] + 2*x)/Sqrt[
2*(1 + Sqrt[2])]])/4 - Log[Sqrt[2] - Sqrt[2*(-1 + Sqrt[2])]*x + x^2]/(8*Sqr
t[-1 + Sqrt[2]]) + Log[Sqrt[2] + Sqrt[2*(-1 + Sqrt[2])]*x + x^2]/(8*Sqrt[-1
 + Sqrt[2]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{2+2x^2+x^4}\,dx = \frac{\int \frac{\sqrt{2\left(-1+\sqrt{2}\right)}-x}{\sqrt{2}-\sqrt{2\left(-1+\sqrt{2}\right)}\,x+x^2}\,dx}{4\sqrt{-1+\sqrt{2}}} + \frac{\int \frac{\sqrt{2\left(-1+\sqrt{2}\right)}+x}{\sqrt{2}+\sqrt{2\left(-1+\sqrt{2}\right)}\,x+x^2}\,dx}{4\sqrt{-1+\sqrt{2}}}$$

$$= \frac{\int \frac{1}{\sqrt{2}-\sqrt{2\left(-1+\sqrt{2}\right)}\,x+x^2}\,dx}{4\sqrt{2}} + \frac{\int \frac{1}{\sqrt{2}+\sqrt{2\left(-1+\sqrt{2}\right)}\,x+x^2}\,dx}{4\sqrt{2}} - \frac{\int \frac{-\sqrt{2\left(-1+\sqrt{2}\right)}+2x}{\sqrt{2}-\sqrt{2\left(-1+\sqrt{2}\right)}\,x+x^2}\,dx}{8\sqrt{-1+\sqrt{2}}} + \cdots$$

$$= -\frac{\log\left(\sqrt{2}-\sqrt{2\left(-1+\sqrt{2}\right)}\,x+x^2\right)}{8\sqrt{-1+\sqrt{2}}} + \frac{\log\left(\sqrt{2}+\sqrt{2\left(-1+\sqrt{2}\right)}\,x+x^2\right)}{8\sqrt{-1+\sqrt{2}}} - \cdots$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}-2x}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)}{4\sqrt{1+\sqrt{2}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}+2x}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)}{4\sqrt{1+\sqrt{2}}} - \frac{\log\left(\sqrt{2}-\sqrt{2\left(-1+\sqrt{2}\right)}\,x+x^2\right)}{8\sqrt{-1+\sqrt{2}}} + \cdots$$

Mathematica [C] time = 0.04, size = 41, normalized size = 0.23

$$\frac{1}{4}\left((1-i)^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{1-i}}\right)+(1+i)^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{1+i}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x^2 + x^4)^(-1),x]
```

```
[Out] ((1 - I)^(3/2)*ArcTan[x/Sqrt[1 - I]] + (1 + I)^(3/2)*ArcTan[x/Sqrt[1 + I]])
/4
```

fricas [A] time = 0.82, size = 247, normalized size = 1.40

$$\frac{1}{16}\cdot 2^{\frac{1}{4}}\left(\sqrt{2}+1\right)\sqrt{-2\sqrt{2}+4}\log\left(2^{\frac{3}{4}}x\sqrt{-2\sqrt{2}+4}+2x^{2}+2\sqrt{2}\right)-\frac{1}{16}\cdot 2^{\frac{1}{4}}\left(\sqrt{2}+1\right)\sqrt{-2\sqrt{2}+4}\log\left(-2^{\frac{3}{4}}x\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+2),x, algorithm="fricas")
```

```
[Out] 1/16*2^(1/4)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4)*log(2^(3/4)*x*sqrt(-2*sqrt(
2) + 4) + 2*x^2 + 2*sqrt(2)) - 1/16*2^(1/4)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) +
 4)*log(-2^(3/4)*x*sqrt(-2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2)) - 1/4*2^(1/4)*
sqrt(-2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*x*sqrt(-2*sqrt(2) + 4) + 1/2*2^(1/
4)*sqrt(2^(3/4)*x*sqrt(-2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2))*sqrt(-2*sqrt(2)
 + 4) - sqrt(2) + 1) - 1/4*2^(1/4)*sqrt(-2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)
*x*sqrt(-2*sqrt(2) + 4) + 1/2*2^(1/4)*sqrt(-2^(3/4)*x*sqrt(-2*sqrt(2) + 4)
+ 2*x^2 + 2*sqrt(2))*sqrt(-2*sqrt(2) + 4) + sqrt(2) - 1)
```

giac [A] time = 0.53, size = 143, normalized size = 0.81

$$\frac{1}{4}\sqrt{\sqrt{2}-1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x+2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\right)}{2\sqrt{\sqrt{2}+2}}\right)+\frac{1}{4}\sqrt{\sqrt{2}-1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x-2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\right)}{2\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{\sqrt{2}+1}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(sqrt(2) - 1)*arctan(1/2*2^(3/4)*(2*x + 2^(1/4)*sqrt(-sqrt(2) + 2))
/sqrt(sqrt(2) + 2)) + 1/4*sqrt(sqrt(2) - 1)*arctan(1/2*2^(3/4)*(2*x - 2^(1/
4)*sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) + 1/8*sqrt(sqrt(2) + 1)*log(x^2 +
 2^(1/4)*x*sqrt(-sqrt(2) + 2) + sqrt(2)) - 1/8*sqrt(sqrt(2) + 1)*log(x^2 -
2^(1/4)*x*sqrt(-sqrt(2) + 2) + sqrt(2))
```

maple [B] time = 0.11, size = 386, normalized size = 2.19

$$-\frac{\left(-2+2\sqrt{2}\right)\sqrt{2}\ \arctan\left(\frac{2x-\sqrt{-2+2\sqrt{2}}}{\sqrt{2+2\sqrt{2}}}\right)}{8\sqrt{2+2\sqrt{2}}}-\frac{\left(-2+2\sqrt{2}\right)\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{2}}}{\sqrt{2+2\sqrt{2}}}\right)}{4\sqrt{2+2\sqrt{2}}}+\frac{\sqrt{2}\ \arctan\left(\frac{2x-\sqrt{-2+2\sqrt{2}}}{\sqrt{2+2\sqrt{2}}}\right)}{2\sqrt{2+2\sqrt{2}}}-\frac{\left(-2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2*x^2+2),x)
```

```
[Out] -1/16*ln(x^2+2^(1/2)-x*(-2+2*2^(1/2))^(1/2))*(-2+2*2^(1/2))^(1/2)*2^(1/2)-1
/8*ln(x^2+2^(1/2)-x*(-2+2*2^(1/2))^(1/2))*(-2+2*2^(1/2))^(1/2)-1/8/(2+2*2^(
1/2))^(1/2)*arctan((2*x-(-2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2))*(-2+2*2^
(1/2))*2^(1/2)-1/4/(2+2*2^(1/2))^(1/2)*arctan((2*x-(-2+2*2^(1/2))^(1/2))/(2
+2*2^(1/2))^(1/2))*(-2+2*2^(1/2))+1/2/(2+2*2^(1/2))^(1/2)*arctan((2*x-(-2+2
*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2))*2^(1/2)+1/16*ln(x^2+2^(1/2)+x*(-2+2*2
^(1/2))^(1/2))*(-2+2*2^(1/2))^(1/2)*2^(1/2)+1/8*ln(x^2+2^(1/2)+x*(-2+2*2^(1
/2))^(1/2))*(-2+2*2^(1/2))^(1/2)-1/8/(2+2*2^(1/2))^(1/2)*arctan((2*x+(-2+2*
2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2))*(-2+2*2^(1/2))*2^(1/2)-1/4/(2+2*2^(1/2
))^(1/2)*arctan((2*x+(-2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2))*(-2+2*2^(1/
2))+1/2/(2+2*2^(1/2))^(1/2)*arctan((2*x+(-2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))
^(1/2))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4+2\,x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + 2*x^2 + 2), x)
```

mupad [B] time = 4.21, size = 210, normalized size = 1.19

$$\operatorname{atanh}\left(\frac{4\sqrt{2}\,x\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}-1}+\frac{4\sqrt{2}\,x\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}-1}\right)\left(2\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}-2\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\right)-\operatorname{atanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 + x^4 + 2),x)
```

```
[Out] atanh((4*2^(1/2)*x*(1/64 - 2^(1/2)/64)^(1/2))/(64*(1/64 - 2^(1/2)/64)^(1/2)
*(2^(1/2)/64 + 1/64)^(1/2) - 1) + (4*2^(1/2)*x*(2^(1/2)/64 + 1/64)^(1/2))/(
64*(1/64 - 2^(1/2)/64)^(1/2)*(2^(1/2)/64 + 1/64)^(1/2) - 1))*(2*(1/64 - 2^(
1/2)/64)^(1/2) - 2*(2^(1/2)/64 + 1/64)^(1/2)) - atanh((4*2^(1/2)*x*(1/64 -
2^(1/2)/64)^(1/2))/(64*(1/64 - 2^(1/2)/64)^(1/2)*(2^(1/2)/64 + 1/64)^(1/2)
+ 1) - (4*2^(1/2)*x*(2^(1/2)/64 + 1/64)^(1/2))/(64*(1/64 - 2^(1/2)/64)^(1/2
)*(2^(1/2)/64 + 1/64)^(1/2) + 1))*(2*(1/64 - 2^(1/2)/64)^(1/2) + 2*(2^(1/2)
/64 + 1/64)^(1/2))
```

sympy [B] time = 1.13, size = 899, normalized size = 5.11

$$\sqrt{\frac{1}{64}+\frac{\sqrt{2}}{64}}\log\left(x^2+x\left(-4\sqrt{2}\sqrt{1+\sqrt{2}}-\sqrt{1+\sqrt{2}}+3\sqrt{1+\sqrt{2}}\sqrt{2\sqrt{2}+3}\right)-15\sqrt{2\sqrt{2}+3}-7\sqrt{2}\sqrt{2\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+2*x**2+2),x)
```

```
[Out] sqrt(1/64 + sqrt(2)/64)*log(x**2 + x*(-4*sqrt(2)*sqrt(1 + sqrt(2)) - sqrt(1
 + sqrt(2)) + 3*sqrt(1 + sqrt(2))*sqrt(2*sqrt(2) + 3)) - 15*sqrt(2*sqrt(2)
+ 3) - 7*sqrt(2)*sqrt(2*sqrt(2) + 3) + 29 + 23*sqrt(2)) - sqrt(1/64 + sqrt(
2)/64)*log(x**2 + x*(-3*sqrt(1 + sqrt(2))*sqrt(2*sqrt(2) + 3) + sqrt(1 + sq
rt(2)) + 4*sqrt(2)*sqrt(1 + sqrt(2))) - 15*sqrt(2*sqrt(2) + 3) - 7*sqrt(2)*
sqrt(2*sqrt(2) + 3) + 29 + 23*sqrt(2)) + 2*sqrt(-sqrt(2*sqrt(2) + 3)/32 + 1
/64 + 3*sqrt(2)/64)*atan(2*x/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2))
+ sqrt(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2))) - 4*sqr
t(2)*sqrt(1 + sqrt(2))/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2)) + sqrt
(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2))) - sqrt(1 + sq
rt(2))/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2)) + sqrt(2*sqrt(2) + 3)*
sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2))) + 3*sqrt(1 + sqrt(2))*sqrt(2*
sqrt(2) + 3)/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2)) + sqrt(2*sqrt(2)
 + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2)))) + 2*sqrt(-sqrt(2*sqrt(
2) + 3)/32 + 1/64 + 3*sqrt(2)/64)*atan(2*x/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1
 + 3*sqrt(2)) + sqrt(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqr
t(2))) - 3*sqrt(1 + sqrt(2))*sqrt(2*sqrt(2) + 3)/(sqrt(-2*sqrt(2*sqrt(2) +
3) + 1 + 3*sqrt(2)) + sqrt(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 +
 3*sqrt(2))) + sqrt(1 + sqrt(2))/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(
2)) + sqrt(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2))) + 4
*sqrt(2)*sqrt(1 + sqrt(2))/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2)) +
sqrt(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2))))
```

3.16 $\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx$

Optimal. Leaf size=10

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right)$$

```
[Out] EllipticF(1/2*x*2^(1/2),I*6^(1/2))
```

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[x/Sqrt[2]], -6]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx &= \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{12-6x^2}\,\sqrt{2+6x^2}}\,dx\\ &= F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right)\end{aligned}$$

Mathematica [C] time = 0.03, size = 65, normalized size = 6.50

$$-\frac{i\sqrt{1-\frac{x^2}{2}}\sqrt{3x^2+1}\,F\left(i\sinh^{-1}\left(\sqrt{3}\,x\right)|-\frac{1}{6}\right)}{\sqrt{3}\sqrt{-3x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2/2]*Sqrt[1 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3]*x], -1/6]
)/(Sqrt[3]*Sqrt[2 + 5*x^2 - 3*x^4])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+5\,x^2+2}}{3\,x^4-5\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 5*x^2 + 2)/(3*x^4 - 5*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 + 2), x)
```

maple [B] time = 0.04, size = 51, normalized size = 5.10

$$\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{3x^2+1}\,\ \text{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{6}\right)}{2\sqrt{-3x^4+5x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*(-2*x^2+4)^(1/2)*(3*x^2+1)^(1/2)/(-3*x^4+5*x^2+2)^(1/2)*Ellipti
cF(1/2*x*2^(1/2),I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.10

$$\int \frac{1}{\sqrt{-3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 3*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 - 3*x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 5*x**2 + 2), x)
```

3.17 $\int \frac{1}{\sqrt{2+4x^2-3x^4}}\,dx$

Optimal. Leaf size=48

$$\sqrt{\frac{1}{6}\left(2+\sqrt{10}\right)}F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(-2+\sqrt{10}\right)}x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)$$

```
[Out] 1/6*EllipticF(1/2*x*(-4+2*10^(1/2))^(1/2),1/3*I*6^(1/2)+1/3*I*15^(1/2))*(12
+6*10^(1/2))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{1}{6}\left(2+\sqrt{10}\right)}F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(-2+\sqrt{10}\right)}x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 4*x^2 - 3*x^4],x]
```

```
[Out] Sqrt[(2 + Sqrt[10])/6]*EllipticF[ArcSin[Sqrt[(-2 + Sqrt[10])/2]*x], (-7 - 2
*Sqrt[10])/3]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+4x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{4+2\sqrt{10}-6x^2}\sqrt{-4+2\sqrt{10}+6x^2}}\,dx$$
$$= \sqrt{\frac{1}{6}\left(2+\sqrt{10}\right)}\,F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(-2+\sqrt{10}\right)}\,x\right)\middle|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)$$

Mathematica [C] time = 0.06, size = 49, normalized size = 1.02

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{1+\sqrt{\frac{5}{2}}}\,x\right)\middle|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2+\sqrt{10}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 4*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[1 + Sqrt[5/2]]*x], (-7 + 2*Sqrt[10])/3])/Sqr
t[2 + Sqrt[10]]
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+4\,x^2+2}}{3\,x^4-4\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+4*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 4*x^2 + 2)/(3*x^4 - 4*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4+4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+4*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 4*x^2 + 2), x)
```

maple [B] time = 0.11, size = 84, normalized size = 1.75

$$\frac{2\sqrt{-\left(-1+\frac{\sqrt{10}}{2}\right)x^{2}+1}\,\sqrt{-\left(-1-\frac{\sqrt{10}}{2}\right)x^{2}+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{-4+2\sqrt{10}}\,x}{2},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{\sqrt{-4+2\sqrt{10}}\,\sqrt{-3x^{4}+4x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+4*x^2+2)^(1/2),x)
```

```
[Out] 2/(-4+2*10^(1/2))^(1/2)*(1-(-1+1/2*10^(1/2))*x^2)^(1/2)*(1-(-1-1/2*10^(1/2)
)*x^2)^(1/2)/(-3*x^4+4*x^2+2)^(1/2)*EllipticF(1/2*x*(-4+2*10^(1/2))^(1/2),1
/3*I*6^(1/2)+1/3*I*15^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^{4}+4\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+4*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 4*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-3\,x^{4}+4\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 - 3*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(4*x^2 - 3*x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^{4}+4x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+4*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 4*x**2 + 2), x)
```

3.18 $\int \frac{1}{\sqrt{2+3x^2-3x^4}}\,dx$

Optimal. Leaf size=48

$$\sqrt{\frac{2}{\sqrt{33}-3}}\,F\left(\sin^{-1}\left(\sqrt{\frac{6}{3+\sqrt{33}}}\,x\right)\middle|\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

```
[Out] EllipticF(x*6^(1/2)/(3+33^(1/2))^(1/2),1/4*I*6^(1/2)+1/4*I*22^(1/2))*2^(1/2
)/(-3+33^(1/2))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{\sqrt{33}-3}}\,F\left(\sin^{-1}\left(\sqrt{\frac{6}{3+\sqrt{33}}}\,x\right)\middle|\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 3*x^2 - 3*x^4],x]
```

```
[Out] Sqrt[2/(-3 + Sqrt[33])]*EllipticF[ArcSin[Sqrt[6/(3 + Sqrt[33])]*x], (-7 - S
qrt[33])/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+3x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{3+\sqrt{33}-6x^2}\,\sqrt{-3+\sqrt{33}+6x^2}}\,dx$$
$$= \sqrt{\frac{2}{-3+\sqrt{33}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{6}{3+\sqrt{33}}}\,x\right)\middle|\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

Mathematica [C] time = 0.06, size = 53, normalized size = 1.10

$$-i\sqrt{\frac{2}{3+\sqrt{33}}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{6}{-3+\sqrt{33}}}\,x\right)\middle|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 3*x^2 - 3*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(3 + Sqrt[33])]*EllipticF[I*ArcSinh[Sqrt[6/(-3 + Sqrt[33])]*x],
 (-7 + Sqrt[33])/4]
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+3\,x^2+2}}{3\,x^4-3\,x^2-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 3*x^2 + 2)/(3*x^4 - 3*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 3*x^2 + 2), x)
```

maple [B] time = 0.10, size = 80, normalized size = 1.67

$$\frac{2\sqrt{-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^{2}+1}\,\sqrt{-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^{2}+1}\,\,\mathrm{EllipticF}\left(\frac{\sqrt{-3+\sqrt{33}}\,x}{2},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{\sqrt{-3+\sqrt{33}}\,\sqrt{-3x^{4}+3x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+3*x^2+2)^(1/2),x)
```

```
[Out] 2/(-3+33^(1/2))^(1/2)*(1-(-3/4+1/4*33^(1/2))*x^2)^(1/2)*(1-(-3/4-1/4*33^(1/
2))*x^2)^(1/2)/(-3*x^4+3*x^2+2)^(1/2)*EllipticF(1/2*x*(-3+33^(1/2))^(1/2),1
/4*I*6^(1/2)+1/4*I*22^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^{4}+3\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 3*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-3\,x^{4}+3\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 - 3*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(3*x^2 - 3*x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^{4}+3x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 3*x**2 + 2), x)
```

3.19 $\int \frac{1}{\sqrt{2+2x^2-3x^4}}\,dx$

Optimal. Leaf size=44

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{\sqrt{7}-1}}$$

```
[Out] EllipticF(x*3^(1/2)/(1+7^(1/2))^(1/2),1/6*I*6^(1/2)+1/6*I*42^(1/2))/(-1+7^(
1/2))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{\sqrt{7}-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 2*x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3/(1 + Sqrt[7])]*x], (-4 - Sqrt[7])/3]/Sqrt[-1 + Sqrt
[7]]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+2x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{2+2\sqrt{7}-6x^2}\sqrt{-2+2\sqrt{7}+6x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{-1+\sqrt{7}}}$$

Mathematica [C] time = 0.05, size = 49, normalized size = 1.11

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{\frac{3}{-1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 2*x^2 - 3*x^4],x]
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[3/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3])/Sqr
t[1 + Sqrt[7]]
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+2\,x^2+2}}{3\,x^4-2\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2*x^2+2)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-3*x^4 + 2*x^2 + 2)/(3*x^4 - 2*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4+2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-3*x^4 + 2*x^2 + 2), x)
```

maple [B] time = 0.10, size = 84, normalized size = 1.91

$$\frac{2\sqrt{-\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^{2}+1}\,\sqrt{-\left(-\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^{2}+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{-2+2\sqrt{7}}\,x}{2},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{\sqrt{-2+2\sqrt{7}}\,\sqrt{-3x^{4}+2x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+2*x^2+2)^(1/2),x)
```

```
[Out] 2/(-2+2*7^(1/2))^(1/2)*(1-(-1/2+1/2*7^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*7^(1/2
))*x^2)^(1/2)/(-3*x^4+2*x^2+2)^(1/2)*EllipticF(1/2*x*(-2+2*7^(1/2))^(1/2),1
/6*I*6^(1/2)+1/6*I*42^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^{4}+2\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 2*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-3\,x^{4}+2\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 - 3*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(2*x^2 - 3*x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^{4}+2x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+2*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 2*x**2 + 2), x)
```

3.20 $\int \frac{1}{\sqrt{2+x^2-3x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(x,1/2*I*6^(1/2))*2^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[x], -3/2]/Sqrt[2]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+x^2-3x^4}}\,dx &= \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{6-6x^2}\sqrt{4+6x^2}}\,dx\\ &= \frac{F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 63, normalized size = 5.25

$$-\frac{i\sqrt{1-x^2}\,\sqrt{3x^2+2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{2}{3}\right)}{\sqrt{3}\,\sqrt{-3x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], -2/3]
)/(Sqrt[3]*Sqrt[2 + x^2 - 3*x^4])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+x^2+2}}{3\,x^4-x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + x^2 + 2)/(3*x^4 - x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-3\,x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + x^2 + 2), x)
```

maple [B] time = 0.02, size = 41, normalized size = 3.42

$$\frac{\sqrt{-x^2+1}\,\sqrt{6x^2+4}\,\ \text{EllipticF}\left(x,\frac{i\sqrt{6}}{2}\right)}{2\sqrt{-3x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+x^2+2)^(1/2),x)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(-3*x^4+x^2+2)^(1/2)*EllipticF(x,1/2*I*6
^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4 + x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{-3\,x^4 + x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 3*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(x^2 - 3*x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4 + x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + x**2 + 2), x)
```

3.21 $\int \frac{1}{\sqrt{2-3x^4}}\,dx$

Optimal. Leaf size=18

$$\frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

[Out] 1/6*EllipticF(1/2*3^(1/4)*2^(3/4)*x,I)*6^(3/4)

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {221}

$$\frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[2 - 3*x^4],x]

[Out] EllipticF[ArcSin[(3/2)^(1/4)*x], -1]/6^(1/4)

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x^4}}\,dx = \frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 1.00

$$\frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[(3/2)^(1/4)*x], -1]/6^(1/4)
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+2}}{3\,x^4-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 2)/(3*x^4 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 2), x)
```

maple [B] time = 0.04, size = 54, normalized size = 3.00

$$\frac{\sqrt{2}\,6^{\frac{3}{4}}\sqrt{-2\sqrt{6}\,x^2+4}\,\sqrt{2\sqrt{6}\,x^2+4}\,\operatorname{EllipticF}\left(\frac{\sqrt{2}\,6^{\frac{1}{4}}x}{2},i\right)}{24\sqrt{-3x^4+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+2)^(1/2),x)
```

```
[Out] 1/24*2^(1/2)*6^(3/4)*(4-2*x^2*6^(1/2))^(1/2)*(4+2*x^2*6^(1/2))^(1/2)/(-3*x^
4+2)^(1/2)*EllipticF(1/2*x*2^(1/2)*6^(1/4),I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 2), x)
```

mupad [B] time = 4.20, size = 16, normalized size = 0.89

$$\frac{\sqrt{2}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ \frac{3\,x^4}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2 - 3*x^4)^(1/2),x)
[Out] (2^(1/2)*x*hypergeom([1/4, 1/2], 5/4, (3*x^4)/2))/2
```

sympy [A] time = 0.69, size = 37, normalized size = 2.06

$$\frac{\sqrt{2}\,x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{3x^4e^{2i\pi}}{2}\right)}{8\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+2)**(1/2),x)
[Out] sqrt(2)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 3*x**4*exp_polar(2*I*pi)/2)/
(8*gamma(5/4))
```

3.22 $\int \frac{1}{\sqrt{2-x^2-3x^4}}\,dx$

Optimal. Leaf size=20

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticF(1/2*x*6^(1/2),1/3*I*6^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{4-6x^2}\sqrt{6+6x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.03, size = 20, normalized size = 1.00

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - x^2 - 3*x^4],x]
[Out] EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4-x^2+2}}{3\,x^4+x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-x^2+2)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-3*x^4 - x^2 + 2)/(3*x^4 + x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-3*x^4 - x^2 + 2), x)
```

maple [B] time = 0.03, size = 49, normalized size = 2.45

$$\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{x^2+1}\;\text{EllipticF}\left(\frac{\sqrt{6}\,x}{2},\frac{i\sqrt{6}}{3}\right)}{6\sqrt{-3x^4-x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-x^2+2)^(1/2),x)
```

```
[Out] 1/6*6^(1/2)*(-6*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-3*x^4-x^2+2)^(1/2)*EllipticF(1
/2*6^(1/2)*x,1/3*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{-3\,x^4-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2 - 3*x^4 - x^2)^(1/2),x)
```

```
[Out] int(1/(2 - 3*x^4 - x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - x**2 + 2), x)
```

3.23 $\int \frac{1}{\sqrt{2-2x^2-3x^4}}\,dx$

Optimal. Leaf size=42

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{-1+\sqrt{7}}}\,x\right)\middle|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

```
[Out] EllipticF(x*3^(1/2)/(-1+7^(1/2))^(1/2),1/6*I*42^(1/2)-1/6*I*6^(1/2))/(1+7^(
1/2))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{-1+\sqrt{7}}}\,x\right)\middle|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 2*x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3]/Sqrt[1 + Sqrt
[7]]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-2x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-2+2\sqrt{7}-6x^2}\,\sqrt{2+2\sqrt{7}+6x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{-1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

Mathematica [C] time = 0.04, size = 51, normalized size = 1.21

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{\frac{3}{1+\sqrt{7}}}\,x\right)|-\frac{4}{3}-\frac{\sqrt{7}}{3}\right)}{\sqrt{\sqrt{7}-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 - 2*x^2 - 3*x^4],x]
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[3/(1 + Sqrt[7])]*x], -4/3 - Sqrt[7]/3])/Sqrt
[-1 + Sqrt[7]]
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-3\,x^4-2\,x^2+2}}{3\,x^4+2\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2*x^2+2)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-3*x^4 - 2*x^2 + 2)/(3*x^4 + 2*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4-2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-3*x^4 - 2*x^2 + 2), x)
```

maple [B] time = 0.09, size = 84, normalized size = 2.00

$$\frac{2\sqrt{-\left(\frac{\sqrt{7}}{2}+\frac{1}{2}\right)x^{2}+1}\,\sqrt{-\left(-\frac{\sqrt{7}}{2}+\frac{1}{2}\right)x^{2}+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{2+2\sqrt{7}}\,x}{2},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{\sqrt{2+2\sqrt{7}}\,\sqrt{-3x^{4}-2x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-2*x^2+2)^(1/2),x)
```

```
[Out] 2/(2+2*7^(1/2))^(1/2)*(1-(1/2*7^(1/2)+1/2)*x^2)^(1/2)*(1-(-1/2*7^(1/2)+1/2)
*x^2)^(1/2)/(-3*x^4-2*x^2+2)^(1/2)*EllipticF(1/2*x*(2+2*7^(1/2))^(1/2),1/6*
I*42^(1/2)-1/6*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^{4}-2\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 2*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-3\,x^{4}-2\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2 - 3*x^4 - 2*x^2)^(1/2),x)
```

```
[Out] int(1/(2 - 3*x^4 - 2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^{4}-2x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-2*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 2*x**2 + 2), x)
```

3.24 $\int \frac{1}{\sqrt{2-3x^2-3x^4}}\,dx$

Optimal. Leaf size=46

$$\sqrt{\frac{2}{3+\sqrt{33}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{6}{-3+\sqrt{33}}}\,x\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

```
[Out] EllipticF(x*6^(1/2)/(-3+33^(1/2))^(1/2),1/4*I*22^(1/2)-1/4*I*6^(1/2))*2^(1/
2)/(3+33^(1/2))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{3+\sqrt{33}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{6}{-3+\sqrt{33}}}\,x\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 3*x^2 - 3*x^4],x]
```

```
[Out] Sqrt[2/(3 + Sqrt[33])]*EllipticF[ArcSin[Sqrt[6/(-3 + Sqrt[33])]*x], (-7 + S
qrt[33])/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-3+\sqrt{33}-6x^2}\sqrt{3+\sqrt{33}+6x^2}}\,dx$$
$$= \sqrt{\frac{2}{3+\sqrt{33}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{6}{-3+\sqrt{33}}}\,x\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

Mathematica [C] time = 0.06, size = 55, normalized size = 1.20

$$-i\sqrt{\frac{2}{\sqrt{33}-3}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{6}{3+\sqrt{33}}}\,x\right)|-\frac{7}{4}-\frac{\sqrt{33}}{4}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 - 3*x^2 - 3*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(-3 + Sqrt[33])]*EllipticF[I*ArcSinh[Sqrt[6/(3 + Sqrt[33])]*x],
 -7/4 - Sqrt[33]/4]
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4-3\,x^2+2}}{3\,x^4+3\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - 3*x^2 + 2)/(3*x^4 + 3*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 3*x^2 + 2), x)
```

maple [B] time = 0.09, size = 80, normalized size = 1.74

$$\frac{2\sqrt{-\left(\frac{\sqrt{33}}{4}+\frac{3}{4}\right)x^{2}+1}\,\sqrt{-\left(-\frac{\sqrt{33}}{4}+\frac{3}{4}\right)x^{2}+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{3+\sqrt{33}}\,x}{2},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{\sqrt{3+\sqrt{33}}\,\sqrt{-3x^{4}-3x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-3*x^2+2)^(1/2),x)
```

```
[Out] 2/(3+33^(1/2))^(1/2)*(1-(1/4*33^(1/2)+3/4)*x^2)^(1/2)*(1-(-1/4*33^(1/2)+3/4
)*x^2)^(1/2)/(-3*x^4-3*x^2+2)^(1/2)*EllipticF(1/2*x*(3+33^(1/2))^(1/2),1/4*
I*22^(1/2)-1/4*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^{4}-3\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 3*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-3\,x^{4}-3\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2 - 3*x^4 - 3*x^2)^(1/2),x)
```

```
[Out] int(1/(2 - 3*x^4 - 3*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^{4}-3x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 3*x**2 + 2), x)
```

3.25 $\int \frac{1}{\sqrt{2-4x^2-3x^4}}\,dx$

Optimal. Leaf size=48

$$\sqrt{\frac{1}{6}\left(\sqrt{10}-2\right)}F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)$$

```
[Out] 1/6*EllipticF(1/2*x*(4+2*10^(1/2))^(1/2),1/3*I*15^(1/2)-1/3*I*6^(1/2))*(-12
+6*10^(1/2))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{1}{6}\left(\sqrt{10}-2\right)}F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 4*x^2 - 3*x^4],x]
```

```
[Out] Sqrt[(-2 + Sqrt[10])/6]*EllipticF[ArcSin[Sqrt[(2 + Sqrt[10])/2]*x], (-7 + 2
*Sqrt[10])/3]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-4x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-4+2\sqrt{10}-6x^2}\sqrt{4+2\sqrt{10}+6x^2}}\,dx$$
$$= \sqrt{\frac{1}{6}\left(-2+\sqrt{10}\right)}F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)$$

Mathematica [C] time = 0.06, size = 49, normalized size = 1.02

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{-1+\sqrt{\frac{5}{2}}}x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{\sqrt{10}-2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 - 4*x^2 - 3*x^4],x]
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[-1 + Sqrt[5/2]]*x], (-7 - 2*Sqrt[10])/3])/Sq
rt[-2 + Sqrt[10]]
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4-4\,x^2+2}}{3\,x^4+4\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-4*x^2+2)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-3*x^4 - 4*x^2 + 2)/(3*x^4 + 4*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-4*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-3*x^4 - 4*x^2 + 2), x)
```

maple [B] time = 0.09, size = 84, normalized size = 1.75

$$\frac{2\sqrt{-\left(1+\frac{\sqrt{10}}{2}\right)x^2+1}\,\sqrt{-\left(-\frac{\sqrt{10}}{2}+1\right)x^2+1}\,\,\mathrm{EllipticF}\left(\frac{\sqrt{4+2\sqrt{10}}\,x}{2},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{\sqrt{4+2\sqrt{10}}\,\sqrt{-3x^4-4x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-4*x^2+2)^(1/2),x)
```

```
[Out] 2/(4+2*10^(1/2))^(1/2)*(1-(1+1/2*10^(1/2))*x^2)^(1/2)*(1-(-1/2*10^(1/2)+1)*
x^2)^(1/2)/(-3*x^4-4*x^2+2)^(1/2)*EllipticF(1/2*x*(4+2*10^(1/2))^(1/2),1/3*
I*15^(1/2)-1/3*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-4*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 4*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-3\,x^4-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2 - 3*x^4 - 4*x^2)^(1/2),x)
```

```
[Out] int(1/(2 - 3*x^4 - 4*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4-4x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-4*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 4*x**2 + 2), x)
```

3.26 $\int \frac{1}{\sqrt{2-5x^2-3x^4}}\,dx$

Optimal. Leaf size=18

$$\frac{F\left(\sin^{-1}\left(\sqrt{3}\,x\right)|-\frac{1}{6}\right)}{\sqrt{6}}$$

```
[Out] 1/6*EllipticF(x*3^(1/2),1/6*I*6^(1/2))*6^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}\left(\sqrt{3}\,x\right)|-\frac{1}{6}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 5*x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3]*x], -1/6]/Sqrt[6]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-5x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{2-6x^2}\sqrt{12+6x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{3}\,x\right)|-\frac{1}{6}\right)}{\sqrt{6}}$$

Mathematica [B] time = 0.02, size = 54, normalized size = 3.00

$$\frac{\sqrt{1-3x^2}\,\sqrt{x^2+2}\,F\left(\sin^{-1}\left(\sqrt{3}\,x\right)|-\frac{1}{6}\right)}{\sqrt{6}\,\sqrt{-3x^4-5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - 5*x^2 - 3*x^4],x]
[Out] (Sqrt[1 - 3*x^2]*Sqrt[2 + x^2]*EllipticF[ArcSin[Sqrt[3]*x], -1/6])/(Sqrt[6]
*Sqrt[2 - 5*x^2 - 3*x^4])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4-5\,x^2+2}}{3\,x^4+5\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-5*x^2+2)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-3*x^4 - 5*x^2 + 2)/(3*x^4 + 5*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-5*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-3*x^4 - 5*x^2 + 2), x)
```

maple [B] time = 0.03, size = 50, normalized size = 2.78

$$\frac{\sqrt{3}\,\sqrt{-3x^2+1}\,\sqrt{2x^2+4}\,\operatorname{EllipticF}\left(\sqrt{3}\,x,\frac{i\sqrt{6}}{6}\right)}{6\sqrt{-3x^4-5x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-5*x^2+2)^(1/2),x)
```

```
[Out] 1/6*3^(1/2)*(-3*x^2+1)^(1/2)*(2*x^2+4)^(1/2)/(-3*x^4-5*x^2+2)^(1/2)*Ellipti
cF(3^(1/2)*x,1/6*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 5*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{-3\,x^4-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2 - 3*x^4 - 5*x^2)^(1/2),x)
```

```
[Out] int(1/(2 - 3*x^4 - 5*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4-5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 5*x**2 + 2), x)
```

3.27 $\int \frac{1}{\sqrt{3+7x^2-2x^4}} dx$

Optimal. Leaf size=45

$$\sqrt{\frac{2}{\sqrt{73}-7}}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)$$

```
[Out] EllipticF(2*x/(7+73^(1/2))^(1/2),7/12*I*6^(1/2)+1/12*I*438^(1/2))*2^(1/2)/(
-7+73^(1/2))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{\sqrt{73}-7}}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 7*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[2/(-7 + Sqrt[73])]*EllipticF[ArcSin[(2*x)/Sqrt[7 + Sqrt[73]]], (-61 -
7*Sqrt[73])/12]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+7x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{7+\sqrt{73}-4x^2}\sqrt{-7+\sqrt{73}+4x^2}}\,dx$$
$$= \sqrt{\frac{2}{-7+\sqrt{73}}}\,F\left(\sin^{-1}\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right)\middle|\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)$$

Mathematica [C] time = 0.05, size = 52, normalized size = 1.16

$$-i\sqrt{\frac{2}{7+\sqrt{73}}}\,F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right)\middle|\frac{1}{12}\left(-61+7\sqrt{73}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 7*x^2 - 2*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(7 + Sqrt[73])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[-7 + Sqrt[73]]],
 (-61 + 7*Sqrt[73])/12]
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+7\,x^2+3}}{2\,x^4-7\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+7*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 7*x^2 + 3)/(2*x^4 - 7*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4+7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+7*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 7*x^2 + 3), x)
```

maple [B] time = 0.11, size = 84, normalized size = 1.87

$$\frac{6\sqrt{-\left(-\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2+1}\,\sqrt{-\left(-\frac{\sqrt{73}}{6}-\frac{7}{6}\right)x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{-42+6\sqrt{73}}\,x}{6},\frac{7i\sqrt{6}}{12}+\frac{i\sqrt{438}}{12}\right)}{\sqrt{-42+6\sqrt{73}}\,\sqrt{-2x^4+7x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+7*x^2+3)^(1/2),x)
[Out] 6/(-42+6*73^(1/2))^(1/2)*(1-(-7/6+1/6*73^(1/2))*x^2)^(1/2)*(1-(-1/6*73^(1/2
)-7/6)*x^2)^(1/2)/(-2*x^4+7*x^2+3)^(1/2)*EllipticF(1/6*x*(-42+6*73^(1/2))^(
1/2),7/12*I*6^(1/2)+1/12*I*438^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4+7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+7*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-2*x^4 + 7*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-2\,x^4+7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7*x^2 - 2*x^4 + 3)^(1/2),x)
[Out] int(1/(7*x^2 - 2*x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4+7x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+7*x**2+3)**(1/2),x)
[Out] Integral(1/sqrt(-2*x**4 + 7*x**2 + 3), x)
```

3.28 $\int \frac{1}{\sqrt{3+6x^2-2x^4}}\,dx$

Optimal. Leaf size=44

$$\sqrt{\frac{1}{6}\left(3+\sqrt{15}\right)}\,F\left(\sin^{-1}\left(\sqrt{\frac{1}{3}\left(-3+\sqrt{15}\right)}\,x\right)\middle|-4-\sqrt{15}\right)$$

```
[Out] 1/6*EllipticF(1/3*x*(-9+3*15^(1/2))^(1/2),1/2*I*6^(1/2)+1/2*I*10^(1/2))*(18
+6*15^(1/2))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{1}{6}\left(3+\sqrt{15}\right)}\,F\left(\sin^{-1}\left(\sqrt{\frac{1}{3}\left(-3+\sqrt{15}\right)}\,x\right)\middle|-4-\sqrt{15}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 6*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[(3 + Sqrt[15])/6]*EllipticF[ArcSin[Sqrt[(-3 + Sqrt[15])/3]*x], -4 - Sq
rt[15]]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+6x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{6+2\sqrt{15}-4x^2}\sqrt{-6+2\sqrt{15}+4x^2}}\,dx$$
$$= \sqrt{\frac{1}{6}\left(3+\sqrt{15}\right)}\,F\left(\sin^{-1}\left(\sqrt{\frac{1}{3}\left(-3+\sqrt{15}\right)}\,x\right)\middle| -4-\sqrt{15}\right)$$

Mathematica [C] time = 0.05, size = 43, normalized size = 0.98

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{1+\sqrt{\frac{5}{3}}}\,x\right)\middle| -4+\sqrt{15}\right)}{\sqrt{3+\sqrt{15}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 6*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[1 + Sqrt[5/3]]*x], -4 + Sqrt[15]])/Sqrt[3 +
Sqrt[15]]
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{\sqrt{-2\,x^4+6\,x^2+3}}{2\,x^4-6\,x^2-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+6*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 6*x^2 + 3)/(2*x^4 - 6*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4+6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+6*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 6*x^2 + 3), x)
```

maple [B] time = 0.11, size = 84, normalized size = 1.91

$$\frac{3\sqrt{-\left(-1+\frac{\sqrt{15}}{3}\right)x^2+1}\,\sqrt{-\left(-1-\frac{\sqrt{15}}{3}\right)x^2+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{-9+3\sqrt{15}}\,x}{3},\frac{i\sqrt{6}}{2}+\frac{i\sqrt{10}}{2}\right)}{\sqrt{-9+3\sqrt{15}}\,\sqrt{-2x^4+6x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+6*x^2+3)^(1/2),x)
```

```
[Out] 3/(-9+3*15^(1/2))^(1/2)*(1-(-1+1/3*15^(1/2))*x^2)^(1/2)*(1-(-1-1/3*15^(1/2)
)*x^2)^(1/2)/(-2*x^4+6*x^2+3)^(1/2)*EllipticF(1/3*x*(-9+3*15^(1/2))^(1/2),1
/2*I*6^(1/2)+1/2*I*10^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4+6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+6*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 6*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-2\,x^4+6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(6*x^2 - 2*x^4 + 3)^(1/2),x)
```

```
[Out] int(1/(6*x^2 - 2*x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4+6x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+6*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 6*x**2 + 3), x)
```

3.29 $\int \frac{1}{\sqrt{3+5x^2-2x^4}}\,dx$

Optimal. Leaf size=10

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-6\right)$$

```
[Out] EllipticF(1/3*x*3^(1/2),I*6^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-6\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 5*x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[x/Sqrt[3]], -6]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3+5x^2-2x^4}}\,dx &= \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{12-4x^2}\sqrt{2+4x^2}}\,dx\\ &= F\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-6\right)\end{aligned}$$

Mathematica [C] time = 0.03, size = 65, normalized size = 6.50

$$-\frac{i\sqrt{1-\frac{x^2}{3}}\sqrt{2x^2+1}\,F\left(i\sinh^{-1}\left(\sqrt{2}\,x\right)|-\frac{1}{6}\right)}{\sqrt{2}\sqrt{-2x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 5*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2/3]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]*x], -1/6]
)/(Sqrt[2]*Sqrt[3 + 5*x^2 - 2*x^4])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+5\,x^2+3}}{2\,x^4-5\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 5*x^2 + 3)/(2*x^4 - 5*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 + 3), x)
```

maple [B] time = 0.03, size = 51, normalized size = 5.10

$$\frac{\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{2x^2+1}\;\text{EllipticF}\left(\frac{\sqrt{3}\,x}{3},i\sqrt{6}\right)}{3\sqrt{-2x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*(-3*x^2+9)^(1/2)*(2*x^2+1)^(1/2)/(-2*x^4+5*x^2+3)^(1/2)*Ellipti
cF(1/3*3^(1/2)*x,I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4 + 5\,x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.10

$$\int \frac{1}{\sqrt{-2\,x^4 + 5\,x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 2*x^4 + 3)^(1/2),x)
[Out] int(1/(5*x^2 - 2*x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4 + 5x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+5*x**2+3)**(1/2),x)
[Out] Integral(1/sqrt(-2*x**4 + 5*x**2 + 3), x)
```

3.30 $\int \frac{1}{\sqrt{3+4x^2-2x^4}}\,dx$

Optimal. Leaf size=44

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{2+\sqrt{10}}}\,x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{\sqrt{10}-2}}$$

```
[Out] EllipticF(x*2^(1/2)/(2+10^(1/2))^(1/2),1/3*I*6^(1/2)+1/3*I*15^(1/2))/(-2+10
^(1/2))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{2+\sqrt{10}}}\,x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{\sqrt{10}-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 4*x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2/(2 + Sqrt[10])]*x], (-7 - 2*Sqrt[10])/3]/Sqrt[-2 +
Sqrt[10]]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+4x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{4+2\sqrt{10}-4x^2}\sqrt{-4+2\sqrt{10}+4x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{2+\sqrt{10}}}\,x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{-2+\sqrt{10}}}$$

Mathematica [C] time = 0.06, size = 51, normalized size = 1.16

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-2+\sqrt{10}}}\,x\right)|-\frac{7}{3}+\frac{2\sqrt{10}}{3}\right)}{\sqrt{2+\sqrt{10}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 4*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[2/(-2 + Sqrt[10])]*x], -7/3 + (2*Sqrt[10])/3
])/Sqrt[2 + Sqrt[10]]
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+4\,x^2+3}}{2\,x^4-4\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+4*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 4*x^2 + 3)/(2*x^4 - 4*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4+4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+4*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 4*x^2 + 3), x)
```

maple [B] time = 0.08, size = 84, normalized size = 1.91

$$\frac{3\sqrt{-\left(-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^{2}+1}\,\sqrt{-\left(-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^{2}+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{-6+3\sqrt{10}}\,x}{3},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{\sqrt{-6+3\sqrt{10}}\,\sqrt{-2x^{4}+4x^{2}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+4*x^2+3)^(1/2),x)
```

```
[Out] 3/(-6+3*10^(1/2))^(1/2)*(1-(-2/3+1/3*10^(1/2))*x^2)^(1/2)*(1-(-2/3-1/3*10^(
1/2))*x^2)^(1/2)/(-2*x^4+4*x^2+3)^(1/2)*EllipticF(1/3*x*(-6+3*10^(1/2))^(1/
2),1/3*I*6^(1/2)+1/3*I*15^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^{4}+4\,x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+4*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 4*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-2\,x^{4}+4\,x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 - 2*x^4 + 3)^(1/2),x)
```

```
[Out] int(1/(4*x^2 - 2*x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^{4}+4x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+4*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 4*x**2 + 3), x)
```

3.31 $\int \frac{1}{\sqrt{3+3x^2-2x^4}}\,dx$

Optimal. Leaf size=45

$$\sqrt{\frac{2}{\sqrt{33}-3}}\,F\left(\sin^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right)\middle|\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

```
[Out] EllipticF(2*x/(3+33^(1/2))^(1/2),1/4*I*6^(1/2)+1/4*I*22^(1/2))*2^(1/2)/(-3+
33^(1/2))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{\sqrt{33}-3}}\,F\left(\sin^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right)\middle|\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 3*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[2/(-3 + Sqrt[33])]*EllipticF[ArcSin[(2*x)/Sqrt[3 + Sqrt[33]]], (-7 - S
qrt[33])/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+3x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{3+\sqrt{33}-4x^2}\sqrt{-3+\sqrt{33}+4x^2}}\,dx$$
$$= \sqrt{\frac{2}{-3+\sqrt{33}}}\,F\left(\sin^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right)\middle|\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

Mathematica [C] time = 0.05, size = 50, normalized size = 1.11

$$-i\sqrt{\frac{2}{3+\sqrt{33}}}\,F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right)\middle|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 3*x^2 - 2*x^4],x]
[Out] (-I)*Sqrt[2/(3 + Sqrt[33])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[-3 + Sqrt[33]]],
 (-7 + Sqrt[33])/4]
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+3\,x^2+3}}{2\,x^4-3\,x^2-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3*x^2+3)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-2*x^4 + 3*x^2 + 3)/(2*x^4 - 3*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4+3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-2*x^4 + 3*x^2 + 3), x)
```

maple [B] time = 0.08, size = 84, normalized size = 1.87

$$\frac{6\sqrt{-\left(-\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^{2}+1}\,\sqrt{-\left(-\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^{2}+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{-18+6\sqrt{33}}\,x}{6},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{\sqrt{-18+6\sqrt{33}}\,\sqrt{-2x^{4}+3x^{2}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+3*x^2+3)^(1/2),x)
```

```
[Out] 6/(-18+6*33^(1/2))^(1/2)*(1-(-1/2+1/6*33^(1/2))*x^2)^(1/2)*(1-(-1/2-1/6*33^
(1/2))*x^2)^(1/2)/(-2*x^4+3*x^2+3)^(1/2)*EllipticF(1/6*x*(-18+6*33^(1/2))^(
1/2),1/4*I*6^(1/2)+1/4*I*22^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^{4}+3\,x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 3*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-2\,x^{4}+3\,x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 - 2*x^4 + 3)^(1/2),x)
```

```
[Out] int(1/(3*x^2 - 2*x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^{4}+3x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+3*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 3*x**2 + 3), x)
```

3.32 $\int \frac{1}{\sqrt{3+2x^2-2x^4}} \, dx$

Optimal. Leaf size=44

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{\sqrt{7}-1}}$$

```
[Out] EllipticF(x*2^(1/2)/(1+7^(1/2))^(1/2),1/6*I*6^(1/2)+1/6*I*42^(1/2))/(-1+7^(
1/2))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{\sqrt{7}-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 2*x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[7])]*x], (-4 - Sqrt[7])/3]/Sqrt[-1 + Sqrt
[7]]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+2x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{2+2\sqrt{7}-4x^2}\sqrt{-2+2\sqrt{7}+4x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{-1+\sqrt{7}}}$$

Mathematica [C] time = 0.04, size = 49, normalized size = 1.11

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 2*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[2/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3])/Sqr
t[1 + Sqrt[7]]
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-2\,x^4+2\,x^2+3}}{2\,x^4-2\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+2*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 2*x^2 + 3)/(2*x^4 - 2*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4+2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+2*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 2*x^2 + 3), x)
```

maple [B] time = 0.08, size = 84, normalized size = 1.91

$$\frac{3\sqrt{-\left(-\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^{2}+1}\,\sqrt{-\left(-\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^{2}+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{-3+3\sqrt{7}}\,x}{3},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{\sqrt{-3+3\sqrt{7}}\,\sqrt{-2x^{4}+2x^{2}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+2*x^2+3)^(1/2),x)
```

```
[Out] 3/(-3+3*7^(1/2))^(1/2)*(1-(-1/3+1/3*7^(1/2))*x^2)^(1/2)*(1-(-1/3-1/3*7^(1/2
))*x^2)^(1/2)/(-2*x^4+2*x^2+3)^(1/2)*EllipticF(1/3*x*(-3+3*7^(1/2))^(1/2),1
/6*I*6^(1/2)+1/6*I*42^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^{4}+2\,x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+2*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 2*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-2\,x^{4}+2\,x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 - 2*x^4 + 3)^(1/2),x)
```

```
[Out] int(1/(2*x^2 - 2*x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^{4}+2x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+2*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 2*x**2 + 3), x)
```

3.33 $\int \frac{1}{\sqrt{3+x^2-2x^4}}\,dx$

Optimal. Leaf size=20

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)|-\frac{3}{2}\right)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(1/3*x*6^(1/2),1/2*I*6^(1/2))*2^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)|-\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2/3]*x], -3/2]/Sqrt[2]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{6-4x^2}\,\sqrt{4+4x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)\middle|-\frac{3}{2}\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.03, size = 20, normalized size = 1.00

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)\middle|-\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + x^2 - 2*x^4],x]
[Out] EllipticF[ArcSin[Sqrt[2/3]*x], -3/2]/Sqrt[2]
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-2\,x^4+x^2+3}}{2\,x^4-x^2-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+x^2+3)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-2*x^4 + x^2 + 3)/(2*x^4 - x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-2*x^4 + x^2 + 3), x)
```

maple [B] time = 0.03, size = 47, normalized size = 2.35

$$\frac{\sqrt{6}\,\sqrt{-6x^2+9}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{6}\,x}{3}, \frac{i\sqrt{6}}{2}\right)}{6\sqrt{-2x^4+x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+x^2+3)^(1/2),x)
```

```
[Out] 1/6*6^(1/2)*(-6*x^2+9)^(1/2)*(x^2+1)^(1/2)/(-2*x^4+x^2+3)^(1/2)*EllipticF(1
/3*6^(1/2)*x,1/2*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{-2\,x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 2*x^4 + 3)^(1/2),x)
```

```
[Out] int(1/(x^2 - 2*x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + x**2 + 3), x)
```

3.34 $\int \frac{1}{\sqrt{3-2x^4}}\,dx$

Optimal. Leaf size=18

$$\frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

[Out] 1/6*EllipticF(1/3*2^(1/4)*3^(3/4)*x,I)*6^(3/4)

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {221}

$$\frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[3 - 2*x^4],x]

[Out] EllipticF[ArcSin[(2/3)^(1/4)*x], -1]/6^(1/4)

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-2x^4}}\,dx = \frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 1.00

$$\frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[(2/3)^(1/4)*x], -1]/6^(1/4)
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+3}}{2\,x^4-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 3)/(2*x^4 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 3), x)
```

maple [B] time = 0.04, size = 54, normalized size = 3.00

$$\frac{\sqrt{3}\;6^{\frac{3}{4}}\sqrt{-3\sqrt{6}\;x^2+9}\;\sqrt{3\sqrt{6}\;x^2+9}\;\;\text{EllipticF}\left(\frac{\sqrt{3}\;6^{\frac{1}{4}}x}{3},i\right)}{54\sqrt{-2x^4+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+3)^(1/2),x)
```

```
[Out] 1/54*3^(1/2)*6^(3/4)*(9-3*6^(1/2)*x^2)^(1/2)*(9+3*6^(1/2)*x^2)^(1/2)/(-2*x^
4+3)^(1/2)*EllipticF(1/3*x*3^(1/2)*6^(1/4),I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 3), x)
```

mupad [B] time = 4.20, size = 16, normalized size = 0.89

$$\frac{\sqrt{3}\, x\, {}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ \frac{2\,x^4}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3 - 2*x^4)^(1/2),x)
```

```
[Out] (3^(1/2)*x*hypergeom([1/4, 1/2], 5/4, (2*x^4)/3))/3
```

sympy [A] time = 0.69, size = 37, normalized size = 2.06

$$\frac{\sqrt{3}\, x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{2x^4e^{2i\pi}}{3}\right)}{12\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+3)**(1/2),x)
```

```
[Out] sqrt(3)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**4*exp_polar(2*I*pi)/3)/
(12*gamma(5/4))
```

3.35 $\int \frac{1}{\sqrt{3-x^2-2x^4}} \, dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticF(x,1/3*I*6^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[x], -2/3]/Sqrt[3]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-x^2-2x^4}} \, dx &= \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{4-4x^2}\sqrt{6+4x^2}} \, dx\\ &= \frac{F\left(\sin^{-1}(x)\middle|-\frac{2}{3}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 65, normalized size = 5.42

$$-\frac{i\sqrt{1-x^2}\,\sqrt{2x^2+3}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)|-\frac{3}{2}\right)}{\sqrt{2}\,\sqrt{-2x^4-x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2]*Sqrt[3 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/3]*x], -3/2]
)/(Sqrt[2]*Sqrt[3 - x^2 - 2*x^4])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4-x^2+3}}{2\,x^4+x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - x^2 + 3)/(2*x^4 + x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - x^2 + 3), x)
```

maple [B] time = 0.01, size = 43, normalized size = 3.58

$$\frac{\sqrt{-x^2+1}\,\sqrt{6x^2+9}\,\ \text{EllipticF}\left(x,\frac{i\sqrt{6}}{3}\right)}{3\sqrt{-2x^4-x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-x^2+3)^(1/2),x)
```

```
[Out] 1/3*(-x^2+1)^(1/2)*(6*x^2+9)^(1/2)/(-2*x^4-x^2+3)^(1/2)*EllipticF(x,1/3*I*6
^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4 - x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{-2\,x^4 - x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3 - 2*x^4 - x^2)^(1/2),x)
```

```
[Out] int(1/(3 - 2*x^4 - x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4 - x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - x**2 + 3), x)
```

3.36 $\int \frac{1}{\sqrt{3-2x^2-2x^4}}\,dx$

Optimal. Leaf size=42

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{7}}}\,x\right)\middle|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

```
[Out] EllipticF(x*2^(1/2)/(-1+7^(1/2))^(1/2),1/6*I*42^(1/2)-1/6*I*6^(1/2))/(1+7^(
1/2))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{7}}}\,x\right)\middle|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 2*x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3]/Sqrt[1 + Sqrt
[7]]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-2x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{-2+2\sqrt{7}-4x^2}\,\sqrt{2+2\sqrt{7}+4x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

Mathematica [C] time = 0.04, size = 51, normalized size = 1.21

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{7}}}\,x\right)|-\frac{4}{3}-\frac{\sqrt{7}}{3}\right)}{\sqrt{\sqrt{7}-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 - 2*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[2/(1 + Sqrt[7])]*x], -4/3 - Sqrt[7]/3])/Sqrt
[-1 + Sqrt[7]]
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4-2\,x^2+3}}{2\,x^4+2\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 2*x^2 + 3)/(2*x^4 + 2*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4-2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-2*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 2*x^2 + 3), x)
```

maple [B] time = 0.08, size = 84, normalized size = 2.00

$$\frac{3\sqrt{-\left(\frac{\sqrt{7}}{3}+\frac{1}{3}\right)x^2+1}\,\sqrt{-\left(-\frac{\sqrt{7}}{3}+\frac{1}{3}\right)x^2+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{3+3\sqrt{7}}\,x}{3},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{\sqrt{3+3\sqrt{7}}\,\sqrt{-2x^4-2x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-2*x^2+3)^(1/2),x)
```

```
[Out] 3/(3+3*7^(1/2))^(1/2)*(1-(1/3*7^(1/2)+1/3)*x^2)^(1/2)*(1-(-1/3*7^(1/2)+1/3)
*x^2)^(1/2)/(-2*x^4-2*x^2+3)^(1/2)*EllipticF(1/3*x*(3+3*7^(1/2))^(1/2),1/6*
I*42^(1/2)-1/6*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4-2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-2*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 2*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-2\,x^4-2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3 - 2*x^4 - 2*x^2)^(1/2),x)
```

```
[Out] int(1/(3 - 2*x^4 - 2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4-2x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-2*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 2*x**2 + 3), x)
```

3.37 $\int \frac{1}{\sqrt{3-3x^2-2x^4}}\,dx$

Optimal. Leaf size=43

$$\sqrt{\frac{2}{3+\sqrt{33}}}\,F\left(\sin^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

[Out] EllipticF(2*x/(-3+33^(1/2))^(1/2),1/4*I*22^(1/2)-1/4*I*6^(1/2))*2^(1/2)/(3+33^(1/2))^(1/2)

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{3+\sqrt{33}}}\,F\left(\sin^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[3 - 3*x^2 - 2*x^4],x]

[Out] Sqrt[2/(3 + Sqrt[33])]*EllipticF[ArcSin[(2*x)/Sqrt[-3 + Sqrt[33]]], (-7 + Sqrt[33])/4]

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-3x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{-3+\sqrt{33}-4x^2}\,\sqrt{3+\sqrt{33}+4x^2}}\,dx$$
$$= \sqrt{\frac{2}{3+\sqrt{33}}}\,F\left(\sin^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right)\middle|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

Mathematica [C] time = 0.04, size = 52, normalized size = 1.21

$$-i\sqrt{\frac{2}{\sqrt{33}-3}}\,F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right)\middle|-\frac{7}{4}-\frac{\sqrt{33}}{4}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 - 3*x^2 - 2*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(-3 + Sqrt[33])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[3 + Sqrt[33]]],
 -7/4 - Sqrt[33]/4]
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4-3\,x^2+3}}{2\,x^4+3\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 3*x^2 + 3)/(2*x^4 + 3*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4-3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 3*x^2 + 3), x)
```

maple [B] time = 0.07, size = 84, normalized size = 1.95

$$\frac{6\sqrt{-\left(\frac{\sqrt{33}}{6}+\frac{1}{2}\right)x^2+1}\,\sqrt{-\left(-\frac{\sqrt{33}}{6}+\frac{1}{2}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{18+6\sqrt{33}}\,x}{6},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{\sqrt{18+6\sqrt{33}}\,\sqrt{-2x^4-3x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-3*x^2+3)^(1/2),x)
```

```
[Out] 6/(18+6*33^(1/2))^(1/2)*(1-(1/6*33^(1/2)+1/2)*x^2)^(1/2)*(1-(-1/6*33^(1/2)+
1/2)*x^2)^(1/2)/(-2*x^4-3*x^2+3)^(1/2)*EllipticF(1/6*x*(18+6*33^(1/2))^(1/2
),1/4*I*22^(1/2)-1/4*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4-3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 3*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-2\,x^4-3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3 - 2*x^4 - 3*x^2)^(1/2),x)
```

```
[Out] int(1/(3 - 2*x^4 - 3*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4-3x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-3*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 3*x**2 + 3), x)
```

3.38 $\int \frac{1}{\sqrt{3-4x^2-2x^4}}\,dx$

Optimal. Leaf size=44

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{-2+\sqrt{10}}}\,x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2+\sqrt{10}}}$$

```
[Out] EllipticF(x*2^(1/2)/(-2+10^(1/2))^(1/2),1/3*I*15^(1/2)-1/3*I*6^(1/2))/(2+10
^(1/2))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{-2+\sqrt{10}}}\,x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2+\sqrt{10}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 4*x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2/(-2 + Sqrt[10])]*x], (-7 + 2*Sqrt[10])/3]/Sqrt[2 +
Sqrt[10]]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-4x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{-4+2\sqrt{10}-4x^2}\sqrt{4+2\sqrt{10}+4x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{-2+\sqrt{10}}}\,x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2+\sqrt{10}}}$$

Mathematica [C] time = 0.05, size = 51, normalized size = 1.16

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{\frac{2}{2+\sqrt{10}}}\,x\right)|-\frac{7}{3}-\frac{2\sqrt{10}}{3}\right)}{\sqrt{\sqrt{10}-2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 - 4*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[2/(2 + Sqrt[10])]*x], -7/3 - (2*Sqrt[10])/3]
)/Sqrt[-2 + Sqrt[10]]
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-2\,x^4-4\,x^2+3}}{2\,x^4+4\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-4*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 4*x^2 + 3)/(2*x^4 + 4*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4-4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-4*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 4*x^2 + 3), x)
```

maple [B] time = 0.08, size = 84, normalized size = 1.91

$$\frac{3\sqrt{-\left(\frac{\sqrt{10}}{3}+\frac{2}{3}\right)x^{2}+1}\,\sqrt{-\left(-\frac{\sqrt{10}}{3}+\frac{2}{3}\right)x^{2}+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{6+3\sqrt{10}}\,x}{3},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{\sqrt{6+3\sqrt{10}}\,\sqrt{-2x^{4}-4x^{2}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-4*x^2+3)^(1/2),x)
```

```
[Out] 3/(6+3*10^(1/2))^(1/2)*(1-(1/3*10^(1/2)+2/3)*x^2)^(1/2)*(1-(-1/3*10^(1/2)+2
/3)*x^2)^(1/2)/(-2*x^4-4*x^2+3)^(1/2)*EllipticF(1/3*x*(6+3*10^(1/2))^(1/2),
1/3*I*15^(1/2)-1/3*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^{4}-4\,x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-4*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 4*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-2\,x^{4}-4\,x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3 - 2*x^4 - 4*x^2)^(1/2),x)
```

```
[Out] int(1/(3 - 2*x^4 - 4*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^{4}-4x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-4*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 4*x**2 + 3), x)
```

3.39 $\int \frac{1}{\sqrt{3-5x^2-2x^4}}\,dx$

Optimal. Leaf size=18

$$\frac{F\left(\sin^{-1}\left(\sqrt{2}\,x\right)|-\frac{1}{6}\right)}{\sqrt{6}}$$

```
[Out] 1/6*EllipticF(x*2^(1/2),1/6*I*6^(1/2))*6^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}\left(\sqrt{2}\,x\right)|-\frac{1}{6}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 5*x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2]*x], -1/6]/Sqrt[6]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-5x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{2-4x^2}\,\sqrt{12+4x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{2}\,x\right)|-\frac{1}{6}\right)}{\sqrt{6}}$$

Mathematica [B] time = 0.03, size = 54, normalized size = 3.00

$$\frac{\sqrt{1-2x^2}\,\sqrt{x^2+3}\,F\left(\sin^{-1}\left(\sqrt{2}\,x\right)|-\frac{1}{6}\right)}{\sqrt{6}\,\sqrt{-2x^4-5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 5*x^2 - 2*x^4],x]
[Out] (Sqrt[1 - 2*x^2]*Sqrt[3 + x^2]*EllipticF[ArcSin[Sqrt[2]*x], -1/6])/(Sqrt[6]
*Sqrt[3 - 5*x^2 - 2*x^4])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-2\,x^4-5\,x^2+3}}{2\,x^4+5\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-5*x^2+3)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-2*x^4 - 5*x^2 + 3)/(2*x^4 + 5*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4-5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-5*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-2*x^4 - 5*x^2 + 3), x)
```

maple [B] time = 0.02, size = 50, normalized size = 2.78

$$\frac{\sqrt{2}\,\sqrt{-2x^2+1}\,\sqrt{3x^2+9}\,\operatorname{EllipticF}\left(\sqrt{2}\,x,\frac{i\sqrt{6}}{6}\right)}{6\sqrt{-2x^4-5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-5*x^2+3)^(1/2),x)
```

```
[Out] 1/6*2^(1/2)*(-2*x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-2*x^4-5*x^2+3)^(1/2)*Ellipti
cF(2^(1/2)*x,1/6*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4 - 5\,x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 5*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{-2\,x^4 - 5\,x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3 - 2*x^4 - 5*x^2)^(1/2),x)
```

```
[Out] int(1/(3 - 2*x^4 - 5*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4 - 5x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-5*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 5*x**2 + 3), x)
```

3.40 $\int \frac{1}{\sqrt{3-6x^2-2x^4}} \, dx$

Optimal. Leaf size=42

$$\sqrt{\frac{1}{6}\left(\sqrt{15}-3\right)}F\left(\sin^{-1}\left(\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}x\right)|-4+\sqrt{15}\right)$$

```
[Out] 1/6*EllipticF(1/3*x*(9+3*15^(1/2))^(1/2),1/2*I*10^(1/2)-1/2*I*6^(1/2))*(-18
+6*15^(1/2))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{1}{6}\left(\sqrt{15}-3\right)}F\left(\sin^{-1}\left(\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}x\right)|-4+\sqrt{15}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 6*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[(-3 + Sqrt[15])/6]*EllipticF[ArcSin[Sqrt[(3 + Sqrt[15])/3]*x], -4 + Sq
rt[15]]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-6x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{-6+2\sqrt{15}-4x^2}\,\sqrt{6+2\sqrt{15}+4x^2}}\,dx$$
$$= \sqrt{\frac{1}{6}\left(-3+\sqrt{15}\right)}\,F\left(\sin^{-1}\left(\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}\,x\right)\middle|-4+\sqrt{15}\right)$$

Mathematica [C] time = 0.06, size = 45, normalized size = 1.07

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{-1+\sqrt{\frac{5}{3}}}\,x\right)\middle|-4-\sqrt{15}\right)}{\sqrt{\sqrt{15}-3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 - 6*x^2 - 2*x^4],x]
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[-1 + Sqrt[5/3]]*x], -4 - Sqrt[15]])/Sqrt[-3
+ Sqrt[15]]
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4-6\,x^2+3}}{2\,x^4+6\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-6*x^2+3)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-2*x^4 - 6*x^2 + 3)/(2*x^4 + 6*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4-6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-6*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-2*x^4 - 6*x^2 + 3), x)
```

maple [B] time = 0.10, size = 84, normalized size = 2.00

$$\frac{3\sqrt{-\left(1+\frac{\sqrt{15}}{3}\right)x^{2}+1}\,\sqrt{-\left(1-\frac{\sqrt{15}}{3}\right)x^{2}+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{9+3\sqrt{15}}\,x}{3},\frac{i\sqrt{10}}{2}-\frac{i\sqrt{6}}{2}\right)}{\sqrt{9+3\sqrt{15}}\,\sqrt{-2x^{4}-6x^{2}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-6*x^2+3)^(1/2),x)
```

```
[Out] 3/(9+3*15^(1/2))^(1/2)*(1-(1+1/3*15^(1/2))*x^2)^(1/2)*(1-(1-1/3*15^(1/2))*x
^2)^(1/2)/(-2*x^4-6*x^2+3)^(1/2)*EllipticF(1/3*x*(9+3*15^(1/2))^(1/2),1/2*I
*10^(1/2)-1/2*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^{4}-6\,x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-6*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 6*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-2\,x^{4}-6\,x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3 - 2*x^4 - 6*x^2)^(1/2),x)
```

```
[Out] int(1/(3 - 2*x^4 - 6*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^{4}-6x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-6*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 6*x**2 + 3), x)
```

3.41 $\int \frac{1}{\sqrt{3-7x^2-2x^4}} \, dx$

Optimal. Leaf size=45

$$\sqrt{\frac{2}{7+\sqrt{73}}}\, F\left(\sin^{-1}\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61+7\sqrt{73}\right)\right)$$

```
[Out] EllipticF(2*x/(-7+73^(1/2))^(1/2),1/12*I*438^(1/2)-7/12*I*6^(1/2))*2^(1/2)/
(7+73^(1/2))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{7+\sqrt{73}}}\, F\left(\sin^{-1}\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61+7\sqrt{73}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 7*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[2/(7 + Sqrt[73])]*EllipticF[ArcSin[(2*x)/Sqrt[-7 + Sqrt[73]]], (-61 +
7*Sqrt[73])/12]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-7x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{-7+\sqrt{73}-4x^2}\sqrt{7+\sqrt{73}+4x^2}}\,dx$$

$$= \sqrt{\frac{2}{7+\sqrt{73}}}\,F\left(\sin^{-1}\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61+7\sqrt{73}\right)\right)$$

Mathematica [C] time = 0.05, size = 52, normalized size = 1.16

$$-i\sqrt{\frac{2}{\sqrt{73}-7}}\,F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 - 7*x^2 - 2*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(-7 + Sqrt[73])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[7 + Sqrt[73]]],
 (-61 - 7*Sqrt[73])/12]
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4-7\,x^2+3}}{2\,x^4+7\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-7*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 7*x^2 + 3)/(2*x^4 + 7*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4-7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-7*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 7*x^2 + 3), x)
```

maple [B] time = 0.09, size = 84, normalized size = 1.87

$$\frac{6\sqrt{-\left(\frac{\sqrt{73}}{6}+\frac{7}{6}\right)x^2+1}\,\sqrt{-\left(\frac{7}{6}-\frac{\sqrt{73}}{6}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{42+6\sqrt{73}}\,x}{6},\frac{i\sqrt{438}}{12}-\frac{7i\sqrt{6}}{12}\right)}{\sqrt{42+6\sqrt{73}}\,\sqrt{-2x^4-7x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-7*x^2+3)^(1/2),x)
```

```
[Out] 6/(42+6*73^(1/2))^(1/2)*(1-(1/6*73^(1/2)+7/6)*x^2)^(1/2)*(1-(7/6-1/6*73^(1/
2))*x^2)^(1/2)/(-2*x^4-7*x^2+3)^(1/2)*EllipticF(1/6*x*(42+6*73^(1/2))^(1/2)
,1/12*I*438^(1/2)-7/12*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4-7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-7*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 7*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-2\,x^4-7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3 - 2*x^4 - 7*x^2)^(1/2),x)
```

```
[Out] int(1/(3 - 2*x^4 - 7*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4-7x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-7*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 7*x**2 + 3), x)
```

3.42 $\int \frac{1}{\sqrt{-2+5x^2+3x^4}}\,dx$

Optimal. Leaf size=67

$$\frac{\sqrt{x^2+2}\,\sqrt{3x^2-1}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{7}{2}}\,x}{\sqrt{3x^2-1}}\right)|\frac{6}{7}\right)}{\sqrt{7}\,\sqrt{3x^4+5x^2-2}}$$

```
[Out] 1/7*EllipticF(1/2*x*14^(1/2)/(3*x^2-1)^(1/2),1/7*42^(1/2))*(x^2+2)^(1/2)*(3
*x^2-1)^(1/2)*7^(1/2)/(3*x^4+5*x^2-2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1097}

$$\frac{\sqrt{x^2+2}\,\sqrt{3x^2-1}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{7}{2}}\,x}{\sqrt{3x^2-1}}\right)|\frac{6}{7}\right)}{\sqrt{7}\,\sqrt{3x^4+5x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 5*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[2 + x^2]*Sqrt[-1 + 3*x^2]*EllipticF[ArcSin[(Sqrt[7/2]*x)/Sqrt[-1 + 3*
x^2]], 6/7])/(Sqrt[7]*Sqrt[-2 + 5*x^2 + 3*x^4])
```

Rule 1097

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[-2*a - (b - q)*x^2]*Sqrt[(2*a + (b + q)*x^2)/q]
*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)])/(2*Sq
rt[-a]*Sqrt[a + b*x^2 + c*x^4]), x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+5x^2+3x^4}}\,dx=\frac{\sqrt{2+x^2}\,\sqrt{-1+3x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{7}{2}}\,x}{\sqrt{-1+3x^2}}\right)|\frac{6}{7}\right)}{\sqrt{7}\,\sqrt{-2+5x^2+3x^4}}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.81

$$\frac{\sqrt{1-3x^2}\sqrt{x^2+2}\,F\left(\sin^{-1}\left(\sqrt{3}\,x\right)|-\frac{1}{6}\right)}{\sqrt{6}\sqrt{3x^4+5x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 5*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[1 - 3*x^2]*Sqrt[2 + x^2]*EllipticF[ArcSin[Sqrt[3]*x], -1/6])/(Sqrt[6]
*Sqrt[-2 + 5*x^2 + 3*x^4])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+5\,x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 5*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{3\,x^4+5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 - 2), x)
```

maple [C] time = 0.01, size = 53, normalized size = 0.79

$$-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-3x^2+1}\;\operatorname{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},i\sqrt{6}\right)}{2\sqrt{3x^4+5x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+5*x^2-2)^(1/2),x)
```

```
[Out] -1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(-3*x^2+1)^(1/2)/(3*x^4+5*x^2-2)^(1/2)*Ellip
ticF(1/2*I*2^(1/2)*x,I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4+5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4+5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 + 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 + 3*x^4 - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4+5x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+5*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 5*x**2 - 2), x)
```

3.43 $\int \frac{1}{\sqrt{-2+4x^2+3x^4}}\,dx$

Optimal. Leaf size=141

$$\frac{\sqrt{\frac{2-\left(2-\sqrt{10}\right)x^2}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{\left(2+\sqrt{10}\right)x^2-2}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}\,x}{\sqrt{\left(2+\sqrt{10}\right)x^2-2}}\right)|\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{\frac{1}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{3x^4+4x^2-2}}$$

```
[Out] 1/20*EllipticF(2^(3/4)*5^(1/4)*x/(-2+x^2*(2+10^(1/2)))^(1/2),1/10*(50+10*10
^(1/2))^(1/2))*((2-x^2*(2-10^(1/2)))/(2-x^2*(2+10^(1/2))))^(1/2)*(-2+x^2*(2
+10^(1/2)))^(1/2)*10^(3/4)/(3*x^4+4*x^2-2)^(1/2)/(1/(2-x^2*(2+10^(1/2))))^(
1/2)
```

Rubi [A] time = 0.03, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{\frac{2-\left(2-\sqrt{10}\right)x^2}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{\left(2+\sqrt{10}\right)x^2-2}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}\,x}{\sqrt{\left(2+\sqrt{10}\right)x^2-2}}\right)|\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{\frac{1}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{3x^4+4x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 4*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[(2 - (2 - Sqrt[10])*x^2)/(2 - (2 + Sqrt[10])*x^2)]*Sqrt[-2 + (2 + Sqr
t[10])*x^2]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-2 + (2 + Sqrt[10])*x
^2]], (5 + Sqrt[10])/10])/(2*10^(1/4)*Sqrt[(2 - (2 + Sqrt[10])*x^2)^(-1)]*S
qrt[-2 + 4*x^2 + 3*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+4x^2+3x^4}}\,dx = \frac{\sqrt{\frac{2-\left(2-\sqrt{10}\right)x^2}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{-2+\left(2+\sqrt{10}\right)x^2}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}\,x}{\sqrt{-2+\left(2+\sqrt{10}\right)x^2}}\right)|\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{\frac{1}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{-2+4x^2+3x^4}}$$

Mathematica [C] time = 0.05, size = 81, normalized size = 0.57

$$-\frac{i\sqrt{-3x^4-4x^2+2}\,F\left(i\sinh^{-1}\left(\sqrt{-1+\sqrt{\frac{5}{2}}}\,x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{\sqrt{10}-2}\sqrt{3x^4+4x^2-2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-2 + 4*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[2 - 4*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[-1 + Sqrt[5/2]]*x],
(-7 - 2*Sqrt[10])/3])/(Sqrt[-2 + Sqrt[10]]*Sqrt[-2 + 4*x^2 + 3*x^4])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+4\,x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+4*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 4*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4+4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+4*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 4*x^2 - 2), x)
```

maple [C] time = 0.03, size = 84, normalized size = 0.60

$$\frac{2\sqrt{-\left(-\frac{\sqrt{10}}{2}+1\right)x^2+1}\,\sqrt{-\left(1+\frac{\sqrt{10}}{2}\right)x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{4-2\sqrt{10}}\,x}{2},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{\sqrt{4-2\sqrt{10}}\,\sqrt{3x^4+4x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+4*x^2-2)^(1/2),x)
```

```
[Out] 2/(4-2*10^(1/2))^(1/2)*(-(-1/2*10^(1/2)+1)*x^2+1)^(1/2)*(-(1+1/2*10^(1/2))*
x^2+1)^(1/2)/(3*x^4+4*x^2-2)^(1/2)*EllipticF(1/2*(4-2*10^(1/2))^(1/2)*x,1/3
*I*6^(1/2)+1/3*I*15^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4+4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+4*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 4*x^2 - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4+4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 + 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(4*x^2 + 3*x^4 - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4+4x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+4*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 4*x**2 - 2), x)
```

3.44 $\int \frac{1}{\sqrt{-2+3x^2+3x^4}}\,dx$

Optimal. Leaf size=146

$$\frac{\sqrt{\frac{4-\left(3-\sqrt{33}\right)x^2}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{\left(3+\sqrt{33}\right)x^2-4}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}\,x}{\sqrt{\left(3+\sqrt{33}\right)x^2-4}}\right)|\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{2\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{3x^4+3x^2-2}}$$

```
[Out] 1/132*EllipticF(33^(1/4)*x*2^(1/2)/(-4+x^2*(3+33^(1/2)))^(1/2),1/22*(242+22
*33^(1/2))^(1/2))*((4-x^2*(3-33^(1/2)))/(4-x^2*(3+33^(1/2))))^(1/2)*(-4+x^2
*(3+33^(1/2)))^(1/2)*33^(3/4)*2^(1/2)/(3*x^4+3*x^2-2)^(1/2)/(1/(4-x^2*(3+33
^(1/2))))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{\frac{4-\left(3-\sqrt{33}\right)x^2}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{\left(3+\sqrt{33}\right)x^2-4}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}\,x}{\sqrt{\left(3+\sqrt{33}\right)x^2-4}}\right)|\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{2\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{3x^4+3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 3*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[(4 - (3 - Sqrt[33])*x^2)/(4 - (3 + Sqrt[33])*x^2)]*Sqrt[-4 + (3 + Sqr
t[33])*x^2]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-4 + (3 + Sqrt[33])*
x^2]], (11 + Sqrt[33])/22])/(2*Sqrt[2]*33^(1/4)*Sqrt[(4 - (3 + Sqrt[33])*x^
2)^(-1)]*Sqrt[-2 + 3*x^2 + 3*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+3x^2+3x^4}}\,dx = \frac{\sqrt{\frac{4-\left(3-\sqrt{33}\right)x^2}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{-4+\left(3+\sqrt{33}\right)x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}\,x}{\sqrt{-4+\left(3+\sqrt{33}\right)x^2}}\right)|\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{2\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{-2+3x^2+3x^4}}$$

Mathematica [C] time = 0.06, size = 83, normalized size = 0.57

$$-\frac{i\sqrt{-6x^4-6x^2+4}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{6}{3+\sqrt{33}}}\,x\right)|-\frac{7}{4}-\frac{\sqrt{33}}{4}\right)}{\sqrt{\sqrt{33}-3}\sqrt{3x^4+3x^2-2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-2 + 3*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[4 - 6*x^2 - 6*x^4]*EllipticF[I*ArcSinh[Sqrt[6/(3 + Sqrt[33])]*x]
, -7/4 - Sqrt[33]/4])/(Sqrt[-3 + Sqrt[33]]*Sqrt[-2 + 3*x^2 + 3*x^4])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+3\,x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+3*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 3*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4+3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+3*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 3*x^2 - 2), x)
```

maple [C] time = 0.04, size = 84, normalized size = 0.58

$$\frac{2\sqrt{-\left(-\frac{\sqrt{33}}{4}+\frac{3}{4}\right)x^2+1}\,\sqrt{-\left(\frac{\sqrt{33}}{4}+\frac{3}{4}\right)x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{3-\sqrt{33}}\,x}{2},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{\sqrt{3-\sqrt{33}}\,\sqrt{3x^4+3x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+3*x^2-2)^(1/2),x)
```

```
[Out] 2/(3-33^(1/2))^(1/2)*(-(-1/4*33^(1/2)+3/4)*x^2+1)^(1/2)*(-(1/4*33^(1/2)+3/4
)*x^2+1)^(1/2)/(3*x^4+3*x^2-2)^(1/2)*EllipticF(1/2*(3-33^(1/2))^(1/2)*x,1/4
*I*6^(1/2)+1/4*I*22^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4+3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+3*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 3*x^2 - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4+3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 + 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(3*x^2 + 3*x^4 - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4+3x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+3*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 3*x**2 - 2), x)
```

3.45 $\int \frac{1}{\sqrt{-2+2x^2+3x^4}}\,dx$

Optimal. Leaf size=141

$$\frac{\sqrt{\frac{2-\left(1-\sqrt{7}\right)x^2}{2-\left(1+\sqrt{7}\right)x^2}}\sqrt{\left(1+\sqrt{7}\right)x^2-2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{\left(1+\sqrt{7}\right)x^2-2}}\right)|\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{2\sqrt[4]{7}\sqrt{\frac{1}{2-\left(1+\sqrt{7}\right)x^2}}\sqrt{3x^4+2x^2-2}}$$

```
[Out] 1/14*EllipticF(7^(1/4)*x*2^(1/2)/(-2+x^2*(1+7^(1/2)))^(1/2),1/14*(98+14*7^(
1/2))^(1/2))*((2-x^2*(1-7^(1/2)))/(2-x^2*(1+7^(1/2))))^(1/2)*(-2+x^2*(1+7^(
1/2)))^(1/2)*7^(3/4)/(3*x^4+2*x^2-2)^(1/2)/(1/(2-x^2*(1+7^(1/2))))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{\frac{2-\left(1-\sqrt{7}\right)x^2}{2-\left(1+\sqrt{7}\right)x^2}}\sqrt{\left(1+\sqrt{7}\right)x^2-2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{\left(1+\sqrt{7}\right)x^2-2}}\right)|\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{2\sqrt[4]{7}\sqrt{\frac{1}{2-\left(1+\sqrt{7}\right)x^2}}\sqrt{3x^4+2x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 2*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[(2 - (1 - Sqrt[7])*x^2)/(2 - (1 + Sqrt[7])*x^2)]*Sqrt[-2 + (1 + Sqrt[
7])*x^2]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-2 + (1 + Sqrt[7])*x^2]]
, (7 + Sqrt[7])/14])/(2*7^(1/4)*Sqrt[(2 - (1 + Sqrt[7])*x^2)^(-1)]*Sqrt[-2
+ 2*x^2 + 3*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+2x^2+3x^4}}\,dx = \frac{\sqrt{\frac{2-(1-\sqrt{7})x^2}{2-(1+\sqrt{7})x^2}}\sqrt{-2+\left(1+\sqrt{7}\right)x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}\,x}{\sqrt{-2+(1+\sqrt{7})x^2}}\right)|\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{2\sqrt[4]{7}\sqrt{\frac{1}{2-(1+\sqrt{7})x^2}}\sqrt{-2+2x^2+3x^4}}$$

Mathematica [C] time = 0.05, size = 83, normalized size = 0.59

$$-\frac{i\sqrt{-3x^4-2x^2+2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{3}{1+\sqrt{7}}}\,x\right)|-\frac{4}{3}-\frac{\sqrt{7}}{3}\right)}{\sqrt{\sqrt{7}-1}\sqrt{3x^4+2x^2-2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-2 + 2*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[2 - 2*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[3/(1 + Sqrt[7])]*x],
 -4/3 - Sqrt[7]/3])/(Sqrt[-1 + Sqrt[7]]*Sqrt[-2 + 2*x^2 + 3*x^4])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+2\,x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 2*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4+2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 2*x^2 - 2), x)
```

maple [C] time = 0.04, size = 84, normalized size = 0.60

$$\frac{2\sqrt{-\left(-\frac{\sqrt{7}}{2}+\frac{1}{2}\right)x^2+1}\,\sqrt{-\left(\frac{\sqrt{7}}{2}+\frac{1}{2}\right)x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{2-2\sqrt{7}}\,x}{2},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{\sqrt{2-2\sqrt{7}}\,\sqrt{3x^4+2x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+2*x^2-2)^(1/2),x)
```

```
[Out] 2/(2-2*7^(1/2))^(1/2)*(-(-1/2*7^(1/2)+1/2)*x^2+1)^(1/2)*(-(1/2*7^(1/2)+1/2)
*x^2+1)^(1/2)/(3*x^4+2*x^2-2)^(1/2)*EllipticF(1/2*(2-2*7^(1/2))^(1/2)*x,1/6
*I*6^(1/2)+1/6*I*42^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4+2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 2*x^2 - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4+2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 + 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(2*x^2 + 3*x^4 - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4+2x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+2*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 2*x**2 - 2), x)
```

3.46 $\int \frac{1}{\sqrt{-2+x^2+3x^4}}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{x^2+1}\,\sqrt{3x^2-2}\,F\left(\sin^{-1}\left(\frac{\sqrt{5}\,x}{\sqrt{3x^2-2}}\right)|\frac{3}{5}\right)}{\sqrt{5}\,\sqrt{3x^4+x^2-2}}$$

```
[Out] 1/5*EllipticF(x*5^(1/2)/(3*x^2-2)^(1/2),1/5*15^(1/2))*(x^2+1)^(1/2)*(3*x^2-
2)^(1/2)*5^(1/2)/(3*x^4+x^2-2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1097}

$$\frac{\sqrt{x^2+1}\,\sqrt{3x^2-2}\,F\left(\sin^{-1}\left(\frac{\sqrt{5}\,x}{\sqrt{3x^2-2}}\right)|\frac{3}{5}\right)}{\sqrt{5}\,\sqrt{3x^4+x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[1 + x^2]*Sqrt[-2 + 3*x^2]*EllipticF[ArcSin[(Sqrt[5]*x)/Sqrt[-2 + 3*x^
2]], 3/5])/(Sqrt[5]*Sqrt[-2 + x^2 + 3*x^4])
```

Rule 1097

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[-2*a - (b - q)*x^2]*Sqrt[(2*a + (b + q)*x^2)/q]
*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)])/(2*Sq
rt[-a]*Sqrt[a + b*x^2 + c*x^4]), x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+x^2+3x^4}}\,dx=\frac{\sqrt{1+x^2}\,\sqrt{-2+3x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{5}\,x}{\sqrt{-2+3x^2}}\right)|\frac{3}{5}\right)}{\sqrt{5}\,\sqrt{-2+x^2+3x^4}}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.76

$$\frac{\sqrt{\left(\frac{2}{3}-x^2\right)\left(x^2+1\right)}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{2}{3}\right)}{\sqrt{3x^4+x^2-2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-2 + x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[(2/3 - x^2)*(1 + x^2)]*EllipticF[ArcSin[Sqrt[3/2]*x], -2/3])/Sqrt[-2
+ x^2 + 3*x^4]
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4+x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + x^2 - 2), x)
```

maple [C] time = 0.01, size = 43, normalized size = 0.68

$$-\frac{i\sqrt{x^2+1}\,\sqrt{-6x^2+4}\,\text{ EllipticF}\left(ix,\frac{i\sqrt{6}}{2}\right)}{2\sqrt{3x^4+x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+x^2-2)^(1/2),x)
```

```
[Out] -1/2*I*(x^2+1)^(1/2)*(-6*x^2+4)^(1/2)/(3*x^4+x^2-2)^(1/2)*EllipticF(I*x,1/2
*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4+x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + x^2 - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{3\,x^4+x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(x^2 + 3*x^4 - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4+x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + x**2 - 2), x)
```

3.47 $\int \frac{1}{\sqrt{-2+3x^4}}\,dx$

Optimal. Leaf size=115

$$\frac{\sqrt{\sqrt{6}\,x^2-2}\,\sqrt{\frac{\sqrt{6}\,x^2+2}{2-\sqrt{6}\,x^2}}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{3}\,x}{\sqrt{\sqrt{6}\,x^2-2}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{\frac{1}{2-\sqrt{6}\,x^2}}\,\sqrt{3x^4-2}}$$

```
[Out] 1/12*EllipticF(2^(3/4)*3^(1/4)*x/(-2+x^2*6^(1/2))^(1/2),1/2*2^(1/2))*(-2+x^
2*6^(1/2))^(1/2)*((2+x^2*6^(1/2))/(2-x^2*6^(1/2)))^(1/2)*6^(3/4)/(3*x^4-2)^
(1/2)/(1/(2-x^2*6^(1/2)))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {223}

$$\frac{\sqrt{\sqrt{6}\,x^2-2}\,\sqrt{\frac{\sqrt{6}\,x^2+2}{2-\sqrt{6}\,x^2}}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{3}\,x}{\sqrt{\sqrt{6}\,x^2-2}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{\frac{1}{2-\sqrt{6}\,x^2}}\,\sqrt{3x^4-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 3*x^4],x]
```

```
[Out] (Sqrt[-2 + Sqrt[6]*x^2]*Sqrt[(2 + Sqrt[6]*x^2)/(2 - Sqrt[6]*x^2)]*EllipticF
[ArcSin[(2^(3/4)*3^(1/4)*x)/Sqrt[-2 + Sqrt[6]*x^2]], 1/2])/(2*6^(1/4)*Sqrt[
(2 - Sqrt[6]*x^2)^(-1)]*Sqrt[-2 + 3*x^4])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[(a - q*x^2)/(a + q*x^2)]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqr
t[(a + q*x^2)/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[a + b*x^4]*Sqrt[a/(a + q*x^2)]),
 x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+3x^4}}\,dx = \frac{\sqrt{-2+\sqrt{6}\,x^2}\sqrt{\frac{2+\sqrt{6}\,x^2}{2-\sqrt{6}\,x^2}}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{3}\,x}{\sqrt{-2+\sqrt{6}\,x^2}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{\frac{1}{2-\sqrt{6}\,x^2}}\sqrt{-2+3x^4}}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.35

$$\frac{\sqrt{2-3x^4}\,F\left(\sin^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)\middle|-1\right)}{\sqrt[4]{6}\sqrt{3x^4-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 3*x^4],x]
```

```
[Out] (Sqrt[2 - 3*x^4]*EllipticF[ArcSin[(3/2)^(1/4)*x], -1])/(6^(1/4)*Sqrt[-2 + 3
*x^4])
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 2), x)
```

maple [C] time = 0.02, size = 56, normalized size = 0.49

$$\frac{\sqrt{2\sqrt{6}\,x^2+4}\,\sqrt{-2\sqrt{6}\,x^2+4}\;\mathrm{EllipticF}\left(\frac{\sqrt{-2\sqrt{6}}\,x}{2},i\right)}{2\sqrt{-2\sqrt{6}}\,\sqrt{3x^4-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-2)^(1/2),x)
```

```
[Out] 1/2/(-2*6^(1/2))^(1/2)*(2*6^(1/2)*x^2+4)^(1/2)*(-2*6^(1/2)*x^2+4)^(1/2)/(3*
x^4-2)^(1/2)*EllipticF(1/2*(-2*6^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 2), x)
```

mupad [B] time = 0.08, size = 31, normalized size = 0.27

$$\frac{x\,\sqrt{4-6\,x^4}\;{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\;\frac{5}{4};\;\frac{3\,x^4}{2}\right)}{2\,\sqrt{3\,x^4-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - 2)^(1/2),x)
```

```
[Out] (x*(4 - 6*x^4)^(1/2)*hypergeom([1/4, 1/2], 5/4, (3*x^4)/2))/(2*(3*x^4 - 2)^
(1/2))
```

sympy [C] time = 0.73, size = 34, normalized size = 0.30

$$-\frac{\sqrt{2}\,ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{3x^4}{2}\right)}{8\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-2)**(1/2),x)
```

```
[Out] -sqrt(2)*I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 3*x**4/2)/(8*gamma(5/4))
```

3.48 $\int \frac{1}{\sqrt{-2-x^2+3x^4}}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{x^2-1}\,\sqrt{3x^2+2}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{5}{2}}\,x}{\sqrt{x^2-1}}\right)|\frac{2}{5}\right)}{\sqrt{5}\,\sqrt{3x^4-x^2-2}}$$

```
[Out] 1/5*EllipticF(1/2*x*10^(1/2)/(x^2-1)^(1/2),1/5*10^(1/2))*(x^2-1)^(1/2)*(3*x
^2+2)^(1/2)*5^(1/2)/(3*x^4-x^2-2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1097}

$$\frac{\sqrt{x^2-1}\,\sqrt{3x^2+2}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{5}{2}}\,x}{\sqrt{x^2-1}}\right)|\frac{2}{5}\right)}{\sqrt{5}\,\sqrt{3x^4-x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[-1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[ArcSin[(Sqrt[5/2]*x)/Sqrt[-1 + x^
2]], 2/5])/(Sqrt[5]*Sqrt[-2 - x^2 + 3*x^4])
```

Rule 1097

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[-2*a - (b - q)*x^2]*Sqrt[(2*a + (b + q)*x^2)/q]
*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)])/(2*Sq
rt[-a]*Sqrt[a + b*x^2 + c*x^4]), x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-x^2+3x^4}}\,dx = \frac{\sqrt{-1+x^2}\,\sqrt{2+3x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{5}{2}}\,x}{\sqrt{-1+x^2}}\right)|\frac{2}{5}\right)}{\sqrt{5}\,\sqrt{-2-x^2+3x^4}}$$

Mathematica [C] time = 0.02, size = 60, normalized size = 0.92

$$-\frac{i\sqrt{1-x^2}\sqrt{3x^2+2}F\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{9x^4-3x^2-6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], -2/3]
)/Sqrt[-6 - 3*x^2 + 9*x^4]
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3x^4-x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4-x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - x^2 - 2), x)
```

maple [C] time = 0.03, size = 53, normalized size = 0.82

$$-\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{-x^2+1}\;\operatorname{EllipticF}\left(\frac{i\sqrt{6}\,x}{2},\frac{i\sqrt{6}}{3}\right)}{6\sqrt{3x^4-x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-x^2-2)^(1/2),x)
```

```
[Out] -1/6*I*6^(1/2)*(6*x^2+4)^(1/2)*(-x^2+1)^(1/2)/(3*x^4-x^2-2)^(1/2)*EllipticF
(1/2*I*x*6^(1/2),1/3*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4-x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - x^2 - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{3\,x^4-x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - x^2 - 2)^(1/2),x)
```

```
[Out] int(1/(3*x^4 - x^2 - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4-x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - x**2 - 2), x)
```

3.49 $\int \frac{1}{\sqrt{-2-2x^2+3x^4}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{-\left(\left(1-\sqrt{7}\right)x^2\right)-2}\sqrt{\frac{\left(1+\sqrt{7}\right)x^2+2}{\left(1-\sqrt{7}\right)x^2+2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-\left(\left(1-\sqrt{7}\right)x^2\right)-2}}\right)|\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{2\sqrt[4]{7}\sqrt{\frac{1}{\left(1-\sqrt{7}\right)x^2+2}}\sqrt{3x^4-2x^2-2}}$$

```
[Out] 1/14*EllipticF(7^(1/4)*x*2^(1/2)/(-2-x^2*(1-7^(1/2)))^(1/2),1/14*(98-14*7^(
1/2))^(1/2))*(-2-x^2*(1-7^(1/2)))^(1/2)*((2+x^2*(1+7^(1/2)))/(2+x^2*(1-7^(1
/2))))^(1/2)*7^(3/4)/(3*x^4-2*x^2-2)^(1/2)/(1/(2+x^2*(1-7^(1/2))))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{-\left(1-\sqrt{7}\right)x^2-2}\sqrt{\frac{\left(1+\sqrt{7}\right)x^2+2}{\left(1-\sqrt{7}\right)x^2+2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-\left(1-\sqrt{7}\right)x^2-2}}\right)|\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{2\sqrt[4]{7}\sqrt{\frac{1}{\left(1-\sqrt{7}\right)x^2+2}}\sqrt{3x^4-2x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 2*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[-2 - (1 - Sqrt[7])*x^2]*Sqrt[(2 + (1 + Sqrt[7])*x^2)/(2 + (1 - Sqrt[7
])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[7])*x^2]]
, (7 - Sqrt[7])/14])/(2*7^(1/4)*Sqrt[(2 + (1 - Sqrt[7])*x^2)^(-1)]*Sqrt[-2
- 2*x^2 + 3*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-2x^2+3x^4}}\,dx = \frac{\sqrt{-2-\left(1-\sqrt{7}\right)x^2}\sqrt{\frac{2+\left(1+\sqrt{7}\right)x^2}{2+\left(1-\sqrt{7}\right)x^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}\,x}{\sqrt{-2-\left(1-\sqrt{7}\right)x^2}}\right)|\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{2\sqrt[4]{7}\sqrt{\frac{1}{2+\left(1-\sqrt{7}\right)x^2}}\sqrt{-2-2x^2+3x^4}}$$

Mathematica [C] time = 0.04, size = 81, normalized size = 0.55

$$-\frac{i\sqrt{-3x^4+2x^2+2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{3}{-1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}\sqrt{3x^4-2x^2-2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-2 - 2*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[2 + 2*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[3/(-1 + Sqrt[7])]*x]
, (-4 + Sqrt[7])/3])/(Sqrt[1 + Sqrt[7]]*Sqrt[-2 - 2*x^2 + 3*x^4])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-2\,x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 2*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4-2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 2*x^2 - 2), x)
```

maple [C] time = 0.04, size = 84, normalized size = 0.57

$$\frac{2\sqrt{-\left(-\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^{2}+1}\,\sqrt{-\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^{2}+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{-2-2\sqrt{7}}\,x}{2},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{\sqrt{-2-2\sqrt{7}}\,\sqrt{3x^{4}-2x^{2}-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-2*x^2-2)^(1/2),x)
```

```
[Out] 2/(-2-2*7^(1/2))^(1/2)*(-(-1/2-1/2*7^(1/2))*x^2+1)^(1/2)*(-(-1/2+1/2*7^(1/2
))*x^2+1)^(1/2)/(3*x^4-2*x^2-2)^(1/2)*EllipticF(1/2*(-2-2*7^(1/2))^(1/2)*x,
1/6*I*42^(1/2)-1/6*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^{4}-2\,x^{2}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 2*x^2 - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^{4}-2\,x^{2}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - 2*x^2 - 2)^(1/2),x)
```

```
[Out] int(1/(3*x^4 - 2*x^2 - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^{4}-2x^{2}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-2*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 2*x**2 - 2), x)
```

3.50 $\int \frac{1}{\sqrt{-2-3x^2+3x^4}}\,dx$

Optimal. Leaf size=153

$$\frac{\sqrt{-\left(\left(3-\sqrt{33}\right)x^2\right)-4}\sqrt{\frac{\left(3+\sqrt{33}\right)x^2+4}{\left(3-\sqrt{33}\right)x^2+4}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{-\left(\left(3-\sqrt{33}\right)x^2\right)-4}}\right)|\frac{1}{22}\left(11-\sqrt{33}\right)\right)}{2\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{\left(3-\sqrt{33}\right)x^2+4}}\sqrt{3x^4-3x^2-2}}$$

```
[Out] 1/132*EllipticF(33^(1/4)*x*2^(1/2)/(-4-x^2*(3-33^(1/2)))^(1/2),1/22*(242-22
*33^(1/2))^(1/2))*(-4-x^2*(3-33^(1/2)))^(1/2)*((4+x^2*(3+33^(1/2)))/(4+x^2*
(3-33^(1/2))))^(1/2)*33^(3/4)*2^(1/2)/(3*x^4-3*x^2-2)^(1/2)/(1/(4+x^2*(3-33
^(1/2))))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{-\left(3-\sqrt{33}\right)x^2-4}\sqrt{\frac{\left(3+\sqrt{33}\right)x^2+4}{\left(3-\sqrt{33}\right)x^2+4}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{-\left(3-\sqrt{33}\right)x^2-4}}\right)|\frac{1}{22}\left(11-\sqrt{33}\right)\right)}{2\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{\left(3-\sqrt{33}\right)x^2+4}}\sqrt{3x^4-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 3*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[-4 - (3 - Sqrt[33])*x^2]*Sqrt[(4 + (3 + Sqrt[33])*x^2)/(4 + (3 - Sqrt
[33])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-4 - (3 - Sqrt[33])*
x^2]], (11 - Sqrt[33])/22])/(2*Sqrt[2]*33^(1/4)*Sqrt[(4 + (3 - Sqrt[33])*x^
2)^(-1)]*Sqrt[-2 - 3*x^2 + 3*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-3x^2+3x^4}}\,dx = \frac{\sqrt{-4-\left(3-\sqrt{33}\right)x^2}\sqrt{\frac{4+\left(3+\sqrt{33}\right)x^2}{4+\left(3-\sqrt{33}\right)x^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}\,x}{\sqrt{-4-\left(3-\sqrt{33}\right)x^2}}\right)|\frac{1}{22}\left(11-\sqrt{33}\right)\right)}{2\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{4+\left(3-\sqrt{33}\right)x^2}}\sqrt{-2-3x^2+3x^4}}$$

Mathematica [C] time = 0.07, size = 81, normalized size = 0.53

$$-\frac{i\sqrt{-6x^4+6x^2+4}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{6}{-3+\sqrt{33}}}\,x\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)}{\sqrt{3+\sqrt{33}}\sqrt{3x^4-3x^2-2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-2 - 3*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[4 + 6*x^2 - 6*x^4]*EllipticF[I*ArcSinh[Sqrt[6/(-3 + Sqrt[33])]*x
], (-7 + Sqrt[33])/4])/(Sqrt[3 + Sqrt[33]]*Sqrt[-2 - 3*x^2 + 3*x^4])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-3\,x^2-2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-3*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 3*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4-3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-3*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 3*x^2 - 2), x)
```

maple [C] time = 0.03, size = 84, normalized size = 0.55

$$\frac{2\sqrt{-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^{2}+1}\,\sqrt{-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^{2}+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{-\sqrt{33}-3}\,x}{2},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{\sqrt{-\sqrt{33}-3}\,\sqrt{3x^{4}-3x^{2}-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-3*x^2-2)^(1/2),x)
```

```
[Out] 2/(-33^(1/2)-3)^(1/2)*(-(-3/4-1/4*33^(1/2))*x^2+1)^(1/2)*(-(-3/4+1/4*33^(1/
2))*x^2+1)^(1/2)/(3*x^4-3*x^2-2)^(1/2)*EllipticF(1/2*(-33^(1/2)-3)^(1/2)*x,
1/4*I*22^(1/2)-1/4*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^{4}-3\,x^{2}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-3*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 3*x^2 - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^{4}-3\,x^{2}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - 3*x^2 - 2)^(1/2),x)
```

```
[Out] int(1/(3*x^4 - 3*x^2 - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^{4}-3x^{2}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-3*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 3*x**2 - 2), x)
```

3.51 $\int \frac{1}{\sqrt{-2-4x^2+3x^4}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{-\left(\left(2-\sqrt{10}\right)x^2\right)-2}\sqrt{\frac{\left(2+\sqrt{10}\right)x^2+2}{\left(2-\sqrt{10}\right)x^2+2}}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{-\left(\left(2-\sqrt{10}\right)x^2\right)-2}}\right)|\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{\frac{1}{\left(2-\sqrt{10}\right)x^2+2}}\sqrt{3x^4-4x^2-2}}$$

```
[Out] 1/20*EllipticF(2^(3/4)*5^(1/4)*x/(-2-x^2*(2-10^(1/2)))^(1/2),1/10*(50-10*10
^(1/2))^(1/2))*(-2-x^2*(2-10^(1/2)))^(1/2)*((2+x^2*(2+10^(1/2)))/(2+x^2*(2-
10^(1/2))))^(1/2)*10^(3/4)/(3*x^4-4*x^2-2)^(1/2)/(1/(2+x^2*(2-10^(1/2))))^(
1/2)
```

Rubi [A] time = 0.03, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{-\left(2-\sqrt{10}\right)x^2-2}\sqrt{\frac{\left(2+\sqrt{10}\right)x^2+2}{\left(2-\sqrt{10}\right)x^2+2}}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{-\left(2-\sqrt{10}\right)x^2-2}}\right)|\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{\frac{1}{\left(2-\sqrt{10}\right)x^2+2}}\sqrt{3x^4-4x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 4*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[-2 - (2 - Sqrt[10])*x^2]*Sqrt[(2 + (2 + Sqrt[10])*x^2)/(2 + (2 - Sqrt
[10])*x^2)]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-2 - (2 - Sqrt[10])*x
^2]], (5 - Sqrt[10])/10])/(2*10^(1/4)*Sqrt[(2 + (2 - Sqrt[10])*x^2)^(-1)]*S
qrt[-2 - 4*x^2 + 3*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-4x^2+3x^4}}\,dx = \frac{\sqrt{-2-\left(2-\sqrt{10}\right)x^2}\sqrt{\frac{2+\left(2+\sqrt{10}\right)x^2}{2+\left(2-\sqrt{10}\right)x^2}}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}\,x}{\sqrt{-2-\left(2-\sqrt{10}\right)x^2}}\right)|\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{\frac{1}{2+\left(2-\sqrt{10}\right)x^2}}\sqrt{-2-4x^2+3x^4}}$$

Mathematica [C] time = 0.06, size = 81, normalized size = 0.55

$$-\frac{i\sqrt{-3x^4+4x^2+2}\,F\left(i\sinh^{-1}\left(\sqrt{1+\sqrt{\frac{5}{2}}}\,x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2+\sqrt{10}}\sqrt{3x^4-4x^2-2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-2 - 4*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[2 + 4*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[1 + Sqrt[5/2]]*x], (
-7 + 2*Sqrt[10])/3])/(Sqrt[2 + Sqrt[10]]*Sqrt[-2 - 4*x^2 + 3*x^4])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-4\,x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-4*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 4*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4-4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-4*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 4*x^2 - 2), x)
```

maple [C] time = 0.03, size = 84, normalized size = 0.57

$$\frac{2\sqrt{-\left(-1-\frac{\sqrt{10}}{2}\right)x^2+1}\,\sqrt{-\left(-1+\frac{\sqrt{10}}{2}\right)x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{-4-2\sqrt{10}}\,x}{2},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{\sqrt{-4-2\sqrt{10}}\,\sqrt{3x^4-4x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-4*x^2-2)^(1/2),x)
```

```
[Out] 2/(-4-2*10^(1/2))^(1/2)*(-(-1-1/2*10^(1/2))*x^2+1)^(1/2)*(-(-1+1/2*10^(1/2)
)*x^2+1)^(1/2)/(3*x^4-4*x^2-2)^(1/2)*EllipticF(1/2*(-4-2*10^(1/2))^(1/2)*x,
1/3*I*15^(1/2)-1/3*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4-4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-4*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 4*x^2 - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4-4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - 4*x^2 - 2)^(1/2),x)
```

```
[Out] int(1/(3*x^4 - 4*x^2 - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4-4x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-4*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 4*x**2 - 2), x)
```

3.52 $\int \frac{1}{\sqrt{-2-5x^2+3x^4}}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{x^2-2}\,\sqrt{3x^2+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{7}\,x}{\sqrt{x^2-2}}\right)|\frac{1}{7}\right)}{\sqrt{7}\,\sqrt{3x^4-5x^2-2}}$$

```
[Out] 1/7*EllipticF(x*7^(1/2)/(x^2-2)^(1/2),1/7*7^(1/2))*(x^2-2)^(1/2)*(3*x^2+1)^
(1/2)*7^(1/2)/(3*x^4-5*x^2-2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1097}

$$\frac{\sqrt{x^2-2}\,\sqrt{3x^2+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{7}\,x}{\sqrt{x^2-2}}\right)|\frac{1}{7}\right)}{\sqrt{7}\,\sqrt{3x^4-5x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 5*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[-2 + x^2]*Sqrt[1 + 3*x^2]*EllipticF[ArcSin[(Sqrt[7]*x)/Sqrt[-2 + x^2]
], 1/7])/(Sqrt[7]*Sqrt[-2 - 5*x^2 + 3*x^4])
```

Rule 1097

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[-2*a - (b - q)*x^2]*Sqrt[(2*a + (b + q)*x^2)/q]
*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)])/(2*Sq
rt[-a]*Sqrt[a + b*x^2 + c*x^4]), x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-5x^2+3x^4}}\,dx=\frac{\sqrt{-2+x^2}\,\sqrt{1+3x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{7}\,x}{\sqrt{-2+x^2}}\right)|\frac{1}{7}\right)}{\sqrt{7}\,\sqrt{-2-5x^2+3x^4}}$$

Mathematica [C] time = 0.03, size = 65, normalized size = 1.03

$$-\frac{i\sqrt{1-\frac{x^2}{2}}\,\sqrt{3x^2+1}\,F\left(i\sinh^{-1}\left(\sqrt{3}\,x\right)|-\frac{1}{6}\right)}{\sqrt{3}\,\sqrt{3x^4-5x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - 5*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2/2]*Sqrt[1 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3]*x], -1/6]
)/(Sqrt[3]*Sqrt[-2 - 5*x^2 + 3*x^4])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-5\,x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-5*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 5*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4-5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-5*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 5*x^2 - 2), x)
```

maple [C] time = 0.02, size = 53, normalized size = 0.84

$$-\frac{i\sqrt{3}\,\sqrt{3x^2+1}\,\sqrt{-2x^2+4}\,\operatorname{EllipticF}\left(i\sqrt{3}\,x,\frac{i\sqrt{6}}{6}\right)}{6\sqrt{3x^4-5x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-5*x^2-2)^(1/2),x)
```

```
[Out] -1/6*I*3^(1/2)*(3*x^2+1)^(1/2)*(-2*x^2+4)^(1/2)/(3*x^4-5*x^2-2)^(1/2)*Ellip
ticF(I*3^(1/2)*x,1/6*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4-5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-5*x^2-2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(3*x^4 - 5*x^2 - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{3\,x^4-5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - 5*x^2 - 2)^(1/2),x)
[Out] int(1/(3*x^4 - 5*x^2 - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4-5x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-5*x**2-2)**(1/2),x)
[Out] Integral(1/sqrt(3*x**4 - 5*x**2 - 2), x)
```

3.53 $\int \frac{1}{\sqrt{-3+7x^2+2x^4}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{\frac{6-\left(7-\sqrt{73}\right)x^2}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{\left(7+\sqrt{73}\right)x^2-6}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{73}\,x}{\sqrt{\left(7+\sqrt{73}\right)x^2-6}}\right)|\frac{1}{146}\left(73+7\sqrt{73}\right)\right)}{2\sqrt{3}\sqrt[4]{73}\sqrt{\frac{1}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{2x^4+7x^2-3}}$$

```
[Out] 1/438*EllipticF(73^(1/4)*x*2^(1/2)/(-6+x^2*(7+73^(1/2)))^(1/2),1/146*(10658
+1022*73^(1/2))^(1/2))*((6-x^2*(7-73^(1/2)))/(6-x^2*(7+73^(1/2))))^(1/2)*(-
6+x^2*(7+73^(1/2)))^(1/2)*73^(3/4)*3^(1/2)/(2*x^4+7*x^2-3)^(1/2)/(1/(6-x^2*
(7+73^(1/2))))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{\frac{6-\left(7-\sqrt{73}\right)x^2}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{\left(7+\sqrt{73}\right)x^2-6}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{73}\,x}{\sqrt{\left(7+\sqrt{73}\right)x^2-6}}\right)|\frac{1}{146}\left(73+7\sqrt{73}\right)\right)}{2\sqrt{3}\sqrt[4]{73}\sqrt{\frac{1}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{2x^4+7x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 7*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(6 - (7 - Sqrt[73])*x^2)/(6 - (7 + Sqrt[73])*x^2)]*Sqrt[-6 + (7 + Sqr
t[73])*x^2]*EllipticF[ArcSin[(Sqrt[2]*73^(1/4)*x)/Sqrt[-6 + (7 + Sqrt[73])*
x^2]], (73 + 7*Sqrt[73])/146])/(2*Sqrt[3]*73^(1/4)*Sqrt[(6 - (7 + Sqrt[73])
*x^2)^(-1)]*Sqrt[-3 + 7*x^2 + 2*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+7x^2+2x^4}}\,dx = \frac{\sqrt{\frac{6-\left(7-\sqrt{73}\right)x^2}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{-6+\left(7+\sqrt{73}\right)x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{73}\,x}{\sqrt{-6+\left(7+\sqrt{73}\right)x^2}}\right)\middle|\frac{1}{146}\left(73+7\sqrt{73}\right)\right)}{2\sqrt{3}\sqrt[4]{73}\sqrt{\frac{1}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{-3+7x^2+2x^4}}$$

Mathematica [C] time = 0.05, size = 80, normalized size = 0.54

$$-\frac{i\sqrt{-4x^4-14x^2+6}\,F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right)\middle|\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)}{\sqrt{\sqrt{73}-7}\sqrt{2x^4+7x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 + 7*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[6 - 14*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/Sqrt[7 + Sqrt[73]]
], (-61 - 7*Sqrt[73])/12])/(Sqrt[-7 + Sqrt[73]]*Sqrt[-3 + 7*x^2 + 2*x^4])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2x^4+7x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+7*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 7*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4+7x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+7*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 7*x^2 - 3), x)
```

maple [C] time = 0.03, size = 84, normalized size = 0.57

$$\frac{6\sqrt{-\left(\frac{7}{6}-\frac{\sqrt{73}}{6}\right)x^{2}+1}\,\sqrt{-\left(\frac{\sqrt{73}}{6}+\frac{7}{6}\right)x^{2}+1}\,\,\mathrm{EllipticF}\left(\frac{\sqrt{42-6\sqrt{73}}\,x}{6},\frac{7i\sqrt{6}}{12}+\frac{i\sqrt{438}}{12}\right)}{\sqrt{42-6\sqrt{73}}\,\sqrt{2x^{4}+7x^{2}-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+7*x^2-3)^(1/2),x)
```

```
[Out] 6/(42-6*73^(1/2))^(1/2)*(-(7/6-1/6*73^(1/2))*x^2+1)^(1/2)*(-(1/6*73^(1/2)+7
/6)*x^2+1)^(1/2)/(2*x^4+7*x^2-3)^(1/2)*EllipticF(1/6*(42-6*73^(1/2))^(1/2)*
x,7/12*I*6^(1/2)+1/12*I*438^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^{4}+7\,x^{2}-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+7*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 7*x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^{4}+7\,x^{2}-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7*x^2 + 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(7*x^2 + 2*x^4 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^{4}+7x^{2}-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+7*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 7*x**2 - 3), x)
```

3.54 $\int \frac{1}{\sqrt{-3+6x^2+2x^4}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{\frac{3-\left(3-\sqrt{15}\right)x^2}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{\left(3+\sqrt{15}\right)x^2-3}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{15}\,x}{\sqrt{\left(3+\sqrt{15}\right)x^2-3}}\right)|\frac{1}{10}\left(5+\sqrt{15}\right)\right)}{\sqrt{2}\,3^{3/4}\sqrt[4]{5}\sqrt{\frac{1}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{2x^4+6x^2-3}}$$

```
[Out] 1/30*EllipticF(15^(1/4)*x*2^(1/2)/(-3+x^2*(3+15^(1/2)))^(1/2),1/10*(50+10*1
5^(1/2))^(1/2))*((3-x^2*(3-15^(1/2)))/(3-x^2*(3+15^(1/2))))^(1/2)*(-3+x^2*(
3+15^(1/2)))^(1/2)*3^(1/4)*5^(3/4)*2^(1/2)/(2*x^4+6*x^2-3)^(1/2)/(1/(3-x^2*
(3+15^(1/2))))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{\frac{3-\left(3-\sqrt{15}\right)x^2}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{\left(3+\sqrt{15}\right)x^2-3}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{15}\,x}{\sqrt{\left(3+\sqrt{15}\right)x^2-3}}\right)|\frac{1}{10}\left(5+\sqrt{15}\right)\right)}{\sqrt{2}\,3^{3/4}\sqrt[4]{5}\sqrt{\frac{1}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{2x^4+6x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 6*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(3 - (3 - Sqrt[15])*x^2)/(3 - (3 + Sqrt[15])*x^2)]*Sqrt[-3 + (3 + Sqr
t[15])*x^2]*EllipticF[ArcSin[(Sqrt[2]*15^(1/4)*x)/Sqrt[-3 + (3 + Sqrt[15])*
x^2]], (5 + Sqrt[15])/10])/(Sqrt[2]*3^(3/4)*5^(1/4)*Sqrt[(3 - (3 + Sqrt[15]
)*x^2)^(-1)]*Sqrt[-3 + 6*x^2 + 2*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+6x^2+2x^4}}\,dx = \frac{\sqrt{\frac{3-\left(3-\sqrt{15}\right)x^2}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{-3+\left(3+\sqrt{15}\right)x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{15}\,x}{\sqrt{-3+\left(3+\sqrt{15}\right)x^2}}\right)|\frac{1}{10}\left(5+\sqrt{15}\right)\right)}{\sqrt{2}\,3^{3/4}\sqrt[4]{5}\sqrt{\frac{1}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{-3+6x^2+2x^4}}$$

Mathematica [C] time = 0.06, size = 77, normalized size = 0.52

$$-\frac{i\sqrt{-2x^4-6x^2+3}\,F\left(i\sinh^{-1}\left(\sqrt{-1+\sqrt{\frac{5}{3}}}\,x\right)|-4-\sqrt{15}\right)}{\sqrt{\sqrt{15}-3}\sqrt{2x^4+6x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 + 6*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[3 - 6*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[-1 + Sqrt[5/3]]*x],
-4 - Sqrt[15]])/(Sqrt[-3 + Sqrt[15]]*Sqrt[-3 + 6*x^2 + 2*x^4])
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+6\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+6*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 6*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+6\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+6*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 6*x^2 - 3), x)
```

maple [C] time = 0.03, size = 84, normalized size = 0.57

$$\frac{3\sqrt{-\left(1-\frac{\sqrt{15}}{3}\right)x^{2}+1}\,\sqrt{-\left(1+\frac{\sqrt{15}}{3}\right)x^{2}+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{9-3\sqrt{15}}\,x}{3},\frac{i\sqrt{6}}{2}+\frac{i\sqrt{10}}{2}\right)}{\sqrt{9-3\sqrt{15}}\,\sqrt{2x^{4}+6x^{2}-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+6*x^2-3)^(1/2),x)
```

```
[Out] 3/(9-3*15^(1/2))^(1/2)*(-(1-1/3*15^(1/2))*x^2+1)^(1/2)*(-(1+1/3*15^(1/2))*x
^2+1)^(1/2)/(2*x^4+6*x^2-3)^(1/2)*EllipticF(1/3*(9-3*15^(1/2))^(1/2)*x,1/2*
I*6^(1/2)+1/2*I*10^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^{4}+6\,x^{2}-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+6*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 6*x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^{4}+6\,x^{2}-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(6*x^2 + 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(6*x^2 + 2*x^4 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^{4}+6x^{2}-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+6*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 6*x**2 - 3), x)
```

3.55 $\int \frac{1}{\sqrt{-3+5x^2+2x^4}}\,dx$

Optimal. Leaf size=67

$$\frac{\sqrt{x^2+3}\,\sqrt{2x^2-1}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{7}{3}}\,x}{\sqrt{2x^2-1}}\right)|\frac{6}{7}\right)}{\sqrt{7}\,\sqrt{2x^4+5x^2-3}}$$

```
[Out] 1/7*EllipticF(1/3*x*21^(1/2)/(2*x^2-1)^(1/2),1/7*42^(1/2))*(x^2+3)^(1/2)*(2
*x^2-1)^(1/2)*7^(1/2)/(2*x^4+5*x^2-3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1097}

$$\frac{\sqrt{x^2+3}\,\sqrt{2x^2-1}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{7}{3}}\,x}{\sqrt{2x^2-1}}\right)|\frac{6}{7}\right)}{\sqrt{7}\,\sqrt{2x^4+5x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 5*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[3 + x^2]*Sqrt[-1 + 2*x^2]*EllipticF[ArcSin[(Sqrt[7/3]*x)/Sqrt[-1 + 2*
x^2]], 6/7])/(Sqrt[7]*Sqrt[-3 + 5*x^2 + 2*x^4])
```

Rule 1097

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[-2*a - (b - q)*x^2]*Sqrt[(2*a + (b + q)*x^2)/q]
*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)])/(2*Sq
rt[-a]*Sqrt[a + b*x^2 + c*x^4]), x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+5x^2+2x^4}}\,dx=\frac{\sqrt{3+x^2}\,\sqrt{-1+2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{7}{3}}\,x}{\sqrt{-1+2x^2}}\right)|\frac{6}{7}\right)}{\sqrt{7}\,\sqrt{-3+5x^2+2x^4}}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.81

$$\frac{\sqrt{1-2x^2}\,\sqrt{x^2+3}\,F\left(\sin^{-1}\left(\sqrt{2}\,x\right)|-\frac{1}{6}\right)}{\sqrt{6}\,\sqrt{2x^4+5x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 5*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[1 - 2*x^2]*Sqrt[3 + x^2]*EllipticF[ArcSin[Sqrt[2]*x], -1/6])/(Sqrt[6]
*Sqrt[-3 + 5*x^2 + 2*x^4])
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+5\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 5*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+5\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 5*x^2 - 3), x)
```

maple [C] time = 0.01, size = 53, normalized size = 0.79

$$-\frac{i\sqrt{3}\,\sqrt{3x^2+9}\,\sqrt{-2x^2+1}\,\text{EllipticF}\left(\frac{i\sqrt{3}\,x}{3},i\sqrt{6}\right)}{3\sqrt{2x^4+5x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+5*x^2-3)^(1/2),x)
```

```
[Out] -1/3*I*3^(1/2)*(3*x^2+9)^(1/2)*(-2*x^2+1)^(1/2)/(2*x^4+5*x^2-3)^(1/2)*Ellip
ticF(1/3*I*3^(1/2)*x,I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+5\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2-3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(2*x^4 + 5*x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4+5\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 + 2*x^4 - 3)^(1/2),x)
[Out] int(1/(5*x^2 + 2*x^4 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4+5x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+5*x**2-3)**(1/2),x)
[Out] Integral(1/sqrt(2*x**4 + 5*x**2 - 3), x)
```

3.56 $\int \frac{1}{\sqrt{-3+4x^2+2x^4}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{\frac{3-\left(2-\sqrt{10}\right)x^2}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{\left(2+\sqrt{10}\right)x^2-3}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}\,x}{\sqrt{\left(2+\sqrt{10}\right)x^2-3}}\right)|\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{2x^4+4x^2-3}}$$

```
[Out] 1/30*EllipticF(2^(3/4)*5^(1/4)*x/(-3+x^2*(2+10^(1/2)))^(1/2),1/10*(50+10*10
^(1/2))^(1/2))*((3-x^2*(2-10^(1/2)))/(3-x^2*(2+10^(1/2))))^(1/2)*(-3+x^2*(2
+10^(1/2)))^(1/2)*2^(1/4)*5^(3/4)*3^(1/2)/(2*x^4+4*x^2-3)^(1/2)/(1/(3-x^2*(
2+10^(1/2))))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{\frac{3-\left(2-\sqrt{10}\right)x^2}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{\left(2+\sqrt{10}\right)x^2-3}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}\,x}{\sqrt{\left(2+\sqrt{10}\right)x^2-3}}\right)|\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{2x^4+4x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 4*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(3 - (2 - Sqrt[10])*x^2)/(3 - (2 + Sqrt[10])*x^2)]*Sqrt[-3 + (2 + Sqr
t[10])*x^2]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-3 + (2 + Sqrt[10])*x
^2]], (5 + Sqrt[10])/10])/(2^(3/4)*Sqrt[3]*5^(1/4)*Sqrt[(3 - (2 + Sqrt[10])
*x^2)^(-1)]*Sqrt[-3 + 4*x^2 + 2*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+4x^2+2x^4}}\,dx = \frac{\sqrt{\frac{3-\left(2-\sqrt{10}\right)x^2}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{-3+\left(2+\sqrt{10}\right)x^2}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}\,x}{\sqrt{-3+\left(2+\sqrt{10}\right)x^2}}\right)|\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{-3+4x^2+2x^4}}$$

Mathematica [C] time = 0.06, size = 83, normalized size = 0.56

$$-\frac{i\sqrt{-2x^4-4x^2+3}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{2+\sqrt{10}}}\,x\right)|-\frac{7}{3}-\frac{2\sqrt{10}}{3}\right)}{\sqrt{\sqrt{10}-2}\sqrt{2x^4+4x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 + 4*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[3 - 4*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(2 + Sqrt[10])]*x]
, -7/3 - (2*Sqrt[10])/3])/(Sqrt[-2 + Sqrt[10]]*Sqrt[-3 + 4*x^2 + 2*x^4])
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{\sqrt{2\,x^4+4\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+4*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 4*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+4\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+4*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 4*x^2 - 3), x)
```

maple [C] time = 0.04, size = 84, normalized size = 0.57

$$\frac{3\sqrt{-\left(-\frac{\sqrt{10}}{3}+\frac{2}{3}\right)x^2+1}\,\sqrt{-\left(\frac{\sqrt{10}}{3}+\frac{2}{3}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{6-3\sqrt{10}}\,x}{3},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{\sqrt{6-3\sqrt{10}}\,\sqrt{2x^4+4x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+4*x^2-3)^(1/2),x)
```

```
[Out] 3/(6-3*10^(1/2))^(1/2)*(-(-1/3*10^(1/2)+2/3)*x^2+1)^(1/2)*(-(1/3*10^(1/2)+2
/3)*x^2+1)^(1/2)/(2*x^4+4*x^2-3)^(1/2)*EllipticF(1/3*(6-3*10^(1/2))^(1/2)*x
,1/3*I*6^(1/2)+1/3*I*15^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+4\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+4*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 4*x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4+4\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 + 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(4*x^2 + 2*x^4 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4+4x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+4*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 4*x**2 - 3), x)
```

3.57 $\int \frac{1}{\sqrt{-3+3x^2+2x^4}}\,dx$

Optimal. Leaf size=146

$$\frac{\sqrt{\frac{6-\left(3-\sqrt{33}\right)x^2}{6-\left(3+\sqrt{33}\right)x^2}}\sqrt{\left(3+\sqrt{33}\right)x^2-6}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}\,x}{\sqrt{\left(3+\sqrt{33}\right)x^2-6}}\right)|\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{2\,3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{6-\left(3+\sqrt{33}\right)x^2}}\sqrt{2x^4+3x^2-3}}$$

```
[Out] 1/66*EllipticF(33^(1/4)*x*2^(1/2)/(-6+x^2*(3+33^(1/2)))^(1/2),1/22*(242+22*
33^(1/2))^(1/2))*((6-x^2*(3-33^(1/2)))/(6-x^2*(3+33^(1/2))))^(1/2)*(-6+x^2*
(3+33^(1/2)))^(1/2)*3^(1/4)*11^(3/4)/(2*x^4+3*x^2-3)^(1/2)/(1/(6-x^2*(3+33^
(1/2))))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{\frac{6-\left(3-\sqrt{33}\right)x^2}{6-\left(3+\sqrt{33}\right)x^2}}\sqrt{\left(3+\sqrt{33}\right)x^2-6}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}\,x}{\sqrt{\left(3+\sqrt{33}\right)x^2-6}}\right)|\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{2\,3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{6-\left(3+\sqrt{33}\right)x^2}}\sqrt{2x^4+3x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 3*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(6 - (3 - Sqrt[33])*x^2)/(6 - (3 + Sqrt[33])*x^2)]*Sqrt[-6 + (3 + Sqr
t[33])*x^2]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-6 + (3 + Sqrt[33])*
x^2]], (11 + Sqrt[33])/22])/(2*3^(3/4)*11^(1/4)*Sqrt[(6 - (3 + Sqrt[33])*x^
2)^(-1)]*Sqrt[-3 + 3*x^2 + 2*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+3x^2+2x^4}}\,dx = \frac{\sqrt{\frac{6-(3-\sqrt{33})x^2}{6-(3+\sqrt{33})x^2}}\sqrt{-6+(3+\sqrt{33})\,x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}\,x}{\sqrt{-6+(3+\sqrt{33})x^2}}\right)|\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{2\,3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{6-(3+\sqrt{33})x^2}}\sqrt{-3+3x^2+2x^4}}$$

Mathematica [C] time = 0.07, size = 80, normalized size = 0.55

$$-\frac{i\sqrt{-4x^4-6x^2+6}\,F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right)|-\frac{7}{4}-\frac{\sqrt{33}}{4}\right)}{\sqrt{\sqrt{33}-3}\sqrt{2x^4+3x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 + 3*x^2 + 2*x^4],x]
[Out] ((-I)*Sqrt[6 - 6*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/Sqrt[3 + Sqrt[33]]]
, -7/4 - Sqrt[33]/4])/(Sqrt[-3 + Sqrt[33]]*Sqrt[-3 + 3*x^2 + 2*x^4])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+3\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3*x^2-3)^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(2*x^4 + 3*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3*x^2-3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(2*x^4 + 3*x^2 - 3), x)
```

maple [C] time = 0.03, size = 84, normalized size = 0.58

$$\frac{6\sqrt{-\left(-\frac{\sqrt{33}}{6}+\frac{1}{2}\right)x^2+1}\,\sqrt{-\left(\frac{\sqrt{33}}{6}+\frac{1}{2}\right)x^2+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{18-6\sqrt{33}}\,x}{6},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{\sqrt{18-6\sqrt{33}}\,\sqrt{2x^4+3x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+3*x^2-3)^(1/2),x)
```

```
[Out] 6/(18-6*33^(1/2))^(1/2)*(-(-1/6*33^(1/2)+1/2)*x^2+1)^(1/2)*(-(1/6*33^(1/2)+
1/2)*x^2+1)^(1/2)/(2*x^4+3*x^2-3)^(1/2)*EllipticF(1/6*(18-6*33^(1/2))^(1/2)
*x,1/4*I*6^(1/2)+1/4*I*22^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 3*x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4+3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 + 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(3*x^2 + 2*x^4 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4+3x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+3*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 3*x**2 - 3), x)
```

3.58 $\int \frac{1}{\sqrt{-3+2x^2+2x^4}}\,dx$

Optimal. Leaf size=143

$$\frac{\sqrt{\frac{3-\left(1-\sqrt{7}\right)x^2}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{\left(1+\sqrt{7}\right)x^2-3}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{\left(1+\sqrt{7}\right)x^2-3}}\right)|\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{2x^4+2x^2-3}}$$

```
[Out] 1/42*EllipticF(7^(1/4)*x*2^(1/2)/(-3+x^2*(1+7^(1/2)))^(1/2),1/14*(98+14*7^(
1/2))^(1/2))*((3-x^2*(1-7^(1/2)))/(3-x^2*(1+7^(1/2))))^(1/2)*(-3+x^2*(1+7^(
1/2)))^(1/2)*7^(3/4)*6^(1/2)/(2*x^4+2*x^2-3)^(1/2)/(1/(3-x^2*(1+7^(1/2))))^
(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{\frac{3-\left(1-\sqrt{7}\right)x^2}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{\left(1+\sqrt{7}\right)x^2-3}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{\left(1+\sqrt{7}\right)x^2-3}}\right)|\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{2x^4+2x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 2*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(3 - (1 - Sqrt[7])*x^2)/(3 - (1 + Sqrt[7])*x^2)]*Sqrt[-3 + (1 + Sqrt[
7])*x^2]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-3 + (1 + Sqrt[7])*x^2]]
, (7 + Sqrt[7])/14])/(Sqrt[6]*7^(1/4)*Sqrt[(3 - (1 + Sqrt[7])*x^2)^(-1)]*Sq
rt[-3 + 2*x^2 + 2*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+2x^2+2x^4}}\,dx = \frac{\sqrt{\frac{3-\left(1-\sqrt{7}\right)x^2}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{-3+\left(1+\sqrt{7}\right)x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}\,x}{\sqrt{-3+\left(1+\sqrt{7}\right)x^2}}\right)|\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{-3+2x^2+2x^4}}$$

Mathematica [C] time = 0.06, size = 83, normalized size = 0.58

$$-\frac{i\sqrt{-2x^4-2x^2+3}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{7}}}\,x\right)|-\frac{4}{3}-\frac{\sqrt{7}}{3}\right)}{\sqrt{\sqrt{7}-1}\sqrt{2x^4+2x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 + 2*x^2 + 2*x^4],x]
[Out] ((-I)*Sqrt[3 - 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(1 + Sqrt[7])]*x],
 -4/3 - Sqrt[7]/3])/(Sqrt[-1 + Sqrt[7]]*Sqrt[-3 + 2*x^2 + 2*x^4])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+2\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+2*x^2-3)^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(2*x^4 + 2*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+2*x^2-3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(2*x^4 + 2*x^2 - 3), x)
```

maple [C] time = 0.03, size = 84, normalized size = 0.59

$$\frac{3\sqrt{-\left(-\frac{\sqrt{7}}{3}+\frac{1}{3}\right)x^{2}+1}\,\sqrt{-\left(\frac{\sqrt{7}}{3}+\frac{1}{3}\right)x^{2}+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{3-3\sqrt{7}}\,x}{3},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{\sqrt{3-3\sqrt{7}}\,\sqrt{2x^{4}+2x^{2}-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+2*x^2-3)^(1/2),x)
```

```
[Out] 3/(3-3*7^(1/2))^(1/2)*(-(-1/3*7^(1/2)+1/3)*x^2+1)^(1/2)*(-(1/3*7^(1/2)+1/3)
*x^2+1)^(1/2)/(2*x^4+2*x^2-3)^(1/2)*EllipticF(1/3*(3-3*7^(1/2))^(1/2)*x,1/6
*I*6^(1/2)+1/6*I*42^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^{4}+2\,x^{2}-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+2*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 2*x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^{4}+2\,x^{2}-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 + 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(2*x^2 + 2*x^4 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^{4}+2x^{2}-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+2*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 2*x**2 - 3), x)
```

3.59 $\int \frac{1}{\sqrt{-3+x^2+2x^4}}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{x^2-1}\,\sqrt{2x^2+3}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{x^2-1}}\right)|\frac{3}{5}\right)}{\sqrt{5}\,\sqrt{2x^4+x^2-3}}$$

```
[Out] 1/5*EllipticF(1/3*x*15^(1/2)/(x^2-1)^(1/2),1/5*15^(1/2))*(x^2-1)^(1/2)*(2*x
^2+3)^(1/2)*5^(1/2)/(2*x^4+x^2-3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1097}

$$\frac{\sqrt{x^2-1}\,\sqrt{2x^2+3}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{x^2-1}}\right)|\frac{3}{5}\right)}{\sqrt{5}\,\sqrt{2x^4+x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[-1 + x^2]*Sqrt[3 + 2*x^2]*EllipticF[ArcSin[(Sqrt[5/3]*x)/Sqrt[-1 + x^
2]], 3/5])/(Sqrt[5]*Sqrt[-3 + x^2 + 2*x^4])
```

Rule 1097

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[-2*a - (b - q)*x^2]*Sqrt[(2*a + (b + q)*x^2)/q]
*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)])/(2*Sq
rt[-a]*Sqrt[a + b*x^2 + c*x^4]), x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+x^2+2x^4}}\,dx = \frac{\sqrt{-1+x^2}\,\sqrt{3+2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{-1+x^2}}\right)|\frac{3}{5}\right)}{\sqrt{5}\,\sqrt{-3+x^2+2x^4}}$$

Mathematica [C] time = 0.02, size = 63, normalized size = 1.00

$$-\frac{i\sqrt{1-x^2}\sqrt{2x^2+3}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)|-\frac{3}{2}\right)}{\sqrt{2}\sqrt{2x^4+x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2]*Sqrt[3 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/3]*x], -3/2]
)/(Sqrt[2]*Sqrt[-3 + x^2 + 2*x^4])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + x^2 - 3), x)
```

maple [C] time = 0.03, size = 51, normalized size = 0.81

$$-\frac{i\sqrt{6}\,\sqrt{6x^2+9}\,\sqrt{-x^2+1}\,\operatorname{EllipticF}\left(\frac{i\sqrt{6}\,x}{3},\frac{i\sqrt{6}}{2}\right)}{6\sqrt{2x^4+x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+x^2-3)^(1/2),x)
```

```
[Out] -1/6*I*6^(1/2)*(6*x^2+9)^(1/2)*(-x^2+1)^(1/2)/(2*x^4+x^2-3)^(1/2)*EllipticF
(1/3*I*x*6^(1/2),1/2*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+x^2-3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(2*x^4 + x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{2\,x^4+x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 2*x^4 - 3)^(1/2),x)
[Out] int(1/(x^2 + 2*x^4 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4+x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+x**2-3)**(1/2),x)
[Out] Integral(1/sqrt(2*x**4 + x**2 - 3), x)
```

3.60 $\int \frac{1}{\sqrt{-3+2x^4}} \, dx$

Optimal. Leaf size=112

$$\frac{\sqrt{\sqrt{6}\,x^2-3}\,\sqrt{\frac{\sqrt{6}\,x^2+3}{3-\sqrt{6}\,x^2}}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{3}\,x}{\sqrt{\sqrt{6}\,x^2-3}}\right)|\frac{1}{2}\right)}{6^{3/4}\sqrt{\frac{1}{3-\sqrt{6}\,x^2}}\,\sqrt{2x^4-3}}$$

```
[Out] 1/6*EllipticF(2^(3/4)*3^(1/4)*x/(-3+x^2*6^(1/2))^(1/2),1/2*2^(1/2))*(-3+x^2
*6^(1/2))^(1/2)*((3+x^2*6^(1/2))/(3-x^2*6^(1/2)))^(1/2)*6^(1/4)/(2*x^4-3)^(
1/2)/(1/(3-x^2*6^(1/2)))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {223}

$$\frac{\sqrt{\sqrt{6}\,x^2-3}\,\sqrt{\frac{\sqrt{6}\,x^2+3}{3-\sqrt{6}\,x^2}}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{3}\,x}{\sqrt{\sqrt{6}\,x^2-3}}\right)|\frac{1}{2}\right)}{6^{3/4}\sqrt{\frac{1}{3-\sqrt{6}\,x^2}}\,\sqrt{2x^4-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 2*x^4],x]
```

```
[Out] (Sqrt[-3 + Sqrt[6]*x^2]*Sqrt[(3 + Sqrt[6]*x^2)/(3 - Sqrt[6]*x^2)]*EllipticF
[ArcSin[(2^(3/4)*3^(1/4)*x)/Sqrt[-3 + Sqrt[6]*x^2]], 1/2])/(6^(3/4)*Sqrt[(3
 - Sqrt[6]*x^2)^(-1)]*Sqrt[-3 + 2*x^4])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[(a - q*x^2)/(a + q*x^2)]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqr
t[(a + q*x^2)/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[a + b*x^4]*Sqrt[a/(a + q*x^2)]),
 x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+2x^4}}\,dx = \frac{\sqrt{-3+\sqrt{6}x^2}\sqrt{\frac{3+\sqrt{6}x^2}{3-\sqrt{6}x^2}}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt{-3+\sqrt{6}x^2}}\right)|\frac{1}{2}\right)}{6^{3/4}\sqrt{\frac{1}{3-\sqrt{6}x^2}}\sqrt{-3+2x^4}}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.36

$$\frac{\sqrt{3-2x^4}\,F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)\middle|-1\right)}{\sqrt[4]{6}\sqrt{2x^4-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 2*x^4],x]
```

```
[Out] (Sqrt[3 - 2*x^4]*EllipticF[ArcSin[(2/3)^(1/4)*x], -1])/(6^(1/4)*Sqrt[-3 + 2
*x^4])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2x^4-3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 3), x)
```

maple [C] time = 0.02, size = 56, normalized size = 0.50

$$\frac{\sqrt{3\sqrt{6}\,x^2+9}\,\sqrt{-3\sqrt{6}\,x^2+9}\;\mathrm{EllipticF}\left(\frac{\sqrt{-3\sqrt{6}}\,x}{3}, i\right)}{3\sqrt{-3\sqrt{6}}\,\sqrt{2x^4-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-3)^(1/2),x)
```

```
[Out] 1/3/(-3*6^(1/2))^(1/2)*(3*6^(1/2)*x^2+9)^(1/2)*(-3*6^(1/2)*x^2+9)^(1/2)/(2*
x^4-3)^(1/2)*EllipticF(1/3*(-3*6^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 3), x)
```

mupad [B] time = 0.08, size = 31, normalized size = 0.28

$$\frac{x\,\sqrt{9-6\,x^4}\;{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\;\frac{5}{4};\;\frac{2\,x^4}{3}\right)}{3\,\sqrt{2\,x^4-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - 3)^(1/2),x)
```

```
[Out] (x*(9 - 6*x^4)^(1/2)*hypergeom([1/4, 1/2], 5/4, (2*x^4)/3))/(3*(2*x^4 - 3)^
(1/2))
```

sympy [C] time = 0.73, size = 34, normalized size = 0.30

$$-\frac{\sqrt{3}\,ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{2x^4}{3}\right)}{12\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-3)**(1/2),x)
```

```
[Out] -sqrt(3)*I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**4/3)/(12*gamma(5/4))
```

3.61 $\int \frac{1}{\sqrt{-3-x^2+2x^4}}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{x^2+1}\,\sqrt{2x^2-3}\,F\left(\sin^{-1}\left(\frac{\sqrt{5}\,x}{\sqrt{2x^2-3}}\right)|\frac{2}{5}\right)}{\sqrt{5}\,\sqrt{2x^4-x^2-3}}$$

```
[Out] 1/5*EllipticF(x*5^(1/2)/(2*x^2-3)^(1/2),1/5*10^(1/2))*(x^2+1)^(1/2)*(2*x^2-
3)^(1/2)*5^(1/2)/(2*x^4-x^2-3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1097}

$$\frac{\sqrt{x^2+1}\,\sqrt{2x^2-3}\,F\left(\sin^{-1}\left(\frac{\sqrt{5}\,x}{\sqrt{2x^2-3}}\right)|\frac{2}{5}\right)}{\sqrt{5}\,\sqrt{2x^4-x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[1 + x^2]*Sqrt[-3 + 2*x^2]*EllipticF[ArcSin[(Sqrt[5]*x)/Sqrt[-3 + 2*x^
2]], 2/5])/(Sqrt[5]*Sqrt[-3 - x^2 + 2*x^4])
```

Rule 1097

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[-2*a - (b - q)*x^2]*Sqrt[(2*a + (b + q)*x^2)/q]
*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)])/(2*Sq
rt[-a]*Sqrt[a + b*x^2 + c*x^4]), x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-x^2+2x^4}}\,dx = \frac{\sqrt{1+x^2}\,\sqrt{-3+2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{5}\,x}{\sqrt{-3+2x^2}}\right)|\frac{2}{5}\right)}{\sqrt{5}\,\sqrt{-3-x^2+2x^4}}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.78

$$\frac{\sqrt{3-2x^2}\,\sqrt{x^2+1}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)|-\frac{3}{2}\right)}{\sqrt{4x^4-2x^2-6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[3 - 2*x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[2/3]*x], -3/2])/Sqrt[-
6 - 2*x^2 + 4*x^4]
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4 - x^2 - 3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4 - x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - x^2 - 3), x)
```

maple [C] time = 0.01, size = 45, normalized size = 0.69

$$-\frac{i\sqrt{x^2+1}\,\sqrt{-6x^2+9}\,\operatorname{EllipticF}\left(ix, \frac{i\sqrt{6}}{3}\right)}{3\sqrt{2x^4-x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-x^2-3)^(1/2),x)
```

```
[Out] -1/3*I*(x^2+1)^(1/2)*(-6*x^2+9)^(1/2)/(2*x^4-x^2-3)^(1/2)*EllipticF(I*x,1/3
*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4 - x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{2\,x^4-x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - x^2 - 3)^(1/2),x)
```

```
[Out] int(1/(2*x^4 - x^2 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4-x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - x**2 - 3), x)
```

3.62 $\int \frac{1}{\sqrt{-3-2x^2+2x^4}}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{-\left(\left(1-\sqrt{7}\right)x^2\right)-3}\sqrt{\frac{\left(1+\sqrt{7}\right)x^2+3}{\left(1-\sqrt{7}\right)x^2+3}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-\left(\left(1-\sqrt{7}\right)x^2\right)-3}}\right)|\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{\left(1-\sqrt{7}\right)x^2+3}}\sqrt{2x^4-2x^2-3}}$$

```
[Out] 1/42*EllipticF(7^(1/4)*x*2^(1/2)/(-3-x^2*(1-7^(1/2)))^(1/2),1/14*(98-14*7^(
1/2))^(1/2))*(-3-x^2*(1-7^(1/2)))^(1/2)*((3+x^2*(1+7^(1/2)))/(3+x^2*(1-7^(1
/2))))^(1/2)*7^(3/4)*6^(1/2)/(2*x^4-2*x^2-3)^(1/2)/(1/(3+x^2*(1-7^(1/2))))^
(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{-\left(1-\sqrt{7}\right)x^2-3}\sqrt{\frac{\left(1+\sqrt{7}\right)x^2+3}{\left(1-\sqrt{7}\right)x^2+3}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-\left(1-\sqrt{7}\right)x^2-3}}\right)|\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{\left(1-\sqrt{7}\right)x^2+3}}\sqrt{2x^4-2x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 2*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[-3 - (1 - Sqrt[7])*x^2]*Sqrt[(3 + (1 + Sqrt[7])*x^2)/(3 + (1 - Sqrt[7
])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-3 - (1 - Sqrt[7])*x^2]]
, (7 - Sqrt[7])/14])/(Sqrt[6]*7^(1/4)*Sqrt[(3 + (1 - Sqrt[7])*x^2)^(-1)]*Sq
rt[-3 - 2*x^2 + 2*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-2x^2+2x^4}}\,dx = \frac{\sqrt{-3-\left(1-\sqrt{7}\right)x^2}\sqrt{\frac{3+\left(1+\sqrt{7}\right)x^2}{3+\left(1-\sqrt{7}\right)x^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}\,x}{\sqrt{-3-\left(1-\sqrt{7}\right)x^2}}\right)|\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{3+\left(1-\sqrt{7}\right)x^2}}\sqrt{-3-2x^2+2x^4}}$$

Mathematica [C] time = 0.05, size = 81, normalized size = 0.54

$$-\frac{i\sqrt{-2x^4+2x^2+3}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}\sqrt{2x^4-2x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 - 2*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[3 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(-1 + Sqrt[7])]*x]
, (-4 + Sqrt[7])/3])/(Sqrt[1 + Sqrt[7]]*Sqrt[-3 - 2*x^2 + 2*x^4])
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-2\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-2*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 2*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4-2\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-2*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 2*x^2 - 3), x)
```

maple [C] time = 0.03, size = 84, normalized size = 0.56

$$\frac{3\sqrt{-\left(-\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^{2}+1}\,\sqrt{-\left(-\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^{2}+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{-3-3\sqrt{7}}\,x}{3},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{\sqrt{-3-3\sqrt{7}}\,\sqrt{2x^{4}-2x^{2}-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-2*x^2-3)^(1/2),x)
```

```
[Out] 3/(-3-3*7^(1/2))^(1/2)*(-(-1/3-1/3*7^(1/2))*x^2+1)^(1/2)*(-(-1/3+1/3*7^(1/2
))*x^2+1)^(1/2)/(2*x^4-2*x^2-3)^(1/2)*EllipticF(1/3*(-3-3*7^(1/2))^(1/2)*x,
1/6*I*42^(1/2)-1/6*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^{4}-2\,x^{2}-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-2*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 2*x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^{4}-2\,x^{2}-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - 2*x^2 - 3)^(1/2),x)
```

```
[Out] int(1/(2*x^4 - 2*x^2 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^{4}-2x^{2}-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-2*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 2*x**2 - 3), x)
```

3.63 $\int \frac{1}{\sqrt{-3-3x^2+2x^4}}\,dx$

Optimal. Leaf size=153

$$\frac{\sqrt{-\left(\left(3-\sqrt{33}\right)x^2\right)-6}\sqrt{\frac{\left(3+\sqrt{33}\right)x^2+6}{\left(3-\sqrt{33}\right)x^2+6}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{-\left(\left(3-\sqrt{33}\right)x^2\right)-6}}\right)|\frac{1}{22}\left(11-\sqrt{33}\right)\right)}{2\,3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{\left(3-\sqrt{33}\right)x^2+6}}\sqrt{2x^4-3x^2-3}}$$

```
[Out] 1/66*EllipticF(33^(1/4)*x*2^(1/2)/(-6-x^2*(3-33^(1/2)))^(1/2),1/22*(242-22*
33^(1/2))^(1/2))*(-6-x^2*(3-33^(1/2)))^(1/2)*((6+x^2*(3+33^(1/2)))/(6+x^2*(
3-33^(1/2))))^(1/2)*3^(1/4)*11^(3/4)/(2*x^4-3*x^2-3)^(1/2)/(1/(6+x^2*(3-33^
(1/2))))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{-\left(3-\sqrt{33}\right)x^2-6}\sqrt{\frac{\left(3+\sqrt{33}\right)x^2+6}{\left(3-\sqrt{33}\right)x^2+6}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{-\left(3-\sqrt{33}\right)x^2-6}}\right)|\frac{1}{22}\left(11-\sqrt{33}\right)\right)}{2\,3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{\left(3-\sqrt{33}\right)x^2+6}}\sqrt{2x^4-3x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 3*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[-6 - (3 - Sqrt[33])*x^2]*Sqrt[(6 + (3 + Sqrt[33])*x^2)/(6 + (3 - Sqrt
[33])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-6 - (3 - Sqrt[33])*
x^2]], (11 - Sqrt[33])/22])/(2*3^(3/4)*11^(1/4)*Sqrt[(6 + (3 - Sqrt[33])*x^
2)^(-1)]*Sqrt[-3 - 3*x^2 + 2*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-3x^2+2x^4}}\,dx = \frac{\sqrt{-6-\left(3-\sqrt{33}\right)x^2}\sqrt{\frac{6+\left(3+\sqrt{33}\right)x^2}{6+\left(3-\sqrt{33}\right)x^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}\,x}{\sqrt{-6-\left(3-\sqrt{33}\right)x^2}}\right)\middle|\frac{1}{22}\left(11-\sqrt{33}\right)\right)}{2\,3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{6+\left(3-\sqrt{33}\right)x^2}}\sqrt{-3-3x^2+2x^4}}$$

Mathematica [C] time = 0.06, size = 78, normalized size = 0.51

$$-\frac{i\sqrt{-4x^4+6x^2+6}\,F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right)\middle|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)}{\sqrt{3+\sqrt{33}}\sqrt{2x^4-3x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 - 3*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[6 + 6*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/Sqrt[-3 + Sqrt[33]]
], (-7 + Sqrt[33])/4])/(Sqrt[3 + Sqrt[33]]*Sqrt[-3 - 3*x^2 + 2*x^4])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-3\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 3*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4-3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 3*x^2 - 3), x)
```

maple [C] time = 0.03, size = 84, normalized size = 0.55

$$\frac{6\sqrt{-\left(-\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2+1}\,\sqrt{-\left(-\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{-18-6\sqrt{33}}\,x}{6},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{\sqrt{-18-6\sqrt{33}}\,\sqrt{2x^4-3x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-3*x^2-3)^(1/2),x)
```

```
[Out] 6/(-18-6*33^(1/2))^(1/2)*(-(-1/2-1/6*33^(1/2))*x^2+1)^(1/2)*(-(-1/2+1/6*33^
(1/2))*x^2+1)^(1/2)/(2*x^4-3*x^2-3)^(1/2)*EllipticF(1/6*(-18-6*33^(1/2))^(1
/2)*x,1/4*I*22^(1/2)-1/4*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4-3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 3*x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4-3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - 3*x^2 - 3)^(1/2),x)
```

```
[Out] int(1/(2*x^4 - 3*x^2 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4-3x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-3*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 3*x**2 - 3), x)
```

3.64 $\int \frac{1}{\sqrt{-3-4x^2+2x^4}}\,dx$

Optimal. Leaf size=155

$$\frac{\sqrt{-\left(\left(2-\sqrt{10}\right)x^2\right)-3}\sqrt{\frac{\left(2+\sqrt{10}\right)x^2+3}{\left(2-\sqrt{10}\right)x^2+3}}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{-\left(\left(2-\sqrt{10}\right)x^2\right)-3}}\right)|\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{\left(2-\sqrt{10}\right)x^2+3}}\sqrt{2x^4-4x^2-3}}$$

```
[Out] 1/30*EllipticF(2^(3/4)*5^(1/4)*x/(-3-x^2*(2-10^(1/2)))^(1/2),1/10*(50-10*10
^(1/2))^(1/2))*(-3-x^2*(2-10^(1/2)))^(1/2)*((3+x^2*(2+10^(1/2)))/(3+x^2*(2-
10^(1/2))))^(1/2)*2^(1/4)*5^(3/4)*3^(1/2)/(2*x^4-4*x^2-3)^(1/2)/(1/(3+x^2*(
2-10^(1/2))))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{-\left(2-\sqrt{10}\right)x^2-3}\sqrt{\frac{\left(2+\sqrt{10}\right)x^2+3}{\left(2-\sqrt{10}\right)x^2+3}}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{-\left(2-\sqrt{10}\right)x^2-3}}\right)|\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{\left(2-\sqrt{10}\right)x^2+3}}\sqrt{2x^4-4x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 4*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[-3 - (2 - Sqrt[10])*x^2]*Sqrt[(3 + (2 + Sqrt[10])*x^2)/(3 + (2 - Sqrt
[10])*x^2)]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-3 - (2 - Sqrt[10])*x
^2]], (5 - Sqrt[10])/10])/(2^(3/4)*Sqrt[3]*5^(1/4)*Sqrt[(3 + (2 - Sqrt[10])
*x^2)^(-1)]*Sqrt[-3 - 4*x^2 + 2*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-4x^2+2x^4}}\,dx = \frac{\sqrt{-3-\left(2-\sqrt{10}\right)x^2}\sqrt{\frac{3+\left(2+\sqrt{10}\right)x^2}{3+\left(2-\sqrt{10}\right)x^2}}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}\,x}{\sqrt{-3-\left(2-\sqrt{10}\right)x^2}}\right)|\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{3+\left(2-\sqrt{10}\right)x^2}}\sqrt{-3-4x^2+2x^4}}$$

Mathematica [C] time = 0.06, size = 83, normalized size = 0.54

$$-\frac{i\sqrt{-2x^4+4x^2+3}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-2+\sqrt{10}}}\,x\right)|-\frac{7}{3}+\frac{2\sqrt{10}}{3}\right)}{\sqrt{2+\sqrt{10}}\sqrt{2x^4-4x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 - 4*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[3 + 4*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(-2 + Sqrt[10])]*x
], -7/3 + (2*Sqrt[10])/3])/(Sqrt[2 + Sqrt[10]]*Sqrt[-3 - 4*x^2 + 2*x^4])
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2x^4-4x^2-3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-4*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 4*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4-4x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-4*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 4*x^2 - 3), x)
```

maple [C] time = 0.03, size = 84, normalized size = 0.54

$$\frac{3\sqrt{-\left(-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2+1}\,\sqrt{-\left(-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{-6-3\sqrt{10}}\,x}{3},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{\sqrt{-6-3\sqrt{10}}\,\sqrt{2x^4-4x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-4*x^2-3)^(1/2),x)
```

```
[Out] 3/(-6-3*10^(1/2))^(1/2)*(-(-2/3-1/3*10^(1/2))*x^2+1)^(1/2)*(-(-2/3+1/3*10^(
1/2))*x^2+1)^(1/2)/(2*x^4-4*x^2-3)^(1/2)*EllipticF(1/3*(-6-3*10^(1/2))^(1/2
)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4-4\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-4*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 4*x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4-4\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - 4*x^2 - 3)^(1/2),x)
```

```
[Out] int(1/(2*x^4 - 4*x^2 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4-4x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-4*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 4*x**2 - 3), x)
```

3.65 $\int \frac{1}{\sqrt{-3-5x^2+2x^4}}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{x^2-3}\,\sqrt{2x^2+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{7}\,x}{\sqrt{x^2-3}}\right)|\frac{1}{7}\right)}{\sqrt{7}\,\sqrt{2x^4-5x^2-3}}$$

```
[Out] 1/7*EllipticF(x*7^(1/2)/(x^2-3)^(1/2),1/7*7^(1/2))*(x^2-3)^(1/2)*(2*x^2+1)^
(1/2)*7^(1/2)/(2*x^4-5*x^2-3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1097}

$$\frac{\sqrt{x^2-3}\,\sqrt{2x^2+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{7}\,x}{\sqrt{x^2-3}}\right)|\frac{1}{7}\right)}{\sqrt{7}\,\sqrt{2x^4-5x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 5*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[-3 + x^2]*Sqrt[1 + 2*x^2]*EllipticF[ArcSin[(Sqrt[7]*x)/Sqrt[-3 + x^2]
], 1/7])/(Sqrt[7]*Sqrt[-3 - 5*x^2 + 2*x^4])
```

Rule 1097

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[-2*a - (b - q)*x^2]*Sqrt[(2*a + (b + q)*x^2)/q]
*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)])/(2*Sq
rt[-a]*Sqrt[a + b*x^2 + c*x^4]), x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-5x^2+2x^4}}\,dx=\frac{\sqrt{-3+x^2}\,\sqrt{1+2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{7}\,x}{\sqrt{-3+x^2}}\right)|\frac{1}{7}\right)}{\sqrt{7}\,\sqrt{-3-5x^2+2x^4}}$$

Mathematica [C] time = 0.02, size = 65, normalized size = 1.03

$$-\frac{i\sqrt{1-\frac{x^2}{3}}\,\sqrt{2x^2+1}\,F\left(i\sinh^{-1}\left(\sqrt{2}\,x\right)|-\frac{1}{6}\right)}{\sqrt{2}\,\sqrt{2x^4-5x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - 5*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2/3]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]*x], -1/6]
)/(Sqrt[2]*Sqrt[-3 - 5*x^2 + 2*x^4])
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-5\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-5*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 5*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4-5\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-5*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 5*x^2 - 3), x)
```

maple [C] time = 0.02, size = 53, normalized size = 0.84

$$-\frac{i\sqrt{2}\,\sqrt{2x^2+1}\,\sqrt{-3x^2+9}\,\operatorname{EllipticF}\left(i\sqrt{2}\,x,\frac{i\sqrt{6}}{6}\right)}{6\sqrt{2x^4-5x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-5*x^2-3)^(1/2),x)
```

```
[Out] -1/6*I*2^(1/2)*(2*x^2+1)^(1/2)*(-3*x^2+9)^(1/2)/(2*x^4-5*x^2-3)^(1/2)*Ellip
ticF(I*2^(1/2)*x,1/6*I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4-5\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-5*x^2-3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(2*x^4 - 5*x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{2\,x^4-5\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - 5*x^2 - 3)^(1/2),x)
[Out] int(1/(2*x^4 - 5*x^2 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4-5x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-5*x**2-3)**(1/2),x)
[Out] Integral(1/sqrt(2*x**4 - 5*x**2 - 3), x)
```

3.66 $\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx$

Optimal. Leaf size=52

$$\frac{\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{3x^4+5x^2+2}}$$

```
[Out] 1/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*I*2^(1/2)
)*((3*x^2+2)/(x^2+1))^(1/2)*2^(1/2)/(3*x^4+5*x^2+2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1100}

$$\frac{\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{3x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

```
[Out] ((1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]
*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rule 1100

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b - q)*x^2)*Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]*EllipticF[ArcTan[Rt[(b - q)/(2*a), 2]*x], (-2*q)/(b - q)])/(2
*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx=\frac{\left(1+x^2\right)\sqrt{\frac{2+3x^2}{1+x^2}}\,F\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{2+5x^2+3x^4}}$$

Mathematica [C] time = 0.02, size = 58, normalized size = 1.12

$$-\frac{i\sqrt{x^2+1}\,\sqrt{3x^2+2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|\frac{2}{3}\right)}{\sqrt{9x^4+15x^2+6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])
/Sqrt[6 + 15*x^2 + 9*x^4]
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

maple [A] time = 0.02, size = 44, normalized size = 0.85

$$-\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/
2*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 + 3*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 + 3*x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 5*x**2 + 2), x)
```

3.67 $\int \frac{1}{\sqrt{2+4x^2+3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+4x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{3x^4+4x^2+2}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18-6*6^(1
/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4+4*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(3*x^4+4*x^2+2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+4x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{3x^4+4x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 4*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[2 + 4*x^2 + 3*x^
4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+4x^2+3x^4}}\,dx=\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+4x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{2+4x^2+3x^4}}$$

Mathematica [C] time = 0.08, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{3x^2}{-2-i\sqrt{2}}}\sqrt{1-\frac{3x^2}{-2+i\sqrt{2}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{-2-i\sqrt{2}}}x\right)|\frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right)}{\sqrt{3}\sqrt{-\frac{1}{-2-i\sqrt{2}}}\sqrt{3x^4+4x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 4*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(-2 - I*Sqrt[2])]*Sqrt[1 - (3*x^2)/(-2 + I*Sqrt[2])]
*EllipticF[I*ArcSinh[Sqrt[-3/(-2 - I*Sqrt[2])]*x], (-2 - I*Sqrt[2])/(-2 + I
*Sqrt[2])])/(Sqrt[3]*Sqrt[-(-2 - I*Sqrt[2])^(-1)]*Sqrt[2 + 4*x^2 + 3*x^4])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+4\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+4*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 4*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4+4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+4*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 4*x^2 + 2), x)
```

maple [C] time = 0.16, size = 87, normalized size = 0.97

$$\frac{2\sqrt{-\left(-1+\frac{i\sqrt{2}}{2}\right)x^2+1}\,\sqrt{-\left(-1-\frac{i\sqrt{2}}{2}\right)x^2+1}\,\text{EllipticF}\left(\frac{\sqrt{-4+2i\sqrt{2}}\,x}{2},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{\sqrt{-4+2i\sqrt{2}}\,\sqrt{3x^4+4x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+4*x^2+2)^(1/2),x)
```

```
[Out] 2/(-4+2*I*2^(1/2))^(1/2)*(1-(-1+1/2*I*2^(1/2))*x^2)^(1/2)*(1-(-1-1/2*I*2^(1
/2))*x^2)^(1/2)/(3*x^4+4*x^2+2)^(1/2)*EllipticF(1/2*x*(-4+2*I*2^(1/2))^(1/2
),1/3*(3+6*I*2^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4+4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+4*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 4*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4+4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 + 3*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(4*x^2 + 3*x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4+4x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+4*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 4*x**2 + 2), x)
```

3.68 $\int \frac{1}{\sqrt{2+3x^2+3x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+3x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4+3x^2+2}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/4*(8-2*6^(1/
2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4+3*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(3*x^4+3*x^2+2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+3x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 3*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*Sqrt[2 + 3*x^2 + 3*x^
4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+3x^2+3x^4}}\,dx=\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+3x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2+3x^2+3x^4}}$$

Mathematica [C] time = 0.11, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{6x^2}{-3-i\sqrt{15}}}\sqrt{1-\frac{6x^2}{-3+i\sqrt{15}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{-3-i\sqrt{15}}}x\right)|\frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right)}{\sqrt{6}\sqrt{-\frac{1}{-3-i\sqrt{15}}}\sqrt{3x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 3*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(-3 - I*Sqrt[15])]*Sqrt[1 - (6*x^2)/(-3 + I*Sqrt[15]
)]*EllipticF[I*ArcSinh[Sqrt[-6/(-3 - I*Sqrt[15])]*x], (-3 - I*Sqrt[15])/(-3
 + I*Sqrt[15])])/(Sqrt[6]*Sqrt[-(-3 - I*Sqrt[15])^(-1)]*Sqrt[2 + 3*x^2 + 3*
x^4])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 3*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 3*x^2 + 2), x)
```

maple [C] time = 0.13, size = 87, normalized size = 0.95

$$\frac{2\sqrt{-\left(-\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2+1}\,\sqrt{-\left(-\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2+1}\,\,\text{EllipticF}\left(\frac{\sqrt{-3+i\sqrt{15}}\,x}{2},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{\sqrt{-3+i\sqrt{15}}\,\sqrt{3x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+3*x^2+2)^(1/2),x)
```

```
[Out] 2/(-3+I*15^(1/2))^(1/2)*(1-(-3/4+1/4*I*15^(1/2))*x^2)^(1/2)*(1-(-3/4-1/4*I*
15^(1/2))*x^2)^(1/2)/(3*x^4+3*x^2+2)^(1/2)*EllipticF(1/2*x*(-3+I*15^(1/2))^
(1/2),1/2*(-1+I*15^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 3*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 + 3*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(3*x^2 + 3*x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4+3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 3*x**2 + 2), x)
```

3.69 $\int \frac{1}{\sqrt{2+2x^2+3x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+2x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4+2x^2+2}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18-3*6^(1
/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4+2*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(3*x^4+2*x^2+2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+2x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4+2x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 2*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*Sqrt[2 + 2*x^2 + 3*x
^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+2x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+2x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2+2x^2+3x^4}}$$

Mathematica [C] time = 0.08, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{3x^2}{-1-i\sqrt{5}}}\sqrt{1-\frac{3x^2}{-1+i\sqrt{5}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{-1-i\sqrt{5}}}\,x\right)|\frac{-1-i\sqrt{5}}{-1+i\sqrt{5}}\right)}{\sqrt{3}\sqrt{-\frac{1}{-1-i\sqrt{5}}}\sqrt{3x^4+2x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 2*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(-1 - I*Sqrt[5])]*Sqrt[1 - (3*x^2)/(-1 + I*Sqrt[5])]
*EllipticF[I*ArcSinh[Sqrt[-3/(-1 - I*Sqrt[5])]*x], (-1 - I*Sqrt[5])/(-1 + I
*Sqrt[5])])/(Sqrt[3]*Sqrt[-(-1 - I*Sqrt[5])^(-1)]*Sqrt[2 + 2*x^2 + 3*x^4])
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+2\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 2*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{3\,x^4+2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 2*x^2 + 2), x)
```

maple [C] time = 0.14, size = 87, normalized size = 0.95

$$\frac{2\sqrt{-\left(-\frac{1}{2}+\frac{i\sqrt{5}}{2}\right)x^2+1}\,\sqrt{-\left(-\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{-2+2i\sqrt{5}}\;x}{2},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{\sqrt{-2+2i\sqrt{5}}\;\sqrt{3x^4+2x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+2*x^2+2)^(1/2),x)
```

```
[Out] 2/(-2+2*I*5^(1/2))^(1/2)*(1-(-1/2+1/2*I*5^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*
5^(1/2))*x^2)^(1/2)/(3*x^4+2*x^2+2)^(1/2)*EllipticF(1/2*x*(-2+2*I*5^(1/2))^
(1/2),1/3*(-6+3*I*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4+2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 2*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4+2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 + 3*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(2*x^2 + 3*x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4+2x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+2*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 2*x**2 + 2), x)
```

3.70 $\int \frac{1}{\sqrt{2+x^2+3x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4+x^2+2}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/12*(72-6*6^(
1/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4+x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(3*x^4+x^2+2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(3/2)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*Sqrt[2 + x^2 + 3*x^4]
)
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2+x^2+3x^4}}$$

Mathematica [C] time = 0.08, size = 142, normalized size = 1.61

$$-\frac{i\sqrt{1-\frac{6x^2}{-1-i\sqrt{23}}}\sqrt{1-\frac{6x^2}{-1+i\sqrt{23}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{-1-i\sqrt{23}}}x\right)|\frac{-1-i\sqrt{23}}{-1+i\sqrt{23}}\right)}{\sqrt{6}\sqrt{-\frac{1}{-1-i\sqrt{23}}}\sqrt{3x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(-1 - I*Sqrt[23])]*Sqrt[1 - (6*x^2)/(-1 + I*Sqrt[23]
)]*EllipticF[I*ArcSinh[Sqrt[-6/(-1 - I*Sqrt[23])]*x], (-1 - I*Sqrt[23])/(-1
 + I*Sqrt[23])])/(Sqrt[6]*Sqrt[-(-1 - I*Sqrt[23])^(-1)]*Sqrt[2 + x^2 + 3*x^
4])
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{3\,x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + x^2 + 2), x)
```

maple [C] time = 0.13, size = 85, normalized size = 0.97

$$\frac{2\sqrt{-\left(-\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2+1}\,\sqrt{-\left(-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{-1+i\sqrt{23}}\,x}{2},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{\sqrt{-1+i\sqrt{23}}\,\sqrt{3x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+x^2+2)^(1/2),x)
```

```
[Out] 2/(-1+I*23^(1/2))^(1/2)*(1-(-1/4+1/4*I*23^(1/2))*x^2)^(1/2)*(1-(-1/4-1/4*I*
23^(1/2))*x^2)^(1/2)/(3*x^4+x^2+2)^(1/2)*EllipticF(1/2*x*(-1+I*23^(1/2))^(1
/2),1/6*(-33+3*I*23^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4 + x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4 + x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 3*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(x^2 + 3*x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4 + x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + x**2 + 2), x)
```

3.71 $\int \frac{1}{\sqrt{2+3x^4}}\,dx$

Optimal. Leaf size=72

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{3x^4+2}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/2*2^(1/2))*(
2+x^2*6^(1/2))*((3*x^4+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/4)/(3*x^4+2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {220}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{3x^4+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan
[(3/2)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[2 + 3*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+3x^4}}\,dx=\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{2+3x^4}}$$

Mathematica [C] time = 0.03, size = 25, normalized size = 0.35

$$-\sqrt[4]{-\frac{1}{6}}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{3}{2}}\,x\right)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 3*x^4],x]
[Out] -((-1/6)^(1/4)*EllipticF[I*ArcSinh[(-3/2)^(1/4)*x], -1])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2)^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(3*x^4 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(3*x^4 + 2), x)
```

maple [C] time = 0.06, size = 66, normalized size = 0.92

$$\frac{\sqrt{2}\,\sqrt{-2i\sqrt{6}\,x^2+4}\,\sqrt{2i\sqrt{6}\,x^2+4}\,\operatorname{EllipticF}\left(\frac{\sqrt{2}\,\sqrt{i\sqrt{6}}\,x}{2},i\right)}{4\sqrt{i\sqrt{6}}\,\sqrt{3x^4+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+2)^(1/2),x)
[Out] 1/4*2^(1/2)/(I*6^(1/2))^(1/2)*(4-2*I*6^(1/2)*x^2)^(1/2)*(4+2*I*6^(1/2)*x^2)
^(1/2)/(3*x^4+2)^(1/2)*EllipticF(1/2*x*2^(1/2)*(I*6^(1/2))^(1/2),I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 2), x)
```

mupad [B] time = 0.09, size = 16, normalized size = 0.22

$$\frac{\sqrt{2}\,x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{3\,x^4}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 + 2)^(1/2),x)
```

```
[Out] (2^(1/2)*x*hypergeom([1/4, 1/2], 5/4, -(3*x^4)/2))/2
```

sympy [C] time = 0.71, size = 36, normalized size = 0.50

$$\frac{\sqrt{2}\,x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{3x^4e^{i\pi}}{2}\right)}{8\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+2)**(1/2),x)
```

```
[Out] sqrt(2)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 3*x**4*exp_polar(I*pi)/2)/(8
*gamma(5/4))
```

3.72 $\int \frac{1}{\sqrt{2-x^2+3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4-x^2+2}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/12*(72+6*6^(
1/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4-x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(3*x^4-x^2+2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4-x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(3/2)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*Sqrt[2 - x^2 + 3*x^4]
)
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-x^2+3x^4}}\,dx=\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2-x^2+3x^4}}$$

Mathematica [C] time = 0.08, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{6x^2}{1-i\sqrt{23}}}\sqrt{1-\frac{6x^2}{1+i\sqrt{23}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{1-i\sqrt{23}}}x\right)|\frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right)}{\sqrt{6}\sqrt{-\frac{1}{1-i\sqrt{23}}}\sqrt{3x^4-x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(1 - I*Sqrt[23])]*Sqrt[1 - (6*x^2)/(1 + I*Sqrt[23])]
*EllipticF[I*ArcSinh[Sqrt[-6/(1 - I*Sqrt[23])]*x], (1 - I*Sqrt[23])/(1 + I*
Sqrt[23])])/(Sqrt[6]*Sqrt[-(1 - I*Sqrt[23])^(-1)]*Sqrt[2 - x^2 + 3*x^4])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3x^4-x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{3x^4-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - x^2 + 2), x)
```

maple [C] time = 0.12, size = 87, normalized size = 0.97

$$\frac{2\sqrt{-\left(\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2+1}\sqrt{-\left(\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2+1}\ \text{EllipticF}\left(\frac{\sqrt{i\sqrt{23}+1}\,x}{2},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{\sqrt{i\sqrt{23}+1}\sqrt{3x^4-x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-x^2+2)^(1/2),x)
```

```
[Out] 2/(I*23^(1/2)+1)^(1/2)*(1-(1/4+1/4*I*23^(1/2))*x^2)^(1/2)*(1-(1/4-1/4*I*23^
(1/2))*x^2)^(1/2)/(3*x^4-x^2+2)^(1/2)*EllipticF(1/2*x*(I*23^(1/2)+1)^(1/2),
1/6*(-33-3*I*23^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4 - x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4 - x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - x^2 + 2)^(1/2),x)
```

```
[Out] int(1/(3*x^4 - x^2 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4 - x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - x**2 + 2), x)
```

3.73 $\int \frac{1}{\sqrt{2-2x^2+3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-2x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4-2x^2+2}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18+3*6^(1
/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4-2*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(3*x^4-2*x^2+2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-2x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4-2x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 2*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*Sqrt[2 - 2*x^2 + 3*x
^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-2x^2+3x^4}}\,dx=\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-2x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2-2x^2+3x^4}}$$

Mathematica [C] time = 0.08, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{3x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{3x^2}{1+i\sqrt{5}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{1-i\sqrt{5}}}\,x\right)|\frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right)}{\sqrt{3}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{3x^4-2x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - 2*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(1 - I*Sqrt[5])]*Sqrt[1 - (3*x^2)/(1 + I*Sqrt[5])]*E
llipticF[I*ArcSinh[Sqrt[-3/(1 - I*Sqrt[5])]*x], (1 - I*Sqrt[5])/(1 + I*Sqrt
[5])])/(Sqrt[3]*Sqrt[-(1 - I*Sqrt[5])^(-1)]*Sqrt[2 - 2*x^2 + 3*x^4])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-2\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 2*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{3\,x^4-2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 2*x^2 + 2), x)
```

maple [C] time = 0.12, size = 87, normalized size = 0.97

$$\frac{2\sqrt{-\left(\frac{1}{2}+\frac{i\sqrt{5}}{2}\right)x^2+1}\,\sqrt{-\left(\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2+1}\,\,\mathrm{EllipticF}\left(\frac{\sqrt{2+2i\sqrt{5}}\,x}{2},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{\sqrt{2+2i\sqrt{5}}\,\sqrt{3x^4-2x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-2*x^2+2)^(1/2),x)
```

```
[Out] 2/(2+2*I*5^(1/2))^(1/2)*(1-(1/2+1/2*I*5^(1/2))*x^2)^(1/2)*(1-(1/2-1/2*I*5^(
1/2))*x^2)^(1/2)/(3*x^4-2*x^2+2)^(1/2)*EllipticF(1/2*x*(2+2*I*5^(1/2))^(1/2
),1/3*(-6-3*I*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4 - 2\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 2*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4 - 2\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - 2*x^2 + 2)^(1/2),x)
```

```
[Out] int(1/(3*x^4 - 2*x^2 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4 - 2x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-2*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 2*x**2 + 2), x)
```

3.74 $\int \frac{1}{\sqrt{2-3x^2+3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-3x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4-3x^2+2}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/4*(8+2*6^(1/
2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4-3*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(3*x^4-3*x^2+2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-3x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4-3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 3*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*Sqrt[2 - 3*x^2 + 3*x^
4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x^2+3x^4}}\,dx=\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-3x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2-3x^2+3x^4}}$$

Mathematica [C] time = 0.11, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{6x^2}{3-i\sqrt{15}}}\sqrt{1-\frac{6x^2}{3+i\sqrt{15}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{3-i\sqrt{15}}}x\right)|\frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right)}{\sqrt{6}\sqrt{-\frac{1}{3-i\sqrt{15}}}\sqrt{3x^4-3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - 3*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(3 - I*Sqrt[15])]*Sqrt[1 - (6*x^2)/(3 + I*Sqrt[15])]
*EllipticF[I*ArcSinh[Sqrt[-6/(3 - I*Sqrt[15])]*x], (3 - I*Sqrt[15])/(3 + I*
Sqrt[15])])/(Sqrt[6]*Sqrt[-(3 - I*Sqrt[15])^(-1)]*Sqrt[2 - 3*x^2 + 3*x^4])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 3*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 3*x^2 + 2), x)
```

maple [C] time = 0.12, size = 87, normalized size = 0.97

$$\frac{2\sqrt{-\left(\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2+1}\,\sqrt{-\left(\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2+1}\ \text{EllipticF}\left(\frac{\sqrt{i\sqrt{15}+3}\,x}{2},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{\sqrt{i\sqrt{15}+3}\,\sqrt{3x^4-3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-3*x^2+2)^(1/2),x)
```

```
[Out] 2/(I*15^(1/2)+3)^(1/2)*(1-(3/4+1/4*I*15^(1/2))*x^2)^(1/2)*(1-(3/4-1/4*I*15^
(1/2))*x^2)^(1/2)/(3*x^4-3*x^2+2)^(1/2)*EllipticF(1/2*x*(I*15^(1/2)+3)^(1/2
),1/2*(-1-I*15^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4 - 3\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 3*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4 - 3\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - 3*x^2 + 2)^(1/2),x)
```

```
[Out] int(1/(3*x^4 - 3*x^2 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4 - 3x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 3*x**2 + 2), x)
```

3.75 $\int \frac{1}{\sqrt{2-4x^2+3x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-4x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{3x^4-4x^2+2}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18+6*6^(1
/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4-4*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(3*x^4-4*x^2+2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-4x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{3x^4-4x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 4*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[2 - 4*x^2 + 3*x^
4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-4x^2+3x^4}}\,dx=\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-4x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{2-4x^2+3x^4}}$$

Mathematica [C] time = 0.08, size = 144, normalized size = 1.64

$$-\frac{i\sqrt{1-\frac{3x^2}{2-i\sqrt{2}}}\sqrt{1-\frac{3x^2}{2+i\sqrt{2}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{2-i\sqrt{2}}}\,x\right)|\frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right)}{\sqrt{3}\sqrt{-\frac{1}{2-i\sqrt{2}}}\sqrt{3x^4-4x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - 4*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(2 - I*Sqrt[2])]*Sqrt[1 - (3*x^2)/(2 + I*Sqrt[2])]*E
llipticF[I*ArcSinh[Sqrt[-3/(2 - I*Sqrt[2])]*x], (2 - I*Sqrt[2])/(2 + I*Sqrt
[2])])/(Sqrt[3]*Sqrt[-(2 - I*Sqrt[2])^(-1)]*Sqrt[2 - 4*x^2 + 3*x^4])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{\sqrt{3\,x^4-4\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-4*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 4*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-4*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 4*x^2 + 2), x)
```

maple [C] time = 0.12, size = 87, normalized size = 0.99

$$\frac{2\sqrt{-\left(1+\frac{i\sqrt{2}}{2}\right)x^2+1}\,\sqrt{-\left(1-\frac{i\sqrt{2}}{2}\right)x^2+1}\,\,\mathrm{EllipticF}\left(\frac{\sqrt{4+2i\sqrt{2}}\,x}{2},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{\sqrt{4+2i\sqrt{2}}\,\sqrt{3x^4-4x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-4*x^2+2)^(1/2),x)
```

```
[Out] 2/(4+2*I*2^(1/2))^(1/2)*(1-(1+1/2*I*2^(1/2))*x^2)^(1/2)*(1-(1-1/2*I*2^(1/2)
)*x^2)^(1/2)/(3*x^4-4*x^2+2)^(1/2)*EllipticF(1/2*x*(4+2*I*2^(1/2))^(1/2),1/
3*(3-6*I*2^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-4*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 4*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - 4*x^2 + 2)^(1/2),x)
```

```
[Out] int(1/(3*x^4 - 4*x^2 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4-4x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-4*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 4*x**2 + 2), x)
```

3.76 $\int \frac{1}{\sqrt{2-5x^2+3x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-5x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4-5x^2+2}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/12*(72+30*6^
(1/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4-5*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(
3/4)/(3*x^4-5*x^2+2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1096}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-5x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4-5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 5*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 5*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (12 + 5*Sqrt[6])/24])/(2*6^(1/4)*Sqrt[2 - 5*x^2 +
3*x^4])
```

Rule 1096

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-5x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-5x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2-5x^2+3x^4}}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.58

$$\frac{\sqrt{2-3x^2}\,\sqrt{1-x^2}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|\frac{2}{3}\right)}{\sqrt{9x^4-15x^2+6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - 5*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], 2/3])/Sqrt[6
- 15*x^2 + 9*x^4]
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{\sqrt{3\,x^4-5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 5*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 5*x^2 + 2), x)
```

maple [A] time = 0.01, size = 42, normalized size = 0.46

$$\frac{\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\,\;\mathrm{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4-5x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-5*x^2+2)^(1/2),x)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*(-6*x^2+4)^(1/2)/(3*x^4-5*x^2+2)^(1/2)*EllipticF(x,1/2*6
^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^{4}-5\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 5*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^{4}-5\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - 5*x^2 + 2)^(1/2),x)
```

```
[Out] int(1/(3*x^4 - 5*x^2 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^{4}-5x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 5*x**2 + 2), x)
```

3.77 $\int \frac{1}{\sqrt{2-6x^2+3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-6x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)\middle|\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4-6x^2+2}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/2*(2+6^(1/2)
)^(1/2))*(2+x^2*6^(1/2))*((3*x^4-6*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/4)/
(3*x^4-6*x^2+2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1096}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-6x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)\middle|\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4-6x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 6*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 6*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (2 + Sqrt[6])/4])/(2*6^(1/4)*Sqrt[2 - 6*x^2 + 3*x^
4])
```

Rule 1096

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-6x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-6x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2-6x^2+3x^4}}$$

Mathematica [A] time = 0.08, size = 85, normalized size = 0.94

$$\frac{\sqrt{-3x^2-\sqrt{3}+3}\sqrt{\left(\sqrt{3}-3\right)x^2+2}\,F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{3}\right)}x\right)|2-\sqrt{3}\right)}{\sqrt{6}\sqrt{3x^4-6x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - 6*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[3 - Sqrt[3] - 3*x^2]*Sqrt[2 + (-3 + Sqrt[3])*x^2]*EllipticF[ArcSin[Sq
rt[(3 + Sqrt[3])/2]*x], 2 - Sqrt[3]])/(Sqrt[6]*Sqrt[2 - 6*x^2 + 3*x^4])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-6\,x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-6*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 6*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4-6\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-6*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 6*x^2 + 2), x)
```

maple [A] time = 0.08, size = 82, normalized size = 0.91

$$\frac{2\sqrt{-\left(\frac{\sqrt{3}}{2}+\frac{3}{2}\right)x^2+1}\,\sqrt{-\left(-\frac{\sqrt{3}}{2}+\frac{3}{2}\right)x^2+1}\;\operatorname{EllipticF}\left(\frac{\sqrt{6+2\sqrt{3}}\,x}{2}, \frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{6+2\sqrt{3}}\,\sqrt{3x^4-6x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-6*x^2+2)^(1/2),x)
```

```
[Out] 2/(6+2*3^(1/2))^(1/2)*(1-(1/2*3^(1/2)+3/2)*x^2)^(1/2)*(1-(-1/2*3^(1/2)+3/2)
*x^2)^(1/2)/(3*x^4-6*x^2+2)^(1/2)*EllipticF(1/2*x*(6+2*3^(1/2))^(1/2),1/2*6
^(1/2)-1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4-6\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-6*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 6*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4-6\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - 6*x^2 + 2)^(1/2),x)
```

```
[Out] int(1/(3*x^4 - 6*x^2 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4-6x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-6*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 6*x**2 + 2), x)
```

3.78 $\int \frac{1}{\sqrt{3+9x^2+2x^4}}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{\frac{(9-\sqrt{57})x^2+6}{(9+\sqrt{57})x^2+6}}\left(\left(9+\sqrt{57}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(9+\sqrt{57}\right)}x\right)|\frac{1}{4}\left(-19+3\sqrt{57}\right)\right)}{\sqrt{6\left(9+\sqrt{57}\right)}\sqrt{2x^4+9x^2+3}}$$

```
[Out] (1/(36+x^2*(54+6*57^(1/2))))^(1/2)*(36+x^2*(54+6*57^(1/2)))^(1/2)*EllipticF
(x*(54+6*57^(1/2))^(1/2)/(36+x^2*(54+6*57^(1/2)))^(1/2),1/2*(-19+3*57^(1/2)
)^(1/2))*(6+x^2*(57^(1/2)+9))*((6+x^2*(9-57^(1/2)))/(6+x^2*(57^(1/2)+9)))^(
1/2)/(2*x^4+9*x^2+3)^(1/2)/(54+6*57^(1/2))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1099}

$$\frac{\sqrt{\frac{(9-\sqrt{57})x^2+6}{(9+\sqrt{57})x^2+6}}\left(\left(9+\sqrt{57}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(9+\sqrt{57}\right)}x\right)|\frac{1}{4}\left(-19+3\sqrt{57}\right)\right)}{\sqrt{6\left(9+\sqrt{57}\right)}\sqrt{2x^4+9x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 9*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(6 + (9 - Sqrt[57])*x^2)/(6 + (9 + Sqrt[57])*x^2)]*(6 + (9 + Sqrt[57]
)*x^2)*EllipticF[ArcTan[Sqrt[(9 + Sqrt[57])/6]*x], (-19 + 3*Sqrt[57])/4])/(
Sqrt[6*(9 + Sqrt[57])]*Sqrt[3 + 9*x^2 + 2*x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+9x^2+2x^4}}\,dx = \frac{\sqrt{\frac{6+\left(9-\sqrt{57}\right)x^2}{6+\left(9+\sqrt{57}\right)x^2}}\left(6+\left(9+\sqrt{57}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(9+\sqrt{57}\right)}x\right)|\frac{1}{4}\left(-19+3\sqrt{57}\right)\right)}{\sqrt{6\left(9+\sqrt{57}\right)}\sqrt{3+9x^2+2x^4}}$$

Mathematica [C]   time = 0.07, size = 97, normalized size = 0.88

$$-\frac{i\sqrt{\frac{-4x^2+\sqrt{57}-9}{\sqrt{57}-9}}\sqrt{4x^2+\sqrt{57}+9}\,F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{9+\sqrt{57}}}\right)|\frac{23}{4}+\frac{3\sqrt{57}}{4}\right)}{2\sqrt{2x^4+9x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 9*x^2 + 2*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[(-9 + Sqrt[57] - 4*x^2)/(-9 + Sqrt[57])]*Sqrt[9 + Sqrt[57] +
 4*x^2]*EllipticF[I*ArcSinh[(2*x)/Sqrt[9 + Sqrt[57]]], 23/4 + (3*Sqrt[57])/
4])/Sqrt[3 + 9*x^2 + 2*x^4]
```

fricas [F]   time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+9\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+9*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 9*x^2 + 3), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+9\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+9*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 9*x^2 + 3), x)
```

maple [A] time = 0.10, size = 82, normalized size = 0.75

$$\frac{6\sqrt{-\left(-\frac{3}{2}+\frac{\sqrt{57}}{6}\right)x^2+1}\,\sqrt{-\left(-\frac{3}{2}-\frac{\sqrt{57}}{6}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{-54+6\sqrt{57}}\,x}{6},\frac{3\sqrt{6}}{4}+\frac{\sqrt{38}}{4}\right)}{\sqrt{-54+6\sqrt{57}}\,\sqrt{2x^4+9x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+9*x^2+3)^(1/2),x)
[Out] 6/(-54+6*57^(1/2))^(1/2)*(1-(-3/2+1/6*57^(1/2))*x^2)^(1/2)*(1-(-3/2-1/6*57^
(1/2))*x^2)^(1/2)/(2*x^4+9*x^2+3)^(1/2)*EllipticF(1/6*x*(-54+6*57^(1/2))^(1
/2),3/4*6^(1/2)+1/4*38^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+9\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+9*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(2*x^4 + 9*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4+9\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2 + 2*x^4 + 3)^(1/2),x)
[Out] int(1/(9*x^2 + 2*x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4+9x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+9*x**2+3)**(1/2),x)
[Out] Integral(1/sqrt(2*x**4 + 9*x**2 + 3), x)
```

3.79 $\int \frac{1}{\sqrt{3+8x^2+2x^4}}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{\frac{\left(4-\sqrt{10}\right)x^2+3}{\left(4+\sqrt{10}\right)x^2+3}}\left(\left(4+\sqrt{10}\right)x^2+3\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{3}\left(4+\sqrt{10}\right)}x\right)|-\frac{2}{3}\left(5-2\sqrt{10}\right)\right)}{\sqrt{3\left(4+\sqrt{10}\right)}\sqrt{2x^4+8x^2+3}}$$

```
[Out] (1/(9+x^2*(12+3*10^(1/2))))^(1/2)*(9+x^2*(12+3*10^(1/2)))^(1/2)*EllipticF(x
*(12+3*10^(1/2))^(1/2)/(9+x^2*(12+3*10^(1/2)))^(1/2),1/3*(-30+12*10^(1/2))^
(1/2))*(3+x^2*(10^(1/2)+4))*((3+x^2*(4-10^(1/2)))/(3+x^2*(10^(1/2)+4)))^(1/
2)/(2*x^4+8*x^2+3)^(1/2)/(12+3*10^(1/2))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1099}

$$\frac{\sqrt{\frac{\left(4-\sqrt{10}\right)x^2+3}{\left(4+\sqrt{10}\right)x^2+3}}\left(\left(4+\sqrt{10}\right)x^2+3\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{3}\left(4+\sqrt{10}\right)}x\right)|-\frac{2}{3}\left(5-2\sqrt{10}\right)\right)}{\sqrt{3\left(4+\sqrt{10}\right)}\sqrt{2x^4+8x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 8*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(3 + (4 - Sqrt[10])*x^2)/(3 + (4 + Sqrt[10])*x^2)]*(3 + (4 + Sqrt[10]
)*x^2)*EllipticF[ArcTan[Sqrt[(4 + Sqrt[10])/3]*x], (-2*(5 - 2*Sqrt[10]))/3]
)/(Sqrt[3*(4 + Sqrt[10])]*Sqrt[3 + 8*x^2 + 2*x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+8x^2+2x^4}}\,dx = \frac{\sqrt{\frac{3+\left(4-\sqrt{10}\right)x^2}{3+\left(4+\sqrt{10}\right)x^2}}\left(3+\left(4+\sqrt{10}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{3}\left(4+\sqrt{10}\right)}x\right)\middle|-\frac{2}{3}\left(5-2\sqrt{10}\right)\right)}{\sqrt{3\left(4+\sqrt{10}\right)}\sqrt{3+8x^2+2x^4}}$$

Mathematica [C] time = 0.08, size = 98, normalized size = 0.89

$$-\frac{i\sqrt{\frac{-2x^2+\sqrt{10}-4}{\sqrt{10}-4}}\sqrt{2x^2+\sqrt{10}+4}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{4+\sqrt{10}}}x\right)\middle|\frac{13}{3}+\frac{4\sqrt{10}}{3}\right)}{\sqrt{4x^4+16x^2+6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 8*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[(-4 + Sqrt[10] - 2*x^2)/(-4 + Sqrt[10])]*Sqrt[4 + Sqrt[10] + 2*x
^2]*EllipticF[I*ArcSinh[Sqrt[2/(4 + Sqrt[10])]*x], 13/3 + (4*Sqrt[10])/3])/
Sqrt[6 + 16*x^2 + 4*x^4]
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+8\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+8*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 8*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+8\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+8*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 8*x^2 + 3), x)
```

maple [A] time = 0.09, size = 82, normalized size = 0.75

$$\frac{3\sqrt{-\left(-\frac{4}{3}+\frac{\sqrt{10}}{3}\right)x^{2}+1}\,\sqrt{-\left(-\frac{4}{3}-\frac{\sqrt{10}}{3}\right)x^{2}+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{-12+3\sqrt{10}}\,x}{3},\frac{2\sqrt{6}}{3}+\frac{\sqrt{15}}{3}\right)}{\sqrt{-12+3\sqrt{10}}\,\sqrt{2x^{4}+8x^{2}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+8*x^2+3)^(1/2),x)
```

```
[Out] 3/(-12+3*10^(1/2))^(1/2)*(1-(-4/3+1/3*10^(1/2))*x^2)^(1/2)*(1-(-4/3-1/3*10^
(1/2))*x^2)^(1/2)/(2*x^4+8*x^2+3)^(1/2)*EllipticF(1/3*x*(-12+3*10^(1/2))^(1
/2),2/3*6^(1/2)+1/3*15^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^{4}+8\,x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+8*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 8*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^{4}+8\,x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^2 + 2*x^4 + 3)^(1/2),x)
```

```
[Out] int(1/(8*x^2 + 2*x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^{4}+8x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+8*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 8*x**2 + 3), x)
```

3.80 $\int \frac{1}{\sqrt{3+7x^2+2x^4}}\,dx$

Optimal. Leaf size=60

$$\frac{\sqrt{\frac{x^2+3}{2x^2+1}}\,(2x^2+1)\,F\left(\tan^{-1}\left(\sqrt{2}\,x\right)\middle|\frac{5}{6}\right)}{\sqrt{6}\,\sqrt{2x^4+7x^2+3}}$$

```
[Out] 1/6*(2*x^2+1)^(3/2)*(1/(2*x^2+1))^(1/2)*EllipticF(x*2^(1/2)/(2*x^2+1)^(1/2)
,1/6*30^(1/2))*((x^2+3)/(2*x^2+1))^(1/2)*6^(1/2)/(2*x^4+7*x^2+3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1099}

$$\frac{\sqrt{\frac{x^2+3}{2x^2+1}}\,(2x^2+1)\,F\left(\tan^{-1}\left(\sqrt{2}\,x\right)\middle|\frac{5}{6}\right)}{\sqrt{6}\,\sqrt{2x^4+7x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 7*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(3 + x^2)/(1 + 2*x^2)]*(1 + 2*x^2)*EllipticF[ArcTan[Sqrt[2]*x], 5/6])
/(Sqrt[6]*Sqrt[3 + 7*x^2 + 2*x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+7x^2+2x^4}}\,dx = \frac{\sqrt{\frac{3+x^2}{1+2x^2}}\,(1+2x^2)\,F\left(\tan^{-1}\left(\sqrt{2}\,x\right)\middle|\frac{5}{6}\right)}{\sqrt{6}\,\sqrt{3+7x^2+2x^4}}$$

Mathematica [C] time = 0.03, size = 61, normalized size = 1.02

$$-\frac{i\sqrt{x^2+3}\sqrt{2x^2+1}F\left(i\sinh^{-1}\left(\sqrt{2}x\right)|\frac{1}{6}\right)}{\sqrt{6}\sqrt{2x^4+7x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 7*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[3 + x^2]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]*x], 1/6])/(
Sqrt[6]*Sqrt[3 + 7*x^2 + 2*x^4])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2x^4+7x^2+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+7*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 7*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4+7x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+7*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 7*x^2 + 3), x)
```

maple [C] time = 0.02, size = 50, normalized size = 0.83

$$-\frac{i\sqrt{3}\,\sqrt{3x^2+9}\,\sqrt{2x^2+1}\,\operatorname{EllipticF}\left(\frac{i\sqrt{3}\,x}{3}, \sqrt{6}\right)}{3\sqrt{2x^4+7x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+7*x^2+3)^(1/2),x)
```

```
[Out] -1/3*I*3^(1/2)*(3*x^2+9)^(1/2)*(2*x^2+1)^(1/2)/(2*x^4+7*x^2+3)^(1/2)*Ellipt
icF(1/3*I*3^(1/2)*x,6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+7*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 7*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{2\,x^4+7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7*x^2 + 2*x^4 + 3)^(1/2),x)
```

```
[Out] int(1/(7*x^2 + 2*x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4+7x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+7*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 7*x**2 + 3), x)
```

3.81 $\int \frac{1}{\sqrt{3+6x^2+2x^4}} \, dx$

Optimal. Leaf size=104

$$\frac{\sqrt{\frac{\left(3-\sqrt{3}\right)x^2+3}{\left(3+\sqrt{3}\right)x^2+3}}\left(\left(3+\sqrt{3}\right)x^2+3\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}x\right)|-1+\sqrt{3}\right)}{\sqrt{3\left(3+\sqrt{3}\right)}\sqrt{2x^4+6x^2+3}}$$

```
[Out] (1/(9+x^2*(9+3*3^(1/2))))^(1/2)*(9+x^2*(9+3*3^(1/2)))^(1/2)*EllipticF(x*(9+
3*3^(1/2))^(1/2)/(9+x^2*(9+3*3^(1/2)))^(1/2),(3^(1/2)-1)^(1/2))*(3+x^2*(3^(
1/2)+3))*((3+x^2*(3-3^(1/2)))/(3+x^2*(3^(1/2)+3)))^(1/2)/(2*x^4+6*x^2+3)^(1
/2)/(9+3*3^(1/2))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1099}

$$\frac{\sqrt{\frac{\left(3-\sqrt{3}\right)x^2+3}{\left(3+\sqrt{3}\right)x^2+3}}\left(\left(3+\sqrt{3}\right)x^2+3\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}x\right)|-1+\sqrt{3}\right)}{\sqrt{3\left(3+\sqrt{3}\right)}\sqrt{2x^4+6x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 6*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(3 + (3 - Sqrt[3])*x^2)/(3 + (3 + Sqrt[3])*x^2)]*(3 + (3 + Sqrt[3])*x
^2)*EllipticF[ArcTan[Sqrt[(3 + Sqrt[3])/3]*x], -1 + Sqrt[3]])/(Sqrt[3*(3 +
Sqrt[3])]*Sqrt[3 + 6*x^2 + 2*x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+6x^2+2x^4}}\,dx = \frac{\sqrt{\frac{3+\left(3-\sqrt{3}\right)x^2}{3+\left(3+\sqrt{3}\right)x^2}}\left(3+\left(3+\sqrt{3}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}x\right)\middle|-1+\sqrt{3}\right)}{\sqrt{3\left(3+\sqrt{3}\right)}\sqrt{3+6x^2+2x^4}}$$

Mathematica [C] time = 0.06, size = 90, normalized size = 0.87

$$-\frac{i\sqrt{\frac{-2x^2+\sqrt{3}-3}{\sqrt{3}-3}}\sqrt{2x^2+\sqrt{3}+3}\,F\left(i\sinh^{-1}\left(\sqrt{1-\frac{1}{\sqrt{3}}}x\right)\middle|2+\sqrt{3}\right)}{\sqrt{4x^4+12x^2+6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 6*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[(-3 + Sqrt[3] - 2*x^2)/(-3 + Sqrt[3])]*Sqrt[3 + Sqrt[3] + 2*x^2]
*EllipticF[I*ArcSinh[Sqrt[1 - 1/Sqrt[3]]*x], 2 + Sqrt[3]])/Sqrt[6 + 12*x^2
+ 4*x^4]
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+6\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+6*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 6*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+6*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 6*x^2 + 3), x)
```

maple [A] time = 0.08, size = 82, normalized size = 0.79

$$\frac{3\sqrt{-\left(-1+\frac{\sqrt{3}}{3}\right)x^{2}+1}\,\sqrt{-\left(-1-\frac{\sqrt{3}}{3}\right)x^{2}+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{-9+3\sqrt{3}}\,x}{3},\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{-9+3\sqrt{3}}\,\sqrt{2x^{4}+6x^{2}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+6*x^2+3)^(1/2),x)
```

```
[Out] 3/(-9+3*3^(1/2))^(1/2)*(1-(-1+1/3*3^(1/2))*x^2)^(1/2)*(1-(-1-1/3*3^(1/2))*x
^2)^(1/2)/(2*x^4+6*x^2+3)^(1/2)*EllipticF(1/3*x*(-9+3*3^(1/2))^(1/2),1/2*6^
(1/2)+1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^{4}+6\,x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+6*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 6*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^{4}+6\,x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(6*x^2 + 2*x^4 + 3)^(1/2),x)
```

```
[Out] int(1/(6*x^2 + 2*x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^{4}+6x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+6*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 6*x**2 + 3), x)
```

3.82 $\int \frac{1}{\sqrt{3+5x^2+2x^4}}\,dx$

Optimal. Leaf size=52

$$\frac{\left(x^2+1\right)\sqrt{\frac{2x^2+3}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{3}\right)}{\sqrt{3}\sqrt{2x^4+5x^2+3}}$$

```
[Out] 1/3*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/3*3^(1/2))*
((2*x^2+3)/(x^2+1))^(1/2)*3^(1/2)/(2*x^4+5*x^2+3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1099}

$$\frac{\left(x^2+1\right)\sqrt{\frac{2x^2+3}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{3}\right)}{\sqrt{3}\sqrt{2x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 5*x^2 + 2*x^4],x]
```

```
[Out] ((1 + x^2)*Sqrt[(3 + 2*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/3])/(Sqrt[3]*
Sqrt[3 + 5*x^2 + 2*x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+5x^2+2x^4}}\,dx=\frac{\left(1+x^2\right)\sqrt{\frac{3+2x^2}{1+x^2}}\,F\left(\tan^{-1}(x)|\frac{1}{3}\right)}{\sqrt{3}\sqrt{3+5x^2+2x^4}}$$

Mathematica [C] time = 0.02, size = 58, normalized size = 1.12

$$-\frac{i\sqrt{x^2+1}\,\sqrt{2x^2+3}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)|\frac{3}{2}\right)}{\sqrt{4x^4+10x^2+6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 5*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[3 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/3]*x], 3/2])
/Sqrt[6 + 10*x^2 + 4*x^4]
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+5\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 5*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{2\,x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 5*x^2 + 3), x)
```

maple [C] time = 0.02, size = 50, normalized size = 0.96

$$-\frac{i\sqrt{6}\,\sqrt{6x^2+9}\,\sqrt{x^2+1}\;\text{EllipticF}\left(\frac{i\sqrt{6}\,x}{3},\frac{\sqrt{6}}{2}\right)}{6\sqrt{2x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -1/6*I*6^(1/2)*(6*x^2+9)^(1/2)*(x^2+1)^(1/2)/(2*x^4+5*x^2+3)^(1/2)*Elliptic
F(1/3*I*6^(1/2)*x,1/2*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4 + 5\,x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 5*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{2\,x^4 + 5\,x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 + 2*x^4 + 3)^(1/2),x)
```

```
[Out] int(1/(5*x^2 + 2*x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4 + 5x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 5*x**2 + 3), x)
```

3.83 $\int \frac{1}{\sqrt{3+4x^2+2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+4x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{2x^4+4x^2+3}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18-6*6^(1
/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4+4*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(2*x^4+4*x^2+3)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+4x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{2x^4+4x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 4*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[3 + 4*x^2 + 2*x^
4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+4x^2+2x^4}}\,dx=\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+4x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{3+4x^2+2x^4}}$$

Mathematica [C] time = 0.08, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{2x^2}{-2-i\sqrt{2}}}\sqrt{1-\frac{2x^2}{-2+i\sqrt{2}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{-2-i\sqrt{2}}}x\right)|\frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-2-i\sqrt{2}}}\sqrt{2x^4+4x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 4*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(-2 - I*Sqrt[2])]*Sqrt[1 - (2*x^2)/(-2 + I*Sqrt[2])]
*EllipticF[I*ArcSinh[Sqrt[-2/(-2 - I*Sqrt[2])]*x], (-2 - I*Sqrt[2])/(-2 + I
*Sqrt[2])])/(Sqrt[2]*Sqrt[-(-2 - I*Sqrt[2])^(-1)]*Sqrt[3 + 4*x^2 + 2*x^4])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+4\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+4*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 4*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+4*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 4*x^2 + 3), x)
```

maple [C] time = 0.10, size = 87, normalized size = 0.97

$$\frac{3\sqrt{-\left(-\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2+1}\,\sqrt{-\left(-\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{-6+3i\sqrt{2}}\,x}{3},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{\sqrt{-6+3i\sqrt{2}}\,\sqrt{2x^4+4x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+4*x^2+3)^(1/2),x)
```

```
[Out] 3/(-6+3*I*2^(1/2))^(1/2)*(1-(-2/3+1/3*I*2^(1/2))*x^2)^(1/2)*(1-(-2/3-1/3*I*
2^(1/2))*x^2)^(1/2)/(2*x^4+4*x^2+3)^(1/2)*EllipticF(1/3*x*(-6+3*I*2^(1/2))^
(1/2),1/3*(3+6*I*2^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+4*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 4*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4+4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 + 2*x^4 + 3)^(1/2),x)
```

```
[Out] int(1/(4*x^2 + 2*x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4+4x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+4*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 4*x**2 + 3), x)
```

3.84 $\int \frac{1}{\sqrt{3+3x^2+2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+3x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4+3x^2+3}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/4*(8-2*6^(1/
2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4+3*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(2*x^4+3*x^2+3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+3x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4+3x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 3*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*Sqrt[3 + 3*x^2 + 2*x^
4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+3x^2+2x^4}}\,dx=\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+3x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3+3x^2+2x^4}}$$

Mathematica [C] time = 0.10, size = 142, normalized size = 1.54

$$-\frac{i\sqrt{1-\frac{4x^2}{-3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{-3+i\sqrt{15}}}\,F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{-3-i\sqrt{15}}}\,x\right)|\frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right)}{2\sqrt{-\frac{1}{-3-i\sqrt{15}}}\sqrt{2x^4+3x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 3*x^2 + 2*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 - (4*x^2)/(-3 - I*Sqrt[15])]*Sqrt[1 - (4*x^2)/(-3 + I*Sqrt
[15])]*EllipticF[I*ArcSinh[2*Sqrt[-(-3 - I*Sqrt[15])^(-1)]*x], (-3 - I*Sqrt
[15])/(-3 + I*Sqrt[15])]])/(Sqrt[-(-3 - I*Sqrt[15])^(-1)]*Sqrt[3 + 3*x^2 + 2
*x^4])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+3\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 3*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 3*x^2 + 3), x)
```

maple [C] time = 0.10, size = 87, normalized size = 0.95

$$\frac{6\sqrt{-\left(-\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2+1}\,\sqrt{-\left(-\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2+1}\,\text{EllipticF}\left(\frac{\sqrt{-18+6i\sqrt{15}}\,x}{6},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{\sqrt{-18+6i\sqrt{15}}\,\sqrt{2x^4+3x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+3*x^2+3)^(1/2),x)
```

```
[Out] 6/(-18+6*I*15^(1/2))^(1/2)*(1-(-1/2+1/6*I*15^(1/2))*x^2)^(1/2)*(1-(-1/2-1/6
*I*15^(1/2))*x^2)^(1/2)/(2*x^4+3*x^2+3)^(1/2)*EllipticF(1/6*x*(-18+6*I*15^(
1/2))^(1/2),1/2*(-1+I*15^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 3*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4+3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 + 2*x^4 + 3)^(1/2),x)
```

```
[Out] int(1/(3*x^2 + 2*x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4+3x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+3*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 3*x**2 + 3), x)
```

3.85 $\int \frac{1}{\sqrt{3+2x^2+2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+2x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4+2x^2+3}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18-3*6^(1
/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4+2*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(2*x^4+2*x^2+3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+2x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4+2x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 2*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*Sqrt[3 + 2*x^2 + 2*x
^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+2x^2+2x^4}}\,dx=\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+2x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3+2x^2+2x^4}}$$

Mathematica [C] time = 0.08, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{2x^2}{-1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{-1+i\sqrt{5}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{-1-i\sqrt{5}}}x\right)|\frac{-1-i\sqrt{5}}{-1+i\sqrt{5}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-1-i\sqrt{5}}}\sqrt{2x^4+2x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 2*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(-1 - I*Sqrt[5])]*Sqrt[1 - (2*x^2)/(-1 + I*Sqrt[5])]
*EllipticF[I*ArcSinh[Sqrt[-2/(-1 - I*Sqrt[5])]*x], (-1 - I*Sqrt[5])/(-1 + I
*Sqrt[5])])/(Sqrt[2]*Sqrt[-(-1 - I*Sqrt[5])^(-1)]*Sqrt[3 + 2*x^2 + 2*x^4])
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+2\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+2*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 2*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+2*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 2*x^2 + 3), x)
```

maple [C] time = 0.10, size = 87, normalized size = 0.95

$$\frac{3\sqrt{-\left(-\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2+1}\,\sqrt{-\left(-\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{-3+3i\sqrt{5}}\,x}{3},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{\sqrt{-3+3i\sqrt{5}}\,\sqrt{2x^4+2x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+2*x^2+3)^(1/2),x)
```

```
[Out] 3/(-3+3*I*5^(1/2))^(1/2)*(1-(-1/3+1/3*I*5^(1/2))*x^2)^(1/2)*(1-(-1/3-1/3*I*
5^(1/2))*x^2)^(1/2)/(2*x^4+2*x^2+3)^(1/2)*EllipticF(1/3*x*(-3+3*I*5^(1/2))^
(1/2),1/3*(-6+3*I*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+2*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 2*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 + 2*x^4 + 3)^(1/2),x)
```

```
[Out] int(1/(2*x^2 + 2*x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4+2x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+2*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 2*x**2 + 3), x)
```

3.86 $\int \frac{1}{\sqrt{3+x^2+2x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4+x^2+3}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/12*(72-6*6^(
1/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4+x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(2*x^4+x^2+3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4+x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(2/3)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*Sqrt[3 + x^2 + 2*x^4]
)
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3+x^2+2x^4}}$$

Mathematica [C] time = 0.07, size = 140, normalized size = 1.59

$$-\frac{i\sqrt{1-\frac{4x^2}{-1-i\sqrt{23}}}\sqrt{1-\frac{4x^2}{-1+i\sqrt{23}}}\,F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{-1-i\sqrt{23}}}\,x\right)|\frac{-1-i\sqrt{23}}{-1+i\sqrt{23}}\right)}{2\sqrt{-\frac{1}{-1-i\sqrt{23}}}\sqrt{2x^4+x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + x^2 + 2*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 - (4*x^2)/(-1 - I*Sqrt[23])]*Sqrt[1 - (4*x^2)/(-1 + I*Sqrt
[23])]*EllipticF[I*ArcSinh[2*Sqrt[-(-1 - I*Sqrt[23])^(-1)]*x], (-1 - I*Sqrt
[23])/(-1 + I*Sqrt[23])]])/(Sqrt[-(-1 - I*Sqrt[23])^(-1)]*Sqrt[3 + x^2 + 2*x
^4])
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{2\,x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + x^2 + 3), x)
```

maple [C] time = 0.11, size = 85, normalized size = 0.97

$$\frac{6\sqrt{-\left(-\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2+1}\,\sqrt{-\left(-\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{-6+6i\sqrt{23}}\,x}{6},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{\sqrt{-6+6i\sqrt{23}}\,\sqrt{2x^4+x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+x^2+3)^(1/2),x)
```

```
[Out] 6/(-6+6*I*23^(1/2))^(1/2)*(1-(-1/6+1/6*I*23^(1/2))*x^2)^(1/2)*(1-(-1/6-1/6*
I*23^(1/2))*x^2)^(1/2)/(2*x^4+x^2+3)^(1/2)*EllipticF(1/6*x*(-6+6*I*23^(1/2)
)^(1/2),1/6*(-33+3*I*23^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4 + x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4 + x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 2*x^4 + 3)^(1/2),x)
```

```
[Out] int(1/(x^2 + 2*x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4 + x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + x**2 + 3), x)
```

3.87 $\int \frac{1}{\sqrt{3+2x^4}} \, dx$

Optimal. Leaf size=72

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{2x^4+3}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/2*2^(1/2))*(
3+x^2*6^(1/2))*((2*x^4+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/4)/(2*x^4+3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {220}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{2x^4+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan
[(2/3)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[3 + 2*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+2x^4}} \, dx = \frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{3+2x^4}}$$

Mathematica [C] time = 0.03, size = 25, normalized size = 0.35

$$-\sqrt[4]{-\frac{1}{6}}\, F\left(i \sinh^{-1}\left(\sqrt[4]{-\frac{2}{3}}\, x\right) \middle| -1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 2*x^4],x]
[Out] -((-1/6)^(1/4)*EllipticF[I*ArcSinh[(-2/3)^(1/4)*x], -1])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3)^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(2*x^4 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(2*x^4 + 3), x)
```

maple [C] time = 0.05, size = 66, normalized size = 0.92

$$\frac{\sqrt{3}\,\sqrt{-3i\sqrt{6}\,x^2+9}\,\sqrt{3i\sqrt{6}\,x^2+9}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\sqrt{6}}\,x}{3}, i\right)}{9\sqrt{i\sqrt{6}}\,\sqrt{2x^4+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+3)^(1/2),x)
[Out] 1/9*3^(1/2)/(I*6^(1/2))^(1/2)*(9-3*I*6^(1/2)*x^2)^(1/2)*(9+3*I*6^(1/2)*x^2)
^(1/2)/(2*x^4+3)^(1/2)*EllipticF(1/3*x*3^(1/2)*(I*6^(1/2))^(1/2),I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 3), x)
```

mupad [B] time = 0.09, size = 16, normalized size = 0.22

$$\frac{\sqrt{3}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{2\,x^4}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 + 3)^(1/2),x)
```

```
[Out] (3^(1/2)*x*hypergeom([1/4, 1/2], 5/4, -(2*x^4)/3))/3
```

sympy [C] time = 1.19, size = 36, normalized size = 0.50

$$\frac{\sqrt{3}\,x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{2x^4e^{i\pi}}{3}\right)}{12\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+3)**(1/2),x)
```

```
[Out] sqrt(3)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**4*exp_polar(I*pi)/3)/(1
2*gamma(5/4))
```

3.88 $\int \frac{1}{\sqrt{3-x^2+2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4-x^2+3}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/12*(72+6*6^(
1/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4-x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(2*x^4-x^2+3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4-x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(2/3)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*Sqrt[3 - x^2 + 2*x^4]
)
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-x^2+2x^4}}\,dx=\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3-x^2+2x^4}}$$

Mathematica [C] time = 0.07, size = 142, normalized size = 1.58

$$-\frac{i\sqrt{1-\frac{4x^2}{1-i\sqrt{23}}}\sqrt{1-\frac{4x^2}{1+i\sqrt{23}}}\,F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{1-i\sqrt{23}}}\,x\right)|\frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right)}{2\sqrt{-\frac{1}{1-i\sqrt{23}}}\sqrt{2x^4-x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - x^2 + 2*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 - (4*x^2)/(1 - I*Sqrt[23])]*Sqrt[1 - (4*x^2)/(1 + I*Sqrt[2
3])]*EllipticF[I*ArcSinh[2*Sqrt[-(1 - I*Sqrt[23])^(-1)]*x], (1 - I*Sqrt[23]
)/(1 + I*Sqrt[23])])/(Sqrt[-(1 - I*Sqrt[23])^(-1)]*Sqrt[3 - x^2 + 2*x^4])
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - x^2 + 3), x)
```

maple [C] time = 0.10, size = 87, normalized size = 0.97

$$\frac{6\sqrt{-\left(\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2+1}\,\sqrt{-\left(\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{6+6i\sqrt{23}}\,x}{6},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{\sqrt{6+6i\sqrt{23}}\,\sqrt{2x^4-x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-x^2+3)^(1/2),x)
```

```
[Out] 6/(6+6*I*23^(1/2))^(1/2)*(1-(1/6+1/6*I*23^(1/2))*x^2)^(1/2)*(1-(1/6-1/6*I*2
3^(1/2))*x^2)^(1/2)/(2*x^4-x^2+3)^(1/2)*EllipticF(1/6*x*(6+6*I*23^(1/2))^(1
/2),1/6*(-33-3*I*23^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - x^2 + 3)^(1/2),x)
```

```
[Out] int(1/(2*x^4 - x^2 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - x**2 + 3), x)
```

3.89 $\int \frac{1}{\sqrt{3-2x^2+2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-2x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4-2x^2+3}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18+3*6^(1
/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4-2*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(2*x^4-2*x^2+3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-2x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4-2x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 2*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*Sqrt[3 - 2*x^2 + 2*x
^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-2x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-2x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3-2x^2+2x^4}}$$

Mathematica [C] time = 0.07, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{2x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{1+i\sqrt{5}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{1-i\sqrt{5}}}x\right)|\frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right)}{\sqrt{2}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{2x^4-2x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 2*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(1 - I*Sqrt[5])]*Sqrt[1 - (2*x^2)/(1 + I*Sqrt[5])]*E
llipticF[I*ArcSinh[Sqrt[-2/(1 - I*Sqrt[5])]*x], (1 - I*Sqrt[5])/(1 + I*Sqrt
[5])])/(Sqrt[2]*Sqrt[-(1 - I*Sqrt[5])^(-1)]*Sqrt[3 - 2*x^2 + 2*x^4])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-2\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 2*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4-2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-2*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 2*x^2 + 3), x)
```

maple [C] time = 0.10, size = 87, normalized size = 0.97

$$\frac{3\sqrt{-\left(\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2+1}\,\sqrt{-\left(\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2+1}\,\,\text{EllipticF}\left(\frac{\sqrt{3+3i\sqrt{5}}\,x}{3},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{\sqrt{3+3i\sqrt{5}}\,\sqrt{2x^4-2x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-2*x^2+3)^(1/2),x)
```

```
[Out] 3/(3+3*I*5^(1/2))^(1/2)*(1-(1/3+1/3*I*5^(1/2))*x^2)^(1/2)*(1-(1/3-1/3*I*5^(
1/2))*x^2)^(1/2)/(2*x^4-2*x^2+3)^(1/2)*EllipticF(1/3*x*(3+3*I*5^(1/2))^(1/2
),1/3*(-6-3*I*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4-2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-2*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 2*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4-2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - 2*x^2 + 3)^(1/2),x)
```

```
[Out] int(1/(2*x^4 - 2*x^2 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4-2x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-2*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 2*x**2 + 3), x)
```

3.90 $\int \frac{1}{\sqrt{3-3x^2+2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-3x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4-3x^2+3}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/4*(8+2*6^(1/
2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4-3*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(2*x^4-3*x^2+3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-3x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4-3x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 3*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*Sqrt[3 - 3*x^2 + 2*x^
4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-3x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-3x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3-3x^2+2x^4}}$$

Mathematica [C] time = 0.10, size = 142, normalized size = 1.58

$$-\frac{i\sqrt{1-\frac{4x^2}{3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{3+i\sqrt{15}}}\,F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{3-i\sqrt{15}}}\,x\right)|\frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right)}{2\sqrt{-\frac{1}{3-i\sqrt{15}}}\sqrt{2x^4-3x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 3*x^2 + 2*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 - (4*x^2)/(3 - I*Sqrt[15])]*Sqrt[1 - (4*x^2)/(3 + I*Sqrt[1
5])]*EllipticF[I*ArcSinh[2*Sqrt[-(3 - I*Sqrt[15])^(-1)]*x], (3 - I*Sqrt[15]
)/(3 + I*Sqrt[15])])/(Sqrt[-(3 - I*Sqrt[15])^(-1)]*Sqrt[3 - 3*x^2 + 2*x^4])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-3\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 3*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4-3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 3*x^2 + 3), x)
```

maple [C] time = 0.10, size = 87, normalized size = 0.97

$$\frac{6\sqrt{-\left(\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2+1}\,\sqrt{-\left(\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2+1}\,\,\mathrm{EllipticF}\left(\frac{\sqrt{18+6i\sqrt{15}}\,x}{6},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{\sqrt{18+6i\sqrt{15}}\,\sqrt{2x^4-3x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-3*x^2+3)^(1/2),x)
```

```
[Out] 6/(18+6*I*15^(1/2))^(1/2)*(1-(1/2+1/6*I*15^(1/2))*x^2)^(1/2)*(1-(1/2-1/6*I*
15^(1/2))*x^2)^(1/2)/(2*x^4-3*x^2+3)^(1/2)*EllipticF(1/6*x*(18+6*I*15^(1/2)
)^(1/2),1/2*(-1-I*15^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4-3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 3*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4-3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - 3*x^2 + 3)^(1/2),x)
```

```
[Out] int(1/(2*x^4 - 3*x^2 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4-3x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-3*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 3*x**2 + 3), x)
```

3.91 $\int \frac{1}{\sqrt{3-4x^2+2x^4}} \, dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-4x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{2x^4-4x^2+3}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18+6*6^(1
/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4-4*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(2*x^4-4*x^2+3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-4x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{2x^4-4x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 4*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[3 - 4*x^2 + 2*x^
4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-4x^2+2x^4}} \, dx = \frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-4x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{3-4x^2+2x^4}}$$

Mathematica [C] time = 0.08, size = 144, normalized size = 1.64

$$-\frac{i\sqrt{1-\frac{2x^2}{2-i\sqrt{2}}}\sqrt{1-\frac{2x^2}{2+i\sqrt{2}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{2-i\sqrt{2}}}\,x\right)|\frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right)}{\sqrt{2}\sqrt{-\frac{1}{2-i\sqrt{2}}}\sqrt{2x^4-4x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 4*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(2 - I*Sqrt[2])]*Sqrt[1 - (2*x^2)/(2 + I*Sqrt[2])]*E
llipticF[I*ArcSinh[Sqrt[-2/(2 - I*Sqrt[2])]*x], (2 - I*Sqrt[2])/(2 + I*Sqrt
[2])])/(Sqrt[2]*Sqrt[-(2 - I*Sqrt[2])^(-1)]*Sqrt[3 - 4*x^2 + 2*x^4])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-4\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-4*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 4*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4-4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-4*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 4*x^2 + 3), x)
```

maple [C] time = 0.10, size = 87, normalized size = 0.99

$$\frac{3\sqrt{-\left(\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2+1}\,\sqrt{-\left(\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{6+3i\sqrt{2}}\,x}{3},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{\sqrt{6+3i\sqrt{2}}\,\sqrt{2x^4-4x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-4*x^2+3)^(1/2),x)
```

```
[Out] 3/(6+3*I*2^(1/2))^(1/2)*(1-(2/3+1/3*I*2^(1/2))*x^2)^(1/2)*(1-(2/3-1/3*I*2^(
1/2))*x^2)^(1/2)/(2*x^4-4*x^2+3)^(1/2)*EllipticF(1/3*x*(6+3*I*2^(1/2))^(1/2
),1/3*(3-6*I*2^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4-4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-4*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 4*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4-4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - 4*x^2 + 3)^(1/2),x)
```

```
[Out] int(1/(2*x^4 - 4*x^2 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4-4x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-4*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 4*x**2 + 3), x)
```

3.92 $\int \frac{1}{\sqrt{3-5x^2+2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-5x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4-5x^2+3}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/12*(72+30*6^
(1/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4-5*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(
3/4)/(2*x^4-5*x^2+3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1096}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-5x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4-5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 5*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 5*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (12 + 5*Sqrt[6])/24])/(2*6^(1/4)*Sqrt[3 - 5*x^2 +
2*x^4])
```

Rule 1096

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-5x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-5x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3-5x^2+2x^4}}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.58

$$\frac{\sqrt{3-2x^2}\,\sqrt{1-x^2}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)\middle|\frac{3}{2}\right)}{\sqrt{4x^4-10x^2+6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 5*x^2 + 2*x^4],x]
[Out] (Sqrt[3 - 2*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[2/3]*x], 3/2])/Sqrt[6
- 10*x^2 + 4*x^4]
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-5\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-5*x^2+3)^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(2*x^4 - 5*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4-5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-5*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(2*x^4 - 5*x^2 + 3), x)
```

maple [A] time = 0.01, size = 42, normalized size = 0.46

$$\frac{\sqrt{-x^2+1}\,\sqrt{-6x^2+9}\,\operatorname{EllipticF}\left(x,\frac{\sqrt{6}}{3}\right)}{3\sqrt{2x^4-5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-5*x^2+3)^(1/2),x)
```

```
[Out] 1/3*(-x^2+1)^(1/2)*(-6*x^2+9)^(1/2)/(2*x^4-5*x^2+3)^(1/2)*EllipticF(x,1/3*6
^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4-5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 5*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4-5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - 5*x^2 + 3)^(1/2),x)
```

```
[Out] int(1/(2*x^4 - 5*x^2 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4-5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-5*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 5*x**2 + 3), x)
```

3.93 $\int \frac{1}{\sqrt{3-6x^2+2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-6x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4-6x^2+3}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/2*(2+6^(1/2)
)^(1/2))*(3+x^2*6^(1/2))*((2*x^4-6*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/4)/
(2*x^4-6*x^2+3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1096}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-6x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4-6x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 6*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 6*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (2 + Sqrt[6])/4])/(2*6^(1/4)*Sqrt[3 - 6*x^2 + 2*x^
4])
```

Rule 1096

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-6x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-6x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3-6x^2+2x^4}}$$

Mathematica [A] time = 0.08, size = 81, normalized size = 0.90

$$\frac{\sqrt{-2x^2-\sqrt{3}+3}\,\sqrt{\left(\sqrt{3}-3\right)x^2+3}\,F\left(\sin^{-1}\left(\sqrt{1+\frac{1}{\sqrt{3}}}\,x\right)|2-\sqrt{3}\right)}{\sqrt{6}\,\sqrt{2x^4-6x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 6*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[3 - Sqrt[3] - 2*x^2]*Sqrt[3 + (-3 + Sqrt[3])*x^2]*EllipticF[ArcSin[Sq
rt[1 + 1/Sqrt[3]]*x], 2 - Sqrt[3]])/(Sqrt[6]*Sqrt[3 - 6*x^2 + 2*x^4])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-6\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-6*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 6*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4-6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-6*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 6*x^2 + 3), x)
```

maple [A] time = 0.07, size = 82, normalized size = 0.91

$$\frac{3\sqrt{-\left(1+\frac{\sqrt{3}}{3}\right)x^2+1}\,\sqrt{-\left(1-\frac{\sqrt{3}}{3}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{9+3\sqrt{3}}\,x}{3},\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{9+3\sqrt{3}}\,\sqrt{2x^4-6x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-6*x^2+3)^(1/2),x)
```

```
[Out] 3/(9+3*3^(1/2))^(1/2)*(1-(1+1/3*3^(1/2))*x^2)^(1/2)*(1-(1-1/3*3^(1/2))*x^2)
^(1/2)/(2*x^4-6*x^2+3)^(1/2)*EllipticF(1/3*x*(9+3*3^(1/2))^(1/2),1/2*6^(1/2
)-1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4-6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-6*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 6*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4-6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - 6*x^2 + 3)^(1/2),x)
```

```
[Out] int(1/(2*x^4 - 6*x^2 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4-6x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-6*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 6*x**2 + 3), x)
```

3.94 $\int \frac{1}{\sqrt{3-7x^2+2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-7x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12+7\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4-7x^2+3}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/12*(72+42*6^
(1/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4-7*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(
3/4)/(2*x^4-7*x^2+3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1096}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-7x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12+7\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4-7x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 7*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 7*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (12 + 7*Sqrt[6])/24])/(2*6^(1/4)*Sqrt[3 - 7*x^2 +
2*x^4])
```

Rule 1096

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-7x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-7x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12+7\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3-7x^2+2x^4}}$$

Mathematica [A] time = 0.02, size = 58, normalized size = 0.63

$$\frac{\sqrt{1-2x^2}\,\sqrt{1-\frac{x^2}{3}}\,F\left(\sin^{-1}\left(\sqrt{2}\,x\right)|\frac{1}{6}\right)}{\sqrt{2}\,\sqrt{2x^4-7x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 7*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[1 - 2*x^2]*Sqrt[1 - x^2/3]*EllipticF[ArcSin[Sqrt[2]*x], 1/6])/(Sqrt[2
]*Sqrt[3 - 7*x^2 + 2*x^4])
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-7\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-7*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 7*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4-7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-7*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 7*x^2 + 3), x)
```

maple [A] time = 0.01, size = 49, normalized size = 0.53

$$\frac{\sqrt{2}\,\sqrt{-2x^2+1}\,\sqrt{-3x^2+9}\,\,\mathrm{EllipticF}\left(\sqrt{2}\,x,\frac{\sqrt{6}}{6}\right)}{6\sqrt{2x^4-7x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-7*x^2+3)^(1/2),x)
```

```
[Out] 1/6*2^(1/2)*(-2*x^2+1)^(1/2)*(-3*x^2+9)^(1/2)/(2*x^4-7*x^2+3)^(1/2)*Ellipti
cF(2^(1/2)*x,1/6*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4-7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-7*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 7*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4-7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - 7*x^2 + 3)^(1/2),x)
```

```
[Out] int(1/(2*x^4 - 7*x^2 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4-7x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-7*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 7*x**2 + 3), x)
```

3.95 $\int \frac{1}{\sqrt{-3+7x^2-2x^4}}\,dx$

Optimal. Leaf size=19

$$-\frac{F\left(\cos^{-1}\left(\frac{x}{\sqrt{3}}\right)|\frac{6}{5}\right)}{\sqrt{5}}$$

```
[Out] -1/5*(x^2)^(1/2)/x*EllipticF(1/3*(-3*x^2+9)^(1/2),1/5*30^(1/2))*5^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 420}

$$-\frac{F\left(\cos^{-1}\left(\frac{x}{\sqrt{3}}\right)|\frac{6}{5}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 7*x^2 - 2*x^4],x]
```

```
[Out] -(EllipticF[ArcCos[x/Sqrt[3]], 6/5]/Sqrt[5])
```

Rule 420

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> -
Simp[EllipticF[ArcCos[Rt[-(d/c), 2]*x], (b*c)/(b*c - a*d)]/(Sqrt[c]*Rt[-(d/
c), 2]*Sqrt[a - (b*c)/d]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ
[c, 0] && GtQ[a - (b*c)/d, 0]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+7x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{12-4x^2}\,\sqrt{-2+4x^2}}\,dx$$
$$= -\frac{F\left(\cos^{-1}\left(\frac{x}{\sqrt{3}}\right)|\frac{6}{5}\right)}{\sqrt{5}}$$

Mathematica [B] time = 0.02, size = 58, normalized size = 3.05

$$\frac{\sqrt{1-2x^2}\,\sqrt{1-\frac{x^2}{3}}\,F\left(\sin^{-1}\left(\sqrt{2}\,x\right)|\frac{1}{6}\right)}{\sqrt{2}\,\sqrt{-2x^4+7x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 7*x^2 - 2*x^4],x]
[Out] (Sqrt[1 - 2*x^2]*Sqrt[1 - x^2/3]*EllipticF[ArcSin[Sqrt[2]*x], 1/6])/(Sqrt[2
]*Sqrt[-3 + 7*x^2 - 2*x^4])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+7\,x^2-3}}{2\,x^4-7\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+7*x^2-3)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-2*x^4 + 7*x^2 - 3)/(2*x^4 - 7*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4+7\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+7*x^2-3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-2*x^4 + 7*x^2 - 3), x)
```

maple [A] time = 0.01, size = 48, normalized size = 2.53

$$\frac{\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{-2x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,x}{3},\sqrt{6}\right)}{3\sqrt{-2x^4+7x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+7*x^2-3)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*(-3*x^2+9)^(1/2)*(-2*x^2+1)^(1/2)/(-2*x^4+7*x^2-3)^(1/2)*Ellipt
icF(1/3*3^(1/2)*x,6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4+7\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+7*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 7*x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{-2\,x^4+7\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7*x^2 - 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(7*x^2 - 2*x^4 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4+7x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+7*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 7*x**2 - 3), x)
```

3.96 $\int \frac{1}{\sqrt{-3+6x^2-2x^4}}\,dx$

Optimal. Leaf size=44

$$-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\sqrt[4]{3}}$$

```
[Out] -1/6*(x^2*(9-3*3^(1/2)))^(1/2)/x/(9-3*3^(1/2))^(1/2)*EllipticF(1/3*(9-x^2*(
9-3*3^(1/2)))^(1/2),1/2*(2+2*3^(1/2))^(1/2))*3^(3/4)*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 420}

$$-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 6*x^2 - 2*x^4],x]
```

```
[Out] -(EllipticF[ArcCos[Sqrt[(3 - Sqrt[3])/3]*x], (1 + Sqrt[3])/2]/(Sqrt[2]*3^(1
/4)))
```

Rule 420

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> -
Simp[EllipticF[ArcCos[Rt[-(d/c), 2]*x], (b*c)/(b*c - a*d)]/(Sqrt[c]*Rt[-(d/
c), 2]*Sqrt[a - (b*c)/d]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ
[c, 0] && GtQ[a - (b*c)/d, 0]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+6x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{6+2\sqrt{3}-4x^2}\sqrt{-6+2\sqrt{3}+4x^2}}\,dx$$
$$= -\frac{F\left(\cos^{-1}\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}\,x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\sqrt[4]{3}}$$

Mathematica [A] time = 0.03, size = 81, normalized size = 1.84

$$\frac{\sqrt{-2x^2-\sqrt{3}+3}\sqrt{\left(\sqrt{3}-3\right)x^2+3}\,F\left(\sin^{-1}\left(\sqrt{1+\frac{1}{\sqrt{3}}}\,x\right)|2-\sqrt{3}\right)}{\sqrt{6}\sqrt{-2x^4+6x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 6*x^2 - 2*x^4],x]
```

```
[Out] (Sqrt[3 - Sqrt[3] - 2*x^2]*Sqrt[3 + (-3 + Sqrt[3])*x^2]*EllipticF[ArcSin[Sq
rt[1 + 1/Sqrt[3]]*x], 2 - Sqrt[3]])/(Sqrt[6]*Sqrt[-3 + 6*x^2 - 2*x^4])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+6\,x^2-3}}{2\,x^4-6\,x^2+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+6*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 6*x^2 - 3)/(2*x^4 - 6*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4+6\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+6*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 6*x^2 - 3), x)
```

maple [A] time = 0.02, size = 82, normalized size = 1.86

$$\frac{3\sqrt{-\left(1-\frac{\sqrt{3}}{3}\right)x^2+1}\,\sqrt{-\left(1+\frac{\sqrt{3}}{3}\right)x^2+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{9-3\sqrt{3}}\,x}{3},\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{9-3\sqrt{3}}\,\sqrt{-2x^4+6x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+6*x^2-3)^(1/2),x)
```

```
[Out] 3/(9-3*3^(1/2))^(1/2)*(-(1-1/3*3^(1/2))*x^2+1)^(1/2)*(-(1+1/3*3^(1/2))*x^2+
1)^(1/2)/(-2*x^4+6*x^2-3)^(1/2)*EllipticF(1/3*x*(9-3*3^(1/2))^(1/2),1/2*6^(
1/2)+1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4+6\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+6*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 6*x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-2\,x^4+6\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(6*x^2 - 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(6*x^2 - 2*x^4 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4+6x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+6*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 6*x**2 - 3), x)
```

3.97 $\int \frac{1}{\sqrt{-3+5x^2-2x^4}}\,dx$

Optimal. Leaf size=14

$$-F\left(\cos^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)\middle|\,3\right)$$

```
[Out] -(x^2)^(1/2)/x*EllipticF(1/3*(-6*x^2+9)^(1/2),3^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 420}

$$-F\left(\cos^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)\middle|\,3\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 5*x^2 - 2*x^4],x]
```

```
[Out] -EllipticF[ArcCos[Sqrt[2/3]*x], 3]
```

Rule 420

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> -
Simp[EllipticF[ArcCos[Rt[-(d/c), 2]*x], (b*c)/(b*c - a*d)]/(Sqrt[c]*Rt[-(d/
c), 2]*Sqrt[a - (b*c)/d]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ
[c, 0] && GtQ[a - (b*c)/d, 0]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3+5x^2-2x^4}}\,dx &= \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{6-4x^2}\,\sqrt{-4+4x^2}}\,dx\\ &= -F\left(\cos^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)\middle|\,3\right)\end{aligned}$$

Mathematica [B] time = 0.02, size = 53, normalized size = 3.79

$$\frac{\sqrt{3-2x^2}\,\sqrt{1-x^2}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)|\frac{3}{2}\right)}{\sqrt{-4x^4+10x^2-6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 5*x^2 - 2*x^4],x]
```

```
[Out] (Sqrt[3 - 2*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[2/3]*x], 3/2])/Sqrt[-6
 + 10*x^2 - 4*x^4]
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-2\,x^4+5\,x^2-3}}{2\,x^4-5\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 5*x^2 - 3)/(2*x^4 - 5*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4+5\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 - 3), x)
```

maple [A] time = 0.01, size = 50, normalized size = 3.57

$$\frac{\sqrt{6}\,\sqrt{-6x^2+9}\,\sqrt{-x^2+1}\,\ \mathrm{EllipticF}\left(\frac{\sqrt{6}\,x}{3},\frac{\sqrt{6}}{2}\right)}{6\sqrt{-2x^4+5x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+5*x^2-3)^(1/2),x)
```

```
[Out] 1/6*6^(1/2)*(-6*x^2+9)^(1/2)*(-x^2+1)^(1/2)/(-2*x^4+5*x^2-3)^(1/2)*Elliptic
F(1/3*6^(1/2)*x,1/2*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4 + 5\,x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2-3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{\sqrt{-2\,x^4 + 5\,x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 2*x^4 - 3)^(1/2),x)
[Out] int(1/(5*x^2 - 2*x^4 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4 + 5x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+5*x**2-3)**(1/2),x)
[Out] Integral(1/sqrt(-2*x**4 + 5*x**2 - 3), x)
```

3.98 $\int \frac{1}{\sqrt{-3+4x^2-2x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-4x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4+4x^2-3}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18+6*6^(1
/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4-4*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(-2*x^4+4*x^2-3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-4x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4+4x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 4*x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[-3 + 4*x^2 - 2*x
^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+4x^2-2x^4}}\,dx=\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-4x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-3+4x^2-2x^4}}$$

Mathematica [C] time = 0.05, size = 144, normalized size = 1.64

$$-\frac{i\sqrt{1-\frac{2x^2}{2-i\sqrt{2}}}\sqrt{1-\frac{2x^2}{2+i\sqrt{2}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{2-i\sqrt{2}}}x\right)|\frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right)}{\sqrt{2}\sqrt{-\frac{1}{2-i\sqrt{2}}}\sqrt{-2x^4+4x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 4*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(2 - I*Sqrt[2])]*Sqrt[1 - (2*x^2)/(2 + I*Sqrt[2])]*E
llipticF[I*ArcSinh[Sqrt[-2/(2 - I*Sqrt[2])]*x], (2 - I*Sqrt[2])/(2 + I*Sqrt
[2])])/(Sqrt[2]*Sqrt[-(2 - I*Sqrt[2])^(-1)]*Sqrt[-3 + 4*x^2 - 2*x^4])
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2x^4+4x^2-3}}{2x^4-4x^2+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+4*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 4*x^2 - 3)/(2*x^4 - 4*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4+4x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+4*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 4*x^2 - 3), x)
```

maple [C] time = 0.05, size = 87, normalized size = 0.99

$$\frac{3\sqrt{-\left(\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2+1}\sqrt{-\left(\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2+1}\,\text{EllipticF}\left(\frac{\sqrt{6-3i\sqrt{2}}\,x}{3}, \frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{\sqrt{6-3i\sqrt{2}}\sqrt{-2x^4+4x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+4*x^2-3)^(1/2),x)
```

```
[Out] 3/(6-3*I*2^(1/2))^(1/2)*(-(2/3-1/3*I*2^(1/2))*x^2+1)^(1/2)*(-(2/3+1/3*I*2^(
1/2))*x^2+1)^(1/2)/(-2*x^4+4*x^2-3)^(1/2)*EllipticF(1/3*(6-3*I*2^(1/2))^(1/
2)*x,1/3*(3+6*I*2^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4 + 4\,x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+4*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 4*x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-2\,x^4 + 4\,x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 - 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(4*x^2 - 2*x^4 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4 + 4x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+4*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 4*x**2 - 3), x)
```

3.99 $\int \frac{1}{\sqrt{-3+3x^2-2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-3x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4+3x^2-3}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/4*(8+2*6^(1/
2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4-3*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(-2*x^4+3*x^2-3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-3x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4+3x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 3*x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*Sqrt[-3 + 3*x^2 - 2*x
^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+3x^2-2x^4}}\,dx=\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-3x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3+3x^2-2x^4}}$$

Mathematica [C] time = 0.09, size = 142, normalized size = 1.58

$$-\frac{i\sqrt{1-\frac{4x^2}{3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{3+i\sqrt{15}}}F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{3-i\sqrt{15}}}x\right)|\frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right)}{2\sqrt{-\frac{1}{3-i\sqrt{15}}}\sqrt{-2x^4+3x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 3*x^2 - 2*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 - (4*x^2)/(3 - I*Sqrt[15])]*Sqrt[1 - (4*x^2)/(3 + I*Sqrt[1
5])]*EllipticF[I*ArcSinh[2*Sqrt[-(3 - I*Sqrt[15])^(-1)]*x], (3 - I*Sqrt[15]
)/(3 + I*Sqrt[15])])/(Sqrt[-(3 - I*Sqrt[15])^(-1)]*Sqrt[-3 + 3*x^2 - 2*x^4]
)
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+3\,x^2-3}}{2\,x^4-3\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 3*x^2 - 3)/(2*x^4 - 3*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4+3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 3*x^2 - 3), x)
```

maple [C] time = 0.06, size = 87, normalized size = 0.97

$$\frac{6\sqrt{-\left(\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2+1}\,\sqrt{-\left(\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2+1}\,\,\mathrm{EllipticF}\left(\frac{\sqrt{18-6i\sqrt{15}}\,x}{6},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{\sqrt{18-6i\sqrt{15}}\,\sqrt{-2x^4+3x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+3*x^2-3)^(1/2),x)
```

```
[Out] 6/(18-6*I*15^(1/2))^(1/2)*(-(1/2-1/6*I*15^(1/2))*x^2+1)^(1/2)*(-(1/2+1/6*I*
15^(1/2))*x^2+1)^(1/2)/(-2*x^4+3*x^2-3)^(1/2)*EllipticF(1/6*(18-6*I*15^(1/2
))^(1/2)*x,1/2*(-1+I*15^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4+3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 3*x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-2\,x^4+3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 - 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(3*x^2 - 2*x^4 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4+3x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+3*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 3*x**2 - 3), x)
```

3.100 $\int \frac{1}{\sqrt{-3+2x^2-2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-2x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4+2x^2-3}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18+3*6^(1
/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4-2*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(-2*x^4+2*x^2-3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-2x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4+2x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 2*x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*Sqrt[-3 + 2*x^2 - 2*
x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+2x^2-2x^4}}\,dx=\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-2x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3+2x^2-2x^4}}$$

Mathematica [C] time = 0.07, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{2x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{1+i\sqrt{5}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{1-i\sqrt{5}}}\,x\right)|\frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right)}{\sqrt{2}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{-2x^4+2x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 2*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(1 - I*Sqrt[5])]*Sqrt[1 - (2*x^2)/(1 + I*Sqrt[5])]*E
llipticF[I*ArcSinh[Sqrt[-2/(1 - I*Sqrt[5])]*x], (1 - I*Sqrt[5])/(1 + I*Sqrt
[5])])/(Sqrt[2]*Sqrt[-(1 - I*Sqrt[5])^(-1)]*Sqrt[-3 + 2*x^2 - 2*x^4])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+2\,x^2-3}}{2\,x^4-2\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+2*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 2*x^2 - 3)/(2*x^4 - 2*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4+2\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+2*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 2*x^2 - 3), x)
```

maple [C] time = 0.07, size = 87, normalized size = 0.97

$$\frac{3\sqrt{-\left(\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2+1}\,\sqrt{-\left(\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{3-3i\sqrt{5}}\,x}{3},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{\sqrt{3-3i\sqrt{5}}\,\sqrt{-2x^4+2x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+2*x^2-3)^(1/2),x)
```

```
[Out] 3/(3-3*I*5^(1/2))^(1/2)*(-(1/3-1/3*I*5^(1/2))*x^2+1)^(1/2)*(-(1/3+1/3*I*5^(
1/2))*x^2+1)^(1/2)/(-2*x^4+2*x^2-3)^(1/2)*EllipticF(1/3*(3-3*I*5^(1/2))^(1/
2)*x,1/3*(-6+3*I*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4+2\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+2*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 2*x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-2\,x^4+2\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 - 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(2*x^2 - 2*x^4 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4+2x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+2*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 2*x**2 - 3), x)
```

3.101 $\int \frac{1}{\sqrt{-3+x^2-2x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4+x^2-3}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/12*(72+6*6^(
1/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4-x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(-2*x^4+x^2-3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4+x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(2/3)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*Sqrt[-3 + x^2 - 2*x^4
])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+x^2-2x^4}}\,dx=\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3+x^2-2x^4}}$$

Mathematica [C] time = 0.06, size = 140, normalized size = 1.59

$$-\frac{i\sqrt{1-\frac{4x^2}{1-i\sqrt{23}}}\sqrt{1-\frac{4x^2}{1+i\sqrt{23}}}F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{1-i\sqrt{23}}}x\right)|\frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right)}{2\sqrt{-\frac{1}{1-i\sqrt{23}}}\sqrt{-2x^4+x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + x^2 - 2*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 - (4*x^2)/(1 - I*Sqrt[23])]*Sqrt[1 - (4*x^2)/(1 + I*Sqrt[2
3])]*EllipticF[I*ArcSinh[2*Sqrt[-(1 - I*Sqrt[23])^(-1)]*x], (1 - I*Sqrt[23]
)/(1 + I*Sqrt[23])])/(Sqrt[-(1 - I*Sqrt[23])^(-1)]*Sqrt[-3 + x^2 - 2*x^4])
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+x^2-3}}{2\,x^4-x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + x^2 - 3)/(2*x^4 - x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-2\,x^4+x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + x^2 - 3), x)
```

maple [C] time = 0.05, size = 85, normalized size = 0.97

$$\frac{6\sqrt{-\left(\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2+1}\,\sqrt{-\left(\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2+1}\;\text{EllipticF}\left(\frac{\sqrt{6-6i\sqrt{23}}\,x}{6},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{\sqrt{6-6i\sqrt{23}}\,\sqrt{-2x^4+x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+x^2-3)^(1/2),x)
```

```
[Out] 6/(6-6*I*23^(1/2))^(1/2)*(-(1/6-1/6*I*23^(1/2))*x^2+1)^(1/2)*(-(1/6+1/6*I*2
3^(1/2))*x^2+1)^(1/2)/(-2*x^4+x^2-3)^(1/2)*EllipticF(1/6*(6-6*I*23^(1/2))^(
1/2)*x,1/6*(-33+3*I*23^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4 + x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-2\,x^4 + x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(x^2 - 2*x^4 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4 + x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + x**2 - 3), x)
```

3.102 $\int \frac{1}{\sqrt{-3-2x^4}}\,dx$

Optimal. Leaf size=72

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4-3}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/2*2^(1/2))*(
3+x^2*6^(1/2))*((2*x^4+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/4)/(-2*x^4-3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {220}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan
[(2/3)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[-3 - 2*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-2x^4}}\,dx=\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{-3-2x^4}}$$

Mathematica [C] time = 0.03, size = 47, normalized size = 0.65

$$-\frac{\sqrt[4]{-\frac{1}{6}}\sqrt{2x^4+3}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{2}{3}}\,x\right)\middle|-1\right)}{\sqrt{-2x^4-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - 2*x^4],x]
```

```
[Out] -(((-1/6)^(1/4)*Sqrt[3 + 2*x^4]*EllipticF[I*ArcSinh[(-2/3)^(1/4)*x], -1])/S
qrt[-3 - 2*x^4])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4-3}}{2\,x^4+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 3)/(2*x^4 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-2\,x^4-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 3), x)
```

maple [C] time = 0.02, size = 66, normalized size = 0.92

$$\frac{\sqrt{3}\,\sqrt{3i\sqrt{6}\,x^2+9}\,\sqrt{-3i\sqrt{6}\,x^2+9}\;\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{-i\sqrt{6}}\,x}{3},i\right)}{9\sqrt{-i\sqrt{6}}\,\sqrt{-2x^4-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-3)^(1/2),x)
```

[Out] 1/9*3^(1/2)/(-I*6^(1/2))^(1/2)*(3*I*6^(1/2)*x^2+9)^(1/2)*(-3*I*6^(1/2)*x^2+9)^(1/2)/(-2*x^4-3)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*6^(1/2))^(1/2)*x,I)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-2*x^4-3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(-2*x^4 - 3), x)

mupad [B] time = 4.29, size = 31, normalized size = 0.43

$$\frac{x\,\sqrt{6\,x^4+9}\;{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{2\,x^4}{3}\right)}{3\,\sqrt{-2\,x^4-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(- 2*x^4 - 3)^(1/2),x)

[Out] (x*(6*x^4 + 9)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(2*x^4)/3))/(3*(- 2*x^4 - 3)^(1/2))

sympy [C] time = 0.71, size = 39, normalized size = 0.54

$$-\frac{\sqrt{3}\,ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{2x^4e^{i\pi}}{3}\right)}{12\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-2*x**4-3)**(1/2),x)

[Out] -sqrt(3)*I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**4*exp_polar(I*pi)/3)/(12*gamma(5/4))

3.103 $\int \frac{1}{\sqrt{-3-x^2-2x^4}}\, dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4-x^2-3}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/12*(72-6*6^(
1/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4+x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(-2*x^4-x^2-3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4-x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(2/3)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*Sqrt[-3 - x^2 - 2*x^4
])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-x^2-2x^4}}\, dx = \frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3-x^2-2x^4}}$$

Mathematica [C] time = 0.06, size = 142, normalized size = 1.58

$$-\frac{i\sqrt{1-\frac{4x^2}{-1-i\sqrt{23}}}\sqrt{1-\frac{4x^2}{-1+i\sqrt{23}}}\,F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{-1-i\sqrt{23}}}\,x\right)|\frac{-1-i\sqrt{23}}{-1+i\sqrt{23}}\right)}{2\sqrt{-\frac{1}{-1-i\sqrt{23}}}\sqrt{-2x^4-x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - x^2 - 2*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 - (4*x^2)/(-1 - I*Sqrt[23])]*Sqrt[1 - (4*x^2)/(-1 + I*Sqrt
[23])]*EllipticF[I*ArcSinh[2*Sqrt[-(-1 - I*Sqrt[23])^(-1)]*x], (-1 - I*Sqrt
[23])/(-1 + I*Sqrt[23])]])/(Sqrt[-(-1 - I*Sqrt[23])^(-1)]*Sqrt[-3 - x^2 - 2*
x^4])
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4-x^2-3}}{2\,x^4+x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - x^2 - 3)/(2*x^4 + x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4-x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - x^2 - 3), x)
```

maple [C] time = 0.05, size = 87, normalized size = 0.97

$$\frac{6\sqrt{-\left(-\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2+1}\,\sqrt{-\left(-\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{-6-6i\sqrt{23}}\,x}{6},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{\sqrt{-6-6i\sqrt{23}}\,\sqrt{-2x^4-x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-x^2-3)^(1/2),x)
```

```
[Out] 6/(-6-6*I*23^(1/2))^(1/2)*(-(-1/6-1/6*I*23^(1/2))*x^2+1)^(1/2)*(-(-1/6+1/6*
I*23^(1/2))*x^2+1)^(1/2)/(-2*x^4-x^2-3)^(1/2)*EllipticF(1/6*(-6-6*I*23^(1/2
))^(1/2)*x,1/6*(-33-3*I*23^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4 - x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-2\,x^4 - x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- x^2 - 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(- x^2 - 2*x^4 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4 - x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - x**2 - 3), x)
```

3.104 $\int \frac{1}{\sqrt{-3-2x^2-2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+2x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4-2x^2-3}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18-3*6^(1
/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4+2*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(-2*x^4-2*x^2-3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+2x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4-2x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 2*x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*Sqrt[-3 - 2*x^2 - 2*
x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-2x^2-2x^4}}\,dx=\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+2x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3-2x^2-2x^4}}$$

Mathematica [C] time = 0.08, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{2x^2}{-1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{-1+i\sqrt{5}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{-1-i\sqrt{5}}}x\right)|\frac{-1-i\sqrt{5}}{-1+i\sqrt{5}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-1-i\sqrt{5}}}\sqrt{-2x^4-2x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - 2*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(-1 - I*Sqrt[5])]*Sqrt[1 - (2*x^2)/(-1 + I*Sqrt[5])]
*EllipticF[I*ArcSinh[Sqrt[-2/(-1 - I*Sqrt[5])]*x], (-1 - I*Sqrt[5])/(-1 + I
*Sqrt[5])])/(Sqrt[2]*Sqrt[-(-1 - I*Sqrt[5])^(-1)]*Sqrt[-3 - 2*x^2 - 2*x^4])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4-2\,x^2-3}}{2\,x^4+2\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-2*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 2*x^2 - 3)/(2*x^4 + 2*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4-2\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-2*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 2*x^2 - 3), x)
```

maple [C] time = 0.05, size = 87, normalized size = 0.95

$$\frac{3\sqrt{-\left(-\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2+1}\,\sqrt{-\left(-\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2+1}\;\text{EllipticF}\left(\frac{\sqrt{-3-3i\sqrt{5}}\,x}{3},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{\sqrt{-3-3i\sqrt{5}}\,\sqrt{-2x^4-2x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-2*x^2-3)^(1/2),x)
```

```
[Out] 3/(-3-3*I*5^(1/2))^(1/2)*(-(-1/3-1/3*I*5^(1/2))*x^2+1)^(1/2)*(-(-1/3+1/3*I*
5^(1/2))*x^2+1)^(1/2)/(-2*x^4-2*x^2-3)^(1/2)*EllipticF(1/3*(-3-3*I*5^(1/2))
^(1/2)*x,1/3*(-6-3*I*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4-2\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-2*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 2*x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-2\,x^4-2\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 2*x^2 - 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(- 2*x^2 - 2*x^4 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4-2x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-2*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 2*x**2 - 3), x)
```

3.105 $\int \frac{1}{\sqrt{-3-3x^2-2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+3x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4-3x^2-3}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/4*(8-2*6^(1/
2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4+3*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(-2*x^4-3*x^2-3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+3x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4-3x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 3*x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*Sqrt[-3 - 3*x^2 - 2*x
^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-3x^2-2x^4}}\,dx = \frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+3x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3-3x^2-2x^4}}$$

Mathematica [C] time = 0.10, size = 142, normalized size = 1.54

$$-\frac{i\sqrt{1-\frac{4x^2}{-3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{-3+i\sqrt{15}}}F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{-3-i\sqrt{15}}}x\right)|\frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right)}{2\sqrt{-\frac{1}{-3-i\sqrt{15}}}\sqrt{-2x^4-3x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - 3*x^2 - 2*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 - (4*x^2)/(-3 - I*Sqrt[15])]*Sqrt[1 - (4*x^2)/(-3 + I*Sqrt
[15])]*EllipticF[I*ArcSinh[2*Sqrt[-(-3 - I*Sqrt[15])^(-1)]*x], (-3 - I*Sqrt
[15])/(-3 + I*Sqrt[15])]])/(Sqrt[-(-3 - I*Sqrt[15])^(-1)]*Sqrt[-3 - 3*x^2 -
2*x^4])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4-3\,x^2-3}}{2\,x^4+3\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 3*x^2 - 3)/(2*x^4 + 3*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4-3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 3*x^2 - 3), x)
```

maple [C] time = 0.05, size = 87, normalized size = 0.95

$$\frac{6\sqrt{-\left(-\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2+1}\,\sqrt{-\left(-\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2+1}\;\text{EllipticF}\left(\frac{\sqrt{-18-6i\sqrt{15}}\,x}{6},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{\sqrt{-18-6i\sqrt{15}}\,\sqrt{-2x^4-3x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-3*x^2-3)^(1/2),x)
```

```
[Out] 6/(-18-6*I*15^(1/2))^(1/2)*(-(-1/2-1/6*I*15^(1/2))*x^2+1)^(1/2)*(-(-1/2+1/6
*I*15^(1/2))*x^2+1)^(1/2)/(-2*x^4-3*x^2-3)^(1/2)*EllipticF(1/6*(-18-6*I*15^
(1/2))^(1/2)*x,1/2*(-1-I*15^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4-3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 3*x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-2\,x^4-3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 3*x^2 - 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(- 3*x^2 - 2*x^4 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4-3x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-3*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 3*x**2 - 3), x)
```

3.106 $\int \frac{1}{\sqrt{-3-4x^2-2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+4x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4-4x^2-3}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18-6*6^(1
/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4+4*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(-2*x^4-4*x^2-3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+4x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4-4x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 4*x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[-3 - 4*x^2 - 2*x
^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-4x^2-2x^4}}\,dx=\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+4x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-3-4x^2-2x^4}}$$

Mathematica [C] time = 0.09, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{2x^2}{-2-i\sqrt{2}}}\sqrt{1-\frac{2x^2}{-2+i\sqrt{2}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{-2-i\sqrt{2}}}\,x\right)|\frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-2-i\sqrt{2}}}\sqrt{-2x^4-4x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - 4*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(-2 - I*Sqrt[2])]*Sqrt[1 - (2*x^2)/(-2 + I*Sqrt[2])]
*EllipticF[I*ArcSinh[Sqrt[-2/(-2 - I*Sqrt[2])]*x], (-2 - I*Sqrt[2])/(-2 + I
*Sqrt[2])])/(Sqrt[2]*Sqrt[-(-2 - I*Sqrt[2])^(-1)]*Sqrt[-3 - 4*x^2 - 2*x^4])
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4-4\,x^2-3}}{2\,x^4+4\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-4*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 4*x^2 - 3)/(2*x^4 + 4*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4-4\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-4*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 4*x^2 - 3), x)
```

maple [C] time = 0.05, size = 87, normalized size = 0.97

$$\frac{3\sqrt{-\left(-\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2+1}\,\sqrt{-\left(-\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2+1}\;\text{EllipticF}\left(\frac{\sqrt{-6-3i\sqrt{2}}\,x}{3},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{\sqrt{-6-3i\sqrt{2}}\,\sqrt{-2x^4-4x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-4*x^2-3)^(1/2),x)
```

```
[Out] 3/(-6-3*I*2^(1/2))^(1/2)*(-(-2/3-1/3*I*2^(1/2))*x^2+1)^(1/2)*(-(-2/3+1/3*I*
2^(1/2))*x^2+1)^(1/2)/(-2*x^4-4*x^2-3)^(1/2)*EllipticF(1/3*(-6-3*I*2^(1/2))
^(1/2)*x,1/3*(3-6*I*2^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4-4\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-4*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 4*x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-2\,x^4-4\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 4*x^2 - 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(- 4*x^2 - 2*x^4 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4-4x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-4*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 4*x**2 - 3), x)
```

3.107 $\int \frac{1}{\sqrt{-3-5x^2-2x^4}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{2x^2+3}\,F\left(\tan^{-1}(x)|\frac{1}{3}\right)}{\sqrt{3}\,\sqrt{-x^2-1}\,\sqrt{\frac{2x^2+3}{x^2+1}}}$$

```
[Out] 1/3*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/3*3^(1/2))*
(2*x^2+3)^(1/2)*3^(1/2)/(-x^2-1)^(1/2)/((2*x^2+3)/(x^2+1))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 418}

$$\frac{\sqrt{2x^2+3}\,F\left(\tan^{-1}(x)|\frac{1}{3}\right)}{\sqrt{3}\,\sqrt{-x^2-1}\,\sqrt{\frac{2x^2+3}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 5*x^2 - 2*x^4],x]
```

```
[Out] (Sqrt[3 + 2*x^2]*EllipticF[ArcTan[x], 1/3])/(Sqrt[3]*Sqrt[-1 - x^2]*Sqrt[(3
 + 2*x^2)/(1 + x^2)])
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-5x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{-4-4x^2}\,\sqrt{6+4x^2}}\,dx$$
$$= \frac{\sqrt{3+2x^2}\,F\left(\tan^{-1}(x)|\frac{1}{3}\right)}{\sqrt{3}\,\sqrt{-1-x^2}\,\sqrt{\frac{3+2x^2}{1+x^2}}}$$

Mathematica [C] time = 0.02, size = 63, normalized size = 1.19

$$-\frac{i\sqrt{x^2+1}\,\sqrt{2x^2+3}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)|\frac{3}{2}\right)}{\sqrt{2}\,\sqrt{-2x^4-5x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - 5*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[3 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/3]*x], 3/2])
/(Sqrt[2]*Sqrt[-3 - 5*x^2 - 2*x^4])
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4-5\,x^2-3}}{2\,x^4+5\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-5*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 5*x^2 - 3)/(2*x^4 + 5*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4-5\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-5*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 5*x^2 - 3), x)
```

maple [C] time = 0.01, size = 44, normalized size = 0.83

$$-\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+9}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{3}\right)}{3\sqrt{-2x^4-5x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-5*x^2-3)^(1/2),x)
```

```
[Out] -1/3*I*(x^2+1)^(1/2)*(6*x^2+9)^(1/2)/(-2*x^4-5*x^2-3)^(1/2)*EllipticF(I*x,1
/3*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4-5\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-5*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 5*x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-2\,x^4-5\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 5*x^2 - 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(- 5*x^2 - 2*x^4 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4-5x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-5*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 5*x**2 - 3), x)
```

3.108 $\int \frac{1}{\sqrt{-2+6x^2-3x^4}}\,dx$

Optimal. Leaf size=42

$$-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{3}{3+\sqrt{3}}}\,x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\,\sqrt[4]{3}}$$

```
[Out] -1/6*3^(3/4)*(x^2/(3^(1/2)+3))^(1/2)/x*(3^(1/2)+3)^(1/2)*EllipticF((1-3*x^2
/(3^(1/2)+3))^(1/2),1/2*(2+2*3^(1/2))^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 420}

$$-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{3}{3+\sqrt{3}}}\,x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\,\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 6*x^2 - 3*x^4],x]
```

```
[Out] -(EllipticF[ArcCos[Sqrt[3/(3 + Sqrt[3])]*x], (1 + Sqrt[3])/2]/(Sqrt[2]*3^(1
/4)))
```

Rule 420

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> -
Simp[EllipticF[ArcCos[Rt[-(d/c), 2]*x], (b*c)/(b*c - a*d)]/(Sqrt[c]*Rt[-(d/
c), 2]*Sqrt[a - (b*c)/d]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ
[c, 0] && GtQ[a - (b*c)/d, 0]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+6x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{6+2\sqrt{3}-6x^2}\,\sqrt{-6+2\sqrt{3}+6x^2}}\,dx$$
$$= -\frac{F\left(\cos^{-1}\left(\sqrt{\frac{3}{3+\sqrt{3}}}\,x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\,\sqrt[4]{3}}$$

Mathematica [B] time = 0.08, size = 85, normalized size = 2.02

$$\frac{\sqrt{-3x^2-\sqrt{3}+3}\,\sqrt{\left(\sqrt{3}-3\right)x^2+2}\,F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{3}\right)}\,x\right)|2-\sqrt{3}\right)}{\sqrt{6}\,\sqrt{-3x^4+6x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 6*x^2 - 3*x^4],x]
```

```
[Out] (Sqrt[3 - Sqrt[3] - 3*x^2]*Sqrt[2 + (-3 + Sqrt[3])*x^2]*EllipticF[ArcSin[Sq
rt[(3 + Sqrt[3])/2]*x], 2 - Sqrt[3]])/(Sqrt[6]*Sqrt[-2 + 6*x^2 - 3*x^4])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+6\,x^2-2}}{3\,x^4-6\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+6*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 6*x^2 - 2)/(3*x^4 - 6*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4+6\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+6*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 6*x^2 - 2), x)
```

maple [A] time = 0.02, size = 82, normalized size = 1.95

$$\frac{2\sqrt{-\left(-\frac{\sqrt{3}}{2}+\frac{3}{2}\right)x^2+1}\,\sqrt{-\left(\frac{\sqrt{3}}{2}+\frac{3}{2}\right)x^2+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{6-2\sqrt{3}}\,x}{2}, \frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{6-2\sqrt{3}}\,\sqrt{-3x^4+6x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+6*x^2-2)^(1/2),x)
```

```
[Out] 2/(6-2*3^(1/2))^(1/2)*(-(-1/2*3^(1/2)+3/2)*x^2+1)^(1/2)*(-(1/2*3^(1/2)+3/2)
*x^2+1)^(1/2)/(-3*x^4+6*x^2-2)^(1/2)*EllipticF(1/2*(6-2*3^(1/2))^(1/2)*x,1/
2*6^(1/2)+1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4+6\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+6*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 6*x^2 - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-3\,x^4+6\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(6*x^2 - 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(6*x^2 - 3*x^4 - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4+6x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+6*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 6*x**2 - 2), x)
```

3.109 $\int \frac{1}{\sqrt{-2+5x^2-3x^4}} \, dx$

Optimal. Leaf size=6

$$-F\left(\cos^{-1}(x)\middle|3\right)$$

```
[Out] -(x^2)^(1/2)/x*EllipticF((-x^2+1)^(1/2),3^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 420}

$$-F\left(\cos^{-1}(x)\middle|3\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 5*x^2 - 3*x^4],x]
```

```
[Out] -EllipticF[ArcCos[x], 3]
```

Rule 420

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> -
Simp[EllipticF[ArcCos[Rt[-(d/c), 2]*x], (b*c)/(b*c - a*d)]/(Sqrt[c]*Rt[-(d/
c), 2]*Sqrt[a - (b*c)/d]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ
[c, 0] && GtQ[a - (b*c)/d, 0]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-2+5x^2-3x^4}} \, dx &= \left(2\sqrt{3}\right) \int \frac{1}{\sqrt{6-6x^2}\sqrt{-4+6x^2}} \, dx \\ &= -F\left(\cos^{-1}(x)\middle|3\right) \end{aligned}$$

Mathematica [B] time = 0.03, size = 53, normalized size = 8.83

$$\frac{\sqrt{2-3x^2}\sqrt{1-x^2}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|\frac{2}{3}\right)}{\sqrt{-9x^4+15x^2-6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 5*x^2 - 3*x^4],x]
```

```
[Out] (Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], 2/3])/Sqrt[-6
 + 15*x^2 - 9*x^4]
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+5\,x^2-2}}{3\,x^4-5\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 5*x^2 - 2)/(3*x^4 - 5*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4+5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 - 2), x)
```

maple [A] time = 0.01, size = 42, normalized size = 7.00

$$\frac{\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\,\operatorname{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)}{2\sqrt{-3x^4+5x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+5*x^2-2)^(1/2),x)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*(-6*x^2+4)^(1/2)/(-3*x^4+5*x^2-2)^(1/2)*EllipticF(x,1/2*
6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4+5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2-2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.17

$$\int \frac{1}{\sqrt{-3\,x^4+5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 3*x^4 - 2)^(1/2),x)
[Out] int(1/(5*x^2 - 3*x^4 - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4+5x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+5*x**2-2)**(1/2),x)
[Out] Integral(1/sqrt(-3*x**4 + 5*x**2 - 2), x)
```

3.110 $\int \frac{1}{\sqrt{-2+4x^2-3x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-4x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4+4x^2-2}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18+6*6^(1
/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4-4*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(-3*x^4+4*x^2-2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-4x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4+4x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 4*x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[-2 + 4*x^2 - 3*x
^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+4x^2-3x^4}}\,dx=\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-4x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-2+4x^2-3x^4}}$$

Mathematica [C] time = 0.08, size = 144, normalized size = 1.64

$$-\frac{i\sqrt{1-\frac{3x^2}{2-i\sqrt{2}}}\sqrt{1-\frac{3x^2}{2+i\sqrt{2}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{2-i\sqrt{2}}}x\right)|\frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right)}{\sqrt{3}\sqrt{-\frac{1}{2-i\sqrt{2}}}\sqrt{-3x^4+4x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 4*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(2 - I*Sqrt[2])]*Sqrt[1 - (3*x^2)/(2 + I*Sqrt[2])]*E
llipticF[I*ArcSinh[Sqrt[-3/(2 - I*Sqrt[2])]*x], (2 - I*Sqrt[2])/(2 + I*Sqrt
[2])])/(Sqrt[3]*Sqrt[-(2 - I*Sqrt[2])^(-1)]*Sqrt[-2 + 4*x^2 - 3*x^4])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{\sqrt{-3x^4+4x^2-2}}{3x^4-4x^2+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+4*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 4*x^2 - 2)/(3*x^4 - 4*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4+4x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+4*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 4*x^2 - 2), x)
```

maple [C] time = 0.05, size = 87, normalized size = 0.99

$$\frac{2\sqrt{-\left(1-\frac{i\sqrt{2}}{2}\right)x^2+1}\sqrt{-\left(1+\frac{i\sqrt{2}}{2}\right)x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{4-2i\sqrt{2}}\,x}{2}, \frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{\sqrt{4-2i\sqrt{2}}\sqrt{-3x^4+4x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+4*x^2-2)^(1/2),x)
```

```
[Out] 2/(4-2*I*2^(1/2))^(1/2)*(-(1-1/2*I*2^(1/2))*x^2+1)^(1/2)*(-(1+1/2*I*2^(1/2)
)*x^2+1)^(1/2)/(-3*x^4+4*x^2-2)^(1/2)*EllipticF(1/2*(4-2*I*2^(1/2))^(1/2)*x
,1/3*(3+6*I*2^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4+4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+4*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 4*x^2 - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-3\,x^4+4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 - 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(4*x^2 - 3*x^4 - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4+4x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+4*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 4*x**2 - 2), x)
```

3.111 $\int \frac{1}{\sqrt{-2+3x^2-3x^4}} \, dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-3x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4+3x^2-2}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/4*(8+2*6^(1/
2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4-3*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(-3*x^4+3*x^2-2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-3x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4+3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 3*x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*Sqrt[-2 + 3*x^2 - 3*x
^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+3x^2-3x^4}}\,dx=\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-3x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2+3x^2-3x^4}}$$

Mathematica [C] time = 0.11, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{6x^2}{3-i\sqrt{15}}}\sqrt{1-\frac{6x^2}{3+i\sqrt{15}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{3-i\sqrt{15}}}x\right)|\frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right)}{\sqrt{6}\sqrt{-\frac{1}{3-i\sqrt{15}}}\sqrt{-3x^4+3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 3*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(3 - I*Sqrt[15])]*Sqrt[1 - (6*x^2)/(3 + I*Sqrt[15])]
*EllipticF[I*ArcSinh[Sqrt[-6/(3 - I*Sqrt[15])]*x], (3 - I*Sqrt[15])/(3 + I*
Sqrt[15])])/(Sqrt[6]*Sqrt[-(3 - I*Sqrt[15])^(-1)]*Sqrt[-2 + 3*x^2 - 3*x^4])
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+3\,x^2-2}}{3\,x^4-3\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+3*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 3*x^2 - 2)/(3*x^4 - 3*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4+3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+3*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 3*x^2 - 2), x)
```

maple [C] time = 0.05, size = 87, normalized size = 0.97

$$\frac{2\sqrt{-\left(\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2+1}\,\sqrt{-\left(\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{3-i\sqrt{15}}\,x}{2},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{\sqrt{3-i\sqrt{15}}\,\sqrt{-3x^4+3x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+3*x^2-2)^(1/2),x)
```

```
[Out] 2/(3-I*15^(1/2))^(1/2)*(-(3/4-1/4*I*15^(1/2))*x^2+1)^(1/2)*(-(3/4+1/4*I*15^
(1/2))*x^2+1)^(1/2)/(-3*x^4+3*x^2-2)^(1/2)*EllipticF(1/2*(3-I*15^(1/2))^(1/
2)*x,1/2*(-1+I*15^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4+3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+3*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 3*x^2 - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-3\,x^4+3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 - 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(3*x^2 - 3*x^4 - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4+3x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+3*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 3*x**2 - 2), x)
```

3.112 $\int \frac{1}{\sqrt{-2+2x^2-3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-2x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4+2x^2-2}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18+3*6^(1
/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4-2*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(-3*x^4+2*x^2-2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-2x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4+2x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 2*x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*Sqrt[-2 + 2*x^2 - 3*
x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+2x^2-3x^4}}\,dx=\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-2x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2+2x^2-3x^4}}$$

Mathematica [C] time = 0.08, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{3x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{3x^2}{1+i\sqrt{5}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{1-i\sqrt{5}}}\,x\right)|\frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right)}{\sqrt{3}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{-3x^4+2x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 2*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(1 - I*Sqrt[5])]*Sqrt[1 - (3*x^2)/(1 + I*Sqrt[5])]*E
llipticF[I*ArcSinh[Sqrt[-3/(1 - I*Sqrt[5])]*x], (1 - I*Sqrt[5])/(1 + I*Sqrt
[5])])/(Sqrt[3]*Sqrt[-(1 - I*Sqrt[5])^(-1)]*Sqrt[-2 + 2*x^2 - 3*x^4])
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+2\,x^2-2}}{3\,x^4-2\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 2*x^2 - 2)/(3*x^4 - 2*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4+2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 2*x^2 - 2), x)
```

maple [C] time = 0.06, size = 87, normalized size = 0.97

$$\frac{2\sqrt{-\left(\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2+1}\,\sqrt{-\left(\frac{1}{2}+\frac{i\sqrt{5}}{2}\right)x^2+1}\;\text{EllipticF}\left(\frac{\sqrt{2-2i\sqrt{5}}\,x}{2},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{\sqrt{2-2i\sqrt{5}}\,\sqrt{-3x^4+2x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+2*x^2-2)^(1/2),x)
```

```
[Out] 2/(2-2*I*5^(1/2))^(1/2)*(-(1/2-1/2*I*5^(1/2))*x^2+1)^(1/2)*(-(1/2+1/2*I*5^(
1/2))*x^2+1)^(1/2)/(-3*x^4+2*x^2-2)^(1/2)*EllipticF(1/2*(2-2*I*5^(1/2))^(1/
2)*x,1/3*(-6+3*I*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4+2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 2*x^2 - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-3\,x^4+2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 - 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(2*x^2 - 3*x^4 - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4+2x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+2*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 2*x**2 - 2), x)
```

3.113 $\int \frac{1}{\sqrt{-2+x^2-3x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4+x^2-2}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/12*(72+6*6^(
1/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4-x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(-3*x^4+x^2-2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4+x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(3/2)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*Sqrt[-2 + x^2 - 3*x^4
])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+x^2-3x^4}}\,dx = \frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2+x^2-3x^4}}$$

Mathematica [C] time = 0.08, size = 142, normalized size = 1.61

$$-\frac{i\sqrt{1-\frac{6x^2}{1-i\sqrt{23}}}\sqrt{1-\frac{6x^2}{1+i\sqrt{23}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{1-i\sqrt{23}}}x\right)|\frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right)}{\sqrt{6}\sqrt{-\frac{1}{1-i\sqrt{23}}}\sqrt{-3x^4+x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(1 - I*Sqrt[23])]*Sqrt[1 - (6*x^2)/(1 + I*Sqrt[23])]
*EllipticF[I*ArcSinh[Sqrt[-6/(1 - I*Sqrt[23])]*x], (1 - I*Sqrt[23])/(1 + I*
Sqrt[23])])/(Sqrt[6]*Sqrt[-(1 - I*Sqrt[23])^(-1)]*Sqrt[-2 + x^2 - 3*x^4])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+x^2-2}}{3\,x^4-x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + x^2 - 2)/(3*x^4 - x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4+x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + x^2 - 2), x)
```

maple [C] time = 0.05, size = 85, normalized size = 0.97

$$\frac{2\sqrt{-\left(\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2+1}\,\sqrt{-\left(\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2+1}\ \text{EllipticF}\left(\frac{\sqrt{1-i\sqrt{23}}\,x}{2},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{\sqrt{1-i\sqrt{23}}\,\sqrt{-3x^4+x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+x^2-2)^(1/2),x)
```

```
[Out] 2/(1-I*23^(1/2))^(1/2)*(-(1/4-1/4*I*23^(1/2))*x^2+1)^(1/2)*(-(1/4+1/4*I*23^
(1/2))*x^2+1)^(1/2)/(-3*x^4+x^2-2)^(1/2)*EllipticF(1/2*(1-I*23^(1/2))^(1/2)
*x,1/6*(-33+3*I*23^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4 + x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + x^2 - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-3\,x^4 + x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(x^2 - 3*x^4 - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4 + x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + x**2 - 2), x)
```

3.114 $\int \frac{1}{\sqrt{-2-3x^4}} \, dx$

Optimal. Leaf size=72

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4-2}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/2*2^(1/2))*(
2+x^2*6^(1/2))*((3*x^4+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/4)/(-3*x^4-2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {220}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan
[(3/2)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[-2 - 3*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-3x^4}} \, dx = \frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{-2-3x^4}}$$

Mathematica [C] time = 0.03, size = 47, normalized size = 0.65

$$-\frac{\sqrt[4]{-\frac{1}{6}}\sqrt{3x^4+2}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{3}{2}}\,x\right)\middle|-1\right)}{\sqrt{-3x^4-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - 3*x^4],x]
```

```
[Out] -(((-1/6)^(1/4)*Sqrt[2 + 3*x^4]*EllipticF[I*ArcSinh[(-3/2)^(1/4)*x], -1])/S
qrt[-2 - 3*x^4])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4-2}}{3\,x^4+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - 2)/(3*x^4 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 2), x)
```

maple [C] time = 0.02, size = 66, normalized size = 0.92

$$\frac{\sqrt{2}\,\sqrt{2i\sqrt{6}\,x^2+4}\,\sqrt{-2i\sqrt{6}\,x^2+4}\,\operatorname{EllipticF}\left(\frac{\sqrt{2}\,\sqrt{-i\sqrt{6}}\,x}{2},i\right)}{4\sqrt{-i\sqrt{6}}\,\sqrt{-3x^4-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-2)^(1/2),x)
```

[Out] 1/4*2^(1/2)/(-I*6^(1/2))^(1/2)*(2*I*6^(1/2)*x^2+4)^(1/2)*(-2*I*6^(1/2)*x^2+4)^(1/2)/(-3*x^4-2)^(1/2)*EllipticF(1/2*2^(1/2)*(-I*6^(1/2))^(1/2)*x,I)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^4-2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(-3*x^4 - 2), x)

mupad [B] time = 4.18, size = 31, normalized size = 0.43

$$\frac{x\,\sqrt{6\,x^4+4}\;{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{3\,x^4}{2}\right)}{2\,\sqrt{-3\,x^4-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(- 3*x^4 - 2)^(1/2),x)

[Out] (x*(6*x^4 + 4)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(3*x^4)/2))/(2*(- 3*x^4 - 2)^(1/2))

sympy [C] time = 0.70, size = 39, normalized size = 0.54

$$-\frac{\sqrt{2}\,ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{3x^4e^{i\pi}}{2}\right)}{8\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x**4-2)**(1/2),x)

[Out] -sqrt(2)*I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 3*x**4*exp_polar(I*pi)/2)/(8*gamma(5/4))

3.115 $\int \frac{1}{\sqrt{-2-x^2-3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4-x^2-2}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/12*(72-6*6^(
1/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4+x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(-3*x^4-x^2-2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4-x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(3/2)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*Sqrt[-2 - x^2 - 3*x^4
])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-x^2-3x^4}}\,dx=\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2-x^2-3x^4}}$$

Mathematica [C] time = 0.08, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{6x^2}{-1-i\sqrt{23}}}\sqrt{1-\frac{6x^2}{-1+i\sqrt{23}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{-1-i\sqrt{23}}}x\right)|\frac{-1-i\sqrt{23}}{-1+i\sqrt{23}}\right)}{\sqrt{6}\sqrt{-\frac{1}{-1-i\sqrt{23}}}\sqrt{-3x^4-x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(-1 - I*Sqrt[23])]*Sqrt[1 - (6*x^2)/(-1 + I*Sqrt[23]
)]*EllipticF[I*ArcSinh[Sqrt[-6/(-1 - I*Sqrt[23])]*x], (-1 - I*Sqrt[23])/(-1
 + I*Sqrt[23])])/(Sqrt[6]*Sqrt[-(-1 - I*Sqrt[23])^(-1)]*Sqrt[-2 - x^2 - 3*x
^4])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4-x^2-2}}{3\,x^4+x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - x^2 - 2)/(3*x^4 + x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4-x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - x^2 - 2), x)
```

maple [C] time = 0.05, size = 87, normalized size = 0.97

$$\frac{2\sqrt{-\left(-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2+1}\,\sqrt{-\left(-\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2+1}\;\text{EllipticF}\left(\frac{\sqrt{-1-i\sqrt{23}}\,x}{2},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{\sqrt{-1-i\sqrt{23}}\,\sqrt{-3x^4-x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-x^2-2)^(1/2),x)
```

```
[Out] 2/(-1-I*23^(1/2))^(1/2)*(-(-1/4-1/4*I*23^(1/2))*x^2+1)^(1/2)*(-(-1/4+1/4*I*
23^(1/2))*x^2+1)^(1/2)/(-3*x^4-x^2-2)^(1/2)*EllipticF(1/2*(-1-I*23^(1/2))^(
1/2)*x,1/6*(-33-3*I*23^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4 - x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - x^2 - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-3\,x^4 - x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- x^2 - 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(- x^2 - 3*x^4 - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4 - x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - x**2 - 2), x)
```

3.116 $\int \frac{1}{\sqrt{-2-2x^2-3x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+2x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4-2x^2-2}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18-3*6^(1
/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4+2*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(-3*x^4-2*x^2-2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+2x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4-2x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 2*x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*Sqrt[-2 - 2*x^2 - 3*
x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-2x^2-3x^4}}\,dx=\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+2x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2-2x^2-3x^4}}$$

Mathematica [C] time = 0.08, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{3x^2}{-1-i\sqrt{5}}}\sqrt{1-\frac{3x^2}{-1+i\sqrt{5}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{-1-i\sqrt{5}}}x\right)|\frac{-1-i\sqrt{5}}{-1+i\sqrt{5}}\right)}{\sqrt{3}\sqrt{-\frac{1}{-1-i\sqrt{5}}}\sqrt{-3x^4-2x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - 2*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(-1 - I*Sqrt[5])]*Sqrt[1 - (3*x^2)/(-1 + I*Sqrt[5])]
*EllipticF[I*ArcSinh[Sqrt[-3/(-1 - I*Sqrt[5])]*x], (-1 - I*Sqrt[5])/(-1 + I
*Sqrt[5])])/(Sqrt[3]*Sqrt[-(-1 - I*Sqrt[5])^(-1)]*Sqrt[-2 - 2*x^2 - 3*x^4])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4-2\,x^2-2}}{3\,x^4+2\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - 2*x^2 - 2)/(3*x^4 + 2*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4-2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 2*x^2 - 2), x)
```

maple [C] time = 0.05, size = 87, normalized size = 0.95

$$\frac{2\sqrt{-\left(-\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2+1}\,\sqrt{-\left(-\frac{1}{2}+\frac{i\sqrt{5}}{2}\right)x^2+1}\ \text{EllipticF}\left(\frac{\sqrt{-2-2i\sqrt{5}}\,x}{2},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{\sqrt{-2-2i\sqrt{5}}\,\sqrt{-3x^4-2x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-2*x^2-2)^(1/2),x)
```

```
[Out] 2/(-2-2*I*5^(1/2))^(1/2)*(-(-1/2-1/2*I*5^(1/2))*x^2+1)^(1/2)*(-(-1/2+1/2*I*
5^(1/2))*x^2+1)^(1/2)/(-3*x^4-2*x^2-2)^(1/2)*EllipticF(1/2*(-2-2*I*5^(1/2))
^(1/2)*x,1/3*(-6-3*I*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4 - 2\,x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 2*x^2 - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-3\,x^4 - 2\,x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 2*x^2 - 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(- 2*x^2 - 3*x^4 - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4 - 2x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-2*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 2*x**2 - 2), x)
```

3.117 $\int \frac{1}{\sqrt{-2-3x^2-3x^4}} \, dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+3x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4-3x^2-2}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/4*(8-2*6^(1/
2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4+3*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(-3*x^4-3*x^2-2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+3x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 3*x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*Sqrt[-2 - 3*x^2 - 3*x
^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-3x^2-3x^4}} \, dx = \frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+3x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2-3x^2-3x^4}}$$

Mathematica [C] time = 0.11, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{6x^2}{-3-i\sqrt{15}}}\sqrt{1-\frac{6x^2}{-3+i\sqrt{15}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{-3-i\sqrt{15}}}x\right)|\frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right)}{\sqrt{6}\sqrt{-\frac{1}{-3-i\sqrt{15}}}\sqrt{-3x^4-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - 3*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(-3 - I*Sqrt[15])]*Sqrt[1 - (6*x^2)/(-3 + I*Sqrt[15]
)]*EllipticF[I*ArcSinh[Sqrt[-6/(-3 - I*Sqrt[15])]*x], (-3 - I*Sqrt[15])/(-3
 + I*Sqrt[15])])/(Sqrt[6]*Sqrt[-(-3 - I*Sqrt[15])^(-1)]*Sqrt[-2 - 3*x^2 - 3
*x^4])
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4-3\,x^2-2}}{3\,x^4+3\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-3*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - 3*x^2 - 2)/(3*x^4 + 3*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4-3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-3*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 3*x^2 - 2), x)
```

maple [C] time = 0.05, size = 87, normalized size = 0.95

$$\frac{2\sqrt{-\left(-\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2+1}\,\sqrt{-\left(-\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{-3-i\sqrt{15}}\,x}{2},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{\sqrt{-3-i\sqrt{15}}\,\sqrt{-3x^4-3x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-3*x^2-2)^(1/2),x)
```

```
[Out] 2/(-3-I*15^(1/2))^(1/2)*(-(-3/4-1/4*I*15^(1/2))*x^2+1)^(1/2)*(-(-3/4+1/4*I*
15^(1/2))*x^2+1)^(1/2)/(-3*x^4-3*x^2-2)^(1/2)*EllipticF(1/2*(-3-I*15^(1/2))
^(1/2)*x,1/2*(-1-I*15^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4 - 3\,x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-3*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 3*x^2 - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-3\,x^4 - 3\,x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 3*x^2 - 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(- 3*x^2 - 3*x^4 - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4 - 3x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-3*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 3*x**2 - 2), x)
```

3.118 $\int \frac{1}{\sqrt{-2-4x^2-3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+4x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4-4x^2-2}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18-6*6^(1
/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4+4*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(-3*x^4-4*x^2-2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+4x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4-4x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 4*x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[-2 - 4*x^2 - 3*x
^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-4x^2-3x^4}}\,dx=\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+4x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-2-4x^2-3x^4}}$$

Mathematica [C] time = 0.08, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{3x^2}{-2-i\sqrt{2}}}\sqrt{1-\frac{3x^2}{-2+i\sqrt{2}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{-2-i\sqrt{2}}}x\right)|\frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right)}{\sqrt{3}\sqrt{-\frac{1}{-2-i\sqrt{2}}}\sqrt{-3x^4-4x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - 4*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(-2 - I*Sqrt[2])]*Sqrt[1 - (3*x^2)/(-2 + I*Sqrt[2])]
*EllipticF[I*ArcSinh[Sqrt[-3/(-2 - I*Sqrt[2])]*x], (-2 - I*Sqrt[2])/(-2 + I
*Sqrt[2])])/(Sqrt[3]*Sqrt[-(-2 - I*Sqrt[2])^(-1)]*Sqrt[-2 - 4*x^2 - 3*x^4])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4-4\,x^2-2}}{3\,x^4+4\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-4*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - 4*x^2 - 2)/(3*x^4 + 4*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-3\,x^4-4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-4*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 4*x^2 - 2), x)
```

maple [C] time = 0.05, size = 87, normalized size = 0.97

$$\frac{2\sqrt{-\left(-1-\frac{i\sqrt{2}}{2}\right)x^2+1}\,\sqrt{-\left(-1+\frac{i\sqrt{2}}{2}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{-4-2i\sqrt{2}}\,x}{2},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{\sqrt{-4-2i\sqrt{2}}\,\sqrt{-3x^4-4x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-4*x^2-2)^(1/2),x)
```

```
[Out] 2/(-4-2*I*2^(1/2))^(1/2)*(-(-1-1/2*I*2^(1/2))*x^2+1)^(1/2)*(-(-1+1/2*I*2^(1
/2))*x^2+1)^(1/2)/(-3*x^4-4*x^2-2)^(1/2)*EllipticF(1/2*(-4-2*I*2^(1/2))^(1/
2)*x,1/3*(3-6*I*2^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4-4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-4*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 4*x^2 - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-3\,x^4-4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 4*x^2 - 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(- 4*x^2 - 3*x^4 - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4-4x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-4*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 4*x**2 - 2), x)
```

3.119 $\int \frac{1}{\sqrt{-2-5x^2-3x^4}}\,dx$

Optimal. Leaf size=52

$$-\frac{\sqrt{-3x^2-2}\,F\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

```
[Out] -1/2*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*I*2^(1/2))*(-3*x^2-2)^
(1/2)*2^(1/2)/((3*x^2+2)/(x^2+1))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 418}

$$-\frac{\sqrt{-3x^2-2}\,F\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 5*x^2 - 3*x^4],x]
```

```
[Out] -((Sqrt[-2 - 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt
[(2 + 3*x^2)/(1 + x^2)]))
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-5x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-4-6x^2}\,\sqrt{6+6x^2}}\,dx$$
$$= -\frac{\sqrt{-2-3x^2}\,F\left(\tan^{-1}(x)\middle| -\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{1+x^2}\,\sqrt{\frac{2+3x^2}{1+x^2}}}$$

Mathematica [C] time = 0.02, size = 63, normalized size = 1.21

$$-\frac{i\sqrt{x^2+1}\,\sqrt{3x^2+2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|\frac{2}{3}\right)}{\sqrt{3}\,\sqrt{-3x^4-5x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - 5*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])
/(Sqrt[3]*Sqrt[-2 - 5*x^2 - 3*x^4])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4-5\,x^2-2}}{3\,x^4+5\,x^2+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-5*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - 5*x^2 - 2)/(3*x^4 + 5*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4-5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-5*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 5*x^2 - 2), x)
```

maple [A] time = 0.01, size = 50, normalized size = 0.96

$$-\frac{i\sqrt{6}\,\sqrt{6x^{2}+4}\,\sqrt{x^{2}+1}\,\operatorname{EllipticF}\left(\frac{i\sqrt{6}\,x}{2},\frac{\sqrt{6}}{3}\right)}{6\sqrt{-3x^{4}-5x^{2}-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-5*x^2-2)^(1/2),x)
```

```
[Out] -1/6*I*6^(1/2)*(6*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-3*x^4-5*x^2-2)^(1/2)*Ellipti
cF(1/2*I*6^(1/2)*x,1/3*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^{4}-5\,x^{2}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-5*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 5*x^2 - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-3\,x^{4}-5\,x^{2}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 5*x^2 - 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(- 5*x^2 - 3*x^4 - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^{4}-5x^{2}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-5*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 5*x**2 - 2), x)
```

3.120 $\int \frac{1}{\sqrt{2+5x^2+5x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{10}\,x^2+2\right)\sqrt{\frac{5x^4+5x^2+2}{\left(\sqrt{10}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{5}{2}}\,x\right)|\frac{1}{8}\left(4-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{5x^4+5x^2+2}}$$

```
[Out] 1/20*(cos(2*arctan(1/2*5^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*5^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*5^(1/4)*2^(3/4)*x)),1/4*(8-2*10^(1
/2))^(1/2))*(2+x^2*10^(1/2))*((5*x^4+5*x^2+2)/(2+x^2*10^(1/2))^2)^(1/2)*10^
(3/4)/(5*x^4+5*x^2+2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{10}\,x^2+2\right)\sqrt{\frac{5x^4+5x^2+2}{\left(\sqrt{10}\,x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{5}{2}}\,x\right)|\frac{1}{8}\left(4-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{5x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 + 5*x^4],x]
```

```
[Out] ((2 + Sqrt[10]*x^2)*Sqrt[(2 + 5*x^2 + 5*x^4)/(2 + Sqrt[10]*x^2)^2]*Elliptic
F[2*ArcTan[(5/2)^(1/4)*x], (4 - Sqrt[10])/8])/(2*10^(1/4)*Sqrt[2 + 5*x^2 +
5*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2+5x^4}}\,dx=\frac{\left(2+\sqrt{10}\,x^2\right)\sqrt{\frac{2+5x^2+5x^4}{\left(2+\sqrt{10}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{5}{2}}\,x\right)|\frac{1}{8}\left(4-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{2+5x^2+5x^4}}$$

Mathematica [C] time = 0.11, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{10x^2}{-5-i\sqrt{15}}}\sqrt{1-\frac{10x^2}{-5+i\sqrt{15}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{10}{-5-i\sqrt{15}}}x\right)|\frac{-5-i\sqrt{15}}{-5+i\sqrt{15}}\right)}{\sqrt{10}\sqrt{-\frac{1}{-5-i\sqrt{15}}}\sqrt{5x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 + 5*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (10*x^2)/(-5 - I*Sqrt[15])]*Sqrt[1 - (10*x^2)/(-5 + I*Sqrt[1
5])]*EllipticF[I*ArcSinh[Sqrt[-10/(-5 - I*Sqrt[15])]*x], (-5 - I*Sqrt[15])/
(-5 + I*Sqrt[15])])/(Sqrt[10]*Sqrt[-(-5 - I*Sqrt[15])^(-1)]*Sqrt[2 + 5*x^2
+ 5*x^4])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{5\,x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(5*x^4 + 5*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{5\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(5*x^4 + 5*x^2 + 2), x)
```

maple [C] time = 0.12, size = 87, normalized size = 0.95

$$\frac{2\sqrt{-\left(-\frac{5}{4}+\frac{i\sqrt{15}}{4}\right)x^2+1}\,\sqrt{-\left(-\frac{5}{4}-\frac{i\sqrt{15}}{4}\right)x^2+1}\,\ \text{EllipticF}\left(\frac{\sqrt{-5+i\sqrt{15}}\,x}{2},\frac{\sqrt{1+i\sqrt{15}}}{2}\right)}{\sqrt{-5+i\sqrt{15}}\,\sqrt{5x^4+5x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+I*15^(1/2))^(1/2)*(1-(-5/4+1/4*I*15^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*I*
15^(1/2))*x^2)^(1/2)/(5*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+I*15^(1/2))^
(1/2),1/2*(1+I*15^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{5\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(5*x^4 + 5*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{5\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 + 5*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 + 5*x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{5x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(5*x**4 + 5*x**2 + 2), x)
```

3.121 $\int \frac{1}{\sqrt{2+5x^2+4x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{4x^4+5x^2+2}{\left(\sqrt{2}\,x^2+1\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{2}\,x\right)|\frac{1}{16}\left(8-5\sqrt{2}\right)\right)}{2\,2^{3/4}\sqrt{4x^4+5x^2+2}}$$

```
[Out] 1/4*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(s
in(2*arctan(2^(1/4)*x)),1/4*(8-5*2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((4*x^4+5*
x^2+2)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(4*x^4+5*x^2+2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{4x^4+5x^2+2}{\left(\sqrt{2}\,x^2+1\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{2}\,x\right)|\frac{1}{16}\left(8-5\sqrt{2}\right)\right)}{2\,2^{3/4}\sqrt{4x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 + 4*x^4],x]
```

```
[Out] ((1 + Sqrt[2]*x^2)*Sqrt[(2 + 5*x^2 + 4*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[
2*ArcTan[2^(1/4)*x], (8 - 5*Sqrt[2])/16])/(2*2^(3/4)*Sqrt[2 + 5*x^2 + 4*x^4
])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2+4x^4}}\,dx=\frac{\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{2+5x^2+4x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{2}\,x\right)|\frac{1}{16}\left(8-5\sqrt{2}\right)\right)}{2\,2^{3/4}\sqrt{2+5x^2+4x^4}}$$

Mathematica [C] time = 0.08, size = 147, normalized size = 1.63

$$-\frac{i\sqrt{1-\frac{8x^2}{-5-i\sqrt{7}}}\sqrt{1-\frac{8x^2}{-5+i\sqrt{7}}}F\left(i\sinh^{-1}\left(2\sqrt{-\frac{2}{-5-i\sqrt{7}}}x\right)|\frac{-5-i\sqrt{7}}{-5+i\sqrt{7}}\right)}{2\sqrt{2}\sqrt{-\frac{1}{-5-i\sqrt{7}}}\sqrt{4x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 + 4*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 - (8*x^2)/(-5 - I*Sqrt[7])]*Sqrt[1 - (8*x^2)/(-5 + I*Sqrt[
7])]*EllipticF[I*ArcSinh[2*Sqrt[-2/(-5 - I*Sqrt[7])]*x], (-5 - I*Sqrt[7])/(
-5 + I*Sqrt[7])])/(Sqrt[2]*Sqrt[-(-5 - I*Sqrt[7])^(-1)]*Sqrt[2 + 5*x^2 + 4*
x^4])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{4\,x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(4*x^4 + 5*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(4*x^4 + 5*x^2 + 2), x)
```

maple [C] time = 0.12, size = 87, normalized size = 0.97

$$\frac{2\sqrt{-\left(-\frac{5}{4}+\frac{i\sqrt{7}}{4}\right)x^2+1}\,\sqrt{-\left(-\frac{5}{4}-\frac{i\sqrt{7}}{4}\right)x^2+1}\,\,\text{EllipticF}\left(\frac{\sqrt{-5+i\sqrt{7}}\,x}{2},\frac{\sqrt{9+5i\sqrt{7}}}{4}\right)}{\sqrt{-5+i\sqrt{7}}\,\sqrt{4x^4+5x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+I*7^(1/2))^(1/2)*(1-(-5/4+1/4*I*7^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*I*7^
(1/2))*x^2)^(1/2)/(4*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+I*7^(1/2))^(1/2
),1/4*(9+5*I*7^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(4*x^4 + 5*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{4\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 + 4*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 + 4*x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(4*x**4 + 5*x**2 + 2), x)
```

3.122 $\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx$

Optimal. Leaf size=52

$$\frac{\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{3x^4+5x^2+2}}$$

```
[Out] 1/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*I*2^(1/2)
)*((3*x^2+2)/(x^2+1))^(1/2)*2^(1/2)/(3*x^4+5*x^2+2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1100}

$$\frac{\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{3x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

```
[Out] ((1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]
*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rule 1100

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b - q)*x^2)*Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]*EllipticF[ArcTan[Rt[(b - q)/(2*a), 2]*x], (-2*q)/(b - q)])/(2
*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx=\frac{\left(1+x^2\right)\sqrt{\frac{2+3x^2}{1+x^2}}\,F\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{2+5x^2+3x^4}}$$

Mathematica [C] time = 0.02, size = 58, normalized size = 1.12

$$-\frac{i\sqrt{x^2+1}\,\sqrt{3x^2+2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|\frac{2}{3}\right)}{\sqrt{9x^4+15x^2+6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])
/Sqrt[6 + 15*x^2 + 9*x^4]
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

maple [A] time = 0.00, size = 44, normalized size = 0.85

$$-\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/
2*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 + 3*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 + 3*x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 5*x**2 + 2), x)
```

3.123 $\int \frac{1}{\sqrt{2+5x^2+2x^4}} dx$

Optimal. Leaf size=58

$$\frac{\sqrt{\frac{x^2+2}{2x^2+1}}\left(2x^2+1\right)F\left(\tan^{-1}\left(\sqrt{2}x\right)|\frac{3}{4}\right)}{2\sqrt{2x^4+5x^2+2}}$$

```
[Out] 1/2*(2*x^2+1)^(3/2)*(1/(2*x^2+1))^(1/2)*EllipticF(x*2^(1/2)/(2*x^2+1)^(1/2)
,1/2*3^(1/2))*((x^2+2)/(2*x^2+1))^(1/2)/(2*x^4+5*x^2+2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1099}

$$\frac{\sqrt{\frac{x^2+2}{2x^2+1}}\left(2x^2+1\right)F\left(\tan^{-1}\left(\sqrt{2}x\right)|\frac{3}{4}\right)}{2\sqrt{2x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(2 + x^2)/(1 + 2*x^2)]*(1 + 2*x^2)*EllipticF[ArcTan[Sqrt[2]*x], 3/4])
/(2*Sqrt[2 + 5*x^2 + 2*x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2+2x^4}} dx = \frac{\sqrt{\frac{2+x^2}{1+2x^2}}\left(1+2x^2\right)F\left(\tan^{-1}\left(\sqrt{2}x\right)|\frac{3}{4}\right)}{2\sqrt{2+5x^2+2x^4}}$$

Mathematica [C] time = 0.02, size = 58, normalized size = 1.00

$$-\frac{i\sqrt{x^2+2}\,\sqrt{2x^2+1}\,F\left(i\sinh^{-1}\left(\sqrt{2}\,x\right)|\frac{1}{4}\right)}{2\sqrt{2x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 + 2*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[2 + x^2]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]*x], 1/4
])/Sqrt[2 + 5*x^2 + 2*x^4]
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 5*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 5*x^2 + 2), x)
```

maple [C] time = 0.01, size = 48, normalized size = 0.83

$$-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{2x^2+1}\,\operatorname{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},2\right)}{2\sqrt{2x^4+5x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(2*x^2+1)^(1/2)/(2*x^4+5*x^2+2)^(1/2)*Ellipt
icF(1/2*I*2^(1/2)*x,2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 5*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{2\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 + 2*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 + 2*x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 5*x**2 + 2), x)
```

3.124 $\int \frac{1}{\sqrt{2+5x^2+x^4}}\,dx$

Optimal. Leaf size=108

$$\frac{\sqrt{\frac{\left(5-\sqrt{17}\right)x^2+4}{\left(5+\sqrt{17}\right)x^2+4}}\left(\left(5+\sqrt{17}\right)x^2+4\right)F\left(\tan^{-1}\left(\frac{1}{2}\sqrt{5+\sqrt{17}}\,x\right)|\frac{1}{4}\left(-17+5\sqrt{17}\right)\right)}{2\sqrt{5+\sqrt{17}}\sqrt{x^4+5x^2+2}}$$

```
[Out] 1/2*(1/(4+x^2*(5+17^(1/2))))^(1/2)*(4+x^2*(5+17^(1/2)))^(3/2)*EllipticF(x*(
5+17^(1/2))^(1/2)/(4+x^2*(5+17^(1/2)))^(1/2),1/2*(-17+5*17^(1/2))^(1/2))*((
4+x^2*(5-17^(1/2)))/(4+x^2*(5+17^(1/2))))^(1/2)/(x^4+5*x^2+2)^(1/2)/(5+17^(
1/2))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1099}

$$\frac{\sqrt{\frac{\left(5-\sqrt{17}\right)x^2+4}{\left(5+\sqrt{17}\right)x^2+4}}\left(\left(5+\sqrt{17}\right)x^2+4\right)F\left(\tan^{-1}\left(\frac{1}{2}\sqrt{5+\sqrt{17}}\,x\right)|\frac{1}{4}\left(-17+5\sqrt{17}\right)\right)}{2\sqrt{5+\sqrt{17}}\sqrt{x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 + x^4],x]
```

```
[Out] (Sqrt[(4 + (5 - Sqrt[17])*x^2)/(4 + (5 + Sqrt[17])*x^2)]*(4 + (5 + Sqrt[17]
)*x^2)*EllipticF[ArcTan[(Sqrt[5 + Sqrt[17]]*x)/2], (-17 + 5*Sqrt[17])/4])/(
2*Sqrt[5 + Sqrt[17]]*Sqrt[2 + 5*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2+x^4}}\,dx = \frac{\sqrt{\frac{4+\left(5-\sqrt{17}\right)x^2}{4+\left(5+\sqrt{17}\right)x^2}}\left(4+\left(5+\sqrt{17}\right)x^2\right)F\left(\tan^{-1}\left(\frac{1}{2}\sqrt{5+\sqrt{17}}\,x\right)|\frac{1}{4}\left(-17+5\sqrt{17}\right)\right)}{2\sqrt{5+\sqrt{17}}\sqrt{2+5x^2+x^4}}$$

Mathematica [C] time = 0.08, size = 103, normalized size = 0.95

$$-\frac{i\sqrt{2x^2-\sqrt{17}+5}\sqrt{2x^2+\sqrt{17}+5}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{17}}}\,x\right)|\frac{21}{4}+\frac{5\sqrt{17}}{4}\right)}{\sqrt{2\left(5-\sqrt{17}\right)}\sqrt{x^4+5x^2+2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[5 - Sqrt[17] + 2*x^2]*Sqrt[5 + Sqrt[17] + 2*x^2]*EllipticF[I*Arc
Sinh[Sqrt[2/(5 + Sqrt[17])]*x], 21/4 + (5*Sqrt[17])/4])/(Sqrt[2*(5 - Sqrt[1
7])]*Sqrt[2 + 5*x^2 + x^4])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(x^4 + 5*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(x^4 + 5*x^2 + 2), x)
```

maple [A] time = 0.08, size = 76, normalized size = 0.70

$$\frac{2\sqrt{-\left(-\frac{5}{4}+\frac{\sqrt{17}}{4}\right)x^{2}+1}\,\sqrt{-\left(-\frac{5}{4}-\frac{\sqrt{17}}{4}\right)x^{2}+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{-5+\sqrt{17}}\,x}{2},\frac{5\sqrt{2}}{4}+\frac{\sqrt{34}}{4}\right)}{\sqrt{-5+\sqrt{17}}\,\sqrt{x^{4}+5x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+17^(1/2))^(1/2)*(1-(-5/4+1/4*17^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*17^(1/
2))*x^2)^(1/2)/(x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+17^(1/2))^(1/2),5/4*
2^(1/2)+1/4*34^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^{4}+5\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x^4 + 5*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{x^{4}+5\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 + x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 + x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^{4}+5x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**4 + 5*x**2 + 2), x)
```

3.125 $\int \frac{1}{\sqrt{2+5x^2-x^4}}\,dx$

Optimal. Leaf size=48

$$\sqrt{\frac{2}{\sqrt{33}-5}}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{5+\sqrt{33}}}\,x\right)|\frac{1}{4}\left(-29-5\sqrt{33}\right)\right)$$

```
[Out] EllipticF(x*2^(1/2)/(5+33^(1/2))^(1/2),5/4*I*2^(1/2)+1/4*I*66^(1/2))*2^(1/2
)/(-5+33^(1/2))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{\sqrt{33}-5}}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{5+\sqrt{33}}}\,x\right)|\frac{1}{4}\left(-29-5\sqrt{33}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[33])]*EllipticF[ArcSin[Sqrt[2/(5 + Sqrt[33])]*x], (-29 -
5*Sqrt[33])/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2-x^4}}\,dx = 2\int \frac{1}{\sqrt{5+\sqrt{33}-2x^2}\sqrt{-5+\sqrt{33}+2x^2}}\,dx$$
$$= \sqrt{\frac{2}{-5+\sqrt{33}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{5+\sqrt{33}}}\,x\right)\middle|\frac{1}{4}\left(-29-5\sqrt{33}\right)\right)$$

Mathematica [C] time = 0.06, size = 55, normalized size = 1.15

$$-i\sqrt{\frac{2}{5+\sqrt{33}}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-5+\sqrt{33}}}\,x\right)\middle|-\frac{29}{4}+\frac{5\sqrt{33}}{4}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[33])]*EllipticF[I*ArcSinh[Sqrt[2/(-5 + Sqrt[33])]*x],
 -29/4 + (5*Sqrt[33])/4]
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-x^4+5\,x^2+2}}{x^4-5\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 5*x^2 + 2)/(x^4 - 5*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^4 + 5*x^2 + 2), x)
```

maple [B] time = 0.10, size = 80, normalized size = 1.67

$$\frac{2\sqrt{-\left(-\frac{5}{4}+\frac{\sqrt{33}}{4}\right)x^{2}+1}\,\sqrt{-\left(-\frac{5}{4}-\frac{\sqrt{33}}{4}\right)x^{2}+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{-5+\sqrt{33}}\,x}{2},\frac{5i\sqrt{2}}{4}+\frac{i\sqrt{66}}{4}\right)}{\sqrt{-5+\sqrt{33}}\,\sqrt{-x^{4}+5x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+33^(1/2))^(1/2)*(1-(-5/4+1/4*33^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*33^(1/
2))*x^2)^(1/2)/(-x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+33^(1/2))^(1/2),5/4
*I*2^(1/2)+1/4*I*66^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^{4}+5\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-x^4 + 5*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-x^{4}+5\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 - x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^{4}+5x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**4 + 5*x**2 + 2), x)
```

3.126 $\int \frac{1}{\sqrt{2+5x^2-2x^4}}\,dx$

Optimal. Leaf size=45

$$\sqrt{\frac{2}{\sqrt{41}-5}}\,F\left(\sin^{-1}\left(\frac{2x}{\sqrt{5+\sqrt{41}}}\right)|\frac{1}{8}\left(-33-5\sqrt{41}\right)\right)$$

```
[Out] EllipticF(2*x/(5+41^(1/2))^(1/2),5/4*I+1/4*I*41^(1/2))*2^(1/2)/(-5+41^(1/2)
)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{\sqrt{41}-5}}\,F\left(\sin^{-1}\left(\frac{2x}{\sqrt{5+\sqrt{41}}}\right)|\frac{1}{8}\left(-33-5\sqrt{41}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[41])]*EllipticF[ArcSin[(2*x)/Sqrt[5 + Sqrt[41]]], (-33 -
5*Sqrt[41])/8]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{5+\sqrt{41}-4x^2}\,\sqrt{-5+\sqrt{41}+4x^2}}\,dx$$
$$= \sqrt{\frac{2}{-5+\sqrt{41}}}\,F\left(\sin^{-1}\left(\frac{2x}{\sqrt{5+\sqrt{41}}}\right)\middle|\frac{1}{8}\left(-33-5\sqrt{41}\right)\right)$$

Mathematica [C] time = 0.05, size = 52, normalized size = 1.16

$$-i\sqrt{\frac{2}{5+\sqrt{41}}}\,F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{-5+\sqrt{41}}}\right)\middle|-\frac{33}{8}+\frac{5\sqrt{41}}{8}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 2*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[41])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[-5 + Sqrt[41]]],
 -33/8 + (5*Sqrt[41])/8]
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+5\,x^2+2}}{2\,x^4-5\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 5*x^2 + 2)/(2*x^4 - 5*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 + 2), x)
```

maple [B] time = 0.09, size = 76, normalized size = 1.69

$$\frac{2\sqrt{-\left(-\frac{5}{4}+\frac{\sqrt{41}}{4}\right)x^{2}+1}\,\sqrt{-\left(-\frac{5}{4}-\frac{\sqrt{41}}{4}\right)x^{2}+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{-5+\sqrt{41}}\,x}{2},\frac{5i}{4}+\frac{i\sqrt{41}}{4}\right)}{\sqrt{-5+\sqrt{41}}\,\sqrt{-2x^{4}+5x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+41^(1/2))^(1/2)*(1-(-5/4+1/4*41^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*41^(1/
2))*x^2)^(1/2)/(-2*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+41^(1/2))^(1/2),5
/4*I+1/4*I*41^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,x^{4}+5\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-2\,x^{4}+5\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 2*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 - 2*x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^{4}+5x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 5*x**2 + 2), x)
```

3.127 $\int \frac{1}{\sqrt{2+5x^2-3x^4}} dx$

Optimal. Leaf size=10

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right)$$

[Out] EllipticF(1/2*x*2^(1/2),I*6^(1/2))

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[2 + 5*x^2 - 3*x^4],x]

[Out] EllipticF[ArcSin[x/Sqrt[2]], -6]

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{2+5x^2-3x^4}} dx &= \left(2\sqrt{3}\right) \int \frac{1}{\sqrt{12-6x^2}\sqrt{2+6x^2}} dx \\ &= F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right) \end{aligned}$$

Mathematica [C] time = 0.02, size = 65, normalized size = 6.50

$$-\frac{i\sqrt{1-\frac{x^2}{2}}\sqrt{3x^2+1}\,F\left(i\sinh^{-1}\left(\sqrt{3}\,x\right)|-\frac{1}{6}\right)}{\sqrt{3}\sqrt{-3x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2/2]*Sqrt[1 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3]*x], -1/6]
)/(Sqrt[3]*Sqrt[2 + 5*x^2 - 3*x^4])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+5\,x^2+2}}{3\,x^4-5\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 5*x^2 + 2)/(3*x^4 - 5*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 + 2), x)
```

maple [B] time = 0.00, size = 51, normalized size = 5.10

$$\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{3x^2+1}\,\,\text{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{6}\right)}{2\sqrt{-3x^4+5x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*(-2*x^2+4)^(1/2)*(3*x^2+1)^(1/2)/(-3*x^4+5*x^2+2)^(1/2)*Ellipti
cF(1/2*2^(1/2)*x,I*6^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.10

$$\int \frac{1}{\sqrt{-3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 3*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 - 3*x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 5*x**2 + 2), x)
```

3.128 $\int \frac{1}{\sqrt{2+5x^2-4x^4}}\,dx$

Optimal. Leaf size=49

$$\sqrt{\frac{2}{\sqrt{57}-5}}\,F\left(\sin^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{57}}}\,x\right)\middle|\frac{1}{16}\left(-41-5\sqrt{57}\right)\right)$$

```
[Out] EllipticF(2*x*2^(1/2)/(5+57^(1/2))^(1/2),5/8*I*2^(1/2)+1/8*I*114^(1/2))*2^(
1/2)/(-5+57^(1/2))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{\sqrt{57}-5}}\,F\left(\sin^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{57}}}\,x\right)\middle|\frac{1}{16}\left(-41-5\sqrt{57}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 4*x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[57])]*EllipticF[ArcSin[2*Sqrt[2/(5 + Sqrt[57])]*x], (-41
- 5*Sqrt[57])/16]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2-4x^4}}\,dx = 4\int \frac{1}{\sqrt{5+\sqrt{57}-8x^2}\sqrt{-5+\sqrt{57}+8x^2}}\,dx$$
$$= \sqrt{\frac{2}{-5+\sqrt{57}}}\,F\left(\sin^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{57}}}\,x\right)|\frac{1}{16}\left(-41-5\sqrt{57}\right)\right)$$

Mathematica [C] time = 0.06, size = 56, normalized size = 1.14

$$-i\sqrt{\frac{2}{5+\sqrt{57}}}\,F\left(i\sinh^{-1}\left(2\sqrt{\frac{2}{-5+\sqrt{57}}}\,x\right)|\frac{1}{16}\left(-41+5\sqrt{57}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 4*x^4],x]
[Out] (-I)*Sqrt[2/(5 + Sqrt[57])]*EllipticF[I*ArcSinh[2*Sqrt[2/(-5 + Sqrt[57])]*x
], (-41 + 5*Sqrt[57])/16]
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-4\,x^4+5\,x^2+2}}{4\,x^4-5\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-4*x^4 + 5*x^2 + 2)/(4*x^4 - 5*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-4\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-4*x^4 + 5*x^2 + 2), x)
```

maple [B] time = 0.10, size = 80, normalized size = 1.63

$$\frac{2\sqrt{-\left(-\frac{5}{4}+\frac{\sqrt{57}}{4}\right)x^{2}+1}\,\sqrt{-\left(-\frac{5}{4}-\frac{\sqrt{57}}{4}\right)x^{2}+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{-5+\sqrt{57}}\,x}{2},\frac{5i\sqrt{2}}{8}+\frac{i\sqrt{114}}{8}\right)}{\sqrt{-5+\sqrt{57}}\,\sqrt{-4x^{4}+5x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+57^(1/2))^(1/2)*(1-(-5/4+1/4*57^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*57^(1/
2))*x^2)^(1/2)/(-4*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+57^(1/2))^(1/2),5
/8*I*2^(1/2)+1/8*I*114^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-4\,x^{4}+5\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-4*x^4 + 5*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-4\,x^{4}+5\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 4*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 - 4*x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-4x^{4}+5x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-4*x**4 + 5*x**2 + 2), x)
```

3.129 $\int \frac{1}{\sqrt{2+5x^2-5x^4}}\,dx$

Optimal. Leaf size=48

$$\sqrt{\frac{2}{\sqrt{65}-5}}\,F\left(\sin^{-1}\left(\sqrt{\frac{10}{5+\sqrt{65}}}\,x\right)\middle|\frac{1}{4}\left(-9-\sqrt{65}\right)\right)$$

```
[Out] EllipticF(x*10^(1/2)/(5+65^(1/2))^(1/2),1/4*I*10^(1/2)+1/4*I*26^(1/2))*2^(1
/2)/(-5+65^(1/2))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{\sqrt{65}-5}}\,F\left(\sin^{-1}\left(\sqrt{\frac{10}{5+\sqrt{65}}}\,x\right)\middle|\frac{1}{4}\left(-9-\sqrt{65}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 5*x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[65])]*EllipticF[ArcSin[Sqrt[10/(5 + Sqrt[65])]*x], (-9 -
Sqrt[65])/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2-5x^4}}\,dx = \left(2\sqrt{5}\right)\int \frac{1}{\sqrt{5+\sqrt{65}-10x^2}\,\sqrt{-5+\sqrt{65}+10x^2}}\,dx$$
$$= \sqrt{\frac{2}{-5+\sqrt{65}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{10}{5+\sqrt{65}}}\,x\right)|\frac{1}{4}\left(-9-\sqrt{65}\right)\right)$$

Mathematica [C] time = 0.06, size = 52, normalized size = 1.08

$$-i\sqrt{\frac{2}{5+\sqrt{65}}}\,F\left(i\sinh^{-1}\left(\frac{1}{2}\sqrt{5+\sqrt{65}}\,x\right)|\frac{1}{4}\left(-9+\sqrt{65}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 5*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[65])]*EllipticF[I*ArcSinh[(Sqrt[5 + Sqrt[65]]*x)/2],
(-9 + Sqrt[65])/4]
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-5\,x^4+5\,x^2+2}}{5\,x^4-5\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-5*x^4 + 5*x^2 + 2)/(5*x^4 - 5*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-5\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-5*x^4 + 5*x^2 + 2), x)
```

maple [B] time = 0.11, size = 80, normalized size = 1.67

$$\frac{2\sqrt{-\left(-\frac{5}{4}+\frac{\sqrt{65}}{4}\right)x^{2}+1}\,\sqrt{-\left(-\frac{5}{4}-\frac{\sqrt{65}}{4}\right)x^{2}+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{-5+\sqrt{65}}\,x}{2},\frac{i\sqrt{10}}{4}+\frac{i\sqrt{26}}{4}\right)}{\sqrt{-5+\sqrt{65}}\,\sqrt{-5x^{4}+5x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-5*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+65^(1/2))^(1/2)*(1-(-5/4+1/4*65^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*65^(1/
2))*x^2)^(1/2)/(-5*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+65^(1/2))^(1/2),1
/4*I*10^(1/2)+1/4*I*26^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-5\,x^{4}+5\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-5*x^4 + 5*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-5\,x^{4}+5\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 5*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 - 5*x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-5x^{4}+5x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-5*x**4 + 5*x**2 + 2), x)
```

3.130 $\int \frac{1}{\sqrt{2+5x^2-6x^4}}\,dx$

Optimal. Leaf size=49

$$\sqrt{\frac{2}{\sqrt{73}-5}}\,F\left(\sin^{-1}\left(2\sqrt{\frac{3}{5+\sqrt{73}}}\,x\right)|\frac{1}{24}\left(-49-5\sqrt{73}\right)\right)$$

```
[Out] EllipticF(2*x*3^(1/2)/(5+73^(1/2))^(1/2),5/12*I*3^(1/2)+1/12*I*219^(1/2))*2
^(1/2)/(-5+73^(1/2))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{\sqrt{73}-5}}\,F\left(\sin^{-1}\left(2\sqrt{\frac{3}{5+\sqrt{73}}}\,x\right)|\frac{1}{24}\left(-49-5\sqrt{73}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 6*x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[73])]*EllipticF[ArcSin[2*Sqrt[3/(5 + Sqrt[73])]*x], (-49
- 5*Sqrt[73])/24]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2-6x^4}}\,dx = \left(2\sqrt{6}\right)\int \frac{1}{\sqrt{5+\sqrt{73}-12x^2}\sqrt{-5+\sqrt{73}+12x^2}}\,dx$$
$$= \sqrt{\frac{2}{-5+\sqrt{73}}}\,F\left(\sin^{-1}\left(2\sqrt{\frac{3}{5+\sqrt{73}}}\,x\right)|\frac{1}{24}\left(-49-5\sqrt{73}\right)\right)$$

Mathematica [C] time = 0.05, size = 56, normalized size = 1.14

$$-i\sqrt{\frac{2}{5+\sqrt{73}}}\,F\left(i\sinh^{-1}\left(2\sqrt{\frac{3}{-5+\sqrt{73}}}\,x\right)|\frac{1}{24}\left(-49+5\sqrt{73}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 6*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[73])]*EllipticF[I*ArcSinh[2*Sqrt[3/(-5 + Sqrt[73])]*x
], (-49 + 5*Sqrt[73])/24]
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-6\,x^4+5\,x^2+2}}{6\,x^4-5\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-6*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-6*x^4 + 5*x^2 + 2)/(6*x^4 - 5*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-6\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-6*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-6*x^4 + 5*x^2 + 2), x)
```

maple [B] time = 0.10, size = 80, normalized size = 1.63

$$\frac{2\sqrt{-\left(-\frac{5}{4}+\frac{\sqrt{73}}{4}\right)x^{2}+1}\,\sqrt{-\left(-\frac{5}{4}-\frac{\sqrt{73}}{4}\right)x^{2}+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{-5+\sqrt{73}}\,x}{2},\frac{5i\sqrt{3}}{12}+\frac{i\sqrt{219}}{12}\right)}{\sqrt{-5+\sqrt{73}}\,\sqrt{-6x^{4}+5x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-6*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+73^(1/2))^(1/2)*(1-(-5/4+1/4*73^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*73^(1/
2))*x^2)^(1/2)/(-6*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+73^(1/2))^(1/2),5
/12*I*3^(1/2)+1/12*I*219^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-6\,x^{4}+5\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-6*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-6*x^4 + 5*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-6\,x^{4}+5\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 6*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 - 6*x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-6x^{4}+5x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-6*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-6*x**4 + 5*x**2 + 2), x)
```

3.131 $\int \frac{1}{\sqrt{2+5x^2-7x^4}} \, dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{7}{2}\right)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(x,1/2*I*14^(1/2))*2^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{7}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 7*x^4],x]
```

```
[Out] EllipticF[ArcSin[x], -7/2]/Sqrt[2]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{2+5x^2-7x^4}} \, dx &= \left(2\sqrt{7}\right) \int \frac{1}{\sqrt{14-14x^2}\sqrt{4+14x^2}} \, dx \\
&= \frac{F\left(\sin^{-1}(x)\middle|-\frac{7}{2}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 65, normalized size = 5.42

$$-\frac{i\sqrt{1-x^2}\,\sqrt{7x^2+2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{7}{2}}\,x\right)\middle|-\frac{2}{7}\right)}{\sqrt{7}\,\sqrt{-7x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 7*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2]*Sqrt[2 + 7*x^2]*EllipticF[I*ArcSinh[Sqrt[7/2]*x], -2/7]
)/(Sqrt[7]*Sqrt[2 + 5*x^2 - 7*x^4])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-7\,x^4+5\,x^2+2}}{7\,x^4-5\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-7*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-7*x^4 + 5*x^2 + 2)/(7*x^4 - 5*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-7\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-7*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-7*x^4 + 5*x^2 + 2), x)
```

maple [B] time = 0.02, size = 43, normalized size = 3.58

$$\frac{\sqrt{-x^2+1}\,\sqrt{14x^2+4}\,\operatorname{EllipticF}\left(x,\frac{i\sqrt{14}}{2}\right)}{2\sqrt{-7x^4+5x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-7*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*(14*x^2+4)^(1/2)/(-7*x^4+5*x^2+2)^(1/2)*EllipticF(x,1/2*
I*14^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-7\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-7*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-7*x^4 + 5*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{-7\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 7*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 - 7*x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-7x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-7*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-7*x**4 + 5*x**2 + 2), x)
```

3.132 $\int \frac{1}{\sqrt{2+5x^2-8x^4}}\,dx$

Optimal. Leaf size=45

$$\sqrt{\frac{2}{\sqrt{89}-5}}\,F\left(\sin^{-1}\left(\frac{4x}{\sqrt{5+\sqrt{89}}}\right)\middle|\frac{1}{32}\left(-57-5\sqrt{89}\right)\right)$$

```
[Out] EllipticF(4*x/(5+89^(1/2))^(1/2),5/8*I+1/8*I*89^(1/2))*2^(1/2)/(-5+89^(1/2)
)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{\sqrt{89}-5}}\,F\left(\sin^{-1}\left(\frac{4x}{\sqrt{5+\sqrt{89}}}\right)\middle|\frac{1}{32}\left(-57-5\sqrt{89}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 8*x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[89])]*EllipticF[ArcSin[(4*x)/Sqrt[5 + Sqrt[89]]], (-57 -
5*Sqrt[89])/32]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2-8x^4}}\,dx = \left(4\sqrt{2}\right)\int \frac{1}{\sqrt{5+\sqrt{89}-16x^2}\sqrt{-5+\sqrt{89}+16x^2}}\,dx$$
$$= \sqrt{\frac{2}{-5+\sqrt{89}}}\,F\left(\sin^{-1}\left(\frac{4x}{\sqrt{5+\sqrt{89}}}\right)\middle|\frac{1}{32}\left(-57-5\sqrt{89}\right)\right)$$

Mathematica [C] time = 0.04, size = 52, normalized size = 1.16

$$-i\sqrt{\frac{2}{5+\sqrt{89}}}\,F\left(i\sinh^{-1}\left(\frac{4x}{\sqrt{-5+\sqrt{89}}}\right)\middle|\frac{1}{32}\left(-57+5\sqrt{89}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 8*x^4],x]
[Out] (-I)*Sqrt[2/(5 + Sqrt[89])]*EllipticF[I*ArcSinh[(4*x)/Sqrt[-5 + Sqrt[89]]],
 (-57 + 5*Sqrt[89])/32]
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-8\,x^4+5\,x^2+2}}{8\,x^4-5\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-8*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-8*x^4 + 5*x^2 + 2)/(8*x^4 - 5*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-8\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-8*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-8*x^4 + 5*x^2 + 2), x)
```

maple [B] time = 0.10, size = 76, normalized size = 1.69

$$\frac{2\sqrt{-\left(-\frac{5}{4}+\frac{\sqrt{89}}{4}\right)x^2+1}\,\sqrt{-\left(-\frac{5}{4}-\frac{\sqrt{89}}{4}\right)x^2+1}\;\mathrm{EllipticF}\left(\frac{\sqrt{-5+\sqrt{89}}\,x}{2},\frac{5i}{8}+\frac{i\sqrt{89}}{8}\right)}{\sqrt{-5+\sqrt{89}}\,\sqrt{-8x^4+5x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-8*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+89^(1/2))^(1/2)*(1-(-5/4+1/4*89^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*89^(1/
2))*x^2)^(1/2)/(-8*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+89^(1/2))^(1/2),5
/8*I+1/8*I*89^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-8\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-8*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-8*x^4 + 5*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-8\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 8*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 - 8*x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-8x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-8*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-8*x**4 + 5*x**2 + 2), x)
```

3.133 $\int \frac{1}{\sqrt{2+5x^2-9x^4}}\,dx$

Optimal. Leaf size=49

$$\sqrt{\frac{2}{\sqrt{97}-5}}\,F\left(\sin^{-1}\left(3\sqrt{\frac{2}{5+\sqrt{97}}}\,x\right)\middle|\frac{1}{36}\left(-61-5\sqrt{97}\right)\right)$$

```
[Out] EllipticF(3*x*2^(1/2)/(5+97^(1/2))^(1/2),5/12*I*2^(1/2)+1/12*I*194^(1/2))*2
^(1/2)/(-5+97^(1/2))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{\sqrt{97}-5}}\,F\left(\sin^{-1}\left(3\sqrt{\frac{2}{5+\sqrt{97}}}\,x\right)\middle|\frac{1}{36}\left(-61-5\sqrt{97}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 9*x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[97])]*EllipticF[ArcSin[3*Sqrt[2/(5 + Sqrt[97])]*x], (-61
- 5*Sqrt[97])/36]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2-9x^4}}\,dx = 6\int \frac{1}{\sqrt{5+\sqrt{97}-18x^2}\,\sqrt{-5+\sqrt{97}+18x^2}}\,dx$$
$$= \sqrt{\frac{2}{-5+\sqrt{97}}}\,F\left(\sin^{-1}\left(3\sqrt{\frac{2}{5+\sqrt{97}}}\,x\right)|\frac{1}{36}\left(-61-5\sqrt{97}\right)\right)$$

Mathematica [C] time = 0.05, size = 56, normalized size = 1.14

$$-i\sqrt{\frac{2}{5+\sqrt{97}}}\,F\left(i\sinh^{-1}\left(3\sqrt{\frac{2}{-5+\sqrt{97}}}\,x\right)|\frac{1}{36}\left(-61+5\sqrt{97}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 9*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[97])]*EllipticF[I*ArcSinh[3*Sqrt[2/(-5 + Sqrt[97])]*x
], (-61 + 5*Sqrt[97])/36]
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-9\,x^4+5\,x^2+2}}{9\,x^4-5\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-9*x^4 + 5*x^2 + 2)/(9*x^4 - 5*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-9\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-9*x^4 + 5*x^2 + 2), x)
```

maple [B] time = 0.10, size = 80, normalized size = 1.63

$$\frac{2\sqrt{-\left(-\frac{5}{4}+\frac{\sqrt{97}}{4}\right)x^{2}+1}\,\sqrt{-\left(-\frac{5}{4}-\frac{\sqrt{97}}{4}\right)x^{2}+1}\ \mathrm{EllipticF}\left(\frac{\sqrt{-5+\sqrt{97}}\,x}{2},\frac{5i\sqrt{2}}{12}+\frac{i\sqrt{194}}{12}\right)}{\sqrt{-5+\sqrt{97}}\,\sqrt{-9x^{4}+5x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-9*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+97^(1/2))^(1/2)*(1-(-5/4+1/4*97^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*97^(1/
2))*x^2)^(1/2)/(-9*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+97^(1/2))^(1/2),5
/12*I*2^(1/2)+1/12*I*194^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-9\,x^{4}+5\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-9*x^4 + 5*x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-9\,x^{4}+5\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 9*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 - 9*x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-9x^{4}+5x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-9*x**4 + 5*x**2 + 2), x)
```

3.134 $\int x^2 \left(bx^2 + cx^4\right) dx$

Optimal. Leaf size=17

$$\frac{bx^5}{5} + \frac{cx^7}{7}$$

```
[Out] 1/5*b*x^5+1/7*c*x^7
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$\frac{bx^5}{5} + \frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b*x^2 + c*x^4),x]
```

```
[Out] (b*x^5)/5 + (c*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(bx^2 + cx^4\right) dx &= \int \left(bx^4 + cx^6\right) dx\\ &= \frac{bx^5}{5} + \frac{cx^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{bx^5}{5} + \frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b*x^2 + c*x^4),x]
```

```
[Out] (b*x^5)/5 + (c*x^7)/7
```

fricas [A] time = 0.58, size = 13, normalized size = 0.76

$$\frac{1}{7}x^7c + \frac{1}{5}x^5b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/7*x^7*c + 1/5*x^5*b
```

giac [A] time = 0.16, size = 13, normalized size = 0.76

$$\frac{1}{7}\,cx^7 + \frac{1}{5}\,bx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2),x, algorithm="giac")
[Out] 1/7*c*x^7 + 1/5*b*x^5
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{7}c\,x^7 + \frac{1}{5}b\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2),x)
[Out] 1/5*b*x^5+1/7*c*x^7
```

maxima [A] time = 1.33, size = 13, normalized size = 0.76

$$\frac{1}{7}\,cx^7 + \frac{1}{5}\,bx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 1/7*c*x^7 + 1/5*b*x^5
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{c\,x^7}{7} + \frac{b\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2 + c*x^4),x)
```

```
[Out] (b*x^5)/5 + (c*x^7)/7
```

sympy [A] time = 0.07, size = 12, normalized size = 0.71

$$\frac{bx^5}{5} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2),x)
```

```
[Out] b*x**5/5 + c*x**7/7
```

3.135 $\int x\left(bx^2+cx^4\right)\,dx$

Optimal. Leaf size=17

$$\frac{bx^4}{4}+\frac{cx^6}{6}$$

```
[Out] 1/4*b*x^4+1/6*c*x^6
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{bx^4}{4}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b*x^2 + c*x^4),x]
```

```
[Out] (b*x^4)/4 + (c*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(bx^2+cx^4\right)\,dx &= \int \left(bx^3+cx^5\right)\,dx\\ &= \frac{bx^4}{4}+\frac{cx^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{bx^4}{4}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b*x^2 + c*x^4),x]
```

```
[Out] (b*x^4)/4 + (c*x^6)/6
```

fricas [A]   time = 0.49, size = 13, normalized size = 0.76

$$\frac{1}{6}x^6c + \frac{1}{4}x^4b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/6*x^6*c + 1/4*x^4*b
```

giac [A]   time = 0.17, size = 13, normalized size = 0.76

$$\frac{1}{6}cx^6 + \frac{1}{4}bx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2),x, algorithm="giac")
[Out] 1/6*c*x^6 + 1/4*b*x^4
```

maple [A]   time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{6}c\,x^6 + \frac{1}{4}b\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2),x)
[Out] 1/4*b*x^4+1/6*c*x^6
```

maxima [A]   time = 1.30, size = 13, normalized size = 0.76

$$\frac{1}{6}cx^6 + \frac{1}{4}bx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 1/6*c*x^6 + 1/4*b*x^4
```

mupad [B]   time = 0.02, size = 13, normalized size = 0.76

$$\frac{c\,x^6}{6} + \frac{b\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2 + c*x^4),x)
```

```
[Out] (b*x^4)/4 + (c*x^6)/6
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{bx^4}{4} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2),x)
```

```
[Out] b*x**4/4 + c*x**6/6
```

3.136 $\int \left(bx^2+cx^4\right) dx$

Optimal. Leaf size=17

$$\frac{bx^3}{3}+\frac{cx^5}{5}$$

```
[Out] 1/3*b*x^3+1/5*c*x^5
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{bx^3}{3}+\frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[b*x^2 + c*x^4,x]
[Out] (b*x^3)/3 + (c*x^5)/5
```

Rubi steps

$$\int \left(bx^2+cx^4\right) dx=\frac{bx^3}{3}+\frac{cx^5}{5}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{bx^3}{3}+\frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[b*x^2 + c*x^4,x]
[Out] (b*x^3)/3 + (c*x^5)/5
```

fricas [A] time = 0.83, size = 13, normalized size = 0.76

$$\frac{1}{5}x^5c+\frac{1}{3}x^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*c + 1/3*x^3*b
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$\frac{1}{5}\,cx^5 + \frac{1}{3}\,bx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2,x, algorithm="giac")
[Out] 1/5*c*x^5 + 1/3*b*x^3
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{5}c\,x^5 + \frac{1}{3}b\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^4+b*x^2,x)
[Out] 1/3*b*x^3+1/5*c*x^5
```

maxima [A] time = 1.36, size = 13, normalized size = 0.76

$$\frac{1}{5}\,cx^5 + \frac{1}{3}\,bx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2,x, algorithm="maxima")
[Out] 1/5*c*x^5 + 1/3*b*x^3
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{c\,x^5}{5} + \frac{b\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^2 + c*x^4,x)
[Out] (b*x^3)/3 + (c*x^5)/5
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{bx^3}{3} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**4+b*x**2,x)
```

```
[Out] b*x**3/3 + c*x**5/5
```

3.137 $\int \frac{bx^2+cx^4}{x} \, dx$

Optimal. Leaf size=17

$$\frac{bx^2}{2} + \frac{cx^4}{4}$$

```
[Out] 1/2*b*x^2+1/4*c*x^4
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$\frac{bx^2}{2} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x,x]
```

```
[Out] (b*x^2)/2 + (c*x^4)/4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{bx^2 + cx^4}{x} \, dx &= \int \left(bx + cx^3\right) dx \\ &= \frac{bx^2}{2} + \frac{cx^4}{4} \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{bx^2}{2} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x,x]
```

```
[Out] (b*x^2)/2 + (c*x^4)/4
```

fricas [A] time = 0.53, size = 13, normalized size = 0.76

$$\frac{1}{4}cx^4 + \frac{1}{2}bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x,x, algorithm="fricas")
[Out] 1/4*c*x^4 + 1/2*b*x^2
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$\frac{1}{4}cx^4 + \frac{1}{2}bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x,x, algorithm="giac")
[Out] 1/4*c*x^4 + 1/2*b*x^2
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{4}c\,x^4 + \frac{1}{2}b\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x,x)
[Out] 1/2*b*x^2+1/4*c*x^4
```

maxima [A] time = 1.31, size = 13, normalized size = 0.76

$$\frac{1}{4}cx^4 + \frac{1}{2}bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x,x, algorithm="maxima")
[Out] 1/4*c*x^4 + 1/2*b*x^2
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{c\,x^4}{4} + \frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x,x)
```

```
[Out] (b*x^2)/2 + (c*x^4)/4
```

sympy [A] time = 0.07, size = 12, normalized size = 0.71

$$\frac{bx^2}{2} + \frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x,x)
```

```
[Out] b*x**2/2 + c*x**4/4
```

3.138 $\int \frac{bx^2+cx^4}{x^2} \, dx$

Optimal. Leaf size=12

$$bx + \frac{cx^3}{3}$$

```
[Out] b*x+1/3*c*x^3
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$bx + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^2,x]
```

```
[Out] b*x + (c*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{bx^2 + cx^4}{x^2} \, dx &= \int \left(b + cx^2\right) \, dx \\ &= bx + \frac{cx^3}{3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$bx + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^2,x]
```

```
[Out] b*x + (c*x^3)/3
```

fricas [A] time = 0.69, size = 10, normalized size = 0.83

$$\frac{1}{3}cx^3 + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*c*x^3 + b*x
```

giac [A] time = 0.15, size = 10, normalized size = 0.83

$$\frac{1}{3}cx^3 + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + b*x
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\frac{1}{3}c\,x^3 + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^2,x)
```

```
[Out] b*x+1/3*c*x^3
```

maxima [A] time = 1.26, size = 10, normalized size = 0.83

$$\frac{1}{3}cx^3 + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + b*x
```

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$\frac{c\,x^3}{3} + b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^2,x)
```

```
[Out] b*x + (c*x^3)/3
```

sympy [**A**] time = 0.07, size = 8, normalized size = 0.67

$$bx + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**2,x)
```

```
[Out] b*x + c*x**3/3
```

3.139 $\int \frac{bx^2+cx^4}{x^3} \, dx$

Optimal. Leaf size=13

$$b\log(x) + \frac{cx^2}{2}$$

[Out] 1/2*c*x^2+b*ln(x)

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$b\log(x) + \frac{cx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(b*x^2 + c*x^4)/x^3,x]

[Out] (c*x^2)/2 + b*Log[x]

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{bx^2 + cx^4}{x^3} \, dx &= \int \left(\frac{b}{x} + cx\right) \, dx \\ &= \frac{cx^2}{2} + b\log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$b\log(x) + \frac{cx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^2 + c*x^4)/x^3,x]

[Out] (c*x^2)/2 + b*Log[x]

fricas [A] time = 0.83, size = 11, normalized size = 0.85

$$\frac{1}{2}\,cx^2 + b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^3,x, algorithm="fricas")
[Out] 1/2*c*x^2 + b*log(x)
```

giac [A] time = 0.15, size = 14, normalized size = 1.08

$$\frac{1}{2}\,cx^2 + \frac{1}{2}\,b\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^3,x, algorithm="giac")
[Out] 1/2*c*x^2 + 1/2*b*log(x^2)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\frac{c\,x^2}{2} + b\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^3,x)
[Out] 1/2*c*x^2+b*ln(x)
```

maxima [A] time = 1.35, size = 14, normalized size = 1.08

$$\frac{1}{2}\,cx^2 + \frac{1}{2}\,b\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^3,x, algorithm="maxima")
[Out] 1/2*c*x^2 + 1/2*b*log(x^2)
```

mupad [B] time = 0.02, size = 11, normalized size = 0.85

$$\frac{c\,x^2}{2} + b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^3,x)
```

```
[Out] (c*x^2)/2 + b*log(x)
```

sympy [A] time = 0.10, size = 10, normalized size = 0.77

$$b\log(x) + \frac{cx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**3,x)
```

```
[Out] b*log(x) + c*x**2/2
```

3.140 $\int \frac{bx^2+cx^4}{x^4} dx$

Optimal. Leaf size=10

$$cx - \frac{b}{x}$$

```
[Out] -b/x+c*x
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$cx - \frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^4,x]
```

```
[Out] -(b/x) + c*x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2 + cx^4}{x^4} dx &= \int \left(c + \frac{b}{x^2}\right) dx \\ &= -\frac{b}{x} + cx\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$cx - \frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^4,x]
```

```
[Out] -(b/x) + c*x
```

fricas [A] time = 0.79, size = 13, normalized size = 1.30

$$\frac{cx^2 - b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] (c*x^2 - b)/x
```

giac [A] time = 0.16, size = 10, normalized size = 1.00

$$cx - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^4,x, algorithm="giac")
```

```
[Out] c*x - b/x
```

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$cx - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^4,x)
```

```
[Out] -b/x+c*x
```

maxima [A] time = 1.33, size = 10, normalized size = 1.00

$$cx - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^4,x, algorithm="maxima")
```

```
[Out] c*x - b/x
```

mupad [B] time = 0.02, size = 10, normalized size = 1.00

$$c\,x - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^4,x)
```

```
[Out] c*x - b/x
```

sympy [**A**] time = 0.10, size = 5, normalized size = 0.50

$$-\frac{b}{x}+cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**4,x)
```

```
[Out] -b/x + c*x
```

3.141 $\int \frac{bx^2+cx^4}{x^5}\,dx$

Optimal. Leaf size=13

$$c\log(x)-\frac{b}{2x^2}$$

```
[Out] -1/2*b/x^2+c*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$c\log(x)-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^5,x]
```

```
[Out] -b/(2*x^2) + c*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^5}\,dx &= \int \left(\frac{b}{x^3}+\frac{c}{x}\right)dx\\ &= -\frac{b}{2x^2}+c\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$c\log(x)-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^5,x]
```

```
[Out] -1/2*b/x^2 + c*Log[x]
```

fricas [A]   time = 0.77, size = 17, normalized size = 1.31

$$\frac{2\,cx^2\log(x) - b}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/2*(2*c*x^2*log(x) - b)/x^2
```

giac [A]   time = 0.15, size = 20, normalized size = 1.54

$$\frac{1}{2}\,c\log\left(x^2\right) - \frac{cx^2 + b}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*c*log(x^2) - 1/2*(c*x^2 + b)/x^2
```

maple [A]   time = 0.01, size = 12, normalized size = 0.92

$$c\ln(x) - \frac{b}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^5,x)
```

```
[Out] -1/2*b/x^2+c*ln(x)
```

maxima [A]   time = 1.35, size = 14, normalized size = 1.08

$$\frac{1}{2}\,c\log\left(x^2\right) - \frac{b}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*c*log(x^2) - 1/2*b/x^2
```

mupad [B]   time = 0.04, size = 11, normalized size = 0.85

$$c\,\ln(x) - \frac{b}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^5,x)
```

```
[Out] c*log(x) - b/(2*x^2)
```

sympy [A] time = 0.12, size = 10, normalized size = 0.77

$$-\frac{b}{2x^2}+c\log{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**5,x)
```

```
[Out] -b/(2*x**2) + c*log(x)
```

3.142 $\int \frac{bx^2+cx^4}{x^6} \, dx$

Optimal. Leaf size=15

$$-\frac{b}{3x^3}-\frac{c}{x}$$

```
[Out] -1/3*b/x^3-c/x
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$-\frac{b}{3x^3}-\frac{c}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^6,x]
```

```
[Out] -b/(3*x^3) - c/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{bx^2+cx^4}{x^6} \, dx &= \int \left(\frac{b}{x^4}+\frac{c}{x^2}\right) dx \\ &= -\frac{b}{3x^3}-\frac{c}{x} \end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{b}{3x^3}-\frac{c}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^6,x]
```

```
[Out] -1/3*b/x^3 - c/x
```

fricas [A] time = 0.79, size = 13, normalized size = 0.87

$$-\frac{3\,cx^2+b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^6,x, algorithm="fricas")
[Out] -1/3*(3*c*x^2 + b)/x^3
```

giac [A] time = 0.18, size = 13, normalized size = 0.87

$$-\frac{3\,cx^2+b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^6,x, algorithm="giac")
[Out] -1/3*(3*c*x^2 + b)/x^3
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$-\frac{c}{x}-\frac{b}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^6,x)
[Out] -1/3*b/x^3-c/x
```

maxima [A] time = 1.30, size = 13, normalized size = 0.87

$$-\frac{3\,cx^2+b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^6,x, algorithm="maxima")
[Out] -1/3*(3*c*x^2 + b)/x^3
```

mupad [B] time = 0.03, size = 13, normalized size = 0.87

$$-\frac{3\,c\,x^2+b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^6,x)
```

```
[Out] -(b + 3*c*x^2)/(3*x^3)
```

sympy [A] time = 0.12, size = 14, normalized size = 0.93

$$\frac{-b-3cx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**6,x)
```

```
[Out] (-b - 3*c*x**2)/(3*x**3)
```

3.143 $\int \frac{bx^2+cx^4}{x^7} dx$

Optimal. Leaf size=17

$$-\frac{b}{4x^4}-\frac{c}{2x^2}$$

```
[Out] -1/4*b/x^4-1/2*c/x^2
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^7,x]
```

```
[Out] -b/(4*x^4) - c/(2*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{bx^2+cx^4}{x^7} dx &= \int \left(\frac{b}{x^5}+\frac{c}{x^3}\right) dx \\ &= -\frac{b}{4x^4}-\frac{c}{2x^2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^7,x]
```

```
[Out] -1/4*b/x^4 - c/(2*x^2)
```

fricas [A] time = 0.55, size = 13, normalized size = 0.76

$$-\frac{2\,cx^2+b}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/4*(2*c*x^2 + b)/x^4
```

giac [A] time = 0.19, size = 13, normalized size = 0.76

$$-\frac{2\,cx^2+b}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^7,x, algorithm="giac")
```

```
[Out] -1/4*(2*c*x^2 + b)/x^4
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{c}{2x^2}-\frac{b}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^7,x)
```

```
[Out] -1/4*b/x^4-1/2*c/x^2
```

maxima [A] time = 1.31, size = 13, normalized size = 0.76

$$-\frac{2\,cx^2+b}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/4*(2*c*x^2 + b)/x^4
```

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$-\frac{2\,c\,x^2+b}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^7,x)
```

```
[Out] -(b + 2*c*x^2)/(4*x^4)
```

sympy [A] time = 0.13, size = 14, normalized size = 0.82

$$\frac{-b-2cx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**7,x)
```

```
[Out] (-b - 2*c*x**2)/(4*x**4)
```

3.144 $\int \frac{bx^2+cx^4}{x^8} \, dx$

Optimal. Leaf size=17

$$-\frac{b}{5x^5}-\frac{c}{3x^3}$$

```
[Out] -1/5*b/x^5-1/3*c/x^3
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$-\frac{b}{5x^5}-\frac{c}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^8,x]
```

```
[Out] -b/(5*x^5) - c/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^8} \, dx &= \int \left(\frac{b}{x^6}+\frac{c}{x^4}\right) dx\\ &= -\frac{b}{5x^5}-\frac{c}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{b}{5x^5}-\frac{c}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^8,x]
```

```
[Out] -1/5*b/x^5 - c/(3*x^3)
```

fricas [A] time = 0.82, size = 15, normalized size = 0.88

$$-\frac{5\,cx^2+3\,b}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/15*(5*c*x^2 + 3*b)/x^5
```

giac [A] time = 0.17, size = 15, normalized size = 0.88

$$-\frac{5\,cx^2+3\,b}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^8,x, algorithm="giac")
```

```
[Out] -1/15*(5*c*x^2 + 3*b)/x^5
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{c}{3x^3}-\frac{b}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^8,x)
```

```
[Out] -1/5*b/x^5-1/3*c/x^3
```

maxima [A] time = 1.25, size = 15, normalized size = 0.88

$$-\frac{5\,cx^2+3\,b}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/15*(5*c*x^2 + 3*b)/x^5
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$-\frac{5\,c\,x^2+3\,b}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^8,x)
```

```
[Out] -(3*b + 5*c*x^2)/(15*x^5)
```

sympy [**A**] time = 0.15, size = 15, normalized size = 0.88

$$\frac{-3b - 5cx^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**8,x)
```

```
[Out] (-3*b - 5*c*x**2)/(15*x**5)
```

3.145 $\int \left(bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{b^2x^5}{5}+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

```
[Out] 1/5*b^2*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 270}

$$\frac{b^2x^5}{5}+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b^2*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(bx^2+cx^4\right)^2 \, dx &= \int x^4 \left(b+cx^2\right)^2 \, dx\\ &= \int \left(b^2x^4+2bcx^6+c^2x^8\right) dx\\ &= \frac{b^2x^5}{5}+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{b^2x^5}{5}+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b^2*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

fricas [A] time = 0.65, size = 24, normalized size = 0.80

$$\frac{1}{9}x^9c^2+\frac{2}{7}x^7cb+\frac{1}{5}x^5b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^2 + 2/7*x^7*c*b + 1/5*x^5*b^2
```

giac [A] time = 0.16, size = 24, normalized size = 0.80

$$\frac{1}{9}c^2x^9+\frac{2}{7}bcx^7+\frac{1}{5}b^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{9}c^2x^9+\frac{2}{7}bc\,x^7+\frac{1}{5}b^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2,x)
```

```
[Out] 1/5*b^2*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

maxima [A] time = 1.38, size = 24, normalized size = 0.80

$$\frac{1}{9}c^2x^9+\frac{2}{7}bcx^7+\frac{1}{5}b^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5
```

mupad [B] time = 0.04, size = 24, normalized size = 0.80

$$\frac{b^2\,x^5}{5}+\frac{2\,b\,c\,x^7}{7}+\frac{c^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2,x)
```

```
[Out] (b^2*x^5)/5 + (c^2*x^9)/9 + (2*b*c*x^7)/7
```

sympy [A] time = 0.07, size = 26, normalized size = 0.87

$$\frac{b^{2}x^{5}}{5}+\frac{2bcx^{7}}{7}+\frac{c^{2}x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2,x)
```

```
[Out] b**2*x**5/5 + 2*b*c*x**7/7 + c**2*x**9/9
```

3.146 $\int \frac{(bx^2+cx^4)^2}{x}\,dx$

Optimal. Leaf size=30

$$\frac{b^2x^4}{4}+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}$$

```
[Out] 1/4*b^2*x^4+1/3*b*c*x^6+1/8*c^2*x^8
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{b^2x^4}{4}+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x,x]
```

```
[Out] (b^2*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^2}{x}\,dx = \int x^3\left(b+cx^2\right)^2\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int x(b+cx)^2\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(b^2x+2bcx^2+c^2x^3\right)\,dx, x, x^2\right)$$
$$= \frac{b^2x^4}{4}+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{b^2x^4}{4}+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x,x]
```

```
[Out] (b^2*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8
```

fricas [A] time = 0.80, size = 24, normalized size = 0.80

$$\frac{1}{8}\,c^2x^8+\frac{1}{3}\,bcx^6+\frac{1}{4}\,b^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4
```

giac [A] time = 0.19, size = 24, normalized size = 0.80

$$\frac{1}{8}\,c^2x^8+\frac{1}{3}\,bcx^6+\frac{1}{4}\,b^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{8}c^2x^8+\frac{1}{3}bc\,x^6+\frac{1}{4}b^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x,x)
```

```
[Out] 1/4*b^2*x^4+1/3*b*c*x^6+1/8*c^2*x^8
```

maxima [A] time = 1.35, size = 24, normalized size = 0.80

$$\frac{1}{8}\,c^2x^8+\frac{1}{3}\,bcx^6+\frac{1}{4}\,b^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4
```

mupad [B] time = 0.04, size = 24, normalized size = 0.80

$$\frac{b^2\,x^4}{4}+\frac{b\,c\,x^6}{3}+\frac{c^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x,x)
```

```
[Out] (b^2*x^4)/4 + (c^2*x^8)/8 + (b*c*x^6)/3
```

sympy [A] time = 0.07, size = 24, normalized size = 0.80

$$\frac{b^2x^4}{4}+\frac{bcx^6}{3}+\frac{c^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x,x)
```

```
[Out] b**2*x**4/4 + b*c*x**6/3 + c**2*x**8/8
```

3.147 $\int \frac{(bx^2+cx^4)^2}{x^2} \, dx$

Optimal. Leaf size=30

$$\frac{b^2x^3}{3} + \frac{2}{5}bcx^5 + \frac{c^2x^7}{7}$$

```
[Out] 1/3*b^2*x^3+2/5*b*c*x^5+1/7*c^2*x^7
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$\frac{b^2x^3}{3} + \frac{2}{5}bcx^5 + \frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^2,x]
```

```
[Out] (b^2*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^2}{x^2} \, dx &= \int x^2 \left(b+cx^2\right)^2 \, dx \\
&= \int \left(b^2x^2 + 2bcx^4 + c^2x^6\right) dx \\
&= \frac{b^2x^3}{3} + \frac{2}{5}bcx^5 + \frac{c^2x^7}{7}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{b^2x^3}{3} + \frac{2}{5}bcx^5 + \frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^2,x]
[Out] (b^2*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

fricas [A] time = 0.87, size = 24, normalized size = 0.80

$$\frac{1}{7}\,c^2x^7 + \frac{2}{5}\,bcx^5 + \frac{1}{3}\,b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^2,x, algorithm="fricas")
[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3
```

giac [A] time = 0.17, size = 24, normalized size = 0.80

$$\frac{1}{7}\,c^2x^7 + \frac{2}{5}\,bcx^5 + \frac{1}{3}\,b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^2,x, algorithm="giac")
[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{7}c^2x^7 + \frac{2}{5}bc\,x^5 + \frac{1}{3}b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^2,x)
[Out] 1/3*b^2*x^3+2/5*b*c*x^5+1/7*c^2*x^7
```

maxima [A] time = 1.32, size = 24, normalized size = 0.80

$$\frac{1}{7}\,c^2x^7 + \frac{2}{5}\,bcx^5 + \frac{1}{3}\,b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3
```

mupad [B] time = 0.04, size = 24, normalized size = 0.80

$$\frac{b^2\,x^3}{3}+\frac{2\,b\,c\,x^5}{5}+\frac{c^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^2,x)
```

```
[Out] (b^2*x^3)/3 + (c^2*x^7)/7 + (2*b*c*x^5)/5
```

sympy [A] time = 0.07, size = 26, normalized size = 0.87

$$\frac{b^{2} x^{3}}{3} + \frac{2 b c x^{5}}{5} + \frac{c^{2} x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**2,x)
```

```
[Out] b**2*x**3/3 + 2*b*c*x**5/5 + c**2*x**7/7
```

3.148 $\int \frac{(bx^2+cx^4)^2}{x^3}\,dx$

Optimal. Leaf size=16

$$\frac{(b+cx^2)^3}{6c}$$

```
[Out] 1/6*(c*x^2+b)^3/c
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 261}

$$\frac{(b+cx^2)^3}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^3,x]
```

```
[Out] (b + c*x^2)^3/(6*c)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^2}{x^3}\,dx &= \int x(b+cx^2)^2\,dx\\ &= \frac{(b+cx^2)^3}{6c}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{\left(b+cx^2\right)^3}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^3,x]
```

```
[Out] (b + c*x^2)^3/(6*c)
```

fricas [A] time = 0.82, size = 24, normalized size = 1.50

$$\frac{1}{6}c^2x^6+\frac{1}{2}bcx^4+\frac{1}{2}b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2
```

giac [A] time = 0.15, size = 24, normalized size = 1.50

$$\frac{1}{6}c^2x^6+\frac{1}{2}bcx^4+\frac{1}{2}b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2
```

maple [A] time = 0.00, size = 25, normalized size = 1.56

$$\frac{1}{6}c^2x^6+\frac{1}{2}bc\,x^4+\frac{1}{2}b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^3,x)
```

```
[Out] 1/6*c^2*x^6+1/2*b*c*x^4+1/2*b^2*x^2
```

maxima [A] time = 1.31, size = 24, normalized size = 1.50

$$\frac{1}{6}c^2x^6+\frac{1}{2}bcx^4+\frac{1}{2}b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2
```

mupad [B] time = 0.03, size = 24, normalized size = 1.50

$$\frac{b^2\,x^2}{2}+\frac{b\,c\,x^4}{2}+\frac{c^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^3,x)
```

```
[Out] (b^2*x^2)/2 + (c^2*x^6)/6 + (b*c*x^4)/2
```

sympy [B] time = 0.10, size = 24, normalized size = 1.50

$$\frac{b^{2}x^{2}}{2}+\frac{bcx^{4}}{2}+\frac{c^{2}x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**3,x)
```

```
[Out] b**2*x**2/2 + b*c*x**4/2 + c**2*x**6/6
```

3.149 $\int \frac{(bx^2+cx^4)^2}{x^4} dx$

Optimal. Leaf size=25

$$b^2x + \frac{2}{3}bcx^3 + \frac{c^2x^5}{5}$$

```
[Out] b^2*x+2/3*b*c*x^3+1/5*c^2*x^5
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 194}

$$b^2x + \frac{2}{3}bcx^3 + \frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^4,x]
```

```
[Out] b^2*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{(bx^2+cx^4)^2}{x^4} dx &= \int (b+cx^2)^2 dx \\ &= \int (b^2+2bcx^2+c^2x^4) dx \\ &= b^2x + \frac{2}{3}bcx^3 + \frac{c^2x^5}{5} \end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$b^2x + \frac{2}{3}bcx^3 + \frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^4,x]
```

```
[Out] b^2*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

fricas [A] time = 0.78, size = 21, normalized size = 0.84

$$\frac{1}{5}\,c^2x^5 + \frac{2}{3}\,bcx^3 + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x
```

giac [A] time = 0.18, size = 21, normalized size = 0.84

$$\frac{1}{5}\,c^2x^5 + \frac{2}{3}\,bcx^3 + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x
```

maple [A] time = 0.00, size = 22, normalized size = 0.88

$$\frac{1}{5}c^2x^5 + \frac{2}{3}bc\,x^3 + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^4,x)
```

```
[Out] b^2*x+2/3*b*c*x^3+1/5*c^2*x^5
```

maxima [A] time = 1.35, size = 21, normalized size = 0.84

$$\frac{1}{5}\,c^2x^5 + \frac{2}{3}\,bcx^3 + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x
```

mupad [B] time = 0.03, size = 21, normalized size = 0.84

$$b^2\,x+\frac{2\,b\,c\,x^3}{3}+\frac{c^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^4,x)
```

```
[Out] b^2*x + (c^2*x^5)/5 + (2*b*c*x^3)/3
```

sympy [A] time = 0.08, size = 22, normalized size = 0.88

$$b^{2}x+\frac{2bcx^{3}}{3}+\frac{c^{2}x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**4,x)
```

```
[Out] b**2*x + 2*b*c*x**3/3 + c**2*x**5/5
```

3.150 $\int \frac{(bx^2+cx^4)^2}{x^5} dx$

Optimal. Leaf size=23

$$b^2\log(x)+bcx^2+\frac{c^2x^4}{4}$$

```
[Out] b*c*x^2+1/4*c^2*x^4+b^2*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$b^2\log(x)+bcx^2+\frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^5,x]
```

```
[Out] b*c*x^2 + (c^2*x^4)/4 + b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^2}{x^5}\,dx &= \int \frac{(b+cx^2)^2}{x}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(b+cx)^2}{x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(2bc+\frac{b^2}{x}+c^2x\right)dx, x, x^2\right)\\ &= bcx^2+\frac{c^2x^4}{4}+b^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$b^2\log(x)+bcx^2+\frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^5,x]
[Out] b*c*x^2 + (c^2*x^4)/4 + b^2*Log[x]
```

fricas [A] time = 0.80, size = 21, normalized size = 0.91

$$\frac{1}{4}\,c^2x^4+bcx^2+b^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^5,x, algorithm="fricas")
[Out] 1/4*c^2*x^4 + b*c*x^2 + b^2*log(x)
```

giac [A] time = 0.15, size = 24, normalized size = 1.04

$$\frac{1}{4}\,c^2x^4+bcx^2+\frac{1}{2}\,b^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^5,x, algorithm="giac")
[Out] 1/4*c^2*x^4 + b*c*x^2 + 1/2*b^2*log(x^2)
```

maple [A] time = 0.00, size = 22, normalized size = 0.96

$$\frac{c^2x^4}{4} + bc\,x^2 + b^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^5,x)
```

```
[Out] b*c*x^2+1/4*c^2*x^4+b^2*ln(x)
```

maxima [A] time = 1.35, size = 24, normalized size = 1.04

$$\frac{1}{4}\,c^2x^4 + bcx^2 + \frac{1}{2}\,b^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^2 + 1/2*b^2*log(x^2)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.91

$$b^2\,\ln(x) + \frac{c^2\,x^4}{4} + b\,c\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^5,x)
```

```
[Out] b^2*log(x) + (c^2*x^4)/4 + b*c*x^2
```

sympy [A] time = 0.11, size = 20, normalized size = 0.87

$$b^2\log\left(x\right) + bcx^2 + \frac{c^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**5,x)
```

```
[Out] b**2*log(x) + b*c*x**2 + c**2*x**4/4
```

3.151 $\int \frac{(bx^2+cx^4)^2}{x^6}\,dx$

Optimal. Leaf size=24

$$-\frac{b^2}{x}+2bcx+\frac{c^2x^3}{3}$$

```
[Out] -b^2/x+2*b*c*x+1/3*c^2*x^3
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{b^2}{x}+2bcx+\frac{c^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^6,x]
```

```
[Out] -(b^2/x) + 2*b*c*x + (c^2*x^3)/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^2}{x^6}\,dx &= \int \frac{(b+cx^2)^2}{x^2}\,dx\\ &= \int \left(2bc+\frac{b^2}{x^2}+c^2x^2\right)dx\\ &= -\frac{b^2}{x}+2bcx+\frac{c^2x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$-\frac{b^2}{x} + 2bcx + \frac{c^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^6,x]
[Out] -(b^2/x) + 2*b*c*x + (c^2*x^3)/3
```

fricas [A] time = 0.78, size = 25, normalized size = 1.04

$$\frac{c^2x^4 + 6\,bcx^2 - 3\,b^2}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^6,x, algorithm="fricas")
[Out] 1/3*(c^2*x^4 + 6*b*c*x^2 - 3*b^2)/x
```

giac [A] time = 0.15, size = 22, normalized size = 0.92

$$\frac{1}{3}\,c^2x^3 + 2\,bcx - \frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^6,x, algorithm="giac")
[Out] 1/3*c^2*x^3 + 2*b*c*x - b^2/x
```

maple [A] time = 0.00, size = 23, normalized size = 0.96

$$\frac{c^2x^3}{3} + 2bcx - \frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^6,x)
[Out] -b^2/x+2*b*c*x+1/3*c^2*x^3
```

maxima [A] time = 1.31, size = 22, normalized size = 0.92

$$\frac{1}{3}\,c^2x^3 + 2\,bcx - \frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^6,x, algorithm="maxima")
```

```
[Out] 1/3*c^2*x^3 + 2*b*c*x - b^2/x
```

mupad [B] time = 0.04, size = 22, normalized size = 0.92

$$\frac{c^2\,x^3}{3}-\frac{b^2}{x}+2\,b\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^6,x)
```

```
[Out] (c^2*x^3)/3 - b^2/x + 2*b*c*x
```

sympy [A] time = 0.11, size = 19, normalized size = 0.79

$$-\frac{b^2}{x}+2bcx+\frac{c^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**6,x)
```

```
[Out] -b**2/x + 2*b*c*x + c**2*x**3/3
```

3.152 $\int \frac{(bx^2+cx^4)^2}{x^7}\,dx$

Optimal. Leaf size=27

$$-\frac{b^2}{2x^2}+2bc\log(x)+\frac{c^2x^2}{2}$$

```
[Out] -1/2*b^2/x^2+1/2*c^2*x^2+2*b*c*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$-\frac{b^2}{2x^2}+2bc\log(x)+\frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^7,x]
```

```
[Out] -b^2/(2*x^2) + (c^2*x^2)/2 + 2*b*c*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^2}{x^7}\,dx = \int \frac{\left(b+cx^2\right)^2}{x^3}\,dx$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(b+cx)^2}{x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(c^2+\frac{b^2}{x^2}+\frac{2bc}{x}\right)dx, x, x^2\right)$$
$$= -\frac{b^2}{2x^2}+\frac{c^2x^2}{2}+2bc\log(x)$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$-\frac{b^2}{2x^2}+2bc\log(x)+\frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^7,x]
```

```
[Out] -1/2*b^2/x^2 + (c^2*x^2)/2 + 2*b*c*Log[x]
```

fricas [A] time = 0.76, size = 27, normalized size = 1.00

$$\frac{c^2x^4+4\,bcx^2\log(x)-b^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/2*(c^2*x^4 + 4*b*c*x^2*log(x) - b^2)/x^2
```

giac [A] time = 0.17, size = 32, normalized size = 1.19

$$\frac{1}{2}\,c^2x^2+bc\log\left(x^2\right)-\frac{2\,bcx^2+b^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/2*c^2*x^2 + b*c*log(x^2) - 1/2*(2*b*c*x^2 + b^2)/x^2
```

maple [A] time = 0.00, size = 24, normalized size = 0.89

$$\frac{c^2x^2}{2} + 2bc\ln(x) - \frac{b^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^7,x)
[Out] -1/2*b^2/x^2+1/2*c^2*x^2+2*b*c*ln(x)
```

maxima [A] time = 1.33, size = 24, normalized size = 0.89

$$\frac{1}{2}c^2x^2 + bc\log\left(x^2\right) - \frac{b^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^7,x, algorithm="maxima")
[Out] 1/2*c^2*x^2 + b*c*log(x^2) - 1/2*b^2/x^2
```

mupad [B] time = 0.03, size = 23, normalized size = 0.85

$$\frac{c^2\,x^2}{2} - \frac{b^2}{2\,x^2} + 2\,b\,c\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^7,x)
[Out] (c^2*x^2)/2 - b^2/(2*x^2) + 2*b*c*log(x)
```

sympy [A] time = 0.15, size = 24, normalized size = 0.89

$$-\frac{b^2}{2x^2} + 2bc\log\left(x\right) + \frac{c^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**7,x)
[Out] -b**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2/2
```

3.153 $\int \frac{(bx^2+cx^4)^2}{x^8} dx$

Optimal. Leaf size=23

$$-\frac{b^2}{3x^3}-\frac{2bc}{x}+c^2x$$

```
[Out] -1/3*b^2/x^3-2*b*c/x+c^2*x
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{b^2}{3x^3}-\frac{2bc}{x}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^8,x]
```

```
[Out] -b^2/(3*x^3) - (2*b*c)/x + c^2*x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^2}{x^8} dx &= \int \frac{(b+cx^2)^2}{x^4} dx \\
&= \int \left(c^2+\frac{b^2}{x^4}+\frac{2bc}{x^2}\right) dx \\
&= -\frac{b^2}{3x^3}-\frac{2bc}{x}+c^2x
\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{b^2}{3x^3} - \frac{2bc}{x} + c^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^8,x]
[Out] -1/3*b^2/x^3 - (2*b*c)/x + c^2*x
```

fricas [A] time = 0.73, size = 26, normalized size = 1.13

$$\frac{3\,c^2x^4 - 6\,bcx^2 - b^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^8,x, algorithm="fricas")
[Out] 1/3*(3*c^2*x^4 - 6*b*c*x^2 - b^2)/x^3
```

giac [A] time = 0.16, size = 22, normalized size = 0.96

$$c^2x - \frac{6\,bcx^2 + b^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^8,x, algorithm="giac")
[Out] c^2*x - 1/3*(6*b*c*x^2 + b^2)/x^3
```

maple [A] time = 0.01, size = 22, normalized size = 0.96

$$c^2x - \frac{2bc}{x} - \frac{b^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^8,x)
[Out] -1/3*b^2/x^3-2*b*c/x+c^2*x
```

maxima [A] time = 1.29, size = 22, normalized size = 0.96

$$c^2x - \frac{6\,bcx^2 + b^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^8,x, algorithm="maxima")
```

```
[Out] c^2*x - 1/3*(6*b*c*x^2 + b^2)/x^3
```

mupad [B] time = 0.03, size = 24, normalized size = 1.04

$$c^2\,x-\frac{\frac{b^2}{3}+2\,c\,b\,x^2}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^8,x)
```

```
[Out] c^2*x - (b^2/3 + 2*b*c*x^2)/x^3
```

sympy [A] time = 0.17, size = 22, normalized size = 0.96

$$c^2x+\frac{-b^2-6bcx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**8,x)
```

```
[Out] c**2*x + (-b**2 - 6*b*c*x**2)/(3*x**3)
```

3.154 $\int \frac{(bx^2+cx^4)^2}{x^9} dx$

Optimal. Leaf size=24

$$-\frac{b^2}{4x^4}-\frac{bc}{x^2}+c^2\log(x)$$

```
[Out] -1/4*b^2/x^4-b*c/x^2+c^2*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$-\frac{b^2}{4x^4}-\frac{bc}{x^2}+c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^9,x]
```

```
[Out] -b^2/(4*x^4) - (b*c)/x^2 + c^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^2}{x^9}\,dx = \int \frac{(b+cx^2)^2}{x^5}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{(b+cx)^2}{x^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{b^2}{x^3}+\frac{2bc}{x^2}+\frac{c^2}{x}\right)dx, x, x^2\right)$$
$$= -\frac{b^2}{4x^4}-\frac{bc}{x^2}+c^2\log(x)$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$-\frac{b^2}{4x^4}-\frac{bc}{x^2}+c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^9,x]
[Out] -1/4*b^2/x^4 - (b*c)/x^2 + c^2*Log[x]
```

fricas [A] time = 0.85, size = 28, normalized size = 1.17

$$\frac{4\,c^2x^4\log(x)-4\,bcx^2-b^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^9,x, algorithm="fricas")
[Out] 1/4*(4*c^2*x^4*log(x) - 4*b*c*x^2 - b^2)/x^4
```

giac [A] time = 0.18, size = 34, normalized size = 1.42

$$\frac{1}{2}\,c^2\log\left(x^2\right)-\frac{3\,c^2x^4+4\,bcx^2+b^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^9,x, algorithm="giac")
[Out] 1/2*c^2*log(x^2) - 1/4*(3*c^2*x^4 + 4*b*c*x^2 + b^2)/x^4
```

maple [A] time = 0.00, size = 23, normalized size = 0.96

$$c^2\ln(x)-\frac{bc}{x^2}-\frac{b^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^9,x)
```

```
[Out] -1/4*b^2/x^4-b*c/x^2+c^2*ln(x)
```

maxima [A] time = 1.32, size = 26, normalized size = 1.08

$$\frac{1}{2}\,c^2\log\left(x^2\right)-\frac{4\,bcx^2+b^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log(x^2) - 1/4*(4*b*c*x^2 + b^2)/x^4
```

mupad [B] time = 0.05, size = 24, normalized size = 1.00

$$c^2\,\ln(x)-\frac{\frac{b^2}{4}+c\,b\,x^2}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^9,x)
```

```
[Out] c^2*log(x) - (b^2/4 + b*c*x^2)/x^4
```

sympy [A] time = 0.19, size = 24, normalized size = 1.00

$$c^2\log\left(x\right)+\frac{-b^2-4bcx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**9,x)
```

```
[Out] c**2*log(x) + (-b**2 - 4*b*c*x**2)/(4*x**4)
```

3.155 $\int \frac{(bx^2+cx^4)^2}{x^{10}}\,dx$

Optimal. Leaf size=28

$$-\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

```
[Out] -1/5*b^2/x^5-2/3*b*c/x^3-c^2/x
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^10,x]
```

```
[Out] -b^2/(5*x^5) - (2*b*c)/(3*x^3) - c^2/x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2+cx^4\right)^2}{x^{10}}\,dx &= \int \frac{\left(b+cx^2\right)^2}{x^6}\,dx\\ &= \int \left(\frac{b^2}{x^6}+\frac{2bc}{x^4}+\frac{c^2}{x^2}\right)dx\\ &= -\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$-\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^10,x]
[Out] -1/5*b^2/x^5 - (2*b*c)/(3*x^3) - c^2/x
```

fricas [A] time = 0.74, size = 26, normalized size = 0.93

$$-\frac{15\,c^2x^4+10\,bcx^2+3\,b^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^10,x, algorithm="fricas")
[Out] -1/15*(15*c^2*x^4 + 10*b*c*x^2 + 3*b^2)/x^5
```

giac [A] time = 0.17, size = 26, normalized size = 0.93

$$-\frac{15\,c^2x^4+10\,bcx^2+3\,b^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^10,x, algorithm="giac")
[Out] -1/15*(15*c^2*x^4 + 10*b*c*x^2 + 3*b^2)/x^5
```

maple [A] time = 0.00, size = 25, normalized size = 0.89

$$-\frac{c^2}{x}-\frac{2bc}{3x^3}-\frac{b^2}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^10,x)
[Out] -1/5*b^2/x^5-2/3*b*c/x^3-c^2/x
```

maxima [A] time = 1.30, size = 26, normalized size = 0.93

$$-\frac{15\,c^2x^4+10\,bcx^2+3\,b^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^10,x, algorithm="maxima")
```

```
[Out] -1/15*(15*c^2*x^4 + 10*b*c*x^2 + 3*b^2)/x^5
```

mupad [B] time = 0.04, size = 25, normalized size = 0.89

$$-\frac{\frac{b^2}{5}+\frac{2\,b\,c\,x^2}{3}+c^2\,x^4}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^10,x)
```

```
[Out] -(b^2/5 + c^2*x^4 + (2*b*c*x^2)/3)/x^5
```

sympy [A] time = 0.20, size = 27, normalized size = 0.96

$$\frac{-3b^2-10bcx^2-15c^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**10,x)
```

```
[Out] (-3*b**2 - 10*b*c*x**2 - 15*c**2*x**4)/(15*x**5)
```

3.156 $\int \frac{(bx^2+cx^4)^2}{x^{11}} \, dx$

Optimal. Leaf size=19

$$-\frac{(b+cx^2)^3}{6bx^6}$$

```
[Out] -1/6*(c*x^2+b)^3/b/x^6
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 264}

$$-\frac{(b+cx^2)^3}{6bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^11,x]
```

```
[Out] -(b + c*x^2)^3/(6*b*x^6)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{(bx^2+cx^4)^2}{x^{11}} \, dx &= \int \frac{(b+cx^2)^2}{x^7} \, dx \\ &= -\frac{(b+cx^2)^3}{6bx^6} \end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.58

$$-\frac{b^2}{6x^6}-\frac{bc}{2x^4}-\frac{c^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^11,x]
[Out] -1/6*b^2/x^6 - (b*c)/(2*x^4) - c^2/(2*x^2)
```

fricas [A] time = 0.69, size = 24, normalized size = 1.26

$$-\frac{3\,c^2x^4+3\,bcx^2+b^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^11,x, algorithm="fricas")
[Out] -1/6*(3*c^2*x^4 + 3*b*c*x^2 + b^2)/x^6
```

giac [A] time = 0.17, size = 24, normalized size = 1.26

$$-\frac{3\,c^2x^4+3\,bcx^2+b^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^11,x, algorithm="giac")
[Out] -1/6*(3*c^2*x^4 + 3*b*c*x^2 + b^2)/x^6
```

maple [A] time = 0.01, size = 25, normalized size = 1.32

$$-\frac{c^2}{2x^2}-\frac{bc}{2x^4}-\frac{b^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^11,x)
[Out] -1/2*b*c/x^4-1/2*c^2/x^2-1/6*b^2/x^6
```

maxima [A] time = 1.23, size = 24, normalized size = 1.26

$$-\frac{3\,c^2x^4+3\,bcx^2+b^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^11,x, algorithm="maxima")
```

```
[Out] -1/6*(3*c^2*x^4 + 3*b*c*x^2 + b^2)/x^6
```

mupad [B] time = 0.04, size = 26, normalized size = 1.37

$$-\frac{\frac{b^2}{6}+\frac{b\,c\,x^2}{2}+\frac{c^2\,x^4}{2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^11,x)
```

```
[Out] -(b^2/6 + (c^2*x^4)/2 + (b*c*x^2)/2)/x^6
```

sympy [A] time = 0.21, size = 26, normalized size = 1.37

$$\frac{-b^2-3bcx^2-3c^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**11,x)
```

```
[Out] (-b**2 - 3*b*c*x**2 - 3*c**2*x**4)/(6*x**6)
```

3.157 $\int \frac{(bx^2+cx^4)^2}{x^{12}} dx$

Optimal. Leaf size=30

$$-\frac{b^2}{7x^7} - \frac{2bc}{5x^5} - \frac{c^2}{3x^3}$$

```
[Out] -1/7*b^2/x^7-2/5*b*c/x^5-1/3*c^2/x^3
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{b^2}{7x^7} - \frac{2bc}{5x^5} - \frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^12,x]
```

```
[Out] -b^2/(7*x^7) - (2*b*c)/(5*x^5) - c^2/(3*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^2}{x^{12}} dx &= \int \frac{(b+cx^2)^2}{x^8} dx \\
&= \int \left(\frac{b^2}{x^8} + \frac{2bc}{x^6} + \frac{c^2}{x^4}\right) dx \\
&= -\frac{b^2}{7x^7} - \frac{2bc}{5x^5} - \frac{c^2}{3x^3}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{b^2}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^12,x]
[Out] -1/7*b^2/x^7 - (2*b*c)/(5*x^5) - c^2/(3*x^3)
```

fricas [A] time = 0.84, size = 26, normalized size = 0.87

$$-\frac{35\,c^2x^4+42\,bcx^2+15\,b^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^12,x, algorithm="fricas")
[Out] -1/105*(35*c^2*x^4 + 42*b*c*x^2 + 15*b^2)/x^7
```

giac [A] time = 0.15, size = 26, normalized size = 0.87

$$-\frac{35\,c^2x^4+42\,bcx^2+15\,b^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^12,x, algorithm="giac")
[Out] -1/105*(35*c^2*x^4 + 42*b*c*x^2 + 15*b^2)/x^7
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-\frac{c^2}{3x^3}-\frac{2bc}{5x^5}-\frac{b^2}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^12,x)
[Out] -1/7*b^2/x^7-2/5*b*c/x^5-1/3*c^2/x^3
```

maxima [A] time = 1.37, size = 26, normalized size = 0.87

$$-\frac{35\,c^2x^4+42\,bcx^2+15\,b^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^12,x, algorithm="maxima")
```

```
[Out] -1/105*(35*c^2*x^4 + 42*b*c*x^2 + 15*b^2)/x^7
```

mupad [B] time = 0.04, size = 26, normalized size = 0.87

$$-\frac{\frac{b^2}{7}+\frac{2\,b\,c\,x^2}{5}+\frac{c^2\,x^4}{3}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^12,x)
```

```
[Out] -(b^2/7 + (c^2*x^4)/3 + (2*b*c*x^2)/5)/x^7
```

sympy [A] time = 0.23, size = 27, normalized size = 0.90

$$\frac{-15b^2-42bcx^2-35c^2x^4}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**12,x)
```

```
[Out] (-15*b**2 - 42*b*c*x**2 - 35*c**2*x**4)/(105*x**7)
```

3.158 $\int \frac{(bx^2+cx^4)^3}{x^2}\,dx$

Optimal. Leaf size=43

$$\frac{b^3x^5}{5}+\frac{3}{7}b^2cx^7+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

```
[Out] 1/5*b^3*x^5+3/7*b^2*c*x^7+1/3*b*c^2*x^9+1/11*c^3*x^11
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$\frac{3}{7}b^2cx^7+\frac{b^3x^5}{5}+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^2,x]
```

```
[Out] (b^3*x^5)/5 + (3*b^2*c*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2+cx^4\right)^3}{x^2}\,dx &= \int x^4\left(b+cx^2\right)^3\,dx\\ &= \int \left(b^3x^4+3b^2cx^6+3bc^2x^8+c^3x^{10}\right)dx\\ &= \frac{b^3x^5}{5}+\frac{3}{7}b^2cx^7+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{b^3x^5}{5}+\frac{3}{7}b^2cx^7+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^2,x]
[Out] (b^3*x^5)/5 + (3*b^2*c*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

fricas [A] time = 0.65, size = 35, normalized size = 0.81

$$\frac{1}{11}\,c^3x^{11}+\frac{1}{3}\,bc^2x^9+\frac{3}{7}\,b^2cx^7+\frac{1}{5}\,b^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^2,x, algorithm="fricas")
[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 1/5*b^3*x^5
```

giac [A] time = 0.15, size = 35, normalized size = 0.81

$$\frac{1}{11}\,c^3x^{11}+\frac{1}{3}\,bc^2x^9+\frac{3}{7}\,b^2cx^7+\frac{1}{5}\,b^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^2,x, algorithm="giac")
[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 1/5*b^3*x^5
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{11}c^3x^{11}+\frac{1}{3}b\,c^2x^9+\frac{3}{7}b^2c\,x^7+\frac{1}{5}b^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^2,x)
[Out] 1/5*b^3*x^5+3/7*b^2*c*x^7+1/3*b*c^2*x^9+1/11*c^3*x^11
```

maxima [A] time = 1.30, size = 35, normalized size = 0.81

$$\frac{1}{11}\,c^3x^{11}+\frac{1}{3}\,bc^2x^9+\frac{3}{7}\,b^2cx^7+\frac{1}{5}\,b^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 1/5*b^3*x^5
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{b^3\,x^5}{5}+\frac{3\,b^2\,c\,x^7}{7}+\frac{b\,c^2\,x^9}{3}+\frac{c^3\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^2,x)
```

```
[Out] (b^3*x^5)/5 + (c^3*x^11)/11 + (3*b^2*c*x^7)/7 + (b*c^2*x^9)/3
```

sympy [A] time = 0.08, size = 37, normalized size = 0.86

$$\frac{b^3x^5}{5}+\frac{3b^2cx^7}{7}+\frac{bc^2x^9}{3}+\frac{c^3x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**2,x)
```

```
[Out] b**3*x**5/5 + 3*b**2*c*x**7/7 + b*c**2*x**9/3 + c**3*x**11/11
```

3.159 $\int \frac{(bx^2+cx^4)^3}{x^3} dx$

Optimal. Leaf size=34

$$\frac{(b+cx^2)^5}{10c^2} - \frac{b(b+cx^2)^4}{8c^2}$$

[Out] -1/8*b*(c*x^2+b)^4/c^2+1/10*(c*x^2+b)^5/c^2

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{(b+cx^2)^5}{10c^2} - \frac{b(b+cx^2)^4}{8c^2}$$

Antiderivative was successfully verified.

[In] Int[(b*x^2 + c*x^4)^3/x^3,x]

[Out] -(b*(b + c*x^2)^4)/(8*c^2) + (b + c*x^2)^5/(10*c^2)

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^3}{x^3}\,dx &= \int x^3\left(b+cx^2\right)^3\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int x(b+cx)^3\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{b(b+cx)^3}{c}+\frac{(b+cx)^4}{c}\right)dx, x, x^2\right) \\
&= -\frac{b\left(b+cx^2\right)^4}{8c^2}+\frac{\left(b+cx^2\right)^5}{10c^2}
\end{aligned}
$$

Mathematica [**A**] time = 0.00, size = 43, normalized size = 1.26

$$\frac{b^3x^4}{4}+\frac{1}{2}b^2cx^6+\frac{3}{8}bc^2x^8+\frac{c^3x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^3,x]
[Out] (b^3*x^4)/4 + (b^2*c*x^6)/2 + (3*b*c^2*x^8)/8 + (c^3*x^10)/10
```

fricas [**A**] time = 0.79, size = 35, normalized size = 1.03

$$\frac{1}{10}\,c^3x^{10}+\frac{3}{8}\,bc^2x^8+\frac{1}{2}\,b^2cx^6+\frac{1}{4}\,b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^3,x, algorithm="fricas")
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/4*b^3*x^4
```

giac [**A**] time = 0.18, size = 35, normalized size = 1.03

$$\frac{1}{10}\,c^3x^{10}+\frac{3}{8}\,bc^2x^8+\frac{1}{2}\,b^2cx^6+\frac{1}{4}\,b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^3,x, algorithm="giac")
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/4*b^3*x^4
```

maple [A] time = 0.00, size = 36, normalized size = 1.06

$$\frac{1}{10}c^3x^{10}+\frac{3}{8}b\,c^2x^8+\frac{1}{2}b^2c\,x^6+\frac{1}{4}b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^3,x)
```

```
[Out] 1/10*c^3*x^10+3/8*b*c^2*x^8+1/2*b^2*c*x^6+1/4*b^3*x^4
```

maxima [A] time = 1.32, size = 35, normalized size = 1.03

$$\frac{1}{10}\,c^3x^{10}+\frac{3}{8}\,bc^2x^8+\frac{1}{2}\,b^2cx^6+\frac{1}{4}\,b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/4*b^3*x^4
```

mupad [B] time = 0.04, size = 35, normalized size = 1.03

$$\frac{b^3\,x^4}{4}+\frac{b^2\,c\,x^6}{2}+\frac{3\,b\,c^2\,x^8}{8}+\frac{c^3\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^3,x)
```

```
[Out] (b^3*x^4)/4 + (c^3*x^10)/10 + (b^2*c*x^6)/2 + (3*b*c^2*x^8)/8
```

sympy [A] time = 0.08, size = 37, normalized size = 1.09

$$\frac{b^3x^4}{4}+\frac{b^2cx^6}{2}+\frac{3bc^2x^8}{8}+\frac{c^3x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**3,x)
```

```
[Out] b**3*x**4/4 + b**2*c*x**6/2 + 3*b*c**2*x**8/8 + c**3*x**10/10
```

3.160 $\int \frac{(bx^2+cx^4)^3}{x^4} \, dx$

Optimal. Leaf size=43

$$\frac{b^3x^3}{3}+\frac{3}{5}b^2cx^5+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

```
[Out] 1/3*b^3*x^3+3/5*b^2*c*x^5+3/7*b*c^2*x^7+1/9*c^3*x^9
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$\frac{3}{5}b^2cx^5+\frac{b^3x^3}{3}+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^4,x]
```

```
[Out] (b^3*x^3)/3 + (3*b^2*c*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^3}{x^4} \, dx &= \int x^2 \left(b+cx^2\right)^3 \, dx \\
&= \int \left(b^3x^2+3b^2cx^4+3bc^2x^6+c^3x^8\right) dx \\
&= \frac{b^3x^3}{3}+\frac{3}{5}b^2cx^5+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{b^3x^3}{3}+\frac{3}{5}b^2cx^5+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^4,x]
```

```
[Out] (b^3*x^3)/3 + (3*b^2*c*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

fricas [A] time = 0.84, size = 35, normalized size = 0.81

$$\frac{1}{9}c^3x^9+\frac{3}{7}bc^2x^7+\frac{3}{5}b^2cx^5+\frac{1}{3}b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 1/3*b^3*x^3
```

giac [A] time = 0.17, size = 35, normalized size = 0.81

$$\frac{1}{9}c^3x^9+\frac{3}{7}bc^2x^7+\frac{3}{5}b^2cx^5+\frac{1}{3}b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 1/3*b^3*x^3
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{9}c^3x^9+\frac{3}{7}b\,c^2x^7+\frac{3}{5}b^2c\,x^5+\frac{1}{3}b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^4,x)
```

```
[Out] 1/3*b^3*x^3+3/5*b^2*c*x^5+3/7*b*c^2*x^7+1/9*c^3*x^9
```

maxima [A] time = 1.34, size = 35, normalized size = 0.81

$$\frac{1}{9}c^3x^9+\frac{3}{7}bc^2x^7+\frac{3}{5}b^2cx^5+\frac{1}{3}b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 1/3*b^3*x^3
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{b^3\,x^3}{3}+\frac{3\,b^2\,c\,x^5}{5}+\frac{3\,b\,c^2\,x^7}{7}+\frac{c^3\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^4,x)
```

```
[Out] (b^3*x^3)/3 + (c^3*x^9)/9 + (3*b^2*c*x^5)/5 + (3*b*c^2*x^7)/7
```

sympy [A] time = 0.09, size = 39, normalized size = 0.91

$$\frac{b^{3}x^{3}}{3}+\frac{3b^{2}cx^{5}}{5}+\frac{3bc^{2}x^{7}}{7}+\frac{c^{3}x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**4,x)
```

```
[Out] b**3*x**3/3 + 3*b**2*c*x**5/5 + 3*b*c**2*x**7/7 + c**3*x**9/9
```

3.161 $\int \frac{(bx^2+cx^4)^3}{x^5}\,dx$

Optimal. Leaf size=16

$$\frac{(b+cx^2)^4}{8c}$$

[Out] 1/8*(c*x^2+b)^4/c

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 261}

$$\frac{(b+cx^2)^4}{8c}$$

Antiderivative was successfully verified.

[In] Int[(b*x^2 + c*x^4)^3/x^5,x]

[Out] (b + c*x^2)^4/(8*c)

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^3}{x^5}\,dx &= \int x\left(b+cx^2\right)^3\,dx\\ &= \frac{(b+cx^2)^4}{8c}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{\left(b+cx^2\right)^4}{8c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^5,x]
[Out] (b + c*x^2)^4/(8*c)
```

fricas [B] time = 0.78, size = 35, normalized size = 2.19

$$\frac{1}{8}c^3x^8+\frac{1}{2}bc^2x^6+\frac{3}{4}b^2cx^4+\frac{1}{2}b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^5,x, algorithm="fricas")
[Out] 1/8*c^3*x^8 + 1/2*b*c^2*x^6 + 3/4*b^2*c*x^4 + 1/2*b^3*x^2
```

giac [B] time = 0.15, size = 35, normalized size = 2.19

$$\frac{1}{8}c^3x^8+\frac{1}{2}bc^2x^6+\frac{3}{4}b^2cx^4+\frac{1}{2}b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^5,x, algorithm="giac")
[Out] 1/8*c^3*x^8 + 1/2*b*c^2*x^6 + 3/4*b^2*c*x^4 + 1/2*b^3*x^2
```

maple [B] time = 0.00, size = 36, normalized size = 2.25

$$\frac{1}{8}c^3x^8+\frac{1}{2}b\,c^2x^6+\frac{3}{4}b^2c\,x^4+\frac{1}{2}b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^5,x)
[Out] 1/8*c^3*x^8+1/2*b*c^2*x^6+3/4*b^2*c*x^4+1/2*b^3*x^2
```

maxima [B] time = 1.28, size = 35, normalized size = 2.19

$$\frac{1}{8}c^3x^8+\frac{1}{2}bc^2x^6+\frac{3}{4}b^2cx^4+\frac{1}{2}b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*c^3*x^8 + 1/2*b*c^2*x^6 + 3/4*b^2*c*x^4 + 1/2*b^3*x^2
```

mupad [B] time = 0.04, size = 35, normalized size = 2.19

$$\frac{b^3\,x^2}{2}+\frac{3\,b^2\,c\,x^4}{4}+\frac{b\,c^2\,x^6}{2}+\frac{c^3\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^5,x)
```

```
[Out] (b^3*x^2)/2 + (c^3*x^8)/8 + (3*b^2*c*x^4)/4 + (b*c^2*x^6)/2
```

sympy [B] time = 0.08, size = 37, normalized size = 2.31

$$\frac{b^3x^2}{2}+\frac{3b^2cx^4}{4}+\frac{bc^2x^6}{2}+\frac{c^3x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**5,x)
```

```
[Out] b**3*x**2/2 + 3*b**2*c*x**4/4 + b*c**2*x**6/2 + c**3*x**8/8
```

3.162 $\int \frac{\left(bx^2+cx^4\right)^3}{x^6}\,dx$

Optimal. Leaf size=35

$$b^3x+b^2cx^3+\frac{3}{5}bc^2x^5+\frac{c^3x^7}{7}$$

```
[Out] b^3*x+b^2*c*x^3+3/5*b*c^2*x^5+1/7*c^3*x^7
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 194}

$$b^2cx^3+b^3x+\frac{3}{5}bc^2x^5+\frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^6,x]
```

```
[Out] b^3*x + b^2*c*x^3 + (3*b*c^2*x^5)/5 + (c^3*x^7)/7
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2+cx^4\right)^3}{x^6}\,dx &= \int \left(b+cx^2\right)^3\,dx\\ &= \int \left(b^3+3b^2cx^2+3bc^2x^4+c^3x^6\right)dx\\ &= b^3x+b^2cx^3+\frac{3}{5}bc^2x^5+\frac{c^3x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$b^3x + b^2cx^3 + \frac{3}{5}bc^2x^5 + \frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^6,x]
```

```
[Out] b^3*x + b^2*c*x^3 + (3*b*c^2*x^5)/5 + (c^3*x^7)/7
```

fricas [A] time = 0.80, size = 31, normalized size = 0.89

$$\frac{1}{7}c^3x^7 + \frac{3}{5}bc^2x^5 + b^2cx^3 + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^6,x, algorithm="fricas")
```

```
[Out] 1/7*c^3*x^7 + 3/5*b*c^2*x^5 + b^2*c*x^3 + b^3*x
```

giac [A] time = 0.16, size = 31, normalized size = 0.89

$$\frac{1}{7}c^3x^7 + \frac{3}{5}bc^2x^5 + b^2cx^3 + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^6,x, algorithm="giac")
```

```
[Out] 1/7*c^3*x^7 + 3/5*b*c^2*x^5 + b^2*c*x^3 + b^3*x
```

maple [A] time = 0.00, size = 32, normalized size = 0.91

$$\frac{1}{7}c^3x^7 + \frac{3}{5}b\,c^2x^5 + b^2c\,x^3 + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^6,x)
```

```
[Out] b^3*x+b^2*c*x^3+3/5*b*c^2*x^5+1/7*c^3*x^7
```

maxima [A] time = 1.32, size = 31, normalized size = 0.89

$$\frac{1}{7}c^3x^7 + \frac{3}{5}bc^2x^5 + b^2cx^3 + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^6,x, algorithm="maxima")
```

```
[Out] 1/7*c^3*x^7 + 3/5*b*c^2*x^5 + b^2*c*x^3 + b^3*x
```

mupad [B] time = 0.04, size = 31, normalized size = 0.89

$$b^3\,x+b^2\,c\,x^3+\frac{3\,b\,c^2\,x^5}{5}+\frac{c^3\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^6,x)
```

```
[Out] b^3*x + (c^3*x^7)/7 + b^2*c*x^3 + (3*b*c^2*x^5)/5
```

sympy [A] time = 0.08, size = 32, normalized size = 0.91

$$b^3x+b^2cx^3+\frac{3bc^2x^5}{5}+\frac{c^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**6,x)
```

```
[Out] b**3*x + b**2*c*x**3 + 3*b*c**2*x**5/5 + c**3*x**7/7
```

3.163 $\int \frac{(bx^2+cx^4)^3}{x^7} dx$

Optimal. Leaf size=39

$$b^3 \log(x) + \frac{3}{2}b^2cx^2 + \frac{3}{4}bc^2x^4 + \frac{c^3x^6}{6}$$

```
[Out] 3/2*b^2*c*x^2+3/4*b*c^2*x^4+1/6*c^3*x^6+b^3*ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{3}{2}b^2cx^2 + b^3 \log(x) + \frac{3}{4}bc^2x^4 + \frac{c^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^7,x]
```

```
[Out] (3*b^2*c*x^2)/2 + (3*b*c^2*x^4)/4 + (c^3*x^6)/6 + b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^3}{x^7}\,dx &= \int \frac{\left(b+cx^2\right)^3}{x}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(b+cx)^3}{x}\,dx, x, x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(3b^2c+\frac{b^3}{x}+3bc^2x+c^3x^2\right)dx, x, x^2\right) \\
&= \frac{3}{2}b^2cx^2+\frac{3}{4}bc^2x^4+\frac{c^3x^6}{6}+b^3\log(x)
\end{aligned}
$$

Mathematica [A] time = 0.00, size = 39, normalized size = 1.00

$$b^3\log(x)+\frac{3}{2}b^2cx^2+\frac{3}{4}bc^2x^4+\frac{c^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^7,x]
```

```
[Out] (3*b^2*c*x^2)/2 + (3*b*c^2*x^4)/4 + (c^3*x^6)/6 + b^3*Log[x]
```

fricas [A] time = 0.75, size = 33, normalized size = 0.85

$$\frac{1}{6}\,c^3x^6+\frac{3}{4}\,bc^2x^4+\frac{3}{2}\,b^2cx^2+b^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*c^3*x^6 + 3/4*b*c^2*x^4 + 3/2*b^2*c*x^2 + b^3*log(x)
```

giac [A] time = 0.15, size = 36, normalized size = 0.92

$$\frac{1}{6}\,c^3x^6+\frac{3}{4}\,bc^2x^4+\frac{3}{2}\,b^2cx^2+\frac{1}{2}\,b^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^7,x, algorithm="giac")
```

```
[Out] 1/6*c^3*x^6 + 3/4*b*c^2*x^4 + 3/2*b^2*c*x^2 + 1/2*b^3*log(x^2)
```

maple [A] time = 0.00, size = 34, normalized size = 0.87

$$\frac{c^3x^6}{6} + \frac{3b\,c^2x^4}{4} + \frac{3b^2c\,x^2}{2} + b^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^7,x)
[Out] 3/2*b^2*c*x^2+3/4*b*c^2*x^4+1/6*c^3*x^6+b^3*ln(x)
```

maxima [A] time = 1.33, size = 36, normalized size = 0.92

$$\frac{1}{6}\,c^3x^6 + \frac{3}{4}\,bc^2x^4 + \frac{3}{2}\,b^2cx^2 + \frac{1}{2}\,b^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^7,x, algorithm="maxima")
[Out] 1/6*c^3*x^6 + 3/4*b*c^2*x^4 + 3/2*b^2*c*x^2 + 1/2*b^3*log(x^2)
```

mupad [B] time = 0.04, size = 33, normalized size = 0.85

$$b^3\,\ln(x) + \frac{c^3\,x^6}{6} + \frac{3\,b^2\,c\,x^2}{2} + \frac{3\,b\,c^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^7,x)
[Out] b^3*log(x) + (c^3*x^6)/6 + (3*b^2*c*x^2)/2 + (3*b*c^2*x^4)/4
```

sympy [A] time = 0.12, size = 37, normalized size = 0.95

$$b^3\log\left(x\right) + \frac{3b^2cx^2}{2} + \frac{3bc^2x^4}{4} + \frac{c^3x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**7,x)
[Out] b**3*log(x) + 3*b**2*c*x**2/2 + 3*b*c**2*x**4/4 + c**3*x**6/6
```

3.164 $\int \frac{(bx^2+cx^4)^3}{x^8}\,dx$

Optimal. Leaf size=34

$$-\frac{b^3}{x}+3b^2cx+bc^2x^3+\frac{c^3x^5}{5}$$

```
[Out] -b^3/x+3*b^2*c*x+b*c^2*x^3+1/5*c^3*x^5
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$3b^2cx-\frac{b^3}{x}+bc^2x^3+\frac{c^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^8,x]
```

```
[Out] -(b^3/x) + 3*b^2*c*x + b*c^2*x^3 + (c^3*x^5)/5
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^3}{x^8}\,dx &= \int \frac{\left(b+cx^2\right)^3}{x^2}\,dx\\
&= \int \left(3b^2c+\frac{b^3}{x^2}+3bc^2x^2+c^3x^4\right)dx\\
&= -\frac{b^3}{x}+3b^2cx+bc^2x^3+\frac{c^3x^5}{5}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 34, normalized size = 1.00

$$-\frac{b^3}{x}+3b^2cx+bc^2x^3+\frac{c^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^8,x]
[Out] -(b^3/x) + 3*b^2*c*x + b*c^2*x^3 + (c^3*x^5)/5
```

fricas [A] time = 0.83, size = 36, normalized size = 1.06

$$\frac{c^3x^6+5\,bc^2x^4+15\,b^2cx^2-5\,b^3}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^8,x, algorithm="fricas")
[Out] 1/5*(c^3*x^6 + 5*b*c^2*x^4 + 15*b^2*c*x^2 - 5*b^3)/x
```

giac [A] time = 0.15, size = 32, normalized size = 0.94

$$\frac{1}{5}\,c^3x^5+bc^2x^3+3\,b^2cx-\frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^8,x, algorithm="giac")
[Out] 1/5*c^3*x^5 + b*c^2*x^3 + 3*b^2*c*x - b^3/x
```

maple [A] time = 0.00, size = 33, normalized size = 0.97

$$\frac{c^3x^5}{5}+b\,c^2x^3+3b^2cx-\frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^8,x)
[Out] -b^3/x+3*b^2*c*x+b*c^2*x^3+1/5*c^3*x^5
```

maxima [A] time = 1.38, size = 32, normalized size = 0.94

$$\frac{1}{5}\,c^3x^5+bc^2x^3+3\,b^2cx-\frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^8,x, algorithm="maxima")
```

```
[Out] 1/5*c^3*x^5 + b*c^2*x^3 + 3*b^2*c*x - b^3/x
```

mupad [B] time = 0.04, size = 32, normalized size = 0.94

$$\frac{c^3\,x^5}{5}-\frac{b^3}{x}+b\,c^2\,x^3+3\,b^2\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^8,x)
```

```
[Out] (c^3*x^5)/5 - b^3/x + b*c^2*x^3 + 3*b^2*c*x
```

sympy [A] time = 0.12, size = 29, normalized size = 0.85

$$-\frac{b^3}{x}+3b^2cx+bc^2x^3+\frac{c^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**8,x)
```

```
[Out] -b**3/x + 3*b**2*c*x + b*c**2*x**3 + c**3*x**5/5
```

3.165 $\int \frac{(bx^2+cx^4)^3}{x^9}\,dx$

Optimal. Leaf size=40

$$-\frac{b^3}{2x^2}+3b^2c\log(x)+\frac{3}{2}bc^2x^2+\frac{c^3x^4}{4}$$

```
[Out] -1/2*b^3/x^2+3/2*b*c^2*x^2+1/4*c^3*x^4+3*b^2*c*ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$3b^2c\log(x)-\frac{b^3}{2x^2}+\frac{3}{2}bc^2x^2+\frac{c^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^9,x]
```

```
[Out] -b^3/(2*x^2) + (3*b*c^2*x^2)/2 + (c^3*x^4)/4 + 3*b^2*c*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^3}{x^9}\,dx &= \int \frac{\left(b+cx^2\right)^3}{x^3}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(b+cx)^3}{x^2}\,dx,x,x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(3bc^2+\frac{b^3}{x^2}+\frac{3b^2c}{x}+c^3x\right)dx,x,x^2\right) \\
&= -\frac{b^3}{2x^2}+\frac{3}{2}bc^2x^2+\frac{c^3x^4}{4}+3b^2c\log(x)
\end{aligned}
$$

Mathematica [**A**] time = 0.01, size = 40, normalized size = 1.00

$$-\frac{b^3}{2x^2}+3b^2c\log(x)+\frac{3}{2}bc^2x^2+\frac{c^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^9,x]
```

```
[Out] -1/2*b^3/x^2 + (3*b*c^2*x^2)/2 + (c^3*x^4)/4 + 3*b^2*c*Log[x]
```

fricas [**A**] time = 0.81, size = 38, normalized size = 0.95

$$\frac{c^3x^6+6\,bc^2x^4+12\,b^2cx^2\log(x)-2\,b^3}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^9,x, algorithm="fricas")
```

```
[Out] 1/4*(c^3*x^6 + 6*b*c^2*x^4 + 12*b^2*c*x^2*log(x) - 2*b^3)/x^2
```

giac [**A**] time = 0.16, size = 46, normalized size = 1.15

$$\frac{1}{4}\,c^3x^4+\frac{3}{2}\,bc^2x^2+\frac{3}{2}\,b^2c\log\left(x^2\right)-\frac{3\,b^2cx^2+b^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^9,x, algorithm="giac")
```

```
[Out] 1/4*c^3*x^4 + 3/2*b*c^2*x^2 + 3/2*b^2*c*log(x^2) - 1/2*(3*b^2*c*x^2 + b^3)/
x^2
```

maple [A] time = 0.01, size = 35, normalized size = 0.88

$$\frac{c^3x^4}{4}+\frac{3b\,c^2x^2}{2}+3b^2c\ln(x)-\frac{b^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^9,x)
[Out] -1/2*b^3/x^2+3/2*b*c^2*x^2+1/4*c^3*x^4+3*b^2*c*ln(x)
```

maxima [A] time = 1.27, size = 36, normalized size = 0.90

$$\frac{1}{4}\,c^3x^4+\frac{3}{2}\,bc^2x^2+\frac{3}{2}\,b^2c\log\left(x^2\right)-\frac{b^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^9,x, algorithm="maxima")
[Out] 1/4*c^3*x^4 + 3/2*b*c^2*x^2 + 3/2*b^2*c*log(x^2) - 1/2*b^3/x^2
```

mupad [B] time = 0.04, size = 34, normalized size = 0.85

$$\frac{c^3\,x^4}{4}-\frac{b^3}{2\,x^2}+\frac{3\,b\,c^2\,x^2}{2}+3\,b^2\,c\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^9,x)
[Out] (c^3*x^4)/4 - b^3/(2*x^2) + (3*b*c^2*x^2)/2 + 3*b^2*c*log(x)
```

sympy [A] time = 0.17, size = 37, normalized size = 0.92

$$-\frac{b^3}{2x^2}+3b^2c\log\left(x\right)+\frac{3bc^2x^2}{2}+\frac{c^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**9,x)
[Out] -b**3/(2*x**2) + 3*b**2*c*log(x) + 3*b*c**2*x**2/2 + c**3*x**4/4
```

3.166 $\int \frac{(bx^2+cx^4)^3}{x^{10}}\,dx$

Optimal. Leaf size=37

$$-\frac{b^3}{3x^3}-\frac{3b^2c}{x}+3bc^2x+\frac{c^3x^3}{3}$$

```
[Out] -1/3*b^3/x^3-3*b^2*c/x+3*b*c^2*x+1/3*c^3*x^3
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{3b^2c}{x}-\frac{b^3}{3x^3}+3bc^2x+\frac{c^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^10,x]
```

```
[Out] -b^3/(3*x^3) - (3*b^2*c)/x + 3*b*c^2*x + (c^3*x^3)/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^3}{x^{10}}\,dx &= \int \frac{(b+cx^2)^3}{x^4}\,dx\\ &= \int \left(3bc^2+\frac{b^3}{x^4}+\frac{3b^2c}{x^2}+c^3x^2\right)dx\\ &= -\frac{b^3}{3x^3}-\frac{3b^2c}{x}+3bc^2x+\frac{c^3x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 37, normalized size = 1.00

$$-\frac{b^3}{3x^3}-\frac{3b^2c}{x}+3bc^2x+\frac{c^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^10,x]
[Out] -1/3*b^3/x^3 - (3*b^2*c)/x + 3*b*c^2*x + (c^3*x^3)/3
```

fricas [A] time = 0.78, size = 36, normalized size = 0.97

$$\frac{c^3x^6+9\,bc^2x^4-9\,b^2cx^2-b^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^10,x, algorithm="fricas")
[Out] 1/3*(c^3*x^6 + 9*b*c^2*x^4 - 9*b^2*c*x^2 - b^3)/x^3
```

giac [A] time = 0.17, size = 34, normalized size = 0.92

$$\frac{1}{3}\,c^3x^3+3\,bc^2x-\frac{9\,b^2cx^2+b^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^10,x, algorithm="giac")
[Out] 1/3*c^3*x^3 + 3*b*c^2*x - 1/3*(9*b^2*c*x^2 + b^3)/x^3
```

maple [A] time = 0.01, size = 34, normalized size = 0.92

$$\frac{c^3x^3}{3}+3b\,c^2x-\frac{3b^2c}{x}-\frac{b^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^10,x)
[Out] -1/3*b^3/x^3-3*b^2*c/x+3*b*c^2*x+1/3*c^3*x^3
```

maxima [A] time = 1.29, size = 34, normalized size = 0.92

$$\frac{1}{3}\,c^3x^3+3\,bc^2x-\frac{9\,b^2cx^2+b^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^10,x, algorithm="maxima")
[Out] 1/3*c^3*x^3 + 3*b*c^2*x - 1/3*(9*b^2*c*x^2 + b^3)/x^3
```

mupad [B] time = 0.04, size = 36, normalized size = 0.97

$$\frac{c^3\,x^3}{3}-\frac{\frac{b^3}{3}+3\,c\,b^2\,x^2}{x^3}+3\,b\,c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^10,x)
[Out] (c^3*x^3)/3 - (b^3/3 + 3*b^2*c*x^2)/x^3 + 3*b*c^2*x
```

sympy [A] time = 0.17, size = 36, normalized size = 0.97

$$3bc^2x+\frac{c^3x^3}{3}+\frac{-b^3-9b^2cx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**10,x)
[Out] 3*b*c**2*x + c**3*x**3/3 + (-b**3 - 9*b**2*c*x**2)/(3*x**3)
```

3.167 $\int \frac{(bx^2+cx^4)^3}{x^{11}}\,dx$

Optimal. Leaf size=40

$$-\frac{b^3}{4x^4}-\frac{3b^2c}{2x^2}+3bc^2\log(x)+\frac{c^3x^2}{2}$$

```
[Out] -1/4*b^3/x^4-3/2*b^2*c/x^2+1/2*c^3*x^2+3*b*c^2*ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$-\frac{3b^2c}{2x^2}-\frac{b^3}{4x^4}+3bc^2\log(x)+\frac{c^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^11,x]
```

```
[Out] -b^3/(4*x^4) - (3*b^2*c)/(2*x^2) + (c^3*x^2)/2 + 3*b*c^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^3}{x^{11}}\,dx = \int \frac{\left(b+cx^2\right)^3}{x^5}\,dx$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(b+cx)^3}{x^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(c^3+\frac{b^3}{x^3}+\frac{3b^2c}{x^2}+\frac{3bc^2}{x}\right)dx, x, x^2\right)$$
$$= -\frac{b^3}{4x^4}-\frac{3b^2c}{2x^2}+\frac{c^3x^2}{2}+3bc^2\log(x)$$

Mathematica [A] time = 0.00, size = 40, normalized size = 1.00

$$-\frac{b^3}{4x^4}-\frac{3b^2c}{2x^2}+3bc^2\log(x)+\frac{c^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^11,x]
[Out] -1/4*b^3/x^4 - (3*b^2*c)/(2*x^2) + (c^3*x^2)/2 + 3*b*c^2*Log[x]
```

fricas [A] time = 0.62, size = 39, normalized size = 0.98

$$\frac{2\,c^3x^6+12\,bc^2x^4\log(x)-6\,b^2cx^2-b^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^11,x, algorithm="fricas")
[Out] 1/4*(2*c^3*x^6 + 12*b*c^2*x^4*log(x) - 6*b^2*c*x^2 - b^3)/x^4
```

giac [A] time = 0.15, size = 46, normalized size = 1.15

$$\frac{1}{2}\,c^3x^2+\frac{3}{2}\,bc^2\log\left(x^2\right)-\frac{9\,bc^2x^4+6\,b^2cx^2+b^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^11,x, algorithm="giac")
[Out] 1/2*c^3*x^2 + 3/2*b*c^2*log(x^2) - 1/4*(9*b*c^2*x^4 + 6*b^2*c*x^2 + b^3)/x^
4
```

maple [A] time = 0.01, size = 35, normalized size = 0.88

$$\frac{c^3x^2}{2} + 3b\,c^2\ln(x) - \frac{3b^2c}{2x^2} - \frac{b^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^11,x)
[Out] -1/4*b^3/x^4-3/2*b^2*c/x^2+1/2*c^3*x^2+3*b*c^2*ln(x)
```

maxima [A] time = 1.34, size = 37, normalized size = 0.92

$$\frac{1}{2}\,c^3x^2 + \frac{3}{2}\,bc^2\log\left(x^2\right) - \frac{6\,b^2cx^2 + b^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^11,x, algorithm="maxima")
[Out] 1/2*c^3*x^2 + 3/2*b*c^2*log(x^2) - 1/4*(6*b^2*c*x^2 + b^3)/x^4
```

mupad [B] time = 0.03, size = 37, normalized size = 0.92

$$\frac{c^3\,x^2}{2} - \frac{\frac{b^3}{4} + \frac{3\,c\,b^2\,x^2}{2}}{x^4} + 3\,b\,c^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^11,x)
[Out] (c^3*x^2)/2 - (b^3/4 + (3*b^2*c*x^2)/2)/x^4 + 3*b*c^2*log(x)
```

sympy [A] time = 0.23, size = 37, normalized size = 0.92

$$3bc^2\log\left(x\right) + \frac{c^3x^2}{2} + \frac{-b^3 - 6b^2cx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**11,x)
[Out] 3*b*c**2*log(x) + c**3*x**2/2 + (-b**3 - 6*b**2*c*x**2)/(4*x**4)
```

3.168 $\int \frac{(bx^2+cx^4)^3}{x^{12}}\,dx$

Optimal. Leaf size=34

$$-\frac{b^3}{5x^5}-\frac{b^2c}{x^3}-\frac{3bc^2}{x}+c^3x$$

```
[Out] -1/5*b^3/x^5-b^2*c/x^3-3*b*c^2/x+c^3*x
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{b^2c}{x^3}-\frac{b^3}{5x^5}-\frac{3bc^2}{x}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^12,x]
```

```
[Out] -b^3/(5*x^5) - (b^2*c)/x^3 - (3*b*c^2)/x + c^3*x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^3}{x^{12}}\,dx &= \int \frac{(b+cx^2)^3}{x^6}\,dx\\ &= \int \left(c^3+\frac{b^3}{x^6}+\frac{3b^2c}{x^4}+\frac{3bc^2}{x^2}\right)dx\\ &= -\frac{b^3}{5x^5}-\frac{b^2c}{x^3}-\frac{3bc^2}{x}+c^3x\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$-\frac{b^3}{5x^5}-\frac{b^2c}{x^3}-\frac{3bc^2}{x}+c^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^12,x]
[Out] -1/5*b^3/x^5 - (b^2*c)/x^3 - (3*b*c^2)/x + c^3*x
```

fricas [A] time = 0.74, size = 37, normalized size = 1.09

$$\frac{5\,c^3x^6-15\,bc^2x^4-5\,b^2cx^2-b^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^12,x, algorithm="fricas")
[Out] 1/5*(5*c^3*x^6 - 15*b*c^2*x^4 - 5*b^2*c*x^2 - b^3)/x^5
```

giac [A] time = 0.17, size = 33, normalized size = 0.97

$$c^3x-\frac{15\,bc^2x^4+5\,b^2cx^2+b^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^12,x, algorithm="giac")
[Out] c^3*x - 1/5*(15*b*c^2*x^4 + 5*b^2*c*x^2 + b^3)/x^5
```

maple [A] time = 0.01, size = 33, normalized size = 0.97

$$c^3x-\frac{3b\,c^2}{x}-\frac{b^2c}{x^3}-\frac{b^3}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^12,x)
[Out] -1/5*b^3/x^5-b^2*c/x^3-3*b*c^2/x+c^3*x
```

maxima [A] time = 1.30, size = 33, normalized size = 0.97

$$c^3x-\frac{15\,bc^2x^4+5\,b^2cx^2+b^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^12,x, algorithm="maxima")
```

```
[Out] c^3*x - 1/5*(15*b*c^2*x^4 + 5*b^2*c*x^2 + b^3)/x^5
```

mupad [B] time = 0.03, size = 34, normalized size = 1.00

$$c^3\,x-\frac{\frac{b^3}{5}+b^2\,c\,x^2+3\,b\,c^2\,x^4}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^12,x)
```

```
[Out] c^3*x - (b^3/5 + b^2*c*x^2 + 3*b*c^2*x^4)/x^5
```

sympy [A] time = 0.23, size = 34, normalized size = 1.00

$$c^3x+\frac{-b^3-5b^2cx^2-15bc^2x^4}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**12,x)
```

```
[Out] c**3*x + (-b**3 - 5*b**2*c*x**2 - 15*b*c**2*x**4)/(5*x**5)
```

3.169 $\int \frac{\left(bx^2+cx^4\right)^3}{x^{13}}\,dx$

Optimal. Leaf size=39

$$-\frac{b^3}{6x^6}-\frac{3b^2c}{4x^4}-\frac{3bc^2}{2x^2}+c^3\log(x)$$

```
[Out] -1/6*b^3/x^6-3/4*b^2*c/x^4-3/2*b*c^2/x^2+c^3*ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$-\frac{3b^2c}{4x^4}-\frac{b^3}{6x^6}-\frac{3bc^2}{2x^2}+c^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^13,x]
```

```
[Out] -b^3/(6*x^6) - (3*b^2*c)/(4*x^4) - (3*b*c^2)/(2*x^2) + c^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^3}{x^{13}}\,dx &= \int \frac{\left(b+cx^2\right)^3}{x^7}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{(b+cx)^3}{x^4}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{b^3}{x^4}+\frac{3b^2c}{x^3}+\frac{3bc^2}{x^2}+\frac{c^3}{x}\right)dx, x, x^2\right) \\
&= -\frac{b^3}{6x^6}-\frac{3b^2c}{4x^4}-\frac{3bc^2}{2x^2}+c^3\log(x)
\end{aligned}
$$

Mathematica [**A**] time = 0.00, size = 39, normalized size = 1.00

$$
-\frac{b^3}{6x^6}-\frac{3b^2c}{4x^4}-\frac{3bc^2}{2x^2}+c^3\log(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^13,x]
```

```
[Out] -1/6*b^3/x^6 - (3*b^2*c)/(4*x^4) - (3*b*c^2)/(2*x^2) + c^3*Log[x]
```

fricas [**A**] time = 0.77, size = 39, normalized size = 1.00

$$
\frac{12\,c^3x^6\log(x)-18\,bc^2x^4-9\,b^2cx^2-2\,b^3}{12\,x^6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^13,x, algorithm="fricas")
```

```
[Out] 1/12*(12*c^3*x^6*log(x) - 18*b*c^2*x^4 - 9*b^2*c*x^2 - 2*b^3)/x^6
```

giac [**A**] time = 0.16, size = 47, normalized size = 1.21

$$
\frac{1}{2}\,c^3\log\left(x^2\right)-\frac{11\,c^3x^6+18\,bc^2x^4+9\,b^2cx^2+2\,b^3}{12\,x^6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^13,x, algorithm="giac")
```

```
[Out] 1/2*c^3*log(x^2) - 1/12*(11*c^3*x^6 + 18*b*c^2*x^4 + 9*b^2*c*x^2 + 2*b^3)/x
^6
```

maple [A] time = 0.01, size = 34, normalized size = 0.87

$$c^3\ln(x) - \frac{3b\,c^2}{2x^2} - \frac{3b^2c}{4x^4} - \frac{b^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^13,x)
[Out] -1/6*b^3/x^6-3/4*b^2*c/x^4-3/2*b*c^2/x^2+c^3*ln(x)
```

maxima [A] time = 1.32, size = 39, normalized size = 1.00

$$\frac{1}{2}\,c^3\log\left(x^2\right) - \frac{18\,bc^2x^4 + 9\,b^2cx^2 + 2\,b^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^13,x, algorithm="maxima")
[Out] 1/2*c^3*log(x^2) - 1/12*(18*b*c^2*x^4 + 9*b^2*c*x^2 + 2*b^3)/x^6
```

mupad [B] time = 0.05, size = 36, normalized size = 0.92

$$c^3\,\ln(x) - \frac{\frac{b^3}{6} + \frac{3\,b^2\,c\,x^2}{4} + \frac{3\,b\,c^2\,x^4}{2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^13,x)
[Out] c^3*log(x) - (b^3/6 + (3*b^2*c*x^2)/4 + (3*b*c^2*x^4)/2)/x^6
```

sympy [A] time = 0.29, size = 37, normalized size = 0.95

$$c^3\log\left(x\right) + \frac{-2b^3 - 9b^2cx^2 - 18bc^2x^4}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**13,x)
[Out] c**3*log(x) + (-2*b**3 - 9*b**2*c*x**2 - 18*b*c**2*x**4)/(12*x**6)
```

3.170 $\int \frac{(bx^2+cx^4)^3}{x^{14}}\,dx$

Optimal. Leaf size=39

$$-\frac{b^3}{7x^7}-\frac{3b^2c}{5x^5}-\frac{bc^2}{x^3}-\frac{c^3}{x}$$

```
[Out] -1/7*b^3/x^7-3/5*b^2*c/x^5-b*c^2/x^3-c^3/x
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{3b^2c}{5x^5}-\frac{b^3}{7x^7}-\frac{bc^2}{x^3}-\frac{c^3}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^14,x]
```

```
[Out] -b^3/(7*x^7) - (3*b^2*c)/(5*x^5) - (b*c^2)/x^3 - c^3/x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^3}{x^{14}}\,dx &= \int \frac{(b+cx^2)^3}{x^8}\,dx\\ &= \int \left(\frac{b^3}{x^8}+\frac{3b^2c}{x^6}+\frac{3bc^2}{x^4}+\frac{c^3}{x^2}\right)dx\\ &= -\frac{b^3}{7x^7}-\frac{3b^2c}{5x^5}-\frac{bc^2}{x^3}-\frac{c^3}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 39, normalized size = 1.00

$$-\frac{b^3}{7x^7}-\frac{3b^2c}{5x^5}-\frac{bc^2}{x^3}-\frac{c^3}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^14,x]
[Out] -1/7*b^3/x^7 - (3*b^2*c)/(5*x^5) - (b*c^2)/x^3 - c^3/x
```

fricas [A] time = 0.70, size = 37, normalized size = 0.95

$$-\frac{35\,c^3x^6+35\,bc^2x^4+21\,b^2cx^2+5\,b^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^14,x, algorithm="fricas")
[Out] -1/35*(35*c^3*x^6 + 35*b*c^2*x^4 + 21*b^2*c*x^2 + 5*b^3)/x^7
```

giac [A] time = 0.16, size = 37, normalized size = 0.95

$$-\frac{35\,c^3x^6+35\,bc^2x^4+21\,b^2cx^2+5\,b^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^14,x, algorithm="giac")
[Out] -1/35*(35*c^3*x^6 + 35*b*c^2*x^4 + 21*b^2*c*x^2 + 5*b^3)/x^7
```

maple [A] time = 0.01, size = 36, normalized size = 0.92

$$-\frac{c^3}{x}-\frac{b\,c^2}{x^3}-\frac{3b^2c}{5x^5}-\frac{b^3}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^14,x)
[Out] -1/7*b^3/x^7-3/5*b^2*c/x^5-b*c^2/x^3-c^3/x
```

maxima [A] time = 1.35, size = 37, normalized size = 0.95

$$-\frac{35\,c^3x^6+35\,bc^2x^4+21\,b^2cx^2+5\,b^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^14,x, algorithm="maxima")
[Out] -1/35*(35*c^3*x^6 + 35*b*c^2*x^4 + 21*b^2*c*x^2 + 5*b^3)/x^7
```

mupad [B] time = 0.03, size = 35, normalized size = 0.90

$$-\frac{\frac{b^3}{7}+\frac{3\,b^2\,c\,x^2}{5}+b\,c^2\,x^4+c^3\,x^6}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^14,x)
[Out] -(b^3/7 + c^3*x^6 + (3*b^2*c*x^2)/5 + b*c^2*x^4)/x^7
```

sympy [A] time = 0.28, size = 39, normalized size = 1.00

$$\frac{-5b^3-21b^2cx^2-35bc^2x^4-35c^3x^6}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**14,x)
[Out] (-5*b**3 - 21*b**2*c*x**2 - 35*b*c**2*x**4 - 35*c**3*x**6)/(35*x**7)
```

3.171 $\int \frac{(bx^2+cx^4)^3}{x^{15}} \, dx$

Optimal. Leaf size=19

$$-\frac{(b+cx^2)^4}{8bx^8}$$

```
[Out] -1/8*(c*x^2+b)^4/b/x^8
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 264}

$$-\frac{(b+cx^2)^4}{8bx^8}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^15,x]
```

```
[Out] -(b + c*x^2)^4/(8*b*x^8)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{(bx^2+cx^4)^3}{x^{15}} \, dx &= \int \frac{(b+cx^2)^3}{x^9} \, dx \\ &= -\frac{(b+cx^2)^4}{8bx^8} \end{aligned}$$

Mathematica [B] time = 0.01, size = 43, normalized size = 2.26

$$-\frac{b^3}{8x^8}-\frac{b^2c}{2x^6}-\frac{3bc^2}{4x^4}-\frac{c^3}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^15,x]
[Out] -1/8*b^3/x^8 - (b^2*c)/(2*x^6) - (3*b*c^2)/(4*x^4) - c^3/(2*x^2)
```

fricas [B] time = 0.76, size = 35, normalized size = 1.84

$$-\frac{4\,c^3x^6+6\,bc^2x^4+4\,b^2cx^2+b^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^15,x, algorithm="fricas")
[Out] -1/8*(4*c^3*x^6 + 6*b*c^2*x^4 + 4*b^2*c*x^2 + b^3)/x^8
```

giac [B] time = 0.15, size = 35, normalized size = 1.84

$$-\frac{4\,c^3x^6+6\,bc^2x^4+4\,b^2cx^2+b^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^15,x, algorithm="giac")
[Out] -1/8*(4*c^3*x^6 + 6*b*c^2*x^4 + 4*b^2*c*x^2 + b^3)/x^8
```

maple [B] time = 0.00, size = 36, normalized size = 1.89

$$-\frac{c^3}{2x^2}-\frac{3b\,c^2}{4x^4}-\frac{b^2c}{2x^6}-\frac{b^3}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^15,x)
[Out] -1/2*c^3/x^2-1/2*b^2*c/x^6-3/4*b*c^2/x^4-1/8*b^3/x^8
```

maxima [B] time = 1.34, size = 35, normalized size = 1.84

$$-\frac{4\,c^3x^6+6\,bc^2x^4+4\,b^2cx^2+b^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^15,x, algorithm="maxima")
```

```
[Out] -1/8*(4*c^3*x^6 + 6*b*c^2*x^4 + 4*b^2*c*x^2 + b^3)/x^8
```

mupad [B] time = 0.03, size = 37, normalized size = 1.95

$$-\frac{\frac{b^3}{8}+\frac{b^2\,c\,x^2}{2}+\frac{3\,b\,c^2\,x^4}{4}+\frac{c^3\,x^6}{2}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^15,x)
```

```
[Out] -(b^3/8 + (c^3*x^6)/2 + (b^2*c*x^2)/2 + (3*b*c^2*x^4)/4)/x^8
```

sympy [B] time = 0.31, size = 37, normalized size = 1.95

$$\frac{-b^3-4b^2cx^2-6bc^2x^4-4c^3x^6}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**15,x)
```

```
[Out] (-b**3 - 4*b**2*c*x**2 - 6*b*c**2*x**4 - 4*c**3*x**6)/(8*x**8)
```

3.172 $\int \frac{(bx^2+cx^4)^3}{x^{16}}\,dx$

Optimal. Leaf size=43

$$-\frac{b^3}{9x^9}-\frac{3b^2c}{7x^7}-\frac{3bc^2}{5x^5}-\frac{c^3}{3x^3}$$

```
[Out] -1/9*b^3/x^9-3/7*b^2*c/x^7-3/5*b*c^2/x^5-1/3*c^3/x^3
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{3b^2c}{7x^7}-\frac{b^3}{9x^9}-\frac{3bc^2}{5x^5}-\frac{c^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^16,x]
```

```
[Out] -b^3/(9*x^9) - (3*b^2*c)/(7*x^7) - (3*b*c^2)/(5*x^5) - c^3/(3*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^3}{x^{16}}\,dx &= \int \frac{(b+cx^2)^3}{x^{10}}\,dx\\ &= \int \left(\frac{b^3}{x^{10}}+\frac{3b^2c}{x^8}+\frac{3bc^2}{x^6}+\frac{c^3}{x^4}\right)dx\\ &= -\frac{b^3}{9x^9}-\frac{3b^2c}{7x^7}-\frac{3bc^2}{5x^5}-\frac{c^3}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$-\frac{b^3}{9x^9}-\frac{3b^2c}{7x^7}-\frac{3bc^2}{5x^5}-\frac{c^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^16,x]
[Out] -1/9*b^3/x^9 - (3*b^2*c)/(7*x^7) - (3*b*c^2)/(5*x^5) - c^3/(3*x^3)
```

fricas [A] time = 0.81, size = 37, normalized size = 0.86

$$-\frac{105\,c^3x^6+189\,bc^2x^4+135\,b^2cx^2+35\,b^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^16,x, algorithm="fricas")
[Out] -1/315*(105*c^3*x^6 + 189*b*c^2*x^4 + 135*b^2*c*x^2 + 35*b^3)/x^9
```

giac [A] time = 0.17, size = 37, normalized size = 0.86

$$-\frac{105\,c^3x^6+189\,bc^2x^4+135\,b^2cx^2+35\,b^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^16,x, algorithm="giac")
[Out] -1/315*(105*c^3*x^6 + 189*b*c^2*x^4 + 135*b^2*c*x^2 + 35*b^3)/x^9
```

maple [A] time = 0.01, size = 36, normalized size = 0.84

$$-\frac{c^3}{3x^3}-\frac{3b\,c^2}{5x^5}-\frac{3b^2c}{7x^7}-\frac{b^3}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^16,x)
[Out] -1/9*b^3/x^9-3/7*b^2*c/x^7-3/5*b*c^2/x^5-1/3*c^3/x^3
```

maxima [A] time = 1.33, size = 37, normalized size = 0.86

$$-\frac{105\,c^3x^6+189\,bc^2x^4+135\,b^2cx^2+35\,b^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^16,x, algorithm="maxima")
```

```
[Out] -1/315*(105*c^3*x^6 + 189*b*c^2*x^4 + 135*b^2*c*x^2 + 35*b^3)/x^9
```

mupad [B] time = 0.03, size = 37, normalized size = 0.86

$$-\frac{\frac{b^3}{9}+\frac{3\,b^2\,c\,x^2}{7}+\frac{3\,b\,c^2\,x^4}{5}+\frac{c^3\,x^6}{3}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^16,x)
```

```
[Out] -(b^3/9 + (c^3*x^6)/3 + (3*b^2*c*x^2)/7 + (3*b*c^2*x^4)/5)/x^9
```

sympy [A] time = 0.30, size = 39, normalized size = 0.91

$$\frac{-35b^3-135b^2cx^2-189bc^2x^4-105c^3x^6}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**16,x)
```

```
[Out] (-35*b**3 - 135*b**2*c*x**2 - 189*b*c**2*x**4 - 105*c**3*x**6)/(315*x**9)
```

3.173 $\int \frac{(bx^2+cx^4)^3}{x^{17}}\,dx$

Optimal. Leaf size=40

$$\frac{c(b+cx^2)^4}{40b^2x^8}-\frac{(b+cx^2)^4}{10bx^{10}}$$

```
[Out] -1/10*(c*x^2+b)^4/b/x^10+1/40*c*(c*x^2+b)^4/b^2/x^8
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 266, 45, 37}

$$\frac{c(b+cx^2)^4}{40b^2x^8}-\frac{(b+cx^2)^4}{10bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^17,x]
```

```
[Out] -(b + c*x^2)^4/(10*b*x^10) + (c*(b + c*x^2)^4)/(40*b^2*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^3}{x^{17}}\,dx &= \int \frac{\left(b+cx^2\right)^3}{x^{11}}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{(b+cx)^3}{x^6}\,dx, x, x^2\right)\\
&= -\frac{\left(b+cx^2\right)^4}{10bx^{10}} - \frac{c\,\text{Subst}\left(\int \frac{(b+cx)^3}{x^5}\,dx, x, x^2\right)}{10b}\\
&= -\frac{\left(b+cx^2\right)^4}{10bx^{10}} + \frac{c\left(b+cx^2\right)^4}{40b^2x^8}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.08

$$-\frac{b^3}{10x^{10}} - \frac{3b^2c}{8x^8} - \frac{bc^2}{2x^6} - \frac{c^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^17,x]
[Out] -1/10*b^3/x^10 - (3*b^2*c)/(8*x^8) - (b*c^2)/(2*x^6) - c^3/(4*x^4)
```

fricas [A] time = 0.70, size = 37, normalized size = 0.92

$$-\frac{10\,c^3x^6 + 20\,bc^2x^4 + 15\,b^2cx^2 + 4\,b^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^17,x, algorithm="fricas")
[Out] -1/40*(10*c^3*x^6 + 20*b*c^2*x^4 + 15*b^2*c*x^2 + 4*b^3)/x^10
```

giac [A] time = 0.15, size = 37, normalized size = 0.92

$$-\frac{10\,c^3x^6 + 20\,bc^2x^4 + 15\,b^2cx^2 + 4\,b^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^17,x, algorithm="giac")
[Out] -1/40*(10*c^3*x^6 + 20*b*c^2*x^4 + 15*b^2*c*x^2 + 4*b^3)/x^10
```

maple [A] time = 0.00, size = 36, normalized size = 0.90

$$-\frac{c^3}{4x^4}-\frac{b\,c^2}{2x^6}-\frac{3b^2c}{8x^8}-\frac{b^3}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^17,x)
[Out] -3/8*b^2*c/x^8-1/2*b*c^2/x^6-1/4*c^3/x^4-1/10*b^3/x^10
```

maxima [A] time = 1.31, size = 37, normalized size = 0.92

$$-\frac{10\,c^3x^6+20\,bc^2x^4+15\,b^2cx^2+4\,b^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^17,x, algorithm="maxima")
[Out] -1/40*(10*c^3*x^6 + 20*b*c^2*x^4 + 15*b^2*c*x^2 + 4*b^3)/x^10
```

mupad [B] time = 0.03, size = 37, normalized size = 0.92

$$-\frac{\frac{b^3}{10}+\frac{3\,b^2\,c\,x^2}{8}+\frac{b\,c^2\,x^4}{2}+\frac{c^3\,x^6}{4}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^17,x)
[Out] -(b^3/10 + (c^3*x^6)/4 + (3*b^2*c*x^2)/8 + (b*c^2*x^4)/2)/x^10
```

sympy [A] time = 0.32, size = 39, normalized size = 0.98

$$\frac{-4b^3-15b^2cx^2-20bc^2x^4-10c^3x^6}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**17,x)
[Out] (-4*b**3 - 15*b**2*c*x**2 - 20*b*c**2*x**4 - 10*c**3*x**6)/(40*x**10)
```

3.174 $\int \frac{x^{10}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=68

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}}-\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}$$

```
[Out] -b^3*x/c^4+1/3*b^2*x^3/c^3-1/5*b*x^5/c^2+1/7*x^7/c+b^(7/2)*arctan(x*c^(1/2)
/b^(1/2))/c^(9/2)
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 302, 205}

$$\frac{b^2x^3}{3c^3}-\frac{b^3x}{c^4}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(b*x^2 + c*x^4),x]
```

```
[Out] -((b^3*x)/c^4) + (b^2*x^3)/(3*c^3) - (b*x^5)/(5*c^2) + x^7/(7*c) + (b^(7/2)
*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(9/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{10}}{bx^2+cx^4}\,dx = \int \frac{x^8}{b+cx^2}\,dx$$
$$= \int \left(-\frac{b^3}{c^4}+\frac{b^2x^2}{c^3}-\frac{bx^4}{c^2}+\frac{x^6}{c}+\frac{b^4}{c^4\left(b+cx^2\right)}\right)dx$$
$$= -\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}+\frac{b^4\int \frac{1}{b+cx^2}\,dx}{c^4}$$
$$= -\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 1.00

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}}-\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(b*x^2 + c*x^4),x]
```

```
[Out] -((b^3*x)/c^4) + (b^2*x^3)/(3*c^3) - (b*x^5)/(5*c^2) + x^7/(7*c) + (b^(7/2)
*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(9/2)
```

fricas [A] time = 0.78, size = 148, normalized size = 2.18

$$\left[\frac{30\,c^3x^7-42\,bc^2x^5+70\,b^2cx^3+105\,b^3\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)-210\,b^3x}{210\,c^4},\ \frac{15\,c^3x^7-21\,bc^2x^5+35\,b^2cx^3+105}{105\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/210*(30*c^3*x^7 - 42*b*c^2*x^5 + 70*b^2*c*x^3 + 105*b^3*sqrt(-b/c)*log((
c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) - 210*b^3*x)/c^4, 1/105*(15*c^3*
x^7 - 21*b*c^2*x^5 + 35*b^2*c*x^3 + 105*b^3*sqrt(b/c)*arctan(c*x*sqrt(b/c)/
b) - 105*b^3*x)/c^4]
```

giac [A] time = 0.19, size = 65, normalized size = 0.96

$$\frac{b^4\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^4}+\frac{15\,c^6x^7-21\,bc^5x^5+35\,b^2c^4x^3-105\,b^3c^3x}{105\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] b^4*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/105*(15*c^6*x^7 - 21*b*c^5*x^
5 + 35*b^2*c^4*x^3 - 105*b^3*c^3*x)/c^7
```

maple [A] time = 0.01, size = 60, normalized size = 0.88

$$\frac{x^7}{7c} - \frac{b\,x^5}{5c^2} + \frac{b^2x^3}{3c^3} + \frac{b^4\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\;c^4} - \frac{b^3x}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^4+b*x^2),x)
```

```
[Out] 1/7*x^7/c-1/5*b*x^5/c^2+1/3*b^2*x^3/c^3-b^3*x/c^4+b^4/c^4/(b*c)^(1/2)*arcta
n(x*c/(b*c)^(1/2))
```

maxima [A] time = 3.01, size = 60, normalized size = 0.88

$$\frac{b^4\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^4} + \frac{15\,c^3x^7 - 21\,bc^2x^5 + 35\,b^2cx^3 - 105\,b^3x}{105\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] b^4*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/105*(15*c^3*x^7 - 21*b*c^2*x^
5 + 35*b^2*c*x^3 - 105*b^3*x)/c^4
```

mupad [B] time = 0.03, size = 54, normalized size = 0.79

$$\frac{x^7}{7\,c} - \frac{b\,x^5}{5\,c^2} - \frac{b^3\,x}{c^4} + \frac{b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}} + \frac{b^2\,x^3}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2 + c*x^4),x)
```

```
[Out] x^7/(7*c) - (b*x^5)/(5*c^2) - (b^3*x)/c^4 + (b^(7/2)*atan((c^(1/2)*x)/b^(1/
2)))/c^(9/2) + (b^2*x^3)/(3*c^3)
```

sympy [A] time = 0.22, size = 107, normalized size = 1.57

$$-\frac{b^3x}{c^4} + \frac{b^2x^3}{3c^3} - \frac{bx^5}{5c^2} - \frac{\sqrt{-\frac{b^7}{c^9}}\log\left(x - \frac{c^4\sqrt{-\frac{b^7}{c^9}}}{b^3}\right)}{2} + \frac{\sqrt{-\frac{b^7}{c^9}}\log\left(x + \frac{c^4\sqrt{-\frac{b^7}{c^9}}}{b^3}\right)}{2} + \frac{x^7}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**4+b*x**2),x)
```

```
[Out] -b**3*x/c**4 + b**2*x**3/(3*c**3) - b*x**5/(5*c**2) - sqrt(-b**7/c**9)*log(
x - c**4*sqrt(-b**7/c**9)/b**3)/2 + sqrt(-b**7/c**9)*log(x + c**4*sqrt(-b**
7/c**9)/b**3)/2 + x**7/(7*c)
```

3.175 $\int \frac{x^9}{bx^2+cx^4}\,dx$

Optimal. Leaf size=53

$$-\frac{b^3\log\left(b+cx^2\right)}{2c^4}+\frac{b^2x^2}{2c^3}-\frac{bx^4}{4c^2}+\frac{x^6}{6c}$$

```
[Out] 1/2*b^2*x^2/c^3-1/4*b*x^4/c^2+1/6*x^6/c-1/2*b^3*ln(c*x^2+b)/c^4
```

Rubi [A] time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{b^2x^2}{2c^3}-\frac{b^3\log\left(b+cx^2\right)}{2c^4}-\frac{bx^4}{4c^2}+\frac{x^6}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(b*x^2 + c*x^4),x]
```

```
[Out] (b^2*x^2)/(2*c^3) - (b*x^4)/(4*c^2) + x^6/(6*c) - (b^3*Log[b + c*x^2])/(2*c
^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{bx^2+cx^4}\,dx &= \int \frac{x^7}{b+cx^2}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{b+cx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{b^2}{c^3}-\frac{bx}{c^2}+\frac{x^2}{c}-\frac{b^3}{c^3(b+cx)}\right)dx,x,x^2\right)\\ &= \frac{b^2x^2}{2c^3}-\frac{bx^4}{4c^2}+\frac{x^6}{6c}-\frac{b^3\log\left(b+cx^2\right)}{2c^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 1.00

$$-\frac{b^3\log\left(b+cx^2\right)}{2c^4}+\frac{b^2x^2}{2c^3}-\frac{bx^4}{4c^2}+\frac{x^6}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(b*x^2 + c*x^4),x]
```

```
[Out] (b^2*x^2)/(2*c^3) - (b*x^4)/(4*c^2) + x^6/(6*c) - (b^3*Log[b + c*x^2])/(2*c
^4)
```

fricas [A] time = 0.58, size = 45, normalized size = 0.85

$$\frac{2\,c^3x^6-3\,bc^2x^4+6\,b^2cx^2-6\,b^3\log\left(cx^2+b\right)}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/12*(2*c^3*x^6 - 3*b*c^2*x^4 + 6*b^2*c*x^2 - 6*b^3*log(c*x^2 + b))/c^4
```

giac [A] time = 0.16, size = 47, normalized size = 0.89

$$-\frac{b^3\log\left(\left|cx^2+b\right|\right)}{2\,c^4}+\frac{2\,c^2x^6-3\,bcx^4+6\,b^2x^2}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*b^3*log(abs(c*x^2 + b))/c^4 + 1/12*(2*c^2*x^6 - 3*b*c*x^4 + 6*b^2*x^2)
/c^3
```

maple [A] time = 0.00, size = 46, normalized size = 0.87

$$\frac{x^6}{6c} - \frac{b\,x^4}{4c^2} + \frac{b^2x^2}{2c^3} - \frac{b^3\ln\left(c\,x^2+b\right)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2),x)
```

```
[Out] 1/2*b^2*x^2/c^3-1/4*b*x^4/c^2+1/6*x^6/c-1/2*b^3*ln(c*x^2+b)/c^4
```

maxima [A] time = 1.33, size = 46, normalized size = 0.87

$$-\frac{b^3\log\left(cx^2+b\right)}{2\,c^4} + \frac{2\,c^2x^6 - 3\,bcx^4 + 6\,b^2x^2}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log(c*x^2 + b)/c^4 + 1/12*(2*c^2*x^6 - 3*b*c*x^4 + 6*b^2*x^2)/c^3
```

mupad [B] time = 0.05, size = 45, normalized size = 0.85

$$\frac{x^6}{6\,c} - \frac{b\,x^4}{4\,c^2} - \frac{b^3\,\ln\left(c\,x^2+b\right)}{2\,c^4} + \frac{b^2\,x^2}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2 + c*x^4),x)
```

```
[Out] x^6/(6*c) - (b*x^4)/(4*c^2) - (b^3*log(b + c*x^2))/(2*c^4) + (b^2*x^2)/(2*c
^3)
```

sympy [A] time = 0.18, size = 44, normalized size = 0.83

$$-\frac{b^3\log\left(b+cx^2\right)}{2c^4} + \frac{b^2x^2}{2c^3} - \frac{bx^4}{4c^2} + \frac{x^6}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2),x)
```

```
[Out] -b**3*log(b + c*x**2)/(2*c**4) + b**2*x**2/(2*c**3) - b*x**4/(4*c**2) + x**
6/(6*c)
```

3.176 $\int \frac{x^8}{bx^2+cx^4} \, dx$

Optimal. Leaf size=55

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}}+\frac{b^2x}{c^3}-\frac{bx^3}{3c^2}+\frac{x^5}{5c}$$

```
[Out] b^2*x/c^3-1/3*b*x^3/c^2+1/5*x^5/c-b^(5/2)*arctan(x*c^(1/2)/b^(1/2))/c^(7/2)
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 302, 205}

$$\frac{b^2x}{c^3}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}}-\frac{bx^3}{3c^2}+\frac{x^5}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(b*x^2 + c*x^4),x]
```

```
[Out] (b^2*x)/c^3 - (b*x^3)/(3*c^2) + x^5/(5*c) - (b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqr
t[b]])/c^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{bx^2+cx^4}\,dx &= \int \frac{x^6}{b+cx^2}\,dx\\ &= \int \left(\frac{b^2}{c^3}-\frac{bx^2}{c^2}+\frac{x^4}{c}-\frac{b^3}{c^3\left(b+cx^2\right)}\right)dx\\ &= \frac{b^2x}{c^3}-\frac{bx^3}{3c^2}+\frac{x^5}{5c}-\frac{b^3\int\frac{1}{b+cx^2}\,dx}{c^3}\\ &= \frac{b^2x}{c^3}-\frac{bx^3}{3c^2}+\frac{x^5}{5c}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 1.00

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}}+\frac{b^2x}{c^3}-\frac{bx^3}{3c^2}+\frac{x^5}{5c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(b*x^2 + c*x^4),x]
```

```
[Out] (b^2*x)/c^3 - (b*x^3)/(3*c^2) + x^5/(5*c) - (b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqr
t[b]])/c^(7/2)
```

fricas [A] time = 0.83, size = 126, normalized size = 2.29

$$\left[\frac{6\,c^2x^5-10\,bcx^3+15\,b^2\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)+30\,b^2x}{30\,c^3},\ \frac{3\,c^2x^5-5\,bcx^3-15\,b^2\sqrt{\frac{b}{c}}\,\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)+15\,b^2x}{15\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/30*(6*c^2*x^5 - 10*b*c*x^3 + 15*b^2*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-
b/c) - b)/(c*x^2 + b)) + 30*b^2*x)/c^3, 1/15*(3*c^2*x^5 - 5*b*c*x^3 - 15*b^
2*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) + 15*b^2*x)/c^3]
```

giac [A] time = 0.18, size = 55, normalized size = 1.00

$$-\frac{b^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^3}+\frac{3\,c^4x^5-5\,bc^3x^3+15\,b^2c^2x}{15\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -b^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/15*(3*c^4*x^5 - 5*b*c^3*x^3
+ 15*b^2*c^2*x)/c^5
```

maple [A] time = 0.00, size = 49, normalized size = 0.89

$$\frac{x^5}{5c} - \frac{b\,x^3}{3c^2} - \frac{b^3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\;c^3} + \frac{b^2 x}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2),x)
```

```
[Out] 1/5*x^5/c-1/3*b*x^3/c^2+b^2*x/c^3-b^3/c^3/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*
c*x)
```

maxima [A] time = 2.88, size = 50, normalized size = 0.91

$$-\frac{b^3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^3} + \frac{3\,c^2 x^5 - 5\,bcx^3 + 15\,b^2 x}{15\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -b^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/15*(3*c^2*x^5 - 5*b*c*x^3 +
15*b^2*x)/c^3
```

mupad [B] time = 0.05, size = 43, normalized size = 0.78

$$\frac{x^5}{5\,c} - \frac{b\,x^3}{3\,c^2} + \frac{b^2\,x}{c^3} - \frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2 + c*x^4),x)
```

```
[Out] x^5/(5*c) - (b*x^3)/(3*c^2) + (b^2*x)/c^3 - (b^(5/2)*atan((c^(1/2)*x)/b^(1/
2)))/c^(7/2)
```

sympy [A] time = 0.21, size = 95, normalized size = 1.73

$$\frac{b^2x}{c^3}-\frac{bx^3}{3c^2}+\frac{\sqrt{-\frac{b^5}{c^7}}\log\left(x-\frac{c^3\sqrt{-\frac{b^5}{c^7}}}{b^2}\right)}{2}-\frac{\sqrt{-\frac{b^5}{c^7}}\log\left(x+\frac{c^3\sqrt{-\frac{b^5}{c^7}}}{b^2}\right)}{2}+\frac{x^5}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2),x)
```

```
[Out] b**2*x/c**3 - b*x**3/(3*c**2) + sqrt(-b**5/c**7)*log(x - c**3*sqrt(-b**5/c*
*7)/b**2)/2 - sqrt(-b**5/c**7)*log(x + c**3*sqrt(-b**5/c**7)/b**2)/2 + x**5
/(5*c)
```

3.177 $\int \frac{x^7}{bx^2+cx^4}\,dx$

Optimal. Leaf size=40

$$\frac{b^2\log\left(b+cx^2\right)}{2c^3}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

```
[Out] -1/2*b*x^2/c^2+1/4*x^4/c+1/2*b^2*ln(c*x^2+b)/c^3
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{b^2\log\left(b+cx^2\right)}{2c^3}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(b*x^2 + c*x^4),x]
```

```
[Out] -(b*x^2)/(2*c^2) + x^4/(4*c) + (b^2*Log[b + c*x^2])/(2*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^7}{bx^2+cx^4}\,dx = \int \frac{x^5}{b+cx^2}\,dx$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{b+cx}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{b^2}{c^2(b+cx)}\right)dx, x, x^2\right)$$
$$= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{b^2\log\left(b+cx^2\right)}{2c^3}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{b^2\log\left(b+cx^2\right)}{2c^3}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(b*x^2 + c*x^4),x]
[Out] -1/2*(b*x^2)/c^2 + x^4/(4*c) + (b^2*Log[b + c*x^2])/(2*c^3)
```

fricas [A] time = 0.84, size = 33, normalized size = 0.82

$$\frac{c^2x^4-2\,bcx^2+2\,b^2\log\left(cx^2+b\right)}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/4*(c^2*x^4 - 2*b*c*x^2 + 2*b^2*log(c*x^2 + b))/c^3
```

giac [A] time = 0.15, size = 35, normalized size = 0.88

$$\frac{b^2\log\left(\left|cx^2+b\right|\right)}{2\,c^3}+\frac{cx^4-2\,bx^2}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2),x, algorithm="giac")
[Out] 1/2*b^2*log(abs(c*x^2 + b))/c^3 + 1/4*(c*x^4 - 2*b*x^2)/c^2
```

maple [A] time = 0.00, size = 35, normalized size = 0.88

$$\frac{x^4}{4c} - \frac{b\,x^2}{2c^2} + \frac{b^2 \ln\left(c\,x^2 + b\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2),x)
```

```
[Out] -1/2*b*x^2/c^2+1/4*x^4/c+1/2*b^2*ln(c*x^2+b)/c^3
```

maxima [A] time = 1.33, size = 34, normalized size = 0.85

$$\frac{b^2 \log\left(cx^2 + b\right)}{2\,c^3} + \frac{cx^4 - 2\,bx^2}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(c*x^2 + b)/c^3 + 1/4*(c*x^4 - 2*b*x^2)/c^2
```

mupad [B] time = 0.05, size = 33, normalized size = 0.82

$$\frac{2\,b^2\,\ln\left(c\,x^2 + b\right) + c^2\,x^4 - 2\,b\,c\,x^2}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2 + c*x^4),x)
```

```
[Out] (2*b^2*log(b + c*x^2) + c^2*x^4 - 2*b*c*x^2)/(4*c^3)
```

sympy [A] time = 0.17, size = 32, normalized size = 0.80

$$\frac{b^2 \log\left(b + cx^2\right)}{2c^3} - \frac{bx^2}{2c^2} + \frac{x^4}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2),x)
```

```
[Out] b**2*log(b + c*x**2)/(2*c**3) - b*x**2/(2*c**2) + x**4/(4*c)
```

3.178 $\int \frac{x^6}{bx^2+cx^4}\,dx$

Optimal. Leaf size=42

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{5/2}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

```
[Out] -b*x/c^2+1/3*x^3/c+b^(3/2)*arctan(x*c^(1/2)/b^(1/2))/c^(5/2)
```

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 302, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{5/2}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(b*x^2 + c*x^4),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + (b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^6}{bx^2+cx^4}\,dx = \int \frac{x^4}{b+cx^2}\,dx$$
$$= \int \left(-\frac{b}{c^2}+\frac{x^2}{c}+\frac{b^2}{c^2\left(b+cx^2\right)}\right)dx$$
$$= -\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{b^2\int \frac{1}{b+cx^2}\,dx}{c^2}$$
$$= -\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{5/2}}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{5/2}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(b*x^2 + c*x^4),x]
[Out] -((b*x)/c^2) + x^3/(3*c) + (b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(5/2)
```

fricas [A] time = 0.84, size = 99, normalized size = 2.36

$$\left[\frac{2\,cx^3+3\,b\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)-6\,bx}{6\,c^2},\ \frac{cx^3+3\,b\sqrt{\frac{b}{c}}\,\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)-3\,bx}{3\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] [1/6*(2*c*x^3 + 3*b*sqrt(-b/c)*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 +
b)) - 6*b*x)/c^2, 1/3*(c*x^3 + 3*b*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) - 3*b*
x)/c^2]
```

giac [A] time = 0.17, size = 40, normalized size = 0.95

$$\frac{b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^2}+\frac{c^2x^3-3\,bcx}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2),x, algorithm="giac")
[Out] b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/3*(c^2*x^3 - 3*b*c*x)/c^3
```

maple [A] time = 0.00, size = 38, normalized size = 0.90

$$\frac{x^3}{3c}+\frac{b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^2}-\frac{bx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2),x)
[Out] 1/3*x^3/c-b*x/c^2+b^2/c^2/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)
```

maxima [A] time = 2.95, size = 37, normalized size = 0.88

$$\frac{b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^2}+\frac{cx^3-3\,bx}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/3*(c*x^3 - 3*b*x)/c^2
```

mupad [B] time = 0.05, size = 32, normalized size = 0.76

$$\frac{x^3}{3\,c}+\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{5/2}}-\frac{b\,x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2 + c*x^4),x)
[Out] x^3/(3*c) + (b^(3/2)*atan((c^(1/2)*x)/b^(1/2)))/c^(5/2) - (b*x)/c^2
```

sympy [B] time = 0.19, size = 80, normalized size = 1.90

$$-\frac{bx}{c^2}-\frac{\sqrt{-\frac{b^3}{c^5}}\log\left(x-\frac{c^2\sqrt{-\frac{b^3}{c^5}}}{b}\right)}{2}+\frac{\sqrt{-\frac{b^3}{c^5}}\log\left(x+\frac{c^2\sqrt{-\frac{b^3}{c^5}}}{b}\right)}{2}+\frac{x^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2),x)
```

```
[Out] -b*x/c**2 - sqrt(-b**3/c**5)*log(x - c**2*sqrt(-b**3/c**5)/b)/2 + sqrt(-b**
3/c**5)*log(x + c**2*sqrt(-b**3/c**5)/b)/2 + x**3/(3*c)
```

3.179 $\int \frac{x^5}{bx^2+cx^4}\,dx$

Optimal. Leaf size=27

$$\frac{x^2}{2c}-\frac{b\log\left(b+cx^2\right)}{2c^2}$$

```
[Out] 1/2*x^2/c-1/2*b*ln(c*x^2+b)/c^2
```

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{x^2}{2c}-\frac{b\log\left(b+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(b*x^2 + c*x^4),x]
```

```
[Out] x^2/(2*c) - (b*Log[b + c*x^2])/(2*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{bx^2+cx^4}\,dx &= \int \frac{x^3}{b+cx^2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{b+cx}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{c}-\frac{b}{c(b+cx)}\right)dx, x, x^2\right)\\ &= \frac{x^2}{2c}-\frac{b\log\left(b+cx^2\right)}{2c^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{x^2}{2c}-\frac{b\log\left(b+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(b*x^2 + c*x^4),x]
[Out] x^2/(2*c) - (b*Log[b + c*x^2])/(2*c^2)
```

fricas [A] time = 1.06, size = 22, normalized size = 0.81

$$\frac{cx^2-b\log\left(cx^2+b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/2*(c*x^2 - b*log(c*x^2 + b))/c^2
```

giac [A] time = 0.16, size = 24, normalized size = 0.89

$$\frac{x^2}{2\,c}-\frac{b\log\left(\left|cx^2+b\right|\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2),x, algorithm="giac")
[Out] 1/2*x^2/c - 1/2*b*log(abs(c*x^2 + b))/c^2
```

maple [A] time = 0.00, size = 24, normalized size = 0.89

$$\frac{x^2}{2c} - \frac{b\ln\left(c\,x^2 + b\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2),x)
[Out] 1/2*x^2/c-1/2*b*ln(c*x^2+b)/c^2
```

maxima [A] time = 1.31, size = 23, normalized size = 0.85

$$\frac{x^2}{2\,c} - \frac{b\log\left(cx^2 + b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 1/2*x^2/c - 1/2*b*log(c*x^2 + b)/c^2
```

mupad [B] time = 0.04, size = 22, normalized size = 0.81

$$-\frac{b\,\ln\left(c\,x^2 + b\right) - c\,x^2}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2 + c*x^4),x)
[Out] -(b*log(b + c*x^2) - c*x^2)/(2*c^2)
```

sympy [A] time = 0.17, size = 20, normalized size = 0.74

$$-\frac{b\log\left(b + cx^2\right)}{2c^2} + \frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2),x)
[Out] -b*log(b + c*x**2)/(2*c**2) + x**2/(2*c)
```

3.180 $\int \frac{x^4}{bx^2+cx^4} \, dx$

Optimal. Leaf size=31

$$\frac{x}{c} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{3/2}}$$

```
[Out] x/c-arctan(x*c^(1/2)/b^(1/2))*b^(1/2)/c^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 321, 205}

$$\frac{x}{c} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^2 + c*x^4),x]
```

```
[Out] x/c - (Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(3/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^4}{bx^2+cx^4}\,dx = \int \frac{x^2}{b+cx^2}\,dx$$
$$= \frac{x}{c} - \frac{b\int \frac{1}{b+cx^2}\,dx}{c}$$
$$= \frac{x}{c} - \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{3/2}}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{x}{c} - \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^2 + c*x^4),x]
```

```
[Out] x/c - (Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(3/2)
```

fricas [A] time = 0.89, size = 82, normalized size = 2.65

$$\left[\frac{\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)+2\,x}{2\,c},\,-\frac{\sqrt{\frac{b}{c}}\,\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)-x}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 2*x)/c,
-(sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) - x)/c]
```

giac [A] time = 0.17, size = 26, normalized size = 0.84

$$-\frac{b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c} + \frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c) + x/c
```

maple [A] time = 0.00, size = 27, normalized size = 0.87

$$-\frac{b \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\, c}+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2),x)
```

```
[Out] x/c-b/c/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)
```

maxima [A] time = 2.97, size = 26, normalized size = 0.84

$$-\frac{b \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\, c}+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c) + x/c
```

mupad [B] time = 0.04, size = 23, normalized size = 0.74

$$\frac{x}{c}-\frac{\sqrt{b}\, \mathrm{atan}\left(\frac{\sqrt{c}\, x}{\sqrt{b}}\right)}{c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2 + c*x^4),x)
```

```
[Out] x/c - (b^(1/2)*atan((c^(1/2)*x)/b^(1/2)))/c^(3/2)
```

sympy [B] time = 0.17, size = 56, normalized size = 1.81

$$\frac{\sqrt{-\frac{b}{c^3}}\log\left(-c\sqrt{-\frac{b}{c^3}}+x\right)}{2}-\frac{\sqrt{-\frac{b}{c^3}}\log\left(c\sqrt{-\frac{b}{c^3}}+x\right)}{2}+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2),x)
```

```
[Out] sqrt(-b/c**3)*log(-c*sqrt(-b/c**3) + x)/2 - sqrt(-b/c**3)*log(c*sqrt(-b/c**
3) + x)/2 + x/c
```

3.181 $\int \frac{x^3}{bx^2+cx^4}\,dx$

Optimal. Leaf size=15

$$\frac{\log\left(b+cx^2\right)}{2c}$$

```
[Out] 1/2*ln(c*x^2+b)/c
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 260}

$$\frac{\log\left(b+cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x^2 + c*x^4),x]
```

```
[Out] Log[b + c*x^2]/(2*c)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{bx^2+cx^4}\,dx &= \int \frac{x}{b+cx^2}\,dx\\ &= \frac{\log\left(b+cx^2\right)}{2c}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log\left(b+cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^2 + c*x^4),x]
```

```
[Out] Log[b + c*x^2]/(2*c)
```

fricas [A] time = 1.16, size = 13, normalized size = 0.87

$$\frac{\log\left(cx^2+b\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*log(c*x^2 + b)/c
```

giac [A] time = 0.15, size = 14, normalized size = 0.93

$$\frac{\log\left(\left|cx^2+b\right|\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(c*x^2 + b))/c
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\ln\left(c\,x^2+b\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2),x)
```

```
[Out] 1/2*ln(c*x^2+b)/c
```

maxima [A] time = 1.35, size = 13, normalized size = 0.87

$$\frac{\log\left(cx^2+b\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(c*x^2 + b)/c
```

mupad [B] time = 0.03, size = 13, normalized size = 0.87

$$\frac{\ln\left(c\,x^2+b\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2 + c*x^4),x)
[Out] log(b + c*x^2)/(2*c)
```

sympy [A] time = 0.13, size = 10, normalized size = 0.67

$$\frac{\log\left(b+cx^2\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2),x)
[Out] log(b + c*x**2)/(2*c)
```

3.182 $\int \frac{x^2}{bx^2+cx^4}\,dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{c}}$$

```
[Out] arctan(x*c^(1/2)/b^(1/2))/b^(1/2)/c^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^2 + c*x^4),x]
```

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(Sqrt[b]*Sqrt[c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{bx^2+cx^4}\,dx &= \int \frac{1}{b+cx^2}\,dx\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^2 + c*x^4),x]
```

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(Sqrt[b]*Sqrt[c])
```

fricas [A] time = 0.70, size = 67, normalized size = 2.79

$$\left[-\frac{\sqrt{-bc}\,\log\left(\frac{cx^2-2\,\sqrt{-bc}\,x-b}{cx^2+b}\right)}{2\,bc}, \frac{\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)}{bc}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b))/(b*c), sqrt(
b*c)*arctan(sqrt(b*c)*x/b)/(b*c)]
```

giac [A] time = 0.15, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] arctan(c*x/sqrt(b*c))/sqrt(b*c)
```

maple [A] time = 0.00, size = 16, normalized size = 0.67

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2),x)
```

```
[Out] 1/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)
```

maxima [A] time = 2.97, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] arctan(c*x/sqrt(b*c))/sqrt(b*c)
```

mupad [B] time = 4.20, size = 16, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2 + c*x^4),x)
```

```
[Out] atan((c^(1/2)*x)/b^(1/2))/(b^(1/2)*c^(1/2))
```

sympy [B] time = 0.15, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{bc}}\log\left(-b\sqrt{-\frac{1}{bc}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{bc}}\log\left(b\sqrt{-\frac{1}{bc}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2),x)
```

```
[Out] -sqrt(-1/(b*c))*log(-b*sqrt(-1/(b*c)) + x)/2 + sqrt(-1/(b*c))*log(b*sqrt(-1
/(b*c)) + x)/2
```

3.183 $\int \frac{x}{bx^2+cx^4}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{b}-\frac{\log\left(b+cx^2\right)}{2b}$$

```
[Out] ln(x)/b-1/2*ln(c*x^2+b)/b
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1584, 266, 36, 29, 31}

$$\frac{\log(x)}{b}-\frac{\log\left(b+cx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^2 + c*x^4),x]
```

```
[Out] Log[x]/b - Log[b + c*x^2]/(2*b)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{bx^2+cx^4}\,dx &= \int \frac{1}{x\left(b+cx^2\right)}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(b+cx)}\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2b}-\frac{c\,\text{Subst}\left(\int \frac{1}{b+cx}\,dx,x,x^2\right)}{2b}\\
&= \frac{\log(x)}{b}-\frac{\log\left(b+cx^2\right)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log(x)}{b}-\frac{\log\left(b+cx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^2 + c*x^4),x]
[Out] Log[x]/b - Log[b + c*x^2]/(2*b)
```

fricas [A] time = 0.84, size = 18, normalized size = 0.82

$$-\frac{\log\left(cx^2+b\right)-2\,\log(x)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] -1/2*(log(c*x^2 + b) - 2*log(x))/b
```

giac [A] time = 0.17, size = 22, normalized size = 1.00

$$-\frac{\log\left(\left|cx^2+b\right|\right)}{2\,b}+\frac{\log\left(|x|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(c*x^2 + b))/b + log(abs(x))/b
```

maple [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{\ln(x)}{b} - \frac{\ln\left(c\,x^2 + b\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2),x)
```

```
[Out] ln(x)/b-1/2*ln(c*x^2+b)/b
```

maxima [A] time = 1.36, size = 23, normalized size = 1.05

$$-\frac{\log\left(cx^2 + b\right)}{2\,b} + \frac{\log\left(x^2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*log(c*x^2 + b)/b + 1/2*log(x^2)/b
```

mupad [B] time = 0.06, size = 18, normalized size = 0.82

$$-\frac{\ln\left(c\,x^2 + b\right) - 2\,\ln(x)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2 + c*x^4),x)
```

```
[Out] -(log(b + c*x^2) - 2*log(x))/(2*b)
```

sympy [A] time = 0.23, size = 15, normalized size = 0.68

$$\frac{\log{(x)}}{b} - \frac{\log{\left(\frac{b}{c} + x^2\right)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2),x)
```

```
[Out] log(x)/b - log(b/c + x**2)/(2*b)
```

3.184 $\int \frac{1}{bx^2+cx^4}\,dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}$$

```
[Out] -1/b/x-arctan(x*c^(1/2)/b^(1/2))*c^(1/2)/b^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 325, 205}

$$-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(-1),x]
```

```
[Out] -(1/(b*x)) - (Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{bx^2+cx^4}\,dx = \int \frac{1}{x^2\left(b+cx^2\right)}\,dx$$
$$= -\frac{1}{bx} - \frac{c\int \frac{1}{b+cx^2}\,dx}{b}$$
$$= -\frac{1}{bx} - \frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$-\frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}} - \frac{1}{bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(-1),x]
[Out] -(1/(b*x)) - (Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2)
```

fricas [A] time = 0.62, size = 82, normalized size = 2.41

$$\left[\frac{x\sqrt{-\frac{c}{b}}\,\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)-2}{2\,bx},\,-\frac{x\sqrt{\frac{c}{b}}\,\arctan\left(x\sqrt{\frac{c}{b}}\right)+1}{bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] [1/2*(x*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)) - 2)/(b*
x), -(x*sqrt(c/b)*arctan(x*sqrt(c/b)) + 1)/(b*x)]
```

giac [A] time = 0.16, size = 29, normalized size = 0.85

$$-\frac{c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,b} - \frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2),x, algorithm="giac")
[Out] -c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) - 1/(b*x)
```

maple [A] time = 0.00, size = 30, normalized size = 0.88

$$-\frac{c \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\, b}-\frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2),x)
[Out] -c/b/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)-1/b/x
```

maxima [A] time = 2.88, size = 29, normalized size = 0.85

$$-\frac{c \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\, b}-\frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] -c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) - 1/(b*x)
```

mupad [B] time = 4.27, size = 26, normalized size = 0.76

$$-\frac{1}{b\,x}-\frac{\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 + c*x^4),x)
[Out] - 1/(b*x) - (c^(1/2)*atan((c^(1/2)*x)/b^(1/2)))/b^(3/2)
```

sympy [B] time = 0.19, size = 65, normalized size = 1.91

$$\frac{\sqrt{-\frac{c}{b^3}}\log\left(-\frac{b^2\sqrt{-\frac{c}{b^3}}}{c}+x\right)}{2}-\frac{\sqrt{-\frac{c}{b^3}}\log\left(\frac{b^2\sqrt{-\frac{c}{b^3}}}{c}+x\right)}{2}-\frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2),x)
[Out] sqrt(-c/b**3)*log(-b**2*sqrt(-c/b**3)/c + x)/2 - sqrt(-c/b**3)*log(b**2*sqr
t(-c/b**3)/c + x)/2 - 1/(b*x)
```

3.185 $\int \frac{1}{x(bx^2+cx^4)}\,dx$

Optimal. Leaf size=35

$$\frac{c\log\left(b+cx^2\right)}{2b^2}-\frac{c\log(x)}{b^2}-\frac{1}{2bx^2}$$

```
[Out] -1/2/b/x^2-c*ln(x)/b^2+1/2*c*ln(c*x^2+b)/b^2
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$\frac{c\log\left(b+cx^2\right)}{2b^2}-\frac{c\log(x)}{b^2}-\frac{1}{2bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(2*b*x^2) - (c*Log[x])/b^2 + (c*Log[b + c*x^2])/(2*b^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x\left(bx^2+cx^4\right)}\,dx=\int \frac{1}{x^3\left(b+cx^2\right)}\,dx$$
$$=\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(b+cx)}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{bx^2}-\frac{c}{b^2x}+\frac{c^2}{b^2(b+cx)}\right)dx,x,x^2\right)$$
$$=-\frac{1}{2bx^2}-\frac{c\log(x)}{b^2}+\frac{c\log\left(b+cx^2\right)}{2b^2}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{c\log\left(b+cx^2\right)}{2b^2}-\frac{c\log(x)}{b^2}-\frac{1}{2bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^2 + c*x^4)),x]
[Out] -1/2*1/(b*x^2) - (c*Log[x])/b^2 + (c*Log[b + c*x^2])/(2*b^2)
```

fricas [A] time = 0.72, size = 33, normalized size = 0.94

$$\frac{cx^2\log\left(cx^2+b\right)-2\,cx^2\log(x)-b}{2\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/2*(c*x^2*log(c*x^2 + b) - 2*c*x^2*log(x) - b)/(b^2*x^2)
```

giac [A] time = 0.15, size = 43, normalized size = 1.23

$$-\frac{c\log\left(x^2\right)}{2\,b^2}+\frac{c\log\left(\left|cx^2+b\right|\right)}{2\,b^2}+\frac{cx^2-b}{2\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2),x, algorithm="giac")
[Out] -1/2*c*log(x^2)/b^2 + 1/2*c*log(abs(c*x^2 + b))/b^2 + 1/2*(c*x^2 - b)/(b^2*
x^2)
```

maple [A] time = 0.01, size = 32, normalized size = 0.91

$$-\frac{c\ln(x)}{b^2}+\frac{c\ln\left(c\,x^2+b\right)}{2b^2}-\frac{1}{2b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2),x)
```

```
[Out] -1/2/b/x^2-c*ln(x)/b^2+1/2*c*ln(c*x^2+b)/b^2
```

maxima [A] time = 1.37, size = 33, normalized size = 0.94

$$\frac{c\log\left(cx^2+b\right)}{2\,b^2}-\frac{c\log\left(x^2\right)}{2\,b^2}-\frac{1}{2\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*c*log(c*x^2 + b)/b^2 - 1/2*c*log(x^2)/b^2 - 1/2/(b*x^2)
```

mupad [B] time = 0.06, size = 31, normalized size = 0.89

$$\frac{c\,\ln\left(c\,x^2+b\right)}{2\,b^2}-\frac{1}{2\,b\,x^2}-\frac{c\,\ln(x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + c*x^4)),x)
```

```
[Out] (c*log(b + c*x^2))/(2*b^2) - 1/(2*b*x^2) - (c*log(x))/b^2
```

sympy [A] time = 0.28, size = 31, normalized size = 0.89

$$-\frac{1}{2bx^2}-\frac{c\log\left(x\right)}{b^2}+\frac{c\log\left(\frac{b}{c}+x^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2),x)
```

```
[Out] -1/(2*b*x**2) - c*log(x)/b**2 + c*log(b/c + x**2)/(2*b**2)
```

3.186 $\int \frac{1}{x^2(bx^2+cx^4)} dx$

Optimal. Leaf size=43

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{5/2}}+\frac{c}{b^2x}-\frac{1}{3bx^3}$$

```
[Out] -1/3/b/x^3+c/b^2/x+c^(3/2)*arctan(x*c^(1/2)/b^(1/2))/b^(5/2)
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 325, 205}

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{5/2}}+\frac{c}{b^2x}-\frac{1}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(3*b*x^3) + c/(b^2*x) + (c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^2\left(bx^2+cx^4\right)}\,dx = \int \frac{1}{x^4\left(b+cx^2\right)}\,dx$$
$$= -\frac{1}{3bx^3} - \frac{c\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{b}$$
$$= -\frac{1}{3bx^3} + \frac{c}{b^2x} + \frac{c^2\int \frac{1}{b+cx^2}\,dx}{b^2}$$
$$= -\frac{1}{3bx^3} + \frac{c}{b^2x} + \frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{5/2}} + \frac{c}{b^2x} - \frac{1}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/3*1/(b*x^3) + c/(b^2*x) + (c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(5/2)
```

fricas [A] time = 0.74, size = 106, normalized size = 2.47

$$\left[\frac{3\,cx^3\sqrt{-\frac{c}{b}}\,\log\left(\frac{cx^2+2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)+6\,cx^2-2\,b}{6\,b^2x^3},\ \frac{3\,cx^3\sqrt{\frac{c}{b}}\,\arctan\left(x\sqrt{\frac{c}{b}}\right)+3\,cx^2-b}{3\,b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*c*x^3*sqrt(-c/b)*log((c*x^2 + 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)) +
6*c*x^2 - 2*b)/(b^2*x^3), 1/3*(3*c*x^3*sqrt(c/b)*arctan(x*sqrt(c/b)) + 3*c*
x^2 - b)/(b^2*x^3)]
```

giac [A] time = 0.17, size = 40, normalized size = 0.93

$$\frac{c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,b^2} + \frac{3\,cx^2-b}{3\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) + 1/3*(3*c*x^2 - b)/(b^2*x^3)
```

maple [A] time = 0.01, size = 39, normalized size = 0.91

$$\frac{c^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\ b^2} + \frac{c}{b^2 x} - \frac{1}{3b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2),x)
```

```
[Out] c^2/b^2/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)-1/3/b/x^3+c/b^2/x
```

maxima [A] time = 2.96, size = 40, normalized size = 0.93

$$\frac{c^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,b^2} + \frac{3\,cx^2 - b}{3\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) + 1/3*(3*c*x^2 - b)/(b^2*x^3)
```

mupad [B] time = 4.14, size = 37, normalized size = 0.86

$$\frac{c^{3/2} \operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{5/2}} - \frac{\frac{1}{3\,b} - \frac{c\,x^2}{b^2}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x^2 + c*x^4)),x)
```

```
[Out] (c^(3/2)*atan((c^(1/2)*x)/b^(1/2)))/b^(5/2) - (1/(3*b) - (c*x^2)/b^2)/x^3
```

sympy [B] time = 0.25, size = 87, normalized size = 2.02

$$-\frac{\sqrt{-\frac{c^3}{b^5}}\log\left(-\frac{b^3\sqrt{-\frac{c^3}{b^5}}}{c^2} + x\right)}{2} + \frac{\sqrt{-\frac{c^3}{b^5}}\log\left(\frac{b^3\sqrt{-\frac{c^3}{b^5}}}{c^2} + x\right)}{2} + \frac{-b + 3cx^2}{3b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2),x)
```

```
[Out] -sqrt(-c**3/b**5)*log(-b**3*sqrt(-c**3/b**5)/c**2 + x)/2 + sqrt(-c**3/b**5)
*log(b**3*sqrt(-c**3/b**5)/c**2 + x)/2 + (-b + 3*c*x**2)/(3*b**2*x**3)
```

3.187 $\int \frac{1}{x^3(bx^2+cx^4)}\,dx$

Optimal. Leaf size=49

$$-\frac{c^2\log\left(b+cx^2\right)}{2b^3}+\frac{c^2\log(x)}{b^3}+\frac{c}{2b^2x^2}-\frac{1}{4bx^4}$$

```
[Out] -1/4/b/x^4+1/2*c/b^2/x^2+c^2*ln(x)/b^3-1/2*c^2*ln(c*x^2+b)/b^3
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$-\frac{c^2\log\left(b+cx^2\right)}{2b^3}+\frac{c^2\log(x)}{b^3}+\frac{c}{2b^2x^2}-\frac{1}{4bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(4*b*x^4) + c/(2*b^2*x^2) + (c^2*Log[x])/b^3 - (c^2*Log[b + c*x^2])/(2*b
^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(bx^2+cx^4\right)}\,dx &= \int \frac{1}{x^5\left(b+cx^2\right)}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3(b+cx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{bx^3}-\frac{c}{b^2x^2}+\frac{c^2}{b^3x}-\frac{c^3}{b^3(b+cx)}\right)dx,x,x^2\right)\\
&= -\frac{1}{4bx^4}+\frac{c}{2b^2x^2}+\frac{c^2\log(x)}{b^3}-\frac{c^2\log\left(b+cx^2\right)}{2b^3}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$-\frac{c^2\log\left(b+cx^2\right)}{2b^3}+\frac{c^2\log(x)}{b^3}+\frac{c}{2b^2x^2}-\frac{1}{4bx^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*1/(b*x^4) + c/(2*b^2*x^2) + (c^2*Log[x])/b^3 - (c^2*Log[b + c*x^2])/(2
*b^3)
```

fricas [A] time = 0.84, size = 45, normalized size = 0.92

$$-\frac{2\,c^2x^4\log\left(cx^2+b\right)-4\,c^2x^4\log(x)-2\,bcx^2+b^2}{4\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*c^2*x^4*log(c*x^2 + b) - 4*c^2*x^4*log(x) - 2*b*c*x^2 + b^2)/(b^3*x
^4)
```

giac [A] time = 0.17, size = 57, normalized size = 1.16

$$\frac{c^2\log\left(x^2\right)}{2\,b^3}-\frac{c^2\log\left(\left|cx^2+b\right|\right)}{2\,b^3}-\frac{3\,c^2x^4-2\,bcx^2+b^2}{4\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*c^2*log(x^2)/b^3 - 1/2*c^2*log(abs(c*x^2 + b))/b^3 - 1/4*(3*c^2*x^4 - 2
*b*c*x^2 + b^2)/(b^3*x^4)
```

maple [A] time = 0.01, size = 44, normalized size = 0.90

$$\frac{c^2 \ln(x)}{b^3} - \frac{c^2 \ln\left(c\,x^2 + b\right)}{2b^3} + \frac{c}{2b^2 x^2} - \frac{1}{4b\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2),x)
```

```
[Out] -1/4/b/x^4+1/2*c/b^2/x^2+c^2*ln(x)/b^3-1/2*c^2*ln(c*x^2+b)/b^3
```

maxima [A] time = 1.32, size = 47, normalized size = 0.96

$$-\frac{c^2 \log\left(cx^2 + b\right)}{2\,b^3} + \frac{c^2 \log\left(x^2\right)}{2\,b^3} + \frac{2\,cx^2 - b}{4\,b^2 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*c^2*log(c*x^2 + b)/b^3 + 1/2*c^2*log(x^2)/b^3 + 1/4*(2*c*x^2 - b)/(b^2
*x^4)
```

mupad [B] time = 0.06, size = 46, normalized size = 0.94

$$\frac{c^2\,\ln(x)}{b^3} - \frac{c^2\,\ln\left(c\,x^2 + b\right)}{2\,b^3} - \frac{\frac{1}{4\,b} - \frac{c\,x^2}{2\,b^2}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^2 + c*x^4)),x)
```

```
[Out] (c^2*log(x))/b^3 - (c^2*log(b + c*x^2))/(2*b^3) - (1/(4*b) - (c*x^2)/(2*b^2
))/x^4
```

sympy [A] time = 0.36, size = 42, normalized size = 0.86

$$\frac{-b + 2cx^2}{4b^2 x^4} + \frac{c^2 \log{(x)}}{b^3} - \frac{c^2 \log{\left(\frac{b}{c} + x^2\right)}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2),x)
```

```
[Out] (-b + 2*c*x**2)/(4*b**2*x**4) + c**2*log(x)/b**3 - c**2*log(b/c + x**2)/(2*
b**3)
```

3.188 $\int \frac{1}{x^4(bx^2+cx^4)}\,dx$

Optimal. Leaf size=58

$$-\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}-\frac{c^2}{b^3x}+\frac{c}{3b^2x^3}-\frac{1}{5bx^5}$$

```
[Out] -1/5/b/x^5+1/3*c/b^2/x^3-c^2/b^3/x-c^(5/2)*arctan(x*c^(1/2)/b^(1/2))/b^(7/2
)
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 325, 205}

$$-\frac{c^2}{b^3x}-\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}+\frac{c}{3b^2x^3}-\frac{1}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(5*b*x^5) + c/(3*b^2*x^3) - c^2/(b^3*x) - (c^(5/2)*ArcTan[(Sqrt[c]*x)/Sq
rt[b]])/b^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\left(bx^2+cx^4\right)}\,dx &= \int \frac{1}{x^6\left(b+cx^2\right)}\,dx \\
&= -\frac{1}{5bx^5}-\frac{c\int \frac{1}{x^4\left(b+cx^2\right)}\,dx}{b} \\
&= -\frac{1}{5bx^5}+\frac{c}{3b^2x^3}+\frac{c^2\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{b^2} \\
&= -\frac{1}{5bx^5}+\frac{c}{3b^2x^3}-\frac{c^2}{b^3x}-\frac{c^3\int \frac{1}{b+cx^2}\,dx}{b^3} \\
&= -\frac{1}{5bx^5}+\frac{c}{3b^2x^3}-\frac{c^2}{b^3x}-\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 58, normalized size = 1.00

$$
-\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}-\frac{c^2}{b^3x}+\frac{c}{3b^2x^3}-\frac{1}{5bx^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/5*1/(b*x^5) + c/(3*b^2*x^3) - c^2/(b^3*x) - (c^(5/2)*ArcTan[(Sqrt[c]*x)/
Sqrt[b]])/b^(7/2)
```

fricas [A] time = 0.50, size = 132, normalized size = 2.28

$$
\left[\frac{15\,c^2x^5\sqrt{-\frac{c}{b}}\,\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)-30\,c^2x^4+10\,bcx^2-6\,b^2}{30\,b^3x^5},-\frac{15\,c^2x^5\sqrt{\frac{c}{b}}\,\arctan\left(x\sqrt{\frac{c}{b}}\right)+15\,c^2x^4-5\,bcx^2+3}{15\,b^3x^5}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*c^2*x^5*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)
) - 30*c^2*x^4 + 10*b*c*x^2 - 6*b^2)/(b^3*x^5), -1/15*(15*c^2*x^5*sqrt(c/b)
*arctan(x*sqrt(c/b)) + 15*c^2*x^4 - 5*b*c*x^2 + 3*b^2)/(b^3*x^5)]
```

giac [A] time = 0.17, size = 52, normalized size = 0.90

$$-\frac{c^{3}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,b^{3}}-\frac{15\,c^{2}x^{4}-5\,bcx^{2}+3\,b^{2}}{15\,b^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - 1/15*(15*c^2*x^4 - 5*b*c*x^2 +
 3*b^2)/(b^3*x^5)
```

maple [A] time = 0.01, size = 52, normalized size = 0.90

$$-\frac{c^{3}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\;b^{3}}-\frac{c^{2}}{b^{3}x}+\frac{c}{3b^{2}x^{3}}-\frac{1}{5b\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+b*x^2),x)
```

```
[Out] -c^3/b^3/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)-1/5/b/x^5-c^2/b^3/x+1/3*c/b^
2/x^3
```

maxima [A] time = 2.89, size = 52, normalized size = 0.90

$$-\frac{c^{3}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,b^{3}}-\frac{15\,c^{2}x^{4}-5\,bcx^{2}+3\,b^{2}}{15\,b^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - 1/15*(15*c^2*x^4 - 5*b*c*x^2 +
 3*b^2)/(b^3*x^5)
```

mupad [B] time = 0.05, size = 48, normalized size = 0.83

$$-\frac{\frac{1}{5\,b}-\frac{c\,x^{2}}{3\,b^{2}}+\frac{c^{2}\,x^{4}}{b^{3}}}{x^{5}}-\frac{c^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(b*x^2 + c*x^4)),x)
```

```
[Out] - (1/(5*b) - (c*x^2)/(3*b^2) + (c^2*x^4)/b^3)/x^5 - (c^(5/2)*atan((c^(1/2)*
x)/b^(1/2)))/b^(7/2)
```

sympy [B] time = 0.28, size = 100, normalized size = 1.72

$$\frac{\sqrt{-\frac{c^5}{b^7}}\log\left(-\frac{b^4\sqrt{-\frac{c^5}{b^7}}}{c^3}+x\right)}{2}-\frac{\sqrt{-\frac{c^5}{b^7}}\log\left(\frac{b^4\sqrt{-\frac{c^5}{b^7}}}{c^3}+x\right)}{2}+\frac{-3b^2+5bcx^2-15c^2x^4}{15b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+b*x**2),x)
```

```
[Out] sqrt(-c**5/b**7)*log(-b**4*sqrt(-c**5/b**7)/c**3 + x)/2 - sqrt(-c**5/b**7)*
log(b**4*sqrt(-c**5/b**7)/c**3 + x)/2 + (-3*b**2 + 5*b*c*x**2 - 15*c**2*x**
4)/(15*b**3*x**5)
```

3.189 $\int \frac{1}{x^5(bx^2+cx^4)} dx$

Optimal. Leaf size=63

$$\frac{c^3 \log(b+cx^2)}{2b^4} - \frac{c^3 \log(x)}{b^4} - \frac{c^2}{2b^3x^2} + \frac{c}{4b^2x^4} - \frac{1}{6bx^6}$$

```
[Out] -1/6/b/x^6+1/4*c/b^2/x^4-1/2*c^2/b^3/x^2-c^3*ln(x)/b^4+1/2*c^3*ln(c*x^2+b)/
b^4
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$-\frac{c^2}{2b^3x^2} + \frac{c^3 \log(b+cx^2)}{2b^4} - \frac{c^3 \log(x)}{b^4} + \frac{c}{4b^2x^4} - \frac{1}{6bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(6*b*x^6) + c/(4*b^2*x^4) - c^2/(2*b^3*x^2) - (c^3*Log[x])/b^4 + (c^3*Lo
g[b + c*x^2])/(2*b^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^5\left(bx^2+cx^4\right)}\,dx = \int \frac{1}{x^7\left(b+cx^2\right)}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4(b+cx)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{bx^4}-\frac{c}{b^2x^3}+\frac{c^2}{b^3x^2}-\frac{c^3}{b^4x}+\frac{c^4}{b^4(b+cx)}\right)dx, x, x^2\right)$$
$$= -\frac{1}{6bx^6}+\frac{c}{4b^2x^4}-\frac{c^2}{2b^3x^2}-\frac{c^3\log(x)}{b^4}+\frac{c^3\log\left(b+cx^2\right)}{2b^4}$$

Mathematica [A] time = 0.01, size = 63, normalized size = 1.00

$$\frac{c^3\log\left(b+cx^2\right)}{2b^4}-\frac{c^3\log(x)}{b^4}-\frac{c^2}{2b^3x^2}+\frac{c}{4b^2x^4}-\frac{1}{6bx^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/6*1/(b*x^6) + c/(4*b^2*x^4) - c^2/(2*b^3*x^2) - (c^3*Log[x])/b^4 + (c^3*
Log[b + c*x^2])/(2*b^4)
```

fricas [A] time = 0.53, size = 58, normalized size = 0.92

$$\frac{6\,c^3x^6\log\left(cx^2+b\right)-12\,c^3x^6\log(x)-6\,bc^2x^4+3\,b^2cx^2-2\,b^3}{12\,b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/12*(6*c^3*x^6*log(c*x^2 + b) - 12*c^3*x^6*log(x) - 6*b*c^2*x^4 + 3*b^2*c*
x^2 - 2*b^3)/(b^4*x^6)
```

giac [A] time = 0.15, size = 70, normalized size = 1.11

$$-\frac{c^3\log\left(x^2\right)}{2\,b^4}+\frac{c^3\log\left(\left|cx^2+b\right|\right)}{2\,b^4}+\frac{11\,c^3x^6-6\,bc^2x^4+3\,b^2cx^2-2\,b^3}{12\,b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*c^3*log(x^2)/b^4 + 1/2*c^3*log(abs(c*x^2 + b))/b^4 + 1/12*(11*c^3*x^6
- 6*b*c^2*x^4 + 3*b^2*c*x^2 - 2*b^3)/(b^4*x^6)
```

maple [A] time = 0.01, size = 56, normalized size = 0.89

$$-\frac{c^3\ln(x)}{b^4}+\frac{c^3\ln\left(c\,x^2+b\right)}{2b^4}-\frac{c^2}{2b^3x^2}+\frac{c}{4b^2x^4}-\frac{1}{6b\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2),x)
```

```
[Out] -1/6/b/x^6+1/4*c/b^2/x^4-1/2*c^2/b^3/x^2-c^3*ln(x)/b^4+1/2*c^3*ln(c*x^2+b)/
b^4
```

maxima [A] time = 1.32, size = 58, normalized size = 0.92

$$\frac{c^3\log\left(cx^2+b\right)}{2\,b^4}-\frac{c^3\log\left(x^2\right)}{2\,b^4}-\frac{6\,c^2x^4-3\,bcx^2+2\,b^2}{12\,b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*c^3*log(c*x^2 + b)/b^4 - 1/2*c^3*log(x^2)/b^4 - 1/12*(6*c^2*x^4 - 3*b*c
*x^2 + 2*b^2)/(b^3*x^6)
```

mupad [B] time = 0.07, size = 58, normalized size = 0.92

$$\frac{c^3\,\ln\left(c\,x^2+b\right)}{2\,b^4}-\frac{\frac{1}{6\,b}-\frac{c\,x^2}{4\,b^2}+\frac{c^2\,x^4}{2\,b^3}}{x^6}-\frac{c^3\,\ln(x)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(b*x^2 + c*x^4)),x)
```

```
[Out] (c^3*log(b + c*x^2))/(2*b^4) - (1/(6*b) - (c*x^2)/(4*b^2) + (c^2*x^4)/(2*b^
3))/x^6 - (c^3*log(x))/b^4
```

sympy [A] time = 0.40, size = 56, normalized size = 0.89

$$\frac{-2b^2+3bcx^2-6c^2x^4}{12b^3x^6}-\frac{c^3\log(x)}{b^4}+\frac{c^3\log\left(\frac{b}{c}+x^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2),x)
```

```
[Out] (-2*b**2 + 3*b*c*x**2 - 6*c**2*x**4)/(12*b**3*x**6) - c**3*log(x)/b**4 + c*
*3*log(b/c + x**2)/(2*b**4)
```

3.190 $\int \frac{x^{12}}{\left(bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=79

$$-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{9/2}}+\frac{7b^2x}{2c^4}-\frac{7bx^3}{6c^3}-\frac{x^7}{2c\left(b+cx^2\right)}+\frac{7x^5}{10c^2}$$

```
[Out] 7/2*b^2*x/c^4-7/6*b*x^3/c^3+7/10*x^5/c^2-1/2*x^7/c/(c*x^2+b)-7/2*b^(5/2)*ar
ctan(x*c^(1/2)/b^(1/2))/c^(9/2)
```

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 288, 302, 205}

$$\frac{7b^2x}{2c^4}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{9/2}}-\frac{7bx^3}{6c^3}-\frac{x^7}{2c\left(b+cx^2\right)}+\frac{7x^5}{10c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (7*b^2*x)/(2*c^4) - (7*b*x^3)/(6*c^3) + (7*x^5)/(10*c^2) - x^7/(2*c*(b + c*
x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{12}}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^8}{\left(b+cx^2\right)^2}\,dx\\
&= -\frac{x^7}{2c\left(b+cx^2\right)}+\frac{7\int \frac{x^6}{b+cx^2}\,dx}{2c}\\
&= -\frac{x^7}{2c\left(b+cx^2\right)}+\frac{7\int \left(\frac{b^2}{c^3}-\frac{bx^2}{c^2}+\frac{x^4}{c}-\frac{b^3}{c^3\left(b+cx^2\right)}\right)dx}{2c}\\
&= \frac{7b^2x}{2c^4}-\frac{7bx^3}{6c^3}+\frac{7x^5}{10c^2}-\frac{x^7}{2c\left(b+cx^2\right)}-\frac{\left(7b^3\right)\int \frac{1}{b+cx^2}\,dx}{2c^4}\\
&= \frac{7b^2x}{2c^4}-\frac{7bx^3}{6c^3}+\frac{7x^5}{10c^2}-\frac{x^7}{2c\left(b+cx^2\right)}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 71, normalized size = 0.90

$$\frac{x\left(\frac{15b^3}{b+cx^2}+90b^2-20bcx^2+6c^2x^4\right)}{30c^4}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(90*b^2 - 20*b*c*x^2 + 6*c^2*x^4 + (15*b^3)/(b + c*x^2)))/(30*c^4) - (7*
b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(9/2))
```

fricas [A] time = 0.62, size = 190, normalized size = 2.41

$$\left[\frac{12\,c^3x^7-28\,bc^2x^5+140\,b^2cx^3+210\,b^3x+105\left(b^2cx^2+b^3\right)\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)}{60\left(c^5x^2+bc^4\right)},\frac{6\,c^3x^7-14\,bc^2x^5+70\,b^2c\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*c^3*x^7 - 28*b*c^2*x^5 + 140*b^2*c*x^3 + 210*b^3*x + 105*(b^2*c*x
^2 + b^3)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)))/(c^5*
x^2 + b*c^4), 1/30*(6*c^3*x^7 - 14*b*c^2*x^5 + 70*b^2*c*x^3 + 105*b^3*x - 1
05*(b^2*c*x^2 + b^3)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b))/(c^5*x^2 + b*c^4)]
```

giac [A] time = 0.16, size = 73, normalized size = 0.92

$$-\frac{7\,b^{3}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^{4}}+\frac{b^{3}x}{2\,\left(cx^{2}+b\right)c^{4}}+\frac{3\,c^{8}x^{5}-10\,bc^{7}x^{3}+45\,b^{2}c^{6}x}{15\,c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -7/2*b^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/2*b^3*x/((c*x^2 + b)*c^4
) + 1/15*(3*c^8*x^5 - 10*b*c^7*x^3 + 45*b^2*c^6*x)/c^10
```

maple [A] time = 0.01, size = 68, normalized size = 0.86

$$\frac{x^{5}}{5c^{2}}-\frac{2b\,x^{3}}{3c^{3}}+\frac{b^{3}x}{2\left(c\,x^{2}+b\right)c^{4}}-\frac{7b^{3}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;c^{4}}+\frac{3b^{2}x}{c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/5*x^5/c^2-2/3*b*x^3/c^3+3*b^2*x/c^4+1/2/c^4*b^3*x/(c*x^2+b)-7/2/c^4*b^3/(
b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)
```

maxima [A] time = 2.91, size = 71, normalized size = 0.90

$$\frac{b^{3}x}{2\left(c^{5}x^{2}+bc^{4}\right)}-\frac{7\,b^{3}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^{4}}+\frac{3\,c^{2}x^{5}-10\,bcx^{3}+45\,b^{2}x}{15\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x/(c^5*x^2 + b*c^4) - 7/2*b^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4)
 + 1/15*(3*c^2*x^5 - 10*b*c*x^3 + 45*b^2*x)/c^4
```

mupad [B] time = 0.04, size = 66, normalized size = 0.84

$$\frac{x^5}{5\,c^2}-\frac{2\,b\,x^3}{3\,c^3}+\frac{3\,b^2\,x}{c^4}-\frac{7\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,c^{9/2}}+\frac{b^3\,x}{2\,\left(c^5\,x^2+b\,c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^5/(5*c^2) - (2*b*x^3)/(3*c^3) + (3*b^2*x)/c^4 - (7*b^(5/2)*atan((c^(1/2)*
x)/b^(1/2)))/(2*c^(9/2)) + (b^3*x)/(2*(b*c^4 + c^5*x^2))
```

sympy [A] time = 0.36, size = 124, normalized size = 1.57

$$\frac{b^3x}{2bc^4+2c^5x^2}+\frac{3b^2x}{c^4}-\frac{2bx^3}{3c^3}+\frac{7\sqrt{-\frac{b^5}{c^9}}\log\left(x-\frac{c^4\sqrt{-\frac{b^5}{c^9}}}{b^2}\right)}{4}-\frac{7\sqrt{-\frac{b^5}{c^9}}\log\left(x+\frac{c^4\sqrt{-\frac{b^5}{c^9}}}{b^2}\right)}{4}+\frac{x^5}{5c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(c*x**4+b*x**2)**2,x)
```

```
[Out] b**3*x/(2*b*c**4 + 2*c**5*x**2) + 3*b**2*x/c**4 - 2*b*x**3/(3*c**3) + 7*sqr
t(-b**5/c**9)*log(x - c**4*sqrt(-b**5/c**9)/b**2)/4 - 7*sqrt(-b**5/c**9)*lo
g(x + c**4*sqrt(-b**5/c**9)/b**2)/4 + x**5/(5*c**2)
```

3.191 $\int \frac{x^{11}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=57

$$\frac{b^3}{2c^4(b+cx^2)}+\frac{3b^2\log(b+cx^2)}{2c^4}-\frac{bx^2}{c^3}+\frac{x^4}{4c^2}$$

```
[Out] -b*x^2/c^3+1/4*x^4/c^2+1/2*b^3/c^4/(c*x^2+b)+3/2*b^2*ln(c*x^2+b)/c^4
```

Rubi [A] time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{b^3}{2c^4(b+cx^2)}+\frac{3b^2\log(b+cx^2)}{2c^4}-\frac{bx^2}{c^3}+\frac{x^4}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b*x^2)/c^3) + x^4/(4*c^2) + b^3/(2*c^4*(b + c*x^2)) + (3*b^2*Log[b + c*x
^2])/(2*c^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^7}{\left(b+cx^2\right)^2}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{(b+cx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{2b}{c^3}+\frac{x}{c^2}-\frac{b^3}{c^3(b+cx)^2}+\frac{3b^2}{c^3(b+cx)}\right)dx,x,x^2\right)\\ &= -\frac{bx^2}{c^3}+\frac{x^4}{4c^2}+\frac{b^3}{2c^4\left(b+cx^2\right)}+\frac{3b^2\log\left(b+cx^2\right)}{2c^4}\end{aligned}$$

Mathematica [**A**] time = 0.02, size = 49, normalized size = 0.86

$$\frac{\frac{2b^3}{b+cx^2}+6b^2\log\left(b+cx^2\right)-4bcx^2+c^2x^4}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-4*b*c*x^2 + c^2*x^4 + (2*b^3)/(b + c*x^2) + 6*b^2*Log[b + c*x^2])/(4*c^4)
```

fricas [**A**] time = 0.80, size = 70, normalized size = 1.23

$$\frac{c^3x^6-3\,bc^2x^4-4\,b^2cx^2+2\,b^3+6\left(b^2cx^2+b^3\right)\log\left(cx^2+b\right)}{4\left(c^5x^2+bc^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(c^3*x^6 - 3*b*c^2*x^4 - 4*b^2*c*x^2 + 2*b^3 + 6*(b^2*c*x^2 + b^3)*log(
c*x^2 + b))/(c^5*x^2 + b*c^4)
```

giac [**A**] time = 0.18, size = 67, normalized size = 1.18

$$\frac{3\,b^2\log\left(\left|cx^2+b\right|\right)}{2\,c^4}+\frac{c^2x^4-4\,bcx^2}{4\,c^4}-\frac{3\,b^2cx^2+2\,b^3}{2\left(cx^2+b\right)c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 3/2*b^2*log(abs(c*x^2 + b))/c^4 + 1/4*(c^2*x^4 - 4*b*c*x^2)/c^4 - 1/2*(3*b^2*c*x^2 + 2*b^3)/((c*x^2 + b)*c^4)
```

maple [A] time = 0.01, size = 52, normalized size = 0.91

$$\frac{x^4}{4c^2} - \frac{b\,x^2}{c^3} + \frac{b^3}{2\left(c\,x^2+b\right)c^4} + \frac{3b^2\ln\left(c\,x^2+b\right)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^4+b*x^2)^2,x)
```

```
[Out] -b*x^2/c^3+1/4*x^4/c^2+1/2*b^3/c^4/(c*x^2+b)+3/2*b^2*ln(c*x^2+b)/c^4
```

maxima [A] time = 1.32, size = 54, normalized size = 0.95

$$\frac{b^3}{2\left(c^5x^2+bc^4\right)} + \frac{3\,b^2\log\left(cx^2+b\right)}{2\,c^4} + \frac{cx^4-4\,bx^2}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^3/(c^5*x^2 + b*c^4) + 3/2*b^2*log(c*x^2 + b)/c^4 + 1/4*(c*x^4 - 4*b*x^2)/c^3
```

mupad [B] time = 4.14, size = 57, normalized size = 1.00

$$\frac{x^4}{4\,c^2} + \frac{b^3}{2\,c\,\left(c^4\,x^2+b\,c^3\right)} - \frac{b\,x^2}{c^3} + \frac{3\,b^2\,\ln\left(c\,x^2+b\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^4/(4*c^2) + b^3/(2*c*(b*c^3 + c^4*x^2)) - (b*x^2)/c^3 + (3*b^2*log(b + c*x^2))/(2*c^4)
```

sympy [A] time = 0.30, size = 53, normalized size = 0.93

$$\frac{b^3}{2bc^4+2c^5x^2} + \frac{3b^2\log\left(b+cx^2\right)}{2c^4} - \frac{bx^2}{c^3} + \frac{x^4}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**4+b*x**2)**2,x)
```

```
[Out] b**3/(2*b*c**4 + 2*c**5*x**2) + 3*b**2*log(b + c*x**2)/(2*c**4) - b*x**2/c*
*3 + x**4/(4*c**2)
```

3.192 $\int \frac{x^{10}}{\left(bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=66

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{7/2}}-\frac{5bx}{2c^3}-\frac{x^5}{2c\left(b+cx^2\right)}+\frac{5x^3}{6c^2}$$

```
[Out] -5/2*b*x/c^3+5/6*x^3/c^2-1/2*x^5/c/(c*x^2+b)+5/2*b^(3/2)*arctan(x*c^(1/2)/b
^(1/2))/c^(7/2)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 288, 302, 205}

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{7/2}}-\frac{5bx}{2c^3}-\frac{x^5}{2c\left(b+cx^2\right)}+\frac{5x^3}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-5*b*x)/(2*c^3) + (5*x^3)/(6*c^2) - x^5/(2*c*(b + c*x^2)) + (5*b^(3/2)*Arc
Tan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{10}}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^6}{\left(b+cx^2\right)^2}\,dx\\
&= -\frac{x^5}{2c\left(b+cx^2\right)}+\frac{5\int \frac{x^4}{b+cx^2}\,dx}{2c}\\
&= -\frac{x^5}{2c\left(b+cx^2\right)}+\frac{5\int \left(-\frac{b}{c^2}+\frac{x^2}{c}+\frac{b^2}{c^2\left(b+cx^2\right)}\right)dx}{2c}\\
&= -\frac{5bx}{2c^3}+\frac{5x^3}{6c^2}-\frac{x^5}{2c\left(b+cx^2\right)}+\frac{\left(5b^2\right)\int \frac{1}{b+cx^2}\,dx}{2c^3}\\
&= -\frac{5bx}{2c^3}+\frac{5x^3}{6c^2}-\frac{x^5}{2c\left(b+cx^2\right)}+\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.91

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{7/2}}+\frac{x\left(-\frac{3b^2}{b+cx^2}-12b+2cx^2\right)}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(-12*b + 2*c*x^2 - (3*b^2)/(b + c*x^2)))/(6*c^3) + (5*b^(3/2)*ArcTan[(Sq
rt[c]*x)/Sqrt[b]])/(2*c^(7/2))
```

fricas [A] time = 0.72, size = 164, normalized size = 2.48

$$\left[\frac{4\,c^2x^5-20\,bcx^3-30\,b^2x+15\left(bcx^2+b^2\right)\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)}{12\left(c^4x^2+bc^3\right)},\frac{2\,c^2x^5-10\,bcx^3-15\,b^2x+15\left(bcx^2+b^2\right)\sqrt{}}{6\left(c^4x^2+bc^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*c^2*x^5 - 20*b*c*x^3 - 30*b^2*x + 15*(b*c*x^2 + b^2)*sqrt(-b/c)*lo
g((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)))/(c^4*x^2 + b*c^3), 1/6*(2*c^
2*x^5 - 10*b*c*x^3 - 15*b^2*x + 15*(b*c*x^2 + b^2)*sqrt(b/c)*arctan(c*x*sqr
t(b/c)/b))/(c^4*x^2 + b*c^3)]
```

giac [A] time = 0.17, size = 61, normalized size = 0.92

$$\frac{5\,b^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^3} - \frac{b^2 x}{2\left(cx^2+b\right)c^3} + \frac{c^4x^3 - 6\,bc^3x}{3\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 5/2*b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) - 1/2*b^2*x/((c*x^2 + b)*c^3)
 + 1/3*(c^4*x^3 - 6*b*c^3*x)/c^6
```

maple [A] time = 0.01, size = 57, normalized size = 0.86

$$\frac{x^3}{3c^2} - \frac{b^2 x}{2\left(c\,x^2+b\right)c^3} + \frac{5b^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;c^3} - \frac{2bx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/3*x^3/c^2-2*b*x/c^3-1/2/c^3*b^2*x/(c*x^2+b)+5/2/c^3*b^2/(b*c)^(1/2)*arcta
n(1/(b*c)^(1/2)*c*x)
```

maxima [A] time = 2.93, size = 59, normalized size = 0.89

$$-\frac{b^2 x}{2\left(c^4x^2+bc^3\right)} + \frac{5\,b^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^3} + \frac{cx^3 - 6\,bx}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*x/(c^4*x^2 + b*c^3) + 5/2*b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3
) + 1/3*(c*x^3 - 6*b*x)/c^3
```

mupad [B] time = 0.06, size = 56, normalized size = 0.85

$$\frac{x^3}{3\,c^2}+\frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,c^{7/2}}-\frac{b^2\,x}{2\,\left(c^4\,x^2+b\,c^3\right)}-\frac{2\,b\,x}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^3/(3*c^2) + (5*b^(3/2)*atan((c^(1/2)*x)/b^(1/2)))/(2*c^(7/2)) - (b^2*x)/(
2*(b*c^3 + c^4*x^2)) - (2*b*x)/c^3
```

sympy [A] time = 0.33, size = 107, normalized size = 1.62

$$-\frac{b^2x}{2bc^3+2c^4x^2}-\frac{2bx}{c^3}-\frac{5\sqrt{-\frac{b^3}{c^7}}\,\log\left(x-\frac{c^3\sqrt{-\frac{b^3}{c^7}}}{b}\right)}{4}+\frac{5\sqrt{-\frac{b^3}{c^7}}\,\log\left(x+\frac{c^3\sqrt{-\frac{b^3}{c^7}}}{b}\right)}{4}+\frac{x^3}{3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**4+b*x**2)**2,x)
```

```
[Out] -b**2*x/(2*b*c**3 + 2*c**4*x**2) - 2*b*x/c**3 - 5*sqrt(-b**3/c**7)*log(x -
c**3*sqrt(-b**3/c**7)/b)/4 + 5*sqrt(-b**3/c**7)*log(x + c**3*sqrt(-b**3/c**
7)/b)/4 + x**3/(3*c**2)
```

3.193 $\int \frac{x^9}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=44

$$-\frac{b^2}{2c^3\left(b+cx^2\right)}-\frac{b\log\left(b+cx^2\right)}{c^3}+\frac{x^2}{2c^2}$$

```
[Out] 1/2*x^2/c^2-1/2*b^2/c^3/(c*x^2+b)-b*ln(c*x^2+b)/c^3
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$-\frac{b^2}{2c^3\left(b+cx^2\right)}-\frac{b\log\left(b+cx^2\right)}{c^3}+\frac{x^2}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(b*x^2 + c*x^4)^2,x]
```

```
[Out] x^2/(2*c^2) - b^2/(2*c^3*(b + c*x^2)) - (b*Log[b + c*x^2])/c^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^9}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^5}{\left(b+cx^2\right)^2}\,dx$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(b+cx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{c^2}+\frac{b^2}{c^2(b+cx)^2}-\frac{2b}{c^2(b+cx)}\right)dx, x, x^2\right)$$
$$= \frac{x^2}{2c^2}-\frac{b^2}{2c^3\left(b+cx^2\right)}-\frac{b\log\left(b+cx^2\right)}{c^3}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 0.86

$$\frac{-\frac{b^2}{b+cx^2}-2b\log\left(b+cx^2\right)+cx^2}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (c*x^2 - b^2/(b + c*x^2) - 2*b*Log[b + c*x^2])/(2*c^3)
```

fricas [A] time = 0.48, size = 56, normalized size = 1.27

$$\frac{c^2x^4+bcx^2-b^2-2\left(bcx^2+b^2\right)\log\left(cx^2+b\right)}{2\left(c^4x^2+bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(c^2*x^4 + b*c*x^2 - b^2 - 2*(b*c*x^2 + b^2)*log(c*x^2 + b))/(c^4*x^2 +
 b*c^3)
```

giac [A] time = 0.17, size = 49, normalized size = 1.11

$$\frac{x^2}{2\,c^2}-\frac{b\log\left(\left|cx^2+b\right|\right)}{c^3}+\frac{2\,bcx^2+b^2}{2\left(cx^2+b\right)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2/c^2 - b*log(abs(c*x^2 + b))/c^3 + 1/2*(2*b*c*x^2 + b^2)/((c*x^2 + b
)*c^3)
```

maple [A] time = 0.01, size = 41, normalized size = 0.93

$$\frac{x^2}{2c^2} - \frac{b^2}{2\left(c\,x^2+b\right)c^3} - \frac{b\ln\left(c\,x^2+b\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*x^2/c^2-1/2*b^2/c^3/(c*x^2+b)-b*ln(c*x^2+b)/c^3
```

maxima [A] time = 1.32, size = 43, normalized size = 0.98

$$-\frac{b^2}{2\left(c^4x^2+bc^3\right)} + \frac{x^2}{2\,c^2} - \frac{b\log\left(cx^2+b\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b^2/(c^4*x^2 + b*c^3) + 1/2*x^2/c^2 - b*log(c*x^2 + b)/c^3
```

mupad [B] time = 0.04, size = 45, normalized size = 1.02

$$\frac{x^2}{2\,c^2} - \frac{b^2}{2\,\left(c^4\,x^2+b\,c^3\right)} - \frac{b\,\ln\left(c\,x^2+b\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^2/(2*c^2) - b^2/(2*(b*c^3 + c^4*x^2)) - (b*log(b + c*x^2))/c^3
```

sympy [A] time = 0.28, size = 39, normalized size = 0.89

$$-\frac{b^2}{2bc^3+2c^4x^2} - \frac{b\log\left(b+cx^2\right)}{c^3} + \frac{x^2}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2)**2,x)
```

```
[Out] -b**2/(2*b*c**3 + 2*c**4*x**2) - b*log(b + c*x**2)/c**3 + x**2/(2*c**2)
```

3.194 $\int \frac{x^8}{\left(bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=55

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{5/2}}-\frac{x^3}{2c\left(b+cx^2\right)}+\frac{3x}{2c^2}$$

```
[Out] 3/2*x/c^2-1/2*x^3/c/(c*x^2+b)-3/2*arctan(x*c^(1/2)/b^(1/2))*b^(1/2)/c^(5/2)
```

Rubi [A] time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 288, 321, 205}

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{5/2}}-\frac{x^3}{2c\left(b+cx^2\right)}+\frac{3x}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (3*x)/(2*c^2) - x^3/(2*c*(b + c*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[
b]])/(2*c^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^4}{\left(b+cx^2\right)^2}\,dx\\ &= -\frac{x^3}{2c\left(b+cx^2\right)}+\frac{3\int \frac{x^2}{b+cx^2}\,dx}{2c}\\ &= \frac{3x}{2c^2}-\frac{x^3}{2c\left(b+cx^2\right)}-\frac{(3b)\int \frac{1}{b+cx^2}\,dx}{2c^2}\\ &= \frac{3x}{2c^2}-\frac{x^3}{2c\left(b+cx^2\right)}-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.93

$$-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{5/2}}+\frac{bx}{2c^2\left(b+cx^2\right)}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(b*x^2 + c*x^4)^2,x]
```

```
[Out] x/c^2 + (b*x)/(2*c^2*(b + c*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])
/(2*c^(5/2))
```

fricas [A] time = 0.62, size = 136, normalized size = 2.47

$$\left[\frac{4\,cx^3+3\left(cx^2+b\right)\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)+6\,bx}{4\left(c^3x^2+bc^2\right)},\ \frac{2\,cx^3-3\left(cx^2+b\right)\sqrt{\frac{b}{c}}\,\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)+3\,bx}{2\left(c^3x^2+bc^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c*x^3 + 3*(c*x^2 + b)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)
/(c*x^2 + b)) + 6*b*x)/(c^3*x^2 + b*c^2), 1/2*(2*c*x^3 - 3*(c*x^2 + b)*sqrt
(b/c)*arctan(c*x*sqrt(b/c)/b) + 3*b*x)/(c^3*x^2 + b*c^2)]
```

giac [A] time = 0.17, size = 42, normalized size = 0.76

$$-\frac{3\,b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^2}+\frac{bx}{2\left(cx^2+b\right)c^2}+\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/2*b*x/((c*x^2 + b)*c^2) +
x/c^2
```

maple [A] time = 0.01, size = 43, normalized size = 0.78

$$\frac{bx}{2\left(c\,x^2+b\right)c^2}-\frac{3b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;c^2}+\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2)^2,x)
```

```
[Out] x/c^2+1/2/c^2*b*x/(c*x^2+b)-3/2/c^2*b/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)
```

maxima [A] time = 2.94, size = 45, normalized size = 0.82

$$\frac{bx}{2\left(c^3x^2+bc^2\right)}-\frac{3\,b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^2}+\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*x/(c^3*x^2 + b*c^2) - 3/2*b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + x
/c^2
```

mupad [B] time = 4.17, size = 43, normalized size = 0.78

$$\frac{x}{c^2}+\frac{b\,x}{2\,\left(c^3\,x^2+b\,c^2\right)}-\frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x/c^2 + (b*x)/(2*(b*c^2 + c^3*x^2)) - (3*b^(1/2)*atan((c^(1/2)*x)/b^(1/2)))
/(2*c^(5/2))
```

sympy [A] time = 0.28, size = 83, normalized size = 1.51

$$\frac{bx}{2bc^2+2c^3x^2}+\frac{3\sqrt{-\frac{b}{c^5}}\log\left(-c^2\sqrt{-\frac{b}{c^5}}+x\right)}{4}-\frac{3\sqrt{-\frac{b}{c^5}}\log\left(c^2\sqrt{-\frac{b}{c^5}}+x\right)}{4}+\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2)**2,x)
```

```
[Out] b*x/(2*b*c**2 + 2*c**3*x**2) + 3*sqrt(-b/c**5)*log(-c**2*sqrt(-b/c**5) + x)
/4 - 3*sqrt(-b/c**5)*log(c**2*sqrt(-b/c**5) + x)/4 + x/c**2
```

3.195 $\int \frac{x^7}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=33

$$\frac{b}{2c^2\left(b+cx^2\right)}+\frac{\log\left(b+cx^2\right)}{2c^2}$$

```
[Out] 1/2*b/c^2/(c*x^2+b)+1/2*ln(c*x^2+b)/c^2
```

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{b}{2c^2\left(b+cx^2\right)}+\frac{\log\left(b+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(b*x^2 + c*x^4)^2,x]
```

```
[Out] b/(2*c^2*(b + c*x^2)) + Log[b + c*x^2]/(2*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^3}{\left(b+cx^2\right)^2}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(b+cx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{b}{c(b+cx)^2}+\frac{1}{c(b+cx)}\right)dx,x,x^2\right)\\ &= \frac{b}{2c^2\left(b+cx^2\right)}+\frac{\log\left(b+cx^2\right)}{2c^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.82

$$\frac{\frac{b}{b+cx^2}+\log\left(b+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(b*x^2 + c*x^4)^2,x]
[Out] (b/(b + c*x^2) + Log[b + c*x^2])/(2*c^2)
```

fricas [A] time = 0.63, size = 35, normalized size = 1.06

$$\frac{\left(cx^2+b\right)\log\left(cx^2+b\right)+b}{2\left(c^3x^2+bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^2,x, algorithm="fricas")
[Out] 1/2*((c*x^2 + b)*log(c*x^2 + b) + b)/(c^3*x^2 + b*c^2)
```

giac [A] time = 0.15, size = 32, normalized size = 0.97

$$-\frac{x^2}{2\left(cx^2+b\right)c}+\frac{\log\left(\left|cx^2+b\right|\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*x^2/((c*x^2 + b)*c) + 1/2*log(abs(c*x^2 + b))/c^2
```

maple [A] time = 0.01, size = 30, normalized size = 0.91

$$\frac{b}{2\left(c\,x^{2}+b\right)c^{2}}+\frac{\ln\left(c\,x^{2}+b\right)}{2c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*b/c^2/(c*x^2+b)+1/2*ln(c*x^2+b)/c^2
```

maxima [A] time = 1.29, size = 32, normalized size = 0.97

$$\frac{b}{2\left(c^{3}x^{2}+bc^{2}\right)}+\frac{\log\left(cx^{2}+b\right)}{2\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b/(c^3*x^2 + b*c^2) + 1/2*log(c*x^2 + b)/c^2
```

mupad [B] time = 4.18, size = 29, normalized size = 0.88

$$\frac{\ln\left(c\,x^{2}+b\right)}{2\,c^{2}}+\frac{b}{2\,c^{2}\,\left(c\,x^{2}+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2 + c*x^4)^2,x)
```

```
[Out] log(b + c*x^2)/(2*c^2) + b/(2*c^2*(b + c*x^2))
```

sympy [A] time = 0.22, size = 29, normalized size = 0.88

$$\frac{b}{2bc^{2}+2c^{3}x^{2}}+\frac{\log\left(b+cx^{2}\right)}{2c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2)**2,x)
```

```
[Out] b/(2*b*c**2 + 2*c**3*x**2) + log(b + c*x**2)/(2*c**2)
```

3.196 $\int \frac{x^6}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\sqrt{b}\,c^{3/2}}-\frac{x}{2c\left(b+cx^2\right)}$$

```
[Out] -1/2*x/c/(c*x^2+b)+1/2*arctan(x*c^(1/2)/b^(1/2))/c^(3/2)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 288, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\sqrt{b}\,c^{3/2}}-\frac{x}{2c\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -x/(2*c*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*Sqrt[b]*c^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^6}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^2}{\left(b+cx^2\right)^2}\,dx$$
$$= -\frac{x}{2c\left(b+cx^2\right)} + \frac{\int \frac{1}{b+cx^2}\,dx}{2c}$$
$$= -\frac{x}{2c\left(b+cx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\sqrt{b}\,c^{3/2}}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\sqrt{b}\,c^{3/2}} - \frac{x}{2c\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(b*x^2 + c*x^4)^2,x]
[Out] -1/2*x/(c*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*Sqrt[b]*c^(3/2))
```

fricas [A] time = 0.64, size = 120, normalized size = 2.67

$$\left[-\frac{2\,bcx + \left(cx^2+b\right)\sqrt{-bc}\,\log\left(\frac{cx^2-2\,\sqrt{-bc}\,x-b}{cx^2+b}\right)}{4\left(bc^3x^2+b^2c^2\right)}, -\frac{bcx - \left(cx^2+b\right)\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)}{2\left(bc^3x^2+b^2c^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^2,x, algorithm="fricas")
[Out] [-1/4*(2*b*c*x + (c*x^2 + b)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c
*x^2 + b)))/(b*c^3*x^2 + b^2*c^2), -1/2*(b*c*x - (c*x^2 + b)*sqrt(b*c)*arct
an(sqrt(b*c)*x/b))/(b*c^3*x^2 + b^2*c^2)]
```

giac [A] time = 0.17, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c} - \frac{x}{2\left(cx^2+b\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^2,x, algorithm="giac")
[Out] 1/2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c) - 1/2*x/((c*x^2 + b)*c)
```

maple [A] time = 0.01, size = 36, normalized size = 0.80

$$-\frac{x}{2\left(c\,x^{2}+b\right)c}+\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2)^2,x)
[Out] -1/2*x/c/(c*x^2+b)+1/2/c/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)
```

maxima [A] time = 3.02, size = 36, normalized size = 0.80

$$-\frac{x}{2\left(c^{2}x^{2}+bc\right)}+\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^2,x, algorithm="maxima")
[Out] -1/2*x/(c^2*x^2 + b*c) + 1/2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c)
```

mupad [B] time = 4.15, size = 33, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\sqrt{b}\;c^{3/2}}-\frac{x}{2\,c\,\left(c\,x^{2}+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2 + c*x^4)^2,x)
[Out] atan((c^(1/2)*x)/b^(1/2))/(2*b^(1/2)*c^(3/2)) - x/(2*c*(b + c*x^2))
```

sympy [B] time = 0.23, size = 78, normalized size = 1.73

$$-\frac{x}{2bc+2c^{2}x^{2}}-\frac{\sqrt{-\frac{1}{bc^{3}}}\,\log\left(-bc\sqrt{-\frac{1}{bc^{3}}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{bc^{3}}}\,\log\left(bc\sqrt{-\frac{1}{bc^{3}}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2)**2,x)
```

```
[Out] -x/(2*b*c + 2*c**2*x**2) - sqrt(-1/(b*c**3))*log(-b*c*sqrt(-1/(b*c**3)) + x
)/4 + sqrt(-1/(b*c**3))*log(b*c*sqrt(-1/(b*c**3)) + x)/4
```

3.197 $\int \frac{x^5}{\left(bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=16

$$-\frac{1}{2c\left(b+cx^2\right)}$$

```
[Out] -1/2/c/(c*x^2+b)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 261}

$$-\frac{1}{2c\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/(2*c*(b + c*x^2))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(bx^2+cx^4\right)^2} \, dx &= \int \frac{x}{\left(b+cx^2\right)^2} \, dx \\
&= -\frac{1}{2c\left(b+cx^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{2c\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(b*x^2 + c*x^4)^2,x]
[Out] -1/2*1/(c*(b + c*x^2))
```

fricas [A] time = 0.48, size = 15, normalized size = 0.94

$$-\frac{1}{2\left(c^2x^2+bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^2,x, algorithm="fricas")
[Out] -1/2/(c^2*x^2 + b*c)
```

giac [A] time = 0.17, size = 14, normalized size = 0.88

$$-\frac{1}{2\left(cx^2+b\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^2,x, algorithm="giac")
[Out] -1/2/((c*x^2 + b)*c)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{2\left(c\,x^2+b\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2)^2,x)
[Out] -1/2/c/(c*x^2+b)
```

maxima [A] time = 1.32, size = 15, normalized size = 0.94

$$-\frac{1}{2\left(c^2x^2+bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2/(c^2*x^2 + b*c)
```

mupad [B] time = 0.02, size = 14, normalized size = 0.88

$$-\frac{1}{2\,c\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2 + c*x^4)^2,x)
```

```
[Out] -1/(2*c*(b + c*x^2))
```

sympy [A] time = 0.18, size = 15, normalized size = 0.94

$$-\frac{1}{2bc+2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2)**2,x)
```

```
[Out] -1/(2*b*c + 2*c**2*x**2)
```

3.198 $\int \frac{x^4}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}+\frac{x}{2b\left(b+cx^2\right)}$$

```
[Out] 1/2*x/b/(c*x^2+b)+1/2*arctan(x*c^(1/2)/b^(1/2))/b^(3/2)/c^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}+\frac{x}{2b\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^2 + c*x^4)^2,x]
```

```
[Out] x/(2*b*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*b^(3/2)*Sqrt[c])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^4}{(bx^2+cx^4)^2}\,dx = \int \frac{1}{(b+cx^2)^2}\,dx$$
$$= \frac{x}{2b(b+cx^2)} + \frac{\int \frac{1}{b+cx^2}\,dx}{2b}$$
$$= \frac{x}{2b(b+cx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}$$

Mathematica [**A**] time = 0.02, size = 45, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}} + \frac{x}{2b(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^2 + c*x^4)^2,x]
[Out] x/(2*b*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*b^(3/2)*Sqrt[c])
```

fricas [**A**] time = 0.51, size = 120, normalized size = 2.67

$$\left[\frac{2\,bcx - (cx^2+b)\sqrt{-bc}\,\log\left(\frac{cx^2-2\,\sqrt{-bc}\,x-b}{cx^2+b}\right)}{4\,(b^2c^2x^2+b^3c)}, \frac{bcx + (cx^2+b)\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)}{2\,(b^2c^2x^2+b^3c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^2,x, algorithm="fricas")
[Out] [1/4*(2*b*c*x - (c*x^2 + b)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*
x^2 + b)))/(b^2*c^2*x^2 + b^3*c), 1/2*(b*c*x + (c*x^2 + b)*sqrt(b*c)*arctan
(sqrt(b*c)*x/b))/(b^2*c^2*x^2 + b^3*c)]
```

giac [**A**] time = 0.15, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b} + \frac{x}{2\,(cx^2+b)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) + 1/2*x/((c*x^2 + b)*b)
```

maple [A] time = 0.00, size = 36, normalized size = 0.80

$$\frac{x}{2\left(c\,x^{2}+b\right)b}+\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*x/b/(c*x^2+b)+1/2/b/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)
```

maxima [A] time = 3.00, size = 35, normalized size = 0.78

$$\frac{x}{2\left(bcx^{2}+b^{2}\right)}+\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x/(b*c*x^2 + b^2) + 1/2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b)
```

mupad [B] time = 0.04, size = 33, normalized size = 0.73

$$\frac{x}{2\,b\,\left(c\,x^{2}+b\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,b^{3/2}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x/(2*b*(b + c*x^2)) + atan((c^(1/2)*x)/b^(1/2))/(2*b^(3/2)*c^(1/2))
```

sympy [B] time = 0.25, size = 78, normalized size = 1.73

$$\frac{x}{2b^{2}+2bcx^{2}}-\frac{\sqrt{-\frac{1}{b^{3}c}}\log\left(-b^{2}\sqrt{-\frac{1}{b^{3}c}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{b^{3}c}}\log\left(b^{2}\sqrt{-\frac{1}{b^{3}c}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2)**2,x)
```

```
[Out] x/(2*b**2 + 2*b*c*x**2) - sqrt(-1/(b**3*c))*log(-b**2*sqrt(-1/(b**3*c)) + x
)/4 + sqrt(-1/(b**3*c))*log(b**2*sqrt(-1/(b**3*c)) + x)/4
```

3.199 $\int \frac{x^3}{(bx^2+cx^4)^2} dx$

Optimal. Leaf size=38

$$-\frac{\log(b+cx^2)}{2b^2}+\frac{\log(x)}{b^2}+\frac{1}{2b(b+cx^2)}$$

```
[Out] 1/2/b/(c*x^2+b)+ln(x)/b^2-1/2*ln(c*x^2+b)/b^2
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$-\frac{\log(b+cx^2)}{2b^2}+\frac{\log(x)}{b^2}+\frac{1}{2b(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x^2 + c*x^4)^2,x]
```

```
[Out] 1/(2*b*(b + c*x^2)) + Log[x]/b^2 - Log[b + c*x^2]/(2*b^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^3}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{1}{x\left(b+cx^2\right)^2}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(b+cx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{b^2x} - \frac{c}{b(b+cx)^2} - \frac{c}{b^2(b+cx)}\right)dx, x, x^2\right)$$
$$= \frac{1}{2b\left(b+cx^2\right)} + \frac{\log(x)}{b^2} - \frac{\log\left(b+cx^2\right)}{2b^2}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.87

$$\frac{\frac{b}{b+cx^2} - \log\left(b+cx^2\right) + 2\log(x)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b/(b + c*x^2) + 2*Log[x] - Log[b + c*x^2])/(2*b^2)
```

fricas [A] time = 0.67, size = 47, normalized size = 1.24

$$-\frac{\left(cx^2+b\right)\log\left(cx^2+b\right) - 2\left(cx^2+b\right)\log(x) - b}{2\left(b^2cx^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((c*x^2 + b)*log(c*x^2 + b) - 2*(c*x^2 + b)*log(x) - b)/(b^2*c*x^2 + b
^3)
```

giac [A] time = 0.16, size = 36, normalized size = 0.95

$$-\frac{\log\left(\left|cx^2+b\right|\right)}{2\,b^2} + \frac{\log\left(|x|\right)}{b^2} + \frac{1}{2\left(cx^2+b\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

[Out] -1/2*log(abs(c*x^2 + b))/b^2 + log(abs(x))/b^2 + 1/2/((c*x^2 + b)*b)

maple [A] time = 0.01, size = 35, normalized size = 0.92

$$\frac{1}{2\left(c\,x^{2}+b\right)b}+\frac{\ln(x)}{b^{2}}-\frac{\ln\left(c\,x^{2}+b\right)}{2b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(c*x^4+b*x^2)^2,x)

[Out] 1/2/b/(c*x^2+b)+ln(x)/b^2-1/2*ln(c*x^2+b)/b^2

maxima [A] time = 1.30, size = 37, normalized size = 0.97

$$\frac{1}{2\left(bcx^{2}+b^{2}\right)}-\frac{\log\left(cx^{2}+b\right)}{2\,b^{2}}+\frac{\log\left(x^{2}\right)}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^4+b*x^2)^2,x, algorithm="maxima")

[Out] 1/2/(b*c*x^2 + b^2) - 1/2*log(c*x^2 + b)/b^2 + 1/2*log(x^2)/b^2

mupad [B] time = 4.18, size = 34, normalized size = 0.89

$$\frac{\ln(x)}{b^{2}}+\frac{1}{2\,b\,\left(c\,x^{2}+b\right)}-\frac{\ln\left(c\,x^{2}+b\right)}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^2 + c*x^4)^2,x)

[Out] log(x)/b^2 + 1/(2*b*(b + c*x^2)) - log(b + c*x^2)/(2*b^2)

sympy [A] time = 0.34, size = 34, normalized size = 0.89

$$\frac{1}{2b^{2}+2bcx^{2}}+\frac{\log\left(x\right)}{b^{2}}-\frac{\log\left(\frac{b}{c}+x^{2}\right)}{2b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c*x**4+b*x**2)**2,x)

[Out] 1/(2*b**2 + 2*b*c*x**2) + log(x)/b**2 - log(b/c + x**2)/(2*b**2)

3.200 $\int \frac{x^2}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=57

$$-\frac{3\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{5/2}}-\frac{3}{2b^2x}+\frac{1}{2bx\left(b+cx^2\right)}$$

```
[Out] -3/2/b^2/x+1/2/b/x/(c*x^2+b)-3/2*arctan(x*c^(1/2)/b^(1/2))*c^(1/2)/b^(5/2)
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 290, 325, 205}

$$-\frac{3\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{5/2}}-\frac{3}{2b^2x}+\frac{1}{2bx\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -3/(2*b^2*x) + 1/(2*b*x*(b + c*x^2)) - (3*Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b
]])/(2*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{1}{x^2\left(b+cx^2\right)^2}\,dx \\
&= \frac{1}{2bx\left(b+cx^2\right)}+\frac{3\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{2b} \\
&= -\frac{3}{2b^2x}+\frac{1}{2bx\left(b+cx^2\right)}-\frac{(3c)\int \frac{1}{b+cx^2}\,dx}{2b^2} \\
&= -\frac{3}{2b^2x}+\frac{1}{2bx\left(b+cx^2\right)}-\frac{3\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.95

$$
-\frac{3\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{5/2}}-\frac{cx}{2b^2\left(b+cx^2\right)}-\frac{1}{b^2x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -(1/(b^2*x)) - (c*x)/(2*b^2*(b + c*x^2)) - (3*Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sq
rt[b]])/(2*b^(5/2))
```

fricas [A] time = 0.55, size = 136, normalized size = 2.39

$$
\left[-\frac{6\,cx^2-3\left(cx^3+bx\right)\sqrt{-\frac{c}{b}}\,\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)+4\,b}{4\left(b^2cx^3+b^3x\right)},\,-\frac{3\,cx^2+3\left(cx^3+bx\right)\sqrt{\frac{c}{b}}\,\arctan\left(x\sqrt{\frac{c}{b}}\right)+2\,b}{2\left(b^2cx^3+b^3x\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*c*x^2 - 3*(c*x^3 + b*x)*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) -
 b)/(c*x^2 + b)) + 4*b)/(b^2*c*x^3 + b^3*x), -1/2*(3*c*x^2 + 3*(c*x^3 + b*x
)*sqrt(c/b)*arctan(x*sqrt(c/b)) + 2*b)/(b^2*c*x^3 + b^3*x)]
```

giac [A] time = 0.16, size = 47, normalized size = 0.82

$$-\frac{3\,c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^2}-\frac{3\,cx^2+2\,b}{2\,\left(cx^3+bx\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -3/2*c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) - 1/2*(3*c*x^2 + 2*b)/((c*x^3
+ b*x)*b^2)
```

maple [A] time = 0.01, size = 46, normalized size = 0.81

$$-\frac{cx}{2\left(c\,x^2+b\right)b^2}-\frac{3c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\,\,b^2}-\frac{1}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/b^2*c*x/(c*x^2+b)-3/2/b^2*c/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)-1/b^
2/x
```

maxima [A] time = 2.96, size = 49, normalized size = 0.86

$$-\frac{3\,cx^2+2\,b}{2\,\left(b^2cx^3+b^3x\right)}-\frac{3\,c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*c*x^2 + 2*b)/(b^2*c*x^3 + b^3*x) - 3/2*c*arctan(c*x/sqrt(b*c))/(sqr
t(b*c)*b^2)
```

mupad [B] time = 0.06, size = 44, normalized size = 0.77

$$-\frac{\frac{1}{b}+\frac{3\,c\,x^2}{2\,b^2}}{c\,x^3+b\,x}-\frac{3\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2 + c*x^4)^2,x)
```

```
[Out] - (1/b + (3*c*x^2)/(2*b^2))/(b*x + c*x^3) - (3*c^(1/2)*atan((c^(1/2)*x)/b^(
1/2)))/(2*b^(5/2))
```

sympy [A] time = 0.32, size = 92, normalized size = 1.61

$$\frac{3\sqrt{-\frac{c}{b^5}}\log\left(-\frac{b^3\sqrt{-\frac{c}{b^5}}}{c}+x\right)}{4}-\frac{3\sqrt{-\frac{c}{b^5}}\log\left(\frac{b^3\sqrt{-\frac{c}{b^5}}}{c}+x\right)}{4}+\frac{-2b-3cx^2}{2b^3x+2b^2cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2)**2,x)
```

```
[Out] 3*sqrt(-c/b**5)*log(-b**3*sqrt(-c/b**5)/c + x)/4 - 3*sqrt(-c/b**5)*log(b**3
*sqrt(-c/b**5)/c + x)/4 + (-2*b - 3*c*x**2)/(2*b**3*x + 2*b**2*c*x**3)
```

3.201 $\int \frac{x}{\left(bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=49

$$\frac{c \log \left(b+c x^2\right)}{b^3}-\frac{2 c \log (x)}{b^3}-\frac{c}{2 b^2 \left(b+c x^2\right)}-\frac{1}{2 b^2 x^2}$$

```
[Out] -1/2/b^2/x^2-1/2*c/b^2/(c*x^2+b)-2*c*ln(x)/b^3+c*ln(c*x^2+b)/b^3
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1584, 266, 44}

$$-\frac{c}{2 b^2 \left(b+c x^2\right)}+\frac{c \log \left(b+c x^2\right)}{b^3}-\frac{2 c \log (x)}{b^3}-\frac{1}{2 b^2 x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/(2*b^2*x^2) - c/(2*b^2*(b + c*x^2)) - (2*c*Log[x])/b^3 + (c*Log[b + c*x^
2])/b^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x}{\left(bx^2+cx^4\right)^2}\,dx=\int \frac{1}{x^3\left(b+cx^2\right)^2}\,dx$$
$$=\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(b+cx)^2}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{b^2x^2}-\frac{2c}{b^3x}+\frac{c^2}{b^2(b+cx)^2}+\frac{2c^2}{b^3(b+cx)}\right)dx,x,x^2\right)$$
$$=-\frac{1}{2b^2x^2}-\frac{c}{2b^2\left(b+cx^2\right)}-\frac{2c\log(x)}{b^3}+\frac{c\log\left(b+cx^2\right)}{b^3}$$

Mathematica [**A**] time = 0.04, size = 41, normalized size = 0.84

$$-\frac{b\left(\frac{c}{b+cx^2}+\frac{1}{x^2}\right)-2c\log\left(b+cx^2\right)+4c\log(x)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(b*(x^(-2) + c/(b + c*x^2)) + 4*c*Log[x] - 2*c*Log[b + c*x^2])/b^3
```

fricas [**A**] time = 0.63, size = 73, normalized size = 1.49

$$-\frac{2\,bcx^2+b^2-2\left(c^2x^4+bcx^2\right)\log\left(cx^2+b\right)+4\left(c^2x^4+bcx^2\right)\log(x)}{2\left(b^3cx^4+b^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*c*x^2 + b^2 - 2*(c^2*x^4 + b*c*x^2)*log(c*x^2 + b) + 4*(c^2*x^4 +
 b*c*x^2)*log(x))/(b^3*c*x^4 + b^4*x^2)
```

giac [**A**] time = 0.17, size = 50, normalized size = 1.02

$$\frac{c\log\left(\left|cx^2+b\right|\right)}{b^3}-\frac{2\,c\log\left(|x|\right)}{b^3}-\frac{2\,cx^2+b}{2\left(cx^4+bx^2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] c*log(abs(c*x^2 + b))/b^3 - 2*c*log(abs(x))/b^3 - 1/2*(2*c*x^2 + b)/((c*x^4
 + b*x^2)*b^2)
```

maple [A] time = 0.01, size = 46, normalized size = 0.94

$$-\frac{c}{2\left(c\,x^2+b\right)b^2}-\frac{2c\ln(x)}{b^3}+\frac{c\ln\left(c\,x^2+b\right)}{b^3}-\frac{1}{2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/b^2/x^2-1/2*c/b^2/(c*x^2+b)-2*c*ln(x)/b^3+c*ln(c*x^2+b)/b^3
```

maxima [A] time = 1.34, size = 52, normalized size = 1.06

$$-\frac{2\,cx^2+b}{2\left(b^2cx^4+b^3x^2\right)}+\frac{c\log\left(cx^2+b\right)}{b^3}-\frac{c\log\left(x^2\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^2 + b)/(b^2*c*x^4 + b^3*x^2) + c*log(c*x^2 + b)/b^3 - c*log(x^2
)/b^3
```

mupad [B] time = 4.21, size = 51, normalized size = 1.04

$$\frac{c\,\ln\left(c\,x^2+b\right)}{b^3}-\frac{\frac{1}{2\,b}+\frac{c\,x^2}{b^2}}{c\,x^4+b\,x^2}-\frac{2\,c\,\ln(x)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (c*log(b + c*x^2))/b^3 - (1/(2*b) + (c*x^2)/b^2)/(b*x^2 + c*x^4) - (2*c*log
(x))/b^3
```

sympy [A] time = 0.40, size = 51, normalized size = 1.04

$$\frac{-b-2cx^2}{2b^3x^2+2b^2cx^4}-\frac{2c\log\left(x\right)}{b^3}+\frac{c\log\left(\frac{b}{c}+x^2\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2)**2,x)
```

```
[Out] (-b - 2*c*x**2)/(2*b**3*x**2 + 2*b**2*c*x**4) - 2*c*log(x)/b**3 + c*log(b/c
 + x**2)/b**3
```

3.202 $\int \frac{1}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=68

$$\frac{5c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{7/2}}+\frac{5c}{2b^3x}-\frac{5}{6b^2x^3}+\frac{1}{2bx^3\left(b+cx^2\right)}$$

```
[Out] -5/6/b^2/x^3+5/2*c/b^3/x+1/2/b/x^3/(c*x^2+b)+5/2*c^(3/2)*arctan(x*c^(1/2)/b
^(1/2))/b^(7/2)
```

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1593, 290, 325, 205}

$$\frac{5c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{7/2}}+\frac{5c}{2b^3x}-\frac{5}{6b^2x^3}+\frac{1}{2bx^3\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(-2),x]
```

```
[Out] -5/(6*b^2*x^3) + (5*c)/(2*b^3*x) + 1/(2*b*x^3*(b + c*x^2)) + (5*c^(3/2)*Arc
Tan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{1}{x^4\left(b+cx^2\right)^2}\,dx \\
&= \frac{1}{2bx^3\left(b+cx^2\right)} + \frac{5\int \frac{1}{x^4\left(b+cx^2\right)}\,dx}{2b} \\
&= -\frac{5}{6b^2x^3} + \frac{1}{2bx^3\left(b+cx^2\right)} - \frac{(5c)\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{2b^2} \\
&= -\frac{5}{6b^2x^3} + \frac{5c}{2b^3x} + \frac{1}{2bx^3\left(b+cx^2\right)} + \frac{\left(5c^2\right)\int \frac{1}{b+cx^2}\,dx}{2b^3} \\
&= -\frac{5}{6b^2x^3} + \frac{5c}{2b^3x} + \frac{1}{2bx^3\left(b+cx^2\right)} + \frac{5c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 67, normalized size = 0.99

$$
\frac{5c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{7/2}} + \frac{c^2x}{2b^3\left(b+cx^2\right)} + \frac{2c}{b^3x} - \frac{1}{3b^2x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(-2),x]
```

```
[Out] -1/3*1/(b^2*x^3) + (2*c)/(b^3*x) + (c^2*x)/(2*b^3*(b + c*x^2)) + (5*c^(3/2)
*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(7/2))
```

fricas [A] time = 0.54, size = 172, normalized size = 2.53

$$\left[\frac{30\,c^{2}x^{4} + 20\,bcx^{2} + 15\,\left(c^{2}x^{5} + bcx^{3}\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^{2} + 2\,bx\sqrt{-\frac{c}{b}} - b}{cx^{2} + b}\right) - 4\,b^{2}}{12\,\left(b^{3}cx^{5} + b^{4}x^{3}\right)}, \frac{15\,c^{2}x^{4} + 10\,bcx^{2} + 15\,\left(c^{2}x^{5} + bcx^{3}\right)\sqrt{\frac{c}{b}}}{6\,\left(b^{3}cx^{5} + b^{4}x^{3}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(30*c^2*x^4 + 20*b*c*x^2 + 15*(c^2*x^5 + b*c*x^3)*sqrt(-c/b)*log((c*x
^2 + 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)) - 4*b^2)/(b^3*c*x^5 + b^4*x^3), 1/6
*(15*c^2*x^4 + 10*b*c*x^2 + 15*(c^2*x^5 + b*c*x^3)*sqrt(c/b)*arctan(x*sqrt(
c/b)) - 2*b^2)/(b^3*c*x^5 + b^4*x^3)]
```

giac [A] time = 0.15, size = 59, normalized size = 0.87

$$\frac{5\,c^{2}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^{3}} + \frac{c^{2}x}{2\,\left(cx^{2} + b\right)b^{3}} + \frac{6\,cx^{2} - b}{3\,b^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 5/2*c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) + 1/2*c^2*x/((c*x^2 + b)*b^3)
 + 1/3*(6*c*x^2 - b)/(b^3*x^3)
```

maple [A] time = 0.01, size = 59, normalized size = 0.87

$$\frac{c^{2}x}{2\left(c\,x^{2} + b\right)b^{3}} + \frac{5c^{2}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;b^{3}} + \frac{2c}{b^{3}x} - \frac{1}{3b^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2/b^3*c^2*x/(c*x^2+b)+5/2/b^3*c^2/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)-1
/3/b^2/x^3+2*c/b^3/x
```

maxima [A] time = 2.96, size = 64, normalized size = 0.94

$$\frac{15\,c^{2}x^{4} + 10\,bcx^{2} - 2\,b^{2}}{6\,\left(b^{3}cx^{5} + b^{4}x^{3}\right)} + \frac{5\,c^{2}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(15*c^2*x^4 + 10*b*c*x^2 - 2*b^2)/(b^3*c*x^5 + b^4*x^3) + 5/2*c^2*arcta
n(c*x/sqrt(b*c))/(sqrt(b*c)*b^3)
```

mupad [B] time = 4.17, size = 58, normalized size = 0.85

$$\frac{\frac{5\,c\,x^2}{3\,b^2}-\frac{1}{3\,b}+\frac{5\,c^2\,x^4}{2\,b^3}}{c\,x^5+b\,x^3}+\frac{5\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 + c*x^4)^2,x)
```

```
[Out] ((5*c*x^2)/(3*b^2) - 1/(3*b) + (5*c^2*x^4)/(2*b^3))/(b*x^3 + c*x^5) + (5*c^
(3/2)*atan((c^(1/2)*x)/b^(1/2)))/(2*b^(7/2))
```

sympy [A] time = 0.39, size = 114, normalized size = 1.68

$$-\frac{5\sqrt{-\frac{c^3}{b^7}}\log\left(-\frac{b^4\sqrt{-\frac{c^3}{b^7}}}{c^2}+x\right)}{4}+\frac{5\sqrt{-\frac{c^3}{b^7}}\log\left(\frac{b^4\sqrt{-\frac{c^3}{b^7}}}{c^2}+x\right)}{4}+\frac{-2b^2+10bcx^2+15c^2x^4}{6b^4x^3+6b^3cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**2,x)
```

```
[Out] -5*sqrt(-c**3/b**7)*log(-b**4*sqrt(-c**3/b**7)/c**2 + x)/4 + 5*sqrt(-c**3/b
**7)*log(b**4*sqrt(-c**3/b**7)/c**2 + x)/4 + (-2*b**2 + 10*b*c*x**2 + 15*c*
*2*x**4)/(6*b**4*x**3 + 6*b**3*c*x**5)
```

3.203 $\int \frac{1}{x\left(bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=66

$$-\frac{3c^2 \log\left(b+cx^2\right)}{2b^4}+\frac{3c^2 \log(x)}{b^4}+\frac{c^2}{2b^3\left(b+cx^2\right)}+\frac{c}{b^3x^2}-\frac{1}{4b^2x^4}$$

```
[Out] -1/4/b^2/x^4+c/b^3/x^2+1/2*c^2/b^3/(c*x^2+b)+3*c^2*ln(x)/b^4-3/2*c^2*ln(c*x
^2+b)/b^4
```

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$\frac{c^2}{2b^3\left(b+cx^2\right)}-\frac{3c^2 \log\left(b+cx^2\right)}{2b^4}+\frac{3c^2 \log(x)}{b^4}+\frac{c}{b^3x^2}-\frac{1}{4b^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/(4*b^2*x^4) + c/(b^3*x^2) + c^2/(2*b^3*(b + c*x^2)) + (3*c^2*Log[x])/b^4
 - (3*c^2*Log[b + c*x^2])/(2*b^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{1}{x^5\left(b+cx^2\right)^2}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^3(b+cx)^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{b^2x^3}-\frac{2c}{b^3x^2}+\frac{3c^2}{b^4x}-\frac{c^3}{b^3(b+cx)^2}-\frac{3c^3}{b^4(b+cx)}\right)dx, x, x^2\right)\\
&= -\frac{1}{4b^2x^4}+\frac{c}{b^3x^2}+\frac{c^2}{2b^3\left(b+cx^2\right)}+\frac{3c^2\log(x)}{b^4}-\frac{3c^2\log\left(b+cx^2\right)}{2b^4}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.86

$$
\frac{-6c^2\log\left(b+cx^2\right)+b\left(\frac{2c^2}{b+cx^2}-\frac{b}{x^4}+\frac{4c}{x^2}\right)+12c^2\log(x)}{4b^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^2 + c*x^4)^2),x]
```

```
[Out] (b*(-(b/x^4) + (4*c)/x^2 + (2*c^2)/(b + c*x^2)) + 12*c^2*Log[x] - 6*c^2*Log
[b + c*x^2])/(4*b^4)
```

fricas [A] time = 0.65, size = 90, normalized size = 1.36

$$
\frac{6\,bc^2x^4+3\,b^2cx^2-b^3-6\left(c^3x^6+bc^2x^4\right)\log\left(cx^2+b\right)+12\left(c^3x^6+bc^2x^4\right)\log(x)}{4\left(b^4cx^6+b^5x^4\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*b*c^2*x^4 + 3*b^2*c*x^2 - b^3 - 6*(c^3*x^6 + b*c^2*x^4)*log(c*x^2 +
b) + 12*(c^3*x^6 + b*c^2*x^4)*log(x))/(b^4*c*x^6 + b^5*x^4)
```

giac [A] time = 0.15, size = 86, normalized size = 1.30

$$
\frac{3\,c^2\log\left(x^2\right)}{2\,b^4}-\frac{3\,c^2\log\left(\left|cx^2+b\right|\right)}{2\,b^4}+\frac{3\,c^3x^2+4\,bc^2}{2\left(cx^2+b\right)b^4}-\frac{9\,c^2x^4-4\,bcx^2+b^2}{4\,b^4x^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 3/2*c^2*log(x^2)/b^4 - 3/2*c^2*log(abs(c*x^2 + b))/b^4 + 1/2*(3*c^3*x^2 + 4
*b*c^2)/((c*x^2 + b)*b^4) - 1/4*(9*c^2*x^4 - 4*b*c*x^2 + b^2)/(b^4*x^4)
```

maple [A] time = 0.01, size = 61, normalized size = 0.92

$$\frac{c^2}{2\left(c\,x^2+b\right)b^3}+\frac{3c^2\ln(x)}{b^4}-\frac{3c^2\ln\left(c\,x^2+b\right)}{2b^4}+\frac{c}{b^3x^2}-\frac{1}{4b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/4/b^2/x^4+c/b^3/x^2+1/2*c^2/b^3/(c*x^2+b)+3*c^2*ln(x)/b^4-3/2*c^2*ln(c*x
^2+b)/b^4
```

maxima [A] time = 1.35, size = 70, normalized size = 1.06

$$\frac{6\,c^2x^4+3\,bcx^2-b^2}{4\left(b^3cx^6+b^4x^4\right)}-\frac{3\,c^2\log\left(cx^2+b\right)}{2\,b^4}+\frac{3\,c^2\log\left(x^2\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(6*c^2*x^4 + 3*b*c*x^2 - b^2)/(b^3*c*x^6 + b^4*x^4) - 3/2*c^2*log(c*x^2
 + b)/b^4 + 3/2*c^2*log(x^2)/b^4
```

mupad [B] time = 4.17, size = 67, normalized size = 1.02

$$\frac{\frac{3\,c\,x^2}{4\,b^2}-\frac{1}{4\,b}+\frac{3\,c^2\,x^4}{2\,b^3}}{c\,x^6+b\,x^4}-\frac{3\,c^2\,\ln\left(c\,x^2+b\right)}{2\,b^4}+\frac{3\,c^2\,\ln(x)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + c*x^4)^2),x)
```

```
[Out] ((3*c*x^2)/(4*b^2) - 1/(4*b) + (3*c^2*x^4)/(2*b^3))/(b*x^4 + c*x^6) - (3*c^
2*log(b + c*x^2))/(2*b^4) + (3*c^2*log(x))/b^4
```

sympy [A] time = 0.49, size = 68, normalized size = 1.03

$$\frac{-b^2+3bcx^2+6c^2x^4}{4b^4x^4+4b^3cx^6}+\frac{3c^2\log\left(x\right)}{b^4}-\frac{3c^2\log\left(\frac{b}{c}+x^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2)**2,x)
```

```
[Out] (-b**2 + 3*b*c*x**2 + 6*c**2*x**4)/(4*b**4*x**4 + 4*b**3*c*x**6) + 3*c**2*l
og(x)/b**4 - 3*c**2*log(b/c + x**2)/(2*b**4)
```

3.204 $\int \frac{1}{x^2(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=81

$$-\frac{7c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{9/2}}-\frac{7c^2}{2b^4x}+\frac{7c}{6b^3x^3}-\frac{7}{10b^2x^5}+\frac{1}{2bx^5\left(b+cx^2\right)}$$

```
[Out] -7/10/b^2/x^5+7/6*c/b^3/x^3-7/2*c^2/b^4/x+1/2/b/x^5/(c*x^2+b)-7/2*c^(5/2)*a
rctan(x*c^(1/2)/b^(1/2))/b^(9/2)
```

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 290, 325, 205}

$$-\frac{7c^2}{2b^4x}-\frac{7c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{9/2}}+\frac{7c}{6b^3x^3}-\frac{7}{10b^2x^5}+\frac{1}{2bx^5\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -7/(10*b^2*x^5) + (7*c)/(6*b^3*x^3) - (7*c^2)/(2*b^4*x) + 1/(2*b*x^5*(b + c
*x^2)) - (7*c^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{1}{x^6\left(b+cx^2\right)^2}\,dx \\
&= \frac{1}{2bx^5\left(b+cx^2\right)}+\frac{7\int \frac{1}{x^6\left(b+cx^2\right)}\,dx}{2b} \\
&= -\frac{7}{10b^2x^5}+\frac{1}{2bx^5\left(b+cx^2\right)}-\frac{(7c)\int \frac{1}{x^4\left(b+cx^2\right)}\,dx}{2b^2} \\
&= -\frac{7}{10b^2x^5}+\frac{7c}{6b^3x^3}+\frac{1}{2bx^5\left(b+cx^2\right)}+\frac{\left(7c^2\right)\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{2b^3} \\
&= -\frac{7}{10b^2x^5}+\frac{7c}{6b^3x^3}-\frac{7c^2}{2b^4x}+\frac{1}{2bx^5\left(b+cx^2\right)}-\frac{\left(7c^3\right)\int \frac{1}{b+cx^2}\,dx}{2b^4} \\
&= -\frac{7}{10b^2x^5}+\frac{7c}{6b^3x^3}-\frac{7c^2}{2b^4x}+\frac{1}{2bx^5\left(b+cx^2\right)}-\frac{7c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{9/2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 80, normalized size = 0.99

$$
-\frac{7c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{9/2}}-\frac{c^3x}{2b^4\left(b+cx^2\right)}-\frac{3c^2}{b^4x}+\frac{2c}{3b^3x^3}-\frac{1}{5b^2x^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/5*1/(b^2*x^5) + (2*c)/(3*b^3*x^3) - (3*c^2)/(b^4*x) - (c^3*x)/(2*b^4*(b
+ c*x^2)) - (7*c^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(9/2))
```

fricas [A] time = 0.51, size = 198, normalized size = 2.44

$$\left[-\frac{210\,c^3x^6+140\,bc^2x^4-28\,b^2cx^2+12\,b^3-105\left(c^3x^7+bc^2x^5\right)\sqrt{-\frac{c}{b}}\,\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)}{60\left(b^4cx^7+b^5x^5\right)},\,-\frac{105\,c^3x^6+70\,bc^2x^4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(210*c^3*x^6 + 140*b*c^2*x^4 - 28*b^2*c*x^2 + 12*b^3 - 105*(c^3*x^7
+ b*c^2*x^5)*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)))/(b
^4*c*x^7 + b^5*x^5), -1/30*(105*c^3*x^6 + 70*b*c^2*x^4 - 14*b^2*c*x^2 + 6*b
^3 + 105*(c^3*x^7 + b*c^2*x^5)*sqrt(c/b)*arctan(x*sqrt(c/b)))/(b^4*c*x^7 +
b^5*x^5)]
```

giac [A] time = 0.18, size = 70, normalized size = 0.86

$$-\frac{7\,c^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^4}-\frac{c^3x}{2\left(cx^2+b\right)b^4}-\frac{45\,c^2x^4-10\,bcx^2+3\,b^2}{15\,b^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -7/2*c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4) - 1/2*c^3*x/((c*x^2 + b)*b^4
) - 1/15*(45*c^2*x^4 - 10*b*c*x^2 + 3*b^2)/(b^4*x^5)
```

maple [A] time = 0.01, size = 70, normalized size = 0.86

$$-\frac{c^3x}{2\left(c\,x^2+b\right)b^4}-\frac{7c^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\;b^4}-\frac{3c^2}{b^4x}+\frac{2c}{3b^3x^3}-\frac{1}{5b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/b^4*c^3*x/(c*x^2+b)-7/2/b^4*c^3/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)-
1/5/b^2/x^5-3*c^2/b^4/x+2/3*c/b^3/x^3
```

maxima [A] time = 3.03, size = 75, normalized size = 0.93

$$-\frac{105\,c^{3}x^{6}+70\,bc^{2}x^{4}-14\,b^{2}cx^{2}+6\,b^{3}}{30\left(b^{4}cx^{7}+b^{5}x^{5}\right)}-\frac{7\,c^{3}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/30*(105*c^3*x^6 + 70*b*c^2*x^4 - 14*b^2*c*x^2 + 6*b^3)/(b^4*c*x^7 + b^5*
x^5) - 7/2*c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4)
```

mupad [B] time = 4.28, size = 70, normalized size = 0.86

$$-\frac{\frac{1}{5\,b}-\frac{7\,c\,x^{2}}{15\,b^{2}}+\frac{7\,c^{2}\,x^{4}}{3\,b^{3}}+\frac{7\,c^{3}\,x^{6}}{2\,b^{4}}}{c\,x^{7}+b\,x^{5}}-\frac{7\,c^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x^2 + c*x^4)^2),x)
```

```
[Out] - (1/(5*b) - (7*c*x^2)/(15*b^2) + (7*c^2*x^4)/(3*b^3) + (7*c^3*x^6)/(2*b^4)
)/(b*x^5 + c*x^7) - (7*c^(5/2)*atan((c^(1/2)*x)/b^(1/2)))/(2*b^(9/2))
```

sympy [A] time = 0.43, size = 126, normalized size = 1.56

$$\frac{7\sqrt{-\frac{c^{5}}{b^{9}}}\,\log\left(-\frac{b^{5}\sqrt{-\frac{c^{5}}{b^{9}}}}{c^{3}}+x\right)}{4}-\frac{7\sqrt{-\frac{c^{5}}{b^{9}}}\,\log\left(\frac{b^{5}\sqrt{-\frac{c^{5}}{b^{9}}}}{c^{3}}+x\right)}{4}+\frac{-6b^{3}+14b^{2}cx^{2}-70bc^{2}x^{4}-105c^{3}x^{6}}{30b^{5}x^{5}+30b^{4}cx^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2)**2,x)
```

```
[Out] 7*sqrt(-c**5/b**9)*log(-b**5*sqrt(-c**5/b**9)/c**3 + x)/4 - 7*sqrt(-c**5/b*
*9)*log(b**5*sqrt(-c**5/b**9)/c**3 + x)/4 + (-6*b**3 + 14*b**2*c*x**2 - 70*
b*c**2*x**4 - 105*c**3*x**6)/(30*b**5*x**5 + 30*b**4*c*x**7)
```

3.205 $\int \frac{x^{14}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=85

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{9/2}}-\frac{35bx}{8c^4}-\frac{7x^5}{8c^2\left(b+cx^2\right)}-\frac{x^7}{4c\left(b+cx^2\right)^2}+\frac{35x^3}{24c^3}$$

```
[Out] -35/8*b*x/c^4+35/24*x^3/c^3-1/4*x^7/c/(c*x^2+b)^2-7/8*x^5/c^2/(c*x^2+b)+35/
8*b^(3/2)*arctan(x*c^(1/2)/b^(1/2))/c^(9/2)
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 288, 302, 205}

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{9/2}}-\frac{7x^5}{8c^2\left(b+cx^2\right)}-\frac{35bx}{8c^4}-\frac{x^7}{4c\left(b+cx^2\right)^2}+\frac{35x^3}{24c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-35*b*x)/(8*c^4) + (35*x^3)/(24*c^3) - x^7/(4*c*(b + c*x^2)^2) - (7*x^5)/(
8*c^2*(b + c*x^2)) + (35*b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{14}}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{x^8}{\left(b+cx^2\right)^3}\,dx \\
&= -\frac{x^7}{4c\left(b+cx^2\right)^2} + \frac{7\int \frac{x^6}{\left(b+cx^2\right)^2}\,dx}{4c} \\
&= -\frac{x^7}{4c\left(b+cx^2\right)^2} - \frac{7x^5}{8c^2\left(b+cx^2\right)} + \frac{35\int \frac{x^4}{b+cx^2}\,dx}{8c^2} \\
&= -\frac{x^7}{4c\left(b+cx^2\right)^2} - \frac{7x^5}{8c^2\left(b+cx^2\right)} + \frac{35\int \left(-\frac{b}{c^2}+\frac{x^2}{c}+\frac{b^2}{c^2\left(b+cx^2\right)}\right)dx}{8c^2} \\
&= -\frac{35bx}{8c^4} + \frac{35x^3}{24c^3} - \frac{x^7}{4c\left(b+cx^2\right)^2} - \frac{7x^5}{8c^2\left(b+cx^2\right)} + \frac{\left(35b^2\right)\int \frac{1}{b+cx^2}\,dx}{8c^4} \\
&= -\frac{35bx}{8c^4} + \frac{35x^3}{24c^3} - \frac{x^7}{4c\left(b+cx^2\right)^2} - \frac{7x^5}{8c^2\left(b+cx^2\right)} + \frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{9/2}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 77, normalized size = 0.91

$$
\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{9/2}} - \frac{105b^3x+175b^2cx^3+56bc^2x^5-8c^3x^7}{24c^4\left(b+cx^2\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(b*x^2 + c*x^4)^3,x]
```

[Out] -1/24*(105*b^3*x + 175*b^2*c*x^3 + 56*b*c^2*x^5 - 8*c^3*x^7)/(c^4*(b + c*x^2)^2) + (35*b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(9/2))

fricas [A] time = 0.77, size = 230, normalized size = 2.71

$$\left[\frac{16\,c^3x^7 - 112\,bc^2x^5 - 350\,b^2cx^3 - 210\,b^3x + 105\,\left(bc^2x^4 + 2\,b^2cx^2 + b^3\right)\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)}{48\,\left(c^6x^4 + 2\,bc^5x^2 + b^2c^4\right)}, \frac{8\,c^3x^7 - 56\,bc\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*c^3*x^7 - 112*b*c^2*x^5 - 350*b^2*c*x^3 - 210*b^3*x + 105*(b*c^2*x^4 + 2*b^2*c*x^2 + b^3)*sqrt(-b/c)*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)))/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4), 1/24*(8*c^3*x^7 - 56*b*c^2*x^5 - 175*b^2*c*x^3 - 105*b^3*x + 105*(b*c^2*x^4 + 2*b^2*c*x^2 + b^3)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b))/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)]
```

giac [A] time = 0.16, size = 73, normalized size = 0.86

$$\frac{35\,b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,c^4} - \frac{13\,b^2cx^3 + 11\,b^3x}{8\,\left(cx^2+b\right)^2c^4} + \frac{c^6x^3 - 9\,bc^5x}{3\,c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 35/8*b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) - 1/8*(13*b^2*c*x^3 + 11*b^3*x)/((c*x^2 + b)^2*c^4) + 1/3*(c^6*x^3 - 9*b*c^5*x)/c^9
```

maple [A] time = 0.01, size = 77, normalized size = 0.91

$$-\frac{13b^2x^3}{8\left(c\,x^2+b\right)^2c^3} - \frac{11b^3x}{8\left(c\,x^2+b\right)^2c^4} + \frac{x^3}{3c^3} + \frac{35b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\;c^4} - \frac{3bx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/3*x^3/c^3-3*b*x/c^4-13/8/c^3*b^2/(c*x^2+b)^2*x^3-11/8/c^4*b^3/(c*x^2+b)^2*x+35/8/c^4*b^2/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*x)
```

maxima [A] time = 2.94, size = 82, normalized size = 0.96

$$-\frac{13\,b^2cx^3+11\,b^3x}{8\left(c^6x^4+2\,bc^5x^2+b^2c^4\right)}+\frac{35\,b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,c^4}+\frac{cx^3-9\,bx}{3\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(13*b^2*c*x^3 + 11*b^3*x)/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4) + 35/8*b^2
*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/3*(c*x^3 - 9*b*x)/c^4
```

mupad [B] time = 4.21, size = 77, normalized size = 0.91

$$\frac{x^3}{3\,c^3}-\frac{\frac{11\,b^3\,x}{8}+\frac{13\,c\,b^2\,x^3}{8}}{b^2\,c^4+2\,b\,c^5\,x^2+c^6\,x^4}+\frac{35\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,c^{9/2}}-\frac{3\,b\,x}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^2 + c*x^4)^3,x)
```

```
[Out] x^3/(3*c^3) - ((11*b^3*x)/8 + (13*b^2*c*x^3)/8)/(b^2*c^4 + c^6*x^4 + 2*b*c^
5*x^2) + (35*b^(3/2)*atan((c^(1/2)*x)/b^(1/2)))/(8*c^(9/2)) - (3*b*x)/c^4
```

sympy [A] time = 0.50, size = 133, normalized size = 1.56

$$-\frac{3bx}{c^4}-\frac{35\sqrt{-\frac{b^3}{c^9}}\,\log\left(x-\frac{c^4\sqrt{-\frac{b^3}{c^9}}}{b}\right)}{16}+\frac{35\sqrt{-\frac{b^3}{c^9}}\,\log\left(x+\frac{c^4\sqrt{-\frac{b^3}{c^9}}}{b}\right)}{16}+\frac{-11b^3x-13b^2cx^3}{8b^2c^4+16bc^5x^2+8c^6x^4}+\frac{x^3}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*b*x/c**4 - 35*sqrt(-b**3/c**9)*log(x - c**4*sqrt(-b**3/c**9)/b)/16 + 35*
sqrt(-b**3/c**9)*log(x + c**4*sqrt(-b**3/c**9)/b)/16 + (-11*b**3*x - 13*b**
2*c*x**3)/(8*b**2*c**4 + 16*b*c**5*x**2 + 8*c**6*x**4) + x**3/(3*c**3)
```

3.206 $\int \frac{x^{13}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=65

$$\frac{b^3}{4c^4(b+cx^2)^2}-\frac{3b^2}{2c^4(b+cx^2)}-\frac{3b\log(b+cx^2)}{2c^4}+\frac{x^2}{2c^3}$$

```
[Out] 1/2*x^2/c^3+1/4*b^3/c^4/(c*x^2+b)^2-3/2*b^2/c^4/(c*x^2+b)-3/2*b*ln(c*x^2+b)
/c^4
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{b^3}{4c^4(b+cx^2)^2}-\frac{3b^2}{2c^4(b+cx^2)}-\frac{3b\log(b+cx^2)}{2c^4}+\frac{x^2}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(b*x^2 + c*x^4)^3,x]
```

```
[Out] x^2/(2*c^3) + b^3/(4*c^4*(b + c*x^2)^2) - (3*b^2)/(2*c^4*(b + c*x^2)) - (3*
b*Log[b + c*x^2])/(2*c^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{13}}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{x^7}{\left(b+cx^2\right)^3}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{(b+cx)^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{c^3}-\frac{b^3}{c^3(b+cx)^3}+\frac{3b^2}{c^3(b+cx)^2}-\frac{3b}{c^3(b+cx)}\right)dx,x,x^2\right)\\ &= \frac{x^2}{2c^3}+\frac{b^3}{4c^4\left(b+cx^2\right)^2}-\frac{3b^2}{2c^4\left(b+cx^2\right)}-\frac{3b\log\left(b+cx^2\right)}{2c^4}\end{aligned}$$

Mathematica [A] time = 0.06, size = 48, normalized size = 0.74

$$-\frac{\frac{b^2\left(5b+6cx^2\right)}{\left(b+cx^2\right)^2}+6b\log\left(b+cx^2\right)-2cx^2}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(-2*c*x^2 + (b^2*(5*b + 6*c*x^2))/(b + c*x^2)^2 + 6*b*Log[b + c*x^2])/
c^4
```

fricas [A] time = 0.56, size = 91, normalized size = 1.40

$$\frac{2\,c^3x^6+4\,bc^2x^4-4\,b^2cx^2-5\,b^3-6\left(bc^2x^4+2\,b^2cx^2+b^3\right)\log\left(cx^2+b\right)}{4\left(c^6x^4+2\,bc^5x^2+b^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*c^3*x^6 + 4*b*c^2*x^4 - 4*b^2*c*x^2 - 5*b^3 - 6*(b*c^2*x^4 + 2*b^2*c
*x^2 + b^3)*log(c*x^2 + b))/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)
```

giac [A] time = 0.19, size = 62, normalized size = 0.95

$$\frac{x^2}{2\,c^3}-\frac{3\,b\log\left(\left|cx^2+b\right|\right)}{2\,c^4}+\frac{9\,bc^2x^4+12\,b^2cx^2+4\,b^3}{4\left(cx^2+b\right)^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*x^2/c^3 - 3/2*b*log(abs(c*x^2 + b))/c^4 + 1/4*(9*b*c^2*x^4 + 12*b^2*c*x
^2 + 4*b^3)/((c*x^2 + b)^2*c^4)
```

maple [A] time = 0.01, size = 58, normalized size = 0.89

$$\frac{b^3}{4\left(c\,x^2+b\right)^2 c^4}+\frac{x^2}{2c^3}-\frac{3b^2}{2\left(c\,x^2+b\right)c^4}-\frac{3b\ln\left(c\,x^2+b\right)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/2*x^2/c^3+1/4*b^3/c^4/(c*x^2+b)^2-3/2*b^2/c^4/(c*x^2+b)-3/2*b*ln(c*x^2+b)
/c^4
```

maxima [A] time = 1.36, size = 66, normalized size = 1.02

$$-\frac{6\,b^2cx^2+5\,b^3}{4\left(c^6x^4+2\,bc^5x^2+b^2c^4\right)}+\frac{x^2}{2\,c^3}-\frac{3\,b\log\left(cx^2+b\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*b^2*c*x^2 + 5*b^3)/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4) + 1/2*x^2/c^3
- 3/2*b*log(c*x^2 + b)/c^4
```

mupad [B] time = 4.26, size = 68, normalized size = 1.05

$$\frac{x^2}{2\,c^3}-\frac{\frac{5\,b^3}{4\,c}+\frac{3\,b^2\,x^2}{2}}{b^2\,c^3+2\,b\,c^4\,x^2+c^5\,x^4}-\frac{3\,b\,\ln\left(c\,x^2+b\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^2 + c*x^4)^3,x)
```

```
[Out] x^2/(2*c^3) - ((5*b^3)/(4*c) + (3*b^2*x^2)/2)/(b^2*c^3 + c^5*x^4 + 2*b*c^4*
x^2) - (3*b*log(b + c*x^2))/(2*c^4)
```

sympy [A] time = 0.43, size = 68, normalized size = 1.05

$$-\frac{3b\log\left(b+cx^2\right)}{2c^4}+\frac{-5b^3-6b^2cx^2}{4b^2c^4+8bc^5x^2+4c^6x^4}+\frac{x^2}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*b*log(b + c*x**2)/(2*c**4) + (-5*b**3 - 6*b**2*c*x**2)/(4*b**2*c**4 + 8*
b*c**5*x**2 + 4*c**6*x**4) + x**2/(2*c**3)
```

3.207 $\int \frac{x^{12}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=74

$$-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{7/2}}-\frac{5x^3}{8c^2\left(b+cx^2\right)}-\frac{x^5}{4c\left(b+cx^2\right)^2}+\frac{15x}{8c^3}$$

```
[Out] 15/8*x/c^3-1/4*x^5/c/(c*x^2+b)^2-5/8*x^3/c^2/(c*x^2+b)-15/8*arctan(x*c^(1/2
)/b^(1/2))*b^(1/2)/c^(7/2)
```

Rubi [A] time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 288, 321, 205}

$$-\frac{5x^3}{8c^2\left(b+cx^2\right)}-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{7/2}}-\frac{x^5}{4c\left(b+cx^2\right)^2}+\frac{15x}{8c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (15*x)/(8*c^3) - x^5/(4*c*(b + c*x^2)^2) - (5*x^3)/(8*c^2*(b + c*x^2)) - (1
5*Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{12}}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{x^6}{\left(b+cx^2\right)^3}\,dx\\
&= -\frac{x^5}{4c\left(b+cx^2\right)^2}+\frac{5\int \frac{x^4}{\left(b+cx^2\right)^2}\,dx}{4c}\\
&= -\frac{x^5}{4c\left(b+cx^2\right)^2}-\frac{5x^3}{8c^2\left(b+cx^2\right)}+\frac{15\int \frac{x^2}{b+cx^2}\,dx}{8c^2}\\
&= \frac{15x}{8c^3}-\frac{x^5}{4c\left(b+cx^2\right)^2}-\frac{5x^3}{8c^2\left(b+cx^2\right)}-\frac{(15b)\int \frac{1}{b+cx^2}\,dx}{8c^3}\\
&= \frac{15x}{8c^3}-\frac{x^5}{4c\left(b+cx^2\right)^2}-\frac{5x^3}{8c^2\left(b+cx^2\right)}-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.89

$$
\frac{15b^2x+25bcx^3+8c^2x^5}{8c^3\left(b+cx^2\right)^2}-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (15*b^2*x + 25*b*c*x^3 + 8*c^2*x^5)/(8*c^3*(b + c*x^2)^2) - (15*Sqrt[b]*Arc
Tan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(7/2))
```

fricas [A] time = 0.61, size = 202, normalized size = 2.73

$$\left[\frac{16\,c^{2}x^{5}+50\,bcx^{3}+30\,b^{2}x+15\,\left(c^{2}x^{4}+2\,bcx^{2}+b^{2}\right)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^{2}-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^{2}+b}\right)}{16\,\left(c^{5}x^{4}+2\,bc^{4}x^{2}+b^{2}c^{3}\right)},\ \frac{8\,c^{2}x^{5}+25\,bcx^{3}+15\,b^{2}x-15\,(\ldots}{8\,\left(c^{5}x^{4}+\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*c^2*x^5 + 50*b*c*x^3 + 30*b^2*x + 15*(c^2*x^4 + 2*b*c*x^2 + b^2)*
sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)))/(c^5*x^4 + 2*b*
c^4*x^2 + b^2*c^3), 1/8*(8*c^2*x^5 + 25*b*c*x^3 + 15*b^2*x - 15*(c^2*x^4 +
2*b*c*x^2 + b^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b))/(c^5*x^4 + 2*b*c^4*x^2
+ b^2*c^3)]
```

giac [A] time = 0.17, size = 54, normalized size = 0.73

$$-\frac{15\,b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,c^{3}}+\frac{x}{c^{3}}+\frac{9\,bcx^{3}+7\,b^{2}x}{8\,\left(cx^{2}+b\right)^{2}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -15/8*b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + x/c^3 + 1/8*(9*b*c*x^3 + 7*
b^2*x)/((c*x^2 + b)^2*c^3)
```

maple [A] time = 0.01, size = 63, normalized size = 0.85

$$\frac{9b\,x^{3}}{8\left(c\,x^{2}+b\right)^{2}c^{2}}+\frac{7b^{2}x}{8\left(c\,x^{2}+b\right)^{2}c^{3}}-\frac{15b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\;c^{3}}+\frac{x}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(c*x^4+b*x^2)^3,x)
```

```
[Out] x/c^3+9/8/c^2*b/(c*x^2+b)^2*x^3+7/8/c^3*b^2/(c*x^2+b)^2*x-15/8/c^3*b/(b*c)^
(1/2)*arctan(1/(b*c)^(1/2)*c*x)
```

maxima [A] time = 2.82, size = 68, normalized size = 0.92

$$\frac{9\,bcx^{3}+7\,b^{2}x}{8\,\left(c^{5}x^{4}+2\,bc^{4}x^{2}+b^{2}c^{3}\right)}-\frac{15\,b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,c^{3}}+\frac{x}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(9*b*c*x^3 + 7*b^2*x)/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3) - 15/8*b*arctan
(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + x/c^3
```

mupad [B] time = 4.25, size = 64, normalized size = 0.86

$$\frac{\frac{7\,b^2\,x}{8}+\frac{9\,c\,b\,x^3}{8}}{b^2\,c^3+2\,b\,c^4\,x^2+c^5\,x^4}+\frac{x}{c^3}-\frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,c^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((7*b^2*x)/8 + (9*b*c*x^3)/8)/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^2) + x/c^3 - (
15*b^(1/2)*atan((c^(1/2)*x)/b^(1/2)))/(8*c^(7/2))
```

sympy [A] time = 0.46, size = 107, normalized size = 1.45

$$\frac{15\sqrt{-\frac{b}{c^7}}\log\left(-c^3\sqrt{-\frac{b}{c^7}}+x\right)}{16}-\frac{15\sqrt{-\frac{b}{c^7}}\log\left(c^3\sqrt{-\frac{b}{c^7}}+x\right)}{16}+\frac{7b^2x+9bcx^3}{8b^2c^3+16bc^4x^2+8c^5x^4}+\frac{x}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(c*x**4+b*x**2)**3,x)
```

```
[Out] 15*sqrt(-b/c**7)*log(-c**3*sqrt(-b/c**7) + x)/16 - 15*sqrt(-b/c**7)*log(c**
3*sqrt(-b/c**7) + x)/16 + (7*b**2*x + 9*b*c*x**3)/(8*b**2*c**3 + 16*b*c**4*
x**2 + 8*c**5*x**4) + x/c**3
```

3.208 $\int \frac{x^{11}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=49

$$-\frac{b^2}{4c^3\left(b+cx^2\right)^2}+\frac{b}{c^3\left(b+cx^2\right)}+\frac{\log\left(b+cx^2\right)}{2c^3}$$

```
[Out] -1/4*b^2/c^3/(c*x^2+b)^2+b/c^3/(c*x^2+b)+1/2*ln(c*x^2+b)/c^3
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$-\frac{b^2}{4c^3\left(b+cx^2\right)^2}+\frac{b}{c^3\left(b+cx^2\right)}+\frac{\log\left(b+cx^2\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -b^2/(4*c^3*(b + c*x^2)^2) + b/(c^3*(b + c*x^2)) + Log[b + c*x^2]/(2*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{11}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^5}{\left(b+cx^2\right)^3}\,dx$$
$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^2}{(b+cx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{b^2}{c^2(b+cx)^3} - \frac{2b}{c^2(b+cx)^2} + \frac{1}{c^2(b+cx)}\right)dx, x, x^2\right)$$
$$= -\frac{b^2}{4c^3\left(b+cx^2\right)^2} + \frac{b}{c^3\left(b+cx^2\right)} + \frac{\log\left(b+cx^2\right)}{2c^3}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.80

$$\frac{\frac{b\left(3b+4cx^2\right)}{\left(b+cx^2\right)^2} + 2\log\left(b+cx^2\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(b*x^2 + c*x^4)^3,x]
[Out] ((b*(3*b + 4*c*x^2))/(b + c*x^2)^2 + 2*Log[b + c*x^2])/(4*c^3)
```

fricas [A] time = 0.64, size = 69, normalized size = 1.41

$$\frac{4\,bcx^2 + 3\,b^2 + 2\left(c^2x^4 + 2\,bcx^2 + b^2\right)\log\left(cx^2+b\right)}{4\left(c^5x^4 + 2\,bc^4x^2 + b^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^3,x, algorithm="fricas")
[Out] 1/4*(4*b*c*x^2 + 3*b^2 + 2*(c^2*x^4 + 2*b*c*x^2 + b^2)*log(c*x^2 + b))/(c^5
*x^4 + 2*b*c^4*x^2 + b^2*c^3)
```

giac [A] time = 0.18, size = 42, normalized size = 0.86

$$\frac{\log\left(\left|cx^2+b\right|\right)}{2\,c^3} - \frac{3\,cx^4 + 2\,bx^2}{4\left(cx^2+b\right)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(c*x^2 + b))/c^3 - 1/4*(3*c*x^4 + 2*b*x^2)/((c*x^2 + b)^2*c^2)
```

maple [A] time = 0.01, size = 46, normalized size = 0.94

$$-\frac{b^{2}}{4\left(c\,x^{2}+b\right)^{2} c^{3}}+\frac{b}{\left(c\,x^{2}+b\right) c^{3}}+\frac{\ln\left(c\,x^{2}+b\right)}{2c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/4*b^2/c^3/(c*x^2+b)^2+b/c^3/(c*x^2+b)+1/2*ln(c*x^2+b)/c^3
```

maxima [A] time = 1.35, size = 55, normalized size = 1.12

$$\frac{4\,bcx^{2}+3\,b^{2}}{4\left(c^{5}x^{4}+2\,bc^{4}x^{2}+b^{2}c^{3}\right)}+\frac{\log\left(cx^{2}+b\right)}{2\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*b*c*x^2 + 3*b^2)/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3) + 1/2*log(c*x^2 +
 b)/c^3
```

mupad [B] time = 4.18, size = 52, normalized size = 1.06

$$\frac{\frac{3\,b^{2}}{4\,c^{3}}+\frac{b\,x^{2}}{c^{2}}}{b^{2}+2\,b\,c\,x^{2}+c^{2}\,x^{4}}+\frac{\ln\left(c\,x^{2}+b\right)}{2\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((3*b^2)/(4*c^3) + (b*x^2)/c^2)/(b^2 + c^2*x^4 + 2*b*c*x^2) + log(b + c*x^2
)/(2*c^3)
```

sympy [A] time = 0.37, size = 53, normalized size = 1.08

$$\frac{3b^{2}+4bcx^{2}}{4b^{2}c^{3}+8bc^{4}x^{2}+4c^{5}x^{4}}+\frac{\log\left(b+cx^{2}\right)}{2c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**4+b*x**2)**3,x)
```

```
[Out] (3*b**2 + 4*b*c*x**2)/(4*b**2*c**3 + 8*b*c**4*x**2 + 4*c**5*x**4) + log(b +
 c*x**2)/(2*c**3)
```

3.209 $\int \frac{x^{10}}{\left(bx^2+cx^4\right)^3} \, dx$

Optimal. Leaf size=64

$$\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\sqrt{b}\,c^{5/2}}-\frac{3x}{8c^2\left(b+cx^2\right)}-\frac{x^3}{4c\left(b+cx^2\right)^2}$$

```
[Out] -1/4*x^3/c/(c*x^2+b)^2-3/8*x/c^2/(c*x^2+b)+3/8*arctan(x*c^(1/2)/b^(1/2))/c^
(5/2)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 288, 205}

$$-\frac{3x}{8c^2\left(b+cx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\sqrt{b}\,c^{5/2}}-\frac{x^3}{4c\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -x^3/(4*c*(b + c*x^2)^2) - (3*x)/(8*c^2*(b + c*x^2)) + (3*ArcTan[(Sqrt[c]*x
)/Sqrt[b]])/(8*Sqrt[b]*c^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{10}}{\left(bx^2+cx^4\right)^3}\,dx=\int \frac{x^4}{\left(b+cx^2\right)^3}\,dx$$
$$=-\frac{x^3}{4c\left(b+cx^2\right)^2}+\frac{3\int \frac{x^2}{\left(b+cx^2\right)^2}\,dx}{4c}$$
$$=-\frac{x^3}{4c\left(b+cx^2\right)^2}-\frac{3x}{8c^2\left(b+cx^2\right)}+\frac{3\int \frac{1}{b+cx^2}\,dx}{8c^2}$$
$$=-\frac{x^3}{4c\left(b+cx^2\right)^2}-\frac{3x}{8c^2\left(b+cx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\sqrt{b}\,c^{5/2}}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.86

$$\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\sqrt{b}\,c^{5/2}}-\frac{3bx+5cx^3}{8c^2\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/8*(3*b*x + 5*c*x^3)/(c^2*(b + c*x^2)^2) + (3*ArcTan[(Sqrt[c]*x)/Sqrt[b]]
)/(8*Sqrt[b]*c^(5/2))
```

fricas [A] time = 0.94, size = 188, normalized size = 2.94

$$\left[-\frac{10\,bc^2x^3+6\,b^2cx+3\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{-bc}\,\log\left(\frac{cx^2-2\,\sqrt{-bc}\,x-b}{cx^2+b}\right)}{16\left(bc^5x^4+2\,b^2c^4x^2+b^3c^3\right)},-\frac{5\,bc^2x^3+3\,b^2cx-3\left(c^2x^4+2\,bcx^2+b^2\right)}{8\left(bc^5x^4+2\,b^2c^4x^2+b^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(10*b*c^2*x^3 + 6*b^2*c*x + 3*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(-b*c)
*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)))/(b*c^5*x^4 + 2*b^2*c^4*x^2
+ b^3*c^3), -1/8*(5*b*c^2*x^3 + 3*b^2*c*x - 3*(c^2*x^4 + 2*b*c*x^2 + b^2)*s
qrt(b*c)*arctan(sqrt(b*c)*x/b))/(b*c^5*x^4 + 2*b^2*c^4*x^2 + b^3*c^3)]
```

giac [A] time = 0.16, size = 45, normalized size = 0.70

$$\frac{3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\,c^2} - \frac{5\,cx^3 + 3\,bx}{8\left(cx^2 + b\right)^2 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) - 1/8*(5*c*x^3 + 3*b*x)/((c*x^2 +
 b)^2*c^2)
```

maple [A] time = 0.01, size = 47, normalized size = 0.73

$$\frac{3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\;c^2} + \frac{-\frac{5x^3}{8c} - \frac{3bx}{8c^2}}{\left(c\,x^2 + b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^4+b*x^2)^3,x)
```

```
[Out] (-5/8/c*x^3-3/8*b/c^2*x)/(c*x^2+b)^2+3/8/c^2/(b*c)^(1/2)*arctan(1/(b*c)^(1/
2)*c*x)
```

maxima [A] time = 2.91, size = 59, normalized size = 0.92

$$-\frac{5\,cx^3 + 3\,bx}{8\left(c^4x^4 + 2\,bc^3x^2 + b^2c^2\right)} + \frac{3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(5*c*x^3 + 3*b*x)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2) + 3/8*arctan(c*x/s
qrt(b*c))/(sqrt(b*c)*c^2)
```

mupad [B] time = 4.23, size = 56, normalized size = 0.88

$$\frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,\sqrt{b}\,c^{5/2}} - \frac{\frac{5\,x^3}{8\,c} + \frac{3\,b\,x}{8\,c^2}}{b^2 + 2\,b\,c\,x^2 + c^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (3*atan((c^(1/2)*x)/b^(1/2)))/(8*b^(1/2)*c^(5/2)) - ((5*x^3)/(8*c) + (3*b*x
)/(8*c^2))/(b^2 + c^2*x^4 + 2*b*c*x^2)
```

sympy [A] time = 0.37, size = 110, normalized size = 1.72

$$-\frac{3\sqrt{-\frac{1}{bc^5}}\log\left(-bc^2\sqrt{-\frac{1}{bc^5}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{bc^5}}\log\left(bc^2\sqrt{-\frac{1}{bc^5}}+x\right)}{16}+\frac{-3bx-5cx^3}{8b^2c^2+16bc^3x^2+8c^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*sqrt(-1/(b*c**5))*log(-b*c**2*sqrt(-1/(b*c**5)) + x)/16 + 3*sqrt(-1/(b*c
**5))*log(b*c**2*sqrt(-1/(b*c**5)) + x)/16 + (-3*b*x - 5*c*x**3)/(8*b**2*c*
*2 + 16*b*c**3*x**2 + 8*c**4*x**4)
```

3.210 $\int \frac{x^9}{(bx^2+cx^4)^3} dx$

Optimal. Leaf size=19

$$\frac{x^4}{4b(b+cx^2)^2}$$

```
[Out] 1/4*x^4/b/(c*x^2+b)^2
```

Rubi **[A]** time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 264}

$$\frac{x^4}{4b(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(b*x^2 + c*x^4)^3,x]
```

```
[Out] x^4/(4*b*(b + c*x^2)^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{x^9}{(bx^2+cx^4)^3} dx &= \int \frac{x^3}{(b+cx^2)^3} dx \\ &= \frac{x^4}{4b(b+cx^2)^2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.26

$$-\frac{b+2cx^2}{4c^2\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(b*x^2 + c*x^4)^3,x]
[Out] -1/4*(b + 2*c*x^2)/(c^2*(b + c*x^2)^2)
```

fricas [B] time = 0.65, size = 36, normalized size = 1.89

$$-\frac{2\,cx^2+b}{4\left(c^4x^4+2\,bc^3x^2+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^3,x, algorithm="fricas")
[Out] -1/4*(2*c*x^2 + b)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

giac [A] time = 0.16, size = 22, normalized size = 1.16

$$-\frac{2\,cx^2+b}{4\left(cx^2+b\right)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^3,x, algorithm="giac")
[Out] -1/4*(2*c*x^2 + b)/((c*x^2 + b)^2*c^2)
```

maple [A] time = 0.01, size = 31, normalized size = 1.63

$$\frac{b}{4\left(c\,x^2+b\right)^2c^2}-\frac{1}{2\left(c\,x^2+b\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2)^3,x)
[Out] 1/4*b/c^2/(c*x^2+b)^2-1/2/c^2/(c*x^2+b)
```

maxima [B] time = 1.33, size = 36, normalized size = 1.89

$$-\frac{2\,cx^2+b}{4\left(c^4x^4+2\,bc^3x^2+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*c*x^2 + b)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

mupad [B] time = 4.18, size = 37, normalized size = 1.95

$$-\frac{\frac{b}{4\,c^2}+\frac{x^2}{2\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2 + c*x^4)^3,x)
```

```
[Out] -(b/(4*c^2) + x^2/(2*c))/(b^2 + c^2*x^4 + 2*b*c*x^2)
```

sympy [B] time = 0.31, size = 36, normalized size = 1.89

$$\frac{-b-2cx^2}{4b^2c^2+8bc^3x^2+4c^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-b - 2*c*x**2)/(4*b**2*c**2 + 8*b*c**3*x**2 + 4*c**4*x**4)
```

3.211 $\int \frac{x^8}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=65

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{3/2}c^{3/2}}+\frac{x}{8bc\left(b+cx^2\right)}-\frac{x}{4c\left(b+cx^2\right)^2}$$

```
[Out] -1/4*x/c/(c*x^2+b)^2+1/8*x/b/c/(c*x^2+b)+1/8*arctan(x*c^(1/2)/b^(1/2))/b^(3
/2)/c^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 288, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{3/2}c^{3/2}}+\frac{x}{8bc\left(b+cx^2\right)}-\frac{x}{4c\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -x/(4*c*(b + c*x^2)^2) + x/(8*b*c*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]
]/(8*b^(3/2)*c^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{x^2}{\left(b+cx^2\right)^3}\,dx\\ &= -\frac{x}{4c\left(b+cx^2\right)^2}+\frac{\int \frac{1}{\left(b+cx^2\right)^2}\,dx}{4c}\\ &= -\frac{x}{4c\left(b+cx^2\right)^2}+\frac{x}{8bc\left(b+cx^2\right)}+\frac{\int \frac{1}{b+cx^2}\,dx}{8bc}\\ &= -\frac{x}{4c\left(b+cx^2\right)^2}+\frac{x}{8bc\left(b+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{3/2}c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 0.89

$$\frac{\frac{\sqrt{b}\,\sqrt{c}\,x\left(cx^2-b\right)}{\left(b+cx^2\right)^2}+\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{3/2}c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((Sqrt[b]*Sqrt[c]*x*(-b + c*x^2))/(b + c*x^2)^2 + ArcTan[(Sqrt[c]*x)/Sqrt[b
]])/(8*b^(3/2)*c^(3/2))
```

fricas [A] time = 0.71, size = 190, normalized size = 2.92

$$\left[\frac{2\,bc^2x^3-2\,b^2cx-\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{-bc}\,\log\left(\frac{cx^2-2\,\sqrt{-bc}\,x-b}{cx^2+b}\right)}{16\left(b^2c^4x^4+2\,b^3c^3x^2+b^4c^2\right)},\frac{bc^2x^3-b^2cx+\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{bc}\,\text{arcta}}{8\left(b^2c^4x^4+2\,b^3c^3x^2+b^4c^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*b*c^2*x^3 - 2*b^2*c*x - (c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(-b*c)*log
((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)))/(b^2*c^4*x^4 + 2*b^3*c^3*x^2 +
b^4*c^2), 1/8*(b*c^2*x^3 - b^2*c*x + (c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(b*c)*
arctan(sqrt(b*c)*x/b))/(b^2*c^4*x^4 + 2*b^3*c^3*x^2 + b^4*c^2)]
```

giac [A] time = 0.16, size = 50, normalized size = 0.77

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\,bc}+\frac{cx^3-bx}{8\left(cx^2+b\right)^2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b*c) + 1/8*(c*x^3 - b*x)/((c*x^2 + b)^
2*b*c)
```

maple [A] time = 0.01, size = 49, normalized size = 0.75

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\;bc}+\frac{\frac{x^3}{8b}-\frac{x}{8c}}{\left(c\,x^2+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2)^3,x)
```

```
[Out] (1/8/b*x^3-1/8/c*x)/(c*x^2+b)^2+1/8/c/b/(b*c)^(1/2)*arctan(1/(b*c)^(1/2)*c*
x)
```

maxima [A] time = 2.86, size = 62, normalized size = 0.95

$$\frac{cx^3-bx}{8\left(bc^3x^4+2\,b^2c^2x^2+b^3c\right)}+\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(c*x^3 - b*x)/(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c) + 1/8*arctan(c*x/sqrt
(b*c))/(sqrt(b*c)*b*c)
```

mupad [B] time = 4.23, size = 55, normalized size = 0.85

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,b^{3/2}\,c^{3/2}}-\frac{\frac{x}{8\,c}-\frac{x^3}{8\,b}}{b^2+2\,b\,c\,x^2+c^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2 + c*x^4)^3,x)
```

```
[Out] atan((c^(1/2)*x)/b^(1/2))/(8*b^(3/2)*c^(3/2)) - (x/(8*c) - x^3/(8*b))/(b^2
+ c^2*x^4 + 2*b*c*x^2)
```

sympy [B] time = 0.35, size = 110, normalized size = 1.69

$$-\frac{\sqrt{-\frac{1}{b^3c^3}}\log\left(-b^2c\sqrt{-\frac{1}{b^3c^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{b^3c^3}}\log\left(b^2c\sqrt{-\frac{1}{b^3c^3}}+x\right)}{16}+\frac{-bx+cx^3}{8b^3c+16b^2c^2x^2+8bc^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2)**3,x)
```

```
[Out] -sqrt(-1/(b**3*c**3))*log(-b**2*c*sqrt(-1/(b**3*c**3)) + x)/16 + sqrt(-1/(b
**3*c**3))*log(b**2*c*sqrt(-1/(b**3*c**3)) + x)/16 + (-b*x + c*x**3)/(8*b**
3*c + 16*b**2*c**2*x**2 + 8*b*c**3*x**4)
```

3.212 $\int \frac{x^7}{(bx^2+cx^4)^3} dx$

Optimal. Leaf size=16

$$-\frac{1}{4c\left(b+cx^2\right)^2}$$

[Out] -1/4/c/(c*x^2+b)^2

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 261}

$$-\frac{1}{4c\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

[In] Int[x^7/(b*x^2 + c*x^4)^3,x]

[Out] -1/(4*c*(b + c*x^2)^2)

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^7}{\left(bx^2+cx^4\right)^3} dx = \int \frac{x}{\left(b+cx^2\right)^3} dx$$
$$= -\frac{1}{4c\left(b+cx^2\right)^2}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{4c\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*1/(c*(b + c*x^2)^2)
```

fricas [A] time = 0.57, size = 26, normalized size = 1.62

$$-\frac{1}{4\left(c^3x^4+2\,bc^2x^2+b^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4/(c^3*x^4 + 2*b*c^2*x^2 + b^2*c)
```

giac [A] time = 0.15, size = 14, normalized size = 0.88

$$-\frac{1}{4\left(cx^2+b\right)^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/4/((c*x^2 + b)^2*c)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{4\left(c\,x^2+b\right)^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/4/c/(c*x^2+b)^2
```

maxima [A] time = 1.27, size = 26, normalized size = 1.62

$$-\frac{1}{4\left(c^3x^4+2\,bc^2x^2+b^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4/(c^3*x^4 + 2*b*c^2*x^2 + b^2*c)
```

mupad [B] time = 0.03, size = 28, normalized size = 1.75

$$-\frac{1}{4\,b^2\,c+8\,b\,c^2\,x^2+4\,c^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2 + c*x^4)^3,x)
```

```
[Out] -1/(4*b^2*c + 4*c^3*x^4 + 8*b*c^2*x^2)
```

sympy [A] time = 0.27, size = 27, normalized size = 1.69

$$-\frac{1}{4b^2c+8bc^2x^2+4c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2)**3,x)
```

```
[Out] -1/(4*b**2*c + 8*b*c**2*x**2 + 4*c**3*x**4)
```

3.213 $\int \frac{x^6}{\left(bx^2+cx^4\right)^3} dx$

Optimal. Leaf size=62

$$\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{5/2}\sqrt{c}}+\frac{3x}{8b^2\left(b+cx^2\right)}+\frac{x}{4b\left(b+cx^2\right)^2}$$

```
[Out] 1/4*x/b/(c*x^2+b)^2+3/8*x/b^2/(c*x^2+b)+3/8*arctan(x*c^(1/2)/b^(1/2))/b^(5/
2)/c^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 199, 205}

$$\frac{3x}{8b^2\left(b+cx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{5/2}\sqrt{c}}+\frac{x}{4b\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(b*x^2 + c*x^4)^3,x]
```

```
[Out] x/(4*b*(b + c*x^2)^2) + (3*x)/(8*b^2*(b + c*x^2)) + (3*ArcTan[(Sqrt[c]*x)/S
qrt[b]])/(8*b^(5/2)*Sqrt[c])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{1}{\left(b+cx^2\right)^3}\,dx\\
&= \frac{x}{4b\left(b+cx^2\right)^2}+\frac{3\int \frac{1}{\left(b+cx^2\right)^2}\,dx}{4b}\\
&= \frac{x}{4b\left(b+cx^2\right)^2}+\frac{3x}{8b^2\left(b+cx^2\right)}+\frac{3\int \frac{1}{b+cx^2}\,dx}{8b^2}\\
&= \frac{x}{4b\left(b+cx^2\right)^2}+\frac{3x}{8b^2\left(b+cx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{5/2}\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.89

$$\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{5/2}\sqrt{c}}+\frac{5bx+3cx^3}{8b^2\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (5*b*x + 3*c*x^3)/(8*b^2*(b + c*x^2)^2) + (3*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(
8*b^(5/2)*Sqrt[c])
```

fricas [A] time = 0.75, size = 188, normalized size = 3.03

$$\left[\frac{6\,bc^2x^3+10\,b^2cx-3\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{-bc}\,\log\left(\frac{cx^2-2\,\sqrt{-bc}\,x-b}{cx^2+b}\right)}{16\left(b^3c^3x^4+2\,b^4c^2x^2+b^5c\right)},\frac{3\,bc^2x^3+5\,b^2cx+3\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{b}}{8\left(b^3c^3x^4+2\,b^4c^2x^2+b^5c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*b*c^2*x^3 + 10*b^2*c*x - 3*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(-b*c)*
log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)))/(b^3*c^3*x^4 + 2*b^4*c^2*x^2
 + b^5*c), 1/8*(3*b*c^2*x^3 + 5*b^2*c*x + 3*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqr
t(b*c)*arctan(sqrt(b*c)*x/b))/(b^3*c^3*x^4 + 2*b^4*c^2*x^2 + b^5*c)]
```

giac [A] time = 0.20, size = 45, normalized size = 0.73

$$\frac{3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8 \sqrt{bc}\, b^2} + \frac{3\, cx^3 + 5\, bx}{8 \left(cx^2 + b\right)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) + 1/8*(3*c*x^3 + 5*b*x)/((c*x^2 +
 b)^2*b^2)
```

maple [A] time = 0.01, size = 51, normalized size = 0.82

$$\frac{x}{4 \left(c\, x^2 + b\right)^2 b} + \frac{3x}{8 \left(c\, x^2 + b\right) b^2} + \frac{3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\ b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/4*x/b/(c*x^2+b)^2+3/8*x/b^2/(c*x^2+b)+3/8/b^2/(b*c)^(1/2)*arctan(1/(b*c)^
(1/2)*c*x)
```

maxima [A] time = 2.99, size = 58, normalized size = 0.94

$$\frac{3\, cx^3 + 5\, bx}{8 \left(b^2 c^2 x^4 + 2\, b^3 c x^2 + b^4\right)} + \frac{3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8 \sqrt{bc}\, b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*c*x^3 + 5*b*x)/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4) + 3/8*arctan(c*x/sq
rt(b*c))/(sqrt(b*c)*b^2)
```

mupad [B] time = 4.21, size = 55, normalized size = 0.89

$$\frac{\frac{5\,x}{8\,b} + \frac{3\,c\,x^3}{8\,b^2}}{b^2 + 2\,b\,c\,x^2 + c^2\,x^4} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,b^{5/2}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((5*x)/(8*b) + (3*c*x^3)/(8*b^2))/(b^2 + c^2*x^4 + 2*b*c*x^2) + (3*atan((c^
(1/2)*x)/b^(1/2)))/(8*b^(5/2)*c^(1/2))
```

sympy [A] time = 0.36, size = 105, normalized size = 1.69

$$-\frac{3\sqrt{-\frac{1}{b^5c}}\log\left(-b^3\sqrt{-\frac{1}{b^5c}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{b^5c}}\log\left(b^3\sqrt{-\frac{1}{b^5c}}+x\right)}{16}+\frac{5bx+3cx^3}{8b^4+16b^3cx^2+8b^2c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*sqrt(-1/(b**5*c))*log(-b**3*sqrt(-1/(b**5*c)) + x)/16 + 3*sqrt(-1/(b**5*
c))*log(b**3*sqrt(-1/(b**5*c)) + x)/16 + (5*b*x + 3*c*x**3)/(8*b**4 + 16*b*
*3*c*x**2 + 8*b**2*c**2*x**4)
```

3.214 $\int \frac{x^5}{(bx^2+cx^4)^3} dx$

Optimal. Leaf size=54

$$-\frac{\log(b+cx^2)}{2b^3}+\frac{\log(x)}{b^3}+\frac{1}{2b^2(b+cx^2)}+\frac{1}{4b(b+cx^2)^2}$$

```
[Out] 1/4/b/(c*x^2+b)^2+1/2/b^2/(c*x^2+b)+ln(x)/b^3-1/2*ln(c*x^2+b)/b^3
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$\frac{1}{2b^2(b+cx^2)}-\frac{\log(b+cx^2)}{2b^3}+\frac{\log(x)}{b^3}+\frac{1}{4b(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(b*x^2 + c*x^4)^3,x]
```

```
[Out] 1/(4*b*(b + c*x^2)^2) + 1/(2*b^2*(b + c*x^2)) + Log[x]/b^3 - Log[b + c*x^2]
/(2*b^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^5}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{1}{x\left(b+cx^2\right)^3}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(b+cx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{b^3x} - \frac{c}{b(b+cx)^3} - \frac{c}{b^2(b+cx)^2} - \frac{c}{b^3(b+cx)}\right)dx, x, x^2\right)$$
$$= \frac{1}{4b\left(b+cx^2\right)^2} + \frac{1}{2b^2\left(b+cx^2\right)} + \frac{\log(x)}{b^3} - \frac{\log\left(b+cx^2\right)}{2b^3}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 0.80

$$\frac{\frac{b\left(3b+2cx^2\right)}{\left(b+cx^2\right)^2} - 2\log\left(b+cx^2\right) + 4\log(x)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b*(3*b + 2*c*x^2))/(b + c*x^2)^2 + 4*Log[x] - 2*Log[b + c*x^2])/(4*b^3)
```

fricas [A] time = 0.57, size = 90, normalized size = 1.67

$$\frac{2\,bcx^2 + 3\,b^2 - 2\left(c^2x^4 + 2\,bcx^2 + b^2\right)\log\left(cx^2+b\right) + 4\left(c^2x^4 + 2\,bcx^2 + b^2\right)\log(x)}{4\left(b^3c^2x^4 + 2\,b^4cx^2 + b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*x^2 + 3*b^2 - 2*(c^2*x^4 + 2*b*c*x^2 + b^2)*log(c*x^2 + b) + 4*(
c^2*x^4 + 2*b*c*x^2 + b^2)*log(x))/(b^3*c^2*x^4 + 2*b^4*c*x^2 + b^5)
```

giac [A] time = 0.16, size = 59, normalized size = 1.09

$$\frac{\log\left(x^2\right)}{2\,b^3} - \frac{\log\left(\left|cx^2+b\right|\right)}{2\,b^3} + \frac{3\,c^2x^4 + 8\,bcx^2 + 6\,b^2}{4\left(cx^2+b\right)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/b^3 - 1/2*log(abs(c*x^2 + b))/b^3 + 1/4*(3*c^2*x^4 + 8*b*c*x^2
 + 6*b^2)/((c*x^2 + b)^2*b^3)
```

maple [A] time = 0.01, size = 49, normalized size = 0.91

$$\frac{1}{4\left(c\,x^{2}+b\right)^{2} b}+\frac{1}{2\left(c\,x^{2}+b\right) b^{2}}+\frac{\ln(x)}{b^{3}}-\frac{\ln\left(c\,x^{2}+b\right)}{2b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/4/b/(c*x^2+b)^2+1/2/b^2/(c*x^2+b)+ln(x)/b^3-1/2*ln(c*x^2+b)/b^3
```

maxima [A] time = 1.38, size = 60, normalized size = 1.11

$$\frac{2\,cx^{2}+3\,b}{4\left(b^{2}c^{2}x^{4}+2\,b^{3}cx^{2}+b^{4}\right)}-\frac{\log\left(cx^{2}+b\right)}{2\,b^{3}}+\frac{\log\left(x^{2}\right)}{2\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*c*x^2 + 3*b)/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4) - 1/2*log(c*x^2 + b)/
b^3 + 1/2*log(x^2)/b^3
```

mupad [B] time = 0.06, size = 56, normalized size = 1.04

$$\frac{\ln(x)}{b^{3}}+\frac{\frac{3}{4\,b}+\frac{c\,x^{2}}{2\,b^{2}}}{b^{2}+2\,b\,c\,x^{2}+c^{2}\,x^{4}}-\frac{\ln\left(c\,x^{2}+b\right)}{2\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2 + c*x^4)^3,x)
```

```
[Out] log(x)/b^3 + (3/(4*b) + (c*x^2)/(2*b^2))/(b^2 + c^2*x^4 + 2*b*c*x^2) - log(
b + c*x^2)/(2*b^3)
```

sympy [A] time = 0.45, size = 56, normalized size = 1.04

$$\frac{3b+2cx^{2}}{4b^{4}+8b^{3}cx^{2}+4b^{2}c^{2}x^{4}}+\frac{\log\left(x\right)}{b^{3}}-\frac{\log\left(\frac{b}{c}+x^{2}\right)}{2b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2)**3,x)
```

```
[Out] (3*b + 2*c*x**2)/(4*b**4 + 8*b**3*c*x**2 + 4*b**2*c**2*x**4) + log(x)/b**3
- log(b/c + x**2)/(2*b**3)
```

3.215 $\int \frac{x^4}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=76

$$-\frac{15\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{7/2}}-\frac{15}{8b^3x}+\frac{5}{8b^2x\left(b+cx^2\right)}+\frac{1}{4bx\left(b+cx^2\right)^2}$$

```
[Out] -15/8/b^3/x+1/4/b/x/(c*x^2+b)^2+5/8/b^2/x/(c*x^2+b)-15/8*arctan(x*c^(1/2)/b
^(1/2))*c^(1/2)/b^(7/2)
```

Rubi [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 290, 325, 205}

$$\frac{5}{8b^2x\left(b+cx^2\right)}-\frac{15\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{7/2}}-\frac{15}{8b^3x}+\frac{1}{4bx\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -15/(8*b^3*x) + 1/(4*b*x*(b + c*x^2)^2) + 5/(8*b^2*x*(b + c*x^2)) - (15*Sqr
t[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{1}{x^2\left(b+cx^2\right)^3}\,dx \\
&= \frac{1}{4bx\left(b+cx^2\right)^2} + \frac{5\int \frac{1}{x^2\left(b+cx^2\right)^2}\,dx}{4b} \\
&= \frac{1}{4bx\left(b+cx^2\right)^2} + \frac{5}{8b^2x\left(b+cx^2\right)} + \frac{15\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{8b^2} \\
&= -\frac{15}{8b^3x} + \frac{1}{4bx\left(b+cx^2\right)^2} + \frac{5}{8b^2x\left(b+cx^2\right)} - \frac{(15c)\int \frac{1}{b+cx^2}\,dx}{8b^3} \\
&= -\frac{15}{8b^3x} + \frac{1}{4bx\left(b+cx^2\right)^2} + \frac{5}{8b^2x\left(b+cx^2\right)} - \frac{15\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 68, normalized size = 0.89

$$
-\frac{15\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{7/2}} - \frac{8b^2+25bcx^2+15c^2x^4}{8b^3x\left(b+cx^2\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/8*(8*b^2 + 25*b*c*x^2 + 15*c^2*x^4)/(b^3*x*(b + c*x^2)^2) - (15*Sqrt[c]*
ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(7/2))
```

fricas [A] time = 0.64, size = 202, normalized size = 2.66

$$\left[-\frac{30\,c^2x^4+50\,bcx^2-15\left(c^2x^5+2\,bcx^3+b^2x\right)\sqrt{-\frac{c}{b}}\,\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)+16\,b^2}{16\left(b^3c^2x^5+2\,b^4cx^3+b^5x\right)},\,-\frac{15\,c^2x^4+25\,bcx^2+15\left(c^2x^5\right.}{8\left(b^3c^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(30*c^2*x^4 + 50*b*c*x^2 - 15*(c^2*x^5 + 2*b*c*x^3 + b^2*x)*sqrt(-c/
b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)) + 16*b^2)/(b^3*c^2*x^5 +
 2*b^4*c*x^3 + b^5*x), -1/8*(15*c^2*x^4 + 25*b*c*x^2 + 15*(c^2*x^5 + 2*b*c*
x^3 + b^2*x)*sqrt(c/b)*arctan(x*sqrt(c/b)) + 8*b^2)/(b^3*c^2*x^5 + 2*b^4*c*
x^3 + b^5*x)]
```

giac [A] time = 0.17, size = 57, normalized size = 0.75

$$-\frac{15\,c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^3}-\frac{7\,c^2x^3+9\,bcx}{8\left(cx^2+b\right)^2b^3}-\frac{1}{b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -15/8*c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - 1/8*(7*c^2*x^3 + 9*b*c*x)/(
(c*x^2 + b)^2*b^3) - 1/(b^3*x)
```

maple [A] time = 0.01, size = 66, normalized size = 0.87

$$-\frac{7c^2x^3}{8\left(c\,x^2+b\right)^2b^3}-\frac{9cx}{8\left(c\,x^2+b\right)^2b^2}-\frac{15c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\;b^3}-\frac{1}{b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2)^3,x)
```

```
[Out] -7/8/b^3*c^2/(c*x^2+b)^2*x^3-9/8/b^2*c/(c*x^2+b)^2*x-15/8/b^3*c/(b*c)^(1/2)
*arctan(1/(b*c)^(1/2)*c*x)-1/b^3/x
```

maxima [A] time = 3.00, size = 71, normalized size = 0.93

$$-\frac{15\,c^2x^4+25\,bcx^2+8\,b^2}{8\left(b^3c^2x^5+2\,b^4cx^3+b^5x\right)}-\frac{15\,c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(15*c^2*x^4 + 25*b*c*x^2 + 8*b^2)/(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)
- 15/8*c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3)
```

mupad [B] time = 4.26, size = 66, normalized size = 0.87

$$-\frac{\frac{1}{b}+\frac{25\,c\,x^2}{8\,b^2}+\frac{15\,c^2\,x^4}{8\,b^3}}{b^2\,x+2\,b\,c\,x^3+c^2\,x^5}-\frac{15\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2 + c*x^4)^3,x)
```

```
[Out] - (1/b + (25*c*x^2)/(8*b^2) + (15*c^2*x^4)/(8*b^3))/(b^2*x + c^2*x^5 + 2*b*
c*x^3) - (15*c^(1/2)*atan((c^(1/2)*x)/b^(1/2)))/(8*b^(7/2))
```

sympy [A] time = 0.45, size = 116, normalized size = 1.53

$$\frac{15\sqrt{-\frac{c}{b^7}}\log\left(-\frac{b^4\sqrt{-\frac{c}{b^7}}}{c}+x\right)}{16}-\frac{15\sqrt{-\frac{c}{b^7}}\log\left(\frac{b^4\sqrt{-\frac{c}{b^7}}}{c}+x\right)}{16}+\frac{-8b^2-25bcx^2-15c^2x^4}{8b^5x+16b^4cx^3+8b^3c^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2)**3,x)
```

```
[Out] 15*sqrt(-c/b**7)*log(-b**4*sqrt(-c/b**7)/c + x)/16 - 15*sqrt(-c/b**7)*log(b
**4*sqrt(-c/b**7)/c + x)/16 + (-8*b**2 - 25*b*c*x**2 - 15*c**2*x**4)/(8*b**
5*x + 16*b**4*c*x**3 + 8*b**3*c**2*x**5)
```

3.216 $\int \frac{x^3}{(bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=67

$$\frac{3c\log\left(b+cx^2\right)}{2b^4}-\frac{3c\log(x)}{b^4}-\frac{c}{b^3\left(b+cx^2\right)}-\frac{1}{2b^3x^2}-\frac{c}{4b^2\left(b+cx^2\right)^2}$$

```
[Out] -1/2/b^3/x^2-1/4*c/b^2/(c*x^2+b)^2-c/b^3/(c*x^2+b)-3*c*ln(x)/b^4+3/2*c*ln(c
*x^2+b)/b^4
```

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$-\frac{c}{b^3\left(b+cx^2\right)}-\frac{c}{4b^2\left(b+cx^2\right)^2}+\frac{3c\log\left(b+cx^2\right)}{2b^4}-\frac{3c\log(x)}{b^4}-\frac{1}{2b^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/(2*b^3*x^2) - c/(4*b^2*(b + c*x^2)^2) - c/(b^3*(b + c*x^2)) - (3*c*Log[x
])/b^4 + (3*c*Log[b + c*x^2])/(2*b^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(bx^2+cx^4)^3}\,dx &= \int \frac{1}{x^3(b+cx^2)^3}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(b+cx)^3}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{b^3x^2}-\frac{3c}{b^4x}+\frac{c^2}{b^2(b+cx)^3}+\frac{2c^2}{b^3(b+cx)^2}+\frac{3c^2}{b^4(b+cx)}\right)dx, x, x^2\right)\\ &= -\frac{1}{2b^3x^2}-\frac{c}{4b^2(b+cx^2)^2}-\frac{c}{b^3(b+cx^2)}-\frac{3c\log(x)}{b^4}+\frac{3c\log(b+cx^2)}{2b^4}\end{aligned}$$

Mathematica [A] time = 0.06, size = 59, normalized size = 0.88

$$-\frac{\frac{b(2b^2+9bcx^2+6c^2x^4)}{x^2(b+cx^2)^2}-6c\log(b+cx^2)+12c\log(x)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*((b*(2*b^2 + 9*b*c*x^2 + 6*c^2*x^4))/(x^2*(b + c*x^2)^2) + 12*c*Log[x]
 - 6*c*Log[b + c*x^2])/b^4
```

fricas [A] time = 0.48, size = 119, normalized size = 1.78

$$-\frac{6\,bc^2x^4+9\,b^2cx^2+2\,b^3-6\left(c^3x^6+2\,bc^2x^4+b^2cx^2\right)\log\left(cx^2+b\right)+12\left(c^3x^6+2\,bc^2x^4+b^2cx^2\right)\log(x)}{4\left(b^4c^2x^6+2\,b^5cx^4+b^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*b*c^2*x^4 + 9*b^2*c*x^2 + 2*b^3 - 6*(c^3*x^6 + 2*b*c^2*x^4 + b^2*c*
x^2)*log(c*x^2 + b) + 12*(c^3*x^6 + 2*b*c^2*x^4 + b^2*c*x^2)*log(x))/(b^4*c
^2*x^6 + 2*b^5*c*x^4 + b^6*x^2)
```

giac [A] time = 0.17, size = 66, normalized size = 0.99

$$\frac{3\,c\log\left(\left|cx^2+b\right|\right)}{2\,b^4}-\frac{3\,c\log\left(|x|\right)}{b^4}-\frac{6\,bc^2x^4+9\,b^2cx^2+2\,b^3}{4\left(cx^2+b\right)^2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/2*c*log(abs(c*x^2 + b))/b^4 - 3*c*log(abs(x))/b^4 - 1/4*(6*b*c^2*x^4 + 9*
b^2*c*x^2 + 2*b^3)/((c*x^2 + b)^2*b^4*x^2)
```

maple [A] time = 0.02, size = 62, normalized size = 0.93

$$-\frac{c}{4\left(c\,x^2+b\right)^2 b^2}-\frac{c}{\left(c\,x^2+b\right) b^3}-\frac{3c\ln(x)}{b^4}+\frac{3c\ln\left(c\,x^2+b\right)}{2b^4}-\frac{1}{2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/2/b^3/x^2-1/4*c/b^2/(c*x^2+b)^2-c/b^3/(c*x^2+b)-3*c*ln(x)/b^4+3/2*c*ln(c
*x^2+b)/b^4
```

maxima [A] time = 1.38, size = 77, normalized size = 1.15

$$-\frac{6\,c^2x^4+9\,bcx^2+2\,b^2}{4\left(b^3c^2x^6+2\,b^4cx^4+b^5x^2\right)}+\frac{3\,c\log\left(cx^2+b\right)}{2\,b^4}-\frac{3\,c\log\left(x^2\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*c^2*x^4 + 9*b*c*x^2 + 2*b^2)/(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)
+ 3/2*c*log(c*x^2 + b)/b^4 - 3/2*c*log(x^2)/b^4
```

mupad [B] time = 0.06, size = 75, normalized size = 1.12

$$\frac{3\,c\,\ln\left(c\,x^2+b\right)}{2\,b^4}-\frac{\frac{1}{2\,b}+\frac{9\,c\,x^2}{4\,b^2}+\frac{3\,c^2\,x^4}{2\,b^3}}{b^2\,x^2+2\,b\,c\,x^4+c^2\,x^6}-\frac{3\,c\,\ln(x)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (3*c*log(b + c*x^2))/(2*b^4) - (1/(2*b) + (9*c*x^2)/(4*b^2) + (3*c^2*x^4)/(
2*b^3))/(b^2*x^2 + c^2*x^6 + 2*b*c*x^4) - (3*c*log(x))/b^4
```

sympy [A] time = 0.63, size = 80, normalized size = 1.19

$$\frac{-2b^2-9bcx^2-6c^2x^4}{4b^5x^2+8b^4cx^4+4b^3c^2x^6}-\frac{3c\log(x)}{b^4}+\frac{3c\log\left(\frac{b}{c}+x^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-2*b**2 - 9*b*c*x**2 - 6*c**2*x**4)/(4*b**5*x**2 + 8*b**4*c*x**4 + 4*b**3*
c**2*x**6) - 3*c*log(x)/b**4 + 3*c*log(b/c + x**2)/(2*b**4)
```

3.217 $\int \frac{x^2}{(bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=87

$$\frac{35c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{9/2}}+\frac{35c}{8b^4x}-\frac{35}{24b^3x^3}+\frac{7}{8b^2x^3\left(b+cx^2\right)}+\frac{1}{4bx^3\left(b+cx^2\right)^2}$$

```
[Out] -35/24/b^3/x^3+35/8*c/b^4/x+1/4/b/x^3/(c*x^2+b)^2+7/8/b^2/x^3/(c*x^2+b)+35/
8*c^(3/2)*arctan(x*c^(1/2)/b^(1/2))/b^(9/2)
```

Rubi [A] time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 290, 325, 205}

$$\frac{35c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{9/2}}+\frac{7}{8b^2x^3\left(b+cx^2\right)}+\frac{35c}{8b^4x}-\frac{35}{24b^3x^3}+\frac{1}{4bx^3\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -35/(24*b^3*x^3) + (35*c)/(8*b^4*x) + 1/(4*b*x^3*(b + c*x^2)^2) + 7/(8*b^2*
x^3*(b + c*x^2)) + (35*c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^2}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{1}{x^4\left(b+cx^2\right)^3}\,dx$$

$$= \frac{1}{4bx^3\left(b+cx^2\right)^2} + \frac{7\int \frac{1}{x^4\left(b+cx^2\right)^2}\,dx}{4b}$$

$$= \frac{1}{4bx^3\left(b+cx^2\right)^2} + \frac{7}{8b^2x^3\left(b+cx^2\right)} + \frac{35\int \frac{1}{x^4\left(b+cx^2\right)}\,dx}{8b^2}$$

$$= -\frac{35}{24b^3x^3} + \frac{1}{4bx^3\left(b+cx^2\right)^2} + \frac{7}{8b^2x^3\left(b+cx^2\right)} - \frac{(35c)\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{8b^3}$$

$$= -\frac{35}{24b^3x^3} + \frac{35c}{8b^4x} + \frac{1}{4bx^3\left(b+cx^2\right)^2} + \frac{7}{8b^2x^3\left(b+cx^2\right)} + \frac{\left(35c^2\right)\int \frac{1}{b+cx^2}\,dx}{8b^4}$$

$$= -\frac{35}{24b^3x^3} + \frac{35c}{8b^4x} + \frac{1}{4bx^3\left(b+cx^2\right)^2} + \frac{7}{8b^2x^3\left(b+cx^2\right)} + \frac{35c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{9/2}}$$

Mathematica [A] time = 0.04, size = 79, normalized size = 0.91

$$\frac{35c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{9/2}} + \frac{-8b^3+56b^2cx^2+175bc^2x^4+105c^3x^6}{24b^4x^3\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-8*b^3 + 56*b^2*c*x^2 + 175*b*c^2*x^4 + 105*c^3*x^6)/(24*b^4*x^3*(b + c*x^
2)^2) + (35*c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(9/2))
```

fricas [A] time = 0.56, size = 238, normalized size = 2.74

$$\left[\frac{210\,c^3x^6 + 350\,bc^2x^4 + 112\,b^2cx^2 - 16\,b^3 + 105\left(c^3x^7 + 2\,bc^2x^5 + b^2cx^3\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2+2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)}{48\left(b^4c^2x^7 + 2\,b^5cx^5 + b^6x^3\right)}, \frac{105\,c^3x^6 + 1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(210*c^3*x^6 + 350*b*c^2*x^4 + 112*b^2*c*x^2 - 16*b^3 + 105*(c^3*x^7
+ 2*b*c^2*x^5 + b^2*c*x^3)*sqrt(-c/b)*log((c*x^2 + 2*b*x*sqrt(-c/b) - b)/(c
*x^2 + b)))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3), 1/24*(105*c^3*x^6 + 175*
b*c^2*x^4 + 56*b^2*c*x^2 - 8*b^3 + 105*(c^3*x^7 + 2*b*c^2*x^5 + b^2*c*x^3)*
sqrt(c/b)*arctan(x*sqrt(c/b)))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)]
```

giac [A] time = 0.18, size = 71, normalized size = 0.82

$$\frac{35\,c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^4} + \frac{11\,c^3x^3 + 13\,bc^2x}{8\left(cx^2+b\right)^2b^4} + \frac{9\,cx^2 - b}{3\,b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 35/8*c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4) + 1/8*(11*c^3*x^3 + 13*b*c^2
*x)/((c*x^2 + b)^2*b^4) + 1/3*(9*c*x^2 - b)/(b^4*x^3)
```

maple [A] time = 0.01, size = 79, normalized size = 0.91

$$\frac{11c^3x^3}{8\left(c\,x^2+b\right)^2b^4} + \frac{13c^2x}{8\left(c\,x^2+b\right)^2b^3} + \frac{35c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\;b^4} + \frac{3c}{b^4x} - \frac{1}{3b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2)^3,x)
```

```
[Out] 11/8/b^4*c^3/(c*x^2+b)^2*x^3+13/8/b^3*c^2/(c*x^2+b)^2*x+35/8/b^4*c^2/(b*c)^
(1/2)*arctan(1/(b*c)^(1/2)*c*x)-1/3/b^3/x^3+3*c/b^4/x
```

maxima [A] time = 2.93, size = 86, normalized size = 0.99

$$\frac{105\,c^3x^6+175\,bc^2x^4+56\,b^2cx^2-8\,b^3}{24\left(b^4c^2x^7+2\,b^5cx^5+b^6x^3\right)}+\frac{35\,c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(105*c^3*x^6 + 175*b*c^2*x^4 + 56*b^2*c*x^2 - 8*b^3)/(b^4*c^2*x^7 + 2*
b^5*c*x^5 + b^6*x^3) + 35/8*c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4)
```

mupad [B] time = 4.26, size = 80, normalized size = 0.92

$$\frac{\frac{7\,c\,x^2}{3\,b^2}-\frac{1}{3\,b}+\frac{175\,c^2\,x^4}{24\,b^3}+\frac{35\,c^3\,x^6}{8\,b^4}}{b^2\,x^3+2\,b\,c\,x^5+c^2\,x^7}+\frac{35\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((7*c*x^2)/(3*b^2) - 1/(3*b) + (175*c^2*x^4)/(24*b^3) + (35*c^3*x^6)/(8*b^4
))/(b^2*x^3 + c^2*x^7 + 2*b*c*x^5) + (35*c^(3/2)*atan((c^(1/2)*x)/b^(1/2)))
/(8*b^(9/2))
```

sympy [A] time = 0.50, size = 138, normalized size = 1.59

$$-\frac{35\sqrt{-\frac{c^3}{b^9}}\,\log\left(-\frac{b^5\sqrt{-\frac{c^3}{b^9}}}{c^2}+x\right)}{16}+\frac{35\sqrt{-\frac{c^3}{b^9}}\,\log\left(\frac{b^5\sqrt{-\frac{c^3}{b^9}}}{c^2}+x\right)}{16}+\frac{-8b^3+56b^2cx^2+175bc^2x^4+105c^3x^6}{24b^6x^3+48b^5cx^5+24b^4c^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2)**3,x)
```

```
[Out] -35*sqrt(-c**3/b**9)*log(-b**5*sqrt(-c**3/b**9)/c**2 + x)/16 + 35*sqrt(-c**
3/b**9)*log(b**5*sqrt(-c**3/b**9)/c**2 + x)/16 + (-8*b**3 + 56*b**2*c*x**2
+ 175*b*c**2*x**4 + 105*c**3*x**6)/(24*b**6*x**3 + 48*b**5*c*x**5 + 24*b**4
*c**2*x**7)
```

3.218 $\int \frac{x}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=86

$$-\frac{3c^2\log\left(b+cx^2\right)}{b^5}+\frac{6c^2\log(x)}{b^5}+\frac{3c^2}{2b^4\left(b+cx^2\right)}+\frac{3c}{2b^4x^2}+\frac{c^2}{4b^3\left(b+cx^2\right)^2}-\frac{1}{4b^3x^4}$$

```
[Out] -1/4/b^3/x^4+3/2*c/b^4/x^2+1/4*c^2/b^3/(c*x^2+b)^2+3/2*c^2/b^4/(c*x^2+b)+6*
c^2*ln(x)/b^5-3*c^2*ln(c*x^2+b)/b^5
```

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1584, 266, 44}

$$\frac{3c^2}{2b^4\left(b+cx^2\right)}+\frac{c^2}{4b^3\left(b+cx^2\right)^2}-\frac{3c^2\log\left(b+cx^2\right)}{b^5}+\frac{6c^2\log(x)}{b^5}+\frac{3c}{2b^4x^2}-\frac{1}{4b^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/(4*b^3*x^4) + (3*c)/(2*b^4*x^2) + c^2/(4*b^3*(b + c*x^2)^2) + (3*c^2)/(2
*b^4*(b + c*x^2)) + (6*c^2*Log[x])/b^5 - (3*c^2*Log[b + c*x^2])/b^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{1}{x^5\left(b+cx^2\right)^3}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3(b+cx)^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{b^3x^3}-\frac{3c}{b^4x^2}+\frac{6c^2}{b^5x}-\frac{c^3}{b^3(b+cx)^3}-\frac{3c^3}{b^4(b+cx)^2}-\frac{6c^3}{b^5(b+cx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{4b^3x^4}+\frac{3c}{2b^4x^2}+\frac{c^2}{4b^3\left(b+cx^2\right)^2}+\frac{3c^2}{2b^4\left(b+cx^2\right)}+\frac{6c^2\log(x)}{b^5}-\frac{3c^2\log\left(b+cx^2\right)}{b^5}\end{aligned}$$

Mathematica [A] time = 0.05, size = 74, normalized size = 0.86

$$\frac{\frac{b\left(-b^3+4b^2cx^2+18bc^2x^4+12c^3x^6\right)}{x^4\left(b+cx^2\right)^2}-12c^2\log\left(b+cx^2\right)+24c^2\log(x)}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b*(-b^3 + 4*b^2*c*x^2 + 18*b*c^2*x^4 + 12*c^3*x^6))/(x^4*(b + c*x^2)^2) +
 24*c^2*Log[x] - 12*c^2*Log[b + c*x^2])/(4*b^5)
```

fricas [A] time = 0.72, size = 134, normalized size = 1.56

$$\frac{12\,bc^3x^6+18\,b^2c^2x^4+4\,b^3cx^2-b^4-12\left(c^4x^8+2\,bc^3x^6+b^2c^2x^4\right)\log\left(cx^2+b\right)+24\left(c^4x^8+2\,bc^3x^6+b^2c^2x^4\right)\log}{4\left(b^5c^2x^8+2\,b^6cx^6+b^7x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(12*b*c^3*x^6 + 18*b^2*c^2*x^4 + 4*b^3*c*x^2 - b^4 - 12*(c^4*x^8 + 2*b*
c^3*x^6 + b^2*c^2*x^4)*log(c*x^2 + b) + 24*(c^4*x^8 + 2*b*c^3*x^6 + b^2*c^2
*x^4)*log(x))/(b^5*c^2*x^8 + 2*b^6*c*x^6 + b^7*x^4)
```

giac [A] time = 0.16, size = 79, normalized size = 0.92

$$-\frac{3\,c^2\log\left(\left|cx^2+b\right|\right)}{b^5}+\frac{6\,c^2\log\left(|x|\right)}{b^5}+\frac{12\,c^3x^6+18\,bc^2x^4+4\,b^2cx^2-b^3}{4\left(cx^4+bx^2\right)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -3*c^2*log(abs(c*x^2 + b))/b^5 + 6*c^2*log(abs(x))/b^5 + 1/4*(12*c^3*x^6 +
18*b*c^2*x^4 + 4*b^2*c*x^2 - b^3)/((c*x^4 + b*x^2)^2*b^4)
```

maple [A] time = 0.01, size = 79, normalized size = 0.92

$$\frac{c^{2}}{4\left(c\,x^{2}+b\right)^{2} b^{3}}+\frac{3 c^{2}}{2\left(c\,x^{2}+b\right) b^{4}}+\frac{6 c^{2} \ln(x)}{b^{5}}-\frac{3 c^{2} \ln\left(c\,x^{2}+b\right)}{b^{5}}+\frac{3 c}{2 b^{4} x^{2}}-\frac{1}{4 b^{3} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/4/b^3/x^4+3/2*c/b^4/x^2+1/4*c^2/b^3/(c*x^2+b)^2+3/2*c^2/b^4/(c*x^2+b)+6*
c^2*ln(x)/b^5-3*c^2*ln(c*x^2+b)/b^5
```

maxima [A] time = 1.35, size = 92, normalized size = 1.07

$$\frac{12\,c^{3}x^{6}+18\,bc^{2}x^{4}+4\,b^{2}cx^{2}-b^{3}}{4\left(b^{4}c^{2}x^{8}+2\,b^{5}cx^{6}+b^{6}x^{4}\right)}-\frac{3\,c^{2}\log\left(cx^{2}+b\right)}{b^{5}}+\frac{3\,c^{2}\log\left(x^{2}\right)}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(12*c^3*x^6 + 18*b*c^2*x^4 + 4*b^2*c*x^2 - b^3)/(b^4*c^2*x^8 + 2*b^5*c*
x^6 + b^6*x^4) - 3*c^2*log(c*x^2 + b)/b^5 + 3*c^2*log(x^2)/b^5
```

mupad [B] time = 4.25, size = 88, normalized size = 1.02

$$\frac{\frac{c\,x^{2}}{b^{2}}-\frac{1}{4\,b}+\frac{9\,c^{2}\,x^{4}}{2\,b^{3}}+\frac{3\,c^{3}\,x^{6}}{b^{4}}}{b^{2}\,x^{4}+2\,b\,c\,x^{6}+c^{2}\,x^{8}}-\frac{3\,c^{2}\,\ln\left(c\,x^{2}+b\right)}{b^{5}}+\frac{6\,c^{2}\,\ln(x)}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((c*x^2)/b^2 - 1/(4*b) + (9*c^2*x^4)/(2*b^3) + (3*c^3*x^6)/b^4)/(b^2*x^4 +
c^2*x^8 + 2*b*c*x^6) - (3*c^2*log(b + c*x^2))/b^5 + (6*c^2*log(x))/b^5
```

sympy [A] time = 0.57, size = 90, normalized size = 1.05

$$\frac{-b^{3}+4b^{2}cx^{2}+18bc^{2}x^{4}+12c^{3}x^{6}}{4b^{6}x^{4}+8b^{5}cx^{6}+4b^{4}c^{2}x^{8}}+\frac{6c^{2}\log\left(x\right)}{b^{5}}-\frac{3c^{2}\log\left(\frac{b}{c}+x^{2}\right)}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-b**3 + 4*b**2*c*x**2 + 18*b*c**2*x**4 + 12*c**3*x**6)/(4*b**6*x**4 + 8*b*
*5*c*x**6 + 4*b**4*c**2*x**8) + 6*c**2*log(x)/b**5 - 3*c**2*log(b/c + x**2)
/b**5
```

3.219 $\int \frac{1}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=100

$$-\frac{63c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{11/2}}-\frac{63c^2}{8b^5x}+\frac{21c}{8b^4x^3}-\frac{63}{40b^3x^5}+\frac{9}{8b^2x^5\left(b+cx^2\right)}+\frac{1}{4bx^5\left(b+cx^2\right)^2}$$

```
[Out] -63/40/b^3/x^5+21/8*c/b^4/x^3-63/8*c^2/b^5/x+1/4/b/x^5/(c*x^2+b)^2+9/8/b^2/
x^5/(c*x^2+b)-63/8*c^(5/2)*arctan(x*c^(1/2)/b^(1/2))/b^(11/2)
```

Rubi [A] time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1593, 290, 325, 205}

$$-\frac{63c^2}{8b^5x}-\frac{63c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{11/2}}+\frac{21c}{8b^4x^3}+\frac{9}{8b^2x^5\left(b+cx^2\right)}-\frac{63}{40b^3x^5}+\frac{1}{4bx^5\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(-3),x]
```

```
[Out] -63/(40*b^3*x^5) + (21*c)/(8*b^4*x^3) - (63*c^2)/(8*b^5*x) + 1/(4*b*x^5*(b
+ c*x^2)^2) + 9/(8*b^2*x^5*(b + c*x^2)) - (63*c^(5/2)*ArcTan[(Sqrt[c]*x)/Sq
rt[b]])/(8*b^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{1}{x^6\left(b+cx^2\right)^3}\,dx \\
&= \frac{1}{4bx^5\left(b+cx^2\right)^2}+\frac{9\int \frac{1}{x^6\left(b+cx^2\right)^2}\,dx}{4b} \\
&= \frac{1}{4bx^5\left(b+cx^2\right)^2}+\frac{9}{8b^2x^5\left(b+cx^2\right)}+\frac{63\int \frac{1}{x^6\left(b+cx^2\right)}\,dx}{8b^2} \\
&= -\frac{63}{40b^3x^5}+\frac{1}{4bx^5\left(b+cx^2\right)^2}+\frac{9}{8b^2x^5\left(b+cx^2\right)}-\frac{(63c)\int \frac{1}{x^4\left(b+cx^2\right)}\,dx}{8b^3} \\
&= -\frac{63}{40b^3x^5}+\frac{21c}{8b^4x^3}+\frac{1}{4bx^5\left(b+cx^2\right)^2}+\frac{9}{8b^2x^5\left(b+cx^2\right)}+\frac{\left(63c^2\right)\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{8b^4} \\
&= -\frac{63}{40b^3x^5}+\frac{21c}{8b^4x^3}-\frac{63c^2}{8b^5x}+\frac{1}{4bx^5\left(b+cx^2\right)^2}+\frac{9}{8b^2x^5\left(b+cx^2\right)}-\frac{\left(63c^3\right)\int \frac{1}{b+cx^2}\,dx}{8b^5} \\
&= -\frac{63}{40b^3x^5}+\frac{21c}{8b^4x^3}-\frac{63c^2}{8b^5x}+\frac{1}{4bx^5\left(b+cx^2\right)^2}+\frac{9}{8b^2x^5\left(b+cx^2\right)}-\frac{63c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{11/2}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 90, normalized size = 0.90

$$
-\frac{63c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{11/2}}-\frac{8b^4-24b^3cx^2+168b^2c^2x^4+525bc^3x^6+315c^4x^8}{40b^5x^5\left(b+cx^2\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(-3),x]
```

```
[Out] -1/40*(8*b^4 - 24*b^3*c*x^2 + 168*b^2*c^2*x^4 + 525*b*c^3*x^6 + 315*c^4*x^8
)/(b^5*x^5*(b + c*x^2)^2) - (63*c^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(
11/2))
```

fricas [A] time = 0.63, size = 264, normalized size = 2.64

$$\left[-\frac{630\,c^4x^8+1050\,bc^3x^6+336\,b^2c^2x^4-48\,b^3cx^2+16\,b^4-315\left(c^4x^9+2\,bc^3x^7+b^2c^2x^5\right)\sqrt{-\frac{c}{b}}\,\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-}{cx^2+b}\right.}{80\left(b^5c^2x^9+2\,b^6cx^7+b^7x^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/80*(630*c^4*x^8 + 1050*b*c^3*x^6 + 336*b^2*c^2*x^4 - 48*b^3*c*x^2 + 16*
b^4 - 315*(c^4*x^9 + 2*b*c^3*x^7 + b^2*c^2*x^5)*sqrt(-c/b)*log((c*x^2 - 2*b
*x*sqrt(-c/b) - b)/(c*x^2 + b)))/(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5), -1/
40*(315*c^4*x^8 + 525*b*c^3*x^6 + 168*b^2*c^2*x^4 - 24*b^3*c*x^2 + 8*b^4 +
315*(c^4*x^9 + 2*b*c^3*x^7 + b^2*c^2*x^5)*sqrt(c/b)*arctan(x*sqrt(c/b)))/(b
^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)]
```

giac [A] time = 0.18, size = 80, normalized size = 0.80

$$-\frac{63\,c^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^5}-\frac{15\,c^4x^3+17\,bc^3x}{8\left(cx^2+b\right)^2b^5}-\frac{30\,c^2x^4-5\,bcx^2+b^2}{5\,b^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -63/8*c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^5) - 1/8*(15*c^4*x^3 + 17*b*c^
3*x)/((c*x^2 + b)^2*b^5) - 1/5*(30*c^2*x^4 - 5*b*c*x^2 + b^2)/(b^5*x^5)
```

maple [A] time = 0.02, size = 89, normalized size = 0.89

$$-\frac{15c^4x^3}{8\left(c\,x^2+b\right)^2b^5}-\frac{17c^3x}{8\left(c\,x^2+b\right)^2b^4}-\frac{63c^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\;b^5}-\frac{6c^2}{b^5x}+\frac{c}{b^4x^3}-\frac{1}{5b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^3,x)
```

```
[Out] -15/8/b^5*c^4/(c*x^2+b)^2*x^3-17/8/b^4*c^3/(c*x^2+b)^2*x-63/8/b^5*c^3/(b*c)
^(1/2)*arctan(1/(b*c)^(1/2)*c*x)-1/5/b^3/x^5-6*c^2/b^5/x+c/b^4/x^3
```

maxima [A] time = 2.97, size = 97, normalized size = 0.97

$$-\frac{315\,c^4x^8 + 525\,bc^3x^6 + 168\,b^2c^2x^4 - 24\,b^3cx^2 + 8\,b^4}{40\left(b^5c^2x^9 + 2\,b^6cx^7 + b^7x^5\right)} - \frac{63\,c^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/40*(315*c^4*x^8 + 525*b*c^3*x^6 + 168*b^2*c^2*x^4 - 24*b^3*c*x^2 + 8*b^4
)/(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5) - 63/8*c^3*arctan(c*x/sqrt(b*c))/(s
qrt(b*c)*b^5)
```

mupad [B] time = 4.24, size = 92, normalized size = 0.92

$$-\frac{\frac{1}{5\,b} - \frac{3\,c\,x^2}{5\,b^2} + \frac{21\,c^2\,x^4}{5\,b^3} + \frac{105\,c^3\,x^6}{8\,b^4} + \frac{63\,c^4\,x^8}{8\,b^5}}{b^2\,x^5 + 2\,b\,c\,x^7 + c^2\,x^9} - \frac{63\,c^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 + c*x^4)^3,x)
```

```
[Out] - (1/(5*b) - (3*c*x^2)/(5*b^2) + (21*c^2*x^4)/(5*b^3) + (105*c^3*x^6)/(8*b^
4) + (63*c^4*x^8)/(8*b^5))/(b^2*x^5 + c^2*x^9 + 2*b*c*x^7) - (63*c^(5/2)*at
an((c^(1/2)*x)/b^(1/2)))/(8*b^(11/2))
```

sympy [A] time = 0.58, size = 150, normalized size = 1.50

$$\frac{63\sqrt{-\frac{c^5}{b^{11}}}\log\left(-\frac{b^6\sqrt{-\frac{c^5}{b^{11}}}}{c^3} + x\right)}{16} - \frac{63\sqrt{-\frac{c^5}{b^{11}}}\log\left(\frac{b^6\sqrt{-\frac{c^5}{b^{11}}}}{c^3} + x\right)}{16} + \frac{-8b^4 + 24b^3cx^2 - 168b^2c^2x^4 - 525bc^3x^6 - 315c^4x^8}{40b^7x^5 + 80b^6cx^7 + 40b^5c^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**3,x)
```

```
[Out] 63*sqrt(-c**5/b**11)*log(-b**6*sqrt(-c**5/b**11)/c**3 + x)/16 - 63*sqrt(-c*
*5/b**11)*log(b**6*sqrt(-c**5/b**11)/c**3 + x)/16 + (-8*b**4 + 24*b**3*c*x*
*2 - 168*b**2*c**2*x**4 - 525*b*c**3*x**6 - 315*c**4*x**8)/(40*b**7*x**5 +
80*b**6*c*x**7 + 40*b**5*c**2*x**9)
```

3.220 $\int \frac{1}{x(bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=95

$$\frac{5c^3 \log\left(b+cx^2\right)}{b^6} - \frac{10c^3 \log(x)}{b^6} - \frac{2c^3}{b^5\left(b+cx^2\right)} - \frac{3c^2}{b^5x^2} - \frac{c^3}{4b^4\left(b+cx^2\right)^2} + \frac{3c}{4b^4x^4} - \frac{1}{6b^3x^6}$$

```
[Out] -1/6/b^3/x^6+3/4*c/b^4/x^4-3*c^2/b^5/x^2-1/4*c^3/b^4/(c*x^2+b)^2-2*c^3/b^5/
(c*x^2+b)-10*c^3*ln(x)/b^6+5*c^3*ln(c*x^2+b)/b^6
```

Rubi [A] time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$-\frac{2c^3}{b^5\left(b+cx^2\right)} - \frac{c^3}{4b^4\left(b+cx^2\right)^2} - \frac{3c^2}{b^5x^2} + \frac{5c^3 \log\left(b+cx^2\right)}{b^6} - \frac{10c^3 \log(x)}{b^6} + \frac{3c}{4b^4x^4} - \frac{1}{6b^3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^2 + c*x^4)^3),x]
```

```
[Out] -1/(6*b^3*x^6) + (3*c)/(4*b^4*x^4) - (3*c^2)/(b^5*x^2) - c^3/(4*b^4*(b + c*
x^2)^2) - (2*c^3)/(b^5*(b + c*x^2)) - (10*c^3*Log[x])/b^6 + (5*c^3*Log[b +
c*x^2])/b^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{1}{x^7\left(b+cx^2\right)^3}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^4(b+cx)^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{b^3x^4}-\frac{3c}{b^4x^3}+\frac{6c^2}{b^5x^2}-\frac{10c^3}{b^6x}+\frac{c^4}{b^4(b+cx)^3}+\frac{4c^4}{b^5(b+cx)^2}+\frac{10c^4}{b^6(b+cx)}\right)dx,x\right.\\ &= -\frac{1}{6b^3x^6}+\frac{3c}{4b^4x^4}-\frac{3c^2}{b^5x^2}-\frac{c^3}{4b^4\left(b+cx^2\right)^2}-\frac{2c^3}{b^5\left(b+cx^2\right)}-\frac{10c^3\log(x)}{b^6}+\frac{5c^3\log\left(b+cx^2\right.}{b^6}\end{aligned}$$

Mathematica [A] time = 0.07, size = 85, normalized size = 0.89

$$-\frac{\frac{b\left(2b^4-5b^3cx^2+20b^2c^2x^4+90bc^3x^6+60c^4x^8\right)}{x^6\left(b+cx^2\right)^2}-60c^3\log\left(b+cx^2\right)+120c^3\log(x)}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^2 + c*x^4)^3),x]
```

```
[Out] -1/12*((b*(2*b^4 - 5*b^3*c*x^2 + 20*b^2*c^2*x^4 + 90*b*c^3*x^6 + 60*c^4*x^8
))/(x^6*(b + c*x^2)^2) + 120*c^3*Log[x] - 60*c^3*Log[b + c*x^2])/b^6
```

fricas [A] time = 0.59, size = 145, normalized size = 1.53

$$-\frac{60\,bc^4x^8+90\,b^2c^3x^6+20\,b^3c^2x^4-5\,b^4cx^2+2\,b^5-60\left(c^5x^{10}+2\,bc^4x^8+b^2c^3x^6\right)\log\left(cx^2+b\right)+120\left(c^5x^{10}+2\right.}{12\left(b^6c^2x^{10}+2\,b^7cx^8+b^8x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(60*b*c^4*x^8 + 90*b^2*c^3*x^6 + 20*b^3*c^2*x^4 - 5*b^4*c*x^2 + 2*b^5
 - 60*(c^5*x^10 + 2*b*c^4*x^8 + b^2*c^3*x^6)*log(c*x^2 + b) + 120*(c^5*x^10
 + 2*b*c^4*x^8 + b^2*c^3*x^6)*log(x))/(b^6*c^2*x^10 + 2*b^7*c*x^8 + b^8*x^6
)
```

giac [A] time = 0.15, size = 110, normalized size = 1.16

$$-\frac{5\,c^{3}\log\left(x^{2}\right)}{b^{6}}+\frac{5\,c^{3}\log\left(\left|cx^{2}+b\right|\right)}{b^{6}}-\frac{30\,c^{5}x^{4}+68\,bc^{4}x^{2}+39\,b^{2}c^{3}}{4\left(cx^{2}+b\right)^{2}b^{6}}+\frac{110\,c^{3}x^{6}-36\,bc^{2}x^{4}+9\,b^{2}cx^{2}-2\,b^{3}}{12\,b^{6}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -5*c^3*log(x^2)/b^6 + 5*c^3*log(abs(c*x^2 + b))/b^6 - 1/4*(30*c^5*x^4 + 68*
b*c^4*x^2 + 39*b^2*c^3)/((c*x^2 + b)^2*b^6) + 1/12*(110*c^3*x^6 - 36*b*c^2*
x^4 + 9*b^2*c*x^2 - 2*b^3)/(b^6*x^6)
```

maple [A] time = 0.02, size = 90, normalized size = 0.95

$$-\frac{c^{3}}{4\left(c\,x^{2}+b\right)^{2}b^{4}}-\frac{2c^{3}}{\left(c\,x^{2}+b\right)b^{5}}-\frac{10c^{3}\ln(x)}{b^{6}}+\frac{5c^{3}\ln\left(c\,x^{2}+b\right)}{b^{6}}-\frac{3c^{2}}{b^{5}x^{2}}+\frac{3c}{4b^{4}x^{4}}-\frac{1}{6b^{3}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/6/b^3/x^6+3/4*c/b^4/x^4-3*c^2/b^5/x^2-1/4*c^3/b^4/(c*x^2+b)^2-2*c^3/b^5/
(c*x^2+b)-10*c^3*ln(x)/b^6+5*c^3*ln(c*x^2+b)/b^6
```

maxima [A] time = 1.36, size = 103, normalized size = 1.08

$$-\frac{60\,c^{4}x^{8}+90\,bc^{3}x^{6}+20\,b^{2}c^{2}x^{4}-5\,b^{3}cx^{2}+2\,b^{4}}{12\left(b^{5}c^{2}x^{10}+2\,b^{6}cx^{8}+b^{7}x^{6}\right)}+\frac{5\,c^{3}\log\left(cx^{2}+b\right)}{b^{6}}-\frac{5\,c^{3}\log\left(x^{2}\right)}{b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/12*(60*c^4*x^8 + 90*b*c^3*x^6 + 20*b^2*c^2*x^4 - 5*b^3*c*x^2 + 2*b^4)/(b
^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6) + 5*c^3*log(c*x^2 + b)/b^6 - 5*c^3*log
(x^2)/b^6
```

mupad [B] time = 0.10, size = 101, normalized size = 1.06

$$\frac{5\,c^{3}\,\ln\left(c\,x^{2}+b\right)}{b^{6}}-\frac{\frac{1}{6\,b}-\frac{5\,c\,x^{2}}{12\,b^{2}}+\frac{5\,c^{2}\,x^{4}}{3\,b^{3}}+\frac{15\,c^{3}\,x^{6}}{2\,b^{4}}+\frac{5\,c^{4}\,x^{8}}{b^{5}}}{b^{2}\,x^{6}+2\,b\,c\,x^{8}+c^{2}\,x^{10}}-\frac{10\,c^{3}\,\ln(x)}{b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + c*x^4)^3),x)
```

```
[Out] (5*c^3*log(b + c*x^2))/b^6 - (1/(6*b) - (5*c*x^2)/(12*b^2) + (5*c^2*x^4)/(3
*b^3) + (15*c^3*x^6)/(2*b^4) + (5*c^4*x^8)/b^5)/(b^2*x^6 + c^2*x^10 + 2*b*c
*x^8) - (10*c^3*log(x))/b^6
```

sympy [**A**] time = 0.65, size = 104, normalized size = 1.09

$$\frac{-2b^4+5b^3cx^2-20b^2c^2x^4-90bc^3x^6-60c^4x^8}{12b^7x^6+24b^6cx^8+12b^5c^2x^{10}}-\frac{10c^3\log(x)}{b^6}+\frac{5c^3\log\left(\frac{b}{c}+x^2\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-2*b**4 + 5*b**3*c*x**2 - 20*b**2*c**2*x**4 - 90*b*c**3*x**6 - 60*c**4*x**
8)/(12*b**7*x**6 + 24*b**6*c*x**8 + 12*b**5*c**2*x**10) - 10*c**3*log(x)/b*
*6 + 5*c**3*log(b/c + x**2)/b**6
```

3.221 $\int x^5\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=119

$$-\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}}+\frac{5b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3}-\frac{5b\left(bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{8c}$$

```
[Out] -5/48*b*(c*x^4+b*x^2)^(3/2)/c^2+1/8*x^2*(c*x^4+b*x^2)^(3/2)/c-5/128*b^4*arc
tanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(7/2)+5/128*b^2*(2*c*x^2+b)*(c*x^4+
b*x^2)^(1/2)/c^3
```

Rubi [A] time = 0.13, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 670, 640, 612, 620, 206}

$$\frac{5b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3}-\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}}-\frac{5b\left(bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (5*b^2*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(128*c^3) - (5*b*(b*x^2 + c*x^4)^
(3/2))/(48*c^2) + (x^2*(b*x^2 + c*x^4)^(3/2))/(8*c) - (5*b^4*ArcTanh[(Sqrt[
c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(128*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int x^5\sqrt{bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int x^2\sqrt{bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{x^2\left(bx^2+cx^4\right)^{3/2}}{8c} - \frac{(5b)\,\text{Subst}\left(\int x\sqrt{bx+cx^2}\,dx, x, x^2\right)}{16c}$$

$$= -\frac{5b\left(bx^2+cx^4\right)^{3/2}}{48c^2} + \frac{x^2\left(bx^2+cx^4\right)^{3/2}}{8c} + \frac{\left(5b^2\right)\text{Subst}\left(\int \sqrt{bx+cx^2}\,dx, x, x^2\right)}{32c^2}$$

$$= \frac{5b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3} - \frac{5b\left(bx^2+cx^4\right)^{3/2}}{48c^2} + \frac{x^2\left(bx^2+cx^4\right)^{3/2}}{8c} - \frac{\left(5b^4\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+c}}\right.}{256c^3}$$

$$= \frac{5b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3} - \frac{5b\left(bx^2+cx^4\right)^{3/2}}{48c^2} + \frac{x^2\left(bx^2+cx^4\right)^{3/2}}{8c} - \frac{\left(5b^4\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\right.}{128c}$$

$$= \frac{5b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3} - \frac{5b\left(bx^2+cx^4\right)^{3/2}}{48c^2} + \frac{x^2\left(bx^2+cx^4\right)^{3/2}}{8c} - \frac{5b^4\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right.}{128c^{7/2}}$$

Mathematica [A] time = 0.08, size = 114, normalized size = 0.96

$$\frac{x\sqrt{b+cx^2}\left(\sqrt{c}\,x\sqrt{b+cx^2}\left(15b^3-10b^2cx^2+8bc^2x^4+48c^3x^6\right)-15b^4\log\left(\sqrt{c}\,\sqrt{b+cx^2}\,+cx\right)\right)}{384c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(15*b^3 - 10*b^2*c*x^2 + 8*b*
c^2*x^4 + 48*c^3*x^6) - 15*b^4*Log[c*x + Sqrt[c]*Sqrt[b + c*x^2]]))/(384*c^
(7/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.60, size = 188, normalized size = 1.58

$$\left[\frac{15\,b^4\sqrt{c}\,\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\,\right)+2\left(48\,c^4x^6+8\,bc^3x^4-10\,b^2c^2x^2+15\,b^3c\right)\sqrt{cx^4+bx^2}}{768\,c^4},\ \frac{15\,b^4\sqrt{-}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(15*b^4*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) +
2*(48*c^4*x^6 + 8*b*c^3*x^4 - 10*b^2*c^2*x^2 + 15*b^3*c)*sqrt(c*x^4 + b*x^2
))/c^4, 1/384*(15*b^4*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 +
 b)) + (48*c^4*x^6 + 8*b*c^3*x^4 - 10*b^2*c^2*x^2 + 15*b^3*c)*sqrt(c*x^4 +
b*x^2))/c^4]
```

giac [A] time = 0.19, size = 101, normalized size = 0.85

$$\frac{1}{384}\left(2\left(4\left(6\,x^2\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x^2-\frac{5\,b^2\mathrm{sgn}(x)}{c^2}\right)x^2+\frac{15\,b^3\mathrm{sgn}(x)}{c^3}\right)\sqrt{cx^2+b}\,x+\frac{5\,b^4\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\cdots\right.}{128\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*x^2*sgn(x) + b*sgn(x)/c)*x^2 - 5*b^2*sgn(x)/c^2)*x^2 + 15*b^
3*sgn(x)/c^3)*sqrt(c*x^2 + b)*x + 5/128*b^4*log(abs(-sqrt(c)*x + sqrt(c*x^2
 + b)))*sgn(x)/c^(7/2) - 5/256*b^4*log(abs(b))*sgn(x)/c^(7/2)
```

maple [A] time = 0.02, size = 124, normalized size = 1.04

$$\frac{\sqrt{c\,x^{4}+b\,x^{2}}\,\left(48\left(c\,x^{2}+b\right)^{\frac{3}{2}} c^{\frac{5}{2}} x^{5}-40\left(c\,x^{2}+b\right)^{\frac{3}{2}} b\,c^{\frac{3}{2}} x^{3}-15 b^{4}\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)-15\sqrt{c\,x^{2}+b}\,b^{3}\sqrt{c}\,x+3\right.}{384\sqrt{c\,x^{2}+b}\,c^{\frac{7}{2}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/384*(c*x^4+b*x^2)^(1/2)*(48*x^5*(c*x^2+b)^(3/2)*c^(5/2)-40*(c*x^2+b)^(3/2
)*c^(3/2)*x^3*b+30*(c*x^2+b)^(3/2)*c^(1/2)*x*b^2-15*(c*x^2+b)^(1/2)*c^(1/2)
*x*b^3-15*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^4)/x/(c*x^2+b)^(1/2)/c^(7/2)
```

maxima [A] time = 1.46, size = 121, normalized size = 1.02

$$\frac{5\sqrt{cx^{4}+bx^{2}}\,b^{2}x^{2}}{64\,c^{2}}+\frac{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}x^{2}}{8\,c}-\frac{5\,b^{4}\log\left(2\,cx^{2}+b+2\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{256\,c^{\frac{7}{2}}}+\frac{5\sqrt{cx^{4}+bx^{2}}\,b^{3}}{128\,c^{3}}-\frac{5\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}b}{48\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/64*sqrt(c*x^4 + b*x^2)*b^2*x^2/c^2 + 1/8*(c*x^4 + b*x^2)^(3/2)*x^2/c - 5/
256*b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 5/128*sq
rt(c*x^4 + b*x^2)*b^3/c^3 - 5/48*(c*x^4 + b*x^2)^(3/2)*b/c^2
```

mupad [B] time = 4.68, size = 105, normalized size = 0.88

$$\frac{x^{2}\left(c\,x^{4}+b\,x^{2}\right)^{3/2}}{8\,c}-\frac{5\,b\left(\frac{b^{3}\,\ln\left(\frac{2\,c\,x^{2}+b}{\sqrt{c}}+2\,\sqrt{c\,x^{4}+b\,x^{2}}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^{4}+b\,x^{2}}\,\left(-3\,b^{2}+2\,b\,c\,x^{2}+8\,c^{2}\,x^{4}\right)}{24\,c^{2}}\right)}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (x^2*(b*x^2 + c*x^4)^(3/2))/(8*c) - (5*b*((b^3*log((b + 2*c*x^2)/c^(1/2) +
2*(b*x^2 + c*x^4)^(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(8*c^2*x^4
- 3*b^2 + 2*b*c*x^2))/(24*c^2)))/(16*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{5}\sqrt{x^{2}\left(b+cx^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(x**2*(b + c*x**2)), x)
```

3.222 $\int x^3\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=91

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{\left(bx^2+cx^4\right)^{3/2}}{6c}$$

```
[Out] 1/6*(c*x^4+b*x^2)^(3/2)/c+1/16*b^3*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))
/c^(5/2)-1/16*b*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^2
```

Rubi [A] time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 640, 612, 620, 206}

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{\left(bx^2+cx^4\right)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -(b*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(16*c^2) + (b*x^2 + c*x^4)^(3/2)/(6*
c) + (b^3*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x\sqrt{bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{\left(bx^2+cx^4\right)^{3/2}}{6c}-\frac{b\operatorname{Subst}\left(\int\sqrt{bx+cx^2}\,dx,x,x^2\right)}{4c}\\
&= -\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{\left(bx^2+cx^4\right)^{3/2}}{6c}+\frac{b^3\operatorname{Subst}\left(\int\frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{32c^2}\\
&= -\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{\left(bx^2+cx^4\right)^{3/2}}{6c}+\frac{b^3\operatorname{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^2}\\
&= -\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{\left(bx^2+cx^4\right)^{3/2}}{6c}+\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 103, normalized size = 1.13

$$\frac{x\sqrt{b+cx^2}\left(3b^3\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)+\sqrt{c}\,x\sqrt{b+cx^2}\left(-3b^2+2bcx^2+8c^2x^4\right)\right)}{48c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(-3*b^2 + 2*b*c*x^2 + 8*c^2*x
^4) + 3*b^3*Log[c*x + Sqrt[c]*Sqrt[b + c*x^2]]))/(48*c^(5/2)*Sqrt[x^2*(b +
c*x^2)])
```

fricas [A] time = 0.49, size = 167, normalized size = 1.84

$$\left[\frac{3\,b^{3}\sqrt{c}\,\log\left(-2\,cx^{2}-b-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)+2\left(8\,c^{3}x^{4}+2\,bc^{2}x^{2}-3\,b^{2}c\right)\sqrt{cx^{4}+bx^{2}}}{96\,c^{3}},-\frac{3\,b^{3}\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}}{cx^{2}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*b^3*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*
(8*c^3*x^4 + 2*b*c^2*x^2 - 3*b^2*c)*sqrt(c*x^4 + b*x^2))/c^3, -1/48*(3*b^3*
sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (8*c^3*x^4 + 2*
b*c^2*x^2 - 3*b^2*c)*sqrt(c*x^4 + b*x^2))/c^3]
```

giac [A] time = 0.19, size = 85, normalized size = 0.93

$$\frac{1}{48}\left(2\left(4\,x^{2}\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x^{2}-\frac{3\,b^{2}\mathrm{sgn}(x)}{c^{2}}\right)\sqrt{cx^{2}+b}\,x-\frac{b^{3}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^{2}+b}\right|\right)\mathrm{sgn}(x)}{16\,c^{\frac{5}{2}}}+\frac{b^{3}\log\left(|b|\right)\mathrm{sgn}(}{32\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*x^2*sgn(x) + b*sgn(x)/c)*x^2 - 3*b^2*sgn(x)/c^2)*sqrt(c*x^2 + b)
*x - 1/16*b^3*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(5/2) + 1/32*
b^3*log(abs(b))*sgn(x)/c^(5/2)
```

maple [A] time = 0.01, size = 104, normalized size = 1.14

$$\frac{\sqrt{c\,x^{4}+b\,x^{2}}\,\left(8\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{\frac{3}{2}}x^{3}+3b^{3}\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)+3\sqrt{c\,x^{2}+b}\,b^{2}\sqrt{c}\,x-6\left(c\,x^{2}+b\right)^{\frac{3}{2}}b\sqrt{c}\,x\right)}{48\sqrt{c\,x^{2}+b}\,c^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/48*(c*x^4+b*x^2)^(1/2)*(8*x^3*(c*x^2+b)^(3/2)*c^(3/2)-6*c^(1/2)*(c*x^2+b)
^(3/2)*x*b+3*c^(1/2)*(c*x^2+b)^(1/2)*x*b^2+3*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*
b^3)/x/(c*x^2+b)^(1/2)/c^(5/2)
```

maxima [A] time = 1.45, size = 97, normalized size = 1.07

$$-\frac{\sqrt{cx^{4}+bx^{2}}\,bx^{2}}{8\,c}+\frac{b^{3}\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{32\,c^{\frac{5}{2}}}-\frac{\sqrt{cx^{4}+bx^{2}}\,b^{2}}{16\,c^{2}}+\frac{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(c*x^4 + b*x^2)*b*x^2/c + 1/32*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4
+ b*x^2)*sqrt(c))/c^(5/2) - 1/16*sqrt(c*x^4 + b*x^2)*b^2/c^2 + 1/6*(c*x^4 +
 b*x^2)^(3/2)/c
```

mupad [B] time = 4.36, size = 77, normalized size = 0.85

$$\frac{b^3 \ln\left(\frac{2\,c\,x^2+b}{\sqrt{c}}+2\,\sqrt{c\,x^4+b\,x^2}\right)}{32\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{48\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (b^3*log((b + 2*c*x^2)/c^(1/2) + 2*(b*x^2 + c*x^4)^(1/2)))/(32*c^(5/2)) + (
(b*x^2 + c*x^4)^(1/2)*(8*c^2*x^4 - 3*b^2 + 2*b*c*x^2))/(48*c^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{x^2 \left(b + cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(x**2*(b + c*x**2)), x)
```

3.223 $\int x\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=68

$$\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{8c}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}$$

```
[Out] -1/8*b^2*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(3/2)+1/8*(2*c*x^2+b)*(
c*x^4+b*x^2)^(1/2)/c
```

Rubi [A] time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2013, 612, 620, 206}

$$\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{8c}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(8*c) - (b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt
[b*x^2 + c*x^4]])/(8*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \sqrt{bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{8c}-\frac{b^2\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{16c}\\
&= \frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{8c}-\frac{b^2\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{8c}\\
&= \frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{8c}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 90, normalized size = 1.32

$$\frac{x\sqrt{b+cx^2}\left(\sqrt{c}\,x\sqrt{b+cx^2}\,\left(b+2cx^2\right)-b^2\log\left(\sqrt{c}\,\sqrt{b+cx^2}\,+cx\right)\right)}{8c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(b + 2*c*x^2) - b^2*Log[c*x +
 Sqrt[c]*Sqrt[b + c*x^2]]))/(8*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.65, size = 140, normalized size = 2.06

$$\left[\frac{b^2\sqrt{c}\,\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}\left(2\,c^2x^2+bc\right)}{16\,c^2},\frac{b^2\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4}}{8\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(b^2*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*sq
rt(c*x^4 + b*x^2)*(2*c^2*x^2 + b*c))/c^2, 1/8*(b^2*sqrt(-c)*arctan(sqrt(c*x
^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 + b*c))/
c^2]
```

giac [A] time = 0.19, size = 69, normalized size = 1.01

$$\frac{1}{8}\sqrt{cx^2+b}\left(2x^2\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x+\frac{b^2\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)\mathrm{sgn}(x)}{8\,c^{\frac{3}{2}}}-\frac{b^2\log\left(|b|\right)\mathrm{sgn}(x)}{16\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^2 + b)*(2*x^2*sgn(x) + b*sgn(x)/c)*x + 1/8*b^2*log(abs(-sqrt(c
)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(3/2) - 1/16*b^2*log(abs(b))*sgn(x)/c^(3/2
)
```

maple [A] time = 0.01, size = 84, normalized size = 1.24

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-b^2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)-\sqrt{c\,x^2+b}\,b\sqrt{c}\,x+2\left(c\,x^2+b\right)^{\frac{3}{2}}\sqrt{c}\,x\right)}{8\sqrt{c\,x^2+b}\,c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/8*(c*x^4+b*x^2)^(1/2)*(2*x*(c*x^2+b)^(3/2)*c^(1/2)-c^(1/2)*(c*x^2+b)^(1/2
)*x*b-ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^2)/x/(c*x^2+b)^(1/2)/c^(3/2)
```

maxima [A] time = 1.43, size = 73, normalized size = 1.07

$$\frac{1}{4}\sqrt{cx^4+bx^2}\,x^2-\frac{b^2\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{16\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^4+bx^2}\,b}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c*x^4 + b*x^2)*x^2 - 1/16*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x
^2)*sqrt(c))/c^(3/2) + 1/8*sqrt(c*x^4 + b*x^2)*b/c
```

mupad [B] time = 4.37, size = 64, normalized size = 0.94

$$\frac{\left(\frac{b}{4c}+\frac{x^2}{2}\right)\sqrt{c\,x^4+b\,x^2}}{2}-\frac{b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{16\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((b/(4*c) + x^2/2)*(b*x^2 + c*x^4)^(1/2))/2 - (b^2*log((b/2 + c*x^2)/c^(1/2
) + (b*x^2 + c*x^4)^(1/2)))/(16*c^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x^2\left(b+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2*(b + c*x**2)), x)
```

3.224 $\int \frac{\sqrt{bx^2+cx^4}}{x} \, dx$

Optimal. Leaf size=55

$$\frac{1}{2}\sqrt{bx^2+cx^4}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}$$

```
[Out] 1/2*b*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(1/2)+1/2*(c*x^4+b*x^2)^(1
/2)
```

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2018, 664, 620, 206}

$$\frac{1}{2}\sqrt{bx^2+cx^4}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x,x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/2 + (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(2*S
qrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2+cx^4}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\sqrt{bx^2+cx^4}+\frac{1}{4}b\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\sqrt{bx^2+cx^4}+\frac{1}{2}b\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\ &= \frac{1}{2}\sqrt{bx^2+cx^4}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 1.16

$$\frac{1}{2}\sqrt{x^2\left(b+cx^2\right)}\left(\frac{b\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)}{\sqrt{c}\,x\sqrt{b+cx^2}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(1 + (b*Log[c*x + Sqrt[c]*Sqrt[b + c*x^2]])/(Sqrt[c]
*x*Sqrt[b + c*x^2])))/2
```

fricas [A] time = 0.46, size = 115, normalized size = 2.09

$$\left[\frac{b\sqrt{c}\,\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}\,c}{4\,c}, -\frac{b\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)-\sqrt{cx^4+bx^2}\,c}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*sqrt(
c*x^4 + b*x^2)*c)/c, -1/2*(b*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(
c*x^2 + b)) - sqrt(c*x^4 + b*x^2)*c)/c]
```

giac [A] time = 0.17, size = 52, normalized size = 0.95

$$\frac{b\log\left(|b|\right)\operatorname{sgn}(x)}{4\sqrt{c}}+\frac{1}{2}\left(\sqrt{cx^2+b}\,x-\frac{b\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)}{\sqrt{c}}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(b))*sgn(x)/sqrt(c) + 1/2*(sqrt(c*x^2 + b)*x - b*log(abs(-sqrt
(c)*x + sqrt(c*x^2 + b)))/sqrt(c))*sgn(x)
```

maple [A] time = 0.00, size = 64, normalized size = 1.16

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(b\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)+\sqrt{c\,x^2+b}\,\sqrt{c}\,x\right)}{2\sqrt{c\,x^2+b}\,\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x,x)
```

```
[Out] 1/2*(c*x^4+b*x^2)^(1/2)*(x*(c*x^2+b)^(1/2)*c^(1/2)+b*ln(c^(1/2)*x+(c*x^2+b)
^(1/2)))/x/(c*x^2+b)^(1/2)/c^(1/2)
```

maxima [A] time = 1.43, size = 49, normalized size = 0.89

$$\frac{b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{4\,\sqrt{c}}+\frac{1}{2}\,\sqrt{cx^4+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) + 1/2*sqrt(c
*x^4 + b*x^2)
```

mupad [B] time = 4.21, size = 50, normalized size = 0.91

$$\frac{\sqrt{c\,x^4+b\,x^2}}{2}+\frac{b\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{4\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x,x)
```

```
[Out] (b*x^2 + c*x^4)^(1/2)/2 + (b*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1
/2)))/(4*c^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x, x)
```

3.225 $\int \frac{\sqrt{bx^2+cx^4}}{x^3} dx$

Optimal. Leaf size=52

$$\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{\sqrt{bx^2+cx^4}}{x^2}$$

```
[Out] arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))*c^(1/2)-(c*x^4+b*x^2)^(1/2)/x^2
```

Rubi [A] time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2018, 662, 620, 206}

$$\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{\sqrt{bx^2+cx^4}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^3,x]
```

```
[Out] -(Sqrt[b*x^2 + c*x^4]/x^2) + Sqrt[c]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x
^4]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2+cx^4}}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^2}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{bx^2+cx^4}}{x^2}+\frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{bx^2+cx^4}}{x^2}+c\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\ &= -\frac{\sqrt{bx^2+cx^4}}{x^2}+\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)\end{aligned}$$

Mathematica [A] time = 0.09, size = 60, normalized size = 1.15

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\frac{\sqrt{c}\,x\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{\frac{cx^2}{b}+1}}-1\right)}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^3,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-1 + (Sqrt[c]*x*ArcSinh[(Sqrt[c]*x)/Sqrt[b]])/(Sqrt
[b]*Sqrt[1 + (c*x^2)/b])))/x^2
```

fricas [A] time = 1.05, size = 115, normalized size = 2.21

$$\left[\frac{\sqrt{c}\,x^2\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}}{2\,x^2}, -\frac{\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}}{x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*sqr
t(c*x^4 + b*x^2))/x^2, -(sqrt(-c)*x^2*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(
c*x^2 + b)) + sqrt(c*x^4 + b*x^2))/x^2]
```

giac [A] time = 0.27, size = 61, normalized size = 1.17

$$-\frac{1}{2}\sqrt{c}\log\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right)\operatorname{sgn}(x)+\frac{2b\sqrt{c}\operatorname{sgn}(x)}{\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(c)*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)*sgn(x) + 2*b*sqrt(c)*sgn(
x)/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)
```

maple [A] time = 0.01, size = 84, normalized size = 1.62

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(bcx\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)+\sqrt{c\,x^2+b}\,c^{\frac{3}{2}}x^2-\left(c\,x^2+b\right)^{\frac{3}{2}}\sqrt{c}\right)}{\sqrt{c\,x^2+b}\,b\sqrt{c}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^3,x)
```

```
[Out] (c*x^4+b*x^2)^(1/2)*(c^(3/2)*(c*x^2+b)^(1/2)*x^2-(c*x^2+b)^(3/2)*c^(1/2)+ln
(c^(1/2)*x+(c*x^2+b)^(1/2))*x*b*c)/x^2/(c*x^2+b)^(1/2)/b/c^(1/2)
```

maxima [A] time = 1.39, size = 51, normalized size = 0.98

$$\frac{1}{2}\sqrt{c}\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)-\frac{\sqrt{cx^4+bx^2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - sqrt(c*x^4 +
 b*x^2)/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{c\,x^4+b\,x^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^3,x)
```

```
[Out] int((b*x^2 + c*x^4)^(1/2)/x^3, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 \left(b + cx^2\right)}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**3, x)
```

3.226 $\int \frac{\sqrt{bx^2+cx^4}}{x^5} dx$

Optimal. Leaf size=25

$$-\frac{(bx^2+cx^4)^{3/2}}{3bx^6}$$

```
[Out] -1/3*(c*x^4+b*x^2)^(3/2)/b/x^6
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$-\frac{(bx^2+cx^4)^{3/2}}{3bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^5,x]
```

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(3*b*x^6)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^5} dx = -\frac{(bx^2+cx^4)^{3/2}}{3bx^6}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}}{3bx^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^5,x]
```

```
[Out] -1/3*(x^2*(b + c*x^2))^(3/2)/(b*x^6)
```

fricas [A] time = 0.71, size = 28, normalized size = 1.12

$$-\frac{\sqrt{cx^4 + bx^2}\left(cx^2 + b\right)}{3\,bx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/(b*x^4)
```

giac [B] time = 0.23, size = 63, normalized size = 2.52

$$\frac{2\left(3\left(\sqrt{c}\,x - \sqrt{cx^2 + b}\right)^4 c^{\frac{3}{2}}\mathrm{sgn}(x) + b^2 c^{\frac{3}{2}}\mathrm{sgn}(x)\right)}{3\left(\left(\sqrt{c}\,x - \sqrt{cx^2 + b}\right)^2 - b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(c)*x - sqrt(c*x^2 + b))^4*c^(3/2)*sgn(x) + b^2*c^(3/2)*sgn(x))
/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^3
```

maple [A] time = 0.00, size = 29, normalized size = 1.16

$$-\frac{\left(c\,x^2 + b\right)\sqrt{c\,x^4 + b\,x^2}}{3b\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^5,x)
```

```
[Out] -1/3/x^4*(c*x^2+b)/b*(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.44, size = 41, normalized size = 1.64

$$-\frac{\sqrt{cx^4 + bx^2}\,c}{3\,bx^2} - \frac{\sqrt{cx^4 + bx^2}}{3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(c*x^4 + b*x^2)*c/(b*x^2) - 1/3*sqrt(c*x^4 + b*x^2)/x^4
```

mupad [B] time = 4.15, size = 28, normalized size = 1.12

$$-\frac{\left(c\,x^2+b\right)\,\sqrt{c\,x^4+b\,x^2}}{3\,b\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^5,x)
[Out] -((b + c*x^2)*(b*x^2 + c*x^4)^(1/2))/(3*b*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**5,x)
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**5, x)
```

3.227 $\int \frac{\sqrt{bx^2+cx^4}}{x^7} \, dx$

Optimal. Leaf size=52

$$\frac{2c\left(bx^2+cx^4\right)^{3/2}}{15b^2x^6}-\frac{\left(bx^2+cx^4\right)^{3/2}}{5bx^8}$$

```
[Out] -1/5*(c*x^4+b*x^2)^(3/2)/b/x^8+2/15*c*(c*x^4+b*x^2)^(3/2)/b^2/x^6
```

Rubi [A] time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{2c\left(bx^2+cx^4\right)^{3/2}}{15b^2x^6}-\frac{\left(bx^2+cx^4\right)^{3/2}}{5bx^8}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^7,x]
```

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(5*b*x^8) + (2*c*(b*x^2 + c*x^4)^(3/2))/(15*b^2*x^6)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx = -\frac{\left(bx^2+cx^4\right)^{3/2}}{5bx^8} - \frac{(2c)\int \frac{\sqrt{bx^2+cx^4}}{x^5}\,dx}{5b} = -\frac{\left(bx^2+cx^4\right)^{3/2}}{5bx^8} + \frac{2c\left(bx^2+cx^4\right)^{3/2}}{15b^2x^6}$$

Mathematica [**A**] time = 0.01, size = 35, normalized size = 0.67

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(2cx^2-3b\right)}{15b^2x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^7,x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(-3*b + 2*c*x^2))/(15*b^2*x^8)
```

fricas [**A**] time = 0.98, size = 42, normalized size = 0.81

$$\frac{\left(2\,c^2x^4-bcx^2-3\,b^2\right)\sqrt{cx^4+bx^2}}{15\,b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/15*(2*c^2*x^4 - b*c*x^2 - 3*b^2)*sqrt(c*x^4 + b*x^2)/(b^2*x^6)
```

giac [**B**] time = 0.22, size = 120, normalized size = 2.31

$$\frac{4\left(15\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^6c^{\frac{5}{2}}\mathrm{sgn}(x)+5\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4bc^{\frac{5}{2}}\mathrm{sgn}(x)+5\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2b^2c^{\frac{5}{2}}\mathrm{sgn}(x)-b^3c^{\frac{5}{2}}\mathrm{s}}{15\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 4/15*(15*(sqrt(c)*x - sqrt(c*x^2 + b))^6*c^(5/2)*sgn(x) + 5*(sqrt(c)*x - sq
rt(c*x^2 + b))^4*b*c^(5/2)*sgn(x) + 5*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^2*c
^(5/2)*sgn(x) - b^3*c^(5/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^5
```

maple [A] time = 0.00, size = 39, normalized size = 0.75

$$-\frac{\left(c\,x^2+b\right)\left(-2c\,x^2+3b\right)\sqrt{c\,x^4+b\,x^2}}{15b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^7,x)
```

```
[Out] -1/15*(c*x^2+b)*(-2*c*x^2+3*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^6
```

maxima [A] time = 1.42, size = 65, normalized size = 1.25

$$\frac{2\sqrt{cx^4+bx^2}\,c^2}{15\,b^2x^2}-\frac{\sqrt{cx^4+bx^2}\,c}{15\,bx^4}-\frac{\sqrt{cx^4+bx^2}}{5\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] 2/15*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^2) - 1/15*sqrt(c*x^4 + b*x^2)*c/(b*x^4)
 - 1/5*sqrt(c*x^4 + b*x^2)/x^6
```

mupad [B] time = 4.26, size = 41, normalized size = 0.79

$$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(3\,b^2+b\,c\,x^2-2\,c^2\,x^4\right)}{15\,b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^7,x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*(3*b^2 - 2*c^2*x^4 + b*c*x^2))/(15*b^2*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**7, x)
```

3.228 $\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx$

Optimal. Leaf size=80

$$-\frac{8c^2\left(bx^2+cx^4\right)^{3/2}}{105b^3x^6}+\frac{4c\left(bx^2+cx^4\right)^{3/2}}{35b^2x^8}-\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}$$

```
[Out] -1/7*(c*x^4+b*x^2)^(3/2)/b/x^10+4/35*c*(c*x^4+b*x^2)^(3/2)/b^2/x^8-8/105*c^
2*(c*x^4+b*x^2)^(3/2)/b^3/x^6
```

Rubi [A] time = 0.12, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$-\frac{8c^2\left(bx^2+cx^4\right)^{3/2}}{105b^3x^6}+\frac{4c\left(bx^2+cx^4\right)^{3/2}}{35b^2x^8}-\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^9,x]
```

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(7*b*x^10) + (4*c*(b*x^2 + c*x^4)^(3/2))/(35*b^2*x^8
) - (8*c^2*(b*x^2 + c*x^4)^(3/2))/(105*b^3*x^6)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx = -\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}} - \frac{(4c)\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx}{7b}$$
$$= -\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}} + \frac{4c\left(bx^2+cx^4\right)^{3/2}}{35b^2x^8} + \frac{\left(8c^2\right)\int \frac{\sqrt{bx^2+cx^4}}{x^5}\,dx}{35b^2}$$
$$= -\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}} + \frac{4c\left(bx^2+cx^4\right)^{3/2}}{35b^2x^8} - \frac{8c^2\left(bx^2+cx^4\right)^{3/2}}{105b^3x^6}$$

Mathematica [**A**] time = 0.01, size = 46, normalized size = 0.58

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(15b^2-12bcx^2+8c^2x^4\right)}{105b^3x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^9,x]
```

```
[Out] -1/105*((x^2*(b + c*x^2))^(3/2)*(15*b^2 - 12*b*c*x^2 + 8*c^2*x^4))/(b^3*x^1
0)
```

fricas [**A**] time = 0.75, size = 53, normalized size = 0.66

$$-\frac{\left(8\,c^3x^6-4\,bc^2x^4+3\,b^2cx^2+15\,b^3\right)\sqrt{cx^4+bx^2}}{105\,b^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/105*(8*c^3*x^6 - 4*b*c^2*x^4 + 3*b^2*c*x^2 + 15*b^3)*sqrt(c*x^4 + b*x^2)
/(b^3*x^8)
```

giac [**B**] time = 0.24, size = 148, normalized size = 1.85

$$\frac{16\left(70\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^8 c^{\frac{7}{2}}\mathrm{sgn}(x)+35\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^6 bc^{\frac{7}{2}}\mathrm{sgn}(x)+21\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4 b^2c^{\frac{7}{2}}\mathrm{sgn}(x)-\right.}{105\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] 16/105*(70*(sqrt(c)*x - sqrt(c*x^2 + b))^8*c^(7/2)*sgn(x) + 35*(sqrt(c)*x -
 sqrt(c*x^2 + b))^6*b*c^(7/2)*sgn(x) + 21*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b
^2*c^(7/2)*sgn(x) - 7*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^3*c^(7/2)*sgn(x) +
b^4*c^(7/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^7
```

maple [A] time = 0.00, size = 50, normalized size = 0.62

$$-\frac{\left(c\,x^{2}+b\right)\left(8c^{2}x^{4}-12bc\,x^{2}+15b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{105b^{3}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^9,x)
```

```
[Out] -1/105*(c*x^2+b)*(8*c^2*x^4-12*b*c*x^2+15*b^2)*(c*x^4+b*x^2)^(1/2)/x^8/b^3
```

maxima [A] time = 1.43, size = 89, normalized size = 1.11

$$-\frac{8\sqrt{cx^{4}+bx^{2}}\,c^{3}}{105\,b^{3}x^{2}}+\frac{4\sqrt{cx^{4}+bx^{2}}\,c^{2}}{105\,b^{2}x^{4}}-\frac{\sqrt{cx^{4}+bx^{2}}\,c}{35\,bx^{6}}-\frac{\sqrt{cx^{4}+bx^{2}}}{7\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] -8/105*sqrt(c*x^4 + b*x^2)*c^3/(b^3*x^2) + 4/105*sqrt(c*x^4 + b*x^2)*c^2/(b
^2*x^4) - 1/35*sqrt(c*x^4 + b*x^2)*c/(b*x^6) - 1/7*sqrt(c*x^4 + b*x^2)/x^8
```

mupad [B] time = 4.34, size = 89, normalized size = 1.11

$$\frac{4\,c^{2}\,\sqrt{c\,x^{4}+b\,x^{2}}}{105\,b^{2}\,x^{4}}-\frac{c\,\sqrt{c\,x^{4}+b\,x^{2}}}{35\,b\,x^{6}}-\frac{\sqrt{c\,x^{4}+b\,x^{2}}}{7\,x^{8}}-\frac{8\,c^{3}\,\sqrt{c\,x^{4}+b\,x^{2}}}{105\,b^{3}\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^9,x)
```

```
[Out] (4*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b^2*x^4) - (c*(b*x^2 + c*x^4)^(1/2))/(35
*b*x^6) - (b*x^2 + c*x^4)^(1/2)/(7*x^8) - (8*c^3*(b*x^2 + c*x^4)^(1/2))/(10
5*b^3*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{2}\left(b+cx^{2}\right)}}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**9,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**9, x)
```

3.229 $\int \frac{\sqrt{bx^2+cx^4}}{x^{11}}\,dx$

Optimal. Leaf size=108

$$\frac{16c^3\left(bx^2+cx^4\right)^{3/2}}{315b^4x^6}-\frac{8c^2\left(bx^2+cx^4\right)^{3/2}}{105b^3x^8}+\frac{2c\left(bx^2+cx^4\right)^{3/2}}{21b^2x^{10}}-\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}}$$

```
[Out] -1/9*(c*x^4+b*x^2)^(3/2)/b/x^12+2/21*c*(c*x^4+b*x^2)^(3/2)/b^2/x^10-8/105*c
^2*(c*x^4+b*x^2)^(3/2)/b^3/x^8+16/315*c^3*(c*x^4+b*x^2)^(3/2)/b^4/x^6
```

Rubi [A] time = 0.16, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{16c^3\left(bx^2+cx^4\right)^{3/2}}{315b^4x^6}-\frac{8c^2\left(bx^2+cx^4\right)^{3/2}}{105b^3x^8}+\frac{2c\left(bx^2+cx^4\right)^{3/2}}{21b^2x^{10}}-\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^11,x]
```

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(9*b*x^12) + (2*c*(b*x^2 + c*x^4)^(3/2))/(21*b^2*x^1
0) - (8*c^2*(b*x^2 + c*x^4)^(3/2))/(105*b^3*x^8) + (16*c^3*(b*x^2 + c*x^4)^
(3/2))/(315*b^4*x^6)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{11}}\,dx = -\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}} - \frac{(2c)\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx}{3b}$$
$$= -\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}} + \frac{2c\left(bx^2+cx^4\right)^{3/2}}{21b^2x^{10}} + \frac{\left(8c^2\right)\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx}{21b^2}$$
$$= -\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}} + \frac{2c\left(bx^2+cx^4\right)^{3/2}}{21b^2x^{10}} - \frac{8c^2\left(bx^2+cx^4\right)^{3/2}}{105b^3x^8} - \frac{\left(16c^3\right)\int \frac{\sqrt{bx^2+cx^4}}{x^5}\,dx}{105b^3}$$
$$= -\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}} + \frac{2c\left(bx^2+cx^4\right)^{3/2}}{21b^2x^{10}} - \frac{8c^2\left(bx^2+cx^4\right)^{3/2}}{105b^3x^8} + \frac{16c^3\left(bx^2+cx^4\right)^{3/2}}{315b^4x^6}$$

Mathematica [A] time = 0.01, size = 57, normalized size = 0.53

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(-35b^3+30b^2cx^2-24bc^2x^4+16c^3x^6\right)}{315b^4x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^11,x]
[Out] ((x^2*(b + c*x^2))^(3/2)*(-35*b^3 + 30*b^2*c*x^2 - 24*b*c^2*x^4 + 16*c^3*x^
6))/(315*b^4*x^12)
```

fricas [A] time = 0.60, size = 64, normalized size = 0.59

$$\frac{\left(16\,c^4x^8-8\,bc^3x^6+6\,b^2c^2x^4-5\,b^3cx^2-35\,b^4\right)\sqrt{cx^4+bx^2}}{315\,b^4x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="fricas")
[Out] 1/315*(16*c^4*x^8 - 8*b*c^3*x^6 + 6*b^2*c^2*x^4 - 5*b^3*c*x^2 - 35*b^4)*sqr
t(c*x^4 + b*x^2)/(b^4*x^10)
```

giac [A] time = 0.29, size = 178, normalized size = 1.65

$$\frac{32\left(315\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{10}c^{\frac{9}{2}}\mathrm{sgn}(x)+189\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{8}bc^{\frac{9}{2}}\mathrm{sgn}(x)+84\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{6}b^2c^{\frac{9}{2}}\mathrm{sgn}(x}{315\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{2}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] 32/315*(315*(sqrt(c)*x - sqrt(c*x^2 + b))^10*c^(9/2)*sgn(x) + 189*(sqrt(c)*
x - sqrt(c*x^2 + b))^8*b*c^(9/2)*sgn(x) + 84*(sqrt(c)*x - sqrt(c*x^2 + b))^
6*b^2*c^(9/2)*sgn(x) - 36*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^3*c^(9/2)*sgn(x
) + 9*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^4*c^(9/2)*sgn(x) - b^5*c^(9/2)*sgn(
x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^9
```

maple [A] time = 0.01, size = 61, normalized size = 0.56

$$-\frac{\left(c\,x^{2}+b\right)\left(-16c^{3}x^{6}+24b\,c^{2}x^{4}-30b^{2}c\,x^{2}+35b^{3}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{315b^{4}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^11,x)
```

```
[Out] -1/315*(c*x^2+b)*(-16*c^3*x^6+24*b*c^2*x^4-30*b^2*c*x^2+35*b^3)*(c*x^4+b*x^
2)^(1/2)/x^10/b^4
```

maxima [A] time = 1.42, size = 113, normalized size = 1.05

$$\frac{16\,\sqrt{cx^{4}+bx^{2}}\,c^{4}}{315\,b^{4}x^{2}}-\frac{8\,\sqrt{cx^{4}+bx^{2}}\,c^{3}}{315\,b^{3}x^{4}}+\frac{2\,\sqrt{cx^{4}+bx^{2}}\,c^{2}}{105\,b^{2}x^{6}}-\frac{\sqrt{cx^{4}+bx^{2}}\,c}{63\,bx^{8}}-\frac{\sqrt{cx^{4}+bx^{2}}}{9\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] 16/315*sqrt(c*x^4 + b*x^2)*c^4/(b^4*x^2) - 8/315*sqrt(c*x^4 + b*x^2)*c^3/(b
^3*x^4) + 2/105*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^6) - 1/63*sqrt(c*x^4 + b*x^2
)*c/(b*x^8) - 1/9*sqrt(c*x^4 + b*x^2)/x^10
```

mupad [B] time = 4.50, size = 113, normalized size = 1.05

$$\frac{2\,c^{2}\,\sqrt{c\,x^{4}+b\,x^{2}}}{105\,b^{2}\,x^{6}}-\frac{c\,\sqrt{c\,x^{4}+b\,x^{2}}}{63\,b\,x^{8}}-\frac{\sqrt{c\,x^{4}+b\,x^{2}}}{9\,x^{10}}-\frac{8\,c^{3}\,\sqrt{c\,x^{4}+b\,x^{2}}}{315\,b^{3}\,x^{4}}+\frac{16\,c^{4}\,\sqrt{c\,x^{4}+b\,x^{2}}}{315\,b^{4}\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^11,x)
```

```
[Out] (2*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b^2*x^6) - (c*(b*x^2 + c*x^4)^(1/2))/(63
*b*x^8) - (b*x^2 + c*x^4)^(1/2)/(9*x^10) - (8*c^3*(b*x^2 + c*x^4)^(1/2))/(3
15*b^3*x^4) + (16*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^4*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 (b + cx^2)}}{x^{11}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**11,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**11, x)
```

3.230 $\int \frac{\sqrt{bx^2+cx^4}}{x^{13}} dx$

Optimal. Leaf size=136

$$-\frac{128c^4\left(bx^2+cx^4\right)^{3/2}}{3465b^5x^6}+\frac{64c^3\left(bx^2+cx^4\right)^{3/2}}{1155b^4x^8}-\frac{16c^2\left(bx^2+cx^4\right)^{3/2}}{231b^3x^{10}}+\frac{8c\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}}-\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}$$

```
[Out] -1/11*(c*x^4+b*x^2)^(3/2)/b/x^14+8/99*c*(c*x^4+b*x^2)^(3/2)/b^2/x^12-16/231
*c^2*(c*x^4+b*x^2)^(3/2)/b^3/x^10+64/1155*c^3*(c*x^4+b*x^2)^(3/2)/b^4/x^8-1
28/3465*c^4*(c*x^4+b*x^2)^(3/2)/b^5/x^6
```

Rubi **[A]** time = 0.21, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$-\frac{128c^4\left(bx^2+cx^4\right)^{3/2}}{3465b^5x^6}+\frac{64c^3\left(bx^2+cx^4\right)^{3/2}}{1155b^4x^8}-\frac{16c^2\left(bx^2+cx^4\right)^{3/2}}{231b^3x^{10}}+\frac{8c\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}}-\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^13,x]
```

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(11*b*x^14) + (8*c*(b*x^2 + c*x^4)^(3/2))/(99*b^2*x^
12) - (16*c^2*(b*x^2 + c*x^4)^(3/2))/(231*b^3*x^10) + (64*c^3*(b*x^2 + c*x^
4)^(3/2))/(1155*b^4*x^8) - (128*c^4*(b*x^2 + c*x^4)^(3/2))/(3465*b^5*x^6)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{13}}\,dx = -\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}} - \frac{(8c)\int \frac{\sqrt{bx^2+cx^4}}{x^{11}}\,dx}{11b}$$
$$= -\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}} + \frac{8c\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}} + \frac{\left(16c^2\right)\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx}{33b^2}$$
$$= -\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}} + \frac{8c\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}} - \frac{16c^2\left(bx^2+cx^4\right)^{3/2}}{231b^3x^{10}} - \frac{\left(64c^3\right)\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx}{231b^3}$$
$$= -\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}} + \frac{8c\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}} - \frac{16c^2\left(bx^2+cx^4\right)^{3/2}}{231b^3x^{10}} + \frac{64c^3\left(bx^2+cx^4\right)^{3/2}}{1155b^4x^8} + \frac{\left(128c^4\right)}{}$$
$$= -\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}} + \frac{8c\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}} - \frac{16c^2\left(bx^2+cx^4\right)^{3/2}}{231b^3x^{10}} + \frac{64c^3\left(bx^2+cx^4\right)^{3/2}}{1155b^4x^8} - \frac{128c^4\left(b\right.}{34}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 0.50

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(315b^4-280b^3cx^2+240b^2c^2x^4-192bc^3x^6+128c^4x^8\right)}{3465b^5x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^13,x]
```

```
[Out] -1/3465*((x^2*(b + c*x^2))^(3/2)*(315*b^4 - 280*b^3*c*x^2 + 240*b^2*c^2*x^4
 - 192*b*c^3*x^6 + 128*c^4*x^8))/(b^5*x^14)
```

fricas [A] time = 0.65, size = 75, normalized size = 0.55

$$-\frac{\left(128\,c^5x^{10}-64\,bc^4x^8+48\,b^2c^3x^6-40\,b^3c^2x^4+35\,b^4cx^2+315\,b^5\right)\sqrt{cx^4+bx^2}}{3465\,b^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/3465*(128*c^5*x^10 - 64*b*c^4*x^8 + 48*b^2*c^3*x^6 - 40*b^3*c^2*x^4 + 35
*b^4*c*x^2 + 315*b^5)*sqrt(c*x^4 + b*x^2)/(b^5*x^12)
```

giac [A] time = 0.25, size = 206, normalized size = 1.51

$$\frac{256\left(1386\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{12}c^{\frac{11}{2}}\,\mathrm{sgn}(x)+924\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{10}bc^{\frac{11}{2}}\,\mathrm{sgn}(x)+330\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{8}b^2c^{\frac{1}{2}}\right.}{346}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="giac")
```

```
[Out] 256/3465*(1386*(sqrt(c)*x - sqrt(c*x^2 + b))^12*c^(11/2)*sgn(x) + 924*(sqrt
(c)*x - sqrt(c*x^2 + b))^10*b*c^(11/2)*sgn(x) + 330*(sqrt(c)*x - sqrt(c*x^2
 + b))^8*b^2*c^(11/2)*sgn(x) - 165*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^3*c^(1
1/2)*sgn(x) + 55*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^4*c^(11/2)*sgn(x) - 11*(
sqrt(c)*x - sqrt(c*x^2 + b))^2*b^5*c^(11/2)*sgn(x) + b^6*c^(11/2)*sgn(x))/(
(sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^11
```

maple [A] time = 0.01, size = 72, normalized size = 0.53

$$-\frac{\left(c\,x^{2}+b\right)\left(128c^{4}x^{8}-192c^{3}x^{6}b+240c^{2}x^{4}b^{2}-280c\,x^{2}b^{3}+315b^{4}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{3465b^{5}x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^13,x)
```

```
[Out] -1/3465*(c*x^2+b)*(128*c^4*x^8-192*b*c^3*x^6+240*b^2*c^2*x^4-280*b^3*c*x^2+
315*b^4)*(c*x^4+b*x^2)^(1/2)/x^12/b^5
```

maxima [A] time = 1.53, size = 137, normalized size = 1.01

$$-\frac{128\sqrt{cx^{4}+bx^{2}}\,c^{5}}{3465\,b^{5}x^{2}}+\frac{64\sqrt{cx^{4}+bx^{2}}\,c^{4}}{3465\,b^{4}x^{4}}-\frac{16\sqrt{cx^{4}+bx^{2}}\,c^{3}}{1155\,b^{3}x^{6}}+\frac{8\sqrt{cx^{4}+bx^{2}}\,c^{2}}{693\,b^{2}x^{8}}-\frac{\sqrt{cx^{4}+bx^{2}}\,c}{99\,bx^{10}}-\frac{\sqrt{cx^{4}+bx^{2}}}{11\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="maxima")
```

```
[Out] -128/3465*sqrt(c*x^4 + b*x^2)*c^5/(b^5*x^2) + 64/3465*sqrt(c*x^4 + b*x^2)*c
^4/(b^4*x^4) - 16/1155*sqrt(c*x^4 + b*x^2)*c^3/(b^3*x^6) + 8/693*sqrt(c*x^4
 + b*x^2)*c^2/(b^2*x^8) - 1/99*sqrt(c*x^4 + b*x^2)*c/(b*x^10) - 1/11*sqrt(c
*x^4 + b*x^2)/x^12
```

mupad [B] time = 4.62, size = 137, normalized size = 1.01

$$\frac{8\,c^{2}\sqrt{c\,x^{4}+b\,x^{2}}}{693\,b^{2}\,x^{8}}-\frac{c\sqrt{c\,x^{4}+b\,x^{2}}}{99\,b\,x^{10}}-\frac{\sqrt{c\,x^{4}+b\,x^{2}}}{11\,x^{12}}-\frac{16\,c^{3}\sqrt{c\,x^{4}+b\,x^{2}}}{1155\,b^{3}\,x^{6}}+\frac{64\,c^{4}\sqrt{c\,x^{4}+b\,x^{2}}}{3465\,b^{4}\,x^{4}}-\frac{128\,c^{5}\sqrt{c\,x^{4}+b\,x^{2}}}{3465\,b^{5}\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^13,x)
```

```
[Out] (8*c^2*(b*x^2 + c*x^4)^(1/2))/(693*b^2*x^8) - (c*(b*x^2 + c*x^4)^(1/2))/(99
*b*x^10) - (b*x^2 + c*x^4)^(1/2)/(11*x^12) - (16*c^3*(b*x^2 + c*x^4)^(1/2))
```

```
/(1155*b^3*x^6) + (64*c^4*(b*x^2 + c*x^4)^(1/2))/(3465*b^4*x^4) - (128*c^5*
(b*x^2 + c*x^4)^(1/2))/(3465*b^5*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**13,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**13, x)
```

3.231 $\int x^4\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=78

$$\frac{8b^2\left(bx^2+cx^4\right)^{3/2}}{105c^3x^3}-\frac{4b\left(bx^2+cx^4\right)^{3/2}}{35c^2x}+\frac{x\left(bx^2+cx^4\right)^{3/2}}{7c}$$

```
[Out] 8/105*b^2*(c*x^4+b*x^2)^(3/2)/c^3/x^3-4/35*b*(c*x^4+b*x^2)^(3/2)/c^2/x+1/7*
x*(c*x^4+b*x^2)^(3/2)/c
```

Rubi [A] time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2000}

$$\frac{8b^2\left(bx^2+cx^4\right)^{3/2}}{105c^3x^3}-\frac{4b\left(bx^2+cx^4\right)^{3/2}}{35c^2x}+\frac{x\left(bx^2+cx^4\right)^{3/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (8*b^2*(b*x^2 + c*x^4)^(3/2))/(105*c^3*x^3) - (4*b*(b*x^2 + c*x^4)^(3/2))/(
35*c^2*x) + (x*(b*x^2 + c*x^4)^(3/2))/(7*c)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int x^4\sqrt{bx^2+cx^4}\,dx = \frac{x\left(bx^2+cx^4\right)^{3/2}}{7c} - \frac{(4b)\int x^2\sqrt{bx^2+cx^4}\,dx}{7c}$$
$$= -\frac{4b\left(bx^2+cx^4\right)^{3/2}}{35c^2x} + \frac{x\left(bx^2+cx^4\right)^{3/2}}{7c} + \frac{\left(8b^2\right)\int\sqrt{bx^2+cx^4}\,dx}{35c^2}$$
$$= \frac{8b^2\left(bx^2+cx^4\right)^{3/2}}{105c^3x^3} - \frac{4b\left(bx^2+cx^4\right)^{3/2}}{35c^2x} + \frac{x\left(bx^2+cx^4\right)^{3/2}}{7c}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.59

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(8b^2-12bcx^2+15c^2x^4\right)}{105c^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(8*b^2 - 12*b*c*x^2 + 15*c^2*x^4))/(105*c^3*x^3)
```

fricas [A] time = 0.52, size = 53, normalized size = 0.68

$$\frac{\left(15\,c^3x^6+3\,bc^2x^4-4\,b^2cx^2+8\,b^3\right)\sqrt{cx^4+bx^2}}{105\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*c^3*x^6 + 3*b*c^2*x^4 - 4*b^2*c*x^2 + 8*b^3)*sqrt(c*x^4 + b*x^2)/
(c^3*x)
```

giac [A] time = 0.16, size = 60, normalized size = 0.77

$$-\frac{8\,b^{\frac{7}{2}}\mathrm{sgn}(x)}{105\,c^3} + \frac{15\left(cx^2+b\right)^{\frac{7}{2}}\mathrm{sgn}(x) - 42\left(cx^2+b\right)^{\frac{5}{2}}b\,\mathrm{sgn}(x) + 35\left(cx^2+b\right)^{\frac{3}{2}}b^2\mathrm{sgn}(x)}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -8/105*b^(7/2)*sgn(x)/c^3 + 1/105*(15*(c*x^2 + b)^(7/2)*sgn(x) - 42*(c*x^2
+ b)^(5/2)*b*sgn(x) + 35*(c*x^2 + b)^(3/2)*b^2*sgn(x))/c^3
```

maple [A] time = 0.00, size = 50, normalized size = 0.64

$$\frac{\left(c\,x^{2}+b\right)\left(15c^{2}x^{4}-12bc\,x^{2}+8b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{105c^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/105*(c*x^2+b)*(15*c^2*x^4-12*b*c*x^2+8*b^2)*(c*x^4+b*x^2)^(1/2)/c^3/x
```

maxima [A] time = 1.41, size = 46, normalized size = 0.59

$$\frac{\left(15\,c^{3}x^{6}+3\,bc^{2}x^{4}-4\,b^{2}cx^{2}+8\,b^{3}\right)\sqrt{cx^{2}+b}}{105\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*c^3*x^6 + 3*b*c^2*x^4 - 4*b^2*c*x^2 + 8*b^3)*sqrt(c*x^2 + b)/c^3
```

mupad [B] time = 4.23, size = 53, normalized size = 0.68

$$\frac{\sqrt{c\,x^{4}+b\,x^{2}}\,\left(8\,b^{3}-4\,b^{2}\,c\,x^{2}+3\,b\,c^{2}\,x^{4}+15\,c^{3}\,x^{6}\right)}{105\,c^{3}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*(8*b^3 + 15*c^3*x^6 - 4*b^2*c*x^2 + 3*b*c^2*x^4))/(1
05*c^3*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{4}\sqrt{x^{2}\left(b+cx^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(x**2*(b + c*x**2)), x)
```

3.232 $\int x^2\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=52

$$\frac{\left(bx^2+cx^4\right)^{3/2}}{5cx}-\frac{2b\left(bx^2+cx^4\right)^{3/2}}{15c^2x^3}$$

```
[Out] -2/15*b*(c*x^4+b*x^2)^(3/2)/c^2/x^3+1/5*(c*x^4+b*x^2)^(3/2)/c/x
```

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2000}

$$\frac{\left(bx^2+cx^4\right)^{3/2}}{5cx}-\frac{2b\left(bx^2+cx^4\right)^{3/2}}{15c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-2*b*(b*x^2 + c*x^4)^(3/2))/(15*c^2*x^3) + (b*x^2 + c*x^4)^(3/2)/(5*c*x)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{bx^2+cx^4}\,dx&=\frac{\left(bx^2+cx^4\right)^{3/2}}{5cx}-\frac{(2b)\int\sqrt{bx^2+cx^4}\,dx}{5c}\\&=-\frac{2b\left(bx^2+cx^4\right)^{3/2}}{15c^2x^3}+\frac{\left(bx^2+cx^4\right)^{3/2}}{5cx}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.67

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(3cx^2-2b\right)}{15c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[b*x^2 + c*x^4],x]
[Out] ((x^2*(b + c*x^2))^(3/2)*(-2*b + 3*c*x^2))/(15*c^2*x^3)
```

fricas [A] time = 0.74, size = 41, normalized size = 0.79

$$\frac{\left(3\,c^2x^4+bcx^2-2\,b^2\right)\sqrt{cx^4+bx^2}}{15\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/15*(3*c^2*x^4 + b*c*x^2 - 2*b^2)*sqrt(c*x^4 + b*x^2)/(c^2*x)
```

giac [A] time = 0.16, size = 44, normalized size = 0.85

$$\frac{2\,b^{\frac{5}{2}}\mathrm{sgn}(x)}{15\,c^2}+\frac{3\left(cx^2+b\right)^{\frac{5}{2}}\mathrm{sgn}(x)-5\left(cx^2+b\right)^{\frac{3}{2}}b\mathrm{sgn}(x)}{15\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] 2/15*b^(5/2)*sgn(x)/c^2 + 1/15*(3*(c*x^2 + b)^(5/2)*sgn(x) - 5*(c*x^2 + b)^
(3/2)*b*sgn(x))/c^2
```

maple [A] time = 0.00, size = 39, normalized size = 0.75

$$-\frac{\left(c\,x^2+b\right)\left(-3c\,x^2+2b\right)\sqrt{c\,x^4+b\,x^2}}{15c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2)^(1/2),x)
[Out] -1/15*(c*x^2+b)*(-3*c*x^2+2*b)*(c*x^4+b*x^2)^(1/2)/c^2/x
```

maxima [A] time = 1.39, size = 34, normalized size = 0.65

$$\frac{\left(3\,c^{2}x^{4}+bcx^{2}-2\,b^{2}\right)\sqrt{cx^{2}+b}}{15\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/15*(3*c^2*x^4 + b*c*x^2 - 2*b^2)*sqrt(c*x^2 + b)/c^2
```

mupad [B] time = 4.14, size = 41, normalized size = 0.79

$$\frac{\sqrt{c\,x^{4}+b\,x^{2}}\,\left(-2\,b^{2}+b\,c\,x^{2}+3\,c^{2}\,x^{4}\right)}{15\,c^{2}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2 + c*x^4)^(1/2),x)
[Out] ((b*x^2 + c*x^4)^(1/2)*(3*c^2*x^4 - 2*b^2 + b*c*x^2))/(15*c^2*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\sqrt{x^{2}\left(b+cx^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x**2*sqrt(x**2*(b + c*x**2)), x)
```

3.233 $\int \sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=25

$$\frac{\left(bx^2+cx^4\right)^{3/2}}{3cx^3}$$

[Out] 1/3*(c*x^4+b*x^2)^(3/2)/c/x^3

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2000}

$$\frac{\left(bx^2+cx^4\right)^{3/2}}{3cx^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*x^2 + c*x^4],x]

[Out] (b*x^2 + c*x^4)^(3/2)/(3*c*x^3)

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \sqrt{bx^2+cx^4}\,dx=\frac{\left(bx^2+cx^4\right)^{3/2}}{3cx^3}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}}{3cx^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*x^2 + c*x^4],x]

[Out] (x^2*(b + c*x^2))^(3/2)/(3*c*x^3)

fricas [A] time = 0.54, size = 28, normalized size = 1.12

$$\frac{\sqrt{cx^4+bx^2}\left(cx^2+b\right)}{3\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/(c*x)
```

giac [A] time = 0.15, size = 27, normalized size = 1.08

$$\frac{\left(cx^2+b\right)^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,c}-\frac{b^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] 1/3*(c*x^2 + b)^(3/2)*sgn(x)/c - 1/3*b^(3/2)*sgn(x)/c
```

maple [A] time = 0.00, size = 29, normalized size = 1.16

$$\frac{\left(c\,x^2+b\right)\sqrt{c\,x^4+b\,x^2}}{3cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2),x)
[Out] 1/3*(c*x^2+b)/c/x*(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.43, size = 14, normalized size = 0.56

$$\frac{\left(cx^2+b\right)^{\frac{3}{2}}}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*(c*x^2 + b)^(3/2)/c
```

mupad [B] time = 4.14, size = 29, normalized size = 1.16

$$\frac{\left(\frac{b}{3\,c}+\frac{x^2}{3}\right)\sqrt{c\,x^4+b\,x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2),x)
[Out] ((b/(3*c) + x^2/3)*(b*x^2 + c*x^4)^(1/2))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^2 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2),x)
[Out] Integral(sqrt(b*x**2 + c*x**4), x)
```

3.234 $\int \frac{\sqrt{bx^2+cx^4}}{x^2} dx$

Optimal. Leaf size=50

$$\frac{\sqrt{bx^2+cx^4}}{x} - \sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] -arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))*b^(1/2)+(c*x^4+b*x^2)^(1/2)/x
```

Rubi [A] time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2021, 2008, 206}

$$\frac{\sqrt{bx^2+cx^4}}{x} - \sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^2,x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/x - Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx &= \frac{\sqrt{bx^2+cx^4}}{x} + b\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx\\
&= \frac{\sqrt{bx^2+cx^4}}{x} - b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)\\
&= \frac{\sqrt{bx^2+cx^4}}{x} - \sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 60, normalized size = 1.20

$$
\frac{x\left(-\sqrt{b}\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)+b+cx^2\right)}{\sqrt{x^2\left(b+cx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^2,x]
```

```
[Out] (x*(b + c*x^2 - Sqrt[b]*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/
Sqrt[x^2*(b + c*x^2)]
```

fricas [A] time = 0.69, size = 117, normalized size = 2.34

$$
\left[\frac{\sqrt{b}\,x\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}}{2\,x}, \frac{\sqrt{-b}\,x\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}}{x}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b)*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3) +
 2*sqrt(c*x^4 + b*x^2))/x, (sqrt(-b)*x*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/
(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2))/x]
```

giac [A] time = 0.18, size = 69, normalized size = 1.38

$$
\frac{b\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\text{sgn}(x)}{\sqrt{-b}} + \sqrt{cx^2+b}\,\text{sgn}(x) - \frac{\left(b\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+\sqrt{-b}\sqrt{b}\right)\text{sgn}(x)}{\sqrt{-b}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] b*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) + sqrt(c*x^2 + b)*sgn(x)
 - (b*arctan(sqrt(b)/sqrt(-b)) + sqrt(-b)*sqrt(b))*sgn(x)/sqrt(-b)
```

maple [A] time = 0.01, size = 65, normalized size = 1.30

$$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-\sqrt{c\,x^2+b}\right)}{\sqrt{c\,x^2+b}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^2,x)
```

```
[Out] -(c*x^4+b*x^2)^(1/2)*(b^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)-(c*x^2+b)
^(1/2))/x/(c*x^2+b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^2, x)
```

mupad [B] time = 4.31, size = 68, normalized size = 1.36

$$\frac{\sqrt{c\,x^4+b\,x^2}}{x}+\frac{\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,1\mathrm{i}}{\sqrt{c}\,x}\right)\,\sqrt{c\,x^4+b\,x^2}\,1\mathrm{i}}{\sqrt{c}\,x^2\,\sqrt{\frac{b}{c\,x^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^2,x)
```

```
[Out] (b*x^2 + c*x^4)^(1/2)/x + (b^(1/2)*asin((b^(1/2)*1i)/(c^(1/2)*x))*(b*x^2 +
c*x^4)^(1/2)*1i)/(c^(1/2)*x^2*(b/(c*x^2) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**2, x)
```

3.235 $\int \frac{\sqrt{bx^2+cx^4}}{x^4}\,dx$

Optimal. Leaf size=56

$$-\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}-\frac{\sqrt{bx^2+cx^4}}{2x^3}$$

```
[Out] -1/2*c*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(1/2)-1/2*(c*x^4+b*x^2)^(1/
2)/x^3
```

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2020, 2008, 206}

$$-\frac{\sqrt{bx^2+cx^4}}{2x^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^4,x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(2*x^3) - (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])
/(2*Sqrt[b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2+cx^4}}{x^4}\,dx &= -\frac{\sqrt{bx^2+cx^4}}{2x^3}+\frac{1}{2}c\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx\\ &= -\frac{\sqrt{bx^2+cx^4}}{2x^3}-\frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)\\ &= -\frac{\sqrt{bx^2+cx^4}}{2x^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 1.12

$$-\frac{cx^2\sqrt{\frac{cx^2}{b}+1}\,\tanh^{-1}\left(\sqrt{\frac{cx^2}{b}+1}\right)+b+cx^2}{2x\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^4,x]
```

```
[Out] -1/2*(b + c*x^2 + c*x^2*Sqrt[1 + (c*x^2)/b]*ArcTanh[Sqrt[1 + (c*x^2)/b]])/(
x*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.74, size = 134, normalized size = 2.39

$$\left[\frac{\sqrt{b}\,cx^3\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)-2\,\sqrt{cx^4+bx^2}\,b}{4\,bx^3},\ \frac{\sqrt{-b}\,cx^3\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)-\sqrt{cx^4+bx^2}\,b}{2\,bx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*c*x^3*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^
3) - 2*sqrt(c*x^4 + b*x^2)*b)/(b*x^3), 1/2*(sqrt(-b)*c*x^3*arctan(sqrt(c*x^
4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) - sqrt(c*x^4 + b*x^2)*b)/(b*x^3)]
```

giac [A] time = 0.20, size = 50, normalized size = 0.89

$$\frac{\frac{c^2\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}}-\frac{\sqrt{cx^2+b}\,c\,\mathrm{sgn}(x)}{x^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*(c^2*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - sqrt(c*x^2 + b)
*c*sgn(x)/x^2)/c
```

maple [A] time = 0.01, size = 85, normalized size = 1.52

$$-\frac{\sqrt{c\,x^{4}+b\,x^{2}}\,\left(\sqrt{b}\,c\,x^{2}\ln\left(\frac{2b+2\sqrt{c\,x^{2}+b}\,\sqrt{b}}{x}\right)-\sqrt{c\,x^{2}+b}\,c\,x^{2}+\left(c\,x^{2}+b\right)^{\frac{3}{2}}\right)}{2\sqrt{c\,x^{2}+b}\,b\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^4,x)
```

```
[Out] -1/2*(c*x^4+b*x^2)^(1/2)*(b^(1/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^2*c
-(c*x^2+b)^(1/2)*x^2*c+(c*x^2+b)^(3/2))/x^3/(c*x^2+b)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^4,x)
```

```
[Out] int((b*x^2 + c*x^4)^(1/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**4, x)
```

3.236 $\int \frac{\sqrt{bx^2+cx^4}}{x^6}\,dx$

Optimal. Leaf size=84

$$\frac{c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{4x^5}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}$$

```
[Out] 1/8*c^2*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(3/2)-1/4*(c*x^4+b*x^2)^(1
/2)/x^5-1/8*c*(c*x^4+b*x^2)^(1/2)/b/x^3
```

Rubi [A] time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2008, 206}

$$\frac{c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}-\frac{\sqrt{bx^2+cx^4}}{4x^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^6,x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(4*x^5) - (c*Sqrt[b*x^2 + c*x^4])/(8*b*x^3) + (c^2*Arc
Tanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{bx^2+cx^4}}{x^6}\,dx &= -\frac{\sqrt{bx^2+cx^4}}{4x^5}+\frac{1}{4}c\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx\\
&= -\frac{\sqrt{bx^2+cx^4}}{4x^5}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}-\frac{c^2\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b}\\
&= -\frac{\sqrt{bx^2+cx^4}}{4x^5}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}+\frac{c^2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{8b}\\
&= -\frac{\sqrt{bx^2+cx^4}}{4x^5}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}+\frac{c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.55

$$-\frac{c^2\left(x^2\left(b+cx^2\right)\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{cx^2}{b}+1\right)}{3b^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^6,x]
```

```
[Out] -1/3*(c^2*(x^2*(b + c*x^2))^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (c*x^2
)/b])/(b^3*x^3)
```

fricas [A] time = 0.55, size = 159, normalized size = 1.89

$$\left[\frac{\sqrt{b}\,c^2x^5\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)-2\,\sqrt{cx^4+bx^2}\left(bcx^2+2\,b^2\right)}{16\,b^2x^5},-\frac{\sqrt{-b}\,c^2x^5\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+b}}{8\,b^2x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/16*(sqrt(b)*c^2*x^5*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b))
/x^3) - 2*sqrt(c*x^4 + b*x^2)*(b*c*x^2 + 2*b^2))/(b^2*x^5), -1/8*(sqrt(-b)*
c^2*x^5*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b
*x^2)*(b*c*x^2 + 2*b^2))/(b^2*x^5)]
```

giac [A] time = 0.21, size = 78, normalized size = 0.93

$$-\frac{\frac{c^3 \arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}\,b}+\frac{\left(cx^2+b\right)^{\frac{3}{2}}c^3\operatorname{sgn}(x)+\sqrt{cx^2+b}\,bc^3\operatorname{sgn}(x)}{bc^2x^4}}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] -1/8*(c^3*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b) + ((c*x^2 +
b)^(3/2)*c^3*sgn(x) + sqrt(c*x^2 + b)*b*c^3*sgn(x))/(b*c^2*x^4))/c
```

maple [A] time = 0.01, size = 106, normalized size = 1.26

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\sqrt{b}\,c^2x^4\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-\sqrt{c\,x^2+b}\,c^2x^4+\left(c\,x^2+b\right)^{\frac{3}{2}}c\,x^2-2\left(c\,x^2+b\right)^{\frac{3}{2}}b\right)}{8\sqrt{c\,x^2+b}\,b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^6,x)
```

```
[Out] 1/8*(c*x^4+b*x^2)^(1/2)*(b^(1/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^4*c^
2-(c*x^2+b)^(1/2)*x^4*c^2+(c*x^2+b)^(3/2)*x^2*c-2*(c*x^2+b)^(3/2)*b)/x^5/(c
*x^2+b)^(1/2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^6,x)
```

```
[Out] int((b*x^2 + c*x^4)^(1/2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**6, x)
```

3.237 $\int \frac{\sqrt{bx^2+cx^4}}{x^8} \, dx$

Optimal. Leaf size=112

$$-\frac{c^3 \tanh^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{bx^2+cx^4}}\right)}{16b^{5/2}}+\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3}-\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}$$

```
[Out] -1/16*c^3*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(5/2)-1/6*(c*x^4+b*x^2)^
(1/2)/x^7-1/24*c*(c*x^4+b*x^2)^(1/2)/b/x^5+1/16*c^2*(c*x^4+b*x^2)^(1/2)/b^2
/x^3
```

Rubi [A] time = 0.14, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2008, 206}

$$\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3}-\frac{c^3 \tanh^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{bx^2+cx^4}}\right)}{16b^{5/2}}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}-\frac{\sqrt{bx^2+cx^4}}{6x^7}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^8,x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(6*x^7) - (c*Sqrt[b*x^2 + c*x^4])/(24*b*x^5) + (c^2*Sq
rt[b*x^2 + c*x^4])/(16*b^2*x^3) - (c^3*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x
^4]])/(16*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
```

```
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{bx^2+cx^4}}{x^8}\,dx &= -\frac{\sqrt{bx^2+cx^4}}{6x^7}+\frac{1}{6}c\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx\\
&= -\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}-\frac{c^2\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{8b}\\
&= -\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}+\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3}+\frac{c^3\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{16b^2}\\
&= -\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}+\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3}-\frac{c^3\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{16b^2}\\
&= -\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}+\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3}-\frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{16b^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.41

$$\frac{c^3\left(x^2\left(b+cx^2\right)\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{cx^2}{b}+1\right)}{3b^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^8,x]
```

```
[Out] (c^3*(x^2*(b + c*x^2))^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, 1 + (c*x^2)/b])
/(3*b^4*x^3)
```

fricas [A] time = 0.68, size = 185, normalized size = 1.65

$$\left[\frac{3\sqrt{b}\,c^3x^7\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\left(3\,bc^2x^4-2\,b^2cx^2-8\,b^3\right)\sqrt{cx^4+bx^2}}{96\,b^3x^7},\ \frac{3\sqrt{-b}\,c^3x^7\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{\ }}{cx^3+bx}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/96*(3*sqrt(b)*c^3*x^7*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b
))/x^3) + 2*(3*b*c^2*x^4 - 2*b^2*c*x^2 - 8*b^3)*sqrt(c*x^4 + b*x^2))/(b^3*x
^7), 1/48*(3*sqrt(-b)*c^3*x^7*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 +
b*x)) + (3*b*c^2*x^4 - 2*b^2*c*x^2 - 8*b^3)*sqrt(c*x^4 + b*x^2))/(b^3*x^7)]
```

giac [A] time = 0.27, size = 100, normalized size = 0.89

$$\frac{\frac{3\,c^4\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}\,b^2}+\frac{3\left(cx^2+b\right)^{\frac{5}{2}}c^4\mathrm{sgn}(x)-8\left(cx^2+b\right)^{\frac{3}{2}}bc^4\mathrm{sgn}(x)-3\sqrt{cx^2+b}\,b^2c^4\mathrm{sgn}(x)}{b^2c^3x^6}}{48\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/48*(3*c^4*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b^2) + (3*(c*
x^2 + b)^(5/2)*c^4*sgn(x) - 8*(c*x^2 + b)^(3/2)*b*c^4*sgn(x) - 3*sqrt(c*x^2
 + b)*b^2*c^4*sgn(x))/(b^2*c^3*x^6))/c
```

maple [A] time = 0.01, size = 128, normalized size = 1.14

$$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(3\sqrt{b}\,c^3x^6\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-3\sqrt{c\,x^2+b}\,c^3x^6+3\left(c\,x^2+b\right)^{\frac{3}{2}}c^2x^4-6\left(c\,x^2+b\right)^{\frac{3}{2}}bc\,x^2+8\left(c\right.\right.}{48\sqrt{c\,x^2+b}\,b^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^8,x)
```

```
[Out] -1/48*(c*x^4+b*x^2)^(1/2)*(3*b^(1/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^
6*c^3-3*(c*x^2+b)^(1/2)*x^6*c^3+3*(c*x^2+b)^(3/2)*x^4*c^2-6*(c*x^2+b)^(3/2)
*x^2*b*c+8*(c*x^2+b)^(3/2)*b^2)/x^7/(c*x^2+b)^(1/2)/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^8,x)
```

```
[Out] int((b*x^2 + c*x^4)^(1/2)/x^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**8,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**8, x)
```

3.238 $\int x^3 \left(bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=124

$$-\frac{3b^5 \tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{7/2}}+\frac{3b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(bx^2+cx^4\right)^{5/2}}{10c}$$

```
[Out] -1/32*b*(2*c*x^2+b)*(c*x^4+b*x^2)^(3/2)/c^2+1/10*(c*x^4+b*x^2)^(5/2)/c-3/25
6*b^5*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(7/2)+3/256*b^3*(2*c*x^2+b
)*(c*x^4+b*x^2)^(1/2)/c^3
```

Rubi [A] time = 0.13, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 640, 612, 620, 206}

$$\frac{3b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{3b^5 \tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{7/2}}-\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(bx^2+cx^4\right)^{5/2}}{10c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (3*b^3*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(256*c^3) - (b*(b + 2*c*x^2)*(b*x
^2 + c*x^4)^(3/2))/(32*c^2) + (b*x^2 + c*x^4)^(5/2)/(10*c) - (3*b^5*ArcTanh
[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(256*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int x^3\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{1}{2}\text{Subst}\left(\int x\left(bx+cx^2\right)^{3/2}\,dx,x,x^2\right)$$

$$=\frac{\left(bx^2+cx^4\right)^{5/2}}{10c}-\frac{b\,\text{Subst}\left(\int\left(bx+cx^2\right)^{3/2}\,dx,x,x^2\right)}{4c}$$

$$=-\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(bx^2+cx^4\right)^{5/2}}{10c}+\frac{\left(3b^3\right)\text{Subst}\left(\int\sqrt{bx+cx^2}\,dx,x,x^2\right)}{64c^2}$$

$$=\frac{3b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(bx^2+cx^4\right)^{5/2}}{10c}-\frac{\left(3b^5\right)\text{Subs}}{}$$

$$=\frac{3b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(bx^2+cx^4\right)^{5/2}}{10c}-\frac{\left(3b^5\right)\text{Subs}}{}$$

$$=\frac{3b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(bx^2+cx^4\right)^{5/2}}{10c}-\frac{3b^5\tanh^{-1}}{256}$$

Mathematica [A] time = 0.10, size = 126, normalized size = 1.02

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}\,x\sqrt{\frac{cx^2}{b}+1}\left(15b^4-10b^3cx^2+8b^2c^2x^4+176bc^3x^6+128c^4x^8\right)-15b^{9/2}\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\right)}{1280c^{7/2}x\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(15*b^4 - 10*b^3*c*x^
2 + 8*b^2*c^2*x^4 + 176*b*c^3*x^6 + 128*c^4*x^8) - 15*b^(9/2)*ArcSinh[(Sqrt
[c]*x)/Sqrt[b]]))/(1280*c^(7/2)*x*Sqrt[1 + (c*x^2)/b])
```

fricas [A] time = 0.66, size = 210, normalized size = 1.69

$$\left[\frac{15\,b^5\sqrt{c}\,\log\left(-2\,cx^2 - b + 2\,\sqrt{cx^4 + bx^2}\,\sqrt{c}\right) + 2\left(128\,c^5x^8 + 176\,bc^4x^6 + 8\,b^2c^3x^4 - 10\,b^3c^2x^2 + 15\,b^4c\right)\sqrt{cx^4 +}}{2560\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2560*(15*b^5*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) +
 2*(128*c^5*x^8 + 176*b*c^4*x^6 + 8*b^2*c^3*x^4 - 10*b^3*c^2*x^2 + 15*b^4*c
)*sqrt(c*x^4 + b*x^2))/c^4, 1/1280*(15*b^5*sqrt(-c)*arctan(sqrt(c*x^4 + b*x
^2)*sqrt(-c)/(c*x^2 + b)) + (128*c^5*x^8 + 176*b*c^4*x^6 + 8*b^2*c^3*x^4 -
10*b^3*c^2*x^2 + 15*b^4*c)*sqrt(c*x^4 + b*x^2))/c^4]
```

giac [A] time = 0.28, size = 115, normalized size = 0.93

$$\frac{3\,b^5\log\left(\left|-\sqrt{c}\,x + \sqrt{cx^2 + b}\right|\right)\mathrm{sgn}(x)}{256\,c^{\frac{7}{2}}} - \frac{3\,b^5\log\left(|b|\right)\mathrm{sgn}(x)}{512\,c^{\frac{7}{2}}} + \frac{1}{1280}\left(2\left(4\left(2\left(8\,cx^2\mathrm{sgn}(x) + 11\,b\mathrm{sgn}(x)\right)x^2 + \frac{b^2\mathrm{sg}}{c}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3/256*b^5*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(7/2) - 3/512*b^5
*log(abs(b))*sgn(x)/c^(7/2) + 1/1280*(2*(4*(2*(8*c*x^2*sgn(x) + 11*b*sgn(x)
)*x^2 + b^2*sgn(x)/c)*x^2 - 5*b^3*sgn(x)/c^2)*x^2 + 15*b^4*sgn(x)/c^3)*sqrt
(c*x^2 + b)*x
```

maple [A] time = 0.01, size = 142, normalized size = 1.15

$$\frac{\left(c\,x^4 + b\,x^2\right)^{\frac{3}{2}}\left(128\left(c\,x^2 + b\right)^{\frac{5}{2}}c^{\frac{5}{2}}x^5 - 15b^5\ln\left(\sqrt{c}\,x + \sqrt{c\,x^2 + b}\right) - 15\sqrt{c\,x^2 + b}\,b^4\sqrt{c}\,x - 80\left(c\,x^2 + b\right)^{\frac{5}{2}}b\,c^{\frac{3}{2}}x^3\right.}{1280\left(c\,x^2 + b\right)^{\frac{3}{2}}c^{\frac{7}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/1280*(c*x^4+b*x^2)^(3/2)*(128*x^5*(c*x^2+b)^(5/2)*c^(5/2)-80*c^(3/2)*(c*x
^2+b)^(5/2)*x^3*b+40*c^(1/2)*(c*x^2+b)^(5/2)*x*b^2-10*c^(1/2)*(c*x^2+b)^(3/
2)*x*b^3-15*c^(1/2)*(c*x^2+b)^(1/2)*x*b^4-15*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*
b^5)/x^3/(c*x^2+b)^(3/2)/c^(7/2)
```

maxima [A] time = 1.39, size = 142, normalized size = 1.15

$$\frac{3\sqrt{cx^4+bx^2}\,b^3x^2}{128\,c^2}-\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}bx^2}{16\,c}-\frac{3\,b^5\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{512\,c^{\frac{7}{2}}}+\frac{3\sqrt{cx^4+bx^2}\,b^4}{256\,c^3}-\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}b^2}{32\,c^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/128*sqrt(c*x^4 + b*x^2)*b^3*x^2/c^2 - 1/16*(c*x^4 + b*x^2)^(3/2)*b*x^2/c
- 3/512*b^5*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 3/25
6*sqrt(c*x^4 + b*x^2)*b^4/c^3 - 1/32*(c*x^4 + b*x^2)^(3/2)*b^2/c^2 + 1/10*(
c*x^4 + b*x^2)^(5/2)/c
```

mupad [B] time = 4.35, size = 134, normalized size = 1.08

$$\frac{\left(c\,x^4+b\,x^2\right)^{5/2}}{10\,c}-\frac{b\,\left(\frac{x^2\,\left(c\,x^4+b\,x^2\right)^{3/2}}{4}-\frac{3\,b^2\,\left(\frac{\left(2\,c\,x^2+b\right)\,\sqrt{c\,x^4+b\,x^2}}{4\,c}-\frac{b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{8\,c^{3/2}}\right)}{16\,c}+\frac{b\,\left(c\,x^4+b\,x^2\right)^{3/2}}{8\,c}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] (b*x^2 + c*x^4)^(5/2)/(10*c) - (b*((x^2*(b*x^2 + c*x^4)^(3/2))/4 - (3*b^2*(
((b + 2*c*x^2)*(b*x^2 + c*x^4)^(1/2))/(4*c) - (b^2*log((b/2 + c*x^2)/c^(1/2
) + (b*x^2 + c*x^4)^(1/2)))/(8*c^(3/2))))/(16*c) + (b*(b*x^2 + c*x^4)^(3/2)
)/(8*c)))/(4*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**3*(x**2*(b + c*x**2))**(3/2), x)
```

3.239 $\int x\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=101

$$\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}}-\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{16c}$$

```
[Out] 1/16*(2*c*x^2+b)*(c*x^4+b*x^2)^(3/2)/c+3/128*b^4*arctanh(x^2*c^(1/2)/(c*x^4
+b*x^2)^(1/2))/c^(5/2)-3/128*b^2*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^2
```

Rubi [A] time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2013, 612, 620, 206}

$$-\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2}+\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}}+\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{16c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-3*b^2*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(128*c^2) + ((b + 2*c*x^2)*(b*x^
2 + c*x^4)^(3/2))/(16*c) + (3*b^4*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]
])/(128*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\int x\left(bx^2+cx^4\right)^{3/2}dx &= \frac{1}{2}\text{Subst}\left(\int\left(bx+cx^2\right)^{3/2}dx,x,x^2\right)\\
&= \frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{16c}-\frac{\left(3b^2\right)\text{Subst}\left(\int\sqrt{bx+cx^2}\,dx,x,x^2\right)}{32c}\\
&= -\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{16c}+\frac{\left(3b^4\right)\text{Subst}\left(\int\frac{1}{\sqrt{bx+cx^2}}dx,\right.}{256c^2}\\
&= -\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{16c}+\frac{\left(3b^4\right)\text{Subst}\left(\int\frac{1}{1-cx^2}dx,x,\right.}{128c^2}\\
&= -\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{16c}+\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 115, normalized size = 1.14

$$
\frac{\sqrt{x^2\left(b+cx^2\right)}\left(3b^{7/2}\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)+\sqrt{c}\,x\sqrt{\frac{cx^2}{b}+1}\left(-3b^3+2b^2cx^2+24bc^2x^4+16c^3x^6\right)\right)}{128c^{5/2}x\sqrt{\frac{cx^2}{b}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(-3*b^3 + 2*b^2*c*x^2
 + 24*b*c^2*x^4 + 16*c^3*x^6) + 3*b^(7/2)*ArcSinh[(Sqrt[c]*x)/Sqrt[b]]))/(1
28*c^(5/2)*x*Sqrt[1 + (c*x^2)/b])
```

fricas [A] time = 0.72, size = 189, normalized size = 1.87

$$
\left[\frac{3\,b^4\sqrt{c}\,\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\left(16\,c^4x^6+24\,bc^3x^4+2\,b^2c^2x^2-3\,b^3c\right)\sqrt{cx^4+bx^2}}{256\,c^3},-\frac{3\,b^4\sqrt{-c}}{}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/256*(3*b^4*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2
*(16*c^4*x^6 + 24*b*c^3*x^4 + 2*b^2*c^2*x^2 - 3*b^3*c)*sqrt(c*x^4 + b*x^2))
/c^3, -1/128*(3*b^4*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b
)) - (16*c^4*x^6 + 24*b*c^3*x^4 + 2*b^2*c^2*x^2 - 3*b^3*c)*sqrt(c*x^4 + b*x
^2))/c^3]
```

giac [A] time = 0.19, size = 99, normalized size = 0.98

$$-\frac{3\,b^4\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)\mathrm{sgn}(x)}{128\,c^{\frac{5}{2}}}+\frac{3\,b^4\log\left(|b|\right)\mathrm{sgn}(x)}{256\,c^{\frac{5}{2}}}+\frac{1}{128}\left(2\left(4\left(2\,cx^2\mathrm{sgn}(x)+3\,b\mathrm{sgn}(x)\right)x^2+\frac{b^2\mathrm{sgn}(x)}{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -3/128*b^4*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(5/2) + 3/256*b^
4*log(abs(b))*sgn(x)/c^(5/2) + 1/128*(2*(4*(2*c*x^2*sgn(x) + 3*b*sgn(x))*x^
2 + b^2*sgn(x)/c)*x^2 - 3*b^3*sgn(x)/c^2)*sqrt(c*x^2 + b)*x
```

maple [A] time = 0.01, size = 122, normalized size = 1.21

$$\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(3b^4\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+b}\right)+3\sqrt{c\,x^2+b}\;b^3\sqrt{c}\;x+16\left(c\,x^2+b\right)^{\frac{5}{2}}c^{\frac{3}{2}}x^3+2\left(c\,x^2+b\right)^{\frac{3}{2}}b^2\sqrt{c}\;x-8\left(\right.\right.}{128\left(c\,x^2+b\right)^{\frac{3}{2}}c^{\frac{5}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/128*(c*x^4+b*x^2)^(3/2)*(16*x^3*(c*x^2+b)^(5/2)*c^(3/2)-8*c^(1/2)*(c*x^2+
b)^(5/2)*x*b+2*(c*x^2+b)^(3/2)*b^2*c^(1/2)*x+3*(c*x^2+b)^(1/2)*b^3*c^(1/2)*
x+3*b^4*ln(c^(1/2)*x+(c*x^2+b)^(1/2)))/x^3/(c*x^2+b)^(3/2)/c^(5/2)
```

maxima [A] time = 1.47, size = 118, normalized size = 1.17

$$\frac{1}{8}\left(cx^4+bx^2\right)^{\frac{3}{2}}x^2-\frac{3\sqrt{cx^4+bx^2}\,b^2x^2}{64\,c}+\frac{3\,b^4\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{256\,c^{\frac{5}{2}}}-\frac{3\sqrt{cx^4+bx^2}\,b^3}{128\,c^2}+\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}b}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(c*x^4 + b*x^2)^(3/2)*x^2 - 3/64*sqrt(c*x^4 + b*x^2)*b^2*x^2/c + 3/256*
b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2) - 3/128*sqrt(c
*x^4 + b*x^2)*b^3/c^2 + 1/16*(c*x^4 + b*x^2)^(3/2)*b/c
```

mupad [B] time = 4.44, size = 99, normalized size = 0.98

$$\frac{\left(c\,x^4+b\,x^2\right)^{3/2}\,\left(c\,x^2+\frac{b}{2}\right)}{8\,c}-\frac{3\,b^2\,\left(\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\,\sqrt{c\,x^4+b\,x^2}-\frac{b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{8\,c^{3/2}}\right)}{32\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(3/2)*(b/2 + c*x^2))/(8*c) - (3*b^2*((b/(4*c) + x^2/2)*(b*
x^2 + c*x^4)^(1/2) - (b^2*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)
))/(8*c^(3/2))))/(32*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x*(x**2*(b + c*x**2))**(3/2), x)
```

3.240 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=88

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}}+\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c}+\frac{1}{6}\left(bx^2+cx^4\right)^{3/2}$$

```
[Out] 1/6*(c*x^4+b*x^2)^(3/2)-1/16*b^3*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c
^(3/2)+1/16*b*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A] time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 664, 612, 620, 206}

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}}+\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c}+\frac{1}{6}\left(bx^2+cx^4\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x,x]
```

```
[Out] (b*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(16*c) + (b*x^2 + c*x^4)^(3/2)/6 - (b
^3*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x}\,dx,x,x^2\right)\\
&= \frac{1}{6}\left(bx^2+cx^4\right)^{3/2}+\frac{1}{4}b\operatorname{Subst}\left(\int \sqrt{bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c}+\frac{1}{6}\left(bx^2+cx^4\right)^{3/2}-\frac{b^3\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{32c}\\
&= \frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c}+\frac{1}{6}\left(bx^2+cx^4\right)^{3/2}-\frac{b^3\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{16c}\\
&= \frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c}+\frac{1}{6}\left(bx^2+cx^4\right)^{3/2}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 104, normalized size = 1.18

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}\,x\sqrt{\frac{cx^2}{b}+1}\left(3b^2+14bcx^2+8c^2x^4\right)-3b^{5/2}\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\right)}{48c^{3/2}x\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(3*b^2 + 14*b*c*x^2 +
 8*c^2*x^4) - 3*b^(5/2)*ArcSinh[(Sqrt[c]*x)/Sqrt[b]]))/(48*c^(3/2)*x*Sqrt[1
 + (c*x^2)/b])
```

fricas [A] time = 0.65, size = 166, normalized size = 1.89

$$\left[\frac{3\,b^3\sqrt{c}\,\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\left(8\,c^3x^4+14\,bc^2x^2+3\,b^2c\right)\sqrt{cx^4+bx^2}}{96\,c^2},\frac{3\,b^3\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+b\cdots}}{cx^2\cdots}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/96*(3*b^3*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*
(8*c^3*x^4 + 14*b*c^2*x^2 + 3*b^2*c)*sqrt(c*x^4 + b*x^2))/c^2, 1/48*(3*b^3*
sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (8*c^3*x^4 + 14
*b*c^2*x^2 + 3*b^2*c)*sqrt(c*x^4 + b*x^2))/c^2]
```

giac [A] time = 0.21, size = 84, normalized size = 0.95

$$\frac{b^3\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)\mathrm{sgn}(x)}{16\,c^{\frac{3}{2}}}-\frac{b^3\log\left(|b|\right)\mathrm{sgn}(x)}{32\,c^{\frac{3}{2}}}+\frac{1}{48}\left(2\left(4\,cx^2\mathrm{sgn}(x)+7\,b\mathrm{sgn}(x)\right)x^2+\frac{3\,b^2\mathrm{sgn}(x)}{c}\right)\sqrt{cx^2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/16*b^3*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(3/2) - 1/32*b^3*l
og(abs(b))*sgn(x)/c^(3/2) + 1/48*(2*(4*c*x^2*sgn(x) + 7*b*sgn(x))*x^2 + 3*b
^2*sgn(x)/c)*sqrt(c*x^2 + b)*x
```

maple [A] time = 0.01, size = 102, normalized size = 1.16

$$\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(-3b^3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)-3\sqrt{c\,x^2+b}\,b^2\sqrt{c}\,x-2\left(c\,x^2+b\right)^{\frac{3}{2}}b\sqrt{c}\,x+8\left(c\,x^2+b\right)^{\frac{5}{2}}\sqrt{c}\,x\right)}{48\left(c\,x^2+b\right)^{\frac{3}{2}}c^{\frac{3}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x,x)
```

```
[Out] 1/48*(c*x^4+b*x^2)^(3/2)*(8*x*(c*x^2+b)^(5/2)*c^(1/2)-2*(c*x^2+b)^(3/2)*b*c
^(1/2)*x-3*(c*x^2+b)^(1/2)*b^2*c^(1/2)*x-3*b^3*ln(c^(1/2)*x+(c*x^2+b)^(1/2)
))/x^3/(c*x^2+b)^(3/2)/c^(3/2)
```

maxima [A] time = 1.42, size = 91, normalized size = 1.03

$$\frac{1}{8}\sqrt{cx^4+bx^2}\,bx^2-\frac{b^3\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{32\,c^{\frac{3}{2}}}+\frac{1}{6}\left(cx^4+bx^2\right)^{\frac{3}{2}}+\frac{\sqrt{cx^4+bx^2}\,b^2}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2)*b*x^2 - 1/32*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b
*x^2)*sqrt(c))/c^(3/2) + 1/6*(c*x^4 + b*x^2)^(3/2) + 1/16*sqrt(c*x^4 + b*x^
2)*b^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x, x)
```

3.241 $\int \frac{(bx^2+cx^4)^{3/2}}{x^3} dx$

Optimal. Leaf size=80

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}+\frac{3}{8}b\sqrt{bx^2+cx^4}+\frac{(bx^2+cx^4)^{3/2}}{4x^2}$$

```
[Out] 1/4*(c*x^4+b*x^2)^(3/2)/x^2+3/8*b^2*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2)
)/c^(1/2)+3/8*b*(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2018, 664, 620, 206}

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}+\frac{3}{8}b\sqrt{bx^2+cx^4}+\frac{(bx^2+cx^4)^{3/2}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (3*b*Sqrt[b*x^2 + c*x^4])/8 + (b*x^2 + c*x^4)^(3/2)/(4*x^2) + (3*b^2*ArcTan
h[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
```

```
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x^2}\,dx, x, x^2\right)\\
&= \frac{\left(bx^2+cx^4\right)^{3/2}}{4x^2}+\frac{1}{8}(3b)\,\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x}\,dx, x, x^2\right)\\
&= \frac{3}{8}b\sqrt{bx^2+cx^4}+\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^2}+\frac{1}{16}\left(3b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{3}{8}b\sqrt{bx^2+cx^4}+\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^2}+\frac{1}{8}\left(3b^2\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\
&= \frac{3}{8}b\sqrt{bx^2+cx^4}+\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^2}+\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 71, normalized size = 0.89

$$
\frac{1}{8}\sqrt{x^2\left(b+cx^2\right)}\left(\frac{3b^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{c}\,x\sqrt{\frac{cx^2}{b}+1}}+5b+2cx^2\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(5*b + 2*c*x^2 + (3*b^(3/2)*ArcSinh[(Sqrt[c]*x)/Sqrt
[b]])/(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b])))/8
```

fricas [A] time = 0.75, size = 145, normalized size = 1.81

$$\left[\frac{3\,b^{2}\sqrt{c}\,\log\left(-2\,cx^{2}-b-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)+2\,\sqrt{cx^{4}+bx^{2}}\left(2\,c^{2}x^{2}+5\,bc\right)}{16\,c},-\frac{3\,b^{2}\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-c}}{cx^{2}+b}\right)-\sqrt{\ldots}}{8\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*b^2*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*
sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 + 5*b*c))/c, -1/8*(3*b^2*sqrt(-c)*arctan(sqr
t(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 + 5
*b*c))/c]
```

giac [A] time = 0.25, size = 68, normalized size = 0.85

$$-\frac{3\,b^{2}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^{2}+b}\right|\right)\mathrm{sgn}(x)}{8\,\sqrt{c}}+\frac{3\,b^{2}\log\left(|b|\right)\mathrm{sgn}(x)}{16\,\sqrt{c}}+\frac{1}{8}\left(2\,cx^{2}\mathrm{sgn}(x)+5\,b\mathrm{sgn}(x)\right)\sqrt{cx^{2}+b}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -3/8*b^2*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/sqrt(c) + 3/16*b^2*l
og(abs(b))*sgn(x)/sqrt(c) + 1/8*(2*c*x^2*sgn(x) + 5*b*sgn(x))*sqrt(c*x^2 +
b)*x
```

maple [A] time = 0.00, size = 84, normalized size = 1.05

$$\frac{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}\left(3b^{2}\ln\left(\sqrt{c}\;x+\sqrt{c\,x^{2}+b}\right)+3\sqrt{c\,x^{2}+b}\;b\sqrt{c}\;x+2\left(c\,x^{2}+b\right)^{\frac{3}{2}}\sqrt{c}\;x\right)}{8\left(c\,x^{2}+b\right)^{\frac{3}{2}}\sqrt{c}\;x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^3,x)
```

```
[Out] 1/8*(c*x^4+b*x^2)^(3/2)*(2*(c*x^2+b)^(3/2)*c^(1/2)*x+3*(c*x^2+b)^(1/2)*b*c^
(1/2)*x+3*b^2*ln(c^(1/2)*x+(c*x^2+b)^(1/2)))/x^3/(c*x^2+b)^(3/2)/c^(1/2)
```

maxima [A] time = 1.44, size = 70, normalized size = 0.88

$$\frac{3\,b^{2}\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{16\,\sqrt{c}}+\frac{3}{8}\,\sqrt{cx^{4}+bx^{2}}\,b+\frac{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/16*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) + 3/8*sqr
t(c*x^4 + b*x^2)*b + 1/4*(c*x^4 + b*x^2)^(3/2)/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^3,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**3,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**3, x)
```

3.242 $\int \frac{(bx^2+cx^4)^{3/2}}{x^5} \, dx$

Optimal. Leaf size=76

$$-\frac{(bx^2+cx^4)^{3/2}}{x^4}+\frac{3}{2}c\sqrt{bx^2+cx^4}+\frac{3}{2}b\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] -(c*x^4+b*x^2)^(3/2)/x^4+3/2*b*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))*c^(
1/2)+3/2*c*(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 662, 664, 620, 206}

$$-\frac{(bx^2+cx^4)^{3/2}}{x^4}+\frac{3}{2}c\sqrt{bx^2+cx^4}+\frac{3}{2}b\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (3*c*Sqrt[b*x^2 + c*x^4])/2 - (b*x^2 + c*x^4)^(3/2)/x^4 + (3*b*Sqrt[c]*ArcT
anh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{\left(bx^2+cx^4\right)^{3/2}}{x^4}+\frac{1}{2}(3c)\operatorname{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x}\,dx,x,x^2\right)\\
&= \frac{3}{2}c\sqrt{bx^2+cx^4}-\frac{\left(bx^2+cx^4\right)^{3/2}}{x^4}+\frac{1}{4}(3bc)\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\
&= \frac{3}{2}c\sqrt{bx^2+cx^4}-\frac{\left(bx^2+cx^4\right)^{3/2}}{x^4}+\frac{1}{2}(3bc)\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\
&= \frac{3}{2}c\sqrt{bx^2+cx^4}-\frac{\left(bx^2+cx^4\right)^{3/2}}{x^4}+\frac{3}{2}b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.71

$$-\frac{b\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{cx^2}{b}\right)}{x^2\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] -((b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((c*x^2)/b)]
)/(x^2*Sqrt[1 + (c*x^2)/b]))
```

fricas [A] time = 0.55, size = 139, normalized size = 1.83

$$\left[\frac{3\,b\sqrt{c}\,x^2\log\left(-2\,cx^2 - b - 2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right) + 2\,\sqrt{cx^4+bx^2}\left(cx^2 - 2\,b\right)}{4\,x^2}, -\frac{3\,b\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right) - \sqrt{c}}{2\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/4*(3*b*sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2
*sqrt(c*x^4 + b*x^2)*(c*x^2 - 2*b))/x^2, -1/2*(3*b*sqrt(-c)*x^2*arctan(sqrt
(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - sqrt(c*x^4 + b*x^2)*(c*x^2 - 2*b))/
x^2]
```

giac [A] time = 0.27, size = 79, normalized size = 1.04

$$\frac{1}{2}\,\sqrt{cx^2+b}\,cx\mathrm{sgn}(x) - \frac{3}{4}\,b\sqrt{c}\,\log\left(\left(\sqrt{c}\,x - \sqrt{cx^2+b}\right)^2\right)\mathrm{sgn}(x) + \frac{2\,b^2\sqrt{c}\,\mathrm{sgn}(x)}{\left(\sqrt{c}\,x - \sqrt{cx^2+b}\right)^2 - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + b)*c*x*sgn(x) - 3/4*b*sqrt(c)*log((sqrt(c)*x - sqrt(c*x^2
+ b))^2)*sgn(x) + 2*b^2*sqrt(c)*sgn(x)/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b
)
```

maple [A] time = 0.01, size = 107, normalized size = 1.41

$$\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(3b^2cx\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+b}\right)+3\sqrt{c\,x^2+b}\;b\,c^{\frac{3}{2}}x^2+2\left(c\,x^2+b\right)^{\frac{3}{2}}c^{\frac{3}{2}}x^2-2\left(c\,x^2+b\right)^{\frac{5}{2}}\sqrt{c}\right)}{2\left(c\,x^2+b\right)^{\frac{3}{2}}b\sqrt{c}\;x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^5,x)
```

```
[Out] 1/2*(c*x^4+b*x^2)^(3/2)*(2*c^(3/2)*(c*x^2+b)^(3/2)*x^2+3*c^(3/2)*(c*x^2+b)^
(1/2)*x^2*b-2*(c*x^2+b)^(5/2)*c^(1/2)+3*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*x*b^2
*c)/x^4/(c*x^2+b)^(3/2)/b/c^(1/2)
```

maxima [A] time = 1.44, size = 71, normalized size = 0.93

$$\frac{3}{4}\,b\sqrt{c}\,\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-\frac{3\,\sqrt{cx^4+bx^2}\,b}{2\,x^2}+\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 3/4*b*sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 3/2*sqrt(c
*x^4 + b*x^2)*b/x^2 + 1/2*(c*x^4 + b*x^2)^(3/2)/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^5,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**5,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**5, x)
```

3.243 $\int \frac{(bx^2+cx^4)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=75

$$c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{c\sqrt{bx^2+cx^4}}{x^2}-\frac{(bx^2+cx^4)^{3/2}}{3x^6}$$

```
[Out] -1/3*(c*x^4+b*x^2)^(3/2)/x^6+c^(3/2)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2
))-c*(c*x^4+b*x^2)^(1/2)/x^2
```

Rubi [A] time = 0.10, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2018, 662, 620, 206}

$$c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{c\sqrt{bx^2+cx^4}}{x^2}-\frac{(bx^2+cx^4)^{3/2}}{3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] -((c*Sqrt[b*x^2 + c*x^4])/x^2) - (b*x^2 + c*x^4)^(3/2)/(3*x^6) + c^(3/2)*Ar
cTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^7}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x^4}\,dx, x, x^2\right)\\
&= -\frac{\left(bx^2+cx^4\right)^{3/2}}{3x^6}+\frac{1}{2}c\operatorname{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{c\sqrt{bx^2+cx^4}}{x^2}-\frac{\left(bx^2+cx^4\right)^{3/2}}{3x^6}+\frac{1}{2}c^2\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\
&= -\frac{c\sqrt{bx^2+cx^4}}{x^2}-\frac{\left(bx^2+cx^4\right)^{3/2}}{3x^6}+c^2\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\
&= -\frac{c\sqrt{bx^2+cx^4}}{x^2}-\frac{\left(bx^2+cx^4\right)^{3/2}}{3x^6}+c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)
\end{aligned}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.75

$$-\frac{b\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{cx^2}{b}\right)}{3x^4\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] -1/3*(b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, -3/2, -1/2, -((c*x^2)
/b)])/(x^4*Sqrt[1 + (c*x^2)/b])
```

fricas [A] time = 0.58, size = 135, normalized size = 1.80

$$\left[\frac{3\,c^{\frac{3}{2}}x^4\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}\left(4\,cx^2+b\right)}{6\,x^4}, -\frac{3\,\sqrt{-c}\,cx^4\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+\sqrt{c}\cdots}{3\,x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/6*(3*c^(3/2)*x^4*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*s
qrt(c*x^4 + b*x^2)*(4*c*x^2 + b))/x^4, -1/3*(3*sqrt(-c)*c*x^4*arctan(sqrt(c
*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*(4*c*x^2 + b))/x^
4]
```

giac [A] time = 0.44, size = 122, normalized size = 1.63

$$-\frac{1}{2}c^{\frac{3}{2}}\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2\right)\operatorname{sgn}(x)+\frac{4\left(3\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4bc^{\frac{3}{2}}\operatorname{sgn}(x)-3\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2b^2c^{\frac{3}{2}}\operatorname{sgn}(x)\cdots\right.}{3\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/2*c^(3/2)*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)*sgn(x) + 4/3*(3*(sqrt(c)*
x - sqrt(c*x^2 + b))^4*b*c^(3/2)*sgn(x) - 3*(sqrt(c)*x - sqrt(c*x^2 + b))^2
*b^2*c^(3/2)*sgn(x) + 2*b^3*c^(3/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^
2 - b)^3
```

maple [B] time = 0.01, size = 129, normalized size = 1.72

$$\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(3b^2c^2x^3\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+b}\right)+3\sqrt{c\,x^2+b}\;b\,c^{\frac{5}{2}}x^4+2\left(c\,x^2+b\right)^{\frac{3}{2}}c^{\frac{5}{2}}x^4-2\left(c\,x^2+b\right)^{\frac{5}{2}}c^{\frac{3}{2}}x^2-\left(c\,x\cdots\right.\right.}{3\left(c\,x^2+b\right)^{\frac{3}{2}}b^2\sqrt{c}\;x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^7,x)
```

```
[Out] 1/3*(c*x^4+b*x^2)^(3/2)*(2*c^(5/2)*(c*x^2+b)^(3/2)*x^4+3*c^(5/2)*(c*x^2+b)^
(1/2)*x^4*b-2*c^(3/2)*(c*x^2+b)^(5/2)*x^2+3*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*x
^3*b^2*c^2-(c*x^2+b)^(5/2)*b*c^(1/2))/x^6/(c*x^2+b)^(3/2)/b^2/c^(1/2)
```

maxima [A] time = 1.51, size = 89, normalized size = 1.19

$$\frac{1}{2}c^{\frac{3}{2}}\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)-\frac{7\sqrt{cx^4+bx^2}\,c}{6\,x^2}-\frac{\sqrt{cx^4+bx^2}\,b}{6\,x^4}-\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*c^(3/2)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 7/6*sqrt(c*x
^4 + b*x^2)*c/x^2 - 1/6*sqrt(c*x^4 + b*x^2)*b/x^4 - 1/6*(c*x^4 + b*x^2)^(3/
2)/x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^7,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**7,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**7, x)
```

3.244 $\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=25

$$-\frac{(bx^2+cx^4)^{5/2}}{5bx^{10}}$$

```
[Out] -1/5*(c*x^4+b*x^2)^(5/2)/b/x^10
```

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$-\frac{(bx^2+cx^4)^{5/2}}{5bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(5*b*x^10)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx = -\frac{(bx^2+cx^4)^{5/2}}{5bx^{10}}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{(x^2(b+cx^2))^{5/2}}{5bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] -1/5*(x^2*(b + c*x^2))^(5/2)/(b*x^10)
```

fricas [A] time = 0.55, size = 39, normalized size = 1.56

$$-\frac{\left(c^{2} x^{4}+2 \, b c x^{2}+b^{2}\right) \sqrt{c x^{4}+b x^{2}}}{5 \, b x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(c*x^4 + b*x^2)/(b*x^6)
```

giac [B] time = 0.30, size = 92, normalized size = 3.68

$$\frac{2 \left(5 \left(\sqrt{c} x-\sqrt{c x^{2}+b}\right)^{8} c^{\frac{5}{2}} \mathrm{sgn}(x)+10 \left(\sqrt{c} x-\sqrt{c x^{2}+b}\right)^{4} b^{2} c^{\frac{5}{2}} \mathrm{sgn}(x)+b^{4} c^{\frac{5}{2}} \mathrm{sgn}(x)\right)}{5 \left(\left(\sqrt{c} x-\sqrt{c x^{2}+b}\right)^{2}-b\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 2/5*(5*(sqrt(c)*x - sqrt(c*x^2 + b))^8*c^(5/2)*sgn(x) + 10*(sqrt(c)*x - sqr
t(c*x^2 + b))^4*b^2*c^(5/2)*sgn(x) + b^4*c^(5/2)*sgn(x))/((sqrt(c)*x - sqrt
(c*x^2 + b))^2 - b)^5
```

maple [A] time = 0.00, size = 29, normalized size = 1.16

$$-\frac{\left(c \,x^{2}+b \right) \left(c \,x^{4}+b \,x^{2}\right)^{\frac{3}{2}}}{5 b \,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^9,x)
```

```
[Out] -1/5/x^8*(c*x^2+b)/b*(c*x^4+b*x^2)^(3/2)
```

maxima [B] time = 1.41, size = 81, normalized size = 3.24

$$-\frac{\sqrt{c x^{4}+b x^{2}} c^{2}}{5 \, b x^{2}}+\frac{\sqrt{c x^{4}+b x^{2}} c}{10 \, x^{4}}+\frac{3 \, \sqrt{c x^{4}+b x^{2}} b}{10 \, x^{6}}-\frac{\left(c x^{4}+b x^{2}\right)^{\frac{3}{2}}}{2 \, x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(c*x^4 + b*x^2)*c^2/(b*x^2) + 1/10*sqrt(c*x^4 + b*x^2)*c/x^4 + 3/1
0*sqrt(c*x^4 + b*x^2)*b/x^6 - 1/2*(c*x^4 + b*x^2)^(3/2)/x^8
```

mupad [B] time = 4.38, size = 30, normalized size = 1.20

$$-\frac{\left(c\,x^2+b\right)^2\,\sqrt{c\,x^4+b\,x^2}}{5\,b\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^9,x)
```

```
[Out] -((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2))/(5*b*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**9,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**9, x)
```

3.245 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{11}}\,dx$

Optimal. Leaf size=52

$$\frac{2c\left(bx^2+cx^4\right)^{5/2}}{35b^2x^{10}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{7bx^{12}}$$

```
[Out] -1/7*(c*x^4+b*x^2)^(5/2)/b/x^12+2/35*c*(c*x^4+b*x^2)^(5/2)/b^2/x^10
```

Rubi [A] time = 0.09, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{2c\left(bx^2+cx^4\right)^{5/2}}{35b^2x^{10}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{7bx^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^11,x]
```

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(7*b*x^12) + (2*c*(b*x^2 + c*x^4)^(5/2))/(35*b^2*x^1
0)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{11}}\,dx = -\frac{(bx^2+cx^4)^{5/2}}{7bx^{12}} - \frac{(2c)\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx}{7b}$$
$$= -\frac{(bx^2+cx^4)^{5/2}}{7bx^{12}} + \frac{2c\,(bx^2+cx^4)^{5/2}}{35b^2x^{10}}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.67

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}\left(2cx^2-5b\right)}{35b^2x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^11,x]
[Out] ((x^2*(b + c*x^2))^(5/2)*(-5*b + 2*c*x^2))/(35*b^2*x^12)
```

fricas [A] time = 0.62, size = 53, normalized size = 1.02

$$\frac{\left(2\,c^3x^6 - bc^2x^4 - 8\,b^2cx^2 - 5\,b^3\right)\sqrt{cx^4+bx^2}}{35\,b^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="fricas")
[Out] 1/35*(2*c^3*x^6 - b*c^2*x^4 - 8*b^2*c*x^2 - 5*b^3)*sqrt(c*x^4 + b*x^2)/(b^2
*x^8)
```

giac [B] time = 0.25, size = 178, normalized size = 3.42

$$\frac{4\left(35\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{10}c^{\frac{7}{2}}\mathrm{sgn}(x)+35\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{8}bc^{\frac{7}{2}}\mathrm{sgn}(x)+70\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{6}b^2c^{\frac{7}{2}}\mathrm{sgn}(x)+14\cdots\right.}{35\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="giac")
[Out] 4/35*(35*(sqrt(c)*x - sqrt(c*x^2 + b))^10*c^(7/2)*sgn(x) + 35*(sqrt(c)*x -
sqrt(c*x^2 + b))^8*b*c^(7/2)*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^
```

```
2*c^(7/2)*sgn(x) + 14*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^3*c^(7/2)*sgn(x) +
7*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^4*c^(7/2)*sgn(x) - b^5*c^(7/2)*sgn(x))/
((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^7
```

maple [A] time = 0.00, size = 39, normalized size = 0.75

$$-\frac{\left(c\,x^{2}+b\right)\left(-2c\,x^{2}+5b\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{35b^{2}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^11,x)
```

```
[Out] -1/35*(c*x^2+b)*(-2*c*x^2+5*b)*(c*x^4+b*x^2)^(3/2)/x^10/b^2
```

maxima [B] time = 1.46, size = 105, normalized size = 2.02

$$\frac{2\sqrt{cx^{4}+bx^{2}}\,c^{3}}{35\,b^{2}x^{2}}-\frac{\sqrt{cx^{4}+bx^{2}}\,c^{2}}{35\,bx^{4}}+\frac{3\sqrt{cx^{4}+bx^{2}}\,c}{140\,x^{6}}+\frac{3\sqrt{cx^{4}+bx^{2}}\,b}{28\,x^{8}}-\frac{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}{4\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] 2/35*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^2) - 1/35*sqrt(c*x^4 + b*x^2)*c^2/(b*x^
4) + 3/140*sqrt(c*x^4 + b*x^2)*c/x^6 + 3/28*sqrt(c*x^4 + b*x^2)*b/x^8 - 1/4
*(c*x^4 + b*x^2)^(3/2)/x^10
```

mupad [B] time = 4.58, size = 87, normalized size = 1.67

$$\frac{2\,c^{3}\sqrt{c\,x^{4}+b\,x^{2}}}{35\,b^{2}\,x^{2}}-\frac{8\,c\sqrt{c\,x^{4}+b\,x^{2}}}{35\,x^{6}}-\frac{c^{2}\sqrt{c\,x^{4}+b\,x^{2}}}{35\,b\,x^{4}}-\frac{b\sqrt{c\,x^{4}+b\,x^{2}}}{7\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^11,x)
```

```
[Out] (2*c^3*(b*x^2 + c*x^4)^(1/2))/(35*b^2*x^2) - (8*c*(b*x^2 + c*x^4)^(1/2))/(3
5*x^6) - (c^2*(b*x^2 + c*x^4)^(1/2))/(35*b*x^4) - (b*(b*x^2 + c*x^4)^(1/2))
/(7*x^8)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**11,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**11, x)
```

3.246 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=80

$$-\frac{8c^2\left(bx^2+cx^4\right)^{5/2}}{315b^3x^{10}}+\frac{4c\left(bx^2+cx^4\right)^{5/2}}{63b^2x^{12}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}$$

```
[Out] -1/9*(c*x^4+b*x^2)^(5/2)/b/x^14+4/63*c*(c*x^4+b*x^2)^(5/2)/b^2/x^12-8/315*c
^2*(c*x^4+b*x^2)^(5/2)/b^3/x^10
```

Rubi [A] time = 0.14, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$-\frac{8c^2\left(bx^2+cx^4\right)^{5/2}}{315b^3x^{10}}+\frac{4c\left(bx^2+cx^4\right)^{5/2}}{63b^2x^{12}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^13,x]
```

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(9*b*x^14) + (4*c*(b*x^2 + c*x^4)^(5/2))/(63*b^2*x^1
2) - (8*c^2*(b*x^2 + c*x^4)^(5/2))/(315*b^3*x^10)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{13}}\,dx = -\frac{(bx^2+cx^4)^{5/2}}{9bx^{14}} - \frac{(4c)\int \frac{(bx^2+cx^4)^{3/2}}{x^{11}}\,dx}{9b}$$
$$= -\frac{(bx^2+cx^4)^{5/2}}{9bx^{14}} + \frac{4c\,(bx^2+cx^4)^{5/2}}{63b^2x^{12}} + \frac{(8c^2)\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx}{63b^2}$$
$$= -\frac{(bx^2+cx^4)^{5/2}}{9bx^{14}} + \frac{4c\,(bx^2+cx^4)^{5/2}}{63b^2x^{12}} - \frac{8c^2\,(bx^2+cx^4)^{5/2}}{315b^3x^{10}}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.58

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}\left(35b^2-20bcx^2+8c^2x^4\right)}{315b^3x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^13,x]
```

```
[Out] -1/315*((x^2*(b + c*x^2))^(5/2)*(35*b^2 - 20*b*c*x^2 + 8*c^2*x^4))/(b^3*x^1
4)
```

fricas [A] time = 0.48, size = 64, normalized size = 0.80

$$-\frac{\left(8\,c^4x^8-4\,bc^3x^6+3\,b^2c^2x^4+50\,b^3cx^2+35\,b^4\right)\sqrt{cx^4+bx^2}}{315\,b^3x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/315*(8*c^4*x^8 - 4*b*c^3*x^6 + 3*b^2*c^2*x^4 + 50*b^3*c*x^2 + 35*b^4)*sq
rt(c*x^4 + b*x^2)/(b^3*x^10)
```

giac [B] time = 0.27, size = 206, normalized size = 2.58

$$\frac{16\left(210\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{12}c^{\frac{9}{2}}\mathrm{sgn}(x)+315\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{10}bc^{\frac{9}{2}}\mathrm{sgn}(x)+441\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{8}b^2c^{\frac{9}{2}}\mathrm{sgn}(x \cdots}{315\left(\left(\sqrt{\cdots}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] 16/315*(210*(sqrt(c)*x - sqrt(c*x^2 + b))^12*c^(9/2)*sgn(x) + 315*(sqrt(c)*
x - sqrt(c*x^2 + b))^10*b*c^(9/2)*sgn(x) + 441*(sqrt(c)*x - sqrt(c*x^2 + b)
)^8*b^2*c^(9/2)*sgn(x) + 126*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^3*c^(9/2)*sg
n(x) + 36*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^4*c^(9/2)*sgn(x) - 9*(sqrt(c)*x
 - sqrt(c*x^2 + b))^2*b^5*c^(9/2)*sgn(x) + b^6*c^(9/2)*sgn(x))/((sqrt(c)*x
- sqrt(c*x^2 + b))^2 - b)^9
```

maple [A] time = 0.00, size = 50, normalized size = 0.62

$$-\frac{\left(c\,x^{2}+b\right)\left(8c^{2}x^{4}-20bc\,x^{2}+35b^{2}\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{315b^{3}x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^13,x)
```

```
[Out] -1/315*(c*x^2+b)*(8*c^2*x^4-20*b*c*x^2+35*b^2)*(c*x^4+b*x^2)^(3/2)/x^12/b^3
```

maxima [A] time = 1.52, size = 129, normalized size = 1.61

$$-\frac{8\,\sqrt{cx^{4}+bx^{2}}\,c^{4}}{315\,b^{3}x^{2}}+\frac{4\,\sqrt{cx^{4}+bx^{2}}\,c^{3}}{315\,b^{2}x^{4}}-\frac{\sqrt{cx^{4}+bx^{2}}\,c^{2}}{105\,bx^{6}}+\frac{\sqrt{cx^{4}+bx^{2}}\,c}{126\,x^{8}}+\frac{\sqrt{cx^{4}+bx^{2}}\,b}{18\,x^{10}}-\frac{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}{6\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] -8/315*sqrt(c*x^4 + b*x^2)*c^4/(b^3*x^2) + 4/315*sqrt(c*x^4 + b*x^2)*c^3/(b
^2*x^4) - 1/105*sqrt(c*x^4 + b*x^2)*c^2/(b*x^6) + 1/126*sqrt(c*x^4 + b*x^2)
*c/x^8 + 1/18*sqrt(c*x^4 + b*x^2)*b/x^10 - 1/6*(c*x^4 + b*x^2)^(3/2)/x^12
```

mupad [B] time = 4.72, size = 111, normalized size = 1.39

$$\frac{4\,c^{3}\,\sqrt{c\,x^{4}+b\,x^{2}}}{315\,b^{2}\,x^{4}}-\frac{10\,c\,\sqrt{c\,x^{4}+b\,x^{2}}}{63\,x^{8}}-\frac{c^{2}\,\sqrt{c\,x^{4}+b\,x^{2}}}{105\,b\,x^{6}}-\frac{b\,\sqrt{c\,x^{4}+b\,x^{2}}}{9\,x^{10}}-\frac{8\,c^{4}\,\sqrt{c\,x^{4}+b\,x^{2}}}{315\,b^{3}\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^13,x)
```

```
[Out] (4*c^3*(b*x^2 + c*x^4)^(1/2))/(315*b^2*x^4) - (10*c*(b*x^2 + c*x^4)^(1/2))/
(63*x^8) - (c^2*(b*x^2 + c*x^4)^(1/2))/(105*b*x^6) - (b*(b*x^2 + c*x^4)^(1/
2))/(9*x^10) - (8*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^3*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**13,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**13, x)
```

3.247 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{15}} \, dx$

Optimal. Leaf size=108

$$\frac{16c^3\left(bx^2+cx^4\right)^{5/2}}{1155b^4x^{10}}-\frac{8c^2\left(bx^2+cx^4\right)^{5/2}}{231b^3x^{12}}+\frac{2c\left(bx^2+cx^4\right)^{5/2}}{33b^2x^{14}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}$$

```
[Out] -1/11*(c*x^4+b*x^2)^(5/2)/b/x^16+2/33*c*(c*x^4+b*x^2)^(5/2)/b^2/x^14-8/231*
c^2*(c*x^4+b*x^2)^(5/2)/b^3/x^12+16/1155*c^3*(c*x^4+b*x^2)^(5/2)/b^4/x^10
```

Rubi [A] time = 0.18, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{16c^3\left(bx^2+cx^4\right)^{5/2}}{1155b^4x^{10}}-\frac{8c^2\left(bx^2+cx^4\right)^{5/2}}{231b^3x^{12}}+\frac{2c\left(bx^2+cx^4\right)^{5/2}}{33b^2x^{14}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^15,x]
```

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(11*b*x^16) + (2*c*(b*x^2 + c*x^4)^(5/2))/(33*b^2*x^
14) - (8*c^2*(b*x^2 + c*x^4)^(5/2))/(231*b^3*x^12) + (16*c^3*(b*x^2 + c*x^4
)^(5/2))/(1155*b^4*x^10)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{15}}\,dx = -\frac{(bx^2+cx^4)^{5/2}}{11bx^{16}} - \frac{(6c)\int \frac{(bx^2+cx^4)^{3/2}}{x^{13}}\,dx}{11b}$$
$$= -\frac{(bx^2+cx^4)^{5/2}}{11bx^{16}} + \frac{2c(bx^2+cx^4)^{5/2}}{33b^2x^{14}} + \frac{(8c^2)\int \frac{(bx^2+cx^4)^{3/2}}{x^{11}}\,dx}{33b^2}$$
$$= -\frac{(bx^2+cx^4)^{5/2}}{11bx^{16}} + \frac{2c(bx^2+cx^4)^{5/2}}{33b^2x^{14}} - \frac{8c^2(bx^2+cx^4)^{5/2}}{231b^3x^{12}} - \frac{(16c^3)\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx}{231b^3}$$
$$= -\frac{(bx^2+cx^4)^{5/2}}{11bx^{16}} + \frac{2c(bx^2+cx^4)^{5/2}}{33b^2x^{14}} - \frac{8c^2(bx^2+cx^4)^{5/2}}{231b^3x^{12}} + \frac{16c^3(bx^2+cx^4)^{5/2}}{1155b^4x^{10}}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 0.53

$$\frac{(x^2(b+cx^2))^{5/2}(-105b^3+70b^2cx^2-40bc^2x^4+16c^3x^6)}{1155b^4x^{16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^15,x]
```

```
[Out] ((x^2*(b + c*x^2))^(5/2)*(-105*b^3 + 70*b^2*c*x^2 - 40*b*c^2*x^4 + 16*c^3*x
^6))/(1155*b^4*x^16)
```

fricas [A] time = 1.03, size = 75, normalized size = 0.69

$$\frac{(16\,c^5x^{10} - 8\,bc^4x^8 + 6\,b^2c^3x^6 - 5\,b^3c^2x^4 - 140\,b^4cx^2 - 105\,b^5)\sqrt{cx^4+bx^2}}{1155\,b^4x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="fricas")
```

```
[Out] 1/1155*(16*c^5*x^10 - 8*b*c^4*x^8 + 6*b^2*c^3*x^6 - 5*b^3*c^2*x^4 - 140*b^4
*c*x^2 - 105*b^5)*sqrt(c*x^4 + b*x^2)/(b^4*x^12)
```

giac [B] time = 0.28, size = 236, normalized size = 2.19

$$32\left(1155\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{14}c^{\frac{11}{2}}\operatorname{sgn}(x)+2079\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{12}bc^{\frac{11}{2}}\operatorname{sgn}(x)+2541\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{10}b^2c^{\frac{1}{\cdot}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="giac")
```

```
[Out] 32/1155*(1155*(sqrt(c)*x - sqrt(c*x^2 + b))^14*c^(11/2)*sgn(x) + 2079*(sqrt
(c)*x - sqrt(c*x^2 + b))^12*b*c^(11/2)*sgn(x) + 2541*(sqrt(c)*x - sqrt(c*x^
2 + b))^10*b^2*c^(11/2)*sgn(x) + 825*(sqrt(c)*x - sqrt(c*x^2 + b))^8*b^3*c^
(11/2)*sgn(x) + 165*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^4*c^(11/2)*sgn(x) - 5
5*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^5*c^(11/2)*sgn(x) + 11*(sqrt(c)*x - sqr
t(c*x^2 + b))^2*b^6*c^(11/2)*sgn(x) - b^7*c^(11/2)*sgn(x))/((sqrt(c)*x - sq
rt(c*x^2 + b))^2 - b)^11
```

maple [A] time = 0.00, size = 61, normalized size = 0.56

$$-\frac{\left(c\,x^{2}+b\right)\left(-16c^{3}x^{6}+40b\,c^{2}x^{4}-70b^{2}c\,x^{2}+105b^{3}\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{1155b^{4}x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^15,x)
```

```
[Out] -1/1155*(c*x^2+b)*(-16*c^3*x^6+40*b*c^2*x^4-70*b^2*c*x^2+105*b^3)*(c*x^4+b*
x^2)^(3/2)/x^14/b^4
```

maxima [A] time = 1.49, size = 153, normalized size = 1.42

$$\frac{16\,\sqrt{cx^4+bx^2}\,c^5}{1155\,b^4x^2}-\frac{8\,\sqrt{cx^4+bx^2}\,c^4}{1155\,b^3x^4}+\frac{2\,\sqrt{cx^4+bx^2}\,c^3}{385\,b^2x^6}-\frac{\sqrt{cx^4+bx^2}\,c^2}{231\,bx^8}+\frac{\sqrt{cx^4+bx^2}\,c}{264\,x^{10}}+\frac{3\,\sqrt{cx^4+bx^2}\,b}{88\,x^{12}}-\frac{\left(cx^4+bx^2\right.}{8\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="maxima")
```

```
[Out] 16/1155*sqrt(c*x^4 + b*x^2)*c^5/(b^4*x^2) - 8/1155*sqrt(c*x^4 + b*x^2)*c^4/
(b^3*x^4) + 2/385*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^6) - 1/231*sqrt(c*x^4 + b*
x^2)*c^2/(b*x^8) + 1/264*sqrt(c*x^4 + b*x^2)*c/x^10 + 3/88*sqrt(c*x^4 + b*x
^2)*b/x^12 - 1/8*(c*x^4 + b*x^2)^(3/2)/x^14
```

mupad [B] time = 4.99, size = 135, normalized size = 1.25

$$\frac{2\,c^3\,\sqrt{c\,x^4+b\,x^2}}{385\,b^2\,x^6}-\frac{4\,c\,\sqrt{c\,x^4+b\,x^2}}{33\,x^{10}}-\frac{c^2\,\sqrt{c\,x^4+b\,x^2}}{231\,b\,x^8}-\frac{b\,\sqrt{c\,x^4+b\,x^2}}{11\,x^{12}}-\frac{8\,c^4\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^3\,x^4}+\frac{16\,c^5\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^15,x)
```

```
[Out] (2*c^3*(b*x^2 + c*x^4)^(1/2))/(385*b^2*x^6) - (4*c*(b*x^2 + c*x^4)^(1/2))/(
33*x^10) - (c^2*(b*x^2 + c*x^4)^(1/2))/(231*b*x^8) - (b*(b*x^2 + c*x^4)^(1/
2))/(11*x^12) - (8*c^4*(b*x^2 + c*x^4)^(1/2))/(1155*b^3*x^4) + (16*c^5*(b*x
^2 + c*x^4)^(1/2))/(1155*b^4*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**15,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**15, x)
```

3.248 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{17}} \, dx$

Optimal. Leaf size=136

$$-\frac{128c^4\left(bx^2+cx^4\right)^{5/2}}{15015b^5x^{10}}+\frac{64c^3\left(bx^2+cx^4\right)^{5/2}}{3003b^4x^{12}}-\frac{16c^2\left(bx^2+cx^4\right)^{5/2}}{429b^3x^{14}}+\frac{8c\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}}$$

```
[Out] -1/13*(c*x^4+b*x^2)^(5/2)/b/x^18+8/143*c*(c*x^4+b*x^2)^(5/2)/b^2/x^16-16/42
9*c^2*(c*x^4+b*x^2)^(5/2)/b^3/x^14+64/3003*c^3*(c*x^4+b*x^2)^(5/2)/b^4/x^12
-128/15015*c^4*(c*x^4+b*x^2)^(5/2)/b^5/x^10
```

Rubi [A] time = 0.26, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$-\frac{128c^4\left(bx^2+cx^4\right)^{5/2}}{15015b^5x^{10}}+\frac{64c^3\left(bx^2+cx^4\right)^{5/2}}{3003b^4x^{12}}-\frac{16c^2\left(bx^2+cx^4\right)^{5/2}}{429b^3x^{14}}+\frac{8c\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^17,x]
```

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(13*b*x^18) + (8*c*(b*x^2 + c*x^4)^(5/2))/(143*b^2*x
^16) - (16*c^2*(b*x^2 + c*x^4)^(5/2))/(429*b^3*x^14) + (64*c^3*(b*x^2 + c*x
^4)^(5/2))/(3003*b^4*x^12) - (128*c^4*(b*x^2 + c*x^4)^(5/2))/(15015*b^5*x^1
0)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{17}}\,dx = -\frac{\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}} - \frac{(8c)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{15}}\,dx}{13b}$$

$$= -\frac{\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}} + \frac{8c\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}} + \frac{\left(48c^2\right)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{13}}\,dx}{143b^2}$$

$$= -\frac{\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}} + \frac{8c\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}} - \frac{16c^2\left(bx^2+cx^4\right)^{5/2}}{429b^3x^{14}} - \frac{\left(64c^3\right)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{11}}\,dx}{429b^3}$$

$$= -\frac{\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}} + \frac{8c\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}} - \frac{16c^2\left(bx^2+cx^4\right)^{5/2}}{429b^3x^{14}} + \frac{64c^3\left(bx^2+cx^4\right)^{5/2}}{3003b^4x^{12}} + \frac{\left(128c^4\right.}{}$$

$$= -\frac{\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}} + \frac{8c\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}} - \frac{16c^2\left(bx^2+cx^4\right)^{5/2}}{429b^3x^{14}} + \frac{64c^3\left(bx^2+cx^4\right)^{5/2}}{3003b^4x^{12}} - \frac{128c^4\left(\right.}{150}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 0.50

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}\left(1155b^4-840b^3cx^2+560b^2c^2x^4-320bc^3x^6+128c^4x^8\right)}{15015b^5x^{18}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^17,x]
```

```
[Out] -1/15015*((x^2*(b + c*x^2))^(5/2)*(1155*b^4 - 840*b^3*c*x^2 + 560*b^2*c^2*x
^4 - 320*b*c^3*x^6 + 128*c^4*x^8))/(b^5*x^18)
```

fricas [A] time = 0.70, size = 86, normalized size = 0.63

$$-\frac{\left(128\,c^6x^{12}-64\,bc^5x^{10}+48\,b^2c^4x^8-40\,b^3c^3x^6+35\,b^4c^2x^4+1470\,b^5cx^2+1155\,b^6\right)\sqrt{cx^4+bx^2}}{15015\,b^5x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/15015*(128*c^6*x^12 - 64*b*c^5*x^10 + 48*b^2*c^4*x^8 - 40*b^3*c^3*x^6 +
35*b^4*c^2*x^4 + 1470*b^5*c*x^2 + 1155*b^6)*sqrt(c*x^4 + b*x^2)/(b^5*x^14)
```

giac [B] time = 0.29, size = 264, normalized size = 1.94

$$\frac{256\left(6006\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{16}c^{\frac{13}{2}}\operatorname{sgn}(x)+12012\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{14}bc^{\frac{13}{2}}\operatorname{sgn}(x)+13728\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="giac")
```

```
[Out] 256/15015*(6006*(sqrt(c)*x - sqrt(c*x^2 + b))^16*c^(13/2)*sgn(x) + 12012*(s
qrt(c)*x - sqrt(c*x^2 + b))^14*b*c^(13/2)*sgn(x) + 13728*(sqrt(c)*x - sqrt(
c*x^2 + b))^12*b^2*c^(13/2)*sgn(x) + 4719*(sqrt(c)*x - sqrt(c*x^2 + b))^10*
b^3*c^(13/2)*sgn(x) + 715*(sqrt(c)*x - sqrt(c*x^2 + b))^8*b^4*c^(13/2)*sgn(
x) - 286*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^5*c^(13/2)*sgn(x) + 78*(sqrt(c)*
x - sqrt(c*x^2 + b))^4*b^6*c^(13/2)*sgn(x) - 13*(sqrt(c)*x - sqrt(c*x^2 + b
))^2*b^7*c^(13/2)*sgn(x) + b^8*c^(13/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 +
b))^2 - b)^13
```

maple [A] time = 0.01, size = 72, normalized size = 0.53

$$-\frac{\left(c\,x^2+b\right)\left(128c^4x^8-320c^3x^6b+560c^2x^4b^2-840c\,x^2b^3+1155b^4\right)\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}}{15015b^5x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^17,x)
```

```
[Out] -1/15015*(c*x^2+b)*(128*c^4*x^8-320*b*c^3*x^6+560*b^2*c^2*x^4-840*b^3*c*x^2
+1155*b^4)*(c*x^4+b*x^2)^(3/2)/x^16/b^5
```

maxima [A] time = 1.47, size = 177, normalized size = 1.30

$$-\frac{128\sqrt{cx^4+bx^2}\,c^6}{15015\,b^5x^2}+\frac{64\sqrt{cx^4+bx^2}\,c^5}{15015\,b^4x^4}-\frac{16\sqrt{cx^4+bx^2}\,c^4}{5005\,b^3x^6}+\frac{8\sqrt{cx^4+bx^2}\,c^3}{3003\,b^2x^8}-\frac{\sqrt{cx^4+bx^2}\,c^2}{429\,bx^{10}}+\frac{3\sqrt{cx^4+bx^2}\,c}{1430\,x^{12}}+\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="maxima")
```

```
[Out] -128/15015*sqrt(c*x^4 + b*x^2)*c^6/(b^5*x^2) + 64/15015*sqrt(c*x^4 + b*x^2)
*c^5/(b^4*x^4) - 16/5005*sqrt(c*x^4 + b*x^2)*c^4/(b^3*x^6) + 8/3003*sqrt(c*
x^4 + b*x^2)*c^3/(b^2*x^8) - 1/429*sqrt(c*x^4 + b*x^2)*c^2/(b*x^10) + 3/143
```

```
0*sqrt(c*x^4 + b*x^2)*c/x^12 + 3/130*sqrt(c*x^4 + b*x^2)*b/x^14 - 1/10*(c*x
^4 + b*x^2)^(3/2)/x^16
```

mupad [B] time = 5.17, size = 159, normalized size = 1.17

$$\frac{8\,c^3\,\sqrt{c\,x^4+b\,x^2}}{3003\,b^2\,x^8}-\frac{14\,c\,\sqrt{c\,x^4+b\,x^2}}{143\,x^{12}}-\frac{c^2\,\sqrt{c\,x^4+b\,x^2}}{429\,b\,x^{10}}-\frac{b\,\sqrt{c\,x^4+b\,x^2}}{13\,x^{14}}-\frac{16\,c^4\,\sqrt{c\,x^4+b\,x^2}}{5005\,b^3\,x^6}+\frac{64\,c^5\,\sqrt{c\,x^4+b\,x^2}}{15015\,b^4\,x^4}-12$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^17,x)
```

```
[Out] (8*c^3*(b*x^2 + c*x^4)^(1/2))/(3003*b^2*x^8) - (14*c*(b*x^2 + c*x^4)^(1/2))
/(143*x^12) - (c^2*(b*x^2 + c*x^4)^(1/2))/(429*b*x^10) - (b*(b*x^2 + c*x^4)
^(1/2))/(13*x^14) - (16*c^4*(b*x^2 + c*x^4)^(1/2))/(5005*b^3*x^6) + (64*c^5
*(b*x^2 + c*x^4)^(1/2))/(15015*b^4*x^4) - (128*c^6*(b*x^2 + c*x^4)^(1/2))/(
15015*b^5*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2} \left(b + cx^{2}\right)\right)^{\frac{3}{2}}}{x^{17}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**17,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**17, x)
```

3.249 $\int x^6 \left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=134

$$\frac{128b^4\left(bx^2+cx^4\right)^{5/2}}{15015c^5x^5}-\frac{64b^3\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3}+\frac{16b^2\left(bx^2+cx^4\right)^{5/2}}{429c^3x}-\frac{8bx\left(bx^2+cx^4\right)^{5/2}}{143c^2}+\frac{x^3\left(bx^2+cx^4\right)^{5/2}}{13c}$$

```
[Out] 128/15015*b^4*(c*x^4+b*x^2)^(5/2)/c^5/x^5-64/3003*b^3*(c*x^4+b*x^2)^(5/2)/c
^4/x^3+16/429*b^2*(c*x^4+b*x^2)^(5/2)/c^3/x-8/143*b*x*(c*x^4+b*x^2)^(5/2)/c
^2+1/13*x^3*(c*x^4+b*x^2)^(5/2)/c
```

Rubi [A] time = 0.25, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$\frac{128b^4\left(bx^2+cx^4\right)^{5/2}}{15015c^5x^5}-\frac{64b^3\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3}+\frac{16b^2\left(bx^2+cx^4\right)^{5/2}}{429c^3x}-\frac{8bx\left(bx^2+cx^4\right)^{5/2}}{143c^2}+\frac{x^3\left(bx^2+cx^4\right)^{5/2}}{13c}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (128*b^4*(b*x^2 + c*x^4)^(5/2))/(15015*c^5*x^5) - (64*b^3*(b*x^2 + c*x^4)^(
5/2))/(3003*c^4*x^3) + (16*b^2*(b*x^2 + c*x^4)^(5/2))/(429*c^3*x) - (8*b*x*
(b*x^2 + c*x^4)^(5/2))/(143*c^2) + (x^3*(b*x^2 + c*x^4)^(5/2))/(13*c)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
```

```
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int x^6\left(bx^2+cx^4\right)^{3/2}dx &= \frac{x^3\left(bx^2+cx^4\right)^{5/2}}{13c}-\frac{(8b)\int x^4\left(bx^2+cx^4\right)^{3/2}dx}{13c}\\
&= -\frac{8bx\left(bx^2+cx^4\right)^{5/2}}{143c^2}+\frac{x^3\left(bx^2+cx^4\right)^{5/2}}{13c}+\frac{\left(48b^2\right)\int x^2\left(bx^2+cx^4\right)^{3/2}dx}{143c^2}\\
&= \frac{16b^2\left(bx^2+cx^4\right)^{5/2}}{429c^3x}-\frac{8bx\left(bx^2+cx^4\right)^{5/2}}{143c^2}+\frac{x^3\left(bx^2+cx^4\right)^{5/2}}{13c}-\frac{\left(64b^3\right)\int\left(bx^2+cx^4\right)^{3/2}}{429c^3}\\
&= -\frac{64b^3\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3}+\frac{16b^2\left(bx^2+cx^4\right)^{5/2}}{429c^3x}-\frac{8bx\left(bx^2+cx^4\right)^{5/2}}{143c^2}+\frac{x^3\left(bx^2+cx^4\right)^{5/2}}{13c}+\\
&= \frac{128b^4\left(bx^2+cx^4\right)^{5/2}}{15015c^5x^5}-\frac{64b^3\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3}+\frac{16b^2\left(bx^2+cx^4\right)^{5/2}}{429c^3x}-\frac{8bx\left(bx^2+cx^4\right)^{5/2}}{143c^2}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 75, normalized size = 0.56

$$\frac{x\left(b+cx^2\right)^3\left(128b^4-320b^3cx^2+560b^2c^2x^4-840bc^3x^6+1155c^4x^8\right)}{15015c^5\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b + c*x^2)^3*(128*b^4 - 320*b^3*c*x^2 + 560*b^2*c^2*x^4 - 840*b*c^3*x^6
 + 1155*c^4*x^8))/(15015*c^5*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.78, size = 86, normalized size = 0.64

$$\frac{\left(1155\,c^6x^{12}+1470\,bc^5x^{10}+35\,b^2c^4x^8-40\,b^3c^3x^6+48\,b^4c^2x^4-64\,b^5cx^2+128\,b^6\right)\sqrt{cx^4+bx^2}}{15015\,c^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15015*(1155*c^6*x^12 + 1470*b*c^5*x^10 + 35*b^2*c^4*x^8 - 40*b^3*c^3*x^6
+ 48*b^4*c^2*x^4 - 64*b^5*c*x^2 + 128*b^6)*sqrt(c*x^4 + b*x^2)/(c^5*x)
```

giac [A] time = 0.19, size = 92, normalized size = 0.69

$$-\frac{128\,b^{\frac{13}{2}}\operatorname{sgn}(x)}{15015\,c^5}+\frac{1155\left(cx^2+b\right)^{\frac{13}{2}}\operatorname{sgn}(x)-5460\left(cx^2+b\right)^{\frac{11}{2}}b\operatorname{sgn}(x)+10010\left(cx^2+b\right)^{\frac{9}{2}}b^2\operatorname{sgn}(x)-8580\left(cx^2+\right.}{15015\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -128/15015*b^(13/2)*sgn(x)/c^5 + 1/15015*(1155*(c*x^2 + b)^(13/2)*sgn(x) -
5460*(c*x^2 + b)^(11/2)*b*sgn(x) + 10010*(c*x^2 + b)^(9/2)*b^2*sgn(x) - 858
0*(c*x^2 + b)^(7/2)*b^3*sgn(x) + 3003*(c*x^2 + b)^(5/2)*b^4*sgn(x))/c^5
```

maple [A] time = 0.01, size = 72, normalized size = 0.54

$$\frac{\left(c\,x^2+b\right)\left(1155c^4x^8-840c^3x^6b+560c^2x^4b^2-320c\,x^2b^3+128b^4\right)\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}}{15015c^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/15015*(c*x^2+b)*(1155*c^4*x^8-840*b*c^3*x^6+560*b^2*c^2*x^4-320*b^3*c*x^2
+128*b^4)*(c*x^4+b*x^2)^(3/2)/c^5/x^3
```

maxima [A] time = 1.53, size = 79, normalized size = 0.59

$$\frac{\left(1155\,c^6x^{12}+1470\,bc^5x^{10}+35\,b^2c^4x^8-40\,b^3c^3x^6+48\,b^4c^2x^4-64\,b^5cx^2+128\,b^6\right)\sqrt{cx^2+b}}{15015\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/15015*(1155*c^6*x^12 + 1470*b*c^5*x^10 + 35*b^2*c^4*x^8 - 40*b^3*c^3*x^6
+ 48*b^4*c^2*x^4 - 64*b^5*c*x^2 + 128*b^6)*sqrt(c*x^2 + b)/c^5
```

mupad [B] time = 4.48, size = 73, normalized size = 0.54

$$\frac{\left(c\,x^2+b\right)^2\sqrt{c\,x^4+b\,x^2}\,\left(128\,b^4-320\,b^3\,c\,x^2+560\,b^2\,c^2\,x^4-840\,b\,c^3\,x^6+1155\,c^4\,x^8\right)}{15015\,c^5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2)*(128*b^4 + 1155*c^4*x^8 - 320*b^3*c*x^
2 - 840*b*c^3*x^6 + 560*b^2*c^2*x^4))/(15015*c^5*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**6*(x**2*(b + c*x**2))**(3/2), x)
```

3.250 $\int x^4 \left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=106

$$-\frac{16b^3\left(bx^2+cx^4\right)^{5/2}}{1155c^4x^5}+\frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{231c^3x^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{33c^2x}+\frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}$$

```
[Out] -16/1155*b^3*(c*x^4+b*x^2)^(5/2)/c^4/x^5+8/231*b^2*(c*x^4+b*x^2)^(5/2)/c^3/
x^3-2/33*b*(c*x^4+b*x^2)^(5/2)/c^2/x+1/11*x*(c*x^4+b*x^2)^(5/2)/c
```

Rubi [A] time = 0.20, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$-\frac{16b^3\left(bx^2+cx^4\right)^{5/2}}{1155c^4x^5}+\frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{231c^3x^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{33c^2x}+\frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-16*b^3*(b*x^2 + c*x^4)^(5/2))/(1155*c^4*x^5) + (8*b^2*(b*x^2 + c*x^4)^(5/
2))/(231*c^3*x^3) - (2*b*(b*x^2 + c*x^4)^(5/2))/(33*c^2*x) + (x*(b*x^2 + c*
x^4)^(5/2))/(11*c)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
```

```
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int x^4 \left(bx^2+cx^4\right)^{3/2} dx &= \frac{x\left(bx^2+cx^4\right)^{5/2}}{11c} - \frac{(6b)\int x^2\left(bx^2+cx^4\right)^{3/2} dx}{11c} \\
&= -\frac{2b\left(bx^2+cx^4\right)^{5/2}}{33c^2x} + \frac{x\left(bx^2+cx^4\right)^{5/2}}{11c} + \frac{\left(8b^2\right)\int\left(bx^2+cx^4\right)^{3/2} dx}{33c^2} \\
&= \frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{231c^3x^3} - \frac{2b\left(bx^2+cx^4\right)^{5/2}}{33c^2x} + \frac{x\left(bx^2+cx^4\right)^{5/2}}{11c} - \frac{\left(16b^3\right)\int\frac{\left(bx^2+cx^4\right)^{3/2}}{x^2} dx}{231c^3} \\
&= -\frac{16b^3\left(bx^2+cx^4\right)^{5/2}}{1155c^4x^5} + \frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{231c^3x^3} - \frac{2b\left(bx^2+cx^4\right)^{5/2}}{33c^2x} + \frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 0.60

$$\frac{x\left(b+cx^2\right)^3\left(-16b^3+40b^2cx^2-70bc^2x^4+105c^3x^6\right)}{1155c^4\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b + c*x^2)^3*(-16*b^3 + 40*b^2*c*x^2 - 70*b*c^2*x^4 + 105*c^3*x^6))/(11
55*c^4*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.68, size = 75, normalized size = 0.71

$$\frac{\left(105\,c^5x^{10}+140\,bc^4x^8+5\,b^2c^3x^6-6\,b^3c^2x^4+8\,b^4cx^2-16\,b^5\right)\sqrt{cx^4+bx^2}}{1155\,c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1155*(105*c^5*x^10 + 140*b*c^4*x^8 + 5*b^2*c^3*x^6 - 6*b^3*c^2*x^4 + 8*b^
4*c*x^2 - 16*b^5)*sqrt(c*x^4 + b*x^2)/(c^4*x)
```

giac [A] time = 0.16, size = 76, normalized size = 0.72

$$\frac{16\,b^{\frac{11}{2}}\mathrm{sgn}(x)}{1155\,c^4}+\frac{105\left(cx^2+b\right)^{\frac{11}{2}}\mathrm{sgn}(x)-385\left(cx^2+b\right)^{\frac{9}{2}}b\mathrm{sgn}(x)+495\left(cx^2+b\right)^{\frac{7}{2}}b^2\mathrm{sgn}(x)-231\left(cx^2+b\right)^{\frac{5}{2}}b^3\mathrm{sgn}}{1155\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 16/1155*b^(11/2)*sgn(x)/c^4 + 1/1155*(105*(c*x^2 + b)^(11/2)*sgn(x) - 385*(
c*x^2 + b)^(9/2)*b*sgn(x) + 495*(c*x^2 + b)^(7/2)*b^2*sgn(x) - 231*(c*x^2 +
 b)^(5/2)*b^3*sgn(x))/c^4
```

maple [A] time = 0.01, size = 61, normalized size = 0.58

$$-\frac{\left(c\,x^2+b\right)\left(-105c^3x^6+70b\,c^2x^4-40b^2c\,x^2+16b^3\right)\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}}{1155c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/1155*(c*x^2+b)*(-105*c^3*x^6+70*b*c^2*x^4-40*b^2*c*x^2+16*b^3)*(c*x^4+b*
x^2)^(3/2)/c^4/x^3
```

maxima [A] time = 1.50, size = 68, normalized size = 0.64

$$\frac{\left(105\,c^5x^{10}+140\,bc^4x^8+5\,b^2c^3x^6-6\,b^3c^2x^4+8\,b^4cx^2-16\,b^5\right)\sqrt{cx^2+b}}{1155\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/1155*(105*c^5*x^10 + 140*b*c^4*x^8 + 5*b^2*c^3*x^6 - 6*b^3*c^2*x^4 + 8*b^
4*c*x^2 - 16*b^5)*sqrt(c*x^2 + b)/c^4
```

mupad [B] time = 4.30, size = 62, normalized size = 0.58

$$-\frac{\left(c\,x^2+b\right)^2\,\sqrt{c\,x^4+b\,x^2}\,\left(16\,b^3-40\,b^2\,c\,x^2+70\,b\,c^2\,x^4-105\,c^3\,x^6\right)}{1155\,c^4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] -((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2)*(16*b^3 - 105*c^3*x^6 - 40*b^2*c*x^2
+ 70*b*c^2*x^4))/(1155*c^4*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**4*(x**2*(b + c*x**2))**(3/2), x)
```

3.251 $\int x^2 \left(bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=80

$$\frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{315c^3x^5}-\frac{4b\left(bx^2+cx^4\right)^{5/2}}{63c^2x^3}+\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}$$

```
[Out] 8/315*b^2*(c*x^4+b*x^2)^(5/2)/c^3/x^5-4/63*b*(c*x^4+b*x^2)^(5/2)/c^2/x^3+1/
9*(c*x^4+b*x^2)^(5/2)/c/x
```

Rubi [A] time = 0.11, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$\frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{315c^3x^5}-\frac{4b\left(bx^2+cx^4\right)^{5/2}}{63c^2x^3}+\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (8*b^2*(b*x^2 + c*x^4)^(5/2))/(315*c^3*x^5) - (4*b*(b*x^2 + c*x^4)^(5/2))/(
63*c^2*x^3) + (b*x^2 + c*x^4)^(5/2)/(9*c*x)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
```

```
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2 \left(bx^2+cx^4\right)^{3/2}\,dx &= \frac{\left(bx^2+cx^4\right)^{5/2}}{9cx} - \frac{(4b)\int \left(bx^2+cx^4\right)^{3/2}\,dx}{9c} \\
&= -\frac{4b\left(bx^2+cx^4\right)^{5/2}}{63c^2x^3} + \frac{\left(bx^2+cx^4\right)^{5/2}}{9cx} + \frac{\left(8b^2\right)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^2}\,dx}{63c^2} \\
&= \frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{315c^3x^5} - \frac{4b\left(bx^2+cx^4\right)^{5/2}}{63c^2x^3} + \frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.66

$$\frac{x\left(b+cx^2\right)^3\left(8b^2-20bcx^2+35c^2x^4\right)}{315c^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b + c*x^2)^3*(8*b^2 - 20*b*c*x^2 + 35*c^2*x^4))/(315*c^3*Sqrt[x^2*(b +
c*x^2)])
```

fricas [A] time = 0.65, size = 64, normalized size = 0.80

$$\frac{\left(35\,c^4x^8+50\,bc^3x^6+3\,b^2c^2x^4-4\,b^3cx^2+8\,b^4\right)\sqrt{cx^4+bx^2}}{315\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*c^4*x^8 + 50*b*c^3*x^6 + 3*b^2*c^2*x^4 - 4*b^3*c*x^2 + 8*b^4)*sqr
t(c*x^4 + b*x^2)/(c^3*x)
```

giac [A] time = 0.16, size = 60, normalized size = 0.75

$$-\frac{8\,b^{\frac{9}{2}}\mathrm{sgn}(x)}{315\,c^3} + \frac{35\left(cx^2+b\right)^{\frac{9}{2}}\mathrm{sgn}(x) - 90\left(cx^2+b\right)^{\frac{7}{2}}b\mathrm{sgn}(x) + 63\left(cx^2+b\right)^{\frac{5}{2}}b^2\mathrm{sgn}(x)}{315\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -8/315*b^(9/2)*sgn(x)/c^3 + 1/315*(35*(c*x^2 + b)^(9/2)*sgn(x) - 90*(c*x^2
+ b)^(7/2)*b*sgn(x) + 63*(c*x^2 + b)^(5/2)*b^2*sgn(x))/c^3
```

maple [A] time = 0.01, size = 50, normalized size = 0.62

$$\frac{\left(c\,x^{2}+b\right)\left(35c^{2}x^{4}-20bc\,x^{2}+8b^{2}\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{315c^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/315*(c*x^2+b)*(35*c^2*x^4-20*b*c*x^2+8*b^2)*(c*x^4+b*x^2)^(3/2)/c^3/x^3
```

maxima [A] time = 1.45, size = 57, normalized size = 0.71

$$\frac{\left(35\,c^{4}x^{8}+50\,bc^{3}x^{6}+3\,b^{2}c^{2}x^{4}-4\,b^{3}cx^{2}+8\,b^{4}\right)\sqrt{cx^{2}+b}}{315\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/315*(35*c^4*x^8 + 50*b*c^3*x^6 + 3*b^2*c^2*x^4 - 4*b^3*c*x^2 + 8*b^4)*sqr
t(c*x^2 + b)/c^3
```

mupad [B] time = 4.19, size = 51, normalized size = 0.64

$$\frac{\left(c\,x^{2}+b\right)^{2}\sqrt{c\,x^{4}+b\,x^{2}}\,\left(8\,b^{2}-20\,b\,c\,x^{2}+35\,c^{2}\,x^{4}\right)}{315\,c^{3}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2)*(8*b^2 + 35*c^2*x^4 - 20*b*c*x^2))/(31
5*c^3*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(x**2*(b + c*x**2))**(3/2), x)
```

3.252 $\int \left(bx^2+cx^4\right)^{3/2} \, dx$

Optimal. Leaf size=52

$$\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{35c^2x^5}$$

```
[Out] -2/35*b*(c*x^4+b*x^2)^(5/2)/c^2/x^5+1/7*(c*x^4+b*x^2)^(5/2)/c/x^3
```

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2002, 2014}

$$\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{35c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*b*(b*x^2 + c*x^4)^(5/2))/(35*c^2*x^5) + (b*x^2 + c*x^4)^(5/2)/(7*c*x^3)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \left(bx^2+cx^4\right)^{3/2} dx = \frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3} - \frac{(2b)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^2} dx}{7c}$$
$$= -\frac{2b\left(bx^2+cx^4\right)^{5/2}}{35c^2x^5} + \frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.81

$$\frac{x\left(b+cx^2\right)^3\left(5cx^2-2b\right)}{35c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2),x]
[Out] (x*(b + c*x^2)^3*(-2*b + 5*c*x^2))/(35*c^2*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.76, size = 52, normalized size = 1.00

$$\frac{\left(5\,c^3x^6+8\,bc^2x^4+b^2cx^2-2\,b^3\right)\sqrt{cx^4+bx^2}}{35\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
[Out] 1/35*(5*c^3*x^6 + 8*b*c^2*x^4 + b^2*c*x^2 - 2*b^3)*sqrt(c*x^4 + b*x^2)/(c^2
*x)
```

giac [A] time = 0.16, size = 44, normalized size = 0.85

$$\frac{2\,b^{\frac{7}{2}}\mathrm{sgn}(x)}{35\,c^2} + \frac{5\left(cx^2+b\right)^{\frac{7}{2}}\mathrm{sgn}(x) - 7\left(cx^2+b\right)^{\frac{5}{2}}b\mathrm{sgn}(x)}{35\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] 2/35*b^(7/2)*sgn(x)/c^2 + 1/35*(5*(c*x^2 + b)^(7/2)*sgn(x) - 7*(c*x^2 + b)^
(5/2)*b*sgn(x))/c^2
```

maple [A] time = 0.01, size = 39, normalized size = 0.75

$$-\frac{\left(c\,x^{2}+b\right)\left(-5c\,x^{2}+2b\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{35c^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/35*(c*x^2+b)*(-5*c*x^2+2*b)*(c*x^4+b*x^2)^(3/2)/c^2/x^3
```

maxima [A] time = 1.50, size = 45, normalized size = 0.87

$$\frac{\left(5\,c^{3}x^{6}+8\,bc^{2}x^{4}+b^{2}cx^{2}-2\,b^{3}\right)\sqrt{cx^{2}+b}}{35\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/35*(5*c^3*x^6 + 8*b*c^2*x^4 + b^2*c*x^2 - 2*b^3)*sqrt(c*x^2 + b)/c^2
```

mupad [B] time = 4.16, size = 40, normalized size = 0.77

$$-\frac{\left(c\,x^{2}+b\right)^{2}\sqrt{c\,x^{4}+b\,x^{2}}\,\left(2\,b-5\,c\,x^{2}\right)}{35\,c^{2}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] -((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2)*(2*b - 5*c*x^2))/(35*c^2*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^{2}+cx^{4}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((b*x**2 + c*x**4)**(3/2), x)
```

3.253 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=25

$$\frac{\left(bx^2+cx^4\right)^{5/2}}{5cx^5}$$

```
[Out] 1/5*(c*x^4+b*x^2)^(5/2)/c/x^5
```

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$\frac{\left(bx^2+cx^4\right)^{5/2}}{5cx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^2,x]
```

```
[Out] (b*x^2 + c*x^4)^(5/2)/(5*c*x^5)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^2}\,dx=\frac{\left(bx^2+cx^4\right)^{5/2}}{5cx^5}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}}{5cx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^2,x]
```

```
[Out] (x^2*(b + c*x^2))^(5/2)/(5*c*x^5)
```

fricas [A] time = 0.65, size = 39, normalized size = 1.56

$$\frac{\left(c^{2} x^{4} + 2 \, b c x^{2} + b^{2}\right) \sqrt{c x^{4} + b x^{2}}}{5 \, c x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="fricas")
[Out] 1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(c*x^4 + b*x^2)/(c*x)
```

giac [A] time = 0.16, size = 27, normalized size = 1.08

$$\frac{\left(c x^{2} + b\right)^{\frac{5}{2}} \mathrm{sgn}\left(x\right)}{5 \, c} - \frac{b^{\frac{5}{2}} \mathrm{sgn}\left(x\right)}{5 \, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="giac")
[Out] 1/5*(c*x^2 + b)^(5/2)*sgn(x)/c - 1/5*b^(5/2)*sgn(x)/c
```

maple [A] time = 0.00, size = 29, normalized size = 1.16

$$\frac{\left(c \,x^{2}+b \right) \left(c \,x^{4}+b \,x^{2}\right)^{\frac{3}{2}}}{5 c \,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^2,x)
[Out] 1/5*(c*x^2+b)/c/x^3*(c*x^4+b*x^2)^(3/2)
```

maxima [A] time = 1.48, size = 32, normalized size = 1.28

$$\frac{\left(c^{2} x^{4} + 2 \, b c x^{2} + b^{2}\right) \sqrt{c x^{2} + b}}{5 \, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="maxima")
[Out] 1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(c*x^2 + b)/c
```

mupad [B] time = 4.15, size = 30, normalized size = 1.20

$$\frac{\left(c\,x^2+b\right)^2\,\sqrt{c\,x^4+b\,x^2}}{5\,c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^2,x)
```

```
[Out] ((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2))/(5*c*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**2,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**2, x)
```

3.254 $\int \frac{(bx^2+cx^4)^{3/2}}{x^4} dx$

Optimal. Leaf size=73

$$b^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)\right)+\frac{b\sqrt{bx^2+cx^4}}{x}+\frac{(bx^2+cx^4)^{3/2}}{3x^3}$$

```
[Out] 1/3*(c*x^4+b*x^2)^(3/2)/x^3-b^(3/2)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))+
b*(c*x^4+b*x^2)^(1/2)/x
```

Rubi [A] time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2021, 2008, 206}

$$b^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)\right)+\frac{b\sqrt{bx^2+cx^4}}{x}+\frac{(bx^2+cx^4)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] (b*Sqrt[b*x^2 + c*x^4])/x + (b*x^2 + c*x^4)^(3/2)/(3*x^3) - b^(3/2)*ArcTanh
[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
```

```
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^4}\,dx &= \frac{\left(bx^2+cx^4\right)^{3/2}}{3x^3}+b\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx\\ &= \frac{b\sqrt{bx^2+cx^4}}{x}+\frac{\left(bx^2+cx^4\right)^{3/2}}{3x^3}+b^2\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx\\ &= \frac{b\sqrt{bx^2+cx^4}}{x}+\frac{\left(bx^2+cx^4\right)^{3/2}}{3x^3}-b^2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)\\ &= \frac{b\sqrt{bx^2+cx^4}}{x}+\frac{\left(bx^2+cx^4\right)^{3/2}}{3x^3}-b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)\end{aligned}$$

Mathematica [A] time = 0.05, size = 76, normalized size = 1.04

$$\frac{x\left(-3b^{3/2}\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)+4b^2+5bcx^2+c^2x^4\right)}{3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] (x*(4*b^2 + 5*b*c*x^2 + c^2*x^4 - 3*b^(3/2)*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b
+ c*x^2]/Sqrt[b]]))/(3*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.87, size = 140, normalized size = 1.92

$$\left[\frac{3\,b^{\frac{3}{2}}x\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\left(cx^2+4\,b\right)}{6\,x},\frac{3\,\sqrt{-b}\,bx\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}\,(cx^2}{3\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*(3*b^(3/2)*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3)
 + 2*sqrt(c*x^4 + b*x^2)*(c*x^2 + 4*b))/x, 1/3*(3*sqrt(-b)*b*x*arctan(sqrt(
```

```
c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*(c*x^2 + 4*b))
/x]
```

giac [A] time = 0.17, size = 89, normalized size = 1.22

$$\frac{b^2 \arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right) \operatorname{sgn}(x)}{\sqrt{-b}} + \frac{1}{3} \left(cx^2+b\right)^{\frac{3}{2}} \operatorname{sgn}(x) + \sqrt{cx^2+b}\, b \operatorname{sgn}(x) - \frac{\left(3\, b^2 \arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) + 4\, \sqrt{-b}\, b^{\frac{3}{2}}\right) \operatorname{sgn}(x)}{3\, \sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] b^2*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) + 1/3*(c*x^2 + b)^(3/2
)*sgn(x) + sqrt(c*x^2 + b)*b*sgn(x) - 1/3*(3*b^2*arctan(sqrt(b)/sqrt(-b)) +
 4*sqrt(-b)*b^(3/2))*sgn(x)/sqrt(-b)
```

maple [A] time = 0.01, size = 78, normalized size = 1.07

$$-\frac{\left(c\, x^4 + b\, x^2\right)^{\frac{3}{2}} \left(3 b^{\frac{3}{2}} \ln\left(\frac{2 b + 2 \sqrt{c\, x^2 + b}\, \sqrt{b}}{x}\right) - 3 \sqrt{c\, x^2 + b}\, b - \left(c\, x^2 + b\right)^{\frac{3}{2}}\right)}{3 \left(c\, x^2 + b\right)^{\frac{3}{2}} x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^4,x)
```

```
[Out] -1/3*(c*x^4+b*x^2)^(3/2)*(3*b^(3/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)-(c*
x^2+b)^(3/2)-3*(c*x^2+b)^(1/2)*b)/x^3/(c*x^2+b)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\, x^4 + b\, x^2\right)^{3/2}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^4,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**4,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**4, x)
```

3.255 $\int \frac{(bx^2+cx^4)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=79

$$\frac{3c\sqrt{bx^2+cx^4}}{2x}-\frac{3}{2}\sqrt{b}\,c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)-\frac{\left(bx^2+cx^4\right)^{3/2}}{2x^5}$$

```
[Out] -1/2*(c*x^4+b*x^2)^(3/2)/x^5-3/2*c*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))*b
^(1/2)+3/2*c*(c*x^4+b*x^2)^(1/2)/x
```

Rubi [A] time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2021, 2008, 206}

$$-\frac{\left(bx^2+cx^4\right)^{3/2}}{2x^5}+\frac{3c\sqrt{bx^2+cx^4}}{2x}-\frac{3}{2}\sqrt{b}\,c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] (3*c*Sqrt[b*x^2 + c*x^4])/(2*x) - (b*x^2 + c*x^4)^(3/2)/(2*x^5) - (3*Sqrt[b
]*c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
```

```
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^6}\,dx &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{2x^5}+\frac{1}{2}(3c)\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx\\ &= \frac{3c\sqrt{bx^2+cx^4}}{2x}-\frac{\left(bx^2+cx^4\right)^{3/2}}{2x^5}+\frac{1}{2}(3bc)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx\\ &= \frac{3c\sqrt{bx^2+cx^4}}{2x}-\frac{\left(bx^2+cx^4\right)^{3/2}}{2x^5}-\frac{1}{2}(3bc)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)\\ &= \frac{3c\sqrt{bx^2+cx^4}}{2x}-\frac{\left(bx^2+cx^4\right)^{3/2}}{2x^5}-\frac{3}{2}\sqrt{b}\,c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)\end{aligned}$$

Mathematica [C] time = 0.02, size = 44, normalized size = 0.56

$$\frac{c\left(x^2\left(b+cx^2\right)\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{cx^2}{b}+1\right)}{5b^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] (c*(x^2*(b + c*x^2))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (c*x^2)/b])/(
5*b^2*x^5)
```

fricas [A] time = 0.69, size = 147, normalized size = 1.86

$$\left[\frac{3\sqrt{b}\,cx^3\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\left(2\,cx^2-b\right)}{4\,x^3},\frac{3\,\sqrt{-b}\,cx^3\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}}{2\,x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/4*(3*sqrt(b)*c*x^3*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/
x^3) + 2*sqrt(c*x^4 + b*x^2)*(2*c*x^2 - b))/x^3, 1/2*(3*sqrt(-b)*c*x^3*arct
an(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*(2*c*x
^2 - b))/x^3]
```

giac [A] time = 0.20, size = 69, normalized size = 0.87

$$\frac{\frac{3\,bc^2\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}}+2\,\sqrt{cx^2+b}\,c^2\mathrm{sgn}(x)-\frac{\sqrt{cx^2+b}\,bc\mathrm{sgn}(x)}{x^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/2*(3*b*c^2*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) + 2*sqrt(c*x^
2 + b)*c^2*sgn(x) - sqrt(c*x^2 + b)*b*c*sgn(x)/x^2)/c
```

maple [A] time = 0.01, size = 102, normalized size = 1.29

$$-\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(3b^{\frac{3}{2}}c\,x^2\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-3\sqrt{c\,x^2+b}\,bc\,x^2-\left(c\,x^2+b\right)^{\frac{3}{2}}c\,x^2+\left(c\,x^2+b\right)^{\frac{5}{2}}\right)}{2\left(c\,x^2+b\right)^{\frac{3}{2}}b\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^6,x)
```

```
[Out] -1/2*(c*x^4+b*x^2)^(3/2)*(3*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*b^(3/2)*x^2
*c-(c*x^2+b)^(3/2)*c*x^2+(c*x^2+b)^(5/2)-3*(c*x^2+b)^(1/2)*x^2*b*c)/x^5/(c*
x^2+b)^(3/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2 + c*x^4)^(3/2)/x^6,x)

[Out] int((b*x^2 + c*x^4)^(3/2)/x^6, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2)**(3/2)/x**6,x)

[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**6, x)

3.256 $\int \frac{(bx^2+cx^4)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=81

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}-\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}-\frac{3c\sqrt{bx^2+cx^4}}{8x^3}$$

```
[Out] -1/4*(c*x^4+b*x^2)^(3/2)/x^7-3/8*c^2*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))
/b^(1/2)-3/8*c*(c*x^4+b*x^2)^(1/2)/x^3
```

Rubi [A] time = 0.11, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2020, 2008, 206}

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}-\frac{3c\sqrt{bx^2+cx^4}}{8x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] (-3*c*Sqrt[b*x^2 + c*x^4])/(8*x^3) - (b*x^2 + c*x^4)^(3/2)/(4*x^7) - (3*c^2
*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
```

```
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^8}\,dx &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}+\frac{1}{4}(3c)\int \frac{\sqrt{bx^2+cx^4}}{x^4}\,dx\\
&= -\frac{3c\sqrt{bx^2+cx^4}}{8x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}+\frac{1}{8}\left(3c^2\right)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx\\
&= -\frac{3c\sqrt{bx^2+cx^4}}{8x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}-\frac{1}{8}\left(3c^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)\\
&= -\frac{3c\sqrt{bx^2+cx^4}}{8x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 80, normalized size = 0.99

$$
-\frac{2b^2+3c^2x^4\sqrt{\frac{cx^2}{b}+1}\,\tanh^{-1}\left(\sqrt{\frac{cx^2}{b}+1}\right)+7bcx^2+5c^2x^4}{8x^3\sqrt{x^2\left(b+cx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] -1/8*(2*b^2 + 7*b*c*x^2 + 5*c^2*x^4 + 3*c^2*x^4*Sqrt[1 + (c*x^2)/b]*ArcTanh
[Sqrt[1 + (c*x^2)/b]])/(x^3*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.65, size = 164, normalized size = 2.02

$$
\left[\frac{3\sqrt{b}\,c^2x^5\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)-2\,\sqrt{cx^4+bx^2}\left(5\,bcx^2+2\,b^2\right)}{16\,bx^5},\ \frac{3\,\sqrt{-b}\,c^2x^5\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)-\sqrt{cx^4\cdots}}{8\,bx^5}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*c^2*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b
))/x^3) - 2*sqrt(c*x^4 + b*x^2)*(5*b*c*x^2 + 2*b^2))/(b*x^5), 1/8*(3*sqrt(-
```

```
b)*c^2*x^5*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) - sqrt(c*x^4
+ b*x^2)*(5*b*c*x^2 + 2*b^2))/(b*x^5)]
```

giac [A] time = 0.24, size = 76, normalized size = 0.94

$$\frac{\frac{3\,c^{3}\arctan\left(\frac{\sqrt{cx^{2}+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}}-\frac{5\left(cx^{2}+b\right)^{\frac{3}{2}}c^{3}\mathrm{sgn}(x)-3\sqrt{cx^{2}+b}\,bc^{3}\mathrm{sgn}(x)}{c^{2}x^{4}}}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/8*(3*c^3*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - (5*(c*x^2 + b
)^(3/2)*c^3*sgn(x) - 3*sqrt(c*x^2 + b)*b*c^3*sgn(x))/(c^2*x^4))/c
```

maple [A] time = 0.01, size = 125, normalized size = 1.54

$$-\frac{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}\left(3b^{\frac{3}{2}}c^{2}x^{4}\ln\left(\frac{2b+2\sqrt{c\,x^{2}+b}\,\sqrt{b}}{x}\right)-3\sqrt{c\,x^{2}+b}\;b\,c^{2}x^{4}-\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{2}x^{4}+\left(c\,x^{2}+b\right)^{\frac{5}{2}}c\,x^{2}+2\left(c\,x^{2}+\right.\right.}{8\left(c\,x^{2}+b\right)^{\frac{3}{2}}b^{2}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^8,x)
```

```
[Out] -1/8*(c*x^4+b*x^2)^(3/2)*(3*b^(3/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^4
*c^2-(c*x^2+b)^(3/2)*c^2*x^4+(c*x^2+b)^(5/2)*x^2*c-3*(c*x^2+b)^(1/2)*x^4*b*
c^2+2*(c*x^2+b)^(5/2)*b)/x^7/(c*x^2+b)^(3/2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}{x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^8,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**8,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**8, x)
```

3.257 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{10}} dx$

Optimal. Leaf size=109

$$\frac{c^3 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}} - \frac{c^2\sqrt{bx^2+cx^4}}{16bx^3} - \frac{(bx^2+cx^4)^{3/2}}{6x^9} - \frac{c\sqrt{bx^2+cx^4}}{8x^5}$$

```
[Out] -1/6*(c*x^4+b*x^2)^(3/2)/x^9+1/16*c^3*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2)
)/b^(3/2)-1/8*c*(c*x^4+b*x^2)^(1/2)/x^5-1/16*c^2*(c*x^4+b*x^2)^(1/2)/b/x^3
```

Rubi [A] time = 0.16, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2008, 206}

$$\frac{c^3 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}} - \frac{c^2\sqrt{bx^2+cx^4}}{16bx^3} - \frac{c\sqrt{bx^2+cx^4}}{8x^5} - \frac{(bx^2+cx^4)^{3/2}}{6x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^10,x]
```

```
[Out] -(c*Sqrt[b*x^2 + c*x^4])/(8*x^5) - (c^2*Sqrt[b*x^2 + c*x^4])/(16*b*x^3) - (
b*x^2 + c*x^4)^(3/2)/(6*x^9) + (c^3*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]
])/(16*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
```

```
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{10}}\,dx &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{6x^9}+\frac{1}{2}c\int \frac{\sqrt{bx^2+cx^4}}{x^6}\,dx\\
&= -\frac{c\sqrt{bx^2+cx^4}}{8x^5}-\frac{\left(bx^2+cx^4\right)^{3/2}}{6x^9}+\frac{1}{8}c^2\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx\\
&= -\frac{c\sqrt{bx^2+cx^4}}{8x^5}-\frac{c^2\sqrt{bx^2+cx^4}}{16bx^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{6x^9}-\frac{c^3\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{16b}\\
&= -\frac{c\sqrt{bx^2+cx^4}}{8x^5}-\frac{c^2\sqrt{bx^2+cx^4}}{16bx^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{6x^9}+\frac{c^3\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{16b}\\
&= -\frac{c\sqrt{bx^2+cx^4}}{8x^5}-\frac{c^2\sqrt{bx^2+cx^4}}{16bx^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{6x^9}+\frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 46, normalized size = 0.42

$$\frac{c^3\left(x^2\left(b+cx^2\right)\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{cx^2}{b}+1\right)}{5b^4x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^10,x]
```

```
[Out] (c^3*(x^2*(b + c*x^2))^(5/2)*Hypergeometric2F1[5/2, 4, 7/2, 1 + (c*x^2)/b])
/(5*b^4*x^5)
```

fricas [A] time = 0.90, size = 185, normalized size = 1.70

$$\left[\frac{3\sqrt{b}\,c^3x^7\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)-2\left(3\,bc^2x^4+14\,b^2cx^2+8\,b^3\right)\sqrt{cx^4+bx^2}}{96\,b^2x^7},-\frac{3\sqrt{-b}\,c^3x^7\arctan\left(\frac{\sqrt{cx^4+bx^2}}{cx^3+\cdots}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] [1/96*(3*sqrt(b)*c^3*x^7*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b
))/x^3) - 2*(3*b*c^2*x^4 + 14*b^2*c*x^2 + 8*b^3)*sqrt(c*x^4 + b*x^2))/(b^2*
x^7), -1/48*(3*sqrt(-b)*c^3*x^7*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3
+ b*x)) + (3*b*c^2*x^4 + 14*b^2*c*x^2 + 8*b^3)*sqrt(c*x^4 + b*x^2))/(b^2*x^
7)]
```

giac [A] time = 0.24, size = 100, normalized size = 0.92

$$-\frac{\frac{3\,c^4\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}\,b}+\frac{3\left(cx^2+b\right)^{\frac{5}{2}}c^4\mathrm{sgn}(x)+8\left(cx^2+b\right)^{\frac{3}{2}}bc^4\mathrm{sgn}(x)-3\sqrt{cx^2+b}\,b^2c^4\mathrm{sgn}(x)}{bc^3x^6}}{48\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] -1/48*(3*c^4*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b) + (3*(c*x
^2 + b)^(5/2)*c^4*sgn(x) + 8*(c*x^2 + b)^(3/2)*b*c^4*sgn(x) - 3*sqrt(c*x^2
+ b)*b^2*c^4*sgn(x))/(b*c^3*x^6))/c
```

maple [A] time = 0.01, size = 145, normalized size = 1.33

$$\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(3b^{\frac{3}{2}}c^3x^6\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-3\sqrt{c\,x^2+b}\,b\,c^3x^6-\left(c\,x^2+b\right)^{\frac{3}{2}}c^3x^6+\left(c\,x^2+b\right)^{\frac{5}{2}}c^2x^4+2\left(c\,x^2+\cdots\right.\right.}{48\left(c\,x^2+b\right)^{\frac{3}{2}}b^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^10,x)
```

```
[Out] 1/48*(c*x^4+b*x^2)^(3/2)*(3*b^(3/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^6
*c^3-(c*x^2+b)^(3/2)*x^6*c^3+(c*x^2+b)^(5/2)*x^4*c^2-3*(c*x^2+b)^(1/2)*x^6*
b*c^3+2*(c*x^2+b)^(5/2)*x^2*b*c-8*(c*x^2+b)^(5/2)*b^2)/x^9/(c*x^2+b)^(3/2)/
b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^10, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4 + b\,x^2\right)^{3/2}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^10,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^10, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**10,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**10, x)
```

3.258 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{12}}\,dx$

Optimal. Leaf size=137

$$-\frac{3c^4\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}}+\frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}-\frac{c\sqrt{bx^2+cx^4}}{16x^7}$$

```
[Out] -1/8*(c*x^4+b*x^2)^(3/2)/x^11-3/128*c^4*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/
2))/b^(5/2)-1/16*c*(c*x^4+b*x^2)^(1/2)/x^7-1/64*c^2*(c*x^4+b*x^2)^(1/2)/b/x
^5+3/128*c^3*(c*x^4+b*x^2)^(1/2)/b^2/x^3
```

Rubi [A] time = 0.20, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2008, 206}

$$\frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{3c^4\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}-\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^12,x]
```

```
[Out] -(c*Sqrt[b*x^2 + c*x^4])/(16*x^7) - (c^2*Sqrt[b*x^2 + c*x^4])/(64*b*x^5) +
(3*c^3*Sqrt[b*x^2 + c*x^4])/(128*b^2*x^3) - (b*x^2 + c*x^4)^(3/2)/(8*x^11)
- (3*c^4*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(128*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
```

```
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{12}}\,dx &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}+\frac{1}{8}(3c)\int \frac{\sqrt{bx^2+cx^4}}{x^8}\,dx\\
&= -\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}+\frac{1}{16}c^2\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx\\
&= -\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}-\frac{\left(3c^3\right)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{64b}\\
&= -\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}+\frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}+\frac{\left(3c^4\right)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{128b^2}\\
&= -\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}+\frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}-\frac{\left(3c^4\right)\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\right.}{128b}\\
&= -\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}+\frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}-\frac{3c^4\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right.}{128b^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 46, normalized size = 0.34

$$-\frac{c^4\left(x^2\left(b+cx^2\right)\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};\frac{cx^2}{b}+1\right)}{5b^5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^12,x]
```

[Out] -1/5*(c^4*(x^2*(b + c*x^2))^(5/2)*Hypergeometric2F1[5/2, 5, 7/2, 1 + (c*x^2
)/b])/(b^5*x^5)

fricas [A] time = 0.68, size = 207, normalized size = 1.51

$$\left[\frac{3\sqrt{b}c^4x^9\log\left(-\frac{cx^3+2bx-2\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\left(3bc^3x^6-2b^2c^2x^4-24b^3cx^2-16b^4\right)\sqrt{cx^4+bx^2}}{256b^3x^9}, \frac{3\sqrt{-b}c^4x^9\,\text{arcta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="fricas")
```

```
[Out] [1/256*(3*sqrt(b)*c^4*x^9*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(
b))/x^3) + 2*(3*b*c^3*x^6 - 2*b^2*c^2*x^4 - 24*b^3*c*x^2 - 16*b^4)*sqrt(c*x
^4 + b*x^2))/(b^3*x^9), 1/128*(3*sqrt(-b)*c^4*x^9*arctan(sqrt(c*x^4 + b*x^2
)*sqrt(-b)/(c*x^3 + b*x)) + (3*b*c^3*x^6 - 2*b^2*c^2*x^4 - 24*b^3*c*x^2 - 1
6*b^4)*sqrt(c*x^4 + b*x^2))/(b^3*x^9)]
```

giac [A] time = 0.23, size = 119, normalized size = 0.87

$$\frac{\frac{3c^5\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\text{sgn}(x)}{\sqrt{-b}b^2}+\frac{3\left(cx^2+b\right)^{\frac{7}{2}}c^5\text{sgn}(x)-11\left(cx^2+b\right)^{\frac{5}{2}}bc^5\text{sgn}(x)-11\left(cx^2+b\right)^{\frac{3}{2}}b^2c^5\text{sgn}(x)+3\sqrt{cx^2+b}\,b^3c^5\text{sgn}(x)}{b^2c^4x^8}}{128c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="giac")
```

```
[Out] 1/128*(3*c^5*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b^2) + (3*(c
*x^2 + b)^(7/2)*c^5*sgn(x) - 11*(c*x^2 + b)^(5/2)*b*c^5*sgn(x) - 11*(c*x^2
+ b)^(3/2)*b^2*c^5*sgn(x) + 3*sqrt(c*x^2 + b)*b^3*c^5*sgn(x))/(b^2*c^4*x^8)
)/c
```

maple [A] time = 0.02, size = 165, normalized size = 1.20

$$-\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(3b^{\frac{3}{2}}c^4x^8\ln\left(\frac{2b+2\sqrt{cx^2+b}\,\sqrt{b}}{x}\right)-3\sqrt{cx^2+b}\,bc^4x^8-\left(cx^2+b\right)^{\frac{3}{2}}c^4x^8+\left(cx^2+b\right)^{\frac{5}{2}}c^3x^6+2\left(cx^2\right.\right.}{128\left(cx^2+b\right)^{\frac{3}{2}}b^4x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^12,x)
```

```
[Out] -1/128*(c*x^4+b*x^2)^(3/2)*(3*b^(3/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x
^8*c^4-(c*x^2+b)^(3/2)*x^8*c^4+(c*x^2+b)^(5/2)*x^6*c^3-3*(c*x^2+b)^(1/2)*x^
8*b*c^4+2*(c*x^2+b)^(5/2)*x^4*b*c^2-8*(c*x^2+b)^(5/2)*x^2*b^2*c+16*(c*x^2+b
)^(5/2)*b^3)/x^11/(c*x^2+b)^(3/2)/b^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^12, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^12,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^12, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**12,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**12, x)
```

3.259 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{14}} dx$

Optimal. Leaf size=165

$$\frac{3c^5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{256b^{7/2}}-\frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3}+\frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5}-\frac{c^2\sqrt{bx^2+cx^4}}{160bx^7}-\frac{(bx^2+cx^4)^{3/2}}{10x^{13}}-\frac{3c\sqrt{bx^2+cx^4}}{80x^9}$$

```
[Out] -1/10*(c*x^4+b*x^2)^(3/2)/x^13+3/256*c^5*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1
/2))/b^(7/2)-3/80*c*(c*x^4+b*x^2)^(1/2)/x^9-1/160*c^2*(c*x^4+b*x^2)^(1/2)/b
/x^7+1/128*c^3*(c*x^4+b*x^2)^(1/2)/b^2/x^5-3/256*c^4*(c*x^4+b*x^2)^(1/2)/b^
3/x^3
```

Rubi [A] time = 0.25, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2008, 206}

$$-\frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3}+\frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5}+\frac{3c^5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{256b^{7/2}}-\frac{c^2\sqrt{bx^2+cx^4}}{160bx^7}-\frac{3c\sqrt{bx^2+cx^4}}{80x^9}-\frac{(bx^2+cx^4)^{3/2}}{10x^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^14,x]
```

```
[Out] (-3*c*Sqrt[b*x^2 + c*x^4])/(80*x^9) - (c^2*Sqrt[b*x^2 + c*x^4])/(160*b*x^7)
 + (c^3*Sqrt[b*x^2 + c*x^4])/(128*b^2*x^5) - (3*c^4*Sqrt[b*x^2 + c*x^4])/(2
56*b^3*x^3) - (b*x^2 + c*x^4)^(3/2)/(10*x^13) + (3*c^5*ArcTanh[(Sqrt[b]*x)/
Sqrt[b*x^2 + c*x^4]])/(256*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
```

```
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{14}}\,dx = -\frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}}+\frac{1}{10}(3c)\int \frac{\sqrt{bx^2+cx^4}}{x^{10}}\,dx$$

$$= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9}-\frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}}+\frac{1}{80}\left(3c^2\right)\int \frac{1}{x^6\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9}-\frac{c^2\sqrt{bx^2+cx^4}}{160bx^7}-\frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}}-\frac{c^3\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx}{32b}$$

$$= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9}-\frac{c^2\sqrt{bx^2+cx^4}}{160bx^7}+\frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5}-\frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}}+\frac{\left(3c^4\right)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,d}{128b^2}$$

$$= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9}-\frac{c^2\sqrt{bx^2+cx^4}}{160bx^7}+\frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5}-\frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}}-$$

$$= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9}-\frac{c^2\sqrt{bx^2+cx^4}}{160bx^7}+\frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5}-\frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}}+$$

$$= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9}-\frac{c^2\sqrt{bx^2+cx^4}}{160bx^7}+\frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5}-\frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}}+$$

Mathematica [C] time = 0.02, size = 46, normalized size = 0.28

$$\frac{c^5\left(x^2\left(b+cx^2\right)\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},6;\frac{7}{2};\frac{cx^2}{b}+1\right)}{5b^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^14,x]
```

```
[Out] (c^5*(x^2*(b + c*x^2))^(5/2)*Hypergeometric2F1[5/2, 6, 7/2, 1 + (c*x^2)/b])
/(5*b^6*x^5)
```

fricas [A] time = 0.67, size = 229, normalized size = 1.39

$$\left[\frac{15\,\sqrt{b}\,c^5x^{11}\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)-2\left(15\,bc^4x^8-10\,b^2c^3x^6+8\,b^3c^2x^4+176\,b^4cx^2+128\,b^5\right)\sqrt{cx^4+bx^2}}{2560\,b^4x^{11}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="fricas")
```

```
[Out] [1/2560*(15*sqrt(b)*c^5*x^11*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sq
rt(b))/x^3) - 2*(15*b*c^4*x^8 - 10*b^2*c^3*x^6 + 8*b^3*c^2*x^4 + 176*b^4*c*
x^2 + 128*b^5)*sqrt(c*x^4 + b*x^2))/(b^4*x^11), -1/1280*(15*sqrt(-b)*c^5*x^
11*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (15*b*c^4*x^8 - 10*
b^2*c^3*x^6 + 8*b^3*c^2*x^4 + 176*b^4*c*x^2 + 128*b^5)*sqrt(c*x^4 + b*x^2))
/(b^4*x^11)]
```

giac [A] time = 0.38, size = 138, normalized size = 0.84

$$-\frac{\frac{15\,c^6\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}\,b^3}+\frac{15\left(cx^2+b\right)^{\frac{9}{2}}c^6\mathrm{sgn}(x)-70\left(cx^2+b\right)^{\frac{7}{2}}bc^6\mathrm{sgn}(x)+128\left(cx^2+b\right)^{\frac{5}{2}}b^2c^6\mathrm{sgn}(x)+70\left(cx^2+b\right)^{\frac{3}{2}}b^3c^6\mathrm{sgn}(x)-15\,\sqrt{cx^2+b}\,b^4c}{b^3c^5x^{10}}}{1280\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="giac")
```

```
[Out] -1/1280*(15*c^6*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b^3) + (1
5*(c*x^2 + b)^(9/2)*c^6*sgn(x) - 70*(c*x^2 + b)^(7/2)*b*c^6*sgn(x) + 128*(c
*x^2 + b)^(5/2)*b^2*c^6*sgn(x) + 70*(c*x^2 + b)^(3/2)*b^3*c^6*sgn(x) - 15*s
qrt(c*x^2 + b)*b^4*c^6*sgn(x))/(b^3*c^5*x^10))/c
```

maple [A] time = 0.03, size = 186, normalized size = 1.13

$$\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(15b^{\frac{3}{2}}c^5x^{10}\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)-15\sqrt{c\,x^2+b}\;b\,c^5x^{10}-5\left(c\,x^2+b\right)^{\frac{3}{2}}c^5x^{10}+5\left(c\,x^2+b\right)^{\frac{5}{2}}c^4x^8+1\right.}{1280\left(c\,x^2+b\right)^{\frac{3}{2}}b^5x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^14,x)
```

```
[Out] 1/1280*(c*x^4+b*x^2)^(3/2)*(-5*(c*x^2+b)^(3/2)*x^10*c^5+15*b^(3/2)*ln(2*(b+
(c*x^2+b)^(1/2)*b^(1/2))/x)*x^10*c^5+5*(c*x^2+b)^(5/2)*x^8*c^4-15*(c*x^2+b)
^(1/2)*x^10*b*c^5+10*(c*x^2+b)^(5/2)*x^6*b*c^3-40*(c*x^2+b)^(5/2)*x^4*b^2*c
^2+80*(c*x^2+b)^(5/2)*x^2*b^3*c-128*(c*x^2+b)^(5/2)*b^4)/x^13/(c*x^2+b)^(3/
2)/b^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^14, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4 + b\,x^2\right)^{3/2}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^14,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^14, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**14,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**14, x)
```

3.260 $\int \frac{x^7}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=114

$$-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}}+\frac{5b^2\sqrt{bx^2+cx^4}}{16c^3}-\frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2}+\frac{x^4\sqrt{bx^2+cx^4}}{6c}$$

```
[Out] -5/16*b^3*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(7/2)+5/16*b^2*(c*x^4+
b*x^2)^(1/2)/c^3-5/24*b*x^2*(c*x^4+b*x^2)^(1/2)/c^2+1/6*x^4*(c*x^4+b*x^2)^(
1/2)/c
```

Rubi [A] time = 0.13, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 670, 640, 620, 206}

$$\frac{5b^2\sqrt{bx^2+cx^4}}{16c^3}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}}-\frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2}+\frac{x^4\sqrt{bx^2+cx^4}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (5*b^2*Sqrt[b*x^2 + c*x^4])/(16*c^3) - (5*b*x^2*Sqrt[b*x^2 + c*x^4])/(24*c^
2) + (x^4*Sqrt[b*x^2 + c*x^4])/(6*c) - (5*b^3*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*
x^2 + c*x^4]])/(16*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\sqrt{bx^2+cx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{x^4\sqrt{bx^2+cx^4}}{6c} - \frac{(5b)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{12c}\\
&= -\frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{x^4\sqrt{bx^2+cx^4}}{6c} + \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{16c^2}\\
&= \frac{5b^2\sqrt{bx^2+cx^4}}{16c^3} - \frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{x^4\sqrt{bx^2+cx^4}}{6c} - \frac{\left(5b^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{32c^3}\\
&= \frac{5b^2\sqrt{bx^2+cx^4}}{16c^3} - \frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{x^4\sqrt{bx^2+cx^4}}{6c} - \frac{\left(5b^3\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^3}\\
&= \frac{5b^2\sqrt{bx^2+cx^4}}{16c^3} - \frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{x^4\sqrt{bx^2+cx^4}}{6c} - \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 100, normalized size = 0.88

$$\frac{x\left(\sqrt{c}\,x\left(15b^3+5b^2cx^2-2bc^2x^4+8c^3x^6\right)-15b^3\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b+cx^2}}\right)\right)}{48c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(15*b^3 + 5*b^2*c*x^2 - 2*b*c^2*x^4 + 8*c^3*x^6) - 15*b^3*Sqr
t[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b + c*x^2]]))/(48*c^(7/2)*Sqrt[x^2*(b
 + c*x^2)])
```

fricas [A] time = 0.52, size = 166, normalized size = 1.46

$$\left[\frac{15\,b^{3}\sqrt{c}\,\log\left(-2\,cx^{2}-b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)+2\left(8\,c^{3}x^{4}-10\,bc^{2}x^{2}+15\,b^{2}c\right)\sqrt{cx^{4}+bx^{2}}}{96\,c^{4}},\ \frac{15\,b^{3}\sqrt{-c}\,\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*b^3*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2
*(8*c^3*x^4 - 10*b*c^2*x^2 + 15*b^2*c)*sqrt(c*x^4 + b*x^2))/c^4, 1/48*(15*b
^3*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (8*c^3*x^4 -
 10*b*c^2*x^2 + 15*b^2*c)*sqrt(c*x^4 + b*x^2))/c^4]
```

giac [A] time = 0.22, size = 87, normalized size = 0.76

$$\frac{1}{48}\,\sqrt{cx^{4}+bx^{2}}\left(2\,x^{2}\left(\frac{4\,x^{2}}{c}-\frac{5\,b}{c^{2}}\right)+\frac{15\,b^{2}}{c^{3}}\right)+\frac{5\,b^{3}\log\left(\left|-2\left(\sqrt{c}\,x^{2}-\sqrt{cx^{4}+bx^{2}}\right)\sqrt{c}-b\right|\right)}{32\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c*x^4 + b*x^2)*(2*x^2*(4*x^2/c - 5*b/c^2) + 15*b^2/c^3) + 5/32*b^
3*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/c^(7/2)
```

maple [A] time = 0.01, size = 105, normalized size = 0.92

$$\frac{\sqrt{c\,x^{2}+b}\,\left(8\sqrt{c\,x^{2}+b}\,c^{\frac{7}{2}}x^{5}-10\sqrt{c\,x^{2}+b}\,b\,c^{\frac{5}{2}}x^{3}-15b^{3}c\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)+15\sqrt{c\,x^{2}+b}\,b^{2}c^{\frac{3}{2}}x\right)x}{48\sqrt{c\,x^{4}+b\,x^{2}}\,c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/48*x*(c*x^2+b)^(1/2)*(8*x^5*(c*x^2+b)^(1/2)*c^(7/2)-10*c^(5/2)*(c*x^2+b)^
(1/2)*x^3*b+15*c^(3/2)*(c*x^2+b)^(1/2)*x*b^2-15*ln(c^(1/2)*x+(c*x^2+b)^(1/2
))*b^3*c)/(c*x^4+b*x^2)^(1/2)/c^(9/2)
```

maxima [A] time = 1.47, size = 100, normalized size = 0.88

$$\frac{\sqrt{cx^4+bx^2}\,x^4}{6\,c}-\frac{5\,\sqrt{cx^4+bx^2}\,bx^2}{24\,c^2}-\frac{5\,b^3\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{32\,c^{\frac{7}{2}}}+\frac{5\,\sqrt{cx^4+bx^2}\,b^2}{16\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(c*x^4 + b*x^2)*x^4/c - 5/24*sqrt(c*x^4 + b*x^2)*b*x^2/c^2 - 5/32*b
^3*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 5/16*sqrt(c*x
^4 + b*x^2)*b^2/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^7/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**7/sqrt(x**2*(b + c*x**2)), x)
```

3.261 $\int \frac{x^5}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=86

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}-\frac{3b\sqrt{bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{bx^2+cx^4}}{4c}$$

```
[Out] 3/8*b^2*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(5/2)-3/8*b*(c*x^4+b*x^2
)^(1/2)/c^2+1/4*x^2*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A] time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 670, 640, 620, 206}

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}-\frac{3b\sqrt{bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{bx^2+cx^4}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-3*b*Sqrt[b*x^2 + c*x^4])/(8*c^2) + (x^2*Sqrt[b*x^2 + c*x^4])/(4*c) + (3*b
^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\sqrt{bx^2+cx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{x^2\sqrt{bx^2+cx^4}}{4c} - \frac{(3b)\operatorname{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{8c}\\
&= -\frac{3b\sqrt{bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{bx^2+cx^4}}{4c} + \frac{\left(3b^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{16c^2}\\
&= -\frac{3b\sqrt{bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{bx^2+cx^4}}{4c} + \frac{\left(3b^2\right)\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^2}\\
&= -\frac{3b\sqrt{bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{bx^2+cx^4}}{4c} + \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 89, normalized size = 1.03

$$
\frac{x\left(\sqrt{c}\,x\left(-3b^2-bcx^2+2c^2x^4\right)+3b^2\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b+cx^2}}\right)\right)}{8c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(-3*b^2 - b*c*x^2 + 2*c^2*x^4) + 3*b^2*Sqrt[b + c*x^2]*ArcTan
h[(Sqrt[c]*x)/Sqrt[b + c*x^2]]))/(8*c^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.80, size = 145, normalized size = 1.69

$$\left[\frac{3\,b^{2}\sqrt{c}\,\log\left(-2\,cx^{2}-b-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)+2\,\sqrt{cx^{4}+bx^{2}}\left(2\,c^{2}x^{2}-3\,bc\right)}{16\,c^{3}},-\frac{3\,b^{2}\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-c}}{cx^{2}+b}\right)-}{8\,c^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*b^2*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*
sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 - 3*b*c))/c^3, -1/8*(3*b^2*sqrt(-c)*arctan(s
qrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 -
 3*b*c))/c^3]
```

giac [A] time = 0.22, size = 73, normalized size = 0.85

$$\frac{1}{8}\,\sqrt{cx^{4}+bx^{2}}\left(\frac{2\,x^{2}}{c}-\frac{3\,b}{c^{2}}\right)-\frac{3\,b^{2}\log\left(\left|-2\left(\sqrt{c}\,x^{2}-\sqrt{cx^{4}+bx^{2}}\right)\sqrt{c}\,-b\right|\right)}{16\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2)*(2*x^2/c - 3*b/c^2) - 3/16*b^2*log(abs(-2*(sqrt(c)*
x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/c^(5/2)
```

maple [A] time = 0.01, size = 85, normalized size = 0.99

$$\frac{\sqrt{c\,x^{2}+b}\,\left(2\sqrt{c\,x^{2}+b}\;c^{\frac{5}{2}}x^{3}+3b^{2}c\ln\left(\sqrt{c}\;x+\sqrt{c\,x^{2}+b}\right)-3\sqrt{c\,x^{2}+b}\;b\,c^{\frac{3}{2}}x\right)x}{8\sqrt{c\,x^{4}+b\,x^{2}}\;c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/8*x*(c*x^2+b)^(1/2)*(2*x^3*(c*x^2+b)^(1/2)*c^(5/2)-3*c^(3/2)*(c*x^2+b)^(1
/2)*x*b+3*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^2*c)/(c*x^4+b*x^2)^(1/2)/c^(7/2)
```

maxima [A] time = 1.45, size = 76, normalized size = 0.88

$$\frac{\sqrt{cx^4+bx^2}\,x^2}{4\,c}+\frac{3\,b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{16\,c^{\frac{5}{2}}}-\frac{3\,\sqrt{cx^4+bx^2}\,b}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c*x^4 + b*x^2)*x^2/c + 3/16*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b
*x^2)*sqrt(c))/c^(5/2) - 3/8*sqrt(c*x^4 + b*x^2)*b/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^5/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(x**2*(b + c*x**2)), x)
```

3.262 $\int \frac{x^3}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

```
[Out] -1/2*b*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(3/2)+1/2*(c*x^4+b*x^2)^(
1/2)/c
```

Rubi [A] time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2018, 640, 620, 206}

$$\frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/
(2*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{bx^2+cx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\ &= \frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{4c}\\ &= \frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{2c}\\ &= \frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 1.26

$$\frac{x\left(\sqrt{c}\,x\left(b+cx^2\right)-b\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b+cx^2}}\right)\right)}{2c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2) - b*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b +
c*x^2]]))/(2*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.47, size = 114, normalized size = 1.97

$$\left[\frac{b\sqrt{c}\,\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}\,c}{4\,c^2},\frac{b\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}\,c}{2\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*sqrt(
c*x^4 + b*x^2)*c)/c^2, 1/2*(b*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/
(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*c)/c^2]
```

giac [A] time = 0.20, size = 59, normalized size = 1.02

$$\frac{b \log \left(\left| -2 \left(\sqrt{c} x^2 - \sqrt{cx^4 + bx^2} \right) \sqrt{c} - b \right| \right)}{4 c^{\frac{3}{2}}} + \frac{\sqrt{cx^4 + bx^2}}{2 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/c^(3/2)
+ 1/2*sqrt(c*x^4 + b*x^2)/c
```

maple [A] time = 0.01, size = 64, normalized size = 1.10

$$-\frac{\sqrt{c x^2 + b} \left(bc \ln \left(\sqrt{c}\, x + \sqrt{c x^2 + b} \right) - \sqrt{c x^2 + b}\, c^{\frac{3}{2}} x \right) x}{2 \sqrt{c x^4 + b x^2}\, c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/2*x*(c*x^2+b)^(1/2)*(-x*(c*x^2+b)^(1/2)*c^(3/2)+b*ln(c^(1/2)*x+(c*x^2+b)
^(1/2))*c)/(c*x^4+b*x^2)^(1/2)/c^(5/2)
```

maxima [A] time = 1.45, size = 52, normalized size = 0.90

$$-\frac{b \log \left(2 cx^2 + b + 2 \sqrt{cx^4 + bx^2} \sqrt{c} \right)}{4 c^{\frac{3}{2}}} + \frac{\sqrt{cx^4 + bx^2}}{2 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(3/2) + 1/2*sqrt(
c*x^4 + b*x^2)/c
```

mupad [B] time = 4.30, size = 53, normalized size = 0.91

$$\frac{\sqrt{c x^4 + b x^2}}{2 c} - \frac{b \ln \left(\frac{c x^2 + \frac{b}{2}}{\sqrt{c}} + \sqrt{c x^4 + b x^2} \right)}{4 c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (b*x^2 + c*x^4)^(1/2)/(2*c) - (b*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4
)^(1/2)))/(4*c^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^2 \left(b + cx^2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(b + c*x**2)), x)
```

3.263 $\int \frac{x}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

```
[Out] arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2013, 620, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]/Sqrt[c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$
$$= \text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

Mathematica [A] time = 0.01, size = 52, normalized size = 1.68

$$\frac{x\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b+cx^2}}\right)}{\sqrt{c}\,\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[b*x^2 + c*x^4],x]
[Out] (x*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b + c*x^2]])/(Sqrt[c]*Sqrt[x^2*
(b + c*x^2)])
```

fricas [A] time = 0.57, size = 74, normalized size = 2.39

$$\left[\frac{\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{2\,\sqrt{c}}, -\frac{\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] [1/2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c), -sqrt(-c)*a
rctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b))/c]
```

giac [A] time = 0.19, size = 39, normalized size = 1.26

$$-\frac{\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}\,-b\right|\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/sqrt(c)
```

maple [A] time = 0.00, size = 44, normalized size = 1.42

$$\frac{\sqrt{c\,x^{2}+b}\;x\ln\left(\sqrt{c}\;x+\sqrt{c\,x^{2}+b}\right)}{\sqrt{c\,x^{4}+b\,x^{2}}\;\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x*(c*x^2+b)^(1/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))/c^(1/
2)
```

maxima [A] time = 1.46, size = 32, normalized size = 1.03

$$\frac{\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c)
```

mupad [B] time = 4.36, size = 33, normalized size = 1.06

$$\frac{\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}}\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2))/(2*c^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^{2}\left(b+cx^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2*(b + c*x**2)), x)
```

3.264 $\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=23

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

```
[Out] -(c*x^4+b*x^2)^(1/2)/b/x^2
```

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[b*x^2 + c*x^4]/(b*x^2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx=-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[x^2*(b + c*x^2)]/(b*x^2))
```

fricas [A]   time = 0.63, size = 21, normalized size = 0.91

$$-\frac{\sqrt{cx^4 + bx^2}}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] -sqrt(c*x^4 + b*x^2)/(b*x^2)
```

giac [A]   time = 0.18, size = 25, normalized size = 1.09

$$\frac{1}{\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] 1/(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))
```

maple [A]   time = 0.00, size = 26, normalized size = 1.13

$$-\frac{c\,x^2 + b}{\sqrt{c\,x^4 + b\,x^2}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2)^(1/2),x)
[Out] -(c*x^2+b)/b/(c*x^4+b*x^2)^(1/2)
```

maxima [A]   time = 1.47, size = 21, normalized size = 0.91

$$-\frac{\sqrt{cx^4 + bx^2}}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] -sqrt(c*x^4 + b*x^2)/(b*x^2)
```

mupad [B]   time = 4.21, size = 21, normalized size = 0.91

$$-\frac{\sqrt{c\,x^4 + b\,x^2}}{b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] -(b*x^2 + c*x^4)^(1/2)/(b*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**2*(b + c*x**2))), x)
```

3.265 $\int \frac{1}{x^3 \sqrt{bx^2+cx^4}} \, dx$

Optimal. Leaf size=52

$$\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2} - \frac{\sqrt{bx^2+cx^4}}{3bx^4}$$

```
[Out] -1/3*(c*x^4+b*x^2)^(1/2)/b/x^4+2/3*c*(c*x^4+b*x^2)^(1/2)/b^2/x^2
```

Rubi [A] time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2} - \frac{\sqrt{bx^2+cx^4}}{3bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(3*b*x^4) + (2*c*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{bx^2+cx^4}}{3bx^4} - \frac{(2c)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{3b}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{3bx^4} + \frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.67

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(2cx^2-b\right)}{3b^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[b*x^2 + c*x^4]),x]
[Out] (Sqrt[x^2*(b + c*x^2)]*(-b + 2*c*x^2))/(3*b^2*x^4)
```

fricas [A] time = 0.69, size = 31, normalized size = 0.60

$$\frac{\sqrt{cx^4+bx^2}\left(2\,cx^2-b\right)}{3\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(2*c*x^2 - b)/(b^2*x^4)
```

giac [A] time = 0.19, size = 57, normalized size = 1.10

$$\frac{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}\,+b}{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] 1/3*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) + b)/(sqrt(c)*x^2 - sqrt
(c*x^4 + b*x^2))^3
```

maple [A] time = 0.00, size = 37, normalized size = 0.71

$$-\frac{\left(c\,x^{2}+b\right)\left(-2c\,x^{2}+b\right)}{3\sqrt{c\,x^{4}+b\,x^{2}}\;b^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2)^(1/2),x)
[Out] -1/3*(c*x^2+b)*(-2*c*x^2+b)/x^2/b^2/(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.43, size = 44, normalized size = 0.85

$$\frac{2\sqrt{cx^{4}+bx^{2}}\,c}{3\,b^{2}x^{2}}-\frac{\sqrt{cx^{4}+bx^{2}}}{3\,bx^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/3*sqrt(c*x^4 + b*x^2)*c/(b^2*x^2) - 1/3*sqrt(c*x^4 + b*x^2)/(b*x^4)
```

mupad [B] time = 4.26, size = 29, normalized size = 0.56

$$-\frac{\left(b-2\,c\,x^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{3\,b^{2}\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^2 + c*x^4)^(1/2)),x)
[Out] -((b - 2*c*x^2)*(b*x^2 + c*x^4)^(1/2))/(3*b^2*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{3}\sqrt{x^{2}\left(b+cx^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(x**2*(b + c*x**2))), x)
```

3.266 $\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=80

$$-\frac{8c^2\sqrt{bx^2+cx^4}}{15b^3x^2}+\frac{4c\sqrt{bx^2+cx^4}}{15b^2x^4}-\frac{\sqrt{bx^2+cx^4}}{5bx^6}$$

```
[Out] -1/5*(c*x^4+b*x^2)^(1/2)/b/x^6+4/15*c*(c*x^4+b*x^2)^(1/2)/b^2/x^4-8/15*c^2*
(c*x^4+b*x^2)^(1/2)/b^3/x^2
```

Rubi [A] time = 0.13, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$-\frac{8c^2\sqrt{bx^2+cx^4}}{15b^3x^2}+\frac{4c\sqrt{bx^2+cx^4}}{15b^2x^4}-\frac{\sqrt{bx^2+cx^4}}{5bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(5*b*x^6) + (4*c*Sqrt[b*x^2 + c*x^4])/(15*b^2*x^4) - (
8*c^2*Sqrt[b*x^2 + c*x^4])/(15*b^3*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{bx^2+cx^4}}{5bx^6} - \frac{(4c)\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx}{5b}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{5bx^6} + \frac{4c\sqrt{bx^2+cx^4}}{15b^2x^4} + \frac{\left(8c^2\right)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{15b^2}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{5bx^6} + \frac{4c\sqrt{bx^2+cx^4}}{15b^2x^4} - \frac{8c^2\sqrt{bx^2+cx^4}}{15b^3x^2}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.58

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}\left(3b^2-4bcx^2+8c^2x^4\right)}{15b^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/15*(Sqrt[x^2*(b + c*x^2)]*(3*b^2 - 4*b*c*x^2 + 8*c^2*x^4))/(b^3*x^6)
```

fricas [A] time = 0.61, size = 42, normalized size = 0.52

$$-\frac{\left(8\,c^2x^4-4\,bcx^2+3\,b^2\right)\sqrt{cx^4+bx^2}}{15\,b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*c^2*x^4 - 4*b*c*x^2 + 3*b^2)*sqrt(c*x^4 + b*x^2)/(b^3*x^6)
```

giac [A] time = 0.21, size = 90, normalized size = 1.12

$$\frac{20\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^2c+15\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)b\sqrt{c}\,+3\,b^2}{15\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(20*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))^2*c + 15*(sqrt(c)*x^2 - sqrt(c
*x^4 + b*x^2))*b*sqrt(c) + 3*b^2)/(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))^5
```

maple [A] time = 0.00, size = 50, normalized size = 0.62

$$-\frac{\left(c\,x^{2}+b\right)\left(8c^{2}x^{4}-4bc\,x^{2}+3b^{2}\right)}{15\sqrt{c\,x^{4}+b\,x^{2}}\;b^{3}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/15*(c*x^2+b)*(8*c^2*x^4-4*b*c*x^2+3*b^2)/x^4/b^3/(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.43, size = 68, normalized size = 0.85

$$-\frac{8\,\sqrt{cx^{4}+bx^{2}}\,c^{2}}{15\,b^{3}x^{2}}+\frac{4\,\sqrt{cx^{4}+bx^{2}}\,c}{15\,b^{2}x^{4}}-\frac{\sqrt{cx^{4}+bx^{2}}}{5\,bx^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -8/15*sqrt(c*x^4 + b*x^2)*c^2/(b^3*x^2) + 4/15*sqrt(c*x^4 + b*x^2)*c/(b^2*x
^4) - 1/5*sqrt(c*x^4 + b*x^2)/(b*x^6)
```

mupad [B] time = 4.33, size = 42, normalized size = 0.52

$$-\frac{\sqrt{c\,x^{4}+b\,x^{2}}\;\left(3\,b^{2}-4\,b\,c\,x^{2}+8\,c^{2}\,x^{4}\right)}{15\,b^{3}\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*(3*b^2 + 8*c^2*x^4 - 4*b*c*x^2))/(15*b^3*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{5}\sqrt{x^{2}\left(b+cx^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(x**2*(b + c*x**2))), x)
```

3.267 $\int \frac{1}{x^7\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=108

$$\frac{16c^3\sqrt{bx^2+cx^4}}{35b^4x^2}-\frac{8c^2\sqrt{bx^2+cx^4}}{35b^3x^4}+\frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6}-\frac{\sqrt{bx^2+cx^4}}{7bx^8}$$

```
[Out] -1/7*(c*x^4+b*x^2)^(1/2)/b/x^8+6/35*c*(c*x^4+b*x^2)^(1/2)/b^2/x^6-8/35*c^2*
(c*x^4+b*x^2)^(1/2)/b^3/x^4+16/35*c^3*(c*x^4+b*x^2)^(1/2)/b^4/x^2
```

Rubi [A] time = 0.17, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{16c^3\sqrt{bx^2+cx^4}}{35b^4x^2}-\frac{8c^2\sqrt{bx^2+cx^4}}{35b^3x^4}+\frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6}-\frac{\sqrt{bx^2+cx^4}}{7bx^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(7*b*x^8) + (6*c*Sqrt[b*x^2 + c*x^4])/(35*b^2*x^6) - (
8*c^2*Sqrt[b*x^2 + c*x^4])/(35*b^3*x^4) + (16*c^3*Sqrt[b*x^2 + c*x^4])/(35*
b^4*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^7\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{bx^2+cx^4}}{7bx^8} - \frac{(6c)\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx}{7b}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{7bx^8} + \frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6} + \frac{\left(24c^2\right)\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx}{35b^2}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{7bx^8} + \frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6} - \frac{8c^2\sqrt{bx^2+cx^4}}{35b^3x^4} - \frac{\left(16c^3\right)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{35b^3}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{7bx^8} + \frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6} - \frac{8c^2\sqrt{bx^2+cx^4}}{35b^3x^4} + \frac{16c^3\sqrt{bx^2+cx^4}}{35b^4x^2}$$

Mathematica [A] time = 0.01, size = 57, normalized size = 0.53

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(-5b^3+6b^2cx^2-8bc^2x^4+16c^3x^6\right)}{35b^4x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-5*b^3 + 6*b^2*c*x^2 - 8*b*c^2*x^4 + 16*c^3*x^6))/(
35*b^4*x^8)
```

fricas [A] time = 0.64, size = 53, normalized size = 0.49

$$\frac{\left(16\,c^3x^6 - 8\,bc^2x^4 + 6\,b^2cx^2 - 5\,b^3\right)\sqrt{cx^4+bx^2}}{35\,b^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/35*(16*c^3*x^6 - 8*b*c^2*x^4 + 6*b^2*c*x^2 - 5*b^3)*sqrt(c*x^4 + b*x^2)/(
b^4*x^8)
```

giac [A] time = 0.20, size = 123, normalized size = 1.14

$$\frac{70\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^3 c^{\frac{3}{2}} + 84\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^2 bc + 35\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)b^2\sqrt{c} + 5\,b^3}{35\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/35*(70*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))^3*c^(3/2) + 84*(sqrt(c)*x^2 -
sqrt(c*x^4 + b*x^2))^2*b*c + 35*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*b^2*sqr
t(c) + 5*b^3)/(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))^7
```

maple [A] time = 0.01, size = 61, normalized size = 0.56

$$-\frac{\left(c\,x^{2}+b\right)\left(-16c^{3}x^{6}+8b\,c^{2}x^{4}-6b^{2}c\,x^{2}+5b^{3}\right)}{35\sqrt{c\,x^{4}+b\,x^{2}}\;b^{4}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/35*(c*x^2+b)*(-16*c^3*x^6+8*b*c^2*x^4-6*b^2*c*x^2+5*b^3)/x^6/b^4/(c*x^4+
b*x^2)^(1/2)
```

maxima [A] time = 1.41, size = 92, normalized size = 0.85

$$\frac{16\,\sqrt{cx^{4}+bx^{2}}\,c^{3}}{35\,b^{4}x^{2}}-\frac{8\,\sqrt{cx^{4}+bx^{2}}\,c^{2}}{35\,b^{3}x^{4}}+\frac{6\,\sqrt{cx^{4}+bx^{2}}\,c}{35\,b^{2}x^{6}}-\frac{\sqrt{cx^{4}+bx^{2}}}{7\,bx^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 16/35*sqrt(c*x^4 + b*x^2)*c^3/(b^4*x^2) - 8/35*sqrt(c*x^4 + b*x^2)*c^2/(b^3
*x^4) + 6/35*sqrt(c*x^4 + b*x^2)*c/(b^2*x^6) - 1/7*sqrt(c*x^4 + b*x^2)/(b*x
^8)
```

mupad [B] time = 4.27, size = 92, normalized size = 0.85

$$\frac{6\,c\,\sqrt{c\,x^{4}+b\,x^{2}}}{35\,b^{2}\,x^{6}}-\frac{\sqrt{c\,x^{4}+b\,x^{2}}}{7\,b\,x^{8}}-\frac{8\,c^{2}\,\sqrt{c\,x^{4}+b\,x^{2}}}{35\,b^{3}\,x^{4}}+\frac{16\,c^{3}\,\sqrt{c\,x^{4}+b\,x^{2}}}{35\,b^{4}\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] (6*c*(b*x^2 + c*x^4)^(1/2))/(35*b^2*x^6) - (b*x^2 + c*x^4)^(1/2)/(7*b*x^8)
- (8*c^2*(b*x^2 + c*x^4)^(1/2))/(35*b^3*x^4) + (16*c^3*(b*x^2 + c*x^4)^(1/2
))/(35*b^4*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**7*sqrt(x**2*(b + c*x**2))), x)
```

3.268 $\int \frac{x^4}{\sqrt{bx^2+cx^4}} \, dx$

Optimal. Leaf size=50

$$\frac{x\sqrt{bx^2+cx^4}}{3c} - \frac{2b\sqrt{bx^2+cx^4}}{3c^2x}$$

```
[Out] -2/3*b*(c*x^4+b*x^2)^(1/2)/c^2/x+1/3*x*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A] time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 1588}

$$\frac{x\sqrt{bx^2+cx^4}}{3c} - \frac{2b\sqrt{bx^2+cx^4}}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[b*x^2 + c*x^4],x]
[Out] (-2*b*Sqrt[b*x^2 + c*x^4])/(3*c^2*x) + (x*Sqrt[b*x^2 + c*x^4])/(3*c)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx = \frac{x\sqrt{bx^2+cx^4}}{3c} - \frac{(2b)\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{3c}$$
$$= -\frac{2b\sqrt{bx^2+cx^4}}{3c^2x} + \frac{x\sqrt{bx^2+cx^4}}{3c}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.68

$$\frac{(cx^2-2b)\sqrt{x^2(b+cx^2)}}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[b*x^2 + c*x^4],x]
[Out] ((-2*b + c*x^2)*Sqrt[x^2*(b + c*x^2)])/(3*c^2*x)
```

fricas [A] time = 0.64, size = 30, normalized size = 0.60

$$\frac{\sqrt{cx^4+bx^2}\,(cx^2-2\,b)}{3\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 - 2*b)/(c^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/sqrt(c*x^4 + b*x^2), x)
```

maple [A] time = 0.00, size = 37, normalized size = 0.74

$$-\frac{(c\,x^2+b)(-c\,x^2+2b)\,x}{3\sqrt{c\,x^4+b\,x^2}\;c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/3*(c*x^2+b)*(-c*x^2+2*b)*x/c^2/(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.50, size = 34, normalized size = 0.68

$$\frac{c^2x^4 - bcx^2 - 2\,b^2}{3\,\sqrt{cx^2 + b}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(c^2*x^4 - b*c*x^2 - 2*b^2)/(sqrt(c*x^2 + b)*c^2)
```

mupad [B] time = 4.25, size = 33, normalized size = 0.66

$$-\frac{\sqrt{c\,x^4 + b\,x^2}\,\left(\frac{2\,b}{3\,c^2} - \frac{x^2}{3\,c}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*((2*b)/(3*c^2) - x^2/(3*c)))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x**2*(b + c*x**2)), x)
```

3.269 $\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=22

$$\frac{\sqrt{bx^2+cx^4}}{cx}$$

```
[Out] (c*x^4+b*x^2)^(1/2)/c/x
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1588}

$$\frac{\sqrt{bx^2+cx^4}}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/(c*x)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{bx^2+cx^4}}{cx}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{\sqrt{x^2\left(b+cx^2\right)}}{cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[x^2*(b + c*x^2)]/(c*x)
```

fricas [A] time = 0.63, size = 20, normalized size = 0.91

$$\frac{\sqrt{cx^4 + bx^2}}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4 + b*x^2)/(c*x)
```

giac [A] time = 0.19, size = 31, normalized size = 1.41

$$-\frac{2\sqrt{b}}{\left(\sqrt{c+\frac{b}{x^2}}-\frac{\sqrt{b}}{x}\right)^2-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b)/((sqrt(c + b/x^2) - sqrt(b)/x)^2 - c)
```

maple [A] time = 0.00, size = 26, normalized size = 1.18

$$\frac{\left(c\,x^2+b\right)x}{\sqrt{c\,x^4+b\,x^2}\;c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] (c*x^2+b)/c*x/(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.42, size = 13, normalized size = 0.59

$$\frac{\sqrt{cx^2+b}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2 + b)/c
```

mupad [B] time = 4.26, size = 20, normalized size = 0.91

$$\frac{\sqrt{c\,x^4+b\,x^2}}{c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2 + c*x^4)^(1/2),x)
[Out] (b*x^2 + c*x^4)^(1/2)/(c*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x**2/sqrt(x**2*(b + c*x**2)), x)
```

3.270 $\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=30

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

```
[Out] -arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2008, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/Sqrt[b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx &= -\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 52, normalized size = 1.73

$$-\frac{x\sqrt{b+cx^2}\ \tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*x^2 + c*x^4],x]
[Out] -((x*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[x^2*(b
 + c*x^2)]))
```

fricas [A] time = 0.82, size = 80, normalized size = 2.67

$$\left[\frac{\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)}{2\,\sqrt{b}}, \frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] [1/2*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3)/sqrt(b), sqr
t(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x))/b]
```

giac [A] time = 0.17, size = 46, normalized size = 1.53

$$-\frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} + \frac{\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] -arctan(sqrt(b)/sqrt(-b))*sgn(x)/sqrt(-b) + arctan(sqrt(c*x^2 + b)/sqrt(-b)
)/(sqrt(-b)*sgn(x))
```

maple [B] time = 0.00, size = 50, normalized size = 1.67

$$-\frac{\sqrt{c\,x^2+b}\;x\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\,\sqrt{b}}{x}\right)}{\sqrt{c\,x^4+b\,x^2}\;\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/(c*x^4+b*x^2)^(1/2)*x*(c*x^2+b)^(1/2)/b^(1/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^
(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^4 + b*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{c\,x^4 + b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(1/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*x**2 + c*x**4), x)
```

3.271 $\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=59

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

```
[Out] 1/2*c*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(3/2)-1/2*(c*x^4+b*x^2)^(1/2
)/b/x^3
```

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2008, 206}

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(2*b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]
])/(2*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{bx^2+cx^4}}{2bx^3} - \frac{c\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{2b}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{2bx^3} + \frac{c\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)}{2b}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{2bx^3} + \frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}$$

Mathematica [A] time = 0.06, size = 68, normalized size = 1.15

$$\frac{c\sqrt{x^2\left(b+cx^2\right)}\left(\frac{\tanh^{-1}\left(\sqrt{\frac{cx^2}{b}+1}\right)}{2\sqrt{\frac{cx^2}{b}+1}}-\frac{b}{2cx^2}\right)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (c*Sqrt[x^2*(b + c*x^2)]*(-1/2*b/(c*x^2) + ArcTanh[Sqrt[1 + (c*x^2)/b]]/(2*
Sqrt[1 + (c*x^2)/b])))/(b^2*x)
```

fricas [A] time = 0.66, size = 133, normalized size = 2.25

$$\left[\frac{\sqrt{b}\,cx^3\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)-2\,\sqrt{cx^4+bx^2}\,b}{4\,b^2x^3}, -\frac{\sqrt{-b}\,cx^3\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}\,b}{2\,b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*c*x^3*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^
3) - 2*sqrt(c*x^4 + b*x^2)*b)/(b^2*x^3), -1/2*(sqrt(-b)*c*x^3*arctan(sqrt(c
*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*b)/(b^2*x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,
2,4]%%%}] at parameters values [64.3995612673,65,-85]Warning, choosing root
 of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,2,4]%%%}] at para
meters values [66.1769613782,93,91]-1/2/b/x*sqrt(b*(1/x)^2+c)-2*c/4/b/sqrt(
b)*ln(abs(sqrt(b*(1/x)^2+c)-sqrt(b)/x))
```

maple [A] time = 0.01, size = 73, normalized size = 1.24

$$-\frac{\sqrt{c\,x^{2}+b}\,\left(-b c\,x^{2} \ln \left(\frac{2 b+2 \sqrt{c\,x^{2}+b}\,\sqrt{b}}{x}\right)+\sqrt{c\,x^{2}+b}\,b^{\frac{3}{2}}\right)}{2 \sqrt{c\,x^{4}+b\,x^{2}}\,b^{\frac{5}{2}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/2/x*(c*x^2+b)^(1/2)*(-c*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^2*b+(c*x^2
+b)^(1/2)*b^(3/2))/(c*x^4+b*x^2)^(1/2)/b^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^{4}+bx^{2}}\,x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^2), x)
```

mupad [B] time = 4.47, size = 76, normalized size = 1.29

$$-\frac{\left(\frac{\sqrt{c}\,x^{2}\,\sqrt{c+\frac{b}{x^{2}}}}{2\,b}+\frac{c^{3/2}\,x^{3}\,\mathrm{asin}\left(\frac{\sqrt{b}\,1\mathrm{i}}{\sqrt{c}\,x}\right)\,1\mathrm{i}}{2\,b^{3/2}}\right)\,\sqrt{\frac{b}{c\,x^{2}}+1}}{x\,\sqrt{c\,x^{4}+b\,x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] -(((c^(1/2)*x^2*(c + b/x^2)^(1/2))/(2*b) + (c^(3/2)*x^3*asin((b^(1/2)*1i)/(
c^(1/2)*x))*1i)/(2*b^(3/2)))*(b/(c*x^2) + 1)^(1/2))/(x*(b*x^2 + c*x^4)^(1/2
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(b + c*x**2))), x)
```

3.272 $\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=87

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

```
[Out] -3/8*c^2*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(5/2)-1/4*(c*x^4+b*x^2)^(
1/2)/b/x^5+3/8*c*(c*x^4+b*x^2)^(1/2)/b^2/x^3
```

Rubi [A] time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2008, 206}

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(4*b*x^5) + (3*c*Sqrt[b*x^2 + c*x^4])/(8*b^2*x^3) - (3
*c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
```

```
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx &= -\frac{\sqrt{bx^2+cx^4}}{4bx^5}-\frac{(3c)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{4b}\\ &= -\frac{\sqrt{bx^2+cx^4}}{4bx^5}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}+\frac{\left(3c^2\right)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b^2}\\ &= -\frac{\sqrt{bx^2+cx^4}}{4bx^5}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{\left(3c^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{8b^2}\\ &= -\frac{\sqrt{bx^2+cx^4}}{4bx^5}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 44, normalized size = 0.51

$$-\frac{c^2\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{cx^2}{b}+1\right)}{b^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -((c^2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (c*x^2)/b])
/(b^3*x))
```

fricas [A] time = 0.82, size = 163, normalized size = 1.87

$$\left[\frac{3\sqrt{b}\,c^2x^5\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\left(3\,bcx^2-2\,b^2\right)}{16\,b^3x^5},\frac{3\,\sqrt{-b}\,c^2x^5\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx}}{8\,b^3x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*c^2*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b
))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 - 2*b^2))/(b^3*x^5), 1/8*(3*sqrt
```

```
(-b)*c^2*x^5*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^
4 + b*x^2)*(3*b*c*x^2 - 2*b^2))/(b^3*x^5)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,
2,4]%%%}] at parameters values [64.3995612673,65,-85]Warning, choosing root
 of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,2,4]%%%}] at para
meters values [66.1769613782,93,91]2*(-2*b^2/16/b^3/x/x+3*b*c/16/b^3)/x*sqr
t(b*(1/x)^2+c)+6*c^2/16/b^2/sqrt(b)*ln(abs(sqrt(b*(1/x)^2+c)-sqrt(b)/x))
```

maple [A] time = 0.01, size = 94, normalized size = 1.08

$$-\frac{\sqrt{c\,x^{2}+b}\,\left(3 b\,c^{2} x^{4} \ln\left(\frac{2 b+2\sqrt{c\,x^{2}+b}\,\sqrt{b}}{x}\right)-3\sqrt{c\,x^{2}+b}\,b^{\frac{3}{2}} c\,x^{2}+2\sqrt{c\,x^{2}+b}\,b^{\frac{5}{2}}\right)}{8\sqrt{c\,x^{4}+b\,x^{2}}\,b^{\frac{7}{2}} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/8*(c*x^2+b)^(1/2)*(3*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^4*b*c^2-3*b^(
3/2)*(c*x^2+b)^(1/2)*x^2*c+2*(c*x^2+b)^(1/2)*b^(5/2))/x^3/(c*x^4+b*x^2)^(1/
2)/b^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^2}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(b*x^2 + c*x^4)^(1/2)),x)
[Out] int(1/(x^4*(b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(1/(x**4*sqrt(x**2*(b + c*x**2))), x)
```

3.273 $\int \frac{x^9}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=109

$$\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}-\frac{15b\sqrt{bx^2+cx^4}}{8c^3}+\frac{5x^2\sqrt{bx^2+cx^4}}{4c^2}-\frac{x^6}{c\sqrt{bx^2+cx^4}}$$

```
[Out] 15/8*b^2*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(7/2)-x^6/c/(c*x^4+b*x^
2)^(1/2)-15/8*b*(c*x^4+b*x^2)^(1/2)/c^3+5/4*x^2*(c*x^4+b*x^2)^(1/2)/c^2
```

Rubi [A] time = 0.13, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 668, 670, 640, 620, 206}

$$\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}+\frac{5x^2\sqrt{bx^2+cx^4}}{4c^2}-\frac{15b\sqrt{bx^2+cx^4}}{8c^3}-\frac{x^6}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^6/(c*Sqrt[b*x^2 + c*x^4])) - (15*b*Sqrt[b*x^2 + c*x^4])/(8*c^3) + (5*x^
2*Sqrt[b*x^2 + c*x^4])/(4*c^2) + (15*b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
 c*x^4]])/(8*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^9}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{x^6}{c\sqrt{bx^2+cx^4}} + \frac{5\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{2c}$$

$$= -\frac{x^6}{c\sqrt{bx^2+cx^4}} + \frac{5x^2\sqrt{bx^2+cx^4}}{4c^2} - \frac{(15b)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{8c^2}$$

$$= -\frac{x^6}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{8c^3} + \frac{5x^2\sqrt{bx^2+cx^4}}{4c^2} + \frac{\left(15b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{16c^3}$$

$$= -\frac{x^6}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{8c^3} + \frac{5x^2\sqrt{bx^2+cx^4}}{4c^2} + \frac{\left(15b^2\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx}}\right.}{8c^3}$$

$$= -\frac{x^6}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{8c^3} + \frac{5x^2\sqrt{bx^2+cx^4}}{4c^2} + \frac{15b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}$$

Mathematica [A] time = 0.05, size = 88, normalized size = 0.81

$$\frac{x\left(15b^{5/2}\sqrt{\frac{cx^2}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)+\sqrt{c}\,x\left(-15b^2-5bcx^2+2c^2x^4\right)\right)}{8c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(Sqrt[c]*x*(-15*b^2 - 5*b*c*x^2 + 2*c^2*x^4) + 15*b^(5/2)*Sqrt[1 + (c*x^
2)/b]*ArcSinh[(Sqrt[c]*x)/Sqrt[b]]))/(8*c^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.62, size = 209, normalized size = 1.92

$$\left[\frac{15\left(b^2cx^2+b^3\right)\sqrt{c}\,\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\left(2\,c^3x^4-5\,bc^2x^2-15\,b^2c\right)\sqrt{cx^4+bx^2}}{16\left(c^5x^2+bc^4\right)}, -\frac{15\left(b^2cx^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(15*(b^2*c*x^2 + b^3)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2
)*sqrt(c)) + 2*(2*c^3*x^4 - 5*b*c^2*x^2 - 15*b^2*c)*sqrt(c*x^4 + b*x^2))/(c
^5*x^2 + b*c^4), -1/8*(15*(b^2*c*x^2 + b^3)*sqrt(-c)*arctan(sqrt(c*x^4 + b*
x^2)*sqrt(-c)/(c*x^2 + b)) - (2*c^3*x^4 - 5*b*c^2*x^2 - 15*b^2*c)*sqrt(c*x^
4 + b*x^2))/(c^5*x^2 + b*c^4)]
```

giac [A] time = 0.27, size = 114, normalized size = 1.05

$$\frac{1}{8}\sqrt{cx^4+bx^2}\left(\frac{2x^2}{c^2}-\frac{7b}{c^3}\right)-\frac{15b^2\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}-b\right|\right)}{16c^{\frac{7}{2}}}-\frac{b^3}{\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2}\right)c+b\sqrt{c}\right)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2)*(2*x^2/c^2 - 7*b/c^3) - 15/16*b^2*log(abs(-2*(sqrt(
c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/c^(7/2) - b^3/(((sqrt(c)*x^2 -
sqrt(c*x^4 + b*x^2))*c + b*sqrt(c))*c^3)
```

maple [A] time = 0.01, size = 87, normalized size = 0.80

$$\frac{\left(cx^2+b\right)\left(2c^{\frac{7}{2}}x^5-5bc^{\frac{5}{2}}x^3-15b^2c^{\frac{3}{2}}x+15\sqrt{cx^2+b}\,b^2c\ln\left(\sqrt{c}\,x+\sqrt{cx^2+b}\right)\right)x^3}{8\left(cx^4+bx^2\right)^{\frac{3}{2}}c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/8*x^3*(c*x^2+b)*(2*x^5*c^(7/2)-5*c^(5/2)*x^3*b-15*c^(3/2)*x*b^2+15*ln(c^(
1/2)*x+(c*x^2+b)^(1/2))*(c*x^2+b)^(1/2)*b^2*c)/(c*x^4+b*x^2)^(3/2)/c^(9/2)
```

maxima [A] time = 1.48, size = 103, normalized size = 0.94

$$\frac{x^6}{4\sqrt{cx^4+bx^2}\,c}-\frac{5bx^4}{8\sqrt{cx^4+bx^2}\,c^2}-\frac{15b^2x^2}{8\sqrt{cx^4+bx^2}\,c^3}+\frac{15b^2\log\left(2cx^2+b+2\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{16c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*x^6/(sqrt(c*x^4 + b*x^2)*c) - 5/8*b*x^4/(sqrt(c*x^4 + b*x^2)*c^2) - 15/
8*b^2*x^2/(sqrt(c*x^4 + b*x^2)*c^3) + 15/16*b^2*log(2*c*x^2 + b + 2*sqrt(c*
x^4 + b*x^2)*sqrt(c))/c^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^9/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**9/(x**2*(b + c*x**2))**(3/2), x)
```

3.274 $\int \frac{x^7}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{3b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}+\frac{3\sqrt{bx^2+cx^4}}{2c^2}-\frac{x^4}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -3/2*b*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(5/2)-x^4/c/(c*x^4+b*x^2)
^(1/2)+3/2*(c*x^4+b*x^2)^(1/2)/c^2
```

Rubi [A] time = 0.11, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 668, 640, 620, 206}

$$\frac{3\sqrt{bx^2+cx^4}}{2c^2}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}-\frac{x^4}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^4/(c*Sqrt[b*x^2 + c*x^4])) + (3*Sqrt[b*x^2 + c*x^4])/(2*c^2) - (3*b*Arc
Tanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(2*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{x^4}{c\sqrt{bx^2+cx^4}} + \frac{3\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{2c}\\
&= -\frac{x^4}{c\sqrt{bx^2+cx^4}} + \frac{3\sqrt{bx^2+cx^4}}{2c^2} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{4c^2}\\
&= -\frac{x^4}{c\sqrt{bx^2+cx^4}} + \frac{3\sqrt{bx^2+cx^4}}{2c^2} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^2}\\
&= -\frac{x^4}{c\sqrt{bx^2+cx^4}} + \frac{3\sqrt{bx^2+cx^4}}{2c^2} - \frac{3b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 76, normalized size = 0.94

$$
\frac{x\left(\sqrt{c}\,x\left(3b+cx^2\right)-3b^{3/2}\sqrt{\frac{cx^2}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\right)}{2c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(Sqrt[c]*x*(3*b + c*x^2) - 3*b^(3/2)*Sqrt[1 + (c*x^2)/b]*ArcSinh[(Sqrt[c
]*x)/Sqrt[b]]))/(2*c^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.57, size = 180, normalized size = 2.22

$$\left[\frac{3\left(bcx^2+b^2\right)\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}\left(c^2x^2+3\,bc\right)}{4\left(c^4x^2+bc^3\right)},\frac{3\left(bcx^2+b^2\right)\sqrt{-c}\,\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(b*c*x^2 + b^2)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sq
rt(c)) + 2*sqrt(c*x^4 + b*x^2)*(c^2*x^2 + 3*b*c))/(c^4*x^2 + b*c^3), 1/2*(3
*(b*c*x^2 + b^2)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b))
+ sqrt(c*x^4 + b*x^2)*(c^2*x^2 + 3*b*c))/(c^4*x^2 + b*c^3)]
```

giac [A] time = 0.24, size = 99, normalized size = 1.22

$$\frac{3\,b\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}\,-b\right|\right)}{4\,c^{\frac{5}{2}}}+\frac{b^2}{\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)c+b\sqrt{c}\right)c^2}+\frac{\sqrt{cx^4+bx^2}}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3/4*b*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/c^(5/2)
+ b^2/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*c + b*sqrt(c))*c^2) + 1/2*sqrt(
c*x^4 + b*x^2)/c^2
```

maple [A] time = 0.01, size = 73, normalized size = 0.90

$$\frac{\left(c\,x^2+b\right)\left(c^{\frac{5}{2}}x^3+3b\,c^{\frac{3}{2}}x-3\sqrt{c\,x^2+b}\;bc\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+b}\right)\right)x^3}{2\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/2*x^3*(c*x^2+b)*(x^3*c^(5/2)+3*c^(3/2)*x*b-3*ln(c^(1/2)*x+(c*x^2+b)^(1/2)
)*(c*x^2+b)^(1/2)*b*c)/(c*x^4+b*x^2)^(3/2)/c^(7/2)
```

maxima [A] time = 1.51, size = 77, normalized size = 0.95

$$\frac{x^4}{2\sqrt{cx^4+bx^2}\,c}+\frac{3\,bx^2}{2\sqrt{cx^4+bx^2}\,c^2}-\frac{3\,b\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{4\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^4/(sqrt(c*x^4 + b*x^2)*c) + 3/2*b*x^2/(sqrt(c*x^4 + b*x^2)*c^2) - 3/4
*b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^7/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**7/(x**2*(b + c*x**2))**(3/2), x)
```

3.275 $\int \frac{x^5}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}-\frac{x^2}{c\sqrt{bx^2+cx^4}}$$

```
[Out] arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(3/2)-x^2/c/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2018, 652, 620, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}-\frac{x^2}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^2/(c*Sqrt[b*x^2 + c*x^4])) + ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]
/c^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))^2*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[(e*(d + e*x)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[
(e^2*(p + 2))/(c*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && !IntegerQ[p] && LtQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)\\ &= -\frac{x^2}{c\sqrt{bx^2+cx^4}}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{2c}\\ &= -\frac{x^2}{c\sqrt{bx^2+cx^4}}+\frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{c}\\ &= -\frac{x^2}{c\sqrt{bx^2+cx^4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 66, normalized size = 1.20

$$\frac{\sqrt{b}\,x\sqrt{\frac{cx^2}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)-\sqrt{c}\,x^2}{c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-(Sqrt[c]*x^2) + Sqrt[b]*x*Sqrt[1 + (c*x^2)/b]*ArcSinh[(Sqrt[c]*x)/Sqrt[b]
])/(c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.70, size = 150, normalized size = 2.73

$$\left[\frac{\left(cx^2+b\right)\sqrt{c}\,\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}\,c}{2\left(c^3x^2+bc^2\right)},-\frac{\left(cx^2+b\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx}}{c^3x^2+bc^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*x^2 + b)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c))
 - 2*sqrt(c*x^4 + b*x^2)*c)/(c^3*x^2 + b*c^2), -((c*x^2 + b)*sqrt(-c)*arcta
n(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*c)/(c^3*x
^2 + b*c^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%%{-2,[1]%%%},[2,2]%%%}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,3]%
%%}+%%%{-2,[0,4]%%%} / %%%{%%%{1,[2]%%%},[2,0]%%%}+%%%{%%{[%%%{-2,[1]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[1,1]%%%}+%%%{%%%{1,[1]%%%},[0,2]%%%} Error: Bad
Argument Value
```

maple [A] time = 0.01, size = 63, normalized size = 1.15

$$-\frac{\left(c\,x^{2}+b\right)\left(c^{\frac{3}{2}}x-\sqrt{c\,x^{2}+b}\;c\ln\left(\sqrt{c}\;x+\sqrt{c\,x^{2}+b}\right)\right)x^{3}}{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -x^3*(c*x^2+b)*(x*c^(3/2)-ln(c^(1/2)*x+(c*x^2+b)^(1/2))*c*(c*x^2+b)^(1/2))/
(c*x^4+b*x^2)^(3/2)/c^(5/2)
```

maxima [A] time = 1.47, size = 54, normalized size = 0.98

$$-\frac{x^{2}}{\sqrt{cx^{4}+bx^{2}}\,c}+\frac{\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{2\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -x^2/(sqrt(c*x^4 + b*x^2)*c) + 1/2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*
sqrt(c))/c^(3/2)
```

mupad [B] time = 4.33, size = 55, normalized size = 1.00

$$\frac{\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{2\,c^{3/2}}-\frac{x^2}{c\,\sqrt{c\,x^4+b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2))/(2*c^(3/2)) - x^2/(c*(b*
x^2 + c*x^4)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**5/(x**2*(b + c*x**2))**(3/2), x)
```

3.276 $\int \frac{x^3}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=22

$$\frac{x^2}{b\sqrt{bx^2+cx^4}}$$

[Out] x^2/b/(c*x^4+b*x^2)^(1/2)

Rubi [A] time = 0.06, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$\frac{x^2}{b\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(b*x^2 + c*x^4)^(3/2),x]

[Out] x^2/(b*Sqrt[b*x^2 + c*x^4])

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x^2}{b\sqrt{bx^2+cx^4}}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{x^2}{b\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(b*x^2 + c*x^4)^(3/2),x]

[Out] x^2/(b*Sqrt[x^2*(b + c*x^2)])

fricas [A] time = 0.58, size = 26, normalized size = 1.18

$$\frac{\sqrt{cx^4 + bx^2}}{bcx^2 + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")

[Out] sqrt(c*x^4 + b*x^2)/(b*c*x^2 + b^2)

giac [A] time = 0.18, size = 35, normalized size = 1.59

$$\frac{1}{\left(\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2}\right)\sqrt{c} + b\right)\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")

[Out] 1/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) + b)*sqrt(c))

maple [A] time = 0.00, size = 28, normalized size = 1.27

$$\frac{\left(c\,x^2 + b\right)x^4}{\left(c\,x^4 + b\,x^2\right)^{\frac{3}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(c*x^4+b*x^2)^(3/2),x)

[Out] (c*x^2+b)/b*x^4/(c*x^4+b*x^2)^(3/2)

maxima [A] time = 1.45, size = 20, normalized size = 0.91

$$\frac{x^2}{\sqrt{cx^4 + bx^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")

[Out] x^2/(sqrt(c*x^4 + b*x^2)*b)

mupad [B] time = 4.13, size = 26, normalized size = 1.18

$$\frac{\sqrt{c\,x^4+b\,x^2}}{b\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] (b*x^2 + c*x^4)^(1/2)/(b*(b + c*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**3/(x**2*(b + c*x**2))**(3/2), x)
```

3.277 $\int \frac{x}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=28

$$-\frac{b+2cx^2}{b^2\sqrt{bx^2+cx^4}}$$

```
[Out] (-2*c*x^2-b)/b^2/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2013, 613}

$$-\frac{b+2cx^2}{b^2\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((b + 2*c*x^2)/(b^2*Sqrt[b*x^2 + c*x^4]))
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)\\ &= -\frac{b+2cx^2}{b^2\sqrt{bx^2+cx^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.04

$$\frac{-b-2cx^2}{b^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^2 + c*x^4)^(3/2),x]
[Out] (-b - 2*c*x^2)/(b^2*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.64, size = 41, normalized size = 1.46

$$-\frac{\sqrt{cx^4+bx^2}\left(2\,cx^2+b\right)}{b^2cx^4+b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
[Out] -sqrt(c*x^4 + b*x^2)*(2*c*x^2 + b)/(b^2*c*x^4 + b^3*x^2)
```

giac [A] time = 0.19, size = 28, normalized size = 1.00

$$-\frac{\frac{2\,cx^2}{b^2}+\frac{1}{b}}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] -(2*c*x^2/b^2 + 1/b)/sqrt(c*x^4 + b*x^2)
```

maple [A] time = 0.00, size = 37, normalized size = 1.32

$$-\frac{\left(c\,x^2+b\right)\left(2c\,x^2+b\right)x^2}{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2)^(3/2),x)
[Out] -x^2*(c*x^2+b)*(2*c*x^2+b)/b^2/(c*x^4+b*x^2)^(3/2)
```

maxima [A] time = 1.47, size = 41, normalized size = 1.46

$$-\frac{2\,cx^2}{\sqrt{cx^4+bx^2}\,b^2}-\frac{1}{\sqrt{cx^4+bx^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] -2*c*x^2/(sqrt(c*x^4 + b*x^2)*b^2) - 1/(sqrt(c*x^4 + b*x^2)*b)
```

mupad [B] time = 4.13, size = 26, normalized size = 0.93

$$-\frac{2\,c\,x^2+b}{b^2\,\sqrt{c\,x^4+b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2 + c*x^4)^(3/2),x)
[Out] -(b + 2*c*x^2)/(b^2*(b*x^2 + c*x^4)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(x/(x**2*(b + c*x**2))**(3/2), x)
```

3.278 $\int \frac{1}{x\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{8c\sqrt{bx^2+cx^4}}{3b^3x^2}-\frac{4\sqrt{bx^2+cx^4}}{3b^2x^4}+\frac{1}{bx^2\sqrt{bx^2+cx^4}}$$

```
[Out] 1/b/x^2/(c*x^4+b*x^2)^(1/2)-4/3*(c*x^4+b*x^2)^(1/2)/b^2/x^4+8/3*c*(c*x^4+b*
x^2)^(1/2)/b^3/x^2
```

Rubi [A] time = 0.13, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$\frac{8c\sqrt{bx^2+cx^4}}{3b^3x^2}-\frac{4\sqrt{bx^2+cx^4}}{3b^2x^4}+\frac{1}{bx^2\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^2*Sqrt[b*x^2 + c*x^4]) - (4*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^4) + (8*c*
Sqrt[b*x^2 + c*x^4])/(3*b^3*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
```

```
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{bx^2\sqrt{bx^2+cx^4}}+\frac{4\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx}{b}\\ &= \frac{1}{bx^2\sqrt{bx^2+cx^4}}-\frac{4\sqrt{bx^2+cx^4}}{3b^2x^4}-\frac{(8c)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{3b^2}\\ &= \frac{1}{bx^2\sqrt{bx^2+cx^4}}-\frac{4\sqrt{bx^2+cx^4}}{3b^2x^4}+\frac{8c\sqrt{bx^2+cx^4}}{3b^3x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 0.65

$$-\frac{\left(b+cx^2\right)\left(b^2-4bcx^2-8c^2x^4\right)}{3b^3\left(x^2\left(b+cx^2\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -1/3*((b + c*x^2)*(b^2 - 4*b*c*x^2 - 8*c^2*x^4))/(b^3*(x^2*(b + c*x^2))^(3/
2))
```

fricas [A] time = 0.54, size = 54, normalized size = 0.73

$$\frac{\left(8\,c^2x^4+4\,bcx^2-b^2\right)\sqrt{cx^4+bx^2}}{3\left(b^3cx^6+b^4x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(8*c^2*x^4 + 4*b*c*x^2 - b^2)*sqrt(c*x^4 + b*x^2)/(b^3*c*x^6 + b^4*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x), x)
```

maple [A] time = 0.01, size = 45, normalized size = 0.61

$$-\frac{\left(c\,x^{2}+b\right)\left(-8c^{2}x^{4}-4bc\,x^{2}+b^{2}\right)}{3\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/3*(c*x^2+b)*(-8*c^2*x^4-4*b*c*x^2+b^2)/b^3/(c*x^4+b*x^2)^(3/2)
```

maxima [A] time = 1.49, size = 65, normalized size = 0.88

$$\frac{8\,c^{2}x^{2}}{3\,\sqrt{cx^{4}+bx^{2}}\,b^{3}}+\frac{4\,c}{3\,\sqrt{cx^{4}+bx^{2}}\,b^{2}}-\frac{1}{3\,\sqrt{cx^{4}+bx^{2}}\,bx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*c^2*x^2/(sqrt(c*x^4 + b*x^2)*b^3) + 4/3*c/(sqrt(c*x^4 + b*x^2)*b^2) - 1
/3/(sqrt(c*x^4 + b*x^2)*b*x^2)
```

mupad [B] time = 4.24, size = 51, normalized size = 0.69

$$\frac{\sqrt{c\,x^{4}+b\,x^{2}}\,\left(-b^{2}+4\,b\,c\,x^{2}+8\,c^{2}\,x^{4}\right)}{3\,b^{3}\,x^{4}\,\left(c\,x^{2}+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*(8*c^2*x^4 - b^2 + 4*b*c*x^2))/(3*b^3*x^4*(b + c*x^2
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x*(x**2*(b + c*x**2))**(3/2)), x)
```

3.279 $\int \frac{1}{x^3\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=102

$$-\frac{16c^2\sqrt{bx^2+cx^4}}{5b^4x^2}+\frac{8c\sqrt{bx^2+cx^4}}{5b^3x^4}-\frac{6\sqrt{bx^2+cx^4}}{5b^2x^6}+\frac{1}{bx^4\sqrt{bx^2+cx^4}}$$

```
[Out] 1/b/x^4/(c*x^4+b*x^2)^(1/2)-6/5*(c*x^4+b*x^2)^(1/2)/b^2/x^6+8/5*c*(c*x^4+b*
x^2)^(1/2)/b^3/x^4-16/5*c^2*(c*x^4+b*x^2)^(1/2)/b^4/x^2
```

Rubi [A] time = 0.18, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$-\frac{16c^2\sqrt{bx^2+cx^4}}{5b^4x^2}+\frac{8c\sqrt{bx^2+cx^4}}{5b^3x^4}-\frac{6\sqrt{bx^2+cx^4}}{5b^2x^6}+\frac{1}{bx^4\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^4*Sqrt[b*x^2 + c*x^4]) - (6*Sqrt[b*x^2 + c*x^4])/(5*b^2*x^6) + (8*c*
Sqrt[b*x^2 + c*x^4])/(5*b^3*x^4) - (16*c^2*Sqrt[b*x^2 + c*x^4])/(5*b^4*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
```

```
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{bx^4\sqrt{bx^2+cx^4}}+\frac{6\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx}{b}\\
&= \frac{1}{bx^4\sqrt{bx^2+cx^4}}-\frac{6\sqrt{bx^2+cx^4}}{5b^2x^6}-\frac{(24c)\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx}{5b^2}\\
&= \frac{1}{bx^4\sqrt{bx^2+cx^4}}-\frac{6\sqrt{bx^2+cx^4}}{5b^2x^6}+\frac{8c\sqrt{bx^2+cx^4}}{5b^3x^4}+\frac{\left(16c^2\right)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{5b^3}\\
&= \frac{1}{bx^4\sqrt{bx^2+cx^4}}-\frac{6\sqrt{bx^2+cx^4}}{5b^2x^6}+\frac{8c\sqrt{bx^2+cx^4}}{5b^3x^4}-\frac{16c^2\sqrt{bx^2+cx^4}}{5b^4x^2}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 57, normalized size = 0.56

$$
\frac{-b^3+2b^2cx^2-8bc^2x^4-16c^3x^6}{5b^4x^4\sqrt{x^2\left(b+cx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-b^3 + 2*b^2*c*x^2 - 8*b*c^2*x^4 - 16*c^3*x^6)/(5*b^4*x^4*Sqrt[x^2*(b + c*
x^2)])
```

fricas [A] time = 0.65, size = 63, normalized size = 0.62

$$
-\frac{\left(16\,c^3x^6+8\,bc^2x^4-2\,b^2cx^2+b^3\right)\sqrt{cx^4+bx^2}}{5\left(b^4cx^8+b^5x^6\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(16*c^3*x^6 + 8*b*c^2*x^4 - 2*b^2*c*x^2 + b^3)*sqrt(c*x^4 + b*x^2)/(b^
4*c*x^8 + b^5*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4 + bx^2\right)^{\frac{3}{2}} x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^3), x)
```

maple [A] time = 0.00, size = 59, normalized size = 0.58

$$-\frac{\left(c\,x^2 + b\right)\left(16c^3x^6 + 8b\,c^2x^4 - 2b^2c\,x^2 + b^3\right)}{5\left(c\,x^4 + b\,x^2\right)^{\frac{3}{2}} b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/5*(c*x^2+b)*(16*c^3*x^6+8*b*c^2*x^4-2*b^2*c*x^2+b^3)/x^2/b^4/(c*x^4+b*x^
2)^(3/2)
```

maxima [A] time = 1.51, size = 89, normalized size = 0.87

$$-\frac{16\,c^3x^2}{5\,\sqrt{cx^4 + bx^2}\,b^4} - \frac{8\,c^2}{5\,\sqrt{cx^4 + bx^2}\,b^3} + \frac{2\,c}{5\,\sqrt{cx^4 + bx^2}\,b^2x^2} - \frac{1}{5\,\sqrt{cx^4 + bx^2}\,bx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -16/5*c^3*x^2/(sqrt(c*x^4 + b*x^2)*b^4) - 8/5*c^2/(sqrt(c*x^4 + b*x^2)*b^3)
 + 2/5*c/(sqrt(c*x^4 + b*x^2)*b^2*x^2) - 1/5/(sqrt(c*x^4 + b*x^2)*b*x^4)
```

mupad [B] time = 4.31, size = 60, normalized size = 0.59

$$-\frac{\sqrt{c\,x^4 + b\,x^2}\,\left(b^3 - 2\,b^2\,c\,x^2 + 8\,b\,c^2\,x^4 + 16\,c^3\,x^6\right)}{5\,b^4\,x^6\,\left(c\,x^2 + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*(b^3 + 16*c^3*x^6 - 2*b^2*c*x^2 + 8*b*c^2*x^4))/(5*
b^4*x^6*(b + c*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(x**2*(b + c*x**2))**(3/2)), x)
```

3.280 $\int \frac{1}{x^5\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=130

$$\frac{128c^3\sqrt{bx^2+cx^4}}{35b^5x^2}-\frac{64c^2\sqrt{bx^2+cx^4}}{35b^4x^4}+\frac{48c\sqrt{bx^2+cx^4}}{35b^3x^6}-\frac{8\sqrt{bx^2+cx^4}}{7b^2x^8}+\frac{1}{bx^6\sqrt{bx^2+cx^4}}$$

```
[Out] 1/b/x^6/(c*x^4+b*x^2)^(1/2)-8/7*(c*x^4+b*x^2)^(1/2)/b^2/x^8+48/35*c*(c*x^4+
b*x^2)^(1/2)/b^3/x^6-64/35*c^2*(c*x^4+b*x^2)^(1/2)/b^4/x^4+128/35*c^3*(c*x^
4+b*x^2)^(1/2)/b^5/x^2
```

Rubi [A] time = 0.23, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$\frac{128c^3\sqrt{bx^2+cx^4}}{35b^5x^2}-\frac{64c^2\sqrt{bx^2+cx^4}}{35b^4x^4}+\frac{48c\sqrt{bx^2+cx^4}}{35b^3x^6}-\frac{8\sqrt{bx^2+cx^4}}{7b^2x^8}+\frac{1}{bx^6\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^6*Sqrt[b*x^2 + c*x^4]) - (8*Sqrt[b*x^2 + c*x^4])/(7*b^2*x^8) + (48*c
*Sqrt[b*x^2 + c*x^4])/(35*b^3*x^6) - (64*c^2*Sqrt[b*x^2 + c*x^4])/(35*b^4*x
^4) + (128*c^3*Sqrt[b*x^2 + c*x^4])/(35*b^5*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{bx^6\sqrt{bx^2+cx^4}}+\frac{8\int \frac{1}{x^7\sqrt{bx^2+cx^4}}\,dx}{b}\\
&= \frac{1}{bx^6\sqrt{bx^2+cx^4}}-\frac{8\sqrt{bx^2+cx^4}}{7b^2x^8}-\frac{(48c)\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx}{7b^2}\\
&= \frac{1}{bx^6\sqrt{bx^2+cx^4}}-\frac{8\sqrt{bx^2+cx^4}}{7b^2x^8}+\frac{48c\sqrt{bx^2+cx^4}}{35b^3x^6}+\frac{\left(192c^2\right)\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx}{35b^3}\\
&= \frac{1}{bx^6\sqrt{bx^2+cx^4}}-\frac{8\sqrt{bx^2+cx^4}}{7b^2x^8}+\frac{48c\sqrt{bx^2+cx^4}}{35b^3x^6}-\frac{64c^2\sqrt{bx^2+cx^4}}{35b^4x^4}-\frac{\left(128c^3\right)\int \frac{}{x\sqrt{bx}}}{35b^4}\\
&= \frac{1}{bx^6\sqrt{bx^2+cx^4}}-\frac{8\sqrt{bx^2+cx^4}}{7b^2x^8}+\frac{48c\sqrt{bx^2+cx^4}}{35b^3x^6}-\frac{64c^2\sqrt{bx^2+cx^4}}{35b^4x^4}+\frac{128c^3\sqrt{bx^2+}}{35b^5x^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 68, normalized size = 0.52

$$\frac{-5b^4+8b^3cx^2-16b^2c^2x^4+64bc^3x^6+128c^4x^8}{35b^5x^6\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-5*b^4 + 8*b^3*c*x^2 - 16*b^2*c^2*x^4 + 64*b*c^3*x^6 + 128*c^4*x^8)/(35*b^
5*x^6*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.66, size = 76, normalized size = 0.58

$$\frac{\left(128\,c^4x^8+64\,bc^3x^6-16\,b^2c^2x^4+8\,b^3cx^2-5\,b^4\right)\sqrt{cx^4+bx^2}}{35\left(b^5cx^{10}+b^6x^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35*(128*c^4*x^8 + 64*b*c^3*x^6 - 16*b^2*c^2*x^4 + 8*b^3*c*x^2 - 5*b^4)*sq
rt(c*x^4 + b*x^2)/(b^5*c*x^10 + b^6*x^8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4 + bx^2\right)^{\frac{3}{2}} x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^5), x)
```

maple [A] time = 0.01, size = 72, normalized size = 0.55

$$-\frac{\left(c\,x^2 + b\right)\left(-128c^4x^8 - 64c^3x^6b + 16c^2x^4b^2 - 8c\,x^2b^3 + 5b^4\right)}{35\left(c\,x^4 + b\,x^2\right)^{\frac{3}{2}} b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/35*(c*x^2+b)*(-128*c^4*x^8-64*b*c^3*x^6+16*b^2*c^2*x^4-8*b^3*c*x^2+5*b^4
)/x^4/b^5/(c*x^4+b*x^2)^(3/2)
```

maxima [A] time = 1.47, size = 113, normalized size = 0.87

$$\frac{128\,c^4x^2}{35\,\sqrt{cx^4 + bx^2}\,b^5} + \frac{64\,c^3}{35\,\sqrt{cx^4 + bx^2}\,b^4} - \frac{16\,c^2}{35\,\sqrt{cx^4 + bx^2}\,b^3x^2} + \frac{8\,c}{35\,\sqrt{cx^4 + bx^2}\,b^2x^4} - \frac{1}{7\,\sqrt{cx^4 + bx^2}\,bx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 128/35*c^4*x^2/(sqrt(c*x^4 + b*x^2)*b^5) + 64/35*c^3/(sqrt(c*x^4 + b*x^2)*b
^4) - 16/35*c^2/(sqrt(c*x^4 + b*x^2)*b^3*x^2) + 8/35*c/(sqrt(c*x^4 + b*x^2)
*b^2*x^4) - 1/7/(sqrt(c*x^4 + b*x^2)*b*x^6)
```

mupad [B] time = 4.41, size = 114, normalized size = 0.88

$$\frac{13\,c\,\sqrt{c\,x^4 + b\,x^2}}{35\,b^3\,x^6} - \frac{\sqrt{c\,x^4 + b\,x^2}}{7\,b^2\,x^8} - \frac{29\,c^2\,\sqrt{c\,x^4 + b\,x^2}}{35\,b^4\,x^4} + \frac{\sqrt{c\,x^4 + b\,x^2}\,\left(\frac{93\,c^3}{35\,b^4} + \frac{128\,c^4\,x^2}{35\,b^5}\right)}{x^2\,\left(c\,x^2 + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] (13*c*(b*x^2 + c*x^4)^(1/2))/(35*b^3*x^6) - (b*x^2 + c*x^4)^(1/2)/(7*b^2*x^
8) - (29*c^2*(b*x^2 + c*x^4)^(1/2))/(35*b^4*x^4) + ((b*x^2 + c*x^4)^(1/2)*(
(93*c^3)/(35*b^4) + (128*c^4*x^2)/(35*b^5)))/(x^2*(b + c*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**5*(x**2*(b + c*x**2))**(3/2)), x)
```

3.281 $\int \frac{x^6}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=47

$$\frac{2\sqrt{bx^2+cx^4}}{c^2x}-\frac{x^3}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -x^3/c/(c*x^4+b*x^2)^(1/2)+2*(c*x^4+b*x^2)^(1/2)/c^2/x
```

Rubi [A] time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 1588}

$$\frac{2\sqrt{bx^2+cx^4}}{c^2x}-\frac{x^3}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^3/(c*Sqrt[b*x^2 + c*x^4])) + (2*Sqrt[b*x^2 + c*x^4])/(c^2*x)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^6}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{x^3}{c\sqrt{bx^2+cx^4}} + \frac{2\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{c}$$
$$= -\frac{x^3}{c\sqrt{bx^2+cx^4}} + \frac{2\sqrt{bx^2+cx^4}}{c^2x}$$

Mathematica [**A**] time = 0.01, size = 29, normalized size = 0.62

$$\frac{x\left(2b+cx^2\right)}{c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(b*x^2 + c*x^4)^(3/2),x]
[Out] (x*(2*b + c*x^2))/(c^2*Sqrt[x^2*(b + c*x^2)])
```

fricas [**A**] time = 0.63, size = 39, normalized size = 0.83

$$\frac{\sqrt{cx^4+bx^2}\left(cx^2+2\,b\right)}{c^3x^3+bc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
[Out] sqrt(c*x^4 + b*x^2)*(c*x^2 + 2*b)/(c^3*x^3 + b*c^2*x)
```

giac [**A**] time = 0.21, size = 52, normalized size = 1.11

$$-\frac{2\sqrt{b}}{\left(\left(\sqrt{c+\frac{b}{x^2}}-\frac{\sqrt{b}}{x}\right)^2-c\right)c} + \frac{b}{\sqrt{c+\frac{b}{x^2}}\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] -2*sqrt(b)/(((sqrt(c + b/x^2) - sqrt(b)/x)^2 - c)*c) + b/(sqrt(c + b/x^2)*c
^2*x)
```

maple [A] time = 0.00, size = 37, normalized size = 0.79

$$\frac{\left(c\,x^{2}+b\right)\left(c\,x^{2}+2b\right)x^{3}}{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2)^(3/2),x)
[Out] (c*x^2+b)*(c*x^2+2*b)*x^3/c^2/(c*x^4+b*x^2)^(3/2)
```

maxima [A] time = 1.48, size = 22, normalized size = 0.47

$$\frac{cx^{2}+2\,b}{\sqrt{cx^{2}+b}\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] (c*x^2 + 2*b)/(sqrt(c*x^2 + b)*c^2)
```

mupad [B] time = 4.23, size = 38, normalized size = 0.81

$$\frac{\sqrt{c\,x^{4}+b\,x^{2}}\,\left(c\,x^{2}+2\,b\right)}{c^{2}\,x\,\left(c\,x^{2}+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2 + c*x^4)^(3/2),x)
[Out] ((b*x^2 + c*x^4)^(1/2)*(2*b + c*x^2))/(c^2*x*(b + c*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{6}}{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(x**6/(x**2*(b + c*x**2))**(3/2), x)
```

3.282 $\int \frac{x^4}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=21

$$-\frac{x}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -x/c/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1588}

$$-\frac{x}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x/(c*Sqrt[b*x^2 + c*x^4]))
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^4}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{x}{c\sqrt{bx^2+cx^4}}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$-\frac{x}{c\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x/(c*Sqrt[x^2*(b + c*x^2)]))
```

fricas [A] time = 0.67, size = 29, normalized size = 1.38

$$-\frac{\sqrt{cx^4 + bx^2}}{c^2x^3 + bcx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
[Out] -sqrt(c*x^4 + b*x^2)/(c^2*x^3 + b*c*x)
```

giac [A] time = 0.21, size = 17, normalized size = 0.81

$$-\frac{1}{\sqrt{c + \frac{b}{x^2}}\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] -1/(sqrt(c + b/x^2)*c*x)
```

maple [A] time = 0.00, size = 29, normalized size = 1.38

$$-\frac{\left(c\,x^2 + b\right)x^3}{\left(c\,x^4 + b\,x^2\right)^{\frac{3}{2}}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2)^(3/2),x)
[Out] -(c*x^2+b)/c*x^3/(c*x^4+b*x^2)^(3/2)
```

maxima [A] time = 1.44, size = 14, normalized size = 0.67

$$-\frac{1}{\sqrt{cx^2 + b}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/(sqrt(c*x^2 + b)*c)
```

mupad [B] time = 4.15, size = 30, normalized size = 1.43

$$-\frac{\sqrt{c\,x^4+b\,x^2}}{c\,x\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2 + c*x^4)^(3/2),x)
[Out] -(b*x^2 + c*x^4)^(1/2)/(c*x*(b + c*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(x**4/(x**2*(b + c*x**2))**(3/2), x)
```

3.283 $\int \frac{x^2}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{x}{b\sqrt{bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}$$

```
[Out] -arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(3/2)+x/b/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2023, 2008, 206}

$$\frac{x}{b\sqrt{bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] x/(b*Sqrt[b*x^2 + c*x^4]) - ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/b^(3/2
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rubi steps

$$\int \frac{x^2}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{x}{b\sqrt{bx^2+cx^4}} + \frac{\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{b}$$
$$= \frac{x}{b\sqrt{bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)}{b}$$
$$= \frac{x}{b\sqrt{bx^2+cx^4}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.75

$$\frac{x\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{cx^2}{b}+1\right)}{b\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (c*x^2)/b])/(b*Sqrt[x^2*(b + c*x^2)]
)
```

fricas [A] time = 0.59, size = 162, normalized size = 3.18

$$\left[\frac{\left(cx^3+bx\right)\sqrt{b}\,\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\,b}{2\left(b^2cx^3+b^3x\right)}, \frac{\left(cx^3+bx\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx}}{b^2cx^3+b^3x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*x^3 + b*x)*sqrt(b)*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqr
t(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*b)/(b^2*c*x^3 + b^3*x), ((c*x^3 + b*x)*s
qrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b
*x^2)*b)/(b^2*c*x^3 + b^3*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,
2,4]%%%}] at parameters values [64.3995612673,65,-85]Warning, choosing root
 of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,2,4]%%%}] at para
meters values [66.1769613782,93,91]1/b/x*sqrt(b*(1/x)^2+c)/(b*(1/x)^2+c)+1/
b/sqrt(b)*ln(abs(sqrt(b*(1/x)^2+c)-sqrt(b)/x))
```

maple [A] time = 0.01, size = 65, normalized size = 1.27

$$\frac{\left(c\,x^{2}+b\right)\left(-\sqrt{c\,x^{2}+b}\;b\ln\left(\frac{2b+2\sqrt{c\,x^{2}+b}\;\sqrt{b}}{x}\right)+b^{\frac{3}{2}}\right)x^{3}}{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] x^3*(c*x^2+b)*(b^(3/2)-ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*b*(c*x^2+b)^(1/2
))/(c*x^4+b*x^2)^(3/2)/b^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^4 + b*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{2}}{\left(c\,x^{4}+b\,x^{2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^2/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**2/(x**2*(b + c*x**2))**(3/2), x)
```

3.284 $\int \frac{1}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=81

$$\frac{3c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}-\frac{3\sqrt{bx^2+cx^4}}{2b^2x^3}+\frac{1}{bx\sqrt{bx^2+cx^4}}$$

```
[Out] 3/2*c*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(5/2)+1/b/x/(c*x^4+b*x^2)^(1
/2)-3/2*(c*x^4+b*x^2)^(1/2)/b^2/x^3
```

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2006, 2025, 2008, 206}

$$-\frac{3\sqrt{bx^2+cx^4}}{2b^2x^3}+\frac{3c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}+\frac{1}{bx\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(-3/2),x]
```

```
[Out] 1/(b*x*Sqrt[b*x^2 + c*x^4]) - (3*Sqrt[b*x^2 + c*x^4])/(2*b^2*x^3) + (3*c*Ar
cTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2006

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{bx\sqrt{bx^2+cx^4}}+\frac{3\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{b}\\
&= \frac{1}{bx\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{2b^2x^3}-\frac{(3c)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{2b^2}\\
&= \frac{1}{bx\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{2b^2x^3}+\frac{(3c)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{2b^2}\\
&= \frac{1}{bx\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{2b^2x^3}+\frac{3c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.49

$$-\frac{cx\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{cx^2}{b}+1\right)}{b^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(-3/2),x]
```

```
[Out] -((c*x*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (c*x^2)/b])/(b^2*Sqrt[x^2*(b + c
*x^2)]))
```

fricas [A] time = 0.63, size = 199, normalized size = 2.46

$$\left[\frac{3\left(c^2x^5+bcx^3\right)\sqrt{b}\,\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)-2\,\sqrt{cx^4+bx^2}\left(3\,bcx^2+b^2\right)}{4\left(b^3cx^5+b^4x^3\right)},-\frac{3\left(c^2x^5+bcx^3\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{cx^4}}{}\right.}{2\left(b^3c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(c^2*x^5 + b*c*x^3)*sqrt(b)*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*
x^2)*sqrt(b))/x^3) - 2*sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 + b^2))/(b^3*c*x^5 +
b^4*x^3), -1/2*(3*(c^2*x^5 + b*c*x^3)*sqrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*s
qrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 + b^2))/(b^3*c*x^5
+ b^4*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 77, normalized size = 0.95

$$-\frac{\left(c\,x^{2}+b\right)\left(-3\sqrt{c\,x^{2}+b}\;b c\,x^{2}\ln\left(\frac{2b+2\sqrt{c\,x^{2}+b}\;\sqrt{b}}{x}\right)+3b^{\frac{3}{2}}c\,x^{2}+b^{\frac{5}{2}}\right)x}{2\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/2*x*(c*x^2+b)*(3*b^(3/2)*x^2*c-3*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*(c*
x^2+b)^(1/2)*x^2*b*c+b^(5/2))/(c*x^4+b*x^2)^(3/2)/b^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(-3/2), x)
```

mupad [B] time = 4.34, size = 42, normalized size = 0.52

$$-\frac{x\left(\frac{b}{c\,x^{2}}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{b}{c\,x^{2}}\right)}{5\left(c\,x^{4}+b\,x^{2}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] -(x*(b/(c*x^2) + 1)^(3/2)*hypergeom([3/2, 5/2], 7/2, -b/(c*x^2)))/(5*(b*x^2
 + c*x^4)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2 + cx^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((b*x**2 + c*x**4)**(-3/2), x)
```

3.285 $\int \frac{1}{x^2(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=109

$$-\frac{15c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}+\frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}+\frac{1}{bx^3\sqrt{bx^2+cx^4}}$$

```
[Out] -15/8*c^2*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(7/2)+1/b/x^3/(c*x^4+b*x
^2)^(1/2)-5/4*(c*x^4+b*x^2)^(1/2)/b^2/x^5+15/8*c*(c*x^4+b*x^2)^(1/2)/b^3/x^
3
```

Rubi [A] time = 0.15, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2023, 2025, 2008, 206}

$$-\frac{15c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}+\frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}+\frac{1}{bx^3\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^3*Sqrt[b*x^2 + c*x^4]) - (5*Sqrt[b*x^2 + c*x^4])/(4*b^2*x^5) + (15*c
*Sqrt[b*x^2 + c*x^4])/(8*b^3*x^3) - (15*c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2
+ c*x^4]])/(8*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
```

```
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{bx^3\sqrt{bx^2+cx^4}}+\frac{5\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx}{b}\\
&= \frac{1}{bx^3\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}-\frac{(15c)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{4b^2}\\
&= \frac{1}{bx^3\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}+\frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3}+\frac{\left(15c^2\right)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b^3}\\
&= \frac{1}{bx^3\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}+\frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3}-\frac{\left(15c^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{8b^3}\\
&= \frac{1}{bx^3\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}+\frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3}-\frac{15c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.38

$$\frac{c^2x\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{cx^2}{b}+1\right)}{b^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*x^2 + c*x^4)^(3/2)),x]
```

[Out] (c^2*x*Hypergeometric2F1[-1/2, 3, 1/2, 1 + (c*x^2)/b])/(b^3*Sqrt[x^2*(b + c*x^2)])

fricas [A] time = 0.58, size = 229, normalized size = 2.10

$$\left[\frac{15\left(c^3x^7+bc^2x^5\right)\sqrt{b}\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\left(15\,bc^2x^4+5\,b^2cx^2-2\,b^3\right)\sqrt{cx^4+bx^2}}{16\left(b^4cx^7+b^5x^5\right)}, \frac{15\left(c^3x^7+bc^2x^5\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")

[Out] [1/16*(15*(c^3*x^7 + b*c^2*x^5)*sqrt(b)*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3) + 2*(15*b*c^2*x^4 + 5*b^2*c*x^2 - 2*b^3)*sqrt(c*x^4 + b*x^2))/(b^4*c*x^7 + b^5*x^5), 1/8*(15*(c^3*x^7 + b*c^2*x^5)*sqrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (15*b*c^2*x^4 + 5*b^2*c*x^2 - 2*b^3)*sqrt(c*x^4 + b*x^2))/(b^4*c*x^7 + b^5*x^5)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")

[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^2), x)

maple [A] time = 0.01, size = 94, normalized size = 0.86

$$-\frac{\left(c\,x^2+b\right)\left(15\sqrt{c\,x^2+b}\;b\,c^2x^4\ln\left(\frac{2b+2\sqrt{c\,x^2+b}\;\sqrt{b}}{x}\right)-15b^{\frac{3}{2}}c^2x^4-5b^{\frac{5}{2}}c\,x^2+2b^{\frac{7}{2}}\right)}{8\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}b^{\frac{9}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(c*x^4+b*x^2)^(3/2),x)

[Out] -1/8/x*(c*x^2+b)*(15*(c*x^2+b)^(1/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^4*b*c^2-15*b^(3/2)*x^4*c^2-5*b^(5/2)*x^2*c+2*b^(7/2))/(c*x^4+b*x^2)^(3/2)/b^(9/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^2), x)
```

mupad [B] time = 4.64, size = 44, normalized size = 0.40

$$-\frac{\left(\frac{b}{c\,x^2}+1\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{b}{c\,x^2}\right)}{7\,x\,\left(c\,x^4+b\,x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] -((b/(c*x^2) + 1)^(3/2)*hypergeom([3/2, 7/2], 9/2, -b/(c*x^2)))/(7*x*(b*x^2
 + c*x^4)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x**2*(b + c*x**2))**(3/2)), x)
```

3.286 $\int \frac{x^3}{\sqrt{3x^2-4x^4}}\,dx$

Optimal. Leaf size=34

$$-\frac{3}{32}\sin^{-1}\left(1-\frac{8x^2}{3}\right)-\frac{1}{8}\sqrt{3x^2-4x^4}$$

```
[Out] 3/32*arcsin(-1+8/3*x^2)-1/8*(-4*x^4+3*x^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2018, 640, 619, 216}

$$-\frac{1}{8}\sqrt{3x^2-4x^4}-\frac{3}{32}\sin^{-1}\left(1-\frac{8x^2}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[3*x^2 - 4*x^4],x]
```

```
[Out] -Sqrt[3*x^2 - 4*x^4]/8 - (3*ArcSin[1 - (8*x^2)/3])/32
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
```

```
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{3x^2-4x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{3x-4x^2}}\,dx,x,x^2\right)\\ &= -\frac{1}{8}\sqrt{3x^2-4x^4}+\frac{3}{16}\text{Subst}\left(\int \frac{1}{\sqrt{3x-4x^2}}\,dx,x,x^2\right)\\ &= -\frac{1}{8}\sqrt{3x^2-4x^4}-\frac{1}{32}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx,x,3-8x^2\right)\\ &= -\frac{1}{8}\sqrt{3x^2-4x^4}-\frac{3}{32}\sin^{-1}\left(1-\frac{8x^2}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 1.68

$$\frac{x\left(8x^3+3\sqrt{4x^2-3}\,\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-3}}\right)-6x\right)}{16\sqrt{3x^2-4x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[3*x^2 - 4*x^4],x]
[Out] (x*(-6*x + 8*x^3 + 3*Sqrt[-3 + 4*x^2]*ArcTanh[(2*x)/Sqrt[-3 + 4*x^2]]))/(16
*Sqrt[3*x^2 - 4*x^4])
```

fricas [A] time = 0.51, size = 37, normalized size = 1.09

$$-\frac{1}{8}\sqrt{-4\,x^4+3\,x^2}-\frac{3}{16}\arctan\left(\frac{\sqrt{-4\,x^4+3\,x^2}}{2\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4+3*x^2)^(1/2),x, algorithm="fricas")
[Out] -1/8*sqrt(-4*x^4 + 3*x^2) - 3/16*arctan(1/2*sqrt(-4*x^4 + 3*x^2)/x^2)
```

giac [A] time = 0.18, size = 26, normalized size = 0.76

$$-\frac{1}{8}\sqrt{-4\,x^4+3\,x^2}+\frac{3}{32}\arcsin\left(\frac{8}{3}\,x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4+3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-4*x^4 + 3*x^2) + 3/32*arcsin(8/3*x^2 - 1)
```

maple [A] time = 0.01, size = 48, normalized size = 1.41

$$\frac{\sqrt{-4x^2+3}\,\left(-2\sqrt{-4x^2+3}\,x+3\arcsin\left(\frac{2\sqrt{3}\,x}{3}\right)\right)x}{16\sqrt{-4x^4+3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-4*x^4+3*x^2)^(1/2),x)
```

```
[Out] 1/16*x*(-4*x^2+3)^(1/2)*(-2*x*(-4*x^2+3)^(1/2)+3*arcsin(2/3*3^(1/2)*x))/(-4
*x^4+3*x^2)^(1/2)
```

maxima [A] time = 3.03, size = 26, normalized size = 0.76

$$-\frac{1}{8}\sqrt{-4\,x^4+3\,x^2}-\frac{3}{32}\arcsin\left(-\frac{8}{3}\,x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4+3*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(-4*x^4 + 3*x^2) - 3/32*arcsin(-8/3*x^2 + 1)
```

mupad [B] time = 4.33, size = 42, normalized size = 1.24

$$-\frac{\sqrt{3\,x^2-4\,x^4}}{8}-\frac{\ln\left(x^2-\frac{3}{8}-\frac{\sqrt{3-4\,x^2}\,\sqrt{x^2}\,1\mathrm{i}}{2}\right)\,3\mathrm{i}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^2 - 4*x^4)^(1/2),x)
```

```
[Out] - (log(x^2 - ((3 - 4*x^2)^(1/2)*(x^2)^(1/2)*1i)/2 - 3/8)*3i)/32 - (3*x^2 -
4*x^4)^(1/2)/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{-x^2\left(4x^2-3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-4*x**4+3*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(-x**2*(4*x**2 - 3)), x)
```

3.287 $\int \frac{x^3}{\sqrt{-3x^2-4x^4}}\,dx$

Optimal. Leaf size=34

$$-\frac{3}{32}\sin^{-1}\left(\frac{8x^2}{3}+1\right)-\frac{1}{8}\sqrt{-4x^4-3x^2}$$

```
[Out] -3/32*arcsin(1+8/3*x^2)-1/8*(-4*x^4-3*x^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2018, 640, 619, 216}

$$-\frac{1}{8}\sqrt{-4x^4-3x^2}-\frac{3}{32}\sin^{-1}\left(\frac{8x^2}{3}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[-3*x^2 - 4*x^4],x]
```

```
[Out] -Sqrt[-3*x^2 - 4*x^4]/8 - (3*ArcSin[1 + (8*x^2)/3])/32
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
```

```
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{-3x^2-4x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{-3x-4x^2}}\,dx,x,x^2\right)\\
&= -\frac{1}{8}\sqrt{-3x^2-4x^4}-\frac{3}{16}\text{Subst}\left(\int \frac{1}{\sqrt{-3x-4x^2}}\,dx,x,x^2\right)\\
&= -\frac{1}{8}\sqrt{-3x^2-4x^4}+\frac{1}{32}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx,x,-3-8x^2\right)\\
&= -\frac{1}{8}\sqrt{-3x^2-4x^4}-\frac{3}{32}\sin^{-1}\left(1+\frac{8x^2}{3}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 1.53

$$\frac{x\left(8x^3-3\sqrt{4x^2+3}\,\sinh^{-1}\left(\frac{2x}{\sqrt{3}}\right)+6x\right)}{16\sqrt{-x^2\left(4x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[-3*x^2 - 4*x^4],x]
```

```
[Out] (x*(6*x + 8*x^3 - 3*Sqrt[3 + 4*x^2]*ArcSinh[(2*x)/Sqrt[3]]))/(16*Sqrt[-(x^2
*(3 + 4*x^2))])
```

fricas [C] time = 0.65, size = 59, normalized size = 1.74

$$-\frac{1}{8}\sqrt{-4x^2-3}\,x-\frac{3}{32}i\log\left(-\frac{8x+4i\sqrt{-4x^2-3}}{x}\right)+\frac{3}{32}i\log\left(-\frac{8x-4i\sqrt{-4x^2-3}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4-3*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-4*x^2 - 3)*x - 3/32*I*log(-(8*x + 4*I*sqrt(-4*x^2 - 3))/x) + 3/3
2*I*log(-(8*x - 4*I*sqrt(-4*x^2 - 3))/x)
```

giac [A] time = 0.21, size = 27, normalized size = 0.79

$$-\frac{1}{8}\sqrt{4x^4+3x^2}\,i-\frac{3}{32}\arcsin\left(\frac{8}{3}x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4-3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(4*x^4 + 3*x^2)*i - 3/32*arcsin(8/3*x^2 + 1)
```

maple [B] time = 0.01, size = 54, normalized size = 1.59

$$-\frac{\sqrt{-4x^2-3}\,\left(2\sqrt{-4x^2-3}\,x+3\arctan\left(\frac{2x}{\sqrt{-4x^2-3}}\right)\right)x}{16\sqrt{-4x^4-3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-4*x^4-3*x^2)^(1/2),x)
```

```
[Out] -1/16*x*(-4*x^2-3)^(1/2)*(2*x*(-4*x^2-3)^(1/2)+3*arctan(2*x/(-4*x^2-3)^(1/2
)))/(-4*x^4-3*x^2)^(1/2)
```

maxima [A] time = 2.98, size = 26, normalized size = 0.76

$$-\frac{1}{8}\sqrt{-4\,x^4-3\,x^2}+\frac{3}{32}\arcsin\left(-\frac{8}{3}\,x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4-3*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(-4*x^4 - 3*x^2) + 3/32*arcsin(-8/3*x^2 - 1)
```

mupad [B] time = 4.36, size = 41, normalized size = 1.21

$$-\frac{\sqrt{-4\,x^4-3\,x^2}}{8}+\frac{\ln\left(\frac{\sqrt{4\,x^2+3}\,\sqrt{x^2}}{2}+x^2+\frac{3}{8}\right)\,3\mathrm{i}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(- 3*x^2 - 4*x^4)^(1/2),x)
```

```
[Out] (log(((4*x^2 + 3)^(1/2)*(x^2)^(1/2))/2 + x^2 + 3/8)*3i)/32 - (- 3*x^2 - 4*x
^4)^(1/2)/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{-x^2\left(4x^2+3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-4*x**4-3*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(-x**2*(4*x**2 + 3)), x)
```

3.288 $\int \frac{x^3}{\sqrt{3x^2+4x^4}} \, dx$

Optimal. Leaf size=45

$$\frac{1}{8}\sqrt{4x^4+3x^2} - \frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{4x^4+3x^2}}\right)$$

```
[Out] -3/16*arctanh(2*x^2/(4*x^4+3*x^2)^(1/2))+1/8*(4*x^4+3*x^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2018, 640, 620, 206}

$$\frac{1}{8}\sqrt{4x^4+3x^2} - \frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{4x^4+3x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[3*x^2 + 4*x^4],x]
```

```
[Out] Sqrt[3*x^2 + 4*x^4]/8 - (3*ArcTanh[(2*x^2)/Sqrt[3*x^2 + 4*x^4]])/16
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
```

```
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\sqrt{3x^2+4x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{3x+4x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{8}\sqrt{3x^2+4x^4} - \frac{3}{16}\operatorname{Subst}\left(\int \frac{1}{\sqrt{3x+4x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{8}\sqrt{3x^2+4x^4} - \frac{3}{8}\operatorname{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \frac{x^2}{\sqrt{3x^2+4x^4}}\right)\\
&= \frac{1}{8}\sqrt{3x^2+4x^4} - \frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{3x^2+4x^4}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.13

$$\frac{x\left(8x^3-3\sqrt{4x^2+3}\,\sinh^{-1}\left(\frac{2x}{\sqrt{3}}\right)+6x\right)}{16\sqrt{x^2\left(4x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[3*x^2 + 4*x^4],x]
[Out] (x*(6*x + 8*x^3 - 3*Sqrt[3 + 4*x^2]*ArcSinh[(2*x)/Sqrt[3]]))/(16*Sqrt[x^2*(
3 + 4*x^2)])
```

fricas [A] time = 0.61, size = 45, normalized size = 1.00

$$\frac{1}{8}\sqrt{4\,x^4+3\,x^2}+\frac{3}{16}\log\left(-\frac{2\,x^2-\sqrt{4\,x^4+3\,x^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^4+3*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/8*sqrt(4*x^4 + 3*x^2) + 3/16*log(-(2*x^2 - sqrt(4*x^4 + 3*x^2))/x)
```

giac [A] time = 0.17, size = 41, normalized size = 0.91

$$\frac{1}{8}\sqrt{4\,x^4+3\,x^2}+\frac{3}{32}\log\left(8\,x^2-4\sqrt{4\,x^4+3\,x^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^4+3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(4*x^4 + 3*x^2) + 3/32*log(8*x^2 - 4*sqrt(4*x^4 + 3*x^2) + 3)
```

maple [A] time = 0.01, size = 48, normalized size = 1.07

$$-\frac{\sqrt{4x^2+3}\,\left(-2\sqrt{4x^2+3}\,x+3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,x}{3}\right)\right)x}{16\sqrt{4x^4+3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^4+3*x^2)^(1/2),x)
```

```
[Out] -1/16*x*(4*x^2+3)^(1/2)*(-2*x*(4*x^2+3)^(1/2)+3*arcsinh(2/3*3^(1/2)*x))/(4*
x^4+3*x^2)^(1/2)
```

maxima [A] time = 3.05, size = 41, normalized size = 0.91

$$\frac{1}{8}\sqrt{4\,x^4+3\,x^2}-\frac{3}{32}\,\log\left(8\,x^2+4\,\sqrt{4\,x^4+3\,x^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^4+3*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(4*x^4 + 3*x^2) - 3/32*log(8*x^2 + 4*sqrt(4*x^4 + 3*x^2) + 3)
```

mupad [B] time = 4.40, size = 40, normalized size = 0.89

$$\frac{\sqrt{4\,x^4+3\,x^2}}{8}-\frac{3\,\ln\left(\frac{\sqrt{4\,x^2+3}\,\sqrt{x^2}}{2}+x^2+\frac{3}{8}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^2 + 4*x^4)^(1/2),x)
```

```
[Out] (3*x^2 + 4*x^4)^(1/2)/8 - (3*log(((4*x^2 + 3)^(1/2)*(x^2)^(1/2))/2 + x^2 +
3/8))/32
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{x^2\left(4x^2+3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(4*x**4+3*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(4*x**2 + 3)), x)
```

3.289 $\int \frac{x^3}{\sqrt{-3x^2+4x^4}}\,dx$

Optimal. Leaf size=45

$$\frac{1}{8}\sqrt{4x^4-3x^2}+\frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{4x^4-3x^2}}\right)$$

```
[Out] 3/16*arctanh(2*x^2/(4*x^4-3*x^2)^(1/2))+1/8*(4*x^4-3*x^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2018, 640, 620, 206}

$$\frac{1}{8}\sqrt{4x^4-3x^2}+\frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{4x^4-3x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[-3*x^2 + 4*x^4],x]
[Out] Sqrt[-3*x^2 + 4*x^4]/8 + (3*ArcTanh[(2*x^2)/Sqrt[-3*x^2 + 4*x^4]])/16
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
```

```
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{-3x^2+4x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{-3x+4x^2}}\,dx,x,x^2\right)\\ &= \frac{1}{8}\sqrt{-3x^2+4x^4}+\frac{3}{16}\text{Subst}\left(\int \frac{1}{\sqrt{-3x+4x^2}}\,dx,x,x^2\right)\\ &= \frac{1}{8}\sqrt{-3x^2+4x^4}+\frac{3}{8}\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx,x,\frac{x^2}{\sqrt{-3x^2+4x^4}}\right)\\ &= \frac{1}{8}\sqrt{-3x^2+4x^4}+\frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{-3x^2+4x^4}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 57, normalized size = 1.27

$$\frac{x\left(8x^3+3\sqrt{4x^2-3}\,\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-3}}\right)-6x\right)}{16\sqrt{x^2\left(4x^2-3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[-3*x^2 + 4*x^4],x]
[Out] (x*(-6*x + 8*x^3 + 3*Sqrt[-3 + 4*x^2]*ArcTanh[(2*x)/Sqrt[-3 + 4*x^2]]))/(16
*Sqrt[x^2*(-3 + 4*x^2)])
```

fricas [A] time = 0.76, size = 45, normalized size = 1.00

$$\frac{1}{8}\sqrt{4\,x^4-3\,x^2}-\frac{3}{16}\log\left(-\frac{2\,x^2-\sqrt{4\,x^4-3\,x^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^4-3*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/8*sqrt(4*x^4 - 3*x^2) - 3/16*log(-(2*x^2 - sqrt(4*x^4 - 3*x^2))/x)
```

giac [A] time = 0.17, size = 42, normalized size = 0.93

$$\frac{1}{8}\sqrt{4\,x^4-3\,x^2}-\frac{3}{32}\log\left(\left|-8\,x^2+4\,\sqrt{4\,x^4-3\,x^2}+3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^4-3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(4*x^4 - 3*x^2) - 3/32*log(abs(-8*x^2 + 4*sqrt(4*x^4 - 3*x^2) + 3))
```

maple [A] time = 0.01, size = 60, normalized size = 1.33

$$\frac{\sqrt{4x^2-3}\,\left(4\sqrt{4x^2-3}\,x+3\sqrt{4}\,\ln\left(\sqrt{4}\,x+\sqrt{4x^2-3}\right)\right)x}{32\sqrt{4x^4-3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^4-3*x^2)^(1/2),x)
```

```
[Out] 1/32*x*(4*x^2-3)^(1/2)*(3*ln(x*4^(1/2)+(4*x^2-3)^(1/2))*4^(1/2)+4*x*(4*x^2-
3)^(1/2))/(4*x^4-3*x^2)^(1/2)
```

maxima [A] time = 3.04, size = 41, normalized size = 0.91

$$\frac{1}{8}\sqrt{4\,x^4-3\,x^2}+\frac{3}{32}\log\left(8\,x^2+4\,\sqrt{4\,x^4-3\,x^2}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^4-3*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(4*x^4 - 3*x^2) + 3/32*log(8*x^2 + 4*sqrt(4*x^4 - 3*x^2) - 3)
```

mupad [B] time = 4.46, size = 40, normalized size = 0.89

$$\frac{3\,\ln\left(\frac{\sqrt{4\,x^2-3}\,\sqrt{x^2}}{2}+x^2-\frac{3}{8}\right)}{32}+\frac{\sqrt{4\,x^4-3\,x^2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^4 - 3*x^2)^(1/2),x)
```

```
[Out] (3*log(((4*x^2 - 3)^(1/2)*(x^2)^(1/2))/2 + x^2 - 3/8))/32 + (4*x^4 - 3*x^2)
^(1/2)/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^2\left(4x^2-3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(4*x**4-3*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(4*x**2 - 3)), x)
```

3.290 $\int \frac{x^3}{\sqrt{ax^2+bx^4}} \, dx$

Optimal. Leaf size=58

$$\frac{\sqrt{ax^2+bx^4}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^2+bx^4}}\right)}{2b^{3/2}}$$

```
[Out] -1/2*a*arctanh(x^2*b^(1/2)/(b*x^4+a*x^2)^(1/2))/b^(3/2)+1/2*(b*x^4+a*x^2)^(
1/2)/b
```

Rubi [A] time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2018, 640, 620, 206}

$$\frac{\sqrt{ax^2+bx^4}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^2+bx^4}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x^2 + b*x^4],x]
```

```
[Out] Sqrt[a*x^2 + b*x^4]/(2*b) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^2 + b*x^4]])/
(2*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{ax^2+bx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt{ax+bx^2}}\,dx, x, x^2\right)\\ &= \frac{\sqrt{ax^2+bx^4}}{2b} - \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{ax+bx^2}}\,dx, x, x^2\right)}{4b}\\ &= \frac{\sqrt{ax^2+bx^4}}{2b} - \frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{ax^2+bx^4}}\right)}{2b}\\ &= \frac{\sqrt{ax^2+bx^4}}{2b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^2+bx^4}}\right)}{2b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 1.26

$$\frac{x\left(\sqrt{b}\,x\left(a+bx^2\right)-a\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)\right)}{2b^{3/2}\sqrt{x^2\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^2 + b*x^4],x]
```

```
[Out] (x*(Sqrt[b]*x*(a + b*x^2) - a*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]]))/(2*b^(3/2)*Sqrt[x^2*(a + b*x^2)])
```

fricas [A] time = 0.57, size = 114, normalized size = 1.97

$$\left[\frac{a\sqrt{b}\,\log\left(-2\,bx^2-a+2\,\sqrt{bx^4+ax^2}\,\sqrt{b}\right)+2\,\sqrt{bx^4+ax^2}\,b}{4\,b^2},\ \frac{a\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^4+ax^2}\,\sqrt{-b}}{bx^2+a}\right)+\sqrt{bx^4+ax^2}\,b}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(a*sqrt(b)*log(-2*b*x^2 - a + 2*sqrt(b*x^4 + a*x^2)*sqrt(b)) + 2*sqrt(
b*x^4 + a*x^2)*b)/b^2, 1/2*(a*sqrt(-b)*arctan(sqrt(b*x^4 + a*x^2)*sqrt(-b)/
(b*x^2 + a)) + sqrt(b*x^4 + a*x^2)*b)/b^2]
```

giac [A] time = 0.20, size = 59, normalized size = 1.02

$$\frac{a\log\left(\left|-2\left(\sqrt{b}\,x^2-\sqrt{bx^4+ax^2}\right)\sqrt{b}-a\right|\right)}{4\,b^{\frac{3}{2}}}+\frac{\sqrt{bx^4+ax^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*a*log(abs(-2*(sqrt(b)*x^2 - sqrt(b*x^4 + a*x^2))*sqrt(b) - a))/b^(3/2)
+ 1/2*sqrt(b*x^4 + a*x^2)/b
```

maple [A] time = 0.01, size = 64, normalized size = 1.10

$$\frac{\sqrt{b\,x^2+a}\,\left(-ab\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)+\sqrt{b\,x^2+a}\,b^{\frac{3}{2}}x\right)x}{2\sqrt{b\,x^4+a\,x^2}\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(b*x^2+a)^(1/2)*(x*(b*x^2+a)^(1/2)*b^(3/2)-a*ln(b^(1/2)*x+(b*x^2+a)^(
1/2))*b)/(b*x^4+a*x^2)^(1/2)/b^(5/2)
```

maxima [A] time = 1.42, size = 52, normalized size = 0.90

$$-\frac{a\log\left(2\,bx^2+a+2\,\sqrt{bx^4+ax^2}\,\sqrt{b}\right)}{4\,b^{\frac{3}{2}}}+\frac{\sqrt{bx^4+ax^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a*log(2*b*x^2 + a + 2*sqrt(b*x^4 + a*x^2)*sqrt(b))/b^(3/2) + 1/2*sqrt(
b*x^4 + a*x^2)/b
```

mupad [B] time = 4.71, size = 53, normalized size = 0.91

$$\frac{\sqrt{b\,x^4+a\,x^2}}{2\,b}-\frac{a\,\ln\left(\frac{b\,x^2+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^4+a\,x^2}\right)}{4\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^4)^(1/2),x)
```

```
[Out] (a*x^2 + b*x^4)^(1/2)/(2*b) - (a*log((a/2 + b*x^2)/b^(1/2) + (a*x^2 + b*x^4
)^(1/2)))/(4*b^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^2\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(a + b*x**2)), x)
```

3.291 $\int \frac{x^3}{\sqrt{ax^2-bx^4}}\,dx$

Optimal. Leaf size=60

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^2-bx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{ax^2-bx^4}}{2b}$$

```
[Out] 1/2*a*arctan(x^2*b^(1/2)/(-b*x^4+a*x^2)^(1/2))/b^(3/2)-1/2*(-b*x^4+a*x^2)^(
1/2)/b
```

Rubi [A] time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2018, 640, 620, 203}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^2-bx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{ax^2-bx^4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x^2 - b*x^4],x]
```

```
[Out] -Sqrt[a*x^2 - b*x^4]/(2*b) + (a*ArcTan[(Sqrt[b]*x^2)/Sqrt[a*x^2 - b*x^4]])/
(2*b^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\sqrt{ax^2-bx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{ax-bx^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{ax^2-bx^4}}{2b}+\frac{a\operatorname{Subst}\left(\int \frac{1}{\sqrt{ax-bx^2}}\,dx, x, x^2\right)}{4b}\\
&= -\frac{\sqrt{ax^2-bx^4}}{2b}+\frac{a\operatorname{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x^2}{\sqrt{ax^2-bx^4}}\right)}{2b}\\
&= -\frac{\sqrt{ax^2-bx^4}}{2b}+\frac{a\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^2-bx^4}}\right)}{2b^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 77, normalized size = 1.28

$$
\frac{x\left(\sqrt{b}\,x\left(bx^2-a\right)+a\sqrt{a-bx^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)\right)}{2b^{3/2}\sqrt{x^2\left(a-bx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^2 - b*x^4],x]
```

```
[Out] (x*(Sqrt[b]*x*(-a + b*x^2) + a*Sqrt[a - b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a -
b*x^2]]))/(2*b^(3/2)*Sqrt[x^2*(a - b*x^2)])
```

fricas [A] time = 0.72, size = 120, normalized size = 2.00

$$
\left[-\frac{a\sqrt{-b}\,\log\left(2\,bx^2-a-2\,\sqrt{-bx^4+ax^2}\,\sqrt{-b}\right)+2\,\sqrt{-bx^4+ax^2}\,b}{4\,b^2}, -\frac{a\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx^4+ax^2}\,\sqrt{b}}{bx^2-a}\right)+\sqrt{-bx^4+ax^2}\,b}{2\,b^2}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^4+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(a*sqrt(-b)*log(2*b*x^2 - a - 2*sqrt(-b*x^4 + a*x^2)*sqrt(-b)) + 2*sq
rt(-b*x^4 + a*x^2)*b)/b^2, -1/2*(a*sqrt(b)*arctan(sqrt(-b*x^4 + a*x^2)*sqrt
(b)/(b*x^2 - a)) + sqrt(-b*x^4 + a*x^2)*b)/b^2]
```

giac [A] time = 0.20, size = 68, normalized size = 1.13

$$-\frac{a\log\left(\left|2\left(\sqrt{-b}\,x^2-\sqrt{-bx^4+ax^2}\right)\sqrt{-b}+a\right|\right)}{4\sqrt{-b}\,b}-\frac{\sqrt{-bx^4+ax^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^4+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*a*log(abs(2*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a*x^2))*sqrt(-b) + a))/(sqrt
(-b)*b) - 1/2*sqrt(-b*x^4 + a*x^2)/b
```

maple [A] time = 0.01, size = 67, normalized size = 1.12

$$\frac{\sqrt{-b\,x^2+a}\,\left(ab\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^2+a}}\right)-\sqrt{-b\,x^2+a}\,b^{\frac{3}{2}}x\right)x}{2\sqrt{-b\,x^4+a\,x^2}\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^4+a*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(-b*x^2+a)^(1/2)*(-x*(-b*x^2+a)^(1/2)*b^(3/2)+a*arctan(b^(1/2)*x/(-b*
x^2+a)^(1/2))*b)/(-b*x^4+a*x^2)^(1/2)/b^(5/2)
```

maxima [A] time = 3.03, size = 42, normalized size = 0.70

$$-\frac{a\arcsin\left(-\frac{2\,bx^2-a}{a}\right)}{4\,b^{\frac{3}{2}}}-\frac{\sqrt{-bx^4+ax^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^4+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a*arcsin(-(2*b*x^2 - a)/a)/b^(3/2) - 1/2*sqrt(-b*x^4 + a*x^2)/b
```

mupad [B] time = 4.62, size = 60, normalized size = 1.00

$$-\frac{\sqrt{a\,x^2-b\,x^4}}{2\,b}-\frac{a\,\ln\left(\frac{\frac{a}{2}-b\,x^2}{\sqrt{-b}}+\sqrt{a\,x^2-b\,x^4}\right)}{4\,{\left(-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 - b*x^4)^(1/2),x)
```

```
[Out] - (a*x^2 - b*x^4)^(1/2)/(2*b) - (a*log((a/2 - b*x^2)/(-b)^(1/2) + (a*x^2 -
b*x^4)^(1/2)))/(4*(-b)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-x^2\left(-a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**4+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(-x**2*(-a + b*x**2)), x)
```

3.292 $\int x^{7/2} \left(bx^2 + cx^4\right) dx$

Optimal. Leaf size=21

$$\frac{2}{13}bx^{13/2} + \frac{2}{17}cx^{17/2}$$

```
[Out] 2/13*b*x^(13/2)+2/17*c*x^(17/2)
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{13}bx^{13/2} + \frac{2}{17}cx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(13/2))/13 + (2*c*x^(17/2))/17
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{7/2} \left(bx^2 + cx^4\right) dx &= \int \left(bx^{11/2} + cx^{15/2}\right) dx \\ &= \frac{2}{13}bx^{13/2} + \frac{2}{17}cx^{17/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{13}bx^{13/2} + \frac{2}{17}cx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(13/2))/13 + (2*c*x^(17/2))/17
```

fricas [A] time = 0.58, size = 18, normalized size = 0.86

$$\frac{2}{221}\left(13\,cx^{8}+17\,bx^{6}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 2/221*(13*c*x^8 + 17*b*x^6)*sqrt(x)
```

giac [A] time = 0.15, size = 13, normalized size = 0.62

$$\frac{2}{17}\,cx^{\frac{17}{2}}+\frac{2}{13}\,bx^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/17*c*x^(17/2) + 2/13*b*x^(13/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2\left(13c\,x^{2}+17b\right)x^{\frac{13}{2}}}{221}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(c*x^4+b*x^2),x)
```

```
[Out] 2/221*x^(13/2)*(13*c*x^2+17*b)
```

maxima [A] time = 1.30, size = 13, normalized size = 0.62

$$\frac{2}{17}\,cx^{\frac{17}{2}}+\frac{2}{13}\,bx^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2/17*c*x^(17/2) + 2/13*b*x^(13/2)
```

mupad [B] time = 0.04, size = 15, normalized size = 0.71

$$\frac{2\,x^{13/2}\,\left(13\,c\,x^{2}+17\,b\right)}{221}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(13/2)*(17*b + 13*c*x^2))/221
```

sympy [A] time = 11.34, size = 19, normalized size = 0.90

$$\frac{2bx^{\frac{13}{2}}}{13} + \frac{2cx^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(c*x**4+b*x**2),x)
```

```
[Out] 2*b*x**(13/2)/13 + 2*c*x**(17/2)/17
```

3.293 $\int x^{5/2}\left(bx^2+cx^4\right)\,dx$

Optimal. Leaf size=21

$$\frac{2}{11}bx^{11/2}+\frac{2}{15}cx^{15/2}$$

```
[Out] 2/11*b*x^(11/2)+2/15*c*x^(15/2)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{11}bx^{11/2}+\frac{2}{15}cx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(11/2))/11 + (2*c*x^(15/2))/15
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(bx^2+cx^4\right)\,dx&=\int\left(bx^{9/2}+cx^{13/2}\right)\,dx\\&=\frac{2}{11}bx^{11/2}+\frac{2}{15}cx^{15/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{11}bx^{11/2}+\frac{2}{15}cx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(11/2))/11 + (2*c*x^(15/2))/15
```

fricas [A] time = 0.46, size = 18, normalized size = 0.86

$$\frac{2}{165}\left(11\,cx^{7}+15\,bx^{5}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 2/165*(11*c*x^7 + 15*b*x^5)*sqrt(x)
```

giac [A] time = 0.15, size = 13, normalized size = 0.62

$$\frac{2}{15}\,cx^{\frac{15}{2}}+\frac{2}{11}\,bx^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2),x, algorithm="giac")
[Out] 2/15*c*x^(15/2) + 2/11*b*x^(11/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2\left(11c\,x^{2}+15b\right)x^{\frac{11}{2}}}{165}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2),x)
[Out] 2/165*x^(11/2)*(11*c*x^2+15*b)
```

maxima [A] time = 1.36, size = 13, normalized size = 0.62

$$\frac{2}{15}\,cx^{\frac{15}{2}}+\frac{2}{11}\,bx^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 2/15*c*x^(15/2) + 2/11*b*x^(11/2)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.71

$$\frac{2\,x^{11/2}\,\left(11\,c\,x^{2}+15\,b\right)}{165}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(11/2)*(15*b + 11*c*x^2))/165
```

sympy [A] time = 5.57, size = 19, normalized size = 0.90

$$\frac{2bx^{\frac{11}{2}}}{11} + \frac{2cx^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2),x)
```

```
[Out] 2*b*x**(11/2)/11 + 2*c*x**(15/2)/15
```

3.294 $\int x^{3/2} \left(bx^2 + cx^4\right) dx$

Optimal. Leaf size=21

$$\frac{2}{9}bx^{9/2} + \frac{2}{13}cx^{13/2}$$

[Out] 2/9*b*x^(9/2)+2/13*c*x^(13/2)

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{9}bx^{9/2} + \frac{2}{13}cx^{13/2}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)*(b*x^2 + c*x^4),x]

[Out] (2*b*x^(9/2))/9 + (2*c*x^(13/2))/13

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{3/2} \left(bx^2 + cx^4\right) dx &= \int \left(bx^{7/2} + cx^{11/2}\right) dx\\ &= \frac{2}{9}bx^{9/2} + \frac{2}{13}cx^{13/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{9}bx^{9/2} + \frac{2}{13}cx^{13/2}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)*(b*x^2 + c*x^4),x]

[Out] (2*b*x^(9/2))/9 + (2*c*x^(13/2))/13

fricas [A] time = 0.74, size = 18, normalized size = 0.86

$$\frac{2}{117}\left(9\,cx^6+13\,bx^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 2/117*(9*c*x^6 + 13*b*x^4)*sqrt(x)
```

giac [A] time = 0.16, size = 13, normalized size = 0.62

$$\frac{2}{13}\,cx^{\frac{13}{2}}+\frac{2}{9}\,bx^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2),x, algorithm="giac")
[Out] 2/13*c*x^(13/2) + 2/9*b*x^(9/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2\left(9c\,x^2+13b\right)x^{\frac{9}{2}}}{117}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2),x)
[Out] 2/117*x^(9/2)*(9*c*x^2+13*b)
```

maxima [A] time = 1.34, size = 13, normalized size = 0.62

$$\frac{2}{13}\,cx^{\frac{13}{2}}+\frac{2}{9}\,bx^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 2/13*c*x^(13/2) + 2/9*b*x^(9/2)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.71

$$\frac{2\,x^{9/2}\,\left(9\,c\,x^2+13\,b\right)}{117}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(9/2)*(13*b + 9*c*x^2))/117
```

sympy **[A]** time = 2.53, size = 19, normalized size = 0.90

$$\frac{2bx^{\frac{9}{2}}}{9} + \frac{2cx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2),x)
```

```
[Out] 2*b*x**(9/2)/9 + 2*c*x**(13/2)/13
```

3.295 $\int \sqrt{x} \left(bx^2 + cx^4\right) dx$

Optimal. Leaf size=21

$$\frac{2}{7}bx^{7/2} + \frac{2}{11}cx^{11/2}$$

```
[Out] 2/7*b*x^(7/2)+2/11*c*x^(11/2)
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{7}bx^{7/2} + \frac{2}{11}cx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(7/2))/7 + (2*c*x^(11/2))/11
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \sqrt{x} \left(bx^2 + cx^4\right) dx &= \int \left(bx^{5/2} + cx^{9/2}\right) dx \\ &= \frac{2}{7}bx^{7/2} + \frac{2}{11}cx^{11/2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{7}bx^{7/2} + \frac{2}{11}cx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(7/2))/7 + (2*c*x^(11/2))/11
```

fricas [A] time = 0.53, size = 18, normalized size = 0.86

$$\frac{2}{77}\left(7\,cx^5+11\,bx^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 2/77*(7*c*x^5 + 11*b*x^3)*sqrt(x)
```

giac [A] time = 0.21, size = 13, normalized size = 0.62

$$\frac{2}{11}\,cx^{\frac{11}{2}}+\frac{2}{7}\,bx^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2),x, algorithm="giac")
[Out] 2/11*c*x^(11/2) + 2/7*b*x^(7/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2\left(7c\,x^2+11b\right)x^{\frac{7}{2}}}{77}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2),x)
[Out] 2/77*x^(7/2)*(7*c*x^2+11*b)
```

maxima [A] time = 1.33, size = 13, normalized size = 0.62

$$\frac{2}{11}\,cx^{\frac{11}{2}}+\frac{2}{7}\,bx^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 2/11*c*x^(11/2) + 2/7*b*x^(7/2)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.71

$$\frac{2\,x^{7/2}\,\left(7\,c\,x^2+11\,b\right)}{77}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(7/2)*(11*b + 7*c*x^2))/77
```

sympy [A] time = 1.73, size = 19, normalized size = 0.90

$$\frac{2bx^{\frac{7}{2}}}{7} + \frac{2cx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2),x)
```

```
[Out] 2*b*x**(7/2)/7 + 2*c*x**(11/2)/11
```

3.296 $\int \frac{bx^2+cx^4}{\sqrt{x}}\,dx$

Optimal. Leaf size=21

$$\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

```
[Out] 2/5*b*x^(5/2)+2/9*c*x^(9/2)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/Sqrt[x],x]
```

```
[Out] (2*b*x^(5/2))/5 + (2*c*x^(9/2))/9
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{\sqrt{x}}\,dx &= \int \left(bx^{3/2}+cx^{7/2}\right)dx\\ &= \frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/Sqrt[x],x]
```

```
[Out] (2*b*x^(5/2))/5 + (2*c*x^(9/2))/9
```

fricas [A] time = 0.51, size = 18, normalized size = 0.86

$$\frac{2}{45}\left(5\,cx^4+9\,bx^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(1/2),x, algorithm="fricas")
[Out] 2/45*(5*c*x^4 + 9*b*x^2)*sqrt(x)
```

giac [A] time = 0.15, size = 13, normalized size = 0.62

$$\frac{2}{9}\,cx^{\frac{9}{2}}+\frac{2}{5}\,bx^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(1/2),x, algorithm="giac")
[Out] 2/9*c*x^(9/2) + 2/5*b*x^(5/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2\left(5c\,x^2+9b\right)x^{\frac{5}{2}}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^(1/2),x)
[Out] 2/45*x^(5/2)*(5*c*x^2+9*b)
```

maxima [A] time = 1.33, size = 13, normalized size = 0.62

$$\frac{2}{9}\,cx^{\frac{9}{2}}+\frac{2}{5}\,bx^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(1/2),x, algorithm="maxima")
[Out] 2/9*c*x^(9/2) + 2/5*b*x^(5/2)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.71

$$\frac{2\,x^{5/2}\,\left(5\,c\,x^2+9\,b\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^(1/2),x)
```

```
[Out] (2*x^(5/2)*(9*b + 5*c*x^2))/45
```

sympy [A] time = 0.78, size = 19, normalized size = 0.90

$$\frac{2bx^{\frac{5}{2}}}{5} + \frac{2cx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**(1/2),x)
```

```
[Out] 2*b*x**(5/2)/5 + 2*c*x**(9/2)/9
```

3.297 $\int \frac{bx^2+cx^4}{x^{3/2}}\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}$$

```
[Out] 2/3*b*x^(3/2)+2/7*c*x^(7/2)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^(3/2),x]
```

```
[Out] (2*b*x^(3/2))/3 + (2*c*x^(7/2))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^{3/2}}\,dx &= \int \left(b\sqrt{x}+cx^{5/2}\right)\,dx\\ &= \frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^(3/2),x]
```

```
[Out] (2*b*x^(3/2))/3 + (2*c*x^(7/2))/7
```

fricas [A] time = 0.71, size = 16, normalized size = 0.76

$$\frac{2}{21}\left(3\,cx^3+7\,bx\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*c*x^3 + 7*b*x)*sqrt(x)
```

giac [A] time = 0.20, size = 13, normalized size = 0.62

$$\frac{2}{7}\,cx^{\frac{7}{2}}+\frac{2}{3}\,bx^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/7*c*x^(7/2) + 2/3*b*x^(3/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2\left(3c\,x^2+7b\right)x^{\frac{3}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^(3/2),x)
```

```
[Out] 2/21*x^(3/2)*(3*c*x^2+7*b)
```

maxima [A] time = 1.30, size = 13, normalized size = 0.62

$$\frac{2}{7}\,cx^{\frac{7}{2}}+\frac{2}{3}\,bx^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/7*c*x^(7/2) + 2/3*b*x^(3/2)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.71

$$\frac{2\,x^{3/2}\,\left(3\,c\,x^2+7\,b\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^(3/2),x)
```

```
[Out] (2*x^(3/2)*(7*b + 3*c*x^2))/21
```

sympy [A] time = 0.79, size = 19, normalized size = 0.90

$$\frac{2bx^{\frac{3}{2}}}{3} + \frac{2cx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**(3/2),x)
```

```
[Out] 2*b*x**(3/2)/3 + 2*c*x**(7/2)/7
```

3.298 $\int \frac{bx^2+cx^4}{x^{5/2}}\,dx$

Optimal. Leaf size=19

$$2b\sqrt{x} + \frac{2}{5}cx^{5/2}$$

```
[Out] 2/5*c*x^(5/2)+2*b*x^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$2b\sqrt{x} + \frac{2}{5}cx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^(5/2),x]
```

```
[Out] 2*b*Sqrt[x] + (2*c*x^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2 + cx^4}{x^{5/2}}\,dx &= \int \left(\frac{b}{\sqrt{x}} + cx^{3/2}\right)dx\\ &= 2b\sqrt{x} + \frac{2}{5}cx^{5/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$2b\sqrt{x} + \frac{2}{5}cx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^(5/2),x]
```

```
[Out] 2*b*Sqrt[x] + (2*c*x^(5/2))/5
```

fricas [A]   time = 0.64, size = 14, normalized size = 0.74

$$\frac{2}{5}\left(cx^{2}+5\,b\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(5/2),x, algorithm="fricas")
[Out] 2/5*(c*x^2 + 5*b)*sqrt(x)
```

giac [A]   time = 0.15, size = 13, normalized size = 0.68

$$\frac{2}{5}\,cx^{\frac{5}{2}}+2\,b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(5/2),x, algorithm="giac")
[Out] 2/5*c*x^(5/2) + 2*b*sqrt(x)
```

maple [A]   time = 0.00, size = 15, normalized size = 0.79

$$\frac{2\left(c\,x^{2}+5b\right)\sqrt{x}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^(5/2),x)
[Out] 2/5*x^(1/2)*(c*x^2+5*b)
```

maxima [A]   time = 1.31, size = 13, normalized size = 0.68

$$\frac{2}{5}\,cx^{\frac{5}{2}}+2\,b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(5/2),x, algorithm="maxima")
[Out] 2/5*c*x^(5/2) + 2*b*sqrt(x)
```

mupad [B]   time = 0.03, size = 14, normalized size = 0.74

$$\frac{2\,\sqrt{x}\,\left(c\,x^{2}+5\,b\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^(5/2),x)
```

```
[Out] (2*x^(1/2)*(5*b + c*x^2))/5
```

sympy [A] time = 1.00, size = 17, normalized size = 0.89

$$2b\sqrt{x}+\frac{2cx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**(5/2),x)
```

```
[Out] 2*b*sqrt(x) + 2*c*x**(5/2)/5
```

3.299 $\int \frac{bx^2+cx^4}{x^{7/2}}\,dx$

Optimal. Leaf size=19

$$\frac{2}{3}cx^{3/2}-\frac{2b}{\sqrt{x}}$$

```
[Out] 2/3*c*x^(3/2)-2*b/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{3}cx^{3/2}-\frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^(7/2),x]
```

```
[Out] (-2*b)/Sqrt[x] + (2*c*x^(3/2))/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^{7/2}}\,dx &= \int \left(\frac{b}{x^{3/2}}+c\sqrt{x}\right)dx\\ &= -\frac{2b}{\sqrt{x}}+\frac{2}{3}cx^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{2}{3}cx^{3/2}-\frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^(7/2),x]
```

```
[Out] (-2*b)/Sqrt[x] + (2*c*x^(3/2))/3
```

fricas [A] time = 0.53, size = 14, normalized size = 0.74

$$\frac{2\left(cx^2 - 3\,b\right)}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(7/2),x, algorithm="fricas")
[Out] 2/3*(c*x^2 - 3*b)/sqrt(x)
```

giac [A] time = 0.16, size = 13, normalized size = 0.68

$$\frac{2}{3}\,cx^{\frac{3}{2}} - \frac{2\,b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(7/2),x, algorithm="giac")
[Out] 2/3*c*x^(3/2) - 2*b/sqrt(x)
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$-\frac{2\left(-c\,x^2 + 3b\right)}{3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^(7/2),x)
[Out] -2/3/x^(1/2)*(-c*x^2+3*b)
```

maxima [A] time = 1.34, size = 13, normalized size = 0.68

$$\frac{2}{3}\,cx^{\frac{3}{2}} - \frac{2\,b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(7/2),x, algorithm="maxima")
[Out] 2/3*c*x^(3/2) - 2*b/sqrt(x)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.79

$$-\frac{6\,b-2\,c\,x^2}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^(7/2),x)
[Out] -(6*b - 2*c*x^2)/(3*x^(1/2))
```

sympy [A] time = 1.80, size = 17, normalized size = 0.89

$$-\frac{2b}{\sqrt{x}}+\frac{2cx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**(7/2),x)
[Out] -2*b/sqrt(x) + 2*c*x**(3/2)/3
```

3.300 $\int x^{7/2}\left(bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{17}b^2x^{17/2}+\frac{4}{21}bcx^{21/2}+\frac{2}{25}c^2x^{25/2}$$

```
[Out] 2/17*b^2*x^(17/2)+4/21*b*c*x^(21/2)+2/25*c^2*x^(25/2)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{17}b^2x^{17/2}+\frac{4}{21}bcx^{21/2}+\frac{2}{25}c^2x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*b^2*x^(17/2))/17 + (4*b*c*x^(21/2))/21 + (2*c^2*x^(25/2))/25
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(bx^2+cx^4\right)^2\,dx &= \int x^{15/2}\left(b+cx^2\right)^2\,dx\\ &= \int \left(b^2x^{15/2}+2bcx^{19/2}+c^2x^{23/2}\right)dx\\ &= \frac{2}{17}b^2x^{17/2}+\frac{4}{21}bcx^{21/2}+\frac{2}{25}c^2x^{25/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2x^{17/2}(525b^2 + 850bcx^2 + 357c^2x^4)}{8925}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(b*x^2 + c*x^4)^2,x]
[Out] (2*x^(17/2)*(525*b^2 + 850*b*c*x^2 + 357*c^2*x^4))/8925
```

fricas [A] time = 0.68, size = 29, normalized size = 0.81

$$\frac{2}{8925}\left(357\,c^2x^{12} + 850\,bcx^{10} + 525\,b^2x^8\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
[Out] 2/8925*(357*c^2*x^12 + 850*b*c*x^10 + 525*b^2*x^8)*sqrt(x)
```

giac [A] time = 0.15, size = 24, normalized size = 0.67

$$\frac{2}{25}\,c^2x^{\frac{25}{2}} + \frac{4}{21}\,bcx^{\frac{21}{2}} + \frac{2}{17}\,b^2x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^2,x, algorithm="giac")
[Out] 2/25*c^2*x^(25/2) + 4/21*b*c*x^(21/2) + 2/17*b^2*x^(17/2)
```

maple [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{2\left(357c^2x^4 + 850bc\,x^2 + 525b^2\right)x^{\frac{17}{2}}}{8925}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(c*x^4+b*x^2)^2,x)
[Out] 2/8925*x^(17/2)*(357*c^2*x^4+850*b*c*x^2+525*b^2)
```

maxima [A] time = 1.28, size = 24, normalized size = 0.67

$$\frac{2}{25}\,c^2x^{\frac{25}{2}} + \frac{4}{21}\,bcx^{\frac{21}{2}} + \frac{2}{17}\,b^2x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
[Out] 2/25*c^2*x^(25/2) + 4/21*b*c*x^(21/2) + 2/17*b^2*x^(17/2)
```

mupad [B] time = 0.05, size = 25, normalized size = 0.69

$$x^{17/2}\left(\frac{2\,b^2}{17}+\frac{4\,b\,c\,x^2}{21}+\frac{2\,c^2\,x^4}{25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2 + c*x^4)^2,x)
[Out] x^(17/2)*((2*b^2)/17 + (2*c^2*x^4)/25 + (4*b*c*x^2)/21)
```

sympy [A] time = 35.14, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{17}{2}}}{17}+\frac{4bcx^{\frac{21}{2}}}{21}+\frac{2c^2x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(c*x**4+b*x**2)**2,x)
[Out] 2*b**2*x**(17/2)/17 + 4*b*c*x**(21/2)/21 + 2*c**2*x**(25/2)/25
```

3.301 $\int x^{5/2}\left(bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{15}b^2x^{15/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

```
[Out] 2/15*b^2*x^(15/2)+4/19*b*c*x^(19/2)+2/23*c^2*x^(23/2)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{15}b^2x^{15/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*b^2*x^(15/2))/15 + (4*b*c*x^(19/2))/19 + (2*c^2*x^(23/2))/23
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(bx^2+cx^4\right)^2\,dx&=\int x^{13/2}\left(b+cx^2\right)^2\,dx\\&=\int\left(b^2x^{13/2}+2bcx^{17/2}+c^2x^{21/2}\right)dx\\&=\frac{2}{15}b^2x^{15/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2x^{15/2}\left(437b^2+690bcx^2+285c^2x^4\right)}{6555}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(b*x^2 + c*x^4)^2,x]
[Out] (2*x^(15/2)*(437*b^2 + 690*b*c*x^2 + 285*c^2*x^4))/6555
```

fricas [A] time = 0.63, size = 29, normalized size = 0.81

$$\frac{2}{6555}\left(285\,c^2x^{11}+690\,bcx^9+437\,b^2x^7\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
[Out] 2/6555*(285*c^2*x^11 + 690*b*c*x^9 + 437*b^2*x^7)*sqrt(x)
```

giac [A] time = 0.15, size = 24, normalized size = 0.67

$$\frac{2}{23}\,c^2x^{\frac{23}{2}}+\frac{4}{19}\,bcx^{\frac{19}{2}}+\frac{2}{15}\,b^2x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^2,x, algorithm="giac")
[Out] 2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*b^2*x^(15/2)
```

maple [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{2\left(285c^2x^4+690bc\,x^2+437b^2\right)x^{\frac{15}{2}}}{6555}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2)^2,x)
[Out] 2/6555*x^(15/2)*(285*c^2*x^4+690*b*c*x^2+437*b^2)
```

maxima [A] time = 1.35, size = 24, normalized size = 0.67

$$\frac{2}{23}\,c^2x^{\frac{23}{2}}+\frac{4}{19}\,bcx^{\frac{19}{2}}+\frac{2}{15}\,b^2x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*b^2*x^(15/2)
```

mupad [B] time = 0.04, size = 25, normalized size = 0.69

$$x^{15/2}\left(\frac{2\,b^2}{15}+\frac{4\,b\,c\,x^2}{19}+\frac{2\,c^2\,x^4}{23}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(15/2)*((2*b^2)/15 + (2*c^2*x^4)/23 + (4*b*c*x^2)/19)
```

sympy [A] time = 20.81, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{15}{2}}}{15}+\frac{4bcx^{\frac{19}{2}}}{19}+\frac{2c^2x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*b**2*x**(15/2)/15 + 4*b*c*x**(19/2)/19 + 2*c**2*x**(23/2)/23
```

3.302 $\int x^{3/2} \left(bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{13}b^2x^{13/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}$$

```
[Out] 2/13*b^2*x^(13/2)+4/17*b*c*x^(17/2)+2/21*c^2*x^(21/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{13}b^2x^{13/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*b^2*x^(13/2))/13 + (4*b*c*x^(17/2))/17 + (2*c^2*x^(21/2))/21
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(bx^2+cx^4\right)^2\,dx &= \int x^{11/2}\left(b+cx^2\right)^2\,dx\\ &= \int \left(b^2x^{11/2}+2bcx^{15/2}+c^2x^{19/2}\right)dx\\ &= \frac{2}{13}b^2x^{13/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2x^{13/2}\left(357b^2+546bcx^2+221c^2x^4\right)}{4641}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(b*x^2 + c*x^4)^2,x]
[Out] (2*x^(13/2)*(357*b^2 + 546*b*c*x^2 + 221*c^2*x^4))/4641
```

fricas [A] time = 0.56, size = 29, normalized size = 0.81

$$\frac{2}{4641}\left(221\,c^2x^{10}+546\,bcx^8+357\,b^2x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
[Out] 2/4641*(221*c^2*x^10 + 546*b*c*x^8 + 357*b^2*x^6)*sqrt(x)
```

giac [A] time = 0.16, size = 24, normalized size = 0.67

$$\frac{2}{21}\,c^2x^{\frac{21}{2}}+\frac{4}{17}\,bcx^{\frac{17}{2}}+\frac{2}{13}\,b^2x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^2,x, algorithm="giac")
[Out] 2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*b^2*x^(13/2)
```

maple [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{2\left(221c^2x^4+546bc\,x^2+357b^2\right)x^{\frac{13}{2}}}{4641}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2)^2,x)
[Out] 2/4641*x^(13/2)*(221*c^2*x^4+546*b*c*x^2+357*b^2)
```

maxima [A] time = 1.23, size = 24, normalized size = 0.67

$$\frac{2}{21}\,c^2x^{\frac{21}{2}}+\frac{4}{17}\,bcx^{\frac{17}{2}}+\frac{2}{13}\,b^2x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*b^2*x^(13/2)
```

mupad [B] time = 4.27, size = 25, normalized size = 0.69

$$x^{13/2}\left(\frac{2\,b^2}{13}+\frac{4\,b\,c\,x^2}{17}+\frac{2\,c^2\,x^4}{21}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(13/2)*((2*b^2)/13 + (2*c^2*x^4)/21 + (4*b*c*x^2)/17)
```

sympy [A] time = 11.38, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{13}{2}}}{13}+\frac{4bcx^{\frac{17}{2}}}{17}+\frac{2c^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*b**2*x**(13/2)/13 + 4*b*c*x**(17/2)/17 + 2*c**2*x**(21/2)/21
```

3.303 $\int \sqrt{x} \left(bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{11}b^2x^{11/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}$$

```
[Out] 2/11*b^2*x^(11/2)+4/15*b*c*x^(15/2)+2/19*c^2*x^(19/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{11}b^2x^{11/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*b^2*x^(11/2))/11 + (4*b*c*x^(15/2))/15 + (2*c^2*x^(19/2))/19
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x} \left(bx^2+cx^4\right)^2 \, dx &= \int x^{9/2} \left(b+cx^2\right)^2 \, dx \\
&= \int \left(b^2x^{9/2}+2bcx^{13/2}+c^2x^{17/2}\right) dx \\
&= \frac{2}{11}b^2x^{11/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2x^{11/2}\left(285b^2+418bcx^2+165c^2x^4\right)}{3135}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(b*x^2 + c*x^4)^2,x]
[Out] (2*x^(11/2)*(285*b^2 + 418*b*c*x^2 + 165*c^2*x^4))/3135
```

fricas [A] time = 0.75, size = 29, normalized size = 0.81

$$\frac{2}{3135}\left(165\,c^2x^9+418\,bcx^7+285\,b^2x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
[Out] 2/3135*(165*c^2*x^9 + 418*b*c*x^7 + 285*b^2*x^5)*sqrt(x)
```

giac [A] time = 0.14, size = 24, normalized size = 0.67

$$\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{15}\,bcx^{\frac{15}{2}}+\frac{2}{11}\,b^2x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^2,x, algorithm="giac")
[Out] 2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*b^2*x^(11/2)
```

maple [A] time = 0.00, size = 27, normalized size = 0.75

$$\frac{2\left(165c^2x^4+418bc\,x^2+285b^2\right)x^{\frac{11}{2}}}{3135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2)^2,x)
[Out] 2/3135*x^(11/2)*(165*c^2*x^4+418*b*c*x^2+285*b^2)
```

maxima [A] time = 1.36, size = 24, normalized size = 0.67

$$\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{15}\,bcx^{\frac{15}{2}}+\frac{2}{11}\,b^2x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*b^2*x^(11/2)
```

mupad [B] time = 0.04, size = 25, normalized size = 0.69

$$x^{11/2}\left(\frac{2\,b^2}{11}+\frac{4\,b\,c\,x^2}{15}+\frac{2\,c^2\,x^4}{19}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(11/2)*((2*b^2)/11 + (2*c^2*x^4)/19 + (4*b*c*x^2)/15)
```

sympy [A] time = 2.75, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{11}{2}}}{11}+\frac{4bcx^{\frac{15}{2}}}{15}+\frac{2c^2x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*b**2*x**(11/2)/11 + 4*b*c*x**(15/2)/15 + 2*c**2*x**(19/2)/19
```

3.304 $\int \frac{(bx^2+cx^4)^2}{\sqrt{x}} dx$

Optimal. Leaf size=36

$$\frac{2}{9}b^2x^{9/2} + \frac{4}{13}bcx^{13/2} + \frac{2}{17}c^2x^{17/2}$$

```
[Out] 2/9*b^2*x^(9/2)+4/13*b*c*x^(13/2)+2/17*c^2*x^(17/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{9}b^2x^{9/2} + \frac{4}{13}bcx^{13/2} + \frac{2}{17}c^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/Sqrt[x],x]
```

```
[Out] (2*b^2*x^(9/2))/9 + (4*b*c*x^(13/2))/13 + (2*c^2*x^(17/2))/17
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^2}{\sqrt{x}} dx &= \int x^{7/2}\left(b+cx^2\right)^2 dx \\
&= \int \left(b^2x^{7/2} + 2bcx^{11/2} + c^2x^{15/2}\right) dx \\
&= \frac{2}{9}b^2x^{9/2} + \frac{4}{13}bcx^{13/2} + \frac{2}{17}c^2x^{17/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2x^{9/2}\left(221b^2+306bcx^2+117c^2x^4\right)}{1989}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/Sqrt[x],x]
[Out] (2*x^(9/2)*(221*b^2 + 306*b*c*x^2 + 117*c^2*x^4))/1989
```

fricas [A] time = 0.53, size = 29, normalized size = 0.81

$$\frac{2}{1989}\left(117\,c^2x^8+306\,bcx^6+221\,b^2x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(1/2),x, algorithm="fricas")
[Out] 2/1989*(117*c^2*x^8 + 306*b*c*x^6 + 221*b^2*x^4)*sqrt(x)
```

giac [A] time = 0.15, size = 24, normalized size = 0.67

$$\frac{2}{17}\,c^2x^{\frac{17}{2}}+\frac{4}{13}\,bcx^{\frac{13}{2}}+\frac{2}{9}\,b^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(1/2),x, algorithm="giac")
[Out] 2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2)
```

maple [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{2\left(117c^2x^4+306bc\,x^2+221b^2\right)x^{\frac{9}{2}}}{1989}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^(1/2),x)
[Out] 2/1989*x^(9/2)*(117*c^2*x^4+306*b*c*x^2+221*b^2)
```

maxima [A] time = 1.25, size = 24, normalized size = 0.67

$$\frac{2}{17}\,c^2x^{\frac{17}{2}}+\frac{4}{13}\,bcx^{\frac{13}{2}}+\frac{2}{9}\,b^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2)
```

mupad [B] time = 0.04, size = 25, normalized size = 0.69

$$x^{9/2}\left(\frac{2\,b^2}{9}+\frac{4\,b\,c\,x^2}{13}+\frac{2\,c^2\,x^4}{17}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^(1/2),x)
```

```
[Out] x^(9/2)*((2*b^2)/9 + (2*c^2*x^4)/17 + (4*b*c*x^2)/13)
```

sympy [A] time = 4.98, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{9}{2}}}{9}+\frac{4bcx^{\frac{13}{2}}}{13}+\frac{2c^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**(1/2),x)
```

```
[Out] 2*b**2*x**(9/2)/9 + 4*b*c*x**(13/2)/13 + 2*c**2*x**(17/2)/17
```

3.305 $\int \frac{(bx^2+cx^4)^2}{x^{3/2}} dx$

Optimal. Leaf size=36

$$\frac{2}{7}b^2x^{7/2} + \frac{4}{11}bcx^{11/2} + \frac{2}{15}c^2x^{15/2}$$

```
[Out] 2/7*b^2*x^(7/2)+4/11*b*c*x^(11/2)+2/15*c^2*x^(15/2)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{7}b^2x^{7/2} + \frac{4}{11}bcx^{11/2} + \frac{2}{15}c^2x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^(3/2),x]
```

```
[Out] (2*b^2*x^(7/2))/7 + (4*b*c*x^(11/2))/11 + (2*c^2*x^(15/2))/15
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^2}{x^{3/2}} dx &= \int x^{5/2}\left(b+cx^2\right)^2 dx \\
&= \int \left(b^2x^{5/2} + 2bcx^{9/2} + c^2x^{13/2}\right) dx \\
&= \frac{2}{7}b^2x^{7/2} + \frac{4}{11}bcx^{11/2} + \frac{2}{15}c^2x^{15/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2x^{7/2}\left(165b^2+210bcx^2+77c^2x^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^(3/2),x]
[Out] (2*x^(7/2)*(165*b^2 + 210*b*c*x^2 + 77*c^2*x^4))/1155
```

fricas [A] time = 0.70, size = 29, normalized size = 0.81

$$\frac{2}{1155}\left(77\,c^2x^7+210\,bcx^5+165\,b^2x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(3/2),x, algorithm="fricas")
[Out] 2/1155*(77*c^2*x^7 + 210*b*c*x^5 + 165*b^2*x^3)*sqrt(x)
```

giac [A] time = 0.15, size = 24, normalized size = 0.67

$$\frac{2}{15}\,c^2x^{\frac{15}{2}}+\frac{4}{11}\,bcx^{\frac{11}{2}}+\frac{2}{7}\,b^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(3/2),x, algorithm="giac")
[Out] 2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*b^2*x^(7/2)
```

maple [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{2\left(77c^2x^4+210bc\,x^2+165b^2\right)x^{\frac{7}{2}}}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^(3/2),x)
[Out] 2/1155*x^(7/2)*(77*c^2*x^4+210*b*c*x^2+165*b^2)
```

maxima [A] time = 1.33, size = 24, normalized size = 0.67

$$\frac{2}{15}\,c^2x^{\frac{15}{2}}+\frac{4}{11}\,bcx^{\frac{11}{2}}+\frac{2}{7}\,b^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*b^2*x^(7/2)
```

mupad [B] time = 4.44, size = 26, normalized size = 0.72

$$\frac{2\,x^{7/2}\,\left(165\,b^2+210\,b\,c\,x^2+77\,c^2\,x^4\right)}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^(3/2),x)
```

```
[Out] (2*x^(7/2)*(165*b^2 + 77*c^2*x^4 + 210*b*c*x^2))/1155
```

sympy [A] time = 5.22, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{7}{2}}}{7}+\frac{4bcx^{\frac{11}{2}}}{11}+\frac{2c^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**(3/2),x)
```

```
[Out] 2*b**2*x**(7/2)/7 + 4*b*c*x**(11/2)/11 + 2*c**2*x**(15/2)/15
```

3.306 $\int \frac{(bx^2+cx^4)^2}{x^{5/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2}{5}b^2x^{5/2}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}$$

[Out] 2/5*b^2*x^(5/2)+4/9*b*c*x^(9/2)+2/13*c^2*x^(13/2)

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{5}b^2x^{5/2}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}$$

Antiderivative was successfully verified.

[In] Int[(b*x^2 + c*x^4)^2/x^(5/2),x]

[Out] (2*b^2*x^(5/2))/5 + (4*b*c*x^(9/2))/9 + (2*c^2*x^(13/2))/13

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^2}{x^{5/2}}\,dx &= \int x^{3/2}\left(b+cx^2\right)^2\,dx\\ &= \int \left(b^2x^{3/2}+2bcx^{7/2}+c^2x^{11/2}\right)dx\\ &= \frac{2}{5}b^2x^{5/2}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2}{585}x^{5/2}\left(117b^2+130bcx^2+45c^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^(5/2),x]
```

```
[Out] (2*x^(5/2)*(117*b^2 + 130*b*c*x^2 + 45*c^2*x^4))/585
```

fricas [A] time = 0.59, size = 29, normalized size = 0.81

$$\frac{2}{585}\left(45\,c^2x^6+130\,bcx^4+117\,b^2x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*c^2*x^6 + 130*b*c*x^4 + 117*b^2*x^2)*sqrt(x)
```

giac [A] time = 0.15, size = 24, normalized size = 0.67

$$\frac{2}{13}\,c^2x^{\frac{13}{2}}+\frac{4}{9}\,bcx^{\frac{9}{2}}+\frac{2}{5}\,b^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*b^2*x^(5/2)
```

maple [A] time = 0.00, size = 27, normalized size = 0.75

$$\frac{2\left(45c^2x^4+130bc\,x^2+117b^2\right)x^{\frac{5}{2}}}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^(5/2),x)
```

```
[Out] 2/585*x^(5/2)*(45*c^2*x^4+130*b*c*x^2+117*b^2)
```

maxima [A] time = 1.35, size = 24, normalized size = 0.67

$$\frac{2}{13}\,c^2x^{\frac{13}{2}}+\frac{4}{9}\,bcx^{\frac{9}{2}}+\frac{2}{5}\,b^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(5/2),x, algorithm="maxima")
[Out] 2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*b^2*x^(5/2)
```

mupad [B] time = 0.04, size = 26, normalized size = 0.72

$$\frac{2\,x^{5/2}\,\left(117\,b^2+130\,b\,c\,x^2+45\,c^2\,x^4\right)}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^(5/2),x)
[Out] (2*x^(5/2)*(117*b^2 + 45*c^2*x^4 + 130*b*c*x^2))/585
```

sympy [A] time = 5.99, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{5}{2}}}{5}+\frac{4bcx^{\frac{9}{2}}}{9}+\frac{2c^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**(5/2),x)
[Out] 2*b**2*x**(5/2)/5 + 4*b*c*x**(9/2)/9 + 2*c**2*x**(13/2)/13
```

3.307 $\int \frac{(bx^2+cx^4)^2}{x^{7/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2}{3}b^2x^{3/2}+\frac{4}{7}bcx^{7/2}+\frac{2}{11}c^2x^{11/2}$$

```
[Out] 2/3*b^2*x^(3/2)+4/7*b*c*x^(7/2)+2/11*c^2*x^(11/2)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{3}b^2x^{3/2}+\frac{4}{7}bcx^{7/2}+\frac{2}{11}c^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^(7/2),x]
```

```
[Out] (2*b^2*x^(3/2))/3 + (4*b*c*x^(7/2))/7 + (2*c^2*x^(11/2))/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2+cx^4\right)^2}{x^{7/2}}\,dx &= \int \sqrt{x}\left(b+cx^2\right)^2\,dx\\ &= \int \left(b^2\sqrt{x}+2bcx^{5/2}+c^2x^{9/2}\right)dx\\ &= \frac{2}{3}b^2x^{3/2}+\frac{4}{7}bcx^{7/2}+\frac{2}{11}c^2x^{11/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2}{231}x^{3/2}\left(77b^2+66bcx^2+21c^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^(7/2),x]
[Out] (2*x^(3/2)*(77*b^2 + 66*b*c*x^2 + 21*c^2*x^4))/231
```

fricas [A] time = 0.55, size = 27, normalized size = 0.75

$$\frac{2}{231}\left(21\,c^2x^5+66\,bcx^3+77\,b^2x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(7/2),x, algorithm="fricas")
[Out] 2/231*(21*c^2*x^5 + 66*b*c*x^3 + 77*b^2*x)*sqrt(x)
```

giac [A] time = 0.15, size = 24, normalized size = 0.67

$$\frac{2}{11}\,c^2x^{\frac{11}{2}}+\frac{4}{7}\,bcx^{\frac{7}{2}}+\frac{2}{3}\,b^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(7/2),x, algorithm="giac")
[Out] 2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*b^2*x^(3/2)
```

maple [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{2\left(21c^2x^4+66bc\,x^2+77b^2\right)x^{\frac{3}{2}}}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^(7/2),x)
[Out] 2/231*x^(3/2)*(21*c^2*x^4+66*b*c*x^2+77*b^2)
```

maxima [A] time = 1.32, size = 24, normalized size = 0.67

$$\frac{2}{11}\,c^2x^{\frac{11}{2}}+\frac{4}{7}\,bcx^{\frac{7}{2}}+\frac{2}{3}\,b^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*b^2*x^(3/2)
```

mupad [B] time = 0.05, size = 26, normalized size = 0.72

$$\frac{2\,x^{3/2}\,\left(77\,b^2+66\,b\,c\,x^2+21\,c^2\,x^4\right)}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^(7/2),x)
```

```
[Out] (2*x^(3/2)*(77*b^2 + 21*c^2*x^4 + 66*b*c*x^2))/231
```

sympy [A] time = 8.69, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{3}{2}}}{3}+\frac{4bcx^{\frac{7}{2}}}{7}+\frac{2c^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**(7/2),x)
```

```
[Out] 2*b**2*x**(3/2)/3 + 4*b*c*x**(7/2)/7 + 2*c**2*x**(11/2)/11
```

3.308 $\int x^{7/2} \left(bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{2}{21}b^3x^{21/2}+\frac{6}{25}b^2cx^{25/2}+\frac{6}{29}bc^2x^{29/2}+\frac{2}{33}c^3x^{33/2}$$

```
[Out] 2/21*b^3*x^(21/2)+6/25*b^2*c*x^(25/2)+6/29*b*c^2*x^(29/2)+2/33*c^3*x^(33/2)
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{6}{25}b^2cx^{25/2}+\frac{2}{21}b^3x^{21/2}+\frac{6}{29}bc^2x^{29/2}+\frac{2}{33}c^3x^{33/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(21/2))/21 + (6*b^2*c*x^(25/2))/25 + (6*b*c^2*x^(29/2))/29 + (2*c^
3*x^(33/2))/33
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int x^{7/2}\left(bx^2+cx^4\right)^3\,dx &= \int x^{19/2}\left(b+cx^2\right)^3\,dx\\
&= \int \left(b^3x^{19/2}+3b^2cx^{23/2}+3bc^2x^{27/2}+c^3x^{31/2}\right)dx\\
&= \frac{2}{21}b^3x^{21/2}+\frac{6}{25}b^2cx^{25/2}+\frac{6}{29}bc^2x^{29/2}+\frac{2}{33}c^3x^{33/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.00

$$\frac{2}{21}b^3x^{21/2} + \frac{6}{25}b^2cx^{25/2} + \frac{6}{29}bc^2x^{29/2} + \frac{2}{33}c^3x^{33/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(21/2))/21 + (6*b^2*c*x^(25/2))/25 + (6*b*c^2*x^(29/2))/29 + (2*c^
3*x^(33/2))/33
```

fricas [A] time = 0.68, size = 40, normalized size = 0.78

$$\frac{2}{167475}\left(5075\,c^3x^{16} + 17325\,bc^2x^{14} + 20097\,b^2cx^{12} + 7975\,b^3x^{10}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/167475*(5075*c^3*x^16 + 17325*b*c^2*x^14 + 20097*b^2*c*x^12 + 7975*b^3*x^
10)*sqrt(x)
```

giac [A] time = 0.18, size = 35, normalized size = 0.69

$$\frac{2}{33}\,c^3x^{\frac{33}{2}} + \frac{6}{29}\,bc^2x^{\frac{29}{2}} + \frac{6}{25}\,b^2cx^{\frac{25}{2}} + \frac{2}{21}\,b^3x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/33*c^3*x^(33/2) + 6/29*b*c^2*x^(29/2) + 6/25*b^2*c*x^(25/2) + 2/21*b^3*x^
(21/2)
```

maple [A] time = 0.01, size = 38, normalized size = 0.75

$$\frac{2\left(5075c^3x^6 + 17325b\,c^2x^4 + 20097b^2c\,x^2 + 7975b^3\right)x^{\frac{21}{2}}}{167475}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/167475*x^(21/2)*(5075*c^3*x^6+17325*b*c^2*x^4+20097*b^2*c*x^2+7975*b^3)
```

maxima [A] time = 1.34, size = 35, normalized size = 0.69

$$\frac{2}{33}\,c^3x^{\frac{33}{2}} + \frac{6}{29}\,bc^2x^{\frac{29}{2}} + \frac{6}{25}\,b^2cx^{\frac{25}{2}} + \frac{2}{21}\,b^3x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/33*c^3*x^(33/2) + 6/29*b*c^2*x^(29/2) + 6/25*b^2*c*x^(25/2) + 2/21*b^3*x^
(21/2)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^3\,x^{21/2}}{21} + \frac{2\,c^3\,x^{33/2}}{33} + \frac{6\,b^2\,c\,x^{25/2}}{25} + \frac{6\,b\,c^2\,x^{29/2}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] (2*b^3*x^(21/2))/21 + (2*c^3*x^(33/2))/33 + (6*b^2*c*x^(25/2))/25 + (6*b*c^
2*x^(29/2))/29
```

sympy [A] time = 83.18, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{21}{2}}}{21} + \frac{6b^2cx^{\frac{25}{2}}}{25} + \frac{6bc^2x^{\frac{29}{2}}}{29} + \frac{2c^3x^{\frac{33}{2}}}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*b**3*x**(21/2)/21 + 6*b**2*c*x**(25/2)/25 + 6*b*c**2*x**(29/2)/29 + 2*c**
3*x**(33/2)/33
```

3.309 $\int x^{5/2} \left(bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{2}{19}b^3x^{19/2}+\frac{6}{23}b^2cx^{23/2}+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

```
[Out] 2/19*b^3*x^(19/2)+6/23*b^2*c*x^(23/2)+2/9*b*c^2*x^(27/2)+2/31*c^3*x^(31/2)
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{6}{23}b^2cx^{23/2}+\frac{2}{19}b^3x^{19/2}+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(19/2))/19 + (6*b^2*c*x^(23/2))/23 + (2*b*c^2*x^(27/2))/9 + (2*c^3
*x^(31/2))/31
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int x^{5/2}\left(bx^2+cx^4\right)^3\,dx &= \int x^{17/2}\left(b+cx^2\right)^3\,dx\\
&= \int \left(b^3x^{17/2}+3b^2cx^{21/2}+3bc^2x^{25/2}+c^3x^{29/2}\right)dx\\
&= \frac{2}{19}b^3x^{19/2}+\frac{6}{23}b^2cx^{23/2}+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.00

$$\frac{2}{19}b^3x^{19/2}+\frac{6}{23}b^2cx^{23/2}+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(19/2))/19 + (6*b^2*c*x^(23/2))/23 + (2*b*c^2*x^(27/2))/9 + (2*c^3
*x^(31/2))/31
```

fricas [A] time = 0.67, size = 40, normalized size = 0.78

$$\frac{2}{121923}\left(3933\,c^3x^{15}+13547\,bc^2x^{13}+15903\,b^2cx^{11}+6417\,b^3x^9\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/121923*(3933*c^3*x^15 + 13547*b*c^2*x^13 + 15903*b^2*c*x^11 + 6417*b^3*x^
9)*sqrt(x)
```

giac [A] time = 0.17, size = 35, normalized size = 0.69

$$\frac{2}{31}\,c^3x^{\frac{31}{2}}+\frac{2}{9}\,bc^2x^{\frac{27}{2}}+\frac{6}{23}\,b^2cx^{\frac{23}{2}}+\frac{2}{19}\,b^3x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*b^2*c*x^(23/2) + 2/19*b^3*x^(
19/2)
```

maple [A] time = 0.00, size = 38, normalized size = 0.75

$$\frac{2\left(3933c^3x^6+13547b\,c^2x^4+15903b^2c\,x^2+6417b^3\right)x^{\frac{19}{2}}}{121923}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/121923*x^(19/2)*(3933*c^3*x^6+13547*b*c^2*x^4+15903*b^2*c*x^2+6417*b^3)
```

maxima [A] time = 1.35, size = 35, normalized size = 0.69

$$\frac{2}{31}\,c^{3}x^{\frac{31}{2}}+\frac{2}{9}\,bc^{2}x^{\frac{27}{2}}+\frac{6}{23}\,b^{2}cx^{\frac{23}{2}}+\frac{2}{19}\,b^{3}x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*b^2*c*x^(23/2) + 2/19*b^3*x^(
19/2)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^3\,x^{19/2}}{19}+\frac{2\,c^3\,x^{31/2}}{31}+\frac{6\,b^2\,c\,x^{23/2}}{23}+\frac{2\,b\,c^2\,x^{27/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] (2*b^3*x^(19/2))/19 + (2*c^3*x^(31/2))/31 + (6*b^2*c*x^(23/2))/23 + (2*b*c^
2*x^(27/2))/9
```

sympy [A] time = 53.38, size = 49, normalized size = 0.96

$$\frac{2b^{3}x^{\frac{19}{2}}}{19}+\frac{6b^{2}cx^{\frac{23}{2}}}{23}+\frac{2bc^{2}x^{\frac{27}{2}}}{9}+\frac{2c^{3}x^{\frac{31}{2}}}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*b**3*x**(19/2)/19 + 6*b**2*c*x**(23/2)/23 + 2*b*c**2*x**(27/2)/9 + 2*c**3
*x**(31/2)/31
```

3.310 $\int x^{3/2} \left(bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{2}{17}b^3x^{17/2}+\frac{2}{7}b^2cx^{21/2}+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

```
[Out] 2/17*b^3*x^(17/2)+2/7*b^2*c*x^(21/2)+6/25*b*c^2*x^(25/2)+2/29*c^3*x^(29/2)
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{7}b^2cx^{21/2}+\frac{2}{17}b^3x^{17/2}+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(17/2))/17 + (2*b^2*c*x^(21/2))/7 + (6*b*c^2*x^(25/2))/25 + (2*c^3
*x^(29/2))/29
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(bx^2+cx^4\right)^3\,dx &= \int x^{15/2}\left(b+cx^2\right)^3\,dx\\ &= \int \left(b^3x^{15/2}+3b^2cx^{19/2}+3bc^2x^{23/2}+c^3x^{27/2}\right)dx\\ &= \frac{2}{17}b^3x^{17/2}+\frac{2}{7}b^2cx^{21/2}+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.00

$$\frac{2}{17}b^3x^{17/2}+\frac{2}{7}b^2cx^{21/2}+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(17/2))/17 + (2*b^2*c*x^(21/2))/7 + (6*b*c^2*x^(25/2))/25 + (2*c^3
*x^(29/2))/29
```

fricas [A] time = 0.82, size = 40, normalized size = 0.78

$$\frac{2}{86275}\left(2975\,c^3x^{14}+10353\,bc^2x^{12}+12325\,b^2cx^{10}+5075\,b^3x^8\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/86275*(2975*c^3*x^14 + 10353*b*c^2*x^12 + 12325*b^2*c*x^10 + 5075*b^3*x^8
)*sqrt(x)
```

giac [A] time = 0.15, size = 35, normalized size = 0.69

$$\frac{2}{29}\,c^3x^{\frac{29}{2}}+\frac{6}{25}\,bc^2x^{\frac{25}{2}}+\frac{2}{7}\,b^2cx^{\frac{21}{2}}+\frac{2}{17}\,b^3x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*b^2*c*x^(21/2) + 2/17*b^3*x^(
17/2)
```

maple [A] time = 0.00, size = 38, normalized size = 0.75

$$\frac{2\left(2975c^3x^6+10353b\,c^2x^4+12325b^2c\,x^2+5075b^3\right)x^{\frac{17}{2}}}{86275}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/86275*x^(17/2)*(2975*c^3*x^6+10353*b*c^2*x^4+12325*b^2*c*x^2+5075*b^3)
```

maxima [A] time = 1.31, size = 35, normalized size = 0.69

$$\frac{2}{29}\,c^{3}x^{\frac{29}{2}}+\frac{6}{25}\,bc^{2}x^{\frac{25}{2}}+\frac{2}{7}\,b^{2}cx^{\frac{21}{2}}+\frac{2}{17}\,b^{3}x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*b^2*c*x^(21/2) + 2/17*b^3*x^(
17/2)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^{3}\,x^{17/2}}{17}+\frac{2\,c^{3}\,x^{29/2}}{29}+\frac{2\,b^{2}\,c\,x^{21/2}}{7}+\frac{6\,b\,c^{2}\,x^{25/2}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] (2*b^3*x^(17/2))/17 + (2*c^3*x^(29/2))/29 + (2*b^2*c*x^(21/2))/7 + (6*b*c^2
*x^(25/2))/25
```

sympy [A] time = 33.34, size = 49, normalized size = 0.96

$$\frac{2b^{3}x^{\frac{17}{2}}}{17}+\frac{2b^{2}cx^{\frac{21}{2}}}{7}+\frac{6bc^{2}x^{\frac{25}{2}}}{25}+\frac{2c^{3}x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*b**3*x**(17/2)/17 + 2*b**2*c*x**(21/2)/7 + 6*b*c**2*x**(25/2)/25 + 2*c**3
*x**(29/2)/29
```

3.311 $\int \sqrt{x}\left(bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=51

$$\frac{2}{15}b^3x^{15/2}+\frac{6}{19}b^2cx^{19/2}+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

```
[Out] 2/15*b^3*x^(15/2)+6/19*b^2*c*x^(19/2)+6/23*b*c^2*x^(23/2)+2/27*c^3*x^(27/2)
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{6}{19}b^2cx^{19/2}+\frac{2}{15}b^3x^{15/2}+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(15/2))/15 + (6*b^2*c*x^(19/2))/19 + (6*b*c^2*x^(23/2))/23 + (2*c^
3*x^(27/2))/27
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(bx^2+cx^4\right)^3\,dx &= \int x^{13/2}\left(b+cx^2\right)^3\,dx\\ &= \int \left(b^3x^{13/2}+3b^2cx^{17/2}+3bc^2x^{21/2}+c^3x^{25/2}\right)dx\\ &= \frac{2}{15}b^3x^{15/2}+\frac{6}{19}b^2cx^{19/2}+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.80

$$\frac{2x^{15/2}\left(3933b^3+9315b^2cx^2+7695bc^2x^4+2185c^3x^6\right)}{58995}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*x^(15/2)*(3933*b^3 + 9315*b^2*c*x^2 + 7695*b*c^2*x^4 + 2185*c^3*x^6))/58
995
```

fricas [A] time = 0.80, size = 40, normalized size = 0.78

$$\frac{2}{58995}\left(2185\,c^3x^{13}+7695\,bc^2x^{11}+9315\,b^2cx^9+3933\,b^3x^7\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/58995*(2185*c^3*x^13 + 7695*b*c^2*x^11 + 9315*b^2*c*x^9 + 3933*b^3*x^7)*s
qrt(x)
```

giac [A] time = 0.15, size = 35, normalized size = 0.69

$$\frac{2}{27}\,c^3x^{\frac{27}{2}}+\frac{6}{23}\,bc^2x^{\frac{23}{2}}+\frac{6}{19}\,b^2cx^{\frac{19}{2}}+\frac{2}{15}\,b^3x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*b^2*c*x^(19/2) + 2/15*b^3*x^
(15/2)
```

maple [A] time = 0.01, size = 38, normalized size = 0.75

$$\frac{2\left(2185c^3x^6+7695b\,c^2x^4+9315b^2c\,x^2+3933b^3\right)x^{\frac{15}{2}}}{58995}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/58995*x^(15/2)*(2185*c^3*x^6+7695*b*c^2*x^4+9315*b^2*c*x^2+3933*b^3)
```

maxima [A] time = 1.33, size = 35, normalized size = 0.69

$$\frac{2}{27}c^3x^{\frac{27}{2}} + \frac{6}{23}bc^2x^{\frac{23}{2}} + \frac{6}{19}b^2cx^{\frac{19}{2}} + \frac{2}{15}b^3x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*b^2*c*x^(19/2) + 2/15*b^3*x^
(15/2)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^3\,x^{15/2}}{15} + \frac{2\,c^3\,x^{27/2}}{27} + \frac{6\,b^2\,c\,x^{19/2}}{19} + \frac{6\,b\,c^2\,x^{23/2}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] (2*b^3*x^(15/2))/15 + (2*c^3*x^(27/2))/27 + (6*b^2*c*x^(19/2))/19 + (6*b*c^
2*x^(23/2))/23
```

sympy [A] time = 4.32, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{15}{2}}}{15} + \frac{6b^2cx^{\frac{19}{2}}}{19} + \frac{6bc^2x^{\frac{23}{2}}}{23} + \frac{2c^3x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*b**3*x**(15/2)/15 + 6*b**2*c*x**(19/2)/19 + 6*b*c**2*x**(23/2)/23 + 2*c**
3*x**(27/2)/27
```

3.312 $\int \frac{(bx^2+cx^4)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=51

$$\frac{2}{13}b^3x^{13/2}+\frac{6}{17}b^2cx^{17/2}+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

```
[Out] 2/13*b^3*x^(13/2)+6/17*b^2*c*x^(17/2)+2/7*b*c^2*x^(21/2)+2/25*c^3*x^(25/2)
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{6}{17}b^2cx^{17/2}+\frac{2}{13}b^3x^{13/2}+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/Sqrt[x],x]
```

```
[Out] (2*b^3*x^(13/2))/13 + (6*b^2*c*x^(17/2))/17 + (2*b*c^2*x^(21/2))/7 + (2*c^3
*x^(25/2))/25
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2+cx^4\right)^3}{\sqrt{x}}\,dx &= \int x^{11/2}\left(b+cx^2\right)^3\,dx\\ &= \int \left(b^3x^{11/2}+3b^2cx^{15/2}+3bc^2x^{19/2}+c^3x^{23/2}\right)dx\\ &= \frac{2}{13}b^3x^{13/2}+\frac{6}{17}b^2cx^{17/2}+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.80

$$\frac{2x^{13/2}\left(2975b^3+6825b^2cx^2+5525bc^2x^4+1547c^3x^6\right)}{38675}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/Sqrt[x],x]
```

```
[Out] (2*x^(13/2)*(2975*b^3 + 6825*b^2*c*x^2 + 5525*b*c^2*x^4 + 1547*c^3*x^6))/38
675
```

fricas [A] time = 0.91, size = 40, normalized size = 0.78

$$\frac{2}{38675}\left(1547\,c^3x^{12}+5525\,bc^2x^{10}+6825\,b^2cx^8+2975\,b^3x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/38675*(1547*c^3*x^12 + 5525*b*c^2*x^10 + 6825*b^2*c*x^8 + 2975*b^3*x^6)*s
qrt(x)
```

giac [A] time = 0.15, size = 35, normalized size = 0.69

$$\frac{2}{25}\,c^3x^{\frac{25}{2}}+\frac{2}{7}\,bc^2x^{\frac{21}{2}}+\frac{6}{17}\,b^2cx^{\frac{17}{2}}+\frac{2}{13}\,b^3x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 2/13*b^3*x^(
13/2)
```

maple [A] time = 0.00, size = 38, normalized size = 0.75

$$\frac{2\left(1547c^3x^6+5525b\,c^2x^4+6825b^2c\,x^2+2975b^3\right)x^{\frac{13}{2}}}{38675}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^(1/2),x)
```

```
[Out] 2/38675*x^(13/2)*(1547*c^3*x^6+5525*b*c^2*x^4+6825*b^2*c*x^2+2975*b^3)
```

maxima [A] time = 1.31, size = 35, normalized size = 0.69

$$\frac{2}{25}\,c^3x^{\frac{25}{2}} + \frac{2}{7}\,bc^2x^{\frac{21}{2}} + \frac{6}{17}\,b^2cx^{\frac{17}{2}} + \frac{2}{13}\,b^3x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 2/13*b^3*x^(
13/2)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^3\,x^{13/2}}{13} + \frac{2\,c^3\,x^{25/2}}{25} + \frac{6\,b^2\,c\,x^{17/2}}{17} + \frac{2\,b\,c^2\,x^{21/2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^(1/2),x)
```

```
[Out] (2*b^3*x^(13/2))/13 + (2*c^3*x^(25/2))/25 + (6*b^2*c*x^(17/2))/17 + (2*b*c^
2*x^(21/2))/7
```

sympy [A] time = 17.61, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{13}{2}}}{13} + \frac{6b^2cx^{\frac{17}{2}}}{17} + \frac{2bc^2x^{\frac{21}{2}}}{7} + \frac{2c^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**(1/2),x)
```

```
[Out] 2*b**3*x**(13/2)/13 + 6*b**2*c*x**(17/2)/17 + 2*b*c**2*x**(21/2)/7 + 2*c**3
*x**(25/2)/25
```

3.313 $\int \frac{(bx^2+cx^4)^3}{x^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{2}{11}b^3x^{11/2}+\frac{2}{5}b^2cx^{15/2}+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}$$

```
[Out] 2/11*b^3*x^(11/2)+2/5*b^2*c*x^(15/2)+6/19*b*c^2*x^(19/2)+2/23*c^3*x^(23/2)
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{5}b^2cx^{15/2}+\frac{2}{11}b^3x^{11/2}+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^(3/2),x]
```

```
[Out] (2*b^3*x^(11/2))/11 + (2*b^2*c*x^(15/2))/5 + (6*b*c^2*x^(19/2))/19 + (2*c^3
*x^(23/2))/23
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2+cx^4\right)^3}{x^{3/2}}\,dx &= \int x^{9/2}\left(b+cx^2\right)^3\,dx\\ &= \int \left(b^3x^{9/2}+3b^2cx^{13/2}+3bc^2x^{17/2}+c^3x^{21/2}\right)dx\\ &= \frac{2}{11}b^3x^{11/2}+\frac{2}{5}b^2cx^{15/2}+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.80

$$\frac{2x^{11/2}\left(2185b^3 + 4807b^2cx^2 + 3795bc^2x^4 + 1045c^3x^6\right)}{24035}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^(3/2),x]
[Out] (2*x^(11/2)*(2185*b^3 + 4807*b^2*c*x^2 + 3795*b*c^2*x^4 + 1045*c^3*x^6))/24
035
```

fricas [A] time = 0.85, size = 40, normalized size = 0.78

$$\frac{2}{24035}\left(1045\,c^3x^{11} + 3795\,bc^2x^9 + 4807\,b^2cx^7 + 2185\,b^3x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(3/2),x, algorithm="fricas")
[Out] 2/24035*(1045*c^3*x^11 + 3795*b*c^2*x^9 + 4807*b^2*c*x^7 + 2185*b^3*x^5)*sq
rt(x)
```

giac [A] time = 0.15, size = 35, normalized size = 0.69

$$\frac{2}{23}\,c^3x^{\frac{23}{2}} + \frac{6}{19}\,bc^2x^{\frac{19}{2}} + \frac{2}{5}\,b^2cx^{\frac{15}{2}} + \frac{2}{11}\,b^3x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(3/2),x, algorithm="giac")
[Out] 2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*b^2*c*x^(15/2) + 2/11*b^3*x^(
11/2)
```

maple [A] time = 0.01, size = 38, normalized size = 0.75

$$\frac{2\left(1045c^3x^6 + 3795b\,c^2x^4 + 4807b^2c\,x^2 + 2185b^3\right)x^{\frac{11}{2}}}{24035}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^(3/2),x)
[Out] 2/24035*x^(11/2)*(1045*c^3*x^6+3795*b*c^2*x^4+4807*b^2*c*x^2+2185*b^3)
```

maxima [A] time = 1.31, size = 35, normalized size = 0.69

$$\frac{2}{23}\,c^{3}x^{\frac{23}{2}}+\frac{6}{19}\,bc^{2}x^{\frac{19}{2}}+\frac{2}{5}\,b^{2}cx^{\frac{15}{2}}+\frac{2}{11}\,b^{3}x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*b^2*c*x^(15/2) + 2/11*b^3*x^(
11/2)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^{3}\,x^{11/2}}{11}+\frac{2\,c^{3}\,x^{23/2}}{23}+\frac{2\,b^{2}\,c\,x^{15/2}}{5}+\frac{6\,b\,c^{2}\,x^{19/2}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^(3/2),x)
```

```
[Out] (2*b^3*x^(11/2))/11 + (2*c^3*x^(23/2))/23 + (2*b^2*c*x^(15/2))/5 + (6*b*c^2
*x^(19/2))/19
```

sympy [A] time = 19.94, size = 49, normalized size = 0.96

$$\frac{2b^{3}x^{\frac{11}{2}}}{11}+\frac{2b^{2}cx^{\frac{15}{2}}}{5}+\frac{6bc^{2}x^{\frac{19}{2}}}{19}+\frac{2c^{3}x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**(3/2),x)
```

```
[Out] 2*b**3*x**(11/2)/11 + 2*b**2*c*x**(15/2)/5 + 6*b*c**2*x**(19/2)/19 + 2*c**3
*x**(23/2)/23
```

3.314 $\int \frac{(bx^2+cx^4)^3}{x^{5/2}} dx$

Optimal. Leaf size=51

$$\frac{2}{9}b^3x^{9/2} + \frac{6}{13}b^2cx^{13/2} + \frac{6}{17}bc^2x^{17/2} + \frac{2}{21}c^3x^{21/2}$$

```
[Out] 2/9*b^3*x^(9/2)+6/13*b^2*c*x^(13/2)+6/17*b*c^2*x^(17/2)+2/21*c^3*x^(21/2)
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{6}{13}b^2cx^{13/2} + \frac{2}{9}b^3x^{9/2} + \frac{6}{17}bc^2x^{17/2} + \frac{2}{21}c^3x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^(5/2),x]
```

```
[Out] (2*b^3*x^(9/2))/9 + (6*b^2*c*x^(13/2))/13 + (6*b*c^2*x^(17/2))/17 + (2*c^3*
x^(21/2))/21
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^3}{x^{5/2}} dx &= \int x^{7/2}\left(b+cx^2\right)^3 dx \\
&= \int \left(b^3x^{7/2} + 3b^2cx^{11/2} + 3bc^2x^{15/2} + c^3x^{19/2}\right) dx \\
&= \frac{2}{9}b^3x^{9/2} + \frac{6}{13}b^2cx^{13/2} + \frac{6}{17}bc^2x^{17/2} + \frac{2}{21}c^3x^{21/2}
\end{aligned}$$

Mathematica [A]   time = 0.01, size = 41, normalized size = 0.80

$$\frac{2x^{9/2}\left(1547b^3+3213b^2cx^2+2457bc^2x^4+663c^3x^6\right)}{13923}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^(5/2),x]
```

```
[Out] (2*x^(9/2)*(1547*b^3 + 3213*b^2*c*x^2 + 2457*b*c^2*x^4 + 663*c^3*x^6))/1392
3
```

fricas [A]   time = 0.57, size = 40, normalized size = 0.78

$$\frac{2}{13923}\left(663\,c^3x^{10}+2457\,bc^2x^8+3213\,b^2cx^6+1547\,b^3x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*c^3*x^10 + 2457*b*c^2*x^8 + 3213*b^2*c*x^6 + 1547*b^3*x^4)*sqr
t(x)
```

giac [A]   time = 0.15, size = 35, normalized size = 0.69

$$\frac{2}{21}\,c^3x^{\frac{21}{2}}+\frac{6}{17}\,bc^2x^{\frac{17}{2}}+\frac{6}{13}\,b^2cx^{\frac{13}{2}}+\frac{2}{9}\,b^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*b^2*c*x^(13/2) + 2/9*b^3*x^(
9/2)
```

maple [A]   time = 0.01, size = 38, normalized size = 0.75

$$\frac{2\left(663c^3x^6+2457b\,c^2x^4+3213b^2c\,x^2+1547b^3\right)x^{\frac{9}{2}}}{13923}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^(5/2),x)
```

```
[Out] 2/13923*x^(9/2)*(663*c^3*x^6+2457*b*c^2*x^4+3213*b^2*c*x^2+1547*b^3)
```

maxima [A] time = 1.35, size = 35, normalized size = 0.69

$$\frac{2}{21}\,c^3x^{\frac{21}{2}}+\frac{6}{17}\,bc^2x^{\frac{17}{2}}+\frac{6}{13}\,b^2cx^{\frac{13}{2}}+\frac{2}{9}\,b^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*b^2*c*x^(13/2) + 2/9*b^3*x^(
9/2)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^3\,x^{9/2}}{9}+\frac{2\,c^3\,x^{21/2}}{21}+\frac{6\,b^2\,c\,x^{13/2}}{13}+\frac{6\,b\,c^2\,x^{17/2}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^(5/2),x)
```

```
[Out] (2*b^3*x^(9/2))/9 + (2*c^3*x^(21/2))/21 + (6*b^2*c*x^(13/2))/13 + (6*b*c^2*
x^(17/2))/17
```

sympy [A] time = 22.20, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{9}{2}}}{9}+\frac{6b^2cx^{\frac{13}{2}}}{13}+\frac{6bc^2x^{\frac{17}{2}}}{17}+\frac{2c^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**(5/2),x)
```

```
[Out] 2*b**3*x**(9/2)/9 + 6*b**2*c*x**(13/2)/13 + 6*b*c**2*x**(17/2)/17 + 2*c**3*
x**(21/2)/21
```

3.315 $\int \frac{\left(bx^2+cx^4\right)^3}{x^{7/2}}\,dx$

Optimal. Leaf size=51

$$\frac{2}{7}b^3x^{7/2}+\frac{6}{11}b^2cx^{11/2}+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

```
[Out] 2/7*b^3*x^(7/2)+6/11*b^2*c*x^(11/2)+2/5*b*c^2*x^(15/2)+2/19*c^3*x^(19/2)
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{6}{11}b^2cx^{11/2}+\frac{2}{7}b^3x^{7/2}+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^(7/2),x]
```

```
[Out] (2*b^3*x^(7/2))/7 + (6*b^2*c*x^(11/2))/11 + (2*b*c^2*x^(15/2))/5 + (2*c^3*x
^(19/2))/19
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2+cx^4\right)^3}{x^{7/2}}\,dx &= \int x^{5/2}\left(b+cx^2\right)^3\,dx\\ &= \int \left(b^3x^{5/2}+3b^2cx^{9/2}+3bc^2x^{13/2}+c^3x^{17/2}\right)dx\\ &= \frac{2}{7}b^3x^{7/2}+\frac{6}{11}b^2cx^{11/2}+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.80

$$\frac{2x^{7/2}\left(1045b^3+1995b^2cx^2+1463bc^2x^4+385c^3x^6\right)}{7315}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^(7/2),x]
[Out] (2*x^(7/2)*(1045*b^3 + 1995*b^2*c*x^2 + 1463*b*c^2*x^4 + 385*c^3*x^6))/7315
```

fricas [A] time = 0.82, size = 40, normalized size = 0.78

$$\frac{2}{7315}\left(385\,c^3x^9+1463\,bc^2x^7+1995\,b^2cx^5+1045\,b^3x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(7/2),x, algorithm="fricas")
[Out] 2/7315*(385*c^3*x^9 + 1463*b*c^2*x^7 + 1995*b^2*c*x^5 + 1045*b^3*x^3)*sqrt(
x)
```

giac [A] time = 0.18, size = 35, normalized size = 0.69

$$\frac{2}{19}\,c^3x^{\frac{19}{2}}+\frac{2}{5}\,bc^2x^{\frac{15}{2}}+\frac{6}{11}\,b^2cx^{\frac{11}{2}}+\frac{2}{7}\,b^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(7/2),x, algorithm="giac")
[Out] 2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*b^2*c*x^(11/2) + 2/7*b^3*x^(7
/2)
```

maple [A] time = 0.00, size = 38, normalized size = 0.75

$$\frac{2\left(385c^3x^6+1463b\,c^2x^4+1995b^2c\,x^2+1045b^3\right)x^{\frac{7}{2}}}{7315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^(7/2),x)
[Out] 2/7315*x^(7/2)*(385*c^3*x^6+1463*b*c^2*x^4+1995*b^2*c*x^2+1045*b^3)
```

maxima [A] time = 1.37, size = 35, normalized size = 0.69

$$\frac{2}{19}\,c^3x^{\frac{19}{2}}+\frac{2}{5}\,bc^2x^{\frac{15}{2}}+\frac{6}{11}\,b^2cx^{\frac{11}{2}}+\frac{2}{7}\,b^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*b^2*c*x^(11/2) + 2/7*b^3*x^(7
/2)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^3\,x^{7/2}}{7}+\frac{2\,c^3\,x^{19/2}}{19}+\frac{6\,b^2\,c\,x^{11/2}}{11}+\frac{2\,b\,c^2\,x^{15/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^(7/2),x)
```

```
[Out] (2*b^3*x^(7/2))/7 + (2*c^3*x^(19/2))/19 + (6*b^2*c*x^(11/2))/11 + (2*b*c^2*
x^(15/2))/5
```

sympy [A] time = 26.50, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{7}{2}}}{7}+\frac{6b^2cx^{\frac{11}{2}}}{11}+\frac{2bc^2x^{\frac{15}{2}}}{5}+\frac{2c^3x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**(7/2),x)
```

```
[Out] 2*b**3*x**(7/2)/7 + 6*b**2*c*x**(11/2)/11 + 2*b*c**2*x**(15/2)/5 + 2*c**3*x
**(19/2)/19
```

3.316 $\int \frac{x^{13/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=217

$$\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{11/4}}-\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{11/4}}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{11/4}}+\frac{b^{7/4}}{}$$

```
[Out] -2/3*b*x^(3/2)/c^2+2/7*x^(7/2)/c-1/2*b^(7/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/
2)/b^(1/4))/c^(11/4)*2^(1/2)+1/2*b^(7/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b
^(1/4))/c^(11/4)*2^(1/2)+1/4*b^(7/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2
^(1/2)*x^(1/2))/c^(11/4)*2^(1/2)-1/4*b^(7/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c
^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)*2^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{11/4}}-\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{11/4}}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{11/4}}+\frac{b^{7/4}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*b*x^(3/2))/(3*c^2) + (2*x^(7/2))/(7*c) - (b^(7/4)*ArcTan[1 - (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(11/4)) + (b^(7/4)*ArcTan[1 + (Sqrt[2]
*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(11/4)) + (b^(7/4)*Log[Sqrt[b] - Sqr
t[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(11/4)) - (b^(7/4)*
Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(1
1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{13/2}}{bx^2+cx^4}\,dx = \int \frac{x^{9/2}}{b+cx^2}\,dx$$

$$= \frac{2x^{7/2}}{7c} - \frac{b\int \frac{x^{5/2}}{b+cx^2}\,dx}{c}$$

$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} + \frac{b^2\int \frac{\sqrt{x}}{b+cx^2}\,dx}{c^2}$$

$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} + \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2}$$

$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} - \frac{b^2\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{5/2}} + \frac{b^2\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{5/2}}$$

$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^3} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{}\right.}{2c^3}$$

$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} + \frac{b^{7/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{11/4}} - \frac{b^{7/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{2\sqrt{2}\,c^{11/4}}$$

$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} - \frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{11/4}} + \frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{11/4}} + \frac{b^{7/4}\log\left(\sqrt{b}-\sqrt{2}\right.}{2\sqrt{2}}$$

Mathematica [A] time = 0.04, size = 89, normalized size = 0.41

$$\frac{2c^{3/4}x^{3/2}\left(3cx^2-7b\right)+21(-b)^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)+21b(-b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{21c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (2*c^(3/4)*x^(3/2)*(-7*b + 3*c*x^2) + 21*(-b)^(7/4)*ArcTan[(c^(1/4)*Sqrt[x]
)/(-b)^(1/4)] + 21*(-b)^(3/4)*b*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/(21*
c^(11/4))
```

fricas [A] time = 0.85, size = 182, normalized size = 0.84

$$-\frac{84\,c^2\left(-\frac{b^7}{c^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^5c^3\sqrt{x}\left(-\frac{b^7}{c^{11}}\right)^{\frac{1}{4}}-\sqrt{-b^7c^5\sqrt{-\frac{b^7}{c^{11}}}+b^{10}x}\,c^3\left(-\frac{b^7}{c^{11}}\right)^{\frac{1}{4}}}{b^7}\right)-21\,c^2\left(-\frac{b^7}{c^{11}}\right)^{\frac{1}{4}}\log\left(c^8\left(-\frac{b^7}{c^{11}}\right)^{\frac{3}{4}}+b^5\sqrt{x}\right)+21\,c}{42\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/42*(84*c^2*(-b^7/c^11)^(1/4)*arctan(-(b^5*c^3*sqrt(x)*(-b^7/c^11)^(1/4)
- sqrt(-b^7*c^5*sqrt(-b^7/c^11) + b^10*x)*c^3*(-b^7/c^11)^(1/4))/b^7) - 21*
c^2*(-b^7/c^11)^(1/4)*log(c^8*(-b^7/c^11)^(3/4) + b^5*sqrt(x)) + 21*c^2*(-b
^7/c^11)^(1/4)*log(-c^8*(-b^7/c^11)^(3/4) + b^5*sqrt(x)) - 4*(3*c*x^3 - 7*b
*x)*sqrt(x))/c^2
```

giac [A] time = 0.17, size = 197, normalized size = 0.91

$$\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^5}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^5}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}b\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}\right.}{4\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/c^5 + 1/2*sqrt(2)*(b*c^3)^(3/4)*b*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^5 - 1/4*sqrt(2)*(b*c^3)^(3/4
)*b*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 + 1/4*sqrt(2)*(b*c
^3)^(3/4)*b*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 + 2/21*(3
*c^6*x^(7/2) - 7*b*c^5*x^(3/2))/c^7
```

maple [A] time = 0.01, size = 158, normalized size = 0.73

$$\frac{2x^{\frac{7}{2}}}{7c}-\frac{2b\,x^{\frac{3}{2}}}{3c^{2}}+\frac{\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{3}}+\frac{\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{3}}+\frac{\sqrt{2}\,b^{2}\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2),x)
```

```
[Out] 2/7*x^(7/2)/c-2/3*b*x^(3/2)/c^2+1/4*b^2/c^3/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)
^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1
/2)))+1/2*b^2/c^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)
+1/2*b^2/c^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.01, size = 198, normalized size = 0.91

$$\frac{b^{2}\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\,\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\,\sqrt{\cdots}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{4\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*
sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqr
t(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4
)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-s
qrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/c^
2 + 2/21*(3*c*x^(7/2) - 7*b*x^(3/2))/c^2
```

mupad [B] time = 0.11, size = 66, normalized size = 0.30

$$\frac{2\,x^{7/2}}{7\,c}-\frac{2\,b\,x^{3/2}}{3\,c^{2}}+\frac{(-b)^{7/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{c^{11/4}}+\frac{(-b)^{7/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1\mathrm{i}}{(-b)^{1/4}}\right)1\mathrm{i}}{c^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(7/2))/(7*c) - (2*b*x^(3/2))/(3*c^2) + ((-b)^(7/4)*atan((c^(1/4)*x^(1/
2))/(-b)^(1/4)))/c^(11/4) + ((-b)^(7/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4
))*1i)/c^(11/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

3.317 $\int \frac{x^{11/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=215

$$-\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{9/4}}+\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{9/4}}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{9/4}}+\frac{b^{5}}{}$$

```
[Out] 2/5*x^(5/2)/c-1/2*b^(5/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(9/4)
*2^(1/2)+1/2*b^(5/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(9/4)*2^(1
/2)-1/4*b^(5/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(9/
4)*2^(1/2)+1/4*b^(5/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2)
)/c^(9/4)*2^(1/2)-2*b*x^(1/2)/c^2
```

Rubi [A] time = 0.19, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{9/4}}+\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{9/4}}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{9/4}}+\frac{b^{5}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*b*Sqrt[x])/c^2 + (2*x^(5/2))/(5*c) - (b^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)) + (b^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)) - (b^(5/4)*Log[Sqrt[b] - Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(9/4)) + (b^(5/4)*Log[Sqr
t[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
```

```
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{11/2}}{bx^2+cx^4}\,dx = \int \frac{x^{7/2}}{b+cx^2}\,dx$$
$$= \frac{2x^{5/2}}{5c} - \frac{b\int \frac{x^{3/2}}{b+cx^2}\,dx}{c}$$
$$= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} + \frac{b^2\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{c^2}$$
$$= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} + \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2}$$
$$= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} + \frac{b^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2} + \frac{b^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2}$$
$$= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} + \frac{b^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^{5/2}} + \frac{b^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x\right.}{2c^{5/2}}$$
$$= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} - \frac{b^{5/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{9/4}} + \frac{b^{5/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{2\sqrt{2}\,c^{9/4}}$$
$$= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} - \frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{9/4}} + \frac{b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{9/4}} - \frac{b^{5/4}\log\left(\sqrt{b}-\sqrt{2}\right.}{2\sqrt{}}$$

Mathematica [A] time = 0.06, size = 203, normalized size = 0.94

$$\frac{-5\sqrt{2}\,b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)+5\sqrt{2}\,b^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)-10\sqrt{2}\,b^{5/4}\tan^{-1}\left(1\right.}{20c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (-40*b*c^(1/4)*Sqrt[x] + 8*c^(5/4)*x^(5/2) - 10*Sqrt[2]*b^(5/4)*ArcTan[1 -
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 10*Sqrt[2]*b^(5/4)*ArcTan[1 + (Sqrt[2]
```

```
*c^(1/4)*Sqrt[x])/b^(1/4)] - 5*Sqrt[2]*b^(5/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4
)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 5*Sqrt[2]*b^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^
(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(20*c^(9/4))
```

fricas [A] time = 0.56, size = 170, normalized size = 0.79

$$\frac{20\,c^2\left(-\frac{b^5}{c^9}\right)^{\frac14}\arctan\left(-\frac{bc^7\sqrt{x}\left(-\frac{b^5}{c^9}\right)^{\frac34}-\sqrt{c^4\sqrt{-\frac{b^5}{c^9}}+b^2x}\,c^7\left(-\frac{b^5}{c^9}\right)^{\frac34}}{b^5}\right)+5\,c^2\left(-\frac{b^5}{c^9}\right)^{\frac14}\log\left(c^2\left(-\frac{b^5}{c^9}\right)^{\frac14}+b\sqrt{x}\right)-5\,c^2\left(-\frac{b^5}{c^9}\right)^{\frac14}\log}{10\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/10*(20*c^2*(-b^5/c^9)^(1/4)*arctan(-(b*c^7*sqrt(x)*(-b^5/c^9)^(3/4) - sqr
t(c^4*sqrt(-b^5/c^9) + b^2*x)*c^7*(-b^5/c^9)^(3/4))/b^5) + 5*c^2*(-b^5/c^9)
^(1/4)*log(c^2*(-b^5/c^9)^(1/4) + b*sqrt(x)) - 5*c^2*(-b^5/c^9)^(1/4)*log(-
c^2*(-b^5/c^9)^(1/4) + b*sqrt(x)) + 4*(c*x^2 - 5*b)*sqrt(x))/c^2
```

giac [A] time = 0.17, size = 196, normalized size = 0.91

$$\frac{\sqrt{2}\left(bc^3\right)^{\frac14}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac14}}\right)}{2\,c^3}+\frac{\sqrt{2}\left(bc^3\right)^{\frac14}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac14}}\right)}{2\,c^3}+\frac{\sqrt{2}\left(bc^3\right)^{\frac14}b\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac14}\right.}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/c^3 + 1/2*sqrt(2)*(b*c^3)^(1/4)*b*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^3 + 1/4*sqrt(2)*(b*c^3)^(1/4
)*b*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 - 1/4*sqrt(2)*(b*c
^3)^(1/4)*b*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 + 2/5*(c^
4*x^(5/2) - 5*b*c^3*sqrt(x))/c^5
```

maple [A] time = 0.01, size = 152, normalized size = 0.71

$$\frac{2x^{\frac52}}{5c}+\frac{\left(\frac{b}{c}\right)^{\frac14}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac14}}-1\right)}{2c^2}+\frac{\left(\frac{b}{c}\right)^{\frac14}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac14}}+1\right)}{2c^2}+\frac{\left(\frac{b}{c}\right)^{\frac14}\sqrt{2}\,b\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac14}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac14}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4c^2}-\frac{2b\sqrt{x}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2),x)
```

```
[Out] 2/5*x^(5/2)/c-2*b*x^(1/2)/c^2+1/4*b/c^2*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/
4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))
)+1/2*b/c^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2*b
/c^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 2.97, size = 194, normalized size = 0.90

$$\frac{2\left(cx^{\frac{5}{2}}-5\,b\sqrt{x}\right)}{5\,c^2}+\frac{\frac{2\sqrt{2}\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\,b^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\,b^{\frac{5}{4}}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\cdots\right.}{c^{\frac{1}{4}}}}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2/5*(c*x^(5/2) - 5*b*sqrt(x))/c^2 + 1/4*(2*sqrt(2)*b^(3/2)*arctan(1/2*sqrt(
2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqr
t(sqrt(b)*sqrt(c)) + 2*sqrt(2)*b^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)
*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c))
+ sqrt(2)*b^(5/4)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b)
)/c^(1/4) - sqrt(2)*b^(5/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*
x + sqrt(b))/c^(1/4))/c^2
```

mupad [B] time = 4.49, size = 67, normalized size = 0.31

$$\frac{2\,x^{5/2}}{5\,c}-\frac{2\,b\,\sqrt{x}}{c^2}-\frac{{\left(-b\right)}^{5/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{c^{9/4}}+\frac{{\left(-b\right)}^{5/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-b\right)}^{1/4}}\right)\,1{}\mathrm{i}}{c^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(5/2))/(5*c) - (2*b*x^(1/2))/c^2 - ((-b)^(5/4)*atan((c^(1/4)*x^(1/2))/
(-b)^(1/4)))/c^(9/4) + ((-b)^(5/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*1i
)/c^(9/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

3.318 $\int \frac{x^{9/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=204

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{7/4}}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{7/4}}+\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{7/4}}-\cdots$$

```
[Out] 2/3*x^(3/2)/c+1/2*b^(3/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(7/4)
*2^(1/2)-1/2*b^(3/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(7/4)*2^(1
/2)-1/4*b^(3/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(7/
4)*2^(1/2)+1/4*b^(3/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2)
)/c^(7/4)*2^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{7/4}}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{7/4}}+\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{7/4}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(3/2))/(3*c) + (b^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])
/(Sqrt[2]*c^(7/4)) - (b^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
)/(Sqrt[2]*c^(7/4)) - (b^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x])/(2*Sqrt[2]*c^(7/4)) + (b^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4
)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
```

```
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{9/2}}{bx^2+cx^4}\,dx = \int \frac{x^{5/2}}{b+cx^2}\,dx$$

$$= \frac{2x^{3/2}}{3c} - \frac{b\int \frac{\sqrt{x}}{b+cx^2}\,dx}{c}$$

$$= \frac{2x^{3/2}}{3c} - \frac{(2b)\,\mathrm{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c}$$

$$= \frac{2x^{3/2}}{3c} + \frac{b\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{3/2}} - \frac{b\,\mathrm{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{3/2}}$$

$$= \frac{2x^{3/2}}{3c} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^2} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^2} - \frac{b^{3/4}\,\mathrm{Su}}{}$$

$$= \frac{2x^{3/2}}{3c} - \frac{b^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{7/4}} + \frac{b^{3/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{7/4}} - \frac{b^{3/}}{}$$

$$= \frac{2x^{3/2}}{3c} + \frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{7/4}} - \frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{7/4}} - \frac{b^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}\right.}{2\sqrt{2}\,c^{7/4}}$$

Mathematica [A] time = 0.02, size = 78, normalized size = 0.38

$$\frac{(-b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{c^{7/4}} - \frac{(-b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{c^{7/4}} + \frac{2x^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(3/2))/(3*c) + ((-b)^(3/4)*ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/c^(7/
4) - ((-b)^(3/4)*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/c^(7/4)
```

fricas [A] time = 0.74, size = 165, normalized size = 0.81

$$\frac{12\,c\left(-\frac{b^3}{c^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^2c^2\sqrt{x}\left(-\frac{b^3}{c^7}\right)^{\frac{1}{4}}-\sqrt{-b^3c^3\sqrt{-\frac{b^3}{c^7}}+b^4x}\,c^2\left(-\frac{b^3}{c^7}\right)^{\frac{1}{4}}}{b^3}\right)-3\,c\left(-\frac{b^3}{c^7}\right)^{\frac{1}{4}}\log\left(c^5\left(-\frac{b^3}{c^7}\right)^{\frac{3}{4}}+b^2\sqrt{x}\right)+3\,c\left(-\frac{b^3}{c^7}\right)^{\frac{1}{4}}\log}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*(12*c*(-b^3/c^7)^(1/4)*arctan(-(b^2*c^2*sqrt(x)*(-b^3/c^7)^(1/4) - sqrt
(-b^3*c^3*sqrt(-b^3/c^7) + b^4*x)*c^2*(-b^3/c^7)^(1/4))/b^3) - 3*c*(-b^3/c^
7)^(1/4)*log(c^5*(-b^3/c^7)^(3/4) + b^2*sqrt(x)) + 3*c*(-b^3/c^7)^(1/4)*log
(-c^5*(-b^3/c^7)^(3/4) + b^2*sqrt(x)) + 4*x^(3/2))/c
```

giac [A] time = 0.18, size = 178, normalized size = 0.87

$$\frac{2\,x^{\frac{3}{2}}}{3\,c}-\frac{\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^4}-\frac{\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^4}+\frac{\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{b}{c}\right.\right.}{4\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)/c - 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)
^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^4 - 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^4 + 1/4*sqrt(2)*
(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 - 1/4*sq
rt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4
```

maple [A] time = 0.01, size = 143, normalized size = 0.70

$$\frac{2x^{\frac{3}{2}}}{3c}-\frac{\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c^2}-\frac{\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c^2}-\frac{\sqrt{2}\,b\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4\left(\frac{b}{c}\right)^{\frac{1}{4}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2),x)
```

```
[Out] 2/3*x^(3/2)/c-1/4*b/c^2/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/
2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-1/2*b/c^2/(b/c
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2*b/c^2/(b/c)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.12, size = 186, normalized size = 0.91

$$-\frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{\right.}{b^{\frac{1}{4}}c^{\frac{3}{4}}}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*s
qrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt
(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)
*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sq
rt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/c +
 2/3*x^(3/2)/c
```

mupad [B] time = 4.36, size = 54, normalized size = 0.26

$$\frac{2\,x^{3/2}}{3\,c}+\frac{{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{c^{7/4}}-\frac{{\left(-b\right)}^{3/4}\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{c^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(3/2))/(3*c) + ((-b)^(3/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/c^(7/4)
 - ((-b)^(3/4)*atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/c^(7/4)
```

sympy [A] time = 166.49, size = 180, normalized size = 0.88

$$\begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for } b = 0 \wedge c = 0 \\ \frac{2x^{\frac{7}{2}}}{7b} & \text{for } c = 0 \\ \frac{2x^{\frac{3}{2}}}{3c} & \text{for } b = 0 \\ \frac{(-1)^{\frac{3}{4}}b^{\frac{3}{4}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2c^2\sqrt[4]{\frac{1}{c}}} - \frac{(-1)^{\frac{3}{4}}b^{\frac{3}{4}}\log\left(\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2c^2\sqrt[4]{\frac{1}{c}}} - \frac{(-1)^{\frac{3}{4}}b^{\frac{3}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}}\right)}{c^2\sqrt[4]{\frac{1}{c}}} + \frac{2x^{\frac{3}{2}}}{3c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(b, 0) & Eq(c, 0)), (2*x**(7/2)/(7*b), Eq(c, 0))
, (2*x**(3/2)/(3*c), Eq(b, 0)), ((-1)**(3/4)*b**(3/4)*log(-(-1)**(1/4)*b**(
1/4)*(1/c)**(1/4) + sqrt(x))/(2*c**2*(1/c)**(1/4)) - (-1)**(3/4)*b**(3/4)*l
og((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*c**2*(1/c)**(1/4)) - (-1
)**(3/4)*b**(3/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/(c**2*(
1/c)**(1/4)) + 2*x**(3/2)/(3*c), True))
```

3.319 $\int \frac{x^{7/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=202

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{5/4}}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{5/4}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{5/4}}-\frac{\sqrt[4]{b}\ \cdots}{}$$

```
[Out] 1/2*b^(1/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(5/4)*2^(1/2)-1/2*b
^(1/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(5/4)*2^(1/2)+1/4*b^(1/4
)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)*2^(1/2)-1/4
*b^(1/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)*2^(1
/2)+2*x^(1/2)/c
```

Rubi [A] time = 0.18, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{5/4}}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{5/4}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{5/4}}-\frac{\sqrt[4]{b}\ \cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[x])/c + (b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sq
rt[2]*c^(5/4)) - (b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(S
qrt[2]*c^(5/4)) + (b^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
Sqrt[c]*x])/(2*Sqrt[2]*c^(5/4)) - (b^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
```

```
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{7/2}}{bx^2+cx^4}\,dx = \int \frac{x^{3/2}}{b+cx^2}\,dx$$

$$= \frac{2\sqrt{x}}{c} - \frac{b\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{c}$$

$$= \frac{2\sqrt{x}}{c} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c}$$

$$= \frac{2\sqrt{x}}{c} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c}$$

$$= \frac{2\sqrt{x}}{c} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^{3/2}} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^{3/2}} + \frac{\sqrt[4]{b}}{}$$

$$= \frac{2\sqrt{x}}{c} + \frac{\sqrt[4]{b}\,\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{5/4}} - \frac{\sqrt[4]{b}\,\log\left(\sqrt{b}+\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{5/4}} - \frac{\sqrt[4]{b}}{}$$

$$= \frac{2\sqrt{x}}{c} + \frac{\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{5/4}} - \frac{\sqrt[4]{b}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{5/4}} + \frac{\sqrt[4]{b}\,\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}\right.}{2\sqrt{2}\,c^{5/4}}$$

Mathematica [A] time = 0.04, size = 189, normalized size = 0.94

$$\frac{\sqrt{2}\,\sqrt[4]{b}\,\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)-\sqrt{2}\,\sqrt[4]{b}\,\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)+2\sqrt{2}\,\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{4c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (8*c^(1/4)*Sqrt[x] + 2*Sqrt[2]*b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])
/b^(1/4)] - 2*Sqrt[2]*b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
 + Sqrt[2]*b^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*
```

```
x] - Sqrt[2]*b^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x])/(4*c^(5/4))
```

fricas [A] time = 0.87, size = 124, normalized size = 0.61

$$-\frac{4\,c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^2\sqrt{-\frac{b}{c^5}}+x}\,c^4\left(-\frac{b}{c^5}\right)^{\frac{3}{4}}-c^4\sqrt{x}\left(-\frac{b}{c^5}\right)^{\frac{3}{4}}}{b}\right)+c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}\log\left(c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}\log\left(-c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}+\right.}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(4*c*(-b/c^5)^(1/4)*arctan((sqrt(c^2*sqrt(-b/c^5) + x)*c^4*(-b/c^5)^(3
/4) - c^4*sqrt(x)*(-b/c^5)^(3/4))/b) + c*(-b/c^5)^(1/4)*log(c*(-b/c^5)^(1/4
) + sqrt(x)) - c*(-b/c^5)^(1/4)*log(-c*(-b/c^5)^(1/4) + sqrt(x)) - 4*sqrt(x
))/c
```

giac [A] time = 0.16, size = 178, normalized size = 0.88

$$-\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^2}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^2}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x\right.}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/c^2 - 1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^2 - 1/4*sqrt(2)*(b*c^3)^(1/4)*l
og(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^2 + 1/4*sqrt(2)*(b*c^3)^(
1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^2 + 2*sqrt(x)/c
```

maple [A] time = 0.01, size = 140, normalized size = 0.69

$$-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2c}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2c}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4c}+\frac{2\sqrt{x}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2),x)
```

```
[Out] 2*x^(1/2)/c-1/4/c*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/
c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-1/2/c*(b/c)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2/c*(b/c)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.05, size = 185, normalized size = 0.92

$$-\frac{\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\sqrt{b}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}b^{\frac{1}{4}}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{c^{\frac{1}{4}}}-\frac{\sqrt{2}b^{\frac{1}{4}}\log\left(-\right.}{}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqr
t(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + 2*sqrt(2)*sqrt
(b)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(
sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + sqrt(2)*b^(1/4)*log(sqrt(2)*b^(1/
4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4) - sqrt(2)*b^(1/4)*log(-sq
rt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4))/c + 2*sqrt(x)
/c
```

mupad [B] time = 4.36, size = 55, normalized size = 0.27

$$\frac{2\sqrt{x}}{c}-\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)}{c^{5/4}}-\frac{(-b)^{1/4}\,\mathrm{atanh}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)}{c^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(1/2))/c - ((-b)^(1/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/c^(5/4) - (
(-b)^(1/4)*atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/c^(5/4)
```

sympy [A] time = 77.50, size = 172, normalized size = 0.85

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for } b = 0 \wedge c \\ \frac{2x^{\frac{5}{2}}}{5b} & \text{for } c = 0 \\ \frac{2\sqrt{x}}{c} & \text{for } b = 0 \\ \frac{\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2c} - \frac{\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}\log\left(\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2c} + \frac{\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}}\right)}{c} + \frac{2\sqrt{x}}{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(b, 0) & Eq(c, 0)), (2*x**(5/2)/(5*b), Eq(c, 0)),
 (2*sqrt(x)/c, Eq(b, 0)), ((-1)**(1/4)*b**(1/4)*(1/c)**(1/4)*log(-(-1)**(1/
4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*c) - (-1)**(1/4)*b**(1/4)*(1/c)**(1/
4)*log((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*c) + (-1)**(1/4)*b**
(1/4)*(1/c)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/c + 2*
sqrt(x)/c, True))
```

3.320 $\int \frac{x^{5/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=192

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\sqrt[4]{b}\,c^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\sqrt[4]{b}\,c^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\,c^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{\sqrt{2}\sqrt[4]{b}\,c^{3/4}}$$

```
[Out] -1/2*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/c^(3/4)*2^(1/2)+1/2*
arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/c^(3/4)*2^(1/2)+1/4*ln(b^
(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(3/4)*2^(1/2)-1/
4*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(3/4)*2^(
1/2)
```

Rubi [A] time = 0.14, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1584, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\sqrt[4]{b}\,c^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\sqrt[4]{b}\,c^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\,c^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{\sqrt{2}\sqrt[4]{b}\,c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(b*x^2 + c*x^4),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(3/4)))
+ ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(3/4)) +
 Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1
/4)*c^(3/4)) - Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(
2*Sqrt[2]*b^(1/4)*c^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{5/2}}{bx^2+cx^4}\,dx = \int \frac{\sqrt{x}}{b+cx^2}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{c}}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c} + \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}}{}}\right.}{2\sqrt{2}\sqrt[4]{}}$$
$$= \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{3/4}} - \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{3/4}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,\right.}{\sqrt{2}\sqrt[4]{b}}$$
$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{3/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{3/4}} + \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{3/4}} - \frac{\log\left(\sqrt{\right.}}{}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.28

$$\frac{b\left(\tan^{-1}\left(\frac{b\sqrt[4]{c}\sqrt{x}}{(-b)^{5/4}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)\right)}{(-b)^{5/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (b*(ArcTan[(b*c^(1/4)*Sqrt[x])/(-b)^(5/4)] + ArcTanh[(c^(1/4)*Sqrt[x])/(-b)
^(1/4)]))/((-b)^(5/4)*c^(3/4))
```

fricas [A] time = 0.60, size = 126, normalized size = 0.66

$$-2\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-bc\sqrt{-\frac{1}{bc^3}}+xc}\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}-c\sqrt{x}\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}\right)+\frac{1}{2}\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}\log\left(bc^2\left(-\frac{1}{bc^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-\frac{1}{2}\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -2*(-1/(b*c^3))^(1/4)*arctan(sqrt(-b*c*sqrt(-1/(b*c^3)) + x)*c*(-1/(b*c^3))
^(1/4) - c*sqrt(x)*(-1/(b*c^3))^(1/4)) + 1/2*(-1/(b*c^3))^(1/4)*log(b*c^2*(
-1/(b*c^3))^(3/4) + sqrt(x)) - 1/2*(-1/(b*c^3))^(1/4)*log(-b*c^2*(-1/(b*c^3
))^(3/4) + sqrt(x))
```

giac [A] time = 0.19, size = 182, normalized size = 0.95

$$\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^3}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^3}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x\right.}{4\,bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(
x))/(b/c)^(1/4))/(b*c^3) + 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^3) - 1/4*sqrt(2)*(b*c^3)^
(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^3) + 1/4*sqrt(2
)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^3)
```

maple [A] time = 0.00, size = 132, normalized size = 0.69

$$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}c}+\frac{\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4\left(\frac{b}{c}\right)^{\frac{1}{4}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2),x)
```

```
[Out] 1/4/c/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x
+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+1/2/c/(b/c)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2/c/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b
/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.00, size = 172, normalized size = 0.90

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}\,\sqrt{c}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{4\,b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x)
)/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 1/2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*
sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - 1/4*sqrt(2)*log(sqrt(2)*b^(1/4)
*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + 1/4*sqrt(2)*log
(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4))
```

mupad [B] time = 0.08, size = 38, normalized size = 0.20

$$\frac{\operatorname{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)-\operatorname{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)}{{(-b)}^{1/4}\,c^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2 + c*x^4),x)
```

```
[Out] (atan((c^(1/4)*x^(1/2))/(-b)^(1/4)) - atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/
((-b)^(1/4)*c^(3/4))
```

sympy [A] time = 48.27, size = 165, normalized size = 0.86

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\: b = 0 \wedge c = 0 \\ \frac{2 x^{\frac{3}{2}}}{3 b} & \text{for}\: c = 0 \\ - \frac{2}{c \sqrt{x}} & \text{for}\: b = 0 \\ - \frac{(-1)^{\frac{3}{4}} \log{\left(- \sqrt[4]{-1} \sqrt[4]{b} \sqrt[4]{\frac{1}{c}} + \sqrt{x} \right)}}{2 \sqrt[4]{b} c \sqrt[4]{\frac{1}{c}}} + \frac{(-1)^{\frac{3}{4}} \log{\left(\sqrt[4]{-1} \sqrt[4]{b} \sqrt[4]{\frac{1}{c}} + \sqrt{x} \right)}}{2 \sqrt[4]{b} c \sqrt[4]{\frac{1}{c}}} + \frac{(-1)^{\frac{3}{4}} \operatorname{atan}{\left(\frac{(-1)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{b} \sqrt[4]{\frac{1}{c}}} \right)}}{\sqrt[4]{b} c \sqrt[4]{\frac{1}{c}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(b, 0) & Eq(c, 0)), (2*x**(3/2)/(3*b), Eq(c, 0)),
 (-2/(c*sqrt(x)), Eq(b, 0)), (-(-1)**(3/4)*log(-(-1)**(1/4)*b**(1/4)*(1/c)*
*(1/4) + sqrt(x))/(2*b**(1/4)*c*(1/c)**(1/4)) + (-1)**(3/4)*log((-1)**(1/4)
*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(1/4)*c*(1/c)**(1/4)) + (-1)**(3/4)
```

```
*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/(b**(1/4)*c*(1/c)**(1/4)
), True))
```

3.321 $\int \frac{x^{3/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=192

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\ldots}}{\sqrt[4]{b}}\right.}{\sqrt{2}\,b^{3/4}\sqrt[4]{\ldots}}$$

```
[Out] -1/2*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^(1/4)*2^(1/2)+1/2*
arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^(1/4)*2^(1/2)-1/4*ln(b^
(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(1/4)*2^(1/2)+1/
4*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(1/4)*2^(
1/2)
```

Rubi [A] time = 0.14, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1584, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\ldots}}{\sqrt[4]{b}}\right.}{\sqrt{2}\,b^{3/4}\sqrt[4]{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(b*x^2 + c*x^4),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(3/4)*c^(1/4)))
+ ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(3/4)*c^(1/4)) -
 Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(3
/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(
2*Sqrt[2]*b^(3/4)*c^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{3/2}}{bx^2+cx^4}\,dx = \int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)$$
$$= \frac{\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{b}\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{b}\sqrt{c}} - \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}}{}}\right.}{2\sqrt{2}b^{3}}$$
$$= -\frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}\sqrt[4]{c}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx\right.}{\sqrt{2}b^{3}}$$
$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{3/4}\sqrt[4]{c}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{3/4}\sqrt[4]{c}} - \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{}\right.}{}$$

Mathematica [A] time = 0.03, size = 146, normalized size = 0.76

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)+2\tan^{-1}\left(\right.}{2\sqrt{2}b^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)] - Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
Sqrt[c]*x] + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2
*Sqrt[2]*b^(3/4)*c^(1/4))
```

fricas [A] time = 0.87, size = 126, normalized size = 0.66

$$2\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^2\sqrt{-\frac{1}{b^3c}}+x}\,b^2c\left(-\frac{1}{b^3c}\right)^{\frac{3}{4}}-b^2c\sqrt{x}\left(-\frac{1}{b^3c}\right)^{\frac{3}{4}}\right)+\frac{1}{2}\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}\log\left(b\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}+\sqrt{x}\right)-\frac{1}{2}\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 2*(-1/(b^3*c))^(1/4)*arctan(sqrt(b^2*sqrt(-1/(b^3*c)) + x)*b^2*c*(-1/(b^3*c
))^(3/4) - b^2*c*sqrt(x)*(-1/(b^3*c))^(3/4)) + 1/2*(-1/(b^3*c))^(1/4)*log(b
*(-1/(b^3*c))^(1/4) + sqrt(x)) - 1/2*(-1/(b^3*c))^(1/4)*log(-b*(-1/(b^3*c))
^(1/4) + sqrt(x))
```

giac [A] time = 0.16, size = 182, normalized size = 0.95

$$\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc}+\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc}+\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x\right.}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(
x))/(b/c)^(1/4))/(b*c) + 1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c) + 1/4*sqrt(2)*(b*c^3)^(1/4
)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c) - 1/4*sqrt(2)*(b*c
^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c)
```

maple [A] time = 0.01, size = 132, normalized size = 0.69

$$\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2b}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2b}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2),x)
```

```
[Out] 1/4*(b/c)^(1/4)/b*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x
-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+1/2*(b/c)^(1/4)/b*2^(1/2)*arctan
(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2*(b/c)^(1/4)/b*2^(1/2)*arctan(2^(1/2)/(b
/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.02, size = 172, normalized size = 0.90

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{2\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{2\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{4\,b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x)
)/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 1/2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*
sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 1/4*sqrt(2)*log(sqrt(2)*b^(1/4)
*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - 1/4*sqrt(2)*log
(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4))
```

mupad [B] time = 4.43, size = 37, normalized size = 0.19

$$-\frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)+\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)}{{(-b)}^{3/4}\,c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2 + c*x^4),x)
```

```
[Out] -(atan((c^(1/4)*x^(1/2))/(-b)^(1/4)) + atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))
/((-b)^(3/4)*c^(1/4))
```

sympy [A] time = 27.22, size = 160, normalized size = 0.83

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\: b = 0 \wedge c = 0 \\ - \frac{2}{3 c x^{\frac{3}{2}}} & \text{for}\: b = 0 \\ \frac{2 \sqrt{x}}{b} & \text{for}\: c = 0 \\ - \frac{\sqrt[4]{-1} \sqrt[4]{\frac{1}{c}} \log{\left(- \sqrt[4]{-1} \sqrt[4]{b} \sqrt[4]{\frac{1}{c}} + \sqrt{x} \right)}}{2 b^{\frac{3}{4}}} + \frac{\sqrt[4]{-1} \sqrt[4]{\frac{1}{c}} \log{\left(\sqrt[4]{-1} \sqrt[4]{b} \sqrt[4]{\frac{1}{c}} + \sqrt{x} \right)}}{2 b^{\frac{3}{4}}} - \frac{\sqrt[4]{-1} \sqrt[4]{\frac{1}{c}} \operatorname{atan}{\left(\frac{(-1)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{b} \sqrt[4]{\frac{1}{c}}} \right)}}{b^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(b, 0) & Eq(c, 0)), (-2/(3*c*x**(3/2)), Eq(b, 0)
), (2*sqrt(x)/b, Eq(c, 0)), (-(-1)**(1/4)*(1/c)**(1/4)*log(-(-1)**(1/4)*b**
(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(3/4)) + (-1)**(1/4)*(1/c)**(1/4)*log((
-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(3/4)) - (-1)**(1/4)*(1/c
)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/b**(3/4), True))
```

3.322 $\int \frac{\sqrt{x}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=202

$$-\frac{\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{5/4}}+\frac{\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{5/4}}+\frac{\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{5/4}}-\frac{\sqrt[4]{c}\,\mathrm{t}}{}$$

```
[Out] 1/2*c^(1/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)*2^(1/2)-1/2*c
^(1/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)*2^(1/2)-1/4*c^(1/4
)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)*2^(1/2)+1/4
*c^(1/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)*2^(1
/2)-2/b/x^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{5/4}}+\frac{\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{5/4}}+\frac{\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{5/4}}-\frac{\sqrt[4]{c}\,\mathrm{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(b*x^2 + c*x^4),x]
```

```
[Out] -2/(b*Sqrt[x]) + (c^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(S
qrt[2]*b^(5/4)) - (c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(
Sqrt[2]*b^(5/4)) - (c^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
 Sqrt[c]*x])/(2*Sqrt[2]*b^(5/4)) + (c^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c
^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
```

```
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\sqrt{x}}{bx^2+cx^4}\,dx = \int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx$$
$$= -\frac{2}{b\sqrt{x}} - \frac{c\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b}$$
$$= -\frac{2}{b\sqrt{x}} - \frac{(2c)\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{2}{b\sqrt{x}} + \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b} - \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{2}{b\sqrt{x}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b} - \frac{\sqrt[4]{c}\,\text{Subst}}{}$$
$$= -\frac{2}{b\sqrt{x}} - \frac{\sqrt[4]{c}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{5/4}} + \frac{\sqrt[4]{c}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{5/4}} - \frac{\sqrt[4]{c}}{}$$
$$= -\frac{2}{b\sqrt{x}} + \frac{\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{5/4}} - \frac{\sqrt[4]{c}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{5/4}} - \frac{\sqrt[4]{c}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}\right.}{2\sqrt{2}\,b^{5/4}}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.13

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{cx^2}{b}\right)}{b\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(b*x^2 + c*x^4),x]
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, -((c*x^2)/b)])/(b*Sqrt[x])
```

fricas [A] time = 1.13, size = 142, normalized size = 0.70

$$\frac{4\,bx\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{bc\sqrt{x}\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}-\sqrt{-b^3c\sqrt{-\frac{c}{b^5}}+c^2x}\,b\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}}{c}\right)-bx\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}\log\left(b^4\left(-\frac{c}{b^5}\right)^{\frac{3}{4}}+c\sqrt{x}\right)+bx\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}\log\left(-\right.}{2\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*b*x*(-c/b^5)^(1/4)*arctan(-(b*c*sqrt(x)*(-c/b^5)^(1/4) - sqrt(-b^3*c
*sqrt(-c/b^5) + c^2*x)*b*(-c/b^5)^(1/4))/c) - b*x*(-c/b^5)^(1/4)*log(b^4*(-
c/b^5)^(3/4) + c*sqrt(x)) + b*x*(-c/b^5)^(1/4)*log(-b^4*(-c/b^5)^(3/4) + c*
sqrt(x)) - 4*sqrt(x))/(b*x)
```

giac [A] time = 0.17, size = 190, normalized size = 0.94

$$-\frac{2}{b\sqrt{x}}-\frac{\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^2c^2}-\frac{\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^2c^2}+\frac{\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\right.}{4\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -2/(b*sqrt(x)) - 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c
)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) - 1/2*sqrt(2)*(b*c^3)^(3/4)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) +
 1/4*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))
/(b^2*c^2) - 1/4*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x
 + sqrt(b/c))/(b^2*c^2)
```

maple [A] time = 0.01, size = 140, normalized size = 0.69

$$-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}b}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}b}-\frac{\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4\left(\frac{b}{c}\right)^{\frac{1}{4}}b}-\frac{2}{b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2),x)
```

```
[Out] -1/4/b/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(
x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-1/2/b/(b/c)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2/b/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
b/c)^(1/4)*x^(1/2)-1)-2/b/x^(1/2)
```

maxima [A] time = 2.92, size = 186, normalized size = 0.92

$$-\frac{c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}\right.}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*s
qrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt
(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)
*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sq
rt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b -
 2/(b*sqrt(x))
```

mupad [B] time = 4.53, size = 54, normalized size = 0.27

$$\frac{(-c)^{1/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)}{b^{5/4}}-\frac{(-c)^{1/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)}{b^{5/4}}-\frac{2}{b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2 + c*x^4),x)
```

```
[Out] ((-c)^(1/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(5/4) - ((-c)^(1/4)*atan
(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(5/4) - 2/(b*x^(1/2))
```

sympy [A] time = 18.32, size = 170, normalized size = 0.84

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } b = 0 \wedge c = 0 \\ -\frac{2}{5cx^{\frac{5}{2}}} & \text{for } b = 0 \\ -\frac{2}{b\sqrt{x}} & \text{for } c = 0 \\ -\frac{2}{b\sqrt{x}} + \frac{(-1)^{\frac{3}{4}} \log\left(-\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}} + \sqrt{x}\right)}{2b^{\frac{5}{4}}\sqrt[4]{\frac{1}{c}}} - \frac{(-1)^{\frac{3}{4}} \log\left(\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}} + \sqrt{x}\right)}{2b^{\frac{5}{4}}\sqrt[4]{\frac{1}{c}}} - \frac{(-1)^{\frac{3}{4}} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{5}{4}}\sqrt[4]{\frac{1}{c}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(b, 0) & Eq(c, 0)), (-2/(5*c*x**(5/2)), Eq(b, 0)
), (-2/(b*sqrt(x)), Eq(c, 0)), (-2/(b*sqrt(x)) + (-1)**(3/4)*log(-(-1)**(1/
4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(5/4)*(1/c)**(1/4)) - (-1)**(3/4)
*log((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(5/4)*(1/c)**(1/4))
 - (-1)**(3/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/(b**(5/4)*
(1/c)**(1/4)), True))
```

3.323 $\int \frac{1}{\sqrt{x}(bx^2+cx^4)}\,dx$

Optimal. Leaf size=204

$$\frac{c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{7/4}}-\frac{c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{7/4}}+\frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{7/4}}-\frac{c^{3/4}\mathrm{ta}}{}$$

[Out] -2/3/b/x^(3/2)+1/2*c^(3/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4
)*2^(1/2)-1/2*c^(3/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)*2^(
1/2)+1/4*c^(3/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(7
/4)*2^(1/2)-1/4*c^(3/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2
))/b^(7/4)*2^(1/2)

Rubi [A] time = 0.17, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{7/4}}-\frac{c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{7/4}}+\frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{7/4}}-\frac{c^{3/4}\mathrm{ta}}{}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*(b*x^2 + c*x^4)),x]

[Out] -2/(3*b*x^(3/2)) + (c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/
(Sqrt[2]*b^(7/4)) - (c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])
/(Sqrt[2]*b^(7/4)) + (c^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
 + Sqrt[c]*x])/(2*Sqrt[2]*b^(7/4)) - (c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)
*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(7/4))

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)}\,dx = \int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx$$

$$= -\frac{2}{3bx^{3/2}} - \frac{c\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{b}$$

$$= -\frac{2}{3bx^{3/2}} - \frac{(2c)\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b}$$

$$= -\frac{2}{3bx^{3/2}} - \frac{c\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}} - \frac{c\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}}$$

$$= -\frac{2}{3bx^{3/2}} - \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{3/2}} - \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right.}{2b^{3/2}}$$

$$= -\frac{2}{3bx^{3/2}} + \frac{c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{7/4}} - \frac{c^{3/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right.}{2\sqrt{2}\,b^{7/4}}$$

$$= -\frac{2}{3bx^{3/2}} + \frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{7/4}} - \frac{c^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{7/4}} + \frac{c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\right.}{2\sqrt{2}\,b^{7/4}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.14

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{cx^2}{b}\right)}{3bx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(b*x^2 + c*x^4)),x]
[Out] (-2*Hypergeometric2F1[-3/4, 1, 1/4, -((c*x^2)/b)])/(3*b*x^(3/2))
```

fricas [A] time = 0.61, size = 167, normalized size = 0.82

$$-\frac{12\,bx^2\left(-\frac{c^3}{b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^5c\sqrt{x}\left(-\frac{c^3}{b^7}\right)^{\frac{3}{4}}-\sqrt{b^4\sqrt{-\frac{c^3}{b^7}}+c^2x}\,b^5\left(-\frac{c^3}{b^7}\right)^{\frac{3}{4}}}{c^3}\right)+3\,bx^2\left(-\frac{c^3}{b^7}\right)^{\frac{1}{4}}\log\left(b^2\left(-\frac{c^3}{b^7}\right)^{\frac{1}{4}}+c\sqrt{x}\right)-3\,bx^2\left(-\frac{c^3}{b^7}\right. \cdots}{6\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(12*b*x^2*(-c^3/b^7)^(1/4)*arctan(-(b^5*c*sqrt(x)*(-c^3/b^7)^(3/4) - s
qrt(b^4*sqrt(-c^3/b^7) + c^2*x)*b^5*(-c^3/b^7)^(3/4))/c^3) + 3*b*x^2*(-c^3/
b^7)^(1/4)*log(b^2*(-c^3/b^7)^(1/4) + c*sqrt(x)) - 3*b*x^2*(-c^3/b^7)^(1/4)
*log(-b^2*(-c^3/b^7)^(1/4) + c*sqrt(x)) + 4*sqrt(x))/(b*x^2)
```

giac [A] time = 0.16, size = 178, normalized size = 0.87

$$-\frac{\sqrt{2}\,\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^2}-\frac{\sqrt{2}\,\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^2}-\frac{\sqrt{2}\,\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+\cdots\right.}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)/x^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/b^2 - 1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^2 - 1/4*sqrt(2)*(b*c^3)^(1/4)*l
og(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^2 + 1/4*sqrt(2)*(b*c^3)^(
1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^2 - 2/3/(b*x^(3/2)
)
```

maple [A] time = 0.01, size = 143, normalized size = 0.70

$$-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2b^2}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2b^2}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4b^2}-\frac{2}{3b\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)/x^(1/2),x)
```

```
[Out] -1/4*c/b^2*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2
))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-1/2*c/b^2*(b/c)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2*c/b^2*(b/c)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/3/b/x^(3/2)
```

maxima [A] time = 2.99, size = 187, normalized size = 0.92

$$-\frac{\frac{2\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,c^{\frac{3}{4}}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}\,c^{\frac{3}{4}}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\cdots\right.}{b^{\frac{3}{4}}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*c*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*s
qrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*
c*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sq
rt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*c^(3/4)*log(sqrt(
2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2)*c^(3/4)
*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4))/b - 2
/3/(b*x^(3/2))
```

mupad [B] time = 0.10, size = 53, normalized size = 0.26

$$\frac{(-c)^{3/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{b^{7/4}}-\frac{2}{3\,b\,x^{3/2}}+\frac{(-c)^{3/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] ((-c)^(3/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(7/4) - 2/(3*b*x^(3/2)) +
 ((-c)^(3/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(7/4)
```

sympy [A] time = 27.74, size = 178, normalized size = 0.87

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for } b = 0 \wedge c = 0 \\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for } c = 0 \\ -\frac{2}{7cx^{\frac{7}{2}}} & \text{for } b = 0 \\ -\frac{2}{3bx^{\frac{3}{2}}} + \frac{\sqrt[4]{-1}\,c\sqrt[4]{\frac{1}{c}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{7}{4}}} - \frac{\sqrt[4]{-1}\,c\sqrt[4]{\frac{1}{c}}\log\left(\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{7}{4}}} + \frac{\sqrt[4]{-1}\,c\sqrt[4]{\frac{1}{c}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{7}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(b, 0) & Eq(c, 0)), (-2/(3*b*x**(3/2)), Eq(c, 0)
), (-2/(7*c*x**(7/2)), Eq(b, 0)), (-2/(3*b*x**(3/2)) + (-1)**(1/4)*c*(1/c)*
*(1/4)*log(-(-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(7/4)) - (-1
)**(1/4)*c*(1/c)**(1/4)*log((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2
*b**(7/4)) + (-1)**(1/4)*c*(1/c)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*
(1/c)**(1/4)))/b**(7/4), True))
```

3.324 $\int \frac{1}{x^{3/2}(bx^2+cx^4)}\,dx$

Optimal. Leaf size=215

$$\frac{c^{5/4}\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{9/4}}-\frac{c^{5/4}\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{9/4}}-\frac{c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{9/4}}+\frac{c^{5/4}\,\text{ta}}{}$$

```
[Out] -2/5/b/x^(5/2)-1/2*c^(5/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(9/4
)*2^(1/2)+1/2*c^(5/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(9/4)*2^(
1/2)+1/4*c^(5/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9
/4)*2^(1/2)-1/4*c^(5/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2
))/b^(9/4)*2^(1/2)+2*c/b^2/x^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{c^{5/4}\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{9/4}}-\frac{c^{5/4}\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{9/4}}-\frac{c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{9/4}}+\frac{c^{5/4}\,\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] -2/(5*b*x^(5/2)) + (2*c)/(b^2*Sqrt[x]) - (c^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/4)) + (c^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/4)) + (c^(5/4)*Log[Sqrt[b] - Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(9/4)) - (c^(5/4)*Log[Sqr
t[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)}\,dx &= \int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx \\
&= -\frac{2}{5bx^{5/2}} - \frac{c\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{b} \\
&= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} + \frac{c^2\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b^2} \\
&= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} + \frac{\left(2c^2\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2} \\
&= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} - \frac{c^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2} + \frac{c^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right.}{b^2} \\
&= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} + \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^2} + \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x\right.}{2b^2} \\
&= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} + \frac{c^{5/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{9/4}} - \frac{c^{5/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{\right.}}{2\sqrt{2}\,b^{9/4}} \\
&= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} - \frac{c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{9/4}} + \frac{c^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{9/4}} + \frac{c^{5/4}\log\left(\sqrt{b}\right.}{}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.13

$$
-\frac{2\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\frac{cx^2}{b}\right)}{5bx^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 1, -1/4, -((c*x^2)/b)])/(5*b*x^(5/2))
```

fricas [A] time = 0.85, size = 193, normalized size = 0.90

$$-\frac{20\,b^{2}x^{3}\left(-\frac{c^{5}}{b^{9}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^{2}c^{4}\sqrt{x}\left(-\frac{c^{5}}{b^{9}}\right)^{\frac{1}{4}}-\sqrt{-b^{5}c^{5}\sqrt{-\frac{c^{5}}{b^{9}}}+c^{8}x}\,b^{2}\left(-\frac{c^{5}}{b^{9}}\right)^{\frac{1}{4}}}{c^{5}}\right)-5\,b^{2}x^{3}\left(-\frac{c^{5}}{b^{9}}\right)^{\frac{1}{4}}\log\left(b^{7}\left(-\frac{c^{5}}{b^{9}}\right)^{\frac{3}{4}}+c^{4}\sqrt{x}\right)+5\,b}{10\,b^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/10*(20*b^2*x^3*(-c^5/b^9)^(1/4)*arctan(-(b^2*c^4*sqrt(x)*(-c^5/b^9)^(1/4
) - sqrt(-b^5*c^5*sqrt(-c^5/b^9) + c^8*x)*b^2*(-c^5/b^9)^(1/4))/c^5) - 5*b^
2*x^3*(-c^5/b^9)^(1/4)*log(b^7*(-c^5/b^9)^(3/4) + c^4*sqrt(x)) + 5*b^2*x^3*
(-c^5/b^9)^(1/4)*log(-b^7*(-c^5/b^9)^(3/4) + c^4*sqrt(x)) - 4*(5*c*x^2 - b)
*sqrt(x))/(b^2*x^3)
```

giac [A] time = 0.17, size = 200, normalized size = 0.93

$$\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^{3}c}+\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^{3}c}-\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x\right.}{4\,b^{3}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(
x))/(b/c)^(1/4))/(b^3*c) + 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c) - 1/4*sqrt(2)*(b*c^3)^
(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c) + 1/4*sqrt(2
)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c) +
 2/5*(5*c*x^2 - b)/(b^2*x^(5/2))
```

maple [A] time = 0.01, size = 152, normalized size = 0.71

$$\frac{\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{2}}+\frac{\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{2}}+\frac{\sqrt{2}\,c\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{2}}+\frac{2c}{b^{2}\sqrt{x}}-\frac{2}{5b\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+b*x^2),x)
```

```
[Out] 1/4*c/b^2/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)
)/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+1/2*c/b^2/(b/c)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2*c/b^2/(b/c)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/5/b/x^(5/2)+2*c/b^2/x^(1/2)
```

maxima [A] time = 3.13, size = 198, normalized size = 0.92

$$\frac{c^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\cdots\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\cdots\right)}{4\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*c^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*
sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqr
t(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4
)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-s
qrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^
2 + 2/5*(5*c*x^2 - b)/(b^2*x^(5/2))
```

mupad [B] time = 0.09, size = 66, normalized size = 0.31

$$\frac{{\left(-c\right)}^{5/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{b^{9/4}}-\frac{{\left(-c\right)}^{5/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{b^{9/4}}-\frac{\frac{2}{5\,b}-\frac{2\,c\,x^2}{b^2}}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] ((-c)^(5/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(9/4) - ((-c)^(5/4)*atan
(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(9/4) - (2/(5*b) - (2*c*x^2)/b^2)/x^(5/2)
```

sympy [A] time = 48.98, size = 190, normalized size = 0.88

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for } b = 0 \wedge c = 0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } c = 0 \\ -\frac{2}{9cx^{\frac{9}{2}}} & \text{for } b = 0 \\ -\frac{2}{5bx^{\frac{5}{2}}} + \frac{2c}{b^2\sqrt{x}} - \frac{(-1)^{\frac{3}{4}} c \log\left(-\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}} + \sqrt{x}\right)}{2b^{\frac{9}{4}}\sqrt[4]{\frac{1}{c}}} + \frac{(-1)^{\frac{3}{4}} c \log\left(\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}} + \sqrt{x}\right)}{2b^{\frac{9}{4}}\sqrt[4]{\frac{1}{c}}} + \frac{(-1)^{\frac{3}{4}} c \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{9}{4}}\sqrt[4]{\frac{1}{c}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo/x**(9/2), Eq(b, 0) & Eq(c, 0)), (-2/(5*b*x**(5/2)), Eq(c, 0)
), (-2/(9*c*x**(9/2)), Eq(b, 0)), (-2/(5*b*x**(5/2)) + 2*c/(b**2*sqrt(x)) -
 (-1)**(3/4)*c*log(-(-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(9/4
)*(1/c)**(1/4)) + (-1)**(3/4)*c*log((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqr
t(x))/(2*b**(9/4)*(1/c)**(1/4)) + (-1)**(3/4)*c*atan((-1)**(3/4)*sqrt(x)/(b
**(1/4)*(1/c)**(1/4)))/(b**(9/4)*(1/c)**(1/4)), True))
```

3.325 $\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=217

$$-\frac{c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{11/4}}+\frac{c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{11/4}}-\frac{c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{11/4}}+\frac{c^{7/4}}{}$$

```
[Out] -2/7/b/x^(7/2)+2/3*c/b^2/x^(3/2)-1/2*c^(7/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/
2)/b^(1/4))/b^(11/4)*2^(1/2)+1/2*c^(7/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b
^(1/4))/b^(11/4)*2^(1/2)-1/4*c^(7/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2
^(1/2)*x^(1/2))/b^(11/4)*2^(1/2)+1/4*c^(7/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c
^(1/4)*2^(1/2)*x^(1/2))/b^(11/4)*2^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{11/4}}+\frac{c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{11/4}}-\frac{c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{11/4}}+\frac{c^{7/4}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] -2/(7*b*x^(7/2)) + (2*c)/(3*b^2*x^(3/2)) - (c^(7/4)*ArcTan[1 - (Sqrt[2]*c^(
1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(11/4)) + (c^(7/4)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(11/4)) - (c^(7/4)*Log[Sqrt[b] - Sqrt[
2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(11/4)) + (c^(7/4)*Lo
g[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(11/
4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)}\,dx = \int \frac{1}{x^{9/2}\left(b+cx^2\right)}\,dx$$

$$= -\frac{2}{7bx^{7/2}} - \frac{c\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{b}$$

$$= -\frac{2}{7bx^{7/2}} + \frac{2c}{3b^2x^{3/2}} + \frac{c^2\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{b^2}$$

$$= -\frac{2}{7bx^{7/2}} + \frac{2c}{3b^2x^{3/2}} + \frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2}$$

$$= -\frac{2}{7bx^{7/2}} + \frac{2c}{3b^2x^{3/2}} + \frac{c^2\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^{5/2}} + \frac{c^2\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^{5/2}}$$

$$= -\frac{2}{7bx^{7/2}} + \frac{2c}{3b^2x^{3/2}} + \frac{c^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{5/2}} + \frac{c^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x\cdots}\right.}{2b^{5/2}}$$

$$= -\frac{2}{7bx^{7/2}} + \frac{2c}{3b^2x^{3/2}} - \frac{c^{7/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{11/4}} + \frac{c^{7/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\cdots\right.}{2\sqrt{2}\,b^{11/4}}$$

$$= -\frac{2}{7bx^{7/2}} + \frac{2c}{3b^2x^{3/2}} - \frac{c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{11/4}} + \frac{c^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{11/4}} - \frac{c^{7/4}\log\left(\sqrt{b}\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.13

$$-\frac{2\,{}_2F_1\left(-\frac{7}{4},1;-\frac{3}{4};-\frac{cx^2}{b}\right)}{7bx^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*Hypergeometric2F1[-7/4, 1, -3/4, -((c*x^2)/b)])/(7*b*x^(7/2))
```

fricas [A] time = 0.79, size = 189, normalized size = 0.87

$$\frac{84\,b^2x^4\left(-\frac{c^7}{b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^8c^2\sqrt{x}\left(-\frac{c^7}{b^{11}}\right)^{\frac{3}{4}}-\sqrt{b^6\sqrt{-\frac{c^7}{b^{11}}}+c^4x}\,b^8\left(-\frac{c^7}{b^{11}}\right)^{\frac{3}{4}}}{c^7}\right)+21\,b^2x^4\left(-\frac{c^7}{b^{11}}\right)^{\frac{1}{4}}\log\left(b^3\left(-\frac{c^7}{b^{11}}\right)^{\frac{1}{4}}+c^2\sqrt{x}\right)-2\cdots}{42\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/42*(84*b^2*x^4*(-c^7/b^11)^(1/4)*arctan(-(b^8*c^2*sqrt(x)*(-c^7/b^11)^(3/
4) - sqrt(b^6*sqrt(-c^7/b^11) + c^4*x)*b^8*(-c^7/b^11)^(3/4))/c^7) + 21*b^2
*x^4*(-c^7/b^11)^(1/4)*log(b^3*(-c^7/b^11)^(1/4) + c^2*sqrt(x)) - 21*b^2*x^
4*(-c^7/b^11)^(1/4)*log(-b^3*(-c^7/b^11)^(1/4) + c^2*sqrt(x)) + 4*(7*c*x^2
- 3*b)*sqrt(x))/(b^2*x^4)
```

giac [A] time = 0.20, size = 192, normalized size = 0.88

$$\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^3}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^3}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}c\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)\cdots\right.}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(1/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/b^3 + 1/2*sqrt(2)*(b*c^3)^(1/4)*c*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^3 + 1/4*sqrt(2)*(b*c^3)^(1/4
)*c*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 - 1/4*sqrt(2)*(b*c
^3)^(1/4)*c*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 + 2/21*(7
*c*x^2 - 3*b)/(b^2*x^(7/2))
```

maple [A] time = 0.01, size = 158, normalized size = 0.73

$$\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2b^3}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2b^3}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c^2\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4b^3}+\frac{2c}{3b^2x^{\frac{3}{2}}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(c*x^4+b*x^2),x)
```

```
[Out] 1/4*c^2/b^3*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/
2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+1/2*c^2/b^3*(b/c)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2*c^2/b^3*(b/c)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/7/b/x^(7/2)+2/3*c/b^2/x^(3/2)
```

maxima [A] time = 3.12, size = 201, normalized size = 0.93

$$\frac{\frac{2\sqrt{2}c^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}c^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}c^{\frac{7}{4}}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}c^{\frac{7}{4}}\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\cdots\right)}{b^{\frac{3}{4}}}}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*c^2*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*
sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)
*c^2*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt
(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*c^(7/4)*log(sq
rt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2)*c^(7
/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4))/b^
2 + 2/21*(7*c*x^2 - 3*b)/(b^2*x^(7/2))
```

mupad [B] time = 4.39, size = 65, normalized size = 0.30

$$\frac{(-c)^{7/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)}{b^{11/4}}-\frac{\frac{2}{7b}-\frac{2cx^2}{3b^2}}{x^{7/2}}+\frac{(-c)^{7/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)}{b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] ((-c)^(7/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(11/4) - (2/(7*b) - (2*c*
x^2)/(3*b^2))/x^(7/2) + ((-c)^(7/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^
(11/4)
```

sympy [A] time = 106.89, size = 197, normalized size = 0.91

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{11}{2}}} & \text{for } b = \\ -\frac{2}{7bx^{\frac{7}{2}}} & \text{for } c = \\ -\frac{2}{11cx^{\frac{11}{2}}} & \text{for } b = \\ -\frac{2}{7bx^{\frac{7}{2}}} + \frac{2c}{3b^2x^{\frac{3}{2}}} - \frac{\sqrt[4]{-1}\,c^2\sqrt[4]{\frac{1}{c}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{11}{4}}} + \frac{\sqrt[4]{-1}\,c^2\sqrt[4]{\frac{1}{c}}\log\left(\sqrt[4]{-1}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{11}{4}}} - \frac{\sqrt[4]{-1}\,c^2\sqrt[4]{\frac{1}{c}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\,\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{11}{4}}} & \text{other} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo/x**(11/2), Eq(b, 0) & Eq(c, 0)), (-2/(7*b*x**(7/2)), Eq(c, 0
)), (-2/(11*c*x**(11/2)), Eq(b, 0)), (-2/(7*b*x**(7/2)) + 2*c/(3*b**2*x**(3
/2)) - (-1)**(1/4)*c**2*(1/c)**(1/4)*log(-(-1)**(1/4)*b**(1/4)*(1/c)**(1/4)
 + sqrt(x))/(2*b**(11/4)) + (-1)**(1/4)*c**2*(1/c)**(1/4)*log((-1)**(1/4)*b
**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(11/4)) - (-1)**(1/4)*c**2*(1/c)**(1/
4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/b**(11/4), True))
```

3.326 $\int \frac{1}{x^{7/2}(bx^2+cx^4)}\,dx$

Optimal. Leaf size=230

$$-\frac{c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{13/4}}+\frac{c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{13/4}}+\frac{c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{13/4}}-\frac{c^{9/4}}{}$$

```
[Out] -2/9/b/x^(9/2)+2/5*c/b^2/x^(5/2)+1/2*c^(9/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/
2)/b^(1/4))/b^(13/4)*2^(1/2)-1/2*c^(9/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b
^(1/4))/b^(13/4)*2^(1/2)-1/4*c^(9/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2
^(1/2)*x^(1/2))/b^(13/4)*2^(1/2)+1/4*c^(9/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c
^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^(1/2)-2*c^2/b^3/x^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2c^2}{b^3\sqrt{x}}-\frac{c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{13/4}}+\frac{c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{13/4}}+\frac{c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] -2/(9*b*x^(9/2)) + (2*c)/(5*b^2*x^(5/2)) - (2*c^2)/(b^3*Sqrt[x]) + (c^(9/4)
*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(13/4)) - (c^(9/
4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(13/4)) - (c^(
9/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]
*b^(13/4)) + (c^(9/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[
c]*x])/(2*Sqrt[2]*b^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(bx^2+cx^4\right)}\,dx = \int \frac{1}{x^{11/2}\left(b+cx^2\right)}\,dx$$

$$= -\frac{2}{9bx^{9/2}} - \frac{c\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{b}$$

$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} + \frac{c^2\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{b^2}$$

$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} - \frac{c^3\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b^3}$$

$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} - \frac{\left(2c^3\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^3}$$

$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} + \frac{c^{5/2}\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^3} - \frac{c^{5/2}\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\right.}{b^3}$$

$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} - \frac{c^2\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^3} - \frac{c^2\text{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}+\sqrt{2}\cdots}\right.}{2b\cdots}$$

$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} - \frac{c^{9/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{13/4}} + \frac{c^{9/4}\log\left(\sqrt{b}+\sqrt{\cdots}\right.}{2\sqrt{\cdots}}$$

$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} + \frac{c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{13/4}} - \frac{c^{9/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{13/4}} - \frac{c^{9/\cdots}}{\cdots}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.13

$$-\frac{2\,{}_2F_1\left(-\frac{9}{4},1;-\frac{5}{4};-\frac{cx^2}{b}\right)}{9bx^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*Hypergeometric2F1[-9/4, 1, -5/4, -((c*x^2)/b)])/(9*b*x^(9/2))
```

fricas [A] time = 0.86, size = 204, normalized size = 0.89

$$\frac{180\,b^3x^5\left(-\frac{c^9}{b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^3c^7\sqrt{x}\left(-\frac{c^9}{b^{13}}\right)^{\frac{1}{4}}-\sqrt{-b^7c^9\sqrt{-\frac{c^9}{b^{13}}}+c^{14}x}\,b^3\left(-\frac{c^9}{b^{13}}\right)^{\frac{1}{4}}}{c^9}\right)-45\,b^3x^5\left(-\frac{c^9}{b^{13}}\right)^{\frac{1}{4}}\log\left(b^{10}\left(-\frac{c^9}{b^{13}}\right)^{\frac{3}{4}}+c^7\sqrt{x}\right.}{90\,b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/90*(180*b^3*x^5*(-c^9/b^13)^(1/4)*arctan(-(b^3*c^7*sqrt(x)*(-c^9/b^13)^(1
/4) - sqrt(-b^7*c^9*sqrt(-c^9/b^13) + c^14*x)*b^3*(-c^9/b^13)^(1/4))/c^9) -
 45*b^3*x^5*(-c^9/b^13)^(1/4)*log(b^10*(-c^9/b^13)^(3/4) + c^7*sqrt(x)) + 4
5*b^3*x^5*(-c^9/b^13)^(1/4)*log(-b^10*(-c^9/b^13)^(3/4) + c^7*sqrt(x)) - 4*
(45*c^2*x^4 - 9*b*c*x^2 + 5*b^2)*sqrt(x))/(b^3*x^5)
```

giac [A] time = 0.18, size = 199, normalized size = 0.87

$$-\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^4}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^4}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+\right.}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/b^4 - 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^4 + 1/4*sqrt(2)*(b*c^3)^(3/4)*l
og(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 1/4*sqrt(2)*(b*c^3)^(
3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 2/45*(45*c^2*x
^4 - 9*b*c*x^2 + 5*b^2)/(b^3*x^(9/2))
```

maple [A] time = 0.01, size = 169, normalized size = 0.73

$$-\frac{\sqrt{2}\, c^{2} \arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}} b^{3}}-\frac{\sqrt{2}\, c^{2} \arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}} b^{3}}-\frac{\sqrt{2}\, c^{2} \ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{4\left(\frac{b}{c}\right)^{\frac{1}{4}} b^{3}}-\frac{2c^{2}}{b^{3}\sqrt{x}}+\frac{2c}{5b^{2}x^{\frac{5}{2}}}-\frac{2}{9b\,x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(c*x^4+b*x^2),x)
```

```
[Out] -1/4*c^2/b^3/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1
/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-1/2*c^2/b^3/(b/c)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2*c^2/b^3/(b/c)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/9/b/x^(9/2)-2*c^2/b^3/x^(1/2)+2/5
*c/b^2/x^(5/2)
```

maxima [A] time = 3.09, size = 209, normalized size = 0.91

$$-\frac{c^{3}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x} \cdots\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\cdots\right)}{4\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*c^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)
*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sq
rt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/
4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-
sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b
^3 - 2/45*(45*c^2*x^4 - 9*b*c*x^2 + 5*b^2)/(b^3*x^(9/2))
```

mupad [B] time = 4.47, size = 77, normalized size = 0.33

$$\frac{(-c)^{9/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)}{b^{13/4}}-\frac{(-c)^{9/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)}{b^{13/4}}-\frac{\frac{2}{9\,b}-\frac{2\,c\,x^{2}}{5\,b^{2}}+\frac{2\,c^{2}\,x^{4}}{b^{3}}}{x^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] ((-c)^(9/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(13/4) - ((-c)^(9/4)*ata
n(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(13/4) - (2/(9*b) - (2*c*x^2)/(5*b^2) +
(2*c^2*x^4)/b^3)/x^(9/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

3.327 $\int \frac{x^{19/2}}{(bx^2+cx^4)^2} dx$

Optimal. Leaf size=243

$$-\frac{9b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}+\frac{9b^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{13/4}}+$$

```
[Out] 9/10*x^(5/2)/c^2-1/2*x^(9/2)/c/(c*x^2+b)-9/8*b^(5/4)*arctan(1-c^(1/4)*2^(1/
2)*x^(1/2)/b^(1/4))/c^(13/4)*2^(1/2)+9/8*b^(5/4)*arctan(1+c^(1/4)*2^(1/2)*x
^(1/2)/b^(1/4))/c^(13/4)*2^(1/2)-9/16*b^(5/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*
c^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)*2^(1/2)+9/16*b^(5/4)*ln(b^(1/2)+x*c^(1/2)
+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)*2^(1/2)-9/2*b*x^(1/2)/c^3
```

Rubi [A] time = 0.21, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{9b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}+\frac{9b^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{13/4}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^(19/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-9*b*Sqrt[x])/(2*c^3) + (9*x^(5/2))/(10*c^2) - x^(9/2)/(2*c*(b + c*x^2)) -
 (9*b^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(13
/4)) + (9*b^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]
*c^(13/4)) - (9*b^(5/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqr
t[c]*x])/(8*Sqrt[2]*c^(13/4)) + (9*b^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{19/2}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^{11/2}}{\left(b+cx^2\right)^2}\,dx$$
$$= -\frac{x^{9/2}}{2c\left(b+cx^2\right)} + \frac{9\int \frac{x^{7/2}}{b+cx^2}\,dx}{4c}$$
$$= \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c\left(b+cx^2\right)} - \frac{(9b)\int \frac{x^{3/2}}{b+cx^2}\,dx}{4c^2}$$
$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c\left(b+cx^2\right)} + \frac{\left(9b^2\right)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4c^3}$$
$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c\left(b+cx^2\right)} + \frac{\left(9b^2\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c^3}$$
$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c\left(b+cx^2\right)} + \frac{\left(9b^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^3} + \frac{\left(9b^{3/2}\right)\text{Subst}\left(\int\right.}{}$$
$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c\left(b+cx^2\right)} + \frac{\left(9b^{3/2}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{8c^{7/2}} + \frac{\left(9b^{3/2}\right)\text{Sub}}{}$$
$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c\left(b+cx^2\right)} - \frac{9b^{5/4}\log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}\,x\right)}{8\sqrt{2}\,c^{13/4}} + \frac{9b^{5/4}\log\left(\sqrt{b}\right.}{}$$
$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c\left(b+cx^2\right)} - \frac{9b^{5/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{13/4}} + \frac{9b^{5/4}\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{13/4}}$$

Mathematica [A] time = 0.21, size = 220, normalized size = 0.91

$$\frac{-45\sqrt{2}\,b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b} + \sqrt{c}\,x\right) + 45\sqrt{2}\,b^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b} + \sqrt{c}\,x\right) - 90\sqrt{2}\,b^{5/4}\tan^{-1}}{80c^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(19/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((8*c^(1/4)*Sqrt[x]*(-45*b^2 - 36*b*c*x^2 + 4*c^2*x^4))/(b + c*x^2) - 90*Sq
rt[2]*b^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 90*Sqrt[2]*b^
(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 45*Sqrt[2]*b^(5/4)*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 45*Sqrt[2]*b^(5/
4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(80*c^(13/4)
)
```

fricas [A] time = 0.83, size = 227, normalized size = 0.93

$$\frac{180\left(c^4x^2+bc^3\right)\left(-\frac{b^5}{c^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{bc^{10}\sqrt{x}\left(-\frac{b^5}{c^{13}}\right)^{\frac{3}{4}}-\sqrt{c^6\sqrt{-\frac{b^5}{c^{13}}}+b^2x}\,c^{10}\left(-\frac{b^5}{c^{13}}\right)^{\frac{3}{4}}}{b^5}\right)+45\left(c^4x^2+bc^3\right)\left(-\frac{b^5}{c^{13}}\right)^{\frac{1}{4}}\log\left(9\,c^3\left(-\frac{b^5}{c^{13}}\right.\right.}{40\left(c^4x^2+bc\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/40*(180*(c^4*x^2 + b*c^3)*(-b^5/c^13)^(1/4)*arctan(-(b*c^10*sqrt(x)*(-b^5
/c^13)^(3/4) - sqrt(c^6*sqrt(-b^5/c^13) + b^2*x)*c^10*(-b^5/c^13)^(3/4))/b^
5) + 45*(c^4*x^2 + b*c^3)*(-b^5/c^13)^(1/4)*log(9*c^3*(-b^5/c^13)^(1/4) + 9
*b*sqrt(x)) - 45*(c^4*x^2 + b*c^3)*(-b^5/c^13)^(1/4)*log(-9*c^3*(-b^5/c^13)
^(1/4) + 9*b*sqrt(x)) + 4*(4*c^2*x^4 - 36*b*c*x^2 - 45*b^2)*sqrt(x))/(c^4*x
^2 + b*c^3)
```

giac [A] time = 0.20, size = 216, normalized size = 0.89

$$\frac{9\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8c^4}+\frac{9\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8c^4}+\frac{9\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}b\log\left(\sqrt{2}\sqrt{x}\right.}{16c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 9/8*sqrt(2)*(b*c^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/c^4 + 9/8*sqrt(2)*(b*c^3)^(1/4)*b*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^4 + 9/16*sqrt(2)*(b*c^3)^(1/
4)*b*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 - 9/16*sqrt(2)*(b
*c^3)^(1/4)*b*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 - 1/2*b
^2*sqrt(x)/((c*x^2 + b)*c^3) + 2/5*(c^8*x^(5/2) - 10*b*c^7*sqrt(x))/c^10
```

maple [A] time = 0.01, size = 172, normalized size = 0.71

$$\frac{2x^{\frac{5}{2}}}{5c^2}-\frac{b^2\sqrt{x}}{2\left(c\,x^2+b\right)c^3}+\frac{9\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8c^3}+\frac{9\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8c^3}+\frac{9\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,b\ln\left(\frac{x+\left(\frac{b}{c}\right)}{x-\left(\frac{b}{c}\right)}\right.}{16c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/5*x^(5/2)/c^2-4*b*x^(1/2)/c^3-1/2/c^3*b^2*x^(1/2)/(c*x^2+b)+9/16/c^3*b*(b
/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(
1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+9/8/c^3*b*(b/c)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(b/c)^(1/4)*x^(1/2)+1)+9/8/c^3*b*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b
/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.02, size = 217, normalized size = 0.89

$$-\frac{b^2\sqrt{x}}{2\left(c^4x^2+bc^3\right)}+\frac{2\left(cx^{\frac{5}{2}}-10\,b\sqrt{x}\right)}{5\,c^3}+\frac{9\left(\frac{2\,\sqrt{2}\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\,\sqrt{2}\,b^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,b}{}\right.}{16\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*sqrt(x)/(c^4*x^2 + b*c^3) + 2/5*(c*x^(5/2) - 10*b*sqrt(x))/c^3 + 9
/16*(2*sqrt(2)*b^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt
(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + 2*sqrt(2)*b^(3/
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(s
qrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + sqrt(2)*b^(5/4)*log(sqrt(2)*b^(1/4
)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4) - sqrt(2)*b^(5/4)*log(-sqr
t(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4))/c^3
```

mupad [B] time = 0.10, size = 92, normalized size = 0.38

$$\frac{2\,x^{5/2}}{5\,c^2}-\frac{b^2\,\sqrt{x}}{2\,\left(c^4\,x^2+b\,c^3\right)}-\frac{4\,b\,\sqrt{x}}{c^3}-\frac{9\,{\left(-b\right)}^{5/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,c^{13/4}}+\frac{{\left(-b\right)}^{5/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-b\right)}^{1/4}}\right)\,9{}\mathrm{i}}{4\,c^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (2*x^(5/2))/(5*c^2) - (b^2*x^(1/2))/(2*(b*c^3 + c^4*x^2)) - (4*b*x^(1/2))/c
^3 - (9*(-b)^(5/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*c^(13/4)) + ((-b)
^(5/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*9i)/(4*c^(13/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(19/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.328 $\int \frac{x^{17/2}}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=230

$$-\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{11/4}}+\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{11/4}}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{11/4}}$$

```
[Out] 7/6*x^(3/2)/c^2-1/2*x^(7/2)/c/(c*x^2+b)+7/8*b^(3/4)*arctan(1-c^(1/4)*2^(1/2
)*x^(1/2)/b^(1/4))/c^(11/4)*2^(1/2)-7/8*b^(3/4)*arctan(1+c^(1/4)*2^(1/2)*x^
(1/2)/b^(1/4))/c^(11/4)*2^(1/2)-7/16*b^(3/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c
^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)*2^(1/2)+7/16*b^(3/4)*ln(b^(1/2)+x*c^(1/2)+
b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)*2^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 288, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{11/4}}+\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{11/4}}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(17/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (7*x^(3/2))/(6*c^2) - x^(7/2)/(2*c*(b + c*x^2)) + (7*b^(3/4)*ArcTan[1 - (Sq
rt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(11/4)) - (7*b^(3/4)*ArcTan[1
 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(11/4)) - (7*b^(3/4)*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(11/
4)) + (7*b^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]
)/(8*Sqrt[2]*c^(11/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{17/2}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^{9/2}}{\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{x^{7/2}}{2c\left(b+cx^2\right)} + \frac{7\int \frac{x^{5/2}}{b+cx^2}\,dx}{4c}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c\left(b+cx^2\right)} - \frac{(7b)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4c^2}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c\left(b+cx^2\right)} - \frac{(7b)\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c^2}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c\left(b+cx^2\right)} + \frac{(7b)\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^{5/2}} - \frac{(7b)\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{\right.}{4c^{5/2}}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c\left(b+cx^2\right)} - \frac{(7b)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{8c^3} - \frac{(7b)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} + \frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{c}} + x\right.}{8c^3}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c\left(b+cx^2\right)} - \frac{7b^{3/4}\log\left(\sqrt{b} - \sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x} + \sqrt{c}\,x\right)}{8\sqrt{2}\,c^{11/4}} + \frac{7b^{3/4}\log\left(\sqrt{b} + \sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{\right.}{8\sqrt{2}\,c^{11/4}}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c\left(b+cx^2\right)} + \frac{7b^{3/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{11/4}} - \frac{7b^{3/4}\tan^{-1}\left(1 + \frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{11/4}} - \frac{7b^{3/4}\log\left(\right.}{}$$

<u>**Mathematica**</u> **[C]** time = 0.02, size = 57, normalized size = 0.25

$$-\frac{2x^{3/2}\left(7\left(b+cx^2\right){}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right) - 7b - cx^2\right)}{3c^2\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(17/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-2*x^(3/2)*(-7*b - c*x^2 + 7*(b + c*x^2)*Hypergeometric2F1[3/4, 2, 7/4, -(
(c*x^2)/b)]))/(3*c^2*(b + c*x^2))
```

fricas [A] time = 0.64, size = 229, normalized size = 1.00

$$\frac{84\left(c^3x^2+bc^2\right)\left(-\frac{b^3}{c^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{343\,b^2c^3\sqrt{x}\left(-\frac{b^3}{c^{11}}\right)^{\frac{1}{4}}-\sqrt{-117649\,b^3c^5\sqrt{-\frac{b^3}{c^{11}}}+117649\,b^4x\,c^3\left(-\frac{b^3}{c^{11}}\right)^{\frac{1}{4}}}}{343\,b^3}\right)-21\left(c^3x^2+bc^2\right)\left(-\frac{b^3}{c^{11}}\right.}{24\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(84*(c^3*x^2 + b*c^2)*(-b^3/c^11)^(1/4)*arctan(-1/343*(343*b^2*c^3*sqr
t(x)*(-b^3/c^11)^(1/4) - sqrt(-117649*b^3*c^5*sqrt(-b^3/c^11) + 117649*b^4*
x)*c^3*(-b^3/c^11)^(1/4))/b^3) - 21*(c^3*x^2 + b*c^2)*(-b^3/c^11)^(1/4)*log
(343*c^8*(-b^3/c^11)^(3/4) + 343*b^2*sqrt(x)) + 21*(c^3*x^2 + b*c^2)*(-b^3/
c^11)^(1/4)*log(-343*c^8*(-b^3/c^11)^(3/4) + 343*b^2*sqrt(x)) + 4*(4*c*x^3
+ 7*b*x)*sqrt(x))/(c^3*x^2 + b*c^2)
```

giac [A] time = 0.18, size = 196, normalized size = 0.85

$$\frac{bx^{\frac{3}{2}}}{2\left(cx^2+b\right)c^2}+\frac{2\,x^{\frac{3}{2}}}{3\,c^2}-\frac{7\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^5}-\frac{7\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^5}+\frac{7\sqrt{2}\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*b*x^(3/2)/((c*x^2 + b)*c^2) + 2/3*x^(3/2)/c^2 - 7/8*sqrt(2)*(b*c^3)^(3/
4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^5 -
7/8*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt
(x))/(b/c)^(1/4))/c^5 + 7/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c
)^(1/4) + x + sqrt(b/c))/c^5 - 7/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt
(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5
```

maple [A] time = 0.01, size = 161, normalized size = 0.70

$$\frac{b\,x^{\frac{3}{2}}}{2\left(c\,x^2+b\right)c^2}+\frac{2x^{\frac{3}{2}}}{3c^2}-\frac{7\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}c^3}-\frac{7\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}c^3}-\frac{7\sqrt{2}\,b\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{16\left(\frac{b}{c}\right)^{\frac{1}{4}}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/3*x^(3/2)/c^2+1/2*b/c^2*x^(3/2)/(c*x^2+b)-7/16*b/c^3/(b/c)^(1/4)*2^(1/2)*
ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/
2)+(b/c)^(1/2)))-7/8*b/c^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x
^(1/2)+1)-7/8*b/c^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-
1)
```

maxima [A] time = 3.05, size = 207, normalized size = 0.90

$$\frac{bx^{\frac{3}{2}}}{2\left(c^3x^2+bc^2\right)}-\frac{7b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\cdots\right)}{16c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^(3/2)/(c^3*x^2 + b*c^2) - 7/16*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqr
t(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt
(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1
/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt
(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(
b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x
 + sqrt(b))/(b^(1/4)*c^(3/4)))/c^2 + 2/3*x^(3/2)/c^2
```

mupad [B] time = 0.11, size = 80, normalized size = 0.35

$$\frac{2\,x^{3/2}}{3\,c^2}+\frac{7\,{(-b)}^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)}{4\,c^{11/4}}+\frac{b\,x^{3/2}}{2\,\left(c^3\,x^2+b\,c^2\right)}+\frac{{(-b)}^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1\mathrm{i}}{{(-b)}^{1/4}}\right)\,7\mathrm{i}}{4\,c^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (2*x^(3/2))/(3*c^2) + (7*(-b)^(3/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*
c^(11/4)) + ((-b)^(3/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*7i)/(4*c^(11/
4)) + (b*x^(3/2))/(2*(b*c^2 + c^3*x^2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.329 $\int \frac{x^{15/2}}{(bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=230

$$\frac{5\sqrt[4]{b}\,\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{9/4}}-\frac{5\sqrt[4]{b}\,\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{9/4}}+\frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{9/4}}-\ldots$$

```
[Out] -1/2*x^(5/2)/c/(c*x^2+b)+5/8*b^(1/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/
4))/c^(9/4)*2^(1/2)-5/8*b^(1/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c
^(9/4)*2^(1/2)+5/16*b^(1/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^
(1/2))/c^(9/4)*2^(1/2)-5/16*b^(1/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^
(1/2)*x^(1/2))/c^(9/4)*2^(1/2)+5/2*x^(1/2)/c^2
```

Rubi [A] time = 0.18, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{5\sqrt[4]{b}\,\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{9/4}}-\frac{5\sqrt[4]{b}\,\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{9/4}}+\frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{9/4}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[x^(15/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (5*Sqrt[x])/(2*c^2) - x^(5/2)/(2*c*(b + c*x^2)) + (5*b^(1/4)*ArcTan[1 - (Sq
rt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(9/4)) - (5*b^(1/4)*ArcTan[1
+ (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(9/4)) + (5*b^(1/4)*Log[
Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(9/4))
 - (5*b^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(
8*Sqrt[2]*c^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{15/2}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^{7/2}}{\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{x^{5/2}}{2c\left(b+cx^2\right)} + \frac{5\int \frac{x^{3/2}}{b+cx^2}\,dx}{4c}$$

$$= \frac{5\sqrt{x}}{2c^2} - \frac{x^{5/2}}{2c\left(b+cx^2\right)} - \frac{(5b)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4c^2}$$

$$= \frac{5\sqrt{x}}{2c^2} - \frac{x^{5/2}}{2c\left(b+cx^2\right)} - \frac{(5b)\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c^2}$$

$$= \frac{5\sqrt{x}}{2c^2} - \frac{x^{5/2}}{2c\left(b+cx^2\right)} - \frac{\left(5\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^2} - \frac{\left(5\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,d\right.}{4c^2}$$

$$= \frac{5\sqrt{x}}{2c^2} - \frac{x^{5/2}}{2c\left(b+cx^2\right)} - \frac{\left(5\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8c^{5/2}} - \frac{\left(5\sqrt{b}\right)\text{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}+\right.}{8c^{5/2}}$$

$$= \frac{5\sqrt{x}}{2c^2} - \frac{x^{5/2}}{2c\left(b+cx^2\right)} + \frac{5\sqrt[4]{b}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{9/4}} - \frac{5\sqrt[4]{b}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\right.}{8\sqrt{2}\,c^{9/4}}$$

$$= \frac{5\sqrt{x}}{2c^2} - \frac{x^{5/2}}{2c\left(b+cx^2\right)} + \frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{9/4}} - \frac{5\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{9/4}} + \frac{5\sqrt[4]{b}\log}{}$$

Mathematica [A] time = 0.10, size = 221, normalized size = 0.96

$$\frac{\frac{32c^{5/4}x^{5/2}}{b+cx^2} + \frac{40b\sqrt[4]{c}\sqrt{x}}{b+cx^2} + 5\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right) - 5\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{16c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(15/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((40*b*c^(1/4)*Sqrt[x])/(b + c*x^2) + (32*c^(5/4)*x^(5/2))/(b + c*x^2) + 10
*Sqrt[2]*b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 10*Sqrt[2]
*b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 5*Sqrt[2]*b^(1/4)*
```

```
Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - 5*Sqrt[2]*b^(1
/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(16*c^(9/4)
)
```

fricas [A] time = 0.56, size = 192, normalized size = 0.83

$$-\frac{20\left(c^{3}x^{2}+bc^{2}\right)\left(-\frac{b}{c^{9}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^{4}\sqrt{-\frac{b}{c^{9}}}+x}\,c^{7}\left(-\frac{b}{c^{9}}\right)^{\frac{3}{4}}-c^{7}\sqrt{x}\left(-\frac{b}{c^{9}}\right)^{\frac{3}{4}}}{b}\right)+5\left(c^{3}x^{2}+bc^{2}\right)\left(-\frac{b}{c^{9}}\right)^{\frac{1}{4}}\log\left(5\,c^{2}\left(-\frac{b}{c^{9}}\right)^{\frac{1}{4}}+5\sqrt{x}\right)}{8\left(c^{3}x^{2}+bc^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(20*(c^3*x^2 + b*c^2)*(-b/c^9)^(1/4)*arctan((sqrt(c^4*sqrt(-b/c^9) + x
)*c^7*(-b/c^9)^(3/4) - c^7*sqrt(x)*(-b/c^9)^(3/4))/b) + 5*(c^3*x^2 + b*c^2)
*(-b/c^9)^(1/4)*log(5*c^2*(-b/c^9)^(1/4) + 5*sqrt(x)) - 5*(c^3*x^2 + b*c^2)
*(-b/c^9)^(1/4)*log(-5*c^2*(-b/c^9)^(1/4) + 5*sqrt(x)) - 4*(4*c*x^2 + 5*b)*
sqrt(x))/(c^3*x^2 + b*c^2)
```

giac [A] time = 0.20, size = 196, normalized size = 0.85

$$-\frac{5\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^{3}}-\frac{5\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^{3}}-\frac{5\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)\right.}{16\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -5/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/c^3 - 5/8*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^3 - 5/16*sqrt(2)*(b*c^3)^(1/4)*
log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 + 5/16*sqrt(2)*(b*c^3)
^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 + 1/2*b*sqrt(x
)/((c*x^2 + b)*c^2) + 2*sqrt(x)/c^2
```

maple [A] time = 0.01, size = 158, normalized size = 0.69

$$\frac{b\sqrt{x}}{2\left(c\,x^{2}+b\right)c^{2}}-\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8c^{2}}-\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8c^{2}}-\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{}}\right.}{16c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 2*x^(1/2)/c^2+1/2*b/c^2*x^(1/2)/(c*x^2+b)-5/16/c^2*(b/c)^(1/4)*2^(1/2)*ln((
x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(
b/c)^(1/2)))-5/8/c^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)
+1)-5/8/c^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.06, size = 206, normalized size = 0.90

$$\frac{b\sqrt{x}}{2\left(c^{3}x^{2}+bc^{2}\right)}-\frac{5\left(\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\sqrt{b}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\,b^{\frac{1}{4}}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\right.}{c^{\frac{1}{4}}}\right.}{16\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*sqrt(x)/(c^3*x^2 + b*c^2) - 5/16*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2
)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqrt
(sqrt(b)*sqrt(c)) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*
c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) +
 sqrt(2)*b^(1/4)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))
/c^(1/4) - sqrt(2)*b^(1/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x
 + sqrt(b))/c^(1/4))/c^2 + 2*sqrt(x)/c^2
```

mupad [B] time = 4.32, size = 80, normalized size = 0.35

$$\frac{2\sqrt{x}}{c^{2}}-\frac{5\,{(-b)}^{1/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)}{4\,c^{9/4}}+\frac{b\,\sqrt{x}}{2\,\left(c^{3}\,x^{2}+b\,c^{2}\right)}+\frac{{(-b)}^{1/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1\mathrm{i}}{{(-b)}^{1/4}}\right)\,5\mathrm{i}}{4\,c^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (2*x^(1/2))/c^2 - (5*(-b)^(1/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*c^(9
/4)) + ((-b)^(1/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*5i)/(4*c^(9/4)) +
(b*x^(1/2))/(2*(b*c^2 + c^3*x^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.330 $\int \frac{x^{13/2}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=218

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\sqrt[4]{b}\,c^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\sqrt[4]{b}\,c^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}\,c^{7/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}}{}\right.}{4\sqrt{2}}$$

```
[Out] -1/2*x^(3/2)/c/(c*x^2+b)-3/8*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1
/4)/c^(7/4)*2^(1/2)+3/8*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/c
^(7/4)*2^(1/2)+3/16*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b
^(1/4)/c^(7/4)*2^(1/2)-3/16*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^
(1/2))/b^(1/4)/c^(7/4)*2^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\sqrt[4]{b}\,c^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\sqrt[4]{b}\,c^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}\,c^{7/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}}{}\right.}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -x^(3/2)/(2*c*(b + c*x^2)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4
)])/(4*Sqrt[2]*b^(1/4)*c^(7/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b
^(1/4)])/(4*Sqrt[2]*b^(1/4)*c^(7/4)) + (3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(
1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(1/4)*c^(7/4)) - (3*Log[Sqrt[b] + S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(1/4)*c^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{13/2}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^{5/2}}{\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{x^{3/2}}{2c\left(b+cx^2\right)} + \frac{3\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4c}$$

$$= -\frac{x^{3/2}}{2c\left(b+cx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c}$$

$$= -\frac{x^{3/2}}{2c\left(b+cx^2\right)} - \frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^{3/2}} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^{3/2}}$$

$$= -\frac{x^{3/2}}{2c\left(b+cx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8c^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8c^2}$$

$$= -\frac{x^{3/2}}{2c\left(b+cx^2\right)} + \frac{3\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\sqrt[4]{b}\,c^{7/4}} - \frac{3\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\sqrt[4]{b}\,c^{7/4}}$$

$$= -\frac{x^{3/2}}{2c\left(b+cx^2\right)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}\,c^{7/4}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}\,c^{7/4}} + \frac{3\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\ldots\right.}{8\sqrt{2}\sqrt[4]{b}\,c^{7\ldots}}$$

Mathematica [C] time = 0.01, size = 43, normalized size = 0.20

$$\frac{2x^{3/2}\left(\frac{{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{b}-\frac{1}{b+cx^2}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(3/2)*(-(b + c*x^2)^(-1) + Hypergeometric2F1[3/4, 2, 7/4, -((c*x^2)/b)
]/b))/c
```

fricas [A] time = 0.93, size = 185, normalized size = 0.85

$$-\frac{12\left(c^{2}x^{2}+bc\right)\left(-\frac{1}{bc^{7}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-bc^{3}\sqrt{-\frac{1}{bc^{7}}}+x}\,c^{2}\left(-\frac{1}{bc^{7}}\right)^{\frac{1}{4}}-c^{2}\sqrt{x}\left(-\frac{1}{bc^{7}}\right)^{\frac{1}{4}}\right)-3\left(c^{2}x^{2}+bc\right)\left(-\frac{1}{bc^{7}}\right)^{\frac{1}{4}}\log\left(bc^{5}\left(\right.\right.}{8\left(c^{2}x^{2}+bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(12*(c^2*x^2 + b*c)*(-1/(b*c^7))^(1/4)*arctan(sqrt(-b*c^3*sqrt(-1/(b*c
^7)) + x)*c^2*(-1/(b*c^7))^(1/4) - c^2*sqrt(x)*(-1/(b*c^7))^(1/4)) - 3*(c^2
*x^2 + b*c)*(-1/(b*c^7))^(1/4)*log(b*c^5*(-1/(b*c^7))^(3/4) + sqrt(x)) + 3*
(c^2*x^2 + b*c)*(-1/(b*c^7))^(1/4)*log(-b*c^5*(-1/(b*c^7))^(3/4) + sqrt(x))
 + 4*x^(3/2))/(c^2*x^2 + b*c)
```

giac [A] time = 0.18, size = 199, normalized size = 0.91

$$-\frac{x^{\frac{3}{2}}}{2\left(cx^{2}+b\right)c}+\frac{3\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^{4}}+\frac{3\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^{4}}-\frac{3\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*x^(3/2)/((c*x^2 + b)*c) + 3/8*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) + 3/8*sqrt(2)*(b*c^
3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))
/(b*c^4) - 3/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
 sqrt(b/c))/(b*c^4) + 3/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)
^(1/4) + x + sqrt(b/c))/(b*c^4)
```

maple [A] time = 0.01, size = 149, normalized size = 0.68

$$-\frac{x^{\frac{3}{2}}}{2\left(c x^{2}+b\right)c}+\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{2}}+\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{2}}+\frac{3\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{16\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2*x^(3/2)/c/(c*x^2+b)+3/16/c^2/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*2^(
1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+3/8/
c^2/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+3/8/c^2/(b/c)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.00, size = 195, normalized size = 0.89

$$-\frac{x^{\frac{3}{2}}}{2\left(c^{2}x^{2}+bc\right)}+\frac{3\left(\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\right.}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x^(3/2)/(c^2*x^2 + b*c) + 3/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*
b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*s
qrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) -
 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c))
- sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/
4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sq
rt(b))/(b^(1/4)*c^(3/4)))/c
```

mupad [B] time = 0.09, size = 64, normalized size = 0.29

$$\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)}{4\,{(-b)}^{1/4}\,c^{7/4}}-\frac{x^{3/2}}{2\,c\,\left(c\,x^2+b\right)}-\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)}{4\,{(-b)}^{1/4}\,c^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (3*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*(-b)^(1/4)*c^(7/4)) - x^(3/2)/(2*
c*(b + c*x^2)) - (3*atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*(-b)^(1/4)*c^(7
/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.331 $\int \frac{x^{11/2}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=218

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{3/4}c^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{3/4}c^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{3/4}c^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{4\sqrt{2}\,b^{3/4}c^{5/4}}$$

```
[Out] -1/8*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^(5/4)*2^(1/2)+1/8*
arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^(5/4)*2^(1/2)-1/16*ln(b
^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(5/4)*2^(1/2)+1
/16*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(5/4)*2
^(1/2)-1/2*x^(1/2)/c/(c*x^2+b)
```

Rubi [A] time = 0.16, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{3/4}c^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{3/4}c^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{3/4}c^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{4\sqrt{2}\,b^{3/4}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -Sqrt[x]/(2*c*(b + c*x^2)) - ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/
(4*Sqrt[2]*b^(3/4)*c^(5/4)) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
/(4*Sqrt[2]*b^(3/4)*c^(5/4)) - Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x]/(8*Sqrt[2]*b^(3/4)*c^(5/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*
c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(8*Sqrt[2]*b^(3/4)*c^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{11/2}}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^{3/2}}{\left(b+cx^2\right)^2}\,dx\\
&= -\frac{\sqrt{x}}{2c\left(b+cx^2\right)}+\frac{\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4c}\\
&= -\frac{\sqrt{x}}{2c\left(b+cx^2\right)}+\frac{\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{2c}\\
&= -\frac{\sqrt{x}}{2c\left(b+cx^2\right)}+\frac{\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4\sqrt{b}\,c}+\frac{\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4\sqrt{b}\,c}\\
&= -\frac{\sqrt{x}}{2c\left(b+cx^2\right)}+\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{b}\,c^{3/2}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{b}\,c^{3/2}}-\\
&= -\frac{\sqrt{x}}{2c\left(b+cx^2\right)}-\frac{\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{3/4}c^{5/4}}+\frac{\log\left(\sqrt{b}+\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{3/4}c^{5/4}}+\\
&= -\frac{\sqrt{x}}{2c\left(b+cx^2\right)}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{3/4}c^{5/4}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{3/4}c^{5/4}}-\frac{\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}\right.}{8\sqrt{2}\,b^{3/4}c^{5/4}}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 198, normalized size = 0.91

$$
\frac{-\frac{\sqrt{2}\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{b^{3/4}}+\frac{\sqrt{2}\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{b^{3/4}}-\frac{2\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{b^{3/4}}+\frac{2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{b^{3/4}}-\frac{8\sqrt[4]{c}}{b+c}}{16c^{5/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-8*c^(1/4)*Sqrt[x])/(b + c*x^2) - (2*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*
Sqrt[x])/b^(1/4)])/b^(3/4) + (2*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/b^(3/4) - (Sqrt[2]*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x])/b^(3/4) + (Sqrt[2]*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sq
rt[x] + Sqrt[c]*x])/b^(3/4))/(16*c^(5/4))
```

fricas [A] time = 0.89, size = 187, normalized size = 0.86

$$\frac{4\left(c^2x^2+bc\right)\left(-\frac{1}{b^3c^5}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^2c^2\sqrt{-\frac{1}{b^3c^5}}+x}\,b^2c^4\left(-\frac{1}{b^3c^5}\right)^{\frac{3}{4}}-b^2c^4\sqrt{x}\left(-\frac{1}{b^3c^5}\right)^{\frac{3}{4}}\right)+\left(c^2x^2+bc\right)\left(-\frac{1}{b^3c^5}\right)^{\frac{1}{4}}\log\left(b\cdots\right.}{8\left(c^2x^2+bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(c^2*x^2 + b*c)*(-1/(b^3*c^5))^(1/4)*arctan(sqrt(b^2*c^2*sqrt(-1/(b^
3*c^5)) + x)*b^2*c^4*(-1/(b^3*c^5))^(3/4) - b^2*c^4*sqrt(x)*(-1/(b^3*c^5))^
(3/4)) + (c^2*x^2 + b*c)*(-1/(b^3*c^5))^(1/4)*log(b*c*(-1/(b^3*c^5))^(1/4)
+ sqrt(x)) - (c^2*x^2 + b*c)*(-1/(b^3*c^5))^(1/4)*log(-b*c*(-1/(b^3*c^5))^(
1/4) + sqrt(x)) - 4*sqrt(x))/(c^2*x^2 + b*c)
```

giac [A] time = 0.17, size = 199, normalized size = 0.91

$$\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^2}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^2}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x\cdots\right.}{16\,bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(
x))/(b/c)^(1/4))/(b*c^2) + 1/8*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^2) + 1/16*sqrt(2)*(b*c^3)
^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^2) - 1/16*sqrt
(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^2)
 - 1/2*sqrt(x)/((c*x^2 + b)*c)
```

maple [A] time = 0.01, size = 158, normalized size = 0.72

$$-\frac{\sqrt{x}}{2\left(c\,x^{2}+b\right)c}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8bc}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8bc}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{16bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2*x^(1/2)/c/(c*x^2+b)+1/16/c*(b/c)^(1/4)/b*2^(1/2)*ln((x+(b/c)^(1/4)*2^(
1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+1/8/
c*(b/c)^(1/4)/b*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/8/c*(b/c)^(
1/4)/b*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.10, size = 195, normalized size = 0.89

$$\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{}\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqr
t(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b
)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*log(sqrt(2)*b^(1/4)*c
^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*log(-sqrt
(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/c - 1
/2*sqrt(x)/(c^2*x^2 + b*c)
```

mupad [B] time = 4.31, size = 64, normalized size = 0.29

$$-\frac{\sqrt{x}}{2\,c\,\left(c\,x^{2}+b\right)}-\frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{4\,(-b)^{3/4}\,c^{5/4}}-\frac{\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{4\,(-b)^{3/4}\,c^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] - x^(1/2)/(2*c*(b + c*x^2)) - atan((c^(1/4)*x^(1/2))/(-b)^(1/4))/(4*(-b)^(3
/4)*c^(5/4)) - atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))/(4*(-b)^(3/4)*c^(5/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.332 $\int \frac{x^{9/2}}{\left(bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=218

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{5/4}c^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{5/4}c^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{5/4}c^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{4\sqrt{2}\,b^{5/4}c^{3/4}}$$

```
[Out] 1/2*x^(3/2)/b/(c*x^2+b)-1/8*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/
4)/c^(3/4)*2^(1/2)+1/8*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^
(3/4)*2^(1/2)+1/16*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^
(5/4)/c^(3/4)*2^(1/2)-1/16*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(
1/2))/b^(5/4)/c^(3/4)*2^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{5/4}c^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{5/4}c^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{5/4}c^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{4\sqrt{2}\,b^{5/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] x^(3/2)/(2*b*(b + c*x^2)) - ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(
4*Sqrt[2]*b^(5/4)*c^(3/4)) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/
(4*Sqrt[2]*b^(5/4)*c^(3/4)) + Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
 + Sqrt[c]*x]/(8*Sqrt[2]*b^(5/4)*c^(3/4)) - Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c
^(1/4)*Sqrt[x] + Sqrt[c]*x]/(8*Sqrt[2]*b^(5/4)*c^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

```
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{9/2}}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{\sqrt{x}}{\left(b+cx^2\right)^2}\,dx \\
&= \frac{x^{3/2}}{2b\left(b+cx^2\right)} + \frac{\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b} \\
&= \frac{x^{3/2}}{2b\left(b+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b} \\
&= \frac{x^{3/2}}{2b\left(b+cx^2\right)} - \frac{\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b\sqrt{c}} \\
&= \frac{x^{3/2}}{2b\left(b+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8bc} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8bc} + \\
&= \frac{x^{3/2}}{2b\left(b+cx^2\right)} + \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{5/4}c^{3/4}} - \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{5/4}c^{3/4}} + \\
&= \frac{x^{3/2}}{2b\left(b+cx^2\right)} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{5/4}c^{3/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{5/4}c^{3/4}} + \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{8\sqrt{2}\,b^{5/4}c^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.13

$$\frac{2x^{3/2}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((c*x^2)/b)])/(3*b^2)
```

fricas [A] time = 0.86, size = 182, normalized size = 0.83

$$-\frac{4\left(bcx^2+b^2\right)\left(-\frac{1}{b^5c^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-b^3c\sqrt{-\frac{1}{b^5c^3}}+x}\,bc\left(-\frac{1}{b^5c^3}\right)^{\frac{1}{4}}-bc\sqrt{x}\left(-\frac{1}{b^5c^3}\right)^{\frac{1}{4}}\right)-\left(bcx^2+b^2\right)\left(-\frac{1}{b^5c^3}\right)^{\frac{1}{4}}\log\left(b^4\right.}{8\left(bcx^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(b*c*x^2 + b^2)*(-1/(b^5*c^3))^(1/4)*arctan(sqrt(-b^3*c*sqrt(-1/(b^
5*c^3)) + x)*b*c*(-1/(b^5*c^3))^(1/4) - b*c*sqrt(x)*(-1/(b^5*c^3))^(1/4)) -
 (b*c*x^2 + b^2)*(-1/(b^5*c^3))^(1/4)*log(b^4*c^2*(-1/(b^5*c^3))^(3/4) + sq
rt(x)) + (b*c*x^2 + b^2)*(-1/(b^5*c^3))^(1/4)*log(-b^4*c^2*(-1/(b^5*c^3))^(
3/4) + sqrt(x)) - 4*x^(3/2))/(b*c*x^2 + b^2)
```

giac [A] time = 0.18, size = 199, normalized size = 0.91

$$\frac{x^{\frac{3}{2}}}{2\left(cx^2+b\right)b}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^3}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^3}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^(3/2)/((c*x^2 + b)*b) + 1/8*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) + 1/8*sqrt(2)*(b*c
^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4)
)/(b^2*c^3) - 1/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) +
x + sqrt(b/c))/(b^2*c^3) + 1/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*
(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^3)
```

maple [A] time = 0.01, size = 158, normalized size = 0.72

$$\frac{x^{\frac{3}{2}}}{2\left(c\,x^2+b\right)b}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}bc}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}bc}+\frac{\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{16\left(\frac{b}{c}\right)^{\frac{1}{4}}bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*x^(3/2)/b/(c*x^2+b)+1/16/b/c/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*2^(1
/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+1/8/b
/c/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/8/b/c/(b/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.04, size = 194, normalized size = 0.89

$$\frac{x^{\frac{3}{2}}}{2\left(bcx^2+b^2\right)}+\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\cdots}{\cdots}}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^(3/2)/(b*c*x^2 + b^2) + 1/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b
^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sq
rt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) -
2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) -
 sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4
)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqr
t(b))/(b^(1/4)*c^(3/4)))/b
```

mupad [B] time = 4.34, size = 64, normalized size = 0.29

$$\frac{x^{3/2}}{2\,b\,\left(c\,x^2+b\right)}-\frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,{\left(-b\right)}^{5/4}\,c^{3/4}}+\frac{\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,{\left(-b\right)}^{5/4}\,c^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(3/2)/(2*b*(b + c*x^2)) - atan((c^(1/4)*x^(1/2))/(-b)^(1/4))/(4*(-b)^(5/4
)*c^(3/4)) + atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))/(4*(-b)^(5/4)*c^(3/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.333 $\int \frac{x^{7/2}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=218

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\cdot\right)}{4\sqrt{2}}$$

```
[Out] -3/8*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)/c^(1/4)*2^(1/2)+3/8*
arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)/c^(1/4)*2^(1/2)-3/16*ln(b
^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(1/4)*2^(1/2)+3
/16*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(1/4)*2
^(1/2)+1/2*x^(1/2)/b/(c*x^2+b)
```

Rubi [A] time = 0.17, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\cdot\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] Sqrt[x]/(2*b*(b + c*x^2)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)
])/(4*Sqrt[2]*b^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^
(1/4)])/(4*Sqrt[2]*b^(7/4)*c^(1/4)) - (3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1
/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(7/4)*c^(1/4)) + (3*Log[Sqrt[b] + Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(7/4)*c^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{7/2}}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{1}{\sqrt{x}\left(b+cx^2\right)^2}\,dx\\
&= \frac{\sqrt{x}}{2b\left(b+cx^2\right)}+\frac{3\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4b}\\
&= \frac{\sqrt{x}}{2b\left(b+cx^2\right)}+\frac{3\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{2b}\\
&= \frac{\sqrt{x}}{2b\left(b+cx^2\right)}+\frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4b^{3/2}}+\frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4b^{3/2}}\\
&= \frac{\sqrt{x}}{2b\left(b+cx^2\right)}+\frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8b^{3/2}\sqrt{c}}+\frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8b^{3/2}\sqrt{c}}\\
&= \frac{\sqrt{x}}{2b\left(b+cx^2\right)}-\frac{3\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{7/4}\sqrt[4]{c}}\\
&= \frac{\sqrt{x}}{2b\left(b+cx^2\right)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{7/4}\sqrt[4]{c}}-\frac{3\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\right.}{8\sqrt{2}\,b^{7/4}\sqrt[4]{c}}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 199, normalized size = 0.91

$$
\frac{\frac{8b^{3/4}\sqrt{x}}{b+cx^2}-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{\sqrt[4]{c}}+\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{\sqrt[4]{c}}-\frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt[4]{c}}+\frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{\sqrt[4]{c}}}{16b^{7/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((8*b^(3/4)*Sqrt[x])/(b + c*x^2) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*S
qrt[x])/b^(1/4)])/c^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])
/b^(1/4)])/c^(1/4) - (3*Sqrt[2]*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[
x] + Sqrt[c]*x])/c^(1/4) + (3*Sqrt[2]*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)
*Sqrt[x] + Sqrt[c]*x])/c^(1/4))/(16*b^(7/4))
```

fricas [A] time = 0.91, size = 179, normalized size = 0.82

$$\frac{12\left(bcx^2+b^2\right)\left(-\frac{1}{b^7c}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^4\sqrt{-\frac{1}{b^7c}}+x}\,b^5c\left(-\frac{1}{b^7c}\right)^{\frac{3}{4}}-b^5c\sqrt{x}\left(-\frac{1}{b^7c}\right)^{\frac{3}{4}}\right)+3\left(bcx^2+b^2\right)\left(-\frac{1}{b^7c}\right)^{\frac{1}{4}}\log\left(b^2\left(-\cdots\right.\right.}{8\left(bcx^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(12*(b*c*x^2 + b^2)*(-1/(b^7*c))^(1/4)*arctan(sqrt(b^4*sqrt(-1/(b^7*c))
 + x)*b^5*c*(-1/(b^7*c))^(3/4) - b^5*c*sqrt(x)*(-1/(b^7*c))^(3/4)) + 3*(b*c
*x^2 + b^2)*(-1/(b^7*c))^(1/4)*log(b^2*(-1/(b^7*c))^(1/4) + sqrt(x)) - 3*(b
*c*x^2 + b^2)*(-1/(b^7*c))^(1/4)*log(-b^2*(-1/(b^7*c))^(1/4) + sqrt(x)) + 4
*sqrt(x))/(b*c*x^2 + b^2)
```

giac [A] time = 0.19, size = 199, normalized size = 0.91

$$\frac{3\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c}+\frac{3\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c}+\frac{3\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}\cdots\right.}{16\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(
x))/(b/c)^(1/4))/(b^2*c) + 3/8*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c) + 3/16*sqrt(2)*(b*c^3)
^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c) - 3/16*sqrt
(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c)
 + 1/2*sqrt(x)/((c*x^2 + b)*b)
```

maple [A] time = 0.01, size = 149, normalized size = 0.68

$$\frac{\sqrt{x}}{2\left(c\,x^{2}+b\right)b}+\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8b^{2}}+\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8b^{2}}+\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{}}\right.}{16b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*x^(1/2)/b/(c*x^2+b)+3/16/b^2*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1
/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+3/8/b
^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+3/8/b^2*(b/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.05, size = 194, normalized size = 0.89

$$\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right.}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 3/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqr
t(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b
)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*log(sqrt(2)*b^(1/4)*c
^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*log(-sqrt
(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b + 1
/2*sqrt(x)/(b*c*x^2 + b^2)
```

mupad [B] time = 0.10, size = 64, normalized size = 0.29

$$\frac{\sqrt{x}}{2\,b\,\left(c\,x^2+b\right)}+\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,{\left(-b\right)}^{7/4}\,c^{1/4}}+\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,{\left(-b\right)}^{7/4}\,c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(1/2)/(2*b*(b + c*x^2)) + (3*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*(-b)^
(7/4)*c^(1/4)) + (3*atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*(-b)^(7/4)*c^(1
/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.334 $\int \frac{x^{5/2}}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=230

$$-\frac{5\sqrt[4]{c}\,\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{9/4}}+\frac{5\sqrt[4]{c}\,\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{9/4}}+\frac{5\sqrt[4]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{9/4}}-$$

```
[Out] 5/8*c^(1/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(9/4)*2^(1/2)-5/8*c
^(1/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(9/4)*2^(1/2)-5/16*c^(1/
4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)*2^(1/2)+5/
16*c^(1/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)*2^
(1/2)-5/2/b^2/x^(1/2)+1/2/b/(c*x^2+b)/x^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{5\sqrt[4]{c}\,\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{9/4}}+\frac{5\sqrt[4]{c}\,\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{9/4}}+\frac{5\sqrt[4]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{9/4}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -5/(2*b^2*Sqrt[x]) + 1/(2*b*Sqrt[x]*(b + c*x^2)) + (5*c^(1/4)*ArcTan[1 - (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(9/4)) - (5*c^(1/4)*ArcTan[1
 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(9/4)) - (5*c^(1/4)*Log
[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(9/4)
) + (5*c^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/
(8*Sqrt[2]*b^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
```

```
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{1}{x^{3/2}\left(b+cx^2\right)^2}\,dx$$

$$= \frac{1}{2b\sqrt{x}\left(b+cx^2\right)} + \frac{5\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{4b}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\left(b+cx^2\right)} - \frac{(5c)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b^2}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\left(b+cx^2\right)} - \frac{(5c)\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^2}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\left(b+cx^2\right)} + \frac{\left(5\sqrt{c}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^2} - \frac{\left(5\sqrt{c}\right)\text{Subst}\left(\int \frac{\sqrt{b}+}{b+}\right.}{4b^2}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\left(b+cx^2\right)} - \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^2} - \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}}\right.}{8b^2}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\left(b+cx^2\right)} - \frac{5\sqrt[4]{c}\,\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{9/4}} + \frac{5\sqrt[4]{c}\,\log\left(\sqrt{b}+\sqrt{2}\right.}{8\sqrt{2}}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\left(b+cx^2\right)} + \frac{5\sqrt[4]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{9/4}} - \frac{5\sqrt[4]{c}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{9/4}} - 5\sqrt[4]{}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.12

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},2;\frac{3}{4};-\frac{cx^2}{b}\right)}{b^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 2, 3/4, -((c*x^2)/b)])/(b^2*Sqrt[x])
```

fricas [A] time = 0.84, size = 208, normalized size = 0.90

$$\frac{20\left(b^2cx^3+b^3x\right)\left(-\frac{c}{b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{125\,b^2c\sqrt{x}\left(-\frac{c}{b^9}\right)^{\frac{1}{4}}-\sqrt{-15625\,b^5c\sqrt{-\frac{c}{b^9}}+15625\,c^2x}\,b^2\left(-\frac{c}{b^9}\right)^{\frac{1}{4}}}{125\,c}\right)-5\left(b^2cx^3+b^3x\right)\left(-\frac{c}{b^9}\right)^{\frac{1}{4}}\log\cdots}{8\left(b^2cx^3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(20*(b^2*c*x^3 + b^3*x)*(-c/b^9)^(1/4)*arctan(-1/125*(125*b^2*c*sqrt(x)
*(-c/b^9)^(1/4) - sqrt(-15625*b^5*c*sqrt(-c/b^9) + 15625*c^2*x)*b^2*(-c/b^9
)^(1/4))/c) - 5*(b^2*c*x^3 + b^3*x)*(-c/b^9)^(1/4)*log(125*b^7*(-c/b^9)^(3/
4) + 125*c*sqrt(x)) + 5*(b^2*c*x^3 + b^3*x)*(-c/b^9)^(1/4)*log(-125*b^7*(-c
/b^9)^(3/4) + 125*c*sqrt(x)) - 4*(5*c*x^2 + 4*b)*sqrt(x))/(b^2*c*x^3 + b^3*
x)
```

giac [A] time = 0.17, size = 210, normalized size = 0.91

$$-\frac{5\,cx^2+4\,b}{2\left(cx^{\frac{5}{2}}+b\sqrt{x}\right)b^2}-\frac{5\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3c^2}-\frac{5\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3c^2}+\frac{5\sqrt{2}\,(b\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*c*x^2 + 4*b)/((c*x^(5/2) + b*sqrt(x))*b^2) - 5/8*sqrt(2)*(b*c^3)^(3
/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*
c^2) - 5/8*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) -
 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) + 5/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)
*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2) - 5/16*sqrt(2)*(b*c^3)^(3/4
)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2)
```

maple [A] time = 0.02, size = 158, normalized size = 0.69

$$-\frac{c\,x^{\frac{3}{2}}}{2\left(c\,x^2+b\right)b^2}-\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}b^2}-\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}b^2}-\frac{5\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{16\left(\frac{b}{c}\right)^{\frac{1}{4}}b^2}-\frac{2}{b^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/b^2*c*x^(3/2)/(c*x^2+b)-5/16/b^2/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)
*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-
5/8/b^2/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-5/8/b^2/(
b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/b^2/x^(1/2)
```

maxima [A] time = 2.93, size = 208, normalized size = 0.90

$$-\frac{5\,cx^{2}+4\,b}{2\left(b^{2}cx^{\frac{5}{2}}+b^{3}\sqrt{x}\right)}-\frac{5\,c\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\,\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right.}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right.}{16\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(5*c*x^2 + 4*b)/(b^2*c*x^(5/2) + b^3*sqrt(x)) - 5/16*c*(2*sqrt(2)*arct
an(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*s
qrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(s
qrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sq
rt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqr
t(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*
sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^2
```

mupad [B] time = 0.09, size = 77, normalized size = 0.33

$$\frac{5\,{\left(-c\right)}^{1/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{9/4}}-\frac{5\,{\left(-c\right)}^{1/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{9/4}}-\frac{\frac{2}{b}+\frac{5\,c\,x^2}{2\,b^2}}{b\,\sqrt{x}+c\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (5*(-c)^(1/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(9/4)) - (5*(-c)^(1
/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(9/4)) - (2/b + (5*c*x^2)/(2*b
^2))/(b*x^(1/2) + c*x^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.335 $\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=230

$$\frac{7c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{11/4}}-\frac{7c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{11/4}}+\frac{7c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{11/4}}-7c$$

```
[Out] -7/6/b^2/x^(3/2)+1/2/b/x^(3/2)/(c*x^2+b)+7/8*c^(3/4)*arctan(1-c^(1/4)*2^(1/
2)*x^(1/2)/b^(1/4))/b^(11/4)*2^(1/2)-7/8*c^(3/4)*arctan(1+c^(1/4)*2^(1/2)*x
^(1/2)/b^(1/4))/b^(11/4)*2^(1/2)+7/16*c^(3/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*
c^(1/4)*2^(1/2)*x^(1/2))/b^(11/4)*2^(1/2)-7/16*c^(3/4)*ln(b^(1/2)+x*c^(1/2)
+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(11/4)*2^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{11/4}}-\frac{7c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{11/4}}+\frac{7c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{11/4}}-7c$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -7/(6*b^2*x^(3/2)) + 1/(2*b*x^(3/2)*(b + c*x^2)) + (7*c^(3/4)*ArcTan[1 - (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(11/4)) - (7*c^(3/4)*ArcTan[
1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(11/4)) + (7*c^(3/4)*L
og[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(11
/4)) - (7*c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x
])/(8*Sqrt[2]*b^(11/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^2}\,dx=\int \frac{1}{x^{5/2}\left(b+cx^2\right)^2}\,dx$$

$$=\frac{1}{2bx^{3/2}\left(b+cx^2\right)}+\frac{7\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{4b}$$

$$=-\frac{7}{6b^2x^{3/2}}+\frac{1}{2bx^{3/2}\left(b+cx^2\right)}-\frac{(7c)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4b^2}$$

$$=-\frac{7}{6b^2x^{3/2}}+\frac{1}{2bx^{3/2}\left(b+cx^2\right)}-\frac{(7c)\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{2b^2}$$

$$=-\frac{7}{6b^2x^{3/2}}+\frac{1}{2bx^{3/2}\left(b+cx^2\right)}-\frac{(7c)\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4b^{5/2}}-\frac{(7c)\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\right.}{4b^{5/2}}$$

$$=-\frac{7}{6b^2x^{3/2}}+\frac{1}{2bx^{3/2}\left(b+cx^2\right)}-\frac{\left(7\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8b^{5/2}}-\frac{\left(7\sqrt{c}\right)\text{Subst}\left(\right.}{}$$

$$=-\frac{7}{6b^2x^{3/2}}+\frac{1}{2bx^{3/2}\left(b+cx^2\right)}+\frac{7c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{11/4}}-\frac{7c^{3/4}\log\left(\sqrt{b}+\right.}{8}$$

$$=-\frac{7}{6b^2x^{3/2}}+\frac{1}{2bx^{3/2}\left(b+cx^2\right)}+\frac{7c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{11/4}}-\frac{7c^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{11/4}}+$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.13

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},2;\frac{1}{4};-\frac{cx^2}{b}\right)}{3b^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 2, 1/4, -((c*x^2)/b)])/(3*b^2*x^(3/2))
```

fricas [A] time = 0.88, size = 228, normalized size = 0.99

$$-\frac{84\left(b^{2}cx^{4}+b^{3}x^{2}\right)\left(-\frac{c^{3}}{b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^{8}c\sqrt{x}\left(-\frac{c^{3}}{b^{11}}\right)^{\frac{3}{4}}-\sqrt{b^{6}\sqrt{-\frac{c^{3}}{b^{11}}}+c^{2}x}\,b^{8}\left(-\frac{c^{3}}{b^{11}}\right)^{\frac{3}{4}}}{c^{3}}\right)+21\left(b^{2}cx^{4}+b^{3}x^{2}\right)\left(-\frac{c^{3}}{b^{11}}\right)^{\frac{1}{4}}\log\left(7b^{3}\right.}{24\left(b^{2}cx^{4}+b^{3}x^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(84*(b^2*c*x^4 + b^3*x^2)*(-c^3/b^11)^(1/4)*arctan(-(b^8*c*sqrt(x)*(-
c^3/b^11)^(3/4) - sqrt(b^6*sqrt(-c^3/b^11) + c^2*x)*b^8*(-c^3/b^11)^(3/4))/
c^3) + 21*(b^2*c*x^4 + b^3*x^2)*(-c^3/b^11)^(1/4)*log(7*b^3*(-c^3/b^11)^(1/
4) + 7*c*sqrt(x)) - 21*(b^2*c*x^4 + b^3*x^2)*(-c^3/b^11)^(1/4)*log(-7*b^3*(
-c^3/b^11)^(1/4) + 7*c*sqrt(x)) + 4*(7*c*x^2 + 4*b)*sqrt(x))/(b^2*c*x^4 + b
^3*x^2)
```

giac [A] time = 0.18, size = 196, normalized size = 0.85

$$-\frac{7\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8b^{3}}-\frac{7\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8b^{3}}-\frac{7\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right.\right.}{16b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -7/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/b^3 - 7/8*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^3 - 7/16*sqrt(2)*(b*c^3)^(1/4)*
log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 + 7/16*sqrt(2)*(b*c^3)
^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 - 1/2*c*sqrt(x
)/((c*x^2 + b)*b^2) - 2/3/(b^2*x^(3/2))
```

maple [A] time = 0.02, size = 161, normalized size = 0.70

$$-\frac{c\sqrt{x}}{2\left(cx^{2}+b\right)b^{2}}-\frac{7\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8b^{3}}-\frac{7\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8b^{3}}-\frac{7\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}}\right.}{16b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(c*x^4+b*x^2)^2,x)

[Out] -1/2/b^2*c*x^(1/2)/(c*x^2+b)-7/16/b^3*c*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-7/8/b^3*c*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-7/8/b^3*c*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/3/b^2/x^(3/2)

maxima [A] time = 3.03, size = 209, normalized size = 0.91

$$-\frac{7\,cx^2+4\,b}{6\left(b^2cx^{\frac{7}{2}}+b^3x^{\frac{3}{2}}\right)}-\frac{7\left(\frac{2\,\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\,\sqrt{2}\,c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,c^{\frac{3}{4}}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\,\sqrt{x}+\sqrt{c}\,x\cdots\right)}{b^{\frac{3}{4}}}\cdots\right)}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")

[Out] -1/6*(7*c*x^2 + 4*b)/(b^2*c*x^(7/2) + b^3*x^(3/2)) - 7/16*(2*sqrt(2)*c*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*c*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*c^(3/4)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2)*c^(3/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4))/b^2

mupad [B] time = 0.11, size = 77, normalized size = 0.33

$$\frac{7\,{(-c)}^{3/4}\,\mathrm{atan}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{11/4}}-\frac{\frac{2}{3\,b}+\frac{7\,c\,x^2}{6\,b^2}}{b\,x^{3/2}+c\,x^{7/2}}+\frac{7\,{(-c)}^{3/4}\,\mathrm{atanh}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(b*x^2 + c*x^4)^2,x)

[Out] (7*(-c)^(3/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(11/4)) - (2/(3*b) + (7*c*x^2)/(6*b^2))/(b*x^(3/2) + c*x^(7/2)) + (7*(-c)^(3/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(11/4))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.336 $\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=243

$$\frac{9c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}}-\frac{9c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}}-\frac{9c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{13/4}}+$$

```
[Out] -9/10/b^2/x^(5/2)+1/2/b/x^(5/2)/(c*x^2+b)-9/8*c^(5/4)*arctan(1-c^(1/4)*2^(1
/2)*x^(1/2)/b^(1/4))/b^(13/4)*2^(1/2)+9/8*c^(5/4)*arctan(1+c^(1/4)*2^(1/2)*
x^(1/2)/b^(1/4))/b^(13/4)*2^(1/2)+9/16*c^(5/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)
*c^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^(1/2)-9/16*c^(5/4)*ln(b^(1/2)+x*c^(1/2
)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^(1/2)+9/2*c/b^3/x^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{9c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}}-\frac{9c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}}-\frac{9c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{13/4}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -9/(10*b^2*x^(5/2)) + (9*c)/(2*b^3*Sqrt[x]) + 1/(2*b*x^(5/2)*(b + c*x^2)) -
 (9*c^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(13
/4)) + (9*c^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]
*b^(13/4)) + (9*c^(5/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqr
t[c]*x])/(8*Sqrt[2]*b^(13/4)) - (9*c^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
```

```
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{1}{x^{7/2}\left(b+cx^2\right)^2}\,dx$$

$$= \frac{1}{2bx^{5/2}\left(b+cx^2\right)} + \frac{9\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{4b}$$

$$= -\frac{9}{10b^2x^{5/2}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} - \frac{(9c)\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{4b^2}$$

$$= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} + \frac{\left(9c^2\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b^3}$$

$$= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} + \frac{\left(9c^2\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^3}$$

$$= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} - \frac{\left(9c^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^3} + \frac{\left(9c^{3/2}\right)\text{Su}}{}$$

$$= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} + \frac{(9c)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{8b^3} + \frac{(9c)\,\text{Su}}{}$$

$$= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} + \frac{9c^{5/4}\log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}\,x\right)}{8\sqrt{2}\,b^{13/4}} - \frac{9c^{5/4}\,\text{lo}}{}$$

$$= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} - \frac{9c^{5/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{13/4}} + \frac{9c^{5/4}\tan^{-1}\left(1 + \frac{\sqrt{2}}{}\right.}{4\sqrt{2}\,b^{13/4}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.12

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},2;-\frac{1}{4};-\frac{cx^2}{b}\right)}{5b^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 2, -1/4, -((c*x^2)/b)])/(5*b^2*x^(5/2))
```

fricas [A] time = 0.91, size = 251, normalized size = 1.03

$$-\frac{180\left(b^{3}cx^{5}+b^{4}x^{3}\right)\left(-\frac{c^{5}}{b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{729\,b^{3}c^{4}\sqrt{x}\left(-\frac{c^{5}}{b^{13}}\right)^{\frac{1}{4}}-\sqrt{-531441\,b^{7}c^{5}\sqrt{-\frac{c^{5}}{b^{13}}}+531441\,c^{8}x}\,b^{3}\left(-\frac{c^{5}}{b^{13}}\right)^{\frac{1}{4}}}{729\,c^{5}}\right)-45\left(b^{3}cx^{5}+b^{4}x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(180*(b^3*c*x^5 + b^4*x^3)*(-c^5/b^13)^(1/4)*arctan(-1/729*(729*b^3*c
^4*sqrt(x)*(-c^5/b^13)^(1/4) - sqrt(-531441*b^7*c^5*sqrt(-c^5/b^13) + 53144
1*c^8*x)*b^3*(-c^5/b^13)^(1/4))/c^5) - 45*(b^3*c*x^5 + b^4*x^3)*(-c^5/b^13)
^(1/4)*log(729*b^10*(-c^5/b^13)^(3/4) + 729*c^4*sqrt(x)) + 45*(b^3*c*x^5 +
b^4*x^3)*(-c^5/b^13)^(1/4)*log(-729*b^10*(-c^5/b^13)^(3/4) + 729*c^4*sqrt(x
)) - 4*(45*c^2*x^4 + 36*b*c*x^2 - 4*b^2)*sqrt(x))/(b^3*c*x^5 + b^4*x^3)
```

giac [A] time = 0.19, size = 220, normalized size = 0.91

$$\frac{c^{2}x^{\frac{3}{2}}}{2\left(cx^{2}+b\right)b^{3}}+\frac{9\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^{4}c}+\frac{9\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^{4}c}-\frac{9\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*c^2*x^(3/2)/((c*x^2 + b)*b^3) + 9/8*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sq
rt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c) + 9/8*sqrt(2)*
(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(
1/4))/(b^4*c) - 9/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4)
+ x + sqrt(b/c))/(b^4*c) + 9/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*
(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c) + 2/5*(10*c*x^2 - b)/(b^3*x^(5/2))
```

maple [A] time = 0.02, size = 172, normalized size = 0.71

$$\frac{c^{2} x^{\frac{3}{2}}}{2\left(c\,x^{2}+b\right)b^{3}}+\frac{9\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{3}}+\frac{9\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{3}}+\frac{9\sqrt{2}\,c\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{16\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{3}}+\frac{4c}{b^{3}\sqrt{x}}-\frac{}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2/b^3*c^2*x^(3/2)/(c*x^2+b)+9/16/b^3*c/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1
/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)
))+9/8/b^3*c/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+9/8/
b^3*c/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/5/b^2/x^(
5/2)+4*c/b^3/x^(1/2)
```

maxima [A] time = 2.97, size = 221, normalized size = 0.91

$$\frac{45\,c^{2}x^{4}+36\,bcx^{2}-4\,b^{2}}{10\left(b^{3}cx^{\frac{9}{2}}+b^{4}x^{\frac{5}{2}}\right)}+\frac{9\,c^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\right.}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right.}{16\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/10*(45*c^2*x^4 + 36*b*c*x^2 - 4*b^2)/(b^3*c*x^(9/2) + b^4*x^(5/2)) + 9/16
*c^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqr
t(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b
)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c
^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt
(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^3
```

mupad [B] time = 4.37, size = 87, normalized size = 0.36

$$\frac{\frac{18\,c\,x^{2}}{5\,b^{2}}-\frac{2}{5\,b}+\frac{9\,c^{2}\,x^{4}}{2\,b^{3}}}{b\,x^{5/2}+c\,x^{9/2}}-\frac{9\,{(-c)}^{5/4}\,\mathrm{atan}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{13/4}}+\frac{9\,{(-c)}^{5/4}\,\mathrm{atanh}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] ((18*c*x^2)/(5*b^2) - 2/(5*b) + (9*c^2*x^4)/(2*b^3))/(b*x^(5/2) + c*x^(9/2)
) - (9*(-c)^(5/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(13/4)) + (9*(-c
)^(5/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(13/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.337 $\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=243

$$-\frac{11c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{15/4}}+\frac{11c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{15/4}}-\frac{11c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{15/4}}$$

```
[Out] -11/14/b^2/x^(7/2)+11/6*c/b^3/x^(3/2)+1/2/b/x^(7/2)/(c*x^2+b)-11/8*c^(7/4)*
arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)*2^(1/2)+11/8*c^(7/4)*arc
tan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)*2^(1/2)-11/16*c^(7/4)*ln(b^
(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(15/4)*2^(1/2)+11/16*c^(
7/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(15/4)*2^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{11c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{15/4}}+\frac{11c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{15/4}}-\frac{11c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -11/(14*b^2*x^(7/2)) + (11*c)/(6*b^3*x^(3/2)) + 1/(2*b*x^(7/2)*(b + c*x^2))
 - (11*c^(7/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^
(15/4)) + (11*c^(7/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqr
t[2]*b^(15/4)) - (11*c^(7/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
+ Sqrt[c]*x])/(8*Sqrt[2]*b^(15/4)) + (11*c^(7/4)*Log[Sqrt[b] + Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(15/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^2}\,dx = \int \frac{1}{x^{9/2}\left(b+cx^2\right)^2}\,dx$$
$$= \frac{1}{2bx^{7/2}\left(b+cx^2\right)} + \frac{11\int \frac{1}{x^{9/2}\left(b+cx^2\right)}\,dx}{4b}$$
$$= -\frac{11}{14b^2x^{7/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} - \frac{(11c)\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{4b^2}$$
$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} + \frac{\left(11c^2\right)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4b^3}$$
$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} + \frac{\left(11c^2\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^3}$$
$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} + \frac{\left(11c^2\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^{7/2}} + \frac{(11c$$
$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} + \frac{\left(11c^{3/2}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^{7/2}}$$
$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} - \frac{11c^{7/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{15/4}} + \frac{1}{}$$
$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} - \frac{11c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{15/4}} + \frac{11c^{7/4}\tan^{-1}\left(\right.}{4\sqrt{2}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.12

$$-\frac{2\,{}_2F_1\left(-\frac{7}{4},2;-\frac{3}{4};-\frac{cx^2}{b}\right)}{7b^2x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(b*x^2 + c*x^4)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-7/4, 2, -3/4, -((c*x^2)/b)])/(7*b^2*x^(7/2))
```

fricas [A] time = 0.69, size = 245, normalized size = 1.01

$$\frac{924\left(b^{3}cx^{6}+b^{4}x^{4}\right)\left(-\frac{c^{7}}{b^{15}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^{11}c^{2}\sqrt{x}\left(-\frac{c^{7}}{b^{15}}\right)^{\frac{3}{4}}-\sqrt{b^{8}\sqrt{-\frac{c^{7}}{b^{15}}}+c^{4}x}\,b^{11}\left(-\frac{c^{7}}{b^{15}}\right)^{\frac{3}{4}}}{c^{7}}\right)+231\left(b^{3}cx^{6}+b^{4}x^{4}\right)\left(-\frac{c^{7}}{b^{15}}\right)^{\frac{1}{4}}\log\left(1\cdots\right.}{168\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/168*(924*(b^3*c*x^6 + b^4*x^4)*(-c^7/b^15)^(1/4)*arctan(-(b^11*c^2*sqrt(x
)*(-c^7/b^15)^(3/4) - sqrt(b^8*sqrt(-c^7/b^15) + c^4*x)*b^11*(-c^7/b^15)^(3
/4))/c^7) + 231*(b^3*c*x^6 + b^4*x^4)*(-c^7/b^15)^(1/4)*log(11*b^4*(-c^7/b^
15)^(1/4) + 11*c^2*sqrt(x)) - 231*(b^3*c*x^6 + b^4*x^4)*(-c^7/b^15)^(1/4)*l
og(-11*b^4*(-c^7/b^15)^(1/4) + 11*c^2*sqrt(x)) + 4*(77*c^2*x^4 + 44*b*c*x^2
 - 12*b^2)*sqrt(x))/(b^3*c*x^6 + b^4*x^4)
```

giac [A] time = 0.17, size = 212, normalized size = 0.87

$$\frac{11\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^{4}}+\frac{11\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^{4}}+\frac{11\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}c\log\left(\sqrt{2}\cdots\right.}{16\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 11/8*sqrt(2)*(b*c^3)^(1/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sq
rt(x))/(b/c)^(1/4))/b^4 + 11/8*sqrt(2)*(b*c^3)^(1/4)*c*arctan(-1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^4 + 11/16*sqrt(2)*(b*c^3)^
(1/4)*c*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 11/16*sqrt(2
)*(b*c^3)^(1/4)*c*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 + 1
/2*c^2*sqrt(x)/((c*x^2 + b)*b^3) + 2/21*(14*c*x^2 - 3*b)/(b^3*x^(7/2))
```

maple [A] time = 0.02, size = 178, normalized size = 0.73

$$\frac{c^{2}\sqrt{x}}{2\left(cx^{2}+b\right)b^{3}}+\frac{11\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c^{2}\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8b^{4}}+\frac{11\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c^{2}\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8b^{4}}+\frac{11\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c^{2}\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\cdots}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\cdots}\right.}{16b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^2/x^(1/2),x)
```

```
[Out] 1/2/b^3*c^2*x^(1/2)/(c*x^2+b)+11/16/b^4*c^2*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)
^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1
/2)))+11/8/b^4*c^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1
)+11/8/b^4*c^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/
7/b^2/x^(7/2)+4/3*c/b^3/x^(3/2)
```

maxima [A] time = 3.16, size = 224, normalized size = 0.92

$$\frac{77\,c^2x^4+44\,bcx^2-12\,b^2}{42\left(b^3cx^{\frac{11}{2}}+b^4x^{\frac{7}{2}}\right)}+\frac{11\left(\frac{2\,\sqrt{2}\,c^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\,\sqrt{2}\,c^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,c^{\frac{7}{4}}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c\right.}{b}\right.}{16\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/42*(77*c^2*x^4 + 44*b*c*x^2 - 12*b^2)/(b^3*c*x^(11/2) + b^4*x^(7/2)) + 11
/16*(2*sqrt(2)*c^2*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*
sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)
*c^2*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt
(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*c^(7/4)*log(sq
rt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2)*c^(7
/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4))/b^
3
```

mupad [B] time = 0.11, size = 87, normalized size = 0.36

$$\frac{\frac{22\,c\,x^2}{21\,b^2}-\frac{2}{7\,b}+\frac{11\,c^2\,x^4}{6\,b^3}}{b\,x^{7/2}+c\,x^{11/2}}+\frac{11\,{\left(-c\right)}^{7/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{15/4}}+\frac{11\,{\left(-c\right)}^{7/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x^2 + c*x^4)^2),x)
```

```
[Out] ((22*c*x^2)/(21*b^2) - 2/(7*b) + (11*c^2*x^4)/(6*b^3))/(b*x^(7/2) + c*x^(11
/2)) + (11*(-c)^(7/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(15/4)) + (1
1*(-c)^(7/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(15/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**2/x**(1/2),x)
```

```
[Out] Timed out
```

3.338 $\int \frac{1}{x^{3/2}(bx^2+cx^4)^2} dx$

Optimal. Leaf size=258

$$-\frac{13c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}}+\frac{13c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}}+\frac{13c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{}}{\sqrt[4]{b}}\right.}{4\sqrt{2}b^{17/4}}$$

```
[Out] -13/18/b^2/x^(9/2)+13/10*c/b^3/x^(5/2)+1/2/b/x^(9/2)/(c*x^2+b)+13/8*c^(9/4)
*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)-13/8*c^(9/4)*ar
ctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)-13/16*c^(9/4)*ln(b
^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(17/4)*2^(1/2)+13/16*c^
(9/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(17/4)*2^(1/2
)-13/2*c^2/b^4/x^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{13c^2}{2b^4\sqrt{x}}-\frac{13c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}}+\frac{13c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}}+\frac{13c^{9/4}\tan^{-1}\left(1\right.}{4\sqrt{2}b^{1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -13/(18*b^2*x^(9/2)) + (13*c)/(10*b^3*x^(5/2)) - (13*c^2)/(2*b^4*Sqrt[x]) +
 1/(2*b*x^(9/2)*(b + c*x^2)) + (13*c^(9/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt
[x])/b^(1/4)])/(4*Sqrt[2]*b^(17/4)) - (13*c^(9/4)*ArcTan[1 + (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(17/4)) - (13*c^(9/4)*Log[Sqrt[b] - Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(17/4)) + (13*c^(9/4
)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^
(17/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx = \int \frac{1}{x^{11/2}\left(b+cx^2\right)^2}\,dx$$

$$= \frac{1}{2bx^{9/2}\left(b+cx^2\right)} + \frac{13\int \frac{1}{x^{11/2}\left(b+cx^2\right)}\,dx}{4b}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} - \frac{(13c)\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{4b^2}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} + \frac{\left(13c^2\right)\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{4b^3}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} - \frac{\left(13c^3\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b^4}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} - \frac{\left(13c^3\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^4}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} + \frac{\left(13c^{5/2}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x,\right.}{4b^4}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} - \frac{\left(13c^2\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx,\right.}{8b^4}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} - \frac{13c^{9/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{8\sqrt{2}\,b^{17/4}}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} + \frac{13c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{17/4}} - \frac{13c}{}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.11

$$-\frac{2\,{}_2F_1\left(-\frac{9}{4},2;-\frac{5}{4};-\frac{cx^2}{b}\right)}{9b^2x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(b*x^2 + c*x^4)^2),x]
[Out] (-2*Hypergeometric2F1[-9/4, 2, -5/4, -((c*x^2)/b)])/(9*b^2*x^(9/2))
```

fricas [A] time = 0.90, size = 262, normalized size = 1.02

$$\frac{2340\left(b^4cx^7+b^5x^5\right)\left(-\frac{c^9}{b^{17}}\right)^{\frac{1}{4}}\arctan\left(-\frac{2197\,b^4c^7\sqrt{x}\left(-\frac{c^9}{b^{17}}\right)^{\frac{1}{4}}-\sqrt{-4826809\,b^9c^9\sqrt{-\frac{c^9}{b^{17}}}+4826809\,c^{14}x}\,b^4\left(-\frac{c^9}{b^{17}}\right)^{\frac{1}{4}}}{2197\,c^9}\right)-585\left(b^4cx^7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
[Out] 1/360*(2340*(b^4*c*x^7 + b^5*x^5)*(-c^9/b^17)^(1/4)*arctan(-1/2197*(2197*b^
4*c^7*sqrt(x)*(-c^9/b^17)^(1/4) - sqrt(-4826809*b^9*c^9*sqrt(-c^9/b^17) + 4
826809*c^14*x)*b^4*(-c^9/b^17)^(1/4))/c^9) - 585*(b^4*c*x^7 + b^5*x^5)*(-c^
9/b^17)^(1/4)*log(2197*b^13*(-c^9/b^17)^(3/4) + 2197*c^7*sqrt(x)) + 585*(b^
4*c*x^7 + b^5*x^5)*(-c^9/b^17)^(1/4)*log(-2197*b^13*(-c^9/b^17)^(3/4) + 219
7*c^7*sqrt(x)) - 4*(585*c^3*x^6 + 468*b*c^2*x^4 - 52*b^2*c*x^2 + 20*b^3)*sq
rt(x))/(b^4*c*x^7 + b^5*x^5)
```

giac [A] time = 0.17, size = 219, normalized size = 0.85

$$-\frac{c^3x^{\frac{3}{2}}}{2\left(cx^2+b\right)b^4}-\frac{13\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^5}-\frac{13\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^5}+\frac{13\sqrt{2}\left(bc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
[Out] -1/2*c^3*x^(3/2)/((c*x^2 + b)*b^4) - 13/8*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*
sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^5 - 13/8*sqrt(2)*(
b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1
/4))/b^5 + 13/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x
+ sqrt(b/c))/b^5 - 13/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(
1/4) + x + sqrt(b/c))/b^5 - 2/45*(135*c^2*x^4 - 18*b*c*x^2 + 5*b^2)/(b^4*x^
(9/2))
```

maple [A] time = 0.02, size = 189, normalized size = 0.73

$$-\frac{c^{3}x^{\frac{3}{2}}}{2\left(c\,x^{2}+b\right)b^{4}}-\frac{13\sqrt{2}\,c^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{4}}-\frac{13\sqrt{2}\,c^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{4}}-\frac{13\sqrt{2}\,c^{2}\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{16\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{4}}-\frac{6}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/b^4*c^3*x^(3/2)/(c*x^2+b)-13/16/b^4*c^2/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c
)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(
1/2)))-13/8/b^4*c^2/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+
1)-13/8/b^4*c^2/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2
/9/b^2/x^(9/2)-6*c^2/b^4/x^(1/2)+4/5*c/b^3/x^(5/2)
```

maxima [A] time = 3.01, size = 232, normalized size = 0.90

$$-\frac{585\,c^{3}x^{6}+468\,bc^{2}x^{4}-52\,b^{2}cx^{2}+20\,b^{3}}{90\left(b^{4}cx^{\frac{13}{2}}+b^{5}x^{\frac{9}{2}}\right)}-\frac{13\,c^{3}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\cdots\right)}{16\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/90*(585*c^3*x^6 + 468*b*c^2*x^4 - 52*b^2*c*x^2 + 20*b^3)/(b^4*c*x^(13/2)
 + b^5*x^(9/2)) - 13/16*c^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*
c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*
sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(
c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2
)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/
4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(
b^(1/4)*c^(3/4)))/b^4
```

mupad [B] time = 4.37, size = 99, normalized size = 0.38

$$\frac{13\,{\left(-c\right)}^{9/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{17/4}}-\frac{13\,{\left(-c\right)}^{9/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{17/4}}-\frac{\frac{2}{9\,b}-\frac{26\,c\,x^2}{45\,b^2}+\frac{26\,c^2\,x^4}{5\,b^3}+\frac{13\,c^3\,x^6}{2\,b^4}}{b\,x^{9/2}+c\,x^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(b*x^2 + c*x^4)^2),x)
```

```
[Out] (13*(-c)^(9/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(17/4)) - (13*(-c)
^(9/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(17/4)) - (2/(9*b) - (26*c*
x^2)/(45*b^2) + (26*c^2*x^4)/(5*b^3) + (13*c^3*x^6)/(2*b^4))/(b*x^(9/2) + c
*x^(13/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

3.339 $\int \frac{x^{23/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=251

$$\frac{45\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{13/4}}-\frac{45\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{13/4}}+\frac{45\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}c^{13/4}}$$

```
[Out] -1/4*x^(9/2)/c/(c*x^2+b)^2-9/16*x^(5/2)/c^2/(c*x^2+b)+45/64*b^(1/4)*arctan(
1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(13/4)*2^(1/2)-45/64*b^(1/4)*arctan(1+
c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(13/4)*2^(1/2)+45/128*b^(1/4)*ln(b^(1/2)
+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)*2^(1/2)-45/128*b^(1/4)
*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)*2^(1/2)+45/
16*x^(1/2)/c^3
```

Rubi [A] time = 0.21, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{9x^{5/2}}{16c^2\left(b+cx^2\right)}+\frac{45\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{13/4}}-\frac{45\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{13/4}}+\frac{45\sqrt[4]{b}\tan}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(23/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (45*Sqrt[x])/(16*c^3) - x^(9/2)/(4*c*(b + c*x^2)^2) - (9*x^(5/2))/(16*c^2*(
b + c*x^2)) + (45*b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(3
2*Sqrt[2]*c^(13/4)) - (45*b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(32*Sqrt[2]*c^(13/4)) + (45*b^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(
1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*c^(13/4)) - (45*b^(1/4)*Log[Sqrt[b]
+ Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*c^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{23/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^{11/2}}{\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{x^{9/2}}{4c\left(b+cx^2\right)^2} + \frac{9\int \frac{x^{7/2}}{\left(b+cx^2\right)^2}\,dx}{8c}$$

$$= -\frac{x^{9/2}}{4c\left(b+cx^2\right)^2} - \frac{9x^{5/2}}{16c^2\left(b+cx^2\right)} + \frac{45\int \frac{x^{3/2}}{b+cx^2}\,dx}{32c^2}$$

$$= \frac{45\sqrt{x}}{16c^3} - \frac{x^{9/2}}{4c\left(b+cx^2\right)^2} - \frac{9x^{5/2}}{16c^2\left(b+cx^2\right)} - \frac{(45b)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32c^3}$$

$$= \frac{45\sqrt{x}}{16c^3} - \frac{x^{9/2}}{4c\left(b+cx^2\right)^2} - \frac{9x^{5/2}}{16c^2\left(b+cx^2\right)} - \frac{(45b)\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16c^3}$$

$$= \frac{45\sqrt{x}}{16c^3} - \frac{x^{9/2}}{4c\left(b+cx^2\right)^2} - \frac{9x^{5/2}}{16c^2\left(b+cx^2\right)} - \frac{\left(45\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32c^3} - \frac{\left(45\sqrt{b}\right.}{}$$

$$= \frac{45\sqrt{x}}{16c^3} - \frac{x^{9/2}}{4c\left(b+cx^2\right)^2} - \frac{9x^{5/2}}{16c^2\left(b+cx^2\right)} - \frac{\left(45\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{64c^{7/2}} - \frac{(4}{}$$

$$= \frac{45\sqrt{x}}{16c^3} - \frac{x^{9/2}}{4c\left(b+cx^2\right)^2} - \frac{9x^{5/2}}{16c^2\left(b+cx^2\right)} + \frac{45\sqrt[4]{b}\,\log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}\,x\right)}{64\sqrt{2}\,c^{13/4}} - \frac{45\sqrt[4]{}}{}$$

$$= \frac{45\sqrt{x}}{16c^3} - \frac{x^{9/2}}{4c\left(b+cx^2\right)^2} - \frac{9x^{5/2}}{16c^2\left(b+cx^2\right)} + \frac{45\sqrt[4]{b}\,\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,c^{13/4}} - \frac{45\sqrt[4]{b}\,\tan^{-1}\left(1 + \right.}{32\sqrt{2}\,c^{13}}$$

Mathematica [A] time = 0.25, size = 220, normalized size = 0.88

$$\frac{\frac{8\sqrt[4]{c}\sqrt{x}\left(45b^2+81bcx^2+32c^2x^4\right)}{\left(b+cx^2\right)^2} + 45\sqrt{2}\sqrt[4]{b}\,\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b} + \sqrt{c}\,x\right) - 45\sqrt{2}\sqrt[4]{b}\,\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b}\right.}{128c^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(23/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((8*c^(1/4)*Sqrt[x]*(45*b^2 + 81*b*c*x^2 + 32*c^2*x^4))/(b + c*x^2)^2 + 90*
Sqrt[2]*b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 90*Sqrt[2]*
b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 45*Sqrt[2]*b^(1/4)*
Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - 45*Sqrt[2]*b^(
1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(128*c^(13
/4))
```

fricas [A] time = 0.83, size = 247, normalized size = 0.98

$$-\frac{180\left(c^5x^4+2\,bc^4x^2+b^2c^3\right)\left(-\frac{b}{c^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^6\sqrt{-\frac{b}{c^{13}}}+x}\,c^{10}\left(-\frac{b}{c^{13}}\right)^{\frac{3}{4}}-c^{10}\sqrt{x}\left(-\frac{b}{c^{13}}\right)^{\frac{3}{4}}}{b}\right)+45\left(c^5x^4+2\,bc^4x^2+b^2c^3\right)\left(-\cdots\right.}{6\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(180*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-b/c^13)^(1/4)*arctan((sqrt(c
^6*sqrt(-b/c^13) + x)*c^10*(-b/c^13)^(3/4) - c^10*sqrt(x)*(-b/c^13)^(3/4))/
b) + 45*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-b/c^13)^(1/4)*log(45*c^3*(-b/c^
13)^(1/4) + 45*sqrt(x)) - 45*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-b/c^13)^(1
/4)*log(-45*c^3*(-b/c^13)^(1/4) + 45*sqrt(x)) - 4*(32*c^2*x^4 + 81*b*c*x^2
+ 45*b^2)*sqrt(x))/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)
```

giac [A] time = 0.18, size = 208, normalized size = 0.83

$$-\frac{45\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,c^4}-\frac{45\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,c^4}-\frac{45\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\cdots\right.}{128\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -45/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sq
rt(x))/(b/c)^(1/4))/c^4 - 45/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^4 - 45/128*sqrt(2)*(b*c^3)^
(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 + 45/128*sqrt(2)
*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 + 2*sq
rt(x)/c^3 + 1/16*(17*b*c*x^(5/2) + 13*b^2*sqrt(x))/((c*x^2 + b)^2*c^3)
```

maple [A] time = 0.02, size = 178, normalized size = 0.71

$$\frac{17b\,x^{\frac{5}{2}}}{16\left(c\,x^{2}+b\right)^{2}c^{2}}+\frac{13b^{2}\sqrt{x}}{16\left(c\,x^{2}+b\right)^{2}c^{3}}-\frac{45\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64c^{3}}-\frac{45\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64c^{3}}-\frac{45\left(\frac{b}{c}\right)^{\frac{1}{4}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(23/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*x^(1/2)/c^3+17/16/c^2*b/(c*x^2+b)^2*x^(5/2)+13/16/c^3*b^2/(c*x^2+b)^2*x^(
1/2)-45/128/c^3*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)
^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-45/64/c^3*(b/c)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-45/64/c^3*(b/c)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 2.95, size = 229, normalized size = 0.91

$$\frac{17\,bcx^{\frac{5}{2}}+13\,b^{2}\sqrt{x}}{16\left(c^{5}x^{4}+2\,bc^{4}x^{2}+b^{2}c^{3}\right)}-\frac{45\left(\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\sqrt{b}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\,b^{\frac{1}{4}}\log\left(\sqrt{2}\right.}{}\right.}{128\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(17*b*c*x^(5/2) + 13*b^2*sqrt(x))/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3) -
45/128*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*s
qrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + 2*sqrt(2)*sq
rt(b)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqr
t(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + sqrt(2)*b^(1/4)*log(sqrt(2)*b^(
1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4) - sqrt(2)*b^(1/4)*log(-
sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4))/c^3 + 2*sqr
t(x)/c^3
```

mupad [B] time = 4.39, size = 101, normalized size = 0.40

$$\frac{\frac{13\,b^{2}\,\sqrt{x}}{16}+\frac{17\,b\,c\,x^{5/2}}{16}}{b^{2}\,c^{3}+2\,b\,c^{4}\,x^{2}+c^{5}\,x^{4}}+\frac{2\,\sqrt{x}}{c^{3}}-\frac{45\,{\left(-b\right)}^{1/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,c^{13/4}}+\frac{{\left(-b\right)}^{1/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-b\right)}^{1/4}}\right)\,45{}\mathrm{i}}{32\,c^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(23/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((13*b^2*x^(1/2))/16 + (17*b*c*x^(5/2))/16)/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^
2) + (2*x^(1/2))/c^3 - (45*(-b)^(1/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(
32*c^(13/4)) + ((-b)^(1/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*45i)/(32*c
^(13/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(23/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.340 $\int \frac{x^{21/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=239

$$\frac{21\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\sqrt[4]{b}\,c^{11/4}}-\frac{21\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\sqrt[4]{b}\,c^{11/4}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}\,c^{11/4}}+\frac{21\tan^{-1}}{32}$$

```
[Out] -1/4*x^(7/2)/c/(c*x^2+b)^2-7/16*x^(3/2)/c^2/(c*x^2+b)-21/64*arctan(1-c^(1/4
)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/c^(11/4)*2^(1/2)+21/64*arctan(1+c^(1/4)*
2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/c^(11/4)*2^(1/2)+21/128*ln(b^(1/2)+x*c^(1/
2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(11/4)*2^(1/2)-21/128*ln(b^(1
/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(11/4)*2^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{7x^{3/2}}{16c^2\left(b+cx^2\right)}+\frac{21\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\sqrt[4]{b}\,c^{11/4}}-\frac{21\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\sqrt[4]{b}\,c^{11/4}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{}}{}\right)}{32\sqrt{2}\sqrt[4]{b}\,c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(21/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -x^(7/2)/(4*c*(b + c*x^2)^2) - (7*x^(3/2))/(16*c^2*(b + c*x^2)) - (21*ArcTa
n[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(1/4)*c^(11/4)) + (
21*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(1/4)*c^(11
/4)) + (21*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*
Sqrt[2]*b^(1/4)*c^(11/4)) - (21*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[
x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(1/4)*c^(11/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{21/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^{9/2}}{\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2} + \frac{7\int \frac{x^{5/2}}{\left(b+cx^2\right)^2}\,dx}{8c}$$

$$= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2} - \frac{7x^{3/2}}{16c^2\left(b+cx^2\right)} + \frac{21\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32c^2}$$

$$= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2} - \frac{7x^{3/2}}{16c^2\left(b+cx^2\right)} + \frac{21\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16c^2}$$

$$= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2} - \frac{7x^{3/2}}{16c^2\left(b+cx^2\right)} - \frac{21\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32c^{5/2}} + \frac{21\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\right.}{32c^{5/2}}$$

$$= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2} - \frac{7x^{3/2}}{16c^2\left(b+cx^2\right)} + \frac{21\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64c^3} + \frac{21\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2} - \frac{7x^{3/2}}{16c^2\left(b+cx^2\right)} + \frac{21\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{64\sqrt{2}\sqrt[4]{b}\,c^{11/4}} - \frac{21\log\left(\sqrt{b}+\right.}{64}$$

$$= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2} - \frac{7x^{3/2}}{16c^2\left(b+cx^2\right)} - \frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}\,c^{11/4}} + \frac{21\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}\,c^{11/4}} + \cdots$$

Mathematica [C] time = 0.02, size = 66, normalized size = 0.28

$$\frac{2x^{3/2}\left(7\left(b+cx^2\right)^2\ {}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{cx^2}{b}\right)-b\left(7b+5cx^2\right)\right)}{5bc^2\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(21/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*x^(3/2)*(-(b*(7*b + 5*c*x^2)) + 7*(b + c*x^2)^2*Hypergeometric2F1[3/4, 3
, 7/4, -((c*x^2)/b)]))/(5*b*c^2*(b + c*x^2)^2)
```

fricas [A] time = 0.71, size = 248, normalized size = 1.04

$$-\frac{84\left(c^4x^4+2\,bc^3x^2+b^2c^2\right)\left(-\frac{1}{bc^{11}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-bc^5\sqrt{-\frac{1}{bc^{11}}}+x}\,c^3\left(-\frac{1}{bc^{11}}\right)^{\frac{1}{4}}-c^3\sqrt{x}\left(-\frac{1}{bc^{11}}\right)^{\frac{1}{4}}\right)-21\left(c^4x^4+2\,bc^3\right.}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(84*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b*c^11))^(1/4)*arctan(sqrt
(-b*c^5*sqrt(-1/(b*c^11)) + x)*c^3*(-1/(b*c^11))^(1/4) - c^3*sqrt(x)*(-1/(b
*c^11))^(1/4)) - 21*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b*c^11))^(1/4)*l
og(b*c^8*(-1/(b*c^11))^(3/4) + sqrt(x)) + 21*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c
^2)*(-1/(b*c^11))^(1/4)*log(-b*c^8*(-1/(b*c^11))^(3/4) + sqrt(x)) + 4*(11*c
*x^3 + 7*b*x)*sqrt(x))/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

giac [A] time = 0.22, size = 209, normalized size = 0.87

$$-\frac{11\,cx^{\frac{7}{2}}+7\,bx^{\frac{3}{2}}}{16\left(cx^2+b\right)^2c^2}+\frac{21\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^5}+\frac{21\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^5}-\frac{21\,\sqrt{2}\,\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/16*(11*c*x^(7/2) + 7*b*x^(3/2))/((c*x^2 + b)^2*c^2) + 21/64*sqrt(2)*(b*c
^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))
```

```
/(b*c^5) + 21/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(
1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^5) - 21/128*sqrt(2)*(b*c^3)^(3/4)*log(s
qrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^5) + 21/128*sqrt(2)*(b*c^3
)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^5)
```

maple [A] time = 0.02, size = 161, normalized size = 0.67

$$\frac{21\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{3}}+\frac{21\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{3}}+\frac{21\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{128\left(\frac{b}{c}\right)^{\frac{1}{4}}c^{3}}+\frac{-\frac{11x^{\frac{7}{2}}}{16c}-\frac{7bx^{\frac{3}{2}}}{16c^{2}}}{\left(cx^{2}+b\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(21/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(-11/32/c*x^(7/2)-7/32*b/c^2*x^(3/2))/(c*x^2+b)^2+21/128/c^3/(b/c)^(1/4)*
2^(1/2)*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/
2)*x^(1/2)+(b/c)^(1/2)))+21/64/c^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)
^(1/4)*x^(1/2)+1)+21/64/c^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*
x^(1/2)-1)
```

maxima [A] time = 2.95, size = 218, normalized size = 0.91

$$-\frac{11\,cx^{\frac{7}{2}}+7\,bx^{\frac{3}{2}}}{16\left(c^{4}x^{4}+2\,bc^{3}x^{2}+b^{2}c^{2}\right)}+\frac{21\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\cdots\right.}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\cdots\right)}{128\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(11*c*x^(7/2) + 7*b*x^(3/2))/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2) + 21/1
28*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(
x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*
sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(
1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2
)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/c^2
```

mupad [B] time = 4.28, size = 87, normalized size = 0.36

$$\frac{21\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{32\,(-b)^{1/4}\,c^{11/4}}-\frac{\frac{11\,x^{7/2}}{16\,c}+\frac{7\,b\,x^{3/2}}{16\,c^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4}-\frac{21\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{32\,(-b)^{1/4}\,c^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(21/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (21*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(1/4)*c^(11/4)) - ((11*x^(
7/2))/(16*c) + (7*b*x^(3/2))/(16*c^2))/(b^2 + c^2*x^4 + 2*b*c*x^2) - (21*at
anh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(1/4)*c^(11/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(21/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.341 $\int \frac{x^{19/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=239

$$-\frac{5\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{3/4}c^{9/4}}+\frac{5\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{3/4}c^{9/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{3/4}c^{9/4}}+\frac{5\tan^{-1}\left(\right.}{32\sqrt{}}$$

```
[Out] -1/4*x^(5/2)/c/(c*x^2+b)^2-5/64*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b
^(3/4)/c^(9/4)*2^(1/2)+5/64*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/
4)/c^(9/4)*2^(1/2)-5/128*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/
2))/b^(3/4)/c^(9/4)*2^(1/2)+5/128*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1
/2)*x^(1/2))/b^(3/4)/c^(9/4)*2^(1/2)-5/16*x^(1/2)/c^2/(c*x^2+b)
```

Rubi [A] time = 0.19, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1584, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{5\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{3/4}c^{9/4}}+\frac{5\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{3/4}c^{9/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{3/4}c^{9/4}}+\frac{5\tan^{-1}\left(\right.}{32\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(19/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -x^(5/2)/(4*c*(b + c*x^2)^2) - (5*Sqrt[x])/(16*c^2*(b + c*x^2)) - (5*ArcTan
[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(3/4)*c^(9/4)) + (5*
ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(3/4)*c^(9/4))
 - (5*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[
2]*b^(3/4)*c^(9/4)) + (5*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sq
rt[c]*x])/(64*Sqrt[2]*b^(3/4)*c^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{19/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^{7/2}}{\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{x^{5/2}}{4c\left(b+cx^2\right)^2} + \frac{5\int \frac{x^{3/2}}{\left(b+cx^2\right)^2}\,dx}{8c}$$

$$= -\frac{x^{5/2}}{4c\left(b+cx^2\right)^2} - \frac{5\sqrt{x}}{16c^2\left(b+cx^2\right)} + \frac{5\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32c^2}$$

$$= -\frac{x^{5/2}}{4c\left(b+cx^2\right)^2} - \frac{5\sqrt{x}}{16c^2\left(b+cx^2\right)} + \frac{5\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16c^2}$$

$$= -\frac{x^{5/2}}{4c\left(b+cx^2\right)^2} - \frac{5\sqrt{x}}{16c^2\left(b+cx^2\right)} + \frac{5\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32\sqrt{b}\,c^2} + \frac{5\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\right.}{32\sqrt{b}\,c^2}$$

$$= -\frac{x^{5/2}}{4c\left(b+cx^2\right)^2} - \frac{5\sqrt{x}}{16c^2\left(b+cx^2\right)} + \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64\sqrt{b}\,c^{5/2}} + \frac{5\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}+}\right.}{64}$$

$$= -\frac{x^{5/2}}{4c\left(b+cx^2\right)^2} - \frac{5\sqrt{x}}{16c^2\left(b+cx^2\right)} - \frac{5\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{3/4}c^{9/4}} + \frac{5\log\left(\sqrt{b}+\sqrt{2}\right.}{64\sqrt{2}}$$

$$= -\frac{x^{5/2}}{4c\left(b+cx^2\right)^2} - \frac{5\sqrt{x}}{16c^2\left(b+cx^2\right)} - \frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{3/4}c^{9/4}} + \frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{3/4}c^{9/4}} - \frac{5\,\text{lo}}{}$$

Mathematica [A] time = 0.10, size = 242, normalized size = 1.01

$$\frac{-\frac{15\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{3/4}}+\frac{15\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{3/4}}-\frac{30\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{b^{3/4}}+\frac{30\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{b^{3/4}}-}{384c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(19/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-160*b*c^(1/4)*Sqrt[x])/(b + c*x^2)^2 - (256*c^(5/4)*x^(5/2))/(b + c*x^2)
^2 + (40*c^(1/4)*Sqrt[x])/(b + c*x^2) - (30*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(
1/4)*Sqrt[x])/b^(1/4)])/b^(3/4) + (30*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*S
qrt[x])/b^(1/4)])/b^(3/4) - (15*Sqrt[2]*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/
4)*Sqrt[x] + Sqrt[c]*x])/b^(3/4) + (15*Sqrt[2]*Log[Sqrt[b] + Sqrt[2]*b^(1/4
)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(3/4))/(384*c^(9/4))
```

fricas [A] time = 0.83, size = 254, normalized size = 1.06

$$\frac{20\left(c^4x^4+2bc^3x^2+b^2c^2\right)\left(-\frac{1}{b^3c^9}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^2c^4\sqrt{-\frac{1}{b^3c^9}}+x}\,b^2c^7\left(-\frac{1}{b^3c^9}\right)^{\frac{3}{4}}-b^2c^7\sqrt{x}\left(-\frac{1}{b^3c^9}\right)^{\frac{3}{4}}\right)+5\left(c^4x^4+2bc\right.}{64\left(c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(20*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b^3*c^9))^(1/4)*arctan(sqrt
(b^2*c^4*sqrt(-1/(b^3*c^9)) + x)*b^2*c^7*(-1/(b^3*c^9))^(3/4) - b^2*c^7*sqr
t(x)*(-1/(b^3*c^9))^(3/4)) + 5*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b^3*c
^9))^(1/4)*log(b*c^2*(-1/(b^3*c^9))^(1/4) + sqrt(x)) - 5*(c^4*x^4 + 2*b*c^3
*x^2 + b^2*c^2)*(-1/(b^3*c^9))^(1/4)*log(-b*c^2*(-1/(b^3*c^9))^(1/4) + sqrt
(x)) - 4*(9*c*x^2 + 5*b)*sqrt(x))/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

giac [A] time = 0.18, size = 209, normalized size = 0.87

$$\frac{5\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64bc^3}+\frac{5\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64bc^3}+\frac{5\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}\right.}{128bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 5/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/(b*c^3) + 5/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^3) + 5/128*sqrt(2)*(b*c
^3)^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^3) - 5/128*
sqrt(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*
c^3) - 1/16*(9*c*x^(5/2) + 5*b*sqrt(x))/((c*x^2 + b)^2*c^2)
```

maple [A] time = 0.02, size = 170, normalized size = 0.71

$$\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64b\,c^{2}}+\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64b\,c^{2}}+\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{128b\,c^{2}}+\frac{-\frac{9x^{\frac{5}{2}}}{16c}-\frac{5b}{16}}{\left(c\,x^{2}+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(-9/32/c*x^(5/2)-5/32*b/c^2*x^(1/2))/(c*x^2+b)^2+5/128/c^2*(b/c)^(1/4)/b*
2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/
2)*x^(1/2)+(b/c)^(1/2)))+5/64/c^2*(b/c)^(1/4)/b*2^(1/2)*arctan(2^(1/2)/(b/c
)^(1/4)*x^(1/2)+1)+5/64/c^2*(b/c)^(1/4)/b*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4
)*x^(1/2)-1)
```

maxima [A] time = 2.98, size = 218, normalized size = 0.91

$$-\frac{9\,cx^{\frac{5}{2}}+5\,b\sqrt{x}}{16\left(c^{4}x^{4}+2\,bc^{3}x^{2}+b^{2}c^{2}\right)}+\frac{5\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right.}{128\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(9*c*x^(5/2) + 5*b*sqrt(x))/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2) + 5/128
*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x)
)/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sq
```

```
rt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/
4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*log(-sqrt(2)*
b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/c^2
```

mupad [B] time = 0.10, size = 87, normalized size = 0.36

$$-\frac{\frac{9\,x^{5/2}}{16\,c}+\frac{5\,b\,\sqrt{x}}{16\,c^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4}-\frac{5\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{3/4}\,c^{9/4}}-\frac{5\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{3/4}\,c^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] - ((9*x^(5/2))/(16*c) + (5*b*x^(1/2))/(16*c^2))/(b^2 + c^2*x^4 + 2*b*c*x^2)
 - (5*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(3/4)*c^(9/4)) - (5*atan
h((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(3/4)*c^(9/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(19/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.342 $\int \frac{x^{17/2}}{(bx^2+cx^4)^3}\, dx$

Optimal. Leaf size=242

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{5/4}c^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{5/4}c^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{5/4}c^{7/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}}{}\right.}{32\sqrt{2}\,}$$

[Out] -1/4*x^(3/2)/c/(c*x^2+b)^2+3/16*x^(3/2)/b/c/(c*x^2+b)-3/64*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^(7/4)*2^(1/2)+3/64*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^(7/4)*2^(1/2)+3/128*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)/c^(7/4)*2^(1/2)-3/128*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)/c^(7/4)*2^(1/2)

Rubi [A] time = 0.19, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{5/4}c^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{5/4}c^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{5/4}c^{7/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}}{}\right.}{32\sqrt{2}\,}$$

Antiderivative was successfully verified.

```
[In] Int[x^(17/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -x^(3/2)/(4*c*(b + c*x^2)^2) + (3*x^(3/2))/(16*b*c*(b + c*x^2)) - (3*ArcTan
[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(5/4)*c^(7/4)) + (3*
ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(5/4)*c^(7/4))
 + (3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[
2]*b^(5/4)*c^(7/4)) - (3*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sq
rt[c]*x])/(64*Sqrt[2]*b^(5/4)*c^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{17/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^{5/2}}{\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{x^{3/2}}{4c\left(b+cx^2\right)^2} + \frac{3\int \frac{\sqrt{x}}{\left(b+cx^2\right)^2}\,dx}{8c}$$

$$= -\frac{x^{3/2}}{4c\left(b+cx^2\right)^2} + \frac{3x^{3/2}}{16bc\left(b+cx^2\right)} + \frac{3\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32bc}$$

$$= -\frac{x^{3/2}}{4c\left(b+cx^2\right)^2} + \frac{3x^{3/2}}{16bc\left(b+cx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16bc}$$

$$= -\frac{x^{3/2}}{4c\left(b+cx^2\right)^2} + \frac{3x^{3/2}}{16bc\left(b+cx^2\right)} - \frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32bc^{3/2}} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,d\right.}{32bc^{3/2}}$$

$$= -\frac{x^{3/2}}{4c\left(b+cx^2\right)^2} + \frac{3x^{3/2}}{16bc\left(b+cx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64bc^2} + \frac{3\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}+\cdots}\right.}{64\cdots}$$

$$= -\frac{x^{3/2}}{4c\left(b+cx^2\right)^2} + \frac{3x^{3/2}}{16bc\left(b+cx^2\right)} + \frac{3\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{5/4}c^{7/4}} - \frac{3\log\left(\sqrt{b}+\sqrt{2}\cdots\right.}{64\sqrt{2}\cdots}$$

$$= -\frac{x^{3/2}}{4c\left(b+cx^2\right)^2} + \frac{3x^{3/2}}{16bc\left(b+cx^2\right)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{5/4}c^{7/4}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{5/4}c^{7/4}} + \frac{3\log\cdots}{\cdots}$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.19

$$\frac{2x^{3/2}\left(\frac{{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{cx^2}{b}\right)}{b^2}-\frac{1}{\left(b+cx^2\right)^2}\right)}{5c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(17/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*x^(3/2)*(-(b + c*x^2)^(-2) + Hypergeometric2F1[3/4, 3, 7/4, -((c*x^2)/b)
]/b^2))/(5*c)
```

fricas [A] time = 0.84, size = 260, normalized size = 1.07

$$-\frac{12\left(bc^3x^4+2\,b^2c^2x^2+b^3c\right)\left(-\frac{1}{b^5c^7}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-b^3c^3\sqrt{-\frac{1}{b^5c^7}}+x}\,bc^2\left(-\frac{1}{b^5c^7}\right)^{\frac{1}{4}}-bc^2\sqrt{x}\left(-\frac{1}{b^5c^7}\right)^{\frac{1}{4}}\right)-3\left(bc^3x^4+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(12*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-1/(b^5*c^7))^(1/4)*arctan(s
qrt(-b^3*c^3*sqrt(-1/(b^5*c^7)) + x)*b*c^2*(-1/(b^5*c^7))^(1/4) - b*c^2*sqr
t(x)*(-1/(b^5*c^7))^(1/4)) - 3*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-1/(b^5
*c^7))^(1/4)*log(b^4*c^5*(-1/(b^5*c^7))^(3/4) + sqrt(x)) + 3*(b*c^3*x^4 + 2
*b^2*c^2*x^2 + b^3*c)*(-1/(b^5*c^7))^(1/4)*log(-b^4*c^5*(-1/(b^5*c^7))^(3/4
) + sqrt(x)) - 4*(3*c*x^3 - b*x)*sqrt(x))/(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*
c)
```

giac [A] time = 0.18, size = 212, normalized size = 0.88

$$\frac{3\,cx^{\frac{7}{2}}-bx^{\frac{3}{2}}}{16\left(cx^2+b\right)^2bc}+\frac{3\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^4}+\frac{3\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^4}-\frac{3\sqrt{2}\left(bc^3\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*c*x^(7/2) - b*x^(3/2))/((c*x^2 + b)^2*b*c) + 3/64*sqrt(2)*(b*c^3)^(
3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2
*c^4) + 3/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
 - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^4) - 3/128*sqrt(2)*(b*c^3)^(3/4)*log(sqrt
(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^4) + 3/128*sqrt(2)*(b*c^3)^
(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^4)
```

maple [A] time = 0.02, size = 169, normalized size = 0.70

$$\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}} b\,c^{2}}+\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}} b\,c^{2}}+\frac{3\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{128\left(\frac{b}{c}\right)^{\frac{1}{4}} b\,c^{2}}+\frac{\frac{3x^{\frac{7}{2}}}{16b}-\frac{x^{\frac{3}{2}}}{16c}}{\left(c\,x^{2}+b\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(3/32/b*x^(7/2)-1/32/c*x^(3/2))/(c*x^2+b)^2+3/128/c^2/b/(b/c)^(1/4)*2^(1/
2)*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(1/2)*x^
(1/2)+(b/c)^(1/2)))+3/64/c^2/b/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/
4)*x^(1/2)+1)+3/64/c^2/b/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(
1/2)-1)
```

maxima [A] time = 3.09, size = 222, normalized size = 0.92

$$\frac{3\,cx^{\frac{7}{2}}-bx^{\frac{3}{2}}}{16\left(bc^{3}x^{4}+2\,b^{2}c^{2}x^{2}+b^{3}c\right)}+\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\cdots\right.}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\cdots\right.}{128\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*c*x^(7/2) - b*x^(3/2))/(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c) + 3/128*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))
/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqr
t(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4
)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b
^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/(b*c)
```

mupad [B] time = 0.09, size = 85, normalized size = 0.35

$$\frac{\frac{3\,x^{7/2}}{16\,b}-\frac{x^{3/2}}{16\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}-\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{5/4}\,c^{7/4}}+\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{5/4}\,c^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((3*x^(7/2))/(16*b) - x^(3/2)/(16*c))/(b^2 + c^2*x^4 + 2*b*c*x^2) - (3*atan
((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(5/4)*c^(7/4)) + (3*atanh((c^(1/4)
*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(5/4)*c^(7/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.343 $\int \frac{x^{15/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=242

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{7/4}c^{5/4}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{7/4}c^{5/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{7/4}c^{5/4}}+\frac{3\tan^{-1}\left(\cdots\right.}{32\sqrt{2}\cdots}$$

```
[Out] -3/64*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)/c^(5/4)*2^(1/2)+3/6
4*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)/c^(5/4)*2^(1/2)-3/128*l
n(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(5/4)*2^(1/2
)+3/128*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(5/
4)*2^(1/2)-1/4*x^(1/2)/c/(c*x^2+b)^2+1/16*x^(1/2)/b/c/(c*x^2+b)
```

Rubi [A] time = 0.18, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{7/4}c^{5/4}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{7/4}c^{5/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{7/4}c^{5/4}}+\frac{3\tan^{-1}\left(\cdots\right.}{32\sqrt{2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^(15/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -Sqrt[x]/(4*c*(b + c*x^2)^2) + Sqrt[x]/(16*b*c*(b + c*x^2)) - (3*ArcTan[1 -
 (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(7/4)*c^(5/4)) + (3*ArcT
an[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(7/4)*c^(5/4)) - (
3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b
^(7/4)*c^(5/4)) + (3*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x])/(64*Sqrt[2]*b^(7/4)*c^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{15/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^{3/2}}{\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2} + \frac{\int \frac{1}{\sqrt{x}\left(b+cx^2\right)^2}\,dx}{8c}$$

$$= -\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2} + \frac{\sqrt{x}}{16bc\left(b+cx^2\right)} + \frac{3\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32bc}$$

$$= -\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2} + \frac{\sqrt{x}}{16bc\left(b+cx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16bc}$$

$$= -\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2} + \frac{\sqrt{x}}{16bc\left(b+cx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^{3/2}c} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\right.}{32b^{3/2}c}$$

$$= -\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2} + \frac{\sqrt{x}}{16bc\left(b+cx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64b^{3/2}c^{3/2}} + \frac{3\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}+}\right.}{64}$$

$$= -\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2} + \frac{\sqrt{x}}{16bc\left(b+cx^2\right)} - \frac{3\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{7/4}c^{5/4}} + \frac{3\log\left(\sqrt{b}+\sqrt{2}\right.}{64\sqrt{2}}$$

$$= -\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2} + \frac{\sqrt{x}}{16bc\left(b+cx^2\right)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{7/4}c^{5/4}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{7/4}c^{5/4}} - \frac{3\,\text{lo}}{}$$

Mathematica [A] time = 0.11, size = 223, normalized size = 0.92

$$\frac{-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{b^{7/4}} + \frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{b^{7/4}} - \frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{b^{7/4}} + \frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{b^{7/4}} + \frac{8}{b^2}}{128c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(15/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-32*c^(1/4)*Sqrt[x])/(b + c*x^2)^2 + (8*c^(1/4)*Sqrt[x])/(b^2 + b*c*x^2)
- (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/b^(7/4) + (6*Sq
rt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/b^(7/4) - (3*Sqrt[2]*L
og[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(7/4) + (3*Sqr
t[2]*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(7/4))/(
128*c^(5/4))
```

fricas [A] time = 0.83, size = 257, normalized size = 1.06

$$\frac{12\left(bc^3x^4+2\,b^2c^2x^2+b^3c\right)\left(-\frac{1}{b^7c^5}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^4c^2\sqrt{-\frac{1}{b^7c^5}}+x}\,b^5c^4\left(-\frac{1}{b^7c^5}\right)^{\frac{3}{4}}-b^5c^4\sqrt{x}\left(-\frac{1}{b^7c^5}\right)^{\frac{3}{4}}\right)+3\left(bc^3x^4+2\cdots\right.}{64\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(12*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-1/(b^7*c^5))^(1/4)*arctan(sq
rt(b^4*c^2*sqrt(-1/(b^7*c^5)) + x)*b^5*c^4*(-1/(b^7*c^5))^(3/4) - b^5*c^4*s
qrt(x)*(-1/(b^7*c^5))^(3/4)) + 3*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-1/(b
^7*c^5))^(1/4)*log(b^2*c*(-1/(b^7*c^5))^(1/4) + sqrt(x)) - 3*(b*c^3*x^4 + 2
*b^2*c^2*x^2 + b^3*c)*(-1/(b^7*c^5))^(1/4)*log(-b^2*c*(-1/(b^7*c^5))^(1/4)
+ sqrt(x)) + 4*(c*x^2 - 3*b)*sqrt(x))/(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)
```

giac [A] time = 0.18, size = 211, normalized size = 0.87

$$\frac{3\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^2}+\frac{3\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^2}+\frac{3\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}\cdots\right.}{128\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/(b^2*c^2) + 3/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) + 3/128*sqrt(2)*
(b*c^3)^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^2) -
3/128*sqrt(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c
))/(b^2*c^2) + 1/16*(c*x^(5/2) - 3*b*sqrt(x))/((c*x^2 + b)^2*b*c)
```

maple [A] time = 0.02, size = 169, normalized size = 0.70

$$\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64b^{2}c}+\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64b^{2}c}+\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{b}{c}}}\right)}{128b^{2}c}+\frac{\frac{x^{\frac{5}{2}}}{16b}-\frac{3\sqrt{x}}{16c}}{\left(c\,x^{2}+b\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(1/32/b*x^(5/2)-3/32/c*x^(1/2))/(c*x^2+b)^2+3/128/c/b^2*(b/c)^(1/4)*2^(1/
2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1/2)*x^
(1/2)+(b/c)^(1/2)))+3/64/c/b^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/
4)*x^(1/2)+1)+3/64/c/b^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(
1/2)-1)
```

maxima [A] time = 2.96, size = 221, normalized size = 0.91

$$\frac{cx^{\frac{5}{2}}-3\,b\sqrt{x}}{16\left(bc^{3}x^{4}+2\,b^{2}c^{2}x^{2}+b^{3}c\right)}+\frac{3\left(\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right.}{128\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(c*x^(5/2) - 3*b*sqrt(x))/(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c) + 3/128*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))
/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqr
t(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4
)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*log(-sqrt(2)*b
^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/(b*c)
```

mupad [B] time = 0.10, size = 85, normalized size = 0.35

$$\frac{\frac{x^{5/2}}{16\,b}-\frac{3\,\sqrt{x}}{16\,c}}{b^{2}+2\,b\,c\,x^{2}+c^{2}\,x^{4}}+\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{7/4}\,c^{5/4}}+\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{7/4}\,c^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (x^(5/2)/(16*b) - (3*x^(1/2))/(16*c))/(b^2 + c^2*x^4 + 2*b*c*x^2) + (3*atan
((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(7/4)*c^(5/4)) + (3*atanh((c^(1/4)
*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(7/4)*c^(5/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.344 $\int \frac{x^{13/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=239

$$\frac{5\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{9/4}c^{3/4}}-\frac{5\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{9/4}c^{3/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{9/4}c^{3/4}}+\frac{5\tan^{-1}\left(\right.}{32\sqrt{2}}$$

```
[Out] 1/4*x^(3/2)/b/(c*x^2+b)^2+5/16*x^(3/2)/b^2/(c*x^2+b)-5/64*arctan(1-c^(1/4)*
2^(1/2)*x^(1/2)/b^(1/4))/b^(9/4)/c^(3/4)*2^(1/2)+5/64*arctan(1+c^(1/4)*2^(1
/2)*x^(1/2)/b^(1/4))/b^(9/4)/c^(3/4)*2^(1/2)+5/128*ln(b^(1/2)+x*c^(1/2)-b^(
1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)/c^(3/4)*2^(1/2)-5/128*ln(b^(1/2)+x*c^
(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)/c^(3/4)*2^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1584, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{5\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{9/4}c^{3/4}}-\frac{5\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{9/4}c^{3/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{9/4}c^{3/4}}+\frac{5\tan^{-1}\left(\right.}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] x^(3/2)/(4*b*(b + c*x^2)^2) + (5*x^(3/2))/(16*b^2*(b + c*x^2)) - (5*ArcTan[
1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(9/4)*c^(3/4)) + (5*A
rcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(9/4)*c^(3/4))
+ (5*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2
]*b^(9/4)*c^(3/4)) - (5*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqr
t[c]*x])/(64*Sqrt[2]*b^(9/4)*c^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{13/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{\sqrt{x}}{\left(b+cx^2\right)^3}\,dx$$

$$= \frac{x^{3/2}}{4b\left(b+cx^2\right)^2} + \frac{5\int \frac{\sqrt{x}}{\left(b+cx^2\right)^2}\,dx}{8b}$$

$$= \frac{x^{3/2}}{4b\left(b+cx^2\right)^2} + \frac{5x^{3/2}}{16b^2\left(b+cx^2\right)} + \frac{5\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32b^2}$$

$$= \frac{x^{3/2}}{4b\left(b+cx^2\right)^2} + \frac{5x^{3/2}}{16b^2\left(b+cx^2\right)} + \frac{5\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^2}$$

$$= \frac{x^{3/2}}{4b\left(b+cx^2\right)^2} + \frac{5x^{3/2}}{16b^2\left(b+cx^2\right)} - \frac{5\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^2\sqrt{c}} + \frac{5\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,d\right.}{32b^2\sqrt{c}}$$

$$= \frac{x^{3/2}}{4b\left(b+cx^2\right)^2} + \frac{5x^{3/2}}{16b^2\left(b+cx^2\right)} + \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64b^2c} + \frac{5\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}+\right.}{64}$$

$$= \frac{x^{3/2}}{4b\left(b+cx^2\right)^2} + \frac{5x^{3/2}}{16b^2\left(b+cx^2\right)} + \frac{5\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{9/4}c^{3/4}} - \frac{5\log\left(\sqrt{b}+\sqrt{2}\right.}{64\sqrt{2}}$$

$$= \frac{x^{3/2}}{4b\left(b+cx^2\right)^2} + \frac{5x^{3/2}}{16b^2\left(b+cx^2\right)} - \frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{9/4}c^{3/4}} + \frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{9/4}c^{3/4}} + \frac{5\log}{}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.12

$$\frac{2x^{3/2}\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*x^(3/2)*Hypergeometric2F1[3/4, 3, 7/4, -((c*x^2)/b)])/(3*b^3)
```

fricas [A] time = 1.11, size = 250, normalized size = 1.05

$$-\frac{20\left(b^2c^2x^4+2\,b^3cx^2+b^4\right)\left(-\frac{1}{b^9c^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-b^5c\sqrt{-\frac{1}{b^9c^3}}+x}\,b^2c\left(-\frac{1}{b^9c^3}\right)^{\frac{1}{4}}-b^2c\sqrt{x}\left(-\frac{1}{b^9c^3}\right)^{\frac{1}{4}}\right)-5\left(b^2c^2x^4+2\right.}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(20*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^9*c^3))^(1/4)*arctan(sqr
t(-b^5*c*sqrt(-1/(b^9*c^3)) + x)*b^2*c*(-1/(b^9*c^3))^(1/4) - b^2*c*sqrt(x)
*(-1/(b^9*c^3))^(1/4)) - 5*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^9*c^3))
^(1/4)*log(b^7*c^2*(-1/(b^9*c^3))^(3/4) + sqrt(x)) + 5*(b^2*c^2*x^4 + 2*b^3
*c*x^2 + b^4)*(-1/(b^9*c^3))^(1/4)*log(-b^7*c^2*(-1/(b^9*c^3))^(3/4) + sqrt
(x)) - 4*(5*c*x^3 + 9*b*x)*sqrt(x))/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)
```

giac [A] time = 0.20, size = 209, normalized size = 0.87

$$\frac{5\,cx^{\frac{7}{2}}+9\,bx^{\frac{3}{2}}}{16\left(cx^2+b\right)^2b^2}+\frac{5\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^3c^3}+\frac{5\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^3c^3}-\frac{5\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(5*c*x^(7/2) + 9*b*x^(3/2))/((c*x^2 + b)^2*b^2) + 5/64*sqrt(2)*(b*c^3)
^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b
^3*c^3) + 5/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/
4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^3) - 5/128*sqrt(2)*(b*c^3)^(3/4)*log(sq
```

```
rt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^3) + 5/128*sqrt(2)*(b*c^3
)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^3)
```

maple [A] time = 0.01, size = 175, normalized size = 0.73

$$\frac{x^{\frac{3}{2}}}{4\left(c\,x^{2}+b\right)^{2} b}+\frac{5x^{\frac{3}{2}}}{16\left(c\,x^{2}+b\right) b^{2}}+\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}} b^{2} c}+\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}} b^{2} c}+\frac{5\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\cdots}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\cdots}\right)}{128\left(\frac{b}{c}\right)^{\frac{1}{4}} b^{2} c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/4*x^(3/2)/b/(c*x^2+b)^2+5/16*x^(3/2)/b^2/(c*x^2+b)+5/128/b^2/c/(b/c)^(1/4
)*2^(1/2)*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*2^(
1/2)*x^(1/2)+(b/c)^(1/2)))+5/64/b^2/c/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b
/c)^(1/4)*x^(1/2)+1)+5/64/b^2/c/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1
/4)*x^(1/2)-1)
```

maxima [A] time = 3.09, size = 217, normalized size = 0.91

$$\frac{5\,cx^{\frac{7}{2}}+9\,bx^{\frac{3}{2}}}{16\left(b^{2}c^{2}x^{4}+2\,b^{3}cx^{2}+b^{4}\right)}+\frac{5\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\,\sqrt{x}+\cdots\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\cdots\right)}{128\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(5*c*x^(7/2) + 9*b*x^(3/2))/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4) + 5/128*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))
/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqr
t(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4
)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b
^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^2
```

mupad [B] time = 0.09, size = 86, normalized size = 0.36

$$\frac{\frac{9\,x^{3/2}}{16\,b}+\frac{5\,c\,x^{7/2}}{16\,b^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4}+\frac{5\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{9/4}\,c^{3/4}}-\frac{5\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{9/4}\,c^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((9*x^(3/2))/(16*b) + (5*c*x^(7/2))/(16*b^2))/(b^2 + c^2*x^4 + 2*b*c*x^2) +
 (5*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(9/4)*c^(3/4)) - (5*atanh(
(c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(9/4)*c^(3/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.345 $\int \frac{x^{11/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=239

$$-\frac{21\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{11/4}\sqrt[4]{c}}+\frac{21\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{11/4}\sqrt[4]{c}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{11/4}\sqrt[4]{c}}+\frac{21\tan}{3}$$

```
[Out] -21/64*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/4)/c^(1/4)*2^(1/2)+2
1/64*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/4)/c^(1/4)*2^(1/2)-21/
128*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(11/4)/c^(1/4)*
2^(1/2)+21/128*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(11/
4)/c^(1/4)*2^(1/2)+1/4*x^(1/2)/b/(c*x^2+b)^2+7/16*x^(1/2)/b^2/(c*x^2+b)
```

Rubi [A] time = 0.19, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)}-\frac{21\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{11/4}\sqrt[4]{c}}+\frac{21\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{11/4}\sqrt[4]{c}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{32\sqrt{2}\,b^{11/4}\sqrt[4]{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] Sqrt[x]/(4*b*(b + c*x^2)^2) + (7*Sqrt[x])/(16*b^2*(b + c*x^2)) - (21*ArcTan
[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(11/4)*c^(1/4)) + (2
1*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(11/4)*c^(1/
4)) - (21*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*S
qrt[2]*b^(11/4)*c^(1/4)) + (21*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x])/(64*Sqrt[2]*b^(11/4)*c^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{11/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{1}{\sqrt{x}\left(b+cx^2\right)^3}\,dx$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\int \frac{1}{\sqrt{x}\left(b+cx^2\right)^2}\,dx}{8b}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} + \frac{21\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32b^2}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} + \frac{21\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^2}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} + \frac{21\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^{5/2}} + \frac{21\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\right.}{32b^{5/2}}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} + \frac{21\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64b^{5/2}\sqrt{c}} + \frac{21\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}}\right.}{64}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} - \frac{21\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{11/4}\sqrt[4]{c}} + \frac{21\log\left(\sqrt{b}+\sqrt{2}\right.}{64\sqrt{2}}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} - \frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{11/4}\sqrt[4]{c}} + \frac{21\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{11/4}\sqrt[4]{c}} - \frac{21}{}$$

Mathematica [A] time = 0.08, size = 220, normalized size = 0.92

$$\frac{\frac{32b^{7/4}\sqrt{x}}{(b+cx^2)^2}+\frac{56b^{3/4}\sqrt{x}}{b+cx^2}-\frac{21\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{\sqrt[4]{c}}+\frac{21\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{\sqrt[4]{c}}-\frac{42\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt[4]{c}}+\frac{42\sqrt{\cdots}}{\cdots}}{128b^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((32*b^(7/4)*Sqrt[x])/(b + c*x^2)^2 + (56*b^(3/4)*Sqrt[x])/(b + c*x^2) - (4
2*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/c^(1/4) + (42*Sqrt
[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/c^(1/4) - (21*Sqrt[2]*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/c^(1/4) + (21*Sqr
t[2]*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/c^(1/4))/(
128*b^(11/4))
```

fricas [A] time = 0.78, size = 241, normalized size = 1.01

$$\frac{84\left(b^2c^2x^4+2b^3cx^2+b^4\right)\left(-\frac{1}{b^{11}c}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^6\sqrt{-\frac{1}{b^{11}c}}+x}\,b^8c\left(-\frac{1}{b^{11}c}\right)^{\frac{3}{4}}-b^8c\sqrt{x}\left(-\frac{1}{b^{11}c}\right)^{\frac{3}{4}}\right)+21\left(b^2c^2x^4+2b^3c\cdots\right.}{64\left(b^2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(84*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^11*c))^(1/4)*arctan(sqrt(
b^6*sqrt(-1/(b^11*c)) + x)*b^8*c*(-1/(b^11*c))^(3/4) - b^8*c*sqrt(x)*(-1/(b
^11*c))^(3/4)) + 21*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^11*c))^(1/4)*l
og(b^3*(-1/(b^11*c))^(1/4) + sqrt(x)) - 21*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4
)*(-1/(b^11*c))^(1/4)*log(-b^3*(-1/(b^11*c))^(1/4) + sqrt(x)) + 4*(7*c*x^2
+ 11*b)*sqrt(x))/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)
```

giac [A] time = 0.21, size = 209, normalized size = 0.87

$$\frac{21\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64b^3c}+\frac{21\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64b^3c}+\frac{21\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\cdots\right.\right.}{128b^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 21/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/(b^3*c) + 21/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c) + 21/128*sqrt(2)*(
b*c^3)^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c) - 21/
128*sqrt(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))
/(b^3*c) + 1/16*(7*c*x^(5/2) + 11*b*sqrt(x))/((c*x^2 + b)^2*b^2)
```

maple [A] time = 0.01, size = 166, normalized size = 0.69

$$\frac{\sqrt{x}}{4\left(c\,x^2+b\right)^2 b}+\frac{7\sqrt{x}}{16\left(c\,x^2+b\right)b^2}+\frac{21\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64b^3}+\frac{21\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64b^3}+\frac{21\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/4*x^(1/2)/b/(c*x^2+b)^2+7/16*x^(1/2)/b^2/(c*x^2+b)+21/128/b^3*(b/c)^(1/4)
*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*2^(1
/2)*x^(1/2)+(b/c)^(1/2)))+21/64/b^3*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c
)^(1/4)*x^(1/2)+1)+21/64/b^3*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)
*x^(1/2)-1)
```

maxima [A] time = 2.97, size = 217, normalized size = 0.91

$$\frac{7\,cx^{\frac{5}{2}}+11\,b\sqrt{x}}{16\left(b^2c^2x^4+2\,b^3cx^2+b^4\right)}+\frac{21\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right.}{128\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(7*c*x^(5/2) + 11*b*sqrt(x))/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4) + 21/12
8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x
))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*s
```

```
qrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1
/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*log(-sqrt(2)
*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^2
```

mupad [B] time = 4.29, size = 86, normalized size = 0.36

$$\frac{\frac{11\sqrt{x}}{16\,b}+\frac{7\,c\,x^{5/2}}{16\,b^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4}-\frac{21\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{32\,(-b)^{11/4}\,c^{1/4}}-\frac{21\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{32\,(-b)^{11/4}\,c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((11*x^(1/2))/(16*b) + (7*c*x^(5/2))/(16*b^2))/(b^2 + c^2*x^4 + 2*b*c*x^2)
- (21*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(11/4)*c^(1/4)) - (21*at
anh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(11/4)*c^(1/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.346 $\int \frac{x^{9/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=251

$$-\frac{45\sqrt[4]{c}\,\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{13/4}}+\frac{45\sqrt[4]{c}\,\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{13/4}}+\frac{45\sqrt[4]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{}}{\sqrt[4]{b}}\right.}{32\sqrt{2}\,b^{13/4}}$$

```
[Out] 45/64*c^(1/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(13/4)*2^(1/2)-45
/64*c^(1/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(13/4)*2^(1/2)-45/1
28*c^(1/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2
^(1/2)+45/128*c^(1/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))
/b^(13/4)*2^(1/2)-45/16/b^3/x^(1/2)+1/4/b/(c*x^2+b)^2/x^(1/2)+9/16/b^2/(c*x
^2+b)/x^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)}-\frac{45\sqrt[4]{c}\,\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{13/4}}+\frac{45\sqrt[4]{c}\,\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{13/4}}+\frac{45\sqrt[4]{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -45/(16*b^3*Sqrt[x]) + 1/(4*b*Sqrt[x]*(b + c*x^2)^2) + 9/(16*b^2*Sqrt[x]*(b
 + c*x^2)) + (45*c^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32
*Sqrt[2]*b^(13/4)) - (45*c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/
4)])/(32*Sqrt[2]*b^(13/4)) - (45*c^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1
/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(13/4)) + (45*c^(1/4)*Log[Sqrt[b] +
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{9/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{1}{x^{3/2}\left(b+cx^2\right)^3}\,dx$$

$$= \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9\int \frac{1}{x^{3/2}\left(b+cx^2\right)^2}\,dx}{8b}$$

$$= \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)} + \frac{45\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{32b^2}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)} - \frac{(45c)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32b^3}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)} - \frac{(45c)\,\mathrm{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^3}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)} + \frac{\left(45\sqrt{c}\right)\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^3}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)} - \frac{45\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64b^3}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)} - \frac{45\sqrt[4]{c}\,\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\right.}{64\sqrt{2}\,b^{13/4}}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)} + \frac{45\sqrt[4]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{13/4}} - \frac{45\sqrt[4]{c}\,\mathrm{t}}{}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.11

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},3;\frac{3}{4};-\frac{cx^2}{b}\right)}{b^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 3, 3/4, -((c*x^2)/b)])/(b^3*Sqrt[x])
```

fricas [A] time = 0.77, size = 263, normalized size = 1.05

$$\frac{180\left(b^3c^2x^5+2\,b^4cx^3+b^5x\right)\left(-\frac{c}{b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{91125\,b^3c\sqrt{x}\left(-\frac{c}{b^{13}}\right)^{\frac{1}{4}}-\sqrt{-8303765625\,b^7c\sqrt{-\frac{c}{b^{13}}}+8303765625\,c^2x}\,b^3\left(-\frac{c}{b^{13}}\right)^{\frac{1}{4}}}{91125\,c}\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(180*(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)*(-c/b^13)^(1/4)*arctan(-1/911
25*(91125*b^3*c*sqrt(x)*(-c/b^13)^(1/4) - sqrt(-8303765625*b^7*c*sqrt(-c/b^
13) + 8303765625*c^2*x)*b^3*(-c/b^13)^(1/4))/c) - 45*(b^3*c^2*x^5 + 2*b^4*c
*x^3 + b^5*x)*(-c/b^13)^(1/4)*log(91125*b^10*(-c/b^13)^(3/4) + 91125*c*sqrt
(x)) + 45*(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)*(-c/b^13)^(1/4)*log(-91125*b^
10*(-c/b^13)^(3/4) + 91125*c*sqrt(x)) - 4*(45*c^2*x^4 + 81*b*c*x^2 + 32*b^2
)*sqrt(x))/(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)
```

giac [A] time = 0.19, size = 220, normalized size = 0.88

$$-\frac{2}{b^3\sqrt{x}}-\frac{45\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4c^2}-\frac{45\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4c^2}+\frac{45\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -2/(b^3*sqrt(x)) - 45/64*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*
(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c^2) - 45/64*sqrt(2)*(b*c^3)^(3/
4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^4*
c^2) + 45/128*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + s
qrt(b/c))/(b^4*c^2) - 45/128*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/
c)^(1/4) + x + sqrt(b/c))/(b^4*c^2) - 1/16*(13*c^2*x^(7/2) + 17*b*c*x^(3/2)
)/((c*x^2 + b)^2*b^3)
```

maple [A] time = 0.02, size = 178, normalized size = 0.71

$$-\frac{13c^{2}x^{\frac{7}{2}}}{16\left(c\,x^{2}+b\right)^{2}b^{3}}-\frac{17c\,x^{\frac{3}{2}}}{16\left(c\,x^{2}+b\right)^{2}b^{2}}-\frac{45\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{3}}-\frac{45\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}}b^{3}}-\frac{45\sqrt{2}\ \ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{}}\right.}{128\left(\frac{b}{c}\right)^{\frac{1}{4}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -13/16*c^2/b^3/(c*x^2+b)^2*x^(7/2)-17/16*c/b^2/(c*x^2+b)^2*x^(3/2)-45/128/b
^3/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x+(b
/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-45/64/b^3/(b/c)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-45/64/b^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(b/c)^(1/4)*x^(1/2)-1)-2/b^3/x^(1/2)
```

maxima [A] time = 3.14, size = 230, normalized size = 0.92

$$-\frac{45\,c^{2}x^{4}+81\,bcx^{2}+32\,b^{2}}{16\left(b^{3}c^{2}x^{\frac{9}{2}}+2\,b^{4}cx^{\frac{5}{2}}+b^{5}\sqrt{x}\right)}-\frac{45\,c\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}\right.}{}\right.}{128\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(45*c^2*x^4 + 81*b*c*x^2 + 32*b^2)/(b^3*c^2*x^(9/2) + 2*b^4*c*x^(5/2)
 + b^5*sqrt(x)) - 45/128*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c
^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*s
qrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c
)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)
*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4
)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b
^(1/4)*c^(3/4)))/b^3
```

mupad [B] time = 4.37, size = 99, normalized size = 0.39

$$\frac{45\,{\left(-c\right)}^{1/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{13/4}}-\frac{45\,{\left(-c\right)}^{1/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{13/4}}-\frac{\frac{2}{b}+\frac{81\,c\,x^2}{16\,b^2}+\frac{45\,c^2\,x^4}{16\,b^3}}{b^2\,\sqrt{x}+c^2\,x^{9/2}+2\,b\,c\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (45*(-c)^(1/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(13/4)) - (45*(-c
)^(1/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(13/4)) - (2/b + (81*c*x^
2)/(16*b^2) + (45*c^2*x^4)/(16*b^3))/(b^2*x^(1/2) + c^2*x^(9/2) + 2*b*c*x^(
5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.347 $\int \frac{x^{7/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=251

$$\frac{77c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{15/4}}-\frac{77c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{15/4}}+\frac{77c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{15/4}}$$

```
[Out] -77/48/b^3/x^(3/2)+1/4/b/x^(3/2)/(c*x^2+b)^2+11/16/b^2/x^(3/2)/(c*x^2+b)+77
/64*c^(3/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)*2^(1/2)-77/6
4*c^(3/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)*2^(1/2)+77/128
*c^(3/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(15/4)*2^(
1/2)-77/128*c^(3/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b
^(15/4)*2^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{77c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{15/4}}-\frac{77c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{15/4}}+\frac{77c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -77/(48*b^3*x^(3/2)) + 1/(4*b*x^(3/2)*(b + c*x^2)^2) + 11/(16*b^2*x^(3/2)*(
b + c*x^2)) + (77*c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(3
2*Sqrt[2]*b^(15/4)) - (77*c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(32*Sqrt[2]*b^(15/4)) + (77*c^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(
1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(15/4)) - (77*c^(3/4)*Log[Sqrt[b]
+ Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(15/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{1}{x^{5/2}\left(b+cx^2\right)^3}\,dx$$

$$= \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11\int \frac{1}{x^{5/2}\left(b+cx^2\right)^2}\,dx}{8b}$$

$$= \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11}{16b^2x^{3/2}\left(b+cx^2\right)} + \frac{77\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{32b^2}$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11}{16b^2x^{3/2}\left(b+cx^2\right)} - \frac{(77c)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32b^3}$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11}{16b^2x^{3/2}\left(b+cx^2\right)} - \frac{(77c)\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^3}$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11}{16b^2x^{3/2}\left(b+cx^2\right)} - \frac{(77c)\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^{7/2}}$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11}{16b^2x^{3/2}\left(b+cx^2\right)} - \frac{\left(77\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx,\right.}{64b^{7/2}}$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11}{16b^2x^{3/2}\left(b+cx^2\right)} + \frac{77c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{64\sqrt{2}\,b^{15/4}}$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11}{16b^2x^{3/2}\left(b+cx^2\right)} + \frac{77c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{15/4}} - \frac{77c^{3/}}{}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.12

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},3;\frac{1}{4};-\frac{cx^2}{b}\right)}{3b^3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 3, 1/4, -((c*x^2)/b)])/(3*b^3*x^(3/2))
```

fricas [A] time = 0.95, size = 283, normalized size = 1.13

$$-\frac{924\left(b^3c^2x^6+2\,b^4cx^4+b^5x^2\right)\left(-\frac{c^3}{b^{15}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^{11}c\sqrt{x}\left(-\frac{c^3}{b^{15}}\right)^{\frac{3}{4}}-\sqrt{b^8\sqrt{-\frac{c^3}{b^{15}}}+c^2x}\,b^{11}\left(-\frac{c^3}{b^{15}}\right)^{\frac{3}{4}}}{c^3}\right)+231\left(b^3c^2x^6+2\,b^4cx^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(924*(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)*(-c^3/b^15)^(1/4)*arctan(
-(b^11*c*sqrt(x)*(-c^3/b^15)^(3/4) - sqrt(b^8*sqrt(-c^3/b^15) + c^2*x)*b^11
*(-c^3/b^15)^(3/4))/c^3) + 231*(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)*(-c^3/
b^15)^(1/4)*log(77*b^4*(-c^3/b^15)^(1/4) + 77*c*sqrt(x)) - 231*(b^3*c^2*x^6
 + 2*b^4*c*x^4 + b^5*x^2)*(-c^3/b^15)^(1/4)*log(-77*b^4*(-c^3/b^15)^(1/4) +
 77*c*sqrt(x)) + 4*(77*c^2*x^4 + 121*b*c*x^2 + 32*b^2)*sqrt(x))/(b^3*c^2*x^
6 + 2*b^4*c*x^4 + b^5*x^2)
```

giac [A] time = 0.18, size = 208, normalized size = 0.83

$$-\frac{77\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4}-\frac{77\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4}-\frac{77\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\right.}{128\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -77/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sq
rt(x))/(b/c)^(1/4))/b^4 - 77/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^4 - 77/128*sqrt(2)*(b*c^3)^
(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 + 77/128*sqrt(2)
*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 1/16
*(15*c^2*x^(5/2) + 19*b*c*sqrt(x))/((c*x^2 + b)^2*b^3) - 2/3/(b^3*x^(3/2))
```

maple [A] time = 0.02, size = 181, normalized size = 0.72

$$-\frac{15c^2x^{\frac{5}{2}}}{16\left(cx^2+b\right)^2b^3}-\frac{19c\sqrt{x}}{16\left(cx^2+b\right)^2b^2}-\frac{77\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64b^4}-\frac{77\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64b^4}-\frac{77\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -15/16*c^2/b^3/(c*x^2+b)^2*x^(5/2)-19/16*c/b^2/(c*x^2+b)^2*x^(1/2)-77/128*c
/b^4*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(x-
(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-77/64*c/b^4*(b/c)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-77/64*c/b^4*(b/c)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/3/b^3/x^(3/2)
```

maxima [A] time = 3.05, size = 231, normalized size = 0.92

$$-\frac{77\,c^2x^4+121\,bcx^2+32\,b^2}{48\left(b^3c^2x^{\frac{11}{2}}+2\,b^4cx^{\frac{7}{2}}+b^5x^{\frac{3}{2}}\right)}-\frac{77\left(\frac{2\sqrt{2}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}c^{\frac{3}{4}}\log\left(\right.}{}\right.}{128\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(77*c^2*x^4 + 121*b*c*x^2 + 32*b^2)/(b^3*c^2*x^(11/2) + 2*b^4*c*x^(7/
2) + b^5*x^(3/2)) - 77/128*(2*sqrt(2)*c*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)
*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*
sqrt(c))) + 2*sqrt(2)*c*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sq
rt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqr
t(2)*c^(3/4)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(
3/4) - sqrt(2)*c^(3/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + s
qrt(b))/b^(3/4))/b^3
```

mupad [B] time = 0.13, size = 99, normalized size = 0.39

$$\frac{77\,{\left(-c\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{15/4}}-\frac{\frac{2}{3\,b}+\frac{121\,c\,x^2}{48\,b^2}+\frac{77\,c^2\,x^4}{48\,b^3}}{b^2\,x^{3/2}+c^2\,x^{11/2}+2\,b\,c\,x^{7/2}}+\frac{77\,{\left(-c\right)}^{3/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (77*(-c)^(3/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(15/4)) - (2/(3*b)
 + (121*c*x^2)/(48*b^2) + (77*c^2*x^4)/(48*b^3))/(b^2*x^(3/2) + c^2*x^(11/2
) + 2*b*c*x^(7/2)) + (77*(-c)^(3/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(3
2*b^(15/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.348 $\int \frac{x^{5/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=264

$$\frac{117c^{5/4}\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{17/4}}-\frac{117c^{5/4}\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{17/4}}-\frac{117c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{32\sqrt{2}\,b^{17/4}}$$

```
[Out] -117/80/b^3/x^(5/2)+1/4/b/x^(5/2)/(c*x^2+b)^2+13/16/b^2/x^(5/2)/(c*x^2+b)-1
17/64*c^(5/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)+11
7/64*c^(5/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)+117
/128*c^(5/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(17/4)
*2^(1/2)-117/128*c^(5/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/
2))/b^(17/4)*2^(1/2)+117/16*c/b^4/x^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{117c^{5/4}\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{17/4}}-\frac{117c^{5/4}\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{17/4}}-\frac{117c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{32\sqrt{2}\,b^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -117/(80*b^3*x^(5/2)) + (117*c)/(16*b^4*Sqrt[x]) + 1/(4*b*x^(5/2)*(b + c*x^
2)^2) + 13/(16*b^2*x^(5/2)*(b + c*x^2)) - (117*c^(5/4)*ArcTan[1 - (Sqrt[2]*
c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(17/4)) + (117*c^(5/4)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(17/4)) + (117*c^(5/4)*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(17
/4)) - (117*c^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]
*x])/(64*Sqrt[2]*b^(17/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(bx^2+cx^4\right)^3}\,dx=\int \frac{1}{x^{7/2}\left(b+cx^2\right)^3}\,dx$$

$$=\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}+\frac{13\int \frac{1}{x^{7/2}\left(b+cx^2\right)^2}\,dx}{8b}$$

$$=\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}+\frac{13}{16b^2x^{5/2}\left(b+cx^2\right)}+\frac{117\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{32b^2}$$

$$=-\frac{117}{80b^3x^{5/2}}+\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}+\frac{13}{16b^2x^{5/2}\left(b+cx^2\right)}-\frac{(117c)\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{32b^3}$$

$$=-\frac{117}{80b^3x^{5/2}}+\frac{117c}{16b^4\sqrt{x}}+\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}+\frac{13}{16b^2x^{5/2}\left(b+cx^2\right)}+\frac{\left(117c^2\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32b^4}$$

$$=-\frac{117}{80b^3x^{5/2}}+\frac{117c}{16b^4\sqrt{x}}+\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}+\frac{13}{16b^2x^{5/2}\left(b+cx^2\right)}+\frac{\left(117c^2\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx\right.}{16b^4}$$

$$=-\frac{117}{80b^3x^{5/2}}+\frac{117c}{16b^4\sqrt{x}}+\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}+\frac{13}{16b^2x^{5/2}\left(b+cx^2\right)}-\frac{\left(117c^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}}{b+cx^4}\right.}{32b^4}$$

$$=-\frac{117}{80b^3x^{5/2}}+\frac{117c}{16b^4\sqrt{x}}+\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}+\frac{13}{16b^2x^{5/2}\left(b+cx^2\right)}+\frac{(117c)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}}\right.}{64b^4}$$

$$=-\frac{117}{80b^3x^{5/2}}+\frac{117c}{16b^4\sqrt{x}}+\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}+\frac{13}{16b^2x^{5/2}\left(b+cx^2\right)}+\frac{117c^{5/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\right.}{64\sqrt{2}\,b^{17/4}}$$

$$=-\frac{117}{80b^3x^{5/2}}+\frac{117c}{16b^4\sqrt{x}}+\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}+\frac{13}{16b^2x^{5/2}\left(b+cx^2\right)}-\frac{117c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{}}{\sqrt[4]{b}}\right.}{32\sqrt{2}\,b^{17/4}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.11

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},3;-\frac{1}{4};-\frac{cx^2}{b}\right)}{5b^3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 3, -1/4, -((c*x^2)/b)])/(5*b^3*x^(5/2))
```

fricas [A] time = 0.87, size = 306, normalized size = 1.16

$$-\frac{2340\left(b^4c^2x^7+2\,b^5cx^5+b^6x^3\right)\left(-\frac{c^5}{b^{17}}\right)^{\frac{1}{4}}\arctan\left(-\frac{1601613\,b^4c^4\sqrt{x}\left(-\frac{c^5}{b^{17}}\right)^{\frac{1}{4}}-\sqrt{-2565164201769\,b^9c^5\sqrt{-\frac{c^5}{b^{17}}}+2565164201769\,c^8 \cdots}}{1601613\,c^5} \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/320*(2340*(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-c^5/b^17)^(1/4)*arctan
(-1/1601613*(1601613*b^4*c^4*sqrt(x)*(-c^5/b^17)^(1/4) - sqrt(-256516420176
9*b^9*c^5*sqrt(-c^5/b^17) + 2565164201769*c^8*x)*b^4*(-c^5/b^17)^(1/4))/c^5
) - 585*(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-c^5/b^17)^(1/4)*log(1601613
*b^13*(-c^5/b^17)^(3/4) + 1601613*c^4*sqrt(x)) + 585*(b^4*c^2*x^7 + 2*b^5*c
*x^5 + b^6*x^3)*(-c^5/b^17)^(1/4)*log(-1601613*b^13*(-c^5/b^17)^(3/4) + 160
1613*c^4*sqrt(x)) - 4*(585*c^3*x^6 + 1053*b*c^2*x^4 + 416*b^2*c*x^2 - 32*b^
3)*sqrt(x))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)
```

giac [A] time = 0.27, size = 232, normalized size = 0.88

$$\frac{117\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5c}+\frac{117\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5c}-\frac{117\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\cdots\right.}{128\,b^5 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 117/64*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sq
rt(x))/(b/c)^(1/4))/(b^5*c) + 117/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt
(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^5*c) - 117/128*sqrt(2
)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^5*c) +
117/128*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b
/c))/(b^5*c) + 1/16*(21*c^3*x^(7/2) + 25*b*c^2*x^(3/2))/((c*x^2 + b)^2*b^4)
 + 2/5*(15*c*x^2 - b)/(b^4*x^(5/2))
```

maple [A] time = 0.02, size = 192, normalized size = 0.73

$$\frac{21c^3x^{\frac{7}{2}}}{16\left(c\,x^2+b\right)^2 b^4}+\frac{25c^2x^{\frac{3}{2}}}{16\left(c\,x^2+b\right)^2 b^3}+\frac{117\sqrt{2}\;c\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}} b^4}+\frac{117\sqrt{2}\;c\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}} b^4}+\frac{117\sqrt{2}\;c\ln\left(\frac{x-(}{x+(}\right.}{128\left(\frac{b}{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 21/16*c^3/b^4/(c*x^2+b)^2*x^(7/2)+25/16*c^2/b^3/(c*x^2+b)^2*x^(3/2)+117/128
*c/b^4/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))/(
x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+117/64*c/b^4/(b/c)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+117/64*c/b^4/(b/c)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/5/b^3/x^(5/2)+6*c/b^4/x^(1/2)
```

maxima [A] time = 3.14, size = 243, normalized size = 0.92

$$\frac{585\,c^3x^6+1053\,bc^2x^4+416\,b^2cx^2-32\,b^3}{80\left(b^4c^2x^{\frac{13}{2}}+2\,b^5cx^{\frac{9}{2}}+b^6x^{\frac{5}{2}}\right)}+\frac{117\,c^2\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\right.}{128\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/80*(585*c^3*x^6 + 1053*b*c^2*x^4 + 416*b^2*c*x^2 - 32*b^3)/(b^4*c^2*x^(13
/2) + 2*b^5*c*x^(9/2) + b^6*x^(5/2)) + 117/128*c^2*(2*sqrt(2)*arctan(1/2*sq
rt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/
```

```
(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^
(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqr
t(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x +
sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) +
 sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^4
```

mupad [B] time = 0.12, size = 109, normalized size = 0.41

$$\frac{\frac{26\,c\,x^2}{5\,b^2}-\frac{2}{5\,b}+\frac{1053\,c^2\,x^4}{80\,b^3}+\frac{117\,c^3\,x^6}{16\,b^4}}{b^2\,x^{5/2}+c^2\,x^{13/2}+2\,b\,c\,x^{9/2}}-\frac{117\,{\left(-c\right)}^{5/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{17/4}}+\frac{117\,{\left(-c\right)}^{5/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{17/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((26*c*x^2)/(5*b^2) - 2/(5*b) + (1053*c^2*x^4)/(80*b^3) + (117*c^3*x^6)/(16
*b^4))/(b^2*x^(5/2) + c^2*x^(13/2) + 2*b*c*x^(9/2)) - (117*(-c)^(5/4)*atan(
((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(17/4)) + (117*(-c)^(5/4)*atanh(((-c)^
(1/4)*x^(1/2))/b^(1/4)))/(32*b^(17/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.349 $\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=264

$$-\frac{165c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{19/4}}+\frac{165c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{19/4}}-\frac{165c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{32\sqrt{2}\,b^{19/4}}$$

```
[Out] -165/112/b^3/x^(7/2)+55/16*c/b^4/x^(3/2)+1/4/b/x^(7/2)/(c*x^2+b)^2+15/16/b^
2/x^(7/2)/(c*x^2+b)-165/64*c^(7/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4)
)/b^(19/4)*2^(1/2)+165/64*c^(7/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))
/b^(19/4)*2^(1/2)-165/128*c^(7/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1
/2)*x^(1/2))/b^(19/4)*2^(1/2)+165/128*c^(7/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*
c^(1/4)*2^(1/2)*x^(1/2))/b^(19/4)*2^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{165c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{19/4}}+\frac{165c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{19/4}}-\frac{165c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{32\sqrt{2}\,b^{19/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -165/(112*b^3*x^(7/2)) + (55*c)/(16*b^4*x^(3/2)) + 1/(4*b*x^(7/2)*(b + c*x^
2)^2) + 15/(16*b^2*x^(7/2)*(b + c*x^2)) - (165*c^(7/4)*ArcTan[1 - (Sqrt[2]*
c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(19/4)) + (165*c^(7/4)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(19/4)) - (165*c^(7/4)*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(19
/4)) + (165*c^(7/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]
*x])/(64*Sqrt[2]*b^(19/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^3}\,dx=\int \frac{1}{x^{9/2}\left(b+cx^2\right)^3}\,dx$$

$$=\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15\int \frac{1}{x^{9/2}\left(b+cx^2\right)^2}\,dx}{8b}$$

$$=\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}+\frac{165\int \frac{1}{x^{9/2}\left(b+cx^2\right)}\,dx}{32b^2}$$

$$=-\frac{165}{112b^3x^{7/2}}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}-\frac{(165c)\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{32b^3}$$

$$=-\frac{165}{112b^3x^{7/2}}+\frac{55c}{16b^4x^{3/2}}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}+\frac{\left(165c^2\right)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32b^4}$$

$$=-\frac{165}{112b^3x^{7/2}}+\frac{55c}{16b^4x^{3/2}}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}+\frac{\left(165c^2\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\right.}{16b^4}$$

$$=-\frac{165}{112b^3x^{7/2}}+\frac{55c}{16b^4x^{3/2}}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}+\frac{\left(165c^2\right)\text{Subst}\left(\int \frac{\sqrt{b}-}{b+c}\right.}{32b^{9/2}}$$

$$=-\frac{165}{112b^3x^{7/2}}+\frac{55c}{16b^4x^{3/2}}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}+\frac{\left(165c^{3/2}\right)\text{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}}\right.}{64b}$$

$$=-\frac{165}{112b^3x^{7/2}}+\frac{55c}{16b^4x^{3/2}}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}-\frac{165c^{7/4}\log\left(\sqrt{b}-\sqrt{2}\right.}{64\sqrt{2}\,b}$$

$$=-\frac{165}{112b^3x^{7/2}}+\frac{55c}{16b^4x^{3/2}}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}-\frac{165c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{}}{\sqrt[4]{}}\right.}{32\sqrt{2}\,b^{19/4}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.11

$$-\frac{2\,{}_2F_1\left(-\frac{7}{4},3;-\frac{3}{4};-\frac{cx^2}{b}\right)}{7b^3x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-2*Hypergeometric2F1[-7/4, 3, -3/4, -((c*x^2)/b)])/(7*b^3*x^(7/2))
```

fricas [A] time = 0.75, size = 300, normalized size = 1.14

$$\frac{4620\left(b^4c^2x^8+2\,b^5cx^6+b^6x^4\right)\left(-\frac{c^7}{b^{19}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^{14}c^2\sqrt{x}\left(-\frac{c^7}{b^{19}}\right)^{\frac{3}{4}}-\sqrt{b^{10}\sqrt{-\frac{c^7}{b^{19}}}+c^4x}\,b^{14}\left(-\frac{c^7}{b^{19}}\right)^{\frac{3}{4}}}{c^7}\right)+1155\left(b^4c^2x^8+2\,b^5c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/448*(4620*(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-c^7/b^19)^(1/4)*arctan(
-(b^14*c^2*sqrt(x)*(-c^7/b^19)^(3/4) - sqrt(b^10*sqrt(-c^7/b^19) + c^4*x)*b
^14*(-c^7/b^19)^(3/4))/c^7) + 1155*(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-
c^7/b^19)^(1/4)*log(165*b^5*(-c^7/b^19)^(1/4) + 165*c^2*sqrt(x)) - 1155*(b^
4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-c^7/b^19)^(1/4)*log(-165*b^5*(-c^7/b^1
9)^(1/4) + 165*c^2*sqrt(x)) + 4*(385*c^3*x^6 + 605*b*c^2*x^4 + 160*b^2*c*x^
2 - 32*b^3)*sqrt(x))/(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)
```

giac [A] time = 0.21, size = 224, normalized size = 0.85

$$\frac{165\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5}+\frac{165\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5}+\frac{165\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}c\log\left(\cdots\right.}{12\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 165/64*sqrt(2)*(b*c^3)^(1/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*
sqrt(x))/(b/c)^(1/4))/b^5 + 165/64*sqrt(2)*(b*c^3)^(1/4)*c*arctan(-1/2*sqrt
```

```
(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^5 + 165/128*sqrt(2)*(b
*c^3)^(1/4)*c*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^5 - 165/12
8*sqrt(2)*(b*c^3)^(1/4)*c*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))
/b^5 + 1/16*(23*c^3*x^(5/2) + 27*b*c^2*sqrt(x))/((c*x^2 + b)^2*b^4) + 2/7*(
7*c*x^2 - b)/(b^4*x^(7/2))
```

maple [A] time = 0.02, size = 198, normalized size = 0.75

$$\frac{23c^3x^{\frac{5}{2}}}{16\left(c\,x^2+b\right)^2b^4}+\frac{27c^2\sqrt{x}}{16\left(c\,x^2+b\right)^2b^3}+\frac{165\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64b^5}+\frac{165\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64b^5}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 23/16/b^4*c^3/(c*x^2+b)^2*x^(5/2)+27/16/b^3*c^2/(c*x^2+b)^2*x^(1/2)+165/128
/b^5*c^2*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2))
/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))+165/64/b^5*c^2*(b/c)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+165/64/b^5*c^2*(b/c)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/7/b^3/x^(7/2)+2*c/b^4/x^(3/2)
```

maxima [A] time = 2.93, size = 246, normalized size = 0.93

$$\frac{385\,c^3x^6+605\,bc^2x^4+160\,b^2cx^2-32\,b^3}{112\left(b^4c^2x^{\frac{15}{2}}+2\,b^5cx^{\frac{11}{2}}+b^6x^{\frac{7}{2}}\right)}+\frac{165\left(\frac{2\,\sqrt{2}\,c^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\,\sqrt{2}\,c^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\cdots\right)}{128\,b\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/112*(385*c^3*x^6 + 605*b*c^2*x^4 + 160*b^2*c*x^2 - 32*b^3)/(b^4*c^2*x^(15
/2) + 2*b^5*c*x^(11/2) + b^6*x^(7/2)) + 165/128*(2*sqrt(2)*c^2*arctan(1/2*s
qrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))
/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*c^2*arctan(-1/2*sqrt(2)*(sqrt(
2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqr
t(sqrt(b)*sqrt(c))) + sqrt(2)*c^(7/4)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) +
```

```
sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2)*c^(7/4)*log(-sqrt(2)*b^(1/4)*c^(1/4
)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4))/b^4
```

mupad [B] time = 4.35, size = 109, normalized size = 0.41

$$\frac{\frac{10\,c\,x^2}{7\,b^2}-\frac{2}{7\,b}+\frac{605\,c^2\,x^4}{112\,b^3}+\frac{55\,c^3\,x^6}{16\,b^4}}{b^2\,x^{7/2}+c^2\,x^{15/2}+2\,b\,c\,x^{11/2}}+\frac{165\,{\left(-c\right)}^{7/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{19/4}}+\frac{165\,{\left(-c\right)}^{7/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{19/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((10*c*x^2)/(7*b^2) - 2/(7*b) + (605*c^2*x^4)/(112*b^3) + (55*c^3*x^6)/(16*
b^4))/(b^2*x^(7/2) + c^2*x^(15/2) + 2*b*c*x^(11/2)) + (165*(-c)^(7/4)*atan(
((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(19/4)) + (165*(-c)^(7/4)*atanh(((-c)^
(1/4)*x^(1/2))/b^(1/4)))/(32*b^(19/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.350 $\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=279

$$-\frac{221c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{21/4}}+\frac{221c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{21/4}}+\frac{221c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{32\sqrt{2}\,b^{21/4}}$$

```
[Out] -221/144/b^3/x^(9/2)+221/80*c/b^4/x^(5/2)+1/4/b/x^(9/2)/(c*x^2+b)^2+17/16/b
^2/x^(9/2)/(c*x^2+b)+221/64*c^(9/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4
))/b^(21/4)*2^(1/2)-221/64*c^(9/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4)
)/b^(21/4)*2^(1/2)-221/128*c^(9/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(
1/2)*x^(1/2))/b^(21/4)*2^(1/2)+221/128*c^(9/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)
*c^(1/4)*2^(1/2)*x^(1/2))/b^(21/4)*2^(1/2)-221/16*c^2/b^5/x^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{221c^2}{16b^5\sqrt{x}}-\frac{221c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{21/4}}+\frac{221c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{21/4}}+\frac{221c^{9/4}\tan^{-}}{32}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -221/(144*b^3*x^(9/2)) + (221*c)/(80*b^4*x^(5/2)) - (221*c^2)/(16*b^5*Sqrt[
x]) + 1/(4*b*x^(9/2)*(b + c*x^2)^2) + 17/(16*b^2*x^(9/2)*(b + c*x^2)) + (22
1*c^(9/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(21/
4)) - (221*c^(9/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[
2]*b^(21/4)) - (221*c^(9/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
 Sqrt[c]*x])/(64*Sqrt[2]*b^(21/4)) + (221*c^(9/4)*Log[Sqrt[b] + Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(21/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{1}{x^{11/2}\left(b+cx^2\right)^3}\,dx$$

$$= \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17\int \frac{1}{x^{11/2}\left(b+cx^2\right)^2}\,dx}{8b}$$

$$= \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} + \frac{221\int \frac{1}{x^{11/2}\left(b+cx^2\right)}\,dx}{32b^2}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} - \frac{(221c)\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{32b^3}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} + \frac{\left(221c^2\right)\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{32b^4}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} - \frac{\left(221c^3\right)\int \frac{\sqrt{}}{b+}}{32b^5}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} - \frac{\left(221c^3\right)\text{Subs}}{}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} + \frac{\left(221c^{5/2}\right)\text{Sub}}{}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} - \frac{\left(221c^2\right)\text{Subs}}{}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} - \frac{221c^{9/4}\log\left(\right.}{}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} + \frac{221c^{9/4}\tan^{-1}}{32\sqrt{}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.10

$$-\frac{2\,{}_2F_1\left(-\frac{9}{4},3;-\frac{5}{4};-\frac{cx^2}{b}\right)}{9b^3x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-2*Hypergeometric2F1[-9/4, 3, -5/4, -((c*x^2)/b)])/(9*b^3*x^(9/2))
```

fricas [A] time = 0.81, size = 317, normalized size = 1.14

$$39780\left(b^5c^2x^9+2\,b^6cx^7+b^7x^5\right)\left(-\frac{c^9}{b^{21}}\right)^{\frac{1}{4}}\arctan\left(-\frac{10793861\,b^5c^7\sqrt{x}\left(-\frac{c^9}{b^{21}}\right)^{\frac{1}{4}}-\sqrt{-116507435287321\,b^{11}c^9\sqrt{-\frac{c^9}{b^{21}}}+116507435287}}{10793861\,c^9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2880*(39780*(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)*(-c^9/b^21)^(1/4)*arcta
n(-1/10793861*(10793861*b^5*c^7*sqrt(x)*(-c^9/b^21)^(1/4) - sqrt(-116507435
287321*b^11*c^9*sqrt(-c^9/b^21) + 116507435287321*c^14*x)*b^5*(-c^9/b^21)^(
1/4))/c^9) - 9945*(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)*(-c^9/b^21)^(1/4)*l
og(10793861*b^16*(-c^9/b^21)^(3/4) + 10793861*c^7*sqrt(x)) + 9945*(b^5*c^2*
x^9 + 2*b^6*c*x^7 + b^7*x^5)*(-c^9/b^21)^(1/4)*log(-10793861*b^16*(-c^9/b^2
1)^(3/4) + 10793861*c^7*sqrt(x)) - 4*(9945*c^4*x^8 + 17901*b*c^3*x^6 + 7072
*b^2*c^2*x^4 - 544*b^3*c*x^2 + 160*b^4)*sqrt(x))/(b^5*c^2*x^9 + 2*b^6*c*x^7
 + b^7*x^5)
```

giac [A] time = 0.19, size = 231, normalized size = 0.83

$$-\frac{221\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6}-\frac{221\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6}+\frac{221\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{\right.}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -221/64*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*s
qrt(x))/(b/c)^(1/4))/b^6 - 221/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^6 + 221/128*sqrt(2)*(b*c^
3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^6 - 221/128*sqr
t(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^6 -
1/16*(29*c^4*x^(7/2) + 33*b*c^3*x^(3/2))/((c*x^2 + b)^2*b^5) - 2/45*(270*c^
2*x^4 - 27*b*c*x^2 + 5*b^2)/(b^5*x^(9/2))
```

maple [A] time = 0.02, size = 209, normalized size = 0.75

$$-\frac{29c^4x^{\frac{7}{2}}}{16\left(c\,x^2+b\right)^2 b^5}-\frac{33c^3x^{\frac{3}{2}}}{16\left(c\,x^2+b\right)^2 b^4}-\frac{221\sqrt{2}\;c^2\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}} b^5}-\frac{221\sqrt{2}\;c^2\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{b}{c}\right)^{\frac{1}{4}} b^5}-\frac{221\sqrt{2}\;c^2\ln}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -29/16*c^4/b^5/(c*x^2+b)^2*x^(7/2)-33/16*c^3/b^4/(c*x^2+b)^2*x^(3/2)-221/12
8*c^2/b^5/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)
)/(x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-221/64*c^2/b^5/(b/c)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-221/64*c^2/b^5/(b/c)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/9/b^3/x^(9/2)-12*c^2/b^5/x^(1/
2)+6/5*c/b^4/x^(5/2)
```

maxima [A] time = 3.04, size = 254, normalized size = 0.91

$$-\frac{9945\,c^4x^8+17901\,bc^3x^6+7072\,b^2c^2x^4-544\,b^3cx^2+160\,b^4}{720\left(b^5c^2x^{\frac{17}{2}}+2\,b^6cx^{\frac{13}{2}}+b^7x^{\frac{9}{2}}\right)}-\frac{221\,c^3\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(-\right.}{\sqrt{}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/720*(9945*c^4*x^8 + 17901*b*c^3*x^6 + 7072*b^2*c^2*x^4 - 544*b^3*c*x^2 +
 160*b^4)/(b^5*c^2*x^(17/2) + 2*b^6*c*x^(13/2) + b^7*x^(9/2)) - 221/128*c^3
```

```
*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x)
)/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sq
rt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/
4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*
b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^5
```

mupad [B] time = 0.14, size = 121, normalized size = 0.43

$$\frac{221\,(-c)^{9/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{21/4}}-\frac{221\,(-c)^{9/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{21/4}}-\frac{\frac{2}{9\,b}-\frac{34\,c\,x^2}{45\,b^2}+\frac{442\,c^2\,x^4}{45\,b^3}+\frac{1989\,c^3\,x^6}{80\,b^4}+\frac{221\,c^4\,x^8}{16\,b^5}}{b^2\,x^{9/2}+c^2\,x^{17/2}+2\,b\,c\,x^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (221*(-c)^(9/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(21/4)) - (221*(
-c)^(9/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(21/4)) - (2/(9*b) - (3
4*c*x^2)/(45*b^2) + (442*c^2*x^4)/(45*b^3) + (1989*c^3*x^6)/(80*b^4) + (221
*c^4*x^8)/(16*b^5))/(b^2*x^(9/2) + c^2*x^(17/2) + 2*b*c*x^(13/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

3.351 $\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=279

$$\frac{285c^{11/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{23/4}}-\frac{285c^{11/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{23/4}}+\frac{285c^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{32\sqrt{2}\,b^{23/4}}$$

```
[Out] -285/176/b^3/x^(11/2)+285/112*c/b^4/x^(7/2)-95/16*c^2/b^5/x^(3/2)+1/4/b/x^(
11/2)/(c*x^2+b)^2+19/16/b^2/x^(11/2)/(c*x^2+b)+285/64*c^(11/4)*arctan(1-c^(
1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(23/4)*2^(1/2)-285/64*c^(11/4)*arctan(1+c^(
1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(23/4)*2^(1/2)+285/128*c^(11/4)*ln(b^(1/2)+
x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(23/4)*2^(1/2)-285/128*c^(11/4
)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(23/4)*2^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1584, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{95c^2}{16b^5x^{3/2}}+\frac{285c^{11/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{23/4}}-\frac{285c^{11/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{23/4}}+\frac{285c^{11/4}\tan}{32}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(b*x^2 + c*x^4)^3),x]
```

```
[Out] -285/(176*b^3*x^(11/2)) + (285*c)/(112*b^4*x^(7/2)) - (95*c^2)/(16*b^5*x^(3
/2)) + 1/(4*b*x^(11/2)*(b + c*x^2)^2) + 19/(16*b^2*x^(11/2)*(b + c*x^2)) +
(285*c^(11/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^
(23/4)) - (285*c^(11/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*
Sqrt[2]*b^(23/4)) + (285*c^(11/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqr
t[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(23/4)) - (285*c^(11/4)*Log[Sqrt[b] + Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(23/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^3}\,dx=\int \frac{1}{x^{13/2}\left(b+cx^2\right)^3}\,dx$$

$$=\frac{1}{4bx^{11/2}\left(b+cx^2\right)^2}+\frac{19\int \frac{1}{x^{13/2}\left(b+cx^2\right)^2}\,dx}{8b}$$

$$=\frac{1}{4bx^{11/2}\left(b+cx^2\right)^2}+\frac{19}{16b^2x^{11/2}\left(b+cx^2\right)}+\frac{285\int \frac{1}{x^{13/2}\left(b+cx^2\right)}\,dx}{32b^2}$$

$$=-\frac{285}{176b^3x^{11/2}}+\frac{1}{4bx^{11/2}\left(b+cx^2\right)^2}+\frac{19}{16b^2x^{11/2}\left(b+cx^2\right)}-\frac{(285c)\int \frac{1}{x^{9/2}\left(b+cx^2\right)}\,dx}{32b^3}$$

$$=-\frac{285}{176b^3x^{11/2}}+\frac{285c}{112b^4x^{7/2}}+\frac{1}{4bx^{11/2}\left(b+cx^2\right)^2}+\frac{19}{16b^2x^{11/2}\left(b+cx^2\right)}+\frac{\left(285c^2\right)\int \frac{}{x^{5/2}(}}{32b^4}$$

$$=-\frac{285}{176b^3x^{11/2}}+\frac{285c}{112b^4x^{7/2}}-\frac{95c^2}{16b^5x^{3/2}}+\frac{1}{4bx^{11/2}\left(b+cx^2\right)^2}+\frac{19}{16b^2x^{11/2}\left(b+cx^2\right)}-\frac{(28}{}$$

$$=-\frac{285}{176b^3x^{11/2}}+\frac{285c}{112b^4x^{7/2}}-\frac{95c^2}{16b^5x^{3/2}}+\frac{1}{4bx^{11/2}\left(b+cx^2\right)^2}+\frac{19}{16b^2x^{11/2}\left(b+cx^2\right)}-\frac{(28}{}$$

$$=-\frac{285}{176b^3x^{11/2}}+\frac{285c}{112b^4x^{7/2}}-\frac{95c^2}{16b^5x^{3/2}}+\frac{1}{4bx^{11/2}\left(b+cx^2\right)^2}+\frac{19}{16b^2x^{11/2}\left(b+cx^2\right)}-\frac{(28}{}$$

$$=-\frac{285}{176b^3x^{11/2}}+\frac{285c}{112b^4x^{7/2}}-\frac{95c^2}{16b^5x^{3/2}}+\frac{1}{4bx^{11/2}\left(b+cx^2\right)^2}+\frac{19}{16b^2x^{11/2}\left(b+cx^2\right)}-\frac{(28}{}$$

$$=-\frac{285}{176b^3x^{11/2}}+\frac{285c}{112b^4x^{7/2}}-\frac{95c^2}{16b^5x^{3/2}}+\frac{1}{4bx^{11/2}\left(b+cx^2\right)^2}+\frac{19}{16b^2x^{11/2}\left(b+cx^2\right)}+\frac{285}{}$$

$$=-\frac{285}{176b^3x^{11/2}}+\frac{285c}{112b^4x^{7/2}}-\frac{95c^2}{16b^5x^{3/2}}+\frac{1}{4bx^{11/2}\left(b+cx^2\right)^2}+\frac{19}{16b^2x^{11/2}\left(b+cx^2\right)}+\frac{285}{}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.10

$$-\frac{2\,{}_2F_1\left(-\frac{11}{4},3;-\frac{7}{4};-\frac{cx^2}{b}\right)}{11b^3x^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(b*x^2 + c*x^4)^3),x]
```

```
[Out] (-2*Hypergeometric2F1[-11/4, 3, -7/4, -((c*x^2)/b)])/(11*b^3*x^(11/2))
```

fricas [A] time = 0.86, size = 311, normalized size = 1.11

$$-\frac{87780\left(b^5c^2x^{10}+2\,b^6cx^8+b^7x^6\right)\left(-\frac{c^{11}}{b^{23}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^{17}c^3\sqrt{x}\left(-\frac{c^{11}}{b^{23}}\right)^{\frac{3}{4}}-\sqrt{b^{12}\sqrt{-\frac{c^{11}}{b^{23}}}+c^6x}\,b^{17}\left(-\frac{c^{11}}{b^{23}}\right)^{\frac{3}{4}}}{c^{11}}\right)+21945\left(b^5c^2x^{10}+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4928*(87780*(b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6)*(-c^11/b^23)^(1/4)*ar
ctan(-(b^17*c^3*sqrt(x)*(-c^11/b^23)^(3/4) - sqrt(b^12*sqrt(-c^11/b^23) + c
^6*x)*b^17*(-c^11/b^23)^(3/4))/c^11) + 21945*(b^5*c^2*x^10 + 2*b^6*c*x^8 +
b^7*x^6)*(-c^11/b^23)^(1/4)*log(285*b^6*(-c^11/b^23)^(1/4) + 285*c^3*sqrt(x
)) - 21945*(b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6)*(-c^11/b^23)^(1/4)*log(-2
85*b^6*(-c^11/b^23)^(1/4) + 285*c^3*sqrt(x)) + 4*(7315*c^4*x^8 + 11495*b*c^
3*x^6 + 3040*b^2*c^2*x^4 - 608*b^3*c*x^2 + 224*b^4)*sqrt(x))/(b^5*c^2*x^10
+ 2*b^6*c*x^8 + b^7*x^6)
```

giac [A] time = 0.17, size = 243, normalized size = 0.87

$$-\frac{285\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}c^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6}-\frac{285\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}c^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6}-\frac{285\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}c^2\,\mathrm{l}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] -285/64*sqrt(2)*(b*c^3)^(1/4)*c^2*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) +
 2*sqrt(x))/(b/c)^(1/4))/b^6 - 285/64*sqrt(2)*(b*c^3)^(1/4)*c^2*arctan(-1/2
*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^6 - 285/128*sqrt(
2)*(b*c^3)^(1/4)*c^2*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^6 +
 285/128*sqrt(2)*(b*c^3)^(1/4)*c^2*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + s
qrt(b/c))/b^6 - 1/16*(31*c^4*x^(5/2) + 35*b*c^3*sqrt(x))/((c*x^2 + b)^2*b^5
) - 2/77*(154*c^2*x^4 - 33*b*c*x^2 + 7*b^2)/(b^5*x^(11/2))
```

maple [A] time = 0.02, size = 209, normalized size = 0.75

$$-\frac{31c^4x^{\frac{5}{2}}}{16\left(c\,x^2+b\right)^2 b^5}-\frac{35c^3\sqrt{x}}{16\left(c\,x^2+b\right)^2 b^4}-\frac{285\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{64b^6}-\frac{285\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{64b^6}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^3/x^(1/2),x)
```

```
[Out] -31/16*c^4/b^5/(c*x^2+b)^2*x^(5/2)-35/16*c^3/b^4/(c*x^2+b)^2*x^(1/2)-285/12
8*c^3/b^6*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)
)/(x-(b/c)^(1/4)*2^(1/2)*x^(1/2)+(b/c)^(1/2)))-285/64*c^3/b^6*(b/c)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-285/64*c^3/b^6*(b/c)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/11/b^3/x^(11/2)-4*c^2/b^5/x^(3
/2)+6/7*c/b^4/x^(7/2)
```

maxima [A] time = 3.06, size = 257, normalized size = 0.92

$$-\frac{7315\,c^4x^8+11495\,bc^3x^6+3040\,b^2c^2x^4-608\,b^3cx^2+224\,b^4}{1232\left(b^5c^2x^{\frac{19}{2}}+2\,b^6cx^{\frac{15}{2}}+b^7x^{\frac{11}{2}}\right)}-\frac{285\left(\frac{2\sqrt{2}\,c^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,c^3\,\mathrm{arcta}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/1232*(7315*c^4*x^8 + 11495*b*c^3*x^6 + 3040*b^2*c^2*x^4 - 608*b^3*c*x^2
+ 224*b^4)/(b^5*c^2*x^(19/2) + 2*b^6*c*x^(15/2) + b^7*x^(11/2)) - 285/128*(
2*sqrt(2)*c^3*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(
```

```
x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*c^3*
arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt
(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*c^(11/4)*log(sqrt(2
)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2)*c^(11/4)
*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4))/b^5
```

mupad [B] time = 4.39, size = 121, normalized size = 0.43

$$\frac{285\,{\left(-c\right)}^{11/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{23/4}}-\frac{\frac{2}{11\,b}-\frac{38\,c\,x^2}{77\,b^2}+\frac{190\,c^2\,x^4}{77\,b^3}+\frac{1045\,c^3\,x^6}{112\,b^4}+\frac{95\,c^4\,x^8}{16\,b^5}}{b^2\,x^{11/2}+c^2\,x^{19/2}+2\,b\,c\,x^{15/2}}+\frac{285\,{\left(-c\right)}^{11/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{23/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x^2 + c*x^4)^3),x)
```

```
[Out] (285*(-c)^(11/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(23/4)) - (2/(11
*b) - (38*c*x^2)/(77*b^2) + (190*c^2*x^4)/(77*b^3) + (1045*c^3*x^6)/(112*b^
4) + (95*c^4*x^8)/(16*b^5))/(b^2*x^(11/2) + c^2*x^(19/2) + 2*b*c*x^(15/2))
+ (285*(-c)^(11/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(23/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**3/x**(1/2),x)
```

```
[Out] Timed out
```

3.352 $\int x^{7/2}\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=323

$$\frac{14b^{13/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}}-\frac{28b^{13/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right.\right.}{195c^{11/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 28/195*b^3*x^(3/2)*(c*x^2+b)/c^(5/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2
)+4/117*b*x^(5/2)*(c*x^4+b*x^2)^(1/2)/c+2/13*x^(9/2)*(c*x^4+b*x^2)^(1/2)-28
/585*b^2*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c^2-28/195*b^(13/4)*x*(cos(2*arctan(c^
(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ell
ipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/
2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)+14
/195*b^(13/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)
)),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)
/c^(11/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2021, 2024, 2032, 329, 305, 220, 1196}

$$\frac{28b^3x^{3/2}\left(b+cx^2\right)}{195c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{28b^2\sqrt{x}\,\sqrt{bx^2+cx^4}}{585c^2}+\frac{14b^{13/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\right.}{195c^{11/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (28*b^3*x^(3/2)*(b + c*x^2))/(195*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2
+ c*x^4]) - (28*b^2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(585*c^2) + (4*b*x^(5/2)*S
qrt[b*x^2 + c*x^4])/(117*c) + (2*x^(9/2)*Sqrt[b*x^2 + c*x^4])/13 - (28*b^(1
3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elli
pticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(195*c^(11/4)*Sqrt[b*x^2 +
 c*x^4]) + (14*b^(13/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
 Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(195*c^
(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
```

```
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^{7/2}\sqrt{bx^2+cx^4}\,dx = \frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}+\frac{1}{13}(2b)\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{4bx^{5/2}\sqrt{bx^2+cx^4}}{117c}+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}-\frac{\left(14b^2\right)\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{117c}$$

$$= -\frac{28b^2\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2}+\frac{4bx^{5/2}\sqrt{bx^2+cx^4}}{117c}+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}+\frac{\left(14b^3\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{195c^2}$$

$$= -\frac{28b^2\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2}+\frac{4bx^{5/2}\sqrt{bx^2+cx^4}}{117c}+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}+\frac{\left(14b^3x\sqrt{b+cx^2}\right)\int}{195c^2\sqrt{bx^2+}}$$

$$= -\frac{28b^2\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2}+\frac{4bx^{5/2}\sqrt{bx^2+cx^4}}{117c}+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}+\frac{\left(28b^3x\sqrt{b+cx^2}\right)\mathrm{Su}}{195c^2}$$

$$= -\frac{28b^2\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2}+\frac{4bx^{5/2}\sqrt{bx^2+cx^4}}{117c}+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}+\frac{\left(28b^{7/2}x\sqrt{b+cx^2}\right)}{195c^{5/}}$$

$$= \frac{28b^3x^{3/2}\left(b+cx^2\right)}{195c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{28b^2\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2}+\frac{4bx^{5/2}\sqrt{bx^2+cx^4}}{117c}+\frac{2}{13}x^{9/2}$$

Mathematica [C] time = 0.07, size = 102, normalized size = 0.32

$$\frac{2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{\frac{cx^2}{b}+1}\left(-7b^2+2bcx^2+9c^2x^4\right)+7b^2\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{117c^2\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(Sqrt[1 + (c*x^2)/b]*(-7*b^2 + 2*b*c*x^2 +
 9*c^2*x^4) + 7*b^2*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^2)/b)]))/(117*
c^2*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^4+bx^2}\,x^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*x^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4+bx^2}\,x^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(7/2), x)
```

maple [A] time = 0.06, size = 237, normalized size = 0.73

$$\frac{2\sqrt{c\,x^4+b\,x^2}\,\left(45c^4x^8+55b\,c^3x^6-4b^2c^2x^4-14b^3c\,x^2+42\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b^4\,\mathrm{EllipticE}\left(\sqrt{\right.\right.}{585\left(c\,x^2+b\right)c^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 2/585*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)/c^3*(45*c^4*x^8+55*c^3*x^6*b+42
*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-21*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(
1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-4*c^2*x
^4*b^2-14*c*x^2*b^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4+bx^2}\,x^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{7/2}\sqrt{c\,x^4+b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^(7/2)*(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{7}{2}}\sqrt{x^2\left(b+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(7/2)*sqrt(x**2*(b + c*x**2)), x)
```

3.353 $\int x^{5/2}\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=176

$$\frac{10b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}-\frac{20b^2\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}+\frac{4bx^{3/2}\sqrt{bx^2+c}}{77c}$$

```
[Out] 4/77*b*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c+2/11*x^(7/2)*(c*x^4+b*x^2)^(1/2)-20/23
1*b^2*(c*x^4+b*x^2)^(1/2)/c^2/x^(1/2)+10/231*b^(11/4)*x*(cos(2*arctan(c^(1/
4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ellipt
icF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))
*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2021, 2024, 2032, 329, 220}

$$-\frac{20b^2\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{10b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}+\frac{4bx^{3/2}\sqrt{bx^2+}}{77c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-20*b^2*Sqrt[b*x^2 + c*x^4])/(231*c^2*Sqrt[x]) + (4*b*x^(3/2)*Sqrt[b*x^2 +
 c*x^4])/(77*c) + (2*x^(7/2)*Sqrt[b*x^2 + c*x^4])/11 + (10*b^(11/4)*x*(Sqrt
[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcT
an[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(231*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^{5/2}\sqrt{bx^2+cx^4}\,dx = \frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4} + \frac{1}{11}(2b)\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{4bx^{3/2}\sqrt{bx^2+cx^4}}{77c} + \frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4} - \frac{\left(10b^2\right)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{77c}$$

$$= -\frac{20b^2\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} + \frac{4bx^{3/2}\sqrt{bx^2+cx^4}}{77c} + \frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4} + \frac{\left(10b^3\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{231c^2}$$

$$= -\frac{20b^2\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} + \frac{4bx^{3/2}\sqrt{bx^2+cx^4}}{77c} + \frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4} + \frac{\left(10b^3x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{\cdots}}}{231c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{20b^2\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} + \frac{4bx^{3/2}\sqrt{bx^2+cx^4}}{77c} + \frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4} + \frac{\left(20b^3x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\cdots\right.}{231c^2\sqrt{bx^2\cdots}}$$

$$= -\frac{20b^2\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} + \frac{4bx^{3/2}\sqrt{bx^2+cx^4}}{77c} + \frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4} + \frac{10b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\cdots}}{231c^{9/\cdots}}$$

Mathematica [C] time = 0.06, size = 102, normalized size = 0.58

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{\frac{cx^2}{b}+1}\left(-5b^2+2bcx^2+7c^2x^4\right)+5b^2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{77c^2\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(Sqrt[1 + (c*x^2)/b]*(-5*b^2 + 2*b*c*x^2 + 7*c^2*x
^4) + 5*b^2*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)]))/(77*c^2*Sqrt[
x]*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{cx^4+bx^2}\,x^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*x^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4 + bx^2}\, x^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(5/2), x)
```

maple [A] time = 0.03, size = 157, normalized size = 0.89

$$\frac{2\sqrt{c\,x^4 + b\,x^2}\,\left(21c^4x^7 + 27b\,c^3x^5 - 4b^2c^2x^3 - 10b^3cx + 5\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b^3\,\mathrm{Elliptic}}{231\left(c\,x^2 + b\right)c^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2)^(1/2),x)
[Out] 2/231*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(21*x^7*c^4+5*b^3*(-b*c)^(1/2)*
((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2),1/2*2^(1/2))+27*x^5*b*c^3-4*x^3*b^2*c^2-10*x*b^3*c)/c^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4 + bx^2}\, x^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,\sqrt{c\,x^4 + b\,x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^(5/2)*(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}}\sqrt{x^2\left(b+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(5/2)*sqrt(x**2*(b + c*x**2)), x)
```

3.354 $\int x^{3/2}\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=293

$$-\frac{2b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}+\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -4/15*b^2*x^(3/2)*(c*x^2+b)/c^(3/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)
+2/9*x^(5/2)*(c*x^4+b*x^2)^(1/2)+4/45*b*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c+4/15*
b^(9/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(
1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2
*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/
4)/(c*x^4+b*x^2)^(1/2)-2/15*b^(9/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)
))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c
^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/
2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2021, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{4b^2x^{3/2}\left(b+cx^2\right)}{15c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}+\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)}{1}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-4*b^2*x^(3/2)*(b + c*x^2))/(15*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) + (4*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(45*c) + (2*x^(5/2)*Sqrt[b*x^2
 + c*x^4])/9 + (4*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
 + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c
^(7/4)*Sqrt[b*x^2 + c*x^4]) - (2*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4
)], 1/2])/(15*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^{3/2}\sqrt{bx^2+cx^4}\,dx = \frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}+\frac{1}{9}(2b)\int\frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{4b\sqrt{x}\sqrt{bx^2+cx^4}}{45c}+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}-\frac{\left(2b^2\right)\int\frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{15c}$$

$$= \frac{4b\sqrt{x}\sqrt{bx^2+cx^4}}{45c}+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}-\frac{\left(2b^2x\sqrt{b+cx^2}\right)\int\frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{15c\sqrt{bx^2+cx^4}}$$

$$= \frac{4b\sqrt{x}\sqrt{bx^2+cx^4}}{45c}+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}-\frac{\left(4b^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{15c\sqrt{bx^2+cx^4}}$$

$$= \frac{4b\sqrt{x}\sqrt{bx^2+cx^4}}{45c}+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}-\frac{\left(4b^{5/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{15c^{3/2}\sqrt{bx^2+cx^4}}$$

$$= -\frac{4b^2x^{3/2}\left(b+cx^2\right)}{15c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{4b\sqrt{x}\sqrt{bx^2+cx^4}}{45c}+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}+\frac{4b^{9/4}x\left(\sqrt{\right.}{}$$

Mathematica [C] time = 0.04, size = 86, normalized size = 0.29

$$\frac{2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\left(\left(b+cx^2\right)\sqrt{\frac{cx^2}{b}+1}-b\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{9c\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[b*x^2 + c*x^4],x]
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)*Sqrt[1 + (c*x^2)/b] - b*Hyper
geometric2F1[-1/2, 3/4, 7/4, -((c*x^2)/b)]))/(9*c*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^4+bx^2}\,x^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*x^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4 + bx^2}\, x^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(3/2), x)
```

maple [A] time = 0.03, size = 226, normalized size = 0.77

$$-\frac{2\sqrt{c\,x^4+b\,x^2}\,\left(-5c^3x^6-7b\,c^2x^4-2b^2c\,x^2+6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b^3\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right.\right.}{45\left(c\,x^2+b\right)c^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -2/45*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)/c^2*(-5*c^3*x^6+6*b^3*((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2),1/2*2^(1/2))-3*b^3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*
((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*Ellipt
icF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-7*b*c^2*x^4-2*b^2*
c*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4 + bx^2}\, x^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2}\,\sqrt{c\,x^4+b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^(3/2)*(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\sqrt{x^2\left(b+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(3/2)*sqrt(x**2*(b + c*x**2)), x)
```

3.355 $\int \sqrt{x}\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=146

$$-\frac{2b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{5/4}\sqrt{bx^2+cx^4}}+\frac{4b\sqrt{bx^2+cx^4}}{21c\sqrt{x}}+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}$$

```
[Out] 2/7*x^(3/2)*(c*x^4+b*x^2)^(1/2)+4/21*b*(c*x^4+b*x^2)^(1/2)/c/x^(1/2)-2/21*b
^(7/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1
/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*
2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(5/4
)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2021, 2024, 2032, 329, 220}

$$-\frac{2b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{5/4}\sqrt{bx^2+cx^4}}+\frac{4b\sqrt{bx^2+cx^4}}{21c\sqrt{x}}+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (4*b*Sqrt[b*x^2 + c*x^4])/(21*c*Sqrt[x]) + (2*x^(3/2)*Sqrt[b*x^2 + c*x^4])/
7 - (2*b^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*
x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(21*c^(5/4)*Sqrt
[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \sqrt{x}\sqrt{bx^2+cx^4}\,dx = \frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4} + \frac{1}{7}(2b)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{4b\sqrt{bx^2+cx^4}}{21c\sqrt{x}} + \frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4} - \frac{(2b^2)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{21c}$$
$$= \frac{4b\sqrt{bx^2+cx^4}}{21c\sqrt{x}} + \frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4} - \frac{\left(2b^2x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{21c\sqrt{bx^2+cx^4}}$$
$$= \frac{4b\sqrt{bx^2+cx^4}}{21c\sqrt{x}} + \frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4} - \frac{\left(4b^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{21c\sqrt{bx^2+cx^4}}$$
$$= \frac{4b\sqrt{bx^2+cx^4}}{21c\sqrt{x}} + \frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4} - \frac{2b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\right.}{21c^{5/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.03, size = 86, normalized size = 0.59

$$\frac{2\sqrt{x^2(b+cx^2)}\left((b+cx^2)\sqrt{\frac{cx^2}{b}+1} - b\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{7c\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)*Sqrt[1 + (c*x^2)/b] - b*Hypergeometri
c2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)]))/(7*c*Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^4+bx^2}\sqrt{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4+bx^2}\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(c*x^4 + b*x^2)*sqrt(x), x)

maple [A] time = 0.03, size = 145, normalized size = 0.99

$$-\frac{2\sqrt{c\,x^4+b\,x^2}\left(-3c^3x^5-5b\,c^2x^3-2b^2cx+\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b^2\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.\right.}{21\left(c\,x^2+b\right)c^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)*(c*x^4+b*x^2)^(1/2),x)

[Out] -2/21*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(b^2*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-3*c^3*x^5-5*b*c^2*x^3-2*b^2*c*x)/c^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{cx^4+bx^2}\,\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^4 + b*x^2)*sqrt(x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{x}\,\sqrt{c\,x^4+b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)*(b*x^2 + c*x^4)^(1/2),x)

[Out] int(x^(1/2)*(b*x^2 + c*x^4)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x}\,\sqrt{x^2\left(b+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(sqrt(x)*sqrt(x**2*(b + c*x**2)), x)
```

3.356 $\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$

Optimal. Leaf size=263

$$\frac{2b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}}-\frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 4/5*b*x^(3/2)*(c*x^2+b)/c^(1/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+2/5
*x^(1/2)*(c*x^4+b*x^2)^(1/2)-4/5*b^(5/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(
b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2)^(1/2)+2/5*b^(5/4)*x*(cos(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1
/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2
)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2021, 2032, 329, 305, 220, 1196}

$$\frac{2b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}}-\frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/Sqrt[x],x]
```

```
[Out] (4*b*x^(3/2)*(b + c*x^2))/(5*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x
^4]) + (2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/5 - (4*b^(5/4)*x*(Sqrt[b] + Sqrt[c]*
x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sq
rt[x])/b^(1/4)], 1/2])/(5*c^(3/4)*Sqrt[b*x^2 + c*x^4]) + (2*b^(5/4)*x*(Sqrt
[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcT
an[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx = \frac{2}{5}\sqrt{x}\,\sqrt{bx^2+cx^4} + \frac{1}{5}(2b)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{2}{5}\sqrt{x}\,\sqrt{bx^2+cx^4} + \frac{\left(2bx\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5\sqrt{bx^2+cx^4}}$$

$$= \frac{2}{5}\sqrt{x}\,\sqrt{bx^2+cx^4} + \frac{\left(4bx\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{bx^2+cx^4}}$$

$$= \frac{2}{5}\sqrt{x}\,\sqrt{bx^2+cx^4} + \frac{\left(4b^{3/2}x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{c}\,\sqrt{bx^2+cx^4}} - \frac{\left(4b^{3/2}x\sqrt{b+cx^2}\right)\mathrm{Sub}\cdots}{5\sqrt{c}\,\sqrt{\cdots}}$$

$$= \frac{4bx^{3/2}\left(b+cx^2\right)}{5\sqrt{c}\,\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{2}{5}\sqrt{x}\,\sqrt{bx^2+cx^4} - \frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\cdots\right.}{5c^{3/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.22

$$\frac{2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)}{3\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^2
)/b)])/(3*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}}{\sqrt{x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)/sqrt(x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/sqrt(x), x)
```

maple [A] time = 0.03, size = 213, normalized size = 0.81

$$\frac{2\sqrt{c\,x^4 + b\,x^2}\,\left(c^2x^4 + bc\,x^2 + 2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b^2\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}, \frac{\sqrt{2}}{2}\right) - \sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.}{5\left(c\,x^2 + b\right)c\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^(1/2),x)
```

```
[Out] 2/5*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)/c*(2*b^2*((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)
^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(
1/2))-b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+c^2*x^4+b*c*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/sqrt(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4 + b\,x^2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^(1/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(1/2)/x^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(1/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/sqrt(x), x)
```

3.357 $\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}} dx$

Optimal. Leaf size=118

$$\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}$$

```
[Out] 2/3*(c*x^4+b*x^2)^(1/2)/x^(1/2)+2/3*b^(3/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)
/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*
arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b
)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(1/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2021, 2032, 329, 220}

$$\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^(3/2),x]
```

```
[Out] (2*Sqrt[b*x^2 + c*x^4])/(3*Sqrt[x]) + (2*b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x]
)/b^(1/4)], 1/2])/(3*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx &= \frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}+\frac{1}{3}(2b)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx\\
&= \frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}+\frac{\left(2bx\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{3\sqrt{bx^2+cx^4}}\\
&= \frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}+\frac{\left(4bx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{3\sqrt{bx^2+cx^4}}\\
&= \frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}+\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{bx^2+cx^4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.47

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)}{\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)])/(
Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}}{x^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)/x^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(3/2), x)
```

maple [A] time = 0.03, size = 130, normalized size = 1.10

$$\frac{2\sqrt{c\,x^4+b\,x^2}\,\left(c^2x^3+bcx+\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\right)}{3\left(c\,x^2+b\right)c\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^(3/2),x)
```

```
[Out] 2/3*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(b*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1
/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),
1/2*2^(1/2))+c^2*x^3+b*c*x)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^(3/2),x)
[Out] int((b*x^2 + c*x^4)^(1/2)/x^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(3/2),x)
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**(3/2), x)
```

3.358 $\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx$

Optimal. Leaf size=254

$$\frac{2\sqrt[4]{b}\,\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{bx^2+cx^4}}-\frac{4\sqrt[4]{b}\,\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)\right.}{\sqrt{bx^2+cx^4}}$$

```
[Out] 4*x^(3/2)*(c*x^2+b)*c^(1/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2*(c*x^
4+b*x^2)^(1/2)/x^(3/2)-4*b^(1/4)*c^(1/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(
b^(1/2)+x*c^(1/2))^2)^(1/2)/(c*x^4+b*x^2)^(1/2)+2*b^(1/4)*c^(1/4)*x*(cos(2*
arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1
/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2
)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2020, 2032, 329, 305, 220, 1196}

$$\frac{4\sqrt{c}\,x^{3/2}\left(b+cx^2\right)}{\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}+\frac{2\sqrt[4]{b}\,\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{bx^2+cx^4}}-\frac{4\sqrt[4]{b}\,\sqrt[4]{c}\,x\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^(5/2),x]
```

```
[Out] (4*Sqrt[c]*x^(3/2)*(b + c*x^2))/((Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4])
 - (2*Sqrt[b*x^2 + c*x^4])/x^(3/2) - (4*b^(1/4)*c^(1/4)*x*(Sqrt[b] + Sqrt[c
]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*
Sqrt[x])/b^(1/4)], 1/2])/Sqrt[b*x^2 + c*x^4] + (2*b^(1/4)*c^(1/4)*x*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/Sqrt[b*x^2 + c*x^4]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}} + (2c)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}} + \frac{\left(2cx\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}} + \frac{\left(4cx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}} + \frac{\left(4\sqrt{b}\,\sqrt{c}\,x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{bx^2+cx^4}} - \frac{\left(4\sqrt{b}\,\sqrt{c}\,x\sqrt{b+cx^2}\right)\text{S}}{\sqrt{b}}$$

$$= \frac{4\sqrt{c}\,x^{3/2}\left(b+cx^2\right)}{\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{x^{3/2}} - \frac{4\sqrt[4]{b}\,\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\right.\right.}{\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.22

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{cx^2}{b}\right)}{x^{3/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^(5/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-1/2, -1/4, 3/4, -((c*x^2)/b)])
/(x^(3/2)*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}}{x^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)/x^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(5/2), x)
```

maple [A] time = 0.03, size = 202, normalized size = 0.80

$$\frac{2\sqrt{c\,x^4+b\,x^2}\left(-c\,x^2+2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\cdots}\right.}{\left(c\,x^2+b\right)x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^(5/2),x)
```

```
[Out] 2*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c
*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b-
((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2),1/2*2^(1/2))*b-c*x^2-b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^(5/2),x)
[Out] int((b*x^2 + c*x^4)^(1/2)/x^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 \left(b + cx^2\right)}}{x^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(5/2),x)
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**(5/2), x)
```

3.359 $\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx$

Optimal. Leaf size=118

$$\frac{2c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\,\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}$$

```
[Out] -2/3*(c*x^4+b*x^2)^(1/2)/x^(5/2)+2/3*c^(3/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2
*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+
b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(1/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2020, 2032, 329, 220}

$$\frac{2c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\,\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^(7/2),x]
```

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(3*x^(5/2)) + (2*c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx &= -\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}+\frac{1}{3}(2c)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx\\ &= -\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}+\frac{\left(2cx\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{3\sqrt{bx^2+cx^4}}\\ &= -\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}+\frac{\left(4cx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{3\sqrt{bx^2+cx^4}}\\ &= -\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}+\frac{2c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{bx^2+cx^4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.48

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{cx^2}{b}\right)}{3x^{5/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^(7/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/4, -1/2, 1/4, -((c*x^2)/b)])
/(3*x^(5/2)*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}}{x^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^4 + b*x^2)/x^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{cx^4+bx^2}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(7/2), x)
```

maple [A] time = 0.03, size = 125, normalized size = 1.06

$$\frac{2\sqrt{c\,x^4+b\,x^2}\,\left(-c\,x^2+\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,x\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-b\right)}{3\left(c\,x^2+b\right)x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^(7/2),x)
[Out] 2/3*(c*x^4+b*x^2)^(1/2)/x^(5/2)/(c*x^2+b)*((-b*c)^(1/2)*((c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(
-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/
2*2^(1/2))*x-c*x^2-b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{cx^4+bx^2}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^(7/2),x)
[Out] int((b*x^2 + c*x^4)^(1/2)/x^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(7/2),x)
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**(7/2), x)
```

3.360 $\int \frac{\sqrt{bx^2+cx^4}}{x^{9/2}} \, dx$

Optimal. Leaf size=293

$$\frac{2c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}-\frac{4c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 4/5*c^(3/2)*x^(3/2)*(c*x^2+b)/b/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2/5
*(c*x^4+b*x^2)^(1/2)/x^(7/2)-4/5*c*(c*x^4+b*x^2)^(1/2)/b/x^(3/2)-4/5*c^(5/4
)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x
^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/
2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/(c*
x^4+b*x^2)^(1/2)+2/5*c^(5/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(
1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*
x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^
(1/2))^2)^(1/2)/b^(3/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2020, 2025, 2032, 329, 305, 220, 1196}

$$\frac{2c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}-\frac{4c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^(9/2),x]
```

```
[Out] (4*c^(3/2)*x^(3/2)*(b + c*x^2))/(5*b*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x
^4]) - (2*Sqrt[b*x^2 + c*x^4])/(5*x^(7/2)) - (4*c*Sqrt[b*x^2 + c*x^4])/(5*b
*x^(3/2)) - (4*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(3/
4)*Sqrt[b*x^2 + c*x^4]) + (2*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^
2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
1/2])/(5*b^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
```

```
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}} + \frac{1}{5}(2c)\int \frac{1}{\sqrt{x}\,\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}} - \frac{4c\sqrt{bx^2+cx^4}}{5bx^{3/2}} + \frac{\left(2c^2\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{5b}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}} - \frac{4c\sqrt{bx^2+cx^4}}{5bx^{3/2}} + \frac{\left(2c^2x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}} - \frac{4c\sqrt{bx^2+cx^4}}{5bx^{3/2}} + \frac{\left(4c^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}} - \frac{4c\sqrt{bx^2+cx^4}}{5bx^{3/2}} + \frac{\left(4c^{3/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{b}\,\sqrt{bx^2+cx^4}} - \frac{\left(4c^{3/2}x\right.}{}$$

$$= \frac{4c^{3/2}x^{3/2}\left(b+cx^2\right)}{5b\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}} - \frac{4c\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{4c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b}{(\sqrt{b}}}}{5b^{3/4}\sqrt{}}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.19

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\frac{cx^2}{b}\right)}{5x^{7/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^(9/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((c*x^2)/b)]
)/(5*x^(7/2)*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}}{x^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)/x^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(9/2), x)
```

maple [A] time = 0.04, size = 224, normalized size = 0.76

$$\frac{2\sqrt{c\,x^4+b\,x^2}\,\left(-2c^2x^4+2\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,bc\,x^2\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{\frac{cx \cdots}{\cdots}}\cdots\right.}{5\left(c\,x^2+b\right)b\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^(9/2),x)
```

```
[Out] 2/5*(c*x^4+b*x^2)^(1/2)/x^(7/2)/(c*x^2+b)*(2*(-1/(-b*c)^(1/2)*c*x)^(1/2)*El
lipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*
x^2*b*c-(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-
c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*x^2*b*c-2*c^2*x^4-3*b*c*x^2-b^2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^(9/2),x)
[Out] int((b*x^2 + c*x^4)^(1/2)/x^(9/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(9/2),x)
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**(9/2), x)
```

3.361 $\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{2c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{21bx^{5/2}}-\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}$$

```
[Out] -2/7*(c*x^4+b*x^2)^(1/2)/x^(9/2)-4/21*c*(c*x^4+b*x^2)^(1/2)/b/x^(5/2)-2/21*
c^(7/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(
1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2
*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(5/
4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2020, 2025, 2032, 329, 220}

$$-\frac{2c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{21bx^{5/2}}-\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^(11/2),x]
```

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(7*x^(9/2)) - (4*c*Sqrt[b*x^2 + c*x^4])/(21*b*x^(5
/2)) - (2*c^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[
c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(21*b^(5/4)*S
qrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx &= -\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}+\frac{1}{7}(2c)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx\\
&= -\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}-\frac{4c\sqrt{bx^2+cx^4}}{21bx^{5/2}}-\frac{\left(2c^2\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{21b}\\
&= -\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}-\frac{4c\sqrt{bx^2+cx^4}}{21bx^{5/2}}-\frac{\left(2c^2x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{21b\sqrt{bx^2+cx^4}}\\
&= -\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}-\frac{4c\sqrt{bx^2+cx^4}}{21bx^{5/2}}-\frac{\left(4c^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{21b\sqrt{bx^2+cx^4}}\\
&= -\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}-\frac{4c\sqrt{bx^2+cx^4}}{21bx^{5/2}}-\frac{2c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.39

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{2};-\frac{3}{4};-\frac{cx^2}{b}\right)}{7x^{9/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^(11/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-7/4, -1/2, -3/4, -((c*x^2)/b)]
)/(7*x^(9/2)*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}}{x^{\frac{11}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)/x^(11/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(11/2), x)
```

maple [A] time = 0.03, size = 142, normalized size = 0.97

$$-\frac{2\sqrt{c\,x^4+b\,x^2}\,\left(2c^2x^4+\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,c\,x^3\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)+5bc\,x^2+3}{21\left(c\,x^2+b\right)b\,x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^(11/2),x)
```

```
[Out] -2/21*(c*x^4+b*x^2)^(1/2)/x^(9/2)/(c*x^2+b)*(((-c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2)*2^(1/2)*x^3*c+2*c^2*x^4+5*b*c*x^2+3*b^2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(11/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^(11/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(1/2)/x^(11/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(11/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**(11/2), x)
```

3.362 $\int \frac{\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx$

Optimal. Leaf size=323

$$-\frac{2c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}}+\frac{4c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -4/15*c^(5/2)*x^(3/2)*(c*x^2+b)/b^2/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)
-2/9*(c*x^4+b*x^2)^(1/2)/x^(11/2)-4/45*c*(c*x^4+b*x^2)^(1/2)/b/x^(7/2)+4/15
*c^2*(c*x^4+b*x^2)^(1/2)/b^2/x^(3/2)+4/15*c^(9/4)*x*(cos(2*arctan(c^(1/4)*x
^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(
sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c
*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(7/4)/(c*x^4+b*x^2)^(1/2)-2/15*c^(9/
4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*
x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1
/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(7/4)/(c
*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2020, 2025, 2032, 329, 305, 220, 1196}

$$-\frac{4c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{3/2}}-\frac{2c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}}+\frac{4c^{9/4}x}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^(13/2),x]
```

```
[Out] (-4*c^(5/2)*x^(3/2)*(b + c*x^2))/(15*b^2*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/(9*x^(11/2)) - (4*c*Sqrt[b*x^2 + c*x^4])
/(45*b*x^(7/2)) + (4*c^2*Sqrt[b*x^2 + c*x^4])/(15*b^2*x^(3/2)) + (4*c^(9/4)
*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
E[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[b*x^2 + c*x^4
]) - (2*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]
*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(7/4)*Sqr
t[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
```

```
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}} + \frac{1}{9}(2c)\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}} - \frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}} - \frac{\left(2c^2\right)\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{15b}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}} - \frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}} + \frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{\left(2c^3\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{15b^2}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}} - \frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}} + \frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{\left(2c^3x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{15b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}} - \frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}} + \frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{\left(4c^3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x\right.}{15b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}} - \frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}} + \frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{\left(4c^{5/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,\right.}{15b^{3/2}\sqrt{bx^2+cx^4}}$$

$$= -\frac{4c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}} - \frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}} + \frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} + \frac{4c^{9/4}}{}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.18

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{9}{4},-\frac{1}{2};-\frac{5}{4};-\frac{cx^2}{b}\right)}{9x^{11/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^(13/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-9/4, -1/2, -5/4, -((c*x^2)/b)]
)/(9*x^(11/2)*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}}{x^{\frac{13}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)/x^(13/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(13/2), x)
```

maple [A] time = 0.04, size = 239, normalized size = 0.74

$$-\frac{2\sqrt{c\,x^4+b\,x^2}\,\left(-6c^3x^6+6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\;b\,c^2x^4\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-3\sqrt{\frac{cx+\sqrt{-b}}{\sqrt{-bc}}}\right.}{45\left(c\,x^2+b\right)b^2x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^(13/2),x)
```

```
[Out] -2/45*(c*x^4+b*x^2)^(1/2)/x^(11/2)/(c*x^2+b)*(6*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1
/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*x^4*b*c^2-3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b*c^2-6*c^3*x^6-4*b*c^2*x^
4+7*b^2*c*x^2+5*b^3)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(13/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^(13/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(1/2)/x^(13/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(13/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**(13/2), x)
```

3.363 $\int \frac{\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx$

Optimal. Leaf size=176

$$\frac{10c^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}+\frac{20c^2\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}-\frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}}-\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}$$

```
[Out] -2/11*(c*x^4+b*x^2)^(1/2)/x^(13/2)-4/77*c*(c*x^4+b*x^2)^(1/2)/b/x^(9/2)+20/
231*c^2*(c*x^4+b*x^2)^(1/2)/b^2/x^(5/2)+10/231*c^(11/4)*x*(cos(2*arctan(c^(
1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Elli
pticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2
))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2020, 2025, 2032, 329, 220}

$$\frac{20c^2\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}+\frac{10c^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}}-\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^(15/2),x]
```

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(11*x^(13/2)) - (4*c*Sqrt[b*x^2 + c*x^4])/(77*b*x^
(9/2)) + (20*c^2*Sqrt[b*x^2 + c*x^4])/(231*b^2*x^(5/2)) + (10*c^(11/4)*x*(S
qrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*A
rcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(231*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}} + \frac{1}{11}(2c)\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}} - \frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}} - \frac{\left(10c^2\right)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{77b}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}} - \frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}} + \frac{20c^2\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} + \frac{\left(10c^3\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{231b^2}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}} - \frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}} + \frac{20c^2\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} + \frac{\left(10c^3x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{231b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}} - \frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}} + \frac{20c^2\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} + \frac{\left(20c^3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,d\right.}{231b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}} - \frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}} + \frac{20c^2\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} + \frac{10c^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F}{231b^{9/4}\sqrt{bx^2+c}}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.32

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{11}{4},-\frac{1}{2};-\frac{7}{4};-\frac{cx^2}{b}\right)}{11x^{13/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^(15/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-11/4, -1/2, -7/4, -((c*x^2)/b)
])/(11*x^(13/2)*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}}{x^{\frac{15}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)/x^(15/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(15/2), x)
```

maple [A] time = 0.04, size = 156, normalized size = 0.89

$$\frac{2\sqrt{c\,x^4+b\,x^2}\,\left(10c^3x^6+5\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{-bc}\,c^2x^5\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)+4b\,c^2x^4-}{231\left(c\,x^2+b\right)b^2x^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^(15/2),x)
```

```
[Out] 2/231*(c*x^4+b*x^2)^(1/2)/x^(13/2)/(c*x^2+b)*(5*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1
/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*(-b*c)^(1/2)*x^5*c^2+10*c^3*x^6+4*b*c^2*x^4-27*b^2*c*x^2-21*b^3)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(15/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^{15/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^(15/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(1/2)/x^(15/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(15/2),x)
```

```
[Out] Timed out
```

3.364 $\int x^{3/2}\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=350

$$\frac{28b^{17/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105c^{11/4}\sqrt{bx^2+cx^4}}-\frac{56b^{17/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)\right.}{1105c^{11/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/17*x^(5/2)*(c*x^4+b*x^2)^(3/2)+56/1105*b^4*x^(3/2)*(c*x^2+b)/c^(5/2)/(b^(
1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+8/663*b^2*x^(5/2)*(c*x^4+b*x^2)^(1/2)/c
+12/221*b*x^(9/2)*(c*x^4+b*x^2)^(1/2)-56/3315*b^3*x^(1/2)*(c*x^4+b*x^2)^(1/
2)/c^2-56/1105*b^(17/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/
cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2)
)^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)+28/1105*b^(17/4)*x*(cos(2*arctan(c^
(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ell
ipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/
2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2021, 2024, 2032, 329, 305, 220, 1196}

$$\frac{56b^4x^{3/2}\left(b+cx^2\right)}{1105c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{56b^3\sqrt{x}\,\sqrt{bx^2+cx^4}}{3315c^2}+\frac{28b^{17/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105c^{11/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (56*b^4*x^(3/2)*(b + c*x^2))/(1105*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2
 + c*x^4]) - (56*b^3*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(3315*c^2) + (8*b^2*x^(5/
2)*Sqrt[b*x^2 + c*x^4])/(663*c) + (12*b*x^(9/2)*Sqrt[b*x^2 + c*x^4])/221 +
(2*x^(5/2)*(b*x^2 + c*x^4)^(3/2))/17 - (56*b^(17/4)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(1105*c^(11/4)*Sqrt[b*x^2 + c*x^4]) + (28*b^(17/4)*x*(
Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*
ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(1105*c^(11/4)*Sqrt[b*x^2 + c*x^4]
)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int x^{3/2}\left(bx^2+cx^4\right)^{3/2}\,dx &= \frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}+\frac{1}{17}(6b)\int x^{7/2}\sqrt{bx^2+cx^4}\,dx\\
&= \frac{12}{221}bx^{9/2}\sqrt{bx^2+cx^4}+\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}+\frac{1}{221}\left(12b^2\right)\int\frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx\\
&= \frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}}{663c}+\frac{12}{221}bx^{9/2}\sqrt{bx^2+cx^4}+\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}-\frac{\left(28b^3\right)\int\frac{x^{7/2}}{\sqrt{bx^2+cx}}}{663c}\\
&= -\frac{56b^3\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2}+\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}}{663c}+\frac{12}{221}bx^{9/2}\sqrt{bx^2+cx^4}+\frac{2}{17}x^{5/2}\left(bx^2+c\right.\\
&= -\frac{56b^3\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2}+\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}}{663c}+\frac{12}{221}bx^{9/2}\sqrt{bx^2+cx^4}+\frac{2}{17}x^{5/2}\left(bx^2+c\right.\\
&= -\frac{56b^3\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2}+\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}}{663c}+\frac{12}{221}bx^{9/2}\sqrt{bx^2+cx^4}+\frac{2}{17}x^{5/2}\left(bx^2+c\right.\\
&= -\frac{56b^3\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2}+\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}}{663c}+\frac{12}{221}bx^{9/2}\sqrt{bx^2+cx^4}+\frac{2}{17}x^{5/2}\left(bx^2+c\right.\\
&= \frac{56b^4x^{3/2}\left(b+cx^2\right)}{1105c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{56b^3\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2}+\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}}{663c}+\frac{12}{221}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 101, normalized size = 0.29

$$\frac{2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\left(7b^3\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)-\left(7b-13cx^2\right)\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}\right)}{221c^2\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(-((7*b - 13*c*x^2)*(b + c*x^2)^2*Sqrt[1 +
 (c*x^2)/b]) + 7*b^3*Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*x^2)/b)]))/(221
*c^2*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^5 + bx^3\right)\sqrt{cx^4 + bx^2}\,\sqrt{x}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^5 + b*x^3)*sqrt(c*x^4 + b*x^2)*sqrt(x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2\right)^{\frac{3}{2}} x^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*x^(3/2), x)
```

maple [A] time = 0.03, size = 248, normalized size = 0.71

$$\frac{2\left(c\,x^4 + b\,x^2\right)^{\frac{3}{2}}\left(195c^5x^{10} + 480b\,c^4x^8 + 305b^2c^3x^6 - 8b^3c^2x^4 - 28b^4c\,x^2 + 84\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{c}{\sqrt{-}}}\right.}{3315\left(c\,x^2 + b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 2/3315*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c^3*(195*x^10*c^5+480*x^8*b*
c^4+305*x^6*b^2*c^3+84*b^5*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*
((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*Ellipt
icE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-42*b^5*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2),1/2*2^(1/2))-8*x^4*b^3*c^2-28*x^2*b^4*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2\right)^{\frac{3}{2}} x^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*x^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2} \left(c\, x^4 + b\, x^2\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^(3/2)*(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**(3/2)*(x**2*(b + c*x**2))**(3/2), x)
```

3.365 $\int \sqrt{x}\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=203

$$\frac{4b^{15/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}-\frac{8b^3\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2+}$$

```
[Out] 2/15*x^(3/2)*(c*x^4+b*x^2)^(3/2)+8/385*b^2*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c+4/
55*b*x^(7/2)*(c*x^4+b*x^2)^(1/2)-8/231*b^3*(c*x^4+b*x^2)^(1/2)/c^2/x^(1/2)+
4/231*b^(15/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2
)/c^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2021, 2024, 2032, 329, 220}

$$-\frac{8b^3\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{4b^{15/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}+\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-8*b^3*Sqrt[b*x^2 + c*x^4])/(231*c^2*Sqrt[x]) + (8*b^2*x^(3/2)*Sqrt[b*x^2
+ c*x^4])/(385*c) + (4*b*x^(7/2)*Sqrt[b*x^2 + c*x^4])/55 + (2*x^(3/2)*(b*x^
2 + c*x^4)^(3/2))/15 + (4*b^(15/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)
/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/
2])/(231*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \sqrt{x}\left(bx^2+cx^4\right)^{3/2}dx = \frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}+\frac{1}{5}(2b)\int x^{5/2}\sqrt{bx^2+cx^4}\,dx$$

$$= \frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}+\frac{1}{55}\left(4b^2\right)\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}dx$$

$$= \frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}-\frac{\left(4b^3\right)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}}{77c}$$

$$= -\frac{8b^3\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/}$$

$$= -\frac{8b^3\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/}$$

$$= -\frac{8b^3\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/}$$

$$= -\frac{8b^3\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/}$$

Mathematica [C] time = 0.06, size = 101, normalized size = 0.50

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(5b^3\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)-\left(5b-11cx^2\right)\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}\right)}{165c^2\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-((5*b - 11*c*x^2)*(b + c*x^2)^2*Sqrt[1 + (c*x^2)
/b]) + 5*b^3*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]))/(165*c^2*Sqr
t[x]*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^4+bx^2\right)^{\frac{3}{2}}\sqrt{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="fricas")

[Out] integral((c*x^4 + b*x^2)^(3/2)*sqrt(x), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2\right)^{\frac{3}{2}} \sqrt{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="giac")

[Out] integrate((c*x^4 + b*x^2)^(3/2)*sqrt(x), x)

maple [A] time = 0.03, size = 168, normalized size = 0.83

$$\frac{2\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(77c^5x^9+196b\,c^4x^7+131b^2c^3x^5-8b^3c^2x^3-20b^4cx+10\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{}{\sqrt{}}}\right.}{1155\left(c\,x^2+b\right)^2c^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2)^(3/2)*x^(1/2),x)

[Out] 2/1155*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(77*x^9*c^5+196*x^7*b*c^4+10*b^4*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+131*x^5*b^2*c^3-8*x^3*b^3*c^2-20*x*b^4*c)/c^3

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2\right)^{\frac{3}{2}} \sqrt{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^2)^(3/2)*sqrt(x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{x}\,\left(c\,x^4+b\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^(1/2)*(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*(x**2*(b + c*x**2))**(3/2), x)
```

3.366 $\int \frac{(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=320

$$-\frac{4b^{13/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65c^{7/4}\sqrt{bx^2+cx^4}}+\frac{8b^{13/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\right.}{65c^{7/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/13*(c*x^4+b*x^2)^(3/2)*x^(1/2)-8/65*b^3*x^(3/2)*(c*x^2+b)/c^(3/2)/(b^(1/2
)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+4/39*b*x^(5/2)*(c*x^4+b*x^2)^(1/2)+8/195*b
^2*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c+8/65*b^(13/4)*x*(cos(2*arctan(c^(1/4)*x^(1
/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin
(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^
2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)-4/65*b^(13/4)
*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^
(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2
))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x
^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2021, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{8b^3x^{3/2}\left(b+cx^2\right)}{65c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{4b^{13/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65c^{7/4}\sqrt{bx^2+cx^4}}+\frac{8b^{13/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/Sqrt[x],x]
```

```
[Out] (-8*b^3*x^(3/2)*(b + c*x^2))/(65*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) + (8*b^2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(195*c) + (4*b*x^(5/2)*Sqrt[
b*x^2 + c*x^4])/39 + (2*Sqrt[x]*(b*x^2 + c*x^4)^(3/2))/13 + (8*b^(13/4)*x*(
Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*
ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(65*c^(7/4)*Sqrt[b*x^2 + c*x^4]) -
 (4*b^(13/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)
^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(65*c^(7/4)*Sqrt[b
*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{\sqrt{x}}\,dx = \frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}+\frac{1}{13}(6b)\int x^{3/2}\sqrt{bx^2+cx^4}\,dx$$

$$= \frac{4}{39}bx^{5/2}\sqrt{bx^2+cx^4}+\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}+\frac{1}{39}\left(4b^2\right)\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{8b^2\sqrt{x}\sqrt{bx^2+cx^4}}{195c}+\frac{4}{39}bx^{5/2}\sqrt{bx^2+cx^4}+\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}-\frac{\left(4b^3\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{65c}$$

$$= \frac{8b^2\sqrt{x}\sqrt{bx^2+cx^4}}{195c}+\frac{4}{39}bx^{5/2}\sqrt{bx^2+cx^4}+\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}-\frac{\left(4b^3x\sqrt{b+cx^2}\right)\int \frac{\cdots}{\sqrt{b\cdots}}}{65c\sqrt{bx^2+cx^4}}$$

$$= \frac{8b^2\sqrt{x}\sqrt{bx^2+cx^4}}{195c}+\frac{4}{39}bx^{5/2}\sqrt{bx^2+cx^4}+\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}-\frac{\left(8b^3x\sqrt{b+cx^2}\right)\text{Subs}\cdots}{65c\sqrt{bx\cdots}}$$

$$= \frac{8b^2\sqrt{x}\sqrt{bx^2+cx^4}}{195c}+\frac{4}{39}bx^{5/2}\sqrt{bx^2+cx^4}+\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}-\frac{\left(8b^{7/2}x\sqrt{b+cx^2}\right)\text{Sub}\cdots}{65c^{3/2}\sqrt{\cdots}}$$

$$= -\frac{8b^3x^{3/2}\left(b+cx^2\right)}{65c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{8b^2\sqrt{x}\sqrt{bx^2+cx^4}}{195c}+\frac{4}{39}bx^{5/2}\sqrt{bx^2+cx^4}+\frac{2}{13}\sqrt{x}\left(b\cdots\right.$$

Mathematica [C] time = 0.04, size = 90, normalized size = 0.28

$$\frac{2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\left(\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}-b^2\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{13c\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/Sqrt[x],x]
```

[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] - b^2*Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*x^2)/b)]))/(13*c*Sqrt[1 + (c*x^2)/b])

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^4 + bx^2}\left(cx^3 + bx\right)\sqrt{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^3 + b*x)*sqrt(x), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{\sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="giac")

[Out] integrate((c*x^4 + b*x^2)^(3/2)/sqrt(x), x)

maple [A] time = 0.01, size = 237, normalized size = 0.74

$$-\frac{2\left(c\,x^4 + b\,x^2\right)^{\frac{3}{2}}\left(-15c^4x^8 - 40b\,c^3x^6 - 29b^2c^2x^4 - 4b^3c\,x^2 + 12\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b^4\,\mathrm{Elliptic}\right.}{195\left(c\,x^2 + b\right)^2 c^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2)^(3/2)/x^(1/2),x)

[Out] -2/195*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c^2*(-15*c^4*x^8-40*b*c^3*x^6+12*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-6*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-29*b^2*c^2*x^4-4*b^3*c*x^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{\sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2)^(3/2)/sqrt(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(1/2),x)
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(1/2),x)
[Out] Integral((x**2*(b + c*x**2))**(3/2)/sqrt(x), x)
```

3.367 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=173

$$-\frac{4b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77c^{5/4}\sqrt{bx^2+cx^4}}+\frac{8b^2\sqrt{bx^2+cx^4}}{77c\sqrt{x}}+\frac{2\left(bx^2+cx^4\right)^{3/2}}{11\sqrt{x}}+\frac{12}{77}bx^{3/2}\sqrt{bx^2+c}$$

```
[Out] 2/11*(c*x^4+b*x^2)^(3/2)/x^(1/2)+12/77*b*x^(3/2)*(c*x^4+b*x^2)^(1/2)+8/77*b
^2*(c*x^4+b*x^2)^(1/2)/c/x^(1/2)-4/77*b^(11/4)*x*(cos(2*arctan(c^(1/4)*x^(1
/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin
(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^
2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2021, 2024, 2032, 329, 220}

$$-\frac{4b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77c^{5/4}\sqrt{bx^2+cx^4}}+\frac{8b^2\sqrt{bx^2+cx^4}}{77c\sqrt{x}}+\frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4}+\frac{2\left(bx^2+cx^4\right)}{11\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(3/2),x]
```

```
[Out] (8*b^2*Sqrt[b*x^2 + c*x^4])/(77*c*Sqrt[x]) + (12*b*x^(3/2)*Sqrt[b*x^2 + c*x
^4])/77 + (2*(b*x^2 + c*x^4)^(3/2))/(11*Sqrt[x]) - (4*b^(11/4)*x*(Sqrt[b] +
 Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c
^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(77*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{3/2}}\,dx = \frac{2\left(bx^2+cx^4\right)^{3/2}}{11\sqrt{x}} + \frac{1}{11}(6b)\int \sqrt{x}\,\sqrt{bx^2+cx^4}\,dx$$
$$= \frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4} + \frac{2\left(bx^2+cx^4\right)^{3/2}}{11\sqrt{x}} + \frac{1}{77}\left(12b^2\right)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{8b^2\sqrt{bx^2+cx^4}}{77c\sqrt{x}} + \frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4} + \frac{2\left(bx^2+cx^4\right)^{3/2}}{11\sqrt{x}} - \frac{\left(4b^3\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{77c}$$
$$= \frac{8b^2\sqrt{bx^2+cx^4}}{77c\sqrt{x}} + \frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4} + \frac{2\left(bx^2+cx^4\right)^{3/2}}{11\sqrt{x}} - \frac{\left(4b^3x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{b+cx^2}}}{77c\sqrt{bx^2+cx^4}}$$
$$= \frac{8b^2\sqrt{bx^2+cx^4}}{77c\sqrt{x}} + \frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4} + \frac{2\left(bx^2+cx^4\right)^{3/2}}{11\sqrt{x}} - \frac{\left(8b^3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{77c\sqrt{bx^2+cx}}$$
$$= \frac{8b^2\sqrt{bx^2+cx^4}}{77c\sqrt{x}} + \frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4} + \frac{2\left(bx^2+cx^4\right)^{3/2}}{11\sqrt{x}} - \frac{4b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c}{\left(\sqrt{b}+\right.}}}{77c^{5/4}\sqrt{b}}$$

Mathematica [C] time = 0.04, size = 90, normalized size = 0.52

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}-b^2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{11c\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] - b^2*Hypergeom
etric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]))/(11*c*Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^4+bx^2}\left(cx^2+b\right)\sqrt{x}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)*sqrt(x), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="giac")

[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(3/2), x)

maple [A] time = 0.01, size = 157, normalized size = 0.91

$$-\frac{2\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(-7c^4x^7-20b\,c^3x^5-17b^2c^2x^3-4b^3cx+2\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b^3\,\mathrm{Ellipti}}{77\left(c\,x^2+b\right)^2c^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2)^(3/2)/x^(3/2),x)

[Out] -2/77*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(-7*c^4*x^7+2*b^3*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-20*b*c^3*x^5-17*b^2*c^2*x^3-4*b^3*c*x)/c^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(3/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(3/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**(3/2), x)
```

3.368 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx$

Optimal. Leaf size=290

$$\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}}-\frac{8b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}}+$$

```
[Out] 2/9*(c*x^4+b*x^2)^(3/2)/x^(3/2)+8/15*b^2*x^(3/2)*(c*x^2+b)/c^(1/2)/(b^(1/2)
+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+4/15*b*x^(1/2)*(c*x^4+b*x^2)^(1/2)-8/15*b^(
9/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4
)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^
(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(3/4)/
(c*x^4+b*x^2)^(1/2)+4/15*b^(9/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^
2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1
/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+
x*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2021, 2032, 329, 305, 220, 1196}

$$\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}}-\frac{8b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}}+$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(5/2),x]
```

```
[Out] (8*b^2*x^(3/2)*(b + c*x^2))/(15*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
c*x^4]) + (4*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/15 + (2*(b*x^2 + c*x^4)^(3/2))/
(9*x^(3/2)) - (8*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
+ Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^
(3/4)*Sqrt[b*x^2 + c*x^4]) + (4*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c
*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)
], 1/2])/(15*c^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{5/2}}\,dx = \frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{3/2}} + \frac{1}{3}(2b)\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$$

$$= \frac{4}{15}b\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{3/2}} + \frac{1}{15}\left(4b^2\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{4}{15}b\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{3/2}} + \frac{\left(4b^2x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{15\sqrt{bx^2+cx^4}}$$

$$= \frac{4}{15}b\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{3/2}} + \frac{\left(8b^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15\sqrt{bx^2+cx^4}}$$

$$= \frac{4}{15}b\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{3/2}} + \frac{\left(8b^{5/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15\sqrt{c}\sqrt{bx^2+cx^4}} -$$

$$= \frac{8b^2x^{3/2}\left(b+cx^2\right)}{15\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{4}{15}b\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{3/2}} - \frac{8b^{9/4}x\left(\sqrt{b}+\right.}{}$$

Mathematica [C] time = 0.02, size = 58, normalized size = 0.20

$$\frac{2b\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)}{3\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(5/2),x]
```

```
[Out] (2*b*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*x
^2)/b)])/(3*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(cx^2+b\right)}{\sqrt{x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/sqrt(x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(5/2), x)
```

maple [A] time = 0.01, size = 226, normalized size = 0.78

$$\frac{2\left(c\,x^4 + b\,x^2\right)^{\frac{3}{2}}\left(5c^3x^6 + 16b\,c^2x^4 + 11b^2c\,x^2 + 12\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\;\sqrt{2}\;\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\;\sqrt{-\frac{cx}{\sqrt{-bc}}}\;b^3\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\right.\right.}{45\left(c\,x^2+b\right)^2 c\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(5/2),x)
```

```
[Out] 2/45*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c*(5*c^3*x^6+12*b^3*((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2),1/2*2^(1/2))-6*b^3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*(
(-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*Ellipti
cF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+16*b*c^2*x^4+11*b^2
*c*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(5/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(5/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**(5/2), x)
```

3.369 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx$

Optimal. Leaf size=143

$$\frac{4b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}+\frac{4b\sqrt{bx^2+cx^4}}{7\sqrt{x}}+\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{5/2}}$$

```
[Out] 2/7*(c*x^4+b*x^2)^(3/2)/x^(5/2)+4/7*b*(c*x^4+b*x^2)^(1/2)/x^(1/2)+4/7*b^(7/
4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*
x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1
/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(1/4)/(c
*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2021, 2032, 329, 220}

$$\frac{4b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}+\frac{4b\sqrt{bx^2+cx^4}}{7\sqrt{x}}+\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(7/2),x]
```

```
[Out] (4*b*Sqrt[b*x^2 + c*x^4])/(7*Sqrt[x]) + (2*(b*x^2 + c*x^4)^(3/2))/(7*x^(5/2
)) + (4*b^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]
*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(7*c^(1/4)*Sqrt
[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}\,dx &= \frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{5/2}}+\frac{1}{7}(6b)\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx\\
&= \frac{4b\sqrt{bx^2+cx^4}}{7\sqrt{x}}+\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{5/2}}+\frac{1}{7}\left(4b^2\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx\\
&= \frac{4b\sqrt{bx^2+cx^4}}{7\sqrt{x}}+\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{5/2}}+\frac{\left(4b^2x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{7\sqrt{bx^2+cx^4}}\\
&= \frac{4b\sqrt{bx^2+cx^4}}{7\sqrt{x}}+\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{5/2}}+\frac{\left(8b^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{7\sqrt{bx^2+cx^4}}\\
&= \frac{4b\sqrt{bx^2+cx^4}}{7\sqrt{x}}+\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{5/2}}+\frac{4b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right.}{7\sqrt[4]{c}\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.39

$$\frac{2b\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)}{\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(7/2),x]
```

```
[Out] (2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)])
/(Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(cx^2+b\right)}{x^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(7/2), x)
```

maple [A] time = 0.02, size = 145, normalized size = 1.01

$$\frac{2\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(c^3x^5+4b\,c^2x^3+3b^2cx+2\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b^2\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\right.\right.}{7\left(c\,x^2+b\right)^2c\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(7/2),x)
```

```
[Out] 2/7*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(2*b^2*(-b*c)^(1/2)*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2),1/2*2^(1/2))+c^3*x^5+4*b*c^2*x^3+3*b^2*c*x)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4 + b\,x^2\right)^{3/2}}{x^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(7/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(7/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**(7/2), x)
```

3.370 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{9/2}}\,dx$

Optimal. Leaf size=286

$$\frac{12b^{5/4}\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}}-\frac{24b^{5/4}\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\right.\right.}{5\sqrt{bx^2+cx^4}}$$

```
[Out] -2*(c*x^4+b*x^2)^(3/2)/x^(7/2)+24/5*b*x^(3/2)*(c*x^2+b)*c^(1/2)/(b^(1/2)+x*
c^(1/2))/(c*x^4+b*x^2)^(1/2)+12/5*c*x^(1/2)*(c*x^4+b*x^2)^(1/2)-24/5*b^(5/4
)*c^(1/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c
^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1
/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/(c*
x^4+b*x^2)^(1/2)+12/5*b^(5/4)*c^(1/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/
4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan
(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(
1/2)+x*c^(1/2))^2)^(1/2)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2020, 2021, 2032, 329, 305, 220, 1196}

$$\frac{12b^{5/4}\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}}-\frac{24b^{5/4}\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\right.\right.}{5\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(9/2),x]
```

```
[Out] (24*b*Sqrt[c]*x^(3/2)*(b + c*x^2))/(5*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*
x^4]) + (12*c*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/5 - (2*(b*x^2 + c*x^4)^(3/2))/x^
(7/2) - (24*b^(5/4)*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[
b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*
Sqrt[b*x^2 + c*x^4]) + (12*b^(5/4)*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b
+ c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1
/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{9/2}}\,dx = -\frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}+(6c)\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$$

$$= \frac{12}{5}c\sqrt{x}\,\sqrt{bx^2+cx^4}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}+\frac{1}{5}(12bc)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{12}{5}c\sqrt{x}\,\sqrt{bx^2+cx^4}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}+\frac{\left(12bcx\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5\sqrt{bx^2+cx^4}}$$

$$= \frac{12}{5}c\sqrt{x}\,\sqrt{bx^2+cx^4}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}+\frac{\left(24bcx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{5\sqrt{bx^2+cx^4}}$$

$$= \frac{12}{5}c\sqrt{x}\,\sqrt{bx^2+cx^4}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}+\frac{\left(24b^{3/2}\sqrt{c}\,x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{\right.}$$

$$= \frac{24b\sqrt{c}\,x^{3/2}\left(b+cx^2\right)}{5\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{12}{5}c\sqrt{x}\,\sqrt{bx^2+cx^4}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}-\frac{24b^{5/4}\sqrt[4]{c}\,x\left(\sqrt{b}+\right.}{}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.20

$$-\frac{2b\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{cx^2}{b}\right)}{x^{3/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(9/2),x]
```

```
[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, -1/4, 3/4, -((c*x^2)/b)
])/(x^(3/2)*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(cx^2+b\right)}{x^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(9/2), x)
```

maple [A] time = 0.02, size = 216, normalized size = 0.76

$$\frac{2\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(c^2x^4-4bc\,x^2+12\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b^2\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-6\sqrt{\frac{cx+\sqrt{-}}{\sqrt{-b}}}\right.}{5\left(c\,x^2+b\right)^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(9/2),x)
```

```
[Out] 2/5*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(12*b^2*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c
)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^
(1/2))-6*b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+c^2*x^4-4*b*c*x^2-5*b^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(9/2),x)
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(9/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(9/2),x)
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**(9/2), x)
```

3.371 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{11/2}} \, dx$

Optimal. Leaf size=143

$$\frac{4b^{3/4}c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt{bx^2+cx^4}}+\frac{4c\sqrt{bx^2+cx^4}}{3\sqrt{x}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{3x^{9/2}}$$

```
[Out] -2/3*(c*x^4+b*x^2)^(3/2)/x^(9/2)+4/3*c*(c*x^4+b*x^2)^(1/2)/x^(1/2)+4/3*b^(3
/4)*c^(3/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan
(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))
,1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/(
c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2020, 2021, 2032, 329, 220}

$$\frac{4b^{3/4}c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt{bx^2+cx^4}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{3x^{9/2}}+\frac{4c\sqrt{bx^2+cx^4}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(11/2),x]
```

```
[Out] (4*c*Sqrt[b*x^2 + c*x^4])/(3*Sqrt[x]) - (2*(b*x^2 + c*x^4)^(3/2))/(3*x^(9/2
)) + (4*b^(3/4)*c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
 Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*Sqrt
[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{11/2}}\,dx = -\frac{2\left(bx^2+cx^4\right)^{3/2}}{3x^{9/2}} + (2c)\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx$$
$$= \frac{4c\sqrt{bx^2+cx^4}}{3\sqrt{x}} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{3x^{9/2}} + \frac{1}{3}(4bc)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{4c\sqrt{bx^2+cx^4}}{3\sqrt{x}} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{3x^{9/2}} + \frac{\left(4bcx\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{3\sqrt{bx^2+cx^4}}$$
$$= \frac{4c\sqrt{bx^2+cx^4}}{3\sqrt{x}} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{3x^{9/2}} + \frac{\left(8bcx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{3\sqrt{bx^2+cx^4}}$$
$$= \frac{4c\sqrt{bx^2+cx^4}}{3\sqrt{x}} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{3x^{9/2}} + \frac{4b^{3/4}c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right.\right.}{3\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.02, size = 58, normalized size = 0.41

$$-\frac{2b\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};-\frac{cx^2}{b}\right)}{3x^{5/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(11/2),x]
```

```
[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((c*x^2)/b)
])/(3*x^(5/2)*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(cx^2+b\right)}{x^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(11/2), x)
```

maple [A] time = 0.02, size = 130, normalized size = 0.91

$$\frac{2\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(c^2x^4+2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{-bc}\,bx\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-b^2\right)}{3\left(c\,x^2+b\right)^2x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(11/2),x)
```

```
[Out] 2/3*(c*x^4+b*x^2)^(3/2)/x^(9/2)/(c*x^2+b)^2*(2*((c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/
2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)
)*(-b*c)^(1/2)*x*b+c^2*x^4-b^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(11/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(11/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(11/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(11/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**(11/2), x)
```

3.372 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx$

Optimal. Leaf size=287

$$\frac{12\sqrt[4]{b}\,c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}}-\frac{24\sqrt[4]{b}\,c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\right.\right.}{5\sqrt{bx^2+cx^4}}$$

```
[Out] -2/5*(c*x^4+b*x^2)^(3/2)/x^(11/2)+24/5*c^(3/2)*x^(3/2)*(c*x^2+b)/(b^(1/2)+x
*c^(1/2))/(c*x^4+b*x^2)^(1/2)-12/5*c*(c*x^4+b*x^2)^(1/2)/x^(3/2)-24/5*b^(1/
4)*c^(5/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),
1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/(c
*x^4+b*x^2)^(1/2)+12/5*b^(1/4)*c^(5/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1
/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arcta
n(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^
(1/2)+x*c^(1/2))^2)^(1/2)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2020, 2032, 329, 305, 220, 1196}

$$\frac{24c^{3/2}x^{3/2}\left(b+cx^2\right)}{5\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{12\sqrt[4]{b}\,c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}}-\frac{24\sqrt[4]{b}\,c^{5/4}x\left(\sqrt{b}+\sqrt{c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(13/2),x]
```

```
[Out] (24*c^(3/2)*x^(3/2)*(b + c*x^2))/(5*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^
4]) - (12*c*Sqrt[b*x^2 + c*x^4])/(5*x^(3/2)) - (2*(b*x^2 + c*x^4)^(3/2))/(5
*x^(11/2)) - (24*b^(1/4)*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(
Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2]
)/(5*Sqrt[b*x^2 + c*x^4]) + (12*b^(1/4)*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqr
t[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])
/b^(1/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{13/2}}\,dx = -\frac{2\left(bx^2+cx^4\right)^{3/2}}{5x^{11/2}} + \frac{1}{5}(6c)\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx$$

$$= -\frac{12c\sqrt{bx^2+cx^4}}{5x^{3/2}} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{5x^{11/2}} + \frac{1}{5}\left(12c^2\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{12c\sqrt{bx^2+cx^4}}{5x^{3/2}} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{5x^{11/2}} + \frac{\left(12c^2x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5\sqrt{bx^2+cx^4}}$$

$$= -\frac{12c\sqrt{bx^2+cx^4}}{5x^{3/2}} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{5x^{11/2}} + \frac{\left(24c^2x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{bx^2+cx^4}}$$

$$= -\frac{12c\sqrt{bx^2+cx^4}}{5x^{3/2}} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{5x^{11/2}} + \frac{\left(24\sqrt{b}\,c^{3/2}x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right.}{5\sqrt{bx^2+cx^4}}$$

$$= \frac{24c^{3/2}x^{3/2}\left(b+cx^2\right)}{5\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{12c\sqrt{bx^2+cx^4}}{5x^{3/2}} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{5x^{11/2}} - \frac{24\sqrt[4]{b}\,c^{5/4}x\left(\sqrt{b}+\sqrt{c}\right.}{}$$

Mathematica [C] time = 0.02, size = 58, normalized size = 0.20

$$-\frac{2b\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};-\frac{1}{4};-\frac{cx^2}{b}\right)}{5x^{7/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(13/2),x]
```

```
[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, -5/4, -1/4, -((c*x^2)/b
)])/(5*x^(7/2)*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(cx^2+b\right)}{x^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(9/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="giac")

[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(13/2), x)

maple [A] time = 0.02, size = 221, normalized size = 0.77

$$\frac{2\left(c\,x^4 + b\,x^2\right)^{\frac{3}{2}}\left(-7c^2x^4 + 12\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,bc\,x^2\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}, \frac{\sqrt{2}}{2}\right) - 6\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\cdots\right.}{5\left(c\,x^2 + b\right)^2 x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2)^(3/2)/x^(13/2),x)

[Out] 2/5*(c*x^4+b*x^2)^(3/2)/x^(11/2)/(c*x^2+b)^2*(12*(-1/(-b*c)^(1/2)*c*x)^(1/2
)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*x^2*b*c-6*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/
2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*x^2*b*c-7*c^2*x^4-8*b*c*x^2-b^2
)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(13/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(13/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(13/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(13/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**(13/2), x)
```

3.373 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx$

Optimal. Leaf size=143

$$\frac{4c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\,\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{7x^{5/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{13/2}}$$

```
[Out] -2/7*(c*x^4+b*x^2)^(3/2)/x^(13/2)-4/7*c*(c*x^4+b*x^2)^(1/2)/x^(5/2)+4/7*c^(
7/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4
)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^
(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(1/4)/
(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2020, 2032, 329, 220}

$$\frac{4c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\,\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{7x^{5/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(15/2),x]
```

```
[Out] (-4*c*Sqrt[b*x^2 + c*x^4])/(7*x^(5/2)) - (2*(b*x^2 + c*x^4)^(3/2))/(7*x^(13
/2)) + (4*c^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[
c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(7*b^(1/4)*Sq
rt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{15/2}}\,dx &= -\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{13/2}}+\frac{1}{7}(6c)\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx\\ &= -\frac{4c\sqrt{bx^2+cx^4}}{7x^{5/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{13/2}}+\frac{1}{7}\left(4c^2\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx\\ &= -\frac{4c\sqrt{bx^2+cx^4}}{7x^{5/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{13/2}}+\frac{\left(4c^2x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{b+cx^2}}\,dx}{7\sqrt{bx^2+cx^4}}\\ &= -\frac{4c\sqrt{bx^2+cx^4}}{7x^{5/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{13/2}}+\frac{\left(8c^2x\sqrt{b+cx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{7\sqrt{bx^2+cx^4}}\\ &= -\frac{4c\sqrt{bx^2+cx^4}}{7x^{5/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{13/2}}+\frac{4c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right.\right.}{7\sqrt[4]{b}\,\sqrt{bx^2+cx^4}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 58, normalized size = 0.41

$$-\frac{2b\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{7}{4},-\frac{3}{2};-\frac{3}{4};-\frac{cx^2}{b}\right)}{7x^{9/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(15/2),x]
```

```
[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-7/4, -3/2, -3/4, -((c*x^2)/b
)])/(7*x^(9/2)*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(cx^2+b\right)}{x^{\frac{11}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(11/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(15/2), x)
```

maple [A] time = 0.02, size = 140, normalized size = 0.98

$$\frac{2\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(-3c^2x^4+2\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,c\,x^3\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-4bc\,x^2-\right.}{7\left(c\,x^2+b\right)^2x^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(15/2),x)
```

```
[Out] 2/7*(c*x^4+b*x^2)^(3/2)/x^(13/2)/(c*x^2+b)^2*(2*((-c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*2^(1/2)*x^3*c-3*c^2*x^4-4*b*c*x^2-b^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{15}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(15/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4 + b\,x^2\right)^{3/2}}{x^{15/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(15/2),x)
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(15/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(15/2),x)
[Out] Timed out
```

3.374 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx$

Optimal. Leaf size=320

$$\frac{4c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}}-\frac{8c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}}+$$

```
[Out] -2/9*(c*x^4+b*x^2)^(3/2)/x^(15/2)+8/15*c^(5/2)*x^(3/2)*(c*x^2+b)/b/(b^(1/2)
+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-4/15*c*(c*x^4+b*x^2)^(1/2)/x^(7/2)-8/15*c^2
*(c*x^4+b*x^2)^(1/2)/b/x^(3/2)-8/15*c^(9/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)
/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*
arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b
)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/(c*x^4+b*x^2)^(1/2)+4/15*c^(9/4)*x*(
cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(
b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/(c*x^4+b
*x^2)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2020, 2025, 2032, 329, 305, 220, 1196}

$$\frac{4c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}}-\frac{8c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}}+$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(17/2),x]
```

```
[Out] (8*c^(5/2)*x^(3/2)*(b + c*x^2))/(15*b*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*
x^4]) - (4*c*Sqrt[b*x^2 + c*x^4])/(15*x^(7/2)) - (8*c^2*Sqrt[b*x^2 + c*x^4]
)/(15*b*x^(3/2)) - (2*(b*x^2 + c*x^4)^(3/2))/(9*x^(15/2)) - (8*c^(9/4)*x*(S
qrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*A
rcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[b*x^2 + c*x^4]) +
(4*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[b*x
^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{17/2}}\,dx = -\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{15/2}}+\frac{1}{3}(2c)\int \frac{\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{15/2}}+\frac{1}{15}\left(4c^2\right)\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{15bx^{3/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{15/2}}+\frac{\left(4c^3\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{15b}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{15bx^{3/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{15/2}}+\frac{\left(4c^3x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{15b\sqrt{bx^2+cx^4}}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{15bx^{3/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{15/2}}+\frac{\left(8c^3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\right.}{15b\sqrt{bx^2+cx^4}}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{15bx^{3/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{15/2}}+\frac{\left(8c^{5/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\right.}{15\sqrt{b}\sqrt{bx^2+cx^4}}$$

$$= \frac{8c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{15bx^{3/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{15/2}}-\frac{8c^{9}}{}$$

Mathematica [C] time = 0.02, size = 58, normalized size = 0.18

$$-\frac{2b\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{9}{4},-\frac{3}{2};-\frac{5}{4};-\frac{cx^2}{b}\right)}{9x^{11/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(17/2),x]
```

[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-9/4, -3/2, -5/4, -((c*x^2)/b
)])/(9*x^(11/2)*Sqrt[1 + (c*x^2)/b])

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(cx^2+b\right)}{x^{\frac{13}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(13/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{\frac{17}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="giac")

[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(17/2), x)

maple [A] time = 0.02, size = 239, normalized size = 0.75

$$\frac{2\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(-12c^3x^6+12\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b\,c^2x^4\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-6\sqrt{\frac{cx+\sqrt{}}{\sqrt{-b}}}\right.}{45\left(c\,x^2+b\right)^2 b\,x^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2)^(3/2)/x^(17/2),x)

[Out] 2/45*(c*x^4+b*x^2)^(3/2)/x^(15/2)/(c*x^2+b)^2*(12*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^
(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))*x^4*b*c^2-6*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+
(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b*c^2-12*c^3*x^6-23*b*c^
2*x^4-16*b^2*c*x^2-5*b^3)/b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{17}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(17/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4 + b\,x^2\right)^{3/2}}{x^{17/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(17/2),x)
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(17/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(17/2),x)
[Out] Timed out
```

3.375 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{19/2}}\,dx$

Optimal. Leaf size=173

$$-\frac{4c^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77b^{5/4}\sqrt{bx^2+cx^4}}-\frac{8c^2\sqrt{bx^2+cx^4}}{77bx^{5/2}}-\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{11x^{17/2}}$$

```
[Out] -2/11*(c*x^4+b*x^2)^(3/2)/x^(17/2)-12/77*c*(c*x^4+b*x^2)^(1/2)/x^(9/2)-8/77
*c^2*(c*x^4+b*x^2)^(1/2)/b/x^(5/2)-4/77*c^(11/4)*x*(cos(2*arctan(c^(1/4)*x^
(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(s
in(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*
x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2020, 2025, 2032, 329, 220}

$$-\frac{4c^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77b^{5/4}\sqrt{bx^2+cx^4}}-\frac{8c^2\sqrt{bx^2+cx^4}}{77bx^{5/2}}-\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{11x^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(19/2),x]
```

```
[Out] (-12*c*Sqrt[b*x^2 + c*x^4])/(77*x^(9/2)) - (8*c^2*Sqrt[b*x^2 + c*x^4])/(77*
b*x^(5/2)) - (2*(b*x^2 + c*x^4)^(3/2))/(11*x^(17/2)) - (4*c^(11/4)*x*(Sqrt[
b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTa
n[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(77*b^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{19/2}}\,dx &= -\frac{2\left(bx^2+cx^4\right)^{3/2}}{11x^{17/2}}+\frac{1}{11}(6c)\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx \\
&= -\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{11x^{17/2}}+\frac{1}{77}\left(12c^2\right)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx \\
&= -\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{77bx^{5/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{11x^{17/2}}-\frac{\left(4c^3\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{77b} \\
&= -\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{77bx^{5/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{11x^{17/2}}-\frac{\left(4c^3x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{77b\sqrt{bx^2+cx^4}} \\
&= -\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{77bx^{5/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{11x^{17/2}}-\frac{\left(8c^3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+c\cdots}}\right.}{77b\sqrt{bx^2+cx^4}} \\
&= -\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{77bx^{5/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{11x^{17/2}}-\frac{4c^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\cdots\right.}}}{77b^{5/4}\sqrt{bx^2\cdots}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 58, normalized size = 0.34

$$
-\frac{2b\sqrt{x^2\left(b+cx^2\right)}\;{}_2F_1\left(-\frac{11}{4},-\frac{3}{2};-\frac{7}{4};-\frac{cx^2}{b}\right)}{11x^{13/2}\sqrt{\frac{cx^2}{b}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(19/2),x]
```

```
[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-11/4, -3/2, -7/4, -((c*x^2)/
b)])/(11*x^(13/2)*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(cx^2+b\right)}{x^{\frac{15}{2}}},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(19/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(15/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{19}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(19/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(19/2), x)
```

maple [A] time = 0.02, size = 156, normalized size = 0.90

$$-\frac{2\left(c\,x^4 + b\,x^2\right)^{\frac{3}{2}}\left(4c^3x^6 + 2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{-bc}\,c^2x^5\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}, \frac{\sqrt{2}}{2}\right) + 17b\,c^2x\right.}{77\left(c\,x^2 + b\right)^2 b\,x^{\frac{17}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(19/2),x)
```

```
[Out] -2/77*(c*x^4+b*x^2)^(3/2)/x^(17/2)/(c*x^2+b)^2*(2*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^
(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))*(-b*c)^(1/2)*x^5*c^2+4*c^3*x^6+17*b*c^2*x^4+20*b^2*c*x^2+7*b^3)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{19}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(19/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(19/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4 + b\,x^2\right)^{3/2}}{x^{19/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(19/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(19/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(19/2),x)
```

```
[Out] Timed out
```

3.376 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{21/2}}\,dx$

Optimal. Leaf size=350

$$-\frac{4c^{13/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{bx^2+cx^4}}+\frac{8c^{13/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\right.}{65b^{7/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/13*(c*x^4+b*x^2)^(3/2)/x^(19/2)-8/65*c^(7/2)*x^(3/2)*(c*x^2+b)/b^2/(b^(1
/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-4/39*c*(c*x^4+b*x^2)^(1/2)/x^(11/2)-8/19
5*c^2*(c*x^4+b*x^2)^(1/2)/b/x^(7/2)+8/65*c^3*(c*x^4+b*x^2)^(1/2)/b^2/x^(3/2
)+8/65*c^(13/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*ar
ctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/
4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/
2)/b^(7/4)/(c*x^4+b*x^2)^(1/2)-4/65*c^(13/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2
*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+
b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2020, 2025, 2032, 329, 305, 220, 1196}

$$-\frac{8c^{7/2}x^{3/2}\left(b+cx^2\right)}{65b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{8c^3\sqrt{bx^2+cx^4}}{65b^2x^{3/2}}-\frac{4c^{13/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{bx^2+cx^4}}+\frac{8c^{13/}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(21/2),x]
```

```
[Out] (-8*c^(7/2)*x^(3/2)*(b + c*x^2))/(65*b^2*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) - (4*c*Sqrt[b*x^2 + c*x^4])/(39*x^(11/2)) - (8*c^2*Sqrt[b*x^2 + c*
x^4])/(195*b*x^(7/2)) + (8*c^3*Sqrt[b*x^2 + c*x^4])/(65*b^2*x^(3/2)) - (2*(
b*x^2 + c*x^4)^(3/2))/(13*x^(19/2)) + (8*c^(13/4)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[b*x^2 + c*x^4]) - (4*c^(13/4)*x*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{21/2}}\,dx = -\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}+\frac{1}{13}(6c)\int \frac{\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}+\frac{1}{39}\left(4c^2\right)\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{7/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}-\frac{\left(4c^3\right)\int \frac{1}{\sqrt{x}\,\sqrt{bx^2+cx^4}}\,dx}{65b}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{7/2}}+\frac{8c^3\sqrt{bx^2+cx^4}}{65b^2x^{3/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}-\frac{\left(4c^4\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{7/2}}+\frac{8c^3\sqrt{bx^2+cx^4}}{65b^2x^{3/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}-\frac{\left(4c^4x\sqrt{b+cx^2}\right.}{65b^2\sqrt{bx^2}}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{7/2}}+\frac{8c^3\sqrt{bx^2+cx^4}}{65b^2x^{3/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}-\frac{\left(8c^4x\sqrt{b+cx^2}\right.}{65}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{7/2}}+\frac{8c^3\sqrt{bx^2+cx^4}}{65b^2x^{3/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}-\frac{\left(8c^{7/2}x\sqrt{b+cx^2}\right.}{65}$$

$$= -\frac{8c^{7/2}x^{3/2}\left(b+cx^2\right)}{65b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{7/2}}+\frac{8c^3\sqrt{bx^2+cx^4}}{65b^2x^{3/2}}-2$$

Mathematica [C] time = 0.02, size = 58, normalized size = 0.17

$$-\frac{2b\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{13}{4},-\frac{3}{2};-\frac{9}{4};-\frac{cx^2}{b}\right)}{13x^{15/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(21/2),x]
```

```
[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-13/4, -3/2, -9/4, -((c*x^2)/
b)])/(13*x^(15/2)*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(cx^2+b\right)}{x^{\frac{17}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(21/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(17/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{\frac{21}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(21/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(21/2), x)
```

maple [A] time = 0.04, size = 250, normalized size = 0.71

$$-\frac{2\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(-12c^4x^8+12\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b\,c^3x^6\,\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-6\sqrt{\frac{cx+}{\sqrt{}}}\right.}{195\left(c\,x^2+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(21/2),x)
```

```
[Out] -2/195*(c*x^4+b*x^2)^(3/2)/x^(19/2)/(c*x^2+b)^2*(12*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c
)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^
(1/2))*x^6*b*c^3-6*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*
```

```
x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^6*b*c^3-12*c^4*x^8-8*b*c
^3*x^6+29*b^2*c^2*x^4+40*b^3*c*x^2+15*b^4)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{21}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(21/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(21/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4 + b\,x^2\right)^{3/2}}{x^{21/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(21/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(21/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(21/2),x)
```

```
[Out] Timed out
```

3.377 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{23/2}}\,dx$

Optimal. Leaf size=203

$$\frac{4c^{15/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}+\frac{8c^3\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}}-\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}}-2\left(b\right.$$

```
[Out] -2/15*(c*x^4+b*x^2)^(3/2)/x^(21/2)-4/55*c*(c*x^4+b*x^2)^(1/2)/x^(13/2)-8/38
5*c^2*(c*x^4+b*x^2)^(1/2)/b/x^(9/2)+8/231*c^3*(c*x^4+b*x^2)^(1/2)/b^2/x^(5/
2)+4/231*c^(15/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*
arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(
1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(
1/2)/b^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2020, 2025, 2032, 329, 220}

$$\frac{8c^3\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}+\frac{4c^{15/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}-\frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}}-\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}}-2\left(b\right.$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(23/2),x]
```

```
[Out] (-4*c*Sqrt[b*x^2 + c*x^4])/(55*x^(13/2)) - (8*c^2*Sqrt[b*x^2 + c*x^4])/(385
*b*x^(9/2)) + (8*c^3*Sqrt[b*x^2 + c*x^4])/(231*b^2*x^(5/2)) - (2*(b*x^2 + c
*x^4)^(3/2))/(15*x^(21/2)) + (4*c^(15/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4
)], 1/2])/(231*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{23/2}}\,dx = -\frac{2(bx^2+cx^4)^{3/2}}{15x^{21/2}} + \frac{1}{5}(2c)\int \frac{\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}} - \frac{2(bx^2+cx^4)^{3/2}}{15x^{21/2}} + \frac{1}{55}(4c^2)\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}} - \frac{2(bx^2+cx^4)^{3/2}}{15x^{21/2}} - \frac{(4c^3)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{77b}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}} + \frac{8c^3\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} - \frac{2(bx^2+cx^4)^{3/2}}{15x^{21/2}} + \frac{(4c^4)\int \frac{\sqrt{x}}{\sqrt{bx^2+c}}}{231b^2}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}} + \frac{8c^3\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} - \frac{2(bx^2+cx^4)^{3/2}}{15x^{21/2}} + \frac{(4c^4x\sqrt{b+cx}}{231b^2\sqrt{}}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}} + \frac{8c^3\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} - \frac{2(bx^2+cx^4)^{3/2}}{15x^{21/2}} + \frac{(8c^4x\sqrt{b+cx}}{2}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}} + \frac{8c^3\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} - \frac{2(bx^2+cx^4)^{3/2}}{15x^{21/2}} + \frac{4c^{15/4}x(\sqrt{b}+}{}$$

Mathematica [C] time = 0.02, size = 58, normalized size = 0.29

$$-\frac{2b\sqrt{x^2(b+cx^2)}\,{}_2F_1\left(-\frac{15}{4},-\frac{3}{2};-\frac{11}{4};-\frac{cx^2}{b}\right)}{15x^{17/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(23/2),x]
```

```
[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-15/4, -3/2, -11/4, -((c*x^2)
/b)])/(15*x^(17/2)*Sqrt[1 + (c*x^2)/b])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,(cx^2+b)}{x^{\frac{19}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^(3/2)/x^(23/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(19/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{23}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^(3/2)/x^(23/2),x, algorithm="giac")

[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(23/2), x)

maple [A] time = 0.04, size = 167, normalized size = 0.82

$$\frac{2\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(20c^4x^8+10\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{-bc}\,c^3x^7\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)+8b\,c^3x^6\right.}{1155\left(c\,x^2+b\right)^2 b^2 x^{\frac{21}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2)^(3/2)/x^(23/2),x)

[Out] 2/1155*(c*x^4+b*x^2)^(3/2)/x^(21/2)/(c*x^2+b)^2*(10*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c
)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^
(1/2))*(-b*c)^(1/2)*x^7*c^3+20*c^4*x^8+8*b*c^3*x^6-131*b^2*c^2*x^4-196*b^3*
c*x^2-77*b^4)/b^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{23}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^(3/2)/x^(23/2),x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(23/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{23/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(23/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(23/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(23/2),x)
```

```
[Out] Timed out
```

3.378 $\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=179

$$-\frac{15b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77c^{13/4}\sqrt{bx^2+cx^4}}+\frac{30b^2\sqrt{bx^2+cx^4}}{77c^3\sqrt{x}}-\frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2}+\frac{2x^{7/2}\sqrt{bx^2+}}{11c}$$

```
[Out] -18/77*b*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c^2+2/11*x^(7/2)*(c*x^4+b*x^2)^(1/2)/c
+30/77*b^2*(c*x^4+b*x^2)^(1/2)/c^3/x^(1/2)-15/77*b^(11/4)*x*(cos(2*arctan(c
^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*El
lipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1
/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(13/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2024, 2032, 329, 220}

$$\frac{30b^2\sqrt{bx^2+cx^4}}{77c^3\sqrt{x}}-\frac{15b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77c^{13/4}\sqrt{bx^2+cx^4}}-\frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2}+\frac{2x^{7/2}\sqrt{bx^2+c}}{11c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (30*b^2*Sqrt[b*x^2 + c*x^4])/(77*c^3*Sqrt[x]) - (18*b*x^(3/2)*Sqrt[b*x^2 +
c*x^4])/(77*c^2) + (2*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(11*c) - (15*b^(11/4)*x*
(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2
*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(77*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{(9b)\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx}{11c}$$

$$= -\frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c} + \frac{\left(45b^2\right)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{77c^2}$$

$$= \frac{30b^2\sqrt{bx^2+cx^4}}{77c^3\sqrt{x}} - \frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{\left(15b^3\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{77c^3}$$

$$= \frac{30b^2\sqrt{bx^2+cx^4}}{77c^3\sqrt{x}} - \frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{\left(15b^3x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{b+cx}}}{77c^3\sqrt{bx^2+cx^4}}$$

$$= \frac{30b^2\sqrt{bx^2+cx^4}}{77c^3\sqrt{x}} - \frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{\left(30b^3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\right.}{77c^3\sqrt{bx^2+c}}$$

$$= \frac{30b^2\sqrt{bx^2+cx^4}}{77c^3\sqrt{x}} - \frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{15b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b}{\left(\sqrt{b}\right.}}}{77c^{13/4}\sqrt{}}$$

Mathematica [C] time = 0.04, size = 97, normalized size = 0.54

$$\frac{2x^{3/2}\left(-15b^3\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)+15b^3+6b^2cx^2-2bc^2x^4+7c^3x^6\right)}{77c^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(15*b^3 + 6*b^2*c*x^2 - 2*b*c^2*x^4 + 7*c^3*x^6 - 15*b^3*Sqrt[1
+ (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(77*c^3*Sqrt[
x^2*(b + c*x^2)])
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,x^{\frac{9}{2}}}{cx^2+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*x^(9/2)/(c*x^2 + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{13}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(13/2)/sqrt(c*x^4 + b*x^2), x)
```

maple [A] time = 0.03, size = 148, normalized size = 0.83

$$-\frac{\left(-14c^4x^7+4b\,c^3x^5-12b^2c^2x^3-30b^3cx+15\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b^3\,\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.\right.}{77\sqrt{c\,x^4+b\,x^2}\;c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/77/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(-14*c^4*x^7+15*b^3*(-b*c)^(1/2)*((c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2),1/2*2^(1/2))+4*b*c^3*x^5-12*b^2*c^2*x^3-30*b^3*c*x)/c^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{13}{2}}}{\sqrt{cx^4 + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(13/2)/sqrt(c*x^4 + b*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{13/2}}{\sqrt{c\,x^4 + b\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^(13/2)/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

3.379 $\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}} dx$

Optimal. Leaf size=296

$$\frac{7b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}}-\frac{14b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 14/15*b^2*x^(3/2)*(c*x^2+b)/c^(5/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)
+2/9*x^(5/2)*(c*x^4+b*x^2)^(1/2)/c-14/45*b*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c^2-
14/15*b^(9/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)
)),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)
/c^(11/4)/(c*x^4+b*x^2)^(1/2)+7/15*b^(9/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/
b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*a
rctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)
/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2024, 2032, 329, 305, 220, 1196}

$$\frac{14b^2x^{3/2}\left(b+cx^2\right)}{15c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{7b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}}-\frac{14b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)}{15c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (14*b^2*x^(3/2)*(b + c*x^2))/(15*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) - (14*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(45*c^2) + (2*x^(5/2)*Sqrt[b*
x^2 + c*x^4])/(9*c) - (14*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/
(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2
])/(15*c^(11/4)*Sqrt[b*x^2 + c*x^4]) + (7*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(15*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c} - \frac{(7b)\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{9c}$$

$$= -\frac{14b\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c} + \frac{\left(7b^2\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{15c^2}$$

$$= -\frac{14b\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c} + \frac{\left(7b^2x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{15c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{14b\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c} + \frac{\left(14b^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{14b\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c} + \frac{\left(14b^{5/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15c^{5/2}\sqrt{bx^2+cx^4}} \cdot$$

$$= \frac{14b^2x^{3/2}\left(b+cx^2\right)}{15c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{14b\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c} - \frac{14b^{9/4}x\left(\sqrt{b}+\right.}{}$$

Mathematica [C] time = 0.04, size = 86, normalized size = 0.29

$$\frac{2x^{5/2}\left(7b^2\sqrt{\frac{cx^2}{b}+1}\ {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)-7b^2-2bcx^2+5c^2x^4\right)}{45c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(5/2)*(-7*b^2 - 2*b*c*x^2 + 5*c^2*x^4 + 7*b^2*Sqrt[1 + (c*x^2)/b]*Hype
rgeometric2F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(45*c^2*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,x^{\frac{7}{2}}}{cx^2+b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*x^(7/2)/(c*x^2 + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{11}{2}}}{\sqrt{cx^4 + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(11/2)/sqrt(c*x^4 + b*x^2), x)
```

maple [A] time = 0.03, size = 217, normalized size = 0.73

$$\frac{\left(10c^3x^6 - 4b\,c^2x^4 - 14b^2c\,x^2 + 42\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b^3\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}, \frac{\sqrt{2}}{2}\right) - 21\sqrt{\frac{cx+\cdots}{\sqrt{-\cdots}}}\right.}{45\sqrt{c\,x^4 + b\,x^2}\;c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/45/(c*x^4+b*x^2)^(1/2)*x^(1/2)/c^3*(10*c^3*x^6+42*b^3*((c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(
-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/
2*2^(1/2))-21*b^3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x
+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-4*b*c^2*x^4-14*b^2*c*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{11}{2}}}{\sqrt{cx^4 + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(11/2)/sqrt(c*x^4 + b*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{11/2}}{\sqrt{c\,x^4 + b\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^2 + c*x^4)^(1/2),x)
[Out] int(x^(11/2)/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{11}{2}}}{\sqrt{x^2 \left(b + cx^2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x**(11/2)/sqrt(x**2*(b + c*x**2)), x)
```

3.380 $\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=149

$$\frac{5b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{9/4}\sqrt{bx^2+cx^4}}-\frac{10b\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}}+\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}$$

```
[Out] 2/7*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c-10/21*b*(c*x^4+b*x^2)^(1/2)/c^2/x^(1/2)+5
/21*b^(7/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan
(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))
,1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c
^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2024, 2032, 329, 220}

$$\frac{5b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{9/4}\sqrt{bx^2+cx^4}}-\frac{10b\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}}+\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-10*b*Sqrt[b*x^2 + c*x^4])/(21*c^2*Sqrt[x]) + (2*x^(3/2)*Sqrt[b*x^2 + c*x^
4])/(7*c) + (5*b^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(21*c^(9
/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c} - \frac{(5b)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{7c}$$

$$= -\frac{10b\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{\left(5b^2\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{21c^2}$$

$$= -\frac{10b\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{\left(5b^2x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{b+cx^2}}\,dx}{21c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{10b\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{\left(10b^2x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{21c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{10b\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{5b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\Big(2\tan^{-1}\Big(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\Big)}{21c^{9/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.03, size = 86, normalized size = 0.58

$$\frac{2x^{3/2}\left(5b^2\sqrt{\frac{cx^2}{b}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)-5b^2-2bcx^2+3c^2x^4\right)}{21c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(-5*b^2 - 2*b*c*x^2 + 3*c^2*x^4 + 5*b^2*Sqrt[1 + (c*x^2)/b]*Hype
rgeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(21*c^2*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,x^{\frac{5}{2}}}{cx^2+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*x^(5/2)/(c*x^2 + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{9}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(9/2)/sqrt(c*x^4 + b*x^2), x)
```

maple [A] time = 0.02, size = 137, normalized size = 0.92

$$\frac{\left(6c^3x^5-4b\,c^2x^3-10b^2cx+5\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b^2\,\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\right)\sqrt{x}}{21\sqrt{c\,x^4+b\,x^2}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/21/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(5*b^2*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*
c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2
^(1/2))+6*c^3*x^5-4*b*c^2*x^3-10*b^2*c*x)/c^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{9}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(9/2)/sqrt(c*x^4 + b*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{9/2}}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^(9/2)/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{9}{2}}}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(9/2)/sqrt(x**2*(b + c*x**2)), x)
```

3.381 $\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=266

$$-\frac{3b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}+\frac{6b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\right.}{5c^{7/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -6/5*b*x^(3/2)*(c*x^2+b)/c^(3/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+2/
5*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c+6/5*b^(5/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)
/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*
arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b
)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)-3/5*b^(5/4)*x*(c
os(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)
/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b
^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*
x^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2024, 2032, 329, 305, 220, 1196}

$$-\frac{3b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}+\frac{6b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\right.}{5c^{7/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-6*b*x^(3/2)*(b + c*x^2))/(5*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*
x^4]) + (2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*c) + (6*b^(5/4)*x*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/
4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(7/4)*Sqrt[b*x^2 + c*x^4]) - (3*b^(5/4)*x*
(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2
*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{2\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c} - \frac{(3b)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{5c}$$

$$= \frac{2\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c} - \frac{\left(3bx\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5c\sqrt{bx^2+cx^4}}$$

$$= \frac{2\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c} - \frac{\left(6bx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5c\sqrt{bx^2+cx^4}}$$

$$= \frac{2\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c} - \frac{\left(6b^{3/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5c^{3/2}\sqrt{bx^2+cx^4}} + \frac{\left(6b^{3/2}x\sqrt{b+cx^2}\right)\text{Sul}}{5c^{3/2}\sqrt{}}$$

$$= -\frac{6bx^{3/2}\left(b+cx^2\right)}{5c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{2\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c} + \frac{6b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\Big(}{5c^{7/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.03, size = 70, normalized size = 0.26

$$\frac{2x^{5/2}\left(-b\sqrt{\frac{cx^2}{b}+1}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)+b+cx^2\right)}{5c\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(5/2)*(b + c*x^2 - b*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/2, 3/4, 7
/4, -((c*x^2)/b)]))/(5*c*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,x^{\frac{3}{2}}}{cx^2+b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

[Out] integral(sqrt(c*x^4 + b*x^2)*x^(3/2)/(c*x^2 + b), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")

[Out] integrate(x^(7/2)/sqrt(c*x^4 + b*x^2), x)

maple [A] time = 0.02, size = 206, normalized size = 0.77

$$-\frac{\left(-2c^2x^4 - 2bc\,x^2 + 6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b^2\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}, \frac{\sqrt{2}}{2}\right) - 3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx}{}}\right.}{5\sqrt{c\,x^4 + b\,x^2}\;c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)/(c*x^4+b*x^2)^(1/2),x)

[Out] -1/5/(c*x^4+b*x^2)^(1/2)*x^(1/2)/c^2*(6*b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*
c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-3
b^2((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-2*c^2*x^4-2*b*c*x^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(x^(7/2)/sqrt(c*x^4 + b*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{7/2}}{\sqrt{c\,x^4 + b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^(7/2)/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{x^2 \left(b + cx^2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(7/2)/sqrt(x**2*(b + c*x**2)), x)
```

3.382 $\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=121

$$\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/3*(c*x^4+b*x^2)^(1/2)/c/x^(1/2)-1/3*b^(3/4)*x*(cos(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2
+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2024, 2032, 329, 220}

$$\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - (b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x]
)/b^(1/4)], 1/2])/(3*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx &= \frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{b\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{3c}\\
&= \frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{\left(bx\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{3c\sqrt{bx^2+cx^4}}\\
&= \frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{\left(2bx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{3c\sqrt{bx^2+cx^4}}\\
&= \frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}\end{aligned}$$

Mathematica [C] time = 0.03, size = 70, normalized size = 0.58

$$\frac{2x^{3/2}\left(-b\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)+b+cx^2\right)}{3c\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[b*x^2 + c*x^4],x]
```

[Out] (2*x^(3/2)*(b + c*x^2 - b*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(3*c*Sqrt[x^2*(b + c*x^2)])

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{x}}{cx^2+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c*x^2 + b), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")

[Out] integrate(x^(5/2)/sqrt(c*x^4 + b*x^2), x)

maple [A] time = 0.01, size = 123, normalized size = 1.02

$$-\frac{\left(-2c^2x^3-2bcx+\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b\,\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\right)\sqrt{x}}{3\sqrt{c\,x^4+b\,x^2}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)/(c*x^4+b*x^2)^(1/2),x)

[Out] -1/3/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(b*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-2*c^2*x^3-2*b*c*x)/c^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(5/2)/sqrt(c*x^4 + b*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2 + c*x^4)^(1/2),x)
[Out] int(x^(5/2)/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x**(5/2)/sqrt(x**2*(b + c*x**2)), x)
```

3.383 $\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=231

$$\frac{\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{bx^2+cx^4}}+\cdots$$

```
[Out] 2*x^(3/2)*(c*x^2+b)/c^(1/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2*b^(1/
4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*
x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1
/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(3/4)/(c
*x^4+b*x^2)^(1/2)+b^(1/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2
)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(
1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/
2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2032, 329, 305, 220, 1196}

$$\frac{\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{bx^2+cx^4}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(b + c*x^2))/(Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4])
 - (2*b^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x
)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(3/4)*Sqrt[b*x
^2 + c*x^4]) + (b^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
 Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(3/4
)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{\left(x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{\sqrt{bx^2+cx^4}}$$
$$= \frac{\left(2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{bx^2+cx^4}}$$
$$= \frac{\left(2\sqrt{b}\,x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{c}\,\sqrt{bx^2+cx^4}} - \frac{\left(2\sqrt{b}\,x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx, x, \sqrt{\right.}{\sqrt{c}\,\sqrt{bx^2+cx^4}}$$
$$= \frac{2x^{3/2}\left(b+cx^2\right)}{\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{bx^2+cx^4}} + \ldots$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.25

$$\frac{2x^{5/2}\sqrt{\frac{cx^2}{b}+1}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)}{3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(5/2)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^2)/b
)])/(3*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{x}}{cx^3+bx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c*x^3 + b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/sqrt(c*x^4 + b*x^2), x)
```

maple [A] time = 0.01, size = 131, normalized size = 0.57

$$\frac{\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\right)b\sqrt{x}}{\sqrt{c\,x^{4}+b\,x^{2}}\;c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x^(1/2)*b/c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2
^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)
*(2*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-Elliptic
F(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{cx^{4}+bx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/sqrt(c*x^4 + b*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3/2}}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^(3/2)/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{x^{2}\left(b+cx^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(3/2)/sqrt(x**2*(b + c*x**2)), x)
```

3.384 $\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=90

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}$$

```
[Out] x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(
1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2)
)*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(1/4)/c^(1/
4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2032, 329, 220}

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(1/4)*c^(1/4)*Sqrt[b*x^2 +
c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx &= \frac{\left(x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{b+cx^2}}\,dx}{\sqrt{bx^2+cx^4}}\\ &= \frac{\left(2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{bx^2+cx^4}}\\ &= \frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.61

$$\frac{2x^{3/2}\sqrt{\frac{cx^2}{b}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)}{\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b
)])/Sqrt[x^2*(b + c*x^2)]
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x}}{\sqrt{cx^4+bx^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

maple [A] time = 0.02, size = 106, normalized size = 1.18

$$\frac{\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{x}\,\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{\sqrt{c\,x^4+b\,x^2}\;c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c
*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x}}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^(1/2)/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/sqrt(x**2*(b + c*x**2)), x)
```

3.385 $\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=259

$$\frac{\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{bx^2+cx^4}}+$$

```
[Out] 2*x^(3/2)*(c*x^2+b)*c^(1/2)/b/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2*(c*
x^4+b*x^2)^(1/2)/b/x^(3/2)-2*c^(1/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1
/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/(c*x^4+b*x^2)^(1/2)+c^(1/4)*x*(cos(2*arctan
(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*
EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^
(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2025, 2032, 329, 305, 220, 1196}

$$\frac{\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{bx^2+cx^4}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (2*Sqrt[c]*x^(3/2)*(b + c*x^2))/(b*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4
]) - (2*Sqrt[b*x^2 + c*x^4])/(b*x^(3/2)) - (2*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*
x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sq
rt[x])/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*x^2 + c*x^4]) + (c^(1/4)*x*(Sqrt[b]
+ Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(
c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{c\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{b}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{\left(cx\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{\left(2cx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{\left(2\sqrt{c}\,x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{b}\sqrt{bx^2+cx^4}} - \frac{\left(2\sqrt{c}\,x\sqrt{b+cx^2}\right)\text{Su}}{\sqrt{b}\sqrt{}}$$

$$= \frac{2\sqrt{c}\,x^{3/2}\left(b+cx^2\right)}{b\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}} - \frac{2\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan\right.}{b^{3/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.02, size = 55, normalized size = 0.21

$$-\frac{2\sqrt{x}\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{cx^2}{b}\right)}{\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((c*x^2)
/b)])/Sqrt[x^2*(b + c*x^2)]
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{x}}{cx^5+bx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c*x^5 + b*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^2}\,\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*sqrt(x)), x)
```

maple [A] time = 0.02, size = 195, normalized size = 0.75

$$\frac{\left(-2c\,x^2 + 2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}, \frac{\sqrt{2}}{2}\right) - \sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\cdots}\right.}{\sqrt{c\,x^4 + b\,x^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^
(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*
EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b-((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2),1/2*2^(1/2))*b-2*c*x^2-2*b)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^2}\,\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{x}\,\sqrt{c\,x^4 + b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x^2 + c*x^4)^(1/2)),x)
[Out] int(1/(x^(1/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(1/(sqrt(x)*sqrt(x**2*(b + c*x**2))), x)
```

3.386 $\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=121

$$-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}$$

```
[Out] -2/3*(c*x^4+b*x^2)^(1/2)/b/x^(5/2)-1/3*c^(3/4)*x*(cos(2*arctan(c^(1/4)*x^(1
/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin
(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^
2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2025, 2032, 329, 220}

$$-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(3*b*x^(5/2)) - (c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx &= -\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}} - \frac{c\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{3b} \\
&= -\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}} - \frac{\left(cx\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{b+cx^2}}\,dx}{3b\sqrt{bx^2+cx^4}} \\
&= -\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}} - \frac{\left(2cx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{3b\sqrt{bx^2+cx^4}} \\
&= -\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}} - \frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.47

$$-\frac{2\sqrt{\frac{cx^2}{b}+1}\;{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\frac{cx^2}{b}\right)}{3\sqrt{x}\,\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((c*x^2)/b)])/(3
*Sqrt[x]*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{x}}{cx^6+bx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c*x^6 + b*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+bx^2}\,x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(3/2)), x)
```

maple [A] time = 0.02, size = 119, normalized size = 0.98

$$-\frac{2c\,x^2+\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,x\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)+2b}{3\sqrt{c\,x^4+b\,x^2}\;b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/3/(c*x^4+b*x^2)^(1/2)/x^(1/2)*((-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/
2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)
)*x+2*c*x^2+2*b)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+bx^2}\,x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(b*x^2 + c*x^4)^(1/2)),x)
[Out] int(1/(x^(3/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(1/(x**(3/2)*sqrt(x**2*(b + c*x**2))), x)
```

3.387 $\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=296

$$-\frac{3c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}}+\frac{6c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -6/5*c^(3/2)*x^(3/2)*(c*x^2+b)/b^2/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-
2/5*(c*x^4+b*x^2)^(1/2)/b/x^(7/2)+6/5*c*(c*x^4+b*x^2)^(1/2)/b^2/x^(3/2)+6/5
*c^(5/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^
(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/
2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(7
/4)/(c*x^4+b*x^2)^(1/2)-3/5*c^(5/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)
))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c
^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/
2)+x*c^(1/2))^2)^(1/2)/b^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2025, 2032, 329, 305, 220, 1196}

$$-\frac{6c^{3/2}x^{3/2}\left(b+cx^2\right)}{5b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{3c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}}+\frac{6c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{}{\left(\right.}}}{5b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-6*c^(3/2)*x^(3/2)*(b + c*x^2))/(5*b^2*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/(5*b*x^(7/2)) + (6*c*Sqrt[b*x^2 + c*x^4])
/(5*b^2*x^(3/2)) + (6*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqr
t[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(
5*b^(7/4)*Sqrt[b*x^2 + c*x^4]) - (3*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b
 + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(
1/4)], 1/2])/(5*b^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{(3c)\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{5b}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}} + \frac{6c\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} - \frac{\left(3c^2\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{5b^2}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}} + \frac{6c\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} - \frac{\left(3c^2x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}} + \frac{6c\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} - \frac{\left(6c^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}} + \frac{6c\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} - \frac{\left(6c^{3/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5b^{3/2}\sqrt{bx^2+cx^4}} + \frac{\left(6c^3\right.}{}$$

$$= -\frac{6c^{3/2}x^{3/2}\left(b+cx^2\right)}{5b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}} + \frac{6c\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} + \frac{6c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{}}{5b}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.19

$$-\frac{2\sqrt{\frac{cx^2}{b}+1}\;{}_2F_1\left(-\frac{5}{4},\frac{1}{2};-\frac{1}{4};-\frac{cx^2}{b}\right)}{5x^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-5/4, 1/2, -1/4, -((c*x^2)/b)])/(
5*x^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{x}}{cx^7+bx^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c*x^7 + b*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+bx^2}\,x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(5/2)), x)
```

maple [A] time = 0.02, size = 215, normalized size = 0.73

$$-\frac{-6c^2x^4+6\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,bc\,x^2\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-3\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\cdots}{5\sqrt{c\,x^4+b\,x^2}\,b^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/5/(c*x^4+b*x^2)^(1/2)/x^(3/2)*(6*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE((
(c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*x^2*b*c-3
*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*x^2*b*c-6*c^2*x^4-4*b*c*x^2+2*b^2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+bx^2}\,x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{5/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^(5/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(5/2)*sqrt(x**2*(b + c*x**2))), x)
```

3.388 $\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=149

$$\frac{5c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{bx^2+cx^4}}+\frac{10c\sqrt{bx^2+cx^4}}{21b^2x^{5/2}}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}$$

```
[Out] -2/7*(c*x^4+b*x^2)^(1/2)/b/x^(9/2)+10/21*c*(c*x^4+b*x^2)^(1/2)/b^2/x^(5/2)+
5/21*c^(7/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))
),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/
b^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2025, 2032, 329, 220}

$$\frac{5c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{bx^2+cx^4}}+\frac{10c\sqrt{bx^2+cx^4}}{21b^2x^{5/2}}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(7*b*x^(9/2)) + (10*c*Sqrt[b*x^2 + c*x^4])/(21*b^2
*x^(5/2)) + (5*c^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(21*b^(9
/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx &= -\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}} - \frac{(5c)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{7b} \\
&= -\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}} + \frac{10c\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} + \frac{\left(5c^2\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{21b^2} \\
&= -\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}} + \frac{10c\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} + \frac{\left(5c^2x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{21b^2\sqrt{bx^2+cx^4}} \\
&= -\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}} + \frac{10c\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} + \frac{\left(10c^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{21b^2\sqrt{bx^2+cx^4}} \\
&= -\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}} + \frac{10c\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} + \frac{5c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\right.}{21b^{9/4}\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.38

$$-\frac{2\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(-\frac{7}{4},\frac{1}{2};-\frac{3}{4};-\frac{cx^2}{b}\right)}{7x^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-7/4, 1/2, -3/4, -((c*x^2)/b)])/(
7*x^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{x}}{cx^8+bx^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c*x^8 + b*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+bx^2}\,x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(7/2)), x)
```

maple [A] time = 0.02, size = 134, normalized size = 0.90

$$\frac{10c^2x^4+5\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,c\,x^3\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)+4bc\,x^2-6b^2}{21\sqrt{c\,x^4+b\,x^2}\;b^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/21/(c*x^4+b*x^2)^(1/2)/x^(5/2)*(5*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2),1/2*2^(1/2))*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1
/2)*x^3*c+10*c^2*x^4+4*b*c*x^2-6*b^2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+bx^2}\,x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{7/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^(7/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{7}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(7/2)*sqrt(x**2*(b + c*x**2))), x)
```

3.389 $\int \frac{1}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=326

$$\frac{7c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}}-\frac{14c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 14/15*c^(5/2)*x^(3/2)*(c*x^2+b)/b^3/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)
-2/9*(c*x^4+b*x^2)^(1/2)/b/x^(11/2)+14/45*c*(c*x^4+b*x^2)^(1/2)/b^2/x^(7/2)
-14/15*c^2*(c*x^4+b*x^2)^(1/2)/b^3/x^(3/2)-14/15*c^(9/4)*x*(cos(2*arctan(c^
(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ell
ipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/
2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(11/4)/(c*x^4+b*x^2)^(1/2)+7/
15*c^(9/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),
1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^
(11/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2025, 2032, 329, 305, 220, 1196}

$$\frac{14c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b^3\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{14c^2\sqrt{bx^2+cx^4}}{15b^3x^{3/2}}+\frac{7c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(9/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (14*c^(5/2)*x^(3/2)*(b + c*x^2))/(15*b^3*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/(9*b*x^(11/2)) + (14*c*Sqrt[b*x^2 + c*x^
4])/(45*b^2*x^(7/2)) - (14*c^2*Sqrt[b*x^2 + c*x^4])/(15*b^3*x^(3/2)) - (14*
c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*E
llipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(11/4)*Sqrt[b*x^2
 + c*x^4]) + (7*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
 Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(
11/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{(7c)\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx}{9b}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}} + \frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} + \frac{\left(7c^2\right)\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{15b^2}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}} + \frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} - \frac{14c^2\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} + \frac{\left(7c^3\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{15b^3}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}} + \frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} - \frac{14c^2\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} + \frac{\left(7c^3x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{15b^3\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}} + \frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} - \frac{14c^2\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} + \frac{\left(14c^3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{b+}}\right.}{15b^3\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}} + \frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} - \frac{14c^2\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} + \frac{\left(14c^{5/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{15b^{5/2}\sqrt{bx^2+cx}}$$

$$= \frac{14c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b^3\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}} + \frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} - \frac{14c^2\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} -$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.17

$$-\frac{2\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(-\frac{9}{4},\frac{1}{2};-\frac{5}{4};-\frac{cx^2}{b}\right)}{9x^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(9/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-9/4, 1/2, -5/4, -((c*x^2)/b)])/(
9*x^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\sqrt{x}}{cx^9+bx^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c*x^9 + b*x^7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^2}\, x^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(9/2)), x)
```

maple [A] time = 0.02, size = 230, normalized size = 0.71

$$\frac{-42c^3x^6 + 42\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b\,c^2x^4\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}, \frac{\sqrt{2}}{2}\right) - 21\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-}}{\sqrt{-bc}}}}{45\sqrt{c\,x^4 + b\,x^2}\,b^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(9/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/45/(c*x^4+b*x^2)^(1/2)/x^(7/2)*(42*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1
/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b*c^
2-21*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b*c^2-42*c^3*x^6-28*b*c^2*x^4+4*b^2*
c*x^2-10*b^3)/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^2}\, x^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(9/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{9/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(9/2)*(b*x^2 + c*x^4)^(1/2)),x)
[Out] int(1/(x^(9/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{9}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(9/2)/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(1/(x**(9/2)*sqrt(x**2*(b + c*x**2))), x)
```

3.390 $\int \frac{1}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=179

$$-\frac{15c^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77b^{13/4}\sqrt{bx^2+cx^4}}-\frac{30c^2\sqrt{bx^2+cx^4}}{77b^3x^{5/2}}+\frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}}-\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}}$$

```
[Out] -2/11*(c*x^4+b*x^2)^(1/2)/b/x^(13/2)+18/77*c*(c*x^4+b*x^2)^(1/2)/b^2/x^(9/2
)-30/77*c^2*(c*x^4+b*x^2)^(1/2)/b^3/x^(5/2)-15/77*c^(11/4)*x*(cos(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*E
llipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(
1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(13/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2025, 2032, 329, 220}

$$-\frac{30c^2\sqrt{bx^2+cx^4}}{77b^3x^{5/2}}-\frac{15c^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77b^{13/4}\sqrt{bx^2+cx^4}}+\frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}}-\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(11/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(11*b*x^(13/2)) + (18*c*Sqrt[b*x^2 + c*x^4])/(77*b
^2*x^(9/2)) - (30*c^2*Sqrt[b*x^2 + c*x^4])/(77*b^3*x^(5/2)) - (15*c^(11/4)*
x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF
[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(77*b^(13/4)*Sqrt[b*x^2 + c*x^4
])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}} - \frac{(9c)\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx}{11b}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}} + \frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} + \frac{\left(45c^2\right)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{77b^2}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}} + \frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} - \frac{30c^2\sqrt{bx^2+cx^4}}{77b^3x^{5/2}} - \frac{\left(15c^3\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{77b^3}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}} + \frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} - \frac{30c^2\sqrt{bx^2+cx^4}}{77b^3x^{5/2}} - \frac{\left(15c^3x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}}{77b^3\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}} + \frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} - \frac{30c^2\sqrt{bx^2+cx^4}}{77b^3x^{5/2}} - \frac{\left(30c^3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{b}}\right.}{77b^3\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}} + \frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} - \frac{30c^2\sqrt{bx^2+cx^4}}{77b^3x^{5/2}} - \frac{15c^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c}{\left(\sqrt{b}+\right.}}}{77b^{13/4}\sqrt{bx}}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.32

$$-\frac{2\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(-\frac{11}{4},\frac{1}{2};-\frac{7}{4};-\frac{cx^2}{b}\right)}{11x^{9/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(11/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-11/4, 1/2, -7/4, -((c*x^2)/b)])/
(11*x^(9/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{x}}{cx^{10}+bx^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c*x^10 + b*x^8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+bx^2}\,x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(11/2)), x)
```

maple [A] time = 0.02, size = 147, normalized size = 0.82

$$-\frac{30c^3x^6+15\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{-bc}\,c^2x^5\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)+12b\,c^2x^4-4b^2c\,x^2+1}{77\sqrt{c\,x^4+b\,x^2}\;b^3x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(11/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/77/(c*x^4+b*x^2)^(1/2)/x^(9/2)*(15*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(
1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^
(1/2)*x^5*c^2+30*c^3*x^6+12*b*c^2*x^4-4*b^2*c*x^2+14*b^3)/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+bx^2}\,x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(11/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{11/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(11/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^(11/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{11}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(11/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(11/2)*sqrt(x**2*(b + c*x**2))), x)
```

3.391 $\int \frac{x^{17/2}}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=174

$$\frac{15b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{14c^{13/4}\sqrt{bx^2+cx^4}}-\frac{15b\sqrt{bx^2+cx^4}}{7c^3\sqrt{x}}+\frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2}-\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -x^(11/2)/c/(c*x^4+b*x^2)^(1/2)+9/7*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c^2-15/7*b*
(c*x^4+b*x^2)^(1/2)/c^3/x^(1/2)+15/14*b^(7/4)*x*(cos(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2
+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(13/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2022, 2024, 2032, 329, 220}

$$\frac{15b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{14c^{13/4}\sqrt{bx^2+cx^4}}+\frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2}-\frac{15b\sqrt{bx^2+cx^4}}{7c^3\sqrt{x}}-\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(17/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^(11/2)/(c*Sqrt[b*x^2 + c*x^4])) - (15*b*Sqrt[b*x^2 + c*x^4])/(7*c^3*Sqr
t[x]) + (9*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(7*c^2) + (15*b^(7/4)*x*(Sqrt[b] +
Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^
(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(14*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{17/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}} + \frac{9\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx}{2c}$$

$$= -\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}} + \frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2} - \frac{(45b)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{14c^2}$$

$$= -\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{7c^3\sqrt{x}} + \frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2} + \frac{\left(15b^2\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{14c^3}$$

$$= -\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{7c^3\sqrt{x}} + \frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2} + \frac{\left(15b^2x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{14c^3\sqrt{bx^2+cx^4}}$$

$$= -\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{7c^3\sqrt{x}} + \frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2} + \frac{\left(15b^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\right.}{7c^3\sqrt{bx^2+cx^4}}$$

$$= -\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{7c^3\sqrt{x}} + \frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2} + \frac{15b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}}{14c^{13/4}\sqrt{bx^2+}}$$

Mathematica [C] time = 0.03, size = 86, normalized size = 0.49

$$\frac{x^{3/2}\left(15b^2\sqrt{\frac{cx^2}{b}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)-15b^2-6bcx^2+2c^2x^4\right)}{7c^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(17/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(-15*b^2 - 6*b*c*x^2 + 2*c^2*x^4 + 15*b^2*Sqrt[1 + (c*x^2)/b]*Hype
rgeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(7*c^3*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,x^{\frac{9}{2}}}{c^2x^4+2\,bcx^2+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*x^(9/2)/(c^2*x^4 + 2*b*c*x^2 + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{17}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(17/2)/(c*x^4 + b*x^2)^(3/2), x)
```

maple [A] time = 0.04, size = 144, normalized size = 0.83

$$\frac{\left(c\,x^2+b\right)\left(4c^3x^5-12b\,c^2x^3-30b^2cx+15\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b^2\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\right.\right.}{14\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/14/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(15*b^2*(-b*c)^(1/2)*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2),1/2*2^(1/2))+4*c^3*x^5-12*b*c^2*x^3-30*b^2*c*x)/c^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{17}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(17/2)/(c*x^4 + b*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{17/2}}{\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^(17/2)/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

3.392 $\int \frac{x^{15/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=291

$$-\frac{21b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10c^{11/4}\sqrt{bx^2+cx^4}}+\frac{21b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right.\right.}{5c^{11/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -x^(9/2)/c/(c*x^4+b*x^2)^(1/2)-21/5*b*x^(3/2)*(c*x^2+b)/c^(5/2)/(b^(1/2)+x*
c^(1/2))/(c*x^4+b*x^2)^(1/2)+7/5*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c^2+21/5*b^(5/
4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*
x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1
/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(
c*x^4+b*x^2)^(1/2)-21/10*b^(5/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^
2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1
/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+
x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2022, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{21b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10c^{11/4}\sqrt{bx^2+cx^4}}+\frac{21b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right.\right.}{5c^{11/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(15/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^(9/2)/(c*Sqrt[b*x^2 + c*x^4])) - (21*b*x^(3/2)*(b + c*x^2))/(5*c^(5/2)*
(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) + (7*Sqrt[x]*Sqrt[b*x^2 + c*x^4]
)/(5*c^2) + (21*b^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
 Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(1
1/4)*Sqrt[b*x^2 + c*x^4]) - (21*b^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c
*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)
], 1/2])/(10*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
```

```
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{15/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}} + \frac{7\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{2c}$$

$$= -\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}} + \frac{7\sqrt{x}\sqrt{bx^2+cx^4}}{5c^2} - \frac{(21b)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{10c^2}$$

$$= -\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}} + \frac{7\sqrt{x}\sqrt{bx^2+cx^4}}{5c^2} - \frac{\left(21bx\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{10c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}} + \frac{7\sqrt{x}\sqrt{bx^2+cx^4}}{5c^2} - \frac{\left(21bx\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{5c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}} + \frac{7\sqrt{x}\sqrt{bx^2+cx^4}}{5c^2} - \frac{\left(21b^{3/2}x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{5c^{5/2}\sqrt{bx^2+cx^4}} + \cdots$$

$$= -\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}} - \frac{21bx^{3/2}\left(b+cx^2\right)}{5c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{7\sqrt{x}\sqrt{bx^2+cx^4}}{5c^2} + \frac{21b^{5/4}x\left(\sqrt{b}+\sqrt{c}\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.03, size = 72, normalized size = 0.25

$$\frac{2x^{5/2}\left(7b\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^2}{b}\right)-7b+cx^2\right)}{5c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(15/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^(5/2)*(-7*b + c*x^2 + 7*b*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[3/4, 3
/2, 7/4, -((c*x^2)/b)]))/(5*c^2*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,x^{\frac{7}{2}}}{c^2x^4+2\,bcx^2+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*x^(7/2)/(c^2*x^4 + 2*b*c*x^2 + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{15}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(15/2)/(c*x^4 + b*x^2)^(3/2), x)
```

maple [A] time = 0.04, size = 213, normalized size = 0.73

$$-\frac{\left(c\,x^2+b\right)\left(-4c^2x^4-14bc\,x^2+42\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b^2\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-21\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\cdots\right)}{10\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/10/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(42*b^2*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c
)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^
(1/2))-21*b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-4*c^2*x^4-14*b*c*x^2)/c^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{15}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(15/2)/(c*x^4 + b*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{15/2}}{\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^(15/2)/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

3.393 $\int \frac{x^{13/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{5b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6c^{9/4}\sqrt{bx^2+cx^4}}+\frac{5\sqrt{bx^2+cx^4}}{3c^2\sqrt{x}}-\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -x^(7/2)/c/(c*x^4+b*x^2)^(1/2)+5/3*(c*x^4+b*x^2)^(1/2)/c^2/x^(1/2)-5/6*b^(3
/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)
*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(
1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(9/4)/(
c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2022, 2024, 2032, 329, 220}

$$-\frac{5b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6c^{9/4}\sqrt{bx^2+cx^4}}+\frac{5\sqrt{bx^2+cx^4}}{3c^2\sqrt{x}}-\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^(7/2)/(c*Sqrt[b*x^2 + c*x^4])) + (5*Sqrt[b*x^2 + c*x^4])/(3*c^2*Sqrt[x]
) - (5*b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*
x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(6*c^(9/4)*Sqrt[
b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{13/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}} + \frac{5\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{2c}$$

$$= -\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}} + \frac{5\sqrt{bx^2+cx^4}}{3c^2\sqrt{x}} - \frac{(5b)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{6c^2}$$

$$= -\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}} + \frac{5\sqrt{bx^2+cx^4}}{3c^2\sqrt{x}} - \frac{\left(5bx\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{b+cx^2}}\,dx}{6c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}} + \frac{5\sqrt{bx^2+cx^4}}{3c^2\sqrt{x}} - \frac{\left(5bx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{3c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}} + \frac{5\sqrt{bx^2+cx^4}}{3c^2\sqrt{x}} - \frac{5b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6c^{9/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.03, size = 73, normalized size = 0.50

$$\frac{x^{3/2}\left(-5b\sqrt{\frac{cx^2}{b}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)+5b+2cx^2\right)}{3c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(5*b + 2*c*x^2 - 5*b*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/
2, 5/4, -((c*x^2)/b)]))/(3*c^2*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,x^{\frac{5}{2}}}{c^2x^4+2\,bcx^2+b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*x^(5/2)/(c^2*x^4 + 2*b*c*x^2 + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{13}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(13/2)/(c*x^4 + b*x^2)^(3/2), x)
```

maple [A] time = 0.02, size = 131, normalized size = 0.90

$$-\frac{\left(c\,x^2+b\right)\left(-4c^2x^3-10bcx+5\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\right)x^{\frac{5}{2}}}{6\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/6/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(5*b*(-b*c)^(1/2)*((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*
(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2),1/2*2^(1/2))-4*c^2*x^3-10*b*c*x)/c^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{13}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(13/2)/(c*x^4 + b*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{13/2}}{\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^(13/2)/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

3.394 $\int \frac{x^{11/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=259

$$\frac{3\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2c^{7/4}\sqrt{bx^2+cx^4}}-\frac{3\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{c^{7/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -x^(5/2)/c/(c*x^4+b*x^2)^(1/2)+3*x^(3/2)*(c*x^2+b)/c^(3/2)/(b^(1/2)+x*c^(1/
2))/(c*x^4+b*x^2)^(1/2)-3*b^(1/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))
^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(
1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)
+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)+3/2*b^(1/4)*x*(cos(2*arcta
n(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))
*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c
^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2022, 2032, 329, 305, 220, 1196}

$$\frac{3x^{3/2}\left(b+cx^2\right)}{c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{3\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2c^{7/4}\sqrt{bx^2+cx^4}}-\frac{3\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{\cdots}{\left(\cdots\right.}}}{c^{7/4}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^(5/2)/(c*Sqrt[b*x^2 + c*x^4])) + (3*x^(3/2)*(b + c*x^2))/(c^(3/2)*(Sqrt
[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (3*b^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*
Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[
x])/b^(1/4)], 1/2])/(c^(7/4)*Sqrt[b*x^2 + c*x^4]) + (3*b^(1/4)*x*(Sqrt[b] +
 Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c
^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{11/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{x^{5/2}}{c\sqrt{bx^2+cx^4}}+\frac{3\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{2c}$$

$$= -\frac{x^{5/2}}{c\sqrt{bx^2+cx^4}}+\frac{\left(3x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{2c\sqrt{bx^2+cx^4}}$$

$$= -\frac{x^{5/2}}{c\sqrt{bx^2+cx^4}}+\frac{\left(3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{c\sqrt{bx^2+cx^4}}$$

$$= -\frac{x^{5/2}}{c\sqrt{bx^2+cx^4}}+\frac{\left(3\sqrt{b}\,x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{c^{3/2}\sqrt{bx^2+cx^4}}-\frac{\left(3\sqrt{b}\,x\sqrt{b+cx^2}\right)\text{Sub}}{c^{3/2}\sqrt{b}}$$

$$= -\frac{x^{5/2}}{c\sqrt{bx^2+cx^4}}+\frac{3x^{3/2}\left(b+cx^2\right)}{c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{3\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\,\mathrm{t}\right.}{c^{7/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.24

$$-\frac{2x^{5/2}\left(\sqrt{\frac{cx^2}{b}+1}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^2}{b}\right)-1\right)}{c\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*x^(5/2)*(-1 + Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((c
*x^2)/b)]))/(c*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,x^{\frac{3}{2}}}{c^2x^4+2\,bcx^2+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

[Out] integral(sqrt(c*x^4 + b*x^2)*x^(3/2)/(c^2*x^4 + 2*b*c*x^2 + b^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{11}{2}}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(11/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")

[Out] integrate(x^(11/2)/(c*x^4 + b*x^2)^(3/2), x)

maple [A] time = 0.02, size = 200, normalized size = 0.77

$$\frac{\left(c\,x^2+b\right)\left(-2c\,x^2+6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\cdots}{\sqrt{-\cdots}}}\right.}{2\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(11/2)/(c*x^4+b*x^2)^(3/2),x)

[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(6*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b-3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b-2*c*x^2)/c^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{11}{2}}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(11/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")

[Out] integrate(x^(11/2)/(c*x^4 + b*x^2)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{11/2}}{\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^(11/2)/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{11}{2}}}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**(11/2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.395 $\int \frac{x^{9/2}}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=119

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{b}\,c^{5/4}\sqrt{bx^2+cx^4}}-\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -x^(3/2)/c/(c*x^4+b*x^2)^(1/2)+1/2*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))
)^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^
(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2
)+x*c^(1/2))^2)^(1/2)/b^(1/4)/c^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2022, 2032, 329, 220}

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{b}\,c^{5/4}\sqrt{bx^2+cx^4}}-\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^(3/2)/(c*Sqrt[b*x^2 + c*x^4])) + (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x
^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
 1/2])/(2*b^(1/4)*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{9/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= -\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}+\frac{\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{2c}\\
&= -\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}+\frac{\left(x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{b+cx^2}}\,dx}{2c\sqrt{bx^2+cx^4}}\\
&= -\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}+\frac{\left(x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{c\sqrt{bx^2+cx^4}}\\
&= -\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{b}\,c^{5/4}\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 60, normalized size = 0.50

$$\frac{x^{3/2}\left(\sqrt{\frac{cx^2}{b}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)-1\right)}{c\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(-1 + Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^
2)/b)]))/(c*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{x}}{c^2x^4+2\,bcx^2+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c^2*x^4 + 2*b*c*x^2 + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{9}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(9/2)/(c*x^4 + b*x^2)^(3/2), x)
```

maple [A] time = 0.02, size = 120, normalized size = 1.01

$$\frac{\left(c\,x^2+b\right)\left(-2cx+\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\right)x^{\frac{5}{2}}}{2\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*((-b*c)^(1/2)*((c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(
-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/
2*2^(1/2))-2*c*x)/c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{9}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate(x^(9/2)/(c*x^4 + b*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{9/2}}{\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^2 + c*x^4)^(3/2),x)
[Out] int(x^(9/2)/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{9}{2}}}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(x**(9/2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.396 $\int \frac{x^{7/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=260

$$-\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}+\frac{x^{5/}}{b\sqrt{bx^2}}$$

```
[Out] x^(5/2)/b/(c*x^4+b*x^2)^(1/2)-x^(3/2)*(c*x^2+b)/b/c^(1/2)/(b^(1/2)+x*c^(1/2
))/(c*x^4+b*x^2)^(1/2)+x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/c
os(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))
^2)^(1/2)/b^(3/4)/c^(3/4)/(c*x^4+b*x^2)^(1/2)-1/2*x*(cos(2*arctan(c^(1/4)*x
^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(
sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c
*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/c^(3/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2023, 2032, 329, 305, 220, 1196}

$$-\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}+\frac{x^{5/}}{b\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] x^(5/2)/(b*Sqrt[b*x^2 + c*x^4]) - (x^(3/2)*(b + c*x^2))/(b*Sqrt[c]*(Sqrt[b]
 + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) + (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x
^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
 1/2])/(b^(3/4)*c^(3/4)*Sqrt[b*x^2 + c*x^4]) - (x*(Sqrt[b] + Sqrt[c]*x)*Sqr
t[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])
/b^(1/4)], 1/2])/(2*b^(3/4)*c^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{x^{5/2}}{b\sqrt{bx^2+cx^4}} - \frac{\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{2b}$$

$$= \frac{x^{5/2}}{b\sqrt{bx^2+cx^4}} - \frac{\left(x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{2b\sqrt{bx^2+cx^4}}$$

$$= \frac{x^{5/2}}{b\sqrt{bx^2+cx^4}} - \frac{\left(x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{b\sqrt{bx^2+cx^4}}$$

$$= \frac{x^{5/2}}{b\sqrt{bx^2+cx^4}} - \frac{\left(x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{b}\,\sqrt{c}\,\sqrt{bx^2+cx^4}} + \frac{\left(x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{b}}}{\sqrt{b+cx^4}} \cdots\right.}{\sqrt{b}\,\sqrt{c}\,\sqrt{bx^2+cx^4}}$$

$$= \frac{x^{5/2}}{b\sqrt{bx^2+cx^4}} - \frac{x^{3/2}\left(b+cx^2\right)}{b\sqrt{c}\,\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\cdots\right.\right.}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.02, size = 60, normalized size = 0.23

$$\frac{2x^{5/2}\sqrt{\frac{cx^2}{b}+1}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^2}{b}\right)}{3b\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^(5/2)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((c*x^2)/b
)])/(3*b*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{x}}{c^2x^5+2\,bcx^3+b^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c^2*x^5 + 2*b*c*x^3 + b^2*x), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")

[Out] integrate(x^(7/2)/(c*x^4 + b*x^2)^(3/2), x)

maple [A] time = 0.01, size = 203, normalized size = 0.78

$$-\frac{\left(c\,x^2 + b\right)\left(-2c\,x^2 + 2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}, \frac{\sqrt{2}}{2}\right) - \sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx}{}}\right.}{2\left(c\,x^4 + b\,x^2\right)^{\frac{3}{2}}\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)/(c*x^4+b*x^2)^(3/2),x)

[Out] -1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b-((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b-2*c*x^2)/c/b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")

[Out] integrate(x^(7/2)/(c*x^4 + b*x^2)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{7/2}}{\left(c\,x^4 + b\,x^2\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^(7/2)/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}}}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**(7/2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.397 $\int \frac{x^{5/2}}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{5/4}\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}+\frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}$$

```
[Out] x^(3/2)/b/(c*x^4+b*x^2)^(1/2)+1/2*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))
^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(
1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)
+x*c^(1/2))^2)^(1/2)/b^(5/4)/c^(1/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2023, 2032, 329, 220}

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{5/4}\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}+\frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] x^(3/2)/(b*Sqrt[b*x^2 + c*x^4]) + (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)
/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/
2])/(2*b^(5/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}+\frac{\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{2b}\\
&= \frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}+\frac{\left(x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{b+cx^2}}\,dx}{2b\sqrt{bx^2+cx^4}}\\
&= \frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}+\frac{\left(x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{b\sqrt{bx^2+cx^4}}\\
&= \frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{5/4}\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 60, normalized size = 0.51

$$\frac{x^{3/2}\left(\sqrt{\frac{cx^2}{b}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)+1\right)}{b\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(1 + Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2
)/b)]))/(b*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\sqrt{x}}{c^2x^6+2\,bcx^4+b^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c^2*x^6 + 2*b*c*x^4 + b^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)/(c*x^4 + b*x^2)^(3/2), x)
```

maple [A] time = 0.02, size = 123, normalized size = 1.04

$$\frac{\left(c\,x^2+b\right)\left(2cx+\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\right)x^{\frac{5}{2}}}{2\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*((-b*c)^(1/2)*((c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(
-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/
2*2^(1/2))+2*c*x)/c/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate(x^(5/2)/(c*x^4 + b*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2 + c*x^4)^(3/2),x)
[Out] int(x^(5/2)/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(x**(5/2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.398 $\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=286

$$\frac{3\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{bx^2+cx^4}}-\frac{3\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{7/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 3*x^(3/2)*(c*x^2+b)*c^(1/2)/b^2/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+x^(
1/2)/b/(c*x^4+b*x^2)^(1/2)-3*(c*x^4+b*x^2)^(1/2)/b^2/x^(3/2)-3*c^(1/4)*x*(c
os(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)
/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b
^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(7/4)/(c*x^4+b*
x^2)^(1/2)+3/2*c^(1/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/c
os(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))
^2)^(1/2)/b^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2023, 2025, 2032, 329, 305, 220, 1196}

$$\frac{3\sqrt{c}\,x^{3/2}\left(b+cx^2\right)}{b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{b^2x^{3/2}}+\frac{3\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{bx^2+cx^4}}-\frac{3\sqrt[4]{c}\,x\left(\sqrt{b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] Sqrt[x]/(b*Sqrt[b*x^2 + c*x^4]) + (3*Sqrt[c]*x^(3/2)*(b + c*x^2))/(b^2*(Sqr
t[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (3*Sqrt[b*x^2 + c*x^4])/(b^2*x^(3/
2)) - (3*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c
]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(7/4)*Sqrt[
b*x^2 + c*x^4]) + (3*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt
[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2
*b^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
```

```
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{3\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{2b}$$

$$= \frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}} - \frac{3\sqrt{bx^2+cx^4}}{b^2x^{3/2}} + \frac{(3c)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{2b^2}$$

$$= \frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}} - \frac{3\sqrt{bx^2+cx^4}}{b^2x^{3/2}} + \frac{\left(3cx\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{2b^2\sqrt{bx^2+cx^4}}$$

$$= \frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}} - \frac{3\sqrt{bx^2+cx^4}}{b^2x^{3/2}} + \frac{\left(3cx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{b^2\sqrt{bx^2+cx^4}}$$

$$= \frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}} - \frac{3\sqrt{bx^2+cx^4}}{b^2x^{3/2}} + \frac{\left(3\sqrt{c}\,x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{b^{3/2}\sqrt{bx^2+cx^4}} - \frac{\left(3\sqrt{c}\,x\right.}{}$$

$$= \frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{3\sqrt{c}\,x^{3/2}\left(b+cx^2\right)}{b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{3\sqrt{bx^2+cx^4}}{b^2x^{3/2}} - \frac{3\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{}{(\sqrt{}}}}{b^{7/4}\sqrt{}}$$

Mathematica [C] time = 0.02, size = 58, normalized size = 0.20

$$-\frac{2\sqrt{x}\sqrt{\frac{cx^2}{b}+1}\;{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{cx^2}{b}\right)}{b\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((c*x^2)
/b)])/(b*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{x}}{c^2x^7+2\,bcx^5+b^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c^2*x^7 + 2*b*c*x^5 + b^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/(c*x^4 + b*x^2)^(3/2), x)
```

maple [A] time = 0.02, size = 203, normalized size = 0.71

$$\frac{\left(c\,x^2+b\right)\left(-6c\,x^2+6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b\,\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+}{\sqrt{-}}}\right.}{2\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(6*((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)
*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*
b-3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b-6*c*x^2-4*b)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(c*x^4 + b*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3/2}}{\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^(3/2)/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**(3/2)/(x**2*(b + c*x**2))**(3/2), x)
```

3.399 $\int \frac{\sqrt{x}}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{5c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{3b^2x^{5/2}}+\frac{1}{b\sqrt{x}\,\sqrt{bx^2+cx^4}}$$

```
[Out] 1/b/x^(1/2)/(c*x^4+b*x^2)^(1/2)-5/3*(c*x^4+b*x^2)^(1/2)/b^2/x^(5/2)-5/6*c^(
3/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4
)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^
(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(9/4)/
(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2023, 2025, 2032, 329, 220}

$$-\frac{5c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{3b^2x^{5/2}}+\frac{1}{b\sqrt{x}\,\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] 1/(b*Sqrt[x]*Sqrt[b*x^2 + c*x^4]) - (5*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^(5/2))
 - (5*c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x
)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(6*b^(9/4)*Sqrt[b
*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}} + \frac{5\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{2b}$$

$$= \frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5\sqrt{bx^2+cx^4}}{3b^2x^{5/2}} - \frac{(5c)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{6b^2}$$

$$= \frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5\sqrt{bx^2+cx^4}}{3b^2x^{5/2}} - \frac{\left(5cx\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{6b^2\sqrt{bx^2+cx^4}}$$

$$= \frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5\sqrt{bx^2+cx^4}}{3b^2x^{5/2}} - \frac{\left(5cx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{3b^2\sqrt{bx^2+cx^4}}$$

$$= \frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5\sqrt{bx^2+cx^4}}{3b^2x^{5/2}} - \frac{5c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right.}{6b^{9/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.02, size = 60, normalized size = 0.41

$$-\frac{2\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};-\frac{cx^2}{b}\right)}{3b\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((c*x^2)/b)])/(3
*b*Sqrt[x]*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{x}}{c^2x^8+2\,bcx^6+b^2x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c^2*x^8 + 2*b*c*x^6 + b^2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/(c*x^4 + b*x^2)^(3/2), x)
```

maple [A] time = 0.02, size = 127, normalized size = 0.88

$$-\frac{\left(c\,x^2+b\right)\left(10c\,x^2+5\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,x\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)+4b\right)x^{\frac{3}{2}}}{6\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/6/(c*x^4+b*x^2)^(3/2)*x^(3/2)*(c*x^2+b)*(5*(-b*c)^(1/2)*((c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-
1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
,1/2*2^(1/2))*x+10*c*x^2+4*b)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(c*x^4 + b*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x}}{\left(c\,x^4 + b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^(1/2)/(b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(sqrt(x)/(x**2*(b + c*x**2))**(3/2), x)
```

3.400 $\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=320

$$-\frac{21c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10b^{11/4}\sqrt{bx^2+cx^4}}+\frac{21c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.\right.}{5b^{11/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 1/b/x^(3/2)/(c*x^4+b*x^2)^(1/2)-21/5*c^(3/2)*x^(3/2)*(c*x^2+b)/b^3/(b^(1/2)
+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-7/5*(c*x^4+b*x^2)^(1/2)/b^2/x^(7/2)+21/5*c*
(c*x^4+b*x^2)^(1/2)/b^3/x^(3/2)+21/5*c^(5/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2
*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+
b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(11/4)/(c*x^4+b*x^2)^(1/2)-21/10*c^(5/4)*
x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(
1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2)
)*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(11/4)/(c*x
^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2023, 2025, 2032, 329, 305, 220, 1196}

$$-\frac{21c^{3/2}x^{3/2}\left(b+cx^2\right)}{5b^3\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{21c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10b^{11/4}\sqrt{bx^2+cx^4}}+\frac{21c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^(3/2)*Sqrt[b*x^2 + c*x^4]) - (21*c^(3/2)*x^(3/2)*(b + c*x^2))/(5*b^3
*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (7*Sqrt[b*x^2 + c*x^4])/(5*b^
2*x^(7/2)) + (21*c*Sqrt[b*x^2 + c*x^4])/(5*b^3*x^(3/2)) + (21*c^(5/4)*x*(Sq
rt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*Ar
cTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(11/4)*Sqrt[b*x^2 + c*x^4]) - (
21*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(10*b^(11/4)*Sqrt[b*
x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}+\frac{7\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx}{2b}$$

$$= \frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}-\frac{7\sqrt{bx^2+cx^4}}{5b^2x^{7/2}}-\frac{(21c)\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{10b^2}$$

$$= \frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}-\frac{7\sqrt{bx^2+cx^4}}{5b^2x^{7/2}}+\frac{21c\sqrt{bx^2+cx^4}}{5b^3x^{3/2}}-\frac{\left(21c^2\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{10b^3}$$

$$= \frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}-\frac{7\sqrt{bx^2+cx^4}}{5b^2x^{7/2}}+\frac{21c\sqrt{bx^2+cx^4}}{5b^3x^{3/2}}-\frac{\left(21c^2x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{10b^3\sqrt{bx^2+cx^4}}$$

$$= \frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}-\frac{7\sqrt{bx^2+cx^4}}{5b^2x^{7/2}}+\frac{21c\sqrt{bx^2+cx^4}}{5b^3x^{3/2}}-\frac{\left(21c^2x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \ldots\right.}{5b^3\sqrt{bx^2+cx^4}}$$

$$= \frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}-\frac{7\sqrt{bx^2+cx^4}}{5b^2x^{7/2}}+\frac{21c\sqrt{bx^2+cx^4}}{5b^3x^{3/2}}-\frac{\left(21c^{3/2}x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \ldots\right.}{5b^{5/2}\sqrt{bx^2+cx^4}}$$

$$= \frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}-\frac{21c^{3/2}x^{3/2}\left(b+cx^2\right)}{5b^3\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{7\sqrt{bx^2+cx^4}}{5b^2x^{7/2}}+\frac{21c\sqrt{bx^2+cx^4}}{5b^3x^{3/2}}$$

Mathematica [C] time = 0.02, size = 60, normalized size = 0.19

$$-\frac{2\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(-\frac{5}{4},\frac{3}{2};-\frac{1}{4};-\frac{cx^2}{b}\right)}{5bx^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-5/4, 3/2, -1/4, -((c*x^2)/b)])/(
5*b*x^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{x}}{c^2x^9+2\,bcx^7+b^2x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c^2*x^9 + 2*b*c*x^7 + b^2*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*sqrt(x)), x)
```

maple [A] time = 0.02, size = 222, normalized size = 0.69

$$-\frac{\left(c\,x^2+b\right)\left(-42c^2x^4+42\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,bc\,x^2\,\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-21\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{\cdots}\right.}{10\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^(3/2)/x^(1/2),x)
```

```
[Out] -1/10/(c*x^4+b*x^2)^(3/2)*x^(1/2)*(c*x^2+b)*(42*(-1/(-b*c)^(1/2)*c*x)^(1/2)
*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*x^2*b*c-21*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/
2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*x^2*b*c-42*c^2*x^4-28*b*c*x^2+4
*b^2)/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{x}\,\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x^2 + c*x^4)^(3/2)),x)
[Out] int(1/(x^(1/2)*(b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**(3/2)/x**(1/2),x)
[Out] Integral(1/(sqrt(x)*(x**2*(b + c*x**2))**(3/2)), x)
```

3.401 $\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=173

$$\frac{15c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{14b^{13/4}\sqrt{bx^2+cx^4}}+\frac{15c\sqrt{bx^2+cx^4}}{7b^3x^{5/2}}-\frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}}+\frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}}$$

```
[Out] 1/b/x^(5/2)/(c*x^4+b*x^2)^(1/2)-9/7*(c*x^4+b*x^2)^(1/2)/b^2/x^(9/2)+15/7*c*
(c*x^4+b*x^2)^(1/2)/b^3/x^(5/2)+15/14*c^(7/4)*x*(cos(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2
+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(13/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2023, 2025, 2032, 329, 220}

$$\frac{15c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{14b^{13/4}\sqrt{bx^2+cx^4}}+\frac{15c\sqrt{bx^2+cx^4}}{7b^3x^{5/2}}-\frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}}+\frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^(5/2)*Sqrt[b*x^2 + c*x^4]) - (9*Sqrt[b*x^2 + c*x^4])/(7*b^2*x^(9/2))
 + (15*c*Sqrt[b*x^2 + c*x^4])/(7*b^3*x^(5/2)) + (15*c^(7/4)*x*(Sqrt[b] + Sq
rt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1
/4)*Sqrt[x])/b^(1/4)], 1/2])/(14*b^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}} + \frac{9\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx}{2b}$$

$$= \frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}} - \frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}} - \frac{(45c)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{14b^2}$$

$$= \frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}} - \frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}} + \frac{15c\sqrt{bx^2+cx^4}}{7b^3x^{5/2}} + \frac{\left(15c^2\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{14b^3}$$

$$= \frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}} - \frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}} + \frac{15c\sqrt{bx^2+cx^4}}{7b^3x^{5/2}} + \frac{\left(15c^2x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,d}{14b^3\sqrt{bx^2+cx^4}}$$

$$= \frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}} - \frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}} + \frac{15c\sqrt{bx^2+cx^4}}{7b^3x^{5/2}} + \frac{\left(15c^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+}}\right.}{7b^3\sqrt{bx^2+cx^4}}$$

$$= \frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}} - \frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}} + \frac{15c\sqrt{bx^2+cx^4}}{7b^3x^{5/2}} + \frac{15c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\right.}}}{14b^{13/4}\sqrt{bx^2}}$$

Mathematica [C] time = 0.02, size = 60, normalized size = 0.35

$$-\frac{2\sqrt{\frac{cx^2}{b}+1}\;{}_2F_1\left(-\frac{7}{4},\frac{3}{2};-\frac{3}{4};-\frac{cx^2}{b}\right)}{7bx^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-7/4, 3/2, -3/4, -((c*x^2)/b)])/(
7*b*x^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{x}}{c^2x^{10}+2\,bcx^8+b^2x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c^2*x^10 + 2*b*c*x^8 + b^2*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(3/2)), x)
```

maple [A] time = 0.02, size = 141, normalized size = 0.82

$$\frac{\left(c\,x^2+b\right)\left(30c^2x^4+15\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,c\,x^3\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)+12bc\,x^2-4}{14\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}b^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/14/(c*x^4+b*x^2)^(3/2)/x^(1/2)*(c*x^2+b)*(15*((-c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2)*2^(1/2)*x^3*c+30*c^2*x^4+12*b*c*x^2-4*b^2)/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3/2}\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(1/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**(3/2)*(x**2*(b + c*x**2))**(3/2)), x)
```

3.402 $\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=350

$$\frac{77c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{30b^{15/4}\sqrt{bx^2+cx^4}}-\frac{77c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\right.}{15b^{15/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 1/b/x^(7/2)/(c*x^4+b*x^2)^(1/2)+77/15*c^(5/2)*x^(3/2)*(c*x^2+b)/b^4/(b^(1/2
)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-11/9*(c*x^4+b*x^2)^(1/2)/b^2/x^(11/2)+77/4
5*c*(c*x^4+b*x^2)^(1/2)/b^3/x^(7/2)-77/15*c^2*(c*x^4+b*x^2)^(1/2)/b^4/x^(3/
2)-77/15*c^(9/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*a
rctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1
/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1
/2)/b^(15/4)/(c*x^4+b*x^2)^(1/2)+77/30*c^(9/4)*x*(cos(2*arctan(c^(1/4)*x^(1
/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin
(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^
2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(15/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2023, 2025, 2032, 329, 305, 220, 1196}

$$\frac{77c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b^4\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{77c^2\sqrt{bx^2+cx^4}}{15b^4x^{3/2}}+\frac{77c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{30b^{15/4}\sqrt{bx^2+cx^4}}-\frac{77c}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^(7/2)*Sqrt[b*x^2 + c*x^4]) + (77*c^(5/2)*x^(3/2)*(b + c*x^2))/(15*b^
4*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (11*Sqrt[b*x^2 + c*x^4])/(9*
b^2*x^(11/2)) + (77*c*Sqrt[b*x^2 + c*x^4])/(45*b^3*x^(7/2)) - (77*c^2*Sqrt[
b*x^2 + c*x^4])/(15*b^4*x^(3/2)) - (77*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/
b^(1/4)], 1/2])/(15*b^(15/4)*Sqrt[b*x^2 + c*x^4]) + (77*c^(9/4)*x*(Sqrt[b]
+ Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(
c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(30*b^(15/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{11\int \frac{1}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx}{2b}$$

$$= \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}} - \frac{(77c)\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx}{18b^2}$$

$$= \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}} + \frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} + \frac{\left(77c^2\right)\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{30b^3}$$

$$= \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}} + \frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} - \frac{77c^2\sqrt{bx^2+cx^4}}{15b^4x^{3/2}} + \frac{\left(77c^3\right)\int}{3}$$

$$= \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}} + \frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} - \frac{77c^2\sqrt{bx^2+cx^4}}{15b^4x^{3/2}} + \frac{\left(77c^3x\sqrt{\right.}{30}$$

$$= \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}} + \frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} - \frac{77c^2\sqrt{bx^2+cx^4}}{15b^4x^{3/2}} + \frac{\left(77c^3x\sqrt{\right.}{}$$

$$= \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}} + \frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} - \frac{77c^2\sqrt{bx^2+cx^4}}{15b^4x^{3/2}} + \frac{\left(77c^{5/2}x\right.}{}$$

$$= \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{77c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b^4\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}} + \frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{7/2}}$$

Mathematica [C] time = 0.02, size = 60, normalized size = 0.17

$$-\frac{2\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(-\frac{9}{4},\frac{3}{2};-\frac{5}{4};-\frac{cx^2}{b}\right)}{9bx^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-9/4, 3/2, -5/4, -((c*x^2)/b)])/(
9*b*x^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{x}}{c^2x^{11}+2\,bcx^9+b^2x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c^2*x^11 + 2*b*c*x^9 + b^2*x^7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(5/2)), x)
```

maple [A] time = 0.02, size = 237, normalized size = 0.68

$$\frac{\left(c\,x^2+b\right)\left(-462c^3x^6+462\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,b\,c^2x^4\,\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-231\sqrt{\frac{cx+\sqrt{-b}}{\sqrt{-bc}}}\right.}{90\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}b^4x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/90/(c*x^4+b*x^2)^(3/2)/x^(3/2)*(c*x^2+b)*(462*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1
/2)*c*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*x^4*b*c^2-231*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-1/(-b*c)^(1/2)*c*x)^(1/2)*EllipticF(((c*x+
```

```
(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b*c^2-462*c^3*x^6-308*b*
c^2*x^4+44*b^2*c*x^2-20*b^3)/b^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{5/2}\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x)
[Out] int(1/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}}\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(1/(x**(5/2)*(x**2*(b + c*x**2))**(3/2)), x)
```

3.403 $\int (cx)^m \left(bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=73

$$\frac{b^3x^7(cx)^m}{m+7}+\frac{3b^2cx^9(cx)^m}{m+9}+\frac{3bc^2x^{11}(cx)^m}{m+11}+\frac{c^3x^{13}(cx)^m}{m+13}$$

```
[Out] b^3*x^7*(c*x)^m/(7+m)+3*b^2*c*x^9*(c*x)^m/(9+m)+3*b*c^2*x^11*(c*x)^m/(11+m)
+c^3*x^13*(c*x)^m/(13+m)
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1142, 1584, 270}

$$\frac{3b^2cx^9(cx)^m}{m+9}+\frac{b^3x^7(cx)^m}{m+7}+\frac{3bc^2x^{11}(cx)^m}{m+11}+\frac{c^3x^{13}(cx)^m}{m+13}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (b^3*x^7*(c*x)^m)/(7 + m) + (3*b^2*c*x^9*(c*x)^m)/(9 + m) + (3*b*c^2*x^11*(
c*x)^m)/(11 + m) + (c^3*x^13*(c*x)^m)/(13 + m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int (cx)^m \left(bx^2+cx^4\right)^3 dx = (x^{-m}(cx)^m)\,\text{Subst}\left(\int x^m \left(bx^2+cx^4\right)^3 dx, x, x\right)$$
$$= (x^{-m}(cx)^m)\,\text{Subst}\left(\int x^{6+m} \left(b+cx^2\right)^3 dx, x, x\right)$$
$$= (x^{-m}(cx)^m)\,\text{Subst}\left(\int \left(b^3x^{6+m}+3b^2cx^{8+m}+3bc^2x^{10+m}+c^3x^{12+m}\right) dx, x, x\right)$$
$$= \frac{b^3x^7(cx)^m}{7+m}+\frac{3b^2cx^9(cx)^m}{9+m}+\frac{3bc^2x^{11}(cx)^m}{11+m}+\frac{c^3x^{13}(cx)^m}{13+m}$$

Mathematica [A] time = 0.04, size = 59, normalized size = 0.81

$$x^7(cx)^m\left(\frac{b^3}{m+7}+\frac{3b^2cx^2}{m+9}+\frac{3bc^2x^4}{m+11}+\frac{c^3x^6}{m+13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(b*x^2 + c*x^4)^3,x]
```

```
[Out] x^7*(c*x)^m*(b^3/(7 + m) + (3*b^2*c*x^2)/(9 + m) + (3*b*c^2*x^4)/(11 + m) +
 (c^3*x^6)/(13 + m))
```

fricas [B] time = 0.59, size = 161, normalized size = 2.21

$$\frac{\left(\left(c^3m^3+27\,c^3m^2+239\,c^3m+693\,c^3\right)x^{13}+3\left(bc^2m^3+29\,bc^2m^2+271\,bc^2m+819\,bc^2\right)x^{11}+3\left(b^2cm^3+31\,b^2c\right.\right.}{m^4+40\,m^3+590\,m^2+3800\,m+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] ((c^3*m^3 + 27*c^3*m^2 + 239*c^3*m + 693*c^3)*x^13 + 3*(b*c^2*m^3 + 29*b*c^
2*m^2 + 271*b*c^2*m + 819*b*c^2)*x^11 + 3*(b^2*c*m^3 + 31*b^2*c*m^2 + 311*b
^2*c*m + 1001*b^2*c)*x^9 + (b^3*m^3 + 33*b^3*m^2 + 359*b^3*m + 1287*b^3)*x^
7)*(c*x)^m/(m^4 + 40*m^3 + 590*m^2 + 3800*m + 9009)
```

giac [B] time = 0.18, size = 264, normalized size = 3.62

$$\frac{(cx)^m\,c^3m^3x^{13}+27\,(cx)^m\,c^3m^2x^{13}+3\,(cx)^m\,bc^2m^3x^{11}+239\,(cx)^m\,c^3mx^{13}+87\,(cx)^m\,bc^2m^2x^{11}+693\,(cx)^m\,c^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] ((c*x)^m*c^3*m^3*x^13 + 27*(c*x)^m*c^3*m^2*x^13 + 3*(c*x)^m*b*c^2*m^3*x^11
+ 239*(c*x)^m*c^3*m*x^13 + 87*(c*x)^m*b*c^2*m^2*x^11 + 693*(c*x)^m*c^3*x^13
 + 3*(c*x)^m*b^2*c*m^3*x^9 + 813*(c*x)^m*b*c^2*m*x^11 + 93*(c*x)^m*b^2*c*m^
2*x^9 + 2457*(c*x)^m*b*c^2*x^11 + (c*x)^m*b^3*m^3*x^7 + 933*(c*x)^m*b^2*c*m
*x^9 + 33*(c*x)^m*b^3*m^2*x^7 + 3003*(c*x)^m*b^2*c*x^9 + 359*(c*x)^m*b^3*m*
x^7 + 1287*(c*x)^m*b^3*x^7)/(m^4 + 40*m^3 + 590*m^2 + 3800*m + 9009)
```

maple [B] time = 0.01, size = 181, normalized size = 2.48

$$\frac{\left(c^3m^3x^6 + 27c^3m^2x^6 + 3b\,c^2m^3x^4 + 239c^3m\,x^6 + 87b\,c^2m^2x^4 + 693c^3x^6 + 3b^2c\,m^3x^2 + 813b\,c^2m\,x^4 + 93b^2c\,m^2x^2}{(m+13)\,(m+11)\,(m+9)\,(m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(c*x^4+b*x^2)^3,x)
```

```
[Out] (c*x)^m*(c^3*m^3*x^6+27*c^3*m^2*x^6+3*b*c^2*m^3*x^4+239*c^3*m*x^6+87*b*c^2*
m^2*x^4+693*c^3*x^6+3*b^2*c*m^3*x^2+813*b*c^2*m*x^4+93*b^2*c*m^2*x^2+2457*b
*c^2*x^4+b^3*m^3+933*b^2*c*m*x^2+33*b^3*m^2+3003*b^2*c*x^2+359*b^3*m+1287*b
^3)*x^7/(13+m)/(11+m)/(9+m)/(7+m)
```

maxima [A] time = 1.51, size = 76, normalized size = 1.04

$$\frac{c^{m+3}x^{13}x^m}{m+13} + \frac{3\,bc^{m+2}x^{11}x^m}{m+11} + \frac{3\,b^2c^{m+1}x^9x^m}{m+9} + \frac{b^3c^mx^7x^m}{m+7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] c^(m + 3)*x^13*x^m/(m + 13) + 3*b*c^(m + 2)*x^11*x^m/(m + 11) + 3*b^2*c^(m
+ 1)*x^9*x^m/(m + 9) + b^3*c^m*x^7*x^m/(m + 7)
```

mupad [B] time = 4.29, size = 171, normalized size = 2.34

$$(c\,x)^m\,\left(\frac{b^3\,x^7\,\left(m^3+33\,m^2+359\,m+1287\right)}{m^4+40\,m^3+590\,m^2+3800\,m+9009}+\frac{c^3\,x^{13}\,\left(m^3+27\,m^2+239\,m+693\right)}{m^4+40\,m^3+590\,m^2+3800\,m+9009}+\frac{3\,b\,c^2\,x^{11}\,\left(m^3+29\right.}{m^4+40\,m^3+590\,m}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^2 + c*x^4)^3,x)
```

```
[Out] (c*x)^m*((b^3*x^7*(359*m + 33*m^2 + m^3 + 1287))/(3800*m + 590*m^2 + 40*m^3
 + m^4 + 9009) + (c^3*x^13*(239*m + 27*m^2 + m^3 + 693))/(3800*m + 590*m^2
+ 40*m^3 + m^4 + 9009) + (3*b*c^2*x^11*(271*m + 29*m^2 + m^3 + 819))/(3800*
m + 590*m^2 + 40*m^3 + m^4 + 9009) + (3*b^2*c*x^9*(311*m + 31*m^2 + m^3 + 1
001))/(3800*m + 590*m^2 + 40*m^3 + m^4 + 9009))
```

sympy [A] time = 5.28, size = 758, normalized size = 10.38

$$\begin{cases} \frac{-\frac{b^3}{6x^6}-\frac{3b^2c}{4x^4}-\frac{3bc^2}{2x^2}+c^3\log(x)}{c^{13}} \\ \frac{-\frac{b^3}{4x^4}-\frac{3b^2c}{2x^2}+3bc^2\log(x)+\frac{c^3x^2}{2}}{c^{11}} \\ \frac{-\frac{b^3}{2x^2}+3b^2c\log(x)+\frac{3bc^2x^2}{2}+\frac{c^3x^4}{4}}{c^9} \\ \frac{b^3\log(x)+\frac{3b^2cx^2}{2}+\frac{3bc^2x^4}{4}+\frac{c^3x^6}{6}}{c^7} \\ \frac{b^3c^mm^3x^7x^m}{m^4+40m^3+590m^2+3800m+9009}+\frac{33b^3c^mm^2x^7x^m}{m^4+40m^3+590m^2+3800m+9009}+\frac{359b^3c^mmx^7x^m}{m^4+40m^3+590m^2+3800m+9009}+\frac{1287b^3c^mx^7x^m}{m^4+40m^3+590m^2+3800m+9009} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(c*x**4+b*x**2)**3,x)
```

```
[Out] Piecewise(((-b**3/(6*x**6) - 3*b**2*c/(4*x**4) - 3*b*c**2/(2*x**2) + c**3*l
og(x))/c**13, Eq(m, -13)), ((-b**3/(4*x**4) - 3*b**2*c/(2*x**2) + 3*b*c**2*
log(x) + c**3*x**2/2)/c**11, Eq(m, -11)), ((-b**3/(2*x**2) + 3*b**2*c*log(x
) + 3*b*c**2*x**2/2 + c**3*x**4/4)/c**9, Eq(m, -9)), ((b**3*log(x) + 3*b**2
*c*x**2/2 + 3*b*c**2*x**4/4 + c**3*x**6/6)/c**7, Eq(m, -7)), (b**3*c**m*m**
3*x**7*x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 33*b**3*c**m*m**2
*x**7*x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 359*b**3*c**m*m*x*
*7*x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 1287*b**3*c**m*x**7*x
**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 3*b**2*c*c**m*m**3*x**9*x
**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 93*b**2*c*c**m*m**2*x**9*
x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 933*b**2*c*c**m*m*x**9*x
**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 3003*b**2*c*c**m*x**9*x**
m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 3*b*c**2*c**m*m**3*x**11*x*
*m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 87*b*c**2*c**m*m**2*x**11*
x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 813*b*c**2*c**m*m*x**11*
x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 2457*b*c**2*c**m*x**11*x
**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + c**3*c**m*m**3*x**13*x**m
/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 27*c**3*c**m*m**2*x**13*x**m
/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 239*c**3*c**m*m*x**13*x**m/(
m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 693*c**3*c**m*x**13*x**m/(m**4
 + 40*m**3 + 590*m**2 + 3800*m + 9009), True))
```

3.404 $\int (cx)^m \left(bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=52

$$\frac{b^2x^5(cx)^m}{m+5}+\frac{2bcx^7(cx)^m}{m+7}+\frac{c^2x^9(cx)^m}{m+9}$$

```
[Out] b^2*x^5*(c*x)^m/(5+m)+2*b*c*x^7*(c*x)^m/(7+m)+c^2*x^9*(c*x)^m/(9+m)
```

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1142, 1584, 270}

$$\frac{b^2x^5(cx)^m}{m+5}+\frac{2bcx^7(cx)^m}{m+7}+\frac{c^2x^9(cx)^m}{m+9}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b^2*x^5*(c*x)^m)/(5 + m) + (2*b*c*x^7*(c*x)^m)/(7 + m) + (c^2*x^9*(c*x)^m)
/(9 + m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int (cx)^m \left(bx^2+cx^4\right)^2 dx = (x^{-m}(cx)^m)\,\text{Subst}\left(\int x^m \left(bx^2+cx^4\right)^2 dx, x, x\right)$$
$$= (x^{-m}(cx)^m)\,\text{Subst}\left(\int x^{4+m} \left(b+cx^2\right)^2 dx, x, x\right)$$
$$= (x^{-m}(cx)^m)\,\text{Subst}\left(\int \left(b^2x^{4+m}+2bcx^{6+m}+c^2x^{8+m}\right) dx, x, x\right)$$
$$= \frac{b^2x^5(cx)^m}{5+m}+\frac{2bcx^7(cx)^m}{7+m}+\frac{c^2x^9(cx)^m}{9+m}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.83

$$x^5(cx)^m \left(\frac{b^2}{m+5}+\frac{2bcx^2}{m+7}+\frac{c^2x^4}{m+9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(b*x^2 + c*x^4)^2,x]
```

```
[Out] x^5*(c*x)^m*(b^2/(5 + m) + (2*b*c*x^2)/(7 + m) + (c^2*x^4)/(9 + m))
```

fricas [A] time = 0.90, size = 89, normalized size = 1.71

$$\frac{\left(\left(c^2m^2+12\,c^2m+35\,c^2\right)x^9+2\left(bcm^2+14\,bcm+45\,bc\right)x^7+\left(b^2m^2+16\,b^2m+63\,b^2\right)x^5\right)(cx)^m}{m^3+21\,m^2+143\,m+315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] ((c^2*m^2 + 12*c^2*m + 35*c^2)*x^9 + 2*(b*c*m^2 + 14*b*c*m + 45*b*c)*x^7 +
(b^2*m^2 + 16*b^2*m + 63*b^2)*x^5)*(c*x)^m/(m^3 + 21*m^2 + 143*m + 315)
```

giac [B] time = 0.17, size = 141, normalized size = 2.71

$$\frac{(cx)^m\,c^2m^2x^9+12\,(cx)^m\,c^2mx^9+2\,(cx)^m\,bcm^2x^7+35\,(cx)^m\,c^2x^9+28\,(cx)^m\,bcmx^7+(cx)^m\,b^2m^2x^5+90\,(cx)^m}{m^3+21\,m^2+143\,m+315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] ((c*x)^m*c^2*m^2*x^9 + 12*(c*x)^m*c^2*m*x^9 + 2*(c*x)^m*b*c*m^2*x^7 + 35*(c
*x)^m*c^2*x^9 + 28*(c*x)^m*b*c*m*x^7 + (c*x)^m*b^2*m^2*x^5 + 90*(c*x)^m*b*c
```

```
*x^7 + 16*(c*x)^m*b^2*m*x^5 + 63*(c*x)^m*b^2*x^5)/(m^3 + 21*m^2 + 143*m + 3
15)
```

maple [A] time = 0.01, size = 96, normalized size = 1.85

$$\frac{\left(c^2m^2x^4+12c^2m\,x^4+2bc\,m^2x^2+35c^2x^4+28bcm\,x^2+b^2m^2+90bc\,x^2+16b^2m+63b^2\right)x^5\,(cx)^m}{(m+9)\,(m+7)\,(m+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(c*x^4+b*x^2)^2,x)
```

```
[Out] (c*x)^m*(c^2*m^2*x^4+12*c^2*m*x^4+2*b*c*m^2*x^2+35*c^2*x^4+28*b*c*m*x^2+b^2
*m^2+90*b*c*x^2+16*b^2*m+63*b^2)*x^5/(m+9)/(m+7)/(5+m)
```

maxima [A] time = 1.45, size = 55, normalized size = 1.06

$$\frac{c^{m+2}x^9x^m}{m+9}+\frac{2\,bc^{m+1}x^7x^m}{m+7}+\frac{b^2c^mx^5x^m}{m+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] c^(m + 2)*x^9*x^m/(m + 9) + 2*b*c^(m + 1)*x^7*x^m/(m + 7) + b^2*c^m*x^5*x^m
/(m + 5)
```

mupad [B] time = 4.19, size = 97, normalized size = 1.87

$$(c\,x)^m\,\left(\frac{b^2\,x^5\,\left(m^2+16\,m+63\right)}{m^3+21\,m^2+143\,m+315}+\frac{c^2\,x^9\,\left(m^2+12\,m+35\right)}{m^3+21\,m^2+143\,m+315}+\frac{2\,b\,c\,x^7\,\left(m^2+14\,m+45\right)}{m^3+21\,m^2+143\,m+315}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^2 + c*x^4)^2,x)
```

```
[Out] (c*x)^m*((b^2*x^5*(16*m + m^2 + 63))/(143*m + 21*m^2 + m^3 + 315) + (c^2*x^
9*(12*m + m^2 + 35))/(143*m + 21*m^2 + m^3 + 315) + (2*b*c*x^7*(14*m + m^2
+ 45))/(143*m + 21*m^2 + m^3 + 315))
```

sympy [A] time = 2.29, size = 352, normalized size = 6.77

$$\begin{cases} \frac{-\frac{b^2}{4x^4}-\frac{bc}{x^2}+c^2\log(x)}{c^9} \\ \frac{-\frac{b^2}{2x^2}+2bc\log(x)+\frac{c^2x^2}{2}}{c^7} \\ \frac{b^2\log(x)+bcx^2+\frac{c^2x^4}{4}}{c^5} \\ \frac{b^2c^m m^2x^5x^m}{m^3+21m^2+143m+315}+\frac{16b^2c^m mx^5x^m}{m^3+21m^2+143m+315}+\frac{63b^2c^m x^5x^m}{m^3+21m^2+143m+315}+\frac{2bcc^m m^2x^7x^m}{m^3+21m^2+143m+315}+\frac{28bcc^m mx^7x^m}{m^3+21m^2+143m+315}+\frac{90bcc^m}{m^3+21m^2+1} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(c*x**4+b*x**2)**2,x)
```

```
[Out] Piecewise(((-b**2/(4*x**4) - b*c/x**2 + c**2*log(x))/c**9, Eq(m, -9)), ((-b
**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2/2)/c**7, Eq(m, -7)), ((b**2*log(x)
+ b*c*x**2 + c**2*x**4/4)/c**5, Eq(m, -5)), (b**2*c**m*m**2*x**5*x**m/(m**3
 + 21*m**2 + 143*m + 315) + 16*b**2*c**m*m*x**5*x**m/(m**3 + 21*m**2 + 143*
m + 315) + 63*b**2*c**m*x**5*x**m/(m**3 + 21*m**2 + 143*m + 315) + 2*b*c*c*
*m*m**2*x**7*x**m/(m**3 + 21*m**2 + 143*m + 315) + 28*b*c*c**m*m*x**7*x**m/
(m**3 + 21*m**2 + 143*m + 315) + 90*b*c*c**m*x**7*x**m/(m**3 + 21*m**2 + 14
3*m + 315) + c**2*c**m*m**2*x**9*x**m/(m**3 + 21*m**2 + 143*m + 315) + 12*c
**2*c**m*m*x**9*x**m/(m**3 + 21*m**2 + 143*m + 315) + 35*c**2*c**m*x**9*x**
m/(m**3 + 21*m**2 + 143*m + 315), True))
```

3.405 $\int (cx)^m \left(bx^2+cx^4\right) dx$

Optimal. Leaf size=34

$$\frac{b(cx)^{m+3}}{c^3(m+3)}+\frac{(cx)^{m+5}}{c^4(m+5)}$$

```
[Out] b*(c*x)^(3+m)/c^3/(3+m)+(c*x)^(5+m)/c^4/(5+m)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{b(cx)^{m+3}}{c^3(m+3)}+\frac{(cx)^{m+5}}{c^4(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(b*x^2 + c*x^4),x]
```

```
[Out] (b*(c*x)^(3 + m))/(c^3*(3 + m)) + (c*x)^(5 + m)/(c^4*(5 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (cx)^m \left(bx^2+cx^4\right) dx &= \int \left(\frac{b(cx)^{2+m}}{c^2}+\frac{(cx)^{4+m}}{c^3}\right) dx\\ &= \frac{b(cx)^{3+m}}{c^3(3+m)}+\frac{(cx)^{5+m}}{c^4(5+m)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 0.79

$$x^3(cx)^m\left(\frac{b}{m+3}+\frac{cx^2}{m+5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(b*x^2 + c*x^4),x]
```

```
[Out] x^3*(c*x)^m*(b/(3 + m) + (c*x^2)/(5 + m))
```

fricas [A] time = 0.80, size = 39, normalized size = 1.15

$$\frac{\left((cm+3\,c)x^5+(bm+5\,b)x^3\right)(cx)^m}{m^2+8\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] ((c*m + 3*c)*x^5 + (b*m + 5*b)*x^3)*(c*x)^m/(m^2 + 8*m + 15)
```

giac [A] time = 0.20, size = 56, normalized size = 1.65

$$\frac{(cx)^m\,cmx^5+3\,(cx)^m\,cx^5+(cx)^m\,bmx^3+5\,(cx)^m\,bx^3}{m^2+8\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] ((c*x)^m*c*m*x^5 + 3*(c*x)^m*c*x^5 + (c*x)^m*b*m*x^3 + 5*(c*x)^m*b*x^3)/(m^
2 + 8*m + 15)
```

maple [A] time = 0.00, size = 39, normalized size = 1.15

$$\frac{\left(cm\,x^2+3c\,x^2+bm+5b\right)x^3\,(cx)^m}{(m+5)\,(m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(c*x^4+b*x^2),x)
```

```
[Out] (c*x)^m*(c*m*x^2+3*c*x^2+b*m+5*b)*x^3/(m+5)/(3+m)
```

maxima [A] time = 1.43, size = 34, normalized size = 1.00

$$\frac{c^{m+1}x^5x^m}{m+5}+\frac{bc^mx^3x^m}{m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] c^(m + 1)*x^5*x^m/(m + 5) + b*c^m*x^3*x^m/(m + 3)
```

mupad [B] time = 4.15, size = 38, normalized size = 1.12

$$\frac{x^3\,(c\,x)^m\,\left(5\,b+b\,m+3\,c\,x^2+c\,m\,x^2\right)}{m^2+8\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^2 + c*x^4),x)
```

```
[Out] (x^3*(c*x)^m*(5*b + b*m + 3*c*x^2 + c*m*x^2))/(8*m + m^2 + 15)
```

sympy [A] time = 0.76, size = 119, normalized size = 3.50

$$\begin{cases} \frac{-\frac{b}{2x^2}+c\log(x)}{c^5} & \text{for } m=-5 \\ \frac{b\log(x)+\frac{cx^2}{2}}{c^3} & \text{for } m=-3 \\ \frac{bc^m m x^3 x^m}{m^2+8m+15}+\frac{5bc^m x^3 x^m}{m^2+8m+15}+\frac{cc^m m x^5 x^m}{m^2+8m+15}+\frac{3cc^m x^5 x^m}{m^2+8m+15} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((((-b/(2*x**2) + c*log(x))/c**5, Eq(m, -5)), ((b*log(x) + c*x**2/2
)/c**3, Eq(m, -3)), (b*c**m*m*x**3*x**m/(m**2 + 8*m + 15) + 5*b*c**m*x**3*x
**m/(m**2 + 8*m + 15) + c*c**m*m*x**5*x**m/(m**2 + 8*m + 15) + 3*c*c**m*x**
5*x**m/(m**2 + 8*m + 15), True))
```

3.406 $\int \frac{(cx)^m}{bx^2+cx^4} dx$

Optimal. Leaf size=45

$$-\frac{(cx)^m {}_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\frac{cx^2}{b}\right)}{b(1-m)x}$$

```
[Out] -(c*x)^m*hypergeom([1, -1/2+1/2*m],[1/2+1/2*m],-c*x^2/b)/b/(1-m)/x
```

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.158, Rules used = {1142, 1584, 364}

$$-\frac{(cx)^m {}_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\frac{cx^2}{b}\right)}{b(1-m)x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(b*x^2 + c*x^4),x]
```

```
[Out] -(((c*x)^m*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -((c*x^2)/b)])/(b*(1
 - m)*x))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(cx)^m}{bx^2+cx^4}\,dx = (x^{-m}(cx)^m)\,\text{Subst}\left(\int \frac{x^m}{bx^2+cx^4}\,dx, x, x\right)$$
$$= (x^{-m}(cx)^m)\,\text{Subst}\left(\int \frac{x^{-2+m}}{b+cx^2}\,dx, x, x\right)$$
$$= -\frac{(cx)^m\,{}_2F_1\left(1, \frac{1}{2}(-1+m); \frac{1+m}{2}; -\frac{cx^2}{b}\right)}{b(1-m)x}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.93

$$\frac{(cx)^m\,{}_2F_1\left(1, \frac{m-1}{2}; \frac{m+1}{2}; -\frac{cx^2}{b}\right)}{b(m-1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(b*x^2 + c*x^4),x]
```

```
[Out] ((c*x)^m*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -((c*x^2)/b)])/(b*(-1
+ m)*x)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx)^m}{cx^4+bx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] integral((c*x)^m/(c*x^4 + b*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{cx^4+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/(c*x^4 + b*x^2), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{m}}{c\,x^{4}+b\,x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(c*x^4+b*x^2),x)
[Out] int((c*x)^m/(c*x^4+b*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{m}}{cx^{4}+bx^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] integrate((c*x)^m/(c*x^4 + b*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^{m}}{c\,x^{4}+b\,x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(b*x^2 + c*x^4),x)
[Out] int((c*x)^m/(b*x^2 + c*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{m}}{x^{2}\left(b+cx^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(c*x**4+b*x**2),x)
[Out] Integral((c*x)**m/(x**2*(b + c*x**2)), x)
```

3.407 $\int \frac{(cx)^m}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=45

$$-\frac{(cx)^m\,{}_2F_1\left(2,\frac{m-3}{2};\frac{m-1}{2};-\frac{cx^2}{b}\right)}{b^2(3-m)x^3}$$

```
[Out] -(c*x)^m*hypergeom([2, -3/2+1/2*m],[-1/2+1/2*m],-c*x^2/b)/b^2/(3-m)/x^3
```

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1142, 1584, 364}

$$-\frac{(cx)^m\,{}_2F_1\left(2,\frac{m-3}{2};\frac{m-1}{2};-\frac{cx^2}{b}\right)}{b^2(3-m)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -(((c*x)^m*Hypergeometric2F1[2, (-3 + m)/2, (-1 + m)/2, -((c*x^2)/b)])/(b^2
*(3 - m)*x^3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(cx)^m}{\left(bx^2+cx^4\right)^2}\,dx = (x^{-m}(cx)^m)\,\text{Subst}\left(\int \frac{x^m}{\left(bx^2+cx^4\right)^2}\,dx,x,x\right)$$
$$= (x^{-m}(cx)^m)\,\text{Subst}\left(\int \frac{x^{-4+m}}{\left(b+cx^2\right)^2}\,dx,x,x\right)$$
$$= -\frac{(cx)^m\,{}_2F_1\left(2,\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);-\frac{cx^2}{b}\right)}{b^2(3-m)x^3}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 0.98

$$\frac{(cx)^m\,{}_2F_1\left(2,\frac{m-3}{2};\frac{m-3}{2}+1;-\frac{cx^2}{b}\right)}{b^2(m-3)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((c*x)^m*Hypergeometric2F1[2, (-3 + m)/2, 1 + (-3 + m)/2, -((c*x^2)/b)])/(b
^2*(-3 + m)*x^3)
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx)^m}{c^2x^8+2\,bcx^6+b^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] integral((c*x)^m/(c^2*x^8 + 2*b*c*x^6 + b^2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{\left(cx^4+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/(c*x^4 + b*x^2)^2, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{(c\,x^4 + b\,x^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(c*x^4+b*x^2)^2,x)
[Out] int((c*x)^m/(c*x^4+b*x^2)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{(cx^4 + bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2)^2,x, algorithm="maxima")
[Out] integrate((c*x)^m/(c*x^4 + b*x^2)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^m}{(c\,x^4 + b\,x^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(b*x^2 + c*x^4)^2,x)
[Out] int((c*x)^m/(b*x^2 + c*x^4)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{x^4\,(b + cx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(c*x**4+b*x**2)**2,x)
[Out] Integral((c*x)**m/(x**4*(b + c*x**2)**2), x)
```

3.408 $\int \frac{(cx)^m}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=45

$$-\frac{(cx)^m\,{}_2F_1\left(3,\frac{m-5}{2};\frac{m-3}{2};-\frac{cx^2}{b}\right)}{b^3(5-m)x^5}$$

```
[Out] -(c*x)^m*hypergeom([3, -5/2+1/2*m],[-3/2+1/2*m],-c*x^2/b)/b^3/(5-m)/x^5
```

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1142, 1584, 364}

$$-\frac{(cx)^m\,{}_2F_1\left(3,\frac{m-5}{2};\frac{m-3}{2};-\frac{cx^2}{b}\right)}{b^3(5-m)x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(((c*x)^m*Hypergeometric2F1[3, (-5 + m)/2, (-3 + m)/2, -((c*x^2)/b)])/(b^3
*(5 - m)*x^5))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(cx)^m}{\left(bx^2+cx^4\right)^3}\,dx = \left(x^{-m}(cx)^m\right)\text{Subst}\left(\int \frac{x^m}{\left(bx^2+cx^4\right)^3}\,dx,x,x\right)$$
$$= \left(x^{-m}(cx)^m\right)\text{Subst}\left(\int \frac{x^{-6+m}}{\left(b+cx^2\right)^3}\,dx,x,x\right)$$
$$= -\frac{(cx)^m\,{}_2F_1\left(3,\frac{1}{2}(-5+m);\frac{1}{2}(-3+m);-\frac{cx^2}{b}\right)}{b^3(5-m)x^5}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 0.98

$$\frac{(cx)^m\,{}_2F_1\left(3,\frac{m-5}{2};\frac{m-5}{2}+1;-\frac{cx^2}{b}\right)}{b^3(m-5)x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(b*x^2 + c*x^4)^3,x]
[Out] ((c*x)^m*Hypergeometric2F1[3, (-5 + m)/2, 1 + (-5 + m)/2, -((c*x^2)/b)])/(b
^3*(-5 + m)*x^5)
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx)^m}{c^3x^{12}+3\,bc^2x^{10}+3\,b^2cx^8+b^3x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2)^3,x, algorithm="fricas")
[Out] integral((c*x)^m/(c^3*x^12 + 3*b*c^2*x^10 + 3*b^2*c*x^8 + b^3*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{\left(cx^4+bx^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/(c*x^4 + b*x^2)^3, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{(c\,x^4+b\,x^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(c*x^4+b*x^2)^3,x)
[Out] int((c*x)^m/(c*x^4+b*x^2)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{(cx^4+bx^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2)^3,x, algorithm="maxima")
[Out] integrate((c*x)^m/(c*x^4 + b*x^2)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^m}{(c\,x^4+b\,x^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(b*x^2 + c*x^4)^3,x)
[Out] int((c*x)^m/(b*x^2 + c*x^4)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{x^6\,(b+cx^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(c*x**4+b*x**2)**3,x)
[Out] Integral((c*x)**m/(x**6*(b + c*x**2)**3), x)
```

3.409 $\int x^3 \left(a^2+2abx^2+b^2x^4\right)\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

```
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a^2+2abx^2+b^2x^4\right)\,dx&=\int\left(a^2x^3+2abx^5+b^2x^7\right)\,dx\\&=\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

fricas [A] time = 0.66, size = 24, normalized size = 0.80

$$\frac{1}{8}x^8b^2 + \frac{1}{3}x^6ba + \frac{1}{4}x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b^2 + 1/3*x^6*b*a + 1/4*x^4*a^2
```

giac [A] time = 0.15, size = 24, normalized size = 0.80

$$\frac{1}{8}b^2x^8 + \frac{1}{3}abx^6 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{8}b^2x^8 + \frac{1}{3}ab\,x^6 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

maxima [A] time = 1.36, size = 24, normalized size = 0.80

$$\frac{1}{8}b^2x^8 + \frac{1}{3}abx^6 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

mupad [B] time = 0.04, size = 24, normalized size = 0.80

$$\frac{a^2\,x^4}{4} + \frac{a\,b\,x^6}{3} + \frac{b^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (a^2*x^4)/4 + (b^2*x^8)/8 + (a*b*x^6)/3
```

sympy [A] time = 0.07, size = 24, normalized size = 0.80

$$\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8
```

3.410 $\int x^2 \left(a^2+2abx^2+b^2x^4\right) dx$

Optimal. Leaf size=30

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a^2+2abx^2+b^2x^4\right) dx &= \int \left(a^2x^2+2abx^4+b^2x^6\right) dx\\ &= \frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

fricas [A] time = 0.74, size = 24, normalized size = 0.80

$$\frac{1}{7}x^7b^2 + \frac{2}{5}x^5ba + \frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
[Out] 1/7*x^7*b^2 + 2/5*x^5*b*a + 1/3*x^3*a^2
```

giac [A] time = 0.15, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{7}b^2x^7 + \frac{2}{5}ab\,x^5 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^4+2*a*b*x^2+a^2),x)
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

maxima [A] time = 1.33, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^3}{3} + \frac{2\,a\,b\,x^5}{5} + \frac{b^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
[Out] (a^2*x^3)/3 + (b^2*x^7)/7 + (2*a*b*x^5)/5
```

sympy [A] time = 0.07, size = 26, normalized size = 0.87

$$\frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2),x)
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7
```

3.411 $\int x\left(a^2+2abx^2+b^2x^4\right)\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{b^2x^6}{6}$$

```
[Out] 1/2*a^2*x^2+1/2*a*b*x^4+1/6*b^2*x^6
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {14}

$$\frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + (b^2*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^2+b^2x^4\right)\,dx&=\int\left(a^2x+2abx^3+b^2x^5\right)\,dx\\&=\frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{b^2x^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.53

$$\frac{\left(a+bx^2\right)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a + b*x^2)^3/(6*b)
```

fricas [A] time = 0.66, size = 24, normalized size = 0.80

$$\frac{1}{6}x^6b^2 + \frac{1}{2}x^4ba + \frac{1}{2}x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
[Out] 1/6*x^6*b^2 + 1/2*x^4*b*a + 1/2*x^2*a^2
```

giac [A] time = 0.17, size = 24, normalized size = 0.80

$$\frac{1}{6}\,b^2x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{6}b^2x^6 + \frac{1}{2}ab\,x^4 + \frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2),x)
[Out] 1/2*a^2*x^2+1/2*a*b*x^4+1/6*b^2*x^6
```

maxima [A] time = 1.29, size = 24, normalized size = 0.80

$$\frac{1}{6}\,b^2x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^2}{2} + \frac{a\,b\,x^4}{2} + \frac{b^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
[Out] (a^2*x^2)/2 + (b^2*x^6)/6 + (a*b*x^4)/2
```

sympy [A] time = 0.07, size = 24, normalized size = 0.80

$$\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2),x)
[Out] a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6
```

3.412 $\int \left(a^2+2abx^2+b^2x^4\right) dx$

Optimal. Leaf size=25

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[a^2 + 2*a*b*x^2 + b^2*x^4,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rubi steps

$$\int \left(a^2+2abx^2+b^2x^4\right) dx=a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^2 + 2*a*b*x^2 + b^2*x^4,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

fricas [A] time = 0.69, size = 21, normalized size = 0.84

$$\frac{1}{5}x^5b^2+\frac{2}{3}x^3ba+xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^2*x^4+2*a*b*x^2+a^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*b^2 + 2/3*x^3*b*a + x*a^2
```

giac [A] time = 0.15, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^2*x^4+2*a*b*x^2+a^2,x, algorithm="giac")
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

maple [A] time = 0.00, size = 22, normalized size = 0.88

$$\frac{1}{5}b^2x^5+\frac{2}{3}ab\,x^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b^2*x^4+2*a*b*x^2+a^2,x)
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

maxima [A] time = 1.35, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^2*x^4+2*a*b*x^2+a^2,x, algorithm="maxima")
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

mupad [B] time = 0.03, size = 21, normalized size = 0.84

$$a^2\,x+\frac{2\,a\,b\,x^3}{3}+\frac{b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^2 + b^2*x^4 + 2*a*b*x^2,x)
[Out] a^2*x + (b^2*x^5)/5 + (2*a*b*x^3)/3
```

sympy [A] time = 0.07, size = 22, normalized size = 0.88

$$a^2x+\frac{2abx^3}{3}+\frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b**2*x**4+2*a*b*x**2+a**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

3.413 $\int \frac{a^2+2abx^2+b^2x^4}{x} \, dx$

Optimal. Leaf size=23

$$a^2 \log(x) + abx^2 + \frac{b^2x^4}{4}$$

```
[Out] a*b*x^2+1/4*b^2*x^4+a^2*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$a^2 \log(x) + abx^2 + \frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x,x]
```

```
[Out] a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{a^2 + 2abx^2 + b^2x^4}{x} \, dx &= \int \left(\frac{a^2}{x} + 2abx + b^2x^3\right) dx \\ &= abx^2 + \frac{b^2x^4}{4} + a^2 \log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$a^2 \log(x) + abx^2 + \frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x,x]
```

```
[Out] a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]
```

fricas [A] time = 0.75, size = 21, normalized size = 0.91

$$\frac{1}{4}b^2x^4 + abx^2 + a^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x,x, algorithm="fricas")
[Out] 1/4*b^2*x^4 + a*b*x^2 + a^2*log(x)
```

giac [A] time = 0.17, size = 24, normalized size = 1.04

$$\frac{1}{4}b^2x^4 + abx^2 + \frac{1}{2}a^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x,x, algorithm="giac")
[Out] 1/4*b^2*x^4 + a*b*x^2 + 1/2*a^2*log(x^2)
```

maple [A] time = 0.00, size = 22, normalized size = 0.96

$$\frac{b^2x^4}{4} + ab\,x^2 + a^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x,x)
[Out] a*b*x^2+1/4*b^2*x^4+a^2*ln(x)
```

maxima [A] time = 1.28, size = 24, normalized size = 1.04

$$\frac{1}{4}b^2x^4 + abx^2 + \frac{1}{2}a^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x,x, algorithm="maxima")
[Out] 1/4*b^2*x^4 + a*b*x^2 + 1/2*a^2*log(x^2)
```

mupad [B] time = 4.10, size = 21, normalized size = 0.91

$$a^2\,\ln(x) + \frac{b^2\,x^4}{4} + a\,b\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x,x)
```

```
[Out] a^2*log(x) + (b^2*x^4)/4 + a*b*x^2
```

sympy [A] time = 0.10, size = 20, normalized size = 0.87

$$a^2\log(x) + abx^2 + \frac{b^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x,x)
```

```
[Out] a**2*log(x) + a*b*x**2 + b**2*x**4/4
```

3.414 $\int \frac{a^2+2abx^2+b^2x^4}{x^2} dx$

Optimal. Leaf size=24

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

```
[Out] -a^2/x+2*a*b*x+1/3*b^2*x^3
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^2} dx &= \int \left(2ab+\frac{a^2}{x^2}+b^2x^2\right) dx\\ &= -\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

fricas [A] time = 0.78, size = 25, normalized size = 1.04

$$\frac{b^2x^4 + 6\,abx^2 - 3\,a^2}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^4 + 6*a*b*x^2 - 3*a^2)/x
```

giac [A] time = 0.15, size = 22, normalized size = 0.92

$$\frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

maple [A] time = 0.00, size = 23, normalized size = 0.96

$$\frac{b^2x^3}{3} + 2abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^2,x)
```

```
[Out] -a^2/x+2*a*b*x+1/3*b^2*x^3
```

maxima [A] time = 1.31, size = 22, normalized size = 0.92

$$\frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

mupad [B] time = 0.03, size = 22, normalized size = 0.92

$$\frac{b^2\,x^3}{3} - \frac{a^2}{x} + 2\,a\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^2,x)
```

```
[Out] (b^2*x^3)/3 - a^2/x + 2*a*b*x
```

sympy [A] time = 0.10, size = 19, normalized size = 0.79

$$-\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**2,x)
```

```
[Out] -a**2/x + 2*a*b*x + b**2*x**3/3
```

3.415 $\int \frac{a^2+2abx^2+b^2x^4}{x^3} dx$

Optimal. Leaf size=27

$$-\frac{a^2}{2x^2} + 2ab\log(x) + \frac{b^2x^2}{2}$$

```
[Out] -1/2*a^2/x^2+1/2*b^2*x^2+2*a*b*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$-\frac{a^2}{2x^2} + 2ab\log(x) + \frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^3,x]
```

```
[Out] -a^2/(2*x^2) + (b^2*x^2)/2 + 2*a*b*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^3}\,dx &= \int \left(\frac{a^2}{x^3}+\frac{2ab}{x}+b^2x\right)dx\\ &= -\frac{a^2}{2x^2}+\frac{b^2x^2}{2}+2ab\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$-\frac{a^2}{2x^2} + 2ab\log(x) + \frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^3,x]
```

```
[Out] -1/2*a^2/x^2 + (b^2*x^2)/2 + 2*a*b*Log[x]
```

fricas [A] time = 0.84, size = 27, normalized size = 1.00

$$\frac{b^2x^4 + 4\,abx^2\log(x) - a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^3,x, algorithm="fricas")
[Out] 1/2*(b^2*x^4 + 4*a*b*x^2*log(x) - a^2)/x^2
```

giac [A] time = 0.19, size = 32, normalized size = 1.19

$$\frac{1}{2}\,b^2x^2 + ab\log\left(x^2\right) - \frac{2\,abx^2 + a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^3,x, algorithm="giac")
[Out] 1/2*b^2*x^2 + a*b*log(x^2) - 1/2*(2*a*b*x^2 + a^2)/x^2
```

maple [A] time = 0.01, size = 24, normalized size = 0.89

$$\frac{b^2x^2}{2} + 2ab\ln(x) - \frac{a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^3,x)
[Out] -1/2*a^2/x^2+1/2*b^2*x^2+2*a*b*ln(x)
```

maxima [A] time = 1.32, size = 24, normalized size = 0.89

$$\frac{1}{2}\,b^2x^2 + ab\log\left(x^2\right) - \frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^3,x, algorithm="maxima")
[Out] 1/2*b^2*x^2 + a*b*log(x^2) - 1/2*a^2/x^2
```

mupad [B] time = 0.03, size = 23, normalized size = 0.85

$$\frac{b^2\,x^2}{2} - \frac{a^2}{2\,x^2} + 2\,a\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^3,x)
```

```
[Out] (b^2*x^2)/2 - a^2/(2*x^2) + 2*a*b*log(x)
```

sympy [A] time = 0.14, size = 24, normalized size = 0.89

$$-\frac{a^2}{2x^2} + 2ab\log(x) + \frac{b^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**3,x)
```

```
[Out] -a**2/(2*x**2) + 2*a*b*log(x) + b**2*x**2/2
```

3.416 $\int \frac{a^2+2abx^2+b^2x^4}{x^4}\,dx$

Optimal. Leaf size=23

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

```
[Out] -1/3*a^2/x^3-2*a*b/x+b^2*x
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^4,x]
```

```
[Out] -a^2/(3*x^3) - (2*a*b)/x + b^2*x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^4}\,dx &= \int \left(b^2+\frac{a^2}{x^4}+\frac{2ab}{x^2}\right)dx\\ &= -\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^4,x]
```

```
[Out] -1/3*a^2/x^3 - (2*a*b)/x + b^2*x
```

fricas [A] time = 0.76, size = 26, normalized size = 1.13

$$\frac{3\,b^2x^4 - 6\,abx^2 - a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^2*x^4 - 6*a*b*x^2 - a^2)/x^3
```

giac [A] time = 0.20, size = 22, normalized size = 0.96

$$b^2x - \frac{6\,abx^2 + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^4,x, algorithm="giac")
```

```
[Out] b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

maple [A] time = 0.00, size = 22, normalized size = 0.96

$$b^2x - \frac{2ab}{x} - \frac{a^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^4,x)
```

```
[Out] -1/3*a^2/x^3-2*a*b/x+b^2*x
```

maxima [A] time = 1.33, size = 22, normalized size = 0.96

$$b^2x - \frac{6\,abx^2 + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^4,x, algorithm="maxima")
```

```
[Out] b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

mupad [B] time = 4.11, size = 24, normalized size = 1.04

$$b^2\,x - \frac{\frac{a^2}{3} + 2\,b\,a\,x^2}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^4,x)
```

```
[Out] b^2*x - (a^2/3 + 2*a*b*x^2)/x^3
```

sympy [A] time = 0.14, size = 22, normalized size = 0.96

$$b^2x + \frac{-a^2 - 6abx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**4,x)
```

```
[Out] b**2*x + (-a**2 - 6*a*b*x**2)/(3*x**3)
```

3.417 $\int \frac{a^2+2abx^2+b^2x^4}{x^5}\,dx$

Optimal. Leaf size=24

$$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)$$

```
[Out] -1/4*a^2/x^4-a*b/x^2+b^2*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^5,x]
```

```
[Out] -a^2/(4*x^4) - (a*b)/x^2 + b^2*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^5}\,dx &= \int \left(\frac{a^2}{x^5}+\frac{2ab}{x^3}+\frac{b^2}{x}\right)dx\\ &= -\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^5,x]
```

```
[Out] -1/4*a^2/x^4 - (a*b)/x^2 + b^2*Log[x]
```

fricas [A] time = 0.75, size = 28, normalized size = 1.17

$$\frac{4\,b^2x^4\log(x) - 4\,abx^2 - a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^2*x^4*log(x) - 4*a*b*x^2 - a^2)/x^4
```

giac [A] time = 0.15, size = 34, normalized size = 1.42

$$\frac{1}{2}\,b^2\log\left(x^2\right) - \frac{3\,b^2x^4 + 4\,abx^2 + a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(x^2) - 1/4*(3*b^2*x^4 + 4*a*b*x^2 + a^2)/x^4
```

maple [A] time = 0.01, size = 23, normalized size = 0.96

$$b^2\ln(x) - \frac{ab}{x^2} - \frac{a^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^5,x)
```

```
[Out] -1/4*a^2/x^4-a*b/x^2+b^2*ln(x)
```

maxima [A] time = 1.37, size = 26, normalized size = 1.08

$$\frac{1}{2}\,b^2\log\left(x^2\right) - \frac{4\,abx^2 + a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(x^2) - 1/4*(4*a*b*x^2 + a^2)/x^4
```

mupad [B] time = 0.04, size = 24, normalized size = 1.00

$$b^2\,\ln(x) - \frac{\frac{a^2}{4} + b\,a\,x^2}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^5,x)
```

```
[Out] b^2*log(x) - (a^2/4 + a*b*x^2)/x^4
```

sympy **[A]** time = 0.17, size = 24, normalized size = 1.00

$$b^2 \log(x) + \frac{-a^2 - 4abx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**5,x)
```

```
[Out] b**2*log(x) + (-a**2 - 4*a*b*x**2)/(4*x**4)
```

3.418 $\int \frac{a^2+2abx^2+b^2x^4}{x^6}\,dx$

Optimal. Leaf size=28

$$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}$$

```
[Out] -1/5*a^2/x^5-2/3*a*b/x^3-b^2/x
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^6,x]
```

```
[Out] -a^2/(5*x^5) - (2*a*b)/(3*x^3) - b^2/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^6}\,dx &= \int \left(\frac{a^2}{x^6}+\frac{2ab}{x^4}+\frac{b^2}{x^2}\right)dx\\ &= -\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^6,x]
```

```
[Out] -1/5*a^2/x^5 - (2*a*b)/(3*x^3) - b^2/x
```

fricas [A] time = 0.72, size = 26, normalized size = 0.93

$$-\frac{15\,b^2x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

giac [A] time = 0.17, size = 26, normalized size = 0.93

$$-\frac{15\,b^2x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

maple [A] time = 0.00, size = 25, normalized size = 0.89

$$-\frac{b^2}{x}-\frac{2ab}{3x^3}-\frac{a^2}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^6,x)
```

```
[Out] -1/5*a^2/x^5-2/3*a*b/x^3-b^2/x
```

maxima [A] time = 1.34, size = 26, normalized size = 0.93

$$-\frac{15\,b^2x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

mupad [B] time = 0.04, size = 25, normalized size = 0.89

$$-\frac{\frac{a^2}{5}+\frac{2\,a\,b\,x^2}{3}+b^2\,x^4}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^6,x)
```

```
[Out] -(a^2/5 + b^2*x^4 + (2*a*b*x^2)/3)/x^5
```

sympy [A] time = 0.18, size = 27, normalized size = 0.96

$$\frac{-3a^2 - 10abx^2 - 15b^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**6,x)
```

```
[Out] (-3*a**2 - 10*a*b*x**2 - 15*b**2*x**4)/(15*x**5)
```

3.419 $\int \frac{a^2+2abx^2+b^2x^4}{x^7}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2}{2x^2}$$

[Out] -1/6*a^2/x^6-1/2*a*b/x^4-1/2*b^2/x^2

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$-\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^7,x]

[Out] -a^2/(6*x^6) - (a*b)/(2*x^4) - b^2/(2*x^2)

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^7}\,dx &= \int \left(\frac{a^2}{x^7}+\frac{2ab}{x^5}+\frac{b^2}{x^3}\right)dx\\ &= -\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^7,x]

[Out] -1/6*a^2/x^6 - (a*b)/(2*x^4) - b^2/(2*x^2)

fricas [A] time = 0.69, size = 24, normalized size = 0.80

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^7,x, algorithm="fricas")
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

giac [A] time = 0.18, size = 24, normalized size = 0.80

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^7,x, algorithm="giac")
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-\frac{b^2}{2x^2}-\frac{ab}{2x^4}-\frac{a^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^7,x)
[Out] -1/6*a^2/x^6-1/2*a*b/x^4-1/2*b^2/x^2
```

maxima [A] time = 1.33, size = 24, normalized size = 0.80

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^7,x, algorithm="maxima")
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

mupad [B] time = 0.03, size = 26, normalized size = 0.87

$$-\frac{\frac{a^2}{6}+\frac{a\,b\,x^2}{2}+\frac{b^2\,x^4}{2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^7,x)
```

```
[Out] -(a^2/6 + (b^2*x^4)/2 + (a*b*x^2)/2)/x^6
```

sympy [A] time = 0.20, size = 26, normalized size = 0.87

$$\frac{-a^2 - 3abx^2 - 3b^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**7,x)
```

```
[Out] (-a**2 - 3*a*b*x**2 - 3*b**2*x**4)/(6*x**6)
```

3.420 $\int \frac{a^2+2abx^2+b^2x^4}{x^8} dx$

Optimal. Leaf size=30

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

```
[Out] -1/7*a^2/x^7-2/5*a*b/x^5-1/3*b^2/x^3
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^8,x]
```

```
[Out] -a^2/(7*x^7) - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^8}\,dx &= \int \left(\frac{a^2}{x^8}+\frac{2ab}{x^6}+\frac{b^2}{x^4}\right)dx\\ &= -\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^8,x]
```

```
[Out] -1/7*a^2/x^7 - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

fricas [A] time = 0.59, size = 26, normalized size = 0.87

$$-\frac{35\,b^2x^4+42\,abx^2+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^8,x, algorithm="fricas")
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

giac [A] time = 0.17, size = 26, normalized size = 0.87

$$-\frac{35\,b^2x^4+42\,abx^2+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^8,x, algorithm="giac")
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-\frac{b^2}{3x^3}-\frac{2ab}{5x^5}-\frac{a^2}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^8,x)
[Out] -1/7*a^2/x^7-2/5*a*b/x^5-1/3*b^2/x^3
```

maxima [A] time = 1.35, size = 26, normalized size = 0.87

$$-\frac{35\,b^2x^4+42\,abx^2+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^8,x, algorithm="maxima")
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

mupad [B] time = 0.03, size = 26, normalized size = 0.87

$$-\frac{\frac{a^2}{7}+\frac{2\,a\,b\,x^2}{5}+\frac{b^2\,x^4}{3}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^8,x)
```

```
[Out] -(a^2/7 + (b^2*x^4)/3 + (2*a*b*x^2)/5)/x^7
```

sympy [A] time = 0.21, size = 27, normalized size = 0.90

$$\frac{-15a^2 - 42abx^2 - 35b^2x^4}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**8,x)
```

```
[Out] (-15*a**2 - 42*a*b*x**2 - 35*b**2*x**4)/(105*x**7)
```

3.421 $\int x^6 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=56

$$\frac{a^4x^7}{7}+\frac{4}{9}a^3bx^9+\frac{6}{11}a^2b^2x^{11}+\frac{4}{13}ab^3x^{13}+\frac{b^4x^{15}}{15}$$

```
[Out] 1/7*a^4*x^7+4/9*a^3*b*x^9+6/11*a^2*b^2*x^11+4/13*a*b^3*x^13+1/15*b^4*x^15
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$\frac{6}{11}a^2b^2x^{11}+\frac{4}{9}a^3bx^9+\frac{a^4x^7}{7}+\frac{4}{13}ab^3x^{13}+\frac{b^4x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^7)/7 + (4*a^3*b*x^9)/9 + (6*a^2*b^2*x^11)/11 + (4*a*b^3*x^13)/13 + (
b^4*x^15)/15
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^6 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx &= \frac{\int x^6 \left(ab+b^2x^2\right)^4 \, dx}{b^4}\\ &= \frac{\int \left(a^4b^4x^6+4a^3b^5x^8+6a^2b^6x^{10}+4ab^7x^{12}+b^8x^{14}\right) \, dx}{b^4}\\ &= \frac{a^4x^7}{7}+\frac{4}{9}a^3bx^9+\frac{6}{11}a^2b^2x^{11}+\frac{4}{13}ab^3x^{13}+\frac{b^4x^{15}}{15}\end{aligned}$$

Mathematica [A] time = 0.00, size = 56, normalized size = 1.00

$$\frac{a^4x^7}{7}+\frac{4}{9}a^3bx^9+\frac{6}{11}a^2b^2x^{11}+\frac{4}{13}ab^3x^{13}+\frac{b^4x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
[Out] (a^4*x^7)/7 + (4*a^3*b*x^9)/9 + (6*a^2*b^2*x^11)/11 + (4*a*b^3*x^13)/13 + (
b^4*x^15)/15
```

fricas [A] time = 0.62, size = 46, normalized size = 0.82

$$\frac{1}{15}x^{15}b^4+\frac{4}{13}x^{13}b^3a+\frac{6}{11}x^{11}b^2a^2+\frac{4}{9}x^9ba^3+\frac{1}{7}x^7a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
[Out] 1/15*x^15*b^4 + 4/13*x^13*b^3*a + 6/11*x^11*b^2*a^2 + 4/9*x^9*b*a^3 + 1/7*x
^7*a^4
```

giac [A] time = 0.15, size = 46, normalized size = 0.82

$$\frac{1}{15}\,b^4x^{15}+\frac{4}{13}\,ab^3x^{13}+\frac{6}{11}\,a^2b^2x^{11}+\frac{4}{9}\,a^3bx^9+\frac{1}{7}\,a^4x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
[Out] 1/15*b^4*x^15 + 4/13*a*b^3*x^13 + 6/11*a^2*b^2*x^11 + 4/9*a^3*b*x^9 + 1/7*a
^4*x^7
```

maple [A] time = 0.00, size = 47, normalized size = 0.84

$$\frac{1}{15}b^4x^{15}+\frac{4}{13}a\,b^3x^{13}+\frac{6}{11}a^2b^2x^{11}+\frac{4}{9}a^3b\,x^9+\frac{1}{7}a^4x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
[Out] 1/7*a^4*x^7+4/9*a^3*b*x^9+6/11*a^2*b^2*x^11+4/13*a*b^3*x^13+1/15*b^4*x^15
```

maxima [A] time = 1.33, size = 46, normalized size = 0.82

$$\frac{1}{15}\,b^4x^{15}+\frac{4}{13}\,ab^3x^{13}+\frac{6}{11}\,a^2b^2x^{11}+\frac{4}{9}\,a^3bx^9+\frac{1}{7}\,a^4x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*b^4*x^15 + 4/13*a*b^3*x^13 + 6/11*a^2*b^2*x^11 + 4/9*a^3*b*x^9 + 1/7*a
^4*x^7
```

mupad [B] time = 0.02, size = 46, normalized size = 0.82

$$\frac{a^4\,x^7}{7}+\frac{4\,a^3\,b\,x^9}{9}+\frac{6\,a^2\,b^2\,x^{11}}{11}+\frac{4\,a\,b^3\,x^{13}}{13}+\frac{b^4\,x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (a^4*x^7)/7 + (b^4*x^15)/15 + (4*a^3*b*x^9)/9 + (4*a*b^3*x^13)/13 + (6*a^2*
b^2*x^11)/11
```

sympy [A] time = 0.08, size = 53, normalized size = 0.95

$$\frac{a^{4}x^{7}}{7}+\frac{4a^{3}bx^{9}}{9}+\frac{6a^{2}b^{2}x^{11}}{11}+\frac{4ab^{3}x^{13}}{13}+\frac{b^{4}x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**7/7 + 4*a**3*b*x**9/9 + 6*a**2*b**2*x**11/11 + 4*a*b**3*x**13/13 +
b**4*x**15/15
```

3.422 $\int x^5 \left(a^2 + 2abx^2 + b^2x^4\right)^2 dx$

Optimal. Leaf size=53

$$\frac{a^2 \left(a + bx^2\right)^5}{10b^3} + \frac{\left(a + bx^2\right)^7}{14b^3} - \frac{a \left(a + bx^2\right)^6}{6b^3}$$

```
[Out] 1/10*a^2*(b*x^2+a)^5/b^3-1/6*a*(b*x^2+a)^6/b^3+1/14*(b*x^2+a)^7/b^3
```

Rubi [A] time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a^2 \left(a + bx^2\right)^5}{10b^3} + \frac{\left(a + bx^2\right)^7}{14b^3} - \frac{a \left(a + bx^2\right)^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^2*(a + b*x^2)^5)/(10*b^3) - (a*(a + b*x^2)^6)/(6*b^3) + (a + b*x^2)^7/(1
4*b^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{\int x^5 \left(ab+b^2x^2\right)^4 \, dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int x^2 \left(ab+b^2x\right)^4 \, dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2\left(ab+b^2x\right)^4}{b^2} - \frac{2a\left(ab+b^2x\right)^5}{b^3} + \frac{\left(ab+b^2x\right)^6}{b^4}\right) dx, x, x^2\right)}{2b^4}$$
$$= \frac{a^2\left(a+bx^2\right)^5}{10b^3} - \frac{a\left(a+bx^2\right)^6}{6b^3} + \frac{\left(a+bx^2\right)^7}{14b^3}$$

Mathematica [A] time = 0.00, size = 56, normalized size = 1.06

$$\frac{a^4x^6}{6} + \frac{1}{2}a^3bx^8 + \frac{3}{5}a^2b^2x^{10} + \frac{1}{3}ab^3x^{12} + \frac{b^4x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^6)/6 + (a^3*b*x^8)/2 + (3*a^2*b^2*x^10)/5 + (a*b^3*x^12)/3 + (b^4*x^
14)/14
```

fricas [A] time = 0.69, size = 46, normalized size = 0.87

$$\frac{1}{14}x^{14}b^4 + \frac{1}{3}x^{12}b^3a + \frac{3}{5}x^{10}b^2a^2 + \frac{1}{2}x^8ba^3 + \frac{1}{6}x^6a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^4 + 1/3*x^12*b^3*a + 3/5*x^10*b^2*a^2 + 1/2*x^8*b*a^3 + 1/6*x^6
*a^4
```

giac [A] time = 0.15, size = 46, normalized size = 0.87

$$\frac{1}{14}\,b^4x^{14} + \frac{1}{3}\,ab^3x^{12} + \frac{3}{5}\,a^2b^2x^{10} + \frac{1}{2}\,a^3bx^8 + \frac{1}{6}\,a^4x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/14*b^4*x^14 + 1/3*a*b^3*x^12 + 3/5*a^2*b^2*x^10 + 1/2*a^3*b*x^8 + 1/6*a^4
*x^6
```

maple [A] time = 0.00, size = 47, normalized size = 0.89

$$\frac{1}{14}b^4x^{14}+\frac{1}{3}a\,b^3x^{12}+\frac{3}{5}a^2b^2x^{10}+\frac{1}{2}a^3b\,x^8+\frac{1}{6}a^4x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/14*b^4*x^14+1/3*a*b^3*x^12+3/5*a^2*b^2*x^10+1/2*a^3*b*x^8+1/6*a^4*x^6
```

maxima [A] time = 1.34, size = 46, normalized size = 0.87

$$\frac{1}{14}\,b^4x^{14}+\frac{1}{3}\,ab^3x^{12}+\frac{3}{5}\,a^2b^2x^{10}+\frac{1}{2}\,a^3bx^8+\frac{1}{6}\,a^4x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/14*b^4*x^14 + 1/3*a*b^3*x^12 + 3/5*a^2*b^2*x^10 + 1/2*a^3*b*x^8 + 1/6*a^4
*x^6
```

mupad [B] time = 0.02, size = 46, normalized size = 0.87

$$\frac{a^4\,x^6}{6}+\frac{a^3\,b\,x^8}{2}+\frac{3\,a^2\,b^2\,x^{10}}{5}+\frac{a\,b^3\,x^{12}}{3}+\frac{b^4\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (a^4*x^6)/6 + (b^4*x^14)/14 + (a^3*b*x^8)/2 + (a*b^3*x^12)/3 + (3*a^2*b^2*x
^10)/5
```

sympy [A] time = 0.08, size = 49, normalized size = 0.92

$$\frac{a^4x^6}{6}+\frac{a^3bx^8}{2}+\frac{3a^2b^2x^{10}}{5}+\frac{ab^3x^{12}}{3}+\frac{b^4x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**6/6 + a**3*b*x**8/2 + 3*a**2*b**2*x**10/5 + a*b**3*x**12/3 + b**4*x
**14/14
```

3.423 $\int x^4 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=56

$$\frac{a^4x^5}{5}+\frac{4}{7}a^3bx^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}ab^3x^{11}+\frac{b^4x^{13}}{13}$$

```
[Out] 1/5*a^4*x^5+4/7*a^3*b*x^7+2/3*a^2*b^2*x^9+4/11*a*b^3*x^11+1/13*b^4*x^13
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$\frac{2}{3}a^2b^2x^9+\frac{4}{7}a^3bx^7+\frac{a^4x^5}{5}+\frac{4}{11}ab^3x^{11}+\frac{b^4x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^5)/5 + (4*a^3*b*x^7)/7 + (2*a^2*b^2*x^9)/3 + (4*a*b^3*x^11)/11 + (b^
4*x^13)/13
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a^2+2abx^2+b^2x^4\right)^2\,dx&=\frac{\int x^4\left(ab+b^2x^2\right)^4\,dx}{b^4}\\&=\frac{\int\left(a^4b^4x^4+4a^3b^5x^6+6a^2b^6x^8+4ab^7x^{10}+b^8x^{12}\right)dx}{b^4}\\&=\frac{a^4x^5}{5}+\frac{4}{7}a^3bx^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}ab^3x^{11}+\frac{b^4x^{13}}{13}\end{aligned}$$

Mathematica [A] time = 0.00, size = 56, normalized size = 1.00

$$\frac{a^4x^5}{5}+\frac{4}{7}a^3bx^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}ab^3x^{11}+\frac{b^4x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
[Out] (a^4*x^5)/5 + (4*a^3*b*x^7)/7 + (2*a^2*b^2*x^9)/3 + (4*a*b^3*x^11)/11 + (b^
4*x^13)/13
```

fricas [A] time = 0.54, size = 46, normalized size = 0.82

$$\frac{1}{13}x^{13}b^4+\frac{4}{11}x^{11}b^3a+\frac{2}{3}x^9b^2a^2+\frac{4}{7}x^7ba^3+\frac{1}{5}x^5a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
[Out] 1/13*x^13*b^4 + 4/11*x^11*b^3*a + 2/3*x^9*b^2*a^2 + 4/7*x^7*b*a^3 + 1/5*x^5
*a^4
```

giac [A] time = 0.16, size = 46, normalized size = 0.82

$$\frac{1}{13}\,b^4x^{13}+\frac{4}{11}\,ab^3x^{11}+\frac{2}{3}\,a^2b^2x^9+\frac{4}{7}\,a^3bx^7+\frac{1}{5}\,a^4x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
[Out] 1/13*b^4*x^13 + 4/11*a*b^3*x^11 + 2/3*a^2*b^2*x^9 + 4/7*a^3*b*x^7 + 1/5*a^4
*x^5
```

maple [A] time = 0.00, size = 47, normalized size = 0.84

$$\frac{1}{13}b^4x^{13}+\frac{4}{11}a\,b^3x^{11}+\frac{2}{3}a^2b^2x^9+\frac{4}{7}a^3b\,x^7+\frac{1}{5}a^4x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
[Out] 1/5*a^4*x^5+4/7*a^3*b*x^7+2/3*a^2*b^2*x^9+4/11*a*b^3*x^11+1/13*b^4*x^13
```

maxima [A] time = 1.33, size = 46, normalized size = 0.82

$$\frac{1}{13}\,b^4x^{13}+\frac{4}{11}\,ab^3x^{11}+\frac{2}{3}\,a^2b^2x^9+\frac{4}{7}\,a^3bx^7+\frac{1}{5}\,a^4x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/13*b^4*x^13 + 4/11*a*b^3*x^11 + 2/3*a^2*b^2*x^9 + 4/7*a^3*b*x^7 + 1/5*a^4
*x^5
```

mupad [B] time = 0.02, size = 46, normalized size = 0.82

$$\frac{a^4\,x^5}{5}+\frac{4\,a^3\,b\,x^7}{7}+\frac{2\,a^2\,b^2\,x^9}{3}+\frac{4\,a\,b^3\,x^{11}}{11}+\frac{b^4\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (a^4*x^5)/5 + (b^4*x^13)/13 + (4*a^3*b*x^7)/7 + (4*a*b^3*x^11)/11 + (2*a^2*
b^2*x^9)/3
```

sympy [A] time = 0.08, size = 53, normalized size = 0.95

$$\frac{a^4x^5}{5}+\frac{4a^3bx^7}{7}+\frac{2a^2b^2x^9}{3}+\frac{4ab^3x^{11}}{11}+\frac{b^4x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**5/5 + 4*a**3*b*x**7/7 + 2*a**2*b**2*x**9/3 + 4*a*b**3*x**11/11 + b*
*4*x**13/13
```

3.424 $\int x^3 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=34

$$\frac{\left(a+bx^2\right)^6}{12b^2}-\frac{a\left(a+bx^2\right)^5}{10b^2}$$

```
[Out] -1/10*a*(b*x^2+a)^5/b^2+1/12*(b*x^2+a)^6/b^2
```

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{\left(a+bx^2\right)^6}{12b^2}-\frac{a\left(a+bx^2\right)^5}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(a*(a + b*x^2)^5)/(10*b^2) + (a + b*x^2)^6/(12*b^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^3\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{\int x^3\left(ab+b^2x^2\right)^4\,dx}{b^4}$$
$$=\frac{\text{Subst}\left(\int x\left(ab+b^2x\right)^4\,dx,x,x^2\right)}{2b^4}$$
$$=\frac{\text{Subst}\left(\int\left(-\frac{a\left(ab+b^2x\right)^4}{b}+\frac{\left(ab+b^2x\right)^5}{b^2}\right)dx,x,x^2\right)}{2b^4}$$
$$=-\frac{a\left(a+bx^2\right)^5}{10b^2}+\frac{\left(a+bx^2\right)^6}{12b^2}$$

Mathematica [A] time = 0.00, size = 56, normalized size = 1.65

$$\frac{a^4x^4}{4}+\frac{2}{3}a^3bx^6+\frac{3}{4}a^2b^2x^8+\frac{2}{5}ab^3x^{10}+\frac{b^4x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^4)/4 + (2*a^3*b*x^6)/3 + (3*a^2*b^2*x^8)/4 + (2*a*b^3*x^10)/5 + (b^4
*x^12)/12
```

fricas [A] time = 0.67, size = 46, normalized size = 1.35

$$\frac{1}{12}x^{12}b^4+\frac{2}{5}x^{10}b^3a+\frac{3}{4}x^8b^2a^2+\frac{2}{3}x^6ba^3+\frac{1}{4}x^4a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^4 + 2/5*x^10*b^3*a + 3/4*x^8*b^2*a^2 + 2/3*x^6*b*a^3 + 1/4*x^4*
a^4
```

giac [A] time = 0.15, size = 46, normalized size = 1.35

$$\frac{1}{12}\,b^4x^{12}+\frac{2}{5}\,ab^3x^{10}+\frac{3}{4}\,a^2b^2x^8+\frac{2}{3}\,a^3bx^6+\frac{1}{4}\,a^4x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/12*b^4*x^12 + 2/5*a*b^3*x^10 + 3/4*a^2*b^2*x^8 + 2/3*a^3*b*x^6 + 1/4*a^4*
x^4
```

maple [A] time = 0.00, size = 47, normalized size = 1.38

$$\frac{1}{12}b^4x^{12}+\frac{2}{5}a\,b^3x^{10}+\frac{3}{4}a^2b^2x^8+\frac{2}{3}a^3b\,x^6+\frac{1}{4}a^4x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/12*b^4*x^12+2/5*a*b^3*x^10+3/4*a^2*b^2*x^8+2/3*a^3*b*x^6+1/4*a^4*x^4
```

maxima [A] time = 1.28, size = 46, normalized size = 1.35

$$\frac{1}{12}\,b^4x^{12}+\frac{2}{5}\,ab^3x^{10}+\frac{3}{4}\,a^2b^2x^8+\frac{2}{3}\,a^3bx^6+\frac{1}{4}\,a^4x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^4*x^12 + 2/5*a*b^3*x^10 + 3/4*a^2*b^2*x^8 + 2/3*a^3*b*x^6 + 1/4*a^4*
x^4
```

mupad [B] time = 0.02, size = 46, normalized size = 1.35

$$\frac{a^4\,x^4}{4}+\frac{2\,a^3\,b\,x^6}{3}+\frac{3\,a^2\,b^2\,x^8}{4}+\frac{2\,a\,b^3\,x^{10}}{5}+\frac{b^4\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (a^4*x^4)/4 + (b^4*x^12)/12 + (2*a^3*b*x^6)/3 + (2*a*b^3*x^10)/5 + (3*a^2*b
^2*x^8)/4
```

sympy [A] time = 0.08, size = 53, normalized size = 1.56

$$\frac{a^4x^4}{4}+\frac{2a^3bx^6}{3}+\frac{3a^2b^2x^8}{4}+\frac{2ab^3x^{10}}{5}+\frac{b^4x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**4/4 + 2*a**3*b*x**6/3 + 3*a**2*b**2*x**8/4 + 2*a*b**3*x**10/5 + b**
4*x**12/12
```

3.425 $\int x^2 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=56

$$\frac{a^4x^3}{3}+\frac{4}{5}a^3bx^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}ab^3x^9+\frac{b^4x^{11}}{11}$$

```
[Out] 1/3*a^4*x^3+4/5*a^3*b*x^5+6/7*a^2*b^2*x^7+4/9*a*b^3*x^9+1/11*b^4*x^11
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$\frac{6}{7}a^2b^2x^7+\frac{4}{5}a^3bx^5+\frac{a^4x^3}{3}+\frac{4}{9}ab^3x^9+\frac{b^4x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^3)/3 + (4*a^3*b*x^5)/5 + (6*a^2*b^2*x^7)/7 + (4*a*b^3*x^9)/9 + (b^4*
x^11)/11
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a^2+2abx^2+b^2x^4\right)^2\,dx&=\frac{\int x^2\left(ab+b^2x^2\right)^4\,dx}{b^4}\\&=\frac{\int\left(a^4b^4x^2+4a^3b^5x^4+6a^2b^6x^6+4ab^7x^8+b^8x^{10}\right)dx}{b^4}\\&=\frac{a^4x^3}{3}+\frac{4}{5}a^3bx^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}ab^3x^9+\frac{b^4x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.00, size = 56, normalized size = 1.00

$$\frac{a^4x^3}{3}+\frac{4}{5}a^3bx^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}ab^3x^9+\frac{b^4x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^3)/3 + (4*a^3*b*x^5)/5 + (6*a^2*b^2*x^7)/7 + (4*a*b^3*x^9)/9 + (b^4*
x^11)/11
```

fricas [A] time = 0.67, size = 46, normalized size = 0.82

$$\frac{1}{11}x^{11}b^4+\frac{4}{9}x^9b^3a+\frac{6}{7}x^7b^2a^2+\frac{4}{5}x^5ba^3+\frac{1}{3}x^3a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^4 + 4/9*x^9*b^3*a + 6/7*x^7*b^2*a^2 + 4/5*x^5*b*a^3 + 1/3*x^3*a
^4
```

giac [A] time = 0.15, size = 46, normalized size = 0.82

$$\frac{1}{11}\,b^4x^{11}+\frac{4}{9}\,ab^3x^9+\frac{6}{7}\,a^2b^2x^7+\frac{4}{5}\,a^3bx^5+\frac{1}{3}\,a^4x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/11*b^4*x^11 + 4/9*a*b^3*x^9 + 6/7*a^2*b^2*x^7 + 4/5*a^3*b*x^5 + 1/3*a^4*x
^3
```

maple [A] time = 0.00, size = 47, normalized size = 0.84

$$\frac{1}{11}b^4x^{11}+\frac{4}{9}a\,b^3x^9+\frac{6}{7}a^2b^2x^7+\frac{4}{5}a^3b\,x^5+\frac{1}{3}a^4x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/3*a^4*x^3+4/5*a^3*b*x^5+6/7*a^2*b^2*x^7+4/9*a*b^3*x^9+1/11*b^4*x^11
```

maxima [A] time = 1.32, size = 46, normalized size = 0.82

$$\frac{1}{11}\,b^4x^{11}+\frac{4}{9}\,ab^3x^9+\frac{6}{7}\,a^2b^2x^7+\frac{4}{5}\,a^3bx^5+\frac{1}{3}\,a^4x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^4*x^11 + 4/9*a*b^3*x^9 + 6/7*a^2*b^2*x^7 + 4/5*a^3*b*x^5 + 1/3*a^4*x
^3
```

mupad [B] time = 0.02, size = 46, normalized size = 0.82

$$\frac{a^4\,x^3}{3}+\frac{4\,a^3\,b\,x^5}{5}+\frac{6\,a^2\,b^2\,x^7}{7}+\frac{4\,a\,b^3\,x^9}{9}+\frac{b^4\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (a^4*x^3)/3 + (b^4*x^11)/11 + (4*a^3*b*x^5)/5 + (4*a*b^3*x^9)/9 + (6*a^2*b^
2*x^7)/7
```

sympy [A] time = 0.08, size = 53, normalized size = 0.95

$$\frac{a^4x^3}{3}+\frac{4a^3bx^5}{5}+\frac{6a^2b^2x^7}{7}+\frac{4ab^3x^9}{9}+\frac{b^4x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**3/3 + 4*a**3*b*x**5/5 + 6*a**2*b**2*x**7/7 + 4*a*b**3*x**9/9 + b**4
*x**11/11
```

3.426 $\int x \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^2\right)^5}{10b}$$

```
[Out] 1/10*(b*x^2+a)^5/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {28, 261}

$$\frac{\left(a+bx^2\right)^5}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a + b*x^2)^5/(10*b)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int x \left(a^2+2abx^2+b^2x^4\right)^2 \, dx &= \frac{\int x \left(ab+b^2x^2\right)^4 \, dx}{b^4} \\ &= \frac{\left(a+bx^2\right)^5}{10b} \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{(a+bx^2)^5}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
[Out] (a + b*x^2)^5/(10*b)
```

fricas [B] time = 0.67, size = 44, normalized size = 2.75

$$\frac{1}{10}x^{10}b^4+\frac{1}{2}x^8b^3a+x^6b^2a^2+x^4ba^3+\frac{1}{2}x^2a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
[Out] 1/10*x^10*b^4 + 1/2*x^8*b^3*a + x^6*b^2*a^2 + x^4*b*a^3 + 1/2*x^2*a^4
```

giac [B] time = 0.15, size = 44, normalized size = 2.75

$$\frac{1}{10}b^4x^{10}+\frac{1}{2}ab^3x^8+a^2b^2x^6+a^3bx^4+\frac{1}{2}a^4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
[Out] 1/10*b^4*x^10 + 1/2*a*b^3*x^8 + a^2*b^2*x^6 + a^3*b*x^4 + 1/2*a^4*x^2
```

maple [B] time = 0.00, size = 45, normalized size = 2.81

$$\frac{1}{10}b^4x^{10}+\frac{1}{2}a\,b^3x^8+a^2b^2x^6+a^3b\,x^4+\frac{1}{2}a^4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
[Out] 1/10*b^4*x^10+1/2*a*b^3*x^8+a^2*b^2*x^6+a^3*b*x^4+1/2*a^4*x^2
```

maxima [B] time = 1.33, size = 44, normalized size = 2.75

$$\frac{1}{10}b^4x^{10}+\frac{1}{2}ab^3x^8+a^2b^2x^6+a^3bx^4+\frac{1}{2}a^4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^4*x^10 + 1/2*a*b^3*x^8 + a^2*b^2*x^6 + a^3*b*x^4 + 1/2*a^4*x^2
```

mupad [B] time = 0.02, size = 44, normalized size = 2.75

$$\frac{a^4\,x^2}{2}+a^3\,b\,x^4+a^2\,b^2\,x^6+\frac{a\,b^3\,x^8}{2}+\frac{b^4\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (a^4*x^2)/2 + (b^4*x^10)/10 + a^3*b*x^4 + (a*b^3*x^8)/2 + a^2*b^2*x^6
```

sympy [B] time = 0.08, size = 44, normalized size = 2.75

$$\frac{a^4x^2}{2}+a^3bx^4+a^2b^2x^6+\frac{ab^3x^8}{2}+\frac{b^4x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**2/2 + a**3*b*x**4 + a**2*b**2*x**6 + a*b**3*x**8/2 + b**4*x**10/10
```

3.427 $\int \left(a^2+2abx^2+b^2x^4\right)^2\,dx$

Optimal. Leaf size=51

$$a^4x+\frac{4}{3}a^3bx^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}ab^3x^7+\frac{b^4x^9}{9}$$

```
[Out] a^4*x+4/3*a^3*b*x^3+6/5*a^2*b^2*x^5+4/7*a*b^3*x^7+1/9*b^4*x^9
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {28, 194}

$$\frac{6}{5}a^2b^2x^5+\frac{4}{3}a^3bx^3+a^4x+\frac{4}{7}ab^3x^7+\frac{b^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] a^4*x + (4*a^3*b*x^3)/3 + (6*a^2*b^2*x^5)/5 + (4*a*b^3*x^7)/7 + (b^4*x^9)/9
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2abx^2+b^2x^4\right)^2\,dx &= \frac{\int \left(ab+b^2x^2\right)^4\,dx}{b^4}\\ &= \frac{\int \left(a^4b^4+4a^3b^5x^2+6a^2b^6x^4+4ab^7x^6+b^8x^8\right)dx}{b^4}\\ &= a^4x+\frac{4}{3}a^3bx^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}ab^3x^7+\frac{b^4x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 51, normalized size = 1.00

$$a^4x+\frac{4}{3}a^3bx^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}ab^3x^7+\frac{b^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
[Out] a^4*x + (4*a^3*b*x^3)/3 + (6*a^2*b^2*x^5)/5 + (4*a*b^3*x^7)/7 + (b^4*x^9)/9
```

fricas [A] time = 0.71, size = 43, normalized size = 0.84

$$\frac{1}{9}x^9b^4+\frac{4}{7}x^7b^3a+\frac{6}{5}x^5b^2a^2+\frac{4}{3}x^3ba^3+xa^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
[Out] 1/9*x^9*b^4 + 4/7*x^7*b^3*a + 6/5*x^5*b^2*a^2 + 4/3*x^3*b*a^3 + x*a^4
```

giac [A] time = 0.14, size = 43, normalized size = 0.84

$$\frac{1}{9}b^4x^9+\frac{4}{7}ab^3x^7+\frac{6}{5}a^2b^2x^5+\frac{4}{3}a^3bx^3+a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
[Out] 1/9*b^4*x^9 + 4/7*a*b^3*x^7 + 6/5*a^2*b^2*x^5 + 4/3*a^3*b*x^3 + a^4*x
```

maple [A] time = 0.00, size = 44, normalized size = 0.86

$$\frac{1}{9}b^4x^9+\frac{4}{7}a\,b^3x^7+\frac{6}{5}a^2b^2x^5+\frac{4}{3}a^3b\,x^3+a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2,x)
[Out] a^4*x+4/3*a^3*b*x^3+6/5*a^2*b^2*x^5+4/7*a*b^3*x^7+1/9*b^4*x^9
```

maxima [A] time = 1.28, size = 55, normalized size = 1.08

$$\frac{1}{9}b^4x^9+\frac{4}{7}ab^3x^7+\frac{4}{5}a^2b^2x^5+a^4x+\frac{2}{15}\left(3\,b^2x^5+10\,abx^3\right)a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^4*x^9 + 4/7*a*b^3*x^7 + 4/5*a^2*b^2*x^5 + a^4*x + 2/15*(3*b^2*x^5 + 1
0*a*b*x^3)*a^2
```

mupad [B] time = 0.02, size = 43, normalized size = 0.84

$$a^4\,x+\frac{4\,a^3\,b\,x^3}{3}+\frac{6\,a^2\,b^2\,x^5}{5}+\frac{4\,a\,b^3\,x^7}{7}+\frac{b^4\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] a^4*x + (b^4*x^9)/9 + (4*a^3*b*x^3)/3 + (4*a*b^3*x^7)/7 + (6*a^2*b^2*x^5)/5
```

sympy [A] time = 0.08, size = 49, normalized size = 0.96

$$a^{4}x+\frac{4a^{3}bx^{3}}{3}+\frac{6a^{2}b^{2}x^{5}}{5}+\frac{4ab^{3}x^{7}}{7}+\frac{b^{4}x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x + 4*a**3*b*x**3/3 + 6*a**2*b**2*x**5/5 + 4*a*b**3*x**7/7 + b**4*x**9
/9
```

3.428 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x} dx$

Optimal. Leaf size=50

$$a^4 \log(x) + 2a^3bx^2 + \frac{3}{2}a^2b^2x^4 + \frac{2}{3}ab^3x^6 + \frac{b^4x^8}{8}$$

```
[Out] 2*a^3*b*x^2+3/2*a^2*b^2*x^4+2/3*a*b^3*x^6+1/8*b^4*x^8+a^4*ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{3}{2}a^2b^2x^4 + 2a^3bx^2 + a^4 \log(x) + \frac{2}{3}ab^3x^6 + \frac{b^4x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x,x]
```

```
[Out] 2*a^3*b*x^2 + (3*a^2*b^2*x^4)/2 + (2*a*b^3*x^6)/3 + (b^4*x^8)/8 + a^4*Log[x
]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x}\,dx = \frac{\int \frac{(ab+b^2x^2)^4}{x}\,dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^4}{x}\,dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(4a^3b^5+\frac{a^4b^4}{x}+6a^2b^6x+4ab^7x^2+b^8x^3\right)dx, x, x^2\right)}{2b^4}$$
$$= 2a^3bx^2+\frac{3}{2}a^2b^2x^4+\frac{2}{3}ab^3x^6+\frac{b^4x^8}{8}+a^4\log(x)$$

Mathematica [A] time = 0.00, size = 50, normalized size = 1.00

$$a^4\log(x)+2a^3bx^2+\frac{3}{2}a^2b^2x^4+\frac{2}{3}ab^3x^6+\frac{b^4x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x,x]
```

```
[Out] 2*a^3*b*x^2 + (3*a^2*b^2*x^4)/2 + (2*a*b^3*x^6)/3 + (b^4*x^8)/8 + a^4*Log[x
]
```

fricas [A] time = 0.77, size = 44, normalized size = 0.88

$$\frac{1}{8}b^4x^8+\frac{2}{3}ab^3x^6+\frac{3}{2}a^2b^2x^4+2a^3bx^2+a^4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*b^4*x^8 + 2/3*a*b^3*x^6 + 3/2*a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4*log(x)
```

giac [A] time = 0.15, size = 47, normalized size = 0.94

$$\frac{1}{8}b^4x^8+\frac{2}{3}ab^3x^6+\frac{3}{2}a^2b^2x^4+2a^3bx^2+\frac{1}{2}a^4\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*b^4*x^8 + 2/3*a*b^3*x^6 + 3/2*a^2*b^2*x^4 + 2*a^3*b*x^2 + 1/2*a^4*log(x
^2)
```

maple [A] time = 0.00, size = 45, normalized size = 0.90

$$\frac{b^4x^8}{8}+\frac{2a\,b^3x^6}{3}+\frac{3a^2b^2x^4}{2}+2a^3b\,x^2+a^4\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x,x)
```

```
[Out] 2*a^3*b*x^2+3/2*a^2*b^2*x^4+2/3*a*b^3*x^6+1/8*b^4*x^8+a^4*ln(x)
```

maxima [A] time = 1.34, size = 47, normalized size = 0.94

$$\frac{1}{8}\,b^4x^8+\frac{2}{3}\,ab^3x^6+\frac{3}{2}\,a^2b^2x^4+2\,a^3bx^2+\frac{1}{2}\,a^4\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*b^4*x^8 + 2/3*a*b^3*x^6 + 3/2*a^2*b^2*x^4 + 2*a^3*b*x^2 + 1/2*a^4*log(x
^2)
```

mupad [B] time = 0.03, size = 44, normalized size = 0.88

$$a^4\,\ln(x)+\frac{b^4\,x^8}{8}+2\,a^3\,b\,x^2+\frac{2\,a\,b^3\,x^6}{3}+\frac{3\,a^2\,b^2\,x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x,x)
```

```
[Out] a^4*log(x) + (b^4*x^8)/8 + 2*a^3*b*x^2 + (2*a*b^3*x^6)/3 + (3*a^2*b^2*x^4)/
2
```

sympy [A] time = 0.13, size = 49, normalized size = 0.98

$$a^4\log{(x)}+2a^3bx^2+\frac{3a^2b^2x^4}{2}+\frac{2ab^3x^6}{3}+\frac{b^4x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x,x)
```

```
[Out] a**4*log(x) + 2*a**3*b*x**2 + 3*a**2*b**2*x**4/2 + 2*a*b**3*x**6/3 + b**4*x
**8/8
```

3.429 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^2}\,dx$

Optimal. Leaf size=48

$$-\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4}{5}ab^3x^5+\frac{b^4x^7}{7}$$

```
[Out] -a^4/x+4*a^3*b*x+2*a^2*b^2*x^3+4/5*a*b^3*x^5+1/7*b^4*x^7
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$2a^2b^2x^3+4a^3bx-\frac{a^4}{x}+\frac{4}{5}ab^3x^5+\frac{b^4x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^2,x]
```

```
[Out] -(a^4/x) + 4*a^3*b*x + 2*a^2*b^2*x^3 + (4*a*b^3*x^5)/5 + (b^4*x^7)/7
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^2}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^2}\,dx}{b^4}\\
&= \frac{\int \left(4a^3b^5+\frac{a^4b^4}{x^2}+6a^2b^6x^2+4ab^7x^4+b^8x^6\right)dx}{b^4}\\
&= -\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4}{5}ab^3x^5+\frac{b^4x^7}{7}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.00

$$-\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4}{5}ab^3x^5+\frac{b^4x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^2,x]
```

```
[Out] -(a^4/x) + 4*a^3*b*x + 2*a^2*b^2*x^3 + (4*a*b^3*x^5)/5 + (b^4*x^7)/7
```

fricas [A] time = 0.64, size = 48, normalized size = 1.00

$$\frac{5\,b^4x^8+28\,ab^3x^6+70\,a^2b^2x^4+140\,a^3bx^2-35\,a^4}{35\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/35*(5*b^4*x^8 + 28*a*b^3*x^6 + 70*a^2*b^2*x^4 + 140*a^3*b*x^2 - 35*a^4)/x
```

giac [A] time = 0.15, size = 44, normalized size = 0.92

$$\frac{1}{7}\,b^4x^7+\frac{4}{5}\,ab^3x^5+2\,a^2b^2x^3+4\,a^3bx-\frac{a^4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*b^4*x^7 + 4/5*a*b^3*x^5 + 2*a^2*b^2*x^3 + 4*a^3*b*x - a^4/x
```

maple [A] time = 0.00, size = 45, normalized size = 0.94

$$\frac{b^4x^7}{7}+\frac{4a\,b^3x^5}{5}+2a^2b^2x^3+4a^3bx-\frac{a^4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^2,x)
```

```
[Out] -a^4/x+4*a^3*b*x+2*a^2*b^2*x^3+4/5*a*b^3*x^5+1/7*b^4*x^7
```

maxima [A] time = 1.34, size = 44, normalized size = 0.92

$$\frac{1}{7}\,b^4x^7+\frac{4}{5}\,ab^3x^5+2\,a^2b^2x^3+4\,a^3bx-\frac{a^4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^2,x, algorithm="maxima")
[Out] 1/7*b^4*x^7 + 4/5*a*b^3*x^5 + 2*a^2*b^2*x^3 + 4*a^3*b*x - a^4/x
```

mupad [B] time = 0.02, size = 44, normalized size = 0.92

$$\frac{b^4\,x^7}{7}-\frac{a^4}{x}+\frac{4\,a\,b^3\,x^5}{5}+2\,a^2\,b^2\,x^3+4\,a^3\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^2,x)
[Out] (b^4*x^7)/7 - a^4/x + (4*a*b^3*x^5)/5 + 2*a^2*b^2*x^3 + 4*a^3*b*x
```

sympy [A] time = 0.13, size = 44, normalized size = 0.92

$$-\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4ab^3x^5}{5}+\frac{b^4x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**2,x)
[Out] -a**4/x + 4*a**3*b*x + 2*a**2*b**2*x**3 + 4*a*b**3*x**5/5 + b**4*x**7/7
```

3.430 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^3}\,dx$

Optimal. Leaf size=48

$$-\frac{a^4}{2x^2}+4a^3b\log(x)+3a^2b^2x^2+ab^3x^4+\frac{b^4x^6}{6}$$

```
[Out] -1/2*a^4/x^2+3*a^2*b^2*x^2+a*b^3*x^4+1/6*b^4*x^6+4*a^3*b*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$3a^2b^2x^2+4a^3b\log(x)-\frac{a^4}{2x^2}+ab^3x^4+\frac{b^4x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^3,x]
```

```
[Out] -a^4/(2*x^2) + 3*a^2*b^2*x^2 + a*b^3*x^4 + (b^4*x^6)/6 + 4*a^3*b*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^3}\,dx = \frac{\int \frac{(ab+b^2x^2)^4}{x^3}\,dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^4}{x^2}\,dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(6a^2b^6+\frac{a^4b^4}{x^2}+\frac{4a^3b^5}{x}+4ab^7x+b^8x^2\right)dx, x, x^2\right)}{2b^4}$$
$$= -\frac{a^4}{2x^2}+3a^2b^2x^2+ab^3x^4+\frac{b^4x^6}{6}+4a^3b\log(x)$$

Mathematica [A] time = 0.00, size = 48, normalized size = 1.00

$$-\frac{a^4}{2x^2}+4a^3b\log(x)+3a^2b^2x^2+ab^3x^4+\frac{b^4x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^3,x]
[Out] -1/2*a^4/x^2 + 3*a^2*b^2*x^2 + a*b^3*x^4 + (b^4*x^6)/6 + 4*a^3*b*Log[x]
```

fricas [A] time = 0.86, size = 49, normalized size = 1.02

$$\frac{b^4x^8+6\,ab^3x^6+18\,a^2b^2x^4+24\,a^3bx^2\log(x)-3\,a^4}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^3,x, algorithm="fricas")
[Out] 1/6*(b^4*x^8 + 6*a*b^3*x^6 + 18*a^2*b^2*x^4 + 24*a^3*b*x^2*log(x) - 3*a^4)/
x^2
```

giac [A] time = 0.16, size = 56, normalized size = 1.17

$$\frac{1}{6}\,b^4x^6+ab^3x^4+3\,a^2b^2x^2+2\,a^3b\log\left(x^2\right)-\frac{4\,a^3bx^2+a^4}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^3,x, algorithm="giac")
```

[Out] 1/6*b^4*x^6 + a*b^3*x^4 + 3*a^2*b^2*x^2 + 2*a^3*b*log(x^2) - 1/2*(4*a^3*b*x^2 + a^4)/x^2

maple [A] time = 0.01, size = 45, normalized size = 0.94

$$\frac{b^4x^6}{6} + a\,b^3x^4 + 3a^2b^2x^2 + 4a^3b\ln(x) - \frac{a^4}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^3,x)

[Out] -1/2*a^4/x^2+3*a^2*b^2*x^2+a*b^3*x^4+1/6*b^4*x^6+4*a^3*b*ln(x)

maxima [A] time = 1.34, size = 46, normalized size = 0.96

$$\frac{1}{6}\,b^4x^6 + ab^3x^4 + 3\,a^2b^2x^2 + 2\,a^3b\log\left(x^2\right) - \frac{a^4}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^3,x, algorithm="maxima")

[Out] 1/6*b^4*x^6 + a*b^3*x^4 + 3*a^2*b^2*x^2 + 2*a^3*b*log(x^2) - 1/2*a^4/x^2

mupad [B] time = 0.03, size = 44, normalized size = 0.92

$$\frac{b^4\,x^6}{6} - \frac{a^4}{2\,x^2} + a\,b^3\,x^4 + 4\,a^3\,b\,\ln(x) + 3\,a^2\,b^2\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^3,x)

[Out] (b^4*x^6)/6 - a^4/(2*x^2) + a*b^3*x^4 + 4*a^3*b*log(x) + 3*a^2*b^2*x^2

sympy [A] time = 0.17, size = 46, normalized size = 0.96

$$-\frac{a^4}{2x^2} + 4a^3b\log{(x)} + 3a^2b^2x^2 + ab^3x^4 + \frac{b^4x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**3,x)

[Out] -a**4/(2*x**2) + 4*a**3*b*log(x) + 3*a**2*b**2*x**2 + a*b**3*x**4 + b**4*x**6/6

3.431 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^4}\,dx$

Optimal. Leaf size=50

$$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

```
[Out] -1/3*a^4/x^3-4*a^3*b/x+6*a^2*b^2*x+4/3*a*b^3*x^3+1/5*b^4*x^5
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$6a^2b^2x-\frac{4a^3b}{x}-\frac{a^4}{3x^3}+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^4,x]
```

```
[Out] -a^4/(3*x^3) - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^4}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^4}\,dx}{b^4}\\
&= \frac{\int \left(6a^2b^6+\frac{a^4b^4}{x^4}+\frac{4a^3b^5}{x^2}+4ab^7x^2+b^8x^4\right)dx}{b^4}\\
&= -\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.00

$$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^4,x]
[Out] -1/3*a^4/x^3 - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

fricas [A] time = 0.78, size = 48, normalized size = 0.96

$$\frac{3\,b^4x^8+20\,ab^3x^6+90\,a^2b^2x^4-60\,a^3bx^2-5\,a^4}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^4,x, algorithm="fricas")
[Out] 1/15*(3*b^4*x^8 + 20*a*b^3*x^6 + 90*a^2*b^2*x^4 - 60*a^3*b*x^2 - 5*a^4)/x^3
```

giac [A] time = 0.17, size = 45, normalized size = 0.90

$$\frac{1}{5}\,b^4x^5+\frac{4}{3}\,ab^3x^3+6\,a^2b^2x-\frac{12\,a^3bx^2+a^4}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^4,x, algorithm="giac")
[Out] 1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 1/3*(12*a^3*b*x^2 + a^4)/x^3
```

maple [A] time = 0.00, size = 45, normalized size = 0.90

$$\frac{b^4x^5}{5}+\frac{4a\,b^3x^3}{3}+6a^2b^2x-\frac{4a^3b}{x}-\frac{a^4}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^4,x)
[Out] -1/3*a^4/x^3-4*a^3*b/x+6*a^2*b^2*x+4/3*a*b^3*x^3+1/5*b^4*x^5
```

maxima [A] time = 1.44, size = 45, normalized size = 0.90

$$\frac{1}{5}\,b^4x^5+\frac{4}{3}\,ab^3x^3+6\,a^2b^2x-\frac{12\,a^3bx^2+a^4}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 1/3*(12*a^3*b*x^2 + a^4)/x^3
```

mupad [B] time = 0.04, size = 47, normalized size = 0.94

$$\frac{b^4\,x^5}{5}-\frac{\frac{a^4}{3}+4\,b\,a^3\,x^2}{x^3}+6\,a^2\,b^2\,x+\frac{4\,a\,b^3\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^4,x)
```

```
[Out] (b^4*x^5)/5 - (a^4/3 + 4*a^3*b*x^2)/x^3 + 6*a^2*b^2*x + (4*a*b^3*x^3)/3
```

sympy [A] time = 0.17, size = 49, normalized size = 0.98

$$6a^2b^2x+\frac{4ab^3x^3}{3}+\frac{b^4x^5}{5}+\frac{-a^4-12a^3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**4,x)
```

```
[Out] 6*a**2*b**2*x + 4*a*b**3*x**3/3 + b**4*x**5/5 + (-a**4 - 12*a**3*b*x**2)/(3
*x**3)
```

3.432 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^5}\,dx$

Optimal. Leaf size=49

$$-\frac{a^4}{4x^4}-\frac{2a^3b}{x^2}+6a^2b^2\log(x)+2ab^3x^2+\frac{b^4x^4}{4}$$

```
[Out] -1/4*a^4/x^4-2*a^3*b/x^2+2*a*b^3*x^2+1/4*b^4*x^4+6*a^2*b^2*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$6a^2b^2\log(x)-\frac{2a^3b}{x^2}-\frac{a^4}{4x^4}+2ab^3x^2+\frac{b^4x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^5,x]
```

```
[Out] -a^4/(4*x^4) - (2*a^3*b)/x^2 + 2*a*b^3*x^2 + (b^4*x^4)/4 + 6*a^2*b^2*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^5}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^5}\,dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^4}{x^3}\,dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(4ab^7+\frac{a^4b^4}{x^3}+\frac{4a^3b^5}{x^2}+\frac{6a^2b^6}{x}+b^8x\right)dx, x, x^2\right)}{2b^4}$$
$$= -\frac{a^4}{4x^4}-\frac{2a^3b}{x^2}+2ab^3x^2+\frac{b^4x^4}{4}+6a^2b^2\log(x)$$

Mathematica [**A**] time = 0.00, size = 49, normalized size = 1.00

$$-\frac{a^4}{4x^4}-\frac{2a^3b}{x^2}+6a^2b^2\log(x)+2ab^3x^2+\frac{b^4x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^5,x]
```

```
[Out] -1/4*a^4/x^4 - (2*a^3*b)/x^2 + 2*a*b^3*x^2 + (b^4*x^4)/4 + 6*a^2*b^2*Log[x]
```

fricas [**A**] time = 0.81, size = 49, normalized size = 1.00

$$\frac{b^4x^8+8\,ab^3x^6+24\,a^2b^2x^4\log(x)-8\,a^3bx^2-a^4}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(b^4*x^8 + 8*a*b^3*x^6 + 24*a^2*b^2*x^4*log(x) - 8*a^3*b*x^2 - a^4)/x^4
```

giac [**A**] time = 0.15, size = 59, normalized size = 1.20

$$\frac{1}{4}\,b^4x^4+2\,ab^3x^2+3\,a^2b^2\log\left(x^2\right)-\frac{18\,a^2b^2x^4+8\,a^3bx^2+a^4}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/4*b^4*x^4 + 2*a*b^3*x^2 + 3*a^2*b^2*log(x^2) - 1/4*(18*a^2*b^2*x^4 + 8*a^
3*b*x^2 + a^4)/x^4
```

maple [A] time = 0.01, size = 46, normalized size = 0.94

$$\frac{b^4x^4}{4}+2a\,b^3x^2+6a^2b^2\ln(x)-\frac{2a^3b}{x^2}-\frac{a^4}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^5,x)
```

```
[Out] -1/4*a^4/x^4-2*a^3*b/x^2+2*a*b^3*x^2+1/4*b^4*x^4+6*a^2*b^2*ln(x)
```

maxima [A] time = 1.36, size = 48, normalized size = 0.98

$$\frac{1}{4}\,b^4x^4+2\,ab^3x^2+3\,a^2b^2\log\left(x^2\right)-\frac{8\,a^3bx^2+a^4}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*b^4*x^4 + 2*a*b^3*x^2 + 3*a^2*b^2*log(x^2) - 1/4*(8*a^3*b*x^2 + a^4)/x^
4
```

mupad [B] time = 0.04, size = 48, normalized size = 0.98

$$\frac{b^4\,x^4}{4}-\frac{\frac{a^4}{4}+2\,b\,a^3\,x^2}{x^4}+2\,a\,b^3\,x^2+6\,a^2\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^5,x)
```

```
[Out] (b^4*x^4)/4 - (a^4/4 + 2*a^3*b*x^2)/x^4 + 2*a*b^3*x^2 + 6*a^2*b^2*log(x)
```

sympy [A] time = 0.22, size = 49, normalized size = 1.00

$$6a^2b^2\log\left(x\right)+2ab^3x^2+\frac{b^4x^4}{4}+\frac{-a^4-8a^3bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**5,x)
```

```
[Out] 6*a**2*b**2*log(x) + 2*a*b**3*x**2 + b**4*x**4/4 + (-a**4 - 8*a**3*b*x**2)/
(4*x**4)
```

3.433 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^6} dx$

Optimal. Leaf size=50

$$-\frac{a^4}{5x^5}-\frac{4a^3b}{3x^3}-\frac{6a^2b^2}{x}+4ab^3x+\frac{b^4x^3}{3}$$

```
[Out] -1/5*a^4/x^5-4/3*a^3*b/x^3-6*a^2*b^2/x+4*a*b^3*x+1/3*b^4*x^3
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{6a^2b^2}{x}-\frac{4a^3b}{3x^3}-\frac{a^4}{5x^5}+4ab^3x+\frac{b^4x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^6,x]
```

```
[Out] -a^4/(5*x^5) - (4*a^3*b)/(3*x^3) - (6*a^2*b^2)/x + 4*a*b^3*x + (b^4*x^3)/3
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^6} dx &= \frac{\int \frac{(ab+b^2x^2)^4}{x^6} dx}{b^4}\\ &= \frac{\int \left(4ab^7+\frac{a^4b^4}{x^6}+\frac{4a^3b^5}{x^4}+\frac{6a^2b^6}{x^2}+b^8x^2\right) dx}{b^4}\\ &= -\frac{a^4}{5x^5}-\frac{4a^3b}{3x^3}-\frac{6a^2b^2}{x}+4ab^3x+\frac{b^4x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.00

$$-\frac{a^4}{5x^5}-\frac{4a^3b}{3x^3}-\frac{6a^2b^2}{x}+4ab^3x+\frac{b^4x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^6,x]
[Out] -1/5*a^4/x^5 - (4*a^3*b)/(3*x^3) - (6*a^2*b^2)/x + 4*a*b^3*x + (b^4*x^3)/3
```

fricas [A] time = 0.75, size = 48, normalized size = 0.96

$$\frac{5\,b^4x^8+60\,ab^3x^6-90\,a^2b^2x^4-20\,a^3bx^2-3\,a^4}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^6,x, algorithm="fricas")
[Out] 1/15*(5*b^4*x^8 + 60*a*b^3*x^6 - 90*a^2*b^2*x^4 - 20*a^3*b*x^2 - 3*a^4)/x^5
```

giac [A] time = 0.15, size = 47, normalized size = 0.94

$$\frac{1}{3}\,b^4x^3+4\,ab^3x-\frac{90\,a^2b^2x^4+20\,a^3bx^2+3\,a^4}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^6,x, algorithm="giac")
[Out] 1/3*b^4*x^3 + 4*a*b^3*x - 1/15*(90*a^2*b^2*x^4 + 20*a^3*b*x^2 + 3*a^4)/x^5
```

maple [A] time = 0.01, size = 45, normalized size = 0.90

$$\frac{b^4x^3}{3}+4a\,b^3x-\frac{6a^2b^2}{x}-\frac{4a^3b}{3x^3}-\frac{a^4}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^6,x)
[Out] -1/5*a^4/x^5-4/3*a^3*b/x^3-6*a^2*b^2/x+4*a*b^3*x+1/3*b^4*x^3
```

maxima [A] time = 1.37, size = 47, normalized size = 0.94

$$\frac{1}{3}\,b^4x^3+4\,ab^3x-\frac{90\,a^2b^2x^4+20\,a^3bx^2+3\,a^4}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^6,x, algorithm="maxima")
```

```
[Out] 1/3*b^4*x^3 + 4*a*b^3*x - 1/15*(90*a^2*b^2*x^4 + 20*a^3*b*x^2 + 3*a^4)/x^5
```

mupad [B] time = 0.04, size = 47, normalized size = 0.94

$$\frac{b^4\,x^3}{3}-\frac{\frac{a^4}{5}+\frac{4\,a^3\,b\,x^2}{3}+6\,a^2\,b^2\,x^4}{x^5}+4\,a\,b^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^6,x)
```

```
[Out] (b^4*x^3)/3 - (a^4/5 + (4*a^3*b*x^2)/3 + 6*a^2*b^2*x^4)/x^5 + 4*a*b^3*x
```

sympy [A] time = 0.23, size = 49, normalized size = 0.98

$$4ab^3x+\frac{b^4x^3}{3}+\frac{-3a^4-20a^3bx^2-90a^2b^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**6,x)
```

```
[Out] 4*a*b**3*x + b**4*x**3/3 + (-3*a**4 - 20*a**3*b*x**2 - 90*a**2*b**2*x**4)/(
15*x**5)
```

3.434 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^7} dx$

Optimal. Leaf size=49

$$-\frac{a^4}{6x^6}-\frac{a^3b}{x^4}-\frac{3a^2b^2}{x^2}+4ab^3\log(x)+\frac{b^4x^2}{2}$$

[Out] -1/6*a^4/x^6-a^3*b/x^4-3*a^2*b^2/x^2+1/2*b^4*x^2+4*a*b^3*ln(x)

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{3a^2b^2}{x^2}-\frac{a^3b}{x^4}-\frac{a^4}{6x^6}+4ab^3\log(x)+\frac{b^4x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^7,x]

[Out] -a^4/(6*x^6) - (a^3*b)/x^4 - (3*a^2*b^2)/x^2 + (b^4*x^2)/2 + 4*a*b^3*Log[x]

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^7}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^7}\,dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^4}{x^4}\,dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(b^8+\frac{a^4b^4}{x^4}+\frac{4a^3b^5}{x^3}+\frac{6a^2b^6}{x^2}+\frac{4ab^7}{x}\right)dx, x, x^2\right)}{2b^4}$$
$$= -\frac{a^4}{6x^6}-\frac{a^3b}{x^4}-\frac{3a^2b^2}{x^2}+\frac{b^4x^2}{2}+4ab^3\log(x)$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$-\frac{a^4}{6x^6}-\frac{a^3b}{x^4}-\frac{3a^2b^2}{x^2}+4ab^3\log(x)+\frac{b^4x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^7,x]
```

```
[Out] -1/6*a^4/x^6 - (a^3*b)/x^4 - (3*a^2*b^2)/x^2 + (b^4*x^2)/2 + 4*a*b^3*Log[x]
```

fricas [A] time = 0.77, size = 50, normalized size = 1.02

$$\frac{3\,b^4x^8+24\,ab^3x^6\log(x)-18\,a^2b^2x^4-6\,a^3bx^2-a^4}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^4*x^8 + 24*a*b^3*x^6*log(x) - 18*a^2*b^2*x^4 - 6*a^3*b*x^2 - a^4)/
x^6
```

giac [A] time = 0.17, size = 57, normalized size = 1.16

$$\frac{1}{2}\,b^4x^2+2\,ab^3\log\left(x^2\right)-\frac{22\,ab^3x^6+18\,a^2b^2x^4+6\,a^3bx^2+a^4}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/2*b^4*x^2 + 2*a*b^3*log(x^2) - 1/6*(22*a*b^3*x^6 + 18*a^2*b^2*x^4 + 6*a^3
*b*x^2 + a^4)/x^6
```

maple [A] time = 0.01, size = 46, normalized size = 0.94

$$\frac{b^4x^2}{2}+4a\,b^3\ln(x)-\frac{3a^2b^2}{x^2}-\frac{a^3b}{x^4}-\frac{a^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^7,x)
```

```
[Out] -1/6*a^4/x^6-a^3*b/x^4-3*a^2*b^2/x^2+1/2*b^4*x^2+4*a*b^3*ln(x)
```

maxima [A] time = 1.31, size = 48, normalized size = 0.98

$$\frac{1}{2}\,b^4x^2+2\,ab^3\log\left(x^2\right)-\frac{18\,a^2b^2x^4+6\,a^3bx^2+a^4}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*b^4*x^2 + 2*a*b^3*log(x^2) - 1/6*(18*a^2*b^2*x^4 + 6*a^3*b*x^2 + a^4)/x
^6
```

mupad [B] time = 0.04, size = 47, normalized size = 0.96

$$\frac{b^4\,x^2}{2}-\frac{\frac{a^4}{6}+a^3\,b\,x^2+3\,a^2\,b^2\,x^4}{x^6}+4\,a\,b^3\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^7,x)
```

```
[Out] (b^4*x^2)/2 - (a^4/6 + a^3*b*x^2 + 3*a^2*b^2*x^4)/x^6 + 4*a*b^3*log(x)
```

sympy [A] time = 0.30, size = 49, normalized size = 1.00

$$4ab^3\log\left(x\right)+\frac{b^4x^2}{2}+\frac{-a^4-6a^3bx^2-18a^2b^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**7,x)
```

```
[Out] 4*a*b**3*log(x) + b**4*x**2/2 + (-a**4 - 6*a**3*b*x**2 - 18*a**2*b**2*x**4)
/(6*x**6)
```

3.435 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^8}\,dx$

Optimal. Leaf size=47

$$-\frac{a^4}{7x^7}-\frac{4a^3b}{5x^5}-\frac{2a^2b^2}{x^3}-\frac{4ab^3}{x}+b^4x$$

[Out] -1/7*a^4/x^7-4/5*a^3*b/x^5-2*a^2*b^2/x^3-4*a*b^3/x+b^4*x

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{2a^2b^2}{x^3}-\frac{4a^3b}{5x^5}-\frac{a^4}{7x^7}-\frac{4ab^3}{x}+b^4x$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^8,x]

[Out] -a^4/(7*x^7) - (4*a^3*b)/(5*x^5) - (2*a^2*b^2)/x^3 - (4*a*b^3)/x + b^4*x

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^8}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^8}\,dx}{b^4}\\
&= \frac{\int \left(b^8+\frac{a^4b^4}{x^8}+\frac{4a^3b^5}{x^6}+\frac{6a^2b^6}{x^4}+\frac{4ab^7}{x^2}\right)dx}{b^4}\\
&= -\frac{a^4}{7x^7}-\frac{4a^3b}{5x^5}-\frac{2a^2b^2}{x^3}-\frac{4ab^3}{x}+b^4x
\end{aligned}$$

Mathematica [A] time = 0.01, size = 47, normalized size = 1.00

$$-\frac{a^4}{7x^7}-\frac{4a^3b}{5x^5}-\frac{2a^2b^2}{x^3}-\frac{4ab^3}{x}+b^4x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^8,x]
```

```
[Out] -1/7*a^4/x^7 - (4*a^3*b)/(5*x^5) - (2*a^2*b^2)/x^3 - (4*a*b^3)/x + b^4*x
```

fricas [A] time = 0.85, size = 48, normalized size = 1.02

$$\frac{35\,b^4x^8-140\,ab^3x^6-70\,a^2b^2x^4-28\,a^3bx^2-5\,a^4}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^8,x, algorithm="fricas")
```

```
[Out] 1/35*(35*b^4*x^8 - 140*a*b^3*x^6 - 70*a^2*b^2*x^4 - 28*a^3*b*x^2 - 5*a^4)/x
^7
```

giac [A] time = 0.16, size = 46, normalized size = 0.98

$$b^4x-\frac{140\,ab^3x^6+70\,a^2b^2x^4+28\,a^3bx^2+5\,a^4}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^8,x, algorithm="giac")
```

```
[Out] b^4*x - 1/35*(140*a*b^3*x^6 + 70*a^2*b^2*x^4 + 28*a^3*b*x^2 + 5*a^4)/x^7
```

maple [A] time = 0.01, size = 44, normalized size = 0.94

$$b^4x-\frac{4a\,b^3}{x}-\frac{2a^2b^2}{x^3}-\frac{4a^3b}{5x^5}-\frac{a^4}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^8,x)
```

```
[Out] -1/7*a^4/x^7-4/5*a^3*b/x^5-2*a^2*b^2/x^3-4*a*b^3/x+b^4*x
```

maxima [A] time = 1.45, size = 46, normalized size = 0.98

$$b^4x-\frac{140\,ab^3x^6+70\,a^2b^2x^4+28\,a^3bx^2+5\,a^4}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^8,x, algorithm="maxima")
```

```
[Out] b^4*x - 1/35*(140*a*b^3*x^6 + 70*a^2*b^2*x^4 + 28*a^3*b*x^2 + 5*a^4)/x^7
```

mupad [B] time = 4.19, size = 46, normalized size = 0.98

$$b^4\,x-\frac{\frac{a^4}{7}+\frac{4\,a^3\,b\,x^2}{5}+2\,a^2\,b^2\,x^4+4\,a\,b^3\,x^6}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^8,x)
```

```
[Out] b^4*x - (a^4/7 + (4*a^3*b*x^2)/5 + 4*a*b^3*x^6 + 2*a^2*b^2*x^4)/x^7
```

sympy [A] time = 0.30, size = 48, normalized size = 1.02

$$b^4x+\frac{-5a^4-28a^3bx^2-70a^2b^2x^4-140ab^3x^6}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**8,x)
```

```
[Out] b**4*x + (-5*a**4 - 28*a**3*b*x**2 - 70*a**2*b**2*x**4 - 140*a*b**3*x**6)/(
35*x**7)
```

3.436 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^9} dx$

Optimal. Leaf size=50

$$-\frac{a^4}{8x^8}-\frac{2a^3b}{3x^6}-\frac{3a^2b^2}{2x^4}-\frac{2ab^3}{x^2}+b^4\log(x)$$

[Out] -1/8*a^4/x^8-2/3*a^3*b/x^6-3/2*a^2*b^2/x^4-2*a*b^3/x^2+b^4*ln(x)

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{3a^2b^2}{2x^4}-\frac{2a^3b}{3x^6}-\frac{a^4}{8x^8}-\frac{2ab^3}{x^2}+b^4\log(x)$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^9,x]

[Out] -a^4/(8*x^8) - (2*a^3*b)/(3*x^6) - (3*a^2*b^2)/(2*x^4) - (2*a*b^3)/x^2 + b^4*Log[x]

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^9}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^9}\,dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^4}{x^5}\,dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^4b^4}{x^5}+\frac{4a^3b^5}{x^4}+\frac{6a^2b^6}{x^3}+\frac{4ab^7}{x^2}+\frac{b^8}{x}\right)dx, x, x^2\right)}{2b^4}$$
$$= -\frac{a^4}{8x^8}-\frac{2a^3b}{3x^6}-\frac{3a^2b^2}{2x^4}-\frac{2ab^3}{x^2}+b^4\log(x)$$

Mathematica [A] time = 0.00, size = 50, normalized size = 1.00

$$-\frac{a^4}{8x^8}-\frac{2a^3b}{3x^6}-\frac{3a^2b^2}{2x^4}-\frac{2ab^3}{x^2}+b^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^9,x]
```

```
[Out] -1/8*a^4/x^8 - (2*a^3*b)/(3*x^6) - (3*a^2*b^2)/(2*x^4) - (2*a*b^3)/x^2 + b^
4*Log[x]
```

fricas [A] time = 0.59, size = 50, normalized size = 1.00

$$\frac{24\,b^4x^8\log(x)-48\,ab^3x^6-36\,a^2b^2x^4-16\,a^3bx^2-3\,a^4}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(24*b^4*x^8*log(x) - 48*a*b^3*x^6 - 36*a^2*b^2*x^4 - 16*a^3*b*x^2 - 3*
a^4)/x^8
```

giac [A] time = 0.15, size = 58, normalized size = 1.16

$$\frac{1}{2}\,b^4\log\left(x^2\right)-\frac{25\,b^4x^8+48\,ab^3x^6+36\,a^2b^2x^4+16\,a^3bx^2+3\,a^4}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^9,x, algorithm="giac")
```

```
[Out] 1/2*b^4*log(x^2) - 1/24*(25*b^4*x^8 + 48*a*b^3*x^6 + 36*a^2*b^2*x^4 + 16*a^
3*b*x^2 + 3*a^4)/x^8
```

maple [A] time = 0.00, size = 45, normalized size = 0.90

$$b^4 \ln(x) - \frac{2a\,b^3}{x^2} - \frac{3a^2b^2}{2x^4} - \frac{2a^3b}{3x^6} - \frac{a^4}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^9,x)
```

```
[Out] -1/8*a^4/x^8-2/3*a^3*b/x^6-3/2*a^2*b^2/x^4-2*a*b^3/x^2+b^4*ln(x)
```

maxima [A] time = 1.38, size = 50, normalized size = 1.00

$$\frac{1}{2}\,b^4\log\left(x^2\right) - \frac{48\,ab^3x^6 + 36\,a^2b^2x^4 + 16\,a^3bx^2 + 3\,a^4}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*b^4*log(x^2) - 1/24*(48*a*b^3*x^6 + 36*a^2*b^2*x^4 + 16*a^3*b*x^2 + 3*a
^4)/x^8
```

mupad [B] time = 0.05, size = 47, normalized size = 0.94

$$b^4\,\ln(x) - \frac{\frac{a^4}{8} + \frac{2\,a^3\,b\,x^2}{3} + \frac{3\,a^2\,b^2\,x^4}{2} + 2\,a\,b^3\,x^6}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^9,x)
```

```
[Out] b^4*log(x) - (a^4/8 + (2*a^3*b*x^2)/3 + 2*a*b^3*x^6 + (3*a^2*b^2*x^4)/2)/x^
8
```

sympy [A] time = 0.37, size = 49, normalized size = 0.98

$$b^4\log\left(x\right) + \frac{-3a^4 - 16a^3bx^2 - 36a^2b^2x^4 - 48ab^3x^6}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**9,x)
```

```
[Out] b**4*log(x) + (-3*a**4 - 16*a**3*b*x**2 - 36*a**2*b**2*x**4 - 48*a*b**3*x**
6)/(24*x**8)
```

3.437 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{10}}\,dx$

Optimal. Leaf size=54

$$-\frac{a^4}{9x^9}-\frac{4a^3b}{7x^7}-\frac{6a^2b^2}{5x^5}-\frac{4ab^3}{3x^3}-\frac{b^4}{x}$$

```
[Out] -1/9*a^4/x^9-4/7*a^3*b/x^7-6/5*a^2*b^2/x^5-4/3*a*b^3/x^3-b^4/x
```

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{6a^2b^2}{5x^5}-\frac{4a^3b}{7x^7}-\frac{a^4}{9x^9}-\frac{4ab^3}{3x^3}-\frac{b^4}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^10,x]
```

```
[Out] -a^4/(9*x^9) - (4*a^3*b)/(7*x^7) - (6*a^2*b^2)/(5*x^5) - (4*a*b^3)/(3*x^3)
- b^4/x
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{10}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^{10}}\,dx}{b^4}$$
$$= \frac{\int \left(\frac{a^4b^4}{x^{10}}+\frac{4a^3b^5}{x^8}+\frac{6a^2b^6}{x^6}+\frac{4ab^7}{x^4}+\frac{b^8}{x^2}\right)dx}{b^4}$$
$$= -\frac{a^4}{9x^9}-\frac{4a^3b}{7x^7}-\frac{6a^2b^2}{5x^5}-\frac{4ab^3}{3x^3}-\frac{b^4}{x}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$-\frac{a^4}{9x^9}-\frac{4a^3b}{7x^7}-\frac{6a^2b^2}{5x^5}-\frac{4ab^3}{3x^3}-\frac{b^4}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^10,x]
```

```
[Out] -1/9*a^4/x^9 - (4*a^3*b)/(7*x^7) - (6*a^2*b^2)/(5*x^5) - (4*a*b^3)/(3*x^3)
- b^4/x
```

fricas [A] time = 0.75, size = 48, normalized size = 0.89

$$-\frac{315\,b^4x^8+420\,ab^3x^6+378\,a^2b^2x^4+180\,a^3bx^2+35\,a^4}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(315*b^4*x^8 + 420*a*b^3*x^6 + 378*a^2*b^2*x^4 + 180*a^3*b*x^2 + 35*
a^4)/x^9
```

giac [A] time = 0.17, size = 48, normalized size = 0.89

$$-\frac{315\,b^4x^8+420\,ab^3x^6+378\,a^2b^2x^4+180\,a^3bx^2+35\,a^4}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^10,x, algorithm="giac")
```

```
[Out] -1/315*(315*b^4*x^8 + 420*a*b^3*x^6 + 378*a^2*b^2*x^4 + 180*a^3*b*x^2 + 35*
a^4)/x^9
```

maple [A] time = 0.01, size = 47, normalized size = 0.87

$$-\frac{b^{4}}{x}-\frac{4 a \,b^{3}}{3 x^{3}}-\frac{6 a^{2} b^{2}}{5 x^{5}}-\frac{4 a^{3} b}{7 x^{7}}-\frac{a^{4}}{9 x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^10,x)
```

```
[Out] -1/9*a^4/x^9-4/7*a^3*b/x^7-6/5*a^2*b^2/x^5-4/3*a*b^3/x^3-b^4/x
```

maxima [A] time = 1.43, size = 48, normalized size = 0.89

$$-\frac{315 \, b^{4} x^{8} + 420 \, a b^{3} x^{6} + 378 \, a^{2} b^{2} x^{4} + 180 \, a^{3} b x^{2} + 35 \, a^{4}}{315 \, x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^10,x, algorithm="maxima")
```

```
[Out] -1/315*(315*b^4*x^8 + 420*a*b^3*x^6 + 378*a^2*b^2*x^4 + 180*a^3*b*x^2 + 35*
a^4)/x^9
```

mupad [B] time = 0.03, size = 47, normalized size = 0.87

$$-\frac{\frac{a^4}{9}+\frac{4\,a^3\,b\,x^2}{7}+\frac{6\,a^2\,b^2\,x^4}{5}+\frac{4\,a\,b^3\,x^6}{3}+b^4\,x^8}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^10,x)
```

```
[Out] -(a^4/9 + b^4*x^8 + (4*a^3*b*x^2)/7 + (4*a*b^3*x^6)/3 + (6*a^2*b^2*x^4)/5)/
x^9
```

sympy [A] time = 0.37, size = 51, normalized size = 0.94

$$\frac{- 35 a^{4} - 180 a^{3} b x^{2} - 378 a^{2} b^{2} x^{4} - 420 a b^{3} x^{6} - 315 b^{4} x^{8}}{315 x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**10,x)
```

```
[Out] (-35*a**4 - 180*a**3*b*x**2 - 378*a**2*b**2*x**4 - 420*a*b**3*x**6 - 315*b*
*4*x**8)/(315*x**9)
```

3.438 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{11}}\,dx$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^5}{10ax^{10}}$$

```
[Out] -1/10*(b*x^2+a)^5/a/x^10
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 264}

$$-\frac{(a+bx^2)^5}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^11,x]
```

```
[Out] -(a + b*x^2)^5/(10*a*x^10)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{11}}\,dx &= \frac{\int \frac{(ab+b^2x^2)^4}{x^{11}}\,dx}{b^4}\\ &= -\frac{(a+bx^2)^5}{10ax^{10}}\end{aligned}$$

Mathematica [**B**] time = 0.00, size = 52, normalized size = 2.74

$$-\frac{a^4}{10x^{10}}-\frac{a^3b}{2x^8}-\frac{a^2b^2}{x^6}-\frac{ab^3}{x^4}-\frac{b^4}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^11,x]
```

```
[Out] -1/10*a^4/x^10 - (a^3*b)/(2*x^8) - (a^2*b^2)/x^6 - (a*b^3)/x^4 - b^4/(2*x^2
)
```

fricas [**B**] time = 0.79, size = 46, normalized size = 2.42

$$-\frac{5\,b^4x^8+10\,ab^3x^6+10\,a^2b^2x^4+5\,a^3bx^2+a^4}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^11,x, algorithm="fricas")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/x^10
```

giac [**B**] time = 0.18, size = 46, normalized size = 2.42

$$-\frac{5\,b^4x^8+10\,ab^3x^6+10\,a^2b^2x^4+5\,a^3bx^2+a^4}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^11,x, algorithm="giac")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/x^10
```

maple [**B**] time = 0.00, size = 47, normalized size = 2.47

$$-\frac{b^4}{2x^2}-\frac{a\,b^3}{x^4}-\frac{a^2b^2}{x^6}-\frac{a^3b}{2x^8}-\frac{a^4}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^11,x)
```

```
[Out] -1/2*b^4/x^2-a^2*b^2/x^6-1/2*a^3*b/x^8-1/10*a^4/x^10-a*b^3/x^4
```

maxima [**B**] time = 1.35, size = 46, normalized size = 2.42

$$-\frac{5\,b^4x^8+10\,ab^3x^6+10\,a^2b^2x^4+5\,a^3bx^2+a^4}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^11,x, algorithm="maxima")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/x^10
```

mupad [B] time = 0.03, size = 46, normalized size = 2.42

$$-\frac{\frac{a^4}{10}+\frac{a^3\,b\,x^2}{2}+a^2\,b^2\,x^4+a\,b^3\,x^6+\frac{b^4\,x^8}{2}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^11,x)
```

```
[Out] -(a^4/10 + (b^4*x^8)/2 + (a^3*b*x^2)/2 + a*b^3*x^6 + a^2*b^2*x^4)/x^10
```

sympy [B] time = 0.39, size = 49, normalized size = 2.58

$$\frac{-a^4-5a^3bx^2-10a^2b^2x^4-10ab^3x^6-5b^4x^8}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**11,x)
```

```
[Out] (-a**4 - 5*a**3*b*x**2 - 10*a**2*b**2*x**4 - 10*a*b**3*x**6 - 5*b**4*x**8)/
(10*x**10)
```

3.439 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{12}} dx$

Optimal. Leaf size=56

$$-\frac{a^4}{11x^{11}}-\frac{4a^3b}{9x^9}-\frac{6a^2b^2}{7x^7}-\frac{4ab^3}{5x^5}-\frac{b^4}{3x^3}$$

```
[Out] -1/11*a^4/x^11-4/9*a^3*b/x^9-6/7*a^2*b^2/x^7-4/5*a*b^3/x^5-1/3*b^4/x^3
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{6a^2b^2}{7x^7}-\frac{4a^3b}{9x^9}-\frac{a^4}{11x^{11}}-\frac{4ab^3}{5x^5}-\frac{b^4}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^12,x]
```

```
[Out] -a^4/(11*x^11) - (4*a^3*b)/(9*x^9) - (6*a^2*b^2)/(7*x^7) - (4*a*b^3)/(5*x^5
) - b^4/(3*x^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{12}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^{12}}\,dx}{b^4}$$
$$= \frac{\int \left(\frac{a^4b^4}{x^{12}}+\frac{4a^3b^5}{x^{10}}+\frac{6a^2b^6}{x^8}+\frac{4ab^7}{x^6}+\frac{b^8}{x^4}\right)dx}{b^4}$$
$$= -\frac{a^4}{11x^{11}}-\frac{4a^3b}{9x^9}-\frac{6a^2b^2}{7x^7}-\frac{4ab^3}{5x^5}-\frac{b^4}{3x^3}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 1.00

$$-\frac{a^4}{11x^{11}}-\frac{4a^3b}{9x^9}-\frac{6a^2b^2}{7x^7}-\frac{4ab^3}{5x^5}-\frac{b^4}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^12,x]
```

```
[Out] -1/11*a^4/x^11 - (4*a^3*b)/(9*x^9) - (6*a^2*b^2)/(7*x^7) - (4*a*b^3)/(5*x^5
) - b^4/(3*x^3)
```

fricas [A] time = 0.79, size = 48, normalized size = 0.86

$$-\frac{1155\,b^4x^8+2772\,ab^3x^6+2970\,a^2b^2x^4+1540\,a^3bx^2+315\,a^4}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^12,x, algorithm="fricas")
```

```
[Out] -1/3465*(1155*b^4*x^8 + 2772*a*b^3*x^6 + 2970*a^2*b^2*x^4 + 1540*a^3*b*x^2
+ 315*a^4)/x^11
```

giac [A] time = 0.16, size = 48, normalized size = 0.86

$$-\frac{1155\,b^4x^8+2772\,ab^3x^6+2970\,a^2b^2x^4+1540\,a^3bx^2+315\,a^4}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^12,x, algorithm="giac")
```

```
[Out] -1/3465*(1155*b^4*x^8 + 2772*a*b^3*x^6 + 2970*a^2*b^2*x^4 + 1540*a^3*b*x^2
+ 315*a^4)/x^11
```

maple [A] time = 0.01, size = 47, normalized size = 0.84

$$-\frac{b^4}{3x^3}-\frac{4a\,b^3}{5x^5}-\frac{6a^2b^2}{7x^7}-\frac{4a^3b}{9x^9}-\frac{a^4}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^12,x)
```

```
[Out] -1/11*a^4/x^11-4/9*a^3*b/x^9-6/7*a^2*b^2/x^7-4/5*a*b^3/x^5-1/3*b^4/x^3
```

maxima [A] time = 1.30, size = 48, normalized size = 0.86

$$-\frac{1155\,b^4x^8+2772\,ab^3x^6+2970\,a^2b^2x^4+1540\,a^3bx^2+315\,a^4}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^12,x, algorithm="maxima")
```

```
[Out] -1/3465*(1155*b^4*x^8 + 2772*a*b^3*x^6 + 2970*a^2*b^2*x^4 + 1540*a^3*b*x^2
+ 315*a^4)/x^11
```

mupad [B] time = 4.84, size = 48, normalized size = 0.86

$$-\frac{\frac{a^4}{11}+\frac{4\,a^3\,b\,x^2}{9}+\frac{6\,a^2\,b^2\,x^4}{7}+\frac{4\,a\,b^3\,x^6}{5}+\frac{b^4\,x^8}{3}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^12,x)
```

```
[Out] -(a^4/11 + (b^4*x^8)/3 + (4*a^3*b*x^2)/9 + (4*a*b^3*x^6)/5 + (6*a^2*b^2*x^4
)/7)/x^11
```

sympy [A] time = 0.40, size = 51, normalized size = 0.91

$$\frac{-315a^4-1540a^3bx^2-2970a^2b^2x^4-2772ab^3x^6-1155b^4x^8}{3465x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**12,x)
```

```
[Out] (-315*a**4 - 1540*a**3*b*x**2 - 2970*a**2*b**2*x**4 - 2772*a*b**3*x**6 - 11
55*b**4*x**8)/(3465*x**11)
```

3.440 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{13}} dx$

Optimal. Leaf size=40

$$\frac{b(a+bx^2)^5}{60a^2x^{10}} - \frac{(a+bx^2)^5}{12ax^{12}}$$

```
[Out] -1/12*(b*x^2+a)^5/a/x^12+1/60*b*(b*x^2+a)^5/a^2/x^10
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 266, 45, 37}

$$\frac{b(a+bx^2)^5}{60a^2x^{10}} - \frac{(a+bx^2)^5}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^13,x]
```

```
[Out] -(a + b*x^2)^5/(12*a*x^12) + (b*(a + b*x^2)^5)/(60*a^2*x^10)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{13}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^{13}}\,dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^4}{x^7}\,dx, x, x^2\right)}{2b^4}$$
$$= -\frac{\left(a+bx^2\right)^5}{12ax^{12}} - \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^4}{x^6}\,dx, x, x^2\right)}{12ab^3}$$
$$= -\frac{\left(a+bx^2\right)^5}{12ax^{12}} + \frac{b\left(a+bx^2\right)^5}{60a^2x^{10}}$$

Mathematica [A] time = 0.00, size = 56, normalized size = 1.40

$$-\frac{a^4}{12x^{12}} - \frac{2a^3b}{5x^{10}} - \frac{3a^2b^2}{4x^8} - \frac{2ab^3}{3x^6} - \frac{b^4}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^13,x]
```

```
[Out] -1/12*a^4/x^12 - (2*a^3*b)/(5*x^10) - (3*a^2*b^2)/(4*x^8) - (2*a*b^3)/(3*x^
6) - b^4/(4*x^4)
```

fricas [A] time = 0.57, size = 48, normalized size = 1.20

$$-\frac{15\,b^4x^8 + 40\,ab^3x^6 + 45\,a^2b^2x^4 + 24\,a^3bx^2 + 5\,a^4}{60\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^13,x, algorithm="fricas")
```

```
[Out] -1/60*(15*b^4*x^8 + 40*a*b^3*x^6 + 45*a^2*b^2*x^4 + 24*a^3*b*x^2 + 5*a^4)/x
^12
```

giac [A] time = 0.15, size = 48, normalized size = 1.20

$$-\frac{15\,b^4x^8+40\,ab^3x^6+45\,a^2b^2x^4+24\,a^3bx^2+5\,a^4}{60\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^13,x, algorithm="giac")
```

```
[Out] -1/60*(15*b^4*x^8 + 40*a*b^3*x^6 + 45*a^2*b^2*x^4 + 24*a^3*b*x^2 + 5*a^4)/x
^12
```

maple [A] time = 0.00, size = 47, normalized size = 1.18

$$-\frac{b^4}{4x^4}-\frac{2a\,b^3}{3x^6}-\frac{3a^2b^2}{4x^8}-\frac{2a^3b}{5x^{10}}-\frac{a^4}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^13,x)
```

```
[Out] -2/3*a*b^3/x^6-2/5*a^3*b/x^10-1/12*a^4/x^12-3/4*a^2*b^2/x^8-1/4*b^4/x^4
```

maxima [A] time = 1.35, size = 48, normalized size = 1.20

$$-\frac{15\,b^4x^8+40\,ab^3x^6+45\,a^2b^2x^4+24\,a^3bx^2+5\,a^4}{60\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^13,x, algorithm="maxima")
```

```
[Out] -1/60*(15*b^4*x^8 + 40*a*b^3*x^6 + 45*a^2*b^2*x^4 + 24*a^3*b*x^2 + 5*a^4)/x
^12
```

mupad [B] time = 4.22, size = 48, normalized size = 1.20

$$-\frac{\frac{a^4}{12}+\frac{2\,a^3\,b\,x^2}{5}+\frac{3\,a^2\,b^2\,x^4}{4}+\frac{2\,a\,b^3\,x^6}{3}+\frac{b^4\,x^8}{4}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^13,x)
```

```
[Out] -(a^4/12 + (b^4*x^8)/4 + (2*a^3*b*x^2)/5 + (2*a*b^3*x^6)/3 + (3*a^2*b^2*x^4
)/4)/x^12
```

sympy [A] time = 0.43, size = 51, normalized size = 1.28

$$\frac{-5a^4 - 24a^3bx^2 - 45a^2b^2x^4 - 40ab^3x^6 - 15b^4x^8}{60x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**13,x)
```

```
[Out] (-5*a**4 - 24*a**3*b*x**2 - 45*a**2*b**2*x**4 - 40*a*b**3*x**6 - 15*b**4*x*
*8)/(60*x**12)
```

3.441 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{14}} dx$

Optimal. Leaf size=56

$$-\frac{a^4}{13x^{13}}-\frac{4a^3b}{11x^{11}}-\frac{2a^2b^2}{3x^9}-\frac{4ab^3}{7x^7}-\frac{b^4}{5x^5}$$

```
[Out] -1/13*a^4/x^13-4/11*a^3*b/x^11-2/3*a^2*b^2/x^9-4/7*a*b^3/x^7-1/5*b^4/x^5
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{2a^2b^2}{3x^9}-\frac{4a^3b}{11x^{11}}-\frac{a^4}{13x^{13}}-\frac{4ab^3}{7x^7}-\frac{b^4}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^14,x]
```

```
[Out] -a^4/(13*x^13) - (4*a^3*b)/(11*x^11) - (2*a^2*b^2)/(3*x^9) - (4*a*b^3)/(7*x
^7) - b^4/(5*x^5)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{14}}\,dx = \frac{\int \frac{(ab+b^2x^2)^4}{x^{14}}\,dx}{b^4}$$
$$= \frac{\int \left(\frac{a^4b^4}{x^{14}} + \frac{4a^3b^5}{x^{12}} + \frac{6a^2b^6}{x^{10}} + \frac{4ab^7}{x^8} + \frac{b^8}{x^6}\right)dx}{b^4}$$
$$= -\frac{a^4}{13x^{13}} - \frac{4a^3b}{11x^{11}} - \frac{2a^2b^2}{3x^9} - \frac{4ab^3}{7x^7} - \frac{b^4}{5x^5}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 1.00

$$-\frac{a^4}{13x^{13}} - \frac{4a^3b}{11x^{11}} - \frac{2a^2b^2}{3x^9} - \frac{4ab^3}{7x^7} - \frac{b^4}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^14,x]
```

```
[Out] -1/13*a^4/x^13 - (4*a^3*b)/(11*x^11) - (2*a^2*b^2)/(3*x^9) - (4*a*b^3)/(7*x
^7) - b^4/(5*x^5)
```

fricas [A] time = 0.80, size = 48, normalized size = 0.86

$$-\frac{3003\,b^4x^8 + 8580\,ab^3x^6 + 10010\,a^2b^2x^4 + 5460\,a^3bx^2 + 1155\,a^4}{15015\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^14,x, algorithm="fricas")
```

```
[Out] -1/15015*(3003*b^4*x^8 + 8580*a*b^3*x^6 + 10010*a^2*b^2*x^4 + 5460*a^3*b*x^
2 + 1155*a^4)/x^13
```

giac [A] time = 0.21, size = 48, normalized size = 0.86

$$-\frac{3003\,b^4x^8 + 8580\,ab^3x^6 + 10010\,a^2b^2x^4 + 5460\,a^3bx^2 + 1155\,a^4}{15015\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^14,x, algorithm="giac")
```

```
[Out] -1/15015*(3003*b^4*x^8 + 8580*a*b^3*x^6 + 10010*a^2*b^2*x^4 + 5460*a^3*b*x^
2 + 1155*a^4)/x^13
```

maple [A] time = 0.00, size = 47, normalized size = 0.84

$$-\frac{b^4}{5x^5}-\frac{4a\,b^3}{7x^7}-\frac{2a^2b^2}{3x^9}-\frac{4a^3b}{11x^{11}}-\frac{a^4}{13x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^14,x)
```

```
[Out] -1/13*a^4/x^13-4/11*a^3*b/x^11-2/3*a^2*b^2/x^9-4/7*a*b^3/x^7-1/5*b^4/x^5
```

maxima [A] time = 1.46, size = 48, normalized size = 0.86

$$-\frac{3003\,b^4x^8+8580\,ab^3x^6+10010\,a^2b^2x^4+5460\,a^3bx^2+1155\,a^4}{15015\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^14,x, algorithm="maxima")
```

```
[Out] -1/15015*(3003*b^4*x^8 + 8580*a*b^3*x^6 + 10010*a^2*b^2*x^4 + 5460*a^3*b*x^
2 + 1155*a^4)/x^13
```

mupad [B] time = 0.04, size = 48, normalized size = 0.86

$$-\frac{\frac{a^4}{13}+\frac{4\,a^3\,b\,x^2}{11}+\frac{2\,a^2\,b^2\,x^4}{3}+\frac{4\,a\,b^3\,x^6}{7}+\frac{b^4\,x^8}{5}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^14,x)
```

```
[Out] -(a^4/13 + (b^4*x^8)/5 + (4*a^3*b*x^2)/11 + (4*a*b^3*x^6)/7 + (2*a^2*b^2*x^
4)/3)/x^13
```

sympy [A] time = 0.44, size = 51, normalized size = 0.91

$$\frac{-1155a^4-5460a^3bx^2-10010a^2b^2x^4-8580ab^3x^6-3003b^4x^8}{15015x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**14,x)
```

```
[Out] (-1155*a**4 - 5460*a**3*b*x**2 - 10010*a**2*b**2*x**4 - 8580*a*b**3*x**6 -
3003*b**4*x**8)/(15015*x**13)
```

3.442 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{15}}\,dx$

Optimal. Leaf size=56

$$-\frac{a^4}{14x^{14}}-\frac{a^3b}{3x^{12}}-\frac{3a^2b^2}{5x^{10}}-\frac{ab^3}{2x^8}-\frac{b^4}{6x^6}$$

```
[Out] -1/14*a^4/x^14-1/3*a^3*b/x^12-3/5*a^2*b^2/x^10-1/2*a*b^3/x^8-1/6*b^4/x^6
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{3a^2b^2}{5x^{10}}-\frac{a^3b}{3x^{12}}-\frac{a^4}{14x^{14}}-\frac{ab^3}{2x^8}-\frac{b^4}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^15,x]
```

```
[Out] -a^4/(14*x^14) - (a^3*b)/(3*x^12) - (3*a^2*b^2)/(5*x^10) - (a*b^3)/(2*x^8)
- b^4/(6*x^6)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{15}}\,dx = \frac{\int \frac{(ab+b^2x^2)^4}{x^{15}}\,dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^4}{x^8}\,dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^4b^4}{x^8}+\frac{4a^3b^5}{x^7}+\frac{6a^2b^6}{x^6}+\frac{4ab^7}{x^5}+\frac{b^8}{x^4}\right)dx, x, x^2\right)}{2b^4}$$
$$= -\frac{a^4}{14x^{14}}-\frac{a^3b}{3x^{12}}-\frac{3a^2b^2}{5x^{10}}-\frac{ab^3}{2x^8}-\frac{b^4}{6x^6}$$

Mathematica [A] time = 0.00, size = 56, normalized size = 1.00

$$-\frac{a^4}{14x^{14}}-\frac{a^3b}{3x^{12}}-\frac{3a^2b^2}{5x^{10}}-\frac{ab^3}{2x^8}-\frac{b^4}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^15,x]
```

```
[Out] -1/14*a^4/x^14 - (a^3*b)/(3*x^12) - (3*a^2*b^2)/(5*x^10) - (a*b^3)/(2*x^8)
- b^4/(6*x^6)
```

fricas [A] time = 0.84, size = 48, normalized size = 0.86

$$-\frac{35\,b^4x^8+105\,ab^3x^6+126\,a^2b^2x^4+70\,a^3bx^2+15\,a^4}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^15,x, algorithm="fricas")
```

```
[Out] -1/210*(35*b^4*x^8 + 105*a*b^3*x^6 + 126*a^2*b^2*x^4 + 70*a^3*b*x^2 + 15*a^
4)/x^14
```

giac [A] time = 0.15, size = 48, normalized size = 0.86

$$-\frac{35\,b^4x^8+105\,ab^3x^6+126\,a^2b^2x^4+70\,a^3bx^2+15\,a^4}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^15,x, algorithm="giac")
```

```
[Out] -1/210*(35*b^4*x^8 + 105*a*b^3*x^6 + 126*a^2*b^2*x^4 + 70*a^3*b*x^2 + 15*a^
4)/x^14
```

maple [A] time = 0.01, size = 47, normalized size = 0.84

$$-\frac{b^4}{6x^6}-\frac{a\,b^3}{2x^8}-\frac{3a^2b^2}{5x^{10}}-\frac{a^3b}{3x^{12}}-\frac{a^4}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^15,x)
```

```
[Out] -1/14*a^4/x^14-1/3*a^3*b/x^12-3/5*a^2*b^2/x^10-1/2*a*b^3/x^8-1/6*b^4/x^6
```

maxima [A] time = 1.43, size = 48, normalized size = 0.86

$$-\frac{35\,b^4x^8+105\,ab^3x^6+126\,a^2b^2x^4+70\,a^3bx^2+15\,a^4}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^15,x, algorithm="maxima")
```

```
[Out] -1/210*(35*b^4*x^8 + 105*a*b^3*x^6 + 126*a^2*b^2*x^4 + 70*a^3*b*x^2 + 15*a^
4)/x^14
```

mupad [B] time = 4.33, size = 48, normalized size = 0.86

$$-\frac{\frac{a^4}{14}+\frac{a^3\,b\,x^2}{3}+\frac{3\,a^2\,b^2\,x^4}{5}+\frac{a\,b^3\,x^6}{2}+\frac{b^4\,x^8}{6}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^15,x)
```

```
[Out] -(a^4/14 + (b^4*x^8)/6 + (a^3*b*x^2)/3 + (a*b^3*x^6)/2 + (3*a^2*b^2*x^4)/5)
/x^14
```

sympy [A] time = 0.45, size = 51, normalized size = 0.91

$$\frac{-15a^4-70a^3bx^2-126a^2b^2x^4-105ab^3x^6-35b^4x^8}{210x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**15,x)
```

```
[Out] (-15*a**4 - 70*a**3*b*x**2 - 126*a**2*b**2*x**4 - 105*a*b**3*x**6 - 35*b**4
*x**8)/(210*x**14)
```

3.443 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{16}}\,dx$

Optimal. Leaf size=56

$$-\frac{a^4}{15x^{15}}-\frac{4a^3b}{13x^{13}}-\frac{6a^2b^2}{11x^{11}}-\frac{4ab^3}{9x^9}-\frac{b^4}{7x^7}$$

```
[Out] -1/15*a^4/x^15-4/13*a^3*b/x^13-6/11*a^2*b^2/x^11-4/9*a*b^3/x^9-1/7*b^4/x^7
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{6a^2b^2}{11x^{11}}-\frac{4a^3b}{13x^{13}}-\frac{a^4}{15x^{15}}-\frac{4ab^3}{9x^9}-\frac{b^4}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^16,x]
```

```
[Out] -a^4/(15*x^15) - (4*a^3*b)/(13*x^13) - (6*a^2*b^2)/(11*x^11) - (4*a*b^3)/(9
*x^9) - b^4/(7*x^7)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{16}}\,dx = \frac{\int \frac{(ab+b^2x^2)^4}{x^{16}}\,dx}{b^4}$$
$$= \frac{\int \left(\frac{a^4b^4}{x^{16}} + \frac{4a^3b^5}{x^{14}} + \frac{6a^2b^6}{x^{12}} + \frac{4ab^7}{x^{10}} + \frac{b^8}{x^8}\right)dx}{b^4}$$
$$= -\frac{a^4}{15x^{15}} - \frac{4a^3b}{13x^{13}} - \frac{6a^2b^2}{11x^{11}} - \frac{4ab^3}{9x^9} - \frac{b^4}{7x^7}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 1.00

$$-\frac{a^4}{15x^{15}} - \frac{4a^3b}{13x^{13}} - \frac{6a^2b^2}{11x^{11}} - \frac{4ab^3}{9x^9} - \frac{b^4}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^16,x]
```

```
[Out] -1/15*a^4/x^15 - (4*a^3*b)/(13*x^13) - (6*a^2*b^2)/(11*x^11) - (4*a*b^3)/(9
*x^9) - b^4/(7*x^7)
```

fricas [A] time = 0.84, size = 48, normalized size = 0.86

$$-\frac{6435\,b^4x^8 + 20020\,ab^3x^6 + 24570\,a^2b^2x^4 + 13860\,a^3bx^2 + 3003\,a^4}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(6435*b^4*x^8 + 20020*a*b^3*x^6 + 24570*a^2*b^2*x^4 + 13860*a^3*b*
x^2 + 3003*a^4)/x^15
```

giac [A] time = 0.15, size = 48, normalized size = 0.86

$$-\frac{6435\,b^4x^8 + 20020\,ab^3x^6 + 24570\,a^2b^2x^4 + 13860\,a^3bx^2 + 3003\,a^4}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^16,x, algorithm="giac")
```

```
[Out] -1/45045*(6435*b^4*x^8 + 20020*a*b^3*x^6 + 24570*a^2*b^2*x^4 + 13860*a^3*b*
x^2 + 3003*a^4)/x^15
```

maple [A] time = 0.00, size = 47, normalized size = 0.84

$$-\frac{b^4}{7x^7}-\frac{4a\,b^3}{9x^9}-\frac{6a^2b^2}{11x^{11}}-\frac{4a^3b}{13x^{13}}-\frac{a^4}{15x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^16,x)
```

```
[Out] -1/15*a^4/x^15-4/13*a^3*b/x^13-6/11*a^2*b^2/x^11-4/9*a*b^3/x^9-1/7*b^4/x^7
```

maxima [A] time = 1.38, size = 48, normalized size = 0.86

$$-\frac{6435\,b^4x^8+20020\,ab^3x^6+24570\,a^2b^2x^4+13860\,a^3bx^2+3003\,a^4}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^16,x, algorithm="maxima")
```

```
[Out] -1/45045*(6435*b^4*x^8 + 20020*a*b^3*x^6 + 24570*a^2*b^2*x^4 + 13860*a^3*b*
x^2 + 3003*a^4)/x^15
```

mupad [B] time = 4.35, size = 48, normalized size = 0.86

$$-\frac{\frac{a^4}{15}+\frac{4\,a^3\,b\,x^2}{13}+\frac{6\,a^2\,b^2\,x^4}{11}+\frac{4\,a\,b^3\,x^6}{9}+\frac{b^4\,x^8}{7}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^16,x)
```

```
[Out] -(a^4/15 + (b^4*x^8)/7 + (4*a^3*b*x^2)/13 + (4*a*b^3*x^6)/9 + (6*a^2*b^2*x^
4)/11)/x^15
```

sympy [A] time = 0.46, size = 51, normalized size = 0.91

$$\frac{-3003a^4-13860a^3bx^2-24570a^2b^2x^4-20020ab^3x^6-6435b^4x^8}{45045x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**16,x)
```

```
[Out] (-3003*a**4 - 13860*a**3*b*x**2 - 24570*a**2*b**2*x**4 - 20020*a*b**3*x**6
- 6435*b**4*x**8)/(45045*x**15)
```

3.444 $\int x^8 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=82

$$\frac{a^6x^9}{9}+\frac{6}{11}a^5bx^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}ab^5x^{19}+\frac{b^6x^{21}}{21}$$

```
[Out] 1/9*a^6*x^9+6/11*a^5*b*x^11+15/13*a^4*b^2*x^13+4/3*a^3*b^3*x^15+15/17*a^2*b
^4*x^17+6/19*a*b^5*x^19+1/21*b^6*x^21
```

Rubi [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$\frac{15}{17}a^2b^4x^{17}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{13}a^4b^2x^{13}+\frac{6}{11}a^5bx^{11}+\frac{a^6x^9}{9}+\frac{6}{19}ab^5x^{19}+\frac{b^6x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^9)/9 + (6*a^5*b*x^11)/11 + (15*a^4*b^2*x^13)/13 + (4*a^3*b^3*x^15)/3
 + (15*a^2*b^4*x^17)/17 + (6*a*b^5*x^19)/19 + (b^6*x^21)/21
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{\int x^8 \left(ab+b^2x^2\right)^6 \, dx}{b^6}$$

$$= \frac{\int \left(a^6b^6x^8+6a^5b^7x^{10}+15a^4b^8x^{12}+20a^3b^9x^{14}+15a^2b^{10}x^{16}+6ab^{11}x^{18}+b^{12}x^2\right.}{b^6}$$

$$= \frac{a^6x^9}{9}+\frac{6}{11}a^5bx^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}ab^5x^{19}+\frac{b^6x^{21}}{21}$$

Mathematica [A] time = 0.00, size = 82, normalized size = 1.00

$$\frac{a^6x^9}{9}+\frac{6}{11}a^5bx^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}ab^5x^{19}+\frac{b^6x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^9)/9 + (6*a^5*b*x^11)/11 + (15*a^4*b^2*x^13)/13 + (4*a^3*b^3*x^15)/3
 + (15*a^2*b^4*x^17)/17 + (6*a*b^5*x^19)/19 + (b^6*x^21)/21
```

fricas [A] time = 0.72, size = 68, normalized size = 0.83

$$\frac{1}{21}x^{21}b^6+\frac{6}{19}x^{19}b^5a+\frac{15}{17}x^{17}b^4a^2+\frac{4}{3}x^{15}b^3a^3+\frac{15}{13}x^{13}b^2a^4+\frac{6}{11}x^{11}ba^5+\frac{1}{9}x^9a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/21*x^21*b^6 + 6/19*x^19*b^5*a + 15/17*x^17*b^4*a^2 + 4/3*x^15*b^3*a^3 + 1
5/13*x^13*b^2*a^4 + 6/11*x^11*b*a^5 + 1/9*x^9*a^6
```

giac [A] time = 0.16, size = 68, normalized size = 0.83

$$\frac{1}{21}\,b^6x^{21}+\frac{6}{19}\,ab^5x^{19}+\frac{15}{17}\,a^2b^4x^{17}+\frac{4}{3}\,a^3b^3x^{15}+\frac{15}{13}\,a^4b^2x^{13}+\frac{6}{11}\,a^5bx^{11}+\frac{1}{9}\,a^6x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/21*b^6*x^21 + 6/19*a*b^5*x^19 + 15/17*a^2*b^4*x^17 + 4/3*a^3*b^3*x^15 + 1
5/13*a^4*b^2*x^13 + 6/11*a^5*b*x^11 + 1/9*a^6*x^9
```

maple [A] time = 0.00, size = 69, normalized size = 0.84

$$\frac{1}{21}b^6x^{21}+\frac{6}{19}a\,b^5x^{19}+\frac{15}{17}a^2b^4x^{17}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{13}a^4b^2x^{13}+\frac{6}{11}a^5b\,x^{11}+\frac{1}{9}a^6x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/9*a^6*x^9+6/11*a^5*b*x^11+15/13*a^4*b^2*x^13+4/3*a^3*b^3*x^15+15/17*a^2*b
^4*x^17+6/19*a*b^5*x^19+1/21*b^6*x^21
```

maxima [A] time = 1.35, size = 68, normalized size = 0.83

$$\frac{1}{21}\,b^6x^{21}+\frac{6}{19}\,ab^5x^{19}+\frac{15}{17}\,a^2b^4x^{17}+\frac{4}{3}\,a^3b^3x^{15}+\frac{15}{13}\,a^4b^2x^{13}+\frac{6}{11}\,a^5bx^{11}+\frac{1}{9}\,a^6x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/21*b^6*x^21 + 6/19*a*b^5*x^19 + 15/17*a^2*b^4*x^17 + 4/3*a^3*b^3*x^15 + 1
5/13*a^4*b^2*x^13 + 6/11*a^5*b*x^11 + 1/9*a^6*x^9
```

mupad [B] time = 0.03, size = 68, normalized size = 0.83

$$\frac{a^6\,x^9}{9}+\frac{6\,a^5\,b\,x^{11}}{11}+\frac{15\,a^4\,b^2\,x^{13}}{13}+\frac{4\,a^3\,b^3\,x^{15}}{3}+\frac{15\,a^2\,b^4\,x^{17}}{17}+\frac{6\,a\,b^5\,x^{19}}{19}+\frac{b^6\,x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^9)/9 + (b^6*x^21)/21 + (6*a^5*b*x^11)/11 + (6*a*b^5*x^19)/19 + (15*a
^4*b^2*x^13)/13 + (4*a^3*b^3*x^15)/3 + (15*a^2*b^4*x^17)/17
```

sympy [A] time = 0.09, size = 80, normalized size = 0.98

$$\frac{a^6x^9}{9}+\frac{6a^5bx^{11}}{11}+\frac{15a^4b^2x^{13}}{13}+\frac{4a^3b^3x^{15}}{3}+\frac{15a^2b^4x^{17}}{17}+\frac{6ab^5x^{19}}{19}+\frac{b^6x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**9/9 + 6*a**5*b*x**11/11 + 15*a**4*b**2*x**13/13 + 4*a**3*b**3*x**15
/3 + 15*a**2*b**4*x**17/17 + 6*a*b**5*x**19/19 + b**6*x**21/21
```

3.445 $\int x^7 \left(a^2+2abx^2+b^2x^4\right)^3\,dx$

Optimal. Leaf size=72

$$-\frac{a^3\left(a+bx^2\right)^7}{14b^4}+\frac{3a^2\left(a+bx^2\right)^8}{16b^4}+\frac{\left(a+bx^2\right)^{10}}{20b^4}-\frac{a\left(a+bx^2\right)^9}{6b^4}$$

```
[Out] -1/14*a^3*(b*x^2+a)^7/b^4+3/16*a^2*(b*x^2+a)^8/b^4-1/6*a*(b*x^2+a)^9/b^4+1/
20*(b*x^2+a)^10/b^4
```

Rubi [A] time = 0.12, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{3a^2\left(a+bx^2\right)^8}{16b^4}-\frac{a^3\left(a+bx^2\right)^7}{14b^4}+\frac{\left(a+bx^2\right)^{10}}{20b^4}-\frac{a\left(a+bx^2\right)^9}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(a^3*(a + b*x^2)^7)/(14*b^4) + (3*a^2*(a + b*x^2)^8)/(16*b^4) - (a*(a + b*
x^2)^9)/(6*b^4) + (a + b*x^2)^10/(20*b^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{\int x^7 \left(ab+b^2x^2\right)^6 \, dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int x^3 \left(ab+b^2x\right)^6 \, dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a^3(ab+b^2x)^6}{b^3}+\frac{3a^2(ab+b^2x)^7}{b^4}-\frac{3a(ab+b^2x)^8}{b^5}+\frac{(ab+b^2x)^9}{b^6}\right) dx, x, x^2\right)}{2b^6}$$
$$= -\frac{a^3\left(a+bx^2\right)^7}{14b^4}+\frac{3a^2\left(a+bx^2\right)^8}{16b^4}-\frac{a\left(a+bx^2\right)^9}{6b^4}+\frac{\left(a+bx^2\right)^{10}}{20b^4}$$

Mathematica [A] time = 0.00, size = 82, normalized size = 1.14

$$\frac{a^6x^8}{8}+\frac{3}{5}a^5bx^{10}+\frac{5}{4}a^4b^2x^{12}+\frac{10}{7}a^3b^3x^{14}+\frac{15}{16}a^2b^4x^{16}+\frac{1}{3}ab^5x^{18}+\frac{b^6x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^8)/8 + (3*a^5*b*x^10)/5 + (5*a^4*b^2*x^12)/4 + (10*a^3*b^3*x^14)/7 +
 (15*a^2*b^4*x^16)/16 + (a*b^5*x^18)/3 + (b^6*x^20)/20
```

fricas [A] time = 0.82, size = 68, normalized size = 0.94

$$\frac{1}{20}x^{20}b^6+\frac{1}{3}x^{18}b^5a+\frac{15}{16}x^{16}b^4a^2+\frac{10}{7}x^{14}b^3a^3+\frac{5}{4}x^{12}b^2a^4+\frac{3}{5}x^{10}ba^5+\frac{1}{8}x^8a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/20*x^20*b^6 + 1/3*x^18*b^5*a + 15/16*x^16*b^4*a^2 + 10/7*x^14*b^3*a^3 + 5
/4*x^12*b^2*a^4 + 3/5*x^10*b*a^5 + 1/8*x^8*a^6
```

giac [A] time = 0.15, size = 68, normalized size = 0.94

$$\frac{1}{20}\,b^6x^{20}+\frac{1}{3}\,ab^5x^{18}+\frac{15}{16}\,a^2b^4x^{16}+\frac{10}{7}\,a^3b^3x^{14}+\frac{5}{4}\,a^4b^2x^{12}+\frac{3}{5}\,a^5bx^{10}+\frac{1}{8}\,a^6x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/20*b^6*x^20 + 1/3*a*b^5*x^18 + 15/16*a^2*b^4*x^16 + 10/7*a^3*b^3*x^14 + 5
/4*a^4*b^2*x^12 + 3/5*a^5*b*x^10 + 1/8*a^6*x^8
```

maple [A] time = 0.00, size = 69, normalized size = 0.96

$$\frac{1}{20}b^6x^{20} + \frac{1}{3}a\,b^5x^{18} + \frac{15}{16}a^2b^4x^{16} + \frac{10}{7}a^3b^3x^{14} + \frac{5}{4}a^4b^2x^{12} + \frac{3}{5}a^5b\,x^{10} + \frac{1}{8}a^6x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/20*b^6*x^20+1/3*a*b^5*x^18+15/16*a^2*b^4*x^16+10/7*a^3*b^3*x^14+5/4*a^4*b
^2*x^12+3/5*a^5*b*x^10+1/8*a^6*x^8
```

maxima [A] time = 1.43, size = 68, normalized size = 0.94

$$\frac{1}{20}\,b^6x^{20} + \frac{1}{3}\,ab^5x^{18} + \frac{15}{16}\,a^2b^4x^{16} + \frac{10}{7}\,a^3b^3x^{14} + \frac{5}{4}\,a^4b^2x^{12} + \frac{3}{5}\,a^5bx^{10} + \frac{1}{8}\,a^6x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/20*b^6*x^20 + 1/3*a*b^5*x^18 + 15/16*a^2*b^4*x^16 + 10/7*a^3*b^3*x^14 + 5
/4*a^4*b^2*x^12 + 3/5*a^5*b*x^10 + 1/8*a^6*x^8
```

mupad [B] time = 0.03, size = 68, normalized size = 0.94

$$\frac{a^6\,x^8}{8} + \frac{3\,a^5\,b\,x^{10}}{5} + \frac{5\,a^4\,b^2\,x^{12}}{4} + \frac{10\,a^3\,b^3\,x^{14}}{7} + \frac{15\,a^2\,b^4\,x^{16}}{16} + \frac{a\,b^5\,x^{18}}{3} + \frac{b^6\,x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^8)/8 + (b^6*x^20)/20 + (3*a^5*b*x^10)/5 + (a*b^5*x^18)/3 + (5*a^4*b^
2*x^12)/4 + (10*a^3*b^3*x^14)/7 + (15*a^2*b^4*x^16)/16
```

sympy [A] time = 0.09, size = 78, normalized size = 1.08

$$\frac{a^6x^8}{8} + \frac{3a^5bx^{10}}{5} + \frac{5a^4b^2x^{12}}{4} + \frac{10a^3b^3x^{14}}{7} + \frac{15a^2b^4x^{16}}{16} + \frac{ab^5x^{18}}{3} + \frac{b^6x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**8/8 + 3*a**5*b*x**10/5 + 5*a**4*b**2*x**12/4 + 10*a**3*b**3*x**14/7
 + 15*a**2*b**4*x**16/16 + a*b**5*x**18/3 + b**6*x**20/20
```

3.446 $\int x^6 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=79

$$\frac{a^6x^7}{7}+\frac{2}{3}a^5bx^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}ab^5x^{17}+\frac{b^6x^{19}}{19}$$

```
[Out] 1/7*a^6*x^7+2/3*a^5*b*x^9+15/11*a^4*b^2*x^11+20/13*a^3*b^3*x^13+a^2*b^4*x^1
5+6/17*a*b^5*x^17+1/19*b^6*x^19
```

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$a^2b^4x^{15}+\frac{20}{13}a^3b^3x^{13}+\frac{15}{11}a^4b^2x^{11}+\frac{2}{3}a^5bx^9+\frac{a^6x^7}{7}+\frac{6}{17}ab^5x^{17}+\frac{b^6x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^7)/7 + (2*a^5*b*x^9)/3 + (15*a^4*b^2*x^11)/11 + (20*a^3*b^3*x^13)/13
 + a^2*b^4*x^15 + (6*a*b^5*x^17)/17 + (b^6*x^19)/19
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^6 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx &= \frac{\int x^6 \left(ab+b^2x^2\right)^6 \, dx}{b^6}\\ &= \frac{\int \left(a^6b^6x^6+6a^5b^7x^8+15a^4b^8x^{10}+20a^3b^9x^{12}+15a^2b^{10}x^{14}+6ab^{11}x^{16}+b^{12}x^{18}\right)}{b^6}\\ &= \frac{a^6x^7}{7}+\frac{2}{3}a^5bx^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}ab^5x^{17}+\frac{b^6x^{19}}{19}\end{aligned}$$

Mathematica [A] time = 0.00, size = 79, normalized size = 1.00

$$\frac{a^6x^7}{7}+\frac{2}{3}a^5bx^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}ab^5x^{17}+\frac{b^6x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^7)/7 + (2*a^5*b*x^9)/3 + (15*a^4*b^2*x^11)/11 + (20*a^3*b^3*x^13)/13
 + a^2*b^4*x^15 + (6*a*b^5*x^17)/17 + (b^6*x^19)/19
```

fricas [A] time = 0.76, size = 67, normalized size = 0.85

$$\frac{1}{19}x^{19}b^6+\frac{6}{17}x^{17}b^5a+x^{15}b^4a^2+\frac{20}{13}x^{13}b^3a^3+\frac{15}{11}x^{11}b^2a^4+\frac{2}{3}x^9ba^5+\frac{1}{7}x^7a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/19*x^19*b^6 + 6/17*x^17*b^5*a + x^15*b^4*a^2 + 20/13*x^13*b^3*a^3 + 15/11
*x^11*b^2*a^4 + 2/3*x^9*b*a^5 + 1/7*x^7*a^6
```

giac [A] time = 0.15, size = 67, normalized size = 0.85

$$\frac{1}{19}b^6x^{19}+\frac{6}{17}ab^5x^{17}+a^2b^4x^{15}+\frac{20}{13}a^3b^3x^{13}+\frac{15}{11}a^4b^2x^{11}+\frac{2}{3}a^5bx^9+\frac{1}{7}a^6x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/19*b^6*x^19 + 6/17*a*b^5*x^17 + a^2*b^4*x^15 + 20/13*a^3*b^3*x^13 + 15/11
*a^4*b^2*x^11 + 2/3*a^5*b*x^9 + 1/7*a^6*x^7
```

maple [A] time = 0.00, size = 68, normalized size = 0.86

$$\frac{1}{19}b^6x^{19}+\frac{6}{17}a\,b^5x^{17}+a^2b^4x^{15}+\frac{20}{13}a^3b^3x^{13}+\frac{15}{11}a^4b^2x^{11}+\frac{2}{3}a^5b\,x^9+\frac{1}{7}a^6x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/7*a^6*x^7+2/3*a^5*b*x^9+15/11*a^4*b^2*x^11+20/13*a^3*b^3*x^13+a^2*b^4*x^1
5+6/17*a*b^5*x^17+1/19*b^6*x^19
```

maxima [A]   time = 1.39, size = 67, normalized size = 0.85

$$\frac{1}{19}\,b^{6}x^{19} + \frac{6}{17}\,ab^{5}x^{17} + a^{2}b^{4}x^{15} + \frac{20}{13}\,a^{3}b^{3}x^{13} + \frac{15}{11}\,a^{4}b^{2}x^{11} + \frac{2}{3}\,a^{5}bx^{9} + \frac{1}{7}\,a^{6}x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/19*b^6*x^19 + 6/17*a*b^5*x^17 + a^2*b^4*x^15 + 20/13*a^3*b^3*x^13 + 15/11
*a^4*b^2*x^11 + 2/3*a^5*b*x^9 + 1/7*a^6*x^7
```

mupad [B]   time = 0.03, size = 67, normalized size = 0.85

$$\frac{a^6\,x^7}{7} + \frac{2\,a^5\,b\,x^9}{3} + \frac{15\,a^4\,b^2\,x^{11}}{11} + \frac{20\,a^3\,b^3\,x^{13}}{13} + a^2\,b^4\,x^{15} + \frac{6\,a\,b^5\,x^{17}}{17} + \frac{b^6\,x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^7)/7 + (b^6*x^19)/19 + (2*a^5*b*x^9)/3 + (6*a*b^5*x^17)/17 + (15*a^4
*b^2*x^11)/11 + (20*a^3*b^3*x^13)/13 + a^2*b^4*x^15
```

sympy [A]   time = 0.09, size = 76, normalized size = 0.96

$$\frac{a^{6}x^{7}}{7} + \frac{2a^{5}bx^{9}}{3} + \frac{15a^{4}b^{2}x^{11}}{11} + \frac{20a^{3}b^{3}x^{13}}{13} + a^{2}b^{4}x^{15} + \frac{6ab^{5}x^{17}}{17} + \frac{b^{6}x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**7/7 + 2*a**5*b*x**9/3 + 15*a**4*b**2*x**11/11 + 20*a**3*b**3*x**13/
13 + a**2*b**4*x**15 + 6*a*b**5*x**17/17 + b**6*x**19/19
```

3.447 $\int x^5 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=53

$$\frac{a^2\left(a+bx^2\right)^7}{14b^3}+\frac{\left(a+bx^2\right)^9}{18b^3}-\frac{a\left(a+bx^2\right)^8}{8b^3}$$

```
[Out] 1/14*a^2*(b*x^2+a)^7/b^3-1/8*a*(b*x^2+a)^8/b^3+1/18*(b*x^2+a)^9/b^3
```

Rubi [A] time = 0.09, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a^2\left(a+bx^2\right)^7}{14b^3}+\frac{\left(a+bx^2\right)^9}{18b^3}-\frac{a\left(a+bx^2\right)^8}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^2*(a + b*x^2)^7)/(14*b^3) - (a*(a + b*x^2)^8)/(8*b^3) + (a + b*x^2)^9/(1
8*b^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^3 dx = \frac{\int x^5 \left(ab+b^2x^2\right)^6 dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int x^2 \left(ab+b^2x\right)^6 dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2\left(ab+b^2x\right)^6}{b^2} - \frac{2a\left(ab+b^2x\right)^7}{b^3} + \frac{\left(ab+b^2x\right)^8}{b^4}\right) dx, x, x^2\right)}{2b^6}$$
$$= \frac{a^2\left(a+bx^2\right)^7}{14b^3} - \frac{a\left(a+bx^2\right)^8}{8b^3} + \frac{\left(a+bx^2\right)^9}{18b^3}$$

Mathematica [A] time = 0.00, size = 82, normalized size = 1.55

$$\frac{a^6x^6}{6} + \frac{3}{4}a^5bx^8 + \frac{3}{2}a^4b^2x^{10} + \frac{5}{3}a^3b^3x^{12} + \frac{15}{14}a^2b^4x^{14} + \frac{3}{8}ab^5x^{16} + \frac{b^6x^{18}}{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^6)/6 + (3*a^5*b*x^8)/4 + (3*a^4*b^2*x^10)/2 + (5*a^3*b^3*x^12)/3 + (
15*a^2*b^4*x^14)/14 + (3*a*b^5*x^16)/8 + (b^6*x^18)/18
```

fricas [A] time = 0.55, size = 68, normalized size = 1.28

$$\frac{1}{18}x^{18}b^6 + \frac{3}{8}x^{16}b^5a + \frac{15}{14}x^{14}b^4a^2 + \frac{5}{3}x^{12}b^3a^3 + \frac{3}{2}x^{10}b^2a^4 + \frac{3}{4}x^8ba^5 + \frac{1}{6}x^6a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/18*x^18*b^6 + 3/8*x^16*b^5*a + 15/14*x^14*b^4*a^2 + 5/3*x^12*b^3*a^3 + 3/
2*x^10*b^2*a^4 + 3/4*x^8*b*a^5 + 1/6*x^6*a^6
```

giac [A] time = 0.15, size = 68, normalized size = 1.28

$$\frac{1}{18}\,b^6x^{18} + \frac{3}{8}\,ab^5x^{16} + \frac{15}{14}\,a^2b^4x^{14} + \frac{5}{3}\,a^3b^3x^{12} + \frac{3}{2}\,a^4b^2x^{10} + \frac{3}{4}\,a^5bx^8 + \frac{1}{6}\,a^6x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/18*b^6*x^18 + 3/8*a*b^5*x^16 + 15/14*a^2*b^4*x^14 + 5/3*a^3*b^3*x^12 + 3/
2*a^4*b^2*x^10 + 3/4*a^5*b*x^8 + 1/6*a^6*x^6
```

maple [A] time = 0.00, size = 69, normalized size = 1.30

$$\frac{1}{18}b^{6}x^{18}+\frac{3}{8}a\,b^{5}x^{16}+\frac{15}{14}a^{2}b^{4}x^{14}+\frac{5}{3}a^{3}b^{3}x^{12}+\frac{3}{2}a^{4}b^{2}x^{10}+\frac{3}{4}a^{5}b\,x^{8}+\frac{1}{6}a^{6}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/18*b^6*x^18+3/8*a*b^5*x^16+15/14*a^2*b^4*x^14+5/3*a^3*b^3*x^12+3/2*a^4*b^
2*x^10+3/4*a^5*b*x^8+1/6*a^6*x^6
```

maxima [A] time = 1.36, size = 68, normalized size = 1.28

$$\frac{1}{18}\,b^{6}x^{18}+\frac{3}{8}\,ab^{5}x^{16}+\frac{15}{14}\,a^{2}b^{4}x^{14}+\frac{5}{3}\,a^{3}b^{3}x^{12}+\frac{3}{2}\,a^{4}b^{2}x^{10}+\frac{3}{4}\,a^{5}bx^{8}+\frac{1}{6}\,a^{6}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/18*b^6*x^18 + 3/8*a*b^5*x^16 + 15/14*a^2*b^4*x^14 + 5/3*a^3*b^3*x^12 + 3/
2*a^4*b^2*x^10 + 3/4*a^5*b*x^8 + 1/6*a^6*x^6
```

mupad [B] time = 0.03, size = 68, normalized size = 1.28

$$\frac{a^6\,x^6}{6}+\frac{3\,a^5\,b\,x^8}{4}+\frac{3\,a^4\,b^2\,x^{10}}{2}+\frac{5\,a^3\,b^3\,x^{12}}{3}+\frac{15\,a^2\,b^4\,x^{14}}{14}+\frac{3\,a\,b^5\,x^{16}}{8}+\frac{b^6\,x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^6)/6 + (b^6*x^18)/18 + (3*a^5*b*x^8)/4 + (3*a*b^5*x^16)/8 + (3*a^4*b
^2*x^10)/2 + (5*a^3*b^3*x^12)/3 + (15*a^2*b^4*x^14)/14
```

sympy [A] time = 0.09, size = 80, normalized size = 1.51

$$\frac{a^{6}x^{6}}{6}+\frac{3a^{5}bx^{8}}{4}+\frac{3a^{4}b^{2}x^{10}}{2}+\frac{5a^{3}b^{3}x^{12}}{3}+\frac{15a^{2}b^{4}x^{14}}{14}+\frac{3ab^{5}x^{16}}{8}+\frac{b^{6}x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**6/6 + 3*a**5*b*x**8/4 + 3*a**4*b**2*x**10/2 + 5*a**3*b**3*x**12/3 +
 15*a**2*b**4*x**14/14 + 3*a*b**5*x**16/8 + b**6*x**18/18
```

3.448 $\int x^4 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=82

$$\frac{a^6x^5}{5}+\frac{6}{7}a^5bx^7+\frac{5}{3}a^4b^2x^9+\frac{20}{11}a^3b^3x^{11}+\frac{15}{13}a^2b^4x^{13}+\frac{2}{5}ab^5x^{15}+\frac{b^6x^{17}}{17}$$

```
[Out] 1/5*a^6*x^5+6/7*a^5*b*x^7+5/3*a^4*b^2*x^9+20/11*a^3*b^3*x^11+15/13*a^2*b^4*
x^13+2/5*a*b^5*x^15+1/17*b^6*x^17
```

Rubi [A] time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$\frac{15}{13}a^2b^4x^{13}+\frac{20}{11}a^3b^3x^{11}+\frac{5}{3}a^4b^2x^9+\frac{6}{7}a^5bx^7+\frac{a^6x^5}{5}+\frac{2}{5}ab^5x^{15}+\frac{b^6x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^5)/5 + (6*a^5*b*x^7)/7 + (5*a^4*b^2*x^9)/3 + (20*a^3*b^3*x^11)/11 +
(15*a^2*b^4*x^13)/13 + (2*a*b^5*x^15)/5 + (b^6*x^17)/17
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a^2+2abx^2+b^2x^4\right)^3\,dx&=\frac{\int x^4\left(ab+b^2x^2\right)^6\,dx}{b^6}\\&=\frac{\int\left(a^6b^6x^4+6a^5b^7x^6+15a^4b^8x^8+20a^3b^9x^{10}+15a^2b^{10}x^{12}+6ab^{11}x^{14}+b^{12}x^{16}\right)dx}{b^6}\\&=\frac{a^6x^5}{5}+\frac{6}{7}a^5bx^7+\frac{5}{3}a^4b^2x^9+\frac{20}{11}a^3b^3x^{11}+\frac{15}{13}a^2b^4x^{13}+\frac{2}{5}ab^5x^{15}+\frac{b^6x^{17}}{17}\end{aligned}$$

Mathematica [A] time = 0.00, size = 82, normalized size = 1.00

$$\frac{a^6x^5}{5}+\frac{6}{7}a^5bx^7+\frac{5}{3}a^4b^2x^9+\frac{20}{11}a^3b^3x^{11}+\frac{15}{13}a^2b^4x^{13}+\frac{2}{5}ab^5x^{15}+\frac{b^6x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^5)/5 + (6*a^5*b*x^7)/7 + (5*a^4*b^2*x^9)/3 + (20*a^3*b^3*x^11)/11 +
(15*a^2*b^4*x^13)/13 + (2*a*b^5*x^15)/5 + (b^6*x^17)/17
```

fricas [A] time = 0.63, size = 68, normalized size = 0.83

$$\frac{1}{17}x^{17}b^6+\frac{2}{5}x^{15}b^5a+\frac{15}{13}x^{13}b^4a^2+\frac{20}{11}x^{11}b^3a^3+\frac{5}{3}x^9b^2a^4+\frac{6}{7}x^7ba^5+\frac{1}{5}x^5a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*b^6 + 2/5*x^15*b^5*a + 15/13*x^13*b^4*a^2 + 20/11*x^11*b^3*a^3 +
5/3*x^9*b^2*a^4 + 6/7*x^7*b*a^5 + 1/5*x^5*a^6
```

giac [A] time = 0.17, size = 68, normalized size = 0.83

$$\frac{1}{17}\,b^6x^{17}+\frac{2}{5}\,ab^5x^{15}+\frac{15}{13}\,a^2b^4x^{13}+\frac{20}{11}\,a^3b^3x^{11}+\frac{5}{3}\,a^4b^2x^9+\frac{6}{7}\,a^5bx^7+\frac{1}{5}\,a^6x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/17*b^6*x^17 + 2/5*a*b^5*x^15 + 15/13*a^2*b^4*x^13 + 20/11*a^3*b^3*x^11 +
5/3*a^4*b^2*x^9 + 6/7*a^5*b*x^7 + 1/5*a^6*x^5
```

maple [A] time = 0.00, size = 69, normalized size = 0.84

$$\frac{1}{17}b^6x^{17}+\frac{2}{5}a\,b^5x^{15}+\frac{15}{13}a^2b^4x^{13}+\frac{20}{11}a^3b^3x^{11}+\frac{5}{3}a^4b^2x^9+\frac{6}{7}a^5b\,x^7+\frac{1}{5}a^6x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/5*a^6*x^5+6/7*a^5*b*x^7+5/3*a^4*b^2*x^9+20/11*a^3*b^3*x^11+15/13*a^2*b^4*
x^13+2/5*a*b^5*x^15+1/17*b^6*x^17
```

maxima [A] time = 1.29, size = 68, normalized size = 0.83

$$\frac{1}{17}\,b^6x^{17}+\frac{2}{5}\,ab^5x^{15}+\frac{15}{13}\,a^2b^4x^{13}+\frac{20}{11}\,a^3b^3x^{11}+\frac{5}{3}\,a^4b^2x^9+\frac{6}{7}\,a^5bx^7+\frac{1}{5}\,a^6x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/17*b^6*x^17 + 2/5*a*b^5*x^15 + 15/13*a^2*b^4*x^13 + 20/11*a^3*b^3*x^11 +
5/3*a^4*b^2*x^9 + 6/7*a^5*b*x^7 + 1/5*a^6*x^5
```

mupad [B] time = 0.03, size = 68, normalized size = 0.83

$$\frac{a^6\,x^5}{5}+\frac{6\,a^5\,b\,x^7}{7}+\frac{5\,a^4\,b^2\,x^9}{3}+\frac{20\,a^3\,b^3\,x^{11}}{11}+\frac{15\,a^2\,b^4\,x^{13}}{13}+\frac{2\,a\,b^5\,x^{15}}{5}+\frac{b^6\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^5)/5 + (b^6*x^17)/17 + (6*a^5*b*x^7)/7 + (2*a*b^5*x^15)/5 + (5*a^4*b
^2*x^9)/3 + (20*a^3*b^3*x^11)/11 + (15*a^2*b^4*x^13)/13
```

sympy [A] time = 0.09, size = 80, normalized size = 0.98

$$\frac{a^6x^5}{5}+\frac{6a^5bx^7}{7}+\frac{5a^4b^2x^9}{3}+\frac{20a^3b^3x^{11}}{11}+\frac{15a^2b^4x^{13}}{13}+\frac{2ab^5x^{15}}{5}+\frac{b^6x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**5/5 + 6*a**5*b*x**7/7 + 5*a**4*b**2*x**9/3 + 20*a**3*b**3*x**11/11
+ 15*a**2*b**4*x**13/13 + 2*a*b**5*x**15/5 + b**6*x**17/17
```

3.449 $\int x^3 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=34

$$\frac{\left(a+bx^2\right)^8}{16b^2}-\frac{a\left(a+bx^2\right)^7}{14b^2}$$

```
[Out] -1/14*a*(b*x^2+a)^7/b^2+1/16*(b*x^2+a)^8/b^2
```

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{\left(a+bx^2\right)^8}{16b^2}-\frac{a\left(a+bx^2\right)^7}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(a*(a + b*x^2)^7)/(14*b^2) + (a + b*x^2)^8/(16*b^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^3 dx = \frac{\int x^3 \left(ab+b^2x^2\right)^6 dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int x \left(ab+b^2x\right)^6 dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a(ab+b^2x)^6}{b}+\frac{(ab+b^2x)^7}{b^2}\right) dx, x, x^2\right)}{2b^6}$$
$$= -\frac{a\left(a+bx^2\right)^7}{14b^2}+\frac{\left(a+bx^2\right)^8}{16b^2}$$

Mathematica [B] time = 0.00, size = 77, normalized size = 2.26

$$\frac{a^6x^4}{4}+a^5bx^6+\frac{15}{8}a^4b^2x^8+2a^3b^3x^{10}+\frac{5}{4}a^2b^4x^{12}+\frac{3}{7}ab^5x^{14}+\frac{b^6x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^4)/4 + a^5*b*x^6 + (15*a^4*b^2*x^8)/8 + 2*a^3*b^3*x^10 + (5*a^2*b^4*
x^12)/4 + (3*a*b^5*x^14)/7 + (b^6*x^16)/16
```

fricas [B] time = 0.82, size = 67, normalized size = 1.97

$$\frac{1}{16}x^{16}b^6+\frac{3}{7}x^{14}b^5a+\frac{5}{4}x^{12}b^4a^2+2x^{10}b^3a^3+\frac{15}{8}x^8b^2a^4+x^6ba^5+\frac{1}{4}x^4a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b^6 + 3/7*x^14*b^5*a + 5/4*x^12*b^4*a^2 + 2*x^10*b^3*a^3 + 15/8*x
^8*b^2*a^4 + x^6*b*a^5 + 1/4*x^4*a^6
```

giac [B] time = 0.17, size = 67, normalized size = 1.97

$$\frac{1}{16}b^6x^{16}+\frac{3}{7}ab^5x^{14}+\frac{5}{4}a^2b^4x^{12}+2a^3b^3x^{10}+\frac{15}{8}a^4b^2x^8+a^5bx^6+\frac{1}{4}a^6x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*b^6*x^16 + 3/7*a*b^5*x^14 + 5/4*a^2*b^4*x^12 + 2*a^3*b^3*x^10 + 15/8*a
^4*b^2*x^8 + a^5*b*x^6 + 1/4*a^6*x^4
```

maple [B] time = 0.00, size = 68, normalized size = 2.00

$$\frac{1}{16}b^6x^{16}+\frac{3}{7}a\,b^5x^{14}+\frac{5}{4}a^2b^4x^{12}+2a^3b^3x^{10}+\frac{15}{8}a^4b^2x^8+a^5b\,x^6+\frac{1}{4}a^6x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/16*b^6*x^16+3/7*a*b^5*x^14+5/4*a^2*b^4*x^12+2*a^3*b^3*x^10+15/8*a^4*b^2*x
^8+a^5*b*x^6+1/4*a^6*x^4
```

maxima [B] time = 1.36, size = 67, normalized size = 1.97

$$\frac{1}{16}\,b^6x^{16}+\frac{3}{7}\,ab^5x^{14}+\frac{5}{4}\,a^2b^4x^{12}+2\,a^3b^3x^{10}+\frac{15}{8}\,a^4b^2x^8+a^5bx^6+\frac{1}{4}\,a^6x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^6*x^16 + 3/7*a*b^5*x^14 + 5/4*a^2*b^4*x^12 + 2*a^3*b^3*x^10 + 15/8*a
^4*b^2*x^8 + a^5*b*x^6 + 1/4*a^6*x^4
```

mupad [B] time = 0.03, size = 67, normalized size = 1.97

$$\frac{a^6\,x^4}{4}+a^5\,b\,x^6+\frac{15\,a^4\,b^2\,x^8}{8}+2\,a^3\,b^3\,x^{10}+\frac{5\,a^2\,b^4\,x^{12}}{4}+\frac{3\,a\,b^5\,x^{14}}{7}+\frac{b^6\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^4)/4 + (b^6*x^16)/16 + a^5*b*x^6 + (3*a*b^5*x^14)/7 + (15*a^4*b^2*x^
8)/8 + 2*a^3*b^3*x^10 + (5*a^2*b^4*x^12)/4
```

sympy [B] time = 0.09, size = 75, normalized size = 2.21

$$\frac{a^6x^4}{4}+a^5bx^6+\frac{15a^4b^2x^8}{8}+2a^3b^3x^{10}+\frac{5a^2b^4x^{12}}{4}+\frac{3ab^5x^{14}}{7}+\frac{b^6x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**4/4 + a**5*b*x**6 + 15*a**4*b**2*x**8/8 + 2*a**3*b**3*x**10 + 5*a**
2*b**4*x**12/4 + 3*a*b**5*x**14/7 + b**6*x**16/16
```

3.450 $\int x^2 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=82

$$\frac{a^6x^3}{3}+\frac{6}{5}a^5bx^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}ab^5x^{13}+\frac{b^6x^{15}}{15}$$

```
[Out] 1/3*a^6*x^3+6/5*a^5*b*x^5+15/7*a^4*b^2*x^7+20/9*a^3*b^3*x^9+15/11*a^2*b^4*x
^11+6/13*a*b^5*x^13+1/15*b^6*x^15
```

Rubi [A] time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$\frac{15}{11}a^2b^4x^{11}+\frac{20}{9}a^3b^3x^9+\frac{15}{7}a^4b^2x^7+\frac{6}{5}a^5bx^5+\frac{a^6x^3}{3}+\frac{6}{13}ab^5x^{13}+\frac{b^6x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^3)/3 + (6*a^5*b*x^5)/5 + (15*a^4*b^2*x^7)/7 + (20*a^3*b^3*x^9)/9 + (
15*a^2*b^4*x^11)/11 + (6*a*b^5*x^13)/13 + (b^6*x^15)/15
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx &= \frac{\int x^2 \left(ab+b^2x^2\right)^6 \, dx}{b^6}\\ &= \frac{\int \left(a^6b^6x^2+6a^5b^7x^4+15a^4b^8x^6+20a^3b^9x^8+15a^2b^{10}x^{10}+6ab^{11}x^{12}+b^{12}x^{14}\right)}{b^6}\\ &= \frac{a^6x^3}{3}+\frac{6}{5}a^5bx^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}ab^5x^{13}+\frac{b^6x^{15}}{15}\end{aligned}$$

Mathematica [A] time = 0.00, size = 82, normalized size = 1.00

$$\frac{a^6x^3}{3}+\frac{6}{5}a^5bx^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}ab^5x^{13}+\frac{b^6x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^3)/3 + (6*a^5*b*x^5)/5 + (15*a^4*b^2*x^7)/7 + (20*a^3*b^3*x^9)/9 + (
15*a^2*b^4*x^11)/11 + (6*a*b^5*x^13)/13 + (b^6*x^15)/15
```

fricas [A] time = 0.85, size = 68, normalized size = 0.83

$$\frac{1}{15}x^{15}b^6+\frac{6}{13}x^{13}b^5a+\frac{15}{11}x^{11}b^4a^2+\frac{20}{9}x^9b^3a^3+\frac{15}{7}x^7b^2a^4+\frac{6}{5}x^5ba^5+\frac{1}{3}x^3a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*b^6 + 6/13*x^13*b^5*a + 15/11*x^11*b^4*a^2 + 20/9*x^9*b^3*a^3 + 1
5/7*x^7*b^2*a^4 + 6/5*x^5*b*a^5 + 1/3*x^3*a^6
```

giac [A] time = 0.15, size = 68, normalized size = 0.83

$$\frac{1}{15}b^6x^{15}+\frac{6}{13}ab^5x^{13}+\frac{15}{11}a^2b^4x^{11}+\frac{20}{9}a^3b^3x^9+\frac{15}{7}a^4b^2x^7+\frac{6}{5}a^5bx^5+\frac{1}{3}a^6x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*b^6*x^15 + 6/13*a*b^5*x^13 + 15/11*a^2*b^4*x^11 + 20/9*a^3*b^3*x^9 + 1
5/7*a^4*b^2*x^7 + 6/5*a^5*b*x^5 + 1/3*a^6*x^3
```

maple [A] time = 0.00, size = 69, normalized size = 0.84

$$\frac{1}{15}b^6x^{15}+\frac{6}{13}a\,b^5x^{13}+\frac{15}{11}a^2b^4x^{11}+\frac{20}{9}a^3b^3x^9+\frac{15}{7}a^4b^2x^7+\frac{6}{5}a^5b\,x^5+\frac{1}{3}a^6x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/3*a^6*x^3+6/5*a^5*b*x^5+15/7*a^4*b^2*x^7+20/9*a^3*b^3*x^9+15/11*a^2*b^4*x
^11+6/13*a*b^5*x^13+1/15*b^6*x^15
```

maxima [A] time = 1.36, size = 68, normalized size = 0.83

$$\frac{1}{15}\,b^{6}x^{15} + \frac{6}{13}\,ab^{5}x^{13} + \frac{15}{11}\,a^{2}b^{4}x^{11} + \frac{20}{9}\,a^{3}b^{3}x^{9} + \frac{15}{7}\,a^{4}b^{2}x^{7} + \frac{6}{5}\,a^{5}bx^{5} + \frac{1}{3}\,a^{6}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/15*b^6*x^15 + 6/13*a*b^5*x^13 + 15/11*a^2*b^4*x^11 + 20/9*a^3*b^3*x^9 + 1
5/7*a^4*b^2*x^7 + 6/5*a^5*b*x^5 + 1/3*a^6*x^3
```

mupad [B] time = 0.03, size = 68, normalized size = 0.83

$$\frac{a^6\,x^3}{3} + \frac{6\,a^5\,b\,x^5}{5} + \frac{15\,a^4\,b^2\,x^7}{7} + \frac{20\,a^3\,b^3\,x^9}{9} + \frac{15\,a^2\,b^4\,x^{11}}{11} + \frac{6\,a\,b^5\,x^{13}}{13} + \frac{b^6\,x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^3)/3 + (b^6*x^15)/15 + (6*a^5*b*x^5)/5 + (6*a*b^5*x^13)/13 + (15*a^4
*b^2*x^7)/7 + (20*a^3*b^3*x^9)/9 + (15*a^2*b^4*x^11)/11
```

sympy [A] time = 0.09, size = 80, normalized size = 0.98

$$\frac{a^{6}x^{3}}{3} + \frac{6a^{5}bx^{5}}{5} + \frac{15a^{4}b^{2}x^{7}}{7} + \frac{20a^{3}b^{3}x^{9}}{9} + \frac{15a^{2}b^{4}x^{11}}{11} + \frac{6ab^{5}x^{13}}{13} + \frac{b^{6}x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**3/3 + 6*a**5*b*x**5/5 + 15*a**4*b**2*x**7/7 + 20*a**3*b**3*x**9/9 +
 15*a**2*b**4*x**11/11 + 6*a*b**5*x**13/13 + b**6*x**15/15
```

3.451 $\int x\left(a^2+2abx^2+b^2x^4\right)^3\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^2\right)^7}{14b}$$

```
[Out] 1/14*(b*x^2+a)^7/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {28, 261}

$$\frac{\left(a+bx^2\right)^7}{14b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a + b*x^2)^7/(14*b)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^2+b^2x^4\right)^3\,dx &= \frac{\int x\left(ab+b^2x^2\right)^6\,dx}{b^6}\\ &= \frac{\left(a+bx^2\right)^7}{14b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^7}{14b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a + b*x^2)^7/(14*b)
```

fricas [B] time = 0.56, size = 68, normalized size = 4.25

$$\frac{1}{14}x^{14}b^6+\frac{1}{2}x^{12}b^5a+\frac{3}{2}x^{10}b^4a^2+\frac{5}{2}x^8b^3a^3+\frac{5}{2}x^6b^2a^4+\frac{3}{2}x^4ba^5+\frac{1}{2}x^2a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^6 + 1/2*x^12*b^5*a + 3/2*x^10*b^4*a^2 + 5/2*x^8*b^3*a^3 + 5/2*x
^6*b^2*a^4 + 3/2*x^4*b*a^5 + 1/2*x^2*a^6
```

giac [B] time = 0.15, size = 68, normalized size = 4.25

$$\frac{1}{14}\,b^6x^{14}+\frac{1}{2}\,ab^5x^{12}+\frac{3}{2}\,a^2b^4x^{10}+\frac{5}{2}\,a^3b^3x^8+\frac{5}{2}\,a^4b^2x^6+\frac{3}{2}\,a^5bx^4+\frac{1}{2}\,a^6x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/14*b^6*x^14 + 1/2*a*b^5*x^12 + 3/2*a^2*b^4*x^10 + 5/2*a^3*b^3*x^8 + 5/2*a
^4*b^2*x^6 + 3/2*a^5*b*x^4 + 1/2*a^6*x^2
```

maple [B] time = 0.00, size = 69, normalized size = 4.31

$$\frac{1}{14}b^6x^{14}+\frac{1}{2}a\,b^5x^{12}+\frac{3}{2}a^2b^4x^{10}+\frac{5}{2}a^3b^3x^8+\frac{5}{2}a^4b^2x^6+\frac{3}{2}a^5b\,x^4+\frac{1}{2}a^6x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/14*b^6*x^14+1/2*a*b^5*x^12+3/2*a^2*b^4*x^10+5/2*a^3*b^3*x^8+5/2*a^4*b^2*x
^6+3/2*a^5*b*x^4+1/2*a^6*x^2
```

maxima [B] time = 1.38, size = 68, normalized size = 4.25

$$\frac{1}{14}b^6x^{14} + \frac{1}{2}ab^5x^{12} + \frac{3}{2}a^2b^4x^{10} + \frac{5}{2}a^3b^3x^8 + \frac{5}{2}a^4b^2x^6 + \frac{3}{2}a^5bx^4 + \frac{1}{2}a^6x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/14*b^6*x^14 + 1/2*a*b^5*x^12 + 3/2*a^2*b^4*x^10 + 5/2*a^3*b^3*x^8 + 5/2*a
^4*b^2*x^6 + 3/2*a^5*b*x^4 + 1/2*a^6*x^2
```

mupad [B] time = 0.03, size = 68, normalized size = 4.25

$$\frac{a^6\,x^2}{2} + \frac{3\,a^5\,b\,x^4}{2} + \frac{5\,a^4\,b^2\,x^6}{2} + \frac{5\,a^3\,b^3\,x^8}{2} + \frac{3\,a^2\,b^4\,x^{10}}{2} + \frac{a\,b^5\,x^{12}}{2} + \frac{b^6\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^2)/2 + (b^6*x^14)/14 + (3*a^5*b*x^4)/2 + (a*b^5*x^12)/2 + (5*a^4*b^2
*x^6)/2 + (5*a^3*b^3*x^8)/2 + (3*a^2*b^4*x^10)/2
```

sympy [B] time = 0.09, size = 78, normalized size = 4.88

$$\frac{a^6x^2}{2} + \frac{3a^5bx^4}{2} + \frac{5a^4b^2x^6}{2} + \frac{5a^3b^3x^8}{2} + \frac{3a^2b^4x^{10}}{2} + \frac{ab^5x^{12}}{2} + \frac{b^6x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**2/2 + 3*a**5*b*x**4/2 + 5*a**4*b**2*x**6/2 + 5*a**3*b**3*x**8/2 + 3
*a**2*b**4*x**10/2 + a*b**5*x**12/2 + b**6*x**14/14
```

3.452 $\int \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=73

$$a^6x+2a^5bx^3+3a^4b^2x^5+\frac{20}{7}a^3b^3x^7+\frac{5}{3}a^2b^4x^9+\frac{6}{11}ab^5x^{11}+\frac{b^6x^{13}}{13}$$

```
[Out] a^6*x+2*a^5*b*x^3+3*a^4*b^2*x^5+20/7*a^3*b^3*x^7+5/3*a^2*b^4*x^9+6/11*a*b^5
*x^11+1/13*b^6*x^13
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {28, 194}

$$\frac{5}{3}a^2b^4x^9+\frac{20}{7}a^3b^3x^7+3a^4b^2x^5+2a^5bx^3+a^6x+\frac{6}{11}ab^5x^{11}+\frac{b^6x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] a^6*x + 2*a^5*b*x^3 + 3*a^4*b^2*x^5 + (20*a^3*b^3*x^7)/7 + (5*a^2*b^4*x^9)/
3 + (6*a*b^5*x^11)/11 + (b^6*x^13)/13
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a^2+2abx^2+b^2x^4\right)^3 \, dx &= \frac{\int \left(ab+b^2x^2\right)^6 \, dx}{b^6}\\
&= \frac{\int \left(a^6b^6+6a^5b^7x^2+15a^4b^8x^4+20a^3b^9x^6+15a^2b^{10}x^8+6ab^{11}x^{10}+b^{12}x^{12}\right) \, dx}{b^6}\\
&= a^6x+2a^5bx^3+3a^4b^2x^5+\frac{20}{7}a^3b^3x^7+\frac{5}{3}a^2b^4x^9+\frac{6}{11}ab^5x^{11}+\frac{b^6x^{13}}{13}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 73, normalized size = 1.00

$$a^6x + 2a^5bx^3 + 3a^4b^2x^5 + \frac{20}{7}a^3b^3x^7 + \frac{5}{3}a^2b^4x^9 + \frac{6}{11}ab^5x^{11} + \frac{b^6x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] a^6*x + 2*a^5*b*x^3 + 3*a^4*b^2*x^5 + (20*a^3*b^3*x^7)/7 + (5*a^2*b^4*x^9)/
3 + (6*a*b^5*x^11)/11 + (b^6*x^13)/13
```

fricas [A] time = 0.91, size = 65, normalized size = 0.89

$$\frac{1}{13}x^{13}b^6 + \frac{6}{11}x^{11}b^5a + \frac{5}{3}x^9b^4a^2 + \frac{20}{7}x^7b^3a^3 + 3x^5b^2a^4 + 2x^3ba^5 + xa^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^6 + 6/11*x^11*b^5*a + 5/3*x^9*b^4*a^2 + 20/7*x^7*b^3*a^3 + 3*x^
5*b^2*a^4 + 2*x^3*b*a^5 + x*a^6
```

giac [A] time = 0.16, size = 65, normalized size = 0.89

$$\frac{1}{13}b^6x^{13} + \frac{6}{11}ab^5x^{11} + \frac{5}{3}a^2b^4x^9 + \frac{20}{7}a^3b^3x^7 + 3a^4b^2x^5 + 2a^5bx^3 + a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/13*b^6*x^13 + 6/11*a*b^5*x^11 + 5/3*a^2*b^4*x^9 + 20/7*a^3*b^3*x^7 + 3*a^
4*b^2*x^5 + 2*a^5*b*x^3 + a^6*x
```

maple [A] time = 0.00, size = 66, normalized size = 0.90

$$\frac{1}{13}b^6x^{13} + \frac{6}{11}a\,b^5x^{11} + \frac{5}{3}a^2b^4x^9 + \frac{20}{7}a^3b^3x^7 + 3a^4b^2x^5 + 2a^5b\,x^3 + a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] a^6*x+2*a^5*b*x^3+3*a^4*b^2*x^5+20/7*a^3*b^3*x^7+5/3*a^2*b^4*x^9+6/11*a*b^5
*x^11+1/13*b^6*x^13
```

maxima [A] time = 1.34, size = 100, normalized size = 1.37

$$\frac{1}{13}\,b^6x^{13}+\frac{6}{11}\,ab^5x^{11}+\frac{4}{3}\,a^2b^4x^9+\frac{8}{7}\,a^3b^3x^7+a^6x+\frac{1}{5}\left(3\,b^2x^5+10\,abx^3\right)a^4+\frac{1}{105}\left(35\,b^4x^9+180\,ab^3x^7+252\,a^2b^2x^5\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^6*x^13 + 6/11*a*b^5*x^11 + 4/3*a^2*b^4*x^9 + 8/7*a^3*b^3*x^7 + a^6*x
 + 1/5*(3*b^2*x^5 + 10*a*b*x^3)*a^4 + 1/105*(35*b^4*x^9 + 180*a*b^3*x^7 + 2
52*a^2*b^2*x^5)*a^2
```

mupad [B] time = 0.03, size = 65, normalized size = 0.89

$$a^6\,x+2\,a^5\,b\,x^3+3\,a^4\,b^2\,x^5+\frac{20\,a^3\,b^3\,x^7}{7}+\frac{5\,a^2\,b^4\,x^9}{3}+\frac{6\,a\,b^5\,x^{11}}{11}+\frac{b^6\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] a^6*x + (b^6*x^13)/13 + 2*a^5*b*x^3 + (6*a*b^5*x^11)/11 + 3*a^4*b^2*x^5 + (
20*a^3*b^3*x^7)/7 + (5*a^2*b^4*x^9)/3
```

sympy [A] time = 0.08, size = 73, normalized size = 1.00

$$a^6x+2a^5bx^3+3a^4b^2x^5+\frac{20a^3b^3x^7}{7}+\frac{5a^2b^4x^9}{3}+\frac{6ab^5x^{11}}{11}+\frac{b^6x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x + 2*a**5*b*x**3 + 3*a**4*b**2*x**5 + 20*a**3*b**3*x**7/7 + 5*a**2*b*
*4*x**9/3 + 6*a*b**5*x**11/11 + b**6*x**13/13
```

3.453 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x} dx$

Optimal. Leaf size=76

$$a^6\log(x)+3a^5bx^2+\frac{15}{4}a^4b^2x^4+\frac{10}{3}a^3b^3x^6+\frac{15}{8}a^2b^4x^8+\frac{3}{5}ab^5x^{10}+\frac{b^6x^{12}}{12}$$

```
[Out] 3*a^5*b*x^2+15/4*a^4*b^2*x^4+10/3*a^3*b^3*x^6+15/8*a^2*b^4*x^8+3/5*a*b^5*x^
10+1/12*b^6*x^12+a^6*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{15}{8}a^2b^4x^8+\frac{10}{3}a^3b^3x^6+\frac{15}{4}a^4b^2x^4+3a^5bx^2+a^6\log(x)+\frac{3}{5}ab^5x^{10}+\frac{b^6x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x,x]
```

```
[Out] 3*a^5*b*x^2 + (15*a^4*b^2*x^4)/4 + (10*a^3*b^3*x^6)/3 + (15*a^2*b^4*x^8)/8
+ (3*a*b^5*x^10)/5 + (b^6*x^12)/12 + a^6*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x}\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x}\,dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(6a^5b^7+\frac{a^6b^6}{x}+15a^4b^8x+20a^3b^9x^2+15a^2b^{10}x^3+6ab^{11}x^4+b^{12}x^5\right)\,dx,\right.}{2b^6}$$
$$= 3a^5bx^2+\frac{15}{4}a^4b^2x^4+\frac{10}{3}a^3b^3x^6+\frac{15}{8}a^2b^4x^8+\frac{3}{5}ab^5x^{10}+\frac{b^6x^{12}}{12}+a^6\log(x)$$

Mathematica [A] time = 0.00, size = 76, normalized size = 1.00

$$a^6\log(x)+3a^5bx^2+\frac{15}{4}a^4b^2x^4+\frac{10}{3}a^3b^3x^6+\frac{15}{8}a^2b^4x^8+\frac{3}{5}ab^5x^{10}+\frac{b^6x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x,x]
```

```
[Out] 3*a^5*b*x^2 + (15*a^4*b^2*x^4)/4 + (10*a^3*b^3*x^6)/3 + (15*a^2*b^4*x^8)/8
+ (3*a*b^5*x^10)/5 + (b^6*x^12)/12 + a^6*Log[x]
```

fricas [A] time = 0.63, size = 66, normalized size = 0.87

$$\frac{1}{12}\,b^6x^{12}+\frac{3}{5}\,ab^5x^{10}+\frac{15}{8}\,a^2b^4x^8+\frac{10}{3}\,a^3b^3x^6+\frac{15}{4}\,a^4b^2x^4+3\,a^5bx^2+a^6\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x,x, algorithm="fricas")
```

```
[Out] 1/12*b^6*x^12 + 3/5*a*b^5*x^10 + 15/8*a^2*b^4*x^8 + 10/3*a^3*b^3*x^6 + 15/4
*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6*log(x)
```

giac [A] time = 0.15, size = 69, normalized size = 0.91

$$\frac{1}{12}\,b^6x^{12}+\frac{3}{5}\,ab^5x^{10}+\frac{15}{8}\,a^2b^4x^8+\frac{10}{3}\,a^3b^3x^6+\frac{15}{4}\,a^4b^2x^4+3\,a^5bx^2+\frac{1}{2}\,a^6\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x,x, algorithm="giac")
```

```
[Out] 1/12*b^6*x^12 + 3/5*a*b^5*x^10 + 15/8*a^2*b^4*x^8 + 10/3*a^3*b^3*x^6 + 15/4
*a^4*b^2*x^4 + 3*a^5*b*x^2 + 1/2*a^6*log(x^2)
```

maple [A] time = 0.00, size = 67, normalized size = 0.88

$$\frac{b^6x^{12}}{12}+\frac{3a\,b^5x^{10}}{5}+\frac{15a^2b^4x^8}{8}+\frac{10a^3b^3x^6}{3}+\frac{15a^4b^2x^4}{4}+3a^5b\,x^2+a^6\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x,x)
```

```
[Out] 3*a^5*b*x^2+15/4*a^4*b^2*x^4+10/3*a^3*b^3*x^6+15/8*a^2*b^4*x^8+3/5*a*b^5*x^
10+1/12*b^6*x^12+a^6*ln(x)
```

maxima [A] time = 1.40, size = 69, normalized size = 0.91

$$\frac{1}{12}\,b^6x^{12}+\frac{3}{5}\,ab^5x^{10}+\frac{15}{8}\,a^2b^4x^8+\frac{10}{3}\,a^3b^3x^6+\frac{15}{4}\,a^4b^2x^4+3\,a^5bx^2+\frac{1}{2}\,a^6\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x,x, algorithm="maxima")
```

```
[Out] 1/12*b^6*x^12 + 3/5*a*b^5*x^10 + 15/8*a^2*b^4*x^8 + 10/3*a^3*b^3*x^6 + 15/4
*a^4*b^2*x^4 + 3*a^5*b*x^2 + 1/2*a^6*log(x^2)
```

mupad [B] time = 0.04, size = 66, normalized size = 0.87

$$a^6\,\ln(x)+\frac{b^6\,x^{12}}{12}+3\,a^5\,b\,x^2+\frac{3\,a\,b^5\,x^{10}}{5}+\frac{15\,a^4\,b^2\,x^4}{4}+\frac{10\,a^3\,b^3\,x^6}{3}+\frac{15\,a^2\,b^4\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x,x)
```

```
[Out] a^6*log(x) + (b^6*x^12)/12 + 3*a^5*b*x^2 + (3*a*b^5*x^10)/5 + (15*a^4*b^2*x
^4)/4 + (10*a^3*b^3*x^6)/3 + (15*a^2*b^4*x^8)/8
```

sympy [A] time = 0.17, size = 76, normalized size = 1.00

$$a^6\log\left(x\right)+3a^5bx^2+\frac{15a^4b^2x^4}{4}+\frac{10a^3b^3x^6}{3}+\frac{15a^2b^4x^8}{8}+\frac{3ab^5x^{10}}{5}+\frac{b^6x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x,x)
```

```
[Out] a**6*log(x) + 3*a**5*b*x**2 + 15*a**4*b**2*x**4/4 + 10*a**3*b**3*x**6/3 + 1
5*a**2*b**4*x**8/8 + 3*a*b**5*x**10/5 + b**6*x**12/12
```

3.454 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^2}\,dx$

Optimal. Leaf size=72

$$-\frac{a^6}{x}+6a^5bx+5a^4b^2x^3+4a^3b^3x^5+\frac{15}{7}a^2b^4x^7+\frac{2}{3}ab^5x^9+\frac{b^6x^{11}}{11}$$

```
[Out] -a^6/x+6*a^5*b*x+5*a^4*b^2*x^3+4*a^3*b^3*x^5+15/7*a^2*b^4*x^7+2/3*a*b^5*x^9
+1/11*b^6*x^11
```

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$\frac{15}{7}a^2b^4x^7+4a^3b^3x^5+5a^4b^2x^3+6a^5bx-\frac{a^6}{x}+\frac{2}{3}ab^5x^9+\frac{b^6x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^2,x]
```

```
[Out] -(a^6/x) + 6*a^5*b*x + 5*a^4*b^2*x^3 + 4*a^3*b^3*x^5 + (15*a^2*b^4*x^7)/7 +
 (2*a*b^5*x^9)/3 + (b^6*x^11)/11
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^2}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^2}\,dx}{b^6}$$
$$= \frac{\int \left(6a^5b^7 + \frac{a^6b^6}{x^2} + 15a^4b^8x^2 + 20a^3b^9x^4 + 15a^2b^{10}x^6 + 6ab^{11}x^8 + b^{12}x^{10}\right)dx}{b^6}$$
$$= -\frac{a^6}{x} + 6a^5bx + 5a^4b^2x^3 + 4a^3b^3x^5 + \frac{15}{7}a^2b^4x^7 + \frac{2}{3}ab^5x^9 + \frac{b^6x^{11}}{11}$$

Mathematica [A] time = 0.01, size = 72, normalized size = 1.00

$$-\frac{a^6}{x} + 6a^5bx + 5a^4b^2x^3 + 4a^3b^3x^5 + \frac{15}{7}a^2b^4x^7 + \frac{2}{3}ab^5x^9 + \frac{b^6x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^2,x]
```

```
[Out] -(a^6/x) + 6*a^5*b*x + 5*a^4*b^2*x^3 + 4*a^3*b^3*x^5 + (15*a^2*b^4*x^7)/7 +
 (2*a*b^5*x^9)/3 + (b^6*x^11)/11
```

fricas [A] time = 0.59, size = 70, normalized size = 0.97

$$\frac{21\,b^6x^{12} + 154\,ab^5x^{10} + 495\,a^2b^4x^8 + 924\,a^3b^3x^6 + 1155\,a^4b^2x^4 + 1386\,a^5bx^2 - 231\,a^6}{231\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/231*(21*b^6*x^12 + 154*a*b^5*x^10 + 495*a^2*b^4*x^8 + 924*a^3*b^3*x^6 + 1
155*a^4*b^2*x^4 + 1386*a^5*b*x^2 - 231*a^6)/x
```

giac [A] time = 0.16, size = 66, normalized size = 0.92

$$\frac{1}{11}\,b^6x^{11} + \frac{2}{3}\,ab^5x^9 + \frac{15}{7}\,a^2b^4x^7 + 4\,a^3b^3x^5 + 5\,a^4b^2x^3 + 6\,a^5bx - \frac{a^6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/11*b^6*x^11 + 2/3*a*b^5*x^9 + 15/7*a^2*b^4*x^7 + 4*a^3*b^3*x^5 + 5*a^4*b^
2*x^3 + 6*a^5*b*x - a^6/x
```

maple [A] time = 0.00, size = 67, normalized size = 0.93

$$\frac{b^6x^{11}}{11}+\frac{2a\,b^5x^9}{3}+\frac{15a^2b^4x^7}{7}+4a^3b^3x^5+5a^4b^2x^3+6a^5bx-\frac{a^6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^2,x)
```

```
[Out] -a^6/x+6*a^5*b*x+5*a^4*b^2*x^3+4*a^3*b^3*x^5+15/7*a^2*b^4*x^7+2/3*a*b^5*x^9
+1/11*b^6*x^11
```

maxima [A] time = 1.35, size = 66, normalized size = 0.92

$$\frac{1}{11}\,b^6x^{11}+\frac{2}{3}\,ab^5x^9+\frac{15}{7}\,a^2b^4x^7+4\,a^3b^3x^5+5\,a^4b^2x^3+6\,a^5bx-\frac{a^6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^6*x^11 + 2/3*a*b^5*x^9 + 15/7*a^2*b^4*x^7 + 4*a^3*b^3*x^5 + 5*a^4*b^
2*x^3 + 6*a^5*b*x - a^6/x
```

mupad [B] time = 0.03, size = 66, normalized size = 0.92

$$\frac{b^6\,x^{11}}{11}-\frac{a^6}{x}+\frac{2\,a\,b^5\,x^9}{3}+5\,a^4\,b^2\,x^3+4\,a^3\,b^3\,x^5+\frac{15\,a^2\,b^4\,x^7}{7}+6\,a^5\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^2,x)
```

```
[Out] (b^6*x^11)/11 - a^6/x + (2*a*b^5*x^9)/3 + 5*a^4*b^2*x^3 + 4*a^3*b^3*x^5 + (
15*a^2*b^4*x^7)/7 + 6*a^5*b*x
```

sympy [A] time = 0.16, size = 70, normalized size = 0.97

$$-\frac{a^6}{x}+6a^5bx+5a^4b^2x^3+4a^3b^3x^5+\frac{15a^2b^4x^7}{7}+\frac{2ab^5x^9}{3}+\frac{b^6x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**2,x)
```

```
[Out] -a**6/x + 6*a**5*b*x + 5*a**4*b**2*x**3 + 4*a**3*b**3*x**5 + 15*a**2*b**4*x
**7/7 + 2*a*b**5*x**9/3 + b**6*x**11/11
```

3.455 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^3}\,dx$

Optimal. Leaf size=77

$$-\frac{a^6}{2x^2}+6a^5b\log(x)+\frac{15}{2}a^4b^2x^2+5a^3b^3x^4+\frac{5}{2}a^2b^4x^6+\frac{3}{4}ab^5x^8+\frac{b^6x^{10}}{10}$$

```
[Out] -1/2*a^6/x^2+15/2*a^4*b^2*x^2+5*a^3*b^3*x^4+5/2*a^2*b^4*x^6+3/4*a*b^5*x^8+1
/10*b^6*x^10+6*a^5*b*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{5}{2}a^2b^4x^6+5a^3b^3x^4+\frac{15}{2}a^4b^2x^2+6a^5b\log(x)-\frac{a^6}{2x^2}+\frac{3}{4}ab^5x^8+\frac{b^6x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^3,x]
```

```
[Out] -a^6/(2*x^2) + (15*a^4*b^2*x^2)/2 + 5*a^3*b^3*x^4 + (5*a^2*b^4*x^6)/2 + (3*
a*b^5*x^8)/4 + (b^6*x^10)/10 + 6*a^5*b*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^3}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^3}\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^2}\,dx,x,x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(15a^4b^8+\frac{a^6b^6}{x^2}+\frac{6a^5b^7}{x}+20a^3b^9x+15a^2b^{10}x^2+6ab^{11}x^3+b^{12}x^4\right)dx,x,x\right.}{2b^6}$$
$$= -\frac{a^6}{2x^2}+\frac{15}{2}a^4b^2x^2+5a^3b^3x^4+\frac{5}{2}a^2b^4x^6+\frac{3}{4}ab^5x^8+\frac{b^6x^{10}}{10}+6a^5b\log(x)$$

Mathematica [A] time = 0.01, size = 77, normalized size = 1.00

$$-\frac{a^6}{2x^2}+6a^5b\log(x)+\frac{15}{2}a^4b^2x^2+5a^3b^3x^4+\frac{5}{2}a^2b^4x^6+\frac{3}{4}ab^5x^8+\frac{b^6x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^3,x]
```

```
[Out] -1/2*a^6/x^2 + (15*a^4*b^2*x^2)/2 + 5*a^3*b^3*x^4 + (5*a^2*b^4*x^6)/2 + (3*
a*b^5*x^8)/4 + (b^6*x^10)/10 + 6*a^5*b*Log[x]
```

fricas [A] time = 0.98, size = 72, normalized size = 0.94

$$\frac{2\,b^6x^{12}+15\,ab^5x^{10}+50\,a^2b^4x^8+100\,a^3b^3x^6+150\,a^4b^2x^4+120\,a^5bx^2\log(x)-10\,a^6}{20\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/20*(2*b^6*x^12 + 15*a*b^5*x^10 + 50*a^2*b^4*x^8 + 100*a^3*b^3*x^6 + 150*a
^4*b^2*x^4 + 120*a^5*b*x^2*log(x) - 10*a^6)/x^2
```

giac [A] time = 0.15, size = 79, normalized size = 1.03

$$\frac{1}{10}\,b^6x^{10}+\frac{3}{4}\,ab^5x^8+\frac{5}{2}\,a^2b^4x^6+5\,a^3b^3x^4+\frac{15}{2}\,a^4b^2x^2+3\,a^5b\log\left(x^2\right)-\frac{6\,a^5bx^2+a^6}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/10*b^6*x^10 + 3/4*a*b^5*x^8 + 5/2*a^2*b^4*x^6 + 5*a^3*b^3*x^4 + 15/2*a^4*
b^2*x^2 + 3*a^5*b*log(x^2) - 1/2*(6*a^5*b*x^2 + a^6)/x^2
```

maple [A] time = 0.01, size = 68, normalized size = 0.88

$$\frac{b^6x^{10}}{10}+\frac{3a\,b^5x^8}{4}+\frac{5a^2b^4x^6}{2}+5a^3b^3x^4+\frac{15a^4b^2x^2}{2}+6a^5b\ln(x)-\frac{a^6}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^3,x)
```

```
[Out] -1/2*a^6/x^2+15/2*a^4*b^2*x^2+5*a^3*b^3*x^4+5/2*a^2*b^4*x^6+3/4*a*b^5*x^8+1
/10*b^6*x^10+6*a^5*b*ln(x)
```

maxima [A] time = 1.43, size = 69, normalized size = 0.90

$$\frac{1}{10}\,b^6x^{10}+\frac{3}{4}\,ab^5x^8+\frac{5}{2}\,a^2b^4x^6+5\,a^3b^3x^4+\frac{15}{2}\,a^4b^2x^2+3\,a^5b\log\left(x^2\right)-\frac{a^6}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^6*x^10 + 3/4*a*b^5*x^8 + 5/2*a^2*b^4*x^6 + 5*a^3*b^3*x^4 + 15/2*a^4*
b^2*x^2 + 3*a^5*b*log(x^2) - 1/2*a^6/x^2
```

mupad [B] time = 0.04, size = 67, normalized size = 0.87

$$\frac{b^6\,x^{10}}{10}-\frac{a^6}{2\,x^2}+\frac{3\,a\,b^5\,x^8}{4}+6\,a^5\,b\,\ln(x)+\frac{15\,a^4\,b^2\,x^2}{2}+5\,a^3\,b^3\,x^4+\frac{5\,a^2\,b^4\,x^6}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^3,x)
```

```
[Out] (b^6*x^10)/10 - a^6/(2*x^2) + (3*a*b^5*x^8)/4 + 6*a^5*b*log(x) + (15*a^4*b^
2*x^2)/2 + 5*a^3*b^3*x^4 + (5*a^2*b^4*x^6)/2
```

sympy [A] time = 0.20, size = 76, normalized size = 0.99

$$-\frac{a^6}{2x^2}+6a^5b\log{(x)}+\frac{15a^4b^2x^2}{2}+5a^3b^3x^4+\frac{5a^2b^4x^6}{2}+\frac{3ab^5x^8}{4}+\frac{b^6x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**3,x)
```

```
[Out] -a**6/(2*x**2) + 6*a**5*b*log(x) + 15*a**4*b**2*x**2/2 + 5*a**3*b**3*x**4 +
 5*a**2*b**4*x**6/2 + 3*a*b**5*x**8/4 + b**6*x**10/10
```

3.456 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^4} dx$

Optimal. Leaf size=74

$$-\frac{a^6}{3x^3}-\frac{6a^5b}{x}+15a^4b^2x+\frac{20}{3}a^3b^3x^3+3a^2b^4x^5+\frac{6}{7}ab^5x^7+\frac{b^6x^9}{9}$$

```
[Out] -1/3*a^6/x^3-6*a^5*b/x+15*a^4*b^2*x+20/3*a^3*b^3*x^3+3*a^2*b^4*x^5+6/7*a*b^
5*x^7+1/9*b^6*x^9
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$3a^2b^4x^5+\frac{20}{3}a^3b^3x^3+15a^4b^2x-\frac{6a^5b}{x}-\frac{a^6}{3x^3}+\frac{6}{7}ab^5x^7+\frac{b^6x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^4,x]
```

```
[Out] -a^6/(3*x^3) - (6*a^5*b)/x + 15*a^4*b^2*x + (20*a^3*b^3*x^3)/3 + 3*a^2*b^4*
x^5 + (6*a*b^5*x^7)/7 + (b^6*x^9)/9
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^4}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^4}\,dx}{b^6}$$
$$= \frac{\int \left(15a^4b^8+\frac{a^6b^6}{x^4}+\frac{6a^5b^7}{x^2}+20a^3b^9x^2+15a^2b^{10}x^4+6ab^{11}x^6+b^{12}x^8\right)dx}{b^6}$$
$$= -\frac{a^6}{3x^3}-\frac{6a^5b}{x}+15a^4b^2x+\frac{20}{3}a^3b^3x^3+3a^2b^4x^5+\frac{6}{7}ab^5x^7+\frac{b^6x^9}{9}$$

Mathematica [A] time = 0.01, size = 74, normalized size = 1.00

$$-\frac{a^6}{3x^3}-\frac{6a^5b}{x}+15a^4b^2x+\frac{20}{3}a^3b^3x^3+3a^2b^4x^5+\frac{6}{7}ab^5x^7+\frac{b^6x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^4,x]
```

```
[Out] -1/3*a^6/x^3 - (6*a^5*b)/x + 15*a^4*b^2*x + (20*a^3*b^3*x^3)/3 + 3*a^2*b^4*
x^5 + (6*a*b^5*x^7)/7 + (b^6*x^9)/9
```

fricas [A] time = 0.66, size = 70, normalized size = 0.95

$$\frac{7\,b^6x^{12}+54\,ab^5x^{10}+189\,a^2b^4x^8+420\,a^3b^3x^6+945\,a^4b^2x^4-378\,a^5bx^2-21\,a^6}{63\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/63*(7*b^6*x^12 + 54*a*b^5*x^10 + 189*a^2*b^4*x^8 + 420*a^3*b^3*x^6 + 945*
a^4*b^2*x^4 - 378*a^5*b*x^2 - 21*a^6)/x^3
```

giac [A] time = 0.18, size = 67, normalized size = 0.91

$$\frac{1}{9}\,b^6x^9+\frac{6}{7}\,ab^5x^7+3\,a^2b^4x^5+\frac{20}{3}\,a^3b^3x^3+15\,a^4b^2x-\frac{18\,a^5bx^2+a^6}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/9*b^6*x^9 + 6/7*a*b^5*x^7 + 3*a^2*b^4*x^5 + 20/3*a^3*b^3*x^3 + 15*a^4*b^2
*x - 1/3*(18*a^5*b*x^2 + a^6)/x^3
```

maple [A] time = 0.00, size = 67, normalized size = 0.91

$$\frac{b^{6}x^{9}}{9}+\frac{6a\,b^{5}x^{7}}{7}+3a^{2}b^{4}x^{5}+\frac{20a^{3}b^{3}x^{3}}{3}+15a^{4}b^{2}x-\frac{6a^{5}b}{x}-\frac{a^{6}}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^4,x)
```

```
[Out] -1/3*a^6/x^3-6*a^5*b/x+15*a^4*b^2*x+20/3*a^3*b^3*x^3+3*a^2*b^4*x^5+6/7*a*b^
5*x^7+1/9*b^6*x^9
```

maxima [A] time = 1.29, size = 67, normalized size = 0.91

$$\frac{1}{9}\,b^{6}x^{9}+\frac{6}{7}\,ab^{5}x^{7}+3\,a^{2}b^{4}x^{5}+\frac{20}{3}\,a^{3}b^{3}x^{3}+15\,a^{4}b^{2}x-\frac{18\,a^{5}bx^{2}+a^{6}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*b^6*x^9 + 6/7*a*b^5*x^7 + 3*a^2*b^4*x^5 + 20/3*a^3*b^3*x^3 + 15*a^4*b^2
*x - 1/3*(18*a^5*b*x^2 + a^6)/x^3
```

mupad [B] time = 0.03, size = 69, normalized size = 0.93

$$\frac{b^{6}\,x^{9}}{9}-\frac{\frac{a^{6}}{3}+6\,b\,a^{5}\,x^{2}}{x^{3}}+15\,a^{4}\,b^{2}\,x+\frac{6\,a\,b^{5}\,x^{7}}{7}+\frac{20\,a^{3}\,b^{3}\,x^{3}}{3}+3\,a^{2}\,b^{4}\,x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^4,x)
```

```
[Out] (b^6*x^9)/9 - (a^6/3 + 6*a^5*b*x^2)/x^3 + 15*a^4*b^2*x + (6*a*b^5*x^7)/7 +
(20*a^3*b^3*x^3)/3 + 3*a^2*b^4*x^5
```

sympy [A] time = 0.21, size = 75, normalized size = 1.01

$$15a^{4}b^{2}x+\frac{20a^{3}b^{3}x^{3}}{3}+3a^{2}b^{4}x^{5}+\frac{6ab^{5}x^{7}}{7}+\frac{b^{6}x^{9}}{9}+\frac{-a^{6}-18a^{5}bx^{2}}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**4,x)
```

```
[Out] 15*a**4*b**2*x + 20*a**3*b**3*x**3/3 + 3*a**2*b**4*x**5 + 6*a*b**5*x**7/7 +
 b**6*x**9/9 + (-a**6 - 18*a**5*b*x**2)/(3*x**3)
```

3.457 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^5}\,dx$

Optimal. Leaf size=72

$$-\frac{a^6}{4x^4}-\frac{3a^5b}{x^2}+15a^4b^2\log(x)+10a^3b^3x^2+\frac{15}{4}a^2b^4x^4+ab^5x^6+\frac{b^6x^8}{8}$$

```
[Out] -1/4*a^6/x^4-3*a^5*b/x^2+10*a^3*b^3*x^2+15/4*a^2*b^4*x^4+a*b^5*x^6+1/8*b^6*
x^8+15*a^4*b^2*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{15}{4}a^2b^4x^4+10a^3b^3x^2+15a^4b^2\log(x)-\frac{3a^5b}{x^2}-\frac{a^6}{4x^4}+ab^5x^6+\frac{b^6x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^5,x]
```

```
[Out] -a^6/(4*x^4) - (3*a^5*b)/x^2 + 10*a^3*b^3*x^2 + (15*a^2*b^4*x^4)/4 + a*b^5*
x^6 + (b^6*x^8)/8 + 15*a^4*b^2*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^5}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^5}\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^3}\,dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(20a^3b^9+\frac{a^6b^6}{x^3}+\frac{6a^5b^7}{x^2}+\frac{15a^4b^8}{x}+15a^2b^{10}x+6ab^{11}x^2+b^{12}x^3\right)dx, x, x^2\right)}{2b^6}$$
$$= -\frac{a^6}{4x^4}-\frac{3a^5b}{x^2}+10a^3b^3x^2+\frac{15}{4}a^2b^4x^4+ab^5x^6+\frac{b^6x^8}{8}+15a^4b^2\log(x)$$

Mathematica [A] time = 0.00, size = 72, normalized size = 1.00

$$-\frac{a^6}{4x^4}-\frac{3a^5b}{x^2}+15a^4b^2\log(x)+10a^3b^3x^2+\frac{15}{4}a^2b^4x^4+ab^5x^6+\frac{b^6x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^5,x]
```

```
[Out] -1/4*a^6/x^4 - (3*a^5*b)/x^2 + 10*a^3*b^3*x^2 + (15*a^2*b^4*x^4)/4 + a*b^5*
x^6 + (b^6*x^8)/8 + 15*a^4*b^2*Log[x]
```

fricas [A] time = 0.95, size = 71, normalized size = 0.99

$$\frac{b^6x^{12}+8\,ab^5x^{10}+30\,a^2b^4x^8+80\,a^3b^3x^6+120\,a^4b^2x^4\log(x)-24\,a^5bx^2-2\,a^6}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(b^6*x^12 + 8*a*b^5*x^10 + 30*a^2*b^4*x^8 + 80*a^3*b^3*x^6 + 120*a^4*b^
2*x^4*log(x) - 24*a^5*b*x^2 - 2*a^6)/x^4
```

giac [A] time = 0.16, size = 80, normalized size = 1.11

$$\frac{1}{8}\,b^6x^8+ab^5x^6+\frac{15}{4}\,a^2b^4x^4+10\,a^3b^3x^2+\frac{15}{2}\,a^4b^2\log\left(x^2\right)-\frac{45\,a^4b^2x^4+12\,a^5bx^2+a^6}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^5,x, algorithm="giac")
```

```
[Out] 1/8*b^6*x^8 + a*b^5*x^6 + 15/4*a^2*b^4*x^4 + 10*a^3*b^3*x^2 + 15/2*a^4*b^2*
log(x^2) - 1/4*(45*a^4*b^2*x^4 + 12*a^5*b*x^2 + a^6)/x^4
```

maple [A] time = 0.01, size = 67, normalized size = 0.93

$$\frac{b^6x^8}{8} + a\,b^5x^6 + \frac{15a^2b^4x^4}{4} + 10a^3b^3x^2 + 15a^4b^2\ln(x) - \frac{3a^5b}{x^2} - \frac{a^6}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^5,x)
```

```
[Out] -1/4*a^6/x^4-3*a^5*b/x^2+10*a^3*b^3*x^2+15/4*a^2*b^4*x^4+a*b^5*x^6+1/8*b^6*
x^8+15*a^4*b^2*ln(x)
```

maxima [A] time = 1.37, size = 69, normalized size = 0.96

$$\frac{1}{8}\,b^6x^8 + ab^5x^6 + \frac{15}{4}\,a^2b^4x^4 + 10\,a^3b^3x^2 + \frac{15}{2}\,a^4b^2\log\left(x^2\right) - \frac{12\,a^5bx^2 + a^6}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*b^6*x^8 + a*b^5*x^6 + 15/4*a^2*b^4*x^4 + 10*a^3*b^3*x^2 + 15/2*a^4*b^2*
log(x^2) - 1/4*(12*a^5*b*x^2 + a^6)/x^4
```

mupad [B] time = 0.04, size = 69, normalized size = 0.96

$$\frac{b^6\,x^8}{8} - \frac{\frac{a^6}{4} + 3\,b\,a^5\,x^2}{x^4} + a\,b^5\,x^6 + 10\,a^3\,b^3\,x^2 + \frac{15\,a^2\,b^4\,x^4}{4} + 15\,a^4\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^5,x)
```

```
[Out] (b^6*x^8)/8 - (a^6/4 + 3*a^5*b*x^2)/x^4 + a*b^5*x^6 + 10*a^3*b^3*x^2 + (15*
a^2*b^4*x^4)/4 + 15*a^4*b^2*log(x)
```

sympy [A] time = 0.25, size = 73, normalized size = 1.01

$$15a^4b^2\log{(x)} + 10a^3b^3x^2 + \frac{15a^2b^4x^4}{4} + ab^5x^6 + \frac{b^6x^8}{8} + \frac{-a^6 - 12a^5bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**5,x)
```

```
[Out] 15*a**4*b**2*log(x) + 10*a**3*b**3*x**2 + 15*a**2*b**4*x**4/4 + a*b**5*x**6
 + b**6*x**8/8 + (-a**6 - 12*a**5*b*x**2)/(4*x**4)
```

3.458 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^6}\,dx$

Optimal. Leaf size=72

$$-\frac{a^6}{5x^5}-\frac{2a^5b}{x^3}-\frac{15a^4b^2}{x}+20a^3b^3x+5a^2b^4x^3+\frac{6}{5}ab^5x^5+\frac{b^6x^7}{7}$$

```
[Out] -1/5*a^6/x^5-2*a^5*b/x^3-15*a^4*b^2/x+20*a^3*b^3*x+5*a^2*b^4*x^3+6/5*a*b^5*
x^5+1/7*b^6*x^7
```

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$5a^2b^4x^3+20a^3b^3x-\frac{15a^4b^2}{x}-\frac{2a^5b}{x^3}-\frac{a^6}{5x^5}+\frac{6}{5}ab^5x^5+\frac{b^6x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^6,x]
```

```
[Out] -a^6/(5*x^5) - (2*a^5*b)/x^3 - (15*a^4*b^2)/x + 20*a^3*b^3*x + 5*a^2*b^4*x^
3 + (6*a*b^5*x^5)/5 + (b^6*x^7)/7
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^6}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^6}\,dx}{b^6}$$
$$= \frac{\int \left(20a^3b^9+\frac{a^6b^6}{x^6}+\frac{6a^5b^7}{x^4}+\frac{15a^4b^8}{x^2}+15a^2b^{10}x^2+6ab^{11}x^4+b^{12}x^6\right)dx}{b^6}$$
$$= -\frac{a^6}{5x^5}-\frac{2a^5b}{x^3}-\frac{15a^4b^2}{x}+20a^3b^3x+5a^2b^4x^3+\frac{6}{5}ab^5x^5+\frac{b^6x^7}{7}$$

Mathematica [A] time = 0.01, size = 72, normalized size = 1.00

$$-\frac{a^6}{5x^5}-\frac{2a^5b}{x^3}-\frac{15a^4b^2}{x}+20a^3b^3x+5a^2b^4x^3+\frac{6}{5}ab^5x^5+\frac{b^6x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^6,x]
```

```
[Out] -1/5*a^6/x^5 - (2*a^5*b)/x^3 - (15*a^4*b^2)/x + 20*a^3*b^3*x + 5*a^2*b^4*x^
3 + (6*a*b^5*x^5)/5 + (b^6*x^7)/7
```

fricas [A] time = 0.87, size = 70, normalized size = 0.97

$$\frac{5\,b^6x^{12}+42\,ab^5x^{10}+175\,a^2b^4x^8+700\,a^3b^3x^6-525\,a^4b^2x^4-70\,a^5bx^2-7\,a^6}{35\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^6,x, algorithm="fricas")
```

```
[Out] 1/35*(5*b^6*x^12 + 42*a*b^5*x^10 + 175*a^2*b^4*x^8 + 700*a^3*b^3*x^6 - 525*
a^4*b^2*x^4 - 70*a^5*b*x^2 - 7*a^6)/x^5
```

giac [A] time = 0.16, size = 67, normalized size = 0.93

$$\frac{1}{7}\,b^6x^7+\frac{6}{5}\,ab^5x^5+5\,a^2b^4x^3+20\,a^3b^3x-\frac{75\,a^4b^2x^4+10\,a^5bx^2+a^6}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^6,x, algorithm="giac")
```

```
[Out] 1/7*b^6*x^7 + 6/5*a*b^5*x^5 + 5*a^2*b^4*x^3 + 20*a^3*b^3*x - 1/5*(75*a^4*b^
2*x^4 + 10*a^5*b*x^2 + a^6)/x^5
```

maple [A] time = 0.01, size = 67, normalized size = 0.93

$$\frac{b^6x^7}{7}+\frac{6a\,b^5x^5}{5}+5a^2b^4x^3+20a^3b^3x-\frac{15a^4b^2}{x}-\frac{2a^5b}{x^3}-\frac{a^6}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^6,x)
```

```
[Out] -1/5*a^6/x^5-2*a^5*b/x^3-15*a^4*b^2/x+20*a^3*b^3*x+5*a^2*b^4*x^3+6/5*a*b^5*
x^5+1/7*b^6*x^7
```

maxima [A] time = 1.31, size = 67, normalized size = 0.93

$$\frac{1}{7}\,b^6x^7+\frac{6}{5}\,ab^5x^5+5\,a^2b^4x^3+20\,a^3b^3x-\frac{75\,a^4b^2x^4+10\,a^5bx^2+a^6}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^6,x, algorithm="maxima")
```

```
[Out] 1/7*b^6*x^7 + 6/5*a*b^5*x^5 + 5*a^2*b^4*x^3 + 20*a^3*b^3*x - 1/5*(75*a^4*b^
2*x^4 + 10*a^5*b*x^2 + a^6)/x^5
```

mupad [B] time = 0.03, size = 69, normalized size = 0.96

$$\frac{b^6\,x^7}{7}-\frac{\frac{a^6}{5}+2\,a^5\,b\,x^2+15\,a^4\,b^2\,x^4}{x^5}+20\,a^3\,b^3\,x+\frac{6\,a\,b^5\,x^5}{5}+5\,a^2\,b^4\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^6,x)
```

```
[Out] (b^6*x^7)/7 - (a^6/5 + 2*a^5*b*x^2 + 15*a^4*b^2*x^4)/x^5 + 20*a^3*b^3*x + (
6*a*b^5*x^5)/5 + 5*a^2*b^4*x^3
```

sympy [A] time = 0.26, size = 73, normalized size = 1.01

$$20a^3b^3x+5a^2b^4x^3+\frac{6ab^5x^5}{5}+\frac{b^6x^7}{7}+\frac{-a^6-10a^5bx^2-75a^4b^2x^4}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**6,x)
```

```
[Out] 20*a**3*b**3*x + 5*a**2*b**4*x**3 + 6*a*b**5*x**5/5 + b**6*x**7/7 + (-a**6
- 10*a**5*b*x**2 - 75*a**4*b**2*x**4)/(5*x**5)
```

3.459 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^7}\,dx$

Optimal. Leaf size=79

$$-\frac{a^6}{6x^6}-\frac{3a^5b}{2x^4}-\frac{15a^4b^2}{2x^2}+20a^3b^3\log(x)+\frac{15}{2}a^2b^4x^2+\frac{3}{2}ab^5x^4+\frac{b^6x^6}{6}$$

```
[Out] -1/6*a^6/x^6-3/2*a^5*b/x^4-15/2*a^4*b^2/x^2+15/2*a^2*b^4*x^2+3/2*a*b^5*x^4+
1/6*b^6*x^6+20*a^3*b^3*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{15a^4b^2}{2x^2}+\frac{15}{2}a^2b^4x^2+20a^3b^3\log(x)-\frac{3a^5b}{2x^4}-\frac{a^6}{6x^6}+\frac{3}{2}ab^5x^4+\frac{b^6x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^7,x]
```

```
[Out] -a^6/(6*x^6) - (3*a^5*b)/(2*x^4) - (15*a^4*b^2)/(2*x^2) + (15*a^2*b^4*x^2)/
2 + (3*a*b^5*x^4)/2 + (b^6*x^6)/6 + 20*a^3*b^3*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^7}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^7}\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^4}\,dx,x,x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(15a^2b^{10}+\frac{a^6b^6}{x^4}+\frac{6a^5b^7}{x^3}+\frac{15a^4b^8}{x^2}+\frac{20a^3b^9}{x}+6ab^{11}x+b^{12}x^2\right)dx,x,x^2\right)}{2b^6}$$
$$= -\frac{a^6}{6x^6}-\frac{3a^5b}{2x^4}-\frac{15a^4b^2}{2x^2}+\frac{15}{2}a^2b^4x^2+\frac{3}{2}ab^5x^4+\frac{b^6x^6}{6}+20a^3b^3\log(x)$$

Mathematica [A] time = 0.01, size = 79, normalized size = 1.00

$$-\frac{a^6}{6x^6}-\frac{3a^5b}{2x^4}-\frac{15a^4b^2}{2x^2}+20a^3b^3\log(x)+\frac{15}{2}a^2b^4x^2+\frac{3}{2}ab^5x^4+\frac{b^6x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^7,x]
```

```
[Out] -1/6*a^6/x^6 - (3*a^5*b)/(2*x^4) - (15*a^4*b^2)/(2*x^2) + (15*a^2*b^4*x^2)/
2 + (3*a*b^5*x^4)/2 + (b^6*x^6)/6 + 20*a^3*b^3*Log[x]
```

fricas [A] time = 0.96, size = 71, normalized size = 0.90

$$\frac{b^6x^{12}+9\,ab^5x^{10}+45\,a^2b^4x^8+120\,a^3b^3x^6\log(x)-45\,a^4b^2x^4-9\,a^5bx^2-a^6}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(b^6*x^12 + 9*a*b^5*x^10 + 45*a^2*b^4*x^8 + 120*a^3*b^3*x^6*log(x) - 45
*a^4*b^2*x^4 - 9*a^5*b*x^2 - a^6)/x^6
```

giac [A] time = 0.15, size = 81, normalized size = 1.03

$$\frac{1}{6}\,b^6x^6+\frac{3}{2}\,ab^5x^4+\frac{15}{2}\,a^2b^4x^2+10\,a^3b^3\log\left(x^2\right)-\frac{110\,a^3b^3x^6+45\,a^4b^2x^4+9\,a^5bx^2+a^6}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^7,x, algorithm="giac")
```

```
[Out] 1/6*b^6*x^6 + 3/2*a*b^5*x^4 + 15/2*a^2*b^4*x^2 + 10*a^3*b^3*log(x^2) - 1/6*
(110*a^3*b^3*x^6 + 45*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/x^6
```

maple [A] time = 0.01, size = 68, normalized size = 0.86

$$\frac{b^6x^6}{6}+\frac{3a\,b^5x^4}{2}+\frac{15a^2b^4x^2}{2}+20a^3b^3\ln(x)-\frac{15a^4b^2}{2x^2}-\frac{3a^5b}{2x^4}-\frac{a^6}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^7,x)
```

```
[Out] -1/6*a^6/x^6-3/2*a^5*b/x^4-15/2*a^4*b^2/x^2+15/2*a^2*b^4*x^2+3/2*a*b^5*x^4+
1/6*b^6*x^6+20*a^3*b^3*ln(x)
```

maxima [A] time = 1.39, size = 70, normalized size = 0.89

$$\frac{1}{6}\,b^6x^6+\frac{3}{2}\,ab^5x^4+\frac{15}{2}\,a^2b^4x^2+10\,a^3b^3\log\left(x^2\right)-\frac{45\,a^4b^2x^4+9\,a^5bx^2+a^6}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*b^6*x^6 + 3/2*a*b^5*x^4 + 15/2*a^2*b^4*x^2 + 10*a^3*b^3*log(x^2) - 1/6*
(45*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/x^6
```

mupad [B] time = 4.34, size = 70, normalized size = 0.89

$$\frac{b^6\,x^6}{6}-\frac{\frac{a^6}{6}+\frac{3\,a^5\,b\,x^2}{2}+\frac{15\,a^4\,b^2\,x^4}{2}}{x^6}+\frac{3\,a\,b^5\,x^4}{2}+\frac{15\,a^2\,b^4\,x^2}{2}+20\,a^3\,b^3\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^7,x)
```

```
[Out] (b^6*x^6)/6 - (a^6/6 + (3*a^5*b*x^2)/2 + (15*a^4*b^2*x^4)/2)/x^6 + (3*a*b^5
*x^4)/2 + (15*a^2*b^4*x^2)/2 + 20*a^3*b^3*log(x)
```

sympy [A] time = 0.32, size = 76, normalized size = 0.96

$$20a^3b^3\log{(x)}+\frac{15a^2b^4x^2}{2}+\frac{3ab^5x^4}{2}+\frac{b^6x^6}{6}+\frac{-a^6-9a^5bx^2-45a^4b^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**7,x)
```

```
[Out] 20*a**3*b**3*log(x) + 15*a**2*b**4*x**2/2 + 3*a*b**5*x**4/2 + b**6*x**6/6 +
 (-a**6 - 9*a**5*b*x**2 - 45*a**4*b**2*x**4)/(6*x**6)
```

3.460 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^8}\,dx$

Optimal. Leaf size=72

$$-\frac{a^6}{7x^7}-\frac{6a^5b}{5x^5}-\frac{5a^4b^2}{x^3}-\frac{20a^3b^3}{x}+15a^2b^4x+2ab^5x^3+\frac{b^6x^5}{5}$$

```
[Out] -1/7*a^6/x^7-6/5*a^5*b/x^5-5*a^4*b^2/x^3-20*a^3*b^3/x+15*a^2*b^4*x+2*a*b^5*
x^3+1/5*b^6*x^5
```

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{5a^4b^2}{x^3}-\frac{20a^3b^3}{x}+15a^2b^4x-\frac{6a^5b}{5x^5}-\frac{a^6}{7x^7}+2ab^5x^3+\frac{b^6x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^8,x]
```

```
[Out] -a^6/(7*x^7) - (6*a^5*b)/(5*x^5) - (5*a^4*b^2)/x^3 - (20*a^3*b^3)/x + 15*a^
2*b^4*x + 2*a*b^5*x^3 + (b^6*x^5)/5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^8}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^8}\,dx}{b^6}$$
$$= \frac{\int \left(15a^2b^{10}+\frac{a^6b^6}{x^8}+\frac{6a^5b^7}{x^6}+\frac{15a^4b^8}{x^4}+\frac{20a^3b^9}{x^2}+6ab^{11}x^2+b^{12}x^4\right)dx}{b^6}$$
$$= -\frac{a^6}{7x^7}-\frac{6a^5b}{5x^5}-\frac{5a^4b^2}{x^3}-\frac{20a^3b^3}{x}+15a^2b^4x+2ab^5x^3+\frac{b^6x^5}{5}$$

Mathematica [A] time = 0.01, size = 72, normalized size = 1.00

$$-\frac{a^6}{7x^7}-\frac{6a^5b}{5x^5}-\frac{5a^4b^2}{x^3}-\frac{20a^3b^3}{x}+15a^2b^4x+2ab^5x^3+\frac{b^6x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^8,x]
```

```
[Out] -1/7*a^6/x^7 - (6*a^5*b)/(5*x^5) - (5*a^4*b^2)/x^3 - (20*a^3*b^3)/x + 15*a^
2*b^4*x + 2*a*b^5*x^3 + (b^6*x^5)/5
```

fricas [A] time = 0.72, size = 70, normalized size = 0.97

$$\frac{7\,b^6x^{12}+70\,ab^5x^{10}+525\,a^2b^4x^8-700\,a^3b^3x^6-175\,a^4b^2x^4-42\,a^5bx^2-5\,a^6}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^8,x, algorithm="fricas")
```

```
[Out] 1/35*(7*b^6*x^12 + 70*a*b^5*x^10 + 525*a^2*b^4*x^8 - 700*a^3*b^3*x^6 - 175*
a^4*b^2*x^4 - 42*a^5*b*x^2 - 5*a^6)/x^7
```

giac [A] time = 0.15, size = 69, normalized size = 0.96

$$\frac{1}{5}\,b^6x^5+2\,ab^5x^3+15\,a^2b^4x-\frac{700\,a^3b^3x^6+175\,a^4b^2x^4+42\,a^5bx^2+5\,a^6}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^8,x, algorithm="giac")
```

```
[Out] 1/5*b^6*x^5 + 2*a*b^5*x^3 + 15*a^2*b^4*x - 1/35*(700*a^3*b^3*x^6 + 175*a^4*
b^2*x^4 + 42*a^5*b*x^2 + 5*a^6)/x^7
```

maple [A] time = 0.01, size = 67, normalized size = 0.93

$$\frac{b^6x^5}{5}+2a\,b^5x^3+15a^2b^4x-\frac{20a^3b^3}{x}-\frac{5a^4b^2}{x^3}-\frac{6a^5b}{5x^5}-\frac{a^6}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^8,x)
```

```
[Out] -1/7*a^6/x^7-6/5*a^5*b/x^5-5*a^4*b^2/x^3-20*a^3*b^3/x+15*a^2*b^4*x+2*a*b^5*
x^3+1/5*b^6*x^5
```

maxima [A] time = 1.38, size = 69, normalized size = 0.96

$$\frac{1}{5}\,b^6x^5+2\,ab^5x^3+15\,a^2b^4x-\frac{700\,a^3b^3x^6+175\,a^4b^2x^4+42\,a^5bx^2+5\,a^6}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^8,x, algorithm="maxima")
```

```
[Out] 1/5*b^6*x^5 + 2*a*b^5*x^3 + 15*a^2*b^4*x - 1/35*(700*a^3*b^3*x^6 + 175*a^4*
b^2*x^4 + 42*a^5*b*x^2 + 5*a^6)/x^7
```

mupad [B] time = 0.05, size = 69, normalized size = 0.96

$$\frac{b^6\,x^5}{5}-\frac{\frac{a^6}{7}+\frac{6\,a^5\,b\,x^2}{5}+5\,a^4\,b^2\,x^4+20\,a^3\,b^3\,x^6}{x^7}+15\,a^2\,b^4\,x+2\,a\,b^5\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^8,x)
```

```
[Out] (b^6*x^5)/5 - (a^6/7 + (6*a^5*b*x^2)/5 + 5*a^4*b^2*x^4 + 20*a^3*b^3*x^6)/x^
7 + 15*a^2*b^4*x + 2*a*b^5*x^3
```

sympy [A] time = 0.34, size = 73, normalized size = 1.01

$$15a^2b^4x+2ab^5x^3+\frac{b^6x^5}{5}+\frac{-5a^6-42a^5bx^2-175a^4b^2x^4-700a^3b^3x^6}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**8,x)
```

```
[Out] 15*a**2*b**4*x + 2*a*b**5*x**3 + b**6*x**5/5 + (-5*a**6 - 42*a**5*b*x**2 -
175*a**4*b**2*x**4 - 700*a**3*b**3*x**6)/(35*x**7)
```

3.461 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^9} dx$

Optimal. Leaf size=73

$$-\frac{a^6}{8x^8}-\frac{a^5b}{x^6}-\frac{15a^4b^2}{4x^4}-\frac{10a^3b^3}{x^2}+15a^2b^4\log(x)+3ab^5x^2+\frac{b^6x^4}{4}$$

```
[Out] -1/8*a^6/x^8-a^5*b/x^6-15/4*a^4*b^2/x^4-10*a^3*b^3/x^2+3*a*b^5*x^2+1/4*b^6*
x^4+15*a^2*b^4*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{15a^4b^2}{4x^4}-\frac{10a^3b^3}{x^2}+15a^2b^4\log(x)-\frac{a^5b}{x^6}-\frac{a^6}{8x^8}+3ab^5x^2+\frac{b^6x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^9,x]
```

```
[Out] -a^6/(8*x^8) - (a^5*b)/x^6 - (15*a^4*b^2)/(4*x^4) - (10*a^3*b^3)/x^2 + 3*a*
b^5*x^2 + (b^6*x^4)/4 + 15*a^2*b^4*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^9}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^9}\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^6}{x^5}\,dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(6ab^{11}+\frac{a^6b^6}{x^5}+\frac{6a^5b^7}{x^4}+\frac{15a^4b^8}{x^3}+\frac{20a^3b^9}{x^2}+\frac{15a^2b^{10}}{x}+b^{12}x\right)dx, x, x^2\right)}{2b^6}$$
$$= -\frac{a^6}{8x^8}-\frac{a^5b}{x^6}-\frac{15a^4b^2}{4x^4}-\frac{10a^3b^3}{x^2}+3ab^5x^2+\frac{b^6x^4}{4}+15a^2b^4\log(x)$$

Mathematica [A] time = 0.01, size = 73, normalized size = 1.00

$$-\frac{a^6}{8x^8}-\frac{a^5b}{x^6}-\frac{15a^4b^2}{4x^4}-\frac{10a^3b^3}{x^2}+15a^2b^4\log(x)+3ab^5x^2+\frac{b^6x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^9,x]
```

```
[Out] -1/8*a^6/x^8 - (a^5*b)/x^6 - (15*a^4*b^2)/(4*x^4) - (10*a^3*b^3)/x^2 + 3*a*
b^5*x^2 + (b^6*x^4)/4 + 15*a^2*b^4*Log[x]
```

fricas [A] time = 0.75, size = 72, normalized size = 0.99

$$\frac{2\,b^6x^{12}+24\,ab^5x^{10}+120\,a^2b^4x^8\log(x)-80\,a^3b^3x^6-30\,a^4b^2x^4-8\,a^5bx^2-a^6}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^9,x, algorithm="fricas")
```

```
[Out] 1/8*(2*b^6*x^12 + 24*a*b^5*x^10 + 120*a^2*b^4*x^8*log(x) - 80*a^3*b^3*x^6 -
 30*a^4*b^2*x^4 - 8*a^5*b*x^2 - a^6)/x^8
```

giac [A] time = 0.16, size = 81, normalized size = 1.11

$$\frac{1}{4}\,b^6x^4+3\,ab^5x^2+\frac{15}{2}\,a^2b^4\log\left(x^2\right)-\frac{125\,a^2b^4x^8+80\,a^3b^3x^6+30\,a^4b^2x^4+8\,a^5bx^2+a^6}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^9,x, algorithm="giac")
```

```
[Out] 1/4*b^6*x^4 + 3*a*b^5*x^2 + 15/2*a^2*b^4*log(x^2) - 1/8*(125*a^2*b^4*x^8 +
80*a^3*b^3*x^6 + 30*a^4*b^2*x^4 + 8*a^5*b*x^2 + a^6)/x^8
```

maple [A] time = 0.01, size = 68, normalized size = 0.93

$$\frac{b^6x^4}{4} + 3a\,b^5x^2 + 15a^2b^4\ln(x) - \frac{10a^3b^3}{x^2} - \frac{15a^4b^2}{4x^4} - \frac{a^5b}{x^6} - \frac{a^6}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^9,x)
```

```
[Out] -1/8*a^6/x^8-a^5*b/x^6-15/4*a^4*b^2/x^4-10*a^3*b^3/x^2+3*a*b^5*x^2+1/4*b^6*
x^4+15*a^2*b^4*ln(x)
```

maxima [A] time = 1.31, size = 70, normalized size = 0.96

$$\frac{1}{4}\,b^6x^4 + 3\,ab^5x^2 + \frac{15}{2}\,a^2b^4\log\left(x^2\right) - \frac{80\,a^3b^3x^6 + 30\,a^4b^2x^4 + 8\,a^5bx^2 + a^6}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^9,x, algorithm="maxima")
```

```
[Out] 1/4*b^6*x^4 + 3*a*b^5*x^2 + 15/2*a^2*b^4*log(x^2) - 1/8*(80*a^3*b^3*x^6 + 3
0*a^4*b^2*x^4 + 8*a^5*b*x^2 + a^6)/x^8
```

mupad [B] time = 0.05, size = 69, normalized size = 0.95

$$\frac{b^6\,x^4}{4} - \frac{\frac{a^6}{8} + a^5\,b\,x^2 + \frac{15\,a^4\,b^2\,x^4}{4} + 10\,a^3\,b^3\,x^6}{x^8} + 3\,a\,b^5\,x^2 + 15\,a^2\,b^4\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^9,x)
```

```
[Out] (b^6*x^4)/4 - (a^6/8 + a^5*b*x^2 + (15*a^4*b^2*x^4)/4 + 10*a^3*b^3*x^6)/x^8
 + 3*a*b^5*x^2 + 15*a^2*b^4*log(x)
```

sympy [A] time = 0.42, size = 73, normalized size = 1.00

$$15a^2b^4\log(x) + 3ab^5x^2 + \frac{b^6x^4}{4} + \frac{-a^6 - 8a^5bx^2 - 30a^4b^2x^4 - 80a^3b^3x^6}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**9,x)
```

```
[Out] 15*a**2*b**4*log(x) + 3*a*b**5*x**2 + b**6*x**4/4 + (-a**6 - 8*a**5*b*x**2
- 30*a**4*b**2*x**4 - 80*a**3*b**3*x**6)/(8*x**8)
```

3.462 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{10}}\,dx$

Optimal. Leaf size=74

$$-\frac{a^6}{9x^9}-\frac{6a^5b}{7x^7}-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}+6ab^5x+\frac{b^6x^3}{3}$$

```
[Out] -1/9*a^6/x^9-6/7*a^5*b/x^7-3*a^4*b^2/x^5-20/3*a^3*b^3/x^3-15*a^2*b^4/x+6*a*
b^5*x+1/3*b^6*x^3
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}-\frac{6a^5b}{7x^7}-\frac{a^6}{9x^9}+6ab^5x+\frac{b^6x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^10,x]
```

```
[Out] -a^6/(9*x^9) - (6*a^5*b)/(7*x^7) - (3*a^4*b^2)/x^5 - (20*a^3*b^3)/(3*x^3) -
 (15*a^2*b^4)/x + 6*a*b^5*x + (b^6*x^3)/3
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{10}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{10}}\,dx}{b^6}$$
$$= \frac{\int \left(6ab^{11}+\frac{a^6b^6}{x^{10}}+\frac{6a^5b^7}{x^8}+\frac{15a^4b^8}{x^6}+\frac{20a^3b^9}{x^4}+\frac{15a^2b^{10}}{x^2}+b^{12}x^2\right)dx}{b^6}$$
$$= -\frac{a^6}{9x^9}-\frac{6a^5b}{7x^7}-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}+6ab^5x+\frac{b^6x^3}{3}$$

Mathematica [A] time = 0.01, size = 74, normalized size = 1.00

$$-\frac{a^6}{9x^9}-\frac{6a^5b}{7x^7}-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}+6ab^5x+\frac{b^6x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^10,x]
```

```
[Out] -1/9*a^6/x^9 - (6*a^5*b)/(7*x^7) - (3*a^4*b^2)/x^5 - (20*a^3*b^3)/(3*x^3) -
 (15*a^2*b^4)/x + 6*a*b^5*x + (b^6*x^3)/3
```

fricas [A] time = 0.92, size = 70, normalized size = 0.95

$$\frac{21\,b^6x^{12}+378\,ab^5x^{10}-945\,a^2b^4x^8-420\,a^3b^3x^6-189\,a^4b^2x^4-54\,a^5bx^2-7\,a^6}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^10,x, algorithm="fricas")
```

```
[Out] 1/63*(21*b^6*x^12 + 378*a*b^5*x^10 - 945*a^2*b^4*x^8 - 420*a^3*b^3*x^6 - 18
9*a^4*b^2*x^4 - 54*a^5*b*x^2 - 7*a^6)/x^9
```

giac [A] time = 0.24, size = 69, normalized size = 0.93

$$\frac{1}{3}\,b^6x^3+6\,ab^5x-\frac{945\,a^2b^4x^8+420\,a^3b^3x^6+189\,a^4b^2x^4+54\,a^5bx^2+7\,a^6}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^10,x, algorithm="giac")
```

```
[Out] 1/3*b^6*x^3 + 6*a*b^5*x - 1/63*(945*a^2*b^4*x^8 + 420*a^3*b^3*x^6 + 189*a^4
*b^2*x^4 + 54*a^5*b*x^2 + 7*a^6)/x^9
```

maple [A] time = 0.01, size = 67, normalized size = 0.91

$$\frac{b^6x^3}{3}+6a\,b^5x-\frac{15a^2b^4}{x}-\frac{20a^3b^3}{3x^3}-\frac{3a^4b^2}{x^5}-\frac{6a^5b}{7x^7}-\frac{a^6}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^10,x)
```

```
[Out] -1/9*a^6/x^9-6/7*a^5*b/x^7-3*a^4*b^2/x^5-20/3*a^3*b^3/x^3-15*a^2*b^4/x+6*a*
b^5*x+1/3*b^6*x^3
```

maxima [A] time = 1.41, size = 69, normalized size = 0.93

$$\frac{1}{3}\,b^6x^3+6\,ab^5x-\frac{945\,a^2b^4x^8+420\,a^3b^3x^6+189\,a^4b^2x^4+54\,a^5bx^2+7\,a^6}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^10,x, algorithm="maxima")
```

```
[Out] 1/3*b^6*x^3 + 6*a*b^5*x - 1/63*(945*a^2*b^4*x^8 + 420*a^3*b^3*x^6 + 189*a^4
*b^2*x^4 + 54*a^5*b*x^2 + 7*a^6)/x^9
```

mupad [B] time = 0.05, size = 70, normalized size = 0.95

$$-\frac{\frac{a^6}{9}+\frac{6\,a^5\,b\,x^2}{7}+3\,a^4\,b^2\,x^4+\frac{20\,a^3\,b^3\,x^6}{3}+15\,a^2\,b^4\,x^8-6\,a\,b^5\,x^{10}-\frac{b^6\,x^{12}}{3}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^10,x)
```

```
[Out] -(a^6/9 - (b^6*x^12)/3 + (6*a^5*b*x^2)/7 - 6*a*b^5*x^10 + 3*a^4*b^2*x^4 + (
20*a^3*b^3*x^6)/3 + 15*a^2*b^4*x^8)/x^9
```

sympy [A] time = 0.44, size = 73, normalized size = 0.99

$$6ab^5x+\frac{b^6x^3}{3}+\frac{-7a^6-54a^5bx^2-189a^4b^2x^4-420a^3b^3x^6-945a^2b^4x^8}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**10,x)
```

```
[Out] 6*a*b**5*x + b**6*x**3/3 + (-7*a**6 - 54*a**5*b*x**2 - 189*a**4*b**2*x**4 -
 420*a**3*b**3*x**6 - 945*a**2*b**4*x**8)/(63*x**9)
```

3.463 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{11}}\,dx$

Optimal. Leaf size=77

$$-\frac{a^6}{10x^{10}}-\frac{3a^5b}{4x^8}-\frac{5a^4b^2}{2x^6}-\frac{5a^3b^3}{x^4}-\frac{15a^2b^4}{2x^2}+6ab^5\log(x)+\frac{b^6x^2}{2}$$

```
[Out] -1/10*a^6/x^10-3/4*a^5*b/x^8-5/2*a^4*b^2/x^6-5*a^3*b^3/x^4-15/2*a^2*b^4/x^2
+1/2*b^6*x^2+6*a*b^5*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{5a^4b^2}{2x^6}-\frac{5a^3b^3}{x^4}-\frac{15a^2b^4}{2x^2}-\frac{3a^5b}{4x^8}-\frac{a^6}{10x^{10}}+6ab^5\log(x)+\frac{b^6x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^11,x]
```

```
[Out] -a^6/(10*x^10) - (3*a^5*b)/(4*x^8) - (5*a^4*b^2)/(2*x^6) - (5*a^3*b^3)/x^4
- (15*a^2*b^4)/(2*x^2) + (b^6*x^2)/2 + 6*a*b^5*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{11}}\,dx=\frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{11}}\,dx}{b^6}$$
$$=\frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^6}\,dx,x,x^2\right)}{2b^6}$$
$$=\frac{\text{Subst}\left(\int \left(b^{12}+\frac{a^6b^6}{x^6}+\frac{6a^5b^7}{x^5}+\frac{15a^4b^8}{x^4}+\frac{20a^3b^9}{x^3}+\frac{15a^2b^{10}}{x^2}+\frac{6ab^{11}}{x}\right)dx,x,x^2\right)}{2b^6}$$
$$=-\frac{a^6}{10x^{10}}-\frac{3a^5b}{4x^8}-\frac{5a^4b^2}{2x^6}-\frac{5a^3b^3}{x^4}-\frac{15a^2b^4}{2x^2}+\frac{b^6x^2}{2}+6ab^5\log(x)$$

Mathematica [**A**] time = 0.01, size = 77, normalized size = 1.00

$$-\frac{a^6}{10x^{10}}-\frac{3a^5b}{4x^8}-\frac{5a^4b^2}{2x^6}-\frac{5a^3b^3}{x^4}-\frac{15a^2b^4}{2x^2}+6ab^5\log(x)+\frac{b^6x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^11,x]
```

```
[Out] -1/10*a^6/x^10 - (3*a^5*b)/(4*x^8) - (5*a^4*b^2)/(2*x^6) - (5*a^3*b^3)/x^4
- (15*a^2*b^4)/(2*x^2) + (b^6*x^2)/2 + 6*a*b^5*Log[x]
```

fricas [**A**] time = 0.78, size = 72, normalized size = 0.94

$$\frac{10\,b^6x^{12}+120\,ab^5x^{10}\log(x)-150\,a^2b^4x^8-100\,a^3b^3x^6-50\,a^4b^2x^4-15\,a^5bx^2-2\,a^6}{20\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^11,x, algorithm="fricas")
```

```
[Out] 1/20*(10*b^6*x^12 + 120*a*b^5*x^10*log(x) - 150*a^2*b^4*x^8 - 100*a^3*b^3*x
^6 - 50*a^4*b^2*x^4 - 15*a^5*b*x^2 - 2*a^6)/x^10
```

giac [**A**] time = 0.15, size = 81, normalized size = 1.05

$$\frac{1}{2}\,b^6x^2+3\,ab^5\log\left(x^2\right)-\frac{137\,ab^5x^{10}+150\,a^2b^4x^8+100\,a^3b^3x^6+50\,a^4b^2x^4+15\,a^5bx^2+2\,a^6}{20\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^11,x, algorithm="giac")
```

```
[Out] 1/2*b^6*x^2 + 3*a*b^5*log(x^2) - 1/20*(137*a*b^5*x^10 + 150*a^2*b^4*x^8 + 1
00*a^3*b^3*x^6 + 50*a^4*b^2*x^4 + 15*a^5*b*x^2 + 2*a^6)/x^10
```

maple [A] time = 0.01, size = 68, normalized size = 0.88

$$\frac{b^6x^2}{2}+6a\,b^5\ln(x)-\frac{15a^2b^4}{2x^2}-\frac{5a^3b^3}{x^4}-\frac{5a^4b^2}{2x^6}-\frac{3a^5b}{4x^8}-\frac{a^6}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^11,x)
```

```
[Out] -1/10*a^6/x^10-3/4*a^5*b/x^8-5/2*a^4*b^2/x^6-5*a^3*b^3/x^4-15/2*a^2*b^4/x^2
+1/2*b^6*x^2+6*a*b^5*ln(x)
```

maxima [A] time = 1.38, size = 72, normalized size = 0.94

$$\frac{1}{2}\,b^6x^2+3\,ab^5\log\left(x^2\right)-\frac{150\,a^2b^4x^8+100\,a^3b^3x^6+50\,a^4b^2x^4+15\,a^5bx^2+2\,a^6}{20\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^11,x, algorithm="maxima")
```

```
[Out] 1/2*b^6*x^2 + 3*a*b^5*log(x^2) - 1/20*(150*a^2*b^4*x^8 + 100*a^3*b^3*x^6 +
50*a^4*b^2*x^4 + 15*a^5*b*x^2 + 2*a^6)/x^10
```

mupad [B] time = 4.40, size = 70, normalized size = 0.91

$$\frac{b^6\,x^2}{2}-\frac{\frac{a^6}{10}+\frac{3\,a^5\,b\,x^2}{4}+\frac{5\,a^4\,b^2\,x^4}{2}+5\,a^3\,b^3\,x^6+\frac{15\,a^2\,b^4\,x^8}{2}}{x^{10}}+6\,a\,b^5\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^11,x)
```

```
[Out] (b^6*x^2)/2 - (a^6/10 + (3*a^5*b*x^2)/4 + (5*a^4*b^2*x^4)/2 + 5*a^3*b^3*x^6
 + (15*a^2*b^4*x^8)/2)/x^10 + 6*a*b^5*log(x)
```

sympy [A] time = 0.63, size = 75, normalized size = 0.97

$$6ab^5\log\left(x\right)+\frac{b^6x^2}{2}+\frac{-2a^6-15a^5bx^2-50a^4b^2x^4-100a^3b^3x^6-150a^2b^4x^8}{20x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**11,x)
```

```
[Out] 6*a*b**5*log(x) + b**6*x**2/2 + (-2*a**6 - 15*a**5*b*x**2 - 50*a**4*b**2*x*
*4 - 100*a**3*b**3*x**6 - 150*a**2*b**4*x**8)/(20*x**10)
```

3.464 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{12}}\,dx$

Optimal. Leaf size=71

$$-\frac{a^6}{11x^{11}}-\frac{2a^5b}{3x^9}-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{6ab^5}{x}+b^6x$$

```
[Out] -1/11*a^6/x^11-2/3*a^5*b/x^9-15/7*a^4*b^2/x^7-4*a^3*b^3/x^5-5*a^2*b^4/x^3-6
*a*b^5/x+b^6*x
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{2a^5b}{3x^9}-\frac{a^6}{11x^{11}}-\frac{6ab^5}{x}+b^6x$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^12,x]
```

```
[Out] -a^6/(11*x^11) - (2*a^5*b)/(3*x^9) - (15*a^4*b^2)/(7*x^7) - (4*a^3*b^3)/x^5
 - (5*a^2*b^4)/x^3 - (6*a*b^5)/x + b^6*x
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{12}}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^{12}}\,dx}{b^6}$$
$$= \frac{\int \left(b^{12}+\frac{a^6b^6}{x^{12}}+\frac{6a^5b^7}{x^{10}}+\frac{15a^4b^8}{x^8}+\frac{20a^3b^9}{x^6}+\frac{15a^2b^{10}}{x^4}+\frac{6ab^{11}}{x^2}\right)dx}{b^6}$$
$$= -\frac{a^6}{11x^{11}}-\frac{2a^5b}{3x^9}-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{6ab^5}{x}+b^6x$$

Mathematica [A] time = 0.01, size = 71, normalized size = 1.00

$$-\frac{a^6}{11x^{11}}-\frac{2a^5b}{3x^9}-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{6ab^5}{x}+b^6x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^12,x]
```

```
[Out] -1/11*a^6/x^11 - (2*a^5*b)/(3*x^9) - (15*a^4*b^2)/(7*x^7) - (4*a^3*b^3)/x^5
 - (5*a^2*b^4)/x^3 - (6*a*b^5)/x + b^6*x
```

fricas [A] time = 0.91, size = 70, normalized size = 0.99

$$\frac{231\,b^6x^{12}-1386\,ab^5x^{10}-1155\,a^2b^4x^8-924\,a^3b^3x^6-495\,a^4b^2x^4-154\,a^5bx^2-21\,a^6}{231\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^12,x, algorithm="fricas")
```

```
[Out] 1/231*(231*b^6*x^12 - 1386*a*b^5*x^10 - 1155*a^2*b^4*x^8 - 924*a^3*b^3*x^6
- 495*a^4*b^2*x^4 - 154*a^5*b*x^2 - 21*a^6)/x^11
```

giac [A] time = 0.15, size = 68, normalized size = 0.96

$$b^6x-\frac{1386\,ab^5x^{10}+1155\,a^2b^4x^8+924\,a^3b^3x^6+495\,a^4b^2x^4+154\,a^5bx^2+21\,a^6}{231\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^12,x, algorithm="giac")
```

```
[Out] b^6*x - 1/231*(1386*a*b^5*x^10 + 1155*a^2*b^4*x^8 + 924*a^3*b^3*x^6 + 495*a
^4*b^2*x^4 + 154*a^5*b*x^2 + 21*a^6)/x^11
```

maple [A] time = 0.01, size = 66, normalized size = 0.93

$$b^6x - \frac{6a\,b^5}{x} - \frac{5a^2b^4}{x^3} - \frac{4a^3b^3}{x^5} - \frac{15a^4b^2}{7x^7} - \frac{2a^5b}{3x^9} - \frac{a^6}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^12,x)
```

```
[Out] -1/11*a^6/x^11-2/3*a^5*b/x^9-15/7*a^4*b^2/x^7-4*a^3*b^3/x^5-5*a^2*b^4/x^3-6
*a*b^5/x+b^6*x
```

maxima [A] time = 1.31, size = 68, normalized size = 0.96

$$b^6x - \frac{1386\,ab^5x^{10} + 1155\,a^2b^4x^8 + 924\,a^3b^3x^6 + 495\,a^4b^2x^4 + 154\,a^5bx^2 + 21\,a^6}{231\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^12,x, algorithm="maxima")
```

```
[Out] b^6*x - 1/231*(1386*a*b^5*x^10 + 1155*a^2*b^4*x^8 + 924*a^3*b^3*x^6 + 495*a
^4*b^2*x^4 + 154*a^5*b*x^2 + 21*a^6)/x^11
```

mupad [B] time = 4.30, size = 68, normalized size = 0.96

$$b^6\,x - \frac{\frac{a^6}{11} + \frac{2\,a^5\,b\,x^2}{3} + \frac{15\,a^4\,b^2\,x^4}{7} + 4\,a^3\,b^3\,x^6 + 5\,a^2\,b^4\,x^8 + 6\,a\,b^5\,x^{10}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^12,x)
```

```
[Out] b^6*x - (a^6/11 + (2*a^5*b*x^2)/3 + 6*a*b^5*x^10 + (15*a^4*b^2*x^4)/7 + 4*a
^3*b^3*x^6 + 5*a^2*b^4*x^8)/x^11
```

sympy [A] time = 0.52, size = 71, normalized size = 1.00

$$b^6x + \frac{-21a^6 - 154a^5bx^2 - 495a^4b^2x^4 - 924a^3b^3x^6 - 1155a^2b^4x^8 - 1386ab^5x^{10}}{231x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**12,x)
```

```
[Out] b**6*x + (-21*a**6 - 154*a**5*b*x**2 - 495*a**4*b**2*x**4 - 924*a**3*b**3*x
**6 - 1155*a**2*b**4*x**8 - 1386*a*b**5*x**10)/(231*x**11)
```

3.465 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{13}}\,dx$

Optimal. Leaf size=76

$$-\frac{a^6}{12x^{12}}-\frac{3a^5b}{5x^{10}}-\frac{15a^4b^2}{8x^8}-\frac{10a^3b^3}{3x^6}-\frac{15a^2b^4}{4x^4}-\frac{3ab^5}{x^2}+b^6\log(x)$$

```
[Out] -1/12*a^6/x^12-3/5*a^5*b/x^10-15/8*a^4*b^2/x^8-10/3*a^3*b^3/x^6-15/4*a^2*b^
4/x^4-3*a*b^5/x^2+b^6*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{15a^4b^2}{8x^8}-\frac{10a^3b^3}{3x^6}-\frac{15a^2b^4}{4x^4}-\frac{3a^5b}{5x^{10}}-\frac{a^6}{12x^{12}}-\frac{3ab^5}{x^2}+b^6\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^13,x]
```

```
[Out] -a^6/(12*x^12) - (3*a^5*b)/(5*x^10) - (15*a^4*b^2)/(8*x^8) - (10*a^3*b^3)/(
3*x^6) - (15*a^2*b^4)/(4*x^4) - (3*a*b^5)/x^2 + b^6*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{13}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{13}}\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^7}\,dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^6b^6}{x^7}+\frac{6a^5b^7}{x^6}+\frac{15a^4b^8}{x^5}+\frac{20a^3b^9}{x^4}+\frac{15a^2b^{10}}{x^3}+\frac{6ab^{11}}{x^2}+\frac{b^{12}}{x}\right)dx, x, x^2\right)}{2b^6}$$
$$= -\frac{a^6}{12x^{12}}-\frac{3a^5b}{5x^{10}}-\frac{15a^4b^2}{8x^8}-\frac{10a^3b^3}{3x^6}-\frac{15a^2b^4}{4x^4}-\frac{3ab^5}{x^2}+b^6\log(x)$$

Mathematica [A] time = 0.00, size = 76, normalized size = 1.00

$$-\frac{a^6}{12x^{12}}-\frac{3a^5b}{5x^{10}}-\frac{15a^4b^2}{8x^8}-\frac{10a^3b^3}{3x^6}-\frac{15a^2b^4}{4x^4}-\frac{3ab^5}{x^2}+b^6\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^13,x]
```

```
[Out] -1/12*a^6/x^12 - (3*a^5*b)/(5*x^10) - (15*a^4*b^2)/(8*x^8) - (10*a^3*b^3)/(
3*x^6) - (15*a^2*b^4)/(4*x^4) - (3*a*b^5)/x^2 + b^6*Log[x]
```

fricas [A] time = 0.85, size = 72, normalized size = 0.95

$$\frac{120\,b^6x^{12}\log(x)-360\,ab^5x^{10}-450\,a^2b^4x^8-400\,a^3b^3x^6-225\,a^4b^2x^4-72\,a^5bx^2-10\,a^6}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^13,x, algorithm="fricas")
```

```
[Out] 1/120*(120*b^6*x^12*log(x) - 360*a*b^5*x^10 - 450*a^2*b^4*x^8 - 400*a^3*b^3
*x^6 - 225*a^4*b^2*x^4 - 72*a^5*b*x^2 - 10*a^6)/x^12
```

giac [A] time = 0.15, size = 80, normalized size = 1.05

$$\frac{1}{2}\,b^6\log\left(x^2\right)-\frac{147\,b^6x^{12}+360\,ab^5x^{10}+450\,a^2b^4x^8+400\,a^3b^3x^6+225\,a^4b^2x^4+72\,a^5bx^2+10\,a^6}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^13,x, algorithm="giac")
```

```
[Out] 1/2*b^6*log(x^2) - 1/120*(147*b^6*x^12 + 360*a*b^5*x^10 + 450*a^2*b^4*x^8 +
 400*a^3*b^3*x^6 + 225*a^4*b^2*x^4 + 72*a^5*b*x^2 + 10*a^6)/x^12
```

maple [A] time = 0.01, size = 67, normalized size = 0.88

$$b^6 \ln(x) - \frac{3a\,b^5}{x^2} - \frac{15a^2b^4}{4x^4} - \frac{10a^3b^3}{3x^6} - \frac{15a^4b^2}{8x^8} - \frac{3a^5b}{5x^{10}} - \frac{a^6}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^13,x)
```

```
[Out] -1/12*a^6/x^12-3/5*a^5*b/x^10-15/8*a^4*b^2/x^8-10/3*a^3*b^3/x^6-15/4*a^2*b^
4/x^4-3*a*b^5/x^2+b^6*ln(x)
```

maxima [A] time = 1.40, size = 72, normalized size = 0.95

$$\frac{1}{2}\,b^6 \log\left(x^2\right) - \frac{360\,ab^5x^{10} + 450\,a^2b^4x^8 + 400\,a^3b^3x^6 + 225\,a^4b^2x^4 + 72\,a^5bx^2 + 10\,a^6}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^13,x, algorithm="maxima")
```

```
[Out] 1/2*b^6*log(x^2) - 1/120*(360*a*b^5*x^10 + 450*a^2*b^4*x^8 + 400*a^3*b^3*x^
6 + 225*a^4*b^2*x^4 + 72*a^5*b*x^2 + 10*a^6)/x^12
```

mupad [B] time = 0.07, size = 69, normalized size = 0.91

$$b^6\,\ln(x) - \frac{\frac{a^6}{12} + \frac{3\,a^5\,b\,x^2}{5} + \frac{15\,a^4\,b^2\,x^4}{8} + \frac{10\,a^3\,b^3\,x^6}{3} + \frac{15\,a^2\,b^4\,x^8}{4} + 3\,a\,b^5\,x^{10}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^13,x)
```

```
[Out] b^6*log(x) - (a^6/12 + (3*a^5*b*x^2)/5 + 3*a*b^5*x^10 + (15*a^4*b^2*x^4)/8
+ (10*a^3*b^3*x^6)/3 + (15*a^2*b^4*x^8)/4)/x^12
```

sympy [A] time = 0.62, size = 73, normalized size = 0.96

$$b^6 \log{(x)} + \frac{-10a^6 - 72a^5bx^2 - 225a^4b^2x^4 - 400a^3b^3x^6 - 450a^2b^4x^8 - 360ab^5x^{10}}{120x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**13,x)
```

```
[Out] b**6*log(x) + (-10*a**6 - 72*a**5*b*x**2 - 225*a**4*b**2*x**4 - 400*a**3*b*
*3*x**6 - 450*a**2*b**4*x**8 - 360*a*b**5*x**10)/(120*x**12)
```

3.466 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{14}}\,dx$

Optimal. Leaf size=76

$$-\frac{a^6}{13x^{13}}-\frac{6a^5b}{11x^{11}}-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{2ab^5}{x^3}-\frac{b^6}{x}$$

```
[Out] -1/13*a^6/x^13-6/11*a^5*b/x^11-5/3*a^4*b^2/x^9-20/7*a^3*b^3/x^7-3*a^2*b^4/x
^5-2*a*b^5/x^3-b^6/x
```

Rubi [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{6a^5b}{11x^{11}}-\frac{a^6}{13x^{13}}-\frac{2ab^5}{x^3}-\frac{b^6}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^14,x]
```

```
[Out] -a^6/(13*x^13) - (6*a^5*b)/(11*x^11) - (5*a^4*b^2)/(3*x^9) - (20*a^3*b^3)/(
7*x^7) - (3*a^2*b^4)/x^5 - (2*a*b^5)/x^3 - b^6/x
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{14}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{14}}\,dx}{b^6}$$
$$= \frac{\int \left(\frac{a^6b^6}{x^{14}}+\frac{6a^5b^7}{x^{12}}+\frac{15a^4b^8}{x^{10}}+\frac{20a^3b^9}{x^8}+\frac{15a^2b^{10}}{x^6}+\frac{6ab^{11}}{x^4}+\frac{b^{12}}{x^2}\right)dx}{b^6}$$
$$= -\frac{a^6}{13x^{13}}-\frac{6a^5b}{11x^{11}}-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{2ab^5}{x^3}-\frac{b^6}{x}$$

Mathematica [A] time = 0.01, size = 76, normalized size = 1.00

$$-\frac{a^6}{13x^{13}}-\frac{6a^5b}{11x^{11}}-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{2ab^5}{x^3}-\frac{b^6}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^14,x]
```

```
[Out] -1/13*a^6/x^13 - (6*a^5*b)/(11*x^11) - (5*a^4*b^2)/(3*x^9) - (20*a^3*b^3)/(
7*x^7) - (3*a^2*b^4)/x^5 - (2*a*b^5)/x^3 - b^6/x
```

fricas [A] time = 0.81, size = 70, normalized size = 0.92

$$-\frac{3003\,b^6x^{12}+6006\,ab^5x^{10}+9009\,a^2b^4x^8+8580\,a^3b^3x^6+5005\,a^4b^2x^4+1638\,a^5bx^2+231\,a^6}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^14,x, algorithm="fricas")
```

```
[Out] -1/3003*(3003*b^6*x^12 + 6006*a*b^5*x^10 + 9009*a^2*b^4*x^8 + 8580*a^3*b^3*
x^6 + 5005*a^4*b^2*x^4 + 1638*a^5*b*x^2 + 231*a^6)/x^13
```

giac [A] time = 0.18, size = 70, normalized size = 0.92

$$-\frac{3003\,b^6x^{12}+6006\,ab^5x^{10}+9009\,a^2b^4x^8+8580\,a^3b^3x^6+5005\,a^4b^2x^4+1638\,a^5bx^2+231\,a^6}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^14,x, algorithm="giac")
```

```
[Out] -1/3003*(3003*b^6*x^12 + 6006*a*b^5*x^10 + 9009*a^2*b^4*x^8 + 8580*a^3*b^3*
x^6 + 5005*a^4*b^2*x^4 + 1638*a^5*b*x^2 + 231*a^6)/x^13
```

maple [A] time = 0.00, size = 69, normalized size = 0.91

$$-\frac{b^6}{x}-\frac{2a\,b^5}{x^3}-\frac{3a^2b^4}{x^5}-\frac{20a^3b^3}{7x^7}-\frac{5a^4b^2}{3x^9}-\frac{6a^5b}{11x^{11}}-\frac{a^6}{13x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^14,x)
```

```
[Out] -1/13*a^6/x^13-6/11*a^5*b/x^11-5/3*a^4*b^2/x^9-20/7*a^3*b^3/x^7-3*a^2*b^4/x
^5-2*a*b^5/x^3-b^6/x
```

maxima [A] time = 1.34, size = 70, normalized size = 0.92

$$-\frac{3003\,b^6x^{12}+6006\,ab^5x^{10}+9009\,a^2b^4x^8+8580\,a^3b^3x^6+5005\,a^4b^2x^4+1638\,a^5bx^2+231\,a^6}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^14,x, algorithm="maxima")
```

```
[Out] -1/3003*(3003*b^6*x^12 + 6006*a*b^5*x^10 + 9009*a^2*b^4*x^8 + 8580*a^3*b^3*
x^6 + 5005*a^4*b^2*x^4 + 1638*a^5*b*x^2 + 231*a^6)/x^13
```

mupad [B] time = 0.05, size = 69, normalized size = 0.91

$$-\frac{\frac{a^6}{13}+\frac{6\,a^5\,b\,x^2}{11}+\frac{5\,a^4\,b^2\,x^4}{3}+\frac{20\,a^3\,b^3\,x^6}{7}+3\,a^2\,b^4\,x^8+2\,a\,b^5\,x^{10}+b^6\,x^{12}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^14,x)
```

```
[Out] -(a^6/13 + b^6*x^12 + (6*a^5*b*x^2)/11 + 2*a*b^5*x^10 + (5*a^4*b^2*x^4)/3 +
 (20*a^3*b^3*x^6)/7 + 3*a^2*b^4*x^8)/x^13
```

sympy [A] time = 0.60, size = 75, normalized size = 0.99

$$\frac{-231a^6-1638a^5bx^2-5005a^4b^2x^4-8580a^3b^3x^6-9009a^2b^4x^8-6006ab^5x^{10}-3003b^6x^{12}}{3003x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**14,x)
```

```
[Out] (-231*a**6 - 1638*a**5*b*x**2 - 5005*a**4*b**2*x**4 - 8580*a**3*b**3*x**6 -
 9009*a**2*b**4*x**8 - 6006*a*b**5*x**10 - 3003*b**6*x**12)/(3003*x**13)
```

3.467 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{15}}\,dx$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^7}{14ax^{14}}$$

```
[Out] -1/14*(b*x^2+a)^7/a/x^14
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 264}

$$-\frac{(a+bx^2)^7}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^15,x]
```

```
[Out] -(a + b*x^2)^7/(14*a*x^14)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{15}}\,dx &= \frac{\int \frac{(ab+b^2x^2)^6}{x^{15}}\,dx}{b^6}\\ &= -\frac{(a+bx^2)^7}{14ax^{14}}\end{aligned}$$

Mathematica [B] time = 0.01, size = 82, normalized size = 4.32

$$-\frac{a^6}{14x^{14}}-\frac{a^5b}{2x^{12}}-\frac{3a^4b^2}{2x^{10}}-\frac{5a^3b^3}{2x^8}-\frac{5a^2b^4}{2x^6}-\frac{3ab^5}{2x^4}-\frac{b^6}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^15,x]
```

```
[Out] -1/14*a^6/x^14 - (a^5*b)/(2*x^12) - (3*a^4*b^2)/(2*x^10) - (5*a^3*b^3)/(2*x
^8) - (5*a^2*b^4)/(2*x^6) - (3*a*b^5)/(2*x^4) - b^6/(2*x^2)
```

fricas [B] time = 0.98, size = 68, normalized size = 3.58

$$-\frac{7\,b^6x^{12}+21\,ab^5x^{10}+35\,a^2b^4x^8+35\,a^3b^3x^6+21\,a^4b^2x^4+7\,a^5bx^2+a^6}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^15,x, algorithm="fricas")
```

```
[Out] -1/14*(7*b^6*x^12 + 21*a*b^5*x^10 + 35*a^2*b^4*x^8 + 35*a^3*b^3*x^6 + 21*a^
4*b^2*x^4 + 7*a^5*b*x^2 + a^6)/x^14
```

giac [B] time = 0.16, size = 68, normalized size = 3.58

$$-\frac{7\,b^6x^{12}+21\,ab^5x^{10}+35\,a^2b^4x^8+35\,a^3b^3x^6+21\,a^4b^2x^4+7\,a^5bx^2+a^6}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^15,x, algorithm="giac")
```

```
[Out] -1/14*(7*b^6*x^12 + 21*a*b^5*x^10 + 35*a^2*b^4*x^8 + 35*a^3*b^3*x^6 + 21*a^
4*b^2*x^4 + 7*a^5*b*x^2 + a^6)/x^14
```

maple [B] time = 0.01, size = 69, normalized size = 3.63

$$-\frac{b^6}{2x^2}-\frac{3a\,b^5}{2x^4}-\frac{5a^2b^4}{2x^6}-\frac{5a^3b^3}{2x^8}-\frac{3a^4b^2}{2x^{10}}-\frac{a^5b}{2x^{12}}-\frac{a^6}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^15,x)
```

```
[Out] -1/2*b^6/x^2-3/2*a^4*b^2/x^10-1/14*a^6/x^14-1/2*a^5*b/x^12-5/2*a^2*b^4/x^6-
5/2*a^3*b^3/x^8-3/2*a*b^5/x^4
```

maxima [B] time = 1.38, size = 68, normalized size = 3.58

$$-\frac{7\,b^{6}x^{12}+21\,ab^{5}x^{10}+35\,a^{2}b^{4}x^{8}+35\,a^{3}b^{3}x^{6}+21\,a^{4}b^{2}x^{4}+7\,a^{5}bx^{2}+a^{6}}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^15,x, algorithm="maxima")
```

```
[Out] -1/14*(7*b^6*x^12 + 21*a*b^5*x^10 + 35*a^2*b^4*x^8 + 35*a^3*b^3*x^6 + 21*a^
4*b^2*x^4 + 7*a^5*b*x^2 + a^6)/x^14
```

mupad [B] time = 4.36, size = 70, normalized size = 3.68

$$-\frac{\frac{a^{6}}{14}+\frac{a^{5}\,b\,x^{2}}{2}+\frac{3\,a^{4}\,b^{2}\,x^{4}}{2}+\frac{5\,a^{3}\,b^{3}\,x^{6}}{2}+\frac{5\,a^{2}\,b^{4}\,x^{8}}{2}+\frac{3\,a\,b^{5}\,x^{10}}{2}+\frac{b^{6}\,x^{12}}{2}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^15,x)
```

```
[Out] -(a^6/14 + (b^6*x^12)/2 + (a^5*b*x^2)/2 + (3*a*b^5*x^10)/2 + (3*a^4*b^2*x^4
)/2 + (5*a^3*b^3*x^6)/2 + (5*a^2*b^4*x^8)/2)/x^14
```

sympy [B] time = 0.63, size = 73, normalized size = 3.84

$$\frac{-a^{6}-7a^{5}bx^{2}-21a^{4}b^{2}x^{4}-35a^{3}b^{3}x^{6}-35a^{2}b^{4}x^{8}-21ab^{5}x^{10}-7b^{6}x^{12}}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**15,x)
```

```
[Out] (-a**6 - 7*a**5*b*x**2 - 21*a**4*b**2*x**4 - 35*a**3*b**3*x**6 - 35*a**2*b*
*4*x**8 - 21*a*b**5*x**10 - 7*b**6*x**12)/(14*x**14)
```

3.468 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{16}} \, dx$

Optimal. Leaf size=82

$$-\frac{a^6}{15x^{15}}-\frac{6a^5b}{13x^{13}}-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6ab^5}{5x^5}-\frac{b^6}{3x^3}$$

```
[Out] -1/15*a^6/x^15-6/13*a^5*b/x^13-15/11*a^4*b^2/x^11-20/9*a^3*b^3/x^9-15/7*a^2
*b^4/x^7-6/5*a*b^5/x^5-1/3*b^6/x^3
```

Rubi [A] time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6a^5b}{13x^{13}}-\frac{a^6}{15x^{15}}-\frac{6ab^5}{5x^5}-\frac{b^6}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^16,x]
```

```
[Out] -a^6/(15*x^15) - (6*a^5*b)/(13*x^13) - (15*a^4*b^2)/(11*x^11) - (20*a^3*b^3
)/(9*x^9) - (15*a^2*b^4)/(7*x^7) - (6*a*b^5)/(5*x^5) - b^6/(3*x^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{16}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{16}}\,dx}{b^6}$$
$$= \frac{\int \left(\frac{a^6b^6}{x^{16}}+\frac{6a^5b^7}{x^{14}}+\frac{15a^4b^8}{x^{12}}+\frac{20a^3b^9}{x^{10}}+\frac{15a^2b^{10}}{x^8}+\frac{6ab^{11}}{x^6}+\frac{b^{12}}{x^4}\right)dx}{b^6}$$
$$= -\frac{a^6}{15x^{15}}-\frac{6a^5b}{13x^{13}}-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6ab^5}{5x^5}-\frac{b^6}{3x^3}$$

Mathematica [A] time = 0.01, size = 82, normalized size = 1.00

$$-\frac{a^6}{15x^{15}}-\frac{6a^5b}{13x^{13}}-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6ab^5}{5x^5}-\frac{b^6}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^16,x]
```

```
[Out] -1/15*a^6/x^15 - (6*a^5*b)/(13*x^13) - (15*a^4*b^2)/(11*x^11) - (20*a^3*b^3
)/(9*x^9) - (15*a^2*b^4)/(7*x^7) - (6*a*b^5)/(5*x^5) - b^6/(3*x^3)
```

fricas [A] time = 0.74, size = 70, normalized size = 0.85

$$-\frac{15015\,b^6x^{12}+54054\,ab^5x^{10}+96525\,a^2b^4x^8+100100\,a^3b^3x^6+61425\,a^4b^2x^4+20790\,a^5bx^2+3003\,a^6}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(15015*b^6*x^12 + 54054*a*b^5*x^10 + 96525*a^2*b^4*x^8 + 100100*a^
3*b^3*x^6 + 61425*a^4*b^2*x^4 + 20790*a^5*b*x^2 + 3003*a^6)/x^15
```

giac [A] time = 0.15, size = 70, normalized size = 0.85

$$-\frac{15015\,b^6x^{12}+54054\,ab^5x^{10}+96525\,a^2b^4x^8+100100\,a^3b^3x^6+61425\,a^4b^2x^4+20790\,a^5bx^2+3003\,a^6}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^16,x, algorithm="giac")
```

```
[Out] -1/45045*(15015*b^6*x^12 + 54054*a*b^5*x^10 + 96525*a^2*b^4*x^8 + 100100*a^
3*b^3*x^6 + 61425*a^4*b^2*x^4 + 20790*a^5*b*x^2 + 3003*a^6)/x^15
```

maple [A] time = 0.00, size = 69, normalized size = 0.84

$$-\frac{b^{6}}{3x^{3}}-\frac{6a\,b^{5}}{5x^{5}}-\frac{15a^{2}b^{4}}{7x^{7}}-\frac{20a^{3}b^{3}}{9x^{9}}-\frac{15a^{4}b^{2}}{11x^{11}}-\frac{6a^{5}b}{13x^{13}}-\frac{a^{6}}{15x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^16,x)
```

```
[Out] -1/15*a^6/x^15-6/13*a^5*b/x^13-15/11*a^4*b^2/x^11-20/9*a^3*b^3/x^9-15/7*a^2
*b^4/x^7-6/5*a*b^5/x^5-1/3*b^6/x^3
```

maxima [A] time = 1.37, size = 70, normalized size = 0.85

$$-\frac{15015\,b^{6}x^{12}+54054\,ab^{5}x^{10}+96525\,a^{2}b^{4}x^{8}+100100\,a^{3}b^{3}x^{6}+61425\,a^{4}b^{2}x^{4}+20790\,a^{5}bx^{2}+3003\,a^{6}}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^16,x, algorithm="maxima")
```

```
[Out] -1/45045*(15015*b^6*x^12 + 54054*a*b^5*x^10 + 96525*a^2*b^4*x^8 + 100100*a^
3*b^3*x^6 + 61425*a^4*b^2*x^4 + 20790*a^5*b*x^2 + 3003*a^6)/x^15
```

mupad [B] time = 0.05, size = 70, normalized size = 0.85

$$-\frac{\frac{a^{6}}{15}+\frac{6\,a^{5}\,b\,x^{2}}{13}+\frac{15\,a^{4}\,b^{2}\,x^{4}}{11}+\frac{20\,a^{3}\,b^{3}\,x^{6}}{9}+\frac{15\,a^{2}\,b^{4}\,x^{8}}{7}+\frac{6\,a\,b^{5}\,x^{10}}{5}+\frac{b^{6}\,x^{12}}{3}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^16,x)
```

```
[Out] -(a^6/15 + (b^6*x^12)/3 + (6*a^5*b*x^2)/13 + (6*a*b^5*x^10)/5 + (15*a^4*b^2
*x^4)/11 + (20*a^3*b^3*x^6)/9 + (15*a^2*b^4*x^8)/7)/x^15
```

sympy [A] time = 0.62, size = 75, normalized size = 0.91

$$\frac{-3003a^{6}-20790a^{5}bx^{2}-61425a^{4}b^{2}x^{4}-100100a^{3}b^{3}x^{6}-96525a^{2}b^{4}x^{8}-54054ab^{5}x^{10}-15015b^{6}x^{12}}{45045x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**16,x)
```

```
[Out] (-3003*a**6 - 20790*a**5*b*x**2 - 61425*a**4*b**2*x**4 - 100100*a**3*b**3*x
**6 - 96525*a**2*b**4*x**8 - 54054*a*b**5*x**10 - 15015*b**6*x**12)/(45045*
x**15)
```

3.469 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{17}}\,dx$

Optimal. Leaf size=40

$$\frac{b\left(a+bx^2\right)^7}{112a^2x^{14}}-\frac{\left(a+bx^2\right)^7}{16ax^{16}}$$

```
[Out] -1/16*(b*x^2+a)^7/a/x^16+1/112*b*(b*x^2+a)^7/a^2/x^14
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 266, 45, 37}

$$\frac{b\left(a+bx^2\right)^7}{112a^2x^{14}}-\frac{\left(a+bx^2\right)^7}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^17,x]
```

```
[Out] -(a + b*x^2)^7/(16*a*x^16) + (b*(a + b*x^2)^7)/(112*a^2*x^14)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{17}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{17}}\,dx}{b^6}\\ &= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^9}\,dx, x, x^2\right)}{2b^6}\\ &= -\frac{\left(a+bx^2\right)^7}{16ax^{16}} - \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^8}\,dx, x, x^2\right)}{16ab^5}\\ &= -\frac{\left(a+bx^2\right)^7}{16ax^{16}} + \frac{b\left(a+bx^2\right)^7}{112a^2x^{14}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 78, normalized size = 1.95

$$-\frac{a^6}{16x^{16}} - \frac{3a^5b}{7x^{14}} - \frac{5a^4b^2}{4x^{12}} - \frac{2a^3b^3}{x^{10}} - \frac{15a^2b^4}{8x^8} - \frac{ab^5}{x^6} - \frac{b^6}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^17,x]
```

```
[Out] -1/16*a^6/x^16 - (3*a^5*b)/(7*x^14) - (5*a^4*b^2)/(4*x^12) - (2*a^3*b^3)/x^
10 - (15*a^2*b^4)/(8*x^8) - (a*b^5)/x^6 - b^6/(4*x^4)
```

fricas [A] time = 0.97, size = 70, normalized size = 1.75

$$-\frac{28\,b^6x^{12} + 112\,ab^5x^{10} + 210\,a^2b^4x^8 + 224\,a^3b^3x^6 + 140\,a^4b^2x^4 + 48\,a^5bx^2 + 7\,a^6}{112\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^17,x, algorithm="fricas")
```

```
[Out] -1/112*(28*b^6*x^12 + 112*a*b^5*x^10 + 210*a^2*b^4*x^8 + 224*a^3*b^3*x^6 +
140*a^4*b^2*x^4 + 48*a^5*b*x^2 + 7*a^6)/x^16
```

giac [A] time = 0.17, size = 70, normalized size = 1.75

$$-\frac{28\,b^{6}x^{12}+112\,ab^{5}x^{10}+210\,a^{2}b^{4}x^{8}+224\,a^{3}b^{3}x^{6}+140\,a^{4}b^{2}x^{4}+48\,a^{5}bx^{2}+7\,a^{6}}{112\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^17,x, algorithm="giac")
```

```
[Out] -1/112*(28*b^6*x^12 + 112*a*b^5*x^10 + 210*a^2*b^4*x^8 + 224*a^3*b^3*x^6 +
140*a^4*b^2*x^4 + 48*a^5*b*x^2 + 7*a^6)/x^16
```

maple [A] time = 0.01, size = 69, normalized size = 1.72

$$-\frac{b^{6}}{4x^{4}}-\frac{a\,b^{5}}{x^{6}}-\frac{15a^{2}b^{4}}{8x^{8}}-\frac{2a^{3}b^{3}}{x^{10}}-\frac{5a^{4}b^{2}}{4x^{12}}-\frac{3a^{5}b}{7x^{14}}-\frac{a^{6}}{16x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^17,x)
```

```
[Out] -2*a^3*b^3/x^10-1/16*a^6/x^16-5/4*a^4*b^2/x^12-a*b^5/x^6-15/8*a^2*b^4/x^8-3
/7*a^5*b/x^14-1/4*b^6/x^4
```

maxima [A] time = 1.39, size = 70, normalized size = 1.75

$$-\frac{28\,b^{6}x^{12}+112\,ab^{5}x^{10}+210\,a^{2}b^{4}x^{8}+224\,a^{3}b^{3}x^{6}+140\,a^{4}b^{2}x^{4}+48\,a^{5}bx^{2}+7\,a^{6}}{112\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^17,x, algorithm="maxima")
```

```
[Out] -1/112*(28*b^6*x^12 + 112*a*b^5*x^10 + 210*a^2*b^4*x^8 + 224*a^3*b^3*x^6 +
140*a^4*b^2*x^4 + 48*a^5*b*x^2 + 7*a^6)/x^16
```

mupad [B] time = 4.37, size = 69, normalized size = 1.72

$$-\frac{\frac{a^{6}}{16}+\frac{3\,a^{5}\,b\,x^{2}}{7}+\frac{5\,a^{4}\,b^{2}\,x^{4}}{4}+2\,a^{3}\,b^{3}\,x^{6}+\frac{15\,a^{2}\,b^{4}\,x^{8}}{8}+a\,b^{5}\,x^{10}+\frac{b^{6}\,x^{12}}{4}}{x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^17,x)
```

```
[Out] -(a^6/16 + (b^6*x^12)/4 + (3*a^5*b*x^2)/7 + a*b^5*x^10 + (5*a^4*b^2*x^4)/4
+ 2*a^3*b^3*x^6 + (15*a^2*b^4*x^8)/8)/x^16
```

sympy [B] time = 0.68, size = 75, normalized size = 1.88

$$\frac{-7a^6 - 48a^5bx^2 - 140a^4b^2x^4 - 224a^3b^3x^6 - 210a^2b^4x^8 - 112ab^5x^{10} - 28b^6x^{12}}{112x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**17,x)
```

```
[Out] (-7*a**6 - 48*a**5*b*x**2 - 140*a**4*b**2*x**4 - 224*a**3*b**3*x**6 - 210*a
**2*b**4*x**8 - 112*a*b**5*x**10 - 28*b**6*x**12)/(112*x**16)
```

3.470 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{18}}\,dx$

Optimal. Leaf size=82

$$-\frac{a^6}{17x^{17}}-\frac{2a^5b}{5x^{15}}-\frac{15a^4b^2}{13x^{13}}-\frac{20a^3b^3}{11x^{11}}-\frac{5a^2b^4}{3x^9}-\frac{6ab^5}{7x^7}-\frac{b^6}{5x^5}$$

```
[Out] -1/17*a^6/x^17-2/5*a^5*b/x^15-15/13*a^4*b^2/x^13-20/11*a^3*b^3/x^11-5/3*a^2
*b^4/x^9-6/7*a*b^5/x^7-1/5*b^6/x^5
```

Rubi [A] time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{15a^4b^2}{13x^{13}}-\frac{20a^3b^3}{11x^{11}}-\frac{5a^2b^4}{3x^9}-\frac{2a^5b}{5x^{15}}-\frac{a^6}{17x^{17}}-\frac{6ab^5}{7x^7}-\frac{b^6}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^18,x]
```

```
[Out] -a^6/(17*x^17) - (2*a^5*b)/(5*x^15) - (15*a^4*b^2)/(13*x^13) - (20*a^3*b^3)
/(11*x^11) - (5*a^2*b^4)/(3*x^9) - (6*a*b^5)/(7*x^7) - b^6/(5*x^5)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{18}}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^{18}}\,dx}{b^6}$$
$$= \frac{\int \left(\frac{a^6b^6}{x^{18}}+\frac{6a^5b^7}{x^{16}}+\frac{15a^4b^8}{x^{14}}+\frac{20a^3b^9}{x^{12}}+\frac{15a^2b^{10}}{x^{10}}+\frac{6ab^{11}}{x^8}+\frac{b^{12}}{x^6}\right)dx}{b^6}$$
$$= -\frac{a^6}{17x^{17}}-\frac{2a^5b}{5x^{15}}-\frac{15a^4b^2}{13x^{13}}-\frac{20a^3b^3}{11x^{11}}-\frac{5a^2b^4}{3x^9}-\frac{6ab^5}{7x^7}-\frac{b^6}{5x^5}$$

Mathematica [A] time = 0.01, size = 82, normalized size = 1.00

$$-\frac{a^6}{17x^{17}}-\frac{2a^5b}{5x^{15}}-\frac{15a^4b^2}{13x^{13}}-\frac{20a^3b^3}{11x^{11}}-\frac{5a^2b^4}{3x^9}-\frac{6ab^5}{7x^7}-\frac{b^6}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^18,x]
```

```
[Out] -1/17*a^6/x^17 - (2*a^5*b)/(5*x^15) - (15*a^4*b^2)/(13*x^13) - (20*a^3*b^3)
/(11*x^11) - (5*a^2*b^4)/(3*x^9) - (6*a*b^5)/(7*x^7) - b^6/(5*x^5)
```

fricas [A] time = 0.54, size = 70, normalized size = 0.85

$$-\frac{51051\,b^6x^{12}+218790\,ab^5x^{10}+425425\,a^2b^4x^8+464100\,a^3b^3x^6+294525\,a^4b^2x^4+102102\,a^5bx^2+15015\,a^6}{255255\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^18,x, algorithm="fricas")
```

```
[Out] -1/255255*(51051*b^6*x^12 + 218790*a*b^5*x^10 + 425425*a^2*b^4*x^8 + 464100
*a^3*b^3*x^6 + 294525*a^4*b^2*x^4 + 102102*a^5*b*x^2 + 15015*a^6)/x^17
```

giac [A] time = 0.17, size = 70, normalized size = 0.85

$$-\frac{51051\,b^6x^{12}+218790\,ab^5x^{10}+425425\,a^2b^4x^8+464100\,a^3b^3x^6+294525\,a^4b^2x^4+102102\,a^5bx^2+15015\,a^6}{255255\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^18,x, algorithm="giac")
```

```
[Out] -1/255255*(51051*b^6*x^12 + 218790*a*b^5*x^10 + 425425*a^2*b^4*x^8 + 464100
*a^3*b^3*x^6 + 294525*a^4*b^2*x^4 + 102102*a^5*b*x^2 + 15015*a^6)/x^17
```

maple [A] time = 0.01, size = 69, normalized size = 0.84

$$-\frac{b^{6}}{5x^{5}}-\frac{6a\,b^{5}}{7x^{7}}-\frac{5a^{2}b^{4}}{3x^{9}}-\frac{20a^{3}b^{3}}{11x^{11}}-\frac{15a^{4}b^{2}}{13x^{13}}-\frac{2a^{5}b}{5x^{15}}-\frac{a^{6}}{17x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^18,x)
```

```
[Out] -1/17*a^6/x^17-2/5*a^5*b/x^15-15/13*a^4*b^2/x^13-20/11*a^3*b^3/x^11-5/3*a^2
*b^4/x^9-6/7*a*b^5/x^7-1/5*b^6/x^5
```

maxima [A] time = 1.37, size = 70, normalized size = 0.85

$$-\frac{51051\,b^{6}x^{12}+218790\,ab^{5}x^{10}+425425\,a^{2}b^{4}x^{8}+464100\,a^{3}b^{3}x^{6}+294525\,a^{4}b^{2}x^{4}+102102\,a^{5}bx^{2}+15015\,a^{6}}{255255\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^18,x, algorithm="maxima")
```

```
[Out] -1/255255*(51051*b^6*x^12 + 218790*a*b^5*x^10 + 425425*a^2*b^4*x^8 + 464100
*a^3*b^3*x^6 + 294525*a^4*b^2*x^4 + 102102*a^5*b*x^2 + 15015*a^6)/x^17
```

mupad [B] time = 0.05, size = 70, normalized size = 0.85

$$-\frac{\frac{a^{6}}{17}+\frac{2\,a^{5}\,b\,x^{2}}{5}+\frac{15\,a^{4}\,b^{2}\,x^{4}}{13}+\frac{20\,a^{3}\,b^{3}\,x^{6}}{11}+\frac{5\,a^{2}\,b^{4}\,x^{8}}{3}+\frac{6\,a\,b^{5}\,x^{10}}{7}+\frac{b^{6}\,x^{12}}{5}}{x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^18,x)
```

```
[Out] -(a^6/17 + (b^6*x^12)/5 + (2*a^5*b*x^2)/5 + (6*a*b^5*x^10)/7 + (15*a^4*b^2*
x^4)/13 + (20*a^3*b^3*x^6)/11 + (5*a^2*b^4*x^8)/3)/x^17
```

sympy [A] time = 0.66, size = 75, normalized size = 0.91

$$\frac{-15015a^{6}-102102a^{5}bx^{2}-294525a^{4}b^{2}x^{4}-464100a^{3}b^{3}x^{6}-425425a^{2}b^{4}x^{8}-218790ab^{5}x^{10}-51051b^{6}x^{12}}{255255x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**18,x)
```

```
[Out] (-15015*a**6 - 102102*a**5*b*x**2 - 294525*a**4*b**2*x**4 - 464100*a**3*b**
3*x**6 - 425425*a**2*b**4*x**8 - 218790*a*b**5*x**10 - 51051*b**6*x**12)/(2
55255*x**17)
```

3.471 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{19}}\,dx$

Optimal. Leaf size=62

$$-\frac{b^2\left(a+bx^2\right)^7}{504a^3x^{14}}+\frac{b\left(a+bx^2\right)^7}{72a^2x^{16}}-\frac{\left(a+bx^2\right)^7}{18ax^{18}}$$

```
[Out] -1/18*(b*x^2+a)^7/a/x^18+1/72*b*(b*x^2+a)^7/a^2/x^16-1/504*b^2*(b*x^2+a)^7/
a^3/x^14
```

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 266, 45, 37}

$$-\frac{b^2\left(a+bx^2\right)^7}{504a^3x^{14}}+\frac{b\left(a+bx^2\right)^7}{72a^2x^{16}}-\frac{\left(a+bx^2\right)^7}{18ax^{18}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^19,x]
```

```
[Out] -(a + b*x^2)^7/(18*a*x^18) + (b*(a + b*x^2)^7)/(72*a^2*x^16) - (b^2*(a + b*
x^2)^7)/(504*a^3*x^14)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
```

```
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{19}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{19}}\,dx}{b^6}\\
&= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^{10}}\,dx,x,x^2\right)}{2b^6}\\
&= -\frac{\left(a+bx^2\right)^7}{18ax^{18}}-\frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^9}\,dx,x,x^2\right)}{9ab^5}\\
&= -\frac{\left(a+bx^2\right)^7}{18ax^{18}}+\frac{b\left(a+bx^2\right)^7}{72a^2x^{16}}+\frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^8}\,dx,x,x^2\right)}{72a^2b^4}\\
&= -\frac{\left(a+bx^2\right)^7}{18ax^{18}}+\frac{b\left(a+bx^2\right)^7}{72a^2x^{16}}-\frac{b^2\left(a+bx^2\right)^7}{504a^3x^{14}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 82, normalized size = 1.32

$$-\frac{a^6}{18x^{18}}-\frac{3a^5b}{8x^{16}}-\frac{15a^4b^2}{14x^{14}}-\frac{5a^3b^3}{3x^{12}}-\frac{3a^2b^4}{2x^{10}}-\frac{3ab^5}{4x^8}-\frac{b^6}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^19,x]
```

```
[Out] -1/18*a^6/x^18 - (3*a^5*b)/(8*x^16) - (15*a^4*b^2)/(14*x^14) - (5*a^3*b^3)/
(3*x^12) - (3*a^2*b^4)/(2*x^10) - (3*a*b^5)/(4*x^8) - b^6/(6*x^6)
```

fricas [A] time = 0.86, size = 70, normalized size = 1.13

$$-\frac{84\,b^6x^{12}+378\,ab^5x^{10}+756\,a^2b^4x^8+840\,a^3b^3x^6+540\,a^4b^2x^4+189\,a^5bx^2+28\,a^6}{504\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^19,x, algorithm="fricas")
```

```
[Out] -1/504*(84*b^6*x^12 + 378*a*b^5*x^10 + 756*a^2*b^4*x^8 + 840*a^3*b^3*x^6 +
540*a^4*b^2*x^4 + 189*a^5*b*x^2 + 28*a^6)/x^18
```

giac [A] time = 0.17, size = 70, normalized size = 1.13

$$-\frac{84\,b^{6}x^{12}+378\,ab^{5}x^{10}+756\,a^{2}b^{4}x^{8}+840\,a^{3}b^{3}x^{6}+540\,a^{4}b^{2}x^{4}+189\,a^{5}bx^{2}+28\,a^{6}}{504\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^19,x, algorithm="giac")
```

```
[Out] -1/504*(84*b^6*x^12 + 378*a*b^5*x^10 + 756*a^2*b^4*x^8 + 840*a^3*b^3*x^6 +
540*a^4*b^2*x^4 + 189*a^5*b*x^2 + 28*a^6)/x^18
```

maple [A] time = 0.01, size = 69, normalized size = 1.11

$$-\frac{b^{6}}{6x^{6}}-\frac{3a\,b^{5}}{4x^{8}}-\frac{3a^{2}b^{4}}{2x^{10}}-\frac{5a^{3}b^{3}}{3x^{12}}-\frac{15a^{4}b^{2}}{14x^{14}}-\frac{3a^{5}b}{8x^{16}}-\frac{a^{6}}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^19,x)
```

```
[Out] -3/2*a^2*b^4/x^10-1/6*b^6/x^6-15/14*a^4*b^2/x^14-3/4*a*b^5/x^8-3/8*a^5*b/x^
16-1/18*a^6/x^18-5/3*a^3*b^3/x^12
```

maxima [A] time = 1.34, size = 70, normalized size = 1.13

$$-\frac{84\,b^{6}x^{12}+378\,ab^{5}x^{10}+756\,a^{2}b^{4}x^{8}+840\,a^{3}b^{3}x^{6}+540\,a^{4}b^{2}x^{4}+189\,a^{5}bx^{2}+28\,a^{6}}{504\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^19,x, algorithm="maxima")
```

```
[Out] -1/504*(84*b^6*x^12 + 378*a*b^5*x^10 + 756*a^2*b^4*x^8 + 840*a^3*b^3*x^6 +
540*a^4*b^2*x^4 + 189*a^5*b*x^2 + 28*a^6)/x^18
```

mupad [B] time = 4.31, size = 70, normalized size = 1.13

$$-\frac{\frac{a^6}{18}+\frac{3\,a^5\,b\,x^2}{8}+\frac{15\,a^4\,b^2\,x^4}{14}+\frac{5\,a^3\,b^3\,x^6}{3}+\frac{3\,a^2\,b^4\,x^8}{2}+\frac{3\,a\,b^5\,x^{10}}{4}+\frac{b^6\,x^{12}}{6}}{x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^19,x)
```

```
[Out] -(a^6/18 + (b^6*x^12)/6 + (3*a^5*b*x^2)/8 + (3*a*b^5*x^10)/4 + (15*a^4*b^2*
x^4)/14 + (5*a^3*b^3*x^6)/3 + (3*a^2*b^4*x^8)/2)/x^18
```

sympy [A] time = 0.74, size = 75, normalized size = 1.21

$$\frac{-28a^6 - 189a^5bx^2 - 540a^4b^2x^4 - 840a^3b^3x^6 - 756a^2b^4x^8 - 378ab^5x^{10} - 84b^6x^{12}}{504x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**19,x)
```

```
[Out] (-28*a**6 - 189*a**5*b*x**2 - 540*a**4*b**2*x**4 - 840*a**3*b**3*x**6 - 756
*a**2*b**4*x**8 - 378*a*b**5*x**10 - 84*b**6*x**12)/(504*x**18)
```

3.472 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{20}}\,dx$

Optimal. Leaf size=80

$$-\frac{a^6}{19x^{19}}-\frac{6a^5b}{17x^{17}}-\frac{a^4b^2}{x^{15}}-\frac{20a^3b^3}{13x^{13}}-\frac{15a^2b^4}{11x^{11}}-\frac{2ab^5}{3x^9}-\frac{b^6}{7x^7}$$

```
[Out] -1/19*a^6/x^19-6/17*a^5*b/x^17-a^4*b^2/x^15-20/13*a^3*b^3/x^13-15/11*a^2*b^
4/x^11-2/3*a*b^5/x^9-1/7*b^6/x^7
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{a^4b^2}{x^{15}}-\frac{20a^3b^3}{13x^{13}}-\frac{15a^2b^4}{11x^{11}}-\frac{6a^5b}{17x^{17}}-\frac{a^6}{19x^{19}}-\frac{2ab^5}{3x^9}-\frac{b^6}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^20,x]
```

```
[Out] -a^6/(19*x^19) - (6*a^5*b)/(17*x^17) - (a^4*b^2)/x^15 - (20*a^3*b^3)/(13*x^
13) - (15*a^2*b^4)/(11*x^11) - (2*a*b^5)/(3*x^9) - b^6/(7*x^7)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{20}}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^{20}}\,dx}{b^6}$$
$$= \frac{\int \left(\frac{a^6b^6}{x^{20}} + \frac{6a^5b^7}{x^{18}} + \frac{15a^4b^8}{x^{16}} + \frac{20a^3b^9}{x^{14}} + \frac{15a^2b^{10}}{x^{12}} + \frac{6ab^{11}}{x^{10}} + \frac{b^{12}}{x^8}\right)dx}{b^6}$$
$$= -\frac{a^6}{19x^{19}} - \frac{6a^5b}{17x^{17}} - \frac{a^4b^2}{x^{15}} - \frac{20a^3b^3}{13x^{13}} - \frac{15a^2b^4}{11x^{11}} - \frac{2ab^5}{3x^9} - \frac{b^6}{7x^7}$$

Mathematica [A] time = 0.01, size = 80, normalized size = 1.00

$$-\frac{a^6}{19x^{19}} - \frac{6a^5b}{17x^{17}} - \frac{a^4b^2}{x^{15}} - \frac{20a^3b^3}{13x^{13}} - \frac{15a^2b^4}{11x^{11}} - \frac{2ab^5}{3x^9} - \frac{b^6}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^20,x]
```

```
[Out] -1/19*a^6/x^19 - (6*a^5*b)/(17*x^17) - (a^4*b^2)/x^15 - (20*a^3*b^3)/(13*x^
13) - (15*a^2*b^4)/(11*x^11) - (2*a*b^5)/(3*x^9) - b^6/(7*x^7)
```

fricas [A] time = 0.78, size = 70, normalized size = 0.88

$$-\frac{138567\,b^6x^{12} + 646646\,ab^5x^{10} + 1322685\,a^2b^4x^8 + 1492260\,a^3b^3x^6 + 969969\,a^4b^2x^4 + 342342\,a^5bx^2 + 51051}{969969\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^20,x, algorithm="fricas")
```

```
[Out] -1/969969*(138567*b^6*x^12 + 646646*a*b^5*x^10 + 1322685*a^2*b^4*x^8 + 1492
260*a^3*b^3*x^6 + 969969*a^4*b^2*x^4 + 342342*a^5*b*x^2 + 51051*a^6)/x^19
```

giac [A] time = 0.18, size = 70, normalized size = 0.88

$$-\frac{138567\,b^6x^{12} + 646646\,ab^5x^{10} + 1322685\,a^2b^4x^8 + 1492260\,a^3b^3x^6 + 969969\,a^4b^2x^4 + 342342\,a^5bx^2 + 51051}{969969\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^20,x, algorithm="giac")
```

```
[Out] -1/969969*(138567*b^6*x^12 + 646646*a*b^5*x^10 + 1322685*a^2*b^4*x^8 + 1492
260*a^3*b^3*x^6 + 969969*a^4*b^2*x^4 + 342342*a^5*b*x^2 + 51051*a^6)/x^19
```

maple [A] time = 0.01, size = 69, normalized size = 0.86

$$-\frac{b^6}{7x^7} - \frac{2a\,b^5}{3x^9} - \frac{15a^2b^4}{11x^{11}} - \frac{20a^3b^3}{13x^{13}} - \frac{a^4b^2}{x^{15}} - \frac{6a^5b}{17x^{17}} - \frac{a^6}{19x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^20,x)
```

```
[Out] -1/19*a^6/x^19-6/17*a^5*b/x^17-a^4*b^2/x^15-20/13*a^3*b^3/x^13-15/11*a^2*b^
4/x^11-2/3*a*b^5/x^9-1/7*b^6/x^7
```

maxima [A] time = 1.35, size = 70, normalized size = 0.88

$$-\frac{138567\,b^6x^{12} + 646646\,ab^5x^{10} + 1322685\,a^2b^4x^8 + 1492260\,a^3b^3x^6 + 969969\,a^4b^2x^4 + 342342\,a^5bx^2 + 51051\,a}{969969\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^20,x, algorithm="maxima")
```

```
[Out] -1/969969*(138567*b^6*x^12 + 646646*a*b^5*x^10 + 1322685*a^2*b^4*x^8 + 1492
260*a^3*b^3*x^6 + 969969*a^4*b^2*x^4 + 342342*a^5*b*x^2 + 51051*a^6)/x^19
```

mupad [B] time = 0.05, size = 69, normalized size = 0.86

$$-\frac{\frac{a^6}{19} + \frac{6\,a^5\,b\,x^2}{17} + a^4\,b^2\,x^4 + \frac{20\,a^3\,b^3\,x^6}{13} + \frac{15\,a^2\,b^4\,x^8}{11} + \frac{2\,a\,b^5\,x^{10}}{3} + \frac{b^6\,x^{12}}{7}}{x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^20,x)
```

```
[Out] -(a^6/19 + (b^6*x^12)/7 + (6*a^5*b*x^2)/17 + (2*a*b^5*x^10)/3 + a^4*b^2*x^4
 + (20*a^3*b^3*x^6)/13 + (15*a^2*b^4*x^8)/11)/x^19
```

sympy [A] time = 0.71, size = 75, normalized size = 0.94

$$\frac{-51051a^6 - 342342a^5bx^2 - 969969a^4b^2x^4 - 1492260a^3b^3x^6 - 1322685a^2b^4x^8 - 646646ab^5x^{10} - 138567b^6x^{12}}{969969x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**20,x)
```

```
[Out] (-51051*a**6 - 342342*a**5*b*x**2 - 969969*a**4*b**2*x**4 - 1492260*a**3*b*
*3*x**6 - 1322685*a**2*b**4*x**8 - 646646*a*b**5*x**10 - 138567*b**6*x**12)
/(969969*x**19)
```

3.473 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{21}}\,dx$

Optimal. Leaf size=84

$$\frac{b^3\left(a+bx^2\right)^7}{1680a^4x^{14}}-\frac{b^2\left(a+bx^2\right)^7}{240a^3x^{16}}+\frac{b\left(a+bx^2\right)^7}{60a^2x^{18}}-\frac{\left(a+bx^2\right)^7}{20ax^{20}}$$

```
[Out] -1/20*(b*x^2+a)^7/a/x^20+1/60*b*(b*x^2+a)^7/a^2/x^18-1/240*b^2*(b*x^2+a)^7/
a^3/x^16+1/1680*b^3*(b*x^2+a)^7/a^4/x^14
```

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 266, 45, 37}

$$\frac{b^3\left(a+bx^2\right)^7}{1680a^4x^{14}}-\frac{b^2\left(a+bx^2\right)^7}{240a^3x^{16}}+\frac{b\left(a+bx^2\right)^7}{60a^2x^{18}}-\frac{\left(a+bx^2\right)^7}{20ax^{20}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^21,x]
```

```
[Out] -(a + b*x^2)^7/(20*a*x^20) + (b*(a + b*x^2)^7)/(60*a^2*x^18) - (b^2*(a + b*
x^2)^7)/(240*a^3*x^16) + (b^3*(a + b*x^2)^7)/(1680*a^4*x^14)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
```

```
(EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{21}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{21}}\,dx}{b^6}\\
&= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^{11}}\,dx,x,x^2\right)}{2b^6}\\
&= -\frac{\left(a+bx^2\right)^7}{20ax^{20}}-\frac{3\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^{10}}\,dx,x,x^2\right)}{20ab^5}\\
&= -\frac{\left(a+bx^2\right)^7}{20ax^{20}}+\frac{b\left(a+bx^2\right)^7}{60a^2x^{18}}+\frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^9}\,dx,x,x^2\right)}{30a^2b^4}\\
&= -\frac{\left(a+bx^2\right)^7}{20ax^{20}}+\frac{b\left(a+bx^2\right)^7}{60a^2x^{18}}-\frac{b^2\left(a+bx^2\right)^7}{240a^3x^{16}}-\frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^8}\,dx,x,x^2\right)}{240a^3b^3}\\
&= -\frac{\left(a+bx^2\right)^7}{20ax^{20}}+\frac{b\left(a+bx^2\right)^7}{60a^2x^{18}}-\frac{b^2\left(a+bx^2\right)^7}{240a^3x^{16}}+\frac{b^3\left(a+bx^2\right)^7}{1680a^4x^{14}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 82, normalized size = 0.98

$$-\frac{a^6}{20x^{20}}-\frac{a^5b}{3x^{18}}-\frac{15a^4b^2}{16x^{16}}-\frac{10a^3b^3}{7x^{14}}-\frac{5a^2b^4}{4x^{12}}-\frac{3ab^5}{5x^{10}}-\frac{b^6}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^21,x]
```

```
[Out] -1/20*a^6/x^20 - (a^5*b)/(3*x^18) - (15*a^4*b^2)/(16*x^16) - (10*a^3*b^3)/(
7*x^14) - (5*a^2*b^4)/(4*x^12) - (3*a*b^5)/(5*x^10) - b^6/(8*x^8)
```

fricas [A] time = 0.97, size = 70, normalized size = 0.83

$$-\frac{210\, b^{6} x^{12} + 1008\, a b^{5} x^{10} + 2100\, a^{2} b^{4} x^{8} + 2400\, a^{3} b^{3} x^{6} + 1575\, a^{4} b^{2} x^{4} + 560\, a^{5} b x^{2} + 84\, a^{6}}{1680\, x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^21,x, algorithm="fricas")
```

```
[Out] -1/1680*(210*b^6*x^12 + 1008*a*b^5*x^10 + 2100*a^2*b^4*x^8 + 2400*a^3*b^3*x
^6 + 1575*a^4*b^2*x^4 + 560*a^5*b*x^2 + 84*a^6)/x^20
```

giac [A] time = 0.15, size = 70, normalized size = 0.83

$$-\frac{210\, b^{6} x^{12} + 1008\, a b^{5} x^{10} + 2100\, a^{2} b^{4} x^{8} + 2400\, a^{3} b^{3} x^{6} + 1575\, a^{4} b^{2} x^{4} + 560\, a^{5} b x^{2} + 84\, a^{6}}{1680\, x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^21,x, algorithm="giac")
```

```
[Out] -1/1680*(210*b^6*x^12 + 1008*a*b^5*x^10 + 2100*a^2*b^4*x^8 + 2400*a^3*b^3*x
^6 + 1575*a^4*b^2*x^4 + 560*a^5*b*x^2 + 84*a^6)/x^20
```

maple [A] time = 0.01, size = 69, normalized size = 0.82

$$-\frac{b^{6}}{8x^{8}}-\frac{3a\, b^{5}}{5x^{10}}-\frac{5a^{2} b^{4}}{4x^{12}}-\frac{10a^{3} b^{3}}{7x^{14}}-\frac{15a^{4} b^{2}}{16x^{16}}-\frac{a^{5} b}{3x^{18}}-\frac{a^{6}}{20x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^21,x)
```

```
[Out] -3/5*a*b^5/x^10-1/3*a^5*b/x^18-1/20*a^6/x^20-1/8*b^6/x^8-15/16*a^4*b^2/x^16
-10/7*a^3*b^3/x^14-5/4*a^2*b^4/x^12
```

maxima [A] time = 1.42, size = 70, normalized size = 0.83

$$-\frac{210\, b^{6} x^{12} + 1008\, a b^{5} x^{10} + 2100\, a^{2} b^{4} x^{8} + 2400\, a^{3} b^{3} x^{6} + 1575\, a^{4} b^{2} x^{4} + 560\, a^{5} b x^{2} + 84\, a^{6}}{1680\, x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^21,x, algorithm="maxima")
```

```
[Out] -1/1680*(210*b^6*x^12 + 1008*a*b^5*x^10 + 2100*a^2*b^4*x^8 + 2400*a^3*b^3*x
^6 + 1575*a^4*b^2*x^4 + 560*a^5*b*x^2 + 84*a^6)/x^20
```

mupad [B] time = 0.05, size = 70, normalized size = 0.83

$$-\frac{\frac{a^6}{20}+\frac{a^5\,b\,x^2}{3}+\frac{15\,a^4\,b^2\,x^4}{16}+\frac{10\,a^3\,b^3\,x^6}{7}+\frac{5\,a^2\,b^4\,x^8}{4}+\frac{3\,a\,b^5\,x^{10}}{5}+\frac{b^6\,x^{12}}{8}}{x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^21,x)
```

```
[Out] -(a^6/20 + (b^6*x^12)/8 + (a^5*b*x^2)/3 + (3*a*b^5*x^10)/5 + (15*a^4*b^2*x^
4)/16 + (10*a^3*b^3*x^6)/7 + (5*a^2*b^4*x^8)/4)/x^20
```

sympy [A] time = 0.86, size = 75, normalized size = 0.89

$$\frac{-84a^6-560a^5bx^2-1575a^4b^2x^4-2400a^3b^3x^6-2100a^2b^4x^8-1008ab^5x^{10}-210b^6x^{12}}{1680x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**21,x)
```

```
[Out] (-84*a**6 - 560*a**5*b*x**2 - 1575*a**4*b**2*x**4 - 2400*a**3*b**3*x**6 - 2
100*a**2*b**4*x**8 - 1008*a*b**5*x**10 - 210*b**6*x**12)/(1680*x**20)
```

3.474 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{22}}\,dx$

Optimal. Leaf size=82

$$-\frac{a^6}{21x^{21}}-\frac{6a^5b}{19x^{19}}-\frac{15a^4b^2}{17x^{17}}-\frac{4a^3b^3}{3x^{15}}-\frac{15a^2b^4}{13x^{13}}-\frac{6ab^5}{11x^{11}}-\frac{b^6}{9x^9}$$

```
[Out] -1/21*a^6/x^21-6/19*a^5*b/x^19-15/17*a^4*b^2/x^17-4/3*a^3*b^3/x^15-15/13*a^
2*b^4/x^13-6/11*a*b^5/x^11-1/9*b^6/x^9
```

Rubi [A] time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{15a^4b^2}{17x^{17}}-\frac{4a^3b^3}{3x^{15}}-\frac{15a^2b^4}{13x^{13}}-\frac{6a^5b}{19x^{19}}-\frac{a^6}{21x^{21}}-\frac{6ab^5}{11x^{11}}-\frac{b^6}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^22,x]
```

```
[Out] -a^6/(21*x^21) - (6*a^5*b)/(19*x^19) - (15*a^4*b^2)/(17*x^17) - (4*a^3*b^3)
/(3*x^15) - (15*a^2*b^4)/(13*x^13) - (6*a*b^5)/(11*x^11) - b^6/(9*x^9)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{22}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{22}}\,dx}{b^6}$$
$$= \frac{\int \left(\frac{a^6b^6}{x^{22}}+\frac{6a^5b^7}{x^{20}}+\frac{15a^4b^8}{x^{18}}+\frac{20a^3b^9}{x^{16}}+\frac{15a^2b^{10}}{x^{14}}+\frac{6ab^{11}}{x^{12}}+\frac{b^{12}}{x^{10}}\right)dx}{b^6}$$
$$= -\frac{a^6}{21x^{21}}-\frac{6a^5b}{19x^{19}}-\frac{15a^4b^2}{17x^{17}}-\frac{4a^3b^3}{3x^{15}}-\frac{15a^2b^4}{13x^{13}}-\frac{6ab^5}{11x^{11}}-\frac{b^6}{9x^9}$$

Mathematica [A] time = 0.01, size = 82, normalized size = 1.00

$$-\frac{a^6}{21x^{21}}-\frac{6a^5b}{19x^{19}}-\frac{15a^4b^2}{17x^{17}}-\frac{4a^3b^3}{3x^{15}}-\frac{15a^2b^4}{13x^{13}}-\frac{6ab^5}{11x^{11}}-\frac{b^6}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^22,x]
```

```
[Out] -1/21*a^6/x^21 - (6*a^5*b)/(19*x^19) - (15*a^4*b^2)/(17*x^17) - (4*a^3*b^3)
/(3*x^15) - (15*a^2*b^4)/(13*x^13) - (6*a*b^5)/(11*x^11) - b^6/(9*x^9)
```

fricas [A] time = 0.92, size = 70, normalized size = 0.85

$$-\frac{323323\,b^6x^{12}+1587222\,ab^5x^{10}+3357585\,a^2b^4x^8+3879876\,a^3b^3x^6+2567565\,a^4b^2x^4+918918\,a^5bx^2+1385}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^22,x, algorithm="fricas")
```

```
[Out] -1/2909907*(323323*b^6*x^12 + 1587222*a*b^5*x^10 + 3357585*a^2*b^4*x^8 + 38
79876*a^3*b^3*x^6 + 2567565*a^4*b^2*x^4 + 918918*a^5*b*x^2 + 138567*a^6)/x^
21
```

giac [A] time = 0.17, size = 70, normalized size = 0.85

$$-\frac{323323\,b^6x^{12}+1587222\,ab^5x^{10}+3357585\,a^2b^4x^8+3879876\,a^3b^3x^6+2567565\,a^4b^2x^4+918918\,a^5bx^2+1385}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^22,x, algorithm="giac")
```

```
[Out] -1/2909907*(323323*b^6*x^12 + 1587222*a*b^5*x^10 + 3357585*a^2*b^4*x^8 + 38
79876*a^3*b^3*x^6 + 2567565*a^4*b^2*x^4 + 918918*a^5*b*x^2 + 138567*a^6)/x^
21
```

maple [A] time = 0.01, size = 69, normalized size = 0.84

$$-\frac{b^6}{9x^9}-\frac{6a\,b^5}{11x^{11}}-\frac{15a^2b^4}{13x^{13}}-\frac{4a^3b^3}{3x^{15}}-\frac{15a^4b^2}{17x^{17}}-\frac{6a^5b}{19x^{19}}-\frac{a^6}{21x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^22,x)
```

```
[Out] -1/21*a^6/x^21-6/19*a^5*b/x^19-15/17*a^4*b^2/x^17-4/3*a^3*b^3/x^15-15/13*a^
2*b^4/x^13-6/11*a*b^5/x^11-1/9*b^6/x^9
```

maxima [A] time = 1.34, size = 70, normalized size = 0.85

$$-\frac{323323\,b^6x^{12}+1587222\,ab^5x^{10}+3357585\,a^2b^4x^8+3879876\,a^3b^3x^6+2567565\,a^4b^2x^4+918918\,a^5bx^2+138}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^22,x, algorithm="maxima")
```

```
[Out] -1/2909907*(323323*b^6*x^12 + 1587222*a*b^5*x^10 + 3357585*a^2*b^4*x^8 + 38
79876*a^3*b^3*x^6 + 2567565*a^4*b^2*x^4 + 918918*a^5*b*x^2 + 138567*a^6)/x^
21
```

mupad [B] time = 0.05, size = 70, normalized size = 0.85

$$-\frac{\frac{a^6}{21}+\frac{6\,a^5\,b\,x^2}{19}+\frac{15\,a^4\,b^2\,x^4}{17}+\frac{4\,a^3\,b^3\,x^6}{3}+\frac{15\,a^2\,b^4\,x^8}{13}+\frac{6\,a\,b^5\,x^{10}}{11}+\frac{b^6\,x^{12}}{9}}{x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^22,x)
```

```
[Out] -(a^6/21 + (b^6*x^12)/9 + (6*a^5*b*x^2)/19 + (6*a*b^5*x^10)/11 + (15*a^4*b^
2*x^4)/17 + (4*a^3*b^3*x^6)/3 + (15*a^2*b^4*x^8)/13)/x^21
```

sympy [A] time = 0.76, size = 75, normalized size = 0.91

$$\frac{-138567a^6-918918a^5bx^2-2567565a^4b^2x^4-3879876a^3b^3x^6-3357585a^2b^4x^8-1587222ab^5x^{10}-323323b^6x}{2909907x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**22,x)
```

```
[Out] (-138567*a**6 - 918918*a**5*b*x**2 - 2567565*a**4*b**2*x**4 - 3879876*a**3*
b**3*x**6 - 3357585*a**2*b**4*x**8 - 1587222*a*b**5*x**10 - 323323*b**6*x**
12)/(2909907*x**21)
```

3.475 $\int \frac{x^{11}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=83

$$\frac{a^5}{2b^6\left(a+bx^2\right)}+\frac{5a^4\log\left(a+bx^2\right)}{2b^6}-\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}$$

```
[Out] -2*a^3*x^2/b^5+3/4*a^2*x^4/b^4-1/3*a*x^6/b^3+1/8*x^8/b^2+1/2*a^5/b^6/(b*x^2
+a)+5/2*a^4*ln(b*x^2+a)/b^6
```

Rubi [A] time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{3a^2x^4}{4b^4}-\frac{2a^3x^2}{b^5}+\frac{a^5}{2b^6\left(a+bx^2\right)}+\frac{5a^4\log\left(a+bx^2\right)}{2b^6}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-2*a^3*x^2)/b^5 + (3*a^2*x^4)/(4*b^4) - (a*x^6)/(3*b^3) + x^8/(8*b^2) + a^
5/(2*b^6*(a + b*x^2)) + (5*a^4*Log[a + b*x^2])/(2*b^6)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{x^{11}}{\left(ab+b^2x^2\right)^2}\,dx$$
$$= \frac{1}{2}b^2\,\text{Subst}\left(\int \frac{x^5}{\left(ab+b^2x\right)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^2\,\text{Subst}\left(\int \left(-\frac{4a^3}{b^7}+\frac{3a^2x}{b^6}-\frac{2ax^2}{b^5}+\frac{x^3}{b^4}-\frac{a^5}{b^7(a+bx)^2}+\frac{5a^4}{b^7(a+bx)}\right)dx, x, x^2\right)$$
$$= -\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}+\frac{a^5}{2b^6\left(a+bx^2\right)}+\frac{5a^4\log\left(a+bx^2\right)}{2b^6}$$

Mathematica [A] time = 0.02, size = 72, normalized size = 0.87

$$\frac{\frac{12a^5}{a+bx^2}+60a^4\log\left(a+bx^2\right)-48a^3bx^2+18a^2b^2x^4-8ab^3x^6+3b^4x^8}{24b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-48*a^3*b*x^2 + 18*a^2*b^2*x^4 - 8*a*b^3*x^6 + 3*b^4*x^8 + (12*a^5)/(a + b
*x^2) + 60*a^4*Log[a + b*x^2])/(24*b^6)
```

fricas [A] time = 1.04, size = 93, normalized size = 1.12

$$\frac{3\,b^5x^{10}-5\,ab^4x^8+10\,a^2b^3x^6-30\,a^3b^2x^4-48\,a^4bx^2+12\,a^5+60\left(a^4bx^2+a^5\right)\log\left(bx^2+a\right)}{24\left(b^7x^2+ab^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^5*x^10 - 5*a*b^4*x^8 + 10*a^2*b^3*x^6 - 30*a^3*b^2*x^4 - 48*a^4*b
*x^2 + 12*a^5 + 60*(a^4*b*x^2 + a^5)*log(b*x^2 + a))/(b^7*x^2 + a*b^6)
```

giac [A] time = 0.16, size = 92, normalized size = 1.11

$$\frac{5\,a^4\log\left(\left|bx^2+a\right|\right)}{2\,b^6}-\frac{5\,a^4bx^2+4\,a^5}{2\left(bx^2+a\right)b^6}+\frac{3\,b^6x^8-8\,ab^5x^6+18\,a^2b^4x^4-48\,a^3b^3x^2}{24\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 5/2*a^4*log(abs(b*x^2 + a))/b^6 - 1/2*(5*a^4*b*x^2 + 4*a^5)/((b*x^2 + a)*b^
6) + 1/24*(3*b^6*x^8 - 8*a*b^5*x^6 + 18*a^2*b^4*x^4 - 48*a^3*b^3*x^2)/b^8
```

maple [A] time = 0.01, size = 74, normalized size = 0.89

$$\frac{x^8}{8b^2}-\frac{a\,x^6}{3b^3}+\frac{3a^2x^4}{4b^4}-\frac{2a^3x^2}{b^5}+\frac{a^5}{2\left(b\,x^2+a\right)b^6}+\frac{5a^4\ln\left(b\,x^2+a\right)}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -2*a^3*x^2/b^5+3/4*a^2*x^4/b^4-1/3*a*x^6/b^3+1/8*x^8/b^2+1/2*a^5/b^6/(b*x^2
+a)+5/2*a^4*ln(b*x^2+a)/b^6
```

maxima [A] time = 1.37, size = 77, normalized size = 0.93

$$\frac{a^5}{2\left(b^7x^2+ab^6\right)}+\frac{5\,a^4\log\left(bx^2+a\right)}{2\,b^6}+\frac{3\,b^3x^8-8\,ab^2x^6+18\,a^2bx^4-48\,a^3x^2}{24\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/2*a^5/(b^7*x^2 + a*b^6) + 5/2*a^4*log(b*x^2 + a)/b^6 + 1/24*(3*b^3*x^8 -
8*a*b^2*x^6 + 18*a^2*b*x^4 - 48*a^3*x^2)/b^5
```

mupad [B] time = 4.36, size = 79, normalized size = 0.95

$$\frac{x^8}{8\,b^2}+\frac{a^5}{2\,b\,\left(b^6\,x^2+a\,b^5\right)}-\frac{a\,x^6}{3\,b^3}+\frac{5\,a^4\,\ln\left(b\,x^2+a\right)}{2\,b^6}+\frac{3\,a^2\,x^4}{4\,b^4}-\frac{2\,a^3\,x^2}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x^8/(8*b^2) + a^5/(2*b*(a*b^5 + b^6*x^2)) - (a*x^6)/(3*b^3) + (5*a^4*log(a
+ b*x^2))/(2*b^6) + (3*a^2*x^4)/(4*b^4) - (2*a^3*x^2)/b^5
```

sympy [A] time = 0.32, size = 80, normalized size = 0.96

$$\frac{a^5}{2ab^6+2b^7x^2}+\frac{5a^4\log\left(a+bx^2\right)}{2b^6}-\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**5/(2*a*b**6 + 2*b**7*x**2) + 5*a**4*log(a + b*x**2)/(2*b**6) - 2*a**3*x*
*2/b**5 + 3*a**2*x**4/(4*b**4) - a*x**6/(3*b**3) + x**8/(8*b**2)
```

3.476 $\int \frac{x^9}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=70

$$-\frac{a^4}{2b^5\left(a+bx^2\right)}-\frac{2a^3\log\left(a+bx^2\right)}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}$$

```
[Out] 3/2*a^2*x^2/b^4-1/2*a*x^4/b^3+1/6*x^6/b^2-1/2*a^4/b^5/(b*x^2+a)-2*a^3*ln(b*
x^2+a)/b^5
```

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{3a^2x^2}{2b^4}-\frac{a^4}{2b^5\left(a+bx^2\right)}-\frac{2a^3\log\left(a+bx^2\right)}{b^5}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (3*a^2*x^2)/(2*b^4) - (a*x^4)/(2*b^3) + x^6/(6*b^2) - a^4/(2*b^5*(a + b*x^2
)) - (2*a^3*Log[a + b*x^2])/b^5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^9}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{x^9}{\left(ab+b^2x^2\right)^2}\,dx$$
$$= \frac{1}{2}b^2\,\mathrm{Subst}\left(\int \frac{x^4}{\left(ab+b^2x\right)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^2\,\mathrm{Subst}\left(\int \left(\frac{3a^2}{b^6}-\frac{2ax}{b^5}+\frac{x^2}{b^4}+\frac{a^4}{b^6(a+bx)^2}-\frac{4a^3}{b^6(a+bx)}\right)dx, x, x^2\right)$$
$$= \frac{3a^2x^2}{2b^4}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}-\frac{a^4}{2b^5\left(a+bx^2\right)}-\frac{2a^3\log\left(a+bx^2\right)}{b^5}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 0.86

$$\frac{-\frac{3a^4}{a+bx^2}-12a^3\log\left(a+bx^2\right)+9a^2bx^2-3ab^2x^4+b^3x^6}{6b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (9*a^2*b*x^2 - 3*a*b^2*x^4 + b^3*x^6 - (3*a^4)/(a + b*x^2) - 12*a^3*Log[a +
 b*x^2])/(6*b^5)
```

fricas [A] time = 0.81, size = 81, normalized size = 1.16

$$\frac{b^4x^8-2\,ab^3x^6+6\,a^2b^2x^4+9\,a^3bx^2-3\,a^4-12\left(a^3bx^2+a^4\right)\log\left(bx^2+a\right)}{6\left(b^6x^2+ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/6*(b^4*x^8 - 2*a*b^3*x^6 + 6*a^2*b^2*x^4 + 9*a^3*b*x^2 - 3*a^4 - 12*(a^3*
b*x^2 + a^4)*log(b*x^2 + a))/(b^6*x^2 + a*b^5)
```

giac [A] time = 0.17, size = 80, normalized size = 1.14

$$-\frac{2\,a^3\log\left(\left|bx^2+a\right|\right)}{b^5}+\frac{b^4x^6-3\,ab^3x^4+9\,a^2b^2x^2}{6\,b^6}+\frac{4\,a^3bx^2+3\,a^4}{2\left(bx^2+a\right)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -2*a^3*log(abs(b*x^2 + a))/b^5 + 1/6*(b^4*x^6 - 3*a*b^3*x^4 + 9*a^2*b^2*x^2
)/b^6 + 1/2*(4*a^3*b*x^2 + 3*a^4)/((b*x^2 + a)*b^5)
```

maple [A] time = 0.01, size = 63, normalized size = 0.90

$$\frac{x^{6}}{6b^{2}}-\frac{a\,x^{4}}{2b^{3}}+\frac{3a^{2}x^{2}}{2b^{4}}-\frac{a^{4}}{2\left(b\,x^{2}+a\right)b^{5}}-\frac{2a^{3}\ln\left(b\,x^{2}+a\right)}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 3/2*a^2*x^2/b^4-1/2*a*x^4/b^3+1/6*x^6/b^2-1/2*a^4/b^5/(b*x^2+a)-2*a^3*ln(b*
x^2+a)/b^5
```

maxima [A] time = 1.35, size = 65, normalized size = 0.93

$$-\frac{a^{4}}{2\left(b^{6}x^{2}+ab^{5}\right)}-\frac{2\,a^{3}\log\left(bx^{2}+a\right)}{b^{5}}+\frac{b^{2}x^{6}-3\,abx^{4}+9\,a^{2}x^{2}}{6\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2*a^4/(b^6*x^2 + a*b^5) - 2*a^3*log(b*x^2 + a)/b^5 + 1/6*(b^2*x^6 - 3*a*
b*x^4 + 9*a^2*x^2)/b^4
```

mupad [B] time = 0.04, size = 68, normalized size = 0.97

$$\frac{x^{6}}{6\,b^{2}}-\frac{a^{4}}{2\,b\,\left(b^{5}\,x^{2}+a\,b^{4}\right)}-\frac{a\,x^{4}}{2\,b^{3}}-\frac{2\,a^{3}\,\ln\left(b\,x^{2}+a\right)}{b^{5}}+\frac{3\,a^{2}\,x^{2}}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x^6/(6*b^2) - a^4/(2*b*(a*b^4 + b^5*x^2)) - (a*x^4)/(2*b^3) - (2*a^3*log(a
+ b*x^2))/b^5 + (3*a^2*x^2)/(2*b^4)
```

sympy [A] time = 0.29, size = 66, normalized size = 0.94

$$-\frac{a^{4}}{2ab^{5}+2b^{6}x^{2}}-\frac{2a^{3}\log\left(a+bx^{2}\right)}{b^{5}}+\frac{3a^{2}x^{2}}{2b^{4}}-\frac{ax^{4}}{2b^{3}}+\frac{x^{6}}{6b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -a**4/(2*a*b**5 + 2*b**6*x**2) - 2*a**3*log(a + b*x**2)/b**5 + 3*a**2*x**2/
(2*b**4) - a*x**4/(2*b**3) + x**6/(6*b**2)
```

3.477 $\int \frac{x^7}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=57

$$\frac{a^3}{2b^4\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{2b^4}-\frac{ax^2}{b^3}+\frac{x^4}{4b^2}$$

```
[Out] -a*x^2/b^3+1/4*x^4/b^2+1/2*a^3/b^4/(b*x^2+a)+3/2*a^2*ln(b*x^2+a)/b^4
```

Rubi [A] time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a^3}{2b^4\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{2b^4}-\frac{ax^2}{b^3}+\frac{x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -((a*x^2)/b^3) + x^4/(4*b^2) + a^3/(2*b^4*(a + b*x^2)) + (3*a^2*Log[a + b*x
^2])/(2*b^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{x^7}{\left(ab+b^2x^2\right)^2}\,dx$$
$$= \frac{1}{2}b^2\,\text{Subst}\left(\int \frac{x^3}{\left(ab+b^2x\right)^2}\,dx,x,x^2\right)$$
$$= \frac{1}{2}b^2\,\text{Subst}\left(\int \left(-\frac{2a}{b^5}+\frac{x}{b^4}-\frac{a^3}{b^5(a+bx)^2}+\frac{3a^2}{b^5(a+bx)}\right)dx,x,x^2\right)$$
$$= -\frac{ax^2}{b^3}+\frac{x^4}{4b^2}+\frac{a^3}{2b^4\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{2b^4}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.86

$$\frac{\frac{2a^3}{a+bx^2}+6a^2\log\left(a+bx^2\right)-4abx^2+b^2x^4}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] (-4*a*b*x^2 + b^2*x^4 + (2*a^3)/(a + b*x^2) + 6*a^2*Log[a + b*x^2])/(4*b^4)
```

fricas [A] time = 0.97, size = 70, normalized size = 1.23

$$\frac{b^3x^6-3\,ab^2x^4-4\,a^2bx^2+2\,a^3+6\left(a^2bx^2+a^3\right)\log\left(bx^2+a\right)}{4\left(b^5x^2+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
[Out] 1/4*(b^3*x^6 - 3*a*b^2*x^4 - 4*a^2*b*x^2 + 2*a^3 + 6*(a^2*b*x^2 + a^3)*log(
b*x^2 + a))/(b^5*x^2 + a*b^4)
```

giac [A] time = 0.16, size = 67, normalized size = 1.18

$$\frac{3\,a^2\log\left(\left|bx^2+a\right|\right)}{2\,b^4}+\frac{b^2x^4-4\,abx^2}{4\,b^4}-\frac{3\,a^2bx^2+2\,a^3}{2\left(bx^2+a\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 3/2*a^2*log(abs(b*x^2 + a))/b^4 + 1/4*(b^2*x^4 - 4*a*b*x^2)/b^4 - 1/2*(3*a^
2*b*x^2 + 2*a^3)/((b*x^2 + a)*b^4)
```

maple [A] time = 0.01, size = 52, normalized size = 0.91

$$\frac{x^4}{4b^2} - \frac{a\,x^2}{b^3} + \frac{a^3}{2\left(b\,x^2 + a\right)b^4} + \frac{3a^2\ln\left(b\,x^2 + a\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -a*x^2/b^3+1/4*x^4/b^2+1/2*a^3/b^4/(b*x^2+a)+3/2*a^2*ln(b*x^2+a)/b^4
```

maxima [A] time = 1.35, size = 54, normalized size = 0.95

$$\frac{a^3}{2\left(b^5x^2 + ab^4\right)} + \frac{3\,a^2\log\left(bx^2 + a\right)}{2\,b^4} + \frac{bx^4 - 4\,ax^2}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/2*a^3/(b^5*x^2 + a*b^4) + 3/2*a^2*log(b*x^2 + a)/b^4 + 1/4*(b*x^4 - 4*a*x
^2)/b^3
```

mupad [B] time = 0.05, size = 57, normalized size = 1.00

$$\frac{x^4}{4\,b^2} + \frac{a^3}{2\,b\,\left(b^4\,x^2 + a\,b^3\right)} - \frac{a\,x^2}{b^3} + \frac{3\,a^2\,\ln\left(b\,x^2 + a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x^4/(4*b^2) + a^3/(2*b*(a*b^3 + b^4*x^2)) - (a*x^2)/b^3 + (3*a^2*log(a + b*
x^2))/(2*b^4)
```

sympy [A] time = 0.27, size = 53, normalized size = 0.93

$$\frac{a^3}{2ab^4 + 2b^5x^2} + \frac{3a^2\log\left(a + bx^2\right)}{2b^4} - \frac{ax^2}{b^3} + \frac{x^4}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**3/(2*a*b**4 + 2*b**5*x**2) + 3*a**2*log(a + b*x**2)/(2*b**4) - a*x**2/b*
*3 + x**4/(4*b**2)
```

3.478 $\int \frac{x^5}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=44

$$-\frac{a^2}{2b^3\left(a+bx^2\right)}-\frac{a\log\left(a+bx^2\right)}{b^3}+\frac{x^2}{2b^2}$$

```
[Out] 1/2*x^2/b^2-1/2*a^2/b^3/(b*x^2+a)-a*ln(b*x^2+a)/b^3
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{a^2}{2b^3\left(a+bx^2\right)}-\frac{a\log\left(a+bx^2\right)}{b^3}+\frac{x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] x^2/(2*b^2) - a^2/(2*b^3*(a + b*x^2)) - (a*Log[a + b*x^2])/b^3
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{x^5}{\left(ab+b^2x^2\right)^2}\,dx$$
$$= \frac{1}{2}b^2\,\text{Subst}\left(\int \frac{x^2}{\left(ab+b^2x\right)^2}\,dx,x,x^2\right)$$
$$= \frac{1}{2}b^2\,\text{Subst}\left(\int \left(\frac{1}{b^4}+\frac{a^2}{b^4(a+bx)^2}-\frac{2a}{b^4(a+bx)}\right)dx,x,x^2\right)$$
$$= \frac{x^2}{2b^2}-\frac{a^2}{2b^3\left(a+bx^2\right)}-\frac{a\log\left(a+bx^2\right)}{b^3}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.86

$$\frac{-\frac{a^2}{a+bx^2}-2a\log\left(a+bx^2\right)+bx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] (b*x^2 - a^2/(a + b*x^2) - 2*a*Log[a + b*x^2])/(2*b^3)
```

fricas [A] time = 0.81, size = 56, normalized size = 1.27

$$\frac{b^2x^4+abx^2-a^2-2\left(abx^2+a^2\right)\log\left(bx^2+a\right)}{2\left(b^4x^2+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
[Out] 1/2*(b^2*x^4 + a*b*x^2 - a^2 - 2*(a*b*x^2 + a^2)*log(b*x^2 + a))/(b^4*x^2 +
 a*b^3)
```

giac [A] time = 0.17, size = 49, normalized size = 1.11

$$\frac{x^2}{2\,b^2}-\frac{a\log\left(\left|bx^2+a\right|\right)}{b^3}+\frac{2\,abx^2+a^2}{2\left(bx^2+a\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^2 - a*log(abs(b*x^2 + a))/b^3 + 1/2*(2*a*b*x^2 + a^2)/((b*x^2 + a
)*b^3)
```

maple [A] time = 0.01, size = 41, normalized size = 0.93

$$\frac{x^2}{2b^2}-\frac{a^2}{2\left(b\,x^2+a\right)b^3}-\frac{a\ln\left(b\,x^2+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2*x^2/b^2-1/2*a^2/b^3/(b*x^2+a)-a*ln(b*x^2+a)/b^3
```

maxima [A] time = 1.30, size = 43, normalized size = 0.98

$$-\frac{a^2}{2\left(b^4x^2+ab^3\right)}+\frac{x^2}{2\,b^2}-\frac{a\log\left(bx^2+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2/(b^4*x^2 + a*b^3) + 1/2*x^2/b^2 - a*log(b*x^2 + a)/b^3
```

mupad [B] time = 0.05, size = 45, normalized size = 1.02

$$\frac{x^2}{2\,b^2}-\frac{a^2}{2\,\left(b^4\,x^2+a\,b^3\right)}-\frac{a\,\ln\left(b\,x^2+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x^2/(2*b^2) - a^2/(2*(a*b^3 + b^4*x^2)) - (a*log(a + b*x^2))/b^3
```

sympy [A] time = 0.25, size = 39, normalized size = 0.89

$$-\frac{a^2}{2ab^3+2b^4x^2}-\frac{a\log\left(a+bx^2\right)}{b^3}+\frac{x^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -a**2/(2*a*b**3 + 2*b**4*x**2) - a*log(a + b*x**2)/b**3 + x**2/(2*b**2)
```

3.479 $\int \frac{x^3}{a^2+2abx^2+b^2x^4} dx$

Optimal. Leaf size=33

$$\frac{a}{2b^2(a+bx^2)}+\frac{\log(a+bx^2)}{2b^2}$$

```
[Out] 1/2*a/b^2/(b*x^2+a)+1/2*ln(b*x^2+a)/b^2
```

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a}{2b^2(a+bx^2)}+\frac{\log(a+bx^2)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] a/(2*b^2*(a + b*x^2)) + Log[a + b*x^2]/(2*b^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{x^3}{\left(ab+b^2x^2\right)^2}\,dx$$
$$= \frac{1}{2}b^2\,\text{Subst}\left(\int \frac{x}{\left(ab+b^2x\right)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^2\,\text{Subst}\left(\int \left(-\frac{a}{b^3(a+bx)^2}+\frac{1}{b^3(a+bx)}\right)dx, x, x^2\right)$$
$$= \frac{a}{2b^2\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^2}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.82

$$\frac{\frac{a}{a+bx^2}+\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] (a/(a + b*x^2) + Log[a + b*x^2])/(2*b^2)
```

fricas [A] time = 0.78, size = 35, normalized size = 1.06

$$\frac{\left(bx^2+a\right)\log\left(bx^2+a\right)+a}{2\left(b^3x^2+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
[Out] 1/2*((b*x^2 + a)*log(b*x^2 + a) + a)/(b^3*x^2 + a*b^2)
```

giac [A] time = 0.16, size = 30, normalized size = 0.91

$$\frac{\log\left(\left|bx^2+a\right|\right)}{2\,b^2}+\frac{a}{2\left(bx^2+a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/b^2 + 1/2*a/((b*x^2 + a)*b^2)
```

maple [A] time = 0.01, size = 30, normalized size = 0.91

$$\frac{a}{2\left(b\,x^{2}+a\right)b^{2}}+\frac{\ln\left(b\,x^{2}+a\right)}{2b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^4+2*a*b*x^2+a^2),x)
[Out] 1/2*a/b^2/(b*x^2+a)+1/2*ln(b*x^2+a)/b^2
```

maxima [A] time = 1.41, size = 32, normalized size = 0.97

$$\frac{a}{2\left(b^{3}x^{2}+ab^{2}\right)}+\frac{\log\left(bx^{2}+a\right)}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
[Out] 1/2*a/(b^3*x^2 + a*b^2) + 1/2*log(b*x^2 + a)/b^2
```

mupad [B] time = 0.05, size = 29, normalized size = 0.88

$$\frac{\ln\left(b\,x^{2}+a\right)}{2\,b^{2}}+\frac{a}{2\,b^{2}\,\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
[Out] log(a + b*x^2)/(2*b^2) + a/(2*b^2*(a + b*x^2))
```

sympy [A] time = 0.21, size = 29, normalized size = 0.88

$$\frac{a}{2ab^{2}+2b^{3}x^{2}}+\frac{\log\left(a+bx^{2}\right)}{2b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2),x)
[Out] a/(2*a*b**2 + 2*b**3*x**2) + log(a + b*x**2)/(2*b**2)
```

3.480 $\int \frac{x}{a^2+2abx^2+b^2x^4} \, dx$

Optimal. Leaf size=16

$$-\frac{1}{2b\left(a+bx^2\right)}$$

```
[Out] -1/2/b/(b*x^2+a)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {28, 261}

$$-\frac{1}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -1/(2*b*(a + b*x^2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a^2+2abx^2+b^2x^4} \, dx &= b^2 \int \frac{x}{\left(ab+b^2x^2\right)^2} \, dx\\ &= -\frac{1}{2b\left(a+bx^2\right)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -1/2*1/(b*(a + b*x^2))
```

fricas [A] time = 0.93, size = 15, normalized size = 0.94

$$-\frac{1}{2\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/2/(b^2*x^2 + a*b)
```

giac [A] time = 0.17, size = 14, normalized size = 0.88

$$-\frac{1}{2\left(bx^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/2/((b*x^2 + a)*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{2\left(b\,x^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2/b/(b*x^2+a)
```

maxima [A] time = 1.38, size = 15, normalized size = 0.94

$$-\frac{1}{2\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2/(b^2*x^2 + a*b)
```

mupad [B] time = 4.32, size = 14, normalized size = 0.88

$$-\frac{1}{2\,b\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] -1/(2*b*(a + b*x^2))
```

sympy [A] time = 0.17, size = 15, normalized size = 0.94

$$-\frac{1}{2ab+2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -1/(2*a*b + 2*b**2*x**2)
```

3.481 $\int \frac{1}{x(a^2+2abx^2+b^2x^4)} dx$

Optimal. Leaf size=38

$$-\frac{\log\left(a+bx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a\left(a+bx^2\right)}$$

```
[Out] 1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2
```

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$-\frac{\log\left(a+bx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] 1/(2*a*(a + b*x^2)) + Log[x]/a^2 - Log[a + b*x^2]/(2*a^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)}\,dx = b^2\int \frac{1}{x\left(ab+b^2x^2\right)^2}\,dx$$
$$= \frac{1}{2}b^2\,\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^2\,\text{Subst}\left(\int \left(\frac{1}{a^2b^2x}-\frac{1}{ab(a+bx)^2}-\frac{1}{a^2b(a+bx)}\right)dx, x, x^2\right)$$
$$= \frac{1}{2a\left(a+bx^2\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx^2\right)}{2a^2}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.87

$$\frac{\frac{a}{a+bx^2}-\log\left(a+bx^2\right)+2\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
[Out] (a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2])/(2*a^2)
```

fricas [A] time = 1.26, size = 47, normalized size = 1.24

$$-\frac{\left(bx^2+a\right)\log\left(bx^2+a\right)-2\left(bx^2+a\right)\log(x)-a}{2\left(a^2bx^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
[Out] -1/2*((b*x^2 + a)*log(b*x^2 + a) - 2*(b*x^2 + a)*log(x) - a)/(a^2*b*x^2 + a
^3)
```

giac [A] time = 0.15, size = 47, normalized size = 1.24

$$\frac{\log\left(x^2\right)}{2\,a^2}-\frac{\log\left(\left|bx^2+a\right|\right)}{2\,a^2}+\frac{bx^2+2\,a}{2\left(bx^2+a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^2 - 1/2*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 + 2*a)/((b*x^2
+ a)*a^2)
```

maple [A] time = 0.01, size = 35, normalized size = 0.92

$$\frac{1}{2\left(b\,x^2+a\right)a}+\frac{\ln(x)}{a^2}-\frac{\ln\left(b\,x^2+a\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2
```

maxima [A] time = 1.36, size = 37, normalized size = 0.97

$$\frac{1}{2\left(abx^2+a^2\right)}-\frac{\log\left(bx^2+a\right)}{2\,a^2}+\frac{\log\left(x^2\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/2/(a*b*x^2 + a^2) - 1/2*log(b*x^2 + a)/a^2 + 1/2*log(x^2)/a^2
```

mupad [B] time = 4.39, size = 34, normalized size = 0.89

$$\frac{\ln(x)}{a^2}+\frac{1}{2\,a\,\left(b\,x^2+a\right)}-\frac{\ln\left(b\,x^2+a\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] log(x)/a^2 + 1/(2*a*(a + b*x^2)) - log(a + b*x^2)/(2*a^2)
```

sympy [A] time = 0.32, size = 34, normalized size = 0.89

$$\frac{1}{2a^2+2abx^2}+\frac{\log\left(x\right)}{a^2}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**2)
```

3.482 $\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=49

$$\frac{b\log\left(a+bx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{1}{2a^2x^2}$$

```
[Out] -1/2/a^2/x^2-1/2*b/a^2/(b*x^2+a)-2*b*ln(x)/a^3+b*ln(b*x^2+a)/a^3
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$-\frac{b}{2a^2\left(a+bx^2\right)}+\frac{b\log\left(a+bx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -1/(2*a^2*x^2) - b/(2*a^2*(a + b*x^2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x^
2])/a^3
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)}\,dx &= b^2\int \frac{1}{x^3\left(ab+b^2x^2\right)^2}\,dx \\
&= \frac{1}{2}b^2\,\text{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)^2}\,dx,x,x^2\right) \\
&= \frac{1}{2}b^2\,\text{Subst}\left(\int \left(\frac{1}{a^2b^2x^2}-\frac{2}{a^3bx}+\frac{1}{a^2(a+bx)^2}+\frac{2}{a^3(a+bx)}\right)dx,x,x^2\right) \\
&= -\frac{1}{2a^2x^2}-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{2b\log(x)}{a^3}+\frac{b\log\left(a+bx^2\right)}{a^3}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 41, normalized size = 0.84

$$
-\frac{a\left(\frac{b}{a+bx^2}+\frac{1}{x^2}\right)-2b\log\left(a+bx^2\right)+4b\log(x)}{2a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -1/2*(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2])/a^3
```

fricas [A] time = 1.15, size = 73, normalized size = 1.49

$$
-\frac{2\,abx^2+a^2-2\left(b^2x^4+abx^2\right)\log\left(bx^2+a\right)+4\left(b^2x^4+abx^2\right)\log(x)}{2\left(a^3bx^4+a^4x^2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*x^2 + a^2 - 2*(b^2*x^4 + a*b*x^2)*log(b*x^2 + a) + 4*(b^2*x^4 +
 a*b*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

giac [A] time = 0.16, size = 51, normalized size = 1.04

$$
-\frac{b\log\left(x^2\right)}{a^3}+\frac{b\log\left(\left|bx^2+a\right|\right)}{a^3}-\frac{2\,bx^2+a}{2\left(bx^4+ax^2\right)a^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -b*log(x^2)/a^3 + b*log(abs(b*x^2 + a))/a^3 - 1/2*(2*b*x^2 + a)/((b*x^4 + a
*x^2)*a^2)
```

maple [A] time = 0.01, size = 46, normalized size = 0.94

$$-\frac{b}{2\left(b\,x^{2}+a\right)a^{2}}-\frac{2b\ln(x)}{a^{3}}+\frac{b\ln\left(b\,x^{2}+a\right)}{a^{3}}-\frac{1}{2a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2/a^2/x^2-1/2*b/a^2/(b*x^2+a)-2*b*ln(x)/a^3+b*ln(b*x^2+a)/a^3
```

maxima [A] time = 1.36, size = 52, normalized size = 1.06

$$-\frac{2\,bx^{2}+a}{2\left(a^{2}bx^{4}+a^{3}x^{2}\right)}+\frac{b\log\left(bx^{2}+a\right)}{a^{3}}-\frac{b\log\left(x^{2}\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x^2 + a)/(a^2*b*x^4 + a^3*x^2) + b*log(b*x^2 + a)/a^3 - b*log(x^2
)/a^3
```

mupad [B] time = 0.08, size = 51, normalized size = 1.04

$$\frac{b\,\ln\left(b\,x^{2}+a\right)}{a^{3}}-\frac{\frac{1}{2\,a}+\frac{b\,x^{2}}{a^{2}}}{b\,x^{4}+a\,x^{2}}-\frac{2\,b\,\ln(x)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] (b*log(a + b*x^2))/a^3 - (1/(2*a) + (b*x^2)/a^2)/(a*x^2 + b*x^4) - (2*b*log
(x))/a^3
```

sympy [A] time = 0.39, size = 51, normalized size = 1.04

$$\frac{-a-2bx^{2}}{2a^{3}x^{2}+2a^{2}bx^{4}}-\frac{2b\log\left(x\right)}{a^{3}}+\frac{b\log\left(\frac{a}{b}+x^{2}\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] (-a - 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**3 + b*log(a/b
 + x**2)/a**3
```

3.483 $\int \frac{1}{x^5(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=66

$$-\frac{3b^2\log\left(a+bx^2\right)}{2a^4}+\frac{3b^2\log(x)}{a^4}+\frac{b^2}{2a^3\left(a+bx^2\right)}+\frac{b}{a^3x^2}-\frac{1}{4a^2x^4}$$

```
[Out] -1/4/a^2/x^4+b/a^3/x^2+1/2*b^2/a^3/(b*x^2+a)+3*b^2*ln(x)/a^4-3/2*b^2*ln(b*x
^2+a)/a^4
```

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$\frac{b^2}{2a^3\left(a+bx^2\right)}-\frac{3b^2\log\left(a+bx^2\right)}{2a^4}+\frac{3b^2\log(x)}{a^4}+\frac{b}{a^3x^2}-\frac{1}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -1/(4*a^2*x^4) + b/(a^3*x^2) + b^2/(2*a^3*(a + b*x^2)) + (3*b^2*Log[x])/a^4
 - (3*b^2*Log[a + b*x^2])/(2*a^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\left(a^2+2abx^2+b^2x^4\right)}\,dx=b^2\int \frac{1}{x^5\left(ab+b^2x^2\right)^2}\,dx$$
$$=\frac{1}{2}b^2\,\text{Subst}\left(\int \frac{1}{x^3\left(ab+b^2x\right)^2}\,dx,x,x^2\right)$$
$$=\frac{1}{2}b^2\,\text{Subst}\left(\int \left(\frac{1}{a^2b^2x^3}-\frac{2}{a^3bx^2}+\frac{3}{a^4x}-\frac{b}{a^3(a+bx)^2}-\frac{3b}{a^4(a+bx)}\right)dx,x,x^2\right)$$
$$=-\frac{1}{4a^2x^4}+\frac{b}{a^3x^2}+\frac{b^2}{2a^3\left(a+bx^2\right)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log\left(a+bx^2\right)}{2a^4}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.86

$$\frac{-6b^2\log\left(a+bx^2\right)+a\left(\frac{2b^2}{a+bx^2}-\frac{a}{x^4}+\frac{4b}{x^2}\right)+12b^2\log(x)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (a*(-(a/x^4) + (4*b)/x^2 + (2*b^2)/(a + b*x^2)) + 12*b^2*Log[x] - 6*b^2*Log
[a + b*x^2])/(4*a^4)
```

fricas [A] time = 1.04, size = 90, normalized size = 1.36

$$\frac{6\,ab^2x^4+3\,a^2bx^2-a^3-6\left(b^3x^6+ab^2x^4\right)\log\left(bx^2+a\right)+12\left(b^3x^6+ab^2x^4\right)\log(x)}{4\left(a^4bx^6+a^5x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/4*(6*a*b^2*x^4 + 3*a^2*b*x^2 - a^3 - 6*(b^3*x^6 + a*b^2*x^4)*log(b*x^2 +
a) + 12*(b^3*x^6 + a*b^2*x^4)*log(x))/(a^4*b*x^6 + a^5*x^4)
```

giac [A] time = 0.17, size = 86, normalized size = 1.30

$$\frac{3\,b^2\log\left(x^2\right)}{2\,a^4}-\frac{3\,b^2\log\left(\left|bx^2+a\right|\right)}{2\,a^4}+\frac{3\,b^3x^2+4\,ab^2}{2\left(bx^2+a\right)a^4}-\frac{9\,b^2x^4-4\,abx^2+a^2}{4\,a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 3/2*b^2*log(x^2)/a^4 - 3/2*b^2*log(abs(b*x^2 + a))/a^4 + 1/2*(3*b^3*x^2 + 4
*a*b^2)/((b*x^2 + a)*a^4) - 1/4*(9*b^2*x^4 - 4*a*b*x^2 + a^2)/(a^4*x^4)
```

maple [A] time = 0.01, size = 61, normalized size = 0.92

$$\frac{b^2}{2\left(b\,x^2+a\right)a^3}+\frac{3b^2\ln(x)}{a^4}-\frac{3b^2\ln\left(b\,x^2+a\right)}{2a^4}+\frac{b}{a^3x^2}-\frac{1}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/4/a^2/x^4+b/a^3/x^2+1/2*b^2/a^3/(b*x^2+a)+3*b^2*ln(x)/a^4-3/2*b^2*ln(b*x
^2+a)/a^4
```

maxima [A] time = 1.41, size = 70, normalized size = 1.06

$$\frac{6\,b^2x^4+3\,abx^2-a^2}{4\left(a^3bx^6+a^4x^4\right)}-\frac{3\,b^2\log\left(bx^2+a\right)}{2\,a^4}+\frac{3\,b^2\log\left(x^2\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/4*(6*b^2*x^4 + 3*a*b*x^2 - a^2)/(a^3*b*x^6 + a^4*x^4) - 3/2*b^2*log(b*x^2
 + a)/a^4 + 3/2*b^2*log(x^2)/a^4
```

mupad [B] time = 0.07, size = 67, normalized size = 1.02

$$\frac{\frac{3\,b\,x^2}{4\,a^2}-\frac{1}{4\,a}+\frac{3\,b^2\,x^4}{2\,a^3}}{b\,x^6+a\,x^4}-\frac{3\,b^2\,\ln\left(b\,x^2+a\right)}{2\,a^4}+\frac{3\,b^2\,\ln(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] ((3*b*x^2)/(4*a^2) - 1/(4*a) + (3*b^2*x^4)/(2*a^3))/(a*x^4 + b*x^6) - (3*b^
2*log(a + b*x^2))/(2*a^4) + (3*b^2*log(x))/a^4
```

sympy [A] time = 0.47, size = 68, normalized size = 1.03

$$\frac{-a^2+3abx^2+6b^2x^4}{4a^4x^4+4a^3bx^6}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log\left(\frac{a}{b}+x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] (-a**2 + 3*a*b*x**2 + 6*b**2*x**4)/(4*a**4*x**4 + 4*a**3*b*x**6) + 3*b**2*l
og(x)/a**4 - 3*b**2*log(a/b + x**2)/(2*a**4)
```

3.484 $\int \frac{x^{10}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=92

$$\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}-\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9x^7}{14b^2}$$

```
[Out] -9/2*a^3*x/b^5+3/2*a^2*x^3/b^4-9/10*a*x^5/b^3+9/14*x^7/b^2-1/2*x^9/b/(b*x^2
+a)+9/2*a^(7/2)*arctan(x*b^(1/2)/a^(1/2))/b^(11/2)
```

Rubi [A] time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 302, 205}

$$\frac{3a^2x^3}{2b^4}-\frac{9a^3x}{2b^5}+\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}-\frac{9ax^5}{10b^3}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9x^7}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-9*a^3*x)/(2*b^5) + (3*a^2*x^3)/(2*b^4) - (9*a*x^5)/(10*b^3) + (9*x^7)/(14
*b^2) - x^9/(2*b*(a + b*x^2)) + (9*a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*
b^(11/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{10}}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{x^{10}}{\left(ab+b^2x^2\right)^2}\,dx \\
&= -\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9}{2}\int \frac{x^8}{ab+b^2x^2}\,dx \\
&= -\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9}{2}\int \left(-\frac{a^3}{b^5}+\frac{a^2x^2}{b^4}-\frac{ax^4}{b^3}+\frac{x^6}{b^2}+\frac{a^4}{b^4\left(ab+b^2x^2\right)}\right)dx \\
&= -\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}+\frac{9x^7}{14b^2}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{\left(9a^4\right)\int \frac{1}{ab+b^2x^2}\,dx}{2b^4} \\
&= -\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}+\frac{9x^7}{14b^2}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 82, normalized size = 0.89

$$
\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}+\frac{x\left(-\frac{35a^4}{a+bx^2}-280a^3+70a^2bx^2-28ab^2x^4+10b^3x^6\right)}{70b^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (x*(-280*a^3 + 70*a^2*b*x^2 - 28*a*b^2*x^4 + 10*b^3*x^6 - (35*a^4)/(a + b*x
^2)))/(70*b^5) + (9*a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/2))
```

fricas [A] time = 1.14, size = 212, normalized size = 2.30

$$\left[\frac{20\,b^4x^9 - 36\,ab^3x^7 + 84\,a^2b^2x^5 - 420\,a^3bx^3 - 630\,a^4x + 315\left(a^3bx^2 + a^4\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{140\left(b^6x^2 + ab^5\right)}, \frac{10\,b^4x^9 - 1\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [1/140*(20*b^4*x^9 - 36*a*b^3*x^7 + 84*a^2*b^2*x^5 - 420*a^3*b*x^3 - 630*a^
4*x + 315*(a^3*b*x^2 + a^4)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(
b*x^2 + a)))/(b^6*x^2 + a*b^5), 1/70*(10*b^4*x^9 - 18*a*b^3*x^7 + 42*a^2*b^
2*x^5 - 210*a^3*b*x^3 - 315*a^4*x + 315*(a^3*b*x^2 + a^4)*sqrt(a/b)*arctan(
b*x*sqrt(a/b)/a))/(b^6*x^2 + a*b^5)]
```

giac [A] time = 0.15, size = 84, normalized size = 0.91

$$\frac{9\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^5} - \frac{a^4x}{2\left(bx^2 + a\right)b^5} + \frac{5\,b^{12}x^7 - 14\,ab^{11}x^5 + 35\,a^2b^{10}x^3 - 140\,a^3b^9x}{35\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 9/2*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/2*a^4*x/((b*x^2 + a)*b^5)
 + 1/35*(5*b^12*x^7 - 14*a*b^11*x^5 + 35*a^2*b^10*x^3 - 140*a^3*b^9*x)/b^14
```

maple [A] time = 0.01, size = 78, normalized size = 0.85

$$\frac{x^7}{7b^2} - \frac{2a\,x^5}{5b^3} + \frac{a^2x^3}{b^4} - \frac{a^4x}{2\left(b\,x^2 + a\right)b^5} + \frac{9a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^5} - \frac{4a^3x}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/7*x^7/b^2-2/5*a*x^5/b^3+a^2*x^3/b^4-4*a^3*x/b^5-1/2/b^5*a^4*x/(b*x^2+a)+9
/2/b^5*a^4/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

maxima [A] time = 2.99, size = 82, normalized size = 0.89

$$-\frac{a^4x}{2\left(b^6x^2 + ab^5\right)} + \frac{9\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^5} + \frac{5\,b^3x^7 - 14\,ab^2x^5 + 35\,a^2bx^3 - 140\,a^3x}{35\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2*a^4*x/(b^6*x^2 + a*b^5) + 9/2*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5
) + 1/35*(5*b^3*x^7 - 14*a*b^2*x^5 + 35*a^2*b*x^3 - 140*a^3*x)/b^5
```

mupad [B] time = 0.04, size = 77, normalized size = 0.84

$$\frac{x^7}{7\,b^2}-\frac{2\,a\,x^5}{5\,b^3}-\frac{4\,a^3\,x}{b^5}+\frac{9\,a^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{11/2}}+\frac{a^2\,x^3}{b^4}-\frac{a^4\,x}{2\,\left(b^6\,x^2+a\,b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x^7/(7*b^2) - (2*a*x^5)/(5*b^3) - (4*a^3*x)/b^5 + (9*a^(7/2)*atan((b^(1/2)*
x)/a^(1/2)))/(2*b^(11/2)) + (a^2*x^3)/b^4 - (a^4*x)/(2*(a*b^5 + b^6*x^2))
```

sympy [A] time = 0.35, size = 134, normalized size = 1.46

$$-\frac{a^4x}{2ab^5+2b^6x^2}-\frac{4a^3x}{b^5}+\frac{a^2x^3}{b^4}-\frac{2ax^5}{5b^3}-\frac{9\sqrt{-\frac{a^7}{b^{11}}}\log\left(x-\frac{b^5\sqrt{-\frac{a^7}{b^{11}}}}{a^3}\right)}{4}+\frac{9\sqrt{-\frac{a^7}{b^{11}}}\log\left(x+\frac{b^5\sqrt{-\frac{a^7}{b^{11}}}}{a^3}\right)}{4}+\frac{x^7}{7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -a**4*x/(2*a*b**5 + 2*b**6*x**2) - 4*a**3*x/b**5 + a**2*x**3/b**4 - 2*a*x**
5/(5*b**3) - 9*sqrt(-a**7/b**11)*log(x - b**5*sqrt(-a**7/b**11)/a**3)/4 + 9
*sqrt(-a**7/b**11)*log(x + b**5*sqrt(-a**7/b**11)/a**3)/4 + x**7/(7*b**2)
```

3.485 $\int \frac{x^8}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=79

$$-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}+\frac{7a^2x}{2b^4}-\frac{7ax^3}{6b^3}-\frac{x^7}{2b\left(a+bx^2\right)}+\frac{7x^5}{10b^2}$$

```
[Out] 7/2*a^2*x/b^4-7/6*a*x^3/b^3+7/10*x^5/b^2-1/2*x^7/b/(b*x^2+a)-7/2*a^(5/2)*ar
ctan(x*b^(1/2)/a^(1/2))/b^(9/2)
```

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 302, 205}

$$\frac{7a^2x}{2b^4}-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}-\frac{7ax^3}{6b^3}-\frac{x^7}{2b\left(a+bx^2\right)}+\frac{7x^5}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (7*a^2*x)/(2*b^4) - (7*a*x^3)/(6*b^3) + (7*x^5)/(10*b^2) - x^7/(2*b*(a + b*
x^2)) - (7*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{x^8}{\left(ab+b^2x^2\right)^2}\,dx\\
&= -\frac{x^7}{2b\left(a+bx^2\right)}+\frac{7}{2}\int \frac{x^6}{ab+b^2x^2}\,dx\\
&= -\frac{x^7}{2b\left(a+bx^2\right)}+\frac{7}{2}\int \left(\frac{a^2}{b^4}-\frac{ax^2}{b^3}+\frac{x^4}{b^2}-\frac{a^3}{b^3\left(ab+b^2x^2\right)}\right)dx\\
&= \frac{7a^2x}{2b^4}-\frac{7ax^3}{6b^3}+\frac{7x^5}{10b^2}-\frac{x^7}{2b\left(a+bx^2\right)}-\frac{\left(7a^3\right)\int \frac{1}{ab+b^2x^2}\,dx}{2b^3}\\
&= \frac{7a^2x}{2b^4}-\frac{7ax^3}{6b^3}+\frac{7x^5}{10b^2}-\frac{x^7}{2b\left(a+bx^2\right)}-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 71, normalized size = 0.90

$$\frac{x\left(\frac{15a^3}{a+bx^2}+90a^2-20abx^2+6b^2x^4\right)}{30b^4}-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (x*(90*a^2 - 20*a*b*x^2 + 6*b^2*x^4 + (15*a^3)/(a + b*x^2)))/(30*b^4) - (7*
a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

fricas [A] time = 0.99, size = 190, normalized size = 2.41

$$\left[\frac{12\,b^3x^7-28\,ab^2x^5+140\,a^2bx^3+210\,a^3x+105\left(a^2bx^2+a^3\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{60\left(b^5x^2+ab^4\right)},\frac{6\,b^3x^7-14\,ab^2x^5+70\,a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [1/60*(12*b^3*x^7 - 28*a*b^2*x^5 + 140*a^2*b*x^3 + 210*a^3*x + 105*(a^2*b*x
^2 + a^3)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^5*
x^2 + a*b^4), 1/30*(6*b^3*x^7 - 14*a*b^2*x^5 + 70*a^2*b*x^3 + 105*a^3*x - 1
05*(a^2*b*x^2 + a^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^5*x^2 + a*b^4)]
```

giac [A] time = 0.15, size = 73, normalized size = 0.92

$$-\frac{7\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^4}+\frac{a^3x}{2\,\left(bx^2+a\right)b^4}+\frac{3\,b^8x^5-10\,ab^7x^3+45\,a^2b^6x}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -7/2*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/2*a^3*x/((b*x^2 + a)*b^4
) + 1/15*(3*b^8*x^5 - 10*a*b^7*x^3 + 45*a^2*b^6*x)/b^10
```

maple [A] time = 0.01, size = 68, normalized size = 0.86

$$\frac{x^5}{5b^2}-\frac{2a\,x^3}{3b^3}+\frac{a^3x}{2\left(b\,x^2+a\right)b^4}-\frac{7a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^4}+\frac{3a^2x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/5*x^5/b^2-2/3*a*x^3/b^3+3*a^2*x/b^4+1/2/b^4*a^3*x/(b*x^2+a)-7/2/b^4*a^3/(
a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.95, size = 71, normalized size = 0.90

$$\frac{a^3x}{2\left(b^5x^2+ab^4\right)}-\frac{7\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^4}+\frac{3\,b^2x^5-10\,abx^3+45\,a^2x}{15\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/2*a^3*x/(b^5*x^2 + a*b^4) - 7/2*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4)
 + 1/15*(3*b^2*x^5 - 10*a*b*x^3 + 45*a^2*x)/b^4
```

mupad [B] time = 4.27, size = 66, normalized size = 0.84

$$\frac{x^5}{5\,b^2}-\frac{2\,a\,x^3}{3\,b^3}+\frac{3\,a^2\,x}{b^4}-\frac{7\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{9/2}}+\frac{a^3\,x}{2\,\left(b^5\,x^2+a\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x^5/(5*b^2) - (2*a*x^3)/(3*b^3) + (3*a^2*x)/b^4 - (7*a^(5/2)*atan((b^(1/2)*
x)/a^(1/2)))/(2*b^(9/2)) + (a^3*x)/(2*(a*b^4 + b^5*x^2))
```

sympy [A] time = 0.32, size = 124, normalized size = 1.57

$$\frac{a^3x}{2ab^4+2b^5x^2}+\frac{3a^2x}{b^4}-\frac{2ax^3}{3b^3}+\frac{7\sqrt{-\frac{a^5}{b^9}}\,\log\left(x-\frac{b^4\sqrt{-\frac{a^5}{b^9}}}{a^2}\right)}{4}-\frac{7\sqrt{-\frac{a^5}{b^9}}\,\log\left(x+\frac{b^4\sqrt{-\frac{a^5}{b^9}}}{a^2}\right)}{4}+\frac{x^5}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**3*x/(2*a*b**4 + 2*b**5*x**2) + 3*a**2*x/b**4 - 2*a*x**3/(3*b**3) + 7*sqr
t(-a**5/b**9)*log(x - b**4*sqrt(-a**5/b**9)/a**2)/4 - 7*sqrt(-a**5/b**9)*lo
g(x + b**4*sqrt(-a**5/b**9)/a**2)/4 + x**5/(5*b**2)
```

3.486 $\int \frac{x^6}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=66

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}-\frac{5ax}{2b^3}-\frac{x^5}{2b\left(a+bx^2\right)}+\frac{5x^3}{6b^2}$$

```
[Out] -5/2*a*x/b^3+5/6*x^3/b^2-1/2*x^5/b/(b*x^2+a)+5/2*a^(3/2)*arctan(x*b^(1/2)/a
^(1/2))/b^(7/2)
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 302, 205}

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}-\frac{5ax}{2b^3}-\frac{x^5}{2b\left(a+bx^2\right)}+\frac{5x^3}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-5*a*x)/(2*b^3) + (5*x^3)/(6*b^2) - x^5/(2*b*(a + b*x^2)) + (5*a^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{x^6}{\left(ab+b^2x^2\right)^2}\,dx \\
&= -\frac{x^5}{2b\left(a+bx^2\right)}+\frac{5}{2}\int \frac{x^4}{ab+b^2x^2}\,dx \\
&= -\frac{x^5}{2b\left(a+bx^2\right)}+\frac{5}{2}\int \left(-\frac{a}{b^3}+\frac{x^2}{b^2}+\frac{a^2}{b^2\left(ab+b^2x^2\right)}\right)dx \\
&= -\frac{5ax}{2b^3}+\frac{5x^3}{6b^2}-\frac{x^5}{2b\left(a+bx^2\right)}+\frac{\left(5a^2\right)\int \frac{1}{ab+b^2x^2}\,dx}{2b^2} \\
&= -\frac{5ax}{2b^3}+\frac{5x^3}{6b^2}-\frac{x^5}{2b\left(a+bx^2\right)}+\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.91

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{x\left(-\frac{3a^2}{a+bx^2}-12a+2bx^2\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (x*(-12*a + 2*b*x^2 - (3*a^2)/(a + b*x^2)))/(6*b^3) + (5*a^(3/2)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

fricas [A] time = 0.72, size = 164, normalized size = 2.48

$$\left[\frac{4\,b^2x^5-20\,abx^3-30\,a^2x+15\left(abx^2+a^2\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{12\left(b^4x^2+ab^3\right)},\frac{2\,b^2x^5-10\,abx^3-15\,a^2x+15\left(abx^2+a^2\right)}{6\left(b^4x^2+ab^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*b^2*x^5 - 20*a*b*x^3 - 30*a^2*x + 15*(a*b*x^2 + a^2)*sqrt(-a/b)*lo
g((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^4*x^2 + a*b^3), 1/6*(2*b^
2*x^5 - 10*a*b*x^3 - 15*a^2*x + 15*(a*b*x^2 + a^2)*sqrt(a/b)*arctan(b*x*sqr
t(a/b)/a))/(b^4*x^2 + a*b^3)]
```

giac [A] time = 0.16, size = 61, normalized size = 0.92

$$\frac{5\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3}-\frac{a^2x}{2\left(bx^2+a\right)b^3}+\frac{b^4x^3-6\,ab^3x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 5/2*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*a^2*x/((b*x^2 + a)*b^3)
 + 1/3*(b^4*x^3 - 6*a*b^3*x)/b^6
```

maple [A] time = 0.01, size = 57, normalized size = 0.86

$$\frac{x^3}{3b^2}-\frac{a^2x}{2\left(b\,x^2+a\right)b^3}+\frac{5a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^3}-\frac{2ax}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/3*x^3/b^2-2*a*x/b^3-1/2/b^3*a^2*x/(b*x^2+a)+5/2/b^3*a^2/(a*b)^(1/2)*arcta
n(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.13, size = 59, normalized size = 0.89

$$-\frac{a^2x}{2\left(b^4x^2+ab^3\right)}+\frac{5\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3}+\frac{bx^3-6\,ax}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*x/(b^4*x^2 + a*b^3) + 5/2*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3
) + 1/3*(b*x^3 - 6*a*x)/b^3
```

mupad [B] time = 0.06, size = 56, normalized size = 0.85

$$\frac{x^3}{3\,b^2}+\frac{5\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{7/2}}-\frac{a^2\,x}{2\,\left(b^4\,x^2+a\,b^3\right)}-\frac{2\,a\,x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x^3/(3*b^2) + (5*a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*b^(7/2)) - (a^2*x)/(
2*(a*b^3 + b^4*x^2)) - (2*a*x)/b^3
```

sympy [A] time = 0.30, size = 107, normalized size = 1.62

$$-\frac{a^2x}{2ab^3+2b^4x^2}-\frac{2ax}{b^3}-\frac{5\sqrt{-\frac{a^3}{b^7}}\log\left(x-\frac{b^3\sqrt{-\frac{a^3}{b^7}}}{a}\right)}{4}+\frac{5\sqrt{-\frac{a^3}{b^7}}\log\left(x+\frac{b^3\sqrt{-\frac{a^3}{b^7}}}{a}\right)}{4}+\frac{x^3}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -a**2*x/(2*a*b**3 + 2*b**4*x**2) - 2*a*x/b**3 - 5*sqrt(-a**3/b**7)*log(x -
b**3*sqrt(-a**3/b**7)/a)/4 + 5*sqrt(-a**3/b**7)*log(x + b**3*sqrt(-a**3/b**
7)/a)/4 + x**3/(3*b**2)
```

3.487 $\int \frac{x^4}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=55

$$-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}}-\frac{x^3}{2b\left(a+bx^2\right)}+\frac{3x}{2b^2}$$

```
[Out] 3/2*x/b^2-1/2*x^3/b/(b*x^2+a)-3/2*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(5/2)
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 321, 205}

$$-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}}-\frac{x^3}{2b\left(a+bx^2\right)}+\frac{3x}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (3*x)/(2*b^2) - x^3/(2*b*(a + b*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*b^(5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{x^4}{\left(ab+b^2x^2\right)^2}\,dx\\ &= -\frac{x^3}{2b\left(a+bx^2\right)}+\frac{3}{2}\int \frac{x^2}{ab+b^2x^2}\,dx\\ &= \frac{3x}{2b^2}-\frac{x^3}{2b\left(a+bx^2\right)}-\frac{(3a)\int \frac{1}{ab+b^2x^2}\,dx}{2b}\\ &= \frac{3x}{2b^2}-\frac{x^3}{2b\left(a+bx^2\right)}-\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.93

$$-\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}}+\frac{ax}{2b^2\left(a+bx^2\right)}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] x/b^2 + (a*x)/(2*b^2*(a + b*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(2*b^(5/2))
```

fricas [A] time = 0.88, size = 136, normalized size = 2.47

$$\left[\frac{4\,bx^3+3\left(bx^2+a\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+6\,ax}{4\left(b^3x^2+ab^2\right)},\ \frac{2\,bx^3-3\left(bx^2+a\right)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+3\,ax}{2\left(b^3x^2+ab^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*x^3 + 3*(b*x^2 + a)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)
/(b*x^2 + a)) + 6*a*x)/(b^3*x^2 + a*b^2), 1/2*(2*b*x^3 - 3*(b*x^2 + a)*sqrt
(a/b)*arctan(b*x*sqrt(a/b)/a) + 3*a*x)/(b^3*x^2 + a*b^2)]
```

giac [A] time = 0.15, size = 42, normalized size = 0.76

$$-\frac{3\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}+\frac{ax}{2\left(bx^2+a\right)b^2}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -3/2*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*a*x/((b*x^2 + a)*b^2) +
x/b^2
```

maple [A] time = 0.01, size = 43, normalized size = 0.78

$$\frac{ax}{2\left(b\,x^2+a\right)b^2}-\frac{3a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^2}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] x/b^2+1/2/b^2*a*x/(b*x^2+a)-3/2/b^2*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.08, size = 45, normalized size = 0.82

$$\frac{ax}{2\left(b^3x^2+ab^2\right)}-\frac{3\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/2*a*x/(b^3*x^2 + a*b^2) - 3/2*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + x
/b^2
```

mupad [B] time = 4.29, size = 43, normalized size = 0.78

$$\frac{x}{b^2}+\frac{a\,x}{2\,\left(b^3\,x^2+a\,b^2\right)}-\frac{3\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x/b^2 + (a*x)/(2*(a*b^2 + b^3*x^2)) - (3*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))
/(2*b^(5/2))
```

sympy [A] time = 0.27, size = 83, normalized size = 1.51

$$\frac{ax}{2ab^2+2b^3x^2}+\frac{3\sqrt{-\frac{a}{b^5}}\log\left(-b^2\sqrt{-\frac{a}{b^5}}+x\right)}{4}-\frac{3\sqrt{-\frac{a}{b^5}}\log\left(b^2\sqrt{-\frac{a}{b^5}}+x\right)}{4}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a*x/(2*a*b**2 + 2*b**3*x**2) + 3*sqrt(-a/b**5)*log(-b**2*sqrt(-a/b**5) + x)
/4 - 3*sqrt(-a/b**5)*log(b**2*sqrt(-a/b**5) + x)/4 + x/b**2
```

3.488 $\int \frac{x^2}{a^2+2abx^2+b^2x^4} \, dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{x}{2b\left(a+bx^2\right)}$$

```
[Out] -1/2*x/b/(b*x^2+a)+1/2*arctan(x*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 288, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{x}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -x/(2*b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{x^2}{\left(ab+b^2x^2\right)^2}\,dx$$
$$= -\frac{x}{2b\left(a+bx^2\right)} + \frac{1}{2}\int \frac{1}{ab+b^2x^2}\,dx$$
$$= -\frac{x}{2b\left(a+bx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}} - \frac{x}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -1/2*x/(b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

fricas [A] time = 0.97, size = 120, normalized size = 2.67

$$\left[-\frac{2\,abx + \left(bx^2+a\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\left(ab^3x^2+a^2b^2\right)}, -\frac{abx - \left(bx^2+a\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\left(ab^3x^2+a^2b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*x + (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)))/(a*b^3*x^2 + a^2*b^2), -1/2*(a*b*x - (b*x^2 + a)*sqrt(a*b)*arct
an(sqrt(a*b)*x/a))/(a*b^3*x^2 + a^2*b^2)]
```

giac [A] time = 0.20, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b} - \frac{x}{2\left(bx^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*x/((b*x^2 + a)*b)
```

maple [A] time = 0.01, size = 36, normalized size = 0.80

$$-\frac{x}{2\left(b\,x^{2}+a\right)b}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2),x)
[Out] -1/2*x/b/(b*x^2+a)+1/2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.00, size = 36, normalized size = 0.80

$$-\frac{x}{2\left(b^{2}x^{2}+ab\right)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
[Out] -1/2*x/(b^2*x^2 + a*b) + 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

mupad [B] time = 0.05, size = 33, normalized size = 0.73

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{x}{2\,b\,\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
[Out] atan((b^(1/2)*x)/a^(1/2))/(2*a^(1/2)*b^(3/2)) - x/(2*b*(a + b*x^2))
```

sympy [B] time = 0.22, size = 78, normalized size = 1.73

$$-\frac{x}{2ab+2b^{2}x^{2}}-\frac{\sqrt{-\frac{1}{ab^{3}}}\,\log\left(-ab\sqrt{-\frac{1}{ab^{3}}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{ab^{3}}}\,\log\left(ab\sqrt{-\frac{1}{ab^{3}}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -x/(2*a*b + 2*b**2*x**2) - sqrt(-1/(a*b**3))*log(-a*b*sqrt(-1/(a*b**3)) + x
)/4 + sqrt(-1/(a*b**3))*log(a*b*sqrt(-1/(a*b**3)) + x)/4
```

3.489 $\int \frac{1}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

```
[Out] 1/2*x/a/(b*x^2+a)+1/2*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {28, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1),x]
```

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{1}{\left(ab+b^2x^2\right)^2}\,dx$$
$$= \frac{x}{2a\left(a+bx^2\right)} + \frac{b\int \frac{1}{ab+b^2x^2}\,dx}{2a}$$
$$= \frac{x}{2a\left(a+bx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

Mathematica [**A**] time = 0.02, size = 45, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{x}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1),x]
```

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

fricas [**A**] time = 0.84, size = 120, normalized size = 2.67

$$\left[\frac{2\,abx - \left(bx^2+a\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\left(a^2b^2x^2+a^3b\right)}, \frac{abx + \left(bx^2+a\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\left(a^2b^2x^2+a^3b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*x - (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(a*b*x + (b*x^2 + a)*sqrt(a*b)*arctan
(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]
```

giac [**A**] time = 0.15, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a} + \frac{x}{2\left(bx^2+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*x/((b*x^2 + a)*a)
```

maple [A] time = 0.00, size = 36, normalized size = 0.80

$$\frac{x}{2\left(b\,x^{2}+a\right)a}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.85, size = 35, normalized size = 0.78

$$\frac{x}{2\left(abx^{2}+a^{2}\right)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/2*x/(a*b*x^2 + a^2) + 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a)
```

mupad [B] time = 0.04, size = 33, normalized size = 0.73

$$\frac{x}{2\,a\,\left(b\,x^{2}+a\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x/(2*a*(a + b*x^2)) + atan((b^(1/2)*x)/a^(1/2))/(2*a^(3/2)*b^(1/2))
```

sympy [B] time = 0.22, size = 78, normalized size = 1.73

$$\frac{x}{2a^{2}+2abx^{2}}-\frac{\sqrt{-\frac{1}{a^{3}b}}\log\left(-a^{2}\sqrt{-\frac{1}{a^{3}b}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^{3}b}}\log\left(a^{2}\sqrt{-\frac{1}{a^{3}b}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b)) + x
)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)/4
```

3.490 $\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)}\,dx$

Optimal. Leaf size=57

$$-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}$$

```
[Out] -3/2/a^2/x+1/2/a/x/(b*x^2+a)-3/2*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/2)
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -3/(2*a^2*x) + 1/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(2*a^(5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)}\,dx &= b^2\int \frac{1}{x^2\left(ab+b^2x^2\right)^2}\,dx\\ &= \frac{1}{2ax\left(a+bx^2\right)}+\frac{(3b)\int \frac{1}{x^2\left(ab+b^2x^2\right)}\,dx}{2a}\\ &= -\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}-\frac{\left(3b^2\right)\int \frac{1}{ab+b^2x^2}\,dx}{2a^2}\\ &= -\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.95

$$-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{bx}{2a^2\left(a+bx^2\right)}-\frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(2*a^(5/2))
```

fricas [A] time = 0.91, size = 136, normalized size = 2.39

$$\left[-\frac{6\,bx^2-3\left(bx^3+ax\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+4\,a}{4\left(a^2bx^3+a^3x\right)},-\frac{3\,bx^2+3\left(bx^3+ax\right)\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)+2\,a}{2\left(a^2bx^3+a^3x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(6*b*x^2 - 3*(b*x^3 + a*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)) + 4*a)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 + a*x
)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*a)/(a^2*b*x^3 + a^3*x)]
```

giac [A] time = 0.18, size = 47, normalized size = 0.82

$$-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}-\frac{3\,bx^2+2\,a}{2\,\left(bx^3+ax\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*x^2 + 2*a)/((b*x^3
+ a*x)*a^2)
```

maple [A] time = 0.01, size = 46, normalized size = 0.81

$$-\frac{bx}{2\left(b\,x^2+a\right)a^2}-\frac{3b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^2}-\frac{1}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/a^2/x-1/2/a^2*b*x/(b*x^2+a)-3/2/a^2*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b
*x)
```

maxima [A] time = 2.95, size = 49, normalized size = 0.86

$$-\frac{3\,bx^2+2\,a}{2\,\left(a^2bx^3+a^3x\right)}-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2*(3*b*x^2 + 2*a)/(a^2*b*x^3 + a^3*x) - 3/2*b*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*a^2)
```

mupad [B] time = 4.49, size = 44, normalized size = 0.77

$$-\frac{\frac{1}{a}+\frac{3\,b\,x^2}{2\,a^2}}{b\,x^3+a\,x}-\frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] - (1/a + (3*b*x^2)/(2*a^2))/(a*x + b*x^3) - (3*b^(1/2)*atan((b^(1/2)*x)/a^(
1/2)))/(2*a^(5/2))
```

sympy [A] time = 0.32, size = 92, normalized size = 1.61

$$\frac{3\sqrt{-\frac{b}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}-\frac{3\sqrt{-\frac{b}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}+\frac{-2a-3bx^2}{2a^3x+2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a**5)*log(a**3
*sqrt(-b/a**5)/b + x)/4 + (-2*a - 3*b*x**2)/(2*a**3*x + 2*a**2*b*x**3)
```

3.491 $\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=68

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

```
[Out] -5/6/a^2/x^3+5/2*b/a^3/x+1/2/a/x^3/(b*x^2+a)+5/2*b^(3/2)*arctan(x*b^(1/2)/a
^(1/2))/a^(7/2)
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -5/(6*a^2*x^3) + (5*b)/(2*a^3*x) + 1/(2*a*x^3*(a + b*x^2)) + (5*b^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)}\,dx &= b^2\int \frac{1}{x^4\left(ab+b^2x^2\right)^2}\,dx\\
&= \frac{1}{2ax^3\left(a+bx^2\right)}+\frac{(5b)\int \frac{1}{x^4\left(ab+b^2x^2\right)}\,dx}{2a}\\
&= -\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}-\frac{\left(5b^2\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)}\,dx}{2a^2}\\
&= -\frac{5}{6a^2x^3}+\frac{5b}{2a^3x}+\frac{1}{2ax^3\left(a+bx^2\right)}+\frac{\left(5b^3\right)\int \frac{1}{ab+b^2x^2}\,dx}{2a^3}\\
&= -\frac{5}{6a^2x^3}+\frac{5b}{2a^3x}+\frac{1}{2ax^3\left(a+bx^2\right)}+\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 0.99

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{b^2x}{2a^3\left(a+bx^2\right)}+\frac{2b}{a^3x}-\frac{1}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -1/3*1/(a^2*x^3) + (2*b)/(a^3*x) + (b^2*x)/(2*a^3*(a + b*x^2)) + (5*b^(3/2)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

fricas [A] time = 1.00, size = 172, normalized size = 2.53

$$\left[\frac{30\,b^2x^4+20\,abx^2+15\,\left(b^2x^5+abx^3\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-4\,a^2}{12\,\left(a^3bx^5+a^4x^3\right)},\ \frac{15\,b^2x^4+10\,abx^2+15\,\left(b^2x^5+abx^3\right)\sqrt{\frac{b}{a}}}{6\,\left(a^3bx^5+a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [1/12*(30*b^2*x^4 + 20*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(-b/a)*log((b*x
^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*a^2)/(a^3*b*x^5 + a^4*x^3), 1/6
*(15*b^2*x^4 + 10*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(b/a)*arctan(x*sqrt(
b/a)) - 2*a^2)/(a^3*b*x^5 + a^4*x^3)]
```

giac [A] time = 0.15, size = 59, normalized size = 0.87

$$\frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3}+\frac{b^2x}{2\,\left(bx^2+a\right)a^3}+\frac{6\,bx^2-a}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 5/2*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/2*b^2*x/((b*x^2 + a)*a^3)
 + 1/3*(6*b*x^2 - a)/(a^3*x^3)
```

maple [A] time = 0.01, size = 59, normalized size = 0.87

$$\frac{b^2x}{2\left(b\,x^2+a\right)a^3}+\frac{5b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\ a^3}+\frac{2b}{a^3x}-\frac{1}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/3/a^2/x^3+2*b/a^3/x+1/2/a^3*b^2*x/(b*x^2+a)+5/2/a^3*b^2/(a*b)^(1/2)*arct
an(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.02, size = 64, normalized size = 0.94

$$\frac{15\,b^2x^4+10\,abx^2-2\,a^2}{6\,\left(a^3bx^5+a^4x^3\right)}+\frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/6*(15*b^2*x^4 + 10*a*b*x^2 - 2*a^2)/(a^3*b*x^5 + a^4*x^3) + 5/2*b^2*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

mupad [B] time = 4.43, size = 58, normalized size = 0.85

$$\frac{\frac{5\,b\,x^2}{3\,a^2}-\frac{1}{3\,a}+\frac{5\,b^2\,x^4}{2\,a^3}}{b\,x^5+a\,x^3}+\frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] ((5*b*x^2)/(3*a^2) - 1/(3*a) + (5*b^2*x^4)/(2*a^3))/(a*x^3 + b*x^5) + (5*b^
(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(7/2))
```

sympy [A] time = 0.36, size = 114, normalized size = 1.68

$$-\frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(-\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4}+\frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4}+\frac{-2a^2+10abx^2+15b^2x^4}{6a^4x^3+6a^3bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -5*sqrt(-b**3/a**7)*log(-a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + 5*sqrt(-b**3/a
**7)*log(a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + (-2*a**2 + 10*a*b*x**2 + 15*b*
*2*x**4)/(6*a**4*x**3 + 6*a**3*b*x**5)
```

3.492 $\int \frac{1}{x^6(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=81

$$-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}-\frac{7b^2}{2a^4x}+\frac{7b}{6a^3x^3}-\frac{7}{10a^2x^5}+\frac{1}{2ax^5\left(a+bx^2\right)}$$

```
[Out] -7/10/a^2/x^5+7/6*b/a^3/x^3-7/2*b^2/a^4/x+1/2/a/x^5/(b*x^2+a)-7/2*b^(5/2)*a
rctan(x*b^(1/2)/a^(1/2))/a^(9/2)
```

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$-\frac{7b^2}{2a^4x}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}+\frac{7b}{6a^3x^3}-\frac{7}{10a^2x^5}+\frac{1}{2ax^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -7/(10*a^2*x^5) + (7*b)/(6*a^3*x^3) - (7*b^2)/(2*a^4*x) + 1/(2*a*x^5*(a + b
*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\left(a^2+2abx^2+b^2x^4\right)}\,dx &= b^2\int \frac{1}{x^6\left(ab+b^2x^2\right)^2}\,dx\\
&= \frac{1}{2ax^5\left(a+bx^2\right)}+\frac{(7b)\int \frac{1}{x^6\left(ab+b^2x^2\right)}\,dx}{2a}\\
&= -\frac{7}{10a^2x^5}+\frac{1}{2ax^5\left(a+bx^2\right)}-\frac{\left(7b^2\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)}\,dx}{2a^2}\\
&= -\frac{7}{10a^2x^5}+\frac{7b}{6a^3x^3}+\frac{1}{2ax^5\left(a+bx^2\right)}+\frac{\left(7b^3\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)}\,dx}{2a^3}\\
&= -\frac{7}{10a^2x^5}+\frac{7b}{6a^3x^3}-\frac{7b^2}{2a^4x}+\frac{1}{2ax^5\left(a+bx^2\right)}-\frac{\left(7b^4\right)\int \frac{1}{ab+b^2x^2}\,dx}{2a^4}\\
&= -\frac{7}{10a^2x^5}+\frac{7b}{6a^3x^3}-\frac{7b^2}{2a^4x}+\frac{1}{2ax^5\left(a+bx^2\right)}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 80, normalized size = 0.99

$$-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}-\frac{b^3x}{2a^4\left(a+bx^2\right)}-\frac{3b^2}{a^4x}+\frac{2b}{3a^3x^3}-\frac{1}{5a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -1/5*1/(a^2*x^5) + (2*b)/(3*a^3*x^3) - (3*b^2)/(a^4*x) - (b^3*x)/(2*a^4*(a
+ b*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

fricas [A] time = 1.03, size = 198, normalized size = 2.44

$$\left[-\frac{210\,b^3x^6+140\,ab^2x^4-28\,a^2bx^2+12\,a^3-105\left(b^3x^7+ab^2x^5\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{60\left(a^4bx^7+a^5x^5\right)},-\frac{105\,b^3x^6+70\,ab^2x^4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(210*b^3*x^6 + 140*a*b^2*x^4 - 28*a^2*b*x^2 + 12*a^3 - 105*(b^3*x^7
+ a*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a
^4*b*x^7 + a^5*x^5), -1/30*(105*b^3*x^6 + 70*a*b^2*x^4 - 14*a^2*b*x^2 + 6*a
^3 + 105*(b^3*x^7 + a*b^2*x^5)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b*x^7 +
a^5*x^5)]
```

giac [A] time = 0.15, size = 70, normalized size = 0.86

$$-\frac{7\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^4}-\frac{b^3x}{2\left(bx^2+a\right)a^4}-\frac{45\,b^2x^4-10\,abx^2+3\,a^2}{15\,a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -7/2*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/2*b^3*x/((b*x^2 + a)*a^4
) - 1/15*(45*b^2*x^4 - 10*a*b*x^2 + 3*a^2)/(a^4*x^5)
```

maple [A] time = 0.01, size = 70, normalized size = 0.86

$$-\frac{b^3x}{2\left(b\,x^2+a\right)a^4}-\frac{7b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^4}-\frac{3b^2}{a^4x}+\frac{2b}{3a^3x^3}-\frac{1}{5a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/5/a^2/x^5-3*b^2/a^4/x+2/3*b/a^3/x^3-1/2/a^4*b^3*x/(b*x^2+a)-7/2/a^4*b^3/
(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.08, size = 75, normalized size = 0.93

$$-\frac{105\,b^3x^6+70\,ab^2x^4-14\,a^2bx^2+6\,a^3}{30\left(a^4bx^7+a^5x^5\right)}-\frac{7\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/30*(105*b^3*x^6 + 70*a*b^2*x^4 - 14*a^2*b*x^2 + 6*a^3)/(a^4*b*x^7 + a^5*
x^5) - 7/2*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4)
```

mupad [B] time = 4.71, size = 70, normalized size = 0.86

$$-\frac{\frac{1}{5\,a}-\frac{7\,b\,x^2}{15\,a^2}+\frac{7\,b^2\,x^4}{3\,a^3}+\frac{7\,b^3\,x^6}{2\,a^4}}{b\,x^7+a\,x^5}-\frac{7\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] - (1/(5*a) - (7*b*x^2)/(15*a^2) + (7*b^2*x^4)/(3*a^3) + (7*b^3*x^6)/(2*a^4)
)/(a*x^5 + b*x^7) - (7*b^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(9/2))
```

sympy [A] time = 0.43, size = 126, normalized size = 1.56

$$\frac{7\sqrt{-\frac{b^5}{a^9}}\log\left(-\frac{a^5\sqrt{-\frac{b^5}{a^9}}}{b^3}+x\right)}{4}-\frac{7\sqrt{-\frac{b^5}{a^9}}\log\left(\frac{a^5\sqrt{-\frac{b^5}{a^9}}}{b^3}+x\right)}{4}+\frac{-6a^3+14a^2bx^2-70ab^2x^4-105b^3x^6}{30a^5x^5+30a^4bx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 7*sqrt(-b**5/a**9)*log(-a**5*sqrt(-b**5/a**9)/b**3 + x)/4 - 7*sqrt(-b**5/a*
*9)*log(a**5*sqrt(-b**5/a**9)/b**3 + x)/4 + (-6*a**3 + 14*a**2*b*x**2 - 70*
a*b**2*x**4 - 105*b**3*x**6)/(30*a**5*x**5 + 30*a**4*b*x**7)
```

3.493 $\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=91

$$\frac{a^5}{6b^6\left(a+bx^2\right)^3}-\frac{5a^4}{4b^6\left(a+bx^2\right)^2}+\frac{5a^3}{b^6\left(a+bx^2\right)}+\frac{5a^2\log\left(a+bx^2\right)}{b^6}-\frac{2ax^2}{b^5}+\frac{x^4}{4b^4}$$

```
[Out] -2*a*x^2/b^5+1/4*x^4/b^4+1/6*a^5/b^6/(b*x^2+a)^3-5/4*a^4/b^6/(b*x^2+a)^2+5*
a^3/b^6/(b*x^2+a)+5*a^2*ln(b*x^2+a)/b^6
```

Rubi [A] time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a^5}{6b^6\left(a+bx^2\right)^3}-\frac{5a^4}{4b^6\left(a+bx^2\right)^2}+\frac{5a^3}{b^6\left(a+bx^2\right)}+\frac{5a^2\log\left(a+bx^2\right)}{b^6}-\frac{2ax^2}{b^5}+\frac{x^4}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-2*a*x^2)/b^5 + x^4/(4*b^4) + a^5/(6*b^6*(a + b*x^2)^3) - (5*a^4)/(4*b^6*(
a + b*x^2)^2) + (5*a^3)/(b^6*(a + b*x^2)) + (5*a^2*Log[a + b*x^2])/b^6
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{x^{11}}{\left(ab+b^2x^2\right)^4}\,dx$$
$$= \frac{1}{2}b^4\,\text{Subst}\left(\int \frac{x^5}{\left(ab+b^2x\right)^4}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^4\,\text{Subst}\left(\int \left(-\frac{4a}{b^9}+\frac{x}{b^8}-\frac{a^5}{b^9(a+bx)^4}+\frac{5a^4}{b^9(a+bx)^3}-\frac{10a^3}{b^9(a+bx)^2}+\frac{10a^2}{b^9(a+bx)}\right.\right.$$
$$= -\frac{2ax^2}{b^5}+\frac{x^4}{4b^4}+\frac{a^5}{6b^6\left(a+bx^2\right)^3}-\frac{5a^4}{4b^6\left(a+bx^2\right)^2}+\frac{5a^3}{b^6\left(a+bx^2\right)}+\frac{5a^2\log\left(a+bx^2\right)}{b^6}$$

Mathematica [A] time = 0.03, size = 78, normalized size = 0.86

$$\frac{\frac{2a^5}{\left(a+bx^2\right)^3}-\frac{15a^4}{\left(a+bx^2\right)^2}+\frac{60a^3}{a+bx^2}+60a^2\log\left(a+bx^2\right)-24abx^2+3b^2x^4}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-24*a*b*x^2 + 3*b^2*x^4 + (2*a^5)/(a + b*x^2)^3 - (15*a^4)/(a + b*x^2)^2 +
 (60*a^3)/(a + b*x^2) + 60*a^2*Log[a + b*x^2])/(12*b^6)
```

fricas [A] time = 0.83, size = 137, normalized size = 1.51

$$\frac{3\,b^5x^{10}-15\,ab^4x^8-63\,a^2b^3x^6-9\,a^3b^2x^4+81\,a^4bx^2+47\,a^5+60\left(a^2b^3x^6+3\,a^3b^2x^4+3\,a^4bx^2+a^5\right)\log\left(bx^2+a\right)}{12\left(b^9x^6+3\,ab^8x^4+3\,a^2b^7x^2+a^3b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^10 - 15*a*b^4*x^8 - 63*a^2*b^3*x^6 - 9*a^3*b^2*x^4 + 81*a^4*b
*x^2 + 47*a^5 + 60*(a^2*b^3*x^6 + 3*a^3*b^2*x^4 + 3*a^4*b*x^2 + a^5)*log(b*
x^2 + a))/(b^9*x^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3*b^6)
```

giac [A] time = 0.17, size = 91, normalized size = 1.00

$$\frac{5\,a^2\log\left(\left|bx^2+a\right|\right)}{b^6}+\frac{b^4x^4-8\,ab^3x^2}{4\,b^8}-\frac{110\,a^2b^3x^6+270\,a^3b^2x^4+225\,a^4bx^2+63\,a^5}{12\left(bx^2+a\right)^3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 5*a^2*log(abs(b*x^2 + a))/b^6 + 1/4*(b^4*x^4 - 8*a*b^3*x^2)/b^8 - 1/12*(110
*a^2*b^3*x^6 + 270*a^3*b^2*x^4 + 225*a^4*b*x^2 + 63*a^5)/((b*x^2 + a)^3*b^6
)
```

maple [A] time = 0.01, size = 86, normalized size = 0.95

$$\frac{x^4}{4b^4}+\frac{a^5}{6\left(b\,x^2+a\right)^3 b^6}-\frac{5a^4}{4\left(b\,x^2+a\right)^2 b^6}-\frac{2a\,x^2}{b^5}+\frac{5a^3}{\left(b\,x^2+a\right)b^6}+\frac{5a^2\ln\left(b\,x^2+a\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -2*a*x^2/b^5+1/4*x^4/b^4+1/6*a^5/b^6/(b*x^2+a)^3-5/4*a^4/b^6/(b*x^2+a)^2+5*
a^3/b^6/(b*x^2+a)+5*a^2*ln(b*x^2+a)/b^6
```

maxima [A] time = 1.39, size = 99, normalized size = 1.09

$$\frac{60\,a^3b^2x^4+105\,a^4bx^2+47\,a^5}{12\left(b^9x^6+3\,ab^8x^4+3\,a^2b^7x^2+a^3b^6\right)}+\frac{5\,a^2\log\left(bx^2+a\right)}{b^6}+\frac{bx^4-8\,ax^2}{4\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(60*a^3*b^2*x^4 + 105*a^4*b*x^2 + 47*a^5)/(b^9*x^6 + 3*a*b^8*x^4 + 3*a
^2*b^7*x^2 + a^3*b^6) + 5*a^2*log(b*x^2 + a)/b^6 + 1/4*(b*x^4 - 8*a*x^2)/b^
5
```

mupad [B] time = 4.48, size = 98, normalized size = 1.08

$$\frac{\frac{47\,a^5}{12\,b}+\frac{35\,a^4\,x^2}{4}+5\,a^3\,b\,x^4}{a^3\,b^5+3\,a^2\,b^6\,x^2+3\,a\,b^7\,x^4+b^8\,x^6}+\frac{x^4}{4\,b^4}-\frac{2\,a\,x^2}{b^5}+\frac{5\,a^2\,\ln\left(b\,x^2+a\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((47*a^5)/(12*b) + (35*a^4*x^2)/4 + 5*a^3*b*x^4)/(a^3*b^5 + b^8*x^6 + 3*a*b
^7*x^4 + 3*a^2*b^6*x^2) + x^4/(4*b^4) - (2*a*x^2)/b^5 + (5*a^2*log(a + b*x^
2))/b^6
```

<u>**sympy**</u> **[A]** time = 0.63, size = 100, normalized size = 1.10

$$\frac{5a^2\log\left(a+bx^2\right)}{b^6}-\frac{2ax^2}{b^5}+\frac{47a^5+105a^4bx^2+60a^3b^2x^4}{12a^3b^6+36a^2b^7x^2+36ab^8x^4+12b^9x^6}+\frac{x^4}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 5*a**2*log(a + b*x**2)/b**6 - 2*a*x**2/b**5 + (47*a**5 + 105*a**4*b*x**2 +
60*a**3*b**2*x**4)/(12*a**3*b**6 + 36*a**2*b**7*x**2 + 36*a*b**8*x**4 + 12*
b**9*x**6) + x**4/(4*b**4)
```

3.494 $\int \frac{x^9}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=77

$$-\frac{a^4}{6b^5\left(a+bx^2\right)^3}+\frac{a^3}{b^5\left(a+bx^2\right)^2}-\frac{3a^2}{b^5\left(a+bx^2\right)}-\frac{2a\log\left(a+bx^2\right)}{b^5}+\frac{x^2}{2b^4}$$

```
[Out] 1/2*x^2/b^4-1/6*a^4/b^5/(b*x^2+a)^3+a^3/b^5/(b*x^2+a)^2-3*a^2/b^5/(b*x^2+a)
-2*a*ln(b*x^2+a)/b^5
```

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{a^4}{6b^5\left(a+bx^2\right)^3}+\frac{a^3}{b^5\left(a+bx^2\right)^2}-\frac{3a^2}{b^5\left(a+bx^2\right)}-\frac{2a\log\left(a+bx^2\right)}{b^5}+\frac{x^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] x^2/(2*b^4) - a^4/(6*b^5*(a + b*x^2)^3) + a^3/(b^5*(a + b*x^2)^2) - (3*a^2)
/(b^5*(a + b*x^2)) - (2*a*Log[a + b*x^2])/b^5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^9}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{x^9}{\left(ab+b^2x^2\right)^4}\,dx$$
$$= \frac{1}{2}b^4\,\text{Subst}\left(\int \frac{x^4}{\left(ab+b^2x\right)^4}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^4\,\text{Subst}\left(\int \left(\frac{1}{b^8}+\frac{a^4}{b^8(a+bx)^4}-\frac{4a^3}{b^8(a+bx)^3}+\frac{6a^2}{b^8(a+bx)^2}-\frac{4a}{b^8(a+bx)}\right) dx, x,$$
$$= \frac{x^2}{2b^4}-\frac{a^4}{6b^5\left(a+bx^2\right)^3}+\frac{a^3}{b^5\left(a+bx^2\right)^2}-\frac{3a^2}{b^5\left(a+bx^2\right)}-\frac{2a\log\left(a+bx^2\right)}{b^5}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.77

$$-\frac{\frac{a^2\left(13a^2+30abx^2+18b^2x^4\right)}{\left(a+bx^2\right)^3}+12a\log\left(a+bx^2\right)-3bx^2}{6b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/6*(-3*b*x^2 + (a^2*(13*a^2 + 30*a*b*x^2 + 18*b^2*x^4))/(a + b*x^2)^3 + 1
2*a*Log[a + b*x^2])/b^5
```

fricas [A] time = 0.93, size = 124, normalized size = 1.61

$$\frac{3\,b^4x^8+9\,ab^3x^6-9\,a^2b^2x^4-27\,a^3bx^2-13\,a^4-12\left(ab^3x^6+3\,a^2b^2x^4+3\,a^3bx^2+a^4\right)\log\left(bx^2+a\right)}{6\left(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^4*x^8 + 9*a*b^3*x^6 - 9*a^2*b^2*x^4 - 27*a^3*b*x^2 - 13*a^4 - 12*(
a*b^3*x^6 + 3*a^2*b^2*x^4 + 3*a^3*b*x^2 + a^4)*log(b*x^2 + a))/(b^8*x^6 + 3
*a*b^7*x^4 + 3*a^2*b^6*x^2 + a^3*b^5)
```

giac [A] time = 0.16, size = 73, normalized size = 0.95

$$\frac{x^2}{2\,b^4}-\frac{2\,a\log\left(\left|bx^2+a\right|\right)}{b^5}+\frac{22\,ab^3x^6+48\,a^2b^2x^4+36\,a^3bx^2+9\,a^4}{6\left(bx^2+a\right)^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^4 - 2*a*log(abs(b*x^2 + a))/b^5 + 1/6*(22*a*b^3*x^6 + 48*a^2*b^2*
x^4 + 36*a^3*b*x^2 + 9*a^4)/((b*x^2 + a)^3*b^5)
```

maple [A] time = 0.01, size = 74, normalized size = 0.96

$$-\frac{a^4}{6\left(b\,x^2+a\right)^3 b^5}+\frac{a^3}{\left(b\,x^2+a\right)^2 b^5}+\frac{x^2}{2b^4}-\frac{3a^2}{\left(b\,x^2+a\right)b^5}-\frac{2a\ln\left(b\,x^2+a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/2*x^2/b^4-1/6*a^4/b^5/(b*x^2+a)^3+a^3/b^5/(b*x^2+a)^2-3*a^2/b^5/(b*x^2+a)
-2*a*ln(b*x^2+a)/b^5
```

maxima [A] time = 1.40, size = 88, normalized size = 1.14

$$-\frac{18\,a^2b^2x^4+30\,a^3bx^2+13\,a^4}{6\left(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5\right)}+\frac{x^2}{2\,b^4}-\frac{2\,a\log\left(bx^2+a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(18*a^2*b^2*x^4 + 30*a^3*b*x^2 + 13*a^4)/(b^8*x^6 + 3*a*b^7*x^4 + 3*a^
2*b^6*x^2 + a^3*b^5) + 1/2*x^2/b^4 - 2*a*log(b*x^2 + a)/b^5
```

mupad [B] time = 4.51, size = 88, normalized size = 1.14

$$\frac{x^2}{2\,b^4}-\frac{\frac{13\,a^4}{6\,b}+5\,a^3\,x^2+3\,a^2\,b\,x^4}{a^3\,b^4+3\,a^2\,b^5\,x^2+3\,a\,b^6\,x^4+b^7\,x^6}-\frac{2\,a\,\ln\left(b\,x^2+a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] x^2/(2*b^4) - ((13*a^4)/(6*b) + 5*a^3*x^2 + 3*a^2*b*x^4)/(a^3*b^4 + b^7*x^6
 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2) - (2*a*log(a + b*x^2))/b^5
```

sympy [A] time = 0.59, size = 90, normalized size = 1.17

$$-\frac{2a\log\left(a+bx^2\right)}{b^5}+\frac{-13a^4-30a^3bx^2-18a^2b^2x^4}{6a^3b^5+18a^2b^6x^2+18ab^7x^4+6b^8x^6}+\frac{x^2}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -2*a*log(a + b*x**2)/b**5 + (-13*a**4 - 30*a**3*b*x**2 - 18*a**2*b**2*x**4)
/(6*a**3*b**5 + 18*a**2*b**6*x**2 + 18*a*b**7*x**4 + 6*b**8*x**6) + x**2/(2
*b**4)
```

3.495 $\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=71

$$\frac{a^3}{6b^4\left(a+bx^2\right)^3}-\frac{3a^2}{4b^4\left(a+bx^2\right)^2}+\frac{3a}{2b^4\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^4}$$

```
[Out] 1/6*a^3/b^4/(b*x^2+a)^3-3/4*a^2/b^4/(b*x^2+a)^2+3/2*a/b^4/(b*x^2+a)+1/2*ln(
b*x^2+a)/b^4
```

Rubi [A] time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a^3}{6b^4\left(a+bx^2\right)^3}-\frac{3a^2}{4b^4\left(a+bx^2\right)^2}+\frac{3a}{2b^4\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] a^3/(6*b^4*(a + b*x^2)^3) - (3*a^2)/(4*b^4*(a + b*x^2)^2) + (3*a)/(2*b^4*(a
 + b*x^2)) + Log[a + b*x^2]/(2*b^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{x^7}{\left(ab+b^2x^2\right)^4}\,dx$$
$$= \frac{1}{2}b^4\,\text{Subst}\left(\int \frac{x^3}{\left(ab+b^2x\right)^4}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^4\,\text{Subst}\left(\int \left(-\frac{a^3}{b^7(a+bx)^4}+\frac{3a^2}{b^7(a+bx)^3}-\frac{3a}{b^7(a+bx)^2}+\frac{1}{b^7(a+bx)}\right)dx, x, x^2\right)$$
$$= \frac{a^3}{6b^4\left(a+bx^2\right)^3}-\frac{3a^2}{4b^4\left(a+bx^2\right)^2}+\frac{3a}{2b^4\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^4}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.70

$$\frac{\frac{a\left(11a^2+27abx^2+18b^2x^4\right)}{\left(a+bx^2\right)^3}+6\log\left(a+bx^2\right)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] ((a*(11*a^2 + 27*a*b*x^2 + 18*b^2*x^4))/(a + b*x^2)^3 + 6*Log[a + b*x^2])/(
12*b^4)
```

fricas [A] time = 0.87, size = 102, normalized size = 1.44

$$\frac{18\,ab^2x^4+27\,a^2bx^2+11\,a^3+6\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\log\left(bx^2+a\right)}{12\left(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(18*a*b^2*x^4 + 27*a^2*b*x^2 + 11*a^3 + 6*(b^3*x^6 + 3*a*b^2*x^4 + 3*a
^2*b*x^2 + a^3)*log(b*x^2 + a))/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^
3*b^4)
```

giac [A] time = 0.17, size = 53, normalized size = 0.75

$$\frac{\log\left(\left|bx^2+a\right|\right)}{2\,b^4}-\frac{11\,b^2x^6+15\,abx^4+6\,a^2x^2}{12\left(bx^2+a\right)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/b^4 - 1/12*(11*b^2*x^6 + 15*a*b*x^4 + 6*a^2*x^2)/((
b*x^2 + a)^3*b^3)
```

maple [A] time = 0.01, size = 64, normalized size = 0.90

$$\frac{a^{3}}{6\left(b\,x^{2}+a\right)^{3} b^{4}}-\frac{3a^{2}}{4\left(b\,x^{2}+a\right)^{2} b^{4}}+\frac{3a}{2\left(b\,x^{2}+a\right) b^{4}}+\frac{\ln\left(b\,x^{2}+a\right)}{2b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/6*a^3/b^4/(b*x^2+a)^3-3/4*a^2/b^4/(b*x^2+a)^2+3/2*a/b^4/(b*x^2+a)+1/2*ln(
b*x^2+a)/b^4
```

maxima [A] time = 1.31, size = 77, normalized size = 1.08

$$\frac{18\,ab^{2}x^{4}+27\,a^{2}bx^{2}+11\,a^{3}}{12\left(b^{7}x^{6}+3\,ab^{6}x^{4}+3\,a^{2}b^{5}x^{2}+a^{3}b^{4}\right)}+\frac{\log\left(bx^{2}+a\right)}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(18*a*b^2*x^4 + 27*a^2*b*x^2 + 11*a^3)/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*
b^5*x^2 + a^3*b^4) + 1/2*log(b*x^2 + a)/b^4
```

mupad [B] time = 4.33, size = 75, normalized size = 1.06

$$\frac{\frac{11\,a^{3}}{12\,b^{4}}+\frac{3\,a\,x^{4}}{2\,b^{2}}+\frac{9\,a^{2}\,x^{2}}{4\,b^{3}}}{a^{3}+3\,a^{2}\,b\,x^{2}+3\,a\,b^{2}\,x^{4}+b^{3}\,x^{6}}+\frac{\ln\left(b\,x^{2}+a\right)}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((11*a^3)/(12*b^4) + (3*a*x^4)/(2*b^2) + (9*a^2*x^2)/(4*b^3))/(a^3 + b^3*x^
6 + 3*a^2*b*x^2 + 3*a*b^2*x^4) + log(a + b*x^2)/(2*b^4)
```

sympy [A] time = 0.47, size = 76, normalized size = 1.07

$$\frac{11a^{3}+27a^{2}bx^{2}+18ab^{2}x^{4}}{12a^{3}b^{4}+36a^{2}b^{5}x^{2}+36ab^{6}x^{4}+12b^{7}x^{6}}+\frac{\log\left(a+bx^{2}\right)}{2b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (11*a**3 + 27*a**2*b*x**2 + 18*a*b**2*x**4)/(12*a**3*b**4 + 36*a**2*b**5*x*
*2 + 36*a*b**6*x**4 + 12*b**7*x**6) + log(a + b*x**2)/(2*b**4)
```

3.496 $\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^2} \, dx$

Optimal. Leaf size=19

$$\frac{x^6}{6a\left(a+bx^2\right)^3}$$

```
[Out] 1/6*x^6/a/(b*x^2+a)^3
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 264}

$$\frac{x^6}{6a\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] x^6/(6*a*(a + b*x^2)^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^2} \, dx &= b^4 \int \frac{x^5}{\left(ab+b^2x^2\right)^4} \, dx \\
&= \frac{x^6}{6a\left(a+bx^2\right)^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.84

$$-\frac{a^2+3abx^2+3b^2x^4}{6b^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/6*(a^2 + 3*a*b*x^2 + 3*b^2*x^4)/(b^3*(a + b*x^2)^3)
```

fricas [B] time = 0.91, size = 58, normalized size = 3.05

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\left(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 +
 a^3*b^3)
```

giac [A] time = 0.19, size = 33, normalized size = 1.74

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\left(bx^2+a\right)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/((b*x^2 + a)^3*b^3)
```

maple [B] time = 0.01, size = 48, normalized size = 2.53

$$-\frac{a^2}{6\left(b\,x^2+a\right)^3b^3}+\frac{a}{2\left(b\,x^2+a\right)^2b^3}-\frac{1}{2\left(b\,x^2+a\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/6*a^2/b^3/(b*x^2+a)^3+1/2*a/b^3/(b*x^2+a)^2-1/2/b^3/(b*x^2+a)
```

maxima [B] time = 1.38, size = 58, normalized size = 3.05

$$-\frac{3\,b^{2}x^{4}+3\,abx^{2}+a^{2}}{6\left(b^{6}x^{6}+3\,ab^{5}x^{4}+3\,a^{2}b^{4}x^{2}+a^{3}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 +
 a^3*b^3)
```

mupad [B] time = 4.29, size = 60, normalized size = 3.16

$$-\frac{a^{2}+3\,a\,b\,x^{2}+3\,b^{2}\,x^{4}}{6\,a^{3}\,b^{3}+18\,a^{2}\,b^{4}\,x^{2}+18\,a\,b^{5}\,x^{4}+6\,b^{6}\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] -(a^2 + 3*b^2*x^4 + 3*a*b*x^2)/(6*a^3*b^3 + 6*b^6*x^6 + 18*a*b^5*x^4 + 18*a
^2*b^4*x^2)
```

sympy [B] time = 0.40, size = 60, normalized size = 3.16

$$\frac{-a^{2}-3abx^{2}-3b^{2}x^{4}}{6a^{3}b^{3}+18a^{2}b^{4}x^{2}+18ab^{5}x^{4}+6b^{6}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (-a**2 - 3*a*b*x**2 - 3*b**2*x**4)/(6*a**3*b**3 + 18*a**2*b**4*x**2 + 18*a*
b**5*x**4 + 6*b**6*x**6)
```

3.497 $\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^2} dx$

Optimal. Leaf size=34

$$\frac{a}{6b^2\left(a+bx^2\right)^3}-\frac{1}{4b^2\left(a+bx^2\right)^2}$$

```
[Out] 1/6*a/b^2/(b*x^2+a)^3-1/4/b^2/(b*x^2+a)^2
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a}{6b^2\left(a+bx^2\right)^3}-\frac{1}{4b^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] a/(6*b^2*(a + b*x^2)^3) - 1/(4*b^2*(a + b*x^2)^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{x^3}{\left(ab+b^2x^2\right)^4}\,dx$$
$$= \frac{1}{2}b^4\,\text{Subst}\left(\int \frac{x}{\left(ab+b^2x\right)^4}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^4\,\text{Subst}\left(\int \left(-\frac{a}{b^5(a+bx)^4}+\frac{1}{b^5(a+bx)^3}\right)dx, x, x^2\right)$$
$$= \frac{a}{6b^2\left(a+bx^2\right)^3} - \frac{1}{4b^2\left(a+bx^2\right)^2}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.71

$$-\frac{a+3bx^2}{12b^2\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
[Out] -1/12*(a + 3*b*x^2)/(b^2*(a + b*x^2)^3)
```

fricas [A] time = 1.09, size = 47, normalized size = 1.38

$$-\frac{3\,bx^2+a}{12\left(b^5x^6+3\,ab^4x^4+3\,a^2b^3x^2+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
[Out] -1/12*(3*b*x^2 + a)/(b^5*x^6 + 3*a*b^4*x^4 + 3*a^2*b^3*x^2 + a^3*b^2)
```

giac [A] time = 0.16, size = 22, normalized size = 0.65

$$-\frac{3\,bx^2+a}{12\left(bx^2+a\right)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/12*(3*b*x^2 + a)/((b*x^2 + a)^3*b^2)
```

maple [A] time = 0.01, size = 31, normalized size = 0.91

$$\frac{a}{6\left(b\,x^2+a\right)^3 b^2}-\frac{1}{4\left(b\,x^2+a\right)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/6*a/b^2/(b*x^2+a)^3-1/4/b^2/(b*x^2+a)^2
```

maxima [A] time = 1.35, size = 47, normalized size = 1.38

$$-\frac{3\,bx^2+a}{12\left(b^5x^6+3\,ab^4x^4+3\,a^2b^3x^2+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b*x^2 + a)/(b^5*x^6 + 3*a*b^4*x^4 + 3*a^2*b^3*x^2 + a^3*b^2)
```

mupad [B] time = 4.23, size = 48, normalized size = 1.41

$$-\frac{\frac{a}{12\,b^2}+\frac{x^2}{4\,b}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] -(a/(12*b^2) + x^2/(4*b))/(a^3 + b^3*x^6 + 3*a^2*b*x^2 + 3*a*b^2*x^4)
```

sympy [A] time = 0.37, size = 48, normalized size = 1.41

$$\frac{-a-3bx^2}{12a^3b^2+36a^2b^3x^2+36ab^4x^4+12b^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (-a - 3*b*x**2)/(12*a**3*b**2 + 36*a**2*b**3*x**2 + 36*a*b**4*x**4 + 12*b**
5*x**6)
```

3.498 $\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{6b\left(a+bx^2\right)^3}$$

```
[Out] -1/6/b/(b*x^2+a)^3
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {28, 261}

$$-\frac{1}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/(6*b*(a + b*x^2)^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4\int \frac{x}{\left(ab+b^2x^2\right)^4}\,dx\\ &= -\frac{1}{6b\left(a+bx^2\right)^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/6*1/(b*(a + b*x^2)^3)
```

fricas [B] time = 0.53, size = 37, normalized size = 2.31

$$-\frac{1}{6\left(b^4x^6+3\,ab^3x^4+3\,a^2b^2x^2+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6/(b^4*x^6 + 3*a*b^3*x^4 + 3*a^2*b^2*x^2 + a^3*b)
```

giac [A] time = 0.15, size = 14, normalized size = 0.88

$$-\frac{1}{6\left(bx^2+a\right)^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/6/((b*x^2 + a)^3*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{6\left(b\,x^2+a\right)^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/6/b/(b*x^2+a)^3
```

maxima [B] time = 1.34, size = 37, normalized size = 2.31

$$-\frac{1}{6\left(b^4x^6+3\,ab^3x^4+3\,a^2b^2x^2+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6/(b^4*x^6 + 3*a*b^3*x^4 + 3*a^2*b^2*x^2 + a^3*b)
```

mupad [B] time = 4.28, size = 39, normalized size = 2.44

$$-\frac{1}{6\,a^3\,b+18\,a^2\,b^2\,x^2+18\,a\,b^3\,x^4+6\,b^4\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] -1/(6*a^3*b + 6*b^4*x^6 + 18*a*b^3*x^4 + 18*a^2*b^2*x^2)
```

sympy [B] time = 0.33, size = 39, normalized size = 2.44

$$-\frac{1}{6a^3b+18a^2b^2x^2+18ab^3x^4+6b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -1/(6*a**3*b + 18*a**2*b**2*x**2 + 18*a*b**3*x**4 + 6*b**4*x**6)
```

3.499 $\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^2} dx$

Optimal. Leaf size=70

$$-\frac{\log\left(a+bx^2\right)}{2a^4}+\frac{\log(x)}{a^4}+\frac{1}{2a^3\left(a+bx^2\right)}+\frac{1}{4a^2\left(a+bx^2\right)^2}+\frac{1}{6a\left(a+bx^2\right)^3}$$

```
[Out] 1/6/a/(b*x^2+a)^3+1/4/a^2/(b*x^2+a)^2+1/2/a^3/(b*x^2+a)+ln(x)/a^4-1/2*ln(b*
x^2+a)/a^4
```

Rubi [A] time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$\frac{1}{2a^3\left(a+bx^2\right)}+\frac{1}{4a^2\left(a+bx^2\right)^2}-\frac{\log\left(a+bx^2\right)}{2a^4}+\frac{\log(x)}{a^4}+\frac{1}{6a\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] 1/(6*a*(a + b*x^2)^3) + 1/(4*a^2*(a + b*x^2)^2) + 1/(2*a^3*(a + b*x^2)) + L
og[x]/a^4 - Log[a + b*x^2]/(2*a^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4\int \frac{1}{x\left(ab+b^2x^2\right)^4}\,dx\\ &= \frac{1}{2}b^4\operatorname{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^4}\,dx,x,x^2\right)\\ &= \frac{1}{2}b^4\operatorname{Subst}\left(\int \left(\frac{1}{a^4b^4x}-\frac{1}{ab^3(a+bx)^4}-\frac{1}{a^2b^3(a+bx)^3}-\frac{1}{a^3b^3(a+bx)^2}-\frac{1}{a^4b^3(a\right.\right.\\ &= \frac{1}{6a\left(a+bx^2\right)^3}+\frac{1}{4a^2\left(a+bx^2\right)^2}+\frac{1}{2a^3\left(a+bx^2\right)}+\frac{\log(x)}{a^4}-\frac{\log\left(a+bx^2\right)}{2a^4}\end{aligned}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.77

$$\frac{\frac{a\left(11a^2+15abx^2+6b^2x^4\right)}{\left(a+bx^2\right)^3}-6\log\left(a+bx^2\right)+12\log(x)}{12a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] ((a*(11*a^2 + 15*a*b*x^2 + 6*b^2*x^4))/(a + b*x^2)^3 + 12*Log[x] - 6*Log[a
+ b*x^2])/(12*a^4)
```

fricas [B] time = 1.21, size = 134, normalized size = 1.91

$$\frac{6\,ab^2x^4+15\,a^2bx^2+11\,a^3-6\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\log\left(bx^2+a\right)+12\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right.}{12\left(a^4b^3x^6+3\,a^5b^2x^4+3\,a^6bx^2+a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*a*b^2*x^4 + 15*a^2*b*x^2 + 11*a^3 - 6*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^
2*b*x^2 + a^3)*log(b*x^2 + a) + 12*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a
^3)*log(x))/(a^4*b^3*x^6 + 3*a^5*b^2*x^4 + 3*a^6*b*x^2 + a^7)
```

giac [A] time = 0.15, size = 70, normalized size = 1.00

$$\frac{\log\left(x^2\right)}{2\,a^4}-\frac{\log\left(\left|bx^2+a\right|\right)}{2\,a^4}+\frac{11\,b^3x^6+39\,ab^2x^4+48\,a^2bx^2+22\,a^3}{12\left(bx^2+a\right)^3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^4 - 1/2*log(abs(b*x^2 + a))/a^4 + 1/12*(11*b^3*x^6 + 39*a*b^
2*x^4 + 48*a^2*b*x^2 + 22*a^3)/((b*x^2 + a)^3*a^4)
```

maple [A] time = 0.01, size = 63, normalized size = 0.90

$$\frac{1}{6\left(b\,x^{2}+a\right)^{3} a}+\frac{1}{4\left(b\,x^{2}+a\right)^{2} a^{2}}+\frac{1}{2\left(b\,x^{2}+a\right) a^{3}}+\frac{\ln(x)}{a^{4}}-\frac{\ln\left(b\,x^{2}+a\right)}{2a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/6/a/(b*x^2+a)^3+1/4/a^2/(b*x^2+a)^2+1/2/a^3/(b*x^2+a)+ln(x)/a^4-1/2*ln(b*
x^2+a)/a^4
```

maxima [A] time = 1.42, size = 82, normalized size = 1.17

$$\frac{6\,b^{2}x^{4}+15\,abx^{2}+11\,a^{2}}{12\left(a^{3}b^{3}x^{6}+3\,a^{4}b^{2}x^{4}+3\,a^{5}bx^{2}+a^{6}\right)}-\frac{\log\left(bx^{2}+a\right)}{2\,a^{4}}+\frac{\log\left(x^{2}\right)}{2\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(6*b^2*x^4 + 15*a*b*x^2 + 11*a^2)/(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5
*b*x^2 + a^6) - 1/2*log(b*x^2 + a)/a^4 + 1/2*log(x^2)/a^4
```

mupad [B] time = 4.47, size = 78, normalized size = 1.11

$$\frac{\ln(x)}{a^{4}}+\frac{\frac{11}{12\,a}+\frac{5\,b\,x^{2}}{4\,a^{2}}+\frac{b^{2}\,x^{4}}{2\,a^{3}}}{a^{3}+3\,a^{2}\,b\,x^{2}+3\,a\,b^{2}\,x^{4}+b^{3}\,x^{6}}-\frac{\ln\left(b\,x^{2}+a\right)}{2\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] log(x)/a^4 + (11/(12*a) + (5*b*x^2)/(4*a^2) + (b^2*x^4)/(2*a^3))/(a^3 + b^3
*x^6 + 3*a^2*b*x^2 + 3*a*b^2*x^4) - log(a + b*x^2)/(2*a^4)
```

sympy [A] time = 0.56, size = 80, normalized size = 1.14

$$\frac{11a^{2}+15abx^{2}+6b^{2}x^{4}}{12a^{6}+36a^{5}bx^{2}+36a^{4}b^{2}x^{4}+12a^{3}b^{3}x^{6}}+\frac{\log\left(x\right)}{a^{4}}-\frac{\log\left(\frac{a}{b}+x^{2}\right)}{2a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (11*a**2 + 15*a*b*x**2 + 6*b**2*x**4)/(12*a**6 + 36*a**5*b*x**2 + 36*a**4*b
**2*x**4 + 12*a**3*b**3*x**6) + log(x)/a**4 - log(a/b + x**2)/(2*a**4)
```

3.500 $\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=84

$$\frac{2b\log\left(a+bx^2\right)}{a^5}-\frac{4b\log(x)}{a^5}-\frac{3b}{2a^4\left(a+bx^2\right)}-\frac{1}{2a^4x^2}-\frac{b}{2a^3\left(a+bx^2\right)^2}-\frac{b}{6a^2\left(a+bx^2\right)^3}$$

```
[Out] -1/2/a^4/x^2-1/6*b/a^2/(b*x^2+a)^3-1/2*b/a^3/(b*x^2+a)^2-3/2*b/a^4/(b*x^2+a
)-4*b*ln(x)/a^5+2*b*ln(b*x^2+a)/a^5
```

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$-\frac{3b}{2a^4\left(a+bx^2\right)}-\frac{b}{2a^3\left(a+bx^2\right)^2}-\frac{b}{6a^2\left(a+bx^2\right)^3}+\frac{2b\log\left(a+bx^2\right)}{a^5}-\frac{4b\log(x)}{a^5}-\frac{1}{2a^4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -1/(2*a^4*x^2) - b/(6*a^2*(a + b*x^2)^3) - b/(2*a^3*(a + b*x^2)^2) - (3*b)/
(2*a^4*(a + b*x^2)) - (4*b*Log[x])/a^5 + (2*b*Log[a + b*x^2])/a^5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{x^3\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{1}{2}b^4\,\text{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)^4}\,dx, x, x^2\right)$$

$$= \frac{1}{2}b^4\,\text{Subst}\left(\int \left(\frac{1}{a^4b^4x^2}-\frac{4}{a^5b^3x}+\frac{1}{a^2b^2(a+bx)^4}+\frac{2}{a^3b^2(a+bx)^3}+\frac{3}{a^4b^2(a+b}\right.\right.$$

$$= -\frac{1}{2a^4x^2}-\frac{b}{6a^2\left(a+bx^2\right)^3}-\frac{b}{2a^3\left(a+bx^2\right)^2}-\frac{3b}{2a^4\left(a+bx^2\right)}-\frac{4b\log(x)}{a^5}+\frac{2b\,\text{lo}}{}$$

Mathematica [A] time = 0.07, size = 70, normalized size = 0.83

$$-\frac{\frac{a\left(3a^3+22a^2bx^2+30ab^2x^4+12b^3x^6\right)}{x^2\left(a+bx^2\right)^3}-12b\log\left(a+bx^2\right)+24b\log(x)}{6a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -1/6*((a*(3*a^3 + 22*a^2*b*x^2 + 30*a*b^2*x^4 + 12*b^3*x^6))/(x^2*(a + b*x^
2)^3) + 24*b*Log[x] - 12*b*Log[a + b*x^2])/a^5
```

fricas [B] time = 0.81, size = 163, normalized size = 1.94

$$-\frac{12\,ab^3x^6+30\,a^2b^2x^4+22\,a^3bx^2+3\,a^4-12\left(b^4x^8+3\,ab^3x^6+3\,a^2b^2x^4+a^3bx^2\right)\log\left(bx^2+a\right)+24\left(b^4x^8+3\,a\right.}{6\left(a^5b^3x^8+3\,a^6b^2x^6+3\,a^7bx^4+a^8x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(12*a*b^3*x^6 + 30*a^2*b^2*x^4 + 22*a^3*b*x^2 + 3*a^4 - 12*(b^4*x^8 +
3*a*b^3*x^6 + 3*a^2*b^2*x^4 + a^3*b*x^2)*log(b*x^2 + a) + 24*(b^4*x^8 + 3*a
*b^3*x^6 + 3*a^2*b^2*x^4 + a^3*b*x^2)*log(x))/(a^5*b^3*x^8 + 3*a^6*b^2*x^6
+ 3*a^7*b*x^4 + a^8*x^2)
```

giac [A] time = 0.16, size = 93, normalized size = 1.11

$$-\frac{2\,b\log\left(x^2\right)}{a^5}+\frac{2\,b\log\left(\left|bx^2+a\right|\right)}{a^5}+\frac{4\,bx^2-a}{2\,a^5x^2}-\frac{22\,b^4x^6+75\,ab^3x^4+87\,a^2b^2x^2+35\,a^3b}{6\left(bx^2+a\right)^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -2*b*log(x^2)/a^5 + 2*b*log(abs(b*x^2 + a))/a^5 + 1/2*(4*b*x^2 - a)/(a^5*x^
2) - 1/6*(22*b^4*x^6 + 75*a*b^3*x^4 + 87*a^2*b^2*x^2 + 35*a^3*b)/((b*x^2 +
a)^3*a^5)
```

maple [A] time = 0.02, size = 77, normalized size = 0.92

$$-\frac{b}{6\left(b\,x^2+a\right)^3a^2}-\frac{b}{2\left(b\,x^2+a\right)^2a^3}-\frac{3b}{2\left(b\,x^2+a\right)a^4}-\frac{4b\ln(x)}{a^5}+\frac{2b\ln\left(b\,x^2+a\right)}{a^5}-\frac{1}{2a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/2/a^4/x^2-1/6*b/a^2/(b*x^2+a)^3-1/2*b/a^3/(b*x^2+a)^2-3/2*b/a^4/(b*x^2+a
)-4*b*ln(x)/a^5+2*b*ln(b*x^2+a)/a^5
```

maxima [A] time = 1.40, size = 99, normalized size = 1.18

$$-\frac{12\,b^3x^6+30\,ab^2x^4+22\,a^2bx^2+3\,a^3}{6\left(a^4b^3x^8+3\,a^5b^2x^6+3\,a^6bx^4+a^7x^2\right)}+\frac{2\,b\log\left(bx^2+a\right)}{a^5}-\frac{2\,b\log\left(x^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(12*b^3*x^6 + 30*a*b^2*x^4 + 22*a^2*b*x^2 + 3*a^3)/(a^4*b^3*x^8 + 3*a^
5*b^2*x^6 + 3*a^6*b*x^4 + a^7*x^2) + 2*b*log(b*x^2 + a)/a^5 - 2*b*log(x^2)/
a^5
```

mupad [B] time = 0.15, size = 97, normalized size = 1.15

$$\frac{2\,b\,\ln\left(b\,x^2+a\right)}{a^5}-\frac{\frac{1}{2\,a}+\frac{11\,b\,x^2}{3\,a^2}+\frac{5\,b^2\,x^4}{a^3}+\frac{2\,b^3\,x^6}{a^4}}{a^3\,x^2+3\,a^2\,b\,x^4+3\,a\,b^2\,x^6+b^3\,x^8}-\frac{4\,b\,\ln(x)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] (2*b*log(a + b*x^2))/a^5 - (1/(2*a) + (11*b*x^2)/(3*a^2) + (5*b^2*x^4)/a^3
+ (2*b^3*x^6)/a^4)/(a^3*x^2 + b^3*x^8 + 3*a^2*b*x^4 + 3*a*b^2*x^6) - (4*b*l
og(x))/a^5
```

sympy [A] time = 0.67, size = 102, normalized size = 1.21

$$\frac{-3a^3-22a^2bx^2-30ab^2x^4-12b^3x^6}{6a^7x^2+18a^6bx^4+18a^5b^2x^6+6a^4b^3x^8}-\frac{4b\log(x)}{a^5}+\frac{2b\log\left(\frac{a}{b}+x^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (-3*a**3 - 22*a**2*b*x**2 - 30*a*b**2*x**4 - 12*b**3*x**6)/(6*a**7*x**2 + 1
8*a**6*b*x**4 + 18*a**5*b**2*x**6 + 6*a**4*b**3*x**8) - 4*b*log(x)/a**5 + 2
*b*log(a/b + x**2)/a**5
```

3.501 $\int \frac{1}{x^5\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=101

$$-\frac{5b^2\log\left(a+bx^2\right)}{a^6}+\frac{10b^2\log(x)}{a^6}+\frac{3b^2}{a^5\left(a+bx^2\right)}+\frac{2b}{a^5x^2}+\frac{3b^2}{4a^4\left(a+bx^2\right)^2}-\frac{1}{4a^4x^4}+\frac{b^2}{6a^3\left(a+bx^2\right)^3}$$

```
[Out] -1/4/a^4/x^4+2*b/a^5/x^2+1/6*b^2/a^3/(b*x^2+a)^3+3/4*b^2/a^4/(b*x^2+a)^2+3*
b^2/a^5/(b*x^2+a)+10*b^2*ln(x)/a^6-5*b^2*ln(b*x^2+a)/a^6
```

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$\frac{3b^2}{a^5\left(a+bx^2\right)}+\frac{3b^2}{4a^4\left(a+bx^2\right)^2}+\frac{b^2}{6a^3\left(a+bx^2\right)^3}-\frac{5b^2\log\left(a+bx^2\right)}{a^6}+\frac{10b^2\log(x)}{a^6}+\frac{2b}{a^5x^2}-\frac{1}{4a^4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -1/(4*a^4*x^4) + (2*b)/(a^5*x^2) + b^2/(6*a^3*(a + b*x^2)^3) + (3*b^2)/(4*a
^4*(a + b*x^2)^2) + (3*b^2)/(a^5*(a + b*x^2)) + (10*b^2*Log[x])/a^6 - (5*b^
2*Log[a + b*x^2])/a^6
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{x^5\left(ab+b^2x^2\right)^4}\,dx$$
$$= \frac{1}{2}b^4\,\text{Subst}\left(\int \frac{1}{x^3\left(ab+b^2x\right)^4}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^4\,\text{Subst}\left(\int\left(\frac{1}{a^4b^4x^3}-\frac{4}{a^5b^3x^2}+\frac{10}{a^6b^2x}-\frac{1}{a^3b(a+bx)^4}-\frac{3}{a^4b(a+bx)^3}-\frac{}{a^5b\right.\right.$$
$$= -\frac{1}{4a^4x^4}+\frac{2b}{a^5x^2}+\frac{b^2}{6a^3\left(a+bx^2\right)^3}+\frac{3b^2}{4a^4\left(a+bx^2\right)^2}+\frac{3b^2}{a^5\left(a+bx^2\right)}+\frac{10b^2\log(}{a^6}$$

Mathematica [A] time = 0.06, size = 85, normalized size = 0.84

$$\frac{\frac{a\left(-3a^4+15a^3bx^2+110a^2b^2x^4+150ab^3x^6+60b^4x^8\right)}{x^4\left(a+bx^2\right)^3}-60b^2\log\left(a+bx^2\right)+120b^2\log(x)}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] ((a*(-3*a^4 + 15*a^3*b*x^2 + 110*a^2*b^2*x^4 + 150*a*b^3*x^6 + 60*b^4*x^8))
/(x^4*(a + b*x^2)^3) + 120*b^2*Log[x] - 60*b^2*Log[a + b*x^2])/(12*a^6)
```

fricas [A] time = 1.13, size = 178, normalized size = 1.76

$$\frac{60\,ab^4x^8+150\,a^2b^3x^6+110\,a^3b^2x^4+15\,a^4bx^2-3\,a^5-60\left(b^5x^{10}+3\,ab^4x^8+3\,a^2b^3x^6+a^3b^2x^4\right)\log\left(bx^2+a\right)}{12\left(a^6b^3x^{10}+3\,a^7b^2x^8+3\,a^8bx^6+a^9x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(60*a*b^4*x^8 + 150*a^2*b^3*x^6 + 110*a^3*b^2*x^4 + 15*a^4*b*x^2 - 3*a
^5 - 60*(b^5*x^10 + 3*a*b^4*x^8 + 3*a^2*b^3*x^6 + a^3*b^2*x^4)*log(b*x^2 +
a) + 120*(b^5*x^10 + 3*a*b^4*x^8 + 3*a^2*b^3*x^6 + a^3*b^2*x^4)*log(x))/(a^
6*b^3*x^10 + 3*a^7*b^2*x^8 + 3*a^8*b*x^6 + a^9*x^4)
```

giac [A] time = 0.16, size = 108, normalized size = 1.07

$$\frac{5\,b^2\log\left(x^2\right)}{a^6}-\frac{5\,b^2\log\left(\left|bx^2+a\right|\right)}{a^6}+\frac{110\,b^5x^6+366\,ab^4x^4+411\,a^2b^3x^2+157\,a^3b^2}{12\left(bx^2+a\right)^3a^6}-\frac{30\,b^2x^4-8\,abx^2+a^2}{4\,a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 5*b^2*log(x^2)/a^6 - 5*b^2*log(abs(b*x^2 + a))/a^6 + 1/12*(110*b^5*x^6 + 36
6*a*b^4*x^4 + 411*a^2*b^3*x^2 + 157*a^3*b^2)/((b*x^2 + a)^3*a^6) - 1/4*(30*
b^2*x^4 - 8*a*b*x^2 + a^2)/(a^6*x^4)
```

maple [A] time = 0.02, size = 96, normalized size = 0.95

$$\frac{b^2}{6\left(b\,x^2+a\right)^3a^3}+\frac{3b^2}{4\left(b\,x^2+a\right)^2a^4}+\frac{3b^2}{\left(b\,x^2+a\right)a^5}+\frac{10b^2\ln(x)}{a^6}-\frac{5b^2\ln\left(b\,x^2+a\right)}{a^6}+\frac{2b}{a^5x^2}-\frac{1}{4a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/4/a^4/x^4+2*b/a^5/x^2+1/6*b^2/a^3/(b*x^2+a)^3+3/4*b^2/a^4/(b*x^2+a)^2+3*
b^2/a^5/(b*x^2+a)+10*b^2*ln(x)/a^6-5*b^2*ln(b*x^2+a)/a^6
```

maxima [A] time = 1.42, size = 114, normalized size = 1.13

$$\frac{60\,b^4x^8+150\,ab^3x^6+110\,a^2b^2x^4+15\,a^3bx^2-3\,a^4}{12\left(a^5b^3x^{10}+3\,a^6b^2x^8+3\,a^7bx^6+a^8x^4\right)}-\frac{5\,b^2\log\left(bx^2+a\right)}{a^6}+\frac{5\,b^2\log\left(x^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(60*b^4*x^8 + 150*a*b^3*x^6 + 110*a^2*b^2*x^4 + 15*a^3*b*x^2 - 3*a^4)/
(a^5*b^3*x^10 + 3*a^6*b^2*x^8 + 3*a^7*b*x^6 + a^8*x^4) - 5*b^2*log(b*x^2 +
a)/a^6 + 5*b^2*log(x^2)/a^6
```

mupad [B] time = 4.66, size = 111, normalized size = 1.10

$$\frac{\frac{5\,b\,x^2}{4\,a^2}-\frac{1}{4\,a}+\frac{55\,b^2\,x^4}{6\,a^3}+\frac{25\,b^3\,x^6}{2\,a^4}+\frac{5\,b^4\,x^8}{a^5}}{a^3\,x^4+3\,a^2\,b\,x^6+3\,a\,b^2\,x^8+b^3\,x^{10}}-\frac{5\,b^2\,\ln\left(b\,x^2+a\right)}{a^6}+\frac{10\,b^2\,\ln(x)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] ((5*b*x^2)/(4*a^2) - 1/(4*a) + (55*b^2*x^4)/(6*a^3) + (25*b^3*x^6)/(2*a^4)
+ (5*b^4*x^8)/a^5)/(a^3*x^4 + b^3*x^10 + 3*a^2*b*x^6 + 3*a*b^2*x^8) - (5*b^
2*log(a + b*x^2))/a^6 + (10*b^2*log(x))/a^6
```

sympy [A] time = 0.72, size = 116, normalized size = 1.15

$$\frac{-3a^4 + 15a^3bx^2 + 110a^2b^2x^4 + 150ab^3x^6 + 60b^4x^8}{12a^8x^4 + 36a^7bx^6 + 36a^6b^2x^8 + 12a^5b^3x^{10}} + \frac{10b^2\log(x)}{a^6} - \frac{5b^2\log\left(\frac{a}{b} + x^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (-3*a**4 + 15*a**3*b*x**2 + 110*a**2*b**2*x**4 + 150*a*b**3*x**6 + 60*b**4*
x**8)/(12*a**8*x**4 + 36*a**7*b*x**6 + 36*a**6*b**2*x**8 + 12*a**5*b**3*x**
10) + 10*b**2*log(x)/a**6 - 5*b**2*log(a/b + x**2)/a**6
```

3.502 $\int \frac{x^{12}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=117

$$-\frac{231a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{13/2}}+\frac{231a^2x}{16b^6}-\frac{77ax^3}{16b^5}-\frac{33x^7}{16b^3\left(a+bx^2\right)}-\frac{11x^9}{24b^2\left(a+bx^2\right)^2}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}+\frac{231x^5}{80b^4}$$

```
[Out] 231/16*a^2*x/b^6-77/16*a*x^3/b^5+231/80*x^5/b^4-1/6*x^11/b/(b*x^2+a)^3-11/2
4*x^9/b^2/(b*x^2+a)^2-33/16*x^7/b^3/(b*x^2+a)-231/16*a^(5/2)*arctan(x*b^(1/
2)/a^(1/2))/b^(13/2)
```

Rubi [A] time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 302, 205}

$$\frac{231a^2x}{16b^6}-\frac{231a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{13/2}}-\frac{11x^9}{24b^2\left(a+bx^2\right)^2}-\frac{33x^7}{16b^3\left(a+bx^2\right)}-\frac{77ax^3}{16b^5}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}+\frac{231x^5}{80b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (231*a^2*x)/(16*b^6) - (77*a*x^3)/(16*b^5) + (231*x^5)/(80*b^4) - x^11/(6*b
*(a + b*x^2)^3) - (11*x^9)/(24*b^2*(a + b*x^2)^2) - (33*x^7)/(16*b^3*(a + b
*x^2)) - (231*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*b^(13/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^{12}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{x^{12}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{x^{11}}{6b\left(a+bx^2\right)^3}+\frac{1}{6}\left(11b^2\right)\int \frac{x^{10}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{x^{11}}{6b\left(a+bx^2\right)^3}-\frac{11x^9}{24b^2\left(a+bx^2\right)^2}+\frac{33}{8}\int \frac{x^8}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{x^{11}}{6b\left(a+bx^2\right)^3}-\frac{11x^9}{24b^2\left(a+bx^2\right)^2}-\frac{33x^7}{16b^3\left(a+bx^2\right)}+\frac{231\int \frac{x^6}{ab+b^2x^2}\,dx}{16b^2}$$

$$= -\frac{x^{11}}{6b\left(a+bx^2\right)^3}-\frac{11x^9}{24b^2\left(a+bx^2\right)^2}-\frac{33x^7}{16b^3\left(a+bx^2\right)}+\frac{231\int \left(\frac{a^2}{b^4}-\frac{ax^2}{b^3}+\frac{x^4}{b^2}-\frac{}{b^3(al}\right.}{16b^2}$$

$$= \frac{231a^2x}{16b^6}-\frac{77ax^3}{16b^5}+\frac{231x^5}{80b^4}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}-\frac{11x^9}{24b^2\left(a+bx^2\right)^2}-\frac{33x^7}{16b^3\left(a+bx^2\right)}-$$

$$= \frac{231a^2x}{16b^6}-\frac{77ax^3}{16b^5}+\frac{231x^5}{80b^4}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}-\frac{11x^9}{24b^2\left(a+bx^2\right)^2}-\frac{33x^7}{16b^3\left(a+bx^2\right)}-$$

Mathematica [A] time = 0.06, size = 99, normalized size = 0.85

$$\frac{3465a^5x+9240a^4bx^3+7623a^3b^2x^5+1584a^2b^3x^7-176ab^4x^9+48b^5x^{11}}{240b^6\left(a+bx^2\right)^3}-\frac{231a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (3465*a^5*x + 9240*a^4*b*x^3 + 7623*a^3*b^2*x^5 + 1584*a^2*b^3*x^7 - 176*a*
b^4*x^9 + 48*b^5*x^11)/(240*b^6*(a + b*x^2)^3) - (231*a^(5/2)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(16*b^(13/2))
```

fricas [A] time = 1.09, size = 322, normalized size = 2.75

$$\left[\frac{96\,b^5x^{11} - 352\,ab^4x^9 + 3168\,a^2b^3x^7 + 15246\,a^3b^2x^5 + 18480\,a^4bx^3 + 6930\,a^5x + 3465\left(a^2b^3x^6 + 3\,a^3b^2x^4 + 3\,a^4b\right.}{480\left(b^9x^6 + 3\,ab^8x^4 + 3\,a^2b^7x^2 + a^3b^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/480*(96*b^5*x^11 - 352*a*b^4*x^9 + 3168*a^2*b^3*x^7 + 15246*a^3*b^2*x^5
+ 18480*a^4*b*x^3 + 6930*a^5*x + 3465*(a^2*b^3*x^6 + 3*a^3*b^2*x^4 + 3*a^4*
b*x^2 + a^5)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b
^9*x^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3*b^6), 1/240*(48*b^5*x^11 - 176*a
*b^4*x^9 + 1584*a^2*b^3*x^7 + 7623*a^3*b^2*x^5 + 9240*a^4*b*x^3 + 3465*a^5*
x - 3465*(a^2*b^3*x^6 + 3*a^3*b^2*x^4 + 3*a^4*b*x^2 + a^5)*sqrt(a/b)*arctan
(b*x*sqrt(a/b)/a))/(b^9*x^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3*b^6)]
```

giac [A] time = 0.15, size = 96, normalized size = 0.82

$$-\frac{231\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,b^6} + \frac{267\,a^3b^2x^5 + 472\,a^4bx^3 + 213\,a^5x}{48\left(bx^2 + a\right)^3b^6} + \frac{3\,b^{16}x^5 - 20\,ab^{15}x^3 + 150\,a^2b^{14}x}{15\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -231/16*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) + 1/48*(267*a^3*b^2*x^5 +
 472*a^4*b*x^3 + 213*a^5*x)/((b*x^2 + a)^3*b^6) + 1/15*(3*b^16*x^5 - 20*a*b
^15*x^3 + 150*a^2*b^14*x)/b^20
```

maple [A] time = 0.02, size = 108, normalized size = 0.92

$$\frac{89a^3x^5}{16\left(b\,x^2+a\right)^3b^4}+\frac{59a^4x^3}{6\left(b\,x^2+a\right)^3b^5}+\frac{x^5}{5b^4}+\frac{71a^5x}{16\left(b\,x^2+a\right)^3b^6}-\frac{4a\,x^3}{3b^5}-\frac{231a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\;b^6}+\frac{10a^2x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/5*x^5/b^4-4/3*a*x^3/b^5+10*a^2*x/b^6+89/16/b^4*a^3/(b*x^2+a)^3*x^5+59/6/b
^5*a^4/(b*x^2+a)^3*x^3+71/16/b^6*a^5/(b*x^2+a)^3*x-231/16/b^6*a^3/(a*b)^(1/
2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.98, size = 116, normalized size = 0.99

$$\frac{267\,a^3b^2x^5+472\,a^4bx^3+213\,a^5x}{48\left(b^9x^6+3\,ab^8x^4+3\,a^2b^7x^2+a^3b^6\right)}-\frac{231\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,b^6}+\frac{3\,b^2x^5-20\,abx^3+150\,a^2x}{15\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(267*a^3*b^2*x^5 + 472*a^4*b*x^3 + 213*a^5*x)/(b^9*x^6 + 3*a*b^8*x^4 +
 3*a^2*b^7*x^2 + a^3*b^6) - 231/16*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6
) + 1/15*(3*b^2*x^5 - 20*a*b*x^3 + 150*a^2*x)/b^6
```

mupad [B] time = 0.06, size = 109, normalized size = 0.93

$$\frac{\frac{71\,a^5\,x}{16}+\frac{59\,a^4\,b\,x^3}{6}+\frac{89\,a^3\,b^2\,x^5}{16}}{a^3\,b^6+3\,a^2\,b^7\,x^2+3\,a\,b^8\,x^4+b^9\,x^6}+\frac{x^5}{5\,b^4}-\frac{4\,a\,x^3}{3\,b^5}+\frac{10\,a^2\,x}{b^6}-\frac{231\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,b^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((71*a^5*x)/16 + (59*a^4*b*x^3)/6 + (89*a^3*b^2*x^5)/16)/(a^3*b^6 + b^9*x^6
 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2) + x^5/(5*b^4) - (4*a*x^3)/(3*b^5) + (10*a^2
*x)/b^6 - (231*a^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(16*b^(13/2))
```

sympy [A] time = 0.66, size = 172, normalized size = 1.47

$$\frac{10a^2x}{b^6}-\frac{4ax^3}{3b^5}+\frac{231\sqrt{-\frac{a^5}{b^{13}}}\log\left(x-\frac{b^6\sqrt{-\frac{a^5}{b^{13}}}}{a^2}\right)}{32}-\frac{231\sqrt{-\frac{a^5}{b^{13}}}\log\left(x+\frac{b^6\sqrt{-\frac{a^5}{b^{13}}}}{a^2}\right)}{32}+\frac{213a^5x+472a^4bx^3+267a^3b}{48a^3b^6+144a^2b^7x^2+144ab^8x^4+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 10*a**2*x/b**6 - 4*a*x**3/(3*b**5) + 231*sqrt(-a**5/b**13)*log(x - b**6*sqr
t(-a**5/b**13)/a**2)/32 - 231*sqrt(-a**5/b**13)*log(x + b**6*sqrt(-a**5/b**
13)/a**2)/32 + (213*a**5*x + 472*a**4*b*x**3 + 267*a**3*b**2*x**5)/(48*a**3
*b**6 + 144*a**2*b**7*x**2 + 144*a*b**8*x**4 + 48*b**9*x**6) + x**5/(5*b**4
)
```

3.503 $\int \frac{x^{10}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=104

$$\frac{105a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{11/2}}-\frac{105ax}{16b^5}-\frac{21x^5}{16b^3\left(a+bx^2\right)}-\frac{3x^7}{8b^2\left(a+bx^2\right)^2}-\frac{x^9}{6b\left(a+bx^2\right)^3}+\frac{35x^3}{16b^4}$$

```
[Out] -105/16*a*x/b^5+35/16*x^3/b^4-1/6*x^9/b/(b*x^2+a)^3-3/8*x^7/b^2/(b*x^2+a)^2
-21/16*x^5/b^3/(b*x^2+a)+105/16*a^(3/2)*arctan(x*b^(1/2)/a^(1/2))/b^(11/2)
```

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 302, 205}

$$\frac{105a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{11/2}}-\frac{3x^7}{8b^2\left(a+bx^2\right)^2}-\frac{21x^5}{16b^3\left(a+bx^2\right)}-\frac{105ax}{16b^5}-\frac{x^9}{6b\left(a+bx^2\right)^3}+\frac{35x^3}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-105*a*x)/(16*b^5) + (35*x^3)/(16*b^4) - x^9/(6*b*(a + b*x^2)^3) - (3*x^7)
/(8*b^2*(a + b*x^2)^2) - (21*x^5)/(16*b^3*(a + b*x^2)) + (105*a^(3/2)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(16*b^(11/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^{10}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{x^{10}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{x^9}{6b\left(a+bx^2\right)^3}+\frac{1}{2}\left(3b^2\right)\int \frac{x^8}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{x^9}{6b\left(a+bx^2\right)^3}-\frac{3x^7}{8b^2\left(a+bx^2\right)^2}+\frac{21}{8}\int \frac{x^6}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{x^9}{6b\left(a+bx^2\right)^3}-\frac{3x^7}{8b^2\left(a+bx^2\right)^2}-\frac{21x^5}{16b^3\left(a+bx^2\right)}+\frac{105\int \frac{x^4}{ab+b^2x^2}\,dx}{16b^2}$$

$$= -\frac{x^9}{6b\left(a+bx^2\right)^3}-\frac{3x^7}{8b^2\left(a+bx^2\right)^2}-\frac{21x^5}{16b^3\left(a+bx^2\right)}+\frac{105\int \left(-\frac{a}{b^3}+\frac{x^2}{b^2}+\frac{a^2}{b^2\left(ab+b^2x^2\right)} \cdots\right.}{16b^2}$$

$$= -\frac{105ax}{16b^5}+\frac{35x^3}{16b^4}-\frac{x^9}{6b\left(a+bx^2\right)^3}-\frac{3x^7}{8b^2\left(a+bx^2\right)^2}-\frac{21x^5}{16b^3\left(a+bx^2\right)}+\frac{\left(105a^2\right)\int \cdots}{16b \cdots}$$

$$= -\frac{105ax}{16b^5}+\frac{35x^3}{16b^4}-\frac{x^9}{6b\left(a+bx^2\right)^3}-\frac{3x^7}{8b^2\left(a+bx^2\right)^2}-\frac{21x^5}{16b^3\left(a+bx^2\right)}+\frac{105a^{3/2}\tan \cdots}{16b^{1 \cdots}}$$

Mathematica [A] time = 0.05, size = 89, normalized size = 0.86

$$\frac{315a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+\frac{\sqrt{b}\,x\left(-315a^4-840a^3bx^2-693a^2b^2x^4-144ab^3x^6+16b^4x^8\right)}{\left(a+bx^2\right)^3}}{48b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] ((Sqrt[b]*x*(-315*a^4 - 840*a^3*b*x^2 - 693*a^2*b^2*x^4 - 144*a*b^3*x^6 + 1
6*b^4*x^8))/(a + b*x^2)^3 + 315*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(48*b^
(11/2))
```

fricas [A] time = 1.00, size = 296, normalized size = 2.85

$$\left[\frac{32\,b^4x^9 - 288\,ab^3x^7 - 1386\,a^2b^2x^5 - 1680\,a^3bx^3 - 630\,a^4x + 315\left(ab^3x^6 + 3\,a^2b^2x^4 + 3\,a^3bx^2 + a^4\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2}{\quad}\right.}{96\left(b^8x^6 + 3\,ab^7x^4 + 3\,a^2b^6x^2 + a^3b^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(32*b^4*x^9 - 288*a*b^3*x^7 - 1386*a^2*b^2*x^5 - 1680*a^3*b*x^3 - 630
*a^4*x + 315*(a*b^3*x^6 + 3*a^2*b^2*x^4 + 3*a^3*b*x^2 + a^4)*sqrt(-a/b)*log
((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^8*x^6 + 3*a*b^7*x^4 + 3*a^
2*b^6*x^2 + a^3*b^5), 1/48*(16*b^4*x^9 - 144*a*b^3*x^7 - 693*a^2*b^2*x^5 -
840*a^3*b*x^3 - 315*a^4*x + 315*(a*b^3*x^6 + 3*a^2*b^2*x^4 + 3*a^3*b*x^2 +
a^4)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*
x^2 + a^3*b^5)]
```

giac [A] time = 0.16, size = 84, normalized size = 0.81

$$\frac{105\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,b^5} - \frac{165\,a^2b^2x^5 + 280\,a^3bx^3 + 123\,a^4x}{48\left(bx^2 + a\right)^3 b^5} + \frac{b^8x^3 - 12\,ab^7x}{3\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 105/16*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/48*(165*a^2*b^2*x^5 +
280*a^3*b*x^3 + 123*a^4*x)/((b*x^2 + a)^3*b^5) + 1/3*(b^8*x^3 - 12*a*b^7*x)
/b^12
```

maple [A] time = 0.01, size = 97, normalized size = 0.93

$$-\frac{55a^2x^5}{16\left(b\,x^2+a\right)^3 b^3} - \frac{35a^3x^3}{6\left(b\,x^2+a\right)^3 b^4} - \frac{41a^4x}{16\left(b\,x^2+a\right)^3 b^5} + \frac{x^3}{3b^4} + \frac{105a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\;b^5} - \frac{4ax}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/3*x^3/b^4-4*a*x/b^5-55/16/b^3*a^2/(b*x^2+a)^3*x^5-35/6/b^4*a^3/(b*x^2+a)^
3*x^3-41/16/b^5*a^4/(b*x^2+a)^3*x+105/16/b^5*a^2/(a*b)^(1/2)*arctan(1/(a*b)
^(1/2)*b*x)
```

maxima [A] time = 3.01, size = 104, normalized size = 1.00

$$-\frac{165\,a^2b^2x^5+280\,a^3bx^3+123\,a^4x}{48\left(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5\right)}+\frac{105\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,b^5}+\frac{bx^3-12\,ax}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/48*(165*a^2*b^2*x^5 + 280*a^3*b*x^3 + 123*a^4*x)/(b^8*x^6 + 3*a*b^7*x^4
+ 3*a^2*b^6*x^2 + a^3*b^5) + 105/16*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^
5) + 1/3*(b*x^3 - 12*a*x)/b^5
```

mupad [B] time = 4.36, size = 99, normalized size = 0.95

$$\frac{x^3}{3\,b^4}-\frac{\frac{41\,a^4\,x}{16}+\frac{35\,a^3\,b\,x^3}{6}+\frac{55\,a^2\,b^2\,x^5}{16}}{a^3\,b^5+3\,a^2\,b^6\,x^2+3\,a\,b^7\,x^4+b^8\,x^6}+\frac{105\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,b^{11/2}}-\frac{4\,a\,x}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] x^3/(3*b^4) - ((41*a^4*x)/16 + (35*a^3*b*x^3)/6 + (55*a^2*b^2*x^5)/16)/(a^3
*b^5 + b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2) + (105*a^(3/2)*atan((b^(1/2)*
x)/a^(1/2)))/(16*b^(11/2)) - (4*a*x)/b^5
```

sympy [A] time = 0.63, size = 156, normalized size = 1.50

$$-\frac{4ax}{b^5}-\frac{105\sqrt{-\frac{a^3}{b^{11}}}\log\left(x-\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}}{a}\right)}{32}+\frac{105\sqrt{-\frac{a^3}{b^{11}}}\log\left(x+\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}}{a}\right)}{32}+\frac{-123a^4x-280a^3bx^3-165a^2b^2x^5}{48a^3b^5+144a^2b^6x^2+144ab^7x^4+48b^8x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -4*a*x/b**5 - 105*sqrt(-a**3/b**11)*log(x - b**5*sqrt(-a**3/b**11)/a)/32 +
105*sqrt(-a**3/b**11)*log(x + b**5*sqrt(-a**3/b**11)/a)/32 + (-123*a**4*x -
 280*a**3*b*x**3 - 165*a**2*b**2*x**5)/(48*a**3*b**5 + 144*a**2*b**6*x**2 +
 144*a*b**7*x**4 + 48*b**8*x**6) + x**3/(3*b**4)
```

3.504 $\int \frac{x^8}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=93

$$-\frac{35\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{9/2}}-\frac{35x^3}{48b^3\left(a+bx^2\right)}-\frac{7x^5}{24b^2\left(a+bx^2\right)^2}-\frac{x^7}{6b\left(a+bx^2\right)^3}+\frac{35x}{16b^4}$$

```
[Out] 35/16*x/b^4-1/6*x^7/b/(b*x^2+a)^3-7/24*x^5/b^2/(b*x^2+a)^2-35/48*x^3/b^3/(b
*x^2+a)-35/16*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(9/2)
```

Rubi [A] time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 321, 205}

$$-\frac{7x^5}{24b^2\left(a+bx^2\right)^2}-\frac{35x^3}{48b^3\left(a+bx^2\right)}-\frac{35\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{9/2}}-\frac{x^7}{6b\left(a+bx^2\right)^3}+\frac{35x}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (35*x)/(16*b^4) - x^7/(6*b*(a + b*x^2)^3) - (7*x^5)/(24*b^2*(a + b*x^2)^2)
- (35*x^3)/(48*b^3*(a + b*x^2)) - (35*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(16*b^(9/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4\int \frac{x^8}{\left(ab+b^2x^2\right)^4}\,dx\\
&= -\frac{x^7}{6b\left(a+bx^2\right)^3}+\frac{1}{6}\left(7b^2\right)\int \frac{x^6}{\left(ab+b^2x^2\right)^3}\,dx\\
&= -\frac{x^7}{6b\left(a+bx^2\right)^3}-\frac{7x^5}{24b^2\left(a+bx^2\right)^2}+\frac{35}{24}\int \frac{x^4}{\left(ab+b^2x^2\right)^2}\,dx\\
&= -\frac{x^7}{6b\left(a+bx^2\right)^3}-\frac{7x^5}{24b^2\left(a+bx^2\right)^2}-\frac{35x^3}{48b^3\left(a+bx^2\right)}+\frac{35\int \frac{x^2}{ab+b^2x^2}\,dx}{16b^2}\\
&= \frac{35x}{16b^4}-\frac{x^7}{6b\left(a+bx^2\right)^3}-\frac{7x^5}{24b^2\left(a+bx^2\right)^2}-\frac{35x^3}{48b^3\left(a+bx^2\right)}-\frac{(35a)\int \frac{1}{ab+b^2x^2}\,dx}{16b^3}\\
&= \frac{35x}{16b^4}-\frac{x^7}{6b\left(a+bx^2\right)^3}-\frac{7x^5}{24b^2\left(a+bx^2\right)^2}-\frac{35x^3}{48b^3\left(a+bx^2\right)}-\frac{35\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 0.83

$$\frac{105a^3x+280a^2bx^3+231ab^2x^5+48b^3x^7}{48b^4\left(a+bx^2\right)^3}-\frac{35\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (105*a^3*x + 280*a^2*b*x^3 + 231*a*b^2*x^5 + 48*b^3*x^7)/(48*b^4*(a + b*x^2
)^3) - (35*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*b^(9/2))
```

fricas [A] time = 0.74, size = 268, normalized size = 2.88

$$\left[\frac{96\,b^3x^7 + 462\,ab^2x^5 + 560\,a^2bx^3 + 210\,a^3x + 105\left(b^3x^6 + 3\,ab^2x^4 + 3\,a^2bx^2 + a^3\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{96\left(b^7x^6 + 3\,ab^6x^4 + 3\,a^2b^5x^2 + a^3b^4\right)}, \frac{48\,b \cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(96*b^3*x^7 + 462*a*b^2*x^5 + 560*a^2*b*x^3 + 210*a^3*x + 105*(b^3*x^
6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/
b) - a)/(b*x^2 + a)))/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4), 1/
48*(48*b^3*x^7 + 231*a*b^2*x^5 + 280*a^2*b*x^3 + 105*a^3*x - 105*(b^3*x^6 +
 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^7*x
^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4)]
```

giac [A] time = 0.16, size = 65, normalized size = 0.70

$$-\frac{35\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,b^4} + \frac{x}{b^4} + \frac{87\,ab^2x^5 + 136\,a^2bx^3 + 57\,a^3x}{48\left(bx^2+a\right)^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -35/16*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + x/b^4 + 1/48*(87*a*b^2*x^5
 + 136*a^2*b*x^3 + 57*a^3*x)/((b*x^2 + a)^3*b^4)
```

maple [A] time = 0.01, size = 83, normalized size = 0.89

$$\frac{29a\,x^5}{16\left(b\,x^2+a\right)^3b^2} + \frac{17a^2x^3}{6\left(b\,x^2+a\right)^3b^3} + \frac{19a^3x}{16\left(b\,x^2+a\right)^3b^4} - \frac{35a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\;b^4} + \frac{x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] x/b^4+29/16/b^2*a/(b*x^2+a)^3*x^5+17/6/b^3*a^2/(b*x^2+a)^3*x^3+19/16/b^4*a^
3/(b*x^2+a)^3*x-35/16/b^4*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.85, size = 90, normalized size = 0.97

$$\frac{87\,ab^2x^5+136\,a^2bx^3+57\,a^3x}{48\left(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4\right)}-\frac{35\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,b^4}+\frac{x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(87*a*b^2*x^5 + 136*a^2*b*x^3 + 57*a^3*x)/(b^7*x^6 + 3*a*b^6*x^4 + 3*a
^2*b^5*x^2 + a^3*b^4) - 35/16*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + x/b
^4
```

mupad [B] time = 0.10, size = 86, normalized size = 0.92

$$\frac{x}{b^4}+\frac{\frac{19\,a^3\,x}{16}+\frac{17\,a^2\,b\,x^3}{6}+\frac{29\,a\,b^2\,x^5}{16}}{a^3\,b^4+3\,a^2\,b^5\,x^2+3\,a\,b^6\,x^4+b^7\,x^6}-\frac{35\,\sqrt{a}\,\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] x/b^4 + ((19*a^3*x)/16 + (17*a^2*b*x^3)/6 + (29*a*b^2*x^5)/16)/(a^3*b^4 + b
^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2) - (35*a^(1/2)*atan((b^(1/2)*x)/a^(1/2
)))/(16*b^(9/2))
```

sympy [A] time = 0.57, size = 131, normalized size = 1.41

$$\frac{35\sqrt{-\frac{a}{b^9}}\,\log\left(-b^4\sqrt{-\frac{a}{b^9}}+x\right)}{32}-\frac{35\sqrt{-\frac{a}{b^9}}\,\log\left(b^4\sqrt{-\frac{a}{b^9}}+x\right)}{32}+\frac{57a^3x+136a^2bx^3+87ab^2x^5}{48a^3b^4+144a^2b^5x^2+144ab^6x^4+48b^7x^6}+\frac{x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 35*sqrt(-a/b**9)*log(-b**4*sqrt(-a/b**9) + x)/32 - 35*sqrt(-a/b**9)*log(b**
4*sqrt(-a/b**9) + x)/32 + (57*a**3*x + 136*a**2*b*x**3 + 87*a*b**2*x**5)/(4
8*a**3*b**4 + 144*a**2*b**5*x**2 + 144*a*b**6*x**4 + 48*b**7*x**6) + x/b**4
```

3.505 $\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=83

$$\frac{5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{a}\,b^{7/2}}-\frac{5x}{16b^3\left(a+bx^2\right)}-\frac{5x^3}{24b^2\left(a+bx^2\right)^2}-\frac{x^5}{6b\left(a+bx^2\right)^3}$$

```
[Out] -1/6*x^5/b/(b*x^2+a)^3-5/24*x^3/b^2/(b*x^2+a)^2-5/16*x/b^3/(b*x^2+a)+5/16*a
rctan(x*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 288, 205}

$$-\frac{5x^3}{24b^2\left(a+bx^2\right)^2}-\frac{5x}{16b^3\left(a+bx^2\right)}+\frac{5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{a}\,b^{7/2}}-\frac{x^5}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -x^5/(6*b*(a + b*x^2)^3) - (5*x^3)/(24*b^2*(a + b*x^2)^2) - (5*x)/(16*b^3*(
a + b*x^2)) + (5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*Sqrt[a]*b^(7/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4 \int \frac{x^6}{\left(ab+b^2x^2\right)^4}\,dx\\
&= -\frac{x^5}{6b\left(a+bx^2\right)^3}+\frac{1}{6}\left(5b^2\right)\int \frac{x^4}{\left(ab+b^2x^2\right)^3}\,dx\\
&= -\frac{x^5}{6b\left(a+bx^2\right)^3}-\frac{5x^3}{24b^2\left(a+bx^2\right)^2}+\frac{5}{8}\int \frac{x^2}{\left(ab+b^2x^2\right)^2}\,dx\\
&= -\frac{x^5}{6b\left(a+bx^2\right)^3}-\frac{5x^3}{24b^2\left(a+bx^2\right)^2}-\frac{5x}{16b^3\left(a+bx^2\right)}+\frac{5\int \frac{1}{ab+b^2x^2}\,dx}{16b^2}\\
&= -\frac{x^5}{6b\left(a+bx^2\right)^3}-\frac{5x^3}{24b^2\left(a+bx^2\right)^2}-\frac{5x}{16b^3\left(a+bx^2\right)}+\frac{5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{a}\,b^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 66, normalized size = 0.80

$$
\frac{5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{a}\,b^{7/2}}-\frac{x\left(15a^2+40abx^2+33b^2x^4\right)}{48b^3\left(a+bx^2\right)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/48*(x*(15*a^2 + 40*a*b*x^2 + 33*b^2*x^4))/(b^3*(a + b*x^2)^3) + (5*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(16*Sqrt[a]*b^(7/2))
```

fricas [A] time = 0.91, size = 254, normalized size = 3.06

$$
\left[-\frac{66\,ab^3x^5+80\,a^2b^2x^3+30\,a^3bx+15\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{96\left(ab^7x^6+3\,a^2b^6x^4+3\,a^3b^5x^2+a^4b^4\right)},-\frac{33\,ab^3x^5+4
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/96*(66*a*b^3*x^5 + 80*a^2*b^2*x^3 + 30*a^3*b*x + 15*(b^3*x^6 + 3*a*b^2*
x^4 + 3*a^2*b*x^2 + a^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2
 + a)))/(a*b^7*x^6 + 3*a^2*b^6*x^4 + 3*a^3*b^5*x^2 + a^4*b^4), -1/48*(33*a*
b^3*x^5 + 40*a^2*b^2*x^3 + 15*a^3*b*x - 15*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b
*x^2 + a^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^7*x^6 + 3*a^2*b^6*x^4 + 3
*a^3*b^5*x^2 + a^4*b^4)]
```

giac [A] time = 0.17, size = 56, normalized size = 0.67

$$\frac{5 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16 \sqrt{ab}\, b^3} - \frac{33\, b^2 x^5 + 40\, abx^3 + 15\, a^2 x}{48 \left(bx^2 + a\right)^3 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 5/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/48*(33*b^2*x^5 + 40*a*b*x^3
+ 15*a^2*x)/((b*x^2 + a)^3*b^3)
```

maple [A] time = 0.01, size = 58, normalized size = 0.70

$$\frac{5 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\; b^3} + \frac{-\frac{11 x^5}{16 b} - \frac{5 a\, x^3}{6 b^2} - \frac{5 a^2 x}{16 b^3}}{\left(b\, x^2 + a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] (-11/16/b*x^5-5/6*a/b^2*x^3-5/16*a^2/b^3*x)/(b*x^2+a)^3+5/16/b^3/(a*b)^(1/2
)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.07, size = 81, normalized size = 0.98

$$-\frac{33\, b^2 x^5 + 40\, abx^3 + 15\, a^2 x}{48 \left(b^6 x^6 + 3\, ab^5 x^4 + 3\, a^2 b^4 x^2 + a^3 b^3\right)} + \frac{5 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16 \sqrt{ab}\, b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/48*(33*b^2*x^5 + 40*a*b*x^3 + 15*a^2*x)/(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b
^4*x^2 + a^3*b^3) + 5/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3)
```

mupad [B] time = 4.39, size = 78, normalized size = 0.94

$$\frac{5\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,\sqrt{a}\,b^{7/2}}-\frac{\frac{11\,x^5}{16\,b}+\frac{5\,a\,x^3}{6\,b^2}+\frac{5\,a^2\,x}{16\,b^3}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (5*atan((b^(1/2)*x)/a^(1/2)))/(16*a^(1/2)*b^(7/2)) - ((11*x^5)/(16*b) + (5*
a*x^3)/(6*b^2) + (5*a^2*x)/(16*b^3))/(a^3 + b^3*x^6 + 3*a^2*b*x^2 + 3*a*b^2
*x^4)
```

sympy [A] time = 0.49, size = 134, normalized size = 1.61

$$-\frac{5\sqrt{-\frac{1}{ab^7}}\log\left(-ab^3\sqrt{-\frac{1}{ab^7}}+x\right)}{32}+\frac{5\sqrt{-\frac{1}{ab^7}}\log\left(ab^3\sqrt{-\frac{1}{ab^7}}+x\right)}{32}+\frac{-15a^2x-40abx^3-33b^2x^5}{48a^3b^3+144a^2b^4x^2+144ab^5x^4+48b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -5*sqrt(-1/(a*b**7))*log(-a*b**3*sqrt(-1/(a*b**7)) + x)/32 + 5*sqrt(-1/(a*b
**7))*log(a*b**3*sqrt(-1/(a*b**7)) + x)/32 + (-15*a**2*x - 40*a*b*x**3 - 33
*b**2*x**5)/(48*a**3*b**3 + 144*a**2*b**4*x**2 + 144*a*b**5*x**4 + 48*b**6*
x**6)
```

3.506 $\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=84

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{3/2}b^{5/2}}+\frac{x}{16ab^2\left(a+bx^2\right)}-\frac{x}{8b^2\left(a+bx^2\right)^2}-\frac{x^3}{6b\left(a+bx^2\right)^3}$$

```
[Out] -1/6*x^3/b/(b*x^2+a)^3-1/8*x/b^2/(b*x^2+a)^2+1/16*x/a/b^2/(b*x^2+a)+1/16*ar
ctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(5/2)
```

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{3/2}b^{5/2}}+\frac{x}{16ab^2\left(a+bx^2\right)}-\frac{x}{8b^2\left(a+bx^2\right)^2}-\frac{x^3}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -x^3/(6*b*(a + b*x^2)^3) - x/(8*b^2*(a + b*x^2)^2) + x/(16*a*b^2*(a + b*x^2
)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(16*a^(3/2)*b^(5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{x^4}{\left(ab+b^2x^2\right)^4}\,dx$$
$$= -\frac{x^3}{6b\left(a+bx^2\right)^3}+\frac{1}{2}b^2\int \frac{x^2}{\left(ab+b^2x^2\right)^3}\,dx$$
$$= -\frac{x^3}{6b\left(a+bx^2\right)^3}-\frac{x}{8b^2\left(a+bx^2\right)^2}+\frac{1}{8}\int \frac{1}{\left(ab+b^2x^2\right)^2}\,dx$$
$$= -\frac{x^3}{6b\left(a+bx^2\right)^3}-\frac{x}{8b^2\left(a+bx^2\right)^2}+\frac{x}{16ab^2\left(a+bx^2\right)}+\frac{\int \frac{1}{ab+b^2x^2}\,dx}{16ab}$$
$$= -\frac{x^3}{6b\left(a+bx^2\right)^3}-\frac{x}{8b^2\left(a+bx^2\right)^2}+\frac{x}{16ab^2\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{3/2}b^{5/2}}$$

Mathematica [A] time = 0.04, size = 69, normalized size = 0.82

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{3/2}b^{5/2}}+\frac{-3a^2x-8abx^3+3b^2x^5}{48ab^2\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-3*a^2*x - 8*a*b*x^3 + 3*b^2*x^5)/(48*a*b^2*(a + b*x^2)^3) + ArcTan[(Sqrt[
b]*x)/Sqrt[a]]/(16*a^(3/2)*b^(5/2))
```

fricas [A] time = 1.02, size = 258, normalized size = 3.07

$$\left[\frac{6\,ab^3x^5-16\,a^2b^2x^3-6\,a^3bx-3\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{96\left(a^2b^6x^6+3\,a^3b^5x^4+3\,a^4b^4x^2+a^5b^3\right)},\ \frac{3\,ab^3x^5-8\,a^2b^2x^3\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(6*a*b^3*x^5 - 16*a^2*b^2*x^3 - 6*a^3*b*x - 3*(b^3*x^6 + 3*a*b^2*x^4
+ 3*a^2*b*x^2 + a^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a
)))/(a^2*b^6*x^6 + 3*a^3*b^5*x^4 + 3*a^4*b^4*x^2 + a^5*b^3), 1/48*(3*a*b^3*
x^5 - 8*a^2*b^2*x^3 - 3*a^3*b*x + 3*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 +
a^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^2*b^6*x^6 + 3*a^3*b^5*x^4 + 3*a^4*
b^4*x^2 + a^5*b^3)]
```

giac [A] time = 0.16, size = 62, normalized size = 0.74

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\,ab^2} + \frac{3\,b^2x^5 - 8\,abx^3 - 3\,a^2x}{48\left(bx^2 + a\right)^3 ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2) + 1/48*(3*b^2*x^5 - 8*a*b*x^3
- 3*a^2*x)/((b*x^2 + a)^3*a*b^2)
```

maple [A] time = 0.01, size = 58, normalized size = 0.69

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\;a\,b^2} + \frac{\frac{x^5}{16a} - \frac{x^3}{6b} - \frac{ax}{16b^2}}{\left(b\,x^2 + a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] (1/16/a*x^5-1/6/b*x^3-1/16*a/b^2*x)/(b*x^2+a)^3+1/16/a/b^2/(a*b)^(1/2)*arct
an(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.93, size = 87, normalized size = 1.04

$$\frac{3\,b^2x^5 - 8\,abx^3 - 3\,a^2x}{48\left(ab^5x^6 + 3\,a^2b^4x^4 + 3\,a^3b^3x^2 + a^4b^2\right)} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*b^2*x^5 - 8*a*b*x^3 - 3*a^2*x)/(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b
^3*x^2 + a^4*b^2) + 1/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2)
```

mupad [B] time = 4.35, size = 75, normalized size = 0.89

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{3/2}\,b^{5/2}}-\frac{\frac{x^3}{6\,b}-\frac{x^5}{16\,a}+\frac{a\,x}{16\,b^2}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] atan((b^(1/2)*x)/a^(1/2))/(16*a^(3/2)*b^(5/2)) - (x^3/(6*b) - x^5/(16*a) +
(a*x)/(16*b^2))/(a^3 + b^3*x^6 + 3*a^2*b*x^2 + 3*a*b^2*x^4)
```

sympy [B] time = 0.45, size = 143, normalized size = 1.70

$$-\frac{\sqrt{-\frac{1}{a^3b^5}}\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{a^3b^5}}\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{32}+\frac{-3a^2x-8abx^3+3b^2x^5}{48a^4b^2+144a^3b^3x^2+144a^2b^4x^4+48ab^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -sqrt(-1/(a**3*b**5))*log(-a**2*b**2*sqrt(-1/(a**3*b**5)) + x)/32 + sqrt(-1
/(a**3*b**5))*log(a**2*b**2*sqrt(-1/(a**3*b**5)) + x)/32 + (-3*a**2*x - 8*a
*b*x**3 + 3*b**2*x**5)/(48*a**4*b**2 + 144*a**3*b**3*x**2 + 144*a**2*b**4*x
**4 + 48*a*b**5*x**6)
```

3.507 $\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=85

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x}{16a^2b\left(a+bx^2\right)}+\frac{x}{24ab\left(a+bx^2\right)^2}-\frac{x}{6b\left(a+bx^2\right)^3}$$

```
[Out] -1/6*x/b/(b*x^2+a)^3+1/24*x/a/b/(b*x^2+a)^2+1/16*x/a^2/b/(b*x^2+a)+1/16*arc
tan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x}{16a^2b\left(a+bx^2\right)}+\frac{x}{24ab\left(a+bx^2\right)^2}-\frac{x}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -x/(6*b*(a + b*x^2)^3) + x/(24*a*b*(a + b*x^2)^2) + x/(16*a^2*b*(a + b*x^2)
) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(16*a^(5/2)*b^(3/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4\int \frac{x^2}{\left(ab+b^2x^2\right)^4}\,dx\\
&= -\frac{x}{6b\left(a+bx^2\right)^3}+\frac{1}{6}b^2\int \frac{1}{\left(ab+b^2x^2\right)^3}\,dx\\
&= -\frac{x}{6b\left(a+bx^2\right)^3}+\frac{x}{24ab\left(a+bx^2\right)^2}+\frac{b\int \frac{1}{\left(ab+b^2x^2\right)^2}\,dx}{8a}\\
&= -\frac{x}{6b\left(a+bx^2\right)^3}+\frac{x}{24ab\left(a+bx^2\right)^2}+\frac{x}{16a^2b\left(a+bx^2\right)}+\frac{\int \frac{1}{ab+b^2x^2}\,dx}{16a^2}\\
&= -\frac{x}{6b\left(a+bx^2\right)^3}+\frac{x}{24ab\left(a+bx^2\right)^2}+\frac{x}{16a^2b\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 69, normalized size = 0.81

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{-3a^2x+8abx^3+3b^2x^5}{48a^2b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-3*a^2*x + 8*a*b*x^3 + 3*b^2*x^5)/(48*a^2*b*(a + b*x^2)^3) + ArcTan[(Sqrt[
b]*x)/Sqrt[a]]/(16*a^(5/2)*b^(3/2))
```

fricas [A] time = 0.91, size = 258, normalized size = 3.04

$$\left[\frac{6\,ab^3x^5+16\,a^2b^2x^3-6\,a^3bx-3\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{96\left(a^3b^5x^6+3\,a^4b^4x^4+3\,a^5b^3x^2+a^6b^2\right)},\frac{3\,ab^3x^5+8\,a^2b^2x^3-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(6*a*b^3*x^5 + 16*a^2*b^2*x^3 - 6*a^3*b*x - 3*(b^3*x^6 + 3*a*b^2*x^4
+ 3*a^2*b*x^2 + a^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a
)))/(a^3*b^5*x^6 + 3*a^4*b^4*x^4 + 3*a^5*b^3*x^2 + a^6*b^2), 1/48*(3*a*b^3*
x^5 + 8*a^2*b^2*x^3 - 3*a^3*b*x + 3*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 +
a^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^5*x^6 + 3*a^4*b^4*x^4 + 3*a^5*
b^3*x^2 + a^6*b^2)]
```

giac [A] time = 0.18, size = 62, normalized size = 0.73

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^2b}+\frac{3\,b^2x^5+8\,abx^3-3\,a^2x}{48\left(bx^2+a\right)^3a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/48*(3*b^2*x^5 + 8*a*b*x^3
- 3*a^2*x)/((b*x^2 + a)^3*a^2*b)
```

maple [A] time = 0.01, size = 58, normalized size = 0.68

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\,\,a^2b}+\frac{\frac{b\,x^5}{16a^2}+\frac{x^3}{6a}-\frac{x}{16b}}{\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] (1/16/a^2*b*x^5+1/6/a*x^3-1/16/b*x)/(b*x^2+a)^3+1/16/a^2/b/(a*b)^(1/2)*arct
an(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.98, size = 87, normalized size = 1.02

$$\frac{3\,b^2x^5+8\,abx^3-3\,a^2x}{48\left(a^2b^4x^6+3\,a^3b^3x^4+3\,a^4b^2x^2+a^5b\right)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*b^2*x^5 + 8*a*b*x^3 - 3*a^2*x)/(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4
*b^2*x^2 + a^5*b) + 1/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

mupad [B] time = 4.31, size = 74, normalized size = 0.87

$$\frac{\frac{x^3}{6\,a}-\frac{x}{16\,b}+\frac{b\,x^5}{16\,a^2}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{5/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (x^3/(6*a) - x/(16*b) + (b*x^5)/(16*a^2))/(a^3 + b^3*x^6 + 3*a^2*b*x^2 + 3*
a*b^2*x^4) + atan((b^(1/2)*x)/a^(1/2))/(16*a^(5/2)*b^(3/2))
```

sympy [B] time = 0.44, size = 139, normalized size = 1.64

$$-\frac{\sqrt{-\frac{1}{a^5b^3}}\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{a^5b^3}}\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{32}+\frac{-3a^2x+8abx^3+3b^2x^5}{48a^5b+144a^4b^2x^2+144a^3b^3x^4+48a^2b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/32 + sqrt(-1/(a
**5*b**3))*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)/32 + (-3*a**2*x + 8*a*b*x**
3 + 3*b**2*x**5)/(48*a**5*b + 144*a**4*b**2*x**2 + 144*a**3*b**3*x**4 + 48*
a**2*b**4*x**6)
```

3.508 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^2} \, dx$

Optimal. Leaf size=79

$$\frac{5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}+\frac{5x}{16a^3\left(a+bx^2\right)}+\frac{5x}{24a^2\left(a+bx^2\right)^2}+\frac{x}{6a\left(a+bx^2\right)^3}$$

```
[Out] 1/6*x/a/(b*x^2+a)^3+5/24*x/a^2/(b*x^2+a)^2+5/16*x/a^3/(b*x^2+a)+5/16*arctan
(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {28, 199, 205}

$$\frac{5x}{16a^3\left(a+bx^2\right)}+\frac{5x}{24a^2\left(a+bx^2\right)^2}+\frac{5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}+\frac{x}{6a\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2),x]
```

```
[Out] x/(6*a*(a + b*x^2)^3) + (5*x)/(24*a^2*(a + b*x^2)^2) + (5*x)/(16*a^3*(a + b
*x^2)) + (5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(7/2)*Sqrt[b])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{\left(ab+b^2x^2\right)^4}\,dx$$
$$= \frac{x}{6a\left(a+bx^2\right)^3} + \frac{\left(5b^3\right)\int \frac{1}{\left(ab+b^2x^2\right)^3}\,dx}{6a}$$
$$= \frac{x}{6a\left(a+bx^2\right)^3} + \frac{5x}{24a^2\left(a+bx^2\right)^2} + \frac{\left(5b^2\right)\int \frac{1}{\left(ab+b^2x^2\right)^2}\,dx}{8a^2}$$
$$= \frac{x}{6a\left(a+bx^2\right)^3} + \frac{5x}{24a^2\left(a+bx^2\right)^2} + \frac{5x}{16a^3\left(a+bx^2\right)} + \frac{(5b)\int \frac{1}{ab+b^2x^2}\,dx}{16a^3}$$
$$= \frac{x}{6a\left(a+bx^2\right)^3} + \frac{5x}{24a^2\left(a+bx^2\right)^2} + \frac{5x}{16a^3\left(a+bx^2\right)} + \frac{5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.84

$$\frac{5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}} + \frac{33a^2x+40abx^3+15b^2x^5}{48a^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2),x]
```

```
[Out] (33*a^2*x + 40*a*b*x^3 + 15*b^2*x^5)/(48*a^3*(a + b*x^2)^3) + (5*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(16*a^(7/2)*Sqrt[b])
```

fricas [A] time = 0.92, size = 254, normalized size = 3.22

$$\left[\frac{30\,ab^3x^5+80\,a^2b^2x^3+66\,a^3bx-15\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{96\left(a^4b^4x^6+3\,a^5b^3x^4+3\,a^6b^2x^2+a^7b\right)},\ \frac{15\,ab^3x^5+40\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(30*a*b^3*x^5 + 80*a^2*b^2*x^3 + 66*a^3*b*x - 15*(b^3*x^6 + 3*a*b^2*x
^4 + 3*a^2*b*x^2 + a^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2
+ a)))/(a^4*b^4*x^6 + 3*a^5*b^3*x^4 + 3*a^6*b^2*x^2 + a^7*b), 1/48*(15*a*b^
3*x^5 + 40*a^2*b^2*x^3 + 33*a^3*b*x + 15*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x
^2 + a^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b^4*x^6 + 3*a^5*b^3*x^4 + 3
*a^6*b^2*x^2 + a^7*b)]
```

giac [A] time = 0.15, size = 56, normalized size = 0.71

$$\frac{5 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16 \sqrt{ab}\, a^3} + \frac{15\, b^2x^5 + 40\, abx^3 + 33\, a^2x}{48 \left(bx^2 + a\right)^3 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 5/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/48*(15*b^2*x^5 + 40*a*b*x^3
+ 33*a^2*x)/((b*x^2 + a)^3*a^3)
```

maple [A] time = 0.00, size = 66, normalized size = 0.84

$$\frac{x}{6 \left(b\, x^2 + a\right)^3 a} + \frac{5x}{24 \left(b\, x^2 + a\right)^2 a^2} + \frac{5x}{16 \left(b\, x^2 + a\right) a^3} + \frac{5 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\; a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/6*x/a/(b*x^2+a)^3+5/24*x/a^2/(b*x^2+a)^2+5/16*x/a^3/(b*x^2+a)+5/16/a^3/(a
*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.01, size = 80, normalized size = 1.01

$$\frac{15\, b^2x^5 + 40\, abx^3 + 33\, a^2x}{48 \left(a^3b^3x^6 + 3\, a^4b^2x^4 + 3\, a^5bx^2 + a^6\right)} + \frac{5 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16 \sqrt{ab}\, a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(15*b^2*x^5 + 40*a*b*x^3 + 33*a^2*x)/(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*
a^5*b*x^2 + a^6) + 5/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

mupad [B] time = 4.36, size = 77, normalized size = 0.97

$$\frac{\frac{11\,x}{16\,a}+\frac{5\,b\,x^3}{6\,a^2}+\frac{5\,b^2\,x^5}{16\,a^3}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6}+\frac{5\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{7/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((11*x)/(16*a) + (5*b*x^3)/(6*a^2) + (5*b^2*x^5)/(16*a^3))/(a^3 + b^3*x^6 +
 3*a^2*b*x^2 + 3*a*b^2*x^4) + (5*atan((b^(1/2)*x)/a^(1/2)))/(16*a^(7/2)*b^(
1/2))
```

sympy [A] time = 0.44, size = 129, normalized size = 1.63

$$-\frac{5\sqrt{-\frac{1}{a^7b}}\log\left(-a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{32}+\frac{5\sqrt{-\frac{1}{a^7b}}\log\left(a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{32}+\frac{33a^2x+40abx^3+15b^2x^5}{48a^6+144a^5bx^2+144a^4b^2x^4+48a^3b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -5*sqrt(-1/(a**7*b))*log(-a**4*sqrt(-1/(a**7*b)) + x)/32 + 5*sqrt(-1/(a**7*
b))*log(a**4*sqrt(-1/(a**7*b)) + x)/32 + (33*a**2*x + 40*a*b*x**3 + 15*b**2
*x**5)/(48*a**6 + 144*a**5*b*x**2 + 144*a**4*b**2*x**4 + 48*a**3*b**3*x**6)
```

3.509 $\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=95

$$-\frac{35\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{9/2}}-\frac{35}{16a^4x}+\frac{35}{48a^3x\left(a+bx^2\right)}+\frac{7}{24a^2x\left(a+bx^2\right)^2}+\frac{1}{6ax\left(a+bx^2\right)^3}$$

```
[Out] -35/16/a^4/x+1/6/a/x/(b*x^2+a)^3+7/24/a^2/x/(b*x^2+a)^2+35/48/a^3/x/(b*x^2+
a)-35/16*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(9/2)
```

Rubi [A] time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$\frac{35}{48a^3x\left(a+bx^2\right)}+\frac{7}{24a^2x\left(a+bx^2\right)^2}-\frac{35\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{9/2}}-\frac{35}{16a^4x}+\frac{1}{6ax\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -35/(16*a^4*x) + 1/(6*a*x*(a + b*x^2)^3) + 7/(24*a^2*x*(a + b*x^2)^2) + 35/
(48*a^3*x*(a + b*x^2)) - (35*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(9/
2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

]

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{x^2\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{1}{6ax\left(a+bx^2\right)^3} + \frac{\left(7b^3\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^3}\,dx}{6a}$$

$$= \frac{1}{6ax\left(a+bx^2\right)^3} + \frac{7}{24a^2x\left(a+bx^2\right)^2} + \frac{\left(35b^2\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^2}\,dx}{24a^2}$$

$$= \frac{1}{6ax\left(a+bx^2\right)^3} + \frac{7}{24a^2x\left(a+bx^2\right)^2} + \frac{35}{48a^3x\left(a+bx^2\right)} + \frac{(35b)\int \frac{1}{x^2\left(ab+b^2x^2\right)}\,dx}{16a^3}$$

$$= -\frac{35}{16a^4x} + \frac{1}{6ax\left(a+bx^2\right)^3} + \frac{7}{24a^2x\left(a+bx^2\right)^2} + \frac{35}{48a^3x\left(a+bx^2\right)} - \frac{\left(35b^2\right)\int \frac{}{ab}}{16a}$$

$$= -\frac{35}{16a^4x} + \frac{1}{6ax\left(a+bx^2\right)^3} + \frac{7}{24a^2x\left(a+bx^2\right)^2} + \frac{35}{48a^3x\left(a+bx^2\right)} - \frac{35\sqrt{b}\,\tan^{-}}{16a^{9}}$$

Mathematica [A] time = 0.04, size = 79, normalized size = 0.83

$$-\frac{35\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{9/2}} - \frac{48a^3+231a^2bx^2+280ab^2x^4+105b^3x^6}{48a^4x\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -1/48*(48*a^3 + 231*a^2*b*x^2 + 280*a*b^2*x^4 + 105*b^3*x^6)/(a^4*x*(a + b*
x^2)^3) - (35*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(9/2))
```

fricas [A] time = 0.88, size = 268, normalized size = 2.82

$$\left[-\frac{210\,b^3x^6+560\,ab^2x^4+462\,a^2bx^2+96\,a^3-105\left(b^3x^7+3\,ab^2x^5+3\,a^2bx^3+a^3x\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{96\left(a^4b^3x^7+3\,a^5b^2x^5+3\,a^6bx^3+a^7x\right)},-\text{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/96*(210*b^3*x^6 + 560*a*b^2*x^4 + 462*a^2*b*x^2 + 96*a^3 - 105*(b^3*x^7
 + 3*a*b^2*x^5 + 3*a^2*b*x^3 + a^3*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b
/a) - a)/(b*x^2 + a)))/(a^4*b^3*x^7 + 3*a^5*b^2*x^5 + 3*a^6*b*x^3 + a^7*x),
 -1/48*(105*b^3*x^6 + 280*a*b^2*x^4 + 231*a^2*b*x^2 + 48*a^3 + 105*(b^3*x^7
 + 3*a*b^2*x^5 + 3*a^2*b*x^3 + a^3*x)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b
^3*x^7 + 3*a^5*b^2*x^5 + 3*a^6*b*x^3 + a^7*x)]
```

giac [A] time = 0.16, size = 68, normalized size = 0.72

$$-\frac{35\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^4}-\frac{1}{a^4x}-\frac{57\,b^3x^5+136\,ab^2x^3+87\,a^2bx}{48\left(bx^2+a\right)^3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -35/16*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/(a^4*x) - 1/48*(57*b^3*x
^5 + 136*a*b^2*x^3 + 87*a^2*b*x)/((b*x^2 + a)^3*a^4)
```

maple [A] time = 0.01, size = 86, normalized size = 0.91

$$-\frac{19b^3x^5}{16\left(b\,x^2+a\right)^3a^4}-\frac{17b^2x^3}{6\left(b\,x^2+a\right)^3a^3}-\frac{29bx}{16\left(b\,x^2+a\right)^3a^2}-\frac{35b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\,\,a^4}-\frac{1}{a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/a^4/x-19/16*b^3/a^4/(b*x^2+a)^3*x^5-17/6*b^2/a^3/(b*x^2+a)^3*x^3-29/16*b
/a^2/(b*x^2+a)^3*x-35/16*b/a^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.03, size = 93, normalized size = 0.98

$$-\frac{105\,b^3x^6+280\,ab^2x^4+231\,a^2bx^2+48\,a^3}{48\left(a^4b^3x^7+3\,a^5b^2x^5+3\,a^6bx^3+a^7x\right)}-\frac{35\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/48*(105*b^3*x^6 + 280*a*b^2*x^4 + 231*a^2*b*x^2 + 48*a^3)/(a^4*b^3*x^7 +
 3*a^5*b^2*x^5 + 3*a^6*b*x^3 + a^7*x) - 35/16*b*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*a^4)
```

mupad [B] time = 4.44, size = 88, normalized size = 0.93

$$-\frac{\frac{1}{a}+\frac{77\,b\,x^2}{16\,a^2}+\frac{35\,b^2\,x^4}{6\,a^3}+\frac{35\,b^3\,x^6}{16\,a^4}}{a^3\,x+3\,a^2\,b\,x^3+3\,a\,b^2\,x^5+b^3\,x^7}-\frac{35\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] - (1/a + (77*b*x^2)/(16*a^2) + (35*b^2*x^4)/(6*a^3) + (35*b^3*x^6)/(16*a^4)
)/(a^3*x + b^3*x^7 + 3*a^2*b*x^3 + 3*a*b^2*x^5) - (35*b^(1/2)*atan((b^(1/2)
*x)/a^(1/2)))/(16*a^(9/2))
```

sympy [A] time = 0.58, size = 139, normalized size = 1.46

$$\frac{35\sqrt{-\frac{b}{a^9}}\,\log\left(-\frac{a^5\sqrt{-\frac{b}{a^9}}}{b}+x\right)}{32}-\frac{35\sqrt{-\frac{b}{a^9}}\,\log\left(\frac{a^5\sqrt{-\frac{b}{a^9}}}{b}+x\right)}{32}+\frac{-48a^3-231a^2bx^2-280ab^2x^4-105b^3x^6}{48a^7x+144a^6bx^3+144a^5b^2x^5+48a^4b^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 35*sqrt(-b/a**9)*log(-a**5*sqrt(-b/a**9)/b + x)/32 - 35*sqrt(-b/a**9)*log(a
**5*sqrt(-b/a**9)/b + x)/32 + (-48*a**3 - 231*a**2*b*x**2 - 280*a*b**2*x**4
 - 105*b**3*x**6)/(48*a**7*x + 144*a**6*b*x**3 + 144*a**5*b**2*x**5 + 48*a*
*4*b**3*x**7)
```

3.510 $\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=106

$$\frac{105b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{11/2}}+\frac{105b}{16a^5x}-\frac{35}{16a^4x^3}+\frac{21}{16a^3x^3\left(a+bx^2\right)}+\frac{3}{8a^2x^3\left(a+bx^2\right)^2}+\frac{1}{6ax^3\left(a+bx^2\right)^3}$$

```
[Out] -35/16/a^4/x^3+105/16*b/a^5/x+1/6/a/x^3/(b*x^2+a)^3+3/8/a^2/x^3/(b*x^2+a)^2
+21/16/a^3/x^3/(b*x^2+a)+105/16*b^(3/2)*arctan(x*b^(1/2)/a^(1/2))/a^(11/2)
```

Rubi [A] time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$\frac{105b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{11/2}}+\frac{21}{16a^3x^3\left(a+bx^2\right)}+\frac{3}{8a^2x^3\left(a+bx^2\right)^2}+\frac{105b}{16a^5x}-\frac{35}{16a^4x^3}+\frac{1}{6ax^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -35/(16*a^4*x^3) + (105*b)/(16*a^5*x) + 1/(6*a*x^3*(a + b*x^2)^3) + 3/(8*a^
2*x^3*(a + b*x^2)^2) + 21/(16*a^3*x^3*(a + b*x^2)) + (105*b^(3/2)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(16*a^(11/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

]

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{x^4\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{1}{6ax^3\left(a+bx^2\right)^3} + \frac{\left(3b^3\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^3}\,dx}{2a}$$

$$= \frac{1}{6ax^3\left(a+bx^2\right)^3} + \frac{3}{8a^2x^3\left(a+bx^2\right)^2} + \frac{\left(21b^2\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^2}\,dx}{8a^2}$$

$$= \frac{1}{6ax^3\left(a+bx^2\right)^3} + \frac{3}{8a^2x^3\left(a+bx^2\right)^2} + \frac{21}{16a^3x^3\left(a+bx^2\right)} + \frac{(105b)\int \frac{1}{x^4\left(ab+b^2x^2\right)}$$

$$= -\frac{35}{16a^4x^3} + \frac{1}{6ax^3\left(a+bx^2\right)^3} + \frac{3}{8a^2x^3\left(a+bx^2\right)^2} + \frac{21}{16a^3x^3\left(a+bx^2\right)} - \frac{\left(105b^2\right)}{}$$

$$= -\frac{35}{16a^4x^3} + \frac{105b}{16a^5x} + \frac{1}{6ax^3\left(a+bx^2\right)^3} + \frac{3}{8a^2x^3\left(a+bx^2\right)^2} + \frac{21}{16a^3x^3\left(a+bx^2\right)} +$$

$$= -\frac{35}{16a^4x^3} + \frac{105b}{16a^5x} + \frac{1}{6ax^3\left(a+bx^2\right)^3} + \frac{3}{8a^2x^3\left(a+bx^2\right)^2} + \frac{21}{16a^3x^3\left(a+bx^2\right)} +$$

Mathematica [A] time = 0.05, size = 91, normalized size = 0.86

$$\frac{\frac{\sqrt{a}\left(-16a^4+144a^3bx^2+693a^2b^2x^4+840ab^3x^6+315b^4x^8\right)}{x^3\left(a+bx^2\right)^3}+315b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{48a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] ((Sqrt[a]*(-16*a^4 + 144*a^3*b*x^2 + 693*a^2*b^2*x^4 + 840*a*b^3*x^6 + 315*
b^4*x^8))/(x^3*(a + b*x^2)^3) + 315*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(4
8*a^(11/2))
```

fricas [A] time = 0.94, size = 304, normalized size = 2.87

$$\left[\frac{630\,b^4x^8+1680\,ab^3x^6+1386\,a^2b^2x^4+288\,a^3bx^2-32\,a^4+315\left(b^4x^9+3\,ab^3x^7+3\,a^2b^2x^5+a^3bx^3\right)\sqrt{-\frac{b}{a}}\,\log\left(\right.}{96\left(a^5b^3x^9+3\,a^6b^2x^7+3\,a^7bx^5+a^8x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(630*b^4*x^8 + 1680*a*b^3*x^6 + 1386*a^2*b^2*x^4 + 288*a^3*b*x^2 - 32
*a^4 + 315*(b^4*x^9 + 3*a*b^3*x^7 + 3*a^2*b^2*x^5 + a^3*b*x^3)*sqrt(-b/a)*l
og((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^5*b^3*x^9 + 3*a^6*b^2*x^
7 + 3*a^7*b*x^5 + a^8*x^3), 1/48*(315*b^4*x^8 + 840*a*b^3*x^6 + 693*a^2*b^2
*x^4 + 144*a^3*b*x^2 - 16*a^4 + 315*(b^4*x^9 + 3*a*b^3*x^7 + 3*a^2*b^2*x^5
+ a^3*b*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^5*b^3*x^9 + 3*a^6*b^2*x^7 +
3*a^7*b*x^5 + a^8*x^3)]
```

giac [A] time = 0.17, size = 82, normalized size = 0.77

$$\frac{105\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^5}+\frac{315\,b^4x^8+840\,ab^3x^6+693\,a^2b^2x^4+144\,a^3bx^2-16\,a^4}{48\left(bx^3+ax\right)^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 105/16*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/48*(315*b^4*x^8 + 840*
a*b^3*x^6 + 693*a^2*b^2*x^4 + 144*a^3*b*x^2 - 16*a^4)/((b*x^3 + a*x)^3*a^5)
```

maple [A] time = 0.02, size = 99, normalized size = 0.93

$$\frac{41b^4x^5}{16\left(b\,x^2+a\right)^3a^5}+\frac{35b^3x^3}{6\left(b\,x^2+a\right)^3a^4}+\frac{55b^2x}{16\left(b\,x^2+a\right)^3a^3}+\frac{105b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\,a^5}+\frac{4b}{a^5x}-\frac{1}{3a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/3/a^4/x^3+4*b/a^5/x+41/16*b^4/a^5/(b*x^2+a)^3*x^5+35/6*b^3/a^4/(b*x^2+a)
^3*x^3+55/16*b^2/a^3/(b*x^2+a)^3*x+105/16*b^2/a^5/(a*b)^(1/2)*arctan(1/(a*b
)^(1/2)*b*x)
```

maxima [A] time = 3.05, size = 108, normalized size = 1.02

$$\frac{315\,b^4x^8+840\,ab^3x^6+693\,a^2b^2x^4+144\,a^3bx^2-16\,a^4}{48\left(a^5b^3x^9+3\,a^6b^2x^7+3\,a^7bx^5+a^8x^3\right)}+\frac{105\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(315*b^4*x^8 + 840*a*b^3*x^6 + 693*a^2*b^2*x^4 + 144*a^3*b*x^2 - 16*a^
4)/(a^5*b^3*x^9 + 3*a^6*b^2*x^7 + 3*a^7*b*x^5 + a^8*x^3) + 105/16*b^2*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

mupad [B] time = 4.45, size = 102, normalized size = 0.96

$$\frac{\frac{3\,b\,x^2}{a^2}-\frac{1}{3\,a}+\frac{231\,b^2\,x^4}{16\,a^3}+\frac{35\,b^3\,x^6}{2\,a^4}+\frac{105\,b^4\,x^8}{16\,a^5}}{a^3\,x^3+3\,a^2\,b\,x^5+3\,a\,b^2\,x^7+b^3\,x^9}+\frac{105\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] ((3*b*x^2)/a^2 - 1/(3*a) + (231*b^2*x^4)/(16*a^3) + (35*b^3*x^6)/(2*a^4) +
(105*b^4*x^8)/(16*a^5))/(a^3*x^3 + b^3*x^9 + 3*a^2*b*x^5 + 3*a*b^2*x^7) + (
105*b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(16*a^(11/2))
```

sympy [A] time = 0.63, size = 162, normalized size = 1.53

$$-\frac{105\sqrt{-\frac{b^3}{a^{11}}}\log\left(-\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}}{b^2}+x\right)}{32}+\frac{105\sqrt{-\frac{b^3}{a^{11}}}\log\left(\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}}{b^2}+x\right)}{32}+\frac{-16a^4+144a^3bx^2+693a^2b^2x^4+840ab^3x^6+}{48a^8x^3+144a^7bx^5+144a^6b^2x^7+48a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -105*sqrt(-b**3/a**11)*log(-a**6*sqrt(-b**3/a**11)/b**2 + x)/32 + 105*sqrt(
-b**3/a**11)*log(a**6*sqrt(-b**3/a**11)/b**2 + x)/32 + (-16*a**4 + 144*a**3
*b*x**2 + 693*a**2*b**2*x**4 + 840*a*b**3*x**6 + 315*b**4*x**8)/(48*a**8*x*
*3 + 144*a**7*b*x**5 + 144*a**6*b**2*x**7 + 48*a**5*b**3*x**9)
```

3.511 $\int \frac{1}{x^6(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=119

$$-\frac{231b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{13/2}}-\frac{231b^2}{16a^6x}+\frac{77b}{16a^5x^3}-\frac{231}{80a^4x^5}+\frac{33}{16a^3x^5\left(a+bx^2\right)}+\frac{11}{24a^2x^5\left(a+bx^2\right)^2}+\frac{1}{6ax^5\left(a+bx^2\right)^3}$$

```
[Out] -231/80/a^4/x^5+77/16*b/a^5/x^3-231/16*b^2/a^6/x+1/6/a/x^5/(b*x^2+a)^3+11/2
4/a^2/x^5/(b*x^2+a)^2+33/16/a^3/x^5/(b*x^2+a)-231/16*b^(5/2)*arctan(x*b^(1/
2)/a^(1/2))/a^(13/2)
```

Rubi [A] time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$-\frac{231b^2}{16a^6x}-\frac{231b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{13/2}}+\frac{77b}{16a^5x^3}+\frac{33}{16a^3x^5\left(a+bx^2\right)}+\frac{11}{24a^2x^5\left(a+bx^2\right)^2}-\frac{231}{80a^4x^5}+\frac{1}{6ax^5\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -231/(80*a^4*x^5) + (77*b)/(16*a^5*x^3) - (231*b^2)/(16*a^6*x) + 1/(6*a*x^5
*(a + b*x^2)^3) + 11/(24*a^2*x^5*(a + b*x^2)^2) + 33/(16*a^3*x^5*(a + b*x^2
)) - (231*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(13/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
```

```
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{x^6\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{\left(11b^3\right)\int \frac{1}{x^6\left(ab+b^2x^2\right)^3}\,dx}{6a}$$

$$= \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{\left(33b^2\right)\int \frac{1}{x^6\left(ab+b^2x^2\right)^2}\,dx}{8a^2}$$

$$= \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{33}{16a^3x^5\left(a+bx^2\right)} + \frac{(231b)\int \frac{1}{x^6\left(ab+b^2x^2\right)}\,d}{16a^3}$$

$$= -\frac{231}{80a^4x^5} + \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{33}{16a^3x^5\left(a+bx^2\right)} - \frac{\left(231b^2\right)}{}$$

$$= -\frac{231}{80a^4x^5} + \frac{77b}{16a^5x^3} + \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{33}{16a^3x^5\left(a+bx^2\right)}$$

$$= -\frac{231}{80a^4x^5} + \frac{77b}{16a^5x^3} - \frac{231b^2}{16a^6x} + \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{33}{16a^3x^5\left(a\right.}$$

$$= -\frac{231}{80a^4x^5} + \frac{77b}{16a^5x^3} - \frac{231b^2}{16a^6x} + \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{33}{16a^3x^5\left(a\right.}$$

Mathematica [A] time = 0.05, size = 101, normalized size = 0.85

$$-\frac{231b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{13/2}}-\frac{48a^5-176a^4bx^2+1584a^3b^2x^4+7623a^2b^3x^6+9240ab^4x^8+3465b^5x^{10}}{240a^6x^5\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -1/240*(48*a^5 - 176*a^4*b*x^2 + 1584*a^3*b^2*x^4 + 7623*a^2*b^3*x^6 + 9240
*a*b^4*x^8 + 3465*b^5*x^10)/(a^6*x^5*(a + b*x^2)^3) - (231*b^(5/2)*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(16*a^(13/2))
```

fricas [A] time = 1.05, size = 330, normalized size = 2.77

$$\left[-\frac{6930\,b^5x^{10}+18480\,ab^4x^8+15246\,a^2b^3x^6+3168\,a^3b^2x^4-352\,a^4bx^2+96\,a^5-3465\left(b^5x^{11}+3\,ab^4x^9+3\,a^2b\right.}{480\left(a^6b^3x^{11}+3\,a^7b^2x^9+3\,a^8bx^7+a^9x^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/480*(6930*b^5*x^10 + 18480*a*b^4*x^8 + 15246*a^2*b^3*x^6 + 3168*a^3*b^2
*x^4 - 352*a^4*b*x^2 + 96*a^5 - 3465*(b^5*x^11 + 3*a*b^4*x^9 + 3*a^2*b^3*x^
7 + a^3*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a))
)/(a^6*b^3*x^11 + 3*a^7*b^2*x^9 + 3*a^8*b*x^7 + a^9*x^5), -1/240*(3465*b^5*
x^10 + 9240*a*b^4*x^8 + 7623*a^2*b^3*x^6 + 1584*a^3*b^2*x^4 - 176*a^4*b*x^2
 + 48*a^5 + 3465*(b^5*x^11 + 3*a*b^4*x^9 + 3*a^2*b^3*x^7 + a^3*b^2*x^5)*sqr
t(b/a)*arctan(x*sqrt(b/a)))/(a^6*b^3*x^11 + 3*a^7*b^2*x^9 + 3*a^8*b*x^7 + a
^9*x^5)]
```

giac [A] time = 0.16, size = 93, normalized size = 0.78

$$-\frac{231\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^6}-\frac{213\,b^5x^5+472\,ab^4x^3+267\,a^2b^3x}{48\left(bx^2+a\right)^3a^6}-\frac{150\,b^2x^4-20\,abx^2+3\,a^2}{15\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -231/16*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6) - 1/48*(213*b^5*x^5 + 472
*a*b^4*x^3 + 267*a^2*b^3*x)/((b*x^2 + a)^3*a^6) - 1/15*(150*b^2*x^4 - 20*a*
b*x^2 + 3*a^2)/(a^6*x^5)
```

maple [A] time = 0.02, size = 110, normalized size = 0.92

$$-\frac{71b^5x^5}{16\left(b\,x^2+a\right)^3a^6}-\frac{59b^4x^3}{6\left(b\,x^2+a\right)^3a^5}-\frac{89b^3x}{16\left(b\,x^2+a\right)^3a^4}-\frac{231b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\,a^6}-\frac{10b^2}{a^6x}+\frac{4b}{3a^5x^3}-\frac{1}{5a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/5/a^4/x^5-10*b^2/a^6/x+4/3*b/a^5/x^3-71/16/a^6*b^5/(b*x^2+a)^3*x^5-59/6/
a^5*b^4/(b*x^2+a)^3*x^3-89/16/a^4*b^3/(b*x^2+a)^3*x-231/16/a^6*b^3/(a*b)^(1
/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.01, size = 119, normalized size = 1.00

$$-\frac{3465\,b^5x^{10}+9240\,ab^4x^8+7623\,a^2b^3x^6+1584\,a^3b^2x^4-176\,a^4bx^2+48\,a^5}{240\left(a^6b^3x^{11}+3\,a^7b^2x^9+3\,a^8bx^7+a^9x^5\right)}-\frac{231\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/240*(3465*b^5*x^10 + 9240*a*b^4*x^8 + 7623*a^2*b^3*x^6 + 1584*a^3*b^2*x^
4 - 176*a^4*b*x^2 + 48*a^5)/(a^6*b^3*x^11 + 3*a^7*b^2*x^9 + 3*a^8*b*x^7 + a
^9*x^5) - 231/16*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6)
```

mupad [B] time = 4.46, size = 114, normalized size = 0.96

$$-\frac{\frac{1}{5\,a}-\frac{11\,b\,x^2}{15\,a^2}+\frac{33\,b^2\,x^4}{5\,a^3}+\frac{2541\,b^3\,x^6}{80\,a^4}+\frac{77\,b^4\,x^8}{2\,a^5}+\frac{231\,b^5\,x^{10}}{16\,a^6}}{a^3\,x^5+3\,a^2\,b\,x^7+3\,a\,b^2\,x^9+b^3\,x^{11}}-\frac{231\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] - (1/(5*a) - (11*b*x^2)/(15*a^2) + (33*b^2*x^4)/(5*a^3) + (2541*b^3*x^6)/(8
0*a^4) + (77*b^4*x^8)/(2*a^5) + (231*b^5*x^10)/(16*a^6))/(a^3*x^5 + b^3*x^1
1 + 3*a^2*b*x^7 + 3*a*b^2*x^9) - (231*b^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(1
6*a^(13/2))
```

sympy [A] time = 0.71, size = 173, normalized size = 1.45

$$\frac{231\sqrt{-\frac{b^5}{a^{13}}}\log\left(-\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}}{b^3}+x\right)}{32}-\frac{231\sqrt{-\frac{b^5}{a^{13}}}\log\left(\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}}{b^3}+x\right)}{32}+\frac{-48a^5+176a^4bx^2-1584a^3b^2x^4-7623a^2b^3x^6}{240a^9x^5+720a^8bx^7+720a^7b^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 231*sqrt(-b**5/a**13)*log(-a**7*sqrt(-b**5/a**13)/b**3 + x)/32 - 231*sqrt(-
b**5/a**13)*log(a**7*sqrt(-b**5/a**13)/b**3 + x)/32 + (-48*a**5 + 176*a**4*
b*x**2 - 1584*a**3*b**2*x**4 - 7623*a**2*b**3*x**6 - 9240*a*b**4*x**8 - 346
5*b**5*x**10)/(240*a**9*x**5 + 720*a**8*b*x**7 + 720*a**7*b**2*x**9 + 240*a
**6*b**3*x**11)
```

3.512 $\int \frac{x^{15}}{(a^2+2abx^2+b^2x^4)^3} \, dx$

Optimal. Leaf size=133

$$\frac{a^7}{10b^8(a+bx^2)^5}-\frac{7a^6}{8b^8(a+bx^2)^4}+\frac{7a^5}{2b^8(a+bx^2)^3}-\frac{35a^4}{4b^8(a+bx^2)^2}+\frac{35a^3}{2b^8(a+bx^2)}+\frac{21a^2\log(a+bx^2)}{2b^8}-\frac{3ax^2}{b^7}+\frac{x^4}{4b^6}$$

```
[Out] -3*a*x^2/b^7+1/4*x^4/b^6+1/10*a^7/b^8/(b*x^2+a)^5-7/8*a^6/b^8/(b*x^2+a)^4+7
/2*a^5/b^8/(b*x^2+a)^3-35/4*a^4/b^8/(b*x^2+a)^2+35/2*a^3/b^8/(b*x^2+a)+21/2
*a^2*ln(b*x^2+a)/b^8
```

Rubi [A] time = 0.14, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a^7}{10b^8(a+bx^2)^5}-\frac{7a^6}{8b^8(a+bx^2)^4}+\frac{7a^5}{2b^8(a+bx^2)^3}-\frac{35a^4}{4b^8(a+bx^2)^2}+\frac{35a^3}{2b^8(a+bx^2)}+\frac{21a^2\log(a+bx^2)}{2b^8}-\frac{3ax^2}{b^7}+\frac{x^4}{4b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-3*a*x^2)/b^7 + x^4/(4*b^6) + a^7/(10*b^8*(a + b*x^2)^5) - (7*a^6)/(8*b^8*
(a + b*x^2)^4) + (7*a^5)/(2*b^8*(a + b*x^2)^3) - (35*a^4)/(4*b^8*(a + b*x^2
)^2) + (35*a^3)/(2*b^8*(a + b*x^2)) + (21*a^2*Log[a + b*x^2])/(2*b^8)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{15}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^{15}}{\left(ab+b^2x^2\right)^6}\,dx$$
$$= \frac{1}{2}b^6\,\text{Subst}\left(\int \frac{x^7}{\left(ab+b^2x\right)^6}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^6\,\text{Subst}\left(\int \left(-\frac{6a}{b^{13}}+\frac{x}{b^{12}}-\frac{a^7}{b^{13}(a+bx)^6}+\frac{7a^6}{b^{13}(a+bx)^5}-\frac{21a^5}{b^{13}(a+bx)^4}+\frac{3}{b^{13}(a}\right.\right.$$
$$= -\frac{3ax^2}{b^7}+\frac{x^4}{4b^6}+\frac{a^7}{10b^8\left(a+bx^2\right)^5}-\frac{7a^6}{8b^8\left(a+bx^2\right)^4}+\frac{7a^5}{2b^8\left(a+bx^2\right)^3}-\frac{35a^4}{4b^8\left(a+bx\right.}$$

Mathematica [A] time = 0.03, size = 114, normalized size = 0.86

$$\frac{459a^7+1875a^6bx^2+2700a^5b^2x^4+1300a^4b^3x^6-400a^3b^4x^8-500a^2b^5x^{10}+420a^2\left(a+bx^2\right)^5\log\left(a+bx^2\right)-7}{40b^8\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (459*a^7 + 1875*a^6*b*x^2 + 2700*a^5*b^2*x^4 + 1300*a^4*b^3*x^6 - 400*a^3*b
^4*x^8 - 500*a^2*b^5*x^10 - 70*a*b^6*x^12 + 10*b^7*x^14 + 420*a^2*(a + b*x^
2)^5*Log[a + b*x^2])/(40*b^8*(a + b*x^2)^5)
```

fricas [A] time = 1.01, size = 203, normalized size = 1.53

$$\frac{10\,b^7x^{14}-70\,ab^6x^{12}-500\,a^2b^5x^{10}-400\,a^3b^4x^8+1300\,a^4b^3x^6+2700\,a^5b^2x^4+1875\,a^6bx^2+459\,a^7+420\,\left(a^2b\right.}{40\left(b^{13}x^{10}+5\,ab^{12}x^8+10\,a^2b^{11}x^6+10\,a^3b^{10}x^4+5\,a^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/40*(10*b^7*x^14 - 70*a*b^6*x^12 - 500*a^2*b^5*x^10 - 400*a^3*b^4*x^8 + 13
00*a^4*b^3*x^6 + 2700*a^5*b^2*x^4 + 1875*a^6*b*x^2 + 459*a^7 + 420*(a^2*b^5
*x^10 + 5*a^3*b^4*x^8 + 10*a^4*b^3*x^6 + 10*a^5*b^2*x^4 + 5*a^6*b*x^2 + a^7
```

```
)*log(b*x^2 + a))/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a^3*b^10
*x^4 + 5*a^4*b^9*x^2 + a^5*b^8)
```

giac [A] time = 0.16, size = 113, normalized size = 0.85

$$\frac{21\,a^2\log\left(\left|bx^2+a\right|\right)}{2\,b^8}+\frac{b^6x^4-12\,ab^5x^2}{4\,b^{12}}-\frac{959\,a^2b^5x^{10}+4095\,a^3b^4x^8+7140\,a^4b^3x^6+6300\,a^5b^2x^4+2800\,a^6bx^2+5}{40\left(bx^2+a\right)^5b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 21/2*a^2*log(abs(b*x^2 + a))/b^8 + 1/4*(b^6*x^4 - 12*a*b^5*x^2)/b^12 - 1/40
*(959*a^2*b^5*x^10 + 4095*a^3*b^4*x^8 + 7140*a^4*b^3*x^6 + 6300*a^5*b^2*x^4
 + 2800*a^6*b*x^2 + 500*a^7)/((b*x^2 + a)^5*b^8)
```

maple [A] time = 0.02, size = 120, normalized size = 0.90

$$\frac{a^7}{10\left(b\,x^2+a\right)^5b^8}-\frac{7a^6}{8\left(b\,x^2+a\right)^4b^8}+\frac{x^4}{4b^6}+\frac{7a^5}{2\left(b\,x^2+a\right)^3b^8}-\frac{35a^4}{4\left(b\,x^2+a\right)^2b^8}-\frac{3a\,x^2}{b^7}+\frac{35a^3}{2\left(b\,x^2+a\right)b^8}+\frac{21a^2\ln\left(b\,x^2+\right.}{2b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -3*a*x^2/b^7+1/4*x^4/b^6+1/10*a^7/b^8/(b*x^2+a)^5-7/8*a^6/b^8/(b*x^2+a)^4+7
/2*a^5/b^8/(b*x^2+a)^3-35/4*a^4/b^8/(b*x^2+a)^2+35/2*a^3/b^8/(b*x^2+a)+21/2
*a^2*ln(b*x^2+a)/b^8
```

maxima [A] time = 1.42, size = 143, normalized size = 1.08

$$\frac{700\,a^3b^4x^8+2450\,a^4b^3x^6+3290\,a^5b^2x^4+1995\,a^6bx^2+459\,a^7}{40\left(b^{13}x^{10}+5\,ab^{12}x^8+10\,a^2b^{11}x^6+10\,a^3b^{10}x^4+5\,a^4b^9x^2+a^5b^8\right)}+\frac{21\,a^2\log\left(bx^2+a\right)}{2\,b^8}+\frac{bx^4-12\,ax^2}{4\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/40*(700*a^3*b^4*x^8 + 2450*a^4*b^3*x^6 + 3290*a^5*b^2*x^4 + 1995*a^6*b*x^
2 + 459*a^7)/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a^3*b^10*x^4
+ 5*a^4*b^9*x^2 + a^5*b^8) + 21/2*a^2*log(b*x^2 + a)/b^8 + 1/4*(b*x^4 - 12*
a*x^2)/b^7
```

mupad [B] time = 0.13, size = 142, normalized size = 1.07

$$\frac{\frac{459\,a^7}{40\,b}+\frac{399\,a^6\,x^2}{8}+\frac{329\,a^5\,b\,x^4}{4}+\frac{245\,a^4\,b^2\,x^6}{4}+\frac{35\,a^3\,b^3\,x^8}{2}}{a^5\,b^7+5\,a^4\,b^8\,x^2+10\,a^3\,b^9\,x^4+10\,a^2\,b^{10}\,x^6+5\,a\,b^{11}\,x^8+b^{12}\,x^{10}}+\frac{x^4}{4\,b^6}-\frac{3\,a\,x^2}{b^7}+\frac{21\,a^2\,\ln\left(b\,x^2+a\right)}{2\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((459*a^7)/(40*b) + (399*a^6*x^2)/8 + (329*a^5*b*x^4)/4 + (245*a^4*b^2*x^6)
/4 + (35*a^3*b^3*x^8)/2)/(a^5*b^7 + b^12*x^10 + 5*a*b^11*x^8 + 5*a^4*b^8*x^
2 + 10*a^3*b^9*x^4 + 10*a^2*b^10*x^6) + x^4/(4*b^6) - (3*a*x^2)/b^7 + (21*a
^2*log(a + b*x^2))/(2*b^8)
```

sympy [A] time = 0.99, size = 150, normalized size = 1.13

$$\frac{21a^2\log\left(a+bx^2\right)}{2b^8}-\frac{3ax^2}{b^7}+\frac{459a^7+1995a^6bx^2+3290a^5b^2x^4+2450a^4b^3x^6+700a^3b^4x^8}{40a^5b^8+200a^4b^9x^2+400a^3b^{10}x^4+400a^2b^{11}x^6+200ab^{12}x^8+40b^{13}x^{10}}+\frac{x^4}{4b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 21*a**2*log(a + b*x**2)/(2*b**8) - 3*a*x**2/b**7 + (459*a**7 + 1995*a**6*b*
x**2 + 3290*a**5*b**2*x**4 + 2450*a**4*b**3*x**6 + 700*a**3*b**4*x**8)/(40*
a**5*b**8 + 200*a**4*b**9*x**2 + 400*a**3*b**10*x**4 + 400*a**2*b**11*x**6
+ 200*a*b**12*x**8 + 40*b**13*x**10) + x**4/(4*b**6)
```

3.513 $\int \frac{x^{13}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=118

$$-\frac{a^6}{10b^7\left(a+bx^2\right)^5}+\frac{3a^5}{4b^7\left(a+bx^2\right)^4}-\frac{5a^4}{2b^7\left(a+bx^2\right)^3}+\frac{5a^3}{b^7\left(a+bx^2\right)^2}-\frac{15a^2}{2b^7\left(a+bx^2\right)}-\frac{3a\log\left(a+bx^2\right)}{b^7}+\frac{x^2}{2b^6}$$

```
[Out] 1/2*x^2/b^6-1/10*a^6/b^7/(b*x^2+a)^5+3/4*a^5/b^7/(b*x^2+a)^4-5/2*a^4/b^7/(b
*x^2+a)^3+5*a^3/b^7/(b*x^2+a)^2-15/2*a^2/b^7/(b*x^2+a)-3*a*ln(b*x^2+a)/b^7
```

Rubi [A] time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{a^6}{10b^7\left(a+bx^2\right)^5}+\frac{3a^5}{4b^7\left(a+bx^2\right)^4}-\frac{5a^4}{2b^7\left(a+bx^2\right)^3}+\frac{5a^3}{b^7\left(a+bx^2\right)^2}-\frac{15a^2}{2b^7\left(a+bx^2\right)}-\frac{3a\log\left(a+bx^2\right)}{b^7}+\frac{x^2}{2b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] x^2/(2*b^6) - a^6/(10*b^7*(a + b*x^2)^5) + (3*a^5)/(4*b^7*(a + b*x^2)^4) -
(5*a^4)/(2*b^7*(a + b*x^2)^3) + (5*a^3)/(b^7*(a + b*x^2)^2) - (15*a^2)/(2*b
^7*(a + b*x^2)) - (3*a*Log[a + b*x^2])/b^7
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{13}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^{13}}{\left(ab+b^2x^2\right)^6}\,dx$$
$$= \frac{1}{2}b^6\,\text{Subst}\left(\int \frac{x^6}{\left(ab+b^2x\right)^6}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^6\,\text{Subst}\left(\int \left(\frac{1}{b^{12}}+\frac{a^6}{b^{12}(a+bx)^6}-\frac{6a^5}{b^{12}(a+bx)^5}+\frac{15a^4}{b^{12}(a+bx)^4}-\frac{20a^3}{b^{12}(a+bx)^3}\right.\right.$$
$$= \frac{x^2}{2b^6}-\frac{a^6}{10b^7\left(a+bx^2\right)^5}+\frac{3a^5}{4b^7\left(a+bx^2\right)^4}-\frac{5a^4}{2b^7\left(a+bx^2\right)^3}+\frac{5a^3}{b^7\left(a+bx^2\right)^2}-\frac{1}{2b^7\left(\right.}$$

Mathematica [A] time = 0.03, size = 101, normalized size = 0.86

$$-\frac{87a^6+375a^5bx^2+600a^4b^2x^4+400a^3b^3x^6+50a^2b^4x^8-50ab^5x^{10}+60a\left(a+bx^2\right)^5\log\left(a+bx^2\right)-10b^6x^{12}}{20b^7\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/20*(87*a^6 + 375*a^5*b*x^2 + 600*a^4*b^2*x^4 + 400*a^3*b^3*x^6 + 50*a^2*
b^4*x^8 - 50*a*b^5*x^10 - 10*b^6*x^12 + 60*a*(a + b*x^2)^5*Log[a + b*x^2])/
(b^7*(a + b*x^2)^5)
```

fricas [A] time = 0.99, size = 190, normalized size = 1.61

$$\frac{10\,b^6x^{12}+50\,ab^5x^{10}-50\,a^2b^4x^8-400\,a^3b^3x^6-600\,a^4b^2x^4-375\,a^5bx^2-87\,a^6-60\left(ab^5x^{10}+5\,a^2b^4x^8+10\,a^3\right.}{20\left(b^{12}x^{10}+5\,ab^{11}x^8+10\,a^2b^{10}x^6+10\,a^3b^9x^4+5\,a^4b^8x^2+a^5b^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/20*(10*b^6*x^12 + 50*a*b^5*x^10 - 50*a^2*b^4*x^8 - 400*a^3*b^3*x^6 - 600*
a^4*b^2*x^4 - 375*a^5*b*x^2 - 87*a^6 - 60*(a*b^5*x^10 + 5*a^2*b^4*x^8 + 10*
a^3*b^3*x^6 + 10*a^4*b^2*x^4 + 5*a^5*b*x^2 + a^6)*log(b*x^2 + a))/(b^12*x^1
```

```
0 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b
^7)
```

giac [A] time = 0.23, size = 95, normalized size = 0.81

$$\frac{x^2}{2\,b^6}-\frac{3\,a\log\left(\left|bx^2+a\right|\right)}{b^7}+\frac{137\,ab^5x^{10}+535\,a^2b^4x^8+870\,a^3b^3x^6+720\,a^4b^2x^4+300\,a^5bx^2+50\,a^6}{20\left(bx^2+a\right)^5b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^6 - 3*a*log(abs(b*x^2 + a))/b^7 + 1/20*(137*a*b^5*x^10 + 535*a^2*
b^4*x^8 + 870*a^3*b^3*x^6 + 720*a^4*b^2*x^4 + 300*a^5*b*x^2 + 50*a^6)/((b*x
^2 + a)^5*b^7)
```

maple [A] time = 0.01, size = 109, normalized size = 0.92

$$-\frac{a^6}{10\left(b\,x^2+a\right)^5b^7}+\frac{3a^5}{4\left(b\,x^2+a\right)^4b^7}-\frac{5a^4}{2\left(b\,x^2+a\right)^3b^7}+\frac{5a^3}{\left(b\,x^2+a\right)^2b^7}+\frac{x^2}{2b^6}-\frac{15a^2}{2\left(b\,x^2+a\right)b^7}-\frac{3a\ln\left(b\,x^2+a\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/2*x^2/b^6-1/10*a^6/b^7/(b*x^2+a)^5+3/4*a^5/b^7/(b*x^2+a)^4-5/2*a^4/b^7/(b
*x^2+a)^3+5*a^3/b^7/(b*x^2+a)^2-15/2*a^2/b^7/(b*x^2+a)-3*a*ln(b*x^2+a)/b^7
```

maxima [A] time = 1.47, size = 132, normalized size = 1.12

$$-\frac{150\,a^2b^4x^8+500\,a^3b^3x^6+650\,a^4b^2x^4+385\,a^5bx^2+87\,a^6}{20\left(b^{12}x^{10}+5\,ab^{11}x^8+10\,a^2b^{10}x^6+10\,a^3b^9x^4+5\,a^4b^8x^2+a^5b^7\right)}+\frac{x^2}{2\,b^6}-\frac{3\,a\log\left(bx^2+a\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/20*(150*a^2*b^4*x^8 + 500*a^3*b^3*x^6 + 650*a^4*b^2*x^4 + 385*a^5*b*x^2
+ 87*a^6)/(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*
a^4*b^8*x^2 + a^5*b^7) + 1/2*x^2/b^6 - 3*a*log(b*x^2 + a)/b^7
```

mupad [B] time = 4.60, size = 132, normalized size = 1.12

$$\frac{x^2}{2\,b^6}-\frac{\frac{87\,a^6}{20\,b}+\frac{77\,a^5\,x^2}{4}+\frac{65\,a^4\,b\,x^4}{2}+25\,a^3\,b^2\,x^6+\frac{15\,a^2\,b^3\,x^8}{2}}{a^5\,b^6+5\,a^4\,b^7\,x^2+10\,a^3\,b^8\,x^4+10\,a^2\,b^9\,x^6+5\,a\,b^{10}\,x^8+b^{11}\,x^{10}}-\frac{3\,a\,\ln\left(b\,x^2+a\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] x^2/(2*b^6) - ((87*a^6)/(20*b) + (77*a^5*x^2)/4 + (65*a^4*b*x^4)/2 + 25*a^3
*b^2*x^6 + (15*a^2*b^3*x^8)/2)/(a^5*b^6 + b^11*x^10 + 5*a*b^10*x^8 + 5*a^4*
b^7*x^2 + 10*a^3*b^8*x^4 + 10*a^2*b^9*x^6) - (3*a*log(a + b*x^2))/b^7
```

sympy [A] time = 0.97, size = 138, normalized size = 1.17

$$-\frac{3a\log\left(a+bx^2\right)}{b^7}+\frac{-87a^6-385a^5bx^2-650a^4b^2x^4-500a^3b^3x^6-150a^2b^4x^8}{20a^5b^7+100a^4b^8x^2+200a^3b^9x^4+200a^2b^{10}x^6+100ab^{11}x^8+20b^{12}x^{10}}+\frac{x^2}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -3*a*log(a + b*x**2)/b**7 + (-87*a**6 - 385*a**5*b*x**2 - 650*a**4*b**2*x**
4 - 500*a**3*b**3*x**6 - 150*a**2*b**4*x**8)/(20*a**5*b**7 + 100*a**4*b**8*
x**2 + 200*a**3*b**9*x**4 + 200*a**2*b**10*x**6 + 100*a*b**11*x**8 + 20*b**
12*x**10) + x**2/(2*b**6)
```

3.514 $\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=109

$$\frac{a^5}{10b^6\left(a+bx^2\right)^5}-\frac{5a^4}{8b^6\left(a+bx^2\right)^4}+\frac{5a^3}{3b^6\left(a+bx^2\right)^3}-\frac{5a^2}{2b^6\left(a+bx^2\right)^2}+\frac{5a}{2b^6\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^6}$$

```
[Out] 1/10*a^5/b^6/(b*x^2+a)^5-5/8*a^4/b^6/(b*x^2+a)^4+5/3*a^3/b^6/(b*x^2+a)^3-5/
2*a^2/b^6/(b*x^2+a)^2+5/2*a/b^6/(b*x^2+a)+1/2*ln(b*x^2+a)/b^6
```

Rubi [A] time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a^5}{10b^6\left(a+bx^2\right)^5}-\frac{5a^4}{8b^6\left(a+bx^2\right)^4}+\frac{5a^3}{3b^6\left(a+bx^2\right)^3}-\frac{5a^2}{2b^6\left(a+bx^2\right)^2}+\frac{5a}{2b^6\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] a^5/(10*b^6*(a + b*x^2)^5) - (5*a^4)/(8*b^6*(a + b*x^2)^4) + (5*a^3)/(3*b^6
*(a + b*x^2)^3) - (5*a^2)/(2*b^6*(a + b*x^2)^2) + (5*a)/(2*b^6*(a + b*x^2))
 + Log[a + b*x^2]/(2*b^6)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^{11}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{2}b^6\,\text{Subst}\left(\int \frac{x^5}{\left(ab+b^2x\right)^6}\,dx, x, x^2\right)$$

$$= \frac{1}{2}b^6\,\text{Subst}\left(\int \left(-\frac{a^5}{b^{11}(a+bx)^6}+\frac{5a^4}{b^{11}(a+bx)^5}-\frac{10a^3}{b^{11}(a+bx)^4}+\frac{10a^2}{b^{11}(a+bx)^3}-\frac{}{b^1}\right.\right.$$

$$= \frac{a^5}{10b^6\left(a+bx^2\right)^5}-\frac{5a^4}{8b^6\left(a+bx^2\right)^4}+\frac{5a^3}{3b^6\left(a+bx^2\right)^3}-\frac{5a^2}{2b^6\left(a+bx^2\right)^2}+\frac{5a}{2b^6\left(a+b\right.}$$

Mathematica [A] time = 0.02, size = 72, normalized size = 0.66

$$\frac{\frac{a\left(137a^4+625a^3bx^2+1100a^2b^2x^4+900ab^3x^6+300b^4x^8\right)}{\left(a+bx^2\right)^5}+60\log\left(a+bx^2\right)}{120b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((a*(137*a^4 + 625*a^3*b*x^2 + 1100*a^2*b^2*x^4 + 900*a*b^3*x^6 + 300*b^4*x
^8))/(a + b*x^2)^5 + 60*Log[a + b*x^2])/(120*b^6)
```

fricas [A] time = 0.79, size = 168, normalized size = 1.54

$$\frac{300\,ab^4x^8+900\,a^2b^3x^6+1100\,a^3b^2x^4+625\,a^4bx^2+137\,a^5+60\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a\right.}{120\left(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(300*a*b^4*x^8 + 900*a^2*b^3*x^6 + 1100*a^3*b^2*x^4 + 625*a^4*b*x^2 +
 137*a^5 + 60*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5
*a^4*b*x^2 + a^5)*log(b*x^2 + a))/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^
6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)
```

giac [A] time = 0.16, size = 75, normalized size = 0.69

$$\frac{\log\left(\left|bx^2+a\right|\right)}{2\,b^6}-\frac{137\,b^4x^{10}+385\,ab^3x^8+470\,a^2b^2x^6+270\,a^3bx^4+60\,a^4x^2}{120\,\left(bx^2+a\right)^5b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/b^6 - 1/120*(137*b^4*x^10 + 385*a*b^3*x^8 + 470*a^2
*b^2*x^6 + 270*a^3*b*x^4 + 60*a^4*x^2)/((b*x^2 + a)^5*b^5)
```

maple [A] time = 0.01, size = 98, normalized size = 0.90

$$\frac{a^5}{10\left(b\,x^2+a\right)^5b^6}-\frac{5a^4}{8\left(b\,x^2+a\right)^4b^6}+\frac{5a^3}{3\left(b\,x^2+a\right)^3b^6}-\frac{5a^2}{2\left(b\,x^2+a\right)^2b^6}+\frac{5a}{2\left(b\,x^2+a\right)b^6}+\frac{\ln\left(b\,x^2+a\right)}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/10*a^5/b^6/(b*x^2+a)^5-5/8*a^4/b^6/(b*x^2+a)^4+5/3*a^3/b^6/(b*x^2+a)^3-5/
2*a^2/b^6/(b*x^2+a)^2+5/2*a/b^6/(b*x^2+a)+1/2*ln(b*x^2+a)/b^6
```

maxima [A] time = 1.42, size = 121, normalized size = 1.11

$$\frac{300\,ab^4x^8+900\,a^2b^3x^6+1100\,a^3b^2x^4+625\,a^4bx^2+137\,a^5}{120\,\left(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6\right)}+\frac{\log\left(bx^2+a\right)}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/120*(300*a*b^4*x^8 + 900*a^2*b^3*x^6 + 1100*a^3*b^2*x^4 + 625*a^4*b*x^2 +
 137*a^5)/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a
^4*b^7*x^2 + a^5*b^6) + 1/2*log(b*x^2 + a)/b^6
```

mupad [B] time = 4.37, size = 119, normalized size = 1.09

$$\frac{\frac{137\,a^5}{120\,b^6}+\frac{5\,a\,x^8}{2\,b^2}+\frac{15\,a^2\,x^6}{2\,b^3}+\frac{55\,a^3\,x^4}{6\,b^4}+\frac{125\,a^4\,x^2}{24\,b^5}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}+\frac{\ln\left(b\,x^2+a\right)}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((137*a^5)/(120*b^6) + (5*a*x^8)/(2*b^2) + (15*a^2*x^6)/(2*b^3) + (55*a^3*x
^4)/(6*b^4) + (125*a^4*x^2)/(24*b^5))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b
^4*x^8 + 10*a^3*b^2*x^4 + 10*a^2*b^3*x^6) + log(a + b*x^2)/(2*b^6)
```

sympy [A] time = 0.81, size = 124, normalized size = 1.14

$$\frac{137a^5 + 625a^4bx^2 + 1100a^3b^2x^4 + 900a^2b^3x^6 + 300ab^4x^8}{120a^5b^6 + 600a^4b^7x^2 + 1200a^3b^8x^4 + 1200a^2b^9x^6 + 600ab^{10}x^8 + 120b^{11}x^{10}} + \frac{\log\left(a + bx^2\right)}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (137*a**5 + 625*a**4*b*x**2 + 1100*a**3*b**2*x**4 + 900*a**2*b**3*x**6 + 30
0*a*b**4*x**8)/(120*a**5*b**6 + 600*a**4*b**7*x**2 + 1200*a**3*b**8*x**4 +
1200*a**2*b**9*x**6 + 600*a*b**10*x**8 + 120*b**11*x**10) + log(a + b*x**2)
/(2*b**6)
```

3.515 $\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^3} dx$

Optimal. Leaf size=19

$$\frac{x^{10}}{10a(a+bx^2)^5}$$

```
[Out] 1/10*x^10/a/(b*x^2+a)^5
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 264}

$$\frac{x^{10}}{10a(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] x^10/(10*a*(a + b*x^2)^5)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^3}\,dx &= b^6\int \frac{x^9}{(ab+b^2x^2)^6}\,dx\\ &= \frac{x^{10}}{10a(a+bx^2)^5}\end{aligned}$$

Mathematica [B] time = 0.02, size = 57, normalized size = 3.00

$$-\frac{a^4+5a^3bx^2+10a^2b^2x^4+10ab^3x^6+5b^4x^8}{10b^5\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/10*(a^4 + 5*a^3*b*x^2 + 10*a^2*b^2*x^4 + 10*a*b^3*x^6 + 5*b^4*x^8)/(b^5*
(a + b*x^2)^5)
```

fricas [B] time = 0.97, size = 102, normalized size = 5.37

$$-\frac{5\,b^4x^8+10\,ab^3x^6+10\,a^2b^2x^4+5\,a^3bx^2+a^4}{10\left(b^{10}x^{10}+5\,ab^9x^8+10\,a^2b^8x^6+10\,a^3b^7x^4+5\,a^4b^6x^2+a^5b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/(b^10
*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5
*b^5)
```

giac [B] time = 0.20, size = 55, normalized size = 2.89

$$-\frac{5\,b^4x^8+10\,ab^3x^6+10\,a^2b^2x^4+5\,a^3bx^2+a^4}{10\left(bx^2+a\right)^5b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/((b*x
^2 + a)^5*b^5)
```

maple [B] time = 0.01, size = 81, normalized size = 4.26

$$-\frac{a^4}{10\left(b\,x^2+a\right)^5b^5}+\frac{a^3}{2\left(b\,x^2+a\right)^4b^5}-\frac{a^2}{\left(b\,x^2+a\right)^3b^5}+\frac{a}{\left(b\,x^2+a\right)^2b^5}-\frac{1}{2\left(b\,x^2+a\right)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -a^2/b^5/(b*x^2+a)^3+1/2*a^3/b^5/(b*x^2+a)^4-1/10*a^4/b^5/(b*x^2+a)^5+a/b^5
/(b*x^2+a)^2-1/2/b^5/(b*x^2+a)
```

maxima [B] time = 1.37, size = 102, normalized size = 5.37

$$-\frac{5\,b^4x^8 + 10\,ab^3x^6 + 10\,a^2b^2x^4 + 5\,a^3bx^2 + a^4}{10\left(b^{10}x^{10} + 5\,ab^9x^8 + 10\,a^2b^8x^6 + 10\,a^3b^7x^4 + 5\,a^4b^6x^2 + a^5b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/(b^10
*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5
*b^5)
```

mupad [B] time = 4.45, size = 104, normalized size = 5.47

$$-\frac{a^4 + 5\,a^3\,b\,x^2 + 10\,a^2\,b^2\,x^4 + 10\,a\,b^3\,x^6 + 5\,b^4\,x^8}{10\,a^5\,b^5 + 50\,a^4\,b^6\,x^2 + 100\,a^3\,b^7\,x^4 + 100\,a^2\,b^8\,x^6 + 50\,a\,b^9\,x^8 + 10\,b^{10}\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] -(a^4 + 5*b^4*x^8 + 5*a^3*b*x^2 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4)/(10*a^5*b^
5 + 10*b^10*x^10 + 50*a*b^9*x^8 + 50*a^4*b^6*x^2 + 100*a^3*b^7*x^4 + 100*a^
2*b^8*x^6)
```

sympy [B] time = 0.72, size = 107, normalized size = 5.63

$$\frac{-a^4 - 5a^3bx^2 - 10a^2b^2x^4 - 10ab^3x^6 - 5b^4x^8}{10a^5b^5 + 50a^4b^6x^2 + 100a^3b^7x^4 + 100a^2b^8x^6 + 50ab^9x^8 + 10b^{10}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (-a**4 - 5*a**3*b*x**2 - 10*a**2*b**2*x**4 - 10*a*b**3*x**6 - 5*b**4*x**8)/
(10*a**5*b**5 + 50*a**4*b**6*x**2 + 100*a**3*b**7*x**4 + 100*a**2*b**8*x**6
 + 50*a*b**9*x**8 + 10*b**10*x**10)
```

3.516 $\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^3} dx$

Optimal. Leaf size=39

$$\frac{x^8}{40a^2\left(a+bx^2\right)^4}+\frac{x^8}{10a\left(a+bx^2\right)^5}$$

```
[Out] 1/10*x^8/a/(b*x^2+a)^5+1/40*x^8/a^2/(b*x^2+a)^4
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 266, 45, 37}

$$\frac{x^8}{40a^2\left(a+bx^2\right)^4}+\frac{x^8}{10a\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] x^8/(10*a*(a + b*x^2)^5) + x^8/(40*a^2*(a + b*x^2)^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx &= b^6\int \frac{x^7}{\left(ab+b^2x^2\right)^6}\,dx \\
&= \frac{1}{2}b^6\,\mathrm{Subst}\left(\int \frac{x^3}{\left(ab+b^2x\right)^6}\,dx, x, x^2\right) \\
&= \frac{x^8}{10a\left(a+bx^2\right)^5}+\frac{b^5\,\mathrm{Subst}\left(\int \frac{x^3}{\left(ab+b^2x\right)^5}\,dx, x, x^2\right)}{10a} \\
&= \frac{x^8}{10a\left(a+bx^2\right)^5}+\frac{x^8}{40a^2\left(a+bx^2\right)^4}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.18

$$-\frac{a^3+5a^2bx^2+10ab^2x^4+10b^3x^6}{40b^4\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
[Out] -1/40*(a^3 + 5*a^2*b*x^2 + 10*a*b^2*x^4 + 10*b^3*x^6)/(b^4*(a + b*x^2)^5)
```

fricas [B] time = 1.07, size = 91, normalized size = 2.33

$$-\frac{10\,b^3x^6+10\,ab^2x^4+5\,a^2bx^2+a^3}{40\left(b^9x^{10}+5\,ab^8x^8+10\,a^2b^7x^6+10\,a^3b^6x^4+5\,a^4b^5x^2+a^5b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/40*(10*b^3*x^6 + 10*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/(b^9*x^10 + 5*a*b^8*x
^8 + 10*a^2*b^7*x^6 + 10*a^3*b^6*x^4 + 5*a^4*b^5*x^2 + a^5*b^4)
```

giac [A] time = 0.16, size = 44, normalized size = 1.13

$$-\frac{10\,b^3x^6 + 10\,ab^2x^4 + 5\,a^2bx^2 + a^3}{40\,\left(bx^2 + a\right)^5 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/40*(10*b^3*x^6 + 10*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/((b*x^2 + a)^5*b^4)
```

maple [A] time = 0.01, size = 65, normalized size = 1.67

$$\frac{a^3}{10\left(b\,x^2 + a\right)^5 b^4} - \frac{3a^2}{8\left(b\,x^2 + a\right)^4 b^4} + \frac{a}{2\left(b\,x^2 + a\right)^3 b^4} - \frac{1}{4\left(b\,x^2 + a\right)^2 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/2*a/b^4/(b*x^2+a)^3-3/8*a^2/b^4/(b*x^2+a)^4+1/10*a^3/b^4/(b*x^2+a)^5-1/4/
b^4/(b*x^2+a)^2
```

maxima [B] time = 1.39, size = 91, normalized size = 2.33

$$-\frac{10\,b^3x^6 + 10\,ab^2x^4 + 5\,a^2bx^2 + a^3}{40\left(b^9x^{10} + 5\,ab^8x^8 + 10\,a^2b^7x^6 + 10\,a^3b^6x^4 + 5\,a^4b^5x^2 + a^5b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/40*(10*b^3*x^6 + 10*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/(b^9*x^10 + 5*a*b^8*x
^8 + 10*a^2*b^7*x^6 + 10*a^3*b^6*x^4 + 5*a^4*b^5*x^2 + a^5*b^4)
```

mupad [B] time = 0.05, size = 93, normalized size = 2.38

$$-\frac{a^3 + 5\,a^2\,b\,x^2 + 10\,a\,b^2\,x^4 + 10\,b^3\,x^6}{40\,a^5\,b^4 + 200\,a^4\,b^5\,x^2 + 400\,a^3\,b^6\,x^4 + 400\,a^2\,b^7\,x^6 + 200\,a\,b^8\,x^8 + 40\,b^9\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] -(a^3 + 10*b^3*x^6 + 5*a^2*b*x^2 + 10*a*b^2*x^4)/(40*a^5*b^4 + 40*b^9*x^10
+ 200*a*b^8*x^8 + 200*a^4*b^5*x^2 + 400*a^3*b^6*x^4 + 400*a^2*b^7*x^6)
```

sympy [B] time = 0.67, size = 95, normalized size = 2.44

$$\frac{-a^3-5a^2bx^2-10ab^2x^4-10b^3x^6}{40a^5b^4+200a^4b^5x^2+400a^3b^6x^4+400a^2b^7x^6+200ab^8x^8+40b^9x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (-a**3 - 5*a**2*b*x**2 - 10*a*b**2*x**4 - 10*b**3*x**6)/(40*a**5*b**4 + 200
*a**4*b**5*x**2 + 400*a**3*b**6*x**4 + 400*a**2*b**7*x**6 + 200*a*b**8*x**8
 + 40*b**9*x**10)
```

3.517 $\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=53

$$-\frac{a^2}{10b^3\left(a+bx^2\right)^5}+\frac{a}{4b^3\left(a+bx^2\right)^4}-\frac{1}{6b^3\left(a+bx^2\right)^3}$$

```
[Out] -1/10*a^2/b^3/(b*x^2+a)^5+1/4*a/b^3/(b*x^2+a)^4-1/6/b^3/(b*x^2+a)^3
```

Rubi [A] time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{a^2}{10b^3\left(a+bx^2\right)^5}+\frac{a}{4b^3\left(a+bx^2\right)^4}-\frac{1}{6b^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -a^2/(10*b^3*(a + b*x^2)^5) + a/(4*b^3*(a + b*x^2)^4) - 1/(6*b^3*(a + b*x^2
)^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx &= b^6\int \frac{x^5}{\left(ab+b^2x^2\right)^6}\,dx \\
&= \frac{1}{2}b^6\,\text{Subst}\left(\int \frac{x^2}{\left(ab+b^2x\right)^6}\,dx,x,x^2\right) \\
&= \frac{1}{2}b^6\,\text{Subst}\left(\int \left(\frac{a^2}{b^8(a+bx)^6}-\frac{2a}{b^8(a+bx)^5}+\frac{1}{b^8(a+bx)^4}\right)dx,x,x^2\right) \\
&= -\frac{a^2}{10b^3\left(a+bx^2\right)^5}+\frac{a}{4b^3\left(a+bx^2\right)^4}-\frac{1}{6b^3\left(a+bx^2\right)^3}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.66

$$
-\frac{a^2+5abx^2+10b^2x^4}{60b^3\left(a+bx^2\right)^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/60*(a^2 + 5*a*b*x^2 + 10*b^2*x^4)/(b^3*(a + b*x^2)^5)
```

fricas [A] time = 1.01, size = 80, normalized size = 1.51

$$
-\frac{10\,b^2x^4+5\,abx^2+a^2}{60\left(b^8x^{10}+5\,ab^7x^8+10\,a^2b^6x^6+10\,a^3b^5x^4+5\,a^4b^4x^2+a^5b^3\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/60*(10*b^2*x^4 + 5*a*b*x^2 + a^2)/(b^8*x^10 + 5*a*b^7*x^8 + 10*a^2*b^6*x
^6 + 10*a^3*b^5*x^4 + 5*a^4*b^4*x^2 + a^5*b^3)
```

giac [A] time = 0.19, size = 33, normalized size = 0.62

$$
-\frac{10\,b^2x^4+5\,abx^2+a^2}{60\left(bx^2+a\right)^5b^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/60*(10*b^2*x^4 + 5*a*b*x^2 + a^2)/((b*x^2 + a)^5*b^3)
```

maple [A] time = 0.01, size = 48, normalized size = 0.91

$$-\frac{a^{2}}{10 \left(b \,x^{2}+a \right)^{5} b^{3}}+\frac{a}{4 \left(b \,x^{2}+a \right)^{4} b^{3}}-\frac{1}{6 \left(b \,x^{2}+a \right)^{3} b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/10*a^2/b^3/(b*x^2+a)^5+1/4*a/b^3/(b*x^2+a)^4-1/6/b^3/(b*x^2+a)^3
```

maxima [A] time = 1.35, size = 80, normalized size = 1.51

$$-\frac{10 \, b^{2} x^{4} + 5 \, a b x^{2} + a^{2}}{60 \, {\left(b^{8} x^{10} + 5 \, a b^{7} x^{8} + 10 \, a^{2} b^{6} x^{6} + 10 \, a^{3} b^{5} x^{4} + 5 \, a^{4} b^{4} x^{2} + a^{5} b^{3}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/60*(10*b^2*x^4 + 5*a*b*x^2 + a^2)/(b^8*x^10 + 5*a*b^7*x^8 + 10*a^2*b^6*x
^6 + 10*a^3*b^5*x^4 + 5*a^4*b^4*x^2 + a^5*b^3)
```

mupad [B] time = 4.62, size = 81, normalized size = 1.53

$$-\frac{\frac{a^2}{60\,b^3}+\frac{x^4}{6\,b}+\frac{a\,x^2}{12\,b^2}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] -(a^2/(60*b^3) + x^4/(6*b) + (a*x^2)/(12*b^2))/(a^5 + b^5*x^10 + 5*a^4*b*x^
2 + 5*a*b^4*x^8 + 10*a^3*b^2*x^4 + 10*a^2*b^3*x^6)
```

sympy [A] time = 0.62, size = 83, normalized size = 1.57

$$\frac{- a^{2} - 5 a b x^{2} - 10 b^{2} x^{4}}{60 a^{5} b^{3} + 300 a^{4} b^{4} x^{2} + 600 a^{3} b^{5} x^{4} + 600 a^{2} b^{6} x^{6} + 300 a b^{7} x^{8} + 60 b^{8} x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (-a**2 - 5*a*b*x**2 - 10*b**2*x**4)/(60*a**5*b**3 + 300*a**4*b**4*x**2 + 60
0*a**3*b**5*x**4 + 600*a**2*b**6*x**6 + 300*a*b**7*x**8 + 60*b**8*x**10)
```

3.518 $\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=34

$$\frac{a}{10b^2\left(a+bx^2\right)^5}-\frac{1}{8b^2\left(a+bx^2\right)^4}$$

```
[Out] 1/10*a/b^2/(b*x^2+a)^5-1/8/b^2/(b*x^2+a)^4
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a}{10b^2\left(a+bx^2\right)^5}-\frac{1}{8b^2\left(a+bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] a/(10*b^2*(a + b*x^2)^5) - 1/(8*b^2*(a + b*x^2)^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx &= b^6\int \frac{x^3}{\left(ab+b^2x^2\right)^6}\,dx\\
&= \frac{1}{2}b^6\,\mathrm{Subst}\left(\int \frac{x}{\left(ab+b^2x\right)^6}\,dx,x,x^2\right)\\
&= \frac{1}{2}b^6\,\mathrm{Subst}\left(\int \left(-\frac{a}{b^7(a+bx)^6}+\frac{1}{b^7(a+bx)^5}\right)dx,x,x^2\right)\\
&= \frac{a}{10b^2\left(a+bx^2\right)^5}-\frac{1}{8b^2\left(a+bx^2\right)^4}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.71

$$-\frac{a+5bx^2}{40b^2\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/40*(a + 5*b*x^2)/(b^2*(a + b*x^2)^5)
```

fricas [B] time = 0.84, size = 69, normalized size = 2.03

$$-\frac{5\,bx^2+a}{40\left(b^7x^{10}+5\,ab^6x^8+10\,a^2b^5x^6+10\,a^3b^4x^4+5\,a^4b^3x^2+a^5b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/40*(5*b*x^2 + a)/(b^7*x^10 + 5*a*b^6*x^8 + 10*a^2*b^5*x^6 + 10*a^3*b^4*x
^4 + 5*a^4*b^3*x^2 + a^5*b^2)
```

giac [A] time = 0.16, size = 22, normalized size = 0.65

$$-\frac{5\,bx^2+a}{40\left(bx^2+a\right)^5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/40*(5*b*x^2 + a)/((b*x^2 + a)^5*b^2)
```

maple [A] time = 0.01, size = 31, normalized size = 0.91

$$\frac{a}{10\left(b\,x^{2}+a\right)^{5}b^{2}}-\frac{1}{8\left(b\,x^{2}+a\right)^{4}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/10*a/b^2/(b*x^2+a)^5-1/8/b^2/(b*x^2+a)^4
```

maxima [B] time = 1.35, size = 69, normalized size = 2.03

$$-\frac{5\,bx^{2}+a}{40\left(b^{7}x^{10}+5\,ab^{6}x^{8}+10\,a^{2}b^{5}x^{6}+10\,a^{3}b^{4}x^{4}+5\,a^{4}b^{3}x^{2}+a^{5}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/40*(5*b*x^2 + a)/(b^7*x^10 + 5*a*b^6*x^8 + 10*a^2*b^5*x^6 + 10*a^3*b^4*x
^4 + 5*a^4*b^3*x^2 + a^5*b^2)
```

mupad [B] time = 4.48, size = 70, normalized size = 2.06

$$-\frac{\frac{a}{40\,b^{2}}+\frac{x^{2}}{8\,b}}{a^{5}+5\,a^{4}\,b\,x^{2}+10\,a^{3}\,b^{2}\,x^{4}+10\,a^{2}\,b^{3}\,x^{6}+5\,a\,b^{4}\,x^{8}+b^{5}\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] -(a/(40*b^2) + x^2/(8*b))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10*
a^3*b^2*x^4 + 10*a^2*b^3*x^6)
```

sympy [B] time = 0.58, size = 71, normalized size = 2.09

$$\frac{-a-5bx^{2}}{40a^{5}b^{2}+200a^{4}b^{3}x^{2}+400a^{3}b^{4}x^{4}+400a^{2}b^{5}x^{6}+200ab^{6}x^{8}+40b^{7}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (-a - 5*b*x**2)/(40*a**5*b**2 + 200*a**4*b**3*x**2 + 400*a**3*b**4*x**4 + 4
00*a**2*b**5*x**6 + 200*a*b**6*x**8 + 40*b**7*x**10)
```

3.519 $\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^3} \, dx$

Optimal. Leaf size=16

$$-\frac{1}{10b\left(a+bx^2\right)^5}$$

```
[Out] -1/10/b/(b*x^2+a)^5
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {28, 261}

$$-\frac{1}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/(10*b*(a + b*x^2)^5)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^3} \, dx &= b^6 \int \frac{x}{\left(ab+b^2x^2\right)^6} \, dx \\
&= -\frac{1}{10b\left(a+bx^2\right)^5}
\end{aligned}$$

Mathematica [**A**] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/10*1/(b*(a + b*x^2)^5)
```

fricas [**B**] time = 1.02, size = 59, normalized size = 3.69

$$-\frac{1}{10\left(b^6x^{10}+5\,ab^5x^8+10\,a^2b^4x^6+10\,a^3b^3x^4+5\,a^4b^2x^2+a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/10/(b^6*x^10 + 5*a*b^5*x^8 + 10*a^2*b^4*x^6 + 10*a^3*b^3*x^4 + 5*a^4*b^2
*x^2 + a^5*b)
```

giac [**A**] time = 0.19, size = 14, normalized size = 0.88

$$-\frac{1}{10\left(bx^2+a\right)^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/10/((b*x^2 + a)^5*b)
```

maple [**A**] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{10\left(b\,x^2+a\right)^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/10/b/(b*x^2+a)^5
```

maxima [**B**] time = 1.31, size = 59, normalized size = 3.69

$$-\frac{1}{10\left(b^6x^{10}+5\,ab^5x^8+10\,a^2b^4x^6+10\,a^3b^3x^4+5\,a^4b^2x^2+a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/10/(b^6*x^10 + 5*a*b^5*x^8 + 10*a^2*b^4*x^6 + 10*a^3*b^3*x^4 + 5*a^4*b^2
*x^2 + a^5*b)
```

mupad [**B**] time = 0.06, size = 61, normalized size = 3.81

$$-\frac{1}{10\,a^5\,b+50\,a^4\,b^2\,x^2+100\,a^3\,b^3\,x^4+100\,a^2\,b^4\,x^6+50\,a\,b^5\,x^8+10\,b^6\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] -1/(10*a^5*b + 10*b^6*x^10 + 50*a*b^5*x^8 + 50*a^4*b^2*x^2 + 100*a^3*b^3*x^
4 + 100*a^2*b^4*x^6)
```

sympy [**B**] time = 0.50, size = 63, normalized size = 3.94

$$-\frac{1}{10a^5b+50a^4b^2x^2+100a^3b^3x^4+100a^2b^4x^6+50ab^5x^8+10b^6x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -1/(10*a**5*b + 50*a**4*b**2*x**2 + 100*a**3*b**3*x**4 + 100*a**2*b**4*x**6
 + 50*a*b**5*x**8 + 10*b**6*x**10)
```

3.520 $\int \frac{1}{x(a^2+2abx^2+b^2x^4)^3} dx$

Optimal. Leaf size=102

$$-\frac{\log \left(a+bx^2\right)}{2a^6}+\frac{\log(x)}{a^6}+\frac{1}{2a^5 \left(a+bx^2\right)}+\frac{1}{4a^4 \left(a+bx^2\right)^2}+\frac{1}{6a^3 \left(a+bx^2\right)^3}+\frac{1}{8a^2 \left(a+bx^2\right)^4}+\frac{1}{10a \left(a+bx^2\right)^5}$$

```
[Out] 1/10/a/(b*x^2+a)^5+1/8/a^2/(b*x^2+a)^4+1/6/a^3/(b*x^2+a)^3+1/4/a^4/(b*x^2+a
)^2+1/2/a^5/(b*x^2+a)+ln(x)/a^6-1/2*ln(b*x^2+a)/a^6
```

Rubi [A] time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$\frac{1}{2a^5 \left(a+bx^2\right)}+\frac{1}{4a^4 \left(a+bx^2\right)^2}+\frac{1}{6a^3 \left(a+bx^2\right)^3}+\frac{1}{8a^2 \left(a+bx^2\right)^4}-\frac{\log \left(a+bx^2\right)}{2a^6}+\frac{\log(x)}{a^6}+\frac{1}{10a \left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] 1/(10*a*(a + b*x^2)^5) + 1/(8*a^2*(a + b*x^2)^4) + 1/(6*a^3*(a + b*x^2)^3)
+ 1/(4*a^4*(a + b*x^2)^2) + 1/(2*a^5*(a + b*x^2)) + Log[x]/a^6 - Log[a + b*
x^2]/(2*a^6)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{x\left(ab+b^2x^2\right)^6}\,dx$$
$$= \frac{1}{2}b^6\,\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^6}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^6\,\text{Subst}\left(\int \left(\frac{1}{a^6b^6x}-\frac{1}{ab^5(a+bx)^6}-\frac{1}{a^2b^5(a+bx)^5}-\frac{1}{a^3b^5(a+bx)^4}-\frac{1}{a^4b^5(a\right.\right.$$
$$= \frac{1}{10a\left(a+bx^2\right)^5}+\frac{1}{8a^2\left(a+bx^2\right)^4}+\frac{1}{6a^3\left(a+bx^2\right)^3}+\frac{1}{4a^4\left(a+bx^2\right)^2}+\frac{1}{2a^5\left(a+\right.}$$

Mathematica [A] time = 0.06, size = 76, normalized size = 0.75

$$\frac{\frac{a\left(137a^4+385a^3bx^2+470a^2b^2x^4+270ab^3x^6+60b^4x^8\right)}{\left(a+bx^2\right)^5}-60\log\left(a+bx^2\right)+120\log(x)}{120a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] ((a*(137*a^4 + 385*a^3*b*x^2 + 470*a^2*b^2*x^4 + 270*a*b^3*x^6 + 60*b^4*x^8
))/(a + b*x^2)^5 + 120*Log[x] - 60*Log[a + b*x^2])/(120*a^6)
```

fricas [B] time = 0.90, size = 222, normalized size = 2.18

$$\frac{60\,ab^4x^8+270\,a^2b^3x^6+470\,a^3b^2x^4+385\,a^4bx^2+137\,a^5-60\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4b\right.}{120\left(a^6b^5x^{10}+5\,a^7b^4x^8+10\,a^8b^3x^6+10\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(60*a*b^4*x^8 + 270*a^2*b^3*x^6 + 470*a^3*b^2*x^4 + 385*a^4*b*x^2 + 1
37*a^5 - 60*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a
^4*b*x^2 + a^5)*log(b*x^2 + a) + 120*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x
```

```
^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*log(x))/(a^6*b^5*x^10 + 5*a^7*b^4*
x^8 + 10*a^8*b^3*x^6 + 10*a^9*b^2*x^4 + 5*a^10*b*x^2 + a^11)
```

giac [A] time = 0.15, size = 92, normalized size = 0.90

$$\frac{\log\left(x^2\right)}{2\,a^6}-\frac{\log\left(\left|bx^2+a\right|\right)}{2\,a^6}+\frac{137\,b^5x^{10}+745\,ab^4x^8+1640\,a^2b^3x^6+1840\,a^3b^2x^4+1070\,a^4bx^2+274\,a^5}{120\,\left(bx^2+a\right)^5a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^6 - 1/2*log(abs(b*x^2 + a))/a^6 + 1/120*(137*b^5*x^10 + 745*
a*b^4*x^8 + 1640*a^2*b^3*x^6 + 1840*a^3*b^2*x^4 + 1070*a^4*b*x^2 + 274*a^5)
/((b*x^2 + a)^5*a^6)
```

maple [A] time = 0.02, size = 91, normalized size = 0.89

$$\frac{1}{10\left(b\,x^2+a\right)^5 a}+\frac{1}{8\left(b\,x^2+a\right)^4 a^2}+\frac{1}{6\left(b\,x^2+a\right)^3 a^3}+\frac{1}{4\left(b\,x^2+a\right)^2 a^4}+\frac{1}{2\left(b\,x^2+a\right) a^5}+\frac{\ln(x)}{a^6}-\frac{\ln\left(b\,x^2+a\right)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/10/a/(b*x^2+a)^5+1/8/a^2/(b*x^2+a)^4+1/6/a^3/(b*x^2+a)^3+1/4/a^4/(b*x^2+a
)^2+1/2/a^5/(b*x^2+a)+ln(x)/a^6-1/2*ln(b*x^2+a)/a^6
```

maxima [A] time = 1.45, size = 126, normalized size = 1.24

$$\frac{60\,b^4x^8+270\,ab^3x^6+470\,a^2b^2x^4+385\,a^3bx^2+137\,a^4}{120\,\left(a^5b^5x^{10}+5\,a^6b^4x^8+10\,a^7b^3x^6+10\,a^8b^2x^4+5\,a^9bx^2+a^{10}\right)}-\frac{\log\left(bx^2+a\right)}{2\,a^6}+\frac{\log\left(x^2\right)}{2\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/120*(60*b^4*x^8 + 270*a*b^3*x^6 + 470*a^2*b^2*x^4 + 385*a^3*b*x^2 + 137*a
^4)/(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^2*x^4 + 5*a^9
*b*x^2 + a^10) - 1/2*log(b*x^2 + a)/a^6 + 1/2*log(x^2)/a^6
```

mupad [B] time = 0.24, size = 122, normalized size = 1.20

$$\frac{\ln(x)}{a^6}-\frac{\ln\left(b\,x^2+a\right)}{2\,a^6}+\frac{\frac{137}{120\,a}+\frac{77\,b\,x^2}{24\,a^2}+\frac{47\,b^2\,x^4}{12\,a^3}+\frac{9\,b^3\,x^6}{4\,a^4}+\frac{b^4\,x^8}{2\,a^5}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] log(x)/a^6 - log(a + b*x^2)/(2*a^6) + (137/(120*a) + (77*b*x^2)/(24*a^2) +
(47*b^2*x^4)/(12*a^3) + (9*b^3*x^6)/(4*a^4) + (b^4*x^8)/(2*a^5))/(a^5 + b^5
*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10*a^3*b^2*x^4 + 10*a^2*b^3*x^6)
```

sympy [A] time = 0.81, size = 128, normalized size = 1.25

$$\frac{137a^4 + 385a^3bx^2 + 470a^2b^2x^4 + 270ab^3x^6 + 60b^4x^8}{120a^{10} + 600a^9bx^2 + 1200a^8b^2x^4 + 1200a^7b^3x^6 + 600a^6b^4x^8 + 120a^5b^5x^{10}} + \frac{\log(x)}{a^6} - \frac{\log\left(\frac{a}{b} + x^2\right)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (137*a**4 + 385*a**3*b*x**2 + 470*a**2*b**2*x**4 + 270*a*b**3*x**6 + 60*b**
4*x**8)/(120*a**10 + 600*a**9*b*x**2 + 1200*a**8*b**2*x**4 + 1200*a**7*b**3
*x**6 + 600*a**6*b**4*x**8 + 120*a**5*b**5*x**10) + log(x)/a**6 - log(a/b +
 x**2)/(2*a**6)
```

3.521 $\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=116

$$\frac{3b\log\left(a+bx^2\right)}{a^7}-\frac{6b\log(x)}{a^7}-\frac{5b}{2a^6\left(a+bx^2\right)}-\frac{1}{2a^6x^2}-\frac{b}{a^5\left(a+bx^2\right)^2}-\frac{b}{2a^4\left(a+bx^2\right)^3}-\frac{b}{4a^3\left(a+bx^2\right)^4}-\frac{b}{10a^2\left(a+bx^2\right)^5}$$

```
[Out] -1/2/a^6/x^2-1/10*b/a^2/(b*x^2+a)^5-1/4*b/a^3/(b*x^2+a)^4-1/2*b/a^4/(b*x^2+
a)^3-b/a^5/(b*x^2+a)^2-5/2*b/a^6/(b*x^2+a)-6*b*ln(x)/a^7+3*b*ln(b*x^2+a)/a^
7
```

Rubi [A] time = 0.13, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$-\frac{5b}{2a^6\left(a+bx^2\right)}-\frac{b}{a^5\left(a+bx^2\right)^2}-\frac{b}{2a^4\left(a+bx^2\right)^3}-\frac{b}{4a^3\left(a+bx^2\right)^4}-\frac{b}{10a^2\left(a+bx^2\right)^5}+\frac{3b\log\left(a+bx^2\right)}{a^7}-\frac{6b\log(x)}{a^7}-\frac{1}{2a^6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -1/(2*a^6*x^2) - b/(10*a^2*(a + b*x^2)^5) - b/(4*a^3*(a + b*x^2)^4) - b/(2*
a^4*(a + b*x^2)^3) - b/(a^5*(a + b*x^2)^2) - (5*b)/(2*a^6*(a + b*x^2)) - (6
*b*Log[x])/a^7 + (3*b*Log[a + b*x^2])/a^7
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{x^3\left(ab+b^2x^2\right)^6}\,dx$$
$$= \frac{1}{2}b^6\,\text{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)^6}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^6\,\text{Subst}\left(\int \left(\frac{1}{a^6b^6x^2}-\frac{6}{a^7b^5x}+\frac{1}{a^2b^4(a+bx)^6}+\frac{2}{a^3b^4(a+bx)^5}+\frac{3}{a^4b^4(a+b$$
$$= -\frac{1}{2a^6x^2}-\frac{b}{10a^2\left(a+bx^2\right)^5}-\frac{b}{4a^3\left(a+bx^2\right)^4}-\frac{b}{2a^4\left(a+bx^2\right)^3}-\frac{b}{a^5\left(a+bx^2\right)^2}-$$

Mathematica [A] time = 0.08, size = 92, normalized size = 0.79

$$-\frac{\frac{a\left(10a^5+137a^4bx^2+385a^3b^2x^4+470a^2b^3x^6+270ab^4x^8+60b^5x^{10}\right)}{x^2\left(a+bx^2\right)^5}-60b\log\left(a+bx^2\right)+120b\log(x)}{20a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -1/20*((a*(10*a^5 + 137*a^4*b*x^2 + 385*a^3*b^2*x^4 + 470*a^2*b^3*x^6 + 270
*a*b^4*x^8 + 60*b^5*x^10))/(x^2*(a + b*x^2)^5) + 120*b*Log[x] - 60*b*Log[a
+ b*x^2])/a^7
```

fricas [B] time = 1.02, size = 251, normalized size = 2.16

$$-\frac{60\,ab^5x^{10}+270\,a^2b^4x^8+470\,a^3b^3x^6+385\,a^4b^2x^4+137\,a^5bx^2+10\,a^6-60\left(b^6x^{12}+5\,ab^5x^{10}+10\,a^2b^4x^8+10\right.}{20\left(a^7b^5x^{12}+5\,a^8b^4x^{10}+10\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/20*(60*a*b^5*x^10 + 270*a^2*b^4*x^8 + 470*a^3*b^3*x^6 + 385*a^4*b^2*x^4
+ 137*a^5*b*x^2 + 10*a^6 - 60*(b^6*x^12 + 5*a*b^5*x^10 + 10*a^2*b^4*x^8 + 1
0*a^3*b^3*x^6 + 5*a^4*b^2*x^4 + a^5*b*x^2)*log(b*x^2 + a) + 120*(b^6*x^12 +
```

```
 5*a*b^5*x^10 + 10*a^2*b^4*x^8 + 10*a^3*b^3*x^6 + 5*a^4*b^2*x^4 + a^5*b*x^2
)*log(x))/(a^7*b^5*x^12 + 5*a^8*b^4*x^10 + 10*a^9*b^3*x^8 + 10*a^10*b^2*x^6
 + 5*a^11*b*x^4 + a^12*x^2)
```

giac [A] time = 0.16, size = 115, normalized size = 0.99

$$-\frac{3\,b\log\left(x^2\right)}{a^7}+\frac{3\,b\log\left(\left|bx^2+a\right|\right)}{a^7}+\frac{6\,bx^2-a}{2\,a^7x^2}-\frac{137\,b^6x^{10}+735\,ab^5x^8+1590\,a^2b^4x^6+1740\,a^3b^3x^4+970\,a^4b^2x^2+}{20\left(bx^2+a\right)^5a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -3*b*log(x^2)/a^7 + 3*b*log(abs(b*x^2 + a))/a^7 + 1/2*(6*b*x^2 - a)/(a^7*x^
2) - 1/20*(137*b^6*x^10 + 735*a*b^5*x^8 + 1590*a^2*b^4*x^6 + 1740*a^3*b^3*x
^4 + 970*a^4*b^2*x^2 + 224*a^5*b)/((b*x^2 + a)^5*a^7)
```

maple [A] time = 0.02, size = 107, normalized size = 0.92

$$-\frac{b}{10\left(b\,x^2+a\right)^5a^2}-\frac{b}{4\left(b\,x^2+a\right)^4a^3}-\frac{b}{2\left(b\,x^2+a\right)^3a^4}-\frac{b}{\left(b\,x^2+a\right)^2a^5}-\frac{5b}{2\left(b\,x^2+a\right)a^6}-\frac{6b\ln(x)}{a^7}+\frac{3b\ln\left(b\,x^2+a\right)}{a^7}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/2/a^6/x^2-1/10*b/a^2/(b*x^2+a)^5-1/4*b/a^3/(b*x^2+a)^4-1/2*b/a^4/(b*x^2+
a)^3-b/a^5/(b*x^2+a)^2-5/2*b/a^6/(b*x^2+a)-6*b*ln(x)/a^7+3*b*ln(b*x^2+a)/a^
7
```

maxima [A] time = 1.58, size = 143, normalized size = 1.23

$$-\frac{60\,b^5x^{10}+270\,ab^4x^8+470\,a^2b^3x^6+385\,a^3b^2x^4+137\,a^4bx^2+10\,a^5}{20\left(a^6b^5x^{12}+5\,a^7b^4x^{10}+10\,a^8b^3x^8+10\,a^9b^2x^6+5\,a^{10}bx^4+a^{11}x^2\right)}+\frac{3\,b\log\left(bx^2+a\right)}{a^7}-\frac{3\,b\log\left(x^2\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/20*(60*b^5*x^10 + 270*a*b^4*x^8 + 470*a^2*b^3*x^6 + 385*a^3*b^2*x^4 + 13
7*a^4*b*x^2 + 10*a^5)/(a^6*b^5*x^12 + 5*a^7*b^4*x^10 + 10*a^8*b^3*x^8 + 10*
a^9*b^2*x^6 + 5*a^10*b*x^4 + a^11*x^2) + 3*b*log(b*x^2 + a)/a^7 - 3*b*log(x
^2)/a^7
```

mupad [B] time = 4.68, size = 141, normalized size = 1.22

$$\frac{3\,b\,\ln\left(b\,x^2+a\right)}{a^7}-\frac{\frac{1}{2\,a}+\frac{137\,b\,x^2}{20\,a^2}+\frac{77\,b^2\,x^4}{4\,a^3}+\frac{47\,b^3\,x^6}{2\,a^4}+\frac{27\,b^4\,x^8}{2\,a^5}+\frac{3\,b^5\,x^{10}}{a^6}}{a^5\,x^2+5\,a^4\,b\,x^4+10\,a^3\,b^2\,x^6+10\,a^2\,b^3\,x^8+5\,a\,b^4\,x^{10}+b^5\,x^{12}}-\frac{6\,b\,\ln(x)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] (3*b*log(a + b*x^2))/a^7 - (1/(2*a) + (137*b*x^2)/(20*a^2) + (77*b^2*x^4)/(
4*a^3) + (47*b^3*x^6)/(2*a^4) + (27*b^4*x^8)/(2*a^5) + (3*b^5*x^10)/a^6)/(a
^5*x^2 + b^5*x^12 + 5*a^4*b*x^4 + 5*a*b^4*x^10 + 10*a^3*b^2*x^6 + 10*a^2*b^
3*x^8) - (6*b*log(x))/a^7
```

sympy [A] time = 0.90, size = 150, normalized size = 1.29

$$\frac{-10a^5-137a^4bx^2-385a^3b^2x^4-470a^2b^3x^6-270ab^4x^8-60b^5x^{10}}{20a^{11}x^2+100a^{10}bx^4+200a^9b^2x^6+200a^8b^3x^8+100a^7b^4x^{10}+20a^6b^5x^{12}}-\frac{6b\log(x)}{a^7}+\frac{3b\log\left(\frac{a}{b}+x^2\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (-10*a**5 - 137*a**4*b*x**2 - 385*a**3*b**2*x**4 - 470*a**2*b**3*x**6 - 270
*a*b**4*x**8 - 60*b**5*x**10)/(20*a**11*x**2 + 100*a**10*b*x**4 + 200*a**9*
b**2*x**6 + 200*a**8*b**3*x**8 + 100*a**7*b**4*x**10 + 20*a**6*b**5*x**12)
- 6*b*log(x)/a**7 + 3*b*log(a/b + x**2)/a**7
```

3.522 $\int \frac{1}{x^5\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=140

$$-\frac{21b^2\log\left(a+bx^2\right)}{2a^8}+\frac{21b^2\log(x)}{a^8}+\frac{15b^2}{2a^7\left(a+bx^2\right)}+\frac{3b}{a^7x^2}+\frac{5b^2}{2a^6\left(a+bx^2\right)^2}-\frac{1}{4a^6x^4}+\frac{b^2}{a^5\left(a+bx^2\right)^3}+\frac{3b^2}{8a^4\left(a+bx^2\right)^4}+$$

```
[Out] -1/4/a^6/x^4+3*b/a^7/x^2+1/10*b^2/a^3/(b*x^2+a)^5+3/8*b^2/a^4/(b*x^2+a)^4+b
^2/a^5/(b*x^2+a)^3+5/2*b^2/a^6/(b*x^2+a)^2+15/2*b^2/a^7/(b*x^2+a)+21*b^2*ln
(x)/a^8-21/2*b^2*ln(b*x^2+a)/a^8
```

Rubi [A] time = 0.14, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$\frac{15b^2}{2a^7\left(a+bx^2\right)}+\frac{5b^2}{2a^6\left(a+bx^2\right)^2}+\frac{b^2}{a^5\left(a+bx^2\right)^3}+\frac{3b^2}{8a^4\left(a+bx^2\right)^4}+\frac{b^2}{10a^3\left(a+bx^2\right)^5}-\frac{21b^2\log\left(a+bx^2\right)}{2a^8}+\frac{21b^2\log(x)}{a^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -1/(4*a^6*x^4) + (3*b)/(a^7*x^2) + b^2/(10*a^3*(a + b*x^2)^5) + (3*b^2)/(8*
a^4*(a + b*x^2)^4) + b^2/(a^5*(a + b*x^2)^3) + (5*b^2)/(2*a^6*(a + b*x^2)^2
) + (15*b^2)/(2*a^7*(a + b*x^2)) + (21*b^2*Log[x])/a^8 - (21*b^2*Log[a + b*
x^2])/(2*a^8)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{x^5\left(ab+b^2x^2\right)^6}\,dx$$
$$=\frac{1}{2}b^6\,\text{Subst}\left(\int \frac{1}{x^3\left(ab+b^2x\right)^6}\,dx,x,x^2\right)$$
$$=\frac{1}{2}b^6\,\text{Subst}\left(\int \left(\frac{1}{a^6b^6x^3}-\frac{6}{a^7b^5x^2}+\frac{21}{a^8b^4x}-\frac{1}{a^3b^3(a+bx)^6}-\frac{3}{a^4b^3(a+bx)^5}-\cdots\right.\right.$$
$$=-\frac{1}{4a^6x^4}+\frac{3b}{a^7x^2}+\frac{b^2}{10a^3\left(a+bx^2\right)^5}+\frac{3b^2}{8a^4\left(a+bx^2\right)^4}+\frac{b^2}{a^5\left(a+bx^2\right)^3}+\frac{5b\cdots}{2a^6\left(a\cdots\right.}$$

Mathematica [A] time = 0.06, size = 107, normalized size = 0.76

$$\frac{\frac{a\left(-10a^6+70a^5bx^2+959a^4b^2x^4+2695a^3b^3x^6+3290a^2b^4x^8+1890ab^5x^{10}+420b^6x^{12}\right)}{x^4\left(a+bx^2\right)^5}-420b^2\log\left(a+bx^2\right)+840b^2\log(x)}{40a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] ((a*(-10*a^6 + 70*a^5*b*x^2 + 959*a^4*b^2*x^4 + 2695*a^3*b^3*x^6 + 3290*a^2
*b^4*x^8 + 1890*a*b^5*x^10 + 420*b^6*x^12))/(x^4*(a + b*x^2)^5) + 840*b^2*L
og[x] - 420*b^2*Log[a + b*x^2])/(40*a^8)
```

fricas [B] time = 0.93, size = 266, normalized size = 1.90

$$\frac{420\,ab^6x^{12}+1890\,a^2b^5x^{10}+3290\,a^3b^4x^8+2695\,a^4b^3x^6+959\,a^5b^2x^4+70\,a^6bx^2-10\,a^7-420\left(b^7x^{14}+5\,ab^6x^{1\cdots}\right.}{40\left(a^8b^5x^{14}+5\,a^9b^4x\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/40*(420*a*b^6*x^12 + 1890*a^2*b^5*x^10 + 3290*a^3*b^4*x^8 + 2695*a^4*b^3*
x^6 + 959*a^5*b^2*x^4 + 70*a^6*b*x^2 - 10*a^7 - 420*(b^7*x^14 + 5*a*b^6*x^1
```

```
2 + 10*a^2*b^5*x^10 + 10*a^3*b^4*x^8 + 5*a^4*b^3*x^6 + a^5*b^2*x^4)*log(b*x
^2 + a) + 840*(b^7*x^14 + 5*a*b^6*x^12 + 10*a^2*b^5*x^10 + 10*a^3*b^4*x^8 +
 5*a^4*b^3*x^6 + a^5*b^2*x^4)*log(x))/(a^8*b^5*x^14 + 5*a^9*b^4*x^12 + 10*a
^10*b^3*x^10 + 10*a^11*b^2*x^8 + 5*a^12*b*x^6 + a^13*x^4)
```

giac [A] time = 0.16, size = 130, normalized size = 0.93

$$\frac{21\,b^2\log\left(x^2\right)}{2\,a^8}-\frac{21\,b^2\log\left(\left|bx^2+a\right|\right)}{2\,a^8}-\frac{63\,b^2x^4-12\,abx^2+a^2}{4\,a^8x^4}+\frac{959\,b^7x^{10}+5095\,ab^6x^8+10890\,a^2b^5x^6+11730\,a^3}{40\left(bx^2+a\right)^5a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 21/2*b^2*log(x^2)/a^8 - 21/2*b^2*log(abs(b*x^2 + a))/a^8 - 1/4*(63*b^2*x^4
- 12*a*b*x^2 + a^2)/(a^8*x^4) + 1/40*(959*b^7*x^10 + 5095*a*b^6*x^8 + 10890
*a^2*b^5*x^6 + 11730*a^3*b^4*x^4 + 6390*a^4*b^3*x^2 + 1418*a^5*b^2)/((b*x^2
 + a)^5*a^8)
```

maple [A] time = 0.02, size = 129, normalized size = 0.92

$$\frac{b^2}{10\left(b\,x^2+a\right)^5a^3}+\frac{3b^2}{8\left(b\,x^2+a\right)^4a^4}+\frac{b^2}{\left(b\,x^2+a\right)^3a^5}+\frac{5b^2}{2\left(b\,x^2+a\right)^2a^6}+\frac{15b^2}{2\left(b\,x^2+a\right)a^7}+\frac{21b^2\ln(x)}{a^8}-\frac{21b^2\ln\left(b\,x^2+a\right.}{2a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/4/a^6/x^4+3*b/a^7/x^2+1/10*b^2/a^3/(b*x^2+a)^5+3/8*b^2/a^4/(b*x^2+a)^4+b
^2/a^5/(b*x^2+a)^3+5/2*b^2/a^6/(b*x^2+a)^2+15/2*b^2/a^7/(b*x^2+a)+21*b^2*ln
(x)/a^8-21/2*b^2*ln(b*x^2+a)/a^8
```

maxima [A] time = 1.45, size = 158, normalized size = 1.13

$$\frac{420\,b^6x^{12}+1890\,ab^5x^{10}+3290\,a^2b^4x^8+2695\,a^3b^3x^6+959\,a^4b^2x^4+70\,a^5bx^2-10\,a^6}{40\left(a^7b^5x^{14}+5\,a^8b^4x^{12}+10\,a^9b^3x^{10}+10\,a^{10}b^2x^8+5\,a^{11}bx^6+a^{12}x^4\right)}-\frac{21\,b^2\log\left(bx^2+a\right)}{2\,a^8}+\frac{21\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/40*(420*b^6*x^12 + 1890*a*b^5*x^10 + 3290*a^2*b^4*x^8 + 2695*a^3*b^3*x^6
+ 959*a^4*b^2*x^4 + 70*a^5*b*x^2 - 10*a^6)/(a^7*b^5*x^14 + 5*a^8*b^4*x^12 +
 10*a^9*b^3*x^10 + 10*a^10*b^2*x^8 + 5*a^11*b*x^6 + a^12*x^4) - 21/2*b^2*lo
g(b*x^2 + a)/a^8 + 21/2*b^2*log(x^2)/a^8
```

mupad [B] time = 4.91, size = 155, normalized size = 1.11

$$\frac{\frac{7\,b\,x^2}{4\,a^2}-\frac{1}{4\,a}+\frac{959\,b^2\,x^4}{40\,a^3}+\frac{539\,b^3\,x^6}{8\,a^4}+\frac{329\,b^4\,x^8}{4\,a^5}+\frac{189\,b^5\,x^{10}}{4\,a^6}+\frac{21\,b^6\,x^{12}}{2\,a^7}}{a^5\,x^4+5\,a^4\,b\,x^6+10\,a^3\,b^2\,x^8+10\,a^2\,b^3\,x^{10}+5\,a\,b^4\,x^{12}+b^5\,x^{14}}-\frac{21\,b^2\,\ln\left(b\,x^2+a\right)}{2\,a^8}+\frac{21\,b^2\,\ln(x)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] ((7*b*x^2)/(4*a^2) - 1/(4*a) + (959*b^2*x^4)/(40*a^3) + (539*b^3*x^6)/(8*a^
4) + (329*b^4*x^8)/(4*a^5) + (189*b^5*x^10)/(4*a^6) + (21*b^6*x^12)/(2*a^7)
)/(a^5*x^4 + b^5*x^14 + 5*a^4*b*x^6 + 5*a*b^4*x^12 + 10*a^3*b^2*x^8 + 10*a^
2*b^3*x^10) - (21*b^2*log(a + b*x^2))/(2*a^8) + (21*b^2*log(x))/a^8
```

sympy [A] time = 0.96, size = 165, normalized size = 1.18

$$\frac{-10a^6+70a^5bx^2+959a^4b^2x^4+2695a^3b^3x^6+3290a^2b^4x^8+1890ab^5x^{10}+420b^6x^{12}}{40a^{12}x^4+200a^{11}bx^6+400a^{10}b^2x^8+400a^9b^3x^{10}+200a^8b^4x^{12}+40a^7b^5x^{14}}+\frac{21b^2\log(x)}{a^8}-\frac{21b^2\log\left(\right.}{2a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (-10*a**6 + 70*a**5*b*x**2 + 959*a**4*b**2*x**4 + 2695*a**3*b**3*x**6 + 329
0*a**2*b**4*x**8 + 1890*a*b**5*x**10 + 420*b**6*x**12)/(40*a**12*x**4 + 200
*a**11*b*x**6 + 400*a**10*b**2*x**8 + 400*a**9*b**3*x**10 + 200*a**8*b**4*x
**12 + 40*a**7*b**5*x**14) + 21*b**2*log(x)/a**8 - 21*b**2*log(a/b + x**2)/
(2*a**8)
```

3.523 $\int \frac{x^{16}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=155

$$-\frac{9009a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{17/2}}+\frac{9009a^2x}{256b^8}-\frac{3003ax^3}{256b^7}-\frac{1287x^7}{256b^5\left(a+bx^2\right)}-\frac{143x^9}{128b^4\left(a+bx^2\right)^2}-\frac{13x^{11}}{32b^3\left(a+bx^2\right)^3}-\frac{3x^{13}}{16b^2\left(a+bx^2\right.}$$

```
[Out] 9009/256*a^2*x/b^8-3003/256*a*x^3/b^7+9009/1280*x^5/b^6-1/10*x^15/b/(b*x^2+
a)^5-3/16*x^13/b^2/(b*x^2+a)^4-13/32*x^11/b^3/(b*x^2+a)^3-143/128*x^9/b^4/(
b*x^2+a)^2-1287/256*x^7/b^5/(b*x^2+a)-9009/256*a^(5/2)*arctan(x*b^(1/2)/a^(
1/2))/b^(17/2)
```

Rubi [A] time = 0.11, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 302, 205}

$$\frac{9009a^2x}{256b^8}-\frac{9009a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{17/2}}-\frac{3x^{13}}{16b^2\left(a+bx^2\right)^4}-\frac{13x^{11}}{32b^3\left(a+bx^2\right)^3}-\frac{143x^9}{128b^4\left(a+bx^2\right)^2}-\frac{1287x^7}{256b^5\left(a+bx^2\right)}-\frac{3003ax}{256b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^16/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (9009*a^2*x)/(256*b^8) - (3003*a*x^3)/(256*b^7) + (9009*x^5)/(1280*b^6) - x
^15/(10*b*(a + b*x^2)^5) - (3*x^13)/(16*b^2*(a + b*x^2)^4) - (13*x^11)/(32*
b^3*(a + b*x^2)^3) - (143*x^9)/(128*b^4*(a + b*x^2)^2) - (1287*x^7)/(256*b^
5*(a + b*x^2)) - (9009*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*b^(17/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^{16}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^{16}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{x^{15}}{10b\left(a+bx^2\right)^5} + \frac{1}{2}\left(3b^4\right)\int \frac{x^{14}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{x^{15}}{10b\left(a+bx^2\right)^5} - \frac{3x^{13}}{16b^2\left(a+bx^2\right)^4} + \frac{1}{16}\left(39b^2\right)\int \frac{x^{12}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{x^{15}}{10b\left(a+bx^2\right)^5} - \frac{3x^{13}}{16b^2\left(a+bx^2\right)^4} - \frac{13x^{11}}{32b^3\left(a+bx^2\right)^3} + \frac{143}{32}\int \frac{x^{10}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{x^{15}}{10b\left(a+bx^2\right)^5} - \frac{3x^{13}}{16b^2\left(a+bx^2\right)^4} - \frac{13x^{11}}{32b^3\left(a+bx^2\right)^3} - \frac{143x^9}{128b^4\left(a+bx^2\right)^2} + \frac{1287\int}{}$$

$$= -\frac{x^{15}}{10b\left(a+bx^2\right)^5} - \frac{3x^{13}}{16b^2\left(a+bx^2\right)^4} - \frac{13x^{11}}{32b^3\left(a+bx^2\right)^3} - \frac{143x^9}{128b^4\left(a+bx^2\right)^2} - \frac{12}{256b^5}$$

$$= -\frac{x^{15}}{10b\left(a+bx^2\right)^5} - \frac{3x^{13}}{16b^2\left(a+bx^2\right)^4} - \frac{13x^{11}}{32b^3\left(a+bx^2\right)^3} - \frac{143x^9}{128b^4\left(a+bx^2\right)^2} - \frac{12}{256b^5}$$

$$= \frac{9009a^2x}{256b^8} - \frac{3003ax^3}{256b^7} + \frac{9009x^5}{1280b^6} - \frac{x^{15}}{10b\left(a+bx^2\right)^5} - \frac{3x^{13}}{16b^2\left(a+bx^2\right)^4} - \frac{13x^{11}}{32b^3\left(a+bx\right.}$$

$$= \frac{9009a^2x}{256b^8} - \frac{3003ax^3}{256b^7} + \frac{9009x^5}{1280b^6} - \frac{x^{15}}{10b\left(a+bx^2\right)^5} - \frac{3x^{13}}{16b^2\left(a+bx^2\right)^4} - \frac{13x^{11}}{32b^3\left(a+bx\right.}$$

Mathematica [A] time = 0.06, size = 122, normalized size = 0.79

$$\frac{\frac{\sqrt{b}\,x\left(45045a^7+210210a^6bx^2+384384a^5b^2x^4+338910a^4b^3x^6+137995a^3b^4x^8+16640a^2b^5x^{10}-1280ab^6x^{12}+256b^7x^{14}\right)}{\left(a+bx^2\right)^5} - 45045a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{1280b^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^16/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((Sqrt[b]*x*(45045*a^7 + 210210*a^6*b*x^2 + 384384*a^5*b^2*x^4 + 338910*a^4
*b^3*x^6 + 137995*a^3*b^4*x^8 + 16640*a^2*b^5*x^10 - 1280*a*b^6*x^12 + 256*
```

```
b^7*x^14))/(a + b*x^2)^5 - 45045*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(1280
*b^(17/2))
```

fricas [A] time = 0.85, size = 454, normalized size = 2.93

$$\left[\frac{512\, b^{7} x^{15} - 2560\, a b^{6} x^{13} + 33280\, a^{2} b^{5} x^{11} + 275990\, a^{3} b^{4} x^{9} + 677820\, a^{4} b^{3} x^{7} + 768768\, a^{5} b^{2} x^{5} + 420420\, a^{6} b x^{3}}{2560\left(b^{13} x^{10} + 5\, a b^{12} x^{8} + 10\, a^{2} b^{11} x^{6}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(512*b^7*x^15 - 2560*a*b^6*x^13 + 33280*a^2*b^5*x^11 + 275990*a^3*b
^4*x^9 + 677820*a^4*b^3*x^7 + 768768*a^5*b^2*x^5 + 420420*a^6*b*x^3 + 90090
*a^7*x + 45045*(a^2*b^5*x^10 + 5*a^3*b^4*x^8 + 10*a^4*b^3*x^6 + 10*a^5*b^2*
x^4 + 5*a^6*b*x^2 + a^7)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x
^2 + a)))/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a^3*b^10*x^4 + 5
*a^4*b^9*x^2 + a^5*b^8), 1/1280*(256*b^7*x^15 - 1280*a*b^6*x^13 + 16640*a^2
*b^5*x^11 + 137995*a^3*b^4*x^9 + 338910*a^4*b^3*x^7 + 384384*a^5*b^2*x^5 +
210210*a^6*b*x^3 + 45045*a^7*x - 45045*(a^2*b^5*x^10 + 5*a^3*b^4*x^8 + 10*a
^4*b^3*x^6 + 10*a^5*b^2*x^4 + 5*a^6*b*x^2 + a^7)*sqrt(a/b)*arctan(b*x*sqrt(
a/b)/a))/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a^3*b^10*x^4 + 5*
a^4*b^9*x^2 + a^5*b^8)]
```

giac [A] time = 0.17, size = 117, normalized size = 0.75

$$-\frac{9009\, a^{3} \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\, b^{8}} + \frac{26635\, a^{3} b^{4} x^{9} + 94430\, a^{4} b^{3} x^{7} + 128128\, a^{5} b^{2} x^{5} + 78370\, a^{6} b x^{3} + 18165\, a^{7} x}{1280\left(bx^{2} + a\right)^{5} b^{8}} + \frac{b^{24} x^{5} - 10}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -9009/256*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^8) + 1/1280*(26635*a^3*b^4
*x^9 + 94430*a^4*b^3*x^7 + 128128*a^5*b^2*x^5 + 78370*a^6*b*x^3 + 18165*a^7
*x)/((b*x^2 + a)^5*b^8) + 1/5*(b^24*x^5 - 10*a*b^23*x^3 + 105*a^2*b^22*x)/b
^30
```

maple [A] time = 0.02, size = 148, normalized size = 0.95

$$\frac{5327 a^{3} x^{9}}{256\left(b\,x^{2}+a\right)^{5} b^{4}} + \frac{9443 a^{4} x^{7}}{128\left(b\,x^{2}+a\right)^{5} b^{5}} + \frac{1001 a^{5} x^{5}}{10\left(b\,x^{2}+a\right)^{5} b^{6}} + \frac{7837 a^{6} x^{3}}{128\left(b\,x^{2}+a\right)^{5} b^{7}} + \frac{3633 a^{7} x}{256\left(b\,x^{2}+a\right)^{5} b^{8}} + \frac{x^{5}}{5 b^{6}} - \frac{2 a\,x^{3}}{b^{7}} - \frac{90}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^16/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/5*x^5/b^6-2*a*x^3/b^7+21*a^2*x/b^8+5327/256/b^4*a^3/(b*x^2+a)^5*x^9+9443/
128/b^5*a^4/(b*x^2+a)^5*x^7+1001/10/b^6*a^5/(b*x^2+a)^5*x^5+7837/128/b^7*a^
6/(b*x^2+a)^5*x^3+3633/256/b^8*a^7/(b*x^2+a)^5*x-9009/256/b^8*a^3/(a*b)^(1/
2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.89, size = 159, normalized size = 1.03

$$\frac{26635\,a^3b^4x^9+94430\,a^4b^3x^7+128128\,a^5b^2x^5+78370\,a^6bx^3+18165\,a^7x}{1280\left(b^{13}x^{10}+5\,ab^{12}x^8+10\,a^2b^{11}x^6+10\,a^3b^{10}x^4+5\,a^4b^9x^2+a^5b^8\right)}-\frac{9009\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,b^8}+\frac{b^2x^5-10\,abx}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/1280*(26635*a^3*b^4*x^9 + 94430*a^4*b^3*x^7 + 128128*a^5*b^2*x^5 + 78370*
a^6*b*x^3 + 18165*a^7*x)/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a
^3*b^10*x^4 + 5*a^4*b^9*x^2 + a^5*b^8) - 9009/256*a^3*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*b^8) + 1/5*(b^2*x^5 - 10*a*b*x^3 + 105*a^2*x)/b^8
```

mupad [B] time = 0.11, size = 153, normalized size = 0.99

$$\frac{\frac{3633\,a^7\,x}{256}+\frac{7837\,a^6\,b\,x^3}{128}+\frac{1001\,a^5\,b^2\,x^5}{10}+\frac{9443\,a^4\,b^3\,x^7}{128}+\frac{5327\,a^3\,b^4\,x^9}{256}}{a^5\,b^8+5\,a^4\,b^9\,x^2+10\,a^3\,b^{10}\,x^4+10\,a^2\,b^{11}\,x^6+5\,a\,b^{12}\,x^8+b^{13}\,x^{10}}+\frac{x^5}{5\,b^6}-\frac{2\,a\,x^3}{b^7}+\frac{21\,a^2\,x}{b^8}-\frac{9009\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,b^{17/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^16/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((3633*a^7*x)/256 + (7837*a^6*b*x^3)/128 + (1001*a^5*b^2*x^5)/10 + (9443*a^
4*b^3*x^7)/128 + (5327*a^3*b^4*x^9)/256)/(a^5*b^8 + b^13*x^10 + 5*a*b^12*x^
8 + 5*a^4*b^9*x^2 + 10*a^3*b^10*x^4 + 10*a^2*b^11*x^6) + x^5/(5*b^6) - (2*a
*x^3)/b^7 + (21*a^2*x)/b^8 - (9009*a^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(256*
b^(17/2))
```

sympy [A] time = 1.09, size = 218, normalized size = 1.41

$$\frac{21a^2x}{b^8}-\frac{2ax^3}{b^7}+\frac{9009\sqrt{-\frac{a^5}{b^{17}}}\,\log\left(x-\frac{b^8\sqrt{-\frac{a^5}{b^{17}}}}{a^2}\right)}{512}-\frac{9009\sqrt{-\frac{a^5}{b^{17}}}\,\log\left(x+\frac{b^8\sqrt{-\frac{a^5}{b^{17}}}}{a^2}\right)}{512}+\frac{18165a^7x+78370a^6bx^3+}{1280a^5b^8+6400a^4b^9x^2+12800}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**16/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 21*a**2*x/b**8 - 2*a*x**3/b**7 + 9009*sqrt(-a**5/b**17)*log(x - b**8*sqrt(-
a**5/b**17)/a**2)/512 - 9009*sqrt(-a**5/b**17)*log(x + b**8*sqrt(-a**5/b**1
7)/a**2)/512 + (18165*a**7*x + 78370*a**6*b*x**3 + 128128*a**5*b**2*x**5 +
94430*a**4*b**3*x**7 + 26635*a**3*b**4*x**9)/(1280*a**5*b**8 + 6400*a**4*b*
*9*x**2 + 12800*a**3*b**10*x**4 + 12800*a**2*b**11*x**6 + 6400*a*b**12*x**8
 + 1280*b**13*x**10) + x**5/(5*b**6)
```

3.524 $\int \frac{x^{14}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=142

$$\frac{3003a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{15/2}}-\frac{3003ax}{256b^7}-\frac{3003x^5}{1280b^5\left(a+bx^2\right)}-\frac{429x^7}{640b^4\left(a+bx^2\right)^2}-\frac{143x^9}{480b^3\left(a+bx^2\right)^3}-\frac{13x^{11}}{80b^2\left(a+bx^2\right)^4}-\frac{x\cdots}{10b\left(a\cdots\right.}$$

```
[Out] -3003/256*a*x/b^7+1001/256*x^3/b^6-1/10*x^13/b/(b*x^2+a)^5-13/80*x^11/b^2/(
b*x^2+a)^4-143/480*x^9/b^3/(b*x^2+a)^3-429/640*x^7/b^4/(b*x^2+a)^2-3003/128
0*x^5/b^5/(b*x^2+a)+3003/256*a^(3/2)*arctan(x*b^(1/2)/a^(1/2))/b^(15/2)
```

Rubi [A] time = 0.09, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 302, 205}

$$\frac{3003a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{15/2}}-\frac{13x^{11}}{80b^2\left(a+bx^2\right)^4}-\frac{143x^9}{480b^3\left(a+bx^2\right)^3}-\frac{429x^7}{640b^4\left(a+bx^2\right)^2}-\frac{3003x^5}{1280b^5\left(a+bx^2\right)}-\frac{3003ax}{256b^7}-\frac{x\cdots}{10b\left(a\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-3003*a*x)/(256*b^7) + (1001*x^3)/(256*b^6) - x^13/(10*b*(a + b*x^2)^5) -
(13*x^11)/(80*b^2*(a + b*x^2)^4) - (143*x^9)/(480*b^3*(a + b*x^2)^3) - (429
*x^7)/(640*b^4*(a + b*x^2)^2) - (3003*x^5)/(1280*b^5*(a + b*x^2)) + (3003*a
^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*b^(15/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^{14}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^{14}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{x^{13}}{10b\left(a+bx^2\right)^5}+\frac{1}{10}\left(13b^4\right)\int \frac{x^{12}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{x^{13}}{10b\left(a+bx^2\right)^5}-\frac{13x^{11}}{80b^2\left(a+bx^2\right)^4}+\frac{1}{80}\left(143b^2\right)\int \frac{x^{10}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{x^{13}}{10b\left(a+bx^2\right)^5}-\frac{13x^{11}}{80b^2\left(a+bx^2\right)^4}-\frac{143x^9}{480b^3\left(a+bx^2\right)^3}+\frac{429}{160}\int \frac{x^8}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{x^{13}}{10b\left(a+bx^2\right)^5}-\frac{13x^{11}}{80b^2\left(a+bx^2\right)^4}-\frac{143x^9}{480b^3\left(a+bx^2\right)^3}-\frac{429x^7}{640b^4\left(a+bx^2\right)^2}+ \ldots$$

$$= -\frac{x^{13}}{10b\left(a+bx^2\right)^5}-\frac{13x^{11}}{80b^2\left(a+bx^2\right)^4}-\frac{143x^9}{480b^3\left(a+bx^2\right)^3}-\frac{429x^7}{640b^4\left(a+bx^2\right)^2}- \ldots$$

$$= -\frac{x^{13}}{10b\left(a+bx^2\right)^5}-\frac{13x^{11}}{80b^2\left(a+bx^2\right)^4}-\frac{143x^9}{480b^3\left(a+bx^2\right)^3}-\frac{429x^7}{640b^4\left(a+bx^2\right)^2}- \ldots$$

$$= -\frac{3003ax}{256b^7}+\frac{1001x^3}{256b^6}-\frac{x^{13}}{10b\left(a+bx^2\right)^5}-\frac{13x^{11}}{80b^2\left(a+bx^2\right)^4}-\frac{143x^9}{480b^3\left(a+bx^2\right)^3}- \ldots$$

$$= -\frac{3003ax}{256b^7}+\frac{1001x^3}{256b^6}-\frac{x^{13}}{10b\left(a+bx^2\right)^5}-\frac{13x^{11}}{80b^2\left(a+bx^2\right)^4}-\frac{143x^9}{480b^3\left(a+bx^2\right)^3}- \ldots$$

Mathematica [A] time = 0.06, size = 111, normalized size = 0.78

$$\frac{45045a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+\frac{\sqrt{b}x(-45045a^6-210210a^5bx^2-384384a^4b^2x^4-338910a^3b^3x^6-137995a^2b^4x^8-16640ab^5x^{10}+1280b^6x^{12})}{(a+bx^2)^5}}{3840b^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((Sqrt[b]*x*(-45045*a^6 - 210210*a^5*b*x^2 - 384384*a^4*b^2*x^4 - 338910*a^
3*b^3*x^6 - 137995*a^2*b^4*x^8 - 16640*a*b^5*x^10 + 1280*b^6*x^12))/(a + b*
x^2)^5 + 45045*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3840*b^(15/2))
```

fricas [A] time = 0.87, size = 428, normalized size = 3.01

$$\left[\frac{2560\,b^6x^{13}-33280\,ab^5x^{11}-275990\,a^2b^4x^9-677820\,a^3b^3x^7-768768\,a^4b^2x^5-420420\,a^5bx^3-90090\,a^6x+45}{7680\left(b^{12}x^{10}+5\,ab^{11}x^8+10\,a^2b^{10}x^6+10\,a^3b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/7680*(2560*b^6*x^13 - 33280*a*b^5*x^11 - 275990*a^2*b^4*x^9 - 677820*a^3
*b^3*x^7 - 768768*a^4*b^2*x^5 - 420420*a^5*b*x^3 - 90090*a^6*x + 45045*(a*b
^5*x^10 + 5*a^2*b^4*x^8 + 10*a^3*b^3*x^6 + 10*a^4*b^2*x^4 + 5*a^5*b*x^2 + a
^6)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^12*x^10
+ 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7
), 1/3840*(1280*b^6*x^13 - 16640*a*b^5*x^11 - 137995*a^2*b^4*x^9 - 338910*a
^3*b^3*x^7 - 384384*a^4*b^2*x^5 - 210210*a^5*b*x^3 - 45045*a^6*x + 45045*(a
*b^5*x^10 + 5*a^2*b^4*x^8 + 10*a^3*b^3*x^6 + 10*a^4*b^2*x^4 + 5*a^5*b*x^2 +
 a^6)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2
*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7)]
```

giac [A] time = 0.16, size = 106, normalized size = 0.75

$$\frac{3003\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,b^7}-\frac{35595\,a^2b^4x^9+121310\,a^3b^3x^7+160384\,a^4b^2x^5+96290\,a^5bx^3+22005\,a^6x}{3840\left(bx^2+a\right)^5b^7}+\frac{b^{12}x^3-18\,a}{3\,b^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 3003/256*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^7) - 1/3840*(35595*a^2*b^4*
x^9 + 121310*a^3*b^3*x^7 + 160384*a^4*b^2*x^5 + 96290*a^5*b*x^3 + 22005*a^6
*x)/((b*x^2 + a)^5*b^7) + 1/3*(b^12*x^3 - 18*a*b^11*x)/b^18
```

maple [A] time = 0.02, size = 137, normalized size = 0.96

$$-\frac{2373a^2x^9}{256\left(b\,x^2+a\right)^5b^3}-\frac{12131a^3x^7}{384\left(b\,x^2+a\right)^5b^4}-\frac{1253a^4x^5}{30\left(b\,x^2+a\right)^5b^5}-\frac{9629a^5x^3}{384\left(b\,x^2+a\right)^5b^6}-\frac{1467a^6x}{256\left(b\,x^2+a\right)^5b^7}+\frac{x^3}{3b^6}+\frac{3003a^2}{25\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/3*x^3/b^6-6*a*x/b^7-2373/256/b^3*a^2/(b*x^2+a)^5*x^9-12131/384/b^4*a^3/(b
*x^2+a)^5*x^7-1253/30/b^5*a^4/(b*x^2+a)^5*x^5-9629/384/b^6*a^5/(b*x^2+a)^5*
x^3-1467/256/b^7*a^6/(b*x^2+a)^5*x+3003/256/b^7*a^2/(a*b)^(1/2)*arctan(1/(a
*b)^(1/2)*b*x)
```

maxima [A] time = 2.91, size = 148, normalized size = 1.04

$$-\frac{35595\,a^2b^4x^9+121310\,a^3b^3x^7+160384\,a^4b^2x^5+96290\,a^5bx^3+22005\,a^6x}{3840\left(b^{12}x^{10}+5\,ab^{11}x^8+10\,a^2b^{10}x^6+10\,a^3b^9x^4+5\,a^4b^8x^2+a^5b^7\right)}+\frac{3003\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,b^7}+\frac{bx^3-18\cdots}{3\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/3840*(35595*a^2*b^4*x^9 + 121310*a^3*b^3*x^7 + 160384*a^4*b^2*x^5 + 9629
0*a^5*b*x^3 + 22005*a^6*x)/(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10
*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7) + 3003/256*a^2*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^7) + 1/3*(b*x^3 - 18*a*x)/b^7
```

mupad [B] time = 4.52, size = 143, normalized size = 1.01

$$\frac{x^3}{3\,b^6}-\frac{\frac{1467\,a^6\,x}{256}+\frac{9629\,a^5\,b\,x^3}{384}+\frac{1253\,a^4\,b^2\,x^5}{30}+\frac{12131\,a^3\,b^3\,x^7}{384}+\frac{2373\,a^2\,b^4\,x^9}{256}}{a^5\,b^7+5\,a^4\,b^8\,x^2+10\,a^3\,b^9\,x^4+10\,a^2\,b^{10}\,x^6+5\,a\,b^{11}\,x^8+b^{12}\,x^{10}}+\frac{3003\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,b^{15/2}}-\frac{6\,a\,x}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] x^3/(3*b^6) - ((1467*a^6*x)/256 + (9629*a^5*b*x^3)/384 + (1253*a^4*b^2*x^5)
/30 + (12131*a^3*b^3*x^7)/384 + (2373*a^2*b^4*x^9)/256)/(a^5*b^7 + b^12*x^1
```

```
0 + 5*a*b^11*x^8 + 5*a^4*b^8*x^2 + 10*a^3*b^9*x^4 + 10*a^2*b^10*x^6) + (300
3*a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(256*b^(15/2)) - (6*a*x)/b^7
```

sympy [A] time = 1.03, size = 204, normalized size = 1.44

$$-\frac{6ax}{b^7}-\frac{3003\sqrt{-\frac{a^3}{b^{15}}}\log\left(x-\frac{b^7\sqrt{-\frac{a^3}{b^{15}}}}{a}\right)}{512}+\frac{3003\sqrt{-\frac{a^3}{b^{15}}}\log\left(x+\frac{b^7\sqrt{-\frac{a^3}{b^{15}}}}{a}\right)}{512}+\frac{-22005a^6x-96290a^5bx^3-16038}{3840a^5b^7+19200a^4b^8x^2+38400a^3b^9x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -6*a*x/b**7 - 3003*sqrt(-a**3/b**15)*log(x - b**7*sqrt(-a**3/b**15)/a)/512
+ 3003*sqrt(-a**3/b**15)*log(x + b**7*sqrt(-a**3/b**15)/a)/512 + (-22005*a*
*6*x - 96290*a**5*b*x**3 - 160384*a**4*b**2*x**5 - 121310*a**3*b**3*x**7 -
35595*a**2*b**4*x**9)/(3840*a**5*b**7 + 19200*a**4*b**8*x**2 + 38400*a**3*b
**9*x**4 + 38400*a**2*b**10*x**6 + 19200*a*b**11*x**8 + 3840*b**12*x**10) +
 x**3/(3*b**6)
```

3.525 $\int \frac{x^{12}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=131

$$-\frac{693\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{13/2}}-\frac{231x^3}{256b^5\left(a+bx^2\right)}-\frac{231x^5}{640b^4\left(a+bx^2\right)^2}-\frac{33x^7}{160b^3\left(a+bx^2\right)^3}-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}-\frac{x^{11}}{10b\left(a+bx^2\right)^5}$$

```
[Out] 693/256*x/b^6-1/10*x^11/b/(b*x^2+a)^5-11/80*x^9/b^2/(b*x^2+a)^4-33/160*x^7/
b^3/(b*x^2+a)^3-231/640*x^5/b^4/(b*x^2+a)^2-231/256*x^3/b^5/(b*x^2+a)-693/2
56*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(13/2)
```

Rubi [A] time = 0.08, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 321, 205}

$$-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}-\frac{33x^7}{160b^3\left(a+bx^2\right)^3}-\frac{231x^5}{640b^4\left(a+bx^2\right)^2}-\frac{231x^3}{256b^5\left(a+bx^2\right)}-\frac{693\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{13/2}}-\frac{x^{11}}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (693*x)/(256*b^6) - x^11/(10*b*(a + b*x^2)^5) - (11*x^9)/(80*b^2*(a + b*x^2
)^4) - (33*x^7)/(160*b^3*(a + b*x^2)^3) - (231*x^5)/(640*b^4*(a + b*x^2)^2)
 - (231*x^3)/(256*b^5*(a + b*x^2)) - (693*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(256*b^(13/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{12}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^{12}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{x^{11}}{10b\left(a+bx^2\right)^5}+\frac{1}{10}\left(11b^4\right)\int \frac{x^{10}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{x^{11}}{10b\left(a+bx^2\right)^5}-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}+\frac{1}{80}\left(99b^2\right)\int \frac{x^8}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{x^{11}}{10b\left(a+bx^2\right)^5}-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}-\frac{33x^7}{160b^3\left(a+bx^2\right)^3}+\frac{231}{160}\int \frac{x^6}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{x^{11}}{10b\left(a+bx^2\right)^5}-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}-\frac{33x^7}{160b^3\left(a+bx^2\right)^3}-\frac{231x^5}{640b^4\left(a+bx^2\right)^2}+\frac{231\int}{}$$

$$= -\frac{x^{11}}{10b\left(a+bx^2\right)^5}-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}-\frac{33x^7}{160b^3\left(a+bx^2\right)^3}-\frac{231x^5}{640b^4\left(a+bx^2\right)^2}-\frac{}{256b}$$

$$= \frac{693x}{256b^6}-\frac{x^{11}}{10b\left(a+bx^2\right)^5}-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}-\frac{33x^7}{160b^3\left(a+bx^2\right)^3}-\frac{231x^5}{640b^4\left(a+bx^2\right)^2}$$

$$= \frac{693x}{256b^6}-\frac{x^{11}}{10b\left(a+bx^2\right)^5}-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}-\frac{33x^7}{160b^3\left(a+bx^2\right)^3}-\frac{231x^5}{640b^4\left(a+bx^2\right)^2}$$

Mathematica [A] time = 0.05, size = 100, normalized size = 0.76

$$\frac{\frac{\sqrt{b}x(3465a^5+16170a^4bx^2+29568a^3b^2x^4+26070a^2b^3x^6+10615ab^4x^8+1280b^5x^{10})}{(a+bx^2)^5}-3465\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{1280b^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((Sqrt[b]*x*(3465*a^5 + 16170*a^4*b*x^2 + 29568*a^3*b^2*x^4 + 26070*a^2*b^3
*x^6 + 10615*a*b^4*x^8 + 1280*b^5*x^10))/(a + b*x^2)^5 - 3465*Sqrt[a]*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(1280*b^(13/2))
```

fricas [A] time = 0.86, size = 400, normalized size = 3.05

$$\left[\frac{2560\,b^5x^{11}+21230\,ab^4x^9+52140\,a^2b^3x^7+59136\,a^3b^2x^5+32340\,a^4bx^3+6930\,a^5x+3465\left(b^5x^{10}+5\,ab^4x^8+\right.}{2560\left(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(2560*b^5*x^11 + 21230*a*b^4*x^9 + 52140*a^2*b^3*x^7 + 59136*a^3*b^
2*x^5 + 32340*a^4*b*x^3 + 6930*a^5*x + 3465*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^
2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a/b)*log((b*x^2 - 2*b
*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6
 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6), 1/1280*(1280*b^5*x^11 + 10615
*a*b^4*x^9 + 26070*a^2*b^3*x^7 + 29568*a^3*b^2*x^5 + 16170*a^4*b*x^3 + 3465
*a^5*x - 3465*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5
*a^4*b*x^2 + a^5)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^11*x^10 + 5*a*b^10*
x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)]
```

giac [A] time = 0.17, size = 87, normalized size = 0.66

$$-\frac{693\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,b^6}+\frac{x}{b^6}+\frac{4215\,ab^4x^9+13270\,a^2b^3x^7+16768\,a^3b^2x^5+9770\,a^4bx^3+2185\,a^5x}{1280\left(bx^2+a\right)^5b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -693/256*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) + x/b^6 + 1/1280*(4215*a*b
^4*x^9 + 13270*a^2*b^3*x^7 + 16768*a^3*b^2*x^5 + 9770*a^4*b*x^3 + 2185*a^5*
x)/((b*x^2 + a)^5*b^6)
```

maple [A] time = 0.02, size = 123, normalized size = 0.94

$$\frac{843a\,x^9}{256\left(b\,x^2+a\right)^5 b^2}+\frac{1327a^2x^7}{128\left(b\,x^2+a\right)^5 b^3}+\frac{131a^3x^5}{10\left(b\,x^2+a\right)^5 b^4}+\frac{977a^4x^3}{128\left(b\,x^2+a\right)^5 b^5}+\frac{437a^5x}{256\left(b\,x^2+a\right)^5 b^6}-\frac{693a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\;b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] x/b^6+843/256/b^2*a/(b*x^2+a)^5*x^9+1327/128/b^3*a^2/(b*x^2+a)^5*x^7+131/10
/b^4*a^3/(b*x^2+a)^5*x^5+977/128/b^5*a^4/(b*x^2+a)^5*x^3+437/256/b^6*a^5/(b
*x^2+a)^5*x-693/256/b^6*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.99, size = 134, normalized size = 1.02

$$\frac{4215\,ab^4x^9+13270\,a^2b^3x^7+16768\,a^3b^2x^5+9770\,a^4bx^3+2185\,a^5x}{1280\left(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6\right)}-\frac{693\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,b^6}+\frac{x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/1280*(4215*a*b^4*x^9 + 13270*a^2*b^3*x^7 + 16768*a^3*b^2*x^5 + 9770*a^4*b
*x^3 + 2185*a^5*x)/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*
x^4 + 5*a^4*b^7*x^2 + a^5*b^6) - 693/256*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*b^6) + x/b^6
```

mupad [B] time = 0.16, size = 130, normalized size = 0.99

$$\frac{\frac{437\,a^5\,x}{256}+\frac{977\,a^4\,b\,x^3}{128}+\frac{131\,a^3\,b^2\,x^5}{10}+\frac{1327\,a^2\,b^3\,x^7}{128}+\frac{843\,a\,b^4\,x^9}{256}}{a^5\,b^6+5\,a^4\,b^7\,x^2+10\,a^3\,b^8\,x^4+10\,a^2\,b^9\,x^6+5\,a\,b^{10}\,x^8+b^{11}\,x^{10}}+\frac{x}{b^6}-\frac{693\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,b^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((437*a^5*x)/256 + (977*a^4*b*x^3)/128 + (843*a*b^4*x^9)/256 + (131*a^3*b^2
*x^5)/10 + (1327*a^2*b^3*x^7)/128)/(a^5*b^6 + b^11*x^10 + 5*a*b^10*x^8 + 5*
a^4*b^7*x^2 + 10*a^3*b^8*x^4 + 10*a^2*b^9*x^6) + x/b^6 - (693*a^(1/2)*atan(
(b^(1/2)*x)/a^(1/2)))/(256*b^(13/2))
```

sympy [A] time = 0.95, size = 178, normalized size = 1.36

$$\frac{693\sqrt{-\frac{a}{b^{13}}}\log\left(-b^6\sqrt{-\frac{a}{b^{13}}}+x\right)}{512} - \frac{693\sqrt{-\frac{a}{b^{13}}}\log\left(b^6\sqrt{-\frac{a}{b^{13}}}+x\right)}{512} + \frac{2185a^5x + 9770a^4bx^3 + 16768a^3b^2x}{1280a^5b^6 + 6400a^4b^7x^2 + 12800a^3b^8x^4 + 128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 693*sqrt(-a/b**13)*log(-b**6*sqrt(-a/b**13) + x)/512 - 693*sqrt(-a/b**13)*l
og(b**6*sqrt(-a/b**13) + x)/512 + (2185*a**5*x + 9770*a**4*b*x**3 + 16768*a
**3*b**2*x**5 + 13270*a**2*b**3*x**7 + 4215*a*b**4*x**9)/(1280*a**5*b**6 +
6400*a**4*b**7*x**2 + 12800*a**3*b**8*x**4 + 12800*a**2*b**9*x**6 + 6400*a*
b**10*x**8 + 1280*b**11*x**10) + x/b**6
```

3.526 $\int \frac{x^{10}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=121

$$\frac{63\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\sqrt{a}\,b^{11/2}}-\frac{63x}{256b^5\left(a+bx^2\right)}-\frac{21x^3}{128b^4\left(a+bx^2\right)^2}-\frac{21x^5}{160b^3\left(a+bx^2\right)^3}-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}-\frac{x^9}{10b\left(a+bx^2\right)^5}$$

```
[Out] -1/10*x^9/b/(b*x^2+a)^5-9/80*x^7/b^2/(b*x^2+a)^4-21/160*x^5/b^3/(b*x^2+a)^3
-21/128*x^3/b^4/(b*x^2+a)^2-63/256*x/b^5/(b*x^2+a)+63/256*arctan(x*b^(1/2)/
a^(1/2))/b^(11/2)/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 288, 205}

$$-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}-\frac{21x^5}{160b^3\left(a+bx^2\right)^3}-\frac{21x^3}{128b^4\left(a+bx^2\right)^2}-\frac{63x}{256b^5\left(a+bx^2\right)}+\frac{63\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\sqrt{a}\,b^{11/2}}-\frac{x^9}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -x^9/(10*b*(a + b*x^2)^5) - (9*x^7)/(80*b^2*(a + b*x^2)^4) - (21*x^5)/(160*
b^3*(a + b*x^2)^3) - (21*x^3)/(128*b^4*(a + b*x^2)^2) - (63*x)/(256*b^5*(a
+ b*x^2)) + (63*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*Sqrt[a]*b^(11/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{10}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^{10}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{x^9}{10b\left(a+bx^2\right)^5}+\frac{1}{10}\left(9b^4\right)\int \frac{x^8}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{x^9}{10b\left(a+bx^2\right)^5}-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}+\frac{1}{80}\left(63b^2\right)\int \frac{x^6}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{x^9}{10b\left(a+bx^2\right)^5}-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}-\frac{21x^5}{160b^3\left(a+bx^2\right)^3}+\frac{21}{32}\int \frac{x^4}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{x^9}{10b\left(a+bx^2\right)^5}-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}-\frac{21x^5}{160b^3\left(a+bx^2\right)^3}-\frac{21x^3}{128b^4\left(a+bx^2\right)^2}+\frac{63}{}$$

$$= -\frac{x^9}{10b\left(a+bx^2\right)^5}-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}-\frac{21x^5}{160b^3\left(a+bx^2\right)^3}-\frac{21x^3}{128b^4\left(a+bx^2\right)^2}-\frac{}{256}$$

$$= -\frac{x^9}{10b\left(a+bx^2\right)^5}-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}-\frac{21x^5}{160b^3\left(a+bx^2\right)^3}-\frac{21x^3}{128b^4\left(a+bx^2\right)^2}-\frac{}{256}$$

Mathematica [A] time = 0.05, size = 88, normalized size = 0.73

$$\frac{63\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\sqrt{a}\,b^{11/2}}-\frac{x\left(315a^4+1470a^3bx^2+2688a^2b^2x^4+2370ab^3x^6+965b^4x^8\right)}{1280b^5\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/1280*(x*(315*a^4 + 1470*a^3*b*x^2 + 2688*a^2*b^2*x^4 + 2370*a*b^3*x^6 +
965*b^4*x^8))/(b^5*(a + b*x^2)^5) + (63*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*S
qrt[a]*b^(11/2))
```

fricas [A] time = 0.96, size = 386, normalized size = 3.19

$$\left[-\frac{1930\,ab^{5}x^{9} + 4740\,a^{2}b^{4}x^{7} + 5376\,a^{3}b^{3}x^{5} + 2940\,a^{4}b^{2}x^{3} + 630\,a^{5}bx + 315\left(b^{5}x^{10} + 5\,ab^{4}x^{8} + 10\,a^{2}b^{3}x^{6} + 10\,a^{3}\right.}{2560\left(ab^{11}x^{10} + 5\,a^{2}b^{10}x^{8} + 10\,a^{3}b^{9}x^{6} + 10\,a^{4}b^{8}x^{4} + 5\,a^{5}b^{7}x^{2} + a^{6}b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/2560*(1930*a*b^5*x^9 + 4740*a^2*b^4*x^7 + 5376*a^3*b^3*x^5 + 2940*a^4*b
^2*x^3 + 630*a^5*b*x + 315*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^
3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/
(b*x^2 + a)))/(a*b^11*x^10 + 5*a^2*b^10*x^8 + 10*a^3*b^9*x^6 + 10*a^4*b^8*x
^4 + 5*a^5*b^7*x^2 + a^6*b^6), -1/1280*(965*a*b^5*x^9 + 2370*a^2*b^4*x^7 +
2688*a^3*b^3*x^5 + 1470*a^4*b^2*x^3 + 315*a^5*b*x - 315*(b^5*x^10 + 5*a*b^4
*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arcta
n(sqrt(a*b)*x/a))/(a*b^11*x^10 + 5*a^2*b^10*x^8 + 10*a^3*b^9*x^6 + 10*a^4*b
^8*x^4 + 5*a^5*b^7*x^2 + a^6*b^6)]
```

giac [A] time = 0.19, size = 78, normalized size = 0.64

$$\frac{63\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,b^{5}} - \frac{965\,b^{4}x^{9} + 2370\,ab^{3}x^{7} + 2688\,a^{2}b^{2}x^{5} + 1470\,a^{3}bx^{3} + 315\,a^{4}x}{1280\,\left(bx^{2} + a\right)^{5}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 63/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/1280*(965*b^4*x^9 + 2370*a
*b^3*x^7 + 2688*a^2*b^2*x^5 + 1470*a^3*b*x^3 + 315*a^4*x)/((b*x^2 + a)^5*b^
5)
```

maple [A] time = 0.01, size = 80, normalized size = 0.66

$$\frac{63\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\;b^{5}} + \frac{-\frac{193x^{9}}{256b} - \frac{237a\,x^{7}}{128b^{2}} - \frac{21a^{2}x^{5}}{10b^{3}} - \frac{147a^{3}x^{3}}{128b^{4}} - \frac{63a^{4}x}{256b^{5}}}{\left(b\,x^{2} + a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] (-193/256/b*x^9-237/128*a/b^2*x^7-21/10*a^2/b^3*x^5-147/128*a^3/b^4*x^3-63/
256*a^4/b^5*x)/(b*x^2+a)^5+63/256/b^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.07, size = 125, normalized size = 1.03

$$-\frac{965\,b^4x^9+2370\,ab^3x^7+2688\,a^2b^2x^5+1470\,a^3bx^3+315\,a^4x}{1280\left(b^{10}x^{10}+5\,ab^9x^8+10\,a^2b^8x^6+10\,a^3b^7x^4+5\,a^4b^6x^2+a^5b^5\right)}+\frac{63\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/1280*(965*b^4*x^9 + 2370*a*b^3*x^7 + 2688*a^2*b^2*x^5 + 1470*a^3*b*x^3 +
 315*a^4*x)/(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*
a^4*b^6*x^2 + a^5*b^5) + 63/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5)
```

mupad [B] time = 4.52, size = 122, normalized size = 1.01

$$\frac{63\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,\sqrt{a}\,b^{11/2}}-\frac{\frac{193\,x^9}{256\,b}+\frac{237\,a\,x^7}{128\,b^2}+\frac{63\,a^4\,x}{256\,b^5}+\frac{21\,a^2\,x^5}{10\,b^3}+\frac{147\,a^3\,x^3}{128\,b^4}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (63*atan((b^(1/2)*x)/a^(1/2)))/(256*a^(1/2)*b^(11/2)) - ((193*x^9)/(256*b)
+ (237*a*x^7)/(128*b^2) + (63*a^4*x)/(256*b^5) + (21*a^2*x^5)/(10*b^3) + (1
47*a^3*x^3)/(128*b^4))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10*a^3
*b^2*x^4 + 10*a^2*b^3*x^6)
```

sympy [A] time = 0.79, size = 182, normalized size = 1.50

$$-\frac{63\sqrt{-\frac{1}{ab^{11}}}\,\log\left(-ab^5\sqrt{-\frac{1}{ab^{11}}}+x\right)}{512}+\frac{63\sqrt{-\frac{1}{ab^{11}}}\,\log\left(ab^5\sqrt{-\frac{1}{ab^{11}}}+x\right)}{512}+\frac{-315a^4x-1470a^3bx^3-2688}{1280a^5b^5+6400a^4b^6x^2+12800a^3b^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -63*sqrt(-1/(a*b**11))*log(-a*b**5*sqrt(-1/(a*b**11)) + x)/512 + 63*sqrt(-1
/(a*b**11))*log(a*b**5*sqrt(-1/(a*b**11)) + x)/512 + (-315*a**4*x - 1470*a*
*3*b*x**3 - 2688*a**2*b**2*x**5 - 2370*a*b**3*x**7 - 965*b**4*x**9)/(1280*a
**5*b**5 + 6400*a**4*b**6*x**2 + 12800*a**3*b**7*x**4 + 12800*a**2*b**8*x**
6 + 6400*a*b**9*x**8 + 1280*b**10*x**10)
```

3.527 $\int \frac{x^8}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=122

$$\frac{7\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{3/2}b^{9/2}}+\frac{7x}{256ab^4\left(a+bx^2\right)}-\frac{7x}{128b^4\left(a+bx^2\right)^2}-\frac{7x^3}{96b^3\left(a+bx^2\right)^3}-\frac{7x^5}{80b^2\left(a+bx^2\right)^4}-\frac{x^7}{10b\left(a+bx^2\right)^5}$$

```
[Out] -1/10*x^7/b/(b*x^2+a)^5-7/80*x^5/b^2/(b*x^2+a)^4-7/96*x^3/b^3/(b*x^2+a)^3-7
/128*x/b^4/(b*x^2+a)^2+7/256*x/a/b^4/(b*x^2+a)+7/256*arctan(x*b^(1/2)/a^(1/
2))/a^(3/2)/b^(9/2)
```

Rubi [A] time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 199, 205}

$$\frac{7\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{3/2}b^{9/2}}-\frac{7x^5}{80b^2\left(a+bx^2\right)^4}-\frac{7x^3}{96b^3\left(a+bx^2\right)^3}+\frac{7x}{256ab^4\left(a+bx^2\right)}-\frac{7x}{128b^4\left(a+bx^2\right)^2}-\frac{x^7}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -x^7/(10*b*(a + b*x^2)^5) - (7*x^5)/(80*b^2*(a + b*x^2)^4) - (7*x^3)/(96*b^
3*(a + b*x^2)^3) - (7*x)/(128*b^4*(a + b*x^2)^2) + (7*x)/(256*a*b^4*(a + b*
x^2)) + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(3/2)*b^(9/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^8}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6 \int \frac{x^8}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{x^7}{10b\left(a+bx^2\right)^5} + \frac{1}{10}\left(7b^4\right)\int \frac{x^6}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{x^7}{10b\left(a+bx^2\right)^5} - \frac{7x^5}{80b^2\left(a+bx^2\right)^4} + \frac{1}{16}\left(7b^2\right)\int \frac{x^4}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{x^7}{10b\left(a+bx^2\right)^5} - \frac{7x^5}{80b^2\left(a+bx^2\right)^4} - \frac{7x^3}{96b^3\left(a+bx^2\right)^3} + \frac{7}{32}\int \frac{x^2}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{x^7}{10b\left(a+bx^2\right)^5} - \frac{7x^5}{80b^2\left(a+bx^2\right)^4} - \frac{7x^3}{96b^3\left(a+bx^2\right)^3} - \frac{7x}{128b^4\left(a+bx^2\right)^2} + \frac{7\int \cdots}{\cdots}$$

$$= -\frac{x^7}{10b\left(a+bx^2\right)^5} - \frac{7x^5}{80b^2\left(a+bx^2\right)^4} - \frac{7x^3}{96b^3\left(a+bx^2\right)^3} - \frac{7x}{128b^4\left(a+bx^2\right)^2} + \frac{\cdots}{256a\cdots}$$

$$= -\frac{x^7}{10b\left(a+bx^2\right)^5} - \frac{7x^5}{80b^2\left(a+bx^2\right)^4} - \frac{7x^3}{96b^3\left(a+bx^2\right)^3} - \frac{7x}{128b^4\left(a+bx^2\right)^2} + \frac{\cdots}{256a\cdots}$$

Mathematica [A] time = 0.06, size = 91, normalized size = 0.75

$$\frac{7\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{3/2}b^{9/2}} - \frac{x\left(105a^4+490a^3bx^2+896a^2b^2x^4+790ab^3x^6-105b^4x^8\right)}{3840ab^4\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/3840*(x*(105*a^4 + 490*a^3*b*x^2 + 896*a^2*b^2*x^4 + 790*a*b^3*x^6 - 105
*b^4*x^8))/(a*b^4*(a + b*x^2)^5) + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(
3/2)*b^(9/2))
```

fricas [A] time = 0.97, size = 390, normalized size = 3.20

$$\left[\frac{210\,ab^5x^9 - 1580\,a^2b^4x^7 - 1792\,a^3b^3x^5 - 980\,a^4b^2x^3 - 210\,a^5bx - 105\left(b^5x^{10} + 5\,ab^4x^8 + 10\,a^2b^3x^6 + 10\,a^3b^2x^4 + \cdots\right.}{7680\left(a^2b^{10}x^{10} + 5\,a^3b^9x^8 + 10\,a^4b^8x^6 + 10\,a^5b^7x^4 + 5\,a^6b^6x^2 + a^7b^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/7680*(210*a*b^5*x^9 - 1580*a^2*b^4*x^7 - 1792*a^3*b^3*x^5 - 980*a^4*b^2*
x^3 - 210*a^5*b*x - 105*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b
^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)))/(a^2*b^10*x^10 + 5*a^3*b^9*x^8 + 10*a^4*b^8*x^6 + 10*a^5*b^7*x^4
 + 5*a^6*b^6*x^2 + a^7*b^5), 1/3840*(105*a*b^5*x^9 - 790*a^2*b^4*x^7 - 896*
a^3*b^3*x^5 - 490*a^4*b^2*x^3 - 105*a^5*b*x + 105*(b^5*x^10 + 5*a*b^4*x^8 +
 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arctan(sqrt
(a*b)*x/a))/(a^2*b^10*x^10 + 5*a^3*b^9*x^8 + 10*a^4*b^8*x^6 + 10*a^5*b^7*x^
4 + 5*a^6*b^6*x^2 + a^7*b^5)]
```

giac [A] time = 0.16, size = 84, normalized size = 0.69

$$\frac{7\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,ab^4} + \frac{105\,b^4x^9 - 790\,ab^3x^7 - 896\,a^2b^2x^5 - 490\,a^3bx^3 - 105\,a^4x}{3840\left(bx^2 + a\right)^5ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 7/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^4) + 1/3840*(105*b^4*x^9 - 790*a
*b^3*x^7 - 896*a^2*b^2*x^5 - 490*a^3*b*x^3 - 105*a^4*x)/((b*x^2 + a)^5*a*b^
4)
```

maple [A] time = 0.01, size = 80, normalized size = 0.66

$$\frac{7\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\;a\,b^4} + \frac{\frac{7x^9}{256a} - \frac{79x^7}{384b} - \frac{7a\,x^5}{30b^2} - \frac{49a^2x^3}{384b^3} - \frac{7a^3x}{256b^4}}{\left(b\,x^2 + a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] (7/256/a*x^9-79/384/b*x^7-7/30*a/b^2*x^5-49/384*a^2/b^3*x^3-7/256*a^3/b^4*x
)/(b*x^2+a)^5+7/256/a/b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.03, size = 131, normalized size = 1.07

$$\frac{105\, b^4 x^9 - 790\, ab^3 x^7 - 896\, a^2 b^2 x^5 - 490\, a^3 b x^3 - 105\, a^4 x}{3840\, \left(ab^9 x^{10} + 5\, a^2 b^8 x^8 + 10\, a^3 b^7 x^6 + 10\, a^4 b^6 x^4 + 5\, a^5 b^5 x^2 + a^6 b^4\right)} + \frac{7\, \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\, \sqrt{ab}\, ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3840*(105*b^4*x^9 - 790*a*b^3*x^7 - 896*a^2*b^2*x^5 - 490*a^3*b*x^3 - 105
*a^4*x)/(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a
^5*b^5*x^2 + a^6*b^4) + 7/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^4)
```

mupad [B] time = 4.42, size = 119, normalized size = 0.98

$$\frac{7\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{3/2}\,b^{9/2}} - \frac{\frac{79\,x^7}{384\,b} - \frac{7\,x^9}{256\,a} + \frac{7\,a\,x^5}{30\,b^2} + \frac{7\,a^3\,x}{256\,b^4} + \frac{49\,a^2\,x^3}{384\,b^3}}{a^5 + 5\,a^4\,b\,x^2 + 10\,a^3\,b^2\,x^4 + 10\,a^2\,b^3\,x^6 + 5\,a\,b^4\,x^8 + b^5\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (7*atan((b^(1/2)*x)/a^(1/2)))/(256*a^(3/2)*b^(9/2)) - ((79*x^7)/(384*b) - (
7*x^9)/(256*a) + (7*a*x^5)/(30*b^2) + (7*a^3*x)/(256*b^4) + (49*a^2*x^3)/(3
84*b^3))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10*a^3*b^2*x^4 + 10*
a^2*b^3*x^6)
```

sympy [A] time = 0.76, size = 194, normalized size = 1.59

$$-\frac{7\sqrt{-\frac{1}{a^3 b^9}}\,\log\left(-a^2 b^4\sqrt{-\frac{1}{a^3 b^9}} + x\right)}{512} + \frac{7\sqrt{-\frac{1}{a^3 b^9}}\,\log\left(a^2 b^4\sqrt{-\frac{1}{a^3 b^9}} + x\right)}{512} + \frac{-105a^4 x - 490a^3 b x^3 - 89}{3840a^6 b^4 + 19200a^5 b^5 x^2 + 38400a^4 b^6 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -7*sqrt(-1/(a**3*b**9))*log(-a**2*b**4*sqrt(-1/(a**3*b**9)) + x)/512 + 7*sq
rt(-1/(a**3*b**9))*log(a**2*b**4*sqrt(-1/(a**3*b**9)) + x)/512 + (-105*a**4
```

```
*x - 490*a**3*b*x**3 - 896*a**2*b**2*x**5 - 790*a*b**3*x**7 + 105*b**4*x**9
)/(3840*a**6*b**4 + 19200*a**5*b**5*x**2 + 38400*a**4*b**6*x**4 + 38400*a**
3*b**7*x**6 + 19200*a**2*b**8*x**8 + 3840*a*b**9*x**10)
```

3.528 $\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=123

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{5/2}b^{7/2}}+\frac{3x}{256a^2b^3\left(a+bx^2\right)}+\frac{x}{128ab^3\left(a+bx^2\right)^2}-\frac{x}{32b^3\left(a+bx^2\right)^3}-\frac{x^3}{16b^2\left(a+bx^2\right)^4}-\frac{x^5}{10b\left(a+bx^2\right)^5}$$

```
[Out] -1/10*x^5/b/(b*x^2+a)^5-1/16*x^3/b^2/(b*x^2+a)^4-1/32*x/b^3/(b*x^2+a)^3+1/1
28*x/a/b^3/(b*x^2+a)^2+3/256*x/a^2/b^3/(b*x^2+a)+3/256*arctan(x*b^(1/2)/a^(
1/2))/a^(5/2)/b^(7/2)
```

Rubi [A] time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 199, 205}

$$\frac{3x}{256a^2b^3\left(a+bx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{5/2}b^{7/2}}-\frac{x^3}{16b^2\left(a+bx^2\right)^4}+\frac{x}{128ab^3\left(a+bx^2\right)^2}-\frac{x}{32b^3\left(a+bx^2\right)^3}-\frac{x^5}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -x^5/(10*b*(a + b*x^2)^5) - x^3/(16*b^2*(a + b*x^2)^4) - x/(32*b^3*(a + b*x
^2)^3) + x/(128*a*b^3*(a + b*x^2)^2) + (3*x)/(256*a^2*b^3*(a + b*x^2)) + (3
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(5/2)*b^(7/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^6}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{x^5}{10b\left(a+bx^2\right)^5}+\frac{1}{2}b^4\int \frac{x^4}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{x^5}{10b\left(a+bx^2\right)^5}-\frac{x^3}{16b^2\left(a+bx^2\right)^4}+\frac{1}{16}\left(3b^2\right)\int \frac{x^2}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{x^5}{10b\left(a+bx^2\right)^5}-\frac{x^3}{16b^2\left(a+bx^2\right)^4}-\frac{x}{32b^3\left(a+bx^2\right)^3}+\frac{1}{32}\int \frac{1}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{x^5}{10b\left(a+bx^2\right)^5}-\frac{x^3}{16b^2\left(a+bx^2\right)^4}-\frac{x}{32b^3\left(a+bx^2\right)^3}+\frac{x}{128ab^3\left(a+bx^2\right)^2}+\frac{3\int -}{}$$

$$= -\frac{x^5}{10b\left(a+bx^2\right)^5}-\frac{x^3}{16b^2\left(a+bx^2\right)^4}-\frac{x}{32b^3\left(a+bx^2\right)^3}+\frac{x}{128ab^3\left(a+bx^2\right)^2}+\frac{}{256a}$$

$$= -\frac{x^5}{10b\left(a+bx^2\right)^5}-\frac{x^3}{16b^2\left(a+bx^2\right)^4}-\frac{x}{32b^3\left(a+bx^2\right)^3}+\frac{x}{128ab^3\left(a+bx^2\right)^2}+\frac{}{256a}$$

Mathematica [A] time = 0.06, size = 91, normalized size = 0.74

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{5/2}b^{7/2}}+\frac{-15a^4x-70a^3bx^3-128a^2b^2x^5+70ab^3x^7+15b^4x^9}{1280a^2b^3\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-15*a^4*x - 70*a^3*b*x^3 - 128*a^2*b^2*x^5 + 70*a*b^3*x^7 + 15*b^4*x^9)/(1
280*a^2*b^3*(a + b*x^2)^5) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(5/2)*b
^(7/2))
```

fricas [A] time = 1.03, size = 390, normalized size = 3.17

$$\left[\frac{30\,ab^5x^9 + 140\,a^2b^4x^7 - 256\,a^3b^3x^5 - 140\,a^4b^2x^3 - 30\,a^5bx - 15\left(b^5x^{10} + 5\,ab^4x^8 + 10\,a^2b^3x^6 + 10\,a^3b^2x^4 + 5\,a\right.}{2560\left(a^3b^9x^{10} + 5\,a^4b^8x^8 + 10\,a^5b^7x^6 + 10\,a^6b^6x^4 + 5\,a^7b^5x^2 + a^8b^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(30*a*b^5*x^9 + 140*a^2*b^4*x^7 - 256*a^3*b^3*x^5 - 140*a^4*b^2*x^3
 - 30*a^5*b*x - 15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^
4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 +
 a)))/(a^3*b^9*x^10 + 5*a^4*b^8*x^8 + 10*a^5*b^7*x^6 + 10*a^6*b^6*x^4 + 5*a
^7*b^5*x^2 + a^8*b^4), 1/1280*(15*a*b^5*x^9 + 70*a^2*b^4*x^7 - 128*a^3*b^3*
x^5 - 70*a^4*b^2*x^3 - 15*a^5*b*x + 15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3
*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))
/(a^3*b^9*x^10 + 5*a^4*b^8*x^8 + 10*a^5*b^7*x^6 + 10*a^6*b^6*x^4 + 5*a^7*b^
5*x^2 + a^8*b^4)]
```

giac [A] time = 0.16, size = 84, normalized size = 0.68

$$\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^2b^3} + \frac{15\,b^4x^9 + 70\,ab^3x^7 - 128\,a^2b^2x^5 - 70\,a^3bx^3 - 15\,a^4x}{1280\left(bx^2 + a\right)^5 a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 3/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^3) + 1/1280*(15*b^4*x^9 + 70*a
*b^3*x^7 - 128*a^2*b^2*x^5 - 70*a^3*b*x^3 - 15*a^4*x)/((b*x^2 + a)^5*a^2*b^
3)
```

maple [A] time = 0.01, size = 78, normalized size = 0.63

$$\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\;a^2b^3} + \frac{\frac{3b\,x^9}{256a^2} + \frac{7x^7}{128a} - \frac{x^5}{10b} - \frac{7a\,x^3}{128b^2} - \frac{3a^2x}{256b^3}}{\left(b\,x^2 + a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] (3/256/a^2*b*x^9+7/128/a*x^7-1/10/b*x^5-7/128*a/b^2*x^3-3/256*a^2/b^3*x)/(b
*x^2+a)^5+3/256/a^2/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.96, size = 133, normalized size = 1.08

$$\frac{15\,b^4x^9 + 70\,ab^3x^7 - 128\,a^2b^2x^5 - 70\,a^3bx^3 - 15\,a^4x}{1280\left(a^2b^8x^{10} + 5\,a^3b^7x^8 + 10\,a^4b^6x^6 + 10\,a^5b^5x^4 + 5\,a^6b^4x^2 + a^7b^3\right)} + \frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/1280*(15*b^4*x^9 + 70*a*b^3*x^7 - 128*a^2*b^2*x^5 - 70*a^3*b*x^3 - 15*a^4
*x)/(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x^4 + 5*a^6
*b^4*x^2 + a^7*b^3) + 3/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^3)
```

mupad [B] time = 4.50, size = 117, normalized size = 0.95

$$\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{5/2}\,b^{7/2}} - \frac{\frac{x^5}{10\,b} - \frac{7\,x^7}{128\,a} + \frac{7\,a\,x^3}{128\,b^2} + \frac{3\,a^2\,x}{256\,b^3} - \frac{3\,b\,x^9}{256\,a^2}}{a^5 + 5\,a^4\,b\,x^2 + 10\,a^3\,b^2\,x^4 + 10\,a^2\,b^3\,x^6 + 5\,a\,b^4\,x^8 + b^5\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (3*atan((b^(1/2)*x)/a^(1/2)))/(256*a^(5/2)*b^(7/2)) - (x^5/(10*b) - (7*x^7)
/(128*a) + (7*a*x^3)/(128*b^2) + (3*a^2*x)/(256*b^3) - (3*b*x^9)/(256*a^2))
/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10*a^3*b^2*x^4 + 10*a^2*b^3*
x^6)
```

sympy [A] time = 0.70, size = 196, normalized size = 1.59

$$-\frac{3\sqrt{-\frac{1}{a^5b^7}}\,\log\left(-a^3b^3\sqrt{-\frac{1}{a^5b^7}} + x\right)}{512} + \frac{3\sqrt{-\frac{1}{a^5b^7}}\,\log\left(a^3b^3\sqrt{-\frac{1}{a^5b^7}} + x\right)}{512} + \frac{-15a^4x - 70a^3bx^3 - 128a}{1280a^7b^3 + 6400a^6b^4x^2 + 12800a^5b^5x^4 +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -3*sqrt(-1/(a**5*b**7))*log(-a**3*b**3*sqrt(-1/(a**5*b**7)) + x)/512 + 3*sq
rt(-1/(a**5*b**7))*log(a**3*b**3*sqrt(-1/(a**5*b**7)) + x)/512 + (-15*a**4*
```

```
x - 70*a**3*b*x**3 - 128*a**2*b**2*x**5 + 70*a*b**3*x**7 + 15*b**4*x**9)/(1
280*a**7*b**3 + 6400*a**6*b**4*x**2 + 12800*a**5*b**5*x**4 + 12800*a**4*b**
6*x**6 + 6400*a**3*b**7*x**8 + 1280*a**2*b**8*x**10)
```

3.529 $\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=124

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{7/2}b^{5/2}}+\frac{3x}{256a^3b^2\left(a+bx^2\right)}+\frac{x}{128a^2b^2\left(a+bx^2\right)^2}+\frac{x}{160ab^2\left(a+bx^2\right)^3}-\frac{3x}{80b^2\left(a+bx^2\right)^4}-\frac{x^3}{10b\left(a+bx^2\right)^5}$$

```
[Out] -1/10*x^3/b/(b*x^2+a)^5-3/80*x/b^2/(b*x^2+a)^4+1/160*x/a/b^2/(b*x^2+a)^3+1/
128*x/a^2/b^2/(b*x^2+a)^2+3/256*x/a^3/b^2/(b*x^2+a)+3/256*arctan(x*b^(1/2)/
a^(1/2))/a^(7/2)/b^(5/2)
```

Rubi [A] time = 0.07, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 199, 205}

$$\frac{3x}{256a^3b^2\left(a+bx^2\right)}+\frac{x}{128a^2b^2\left(a+bx^2\right)^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{7/2}b^{5/2}}+\frac{x}{160ab^2\left(a+bx^2\right)^3}-\frac{3x}{80b^2\left(a+bx^2\right)^4}-\frac{x^3}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -x^3/(10*b*(a + b*x^2)^5) - (3*x)/(80*b^2*(a + b*x^2)^4) + x/(160*a*b^2*(a
+ b*x^2)^3) + x/(128*a^2*b^2*(a + b*x^2)^2) + (3*x)/(256*a^3*b^2*(a + b*x^2
)) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(7/2)*b^(5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^4}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{x^3}{10b\left(a+bx^2\right)^5}+\frac{1}{10}\left(3b^4\right)\int \frac{x^2}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{x^3}{10b\left(a+bx^2\right)^5}-\frac{3x}{80b^2\left(a+bx^2\right)^4}+\frac{1}{80}\left(3b^2\right)\int \frac{1}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{x^3}{10b\left(a+bx^2\right)^5}-\frac{3x}{80b^2\left(a+bx^2\right)^4}+\frac{x}{160ab^2\left(a+bx^2\right)^3}+\frac{b\int \frac{1}{\left(ab+b^2x^2\right)^3}\,dx}{32a}$$

$$= -\frac{x^3}{10b\left(a+bx^2\right)^5}-\frac{3x}{80b^2\left(a+bx^2\right)^4}+\frac{x}{160ab^2\left(a+bx^2\right)^3}+\frac{x}{128a^2b^2\left(a+bx^2\right)^2}+$$

$$= -\frac{x^3}{10b\left(a+bx^2\right)^5}-\frac{3x}{80b^2\left(a+bx^2\right)^4}+\frac{x}{160ab^2\left(a+bx^2\right)^3}+\frac{x}{128a^2b^2\left(a+bx^2\right)^2}+$$

$$= -\frac{x^3}{10b\left(a+bx^2\right)^5}-\frac{3x}{80b^2\left(a+bx^2\right)^4}+\frac{x}{160ab^2\left(a+bx^2\right)^3}+\frac{x}{128a^2b^2\left(a+bx^2\right)^2}+$$

Mathematica [A] time = 0.05, size = 91, normalized size = 0.73

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{7/2}b^{5/2}}+\frac{-15a^4x-70a^3bx^3+128a^2b^2x^5+70ab^3x^7+15b^4x^9}{1280a^3b^2\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-15*a^4*x - 70*a^3*b*x^3 + 128*a^2*b^2*x^5 + 70*a*b^3*x^7 + 15*b^4*x^9)/(1
280*a^3*b^2*(a + b*x^2)^5) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(7/2)*b
^(5/2))
```

fricas [A] time = 0.90, size = 390, normalized size = 3.15

$$\left[\frac{30\,ab^5x^9 + 140\,a^2b^4x^7 + 256\,a^3b^3x^5 - 140\,a^4b^2x^3 - 30\,a^5bx - 15\left(b^5x^{10} + 5\,ab^4x^8 + 10\,a^2b^3x^6 + 10\,a^3b^2x^4 + 5\,a^4\right.}{2560\left(a^4b^8x^{10} + 5\,a^5b^7x^8 + 10\,a^6b^6x^6 + 10\,a^7b^5x^4 + 5\,a^8b^4x^2 + a^9b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(30*a*b^5*x^9 + 140*a^2*b^4*x^7 + 256*a^3*b^3*x^5 - 140*a^4*b^2*x^3
 - 30*a^5*b*x - 15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^
4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 +
 a)))/(a^4*b^8*x^10 + 5*a^5*b^7*x^8 + 10*a^6*b^6*x^6 + 10*a^7*b^5*x^4 + 5*a
^8*b^4*x^2 + a^9*b^3), 1/1280*(15*a*b^5*x^9 + 70*a^2*b^4*x^7 + 128*a^3*b^3*
x^5 - 70*a^4*b^2*x^3 - 15*a^5*b*x + 15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3
*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))
/(a^4*b^8*x^10 + 5*a^5*b^7*x^8 + 10*a^6*b^6*x^6 + 10*a^7*b^5*x^4 + 5*a^8*b^
4*x^2 + a^9*b^3)]
```

giac [A] time = 0.16, size = 84, normalized size = 0.68

$$\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^3b^2} + \frac{15\,b^4x^9 + 70\,ab^3x^7 + 128\,a^2b^2x^5 - 70\,a^3bx^3 - 15\,a^4x}{1280\,\left(bx^2 + a\right)^5 a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 3/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^2) + 1/1280*(15*b^4*x^9 + 70*a
*b^3*x^7 + 128*a^2*b^2*x^5 - 70*a^3*b*x^3 - 15*a^4*x)/((b*x^2 + a)^5*a^3*b^
2)
```

maple [A] time = 0.01, size = 78, normalized size = 0.63

$$\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\;a^3b^2} + \frac{\frac{3b^2x^9}{256a^3} + \frac{7b\,x^7}{128a^2} + \frac{x^5}{10a} - \frac{7x^3}{128b} - \frac{3ax}{256b^2}}{\left(b\,x^2 + a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] (3/256/a^3*b^2*x^9+7/128/a^2*b*x^7+1/10/a*x^5-7/128/b*x^3-3/256*a/b^2*x)/(b
*x^2+a)^5+3/256/a^3/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.04, size = 133, normalized size = 1.07

$$\frac{15\,b^4x^9+70\,ab^3x^7+128\,a^2b^2x^5-70\,a^3bx^3-15\,a^4x}{1280\left(a^3b^7x^{10}+5\,a^4b^6x^8+10\,a^5b^5x^6+10\,a^6b^4x^4+5\,a^7b^3x^2+a^8b^2\right)}+\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/1280*(15*b^4*x^9 + 70*a*b^3*x^7 + 128*a^2*b^2*x^5 - 70*a^3*b*x^3 - 15*a^4
*x)/(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a^7
*b^3*x^2 + a^8*b^2) + 3/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^2)
```

mupad [B] time = 4.47, size = 116, normalized size = 0.94

$$\frac{\frac{x^5}{10\,a}-\frac{7\,x^3}{128\,b}+\frac{7\,b\,x^7}{128\,a^2}+\frac{3\,b^2\,x^9}{256\,a^3}-\frac{3\,a\,x}{256\,b^2}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{7/2}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (x^5/(10*a) - (7*x^3)/(128*b) + (7*b*x^7)/(128*a^2) + (3*b^2*x^9)/(256*a^3)
 - (3*a*x)/(256*b^2))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10*a^3*
b^2*x^4 + 10*a^2*b^3*x^6) + (3*atan((b^(1/2)*x)/a^(1/2)))/(256*a^(7/2)*b^(5
/2))
```

sympy [A] time = 0.64, size = 196, normalized size = 1.58

$$-\frac{3\sqrt{-\frac{1}{a^7b^5}}\log\left(-a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{512}+\frac{3\sqrt{-\frac{1}{a^7b^5}}\log\left(a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{512}+\frac{-15a^4x-70a^3bx^3+128}{1280a^8b^2+6400a^7b^3x^2+12800a^6b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -3*sqrt(-1/(a**7*b**5))*log(-a**4*b**2*sqrt(-1/(a**7*b**5)) + x)/512 + 3*sq
rt(-1/(a**7*b**5))*log(a**4*b**2*sqrt(-1/(a**7*b**5)) + x)/512 + (-15*a**4*
```

```
x - 70*a**3*b*x**3 + 128*a**2*b**2*x**5 + 70*a*b**3*x**7 + 15*b**4*x**9)/(1
280*a**8*b**2 + 6400*a**7*b**3*x**2 + 12800*a**6*b**4*x**4 + 12800*a**5*b**
5*x**6 + 6400*a**4*b**6*x**8 + 1280*a**3*b**7*x**10)
```

3.530 $\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^3} \, dx$

Optimal. Leaf size=125

$$\frac{7\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{9/2}b^{3/2}}+\frac{7x}{256a^4b\left(a+bx^2\right)}+\frac{7x}{384a^3b\left(a+bx^2\right)^2}+\frac{7x}{480a^2b\left(a+bx^2\right)^3}+\frac{x}{80ab\left(a+bx^2\right)^4}-\frac{x}{10b\left(a+bx^2\right)^5}$$

```
[Out] -1/10*x/b/(b*x^2+a)^5+1/80*x/a/b/(b*x^2+a)^4+7/480*x/a^2/b/(b*x^2+a)^3+7/38
4*x/a^3/b/(b*x^2+a)^2+7/256*x/a^4/b/(b*x^2+a)+7/256*arctan(x*b^(1/2)/a^(1/2
))/a^(9/2)/b^(3/2)
```

Rubi [A] time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 199, 205}

$$\frac{7\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{9/2}b^{3/2}}+\frac{7x}{256a^4b\left(a+bx^2\right)}+\frac{7x}{384a^3b\left(a+bx^2\right)^2}+\frac{7x}{480a^2b\left(a+bx^2\right)^3}+\frac{x}{80ab\left(a+bx^2\right)^4}-\frac{x}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -x/(10*b*(a + b*x^2)^5) + x/(80*a*b*(a + b*x^2)^4) + (7*x)/(480*a^2*b*(a +
b*x^2)^3) + (7*x)/(384*a^3*b*(a + b*x^2)^2) + (7*x)/(256*a^4*b*(a + b*x^2))
 + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(9/2)*b^(3/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^2}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{x}{10b\left(a+bx^2\right)^5}+\frac{1}{10}b^4\int \frac{1}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{x}{10b\left(a+bx^2\right)^5}+\frac{x}{80ab\left(a+bx^2\right)^4}+\frac{\left(7b^3\right)\int \frac{1}{\left(ab+b^2x^2\right)^4}\,dx}{80a}$$

$$= -\frac{x}{10b\left(a+bx^2\right)^5}+\frac{x}{80ab\left(a+bx^2\right)^4}+\frac{7x}{480a^2b\left(a+bx^2\right)^3}+\frac{\left(7b^2\right)\int \frac{1}{\left(ab+b^2x^2\right)^3}\,dx}{96a^2}$$

$$= -\frac{x}{10b\left(a+bx^2\right)^5}+\frac{x}{80ab\left(a+bx^2\right)^4}+\frac{7x}{480a^2b\left(a+bx^2\right)^3}+\frac{7x}{384a^3b\left(a+bx^2\right)^2}+\frac{(7}{}$$

$$= -\frac{x}{10b\left(a+bx^2\right)^5}+\frac{x}{80ab\left(a+bx^2\right)^4}+\frac{7x}{480a^2b\left(a+bx^2\right)^3}+\frac{7x}{384a^3b\left(a+bx^2\right)^2}+\frac{}{25}$$

$$= -\frac{x}{10b\left(a+bx^2\right)^5}+\frac{x}{80ab\left(a+bx^2\right)^4}+\frac{7x}{480a^2b\left(a+bx^2\right)^3}+\frac{7x}{384a^3b\left(a+bx^2\right)^2}+\frac{}{25}$$

Mathematica [A] time = 0.05, size = 91, normalized size = 0.73

$$\frac{7\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{9/2}b^{3/2}}+\frac{-105a^4x+790a^3bx^3+896a^2b^2x^5+490ab^3x^7+105b^4x^9}{3840a^4b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-105*a^4*x + 790*a^3*b*x^3 + 896*a^2*b^2*x^5 + 490*a*b^3*x^7 + 105*b^4*x^9
)/(3840*a^4*b*(a + b*x^2)^5) + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(9/2)
*b^(3/2))
```

fricas [A] time = 0.71, size = 390, normalized size = 3.12

$$\left[\frac{210\,ab^5x^9+980\,a^2b^4x^7+1792\,a^3b^3x^5+1580\,a^4b^2x^3-210\,a^5bx-105\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x\right.}{7680\left(a^5b^7x^{10}+5\,a^6b^6x^8+10\,a^7b^5x^6+10\,a^8b^4x^4+5\,a^9b^3x^2+a^{10}b^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/7680*(210*a*b^5*x^9 + 980*a^2*b^4*x^7 + 1792*a^3*b^3*x^5 + 1580*a^4*b^2*
x^3 - 210*a^5*b*x - 105*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b
^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)))/(a^5*b^7*x^10 + 5*a^6*b^6*x^8 + 10*a^7*b^5*x^6 + 10*a^8*b^4*x^4
+ 5*a^9*b^3*x^2 + a^10*b^2), 1/3840*(105*a*b^5*x^9 + 490*a^2*b^4*x^7 + 896*
a^3*b^3*x^5 + 790*a^4*b^2*x^3 - 105*a^5*b*x + 105*(b^5*x^10 + 5*a*b^4*x^8 +
 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arctan(sqrt
(a*b)*x/a))/(a^5*b^7*x^10 + 5*a^6*b^6*x^8 + 10*a^7*b^5*x^6 + 10*a^8*b^4*x^4
 + 5*a^9*b^3*x^2 + a^10*b^2)]
```

giac [A] time = 0.17, size = 84, normalized size = 0.67

$$\frac{7\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^4b}+\frac{105\,b^4x^9+490\,ab^3x^7+896\,a^2b^2x^5+790\,a^3bx^3-105\,a^4x}{3840\left(bx^2+a\right)^5a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 7/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*b) + 1/3840*(105*b^4*x^9 + 490*a
*b^3*x^7 + 896*a^2*b^2*x^5 + 790*a^3*b*x^3 - 105*a^4*x)/((b*x^2 + a)^5*a^4*
b)
```

maple [A] time = 0.01, size = 80, normalized size = 0.64

$$\frac{7\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\ a^4b}+\frac{\frac{7b^3x^9}{256a^4}+\frac{49b^2x^7}{384a^3}+\frac{7b\,x^5}{30a^2}+\frac{79x^3}{384a}-\frac{7x}{256b}}{\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] (7/256/a^4*b^3*x^9+49/384/a^3*b^2*x^7+7/30/a^2*b*x^5+79/384/a*x^3-7/256/b*x
)/(b*x^2+a)^5+7/256/a^4/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.97, size = 131, normalized size = 1.05

$$\frac{105\,b^4x^9+490\,ab^3x^7+896\,a^2b^2x^5+790\,a^3bx^3-105\,a^4x}{3840\left(a^4b^6x^{10}+5\,a^5b^5x^8+10\,a^6b^4x^6+10\,a^7b^3x^4+5\,a^8b^2x^2+a^9b\right)}+\frac{7\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3840*(105*b^4*x^9 + 490*a*b^3*x^7 + 896*a^2*b^2*x^5 + 790*a^3*b*x^3 - 105
*a^4*x)/(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5
*a^8*b^2*x^2 + a^9*b) + 7/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*b)
```

mupad [B] time = 4.48, size = 118, normalized size = 0.94

$$\frac{\frac{79\,x^3}{384\,a}-\frac{7\,x}{256\,b}+\frac{7\,b\,x^5}{30\,a^2}+\frac{49\,b^2\,x^7}{384\,a^3}+\frac{7\,b^3\,x^9}{256\,a^4}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}+\frac{7\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{9/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((79*x^3)/(384*a) - (7*x)/(256*b) + (7*b*x^5)/(30*a^2) + (49*b^2*x^7)/(384*
a^3) + (7*b^3*x^9)/(256*a^4))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 +
 10*a^3*b^2*x^4 + 10*a^2*b^3*x^6) + (7*atan((b^(1/2)*x)/a^(1/2)))/(256*a^(9
/2)*b^(3/2))
```

sympy [A] time = 0.62, size = 190, normalized size = 1.52

$$-\frac{7\sqrt{-\frac{1}{a^9b^3}}\log\left(-a^5b\sqrt{-\frac{1}{a^9b^3}}+x\right)}{512}+\frac{7\sqrt{-\frac{1}{a^9b^3}}\log\left(a^5b\sqrt{-\frac{1}{a^9b^3}}+x\right)}{512}+\frac{-105a^4x+790a^3bx^3+896a^2\cdots}{3840a^9b+19200a^8b^2x^2+38400a^7b^3x^4+3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -7*sqrt(-1/(a**9*b**3))*log(-a**5*b*sqrt(-1/(a**9*b**3)) + x)/512 + 7*sqrt(
-1/(a**9*b**3))*log(a**5*b*sqrt(-1/(a**9*b**3)) + x)/512 + (-105*a**4*x + 7
90*a**3*b*x**3 + 896*a**2*b**2*x**5 + 490*a*b**3*x**7 + 105*b**4*x**9)/(384
0*a**9*b + 19200*a**8*b**2*x**2 + 38400*a**7*b**3*x**4 + 38400*a**6*b**4*x*
*6 + 19200*a**5*b**5*x**8 + 3840*a**4*b**6*x**10)
```

3.531 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=113

$$\frac{63\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{11/2}\sqrt{b}}+\frac{63x}{256a^5\left(a+bx^2\right)}+\frac{21x}{128a^4\left(a+bx^2\right)^2}+\frac{21x}{160a^3\left(a+bx^2\right)^3}+\frac{9x}{80a^2\left(a+bx^2\right)^4}+\frac{x}{10a\left(a+bx^2\right)^5}$$

```
[Out] 1/10*x/a/(b*x^2+a)^5+9/80*x/a^2/(b*x^2+a)^4+21/160*x/a^3/(b*x^2+a)^3+21/128
*x/a^4/(b*x^2+a)^2+63/256*x/a^5/(b*x^2+a)+63/256*arctan(x*b^(1/2)/a^(1/2))/
a^(11/2)/b^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {28, 199, 205}

$$\frac{63x}{256a^5\left(a+bx^2\right)}+\frac{21x}{128a^4\left(a+bx^2\right)^2}+\frac{21x}{160a^3\left(a+bx^2\right)^3}+\frac{9x}{80a^2\left(a+bx^2\right)^4}+\frac{63\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{11/2}\sqrt{b}}+\frac{x}{10a\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3),x]
```

```
[Out] x/(10*a*(a + b*x^2)^5) + (9*x)/(80*a^2*(a + b*x^2)^4) + (21*x)/(160*a^3*(a
+ b*x^2)^3) + (21*x)/(128*a^4*(a + b*x^2)^2) + (63*x)/(256*a^5*(a + b*x^2))
 + (63*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(11/2)*Sqrt[b])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{x}{10a\left(a+bx^2\right)^5} + \frac{\left(9b^5\right)\int \frac{1}{\left(ab+b^2x^2\right)^5}\,dx}{10a}$$

$$= \frac{x}{10a\left(a+bx^2\right)^5} + \frac{9x}{80a^2\left(a+bx^2\right)^4} + \frac{\left(63b^4\right)\int \frac{1}{\left(ab+b^2x^2\right)^4}\,dx}{80a^2}$$

$$= \frac{x}{10a\left(a+bx^2\right)^5} + \frac{9x}{80a^2\left(a+bx^2\right)^4} + \frac{21x}{160a^3\left(a+bx^2\right)^3} + \frac{\left(21b^3\right)\int \frac{1}{\left(ab+b^2x^2\right)^3}\,dx}{32a^3}$$

$$= \frac{x}{10a\left(a+bx^2\right)^5} + \frac{9x}{80a^2\left(a+bx^2\right)^4} + \frac{21x}{160a^3\left(a+bx^2\right)^3} + \frac{21x}{128a^4\left(a+bx^2\right)^2} + \frac{(63b \ldots}{\ldots}$$

$$= \frac{x}{10a\left(a+bx^2\right)^5} + \frac{9x}{80a^2\left(a+bx^2\right)^4} + \frac{21x}{160a^3\left(a+bx^2\right)^3} + \frac{21x}{128a^4\left(a+bx^2\right)^2} + \frac{\ldots}{256a \ldots}$$

$$= \frac{x}{10a\left(a+bx^2\right)^5} + \frac{9x}{80a^2\left(a+bx^2\right)^4} + \frac{21x}{160a^3\left(a+bx^2\right)^3} + \frac{21x}{128a^4\left(a+bx^2\right)^2} + \frac{\ldots}{256a \ldots}$$

Mathematica [A] time = 0.05, size = 89, normalized size = 0.79

$$\frac{\frac{\sqrt{a}\,x\left(965a^4+2370a^3bx^2+2688a^2b^2x^4+1470ab^3x^6+315b^4x^8\right)}{\left(a+bx^2\right)^5}+\frac{315\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}}{1280a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3),x]
```

```
[Out] ((Sqrt[a]*x*(965*a^4 + 2370*a^3*b*x^2 + 2688*a^2*b^2*x^4 + 1470*a*b^3*x^6 +
 315*b^4*x^8))/(a + b*x^2)^5 + (315*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])/(
1280*a^(11/2))
```

fricas [A] time = 0.70, size = 386, normalized size = 3.42

$$\left[\frac{630\,ab^5x^9 + 2940\,a^2b^4x^7 + 5376\,a^3b^3x^5 + 4740\,a^4b^2x^3 + 1930\,a^5bx - 315\left(b^5x^{10} + 5\,ab^4x^8 + 10\,a^2b^3x^6 + 10\,a^3b^2\right.}{2560\left(a^6b^6x^{10} + 5\,a^7b^5x^8 + 10\,a^8b^4x^6 + 10\,a^9b^3x^4 + 5\,a^{10}b^2x^2 + a^{11}b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(630*a*b^5*x^9 + 2940*a^2*b^4*x^7 + 5376*a^3*b^3*x^5 + 4740*a^4*b^2
*x^3 + 1930*a^5*b*x - 315*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3
*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a^6*b^6*x^10 + 5*a^7*b^5*x^8 + 10*a^8*b^4*x^6 + 10*a^9*b^3*x^
4 + 5*a^10*b^2*x^2 + a^11*b), 1/1280*(315*a*b^5*x^9 + 1470*a^2*b^4*x^7 + 26
88*a^3*b^3*x^5 + 2370*a^4*b^2*x^3 + 965*a^5*b*x + 315*(b^5*x^10 + 5*a*b^4*x
^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arctan(
sqrt(a*b)*x/a))/(a^6*b^6*x^10 + 5*a^7*b^5*x^8 + 10*a^8*b^4*x^6 + 10*a^9*b^3
*x^4 + 5*a^10*b^2*x^2 + a^11*b)]
```

giac [A] time = 0.17, size = 78, normalized size = 0.69

$$\frac{63\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^5} + \frac{315\,b^4x^9 + 1470\,ab^3x^7 + 2688\,a^2b^2x^5 + 2370\,a^3bx^3 + 965\,a^4x}{1280\,\left(bx^2 + a\right)^5 a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 63/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/1280*(315*b^4*x^9 + 1470*a
*b^3*x^7 + 2688*a^2*b^2*x^5 + 2370*a^3*b*x^3 + 965*a^4*x)/((b*x^2 + a)^5*a^
5)
```

maple [A] time = 0.01, size = 96, normalized size = 0.85

$$\frac{x}{10\left(b\,x^2 + a\right)^5 a} + \frac{9x}{80\left(b\,x^2 + a\right)^4 a^2} + \frac{21x}{160\left(b\,x^2 + a\right)^3 a^3} + \frac{21x}{128\left(b\,x^2 + a\right)^2 a^4} + \frac{63x}{256\left(b\,x^2 + a\right) a^5} + \frac{63\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/10*x/a/(b*x^2+a)^5+9/80*x/a^2/(b*x^2+a)^4+21/160*x/a^3/(b*x^2+a)^3+21/128
*x/a^4/(b*x^2+a)^2+63/256*x/a^5/(b*x^2+a)+63/256/a^5/(a*b)^(1/2)*arctan(1/(
a*b)^(1/2)*b*x)
```

maxima [A] time = 3.04, size = 124, normalized size = 1.10

$$\frac{315\,b^4x^9+1470\,ab^3x^7+2688\,a^2b^2x^5+2370\,a^3bx^3+965\,a^4x}{1280\left(a^5b^5x^{10}+5\,a^6b^4x^8+10\,a^7b^3x^6+10\,a^8b^2x^4+5\,a^9bx^2+a^{10}\right)}+\frac{63\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/1280*(315*b^4*x^9 + 1470*a*b^3*x^7 + 2688*a^2*b^2*x^5 + 2370*a^3*b*x^3 +
965*a^4*x)/(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^2*x^4
+ 5*a^9*b*x^2 + a^10) + 63/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

mupad [B] time = 4.71, size = 121, normalized size = 1.07

$$\frac{\frac{193\,x}{256\,a}+\frac{237\,b\,x^3}{128\,a^2}+\frac{21\,b^2\,x^5}{10\,a^3}+\frac{147\,b^3\,x^7}{128\,a^4}+\frac{63\,b^4\,x^9}{256\,a^5}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}+\frac{63\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{11/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((193*x)/(256*a) + (237*b*x^3)/(128*a^2) + (21*b^2*x^5)/(10*a^3) + (147*b^3
*x^7)/(128*a^4) + (63*b^4*x^9)/(256*a^5))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5
*a*b^4*x^8 + 10*a^3*b^2*x^4 + 10*a^2*b^3*x^6) + (63*atan((b^(1/2)*x)/a^(1/2
)))/(256*a^(11/2)*b^(1/2))
```

sympy [A] time = 0.68, size = 177, normalized size = 1.57

$$-\frac{63\sqrt{-\frac{1}{a^{11}b}}\,\log\left(-a^6\sqrt{-\frac{1}{a^{11}b}}+x\right)}{512}+\frac{63\sqrt{-\frac{1}{a^{11}b}}\,\log\left(a^6\sqrt{-\frac{1}{a^{11}b}}+x\right)}{512}+\frac{965a^4x+2370a^3bx^3+2688a^2}{1280a^{10}+6400a^9bx^2+12800a^8b^2x^4+12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -63*sqrt(-1/(a**11*b))*log(-a**6*sqrt(-1/(a**11*b)) + x)/512 + 63*sqrt(-1/(
a**11*b))*log(a**6*sqrt(-1/(a**11*b)) + x)/512 + (965*a**4*x + 2370*a**3*b*
x**3 + 2688*a**2*b**2*x**5 + 1470*a*b**3*x**7 + 315*b**4*x**9)/(1280*a**10
+ 6400*a**9*b*x**2 + 12800*a**8*b**2*x**4 + 12800*a**7*b**3*x**6 + 6400*a**
6*b**4*x**8 + 1280*a**5*b**5*x**10)
```

3.532 $\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=133

$$-\frac{693\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{13/2}}-\frac{693}{256a^6x}+\frac{231}{256a^5x(a+bx^2)}+\frac{231}{640a^4x(a+bx^2)^2}+\frac{33}{160a^3x(a+bx^2)^3}+\frac{11}{80a^2x(a+bx^2)^4}+\frac{1}{10ax(a+bx^2)^5}$$

[Out] -693/256/a^6/x+1/10/a/x/(b*x^2+a)^5+11/80/a^2/x/(b*x^2+a)^4+33/160/a^3/x/(b*x^2+a)^3+231/640/a^4/x/(b*x^2+a)^2+231/256/a^5/x/(b*x^2+a)-693/256*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(13/2)

Rubi [A] time = 0.09, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$\frac{231}{256a^5x(a+bx^2)}+\frac{231}{640a^4x(a+bx^2)^2}+\frac{33}{160a^3x(a+bx^2)^3}+\frac{11}{80a^2x(a+bx^2)^4}-\frac{693\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{13/2}}-\frac{693}{256a^6x}+\frac{1}{10ax(a+bx^2)^5}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]

[Out] -693/(256*a^6*x) + 1/(10*a*x*(a + b*x^2)^5) + 11/(80*a^2*x*(a + b*x^2)^4) + 33/(160*a^3*x*(a + b*x^2)^3) + 231/(640*a^4*x*(a + b*x^2)^2) + 231/(256*a^5*x*(a + b*x^2)) - (693*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(13/2))

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
```

```
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{x^2\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{\left(11b^5\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^5}\,dx}{10a}$$

$$= \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{\left(99b^4\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^4}\,dx}{80a^2}$$

$$= \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{33}{160a^3x\left(a+bx^2\right)^3} + \frac{\left(231b^3\right)\int \frac{1}{x^2\left(ab+b^2x\right.}}{160a^3}$$

$$= \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{33}{160a^3x\left(a+bx^2\right)^3} + \frac{231}{640a^4x\left(a+bx^2\right)^2}$$

$$= \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{33}{160a^3x\left(a+bx^2\right)^3} + \frac{231}{640a^4x\left(a+bx^2\right)^2}$$

$$= -\frac{693}{256a^6x} + \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{33}{160a^3x\left(a+bx^2\right)^3} + \frac{}{640a^4}$$

$$= -\frac{693}{256a^6x} + \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{33}{160a^3x\left(a+bx^2\right)^3} + \frac{}{640a^4}$$

Mathematica [A] time = 0.06, size = 101, normalized size = 0.76

$$-\frac{693\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{13/2}}-\frac{1280a^5+10615a^4bx^2+26070a^3b^2x^4+29568a^2b^3x^6+16170ab^4x^8+3465b^5x^{10}}{1280a^6x\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -1/1280*(1280*a^5 + 10615*a^4*b*x^2 + 26070*a^3*b^2*x^4 + 29568*a^2*b^3*x^6
 + 16170*a*b^4*x^8 + 3465*b^5*x^10)/(a^6*x*(a + b*x^2)^5) - (693*Sqrt[b]*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(13/2))
```

fricas [A] time = 1.10, size = 400, normalized size = 3.01

$$\left[-\frac{6930\,b^5x^{10}+32340\,ab^4x^8+59136\,a^2b^3x^6+52140\,a^3b^2x^4+21230\,a^4bx^2+2560\,a^5-3465\left(b^5x^{11}+5\,ab^4x^9+\right.}{2560\left(a^6b^5x^{11}+5\,a^7b^4x^9+10\,a^8b^3x^7+10\,a^9b^2x^5+5\,a^{10}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/2560*(6930*b^5*x^10 + 32340*a*b^4*x^8 + 59136*a^2*b^3*x^6 + 52140*a^3*b
^2*x^4 + 21230*a^4*b*x^2 + 2560*a^5 - 3465*(b^5*x^11 + 5*a*b^4*x^9 + 10*a^2
*b^3*x^7 + 10*a^3*b^2*x^5 + 5*a^4*b*x^3 + a^5*x)*sqrt(-b/a)*log((b*x^2 - 2*
a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^6*b^5*x^11 + 5*a^7*b^4*x^9 + 10*a^8*b^
3*x^7 + 10*a^9*b^2*x^5 + 5*a^10*b*x^3 + a^11*x), -1/1280*(3465*b^5*x^10 + 1
6170*a*b^4*x^8 + 29568*a^2*b^3*x^6 + 26070*a^3*b^2*x^4 + 10615*a^4*b*x^2 +
1280*a^5 + 3465*(b^5*x^11 + 5*a*b^4*x^9 + 10*a^2*b^3*x^7 + 10*a^3*b^2*x^5 +
 5*a^4*b*x^3 + a^5*x)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^6*b^5*x^11 + 5*a^7*
b^4*x^9 + 10*a^8*b^3*x^7 + 10*a^9*b^2*x^5 + 5*a^10*b*x^3 + a^11*x)]
```

giac [A] time = 0.16, size = 90, normalized size = 0.68

$$-\frac{693\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^6}-\frac{1}{a^6x}-\frac{2185\,b^5x^9+9770\,ab^4x^7+16768\,a^2b^3x^5+13270\,a^3b^2x^3+4215\,a^4bx}{1280\left(bx^2+a\right)^5a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -693/256*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6) - 1/(a^6*x) - 1/1280*(2185
*b^5*x^9 + 9770*a*b^4*x^7 + 16768*a^2*b^3*x^5 + 13270*a^3*b^2*x^3 + 4215*a^
4*b*x)/((b*x^2 + a)^5*a^6)
```

maple [A] time = 0.02, size = 126, normalized size = 0.95

$$-\frac{437b^5x^9}{256\left(b\,x^2+a\right)^5a^6}-\frac{977b^4x^7}{128\left(b\,x^2+a\right)^5a^5}-\frac{131b^3x^5}{10\left(b\,x^2+a\right)^5a^4}-\frac{1327b^2x^3}{128\left(b\,x^2+a\right)^5a^3}-\frac{843bx}{256\left(b\,x^2+a\right)^5a^2}-\frac{693b\arctan(}{256\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/a^6/x-437/256*b^5/a^6/(b*x^2+a)^5*x^9-977/128*b^4/a^5/(b*x^2+a)^5*x^7-13
1/10*b^3/a^4/(b*x^2+a)^5*x^5-1327/128*b^2/a^3/(b*x^2+a)^5*x^3-843/256*b/a^2
/(b*x^2+a)^5*x-693/256*b/a^6/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.12, size = 137, normalized size = 1.03

$$-\frac{3465\,b^5x^{10}+16170\,ab^4x^8+29568\,a^2b^3x^6+26070\,a^3b^2x^4+10615\,a^4bx^2+1280\,a^5}{1280\left(a^6b^5x^{11}+5\,a^7b^4x^9+10\,a^8b^3x^7+10\,a^9b^2x^5+5\,a^{10}bx^3+a^{11}x\right)}-\frac{693\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/1280*(3465*b^5*x^10 + 16170*a*b^4*x^8 + 29568*a^2*b^3*x^6 + 26070*a^3*b^
2*x^4 + 10615*a^4*b*x^2 + 1280*a^5)/(a^6*b^5*x^11 + 5*a^7*b^4*x^9 + 10*a^8*
b^3*x^7 + 10*a^9*b^2*x^5 + 5*a^10*b*x^3 + a^11*x) - 693/256*b*arctan(b*x/sq
rt(a*b))/(sqrt(a*b)*a^6)
```

mupad [B] time = 4.58, size = 132, normalized size = 0.99

$$-\frac{\frac{1}{a}+\frac{2123\,b\,x^2}{256\,a^2}+\frac{2607\,b^2\,x^4}{128\,a^3}+\frac{231\,b^3\,x^6}{10\,a^4}+\frac{1617\,b^4\,x^8}{128\,a^5}+\frac{693\,b^5\,x^{10}}{256\,a^6}}{a^5\,x+5\,a^4\,b\,x^3+10\,a^3\,b^2\,x^5+10\,a^2\,b^3\,x^7+5\,a\,b^4\,x^9+b^5\,x^{11}}-\frac{693\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] - (1/a + (2123*b*x^2)/(256*a^2) + (2607*b^2*x^4)/(128*a^3) + (231*b^3*x^6)/
(10*a^4) + (1617*b^4*x^8)/(128*a^5) + (693*b^5*x^10)/(256*a^6))/(a^5*x + b^
5*x^11 + 5*a^4*b*x^3 + 5*a*b^4*x^9 + 10*a^3*b^2*x^5 + 10*a^2*b^3*x^7) - (69
3*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(256*a^(13/2))
```

sympy [A] time = 0.83, size = 187, normalized size = 1.41

$$\frac{693\sqrt{-\frac{b}{a^{13}}}\log\left(-\frac{a^7\sqrt{-\frac{b}{a^{13}}}}{b}+x\right)}{512}-\frac{693\sqrt{-\frac{b}{a^{13}}}\log\left(\frac{a^7\sqrt{-\frac{b}{a^{13}}}}{b}+x\right)}{512}+\frac{-1280a^5-10615a^4bx^2-26070a^3b^2x^4-29568}{1280a^{11}x+6400a^{10}bx^3+12800a^9b^2x^5+12800}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 693*sqrt(-b/a**13)*log(-a**7*sqrt(-b/a**13)/b + x)/512 - 693*sqrt(-b/a**13)
*log(a**7*sqrt(-b/a**13)/b + x)/512 + (-1280*a**5 - 10615*a**4*b*x**2 - 260
70*a**3*b**2*x**4 - 29568*a**2*b**3*x**6 - 16170*a*b**4*x**8 - 3465*b**5*x*
*10)/(1280*a**11*x + 6400*a**10*b*x**3 + 12800*a**9*b**2*x**5 + 12800*a**8*
b**3*x**7 + 6400*a**7*b**4*x**9 + 1280*a**6*b**5*x**11)
```

3.533 $\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=144

$$\frac{3003b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{15/2}}+\frac{3003b}{256a^7x}-\frac{1001}{256a^6x^3}+\frac{3003}{1280a^5x^3\left(a+bx^2\right)}+\frac{429}{640a^4x^3\left(a+bx^2\right)^2}+\frac{143}{480a^3x^3\left(a+bx^2\right)^3}+\frac{}{80a^2}$$

```
[Out] -1001/256/a^6/x^3+3003/256*b/a^7/x+1/10/a/x^3/(b*x^2+a)^5+13/80/a^2/x^3/(b*
x^2+a)^4+143/480/a^3/x^3/(b*x^2+a)^3+429/640/a^4/x^3/(b*x^2+a)^2+3003/1280/
a^5/x^3/(b*x^2+a)+3003/256*b^(3/2)*arctan(x*b^(1/2)/a^(1/2))/a^(15/2)
```

Rubi [A] time = 0.10, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$\frac{3003b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{15/2}}+\frac{3003}{1280a^5x^3\left(a+bx^2\right)}+\frac{429}{640a^4x^3\left(a+bx^2\right)^2}+\frac{143}{480a^3x^3\left(a+bx^2\right)^3}+\frac{13}{80a^2x^3\left(a+bx^2\right)^4}+\frac{300}{256a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -1001/(256*a^6*x^3) + (3003*b)/(256*a^7*x) + 1/(10*a*x^3*(a + b*x^2)^5) + 1
3/(80*a^2*x^3*(a + b*x^2)^4) + 143/(480*a^3*x^3*(a + b*x^2)^3) + 429/(640*a
^4*x^3*(a + b*x^2)^2) + 3003/(1280*a^5*x^3*(a + b*x^2)) + (3003*b^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(15/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{x^4\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{\left(13b^5\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^5}\,dx}{10a}$$

$$= \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{13}{80a^2x^3\left(a+bx^2\right)^4} + \frac{\left(143b^4\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^4}\,dx}{80a^2}$$

$$= \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{13}{80a^2x^3\left(a+bx^2\right)^4} + \frac{143}{480a^3x^3\left(a+bx^2\right)^3} + \frac{\left(429b^3\right)\int \frac{}{x^4\left(ab\right.}}{160a^3}$$

$$= \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{13}{80a^2x^3\left(a+bx^2\right)^4} + \frac{143}{480a^3x^3\left(a+bx^2\right)^3} + \frac{429}{640a^4x^3\left(a+b\right.}$$

$$= \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{13}{80a^2x^3\left(a+bx^2\right)^4} + \frac{143}{480a^3x^3\left(a+bx^2\right)^3} + \frac{429}{640a^4x^3\left(a+b\right.}$$

$$= -\frac{1001}{256a^6x^3} + \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{13}{80a^2x^3\left(a+bx^2\right)^4} + \frac{143}{480a^3x^3\left(a+bx^2\right)^3} + \frac{}{64}$$

$$= -\frac{1001}{256a^6x^3} + \frac{3003b}{256a^7x} + \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{13}{80a^2x^3\left(a+bx^2\right)^4} + \frac{143}{480a^3x^3\left(a+\right.}$$

$$= -\frac{1001}{256a^6x^3} + \frac{3003b}{256a^7x} + \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{13}{80a^2x^3\left(a+bx^2\right)^4} + \frac{143}{480a^3x^3\left(a+\right.}$$

Mathematica [A] time = 0.06, size = 113, normalized size = 0.78

$$\frac{\frac{\sqrt{a}\left(-1280a^6+16640a^5bx^2+137995a^4b^2x^4+338910a^3b^3x^6+384384a^2b^4x^8+210210ab^5x^{10}+45045b^6x^{12}\right)}{x^3\left(a+bx^2\right)^5}+45045b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3840a^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

[Out] ((Sqrt[a]*(-1280*a^6 + 16640*a^5*b*x^2 + 137995*a^4*b^2*x^4 + 338910*a^3*b^3*x^6 + 384384*a^2*b^4*x^8 + 210210*a*b^5*x^10 + 45045*b^6*x^12))/(x^3*(a + b*x^2)^5) + 45045*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3840*a^(15/2))

fricas [A] time = 1.02, size = 436, normalized size = 3.03

$$\left[\frac{90090\, b^6x^{12} + 420420\, ab^5x^{10} + 768768\, a^2b^4x^8 + 677820\, a^3b^3x^6 + 275990\, a^4b^2x^4 + 33280\, a^5bx^2 - 2560\, a^6 + 45}{7680\left(a^7b^5x^{13} + 5\, a^8b^4x^{11} + 10\, a^9b^3x^9 + 10\, a^1\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")

[Out] [1/7680*(90090*b^6*x^12 + 420420*a*b^5*x^10 + 768768*a^2*b^4*x^8 + 677820*a^3*b^3*x^6 + 275990*a^4*b^2*x^4 + 33280*a^5*b*x^2 - 2560*a^6 + 45045*(b^6*x^13 + 5*a*b^5*x^11 + 10*a^2*b^4*x^9 + 10*a^3*b^3*x^7 + 5*a^4*b^2*x^5 + a^5*b*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^7*b^5*x^13 + 5*a^8*b^4*x^11 + 10*a^9*b^3*x^9 + 10*a^10*b^2*x^7 + 5*a^11*b*x^5 + a^12*x^3), 1/3840*(45045*b^6*x^12 + 210210*a*b^5*x^10 + 384384*a^2*b^4*x^8 + 338910*a^3*b^3*x^6 + 137995*a^4*b^2*x^4 + 16640*a^5*b*x^2 - 1280*a^6 + 45045*(b^6*x^13 + 5*a*b^5*x^11 + 10*a^2*b^4*x^9 + 10*a^3*b^3*x^7 + 5*a^4*b^2*x^5 + a^5*b*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^7*b^5*x^13 + 5*a^8*b^4*x^11 + 10*a^9*b^3*x^9 + 10*a^10*b^2*x^7 + 5*a^11*b*x^5 + a^12*x^3)]

giac [A] time = 0.16, size = 104, normalized size = 0.72

$$\frac{3003\, b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\, \sqrt{ab}\, a^7} + \frac{18\, bx^2 - a}{3\, a^7x^3} + \frac{22005\, b^6x^9 + 96290\, ab^5x^7 + 160384\, a^2b^4x^5 + 121310\, a^3b^3x^3 + 35595\, a^4b^2x}{3840\left(bx^2 + a\right)^5 a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")

[Out] 3003/256*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7) + 1/3*(18*b*x^2 - a)/(a^7*x^3) + 1/3840*(22005*b^6*x^9 + 96290*a*b^5*x^7 + 160384*a^2*b^4*x^5 + 121310*a^3*b^3*x^3 + 35595*a^4*b^2*x)/((b*x^2 + a)^5*a^7)

maple [A] time = 0.02, size = 139, normalized size = 0.97

$$\frac{1467b^6x^9}{256\left(b\,x^2+a\right)^5 a^7} + \frac{9629b^5x^7}{384\left(b\,x^2+a\right)^5 a^6} + \frac{1253b^4x^5}{30\left(b\,x^2+a\right)^5 a^5} + \frac{12131b^3x^3}{384\left(b\,x^2+a\right)^5 a^4} + \frac{2373b^2x}{256\left(b\,x^2+a\right)^5 a^3} + \frac{3003b^2 \arctan(}{256\sqrt{ab}\ a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/3/a^6/x^3+6*b/a^7/x+1467/256/a^7*b^6/(b*x^2+a)^5*x^9+9629/384/a^6*b^5/(b
*x^2+a)^5*x^7+1253/30/a^5*b^4/(b*x^2+a)^5*x^5+12131/384/a^4*b^3/(b*x^2+a)^5
*x^3+2373/256/a^3*b^2/(b*x^2+a)^5*x+3003/256/a^7*b^2/(a*b)^(1/2)*arctan(1/(
a*b)^(1/2)*b*x)
```

maxima [A] time = 3.10, size = 152, normalized size = 1.06

$$\frac{45045\,b^6x^{12}+210210\,ab^5x^{10}+384384\,a^2b^4x^8+338910\,a^3b^3x^6+137995\,a^4b^2x^4+16640\,a^5bx^2-1280\,a^6}{3840\left(a^7b^5x^{13}+5\,a^8b^4x^{11}+10\,a^9b^3x^9+10\,a^{10}b^2x^7+5\,a^{11}bx^5+a^{12}x^3\right)}+\frac{300}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3840*(45045*b^6*x^12 + 210210*a*b^5*x^10 + 384384*a^2*b^4*x^8 + 338910*a^
3*b^3*x^6 + 137995*a^4*b^2*x^4 + 16640*a^5*b*x^2 - 1280*a^6)/(a^7*b^5*x^13
+ 5*a^8*b^4*x^11 + 10*a^9*b^3*x^9 + 10*a^10*b^2*x^7 + 5*a^11*b*x^5 + a^12*x
^3) + 3003/256*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7)
```

mupad [B] time = 4.62, size = 146, normalized size = 1.01

$$\frac{\frac{13\,b\,x^2}{3\,a^2}-\frac{1}{3\,a}+\frac{27599\,b^2\,x^4}{768\,a^3}+\frac{11297\,b^3\,x^6}{128\,a^4}+\frac{1001\,b^4\,x^8}{10\,a^5}+\frac{7007\,b^5\,x^{10}}{128\,a^6}+\frac{3003\,b^6\,x^{12}}{256\,a^7}}{a^5\,x^3+5\,a^4\,b\,x^5+10\,a^3\,b^2\,x^7+10\,a^2\,b^3\,x^9+5\,a\,b^4\,x^{11}+b^5\,x^{13}}+\frac{3003\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{15/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] ((13*b*x^2)/(3*a^2) - 1/(3*a) + (27599*b^2*x^4)/(768*a^3) + (11297*b^3*x^6)
/(128*a^4) + (1001*b^4*x^8)/(10*a^5) + (7007*b^5*x^10)/(128*a^6) + (3003*b^
6*x^12)/(256*a^7))/(a^5*x^3 + b^5*x^13 + 5*a^4*b*x^5 + 5*a*b^4*x^11 + 10*a^
3*b^2*x^7 + 10*a^2*b^3*x^9) + (3003*b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(256
*a^(15/2))
```

sympy [A] time = 0.89, size = 209, normalized size = 1.45

$$-\frac{3003\sqrt{-\frac{b^3}{a^{15}}}\log\left(-\frac{a^8\sqrt{-\frac{b^3}{a^{15}}}}{b^2}+x\right)}{512}+\frac{3003\sqrt{-\frac{b^3}{a^{15}}}\log\left(\frac{a^8\sqrt{-\frac{b^3}{a^{15}}}}{b^2}+x\right)}{512}+\frac{-1280a^6+16640a^5bx^2+137995a^4b^2x^4+}{3840a^{12}x^3+19200a^{11}bx^5+38400}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -3003*sqrt(-b**3/a**15)*log(-a**8*sqrt(-b**3/a**15)/b**2 + x)/512 + 3003*sq
rt(-b**3/a**15)*log(a**8*sqrt(-b**3/a**15)/b**2 + x)/512 + (-1280*a**6 + 16
640*a**5*b*x**2 + 137995*a**4*b**2*x**4 + 338910*a**3*b**3*x**6 + 384384*a*
*2*b**4*x**8 + 210210*a*b**5*x**10 + 45045*b**6*x**12)/(3840*a**12*x**3 + 1
9200*a**11*b*x**5 + 38400*a**10*b**2*x**7 + 38400*a**9*b**3*x**9 + 19200*a*
*8*b**4*x**11 + 3840*a**7*b**5*x**13)
```

3.534 $\int \frac{1}{x^6\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=157

$$-\frac{9009b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{17/2}}-\frac{9009b^2}{256a^8x}+\frac{3003b}{256a^7x^3}-\frac{9009}{1280a^6x^5}+\frac{1287}{256a^5x^5\left(a+bx^2\right)}+\frac{143}{128a^4x^5\left(a+bx^2\right)^2}+\frac{13}{32a^3x^5\left(a+bx^2\right)^3}$$

```
[Out] -9009/1280/a^6/x^5+3003/256*b/a^7/x^3-9009/256*b^2/a^8/x+1/10/a/x^5/(b*x^2+
a)^5+3/16/a^2/x^5/(b*x^2+a)^4+13/32/a^3/x^5/(b*x^2+a)^3+143/128/a^4/x^5/(b*
x^2+a)^2+1287/256/a^5/x^5/(b*x^2+a)-9009/256*b^(5/2)*arctan(x*b^(1/2)/a^(1/
2))/a^(17/2)
```

Rubi [A] time = 0.12, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$-\frac{9009b^2}{256a^8x}-\frac{9009b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{17/2}}+\frac{3003b}{256a^7x^3}+\frac{1287}{256a^5x^5\left(a+bx^2\right)}+\frac{143}{128a^4x^5\left(a+bx^2\right)^2}+\frac{13}{32a^3x^5\left(a+bx^2\right)^3}+\frac{}{16a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -9009/(1280*a^6*x^5) + (3003*b)/(256*a^7*x^3) - (9009*b^2)/(256*a^8*x) + 1/
(10*a*x^5*(a + b*x^2)^5) + 3/(16*a^2*x^5*(a + b*x^2)^4) + 13/(32*a^3*x^5*(a
 + b*x^2)^3) + 143/(128*a^4*x^5*(a + b*x^2)^2) + 1287/(256*a^5*x^5*(a + b*x
^2)) - (9009*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(17/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{x^6\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{\left(3b^5\right)\int \frac{1}{x^6\left(ab+b^2x^2\right)^5}\,dx}{2a}$$

$$= \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{\left(39b^4\right)\int \frac{1}{x^6\left(ab+b^2x^2\right)^4}\,dx}{16a^2}$$

$$= \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{13}{32a^3x^5\left(a+bx^2\right)^3} + \frac{\left(143b^3\right)\int \frac{1}{x^6(ab+}}{32a^3}$$

$$= \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{13}{32a^3x^5\left(a+bx^2\right)^3} + \frac{143}{128a^4x^5\left(a+bx^2\right.}$$

$$= \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{13}{32a^3x^5\left(a+bx^2\right)^3} + \frac{143}{128a^4x^5\left(a+bx^2\right.}$$

$$= -\frac{9009}{1280a^6x^5} + \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{13}{32a^3x^5\left(a+bx^2\right)^3} + \frac{}{12}$$

$$= -\frac{9009}{1280a^6x^5} + \frac{3003b}{256a^7x^3} + \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{13}{32a^3x^5\left(a+\right.}$$

$$= -\frac{9009}{1280a^6x^5} + \frac{3003b}{256a^7x^3} - \frac{9009b^2}{256a^8x} + \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{}{3}$$

$$= -\frac{9009}{1280a^6x^5} + \frac{3003b}{256a^7x^3} - \frac{9009b^2}{256a^8x} + \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{}{3}$$

Mathematica [A] time = 0.06, size = 123, normalized size = 0.78

$$-\frac{9009b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{17/2}} - \frac{256a^7-1280a^6bx^2+16640a^5b^2x^4+137995a^4b^3x^6+338910a^3b^4x^8+384384a^2b^5x^{10}}{1280a^8x^5\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -1/1280*(256*a^7 - 1280*a^6*b*x^2 + 16640*a^5*b^2*x^4 + 137995*a^4*b^3*x^6
+ 338910*a^3*b^4*x^8 + 384384*a^2*b^5*x^10 + 210210*a*b^6*x^12 + 45045*b^7*
x^14)/(a^8*x^5*(a + b*x^2)^5) - (9009*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(256*a^(17/2))
```

fricas [A] time = 0.94, size = 462, normalized size = 2.94

$$\left[-\frac{90090\,b^{7}x^{14}+420420\,ab^{6}x^{12}+768768\,a^{2}b^{5}x^{10}+677820\,a^{3}b^{4}x^{8}+275990\,a^{4}b^{3}x^{6}+33280\,a^{5}b^{2}x^{4}-2560\,a^{6}bx}{2560\left(a^{8}b^{5}x^{15}+5\,a^{9}b^{4}x^{13}+10\,a^{10}b^{3}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/2560*(90090*b^7*x^14 + 420420*a*b^6*x^12 + 768768*a^2*b^5*x^10 + 677820
*a^3*b^4*x^8 + 275990*a^4*b^3*x^6 + 33280*a^5*b^2*x^4 - 2560*a^6*b*x^2 + 51
2*a^7 - 45045*(b^7*x^15 + 5*a*b^6*x^13 + 10*a^2*b^5*x^11 + 10*a^3*b^4*x^9 +
 5*a^4*b^3*x^7 + a^5*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)
/(b*x^2 + a)))/(a^8*b^5*x^15 + 5*a^9*b^4*x^13 + 10*a^10*b^3*x^11 + 10*a^11*
b^2*x^9 + 5*a^12*b*x^7 + a^13*x^5), -1/1280*(45045*b^7*x^14 + 210210*a*b^6*
x^12 + 384384*a^2*b^5*x^10 + 338910*a^3*b^4*x^8 + 137995*a^4*b^3*x^6 + 1664
0*a^5*b^2*x^4 - 1280*a^6*b*x^2 + 256*a^7 + 45045*(b^7*x^15 + 5*a*b^6*x^13 +
 10*a^2*b^5*x^11 + 10*a^3*b^4*x^9 + 5*a^4*b^3*x^7 + a^5*b^2*x^5)*sqrt(b/a)*
arctan(x*sqrt(b/a)))/(a^8*b^5*x^15 + 5*a^9*b^4*x^13 + 10*a^10*b^3*x^11 + 10
*a^11*b^2*x^9 + 5*a^12*b*x^7 + a^13*x^5)]
```

giac [A] time = 0.16, size = 115, normalized size = 0.73

$$-\frac{9009\,b^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^{8}}-\frac{45045\,b^{7}x^{14}+210210\,ab^{6}x^{12}+384384\,a^{2}b^{5}x^{10}+338910\,a^{3}b^{4}x^{8}+137995\,a^{4}b^{3}x^{6}+166}{1280\left(bx^{3}+ax\right)^{5}a^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -9009/256*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^8) - 1/1280*(45045*b^7*x^1
4 + 210210*a*b^6*x^12 + 384384*a^2*b^5*x^10 + 338910*a^3*b^4*x^8 + 137995*a
^4*b^3*x^6 + 16640*a^5*b^2*x^4 - 1280*a^6*b*x^2 + 256*a^7)/((b*x^3 + a*x)^5
*a^8)
```

maple [A] time = 0.02, size = 150, normalized size = 0.96

$$-\frac{3633b^7x^9}{256\left(b\,x^2+a\right)^5a^8}-\frac{7837b^6x^7}{128\left(b\,x^2+a\right)^5a^7}-\frac{1001b^5x^5}{10\left(b\,x^2+a\right)^5a^6}-\frac{9443b^4x^3}{128\left(b\,x^2+a\right)^5a^5}-\frac{5327b^3x}{256\left(b\,x^2+a\right)^5a^4}-\frac{9009b^3\,\mathrm{arcta}}{256\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/5/a^6/x^5-21*b^2/a^8/x+2*b/a^7/x^3-3633/256*b^7/a^8/(b*x^2+a)^5*x^9-7837
/128*b^6/a^7/(b*x^2+a)^5*x^7-1001/10*b^5/a^6/(b*x^2+a)^5*x^5-9443/128*b^4/a
^5/(b*x^2+a)^5*x^3-5327/256*b^3/a^4/(b*x^2+a)^5*x-9009/256*b^3/a^8/(a*b)^(1
/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.11, size = 163, normalized size = 1.04

$$-\frac{45045\,b^7x^{14}+210210\,ab^6x^{12}+384384\,a^2b^5x^{10}+338910\,a^3b^4x^8+137995\,a^4b^3x^6+16640\,a^5b^2x^4-1280\,a^6bx}{1280\left(a^8b^5x^{15}+5\,a^9b^4x^{13}+10\,a^{10}b^3x^{11}+10\,a^{11}b^2x^9+5\,a^{12}bx^7+a^{13}x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/1280*(45045*b^7*x^14 + 210210*a*b^6*x^12 + 384384*a^2*b^5*x^10 + 338910*
a^3*b^4*x^8 + 137995*a^4*b^3*x^6 + 16640*a^5*b^2*x^4 - 1280*a^6*b*x^2 + 256
*a^7)/(a^8*b^5*x^15 + 5*a^9*b^4*x^13 + 10*a^10*b^3*x^11 + 10*a^11*b^2*x^9 +
 5*a^12*b*x^7 + a^13*x^5) - 9009/256*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a
^8)
```

mupad [B] time = 4.65, size = 158, normalized size = 1.01

$$-\frac{\frac{1}{5\,a}-\frac{b\,x^2}{a^2}+\frac{13\,b^2\,x^4}{a^3}+\frac{27599\,b^3\,x^6}{256\,a^4}+\frac{33891\,b^4\,x^8}{128\,a^5}+\frac{3003\,b^5\,x^{10}}{10\,a^6}+\frac{21021\,b^6\,x^{12}}{128\,a^7}+\frac{9009\,b^7\,x^{14}}{256\,a^8}}{a^5\,x^5+5\,a^4\,b\,x^7+10\,a^3\,b^2\,x^9+10\,a^2\,b^3\,x^{11}+5\,a\,b^4\,x^{13}+b^5\,x^{15}}-\frac{9009\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{17/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] - (1/(5*a) - (b*x^2)/a^2 + (13*b^2*x^4)/a^3 + (27599*b^3*x^6)/(256*a^4) + (
33891*b^4*x^8)/(128*a^5) + (3003*b^5*x^10)/(10*a^6) + (21021*b^6*x^12)/(128
*a^7) + (9009*b^7*x^14)/(256*a^8))/(a^5*x^5 + b^5*x^15 + 5*a^4*b*x^7 + 5*a*
b^4*x^13 + 10*a^3*b^2*x^9 + 10*a^2*b^3*x^11) - (9009*b^(5/2)*atan((b^(1/2)*
x)/a^(1/2)))/(256*a^(17/2))
```

sympy [A] time = 0.95, size = 221, normalized size = 1.41

$$\frac{9009\sqrt{-\frac{b^5}{a^{17}}}\log\left(-\frac{a^9\sqrt{-\frac{b^5}{a^{17}}}}{b^3}+x\right)}{512}-\frac{9009\sqrt{-\frac{b^5}{a^{17}}}\log\left(\frac{a^9\sqrt{-\frac{b^5}{a^{17}}}}{b^3}+x\right)}{512}+\frac{-256a^7+1280a^6bx^2-16640a^5b^2x^4-137995}{1280a^{13}x^5+6400a^{12}bx^7+128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 9009*sqrt(-b**5/a**17)*log(-a**9*sqrt(-b**5/a**17)/b**3 + x)/512 - 9009*sqr
t(-b**5/a**17)*log(a**9*sqrt(-b**5/a**17)/b**3 + x)/512 + (-256*a**7 + 1280
*a**6*b*x**2 - 16640*a**5*b**2*x**4 - 137995*a**4*b**3*x**6 - 338910*a**3*b
**4*x**8 - 384384*a**2*b**5*x**10 - 210210*a*b**6*x**12 - 45045*b**7*x**14)
/(1280*a**13*x**5 + 6400*a**12*b*x**7 + 12800*a**11*b**2*x**9 + 12800*a**10
*b**3*x**11 + 6400*a**9*b**4*x**13 + 1280*a**8*b**5*x**15)
```

3.535 $\int \frac{1}{1+2x^2+x^4} \, dx$

Optimal. Leaf size=19

$$\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\tan^{-1}(x)$$

[Out] 1/2*x/(x^2+1)+1/2*arctan(x)

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {28, 199, 203}

$$\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^2 + x^4)^(-1),x]

[Out] x/(2*(1 + x^2)) + ArcTan[x]/2

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{1+2x^2+x^4}\,dx = \int \frac{1}{\left(1+x^2\right)^2}\,dx$$
$$= \frac{x}{2\left(1+x^2\right)} + \frac{1}{2}\int \frac{1}{1+x^2}\,dx$$
$$= \frac{x}{2\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}(x)$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.84

$$\frac{1}{2}\left(\frac{x}{x^2+1}+\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2 + x^4)^(-1),x]
[Out] (x/(1 + x^2) + ArcTan[x])/2
```

fricas [A] time = 0.99, size = 19, normalized size = 1.00

$$\frac{\left(x^2+1\right)\arctan(x)+x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+1),x, algorithm="fricas")
[Out] 1/2*((x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

giac [A] time = 0.15, size = 15, normalized size = 0.79

$$\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+1),x, algorithm="giac")
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2*x^2+1),x)
```

```
[Out] 1/2/(x^2+1)*x+1/2*arctan(x)
```

maxima [A] time = 3.06, size = 15, normalized size = 0.79

$$\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

mupad [B] time = 0.03, size = 16, normalized size = 0.84

$$\frac{\operatorname{atan}(x)}{2}+\frac{x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 + x^4 + 1),x)
```

```
[Out] atan(x)/2 + x/(2*(x^2 + 1))
```

sympy [A] time = 0.11, size = 12, normalized size = 0.63

$$\frac{x}{2x^2+2}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+2*x**2+1),x)
```

```
[Out] x/(2*x**2 + 2) + atan(x)/2
```

3.536 $\int \frac{x}{1+2x^2+x^4} dx$

Optimal. Leaf size=11

$$-\frac{1}{2\left(x^2+1\right)}$$

```
[Out] -1/2/(x^2+1)
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 261}

$$-\frac{1}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + 2*x^2 + x^4),x]
```

```
[Out] -1/(2*(1 + x^2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1+2x^2+x^4}\,dx &= \int \frac{x}{\left(1+x^2\right)^2}\,dx\\ &= -\frac{1}{2\left(1+x^2\right)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$-\frac{1}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + 2*x^2 + x^4),x]
[Out] -1/2*1/(1 + x^2)
```

fricas [A] time = 0.93, size = 9, normalized size = 0.82

$$-\frac{1}{2\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+1),x, algorithm="fricas")
[Out] -1/2/(x^2 + 1)
```

giac [A] time = 0.15, size = 9, normalized size = 0.82

$$-\frac{1}{2\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+1),x, algorithm="giac")
[Out] -1/2/(x^2 + 1)
```

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$-\frac{1}{2\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+2*x^2+1),x)
[Out] -1/2/(x^2+1)
```

maxima [A] time = 1.35, size = 9, normalized size = 0.82

$$-\frac{1}{2\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2/(x^2 + 1)
```

mupad [B] time = 0.02, size = 11, normalized size = 1.00

$$-\frac{1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*x^2 + x^4 + 1),x)
[Out] -1/(2*(x^2 + 1))
```

sympy [A] time = 0.09, size = 8, normalized size = 0.73

$$-\frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+2*x**2+1),x)
[Out] -1/(2*x**2 + 2)
```

3.537 $\int \frac{x^2}{1+2x^2+x^4} \, dx$

Optimal. Leaf size=19

$$\frac{1}{2}\tan^{-1}(x)-\frac{x}{2\left(x^2+1\right)}$$

```
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 288, 203}

$$\frac{1}{2}\tan^{-1}(x)-\frac{x}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + 2*x^2 + x^4),x]
```

```
[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{1+2x^2+x^4}\,dx = \int \frac{x^2}{\left(1+x^2\right)^2}\,dx$$
$$= -\frac{x}{2\left(1+x^2\right)} + \frac{1}{2}\int \frac{1}{1+x^2}\,dx$$
$$= -\frac{x}{2\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}(x)$$

Mathematica [**A**] time = 0.01, size = 19, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + 2*x^2 + x^4),x]
[Out] -1/2*x/(1 + x^2) + ArcTan[x]/2
```

fricas [**A**] time = 0.81, size = 21, normalized size = 1.11

$$\frac{\left(x^2+1\right)\arctan(x) - x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+2*x^2+1),x, algorithm="fricas")
[Out] 1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)
```

giac [**A**] time = 0.16, size = 15, normalized size = 0.79

$$-\frac{x}{2\left(x^2+1\right)} + \frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+2*x^2+1),x, algorithm="giac")
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

maple [A] time = 0.01, size = 16, normalized size = 0.84

$$-\frac{x}{2\left(x^{2}+1\right)}+\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4+2*x^2+1),x)
[Out] -1/2/(x^2+1)*x+1/2*arctan(x)
```

maxima [A] time = 2.97, size = 15, normalized size = 0.79

$$-\frac{x}{2\left(x^{2}+1\right)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+2*x^2+1),x, algorithm="maxima")
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

mupad [B] time = 0.03, size = 17, normalized size = 0.89

$$\frac{\operatorname{atan}(x)}{2}-\frac{x}{2\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2*x^2 + x^4 + 1),x)
[Out] atan(x)/2 - x/(2*(x^2 + 1))
```

sympy [A] time = 0.11, size = 12, normalized size = 0.63

$$-\frac{x}{2x^{2}+2}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4+2*x**2+1),x)
[Out] -x/(2*x**2 + 2) + atan(x)/2
```

3.538 $\int \frac{x^3}{1+2x^2+x^4} \, dx$

Optimal. Leaf size=22

$$\frac{1}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)$$

```
[Out] 1/2/(x^2+1)+1/2*ln(x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 43}

$$\frac{1}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + 2*x^2 + x^4),x]
```

```
[Out] 1/(2*(1 + x^2)) + Log[1 + x^2]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{1+2x^2+x^4}\,dx = \int \frac{x^3}{\left(1+x^2\right)^2}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(1+x)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{1}{(1+x)^2}+\frac{1}{1+x}\right)dx, x, x^2\right)$$
$$= \frac{1}{2\left(1+x^2\right)}+\frac{1}{2}\log\left(1+x^2\right)$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.82

$$\frac{1}{2}\left(\frac{1}{x^2+1}+\log\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + 2*x^2 + x^4),x]
[Out] ((1 + x^2)^(-1) + Log[1 + x^2])/2
```

fricas [A] time = 0.91, size = 23, normalized size = 1.05

$$\frac{\left(x^2+1\right)\log\left(x^2+1\right)+1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+2*x^2+1),x, algorithm="fricas")
[Out] 1/2*((x^2 + 1)*log(x^2 + 1) + 1)/(x^2 + 1)
```

giac [A] time = 0.20, size = 18, normalized size = 0.82

$$\frac{1}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+2*x^2+1),x, algorithm="giac")
[Out] 1/2/(x^2 + 1) + 1/2*log(x^2 + 1)
```

maple [A] time = 0.00, size = 19, normalized size = 0.86

$$\frac{\ln\left(x^2+1\right)}{2}+\frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4+2*x^2+1),x)
```

```
[Out] 1/2/(x^2+1)+1/2*ln(x^2+1)
```

maxima [A] time = 1.28, size = 18, normalized size = 0.82

$$\frac{1}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2/(x^2 + 1) + 1/2*log(x^2 + 1)
```

mupad [B] time = 0.04, size = 18, normalized size = 0.82

$$\frac{\ln\left(x^2+1\right)}{2}+\frac{1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(2*x^2 + x^4 + 1),x)
```

```
[Out] log(x^2 + 1)/2 + 1/(2*(x^2 + 1))
```

sympy [A] time = 0.09, size = 15, normalized size = 0.68

$$\frac{\log\left(x^2+1\right)}{2}+\frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**4+2*x**2+1),x)
```

```
[Out] log(x**2 + 1)/2 + 1/(2*x**2 + 2)
```

3.539 $\int \frac{x}{81-18x^2+x^4}\,dx$

Optimal. Leaf size=13

$$\frac{1}{2\left(9-x^2\right)}$$

[Out] 1/2/(-x^2+9)

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 261}

$$\frac{1}{2\left(9-x^2\right)}$$

Antiderivative was successfully verified.

[In] Int[x/(81 - 18*x^2 + x^4),x]

[Out] 1/(2*(9 - x^2))

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{81-18x^2+x^4}\,dx &= \int \frac{x}{\left(-9+x^2\right)^2}\,dx\\ &= \frac{1}{2\left(9-x^2\right)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 0.85

$$-\frac{1}{2\left(x^2-9\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(81 - 18*x^2 + x^4),x]
```

```
[Out] -1/2*1/(-9 + x^2)
```

fricas [A] time = 1.01, size = 9, normalized size = 0.69

$$-\frac{1}{2\left(x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-18*x^2+81),x, algorithm="fricas")
```

```
[Out] -1/2/(x^2 - 9)
```

giac [A] time = 0.16, size = 9, normalized size = 0.69

$$-\frac{1}{2\left(x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-18*x^2+81),x, algorithm="giac")
```

```
[Out] -1/2/(x^2 - 9)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$-\frac{1}{2\left(x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4-18*x^2+81),x)
```

```
[Out] -1/2/(x^2-9)
```

maxima [A] time = 1.33, size = 9, normalized size = 0.69

$$-\frac{1}{2\left(x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-18*x^2+81),x, algorithm="maxima")
```

```
[Out] -1/2/(x^2 - 9)
```

mupad [B] time = 0.05, size = 11, normalized size = 0.85

$$-\frac{1}{2\left(x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4 - 18*x^2 + 81),x)
[Out] -1/(2*(x^2 - 9))
```

sympy [A] time = 0.09, size = 8, normalized size = 0.62

$$-\frac{1}{2x^2-18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4-18*x**2+81),x)
[Out] -1/(2*x**2 - 18)
```

3.540 $\int \frac{x^3}{16-8x^2+x^4} \, dx$

Optimal. Leaf size=24

$$\frac{2}{4-x^2} + \frac{1}{2}\log\left(4-x^2\right)$$

[Out] 2/(-x^2+4)+1/2*ln(-x^2+4)

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 43}

$$\frac{2}{4-x^2} + \frac{1}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

[In] Int[x^3/(16 - 8*x^2 + x^4),x]

[Out] 2/(4 - x^2) + Log[4 - x^2]/2

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{16-8x^2+x^4}\,dx = \int \frac{x^3}{\left(-4+x^2\right)^2}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(-4+x)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{4}{(-4+x)^2}+\frac{1}{-4+x}\right)dx, x, x^2\right)$$
$$= \frac{2}{4-x^2}+\frac{1}{2}\log\left(4-x^2\right)$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.83

$$\frac{1}{2}\log\left(x^2-4\right)-\frac{2}{x^2-4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(16 - 8*x^2 + x^4),x]
[Out] -2/(-4 + x^2) + Log[-4 + x^2]/2
```

fricas [A] time = 0.92, size = 23, normalized size = 0.96

$$\frac{\left(x^2-4\right)\log\left(x^2-4\right)-4}{2\left(x^2-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-8*x^2+16),x, algorithm="fricas")
[Out] 1/2*((x^2 - 4)*log(x^2 - 4) - 4)/(x^2 - 4)
```

giac [A] time = 0.15, size = 19, normalized size = 0.79

$$-\frac{2}{x^2-4}+\frac{1}{2}\log\left(\left|x^2-4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-8*x^2+16),x, algorithm="giac")
[Out] -2/(x^2 - 4) + 1/2*log(abs(x^2 - 4))
```

maple [A] time = 0.00, size = 19, normalized size = 0.79

$$\frac{\ln\left(x^2-4\right)}{2}-\frac{2}{x^2-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4-8*x^2+16),x)
[Out] 1/2*ln(x^2-4)-2/(x^2-4)
```

maxima [A] time = 1.32, size = 18, normalized size = 0.75

$$-\frac{2}{x^2-4}+\frac{1}{2}\log\left(x^2-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-8*x^2+16),x, algorithm="maxima")
[Out] -2/(x^2 - 4) + 1/2*log(x^2 - 4)
```

mupad [B] time = 4.23, size = 18, normalized size = 0.75

$$\frac{\ln\left(x^2-4\right)}{2}-\frac{2}{x^2-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4 - 8*x^2 + 16),x)
[Out] log(x^2 - 4)/2 - 2/(x^2 - 4)
```

sympy [A] time = 0.10, size = 14, normalized size = 0.58

$$\frac{\log\left(x^2-4\right)}{2}-\frac{2}{x^2-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**4-8*x**2+16),x)
[Out] log(x**2 - 4)/2 - 2/(x**2 - 4)
```

3.541 $\int x^5\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=79

$$\frac{bx^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}+\frac{ax^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}$$

```
[Out] 1/6*a*x^6*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/8*b*x^8*((b*x^2+a)^2)^(1/2)/(b*x^
2+a)
```

Rubi [A] time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{bx^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}+\frac{ax^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*(a + b*x^2)) + (b*x^8*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
```

```
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{a^2+2abx^2+b^2x^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2\sqrt{a^2+2abx+b^2x^2}\,dx,x,x^2\right)\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int x^2\left(ab+b^2x\right)\,dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(abx^2+b^2x^3\right)\,dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}\\ &= \frac{ax^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}+\frac{bx^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^2\right)^2}\,\left(4ax^6+3bx^8\right)}{24\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(4*a*x^6 + 3*b*x^8))/(24*(a + b*x^2))
```

fricas [A] time = 1.04, size = 13, normalized size = 0.16

$$\frac{1}{8}\,bx^8+\frac{1}{6}\,ax^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/8*b*x^8 + 1/6*a*x^6
```

giac [A] time = 0.16, size = 29, normalized size = 0.37

$$\frac{1}{8}\,bx^8\text{sgn}\left(bx^2+a\right)+\frac{1}{6}\,ax^6\text{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*b*x^8*sgn(b*x^2 + a) + 1/6*a*x^6*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 36, normalized size = 0.46

$$\frac{\left(3b\,x^{2}+4a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}\;x^{6}}{24b\,x^{2}+24a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/24*x^6*(3*b*x^2+4*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

maxima [A] time = 1.35, size = 13, normalized size = 0.16

$$\frac{1}{8}\,bx^{8}+\frac{1}{6}\,ax^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*b*x^8 + 1/6*a*x^6
```

mupad [B] time = 4.45, size = 71, normalized size = 0.90

$$\frac{\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}\;\left(a^{3}-4\,a^{2}\,b\,x^{2}-5\,a\,b^{2}\,x^{4}+3\,b\,x^{2}\,\left(a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}\right)\right)}{24\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(a^3 - 4*a^2*b*x^2 - 5*a*b^2*x^4 + 3*b*x
^2*(a^2 + b^2*x^4 + 2*a*b*x^2)))/(24*b^3)
```

sympy [A] time = 0.10, size = 12, normalized size = 0.15

$$\frac{ax^{6}}{6}+\frac{bx^{8}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x**6/6 + b*x**8/8
```

3.542 $\int x^3\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=67

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{6b^2}-\frac{a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4b^2}$$

```
[Out] 1/6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/b^2-1/4*a*(b*x^2+a)*((b*x^2+a)^2)^(1/2)/b
^2
```

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 640, 609}

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{6b^2}-\frac{a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -(a*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*b^2) + (a^2 + 2*a*b*x^2
 + b^2*x^4)^(3/2)/(6*b^2)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^3\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int x\sqrt{a^2+2abx+b^2x^2}\,dx, x, x^2\right)$$
$$= \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{6b^2} - \frac{a\,\text{Subst}\left(\int \sqrt{a^2+2abx+b^2x^2}\,dx, x, x^2\right)}{2b}$$
$$= -\frac{a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4b^2} + \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{6b^2}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.58

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(3ax^4+2bx^6\right)}{12\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(3*a*x^4 + 2*b*x^6))/(12*(a + b*x^2))
```

fricas [A] time = 0.97, size = 13, normalized size = 0.19

$$\frac{1}{6}\,bx^6 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

giac [A] time = 0.18, size = 23, normalized size = 0.34

$$\frac{1}{12}\left(2\,bx^6+3\,ax^4\right)\text{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/12*(2*b*x^6 + 3*a*x^4)*sgn(b*x^2 + a)
```

maple [A] time = 0.00, size = 36, normalized size = 0.54

$$\frac{\left(2b\,x^2+3a\right)\sqrt{\left(b\,x^2+a\right)^2}\;x^4}{12b\,x^2+12a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/12*x^4*(2*b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

maxima [A] time = 1.28, size = 13, normalized size = 0.19

$$\frac{1}{6}\,bx^6 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

mupad [B] time = 4.31, size = 59, normalized size = 0.88

$$\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(8\,b^2\,\left(a^2+b^2\,x^4\right)-12\,a^2\,b^2+4\,a\,b^3\,x^2\right)}{48\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(8*b^2*(a^2 + b^2*x^4) - 12*a^2*b^2 + 4*
a*b^3*x^2))/(48*b^4)
```

sympy [A] time = 0.10, size = 12, normalized size = 0.18

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x**4/4 + b*x**6/6
```

3.543 $\int x\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4b}$$

```
[Out] 1/4*(b*x^2+a)*((b*x^2+a)^2)^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1107, 609}

$$\frac{\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*b)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a^2+2abx^2+b^2x^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \sqrt{a^2+2abx+b^2x^2}\,dx,x,x^2\right)\\ &= \frac{\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.06

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(2ax^2+bx^4\right)}{4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(2*a*x^2 + b*x^4))/(4*(a + b*x^2))
```

fricas [A] time = 0.79, size = 13, normalized size = 0.36

$$\frac{1}{4}\,bx^4 + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/4*b*x^4 + 1/2*a*x^2
```

giac [A] time = 0.16, size = 22, normalized size = 0.61

$$\frac{1}{4}\left(bx^4 + 2\,ax^2\right)\mathrm{sgn}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/4*(b*x^4 + 2*a*x^2)*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 35, normalized size = 0.97

$$\frac{\left(b\,x^2 + 2a\right)\sqrt{\left(b\,x^2 + a\right)^2}\;x^2}{4b\,x^2 + 4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x^2+a)^2)^(1/2),x)
[Out] 1/4*x^2*(b*x^2+2*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

maxima [A] time = 1.32, size = 14, normalized size = 0.39

$$\frac{\left(bx^2 + a\right)^2}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^2/b
```

mupad [B] time = 4.35, size = 33, normalized size = 0.92

$$\left(\frac{a}{4\,b}+\frac{x^2}{4}\right)\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a + b*x^2)^2)^(1/2),x)
[Out] (a/(4*b) + x^2/4)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)
```

sympy [A] time = 0.10, size = 12, normalized size = 0.33

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x**2+a)**2)**(1/2),x)
[Out] a*x**2/2 + b*x**4/4
```

3.544 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x} dx$

Optimal. Leaf size=75

$$\frac{bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{a\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

```
[Out] 1/2*b*x^2*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+a*ln(x)*((b*x^2+a)^2)^(1/2)/(b*x^2+
a)
```

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$\frac{bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{a\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x,x]
```

```
[Out] (b*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (a*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x}+b^2x\right)dx}{ab+b^2x^2}$$
$$= \frac{bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)} + \frac{a\sqrt{a^2+2abx^2+b^2x^4}\,\log(x)}{a+bx^2}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(2a\log(x)+bx^2\right)}{2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x,x]
[Out] (Sqrt[(a + b*x^2)^2]*(b*x^2 + 2*a*Log[x]))/(2*(a + b*x^2))
```

fricas [A] time = 1.10, size = 11, normalized size = 0.15

$$\frac{1}{2}\,bx^2 + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x,x, algorithm="fricas")
[Out] 1/2*b*x^2 + a*log(x)
```

giac [A] time = 0.16, size = 30, normalized size = 0.40

$$\frac{1}{2}\,bx^2\mathrm{sgn}\left(bx^2+a\right) + \frac{1}{2}\,a\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x,x, algorithm="giac")
[Out] 1/2*b*x^2*sgn(b*x^2 + a) + 1/2*a*log(x^2)*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 34, normalized size = 0.45

$$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(b\,x^{2}+2 a\ln\left(x\right)\right)}{2 b\,x^{2}+2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x,x)
```

```
[Out] 1/2*((b*x^2+a)^2)^(1/2)*(b*x^2+2*a*ln(x))/(b*x^2+a)
```

maxima [A] time = 1.39, size = 14, normalized size = 0.19

$$\frac{1}{2}\,bx^{2}+\frac{1}{2}\,a\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + 1/2*a*log(x^2)
```

mupad [B] time = 4.39, size = 109, normalized size = 1.45

$$\frac{\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{2}-\frac{\ln\left(a\,b+\frac{a^{2}}{x^{2}}+\frac{\sqrt{a^{2}}\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{x^{2}}\right)\,\sqrt{a^{2}}}{2}+\frac{a\,b\,\ln\left(a\,b+\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\sqrt{b^{2}}+b^{2}\,x^{2}\right)}{2\,\sqrt{b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x,x)
```

```
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/2 - (log(a*b + a^2/x^2 + ((a^2)^(1/2)*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/x^2)*(a^2)^(1/2))/2 + (a*b*log(a*b + ((a +
b*x^2)^2)^(1/2)*(b^2)^(1/2) + b^2*x^2))/(2*(b^2)^(1/2))
```

sympy [A] time = 0.12, size = 10, normalized size = 0.13

$$a\log\left(x\right)+\frac{bx^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x,x)
```

```
[Out] a*log(x) + b*x**2/2
```

3.545 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx$

Optimal. Leaf size=75

$$\frac{b\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}$$

```
[Out] -1/2*a*((b*x^2+a)^2)^(1/2)/x^2/(b*x^2+a)+b*ln(x)*((b*x^2+a)^2)^(1/2)/(b*x^2
+a)
```

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$\frac{b\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^3,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^3}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x^3}+\frac{b^2}{x}\right)dx}{ab+b^2x^2}$$
$$= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)} + \frac{b\sqrt{a^2+2abx^2+b^2x^4}\,\log(x)}{a+bx^2}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.52

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a-2bx^2\log(x)\right)}{2x^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^3,x]
[Out] -1/2*(Sqrt[(a + b*x^2)^2]*(a - 2*b*x^2*Log[x]))/(x^2*(a + b*x^2))
```

fricas [A] time = 0.80, size = 17, normalized size = 0.23

$$\frac{2\,bx^2\log(x)-a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="fricas")
[Out] 1/2*(2*b*x^2*log(x) - a)/x^2
```

giac [A] time = 0.23, size = 45, normalized size = 0.60

$$\frac{1}{2}\,b\log\left(x^2\right)\operatorname{sgn}\left(bx^2+a\right)-\frac{bx^2\operatorname{sgn}\left(bx^2+a\right)+a\operatorname{sgn}\left(bx^2+a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="giac")
[Out] 1/2*b*log(x^2)*sgn(b*x^2 + a) - 1/2*(b*x^2*sgn(b*x^2 + a) + a*sgn(b*x^2 + a
))/x^2
```

maple [A] time = 0.01, size = 38, normalized size = 0.51

$$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(2b\,x^{2}\ln(x)-a\right)}{2\left(b\,x^{2}+a\right)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^3,x)
```

```
[Out] 1/2*((b*x^2+a)^2)^(1/2)*(2*b*ln(x)*x^2-a)/(b*x^2+a)/x^2
```

maxima [A] time = 1.40, size = 14, normalized size = 0.19

$$\frac{1}{2}\,b\log\left(x^{2}\right)-\frac{a}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*log(x^2) - 1/2*a/x^2
```

mupad [B] time = 4.45, size = 112, normalized size = 1.49

$$\frac{\ln\left(a\,b+\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\sqrt{b^{2}}+b^{2}\,x^{2}\right)\sqrt{b^{2}}}{2}-\frac{\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{2\,x^{2}}-\frac{a\,b\,\ln\left(a\,b+\frac{a^{2}}{x^{2}}+\frac{\sqrt{a^{2}}\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{x^{2}}\right)}{2\,\sqrt{a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^3,x)
```

```
[Out] (log(a*b + ((a + b*x^2)^2)^(1/2)*(b^2)^(1/2) + b^2*x^2)*(b^2)^(1/2))/2 - (a
^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/(2*x^2) - (a*b*log(a*b + a^2/x^2 + ((a^2)^(
1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/x^2))/(2*(a^2)^(1/2))
```

sympy [A] time = 0.15, size = 10, normalized size = 0.13

$$-\frac{a}{2x^{2}}+b\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**3,x)
```

```
[Out] -a/(2*x**2) + b*log(x)
```

3.546 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx$

Optimal. Leaf size=39

$$-\frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^4}$$

```
[Out] -1/4*(b*x^2+a)*((b*x^2+a)^2)^(1/2)/a/x^4
```

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 37}

$$-\frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^5,x]
```

```
[Out] -((a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*a*x^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^3}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\;\text{Subst}\left(\int \frac{ab+b^2x}{x^3}\,dx, x, x^2\right)}{2\left(ab+b^2x^2\right)}$$
$$= -\frac{\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^4}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.95

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a+2bx^2\right)}{4x^4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^5,x]
[Out] -1/4*(Sqrt[(a + b*x^2)^2]*(a + 2*b*x^2))/(x^4*(a + b*x^2))
```

fricas [A] time = 0.73, size = 13, normalized size = 0.33

$$-\frac{2\,bx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^5,x, algorithm="fricas")
[Out] -1/4*(2*b*x^2 + a)/x^4
```

giac [A] time = 0.16, size = 30, normalized size = 0.77

$$-\frac{2\,bx^2\text{sgn}\left(bx^2+a\right)+a\text{sgn}\left(bx^2+a\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^5,x, algorithm="giac")
[Out] -1/4*(2*b*x^2*sgn(b*x^2 + a) + a*sgn(b*x^2 + a))/x^4
```

maple [A] time = 0.00, size = 34, normalized size = 0.87

$$-\frac{\left(2b\,x^2+a\right)\sqrt{\left(b\,x^2+a\right)^2}}{4\left(b\,x^2+a\right)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^5,x)
```

```
[Out] -1/4*(2*b*x^2+a)*((b*x^2+a)^2)^(1/2)/x^4/(b*x^2+a)
```

maxima [A] time = 1.33, size = 13, normalized size = 0.33

$$-\frac{2\,bx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*x^2 + a)/x^4
```

mupad [B] time = 4.21, size = 33, normalized size = 0.85

$$-\frac{\left(2\,b\,x^2+a\right)\,\sqrt{\left(b\,x^2+a\right)^2}}{4\,x^4\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^5,x)
```

```
[Out] -((a + 2*b*x^2)*((a + b*x^2)^2)^(1/2))/(4*x^4*(a + b*x^2))
```

sympy [A] time = 0.17, size = 14, normalized size = 0.36

$$\frac{-a-2bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**5,x)
```

```
[Out] (-a - 2*b*x**2)/(4*x**4)
```

3.547 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^7}\,dx$

Optimal. Leaf size=72

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{12a^2x^6}-\frac{\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^6}$$

```
[Out] 1/12*(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/a^2/x^6-1/4*(b*x^2+a)*((b*x^2+a)^2)^(1/2
)/a/x^6
```

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1110}

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{12a^2x^6}-\frac{\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^7,x]
```

```
[Out] -((a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*a*x^6) + (a^2 + 2*a*b*x^2
 + b^2*x^4)^(3/2)/(12*a^2*x^6)
```

Rule 1110

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(4*a*d*(p + 1)*(2*p +
1)), x] - Simp[((d*x)^(m + 1)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^p)/(4*a*d*(
2*p + 1)), x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !I
ntegerQ[p] && EqQ[m + 4*p + 5, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^7}\,dx=-\frac{\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^6}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{12a^2x^6}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.54

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(2a+3bx^2\right)}{12x^6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^7,x]
[Out] -1/12*(Sqrt[(a + b*x^2)^2]*(2*a + 3*b*x^2))/(x^6*(a + b*x^2))
```

fricas [A] time = 0.98, size = 15, normalized size = 0.21

$$-\frac{3\,bx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^7,x, algorithm="fricas")
[Out] -1/12*(3*b*x^2 + 2*a)/x^6
```

giac [A] time = 0.16, size = 31, normalized size = 0.43

$$-\frac{3\,bx^2\mathrm{sgn}\left(bx^2+a\right)+2\,a\mathrm{sgn}\left(bx^2+a\right)}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^7,x, algorithm="giac")
[Out] -1/12*(3*b*x^2*sgn(b*x^2 + a) + 2*a*sgn(b*x^2 + a))/x^6
```

maple [A] time = 0.00, size = 36, normalized size = 0.50

$$-\frac{\left(3b\,x^2+2a\right)\sqrt{\left(b\,x^2+a\right)^2}}{12\left(b\,x^2+a\right)x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^7,x)
[Out] -1/12*(3*b*x^2+2*a)*((b*x^2+a)^2)^(1/2)/x^6/(b*x^2+a)
```

maxima [A] time = 1.30, size = 15, normalized size = 0.21

$$-\frac{3\,bx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b*x^2 + 2*a)/x^6
```

mupad [B] time = 4.24, size = 35, normalized size = 0.49

$$-\frac{\left(3\,b\,x^2+2\,a\right)\,\sqrt{{\left(b\,x^2+a\right)}^2}}{12\,x^6\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^7,x)
[Out] -((2*a + 3*b*x^2)*((a + b*x^2)^2)^(1/2))/(12*x^6*(a + b*x^2))
```

sympy [A] time = 0.19, size = 15, normalized size = 0.21

$$\frac{-2a-3bx^2}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**7,x)
[Out] (-2*a - 3*b*x**2)/(12*x**6)
```

3.548 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^9} dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}$$

```
[Out] -1/8*a*((b*x^2+a)^2)^(1/2)/x^8/(b*x^2+a)-1/6*b*((b*x^2+a)^2)^(1/2)/x^6/(b*x
^2+a)
```

Rubi [A] time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^9,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
```

```
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^9}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^5}\,dx,x,x^2\right)\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int \frac{ab+b^2x}{x^5}\,dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int \left(\frac{ab}{x^5}+\frac{b^2}{x^4}\right)dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}\\
&= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(3a+4bx^2\right)}{24x^8\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^9,x]
[Out] -1/24*(Sqrt[(a + b*x^2)^2]*(3*a + 4*b*x^2))/(x^8*(a + b*x^2))
```

fricas [A] time = 0.80, size = 15, normalized size = 0.19

$$-\frac{4\,bx^2+3\,a}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^9,x, algorithm="fricas")
[Out] -1/24*(4*b*x^2 + 3*a)/x^8
```

giac [A] time = 0.16, size = 31, normalized size = 0.39

$$-\frac{4\,bx^2\operatorname{sgn}\left(bx^2+a\right)+3\,a\operatorname{sgn}\left(bx^2+a\right)}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^9,x, algorithm="giac")
[Out] -1/24*(4*b*x^2*sgn(b*x^2 + a) + 3*a*sgn(b*x^2 + a))/x^8
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$-\frac{\left(4b\,x^{2}+3a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{24\left(b\,x^{2}+a\right)x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^9,x)
[Out] -1/24*(4*b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/x^8/(b*x^2+a)
```

maxima [A] time = 1.35, size = 15, normalized size = 0.19

$$-\frac{4\,bx^{2}+3\,a}{24\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^9,x, algorithm="maxima")
[Out] -1/24*(4*b*x^2 + 3*a)/x^8
```

mupad [B] time = 4.24, size = 35, normalized size = 0.44

$$-\frac{\left(4\,b\,x^{2}+3\,a\right)\,\sqrt{\left(b\,x^{2}+a\right)^{2}}}{24\,x^{8}\,\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^9,x)
[Out] -((3*a + 4*b*x^2)*((a + b*x^2)^2)^(1/2))/(24*x^8*(a + b*x^2))
```

sympy [A] time = 0.20, size = 15, normalized size = 0.19

$$\frac{-3a-4bx^{2}}{24x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**9,x)
[Out] (-3*a - 4*b*x**2)/(24*x**8)
```

3.549 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{11}}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}$$

```
[Out] -1/10*a*((b*x^2+a)^2)^(1/2)/x^10/(b*x^2+a)-1/8*b*((b*x^2+a)^2)^(1/2)/x^8/(b
*x^2+a)
```

Rubi [A] time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^11,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (b*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
```

```
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{11}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx,x,x^2\right)\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int \frac{ab+b^2x}{x^6}\,dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int \left(\frac{ab}{x^6}+\frac{b^2}{x^5}\right)dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}\\
&= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(4a+5bx^2\right)}{40x^{10}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^11,x]
[Out] -1/40*(Sqrt[(a + b*x^2)^2]*(4*a + 5*b*x^2))/(x^10*(a + b*x^2))
```

fricas [A] time = 0.74, size = 15, normalized size = 0.19

$$-\frac{5\,bx^2+4\,a}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^11,x, algorithm="fricas")
[Out] -1/40*(5*b*x^2 + 4*a)/x^10
```

giac [A] time = 0.16, size = 31, normalized size = 0.39

$$-\frac{5\,bx^2\operatorname{sgn}\left(bx^2+a\right)+4\,a\operatorname{sgn}\left(bx^2+a\right)}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] -1/40*(5*b*x^2*sgn(b*x^2 + a) + 4*a*sgn(b*x^2 + a))/x^10
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$-\frac{\left(5b\,x^{2}+4a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{40\left(b\,x^{2}+a\right)x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^11,x)
```

```
[Out] -1/40*(5*b*x^2+4*a)*((b*x^2+a)^2)^(1/2)/x^10/(b*x^2+a)
```

maxima [A] time = 1.31, size = 15, normalized size = 0.19

$$-\frac{5\,bx^{2}+4\,a}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] -1/40*(5*b*x^2 + 4*a)/x^10
```

mupad [B] time = 4.21, size = 35, normalized size = 0.44

$$-\frac{\left(5\,b\,x^{2}+4\,a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{40\,x^{10}\,\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^11,x)
```

```
[Out] -((4*a + 5*b*x^2)*((a + b*x^2)^2)^(1/2))/(40*x^10*(a + b*x^2))
```

sympy [A] time = 0.22, size = 15, normalized size = 0.19

$$\frac{-4a-5bx^{2}}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**11,x)
```

```
[Out] (-4*a - 5*b*x**2)/(40*x**10)
```

3.550 $\int x^4\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=79

$$\frac{bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{ax^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}$$

```
[Out] 1/5*a*x^5*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/7*b*x^7*((b*x^2+a)^2)^(1/2)/(b*x^
2+a)
```

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$\frac{bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{ax^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (b*x^7*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^4\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^4\left(ab+b^2x^2\right)dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(abx^4+b^2x^6\right)dx}{ab+b^2x^2}$$
$$= \frac{ax^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)} + \frac{bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(7ax^5+5bx^7\right)}{35\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(7*a*x^5 + 5*b*x^7))/(35*(a + b*x^2))
```

fricas [A] time = 0.81, size = 13, normalized size = 0.16

$$\frac{1}{7}\,bx^7+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/7*b*x^7 + 1/5*a*x^5
```

giac [A] time = 0.17, size = 29, normalized size = 0.37

$$\frac{1}{7}\,bx^7\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{5}\,ax^5\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/7*b*x^7*sgn(b*x^2 + a) + 1/5*a*x^5*sgn(b*x^2 + a)
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$\frac{\left(5b\,x^2+7a\right)\sqrt{\left(b\,x^2+a\right)^2}\;x^5}{35b\,x^2+35a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/35*x^5*(5*b*x^2+7*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

maxima [A] time = 1.35, size = 13, normalized size = 0.16

$$\frac{1}{7}\,bx^7 + \frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*b*x^7 + 1/5*a*x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(x^4*((a + b*x^2)^2)^(1/2), x)
```

sympy [A] time = 0.10, size = 12, normalized size = 0.15

$$\frac{ax^5}{5} + \frac{bx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x**5/5 + b*x**7/7
```

3.551 $\int x^2\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=79

$$\frac{bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{ax^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

```
[Out] 1/3*a*x^3*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/5*b*x^5*((b*x^2+a)^2)^(1/2)/(b*x^
2+a)
```

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$\frac{bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{ax^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (b*x^5*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^2\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^2\left(ab+b^2x^2\right)dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(abx^2+b^2x^4\right)dx}{ab+b^2x^2}$$
$$= \frac{ax^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}$$

Mathematica [**A**] time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(5ax^3+3bx^5\right)}{15\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(5*a*x^3 + 3*b*x^5))/(15*(a + b*x^2))
```

fricas [**A**] time = 0.88, size = 13, normalized size = 0.16

$$\frac{1}{5}\,bx^5+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

giac [**A**] time = 0.15, size = 29, normalized size = 0.37

$$\frac{1}{5}\,bx^5\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{3}\,ax^3\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/5*b*x^5*sgn(b*x^2 + a) + 1/3*a*x^3*sgn(b*x^2 + a)
```

maple [**A**] time = 0.00, size = 36, normalized size = 0.46

$$\frac{\left(3b\,x^2+5a\right)\sqrt{\left(b\,x^2+a\right)^2}\,x^3}{15b\,x^2+15a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/15*x^3*(3*b*x^2+5*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

maxima [A] time = 1.31, size = 13, normalized size = 0.16

$$\frac{1}{5}\,bx^5 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(x^2*((a + b*x^2)^2)^(1/2), x)
```

sympy [A] time = 0.15, size = 12, normalized size = 0.15

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x**3/3 + b*x**5/5
```

3.552 $\int \sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=74

$$\frac{ax\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{bx^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

```
[Out] a*x*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/3*b*x^3*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1088}

$$\frac{bx^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{ax\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (b*x^3*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(3*(a + b*x^2))
```

Rule 1088

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a^2+2abx^2+b^2x^4}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(2ab+2b^2x^2\right)dx}{2ab+2b^2x^2}\\
&= \frac{ax\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{bx^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(3ax+bx^3\right)}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(3*a*x + b*x^3))/(3*(a + b*x^2))
```

fricas [A] time = 0.80, size = 10, normalized size = 0.14

$$\frac{1}{3}\,bx^{3}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*b*x^3 + a*x
```

giac [A] time = 0.18, size = 20, normalized size = 0.27

$$\frac{1}{3}\left(bx^{3}+3\,ax\right)\mathrm{sgn}\left(bx^{2}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/3*(b*x^3 + 3*a*x)*sgn(b*x^2 + a)
```

maple [A] time = 0.00, size = 33, normalized size = 0.45

$$\frac{\left(b\,x^{2}+3a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}\;x}{3b\,x^{2}+3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2),x)
[Out] 1/3*x*(b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

maxima [A] time = 1.32, size = 10, normalized size = 0.14

$$\frac{1}{3}\,bx^{3}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*b*x^3 + a*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\left(b\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(((a + b*x^2)^2)^(1/2), x)
```

sympy [A] time = 0.10, size = 8, normalized size = 0.11

$$ax + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x + b*x**3/3
```

3.553 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^2} dx$

Optimal. Leaf size=72

$$\frac{bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

```
[Out] -a*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)+b*x*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$\frac{bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^2,x]
```

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))) + (b*x*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^2}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^2+\frac{ab}{x^2}\right)dx}{ab+b^2x^2}$$
$$= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)} + \frac{bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.49

$$\frac{\left(bx^2-a\right)\sqrt{\left(a+bx^2\right)^2}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^2,x]
[Out] ((-a + b*x^2)*Sqrt[(a + b*x^2)^2])/(x*(a + b*x^2))
```

fricas [A] time = 1.01, size = 13, normalized size = 0.18

$$\frac{bx^2-a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="fricas")
[Out] (b*x^2 - a)/x
```

giac [A] time = 0.16, size = 26, normalized size = 0.36

$$bx\mathrm{sgn}\left(bx^2+a\right) - \frac{a\mathrm{sgn}\left(bx^2+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="giac")
[Out] b*x*sgn(b*x^2 + a) - a*sgn(b*x^2 + a)/x
```

maple [A] time = 0.00, size = 34, normalized size = 0.47

$$-\frac{\left(-b\,x^{2}+a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{\left(b\,x^{2}+a\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^2,x)
[Out] -(-b*x^2+a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/x
```

maxima [A] time = 1.34, size = 10, normalized size = 0.14

$$bx-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="maxima")
[Out] b*x - a/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^2,x)
[Out] int(((a + b*x^2)^2)^(1/2)/x^2, x)
```

sympy [A] time = 0.13, size = 5, normalized size = 0.07

$$-\frac{a}{x}+bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**2,x)
[Out] -a/x + b*x
```

3.554 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx$

Optimal. Leaf size=77

$$-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

```
[Out] -1/3*a*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)-b*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)
```

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^4,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^4}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x^4}+\frac{b^2}{x^2}\right)dx}{ab+b^2x^2}$$
$$= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)} - \frac{b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.48

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a+3bx^2\right)}{3x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^4,x]
[Out] -1/3*(Sqrt[(a + b*x^2)^2]*(a + 3*b*x^2))/(x^3*(a + b*x^2))
```

fricas [A] time = 0.75, size = 13, normalized size = 0.17

$$-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="fricas")
[Out] -1/3*(3*b*x^2 + a)/x^3
```

giac [A] time = 0.17, size = 30, normalized size = 0.39

$$-\frac{3\,bx^2\mathrm{sgn}\left(bx^2+a\right)+a\mathrm{sgn}\left(bx^2+a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="giac")
[Out] -1/3*(3*b*x^2*sgn(b*x^2 + a) + a*sgn(b*x^2 + a))/x^3
```

maple [A] time = 0.00, size = 34, normalized size = 0.44

$$-\frac{\left(3b\,x^{2}+a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{3\left(b\,x^{2}+a\right)x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^4,x)
[Out] -1/3*(3*b*x^2+a)*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)
```

maxima [A] time = 1.38, size = 13, normalized size = 0.17

$$-\frac{3\,bx^{2}+a}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="maxima")
[Out] -1/3*(3*b*x^2 + a)/x^3
```

mupad [B] time = 4.24, size = 33, normalized size = 0.43

$$-\frac{\left(3\,b\,x^{2}+a\right)\,\sqrt{\left(b\,x^{2}+a\right)^{2}}}{3\,x^{3}\,\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^4,x)
[Out] -((a + 3*b*x^2)*((a + b*x^2)^2)^(1/2))/(3*x^3*(a + b*x^2))
```

sympy [A] time = 0.16, size = 14, normalized size = 0.18

$$\frac{-a-3bx^{2}}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**4,x)
[Out] (-a - 3*b*x**2)/(3*x**3)
```

3.555 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^6} \, dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

```
[Out] -1/5*a*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)-1/3*b*((b*x^2+a)^2)^(1/2)/x^3/(b*x
^2+a)
```

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^6,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^6}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x^6}+\frac{b^2}{x^4}\right)dx}{ab+b^2x^2}$$
$$= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)} - \frac{b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(3a+5bx^2\right)}{15x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^6,x]
[Out] -1/15*(Sqrt[(a + b*x^2)^2]*(3*a + 5*b*x^2))/(x^5*(a + b*x^2))
```

fricas [A] time = 0.77, size = 15, normalized size = 0.19

$$-\frac{5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^6,x, algorithm="fricas")
[Out] -1/15*(5*b*x^2 + 3*a)/x^5
```

giac [A] time = 0.15, size = 31, normalized size = 0.39

$$-\frac{5\,bx^2\mathrm{sgn}\left(bx^2+a\right)+3\,a\mathrm{sgn}\left(bx^2+a\right)}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^6,x, algorithm="giac")
[Out] -1/15*(5*b*x^2*sgn(b*x^2 + a) + 3*a*sgn(b*x^2 + a))/x^5
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$-\frac{\left(5b\,x^{2}+3a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{15\left(b\,x^{2}+a\right)x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^6,x)
[Out] -1/15*(5*b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)
```

maxima [A] time = 1.31, size = 15, normalized size = 0.19

$$-\frac{5\,bx^{2}+3\,a}{15\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^6,x, algorithm="maxima")
[Out] -1/15*(5*b*x^2 + 3*a)/x^5
```

mupad [B] time = 4.21, size = 35, normalized size = 0.44

$$-\frac{\left(5\,b\,x^{2}+3\,a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{15\,x^{5}\,\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^6,x)
[Out] -((3*a + 5*b*x^2)*((a + b*x^2)^2)^(1/2))/(15*x^5*(a + b*x^2))
```

sympy [A] time = 0.18, size = 15, normalized size = 0.19

$$\frac{-3a-5bx^{2}}{15x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**6,x)
[Out] (-3*a - 5*b*x**2)/(15*x**5)
```

3.556 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^8}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}$$

```
[Out] -1/7*a*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)-1/5*b*((b*x^2+a)^2)^(1/2)/x^5/(b*x
^2+a)
```

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^8,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^8}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^8}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x^8}+\frac{b^2}{x^6}\right)dx}{ab+b^2x^2}$$
$$= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)} - \frac{b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(5a+7bx^2\right)}{35x^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^8,x]
```

```
[Out] -1/35*(Sqrt[(a + b*x^2)^2]*(5*a + 7*b*x^2))/(x^7*(a + b*x^2))
```

fricas [A] time = 0.95, size = 15, normalized size = 0.19

$$-\frac{7\,bx^2+5\,a}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/35*(7*b*x^2 + 5*a)/x^7
```

giac [A] time = 0.16, size = 31, normalized size = 0.39

$$-\frac{7\,bx^2\mathrm{sgn}\left(bx^2+a\right)+5\,a\mathrm{sgn}\left(bx^2+a\right)}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] -1/35*(7*b*x^2*sgn(b*x^2 + a) + 5*a*sgn(b*x^2 + a))/x^7
```

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$-\frac{\left(7b\,x^{2}+5a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{35\left(b\,x^{2}+a\right)x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^8,x)
```

```
[Out] -1/35*(7*b*x^2+5*a)*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)
```

maxima [A] time = 1.27, size = 15, normalized size = 0.19

$$-\frac{7\,bx^{2}+5\,a}{35\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/35*(7*b*x^2 + 5*a)/x^7
```

mupad [B] time = 4.18, size = 35, normalized size = 0.44

$$-\frac{\left(7\,b\,x^{2}+5\,a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{35\,x^{7}\,\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^8,x)
```

```
[Out] -((5*a + 7*b*x^2)*((a + b*x^2)^2)^(1/2))/(35*x^7*(a + b*x^2))
```

sympy [A] time = 0.20, size = 15, normalized size = 0.19

$$\frac{-5a-7bx^{2}}{35x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**8,x)
```

```
[Out] (-5*a - 7*b*x**2)/(35*x**7)
```

3.557 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{10}}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}$$

```
[Out] -1/9*a*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)-1/7*b*((b*x^2+a)^2)^(1/2)/x^7/(b*x
^2+a)
```

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^10,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{10}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^{10}}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x^{10}}+\frac{b^2}{x^8}\right)dx}{ab+b^2x^2}$$
$$= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)} - \frac{b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(7a+9bx^2\right)}{63x^9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^10,x]
[Out] -1/63*(Sqrt[(a + b*x^2)^2]*(7*a + 9*b*x^2))/(x^9*(a + b*x^2))
```

fricas [A] time = 0.68, size = 15, normalized size = 0.19

$$-\frac{9\,bx^2+7\,a}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^10,x, algorithm="fricas")
[Out] -1/63*(9*b*x^2 + 7*a)/x^9
```

giac [A] time = 0.16, size = 31, normalized size = 0.39

$$-\frac{9\,bx^2\mathrm{sgn}\left(bx^2+a\right)+7\,a\mathrm{sgn}\left(bx^2+a\right)}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^10,x, algorithm="giac")
[Out] -1/63*(9*b*x^2*sgn(b*x^2 + a) + 7*a*sgn(b*x^2 + a))/x^9
```

maple [A] time = 0.01, size = 36, normalized size = 0.46

$$-\frac{\left(9b\,x^{2}+7a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{63\left(b\,x^{2}+a\right)x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^10,x)
```

```
[Out] -1/63*(9*b*x^2+7*a)*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)
```

maxima [A] time = 1.29, size = 15, normalized size = 0.19

$$-\frac{9\,bx^{2}+7\,a}{63\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] -1/63*(9*b*x^2 + 7*a)/x^9
```

mupad [B] time = 4.20, size = 35, normalized size = 0.44

$$-\frac{\left(9\,b\,x^{2}+7\,a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{63\,x^{9}\,\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^10,x)
```

```
[Out] -((7*a + 9*b*x^2)*((a + b*x^2)^2)^(1/2))/(63*x^9*(a + b*x^2))
```

sympy [A] time = 0.22, size = 15, normalized size = 0.19

$$\frac{-7a-9bx^{2}}{63x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**10,x)
```

```
[Out] (-7*a - 9*b*x**2)/(63*x**9)
```

3.558 $\int x^9 \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{a^2bx^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{b^3x^{16}\sqrt{a^2+2abx^2+b^2x^4}}{16\left(a+bx^2\right)}+\frac{a^3x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}$$

```
[Out] 1/10*a^3*x^10*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/4*a^2*b*x^12*((b*x^2+a)^2)^(1
/2)/(b*x^2+a)+3/14*a*b^2*x^14*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/16*b^3*x^16*(
(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.11, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{b^3x^{16}\sqrt{a^2+2abx^2+b^2x^4}}{16\left(a+bx^2\right)}+\frac{3ab^2x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{a^2bx^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{a^3x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*(a + b*x^2)) + (a^2*b*x^12*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (3*a*b^2*x^14*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/(14*(a + b*x^2)) + (b^3*x^16*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(16*(a + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\begin{aligned}\int x^9\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^4\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx,x,x^2\right)\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int x^4\left(ab+b^2x\right)^3\,dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int\left(a^3b^3x^4+3a^2b^4x^5+3ab^5x^6+b^6x^7\right)dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}\\ &= \frac{a^3x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}+\frac{a^2bx^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{3ab^2x^{14}\sqrt{a^2+2a}}{14\left(a+b\right.}\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^{10}\sqrt{\left(a+bx^2\right)^2}\left(56a^3+140a^2bx^2+120ab^2x^4+35b^3x^6\right)}{560\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
[Out] (x^10*Sqrt[(a + b*x^2)^2]*(56*a^3 + 140*a^2*b*x^2 + 120*a*b^2*x^4 + 35*b^3*
x^6))/(560*(a + b*x^2))
```

fricas [A] time = 1.17, size = 35, normalized size = 0.21

$$\frac{1}{16}\,b^3x^{16}+\frac{3}{14}\,ab^2x^{14}+\frac{1}{4}\,a^2bx^{12}+\frac{1}{10}\,a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
[Out] 1/16*b^3*x^16 + 3/14*a*b^2*x^14 + 1/4*a^2*b*x^12 + 1/10*a^3*x^10
```

giac [A] time = 0.16, size = 67, normalized size = 0.40

$$\frac{1}{16}\,b^3x^{16}\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{14}\,ab^2x^{14}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{4}\,a^2bx^{12}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{10}\,a^3x^{10}\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*b^3*x^16*sgn(b*x^2 + a) + 3/14*a*b^2*x^14*sgn(b*x^2 + a) + 1/4*a^2*b*x
^12*sgn(b*x^2 + a) + 1/10*a^3*x^10*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{\left(35b^3x^6+120a\,b^2x^4+140a^2b\,x^2+56a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}x^{10}}{560\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/560*x^10*(35*b^3*x^6+120*a*b^2*x^4+140*a^2*b*x^2+56*a^3)*((b*x^2+a)^2)^(3
/2)/(b*x^2+a)^3
```

maxima [A] time = 1.31, size = 35, normalized size = 0.21

$$\frac{1}{16}\,b^3x^{16}+\frac{3}{14}\,ab^2x^{14}+\frac{1}{4}\,a^2bx^{12}+\frac{1}{10}\,a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*b^3*x^16 + 3/14*a*b^2*x^14 + 1/4*a^2*b*x^12 + 1/10*a^3*x^10
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^9\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^9*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^9 \left(\left(a + b x^2 \right)^2 \right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**9*((a + b*x**2)**2)**(3/2), x)
```

3.559 $\int x^7 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{ab^2x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}+\frac{b^3x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{a^3x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}$$

```
[Out] 1/8*a^3*x^8*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/10*a^2*b*x^10*((b*x^2+a)^2)^(1/
2)/(b*x^2+a)+1/4*a*b^2*x^12*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/14*b^3*x^14*((b
*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.11, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{b^3x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{ab^2x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}+\frac{a^3x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2)) + (3*a^2*b*x^10*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*(a + b*x^2)) + (a*b^2*x^12*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (b^3*x^14*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(14*(a + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{1}{2}\,\text{Subst}\left(\int x^3 \left(a^2+2abx+b^2x^2\right)^{3/2} dx, x, x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int x^3 \left(ab+b^2x\right)^3 dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(a^3b^3x^3+3a^2b^4x^4+3ab^5x^5+b^6x^6\right) dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= \frac{a^3x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)} + \frac{3a^2bx^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)} + \frac{ab^2x^{12}\sqrt{a^2+2ab}}{4\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^8\sqrt{\left(a+bx^2\right)^2}\left(35a^3+84a^2bx^2+70ab^2x^4+20b^3x^6\right)}{280\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^8*Sqrt[(a + b*x^2)^2]*(35*a^3 + 84*a^2*b*x^2 + 70*a*b^2*x^4 + 20*b^3*x^6
))/(280*(a + b*x^2))
```

fricas [A] time = 0.84, size = 35, normalized size = 0.21

$$\frac{1}{14}\,b^3x^{14} + \frac{1}{4}\,ab^2x^{12} + \frac{3}{10}\,a^2bx^{10} + \frac{1}{8}\,a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/14*b^3*x^14 + 1/4*a*b^2*x^12 + 3/10*a^2*b*x^10 + 1/8*a^3*x^8
```

giac [A] time = 0.16, size = 67, normalized size = 0.40

$$\frac{1}{14}\,b^3x^{14}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{4}\,ab^2x^{12}\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{10}\,a^2bx^{10}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{8}\,a^3x^8\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/14*b^3*x^14*sgn(b*x^2 + a) + 1/4*a*b^2*x^12*sgn(b*x^2 + a) + 3/10*a^2*b*x
^10*sgn(b*x^2 + a) + 1/8*a^3*x^8*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{\left(20b^3x^6+70a\,b^2x^4+84a^2b\,x^2+35a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}x^8}{280\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/280*x^8*(20*b^3*x^6+70*a*b^2*x^4+84*a^2*b*x^2+35*a^3)*((b*x^2+a)^2)^(3/2)
/(b*x^2+a)^3
```

maxima [A] time = 1.32, size = 35, normalized size = 0.21

$$\frac{1}{14}\,b^3x^{14}+\frac{1}{4}\,ab^2x^{12}+\frac{3}{10}\,a^2bx^{10}+\frac{1}{8}\,a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/14*b^3*x^14 + 1/4*a*b^2*x^12 + 3/10*a^2*b*x^10 + 1/8*a^3*x^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^7\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**7*((a + b*x**2)**2)**(3/2), x)
```

3.560 $\int x^5 \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx$

Optimal. Leaf size=106

$$\frac{(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b^3}-\frac{a\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{5b^3}+\frac{a^2(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b^3}$$

```
[Out] 1/8*a^2*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/b^3-1/5*a*(b^2*x^4+2*a*b*x^
2+a^2)^(5/2)/b^3+1/12*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/b^3
```

Rubi [A] time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1111, 645}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^5}{12b^3}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^4}{5b^3}+\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^3}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^2*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*b^3) - (a*(a + b*x^2
)^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*b^3) + ((a + b*x^2)^5*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(12*b^3)
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[ExpandLinearProduct[(b/2 + c*x)^(2*p), (d + e*x)^m, b/2, c
, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[p] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1, 0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{1}{2}\text{Subst}\left(\int x^2 \left(a^2+2abx+b^2x^2\right)^{3/2} dx, x, x^2\right)$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \left(\frac{a^2(ab+b^2x)^3}{b^2} - \frac{2a(ab+b^2x)^4}{b^3} + \frac{(ab+b^2x)^5}{b^4}\right) dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$

$$= \frac{a^2\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}{8b^3} - \frac{a\left(a+bx^2\right)^4\sqrt{a^2+2abx^2+b^2x^4}}{5b^3} + \frac{(a+$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.58

$$\frac{x^6\sqrt{\left(a+bx^2\right)^2}\left(20a^3+45a^2bx^2+36ab^2x^4+10b^3x^6\right)}{120\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x^2)^2]*(20*a^3 + 45*a^2*b*x^2 + 36*a*b^2*x^4 + 10*b^3*x^6
))/(120*(a + b*x^2))
```

fricas [A] time = 0.82, size = 35, normalized size = 0.33

$$\frac{1}{12}b^3x^{12} + \frac{3}{10}ab^2x^{10} + \frac{3}{8}a^2bx^8 + \frac{1}{6}a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6
```

giac [A] time = 0.22, size = 67, normalized size = 0.63

$$\frac{1}{12}b^3x^{12}\text{sgn}\left(bx^2+a\right) + \frac{3}{10}ab^2x^{10}\text{sgn}\left(bx^2+a\right) + \frac{3}{8}a^2bx^8\text{sgn}\left(bx^2+a\right) + \frac{1}{6}a^3x^6\text{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/12*b^3*x^12*sgn(b*x^2 + a) + 3/10*a*b^2*x^10*sgn(b*x^2 + a) + 3/8*a^2*b*x
^8*sgn(b*x^2 + a) + 1/6*a^3*x^6*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.55

$$\frac{\left(10b^3x^6+36a\,b^2x^4+45a^2b\,x^2+20a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}x^6}{120\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/120*x^6*(10*b^3*x^6+36*a*b^2*x^4+45*a^2*b*x^2+20*a^3)*((b*x^2+a)^2)^(3/2)
/(b*x^2+a)^3
```

maxima [A] time = 1.34, size = 35, normalized size = 0.33

$$\frac{1}{12}\,b^3x^{12}+\frac{3}{10}\,ab^2x^{10}+\frac{3}{8}\,a^2bx^8+\frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**5*((a + b*x**2)**2)**(3/2), x)
```

3.561 $\int x^3 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=67

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{10b^2}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b^2}$$

```
[Out] -1/8*a*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/b^2+1/10*(b^2*x^4+2*a*b*x^2+
a^2)^(5/2)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 640, 609}

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{10b^2}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -(a*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(8*b^2) + (a^2 + 2*a*b*x
^2 + b^2*x^4)^(5/2)/(10*b^2)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{1}{2}\text{Subst}\left(\int x \left(a^2+2abx+b^2x^2\right)^{3/2} dx, x, x^2\right)$$
$$= \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{10b^2} - \frac{a\,\text{Subst}\left(\int \left(a^2+2abx+b^2x^2\right)^{3/2} dx, x, x^2\right)}{2b}$$
$$= -\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b^2} + \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{10b^2}$$

Mathematica [**A**] time = 0.02, size = 61, normalized size = 0.91

$$\frac{x^4\sqrt{\left(a+bx^2\right)^2}\left(10a^3+20a^2bx^2+15ab^2x^4+4b^3x^6\right)}{40\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^2)^2]*(10*a^3 + 20*a^2*b*x^2 + 15*a*b^2*x^4 + 4*b^3*x^6)
)/(40*(a + b*x^2))
```

fricas [**A**] time = 0.85, size = 35, normalized size = 0.52

$$\frac{1}{10}b^3x^{10} + \frac{3}{8}ab^2x^8 + \frac{1}{2}a^2bx^6 + \frac{1}{4}a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/10*b^3*x^10 + 3/8*a*b^2*x^8 + 1/2*a^2*b*x^6 + 1/4*a^3*x^4
```

giac [**A**] time = 0.15, size = 45, normalized size = 0.67

$$\frac{1}{40}\left(4\,b^3x^{10}+15\,ab^2x^8+20\,a^2bx^6+10\,a^3x^4\right)\text{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/40*(4*b^3*x^10 + 15*a*b^2*x^8 + 20*a^2*b*x^6 + 10*a^3*x^4)*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.87

$$\frac{\left(4b^{3}x^{6}+15a\,b^{2}x^{4}+20a^{2}b\,x^{2}+10a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}x^{4}}{40\left(b\,x^{2}+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/40*x^4*(4*b^3*x^6+15*a*b^2*x^4+20*a^2*b*x^2+10*a^3)*((b*x^2+a)^2)^(3/2)/(
b*x^2+a)^3
```

maxima [A] time = 1.33, size = 35, normalized size = 0.52

$$\frac{1}{10}\,b^{3}x^{10}+\frac{3}{8}\,ab^{2}x^{8}+\frac{1}{2}\,a^{2}bx^{6}+\frac{1}{4}\,a^{3}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/10*b^3*x^10 + 3/8*a*b^2*x^8 + 1/2*a^2*b*x^6 + 1/4*a^3*x^4
```

mupad [B] time = 4.29, size = 46, normalized size = 0.69

$$\frac{\left(a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}\right)^{3/2}\,\left(-a^{2}+3\,a\,b\,x^{2}+4\,b^{2}\,x^{4}\right)}{40\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)*(4*b^2*x^4 - a^2 + 3*a*b*x^2))/(40*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**3*((a + b*x**2)**2)**(3/2), x)
```

3.562 $\int x \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b}$$

```
[Out] 1/8*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1107, 609}

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(8*b)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int x \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx &= \frac{1}{2}\text{Subst}\left(\int \left(a^2+2abx+b^2x^2\right)^{3/2} \, dx,x,x^2\right)\\ &= \frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^{3/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^(3/2))/(8*b)
```

fricas [A] time = 0.80, size = 35, normalized size = 0.97

$$\frac{1}{8}\,b^3x^8+\frac{1}{2}\,ab^2x^6+\frac{3}{4}\,a^2bx^4+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*b^3*x^8 + 1/2*a*b^2*x^6 + 3/4*a^2*b*x^4 + 1/2*a^3*x^2
```

giac [A] time = 0.15, size = 44, normalized size = 1.22

$$\frac{1}{8}\left(2\left(bx^4+2\,ax^2\right)a^2+\left(bx^4+2\,ax^2\right)^2b\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*(b*x^4 + 2*a*x^2)*a^2 + (b*x^4 + 2*a*x^2)^2*b)*sgn(b*x^2 + a)
```

maple [A] time = 0.00, size = 57, normalized size = 1.58

$$\frac{\left(b^3x^6+4a\,b^2x^4+6a^2b\,x^2+4a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}x^2}{8\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/8*x^2*(b^3*x^6+4*a*b^2*x^4+6*a^2*b*x^2+4*a^3)*((b*x^2+a)^2)^(3/2)/(b*x^2+
a)^3
```

maxima [A] time = 1.36, size = 35, normalized size = 0.97

$$\frac{1}{8}\,b^3x^8+\frac{1}{2}\,ab^2x^6+\frac{3}{4}\,a^2bx^4+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*b^3*x^8 + 1/2*a*b^2*x^6 + 3/4*a^2*b*x^4 + 1/2*a^3*x^2
```

mupad [B] time = 4.25, size = 36, normalized size = 1.00

$$\frac{\left(b^2\,x^2+a\,b\right)\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] ((a*b + b^2*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2))/(8*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x*((a + b*x**2)**2)**(3/2), x)
```

3.563 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x} dx$

Optimal. Leaf size=163

$$\frac{3ab^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{3a^2bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{b^3x^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}+\frac{a^3\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

```
[Out] 3/2*a^2*b*x^2*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/4*a*b^2*x^4*((b*x^2+a)^2)^(1/
2)/(b*x^2+a)+1/6*b^3*x^6*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+a^3*ln(x)*((b*x^2+a)
^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.05, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$\frac{b^3x^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}+\frac{3ab^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{3a^2bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{a^3\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x,x]
```

```
[Out] (3*a^2*b*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (3*a*b^2*x^
4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (b^3*x^6*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(6*(a + b*x^2)) + (a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*
Log[x])/(a + b*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
```

```
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x}\,dx}{b^2\left(ab+b^2x^2\right)} \\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x}\,dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)} \\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \left(3a^2b^4+\frac{a^3b^3}{x}+3ab^5x+b^6x^2\right)dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)} \\
&= \frac{3a^2bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{3ab^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{b^3x^6\sqrt{a^2+2abx^2+}}{6\left(a+bx^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(12a^3\log(x)+bx^2\left(18a^2+9abx^2+2b^2x^4\right)\right)}{12\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(b*x^2*(18*a^2 + 9*a*b*x^2 + 2*b^2*x^4) + 12*a^3*Log[x
]))/(12*(a + b*x^2))
```

fricas [A] time = 0.84, size = 33, normalized size = 0.20

$$\frac{1}{6}\,b^3x^6+\frac{3}{4}\,ab^2x^4+\frac{3}{2}\,a^2bx^2+a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + a^3*log(x)
```

giac [A] time = 0.19, size = 68, normalized size = 0.42

$$\frac{1}{6}\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{4}\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{2}\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{2}\,a^3\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/6*b^3*x^6*sgn(b*x^2 + a) + 3/4*a*b^2*x^4*sgn(b*x^2 + a) + 3/2*a^2*b*x^2*s
gn(b*x^2 + a) + 1/2*a^3*log(x^2)*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 57, normalized size = 0.35

$$\frac{\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}\left(2b^3x^6+9a\,b^2x^4+18a^2b\,x^2+12a^3\ln(x)\right)}{12\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x,x)
```

```
[Out] 1/12*((b*x^2+a)^2)^(3/2)*(2*b^3*x^6+9*a*b^2*x^4+18*a^2*b*x^2+12*a^3*ln(x))/
(b*x^2+a)^3
```

maxima [A] time = 1.33, size = 33, normalized size = 0.20

$$\frac{1}{6}\,b^3x^6+\frac{3}{4}\,ab^2x^4+\frac{3}{2}\,a^2bx^2+a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + a^3*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x, x)
```

3.564 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=164

$$\frac{3ab^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{3a^2b\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{4(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)}$$

```
[Out] -1/2*a^3*((b*x^2+a)^2)^(1/2)/x^2/(b*x^2+a)+3/2*a*b^2*x^2*((b*x^2+a)^2)^(1/2
)/(b*x^2+a)+1/4*b^3*x^4*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+3*a^2*b*ln(x)*((b*x^2
+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.05, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$\frac{b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{4(a+bx^2)}+\frac{3ab^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)}+\frac{3a^2b\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^3,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (3*a*b^2*x^2*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (b^3*x^4*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (3*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
*Log[x])/(a + b*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
```

```
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^3}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^3}\,dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^2}\,dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \left(3ab^5+\frac{a^3b^3}{x^2}+\frac{3a^2b^4}{x}+b^6x\right)dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{3ab^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{b^3x^4\sqrt{a^2+2abx^2+b^2x}}{4\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.38

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-2a^3+12a^2bx^2\log(x)+6ab^2x^4+b^3x^6\right)}{4x^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-2*a^3 + 6*a*b^2*x^4 + b^3*x^6 + 12*a^2*b*x^2*Log[x])
)/(4*x^2*(a + b*x^2))
```

fricas [A] time = 0.80, size = 38, normalized size = 0.23

$$\frac{b^3x^6+6\,ab^2x^4+12\,a^2bx^2\log(x)-2\,a^3}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*x^6 + 6*a*b^2*x^4 + 12*a^2*b*x^2*log(x) - 2*a^3)/x^2
```

giac [A] time = 0.16, size = 87, normalized size = 0.53

$$\frac{1}{4}b^3x^4\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{2}ab^2x^2\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{2}a^2b\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)-\frac{3\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+a^3\mathrm{sgn}\left(bx^2+\right.}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4*sgn(b*x^2 + a) + 3/2*a*b^2*x^2*sgn(b*x^2 + a) + 3/2*a^2*b*log(x
^2)*sgn(b*x^2 + a) - 1/2*(3*a^2*b*x^2*sgn(b*x^2 + a) + a^3*sgn(b*x^2 + a))/
x^2
```

maple [A] time = 0.01, size = 59, normalized size = 0.36

$$\frac{\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}\left(b^3x^6+6a\,b^2x^4+12a^2b\,x^2\ln(x)-2a^3\right)}{4\left(b\,x^2+a\right)^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^3,x)
```

```
[Out] 1/4*((b*x^2+a)^2)^(3/2)*(b^3*x^6+6*a*b^2*x^4+12*a^2*b*ln(x)*x^2-2*a^3)/(b*x
^2+a)^3/x^2
```

maxima [A] time = 1.29, size = 34, normalized size = 0.21

$$\frac{1}{4}b^3x^4+\frac{3}{2}ab^2x^2+3\,a^2b\log(x)-\frac{a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3*a^2*b*log(x) - 1/2*a^3/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^3,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**3,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**3, x)
```

3.565 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^5} dx$

Optimal. Leaf size=164

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{3ab^2\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}$$

```
[Out] -1/4*a^3*((b*x^2+a)^2)^(1/2)/x^4/(b*x^2+a)-3/2*a^2*b*((b*x^2+a)^2)^(1/2)/x^
2/(b*x^2+a)+1/2*b^3*x^2*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+3*a*b^2*ln(x)*((b*x^2
+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.05, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{3ab^2\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^5,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (b^3*x^2*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (3*a*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
*Log[x])/(a + b*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
```

```
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^5}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^5}\,dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^3}\,dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(b^6+\frac{a^3b^3}{x^3}+\frac{3a^2b^4}{x^2}+\frac{3ab^5}{x}\right)dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)} + \frac{b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a^3+6a^2bx^2-12ab^2x^4\log(x)-2b^3x^6\right)}{4x^4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^5,x]
[Out] -1/4*(Sqrt[(a + b*x^2)^2]*(a^3 + 6*a^2*b*x^2 - 2*b^3*x^6 - 12*a*b^2*x^4*Log
[x]))/(x^4*(a + b*x^2))
```

fricas [A] time = 0.83, size = 39, normalized size = 0.24

$$\frac{2\,b^3x^6+12\,ab^2x^4\log(x)-6\,a^2bx^2-a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^5,x, algorithm="fricas")
[Out] 1/4*(2*b^3*x^6 + 12*a*b^2*x^4*log(x) - 6*a^2*b*x^2 - a^3)/x^4
```

giac [A] time = 0.17, size = 87, normalized size = 0.53

$$\frac{1}{2}\,b^3x^2\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{2}\,ab^2\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)-\frac{9\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+6\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+a^3\mathrm{sgn}\left(bx^2+\right.}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2*sgn(b*x^2 + a) + 3/2*a*b^2*log(x^2)*sgn(b*x^2 + a) - 1/4*(9*a*b
^2*x^4*sgn(b*x^2 + a) + 6*a^2*b*x^2*sgn(b*x^2 + a) + a^3*sgn(b*x^2 + a))/x^
4
```

maple [A] time = 0.01, size = 60, normalized size = 0.37

$$\frac{\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}\left(2b^3x^6+12a\,b^2x^4\ln(x)-6a^2b\,x^2-a^3\right)}{4\left(b\,x^2+a\right)^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^5,x)
```

```
[Out] 1/4*((b*x^2+a)^2)^(3/2)*(2*b^3*x^6+12*a*b^2*ln(x)*x^4-6*a^2*b*x^2-a^3)/(b*x
^2+a)^3/x^4
```

maxima [A] time = 1.31, size = 34, normalized size = 0.21

$$\frac{1}{2}\,b^3x^2+3\,ab^2\log(x)-\frac{3\,a^2b}{2\,x^2}-\frac{a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2 + 3*a*b^2*log(x) - 3/2*a^2*b/x^2 - 1/4*a^3/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^5,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**5,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**5, x)
```

3.566 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=163

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{b^3\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}$$

```
[Out] -1/6*a^3*((b*x^2+a)^2)^(1/2)/x^6/(b*x^2+a)-3/4*a^2*b*((b*x^2+a)^2)^(1/2)/x^
4/(b*x^2+a)-3/2*a*b^2*((b*x^2+a)^2)^(1/2)/x^2/(b*x^2+a)+b^3*ln(x)*((b*x^2+a
)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.05, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{b^3\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^7,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
*Log[x])/(a + b*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
```

```
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^7}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^7}\,dx}{b^2\left(ab+b^2x^2\right)} \\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^4}\,dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)} \\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \left(\frac{a^3b^3}{x^4}+\frac{3a^2b^4}{x^3}+\frac{3ab^5}{x^2}+\frac{b^6}{x}\right)dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)} \\
&= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.39

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a\left(2a^2+9abx^2+18b^2x^4\right)-12b^3x^6\log(x)\right)}{12x^6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^7,x]
```

```
[Out] -1/12*(Sqrt[(a + b*x^2)^2]*(a*(2*a^2 + 9*a*b*x^2 + 18*b^2*x^4) - 12*b^3*x^6
*Log[x]))/(x^6*(a + b*x^2))
```

fricas [A] time = 0.95, size = 39, normalized size = 0.24

$$\frac{12\,b^3x^6\log(x)-18\,ab^2x^4-9\,a^2bx^2-2\,a^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(12*b^3*x^6*log(x) - 18*a*b^2*x^4 - 9*a^2*b*x^2 - 2*a^3)/x^6
```

giac [A] time = 0.19, size = 87, normalized size = 0.53

$$\frac{1}{2}\,b^3\log\left(x^2\right)\operatorname{sgn}\left(bx^2+a\right)-\frac{11\,b^3x^6\operatorname{sgn}\left(bx^2+a\right)+18\,ab^2x^4\operatorname{sgn}\left(bx^2+a\right)+9\,a^2bx^2\operatorname{sgn}\left(bx^2+a\right)+2\,a^3\operatorname{sgn}\left(b\right.}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/2*b^3*log(x^2)*sgn(b*x^2 + a) - 1/12*(11*b^3*x^6*sgn(b*x^2 + a) + 18*a*b^
2*x^4*sgn(b*x^2 + a) + 9*a^2*b*x^2*sgn(b*x^2 + a) + 2*a^3*sgn(b*x^2 + a))/x
^6
```

maple [A] time = 0.01, size = 60, normalized size = 0.37

$$\frac{\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}\left(12b^3x^6\ln(x)-18a\,b^2x^4-9a^2b\,x^2-2a^3\right)}{12\left(b\,x^2+a\right)^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^7,x)
```

```
[Out] 1/12*((b*x^2+a)^2)^(3/2)*(12*b^3*ln(x)*x^6-18*a*b^2*x^4-9*a^2*b*x^2-2*a^3)/
(b*x^2+a)^3/x^6
```

maxima [A] time = 1.26, size = 33, normalized size = 0.20

$$b^3\log(x)-\frac{3\,ab^2}{2\,x^2}-\frac{3\,a^2b}{4\,x^4}-\frac{a^3}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] b^3*log(x) - 3/2*a*b^2/x^2 - 3/4*a^2*b/x^4 - 1/6*a^3/x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^7,x)
```

[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^7, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**7,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**7, x)
```

3.567 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=41

$$-\frac{\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}{8ax^8}$$

```
[Out] -1/8*(b*x^2+a)^3*((b*x^2+a)^2)^(1/2)/a/x^8
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 37}

$$-\frac{\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^9,x]
```

```
[Out] -((a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*a*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^9}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^5}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^5}\,dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}{8ax^8}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 1.44

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a^3+4a^2bx^2+6ab^2x^4+4b^3x^6\right)}{8x^8\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^9,x]
```

```
[Out] -1/8*(Sqrt[(a + b*x^2)^2]*(a^3 + 4*a^2*b*x^2 + 6*a*b^2*x^4 + 4*b^3*x^6))/(x
^8*(a + b*x^2))
```

fricas [A] time = 1.00, size = 35, normalized size = 0.85

$$-\frac{4\,b^3x^6+6\,ab^2x^4+4\,a^2bx^2+a^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/x^8
```

giac [B] time = 0.17, size = 68, normalized size = 1.66

$$-\frac{4\,b^3x^6\operatorname{sgn}\left(bx^2+a\right)+6\,ab^2x^4\operatorname{sgn}\left(bx^2+a\right)+4\,a^2bx^2\operatorname{sgn}\left(bx^2+a\right)+a^3\operatorname{sgn}\left(bx^2+a\right)}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^3*x^6*sgn(b*x^2 + a) + 6*a*b^2*x^4*sgn(b*x^2 + a) + 4*a^2*b*x^2*s
gn(b*x^2 + a) + a^3*sgn(b*x^2 + a))/x^8
```

maple [A] time = 0.00, size = 56, normalized size = 1.37

$$-\frac{\left(4b^{3}x^{6}+6a\,b^{2}x^{4}+4a^{2}b\,x^{2}+a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{8\left(b\,x^{2}+a\right)^{3}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^9,x)
```

```
[Out] -1/8*(4*b^3*x^6+6*a*b^2*x^4+4*a^2*b*x^2+a^3)*((b*x^2+a)^2)^(3/2)/x^8/(b*x^2
+a)^3
```

maxima [A] time = 1.36, size = 35, normalized size = 0.85

$$-\frac{b^{3}}{2\,x^{2}}-\frac{3\,ab^{2}}{4\,x^{4}}-\frac{a^{2}b}{2\,x^{6}}-\frac{a^{3}}{8\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/2*b^3/x^2 - 3/4*a*b^2/x^4 - 1/2*a^2*b/x^6 - 1/8*a^3/x^8
```

mupad [B] time = 4.24, size = 151, normalized size = 3.68

$$-\frac{a^{3}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{8\,x^{8}\,\left(b\,x^{2}+a\right)}-\frac{b^{3}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{2\,x^{2}\,\left(b\,x^{2}+a\right)}-\frac{3\,a\,b^{2}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{4\,x^{4}\,\left(b\,x^{2}+a\right)}-\frac{a^{2}\,b\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{2\,x^{6}\,\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^9,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(8*x^8*(a + b*x^2)) - (b^3*(a^2 +
 b^2*x^4 + 2*a*b*x^2)^(1/2))/(2*x^2*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*x^4
+ 2*a*b*x^2)^(1/2))/(4*x^4*(a + b*x^2)) - (a^2*b*(a^2 + b^2*x^4 + 2*a*b*x^2
)^(1/2))/(2*x^6*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**9,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**9, x)
```

3.568 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{11}}\,dx$

Optimal. Leaf size=72

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{40a^2x^{10}}-\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8ax^{10}}$$

```
[Out] -1/8*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/a/x^10+1/40*(b^2*x^4+2*a*b*x^2
+a^2)^(5/2)/a^2/x^10
```

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1110}

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{40a^2x^{10}}-\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^11,x]
```

```
[Out] -((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(8*a*x^10) + (a^2 + 2*a*b*
x^2 + b^2*x^4)^(5/2)/(40*a^2*x^10)
```

Rule 1110

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(4*a*d*(p + 1)*(2*p +
1)), x] - Simp[((d*x)^(m + 1)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^p)/(4*a*d*(
2*p + 1)), x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !I
ntegerQ[p] && EqQ[m + 4*p + 5, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{11}}\,dx=-\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8ax^{10}}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{40a^2x^{10}}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.85

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(4a^3+15a^2bx^2+20ab^2x^4+10b^3x^6\right)}{40x^{10}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^11,x]
```

```
[Out] -1/40*(Sqrt[(a + b*x^2)^2]*(4*a^3 + 15*a^2*b*x^2 + 20*a*b^2*x^4 + 10*b^3*x^
6))/(x^10*(a + b*x^2))
```

fricas [A] time = 0.88, size = 37, normalized size = 0.51

$$-\frac{10\,b^3x^6 + 20\,ab^2x^4 + 15\,a^2bx^2 + 4\,a^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^10
```

giac [A] time = 0.21, size = 69, normalized size = 0.96

$$-\frac{10\,b^3x^6\mathrm{sgn}\left(bx^2 + a\right) + 20\,ab^2x^4\mathrm{sgn}\left(bx^2 + a\right) + 15\,a^2bx^2\mathrm{sgn}\left(bx^2 + a\right) + 4\,a^3\mathrm{sgn}\left(bx^2 + a\right)}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] -1/40*(10*b^3*x^6*sgn(b*x^2 + a) + 20*a*b^2*x^4*sgn(b*x^2 + a) + 15*a^2*b*x
^2*sgn(b*x^2 + a) + 4*a^3*sgn(b*x^2 + a))/x^10
```

maple [A] time = 0.01, size = 58, normalized size = 0.81

$$-\frac{\left(10b^3x^6 + 20a\,b^2x^4 + 15a^2b\,x^2 + 4a^3\right)\left(\left(b\,x^2 + a\right)^2\right)^{\frac{3}{2}}}{40\left(b\,x^2 + a\right)^3 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^11,x)
```

```
[Out] -1/40*(10*b^3*x^6+20*a*b^2*x^4+15*a^2*b*x^2+4*a^3)*((b*x^2+a)^2)^(3/2)/x^10
/(b*x^2+a)^3
```

maxima [A] time = 1.43, size = 35, normalized size = 0.49

$$-\frac{b^3}{4\,x^4} - \frac{ab^2}{2\,x^6} - \frac{3\,a^2b}{8\,x^8} - \frac{a^3}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] -1/4*b^3/x^4 - 1/2*a*b^2/x^6 - 3/8*a^2*b/x^8 - 1/10*a^3/x^10
```

mupad [B] time = 4.20, size = 151, normalized size = 2.10

$$-\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{10\,x^{10}\,\left(b\,x^2+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^4\,\left(b\,x^2+a\right)}-\frac{a\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^6\,\left(b\,x^2+a\right)}-\frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^8\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^11,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(10*x^10*(a + b*x^2)) - (b^3*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(4*x^4*(a + b*x^2)) - (a*b^2*(a^2 + b^2*x^4
+ 2*a*b*x^2)^(1/2))/(2*x^6*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(8*x^8*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**11,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**11, x)
```

3.569 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}\left(a+bx^2\right)}$$

```
[Out] -1/12*a^3*((b*x^2+a)^2)^(1/2)/x^12/(b*x^2+a)-3/10*a^2*b*((b*x^2+a)^2)^(1/2)
/x^10/(b*x^2+a)-3/8*a*b^2*((b*x^2+a)^2)^(1/2)/x^8/(b*x^2+a)-1/6*b^3*((b*x^2
+a)^2)^(1/2)/x^6/(b*x^2+a)
```

Rubi [A] time = 0.10, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^13,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*x^12*(a + b*x^2)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])/(6*x^6*(a + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{13}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^7}\,dx,x,x^2\right)\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^7}\,dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int \left(\frac{a^3b^3}{x^7}+\frac{3a^2b^4}{x^6}+\frac{3ab^5}{x^5}+\frac{b^6}{x^4}\right)dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(10a^3+36a^2bx^2+45ab^2x^4+20b^3x^6\right)}{120x^{12}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^13,x]
```

```
[Out] -1/120*(Sqrt[(a + b*x^2)^2]*(10*a^3 + 36*a^2*b*x^2 + 45*a*b^2*x^4 + 20*b^3*
x^6))/(x^12*(a + b*x^2))
```

fricas [A] time = 0.79, size = 37, normalized size = 0.22

$$-\frac{20\,b^3x^6+45\,ab^2x^4+36\,a^2bx^2+10\,a^3}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^12
```

giac [A] time = 0.16, size = 69, normalized size = 0.41

$$-\frac{20\,b^3x^6\text{sgn}\left(bx^2+a\right)+45\,ab^2x^4\text{sgn}\left(bx^2+a\right)+36\,a^2bx^2\text{sgn}\left(bx^2+a\right)+10\,a^3\text{sgn}\left(bx^2+a\right)}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] -1/120*(20*b^3*x^6*sgn(b*x^2 + a) + 45*a*b^2*x^4*sgn(b*x^2 + a) + 36*a^2*b*
x^2*sgn(b*x^2 + a) + 10*a^3*sgn(b*x^2 + a))/x^12
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(20b^3x^6+45a\,b^2x^4+36a^2b\,x^2+10a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{120\left(b\,x^2+a\right)^3x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^13,x)
```

```
[Out] -1/120*(20*b^3*x^6+45*a*b^2*x^4+36*a^2*b*x^2+10*a^3)*((b*x^2+a)^2)^(3/2)/x^
12/(b*x^2+a)^3
```

maxima [A] time = 1.31, size = 35, normalized size = 0.21

$$-\frac{b^3}{6\,x^6}-\frac{3\,ab^2}{8\,x^8}-\frac{3\,a^2b}{10\,x^{10}}-\frac{a^3}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] -1/6*b^3/x^6 - 3/8*a*b^2/x^8 - 3/10*a^2*b/x^10 - 1/12*a^3/x^12
```

mupad [B] time = 4.21, size = 151, normalized size = 0.90

$$-\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{12\,x^{12}\,\left(b\,x^2+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{6\,x^6\,\left(b\,x^2+a\right)}-\frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^8\,\left(b\,x^2+a\right)}-\frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{10\,x^{10}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^13,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(12*x^12*(a + b*x^2)) - (b^3*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(6*x^6*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(8*x^8*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a*b
*x^2)^(1/2))/(10*x^10*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**13,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**13, x)
```

3.570 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{15}} dx$

Optimal. Leaf size=167

$$-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}(a+bx^2)}$$

```
[Out] -1/14*a^3*((b*x^2+a)^2)^(1/2)/x^14/(b*x^2+a)-1/4*a^2*b*((b*x^2+a)^2)^(1/2)/
x^12/(b*x^2+a)-3/10*a*b^2*((b*x^2+a)^2)^(1/2)/x^10/(b*x^2+a)-1/8*b^3*((b*x^
2+a)^2)^(1/2)/x^8/(b*x^2+a)
```

Rubi [A] time = 0.11, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}(a+bx^2)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^15,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*x^14*(a + b*x^2)) - (a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^12*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])/(8*x^8*(a + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{15}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^8}\,dx,x,x^2\right)\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^8}\,dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int \left(\frac{a^3b^3}{x^8}+\frac{3a^2b^4}{x^7}+\frac{3ab^5}{x^6}+\frac{b^6}{x^5}\right)dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}\left(a+bx^2\right)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(20a^3+70a^2bx^2+84ab^2x^4+35b^3x^6\right)}{280x^{14}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^15,x]
```

```
[Out] -1/280*(Sqrt[(a + b*x^2)^2]*(20*a^3 + 70*a^2*b*x^2 + 84*a*b^2*x^4 + 35*b^3*
x^6))/(x^14*(a + b*x^2))
```

fricas [A] time = 0.88, size = 37, normalized size = 0.22

$$-\frac{35\,b^3x^6+84\,ab^2x^4+70\,a^2bx^2+20\,a^3}{280\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^15,x, algorithm="fricas")
```

```
[Out] -1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^14
```

giac [A] time = 0.16, size = 69, normalized size = 0.41

$$-\frac{35\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+84\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+70\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+20\,a^3\mathrm{sgn}\left(bx^2+a\right)}{280\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^15,x, algorithm="giac")
```

```
[Out] -1/280*(35*b^3*x^6*sgn(b*x^2 + a) + 84*a*b^2*x^4*sgn(b*x^2 + a) + 70*a^2*b*
x^2*sgn(b*x^2 + a) + 20*a^3*sgn(b*x^2 + a))/x^14
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(35b^3x^6+84a\,b^2x^4+70a^2b\,x^2+20a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{280\left(b\,x^2+a\right)^3x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^15,x)
```

```
[Out] -1/280*(35*b^3*x^6+84*a*b^2*x^4+70*a^2*b*x^2+20*a^3)*((b*x^2+a)^2)^(3/2)/x^
14/(b*x^2+a)^3
```

maxima [A] time = 1.13, size = 35, normalized size = 0.21

$$-\frac{b^3}{8\,x^8}-\frac{3\,ab^2}{10\,x^{10}}-\frac{a^2b}{4\,x^{12}}-\frac{a^3}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^15,x, algorithm="maxima")
```

```
[Out] -1/8*b^3/x^8 - 3/10*a*b^2/x^10 - 1/4*a^2*b/x^12 - 1/14*a^3/x^14
```

mupad [B] time = 4.21, size = 151, normalized size = 0.90

$$-\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{14\,x^{14}\,\left(b\,x^2+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^8\,\left(b\,x^2+a\right)}-\frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{10\,x^{10}\,\left(b\,x^2+a\right)}-\frac{a^2\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^{12}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^15,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(14*x^14*(a + b*x^2)) - (b^3*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(8*x^8*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(10*x^10*(a + b*x^2)) - (a^2*b*(a^2 + b^2*x^4 + 2*a*b
*x^2)^(1/2))/(4*x^12*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**15,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**15, x)
```

3.571 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{17}} dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}(a+bx^2)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}(a+bx^2)}$$

```
[Out] -1/16*a^3*((b*x^2+a)^2)^(1/2)/x^16/(b*x^2+a)-3/14*a^2*b*((b*x^2+a)^2)^(1/2)
/x^14/(b*x^2+a)-1/4*a*b^2*((b*x^2+a)^2)^(1/2)/x^12/(b*x^2+a)-1/10*b^3*((b*x
^2+a)^2)^(1/2)/x^10/(b*x^2+a)
```

Rubi [A] time = 0.10, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}(a+bx^2)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}(a+bx^2)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^17,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*x^16*(a + b*x^2)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*x^14*(a + b*x^2)) - (a*b^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(4*x^12*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])/(10*x^10*(a + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{17}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^9}\,dx,x,x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\;\mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^9}\,dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\;\mathrm{Subst}\left(\int \left(\frac{a^3b^3}{x^9}+\frac{3a^2b^4}{x^8}+\frac{3ab^5}{x^7}+\frac{b^6}{x^6}\right)dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}\left(a+bx^2\right)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(35a^3+120a^2bx^2+140ab^2x^4+56b^3x^6\right)}{560x^{16}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^17,x]
```

```
[Out] -1/560*(Sqrt[(a + b*x^2)^2]*(35*a^3 + 120*a^2*b*x^2 + 140*a*b^2*x^4 + 56*b^
3*x^6))/(x^16*(a + b*x^2))
```

fricas [A] time = 0.98, size = 37, normalized size = 0.22

$$-\frac{56\,b^3x^6+140\,ab^2x^4+120\,a^2bx^2+35\,a^3}{560\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^17,x, algorithm="fricas")
```

[Out] -1/560*(56*b^3*x^6 + 140*a*b^2*x^4 + 120*a^2*b*x^2 + 35*a^3)/x^16

giac [A] time = 0.18, size = 69, normalized size = 0.41

$$-\frac{56\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+140\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+120\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+35\,a^3\mathrm{sgn}\left(bx^2+a\right)}{560\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^17,x, algorithm="giac")

[Out] -1/560*(56*b^3*x^6*sgn(b*x^2 + a) + 140*a*b^2*x^4*sgn(b*x^2 + a) + 120*a^2*b*x^2*sgn(b*x^2 + a) + 35*a^3*sgn(b*x^2 + a))/x^16

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(56b^3x^6+140a\,b^2x^4+120a^2b\,x^2+35a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{560\left(b\,x^2+a\right)^3x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^17,x)

[Out] -1/560*(56*b^3*x^6+140*a*b^2*x^4+120*a^2*b*x^2+35*a^3)*((b*x^2+a)^2)^(3/2)/x^16/(b*x^2+a)^3

maxima [A] time = 1.32, size = 35, normalized size = 0.21

$$-\frac{b^3}{10\,x^{10}}-\frac{ab^2}{4\,x^{12}}-\frac{3\,a^2b}{14\,x^{14}}-\frac{a^3}{16\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^17,x, algorithm="maxima")

[Out] -1/10*b^3/x^10 - 1/4*a*b^2/x^12 - 3/14*a^2*b/x^14 - 1/16*a^3/x^16

mupad [B] time = 4.23, size = 151, normalized size = 0.90

$$-\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{16\,x^{16}\left(b\,x^2+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{10\,x^{10}\left(b\,x^2+a\right)}-\frac{a\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^{12}\left(b\,x^2+a\right)}-\frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{14\,x^{14}\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^17,x)

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(16*x^16*(a + b*x^2)) - (b^3*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(10*x^10*(a + b*x^2)) - (a*b^2*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(4*x^12*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a*
b*x^2)^(1/2))/(14*x^14*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**17,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**17, x)
```

3.572 $\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{b^3x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{15\left(a+bx^2\right)}+\frac{a^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}$$

```
[Out] 1/9*a^3*x^9*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/11*a^2*b*x^11*((b*x^2+a)^2)^(1/
2)/(b*x^2+a)+3/13*a*b^2*x^13*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/15*b^3*x^15*((
b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^3x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{15\left(a+bx^2\right)}+\frac{3ab^2x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{a^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (3*a^2*b*x^11*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (3*a*b^2*x^13*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (b^3*x^15*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(15*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^8 \left(ab+b^2x^2\right)^3 dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^3b^3x^8+3a^2b^4x^{10}+3ab^5x^{12}+b^6x^{14}\right) dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{a^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)} + \frac{3a^2bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)} + \frac{3ab^2x^{13}\sqrt{a^2+2a}}{13\left(a+b\right.}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^9\sqrt{\left(a+bx^2\right)^2}\left(715a^3+1755a^2bx^2+1485ab^2x^4+429b^3x^6\right)}{6435\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^9*Sqrt[(a + b*x^2)^2]*(715*a^3 + 1755*a^2*b*x^2 + 1485*a*b^2*x^4 + 429*b
^3*x^6))/(6435*(a + b*x^2))
```

fricas [A] time = 0.96, size = 35, normalized size = 0.21

$$\frac{1}{15}b^3x^{15}+\frac{3}{13}ab^2x^{13}+\frac{3}{11}a^2bx^{11}+\frac{1}{9}a^3x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*b^3*x^15 + 3/13*a*b^2*x^13 + 3/11*a^2*b*x^11 + 1/9*a^3*x^9
```

giac [A] time = 0.16, size = 67, normalized size = 0.40

$$\frac{1}{15}b^3x^{15}\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{13}ab^2x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{11}a^2bx^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{9}a^3x^9\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*b^3*x^15*sgn(b*x^2 + a) + 3/13*a*b^2*x^13*sgn(b*x^2 + a) + 3/11*a^2*b*
x^11*sgn(b*x^2 + a) + 1/9*a^3*x^9*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{\left(429b^3x^6+1485a\,b^2x^4+1755a^2b\,x^2+715a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}x^9}{6435\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/6435*x^9*(429*b^3*x^6+1485*a*b^2*x^4+1755*a^2*b*x^2+715*a^3)*((b*x^2+a)^2
)^(3/2)/(b*x^2+a)^3
```

maxima [A] time = 1.30, size = 35, normalized size = 0.21

$$\frac{1}{15}\,b^3x^{15}+\frac{3}{13}\,ab^2x^{13}+\frac{3}{11}\,a^2bx^{11}+\frac{1}{9}\,a^3x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/15*b^3*x^15 + 3/13*a*b^2*x^13 + 3/11*a^2*b*x^11 + 1/9*a^3*x^9
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^8\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^8*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**8*((a + b*x**2)**2)**(3/2), x)
```

3.573 $\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{a^2bx^9\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{b^3x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{a^3x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}$$

```
[Out] 1/7*a^3*x^7*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/3*a^2*b*x^9*((b*x^2+a)^2)^(1/2)
/(b*x^2+a)+3/11*a*b^2*x^11*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/13*b^3*x^13*((b*
x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^3x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{3ab^2x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{a^2bx^9\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{a^3x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (a^2*b*x^9*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (3*a*b^2*x^11*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (b^3*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])/(13*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^6 \left(ab+b^2x^2\right)^3 dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^3b^3x^6+3a^2b^4x^8+3ab^5x^{10}+b^6x^{12}\right) dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{a^3x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)} + \frac{a^2bx^9\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)} + \frac{3ab^2x^{11}\sqrt{a^2+2abx^2}}{11\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^7\sqrt{\left(a+bx^2\right)^2}\left(429a^3+1001a^2bx^2+819ab^2x^4+231b^3x^6\right)}{3003\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^7*Sqrt[(a + b*x^2)^2]*(429*a^3 + 1001*a^2*b*x^2 + 819*a*b^2*x^4 + 231*b^
3*x^6))/(3003*(a + b*x^2))
```

fricas [A] time = 0.69, size = 35, normalized size = 0.21

$$\frac{1}{13}b^3x^{13} + \frac{3}{11}ab^2x^{11} + \frac{1}{3}a^2bx^9 + \frac{1}{7}a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

giac [A] time = 0.16, size = 67, normalized size = 0.40

$$\frac{1}{13}b^3x^{13}\mathrm{sgn}\left(bx^2+a\right) + \frac{3}{11}ab^2x^{11}\mathrm{sgn}\left(bx^2+a\right) + \frac{1}{3}a^2bx^9\mathrm{sgn}\left(bx^2+a\right) + \frac{1}{7}a^3x^7\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/13*b^3*x^13*sgn(b*x^2 + a) + 3/11*a*b^2*x^11*sgn(b*x^2 + a) + 1/3*a^2*b*x
^9*sgn(b*x^2 + a) + 1/7*a^3*x^7*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{\left(231b^3x^6 + 819a\,b^2x^4 + 1001a^2b\,x^2 + 429a^3\right)\left(\left(b\,x^2 + a\right)^2\right)^{\frac{3}{2}} x^7}{3003\left(b\,x^2 + a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/3003*x^7*(231*b^3*x^6+819*a*b^2*x^4+1001*a^2*b*x^2+429*a^3)*((b*x^2+a)^2)
^(3/2)/(b*x^2+a)^3
```

maxima [A] time = 1.27, size = 35, normalized size = 0.21

$$\frac{1}{13}\,b^3x^{13} + \frac{3}{11}\,ab^2x^{11} + \frac{1}{3}\,a^2bx^9 + \frac{1}{7}\,a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^6\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**6*((a + b*x**2)**2)**(3/2), x)
```

3.574 $\int x^4 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{ab^2x^9\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{b^3x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{a^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}$$

```
[Out] 1/5*a^3*x^5*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/7*a^2*b*x^7*((b*x^2+a)^2)^(1/2)
/(b*x^2+a)+1/3*a*b^2*x^9*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/11*b^3*x^11*((b*x^
2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^3x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{ab^2x^9\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{a^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (3*a^2*b*x^7*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (a*b^2*x^9*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (b^3*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(11*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^4\left(ab+b^2x^2\right)^3 dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a^3b^3x^4+3a^2b^4x^6+3ab^5x^8+b^6x^{10}\right) dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{a^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)} + \frac{3a^2bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)} + \frac{ab^2x^9\sqrt{a^2+2abx^2}}{3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^5\sqrt{\left(a+bx^2\right)^2}\left(231a^3+495a^2bx^2+385ab^2x^4+105b^3x^6\right)}{1155\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^2)^2]*(231*a^3 + 495*a^2*b*x^2 + 385*a*b^2*x^4 + 105*b^3
*x^6))/(1155*(a + b*x^2))
```

fricas [A] time = 0.68, size = 35, normalized size = 0.21

$$\frac{1}{11}b^3x^{11} + \frac{1}{3}ab^2x^9 + \frac{3}{7}a^2bx^7 + \frac{1}{5}a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

giac [A] time = 0.18, size = 67, normalized size = 0.40

$$\frac{1}{11}b^3x^{11}\mathrm{sgn}\left(bx^2+a\right) + \frac{1}{3}ab^2x^9\mathrm{sgn}\left(bx^2+a\right) + \frac{3}{7}a^2bx^7\mathrm{sgn}\left(bx^2+a\right) + \frac{1}{5}a^3x^5\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/11*b^3*x^11*sgn(b*x^2 + a) + 1/3*a*b^2*x^9*sgn(b*x^2 + a) + 3/7*a^2*b*x^7
*sgn(b*x^2 + a) + 1/5*a^3*x^5*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{\left(105b^3x^6+385a\,b^2x^4+495a^2b\,x^2+231a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}x^5}{1155\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/1155*x^5*(105*b^3*x^6+385*a*b^2*x^4+495*a^2*b*x^2+231*a^3)*((b*x^2+a)^2)^
(3/2)/(b*x^2+a)^3
```

maxima [A] time = 1.39, size = 35, normalized size = 0.21

$$\frac{1}{11}\,b^3x^{11}+\frac{1}{3}\,ab^2x^9+\frac{3}{7}\,a^2bx^7+\frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**4*((a + b*x**2)**2)**(3/2), x)
```

3.575 $\int x^2 \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{3a^2bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{b^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

```
[Out] 1/3*a^3*x^3*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/5*a^2*b*x^5*((b*x^2+a)^2)^(1/2)
/(b*x^2+a)+3/7*a*b^2*x^7*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/9*b^3*x^9*((b*x^2+
a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{3ab^2x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{3a^2bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (3*a^2*b*x^5*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (3*a*b^2*x^7*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (b^3*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])/(9*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^2 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^2 \left(ab+b^2x^2\right)^3 dx}{b^2 \left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^3b^3x^2+3a^2b^4x^4+3ab^5x^6+b^6x^8\right) dx}{b^2 \left(ab+b^2x^2\right)}$$
$$= \frac{a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)} + \frac{3a^2bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)} + \frac{3ab^2x^7\sqrt{a^2+2abx^2}}{7\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^2\right)^2} \left(105a^3x^3+189a^2bx^5+135ab^2x^7+35b^3x^9\right)}{315\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(105*a^3*x^3 + 189*a^2*b*x^5 + 135*a*b^2*x^7 + 35*b^3*
x^9))/(315*(a + b*x^2))
```

fricas [A] time = 0.91, size = 35, normalized size = 0.21

$$\frac{1}{9}b^3x^9+\frac{3}{7}ab^2x^7+\frac{3}{5}a^2bx^5+\frac{1}{3}a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

giac [A] time = 0.16, size = 67, normalized size = 0.40

$$\frac{1}{9}b^3x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{7}ab^2x^7\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{5}a^2bx^5\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{3}a^3x^3\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/9*b^3*x^9*sgn(b*x^2 + a) + 3/7*a*b^2*x^7*sgn(b*x^2 + a) + 3/5*a^2*b*x^5*s
gn(b*x^2 + a) + 1/3*a^3*x^3*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{\left(35b^3x^6+135a\,b^2x^4+189a^2b\,x^2+105a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}x^3}{315\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/315*x^3*(35*b^3*x^6+135*a*b^2*x^4+189*a^2*b*x^2+105*a^3)*((b*x^2+a)^2)^(3
/2)/(b*x^2+a)^3
```

maxima [A] time = 1.35, size = 35, normalized size = 0.21

$$\frac{1}{9}\,b^3x^9+\frac{3}{7}\,ab^2x^7+\frac{3}{5}\,a^2bx^5+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**2*((a + b*x**2)**2)**(3/2), x)
```

3.576 $\int \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx$

Optimal. Leaf size=159

$$\frac{3ab^2x^5\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{5\left(a+bx^2\right)^3}+\frac{a^2bx^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\left(a+bx^2\right)^3}+\frac{b^3x^7\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{7\left(a+bx^2\right)^3}+\frac{a^3x\left(a^2+2abx^2+b^2\right.}{\left(a+bx^2\right)^3}$$

```
[Out] a^3*x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(b*x^2+a)^3+a^2*b*x^3*(b^2*x^4+2*a*b*x^
2+a^2)^(3/2)/(b*x^2+a)^3+3/5*a*b^2*x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(b*x^2
+a)^3+1/7*b^3*x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(b*x^2+a)^3
```

Rubi [A] time = 0.03, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1088, 194}

$$\frac{b^3x^7\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{7\left(a+bx^2\right)^3}+\frac{3ab^2x^5\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{5\left(a+bx^2\right)^3}+\frac{a^2bx^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\left(a+bx^2\right)^3}+\frac{a^3x\left(a^2+2abx^2+b^2\right.}{\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(a + b*x^2)^3 + (a^2*b*x^3*(a^2 +
 2*a*b*x^2 + b^2*x^4)^(3/2))/(a + b*x^2)^3 + (3*a*b^2*x^5*(a^2 + 2*a*b*x^2
+ b^2*x^4)^(3/2))/(5*(a + b*x^2)^3) + (b^3*x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^
(3/2))/(7*(a + b*x^2)^3)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1088

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx = \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}\int \left(2ab+2b^2x^2\right)^3\,dx}{\left(2ab+2b^2x^2\right)^3}$$
$$= \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}\int \left(8a^3b^3+24a^2b^4x^2+24ab^5x^4+8b^6x^6\right)dx}{\left(2ab+2b^2x^2\right)^3}$$
$$= \frac{a^3x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\left(a+bx^2\right)^3}+\frac{a^2bx^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\left(a+bx^2\right)^3}+\frac{3ab^2x^5\left(a^2+2abx\right.}{5\left(a+bx\right.}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(35a^3x+35a^2bx^3+21ab^2x^5+5b^3x^7\right)}{35\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
[Out] (Sqrt[(a + b*x^2)^2]*(35*a^3*x + 35*a^2*b*x^3 + 21*a*b^2*x^5 + 5*b^3*x^7))/
(35*(a + b*x^2))
```

fricas [A] time = 0.85, size = 31, normalized size = 0.19

$$\frac{1}{7}b^3x^7+\frac{3}{5}ab^2x^5+a^2bx^3+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
[Out] 1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

giac [A] time = 0.18, size = 63, normalized size = 0.40

$$\frac{1}{7}b^3x^7\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{5}ab^2x^5\mathrm{sgn}\left(bx^2+a\right)+a^2bx^3\mathrm{sgn}\left(bx^2+a\right)+a^3x\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/7*b^3*x^7*sgn(b*x^2 + a) + 3/5*a*b^2*x^5*sgn(b*x^2 + a) + a^2*b*x^3*sgn(b
*x^2 + a) + a^3*x*sgn(b*x^2 + a)
```

maple [A] time = 0.00, size = 56, normalized size = 0.35

$$\frac{\left(5b^3x^6 + 21a\,b^2x^4 + 35a^2b\,x^2 + 35a^3\right)\left(\left(b\,x^2 + a\right)^2\right)^{\frac{3}{2}} x}{35\left(b\,x^2 + a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/35*x*(5*b^3*x^6+21*a*b^2*x^4+35*a^2*b*x^2+35*a^3)*((b*x^2+a)^2)^(3/2)/(b*
x^2+a)^3
```

maxima [A] time = 1.36, size = 31, normalized size = 0.19

$$\frac{1}{7}\,b^3x^7 + \frac{3}{5}\,ab^2x^5 + a^2bx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2abx^2 + b^2x^4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2), x)
```

3.577 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^2} \, dx$

Optimal. Leaf size=158

$$\frac{3a^2bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{ab^2x^3\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

```
[Out] -a^3*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)+3*a^2*b*x*((b*x^2+a)^2)^(1/2)/(b*x^2+a
)+a*b^2*x^3*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/5*b^3*x^5*((b*x^2+a)^2)^(1/2)/(
b*x^2+a)
```

Rubi [A] time = 0.04, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{ab^2x^3\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{3a^2bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^2,x]
```

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))) + (3*a^2*b*x*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (a*b^2*x^3*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(a + b*x^2) + (b^3*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a +
b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^2}\,dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(3a^2b^4+\frac{a^3b^3}{x^2}+3ab^5x^2+b^6x^4\right)dx}{b^2\left(ab+b^2x^2\right)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{3a^2bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{ab^2x^3\sqrt{a^2+2abx^2+b^2x}}{a+bx^2}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 0.38

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-5a^3+15a^2bx^2+5ab^2x^4+b^3x^6\right)}{5x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-5*a^3 + 15*a^2*b*x^2 + 5*a*b^2*x^4 + b^3*x^6))/(5*x*
(a + b*x^2))
```

fricas [A] time = 0.67, size = 36, normalized size = 0.23

$$\frac{b^3x^6+5\,ab^2x^4+15\,a^2bx^2-5\,a^3}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/5*(b^3*x^6 + 5*a*b^2*x^4 + 15*a^2*b*x^2 - 5*a^3)/x
```

giac [A] time = 0.16, size = 64, normalized size = 0.41

$$\frac{1}{5}\,b^3x^5\mathrm{sgn}\left(bx^2+a\right)+ab^2x^3\mathrm{sgn}\left(bx^2+a\right)+3\,a^2bx\mathrm{sgn}\left(bx^2+a\right)-\frac{a^3\mathrm{sgn}\left(bx^2+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*b^3*x^5*sgn(b*x^2 + a) + a*b^2*x^3*sgn(b*x^2 + a) + 3*a^2*b*x*sgn(b*x^2
 + a) - a^3*sgn(b*x^2 + a)/x
```

maple [A] time = 0.01, size = 58, normalized size = 0.37

$$-\frac{\left(-b^{3} x^{6}-5 a \,b^{2} x^{4}-15 a^{2} b \,x^{2}+5 a^{3}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{5 \left(b \,x^{2}+a \right)^{3} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^2,x)
```

```
[Out] -1/5*(-b^3*x^6-5*a*b^2*x^4-15*a^2*b*x^2+5*a^3)*((b*x^2+a)^2)^(3/2)/x/(b*x^2
+a)^3
```

maxima [A] time = 1.34, size = 32, normalized size = 0.20

$$\frac{1}{5} b^{3} x^{5} + a b^{2} x^{3} + 3 \, a^{2} b x - \frac{a^{3}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^3*x^5 + a*b^2*x^3 + 3*a^2*b*x - a^3/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^2,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + b x^{2}\right)^{2}\right)^{\frac{3}{2}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**2,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**2, x)
```

3.578 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=161

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{3ab^2x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

```
[Out] -1/3*a^3*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)-3*a^2*b*((b*x^2+a)^2)^(1/2)/x/(b
*x^2+a)+3*a*b^2*x*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/3*b^3*x^3*((b*x^2+a)^2)^(
1/2)/(b*x^2+a)
```

Rubi [A] time = 0.04, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{3ab^2x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^4,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (3*a*b^2*x*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(a + b*x^2) + (b^3*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(
a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^4}\,dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(3ab^5+\frac{a^3b^3}{x^4}+\frac{3a^2b^4}{x^2}+b^6x^2\right)dx}{b^2\left(ab+b^2x^2\right)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)} + \frac{3ab^2x\sqrt{a^2+2abx^2+b^2}}{a+bx^2}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a^3+9a^2bx^2-9ab^2x^4-b^3x^6\right)}{3x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^4,x]
```

```
[Out] -1/3*(Sqrt[(a + b*x^2)^2]*(a^3 + 9*a^2*b*x^2 - 9*a*b^2*x^4 - b^3*x^6))/(x^3
*(a + b*x^2))
```

fricas [A] time = 0.94, size = 36, normalized size = 0.22

$$\frac{b^3x^6+9\,ab^2x^4-9\,a^2bx^2-a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(b^3*x^6 + 9*a*b^2*x^4 - 9*a^2*b*x^2 - a^3)/x^3
```

giac [A] time = 0.16, size = 67, normalized size = 0.42

$$\frac{1}{3}\,b^3x^3\mathrm{sgn}\left(bx^2+a\right)+3\,ab^2x\mathrm{sgn}\left(bx^2+a\right)-\frac{9\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+a^3\mathrm{sgn}\left(bx^2+a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3*sgn(b*x^2 + a) + 3*a*b^2*x*sgn(b*x^2 + a) - 1/3*(9*a^2*b*x^2*sg
n(b*x^2 + a) + a^3*sgn(b*x^2 + a))/x^3
```

maple [A] time = 0.01, size = 56, normalized size = 0.35

$$-\frac{\left(-b^{3}x^{6}-9a\,b^{2}x^{4}+9a^{2}b\,x^{2}+a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{3\left(b\,x^{2}+a\right)^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^4,x)
```

```
[Out] -1/3*(-b^3*x^6-9*a*b^2*x^4+9*a^2*b*x^2+a^3)*((b*x^2+a)^2)^(3/2)/x^3/(b*x^2+
a)^3
```

maxima [A] time = 1.29, size = 33, normalized size = 0.20

$$\frac{1}{3}\,b^{3}x^{3}+3\,ab^{2}x-\frac{3\,a^{2}b}{x}-\frac{a^{3}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3 + 3*a*b^2*x - 3*a^2*b/x - 1/3*a^3/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^4,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**4,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**4, x)
```

3.579 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=158

$$-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x^3\left(a+bx^2\right)}+\frac{b^3x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}$$

```
[Out] -1/5*a^3*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)-a^2*b*((b*x^2+a)^2)^(1/2)/x^3/(b
*x^2+a)-3*a*b^2*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)+b^3*x*((b*x^2+a)^2)^(1/2)/(
b*x^2+a)
```

Rubi [A] time = 0.04, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x^3\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{b^3x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^6,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (a^2*b*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(x^3*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(x*(a + b*x^2)) + (b^3*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a +
 b*x^2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^6}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^6}\,dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^6+\frac{a^3b^3}{x^6}+\frac{3a^2b^4}{x^4}+\frac{3ab^5}{x^2}\right)dx}{b^2\left(ab+b^2x^2\right)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)} - \frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x^3\left(a+bx^2\right)} - \frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)} +$$

Mathematica [**A**] time = 0.01, size = 59, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a^3+5a^2bx^2+15ab^2x^4-5b^3x^6\right)}{5x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^6,x]
```

```
[Out] -1/5*(Sqrt[(a + b*x^2)^2]*(a^3 + 5*a^2*b*x^2 + 15*a*b^2*x^4 - 5*b^3*x^6))/(
x^5*(a + b*x^2))
```

fricas [**A**] time = 0.97, size = 37, normalized size = 0.23

$$\frac{5\,b^3x^6-15\,ab^2x^4-5\,a^2bx^2-a^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/5*(5*b^3*x^6 - 15*a*b^2*x^4 - 5*a^2*b*x^2 - a^3)/x^5
```

giac [**A**] time = 0.19, size = 66, normalized size = 0.42

$$b^3x\operatorname{sgn}\left(bx^2+a\right) - \frac{15\,ab^2x^4\operatorname{sgn}\left(bx^2+a\right)+5\,a^2bx^2\operatorname{sgn}\left(bx^2+a\right)+a^3\operatorname{sgn}\left(bx^2+a\right)}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] b^3*x*sgn(b*x^2 + a) - 1/5*(15*a*b^2*x^4*sgn(b*x^2 + a) + 5*a^2*b*x^2*sgn(b
*x^2 + a) + a^3*sgn(b*x^2 + a))/x^5
```

maple [A] time = 0.01, size = 56, normalized size = 0.35

$$-\frac{\left(-5b^{3}x^{6}+15a\,b^{2}x^{4}+5a^{2}b\,x^{2}+a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{5\left(b\,x^{2}+a\right)^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^6,x)
```

```
[Out] -1/5*(-5*b^3*x^6+15*a*b^2*x^4+5*a^2*b*x^2+a^3)*((b*x^2+a)^2)^(3/2)/x^5/(b*x
^2+a)^3
```

maxima [A] time = 1.32, size = 32, normalized size = 0.20

$$b^{3}x-\frac{3\,ab^{2}}{x}-\frac{a^{2}b}{x^{3}}-\frac{a^{3}}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] b^3*x - 3*a*b^2/x - a^2*b/x^3 - 1/5*a^3/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}\right)^{3/2}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^6,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**6,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**6, x)
```

3.580 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=163

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x^3\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}$$

```
[Out] -1/7*a^3*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)-3/5*a^2*b*((b*x^2+a)^2)^(1/2)/x^
5/(b*x^2+a)-a*b^2*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)-b^3*((b*x^2+a)^2)^(1/2)
/x/(b*x^2+a)
```

Rubi [A] time = 0.04, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x^3\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^8,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (a*b^2*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(x^3*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x
*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^8}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^8}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{x^8}+\frac{3a^2b^4}{x^6}+\frac{3ab^5}{x^4}+\frac{b^6}{x^2}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)} - \frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x^3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(5a^3+21a^2bx^2+35ab^2x^4+35b^3x^6\right)}{35x^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^8,x]
```

```
[Out] -1/35*(Sqrt[(a + b*x^2)^2]*(5*a^3 + 21*a^2*b*x^2 + 35*a*b^2*x^4 + 35*b^3*x^
6))/(x^7*(a + b*x^2))
```

fricas [A] time = 1.09, size = 37, normalized size = 0.23

$$-\frac{35\,b^3x^6+35\,ab^2x^4+21\,a^2bx^2+5\,a^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/35*(35*b^3*x^6 + 35*a*b^2*x^4 + 21*a^2*b*x^2 + 5*a^3)/x^7
```

giac [A] time = 0.22, size = 69, normalized size = 0.42

$$-\frac{35\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+35\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+21\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+5\,a^3\mathrm{sgn}\left(bx^2+a\right)}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] -1/35*(35*b^3*x^6*sgn(b*x^2 + a) + 35*a*b^2*x^4*sgn(b*x^2 + a) + 21*a^2*b*x
^2*sgn(b*x^2 + a) + 5*a^3*sgn(b*x^2 + a))/x^7
```

maple [A] time = 0.01, size = 58, normalized size = 0.36

$$-\frac{\left(35b^3x^6+35a\,b^2x^4+21a^2b\,x^2+5a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{35\left(b\,x^2+a\right)^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^8,x)
```

```
[Out] -1/35*(35*b^3*x^6+35*a*b^2*x^4+21*a^2*b*x^2+5*a^3)*((b*x^2+a)^2)^(3/2)/x^7/
(b*x^2+a)^3
```

maxima [A] time = 1.27, size = 35, normalized size = 0.21

$$-\frac{b^3}{x}-\frac{ab^2}{x^3}-\frac{3\,a^2b}{5\,x^5}-\frac{a^3}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] -b^3/x - a*b^2/x^3 - 3/5*a^2*b/x^5 - 1/7*a^3/x^7
```

mupad [B] time = 4.25, size = 151, normalized size = 0.93

$$-\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^7\left(b\,x^2+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x\left(b\,x^2+a\right)}-\frac{a\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x^3\left(b\,x^2+a\right)}-\frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{5\,x^5\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^8,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(7*x^7*(a + b*x^2)) - (b^3*(a^2 +
 b^2*x^4 + 2*a*b*x^2)^(1/2))/(x*(a + b*x^2)) - (a*b^2*(a^2 + b^2*x^4 + 2*a*
b*x^2)^(1/2))/(x^3*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2
))/(5*x^5*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**8,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**8, x)
```

3.581 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{10}}\,dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}$$

```
[Out] -1/9*a^3*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)-3/7*a^2*b*((b*x^2+a)^2)^(1/2)/x^
7/(b*x^2+a)-3/5*a*b^2*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)-1/3*b^3*((b*x^2+a)^
2)^(1/2)/x^3/(b*x^2+a)
```

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^10,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
)/(3*x^3*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{10}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^{10}}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{x^{10}}+\frac{3a^2b^4}{x^8}+\frac{3ab^5}{x^6}+\frac{b^6}{x^4}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)} - \frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(35a^3+135a^2bx^2+189ab^2x^4+105b^3x^6\right)}{315x^9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^10,x]
```

```
[Out] -1/315*(Sqrt[(a + b*x^2)^2]*(35*a^3 + 135*a^2*b*x^2 + 189*a*b^2*x^4 + 105*b
^3*x^6))/(x^9*(a + b*x^2))
```

fricas [A] time = 0.82, size = 37, normalized size = 0.22

$$-\frac{105\,b^3x^6+189\,ab^2x^4+135\,a^2bx^2+35\,a^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9
```

giac [A] time = 0.19, size = 69, normalized size = 0.41

$$-\frac{105\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+189\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+135\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+35\,a^3\mathrm{sgn}\left(bx^2+a\right)}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] -1/315*(105*b^3*x^6*sgn(b*x^2 + a) + 189*a*b^2*x^4*sgn(b*x^2 + a) + 135*a^2
*b*x^2*sgn(b*x^2 + a) + 35*a^3*sgn(b*x^2 + a))/x^9
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(105b^3x^6+189a\,b^2x^4+135a^2b\,x^2+35a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{315\left(b\,x^2+a\right)^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^10,x)
```

```
[Out] -1/315*(105*b^3*x^6+189*a*b^2*x^4+135*a^2*b*x^2+35*a^3)*((b*x^2+a)^2)^(3/2)
/x^9/(b*x^2+a)^3
```

maxima [A] time = 1.34, size = 35, normalized size = 0.21

$$-\frac{b^3}{3\,x^3}-\frac{3\,ab^2}{5\,x^5}-\frac{3\,a^2b}{7\,x^7}-\frac{a^3}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] -1/3*b^3/x^3 - 3/5*a*b^2/x^5 - 3/7*a^2*b/x^7 - 1/9*a^3/x^9
```

mupad [B] time = 4.26, size = 151, normalized size = 0.90

$$-\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{9\,x^9\,\left(b\,x^2+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^3\,\left(b\,x^2+a\right)}-\frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{5\,x^5\,\left(b\,x^2+a\right)}-\frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^7\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^10,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(9*x^9*(a + b*x^2)) - (b^3*(a^2 +
 b^2*x^4 + 2*a*b*x^2)^(1/2))/(3*x^3*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*x^4
+ 2*a*b*x^2)^(1/2))/(5*x^5*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(7*x^7*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**10,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**10, x)
```

3.582 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{12}}\,dx$

Optimal. Leaf size=167

$$-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{3x^9(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)}$$

```
[Out] -1/11*a^3*((b*x^2+a)^2)^(1/2)/x^11/(b*x^2+a)-1/3*a^2*b*((b*x^2+a)^2)^(1/2)/
x^9/(b*x^2+a)-3/7*a*b^2*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)-1/5*b^3*((b*x^2+a
)^2)^(1/2)/x^5/(b*x^2+a)
```

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{3x^9(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^12,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^9*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
)/(5*x^5*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{12}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^{12}}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{x^{12}}+\frac{3a^2b^4}{x^{10}}+\frac{3ab^5}{x^8}+\frac{b^6}{x^6}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)} - \frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{3x^9\left(a+bx^2\right)} - \frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(105a^3+385a^2bx^2+495ab^2x^4+231b^3x^6\right)}{1155x^{11}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^12,x]
```

```
[Out] -1/1155*(Sqrt[(a + b*x^2)^2]*(105*a^3 + 385*a^2*b*x^2 + 495*a*b^2*x^4 + 231
*b^3*x^6))/(x^11*(a + b*x^2))
```

fricas [A] time = 0.84, size = 37, normalized size = 0.22

$$-\frac{231\,b^3x^6+495\,ab^2x^4+385\,a^2bx^2+105\,a^3}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^11
```

giac [A] time = 0.16, size = 69, normalized size = 0.41

$$-\frac{231\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+495\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+385\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+105\,a^3\mathrm{sgn}\left(bx^2+a\right)}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^12,x, algorithm="giac")
```

```
[Out] -1/1155*(231*b^3*x^6*sgn(b*x^2 + a) + 495*a*b^2*x^4*sgn(b*x^2 + a) + 385*a^
2*b*x^2*sgn(b*x^2 + a) + 105*a^3*sgn(b*x^2 + a))/x^11
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(231b^{3}x^{6}+495a\,b^{2}x^{4}+385a^{2}b\,x^{2}+105a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{1155\left(b\,x^{2}+a\right)^{3}x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^12,x)
```

```
[Out] -1/1155*(231*b^3*x^6+495*a*b^2*x^4+385*a^2*b*x^2+105*a^3)*((b*x^2+a)^2)^(3/
2)/x^11/(b*x^2+a)^3
```

maxima [A] time = 1.30, size = 35, normalized size = 0.21

$$-\frac{b^{3}}{5\,x^{5}}-\frac{3\,ab^{2}}{7\,x^{7}}-\frac{a^{2}b}{3\,x^{9}}-\frac{a^{3}}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^12,x, algorithm="maxima")
```

```
[Out] -1/5*b^3/x^5 - 3/7*a*b^2/x^7 - 1/3*a^2*b/x^9 - 1/11*a^3/x^11
```

mupad [B] time = 4.55, size = 151, normalized size = 0.90

$$-\frac{a^{3}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{11\,x^{11}\,\left(b\,x^{2}+a\right)}-\frac{b^{3}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{5\,x^{5}\,\left(b\,x^{2}+a\right)}-\frac{3\,a\,b^{2}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{7\,x^{7}\,\left(b\,x^{2}+a\right)}-\frac{a^{2}\,b\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{3\,x^{9}\,\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^12,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(11*x^11*(a + b*x^2)) - (b^3*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(5*x^5*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(7*x^7*(a + b*x^2)) - (a^2*b*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(3*x^9*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**12,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**12, x)
```

3.583 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{14}}\,dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^9\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}$$

```
[Out] -1/13*a^3*((b*x^2+a)^2)^(1/2)/x^13/(b*x^2+a)-3/11*a^2*b*((b*x^2+a)^2)^(1/2)
/x^11/(b*x^2+a)-1/3*a*b^2*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)-1/7*b^3*((b*x^2
+a)^2)^(1/2)/x^7/(b*x^2+a)
```

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^9\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^14,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (a*b^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(3*x^9*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(7*x^7*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{14}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^{14}}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{x^{14}}+\frac{3a^2b^4}{x^{12}}+\frac{3ab^5}{x^{10}}+\frac{b^6}{x^8}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)} - \frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^9\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(231a^3+819a^2bx^2+1001ab^2x^4+429b^3x^6\right)}{3003x^{13}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^14,x]
```

```
[Out] -1/3003*(Sqrt[(a + b*x^2)^2]*(231*a^3 + 819*a^2*b*x^2 + 1001*a*b^2*x^4 + 42
9*b^3*x^6))/(x^13*(a + b*x^2))
```

fricas [A] time = 0.73, size = 37, normalized size = 0.22

$$-\frac{429\,b^3x^6+1001\,ab^2x^4+819\,a^2bx^2+231\,a^3}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^14,x, algorithm="fricas")
```

```
[Out] -1/3003*(429*b^3*x^6 + 1001*a*b^2*x^4 + 819*a^2*b*x^2 + 231*a^3)/x^13
```

giac [A] time = 0.16, size = 69, normalized size = 0.41

$$-\frac{429\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+1001\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+819\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+231\,a^3\mathrm{sgn}\left(bx^2+a\right)}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^14,x, algorithm="giac")
```

```
[Out] -1/3003*(429*b^3*x^6*sgn(b*x^2 + a) + 1001*a*b^2*x^4*sgn(b*x^2 + a) + 819*a
^2*b*x^2*sgn(b*x^2 + a) + 231*a^3*sgn(b*x^2 + a))/x^13
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(429b^3x^6+1001a\,b^2x^4+819a^2b\,x^2+231a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{3003\left(b\,x^2+a\right)^3x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^14,x)
```

```
[Out] -1/3003*(429*b^3*x^6+1001*a*b^2*x^4+819*a^2*b*x^2+231*a^3)*((b*x^2+a)^2)^(3
/2)/x^13/(b*x^2+a)^3
```

maxima [A] time = 1.39, size = 35, normalized size = 0.21

$$-\frac{b^3}{7\,x^7}-\frac{ab^2}{3\,x^9}-\frac{3\,a^2b}{11\,x^{11}}-\frac{a^3}{13\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^14,x, algorithm="maxima")
```

```
[Out] -1/7*b^3/x^7 - 1/3*a*b^2/x^9 - 3/11*a^2*b/x^11 - 1/13*a^3/x^13
```

mupad [B] time = 4.63, size = 151, normalized size = 0.90

$$-\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13\,x^{13}\,\left(b\,x^2+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^7\,\left(b\,x^2+a\right)}-\frac{a\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^9\,\left(b\,x^2+a\right)}-\frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^14,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(13*x^13*(a + b*x^2)) - (b^3*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(7*x^7*(a + b*x^2)) - (a*b^2*(a^2 + b^2*x^4
+ 2*a*b*x^2)^(1/2))/(3*x^9*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(11*x^11*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**14,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**14, x)
```

3.584 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{16}}\,dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)}$$

```
[Out] -1/15*a^3*((b*x^2+a)^2)^(1/2)/x^15/(b*x^2+a)-3/13*a^2*b*((b*x^2+a)^2)^(1/2)
/x^13/(b*x^2+a)-3/11*a*b^2*((b*x^2+a)^2)^(1/2)/x^11/(b*x^2+a)-1/9*b^3*((b*x
^2+a)^2)^(1/2)/x^9/(b*x^2+a)
```

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^16,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*x^15*(a + b*x^2)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(9*x^9*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{16}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^{16}}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{x^{16}}+\frac{3a^2b^4}{x^{14}}+\frac{3ab^5}{x^{12}}+\frac{b^6}{x^{10}}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)} - \frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(429a^3+1485a^2bx^2+1755ab^2x^4+715b^3x^6\right)}{6435x^{15}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^16,x]
[Out] -1/6435*(Sqrt[(a + b*x^2)^2]*(429*a^3 + 1485*a^2*b*x^2 + 1755*a*b^2*x^4 + 7
15*b^3*x^6))/(x^15*(a + b*x^2))
```

fricas [A] time = 0.84, size = 37, normalized size = 0.22

$$-\frac{715\,b^3x^6+1755\,ab^2x^4+1485\,a^2bx^2+429\,a^3}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^16,x, algorithm="fricas")
[Out] -1/6435*(715*b^3*x^6 + 1755*a*b^2*x^4 + 1485*a^2*b*x^2 + 429*a^3)/x^15
```

giac [A] time = 0.21, size = 69, normalized size = 0.41

$$-\frac{715\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+1755\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+1485\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+429\,a^3\mathrm{sgn}\left(bx^2+a\right)}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^16,x, algorithm="giac")
```

```
[Out] -1/6435*(715*b^3*x^6*sgn(b*x^2 + a) + 1755*a*b^2*x^4*sgn(b*x^2 + a) + 1485*
a^2*b*x^2*sgn(b*x^2 + a) + 429*a^3*sgn(b*x^2 + a))/x^15
```

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{\left(715b^{3}x^{6}+1755a\,b^{2}x^{4}+1485a^{2}b\,x^{2}+429a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{6435\left(b\,x^{2}+a\right)^{3}x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^16,x)
```

```
[Out] -1/6435*(715*b^3*x^6+1755*a*b^2*x^4+1485*a^2*b*x^2+429*a^3)*((b*x^2+a)^2)^(
3/2)/x^15/(b*x^2+a)^3
```

maxima [A] time = 1.29, size = 35, normalized size = 0.21

$$-\frac{b^{3}}{9\,x^{9}}-\frac{3\,ab^{2}}{11\,x^{11}}-\frac{3\,a^{2}b}{13\,x^{13}}-\frac{a^{3}}{15\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^16,x, algorithm="maxima")
```

```
[Out] -1/9*b^3/x^9 - 3/11*a*b^2/x^11 - 3/13*a^2*b/x^13 - 1/15*a^3/x^15
```

mupad [B] time = 4.30, size = 151, normalized size = 0.90

$$-\frac{a^{3}\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{15\,x^{15}\,\left(b\,x^{2}+a\right)}-\frac{b^{3}\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{9\,x^{9}\,\left(b\,x^{2}+a\right)}-\frac{3\,a\,b^{2}\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{11\,x^{11}\,\left(b\,x^{2}+a\right)}-\frac{3\,a^{2}\,b\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{13\,x^{13}\,\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^16,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(15*x^15*(a + b*x^2)) - (b^3*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(9*x^9*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(11*x^11*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a
*b*x^2)^(1/2))/(13*x^13*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**16,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**16, x)
```

3.585 $\int x^{13} \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{24}\sqrt{a^2+2abx^2+b^2x^4}}{24\left(a+bx^2\right)}+\frac{5ab^4x^{22}\sqrt{a^2+2abx^2+b^2x^4}}{22\left(a+bx^2\right)}+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{a^5x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}$$

```
[Out] 1/14*a^5*x^14*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/16*a^4*b*x^16*((b*x^2+a)^2)^(
1/2)/(b*x^2+a)+5/9*a^3*b^2*x^18*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/2*a^2*b^3*x
^20*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/22*a*b^4*x^22*((b*x^2+a)^2)^(1/2)/(b*x^
2+a)+1/24*b^5*x^24*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.16, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{b^5x^{24}\sqrt{a^2+2abx^2+b^2x^4}}{24\left(a+bx^2\right)}+\frac{5ab^4x^{22}\sqrt{a^2+2abx^2+b^2x^4}}{22\left(a+bx^2\right)}+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{5a^3b^2x^{18}\sqrt{a^2+2abx^2+}}{9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^14*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*(a + b*x^2)) + (5*a^4*b*x^16
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*(a + b*x^2)) + (5*a^3*b^2*x^18*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (a^2*b^3*x^20*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a*b^4*x^22*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(22*(a + b*x^2)) + (b^5*x^24*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(24*(a
 + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^{13}\left(a^2+2abx^2+b^2x^4\right)^{5/2}dx=\frac{1}{2}\text{Subst}\left(\int x^6\left(a^2+2abx+b^2x^2\right)^{5/2}dx,x,x^2\right)$$

$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int x^6\left(ab+b^2x\right)^5dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int\left(a^5b^5x^6+5a^4b^6x^7+10a^3b^7x^8+10a^2b^8x^9+5\right.\right.}{2b^4\left(ab+b^2x^2\right)}$$

$$=\frac{a^5x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{5a^4bx^{16}\sqrt{a^2+2abx^2+b^2x^4}}{16\left(a+bx^2\right)}+\frac{5a^3b^2x^{18}\sqrt{a^2+}}{9\left(a+\right.}$$

Mathematica [A] time = 0.03, size = 83, normalized size = 0.33

$$\frac{x^{14}\sqrt{\left(a+bx^2\right)^2}\left(792a^5+3465a^4bx^2+6160a^3b^2x^4+5544a^2b^3x^6+2520ab^4x^8+462b^5x^{10}\right)}{11088\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^14*Sqrt[(a + b*x^2)^2]*(792*a^5 + 3465*a^4*b*x^2 + 6160*a^3*b^2*x^4 + 55
44*a^2*b^3*x^6 + 2520*a*b^4*x^8 + 462*b^5*x^10))/(11088*(a + b*x^2))
```

fricas [A] time = 0.65, size = 57, normalized size = 0.22

$$\frac{1}{24}b^5x^{24}+\frac{5}{22}ab^4x^{22}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{9}a^3b^2x^{18}+\frac{5}{16}a^4bx^{16}+\frac{1}{14}a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*b^5*x^24 + 5/22*a*b^4*x^22 + 1/2*a^2*b^3*x^20 + 5/9*a^3*b^2*x^18 + 5/1
6*a^4*b*x^16 + 1/14*a^5*x^14
```

giac [A] time = 0.16, size = 105, normalized size = 0.41

$$\frac{1}{24}\,b^5x^{24}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{22}\,ab^4x^{22}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{2}\,a^2b^3x^{20}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{9}\,a^3b^2x^{18}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{16}\,a^4bx^{16}\mathrm{sgn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*b^5*x^24*sgn(b*x^2 + a) + 5/22*a*b^4*x^22*sgn(b*x^2 + a) + 1/2*a^2*b^3
*x^20*sgn(b*x^2 + a) + 5/9*a^3*b^2*x^18*sgn(b*x^2 + a) + 5/16*a^4*b*x^16*sg
n(b*x^2 + a) + 1/14*a^5*x^14*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(462b^5x^{10}+2520a\,b^4x^8+5544a^2b^3x^6+6160a^3b^2x^4+3465a^4b\,x^2+792a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x^{14}}{11088\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/11088*x^14*(462*b^5*x^10+2520*a*b^4*x^8+5544*a^2*b^3*x^6+6160*a^3*b^2*x^4
+3465*a^4*b*x^2+792*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.32, size = 57, normalized size = 0.22

$$\frac{1}{24}\,b^5x^{24}+\frac{5}{22}\,ab^4x^{22}+\frac{1}{2}\,a^2b^3x^{20}+\frac{5}{9}\,a^3b^2x^{18}+\frac{5}{16}\,a^4bx^{16}+\frac{1}{14}\,a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*b^5*x^24 + 5/22*a*b^4*x^22 + 1/2*a^2*b^3*x^20 + 5/9*a^3*b^2*x^18 + 5/1
6*a^4*b*x^16 + 1/14*a^5*x^14
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{13}\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^13*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{13} \left(\left(a + b x^2 \right)^2 \right)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**13*((a + b*x**2)**2)**(5/2), x)
```

3.586 $\int x^{11} \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx$

Optimal. Leaf size=255

$$\frac{b^5x^{22}\sqrt{a^2+2abx^2+b^2x^4}}{22\left(a+bx^2\right)}+\frac{ab^4x^{20}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{5a^2b^3x^{18}\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{a^5x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{12\left(a+bx^2\right)}$$

```
[Out] 1/12*a^5*x^12*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/14*a^4*b*x^14*((b*x^2+a)^2)^(
1/2)/(b*x^2+a)+5/8*a^3*b^2*x^16*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/9*a^2*b^3*x
^18*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/4*a*b^4*x^20*((b*x^2+a)^2)^(1/2)/(b*x^2
+a)+1/22*b^5*x^22*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.16, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{b^5x^{22}\sqrt{a^2+2abx^2+b^2x^4}}{22\left(a+bx^2\right)}+\frac{ab^4x^{20}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{5a^2b^3x^{18}\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{5a^3b^2x^{16}\sqrt{a^2+2abx^2+}}{8\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^12*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*(a + b*x^2)) + (5*a^4*b*x^14
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*(a + b*x^2)) + (5*a^3*b^2*x^16*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2)) + (5*a^2*b^3*x^18*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (a*b^4*x^20*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(4*(a + b*x^2)) + (b^5*x^22*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(22*(a
+ b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^{11}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{2}\operatorname{Subst}\left(\int x^5\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx,x,x^2\right)$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int x^5\left(ab+b^2x\right)^5\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int\left(a^5b^5x^5+5a^4b^6x^6+10a^3b^7x^7+10a^2b^8x^8+5\right.\right.}{2b^4\left(ab+b^2x^2\right)}$$
$$=\frac{a^5x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{12\left(a+bx^2\right)}+\frac{5a^4bx^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{5a^3b^2x^{16}\sqrt{a^2+}}{8\left(a+\right.}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^{12}\sqrt{\left(a+bx^2\right)^2}\left(462a^5+1980a^4bx^2+3465a^3b^2x^4+3080a^2b^3x^6+1386ab^4x^8+252b^5x^{10}\right)}{5544\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^12*Sqrt[(a + b*x^2)^2]*(462*a^5 + 1980*a^4*b*x^2 + 3465*a^3*b^2*x^4 + 30
80*a^2*b^3*x^6 + 1386*a*b^4*x^8 + 252*b^5*x^10))/(5544*(a + b*x^2))
```

fricas [A] time = 1.05, size = 57, normalized size = 0.22

$$\frac{1}{22}\,b^5x^{22}+\frac{1}{4}\,ab^4x^{20}+\frac{5}{9}\,a^2b^3x^{18}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{12}\,a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

[Out] 1/22*b^5*x^22 + 1/4*a*b^4*x^20 + 5/9*a^2*b^3*x^18 + 5/8*a^3*b^2*x^16 + 5/14 *a^4*b*x^14 + 1/12*a^5*x^12

giac [A] time = 0.15, size = 105, normalized size = 0.41

$$\frac{1}{22}\,b^5x^{22}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{4}\,ab^4x^{20}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{9}\,a^2b^3x^{18}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{8}\,a^3b^2x^{16}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{14}\,a^4bx^{14}\mathrm{sgn}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")

[Out] 1/22*b^5*x^22*sgn(b*x^2 + a) + 1/4*a*b^4*x^20*sgn(b*x^2 + a) + 5/9*a^2*b^3* x^18*sgn(b*x^2 + a) + 5/8*a^3*b^2*x^16*sgn(b*x^2 + a) + 5/14*a^4*b*x^14*sgn (b*x^2 + a) + 1/12*a^5*x^12*sgn(b*x^2 + a)

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(252b^5x^{10}+1386a\,b^4x^8+3080a^2b^3x^6+3465a^3b^2x^4+1980a^4b\,x^2+462a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x^{12}}{5544\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)

[Out] 1/5544*x^12*(252*b^5*x^10+1386*a*b^4*x^8+3080*a^2*b^3*x^6+3465*a^3*b^2*x^4+ 1980*a^4*b*x^2+462*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5

maxima [A] time = 1.37, size = 57, normalized size = 0.22

$$\frac{1}{22}\,b^5x^{22}+\frac{1}{4}\,ab^4x^{20}+\frac{5}{9}\,a^2b^3x^{18}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{12}\,a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")

[Out] 1/22*b^5*x^22 + 1/4*a*b^4*x^20 + 5/9*a^2*b^3*x^18 + 5/8*a^3*b^2*x^16 + 5/14 *a^4*b*x^14 + 1/12*a^5*x^12

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{11}\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^11*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{11}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**11*((a + b*x**2)**2)**(5/2), x)
```

3.587 $\int x^9 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=201

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^9}{20b^5}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^8}{9b^5}+\frac{3a^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^7}{8b^5}+\frac{a^4\sqrt{a^2}}{}$$

```
[Out] 1/12*a^4*(b*x^2+a)^5*((b*x^2+a)^2)^(1/2)/b^5-2/7*a^3*(b*x^2+a)^6*((b*x^2+a)
^2)^(1/2)/b^5+3/8*a^2*(b*x^2+a)^7*((b*x^2+a)^2)^(1/2)/b^5-2/9*a*(b*x^2+a)^8
*((b*x^2+a)^2)^(1/2)/b^5+1/20*(b*x^2+a)^9*((b*x^2+a)^2)^(1/2)/b^5
```

Rubi [A] time = 0.13, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1111, 645}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^9}{20b^5}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^8}{9b^5}+\frac{3a^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^7}{8b^5}-\frac{2a^3\sqrt{a^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^4*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*b^5) - (2*a^3*(a +
b*x^2)^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*b^5) + (3*a^2*(a + b*x^2)^7*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*b^5) - (2*a*(a + b*x^2)^8*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(9*b^5) + ((a + b*x^2)^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
/(20*b^5)
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[ExpandLinearProduct[(b/2 + c*x)^(2*p), (d + e*x)^m, b/2, c
, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[p] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1, 0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{1}{2}\,\text{Subst}\left(\int x^4 \left(a^2+2abx+b^2x^2\right)^{5/2} dx, x, x^2\right)$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \left(\frac{a^4(ab+b^2x)^5}{b^4} - \frac{4a^3(ab+b^2x)^6}{b^5} + \frac{6a^2(ab+b^2x)^7}{b^6} - \frac{4a(ab+}{b^7}\right.\right.}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{a^4\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{12b^5} - \frac{2a^3\left(a+bx^2\right)^6\sqrt{a^2+2abx^2+b^2x^4}}{7b^5} + \frac{3}{}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.41

$$\frac{x^{10}\sqrt{\left(a+bx^2\right)^2}\left(252a^5+1050a^4bx^2+1800a^3b^2x^4+1575a^2b^3x^6+700ab^4x^8+126b^5x^{10}\right)}{2520\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^10*Sqrt[(a + b*x^2)^2]*(252*a^5 + 1050*a^4*b*x^2 + 1800*a^3*b^2*x^4 + 15
75*a^2*b^3*x^6 + 700*a*b^4*x^8 + 126*b^5*x^10))/(2520*(a + b*x^2))
```

fricas [A] time = 0.73, size = 57, normalized size = 0.28

$$\frac{1}{20}\,b^5x^{20} + \frac{5}{18}\,ab^4x^{18} + \frac{5}{8}\,a^2b^3x^{16} + \frac{5}{7}\,a^3b^2x^{14} + \frac{5}{12}\,a^4bx^{12} + \frac{1}{10}\,a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20*b^5*x^20 + 5/18*a*b^4*x^18 + 5/8*a^2*b^3*x^16 + 5/7*a^3*b^2*x^14 + 5/1
2*a^4*b*x^12 + 1/10*a^5*x^10
```

giac [A] time = 0.16, size = 105, normalized size = 0.52

$$\frac{1}{20}\,b^5x^{20}\text{sgn}\left(bx^2+a\right)+\frac{5}{18}\,ab^4x^{18}\text{sgn}\left(bx^2+a\right)+\frac{5}{8}\,a^2b^3x^{16}\text{sgn}\left(bx^2+a\right)+\frac{5}{7}\,a^3b^2x^{14}\text{sgn}\left(bx^2+a\right)+\frac{5}{12}\,a^4bx^{12}\text{sg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/20*b^5*x^20*sgn(b*x^2 + a) + 5/18*a*b^4*x^18*sgn(b*x^2 + a) + 5/8*a^2*b^3
*x^16*sgn(b*x^2 + a) + 5/7*a^3*b^2*x^14*sgn(b*x^2 + a) + 5/12*a^4*b*x^12*sg
n(b*x^2 + a) + 1/10*a^5*x^10*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.40

$$\frac{\left(126b^5x^{10}+700a\,b^4x^8+1575a^2b^3x^6+1800a^3b^2x^4+1050a^4b\,x^2+252a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x^{10}}{2520\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/2520*x^10*(126*b^5*x^10+700*a*b^4*x^8+1575*a^2*b^3*x^6+1800*a^3*b^2*x^4+1
050*a^4*b*x^2+252*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.43, size = 57, normalized size = 0.28

$$\frac{1}{20}\,b^5x^{20}+\frac{5}{18}\,ab^4x^{18}+\frac{5}{8}\,a^2b^3x^{16}+\frac{5}{7}\,a^3b^2x^{14}+\frac{5}{12}\,a^4bx^{12}+\frac{1}{10}\,a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20*b^5*x^20 + 5/18*a*b^4*x^18 + 5/8*a^2*b^3*x^16 + 5/7*a^3*b^2*x^14 + 5/1
2*a^4*b*x^12 + 1/10*a^5*x^10
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^9\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^9*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{9}\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**9*((a + b*x**2)**2)**(5/2), x)
```

3.588 $\int x^7 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=160

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^8}{18b^4}-\frac{3a\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^7}{16b^4}+\frac{3a^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^6}{14b^4}-\frac{a^3\sqrt{a^2}}{}$$

```
[Out] -1/12*a^3*(b*x^2+a)^5*((b*x^2+a)^2)^(1/2)/b^4+3/14*a^2*(b*x^2+a)^6*((b*x^2+
a)^2)^(1/2)/b^4-3/16*a*(b*x^2+a)^7*((b*x^2+a)^2)^(1/2)/b^4+1/18*(b*x^2+a)^8
*((b*x^2+a)^2)^(1/2)/b^4
```

Rubi [A] time = 0.12, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^8}{18b^4}-\frac{3a\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^7}{16b^4}+\frac{3a^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^6}{14b^4}-\frac{a^3\sqrt{a^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] -(a^3*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*b^4) + (3*a^2*(a +
 b*x^2)^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*b^4) - (3*a*(a + b*x^2)^7*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*b^4) + ((a + b*x^2)^8*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(18*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{1}{2}\text{Subst}\left(\int x^3 \left(a^2+2abx+b^2x^2\right)^{5/2} dx, x, x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int x^3 \left(ab+b^2x\right)^5 dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \left(-\frac{a^3\left(ab+b^2x\right)^5}{b^3}+\frac{3a^2\left(ab+b^2x\right)^6}{b^4}-\frac{3a\left(ab+b^2x\right)^7}{b^5}+\frac{\left(ab+b^2x\right)}{b^6}\right.\right.}{2b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{12b^4}+\frac{3a^2\left(a+bx^2\right)^6\sqrt{a^2+2abx^2+b^2x^4}}{14b^4}-\frac{3}{}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.52

$$\frac{x^8\sqrt{\left(a+bx^2\right)^2}\left(126a^5+504a^4bx^2+840a^3b^2x^4+720a^2b^3x^6+315ab^4x^8+56b^5x^{10}\right)}{1008\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
[Out] (x^8*Sqrt[(a + b*x^2)^2]*(126*a^5 + 504*a^4*b*x^2 + 840*a^3*b^2*x^4 + 720*a
^2*b^3*x^6 + 315*a*b^4*x^8 + 56*b^5*x^10))/(1008*(a + b*x^2))
```

fricas [A] time = 0.98, size = 57, normalized size = 0.36

$$\frac{1}{18}b^5x^{18}+\frac{5}{16}ab^4x^{16}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{6}a^3b^2x^{12}+\frac{1}{2}a^4bx^{10}+\frac{1}{8}a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
[Out] 1/18*b^5*x^18 + 5/16*a*b^4*x^16 + 5/7*a^2*b^3*x^14 + 5/6*a^3*b^2*x^12 + 1/2
*a^4*b*x^10 + 1/8*a^5*x^8
```

giac [A] time = 0.16, size = 105, normalized size = 0.66

$$\frac{1}{18}\,b^5x^{18}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{16}\,ab^4x^{16}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{7}\,a^2b^3x^{14}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{6}\,a^3b^2x^{12}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{2}\,a^4bx^{10}\mathrm{sgr}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/18*b^5*x^18*sgn(b*x^2 + a) + 5/16*a*b^4*x^16*sgn(b*x^2 + a) + 5/7*a^2*b^3
*x^14*sgn(b*x^2 + a) + 5/6*a^3*b^2*x^12*sgn(b*x^2 + a) + 1/2*a^4*b*x^10*sgn
(b*x^2 + a) + 1/8*a^5*x^8*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.50

$$\frac{\left(56b^5x^{10}+315a\,b^4x^8+720a^2b^3x^6+840a^3b^2x^4+504a^4b\,x^2+126a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x^8}{1008\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/1008*x^8*(56*b^5*x^10+315*a*b^4*x^8+720*a^2*b^3*x^6+840*a^3*b^2*x^4+504*a
^4*b*x^2+126*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.40, size = 57, normalized size = 0.36

$$\frac{1}{18}\,b^5x^{18}+\frac{5}{16}\,ab^4x^{16}+\frac{5}{7}\,a^2b^3x^{14}+\frac{5}{6}\,a^3b^2x^{12}+\frac{1}{2}\,a^4bx^{10}+\frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/18*b^5*x^18 + 5/16*a*b^4*x^16 + 5/7*a^2*b^3*x^14 + 5/6*a^3*b^2*x^12 + 1/2
*a^4*b*x^10 + 1/8*a^5*x^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^7\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**7*((a + b*x**2)**2)**(5/2), x)
```

3.589 $\int x^5 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=119

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^7}{16b^3}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^6}{7b^3}+\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{12b^3}$$

```
[Out] 1/12*a^2*(b*x^2+a)^5*((b*x^2+a)^2)^(1/2)/b^3-1/7*a*(b*x^2+a)^6*((b*x^2+a)^2
)^(1/2)/b^3+1/16*(b*x^2+a)^7*((b*x^2+a)^2)^(1/2)/b^3
```

Rubi [A] time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^7}{16b^3}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^6}{7b^3}+\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{12b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^2*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*b^3) - (a*(a + b*x^
2)^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*b^3) + ((a + b*x^2)^7*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(16*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
```

```
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\begin{aligned}\int x^5\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int x^2\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx,x,x^2\right)\\&=\frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int x^2\left(ab+b^2x\right)^5\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}\\&=\frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int\left(\frac{a^2\left(ab+b^2x\right)^5}{b^2}-\frac{2a\left(ab+b^2x\right)^6}{b^3}+\frac{\left(ab+b^2x\right)^7}{b^4}\right)dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}\\&=\frac{a^2\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{12b^3}-\frac{a\left(a+bx^2\right)^6\sqrt{a^2+2abx^2+b^2x^4}}{7b^3}+\frac{(a+\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.70

$$\frac{x^6\sqrt{\left(a+bx^2\right)^2}\left(56a^5+210a^4bx^2+336a^3b^2x^4+280a^2b^3x^6+120ab^4x^8+21b^5x^{10}\right)}{336\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x^2)^2]*(56*a^5 + 210*a^4*b*x^2 + 336*a^3*b^2*x^4 + 280*a^
2*b^3*x^6 + 120*a*b^4*x^8 + 21*b^5*x^10))/(336*(a + b*x^2))
```

fricas [A] time = 0.63, size = 56, normalized size = 0.47

$$\frac{1}{16}\,b^5x^{16}+\frac{5}{14}\,ab^4x^{14}+\frac{5}{6}\,a^2b^3x^{12}+a^3b^2x^{10}+\frac{5}{8}\,a^4bx^8+\frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*b^5*x^16 + 5/14*a*b^4*x^14 + 5/6*a^2*b^3*x^12 + a^3*b^2*x^10 + 5/8*a^4
*b*x^8 + 1/6*a^5*x^6
```

giac [A] time = 0.19, size = 104, normalized size = 0.87

$$\frac{1}{16}\,b^5x^{16}\operatorname{sgn}\left(bx^2+a\right)+\frac{5}{14}\,ab^4x^{14}\operatorname{sgn}\left(bx^2+a\right)+\frac{5}{6}\,a^2b^3x^{12}\operatorname{sgn}\left(bx^2+a\right)+a^3b^2x^{10}\operatorname{sgn}\left(bx^2+a\right)+\frac{5}{8}\,a^4bx^8\operatorname{sgn}\left(bx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*b^5*x^16*sgn(b*x^2 + a) + 5/14*a*b^4*x^14*sgn(b*x^2 + a) + 5/6*a^2*b^3
*x^12*sgn(b*x^2 + a) + a^3*b^2*x^10*sgn(b*x^2 + a) + 5/8*a^4*b*x^8*sgn(b*x^
2 + a) + 1/6*a^5*x^6*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.67

$$\frac{\left(21 b^{5} x^{10}+120 a\,b^{4} x^{8}+280 a^{2} b^{3} x^{6}+336 a^{3} b^{2} x^{4}+210 a^{4} b\,x^{2}+56 a^{5}\right) \left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}} x^{6}}{336 \left(b\,x^{2}+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/336*x^6*(21*b^5*x^10+120*a*b^4*x^8+280*a^2*b^3*x^6+336*a^3*b^2*x^4+210*a^
4*b*x^2+56*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.34, size = 56, normalized size = 0.47

$$\frac{1}{16}\,b^{5} x^{16}+\frac{5}{14}\,a b^{4} x^{14}+\frac{5}{6}\,a^{2} b^{3} x^{12}+a^{3} b^{2} x^{10}+\frac{5}{8}\,a^{4} b x^{8}+\frac{1}{6}\,a^{5} x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/16*b^5*x^16 + 5/14*a*b^4*x^14 + 5/6*a^2*b^3*x^12 + a^3*b^2*x^10 + 5/8*a^4
*b*x^8 + 1/6*a^5*x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{5} \left(\left(a + b x^{2}\right)^{2}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral(x**5*((a + b*x**2)**2)**(5/2), x)
```

3.590 $\int x^3 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=67

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{14b^2}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b^2}$$

```
[Out] -1/12*a*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/b^2+1/14*(b^2*x^4+2*a*b*x^2
+a^2)^(7/2)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 640, 609}

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{14b^2}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] -(a*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(12*b^2) + (a^2 + 2*a*b*
x^2 + b^2*x^4)^(7/2)/(14*b^2)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{1}{2}\text{Subst}\left(\int x \left(a^2+2abx+b^2x^2\right)^{5/2} dx, x, x^2\right)$$
$$= \frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{14b^2} - \frac{a\,\text{Subst}\left(\int \left(a^2+2abx+b^2x^2\right)^{5/2} dx, x, x^2\right)}{2b}$$
$$= -\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b^2} + \frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{14b^2}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 1.24

$$\frac{x^4\sqrt{\left(a+bx^2\right)^2}\left(21a^5+70a^4bx^2+105a^3b^2x^4+84a^2b^3x^6+35ab^4x^8+6b^5x^{10}\right)}{84\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^2)^2]*(21*a^5 + 70*a^4*b*x^2 + 105*a^3*b^2*x^4 + 84*a^2*
b^3*x^6 + 35*a*b^4*x^8 + 6*b^5*x^10))/(84*(a + b*x^2))
```

fricas [A] time = 1.16, size = 56, normalized size = 0.84

$$\frac{1}{14}b^5x^{14} + \frac{5}{12}ab^4x^{12} + a^2b^3x^{10} + \frac{5}{4}a^3b^2x^8 + \frac{5}{6}a^4bx^6 + \frac{1}{4}a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/14*b^5*x^14 + 5/12*a*b^4*x^12 + a^2*b^3*x^10 + 5/4*a^3*b^2*x^8 + 5/6*a^4*
b*x^6 + 1/4*a^5*x^4
```

giac [A] time = 0.20, size = 67, normalized size = 1.00

$$\frac{1}{84}\left(6\,b^5x^{14}+35\,ab^4x^{12}+84\,a^2b^3x^{10}+105\,a^3b^2x^8+70\,a^4bx^6+21\,a^5x^4\right)\text{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/84*(6*b^5*x^14 + 35*a*b^4*x^12 + 84*a^2*b^3*x^10 + 105*a^3*b^2*x^8 + 70*a
^4*b*x^6 + 21*a^5*x^4)*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 1.19

$$\frac{\left(6b^{5}x^{10}+35a\,b^{4}x^{8}+84a^{2}b^{3}x^{6}+105a^{3}b^{2}x^{4}+70a^{4}b\,x^{2}+21a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}x^{4}}{84\left(b\,x^{2}+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/84*x^4*(6*b^5*x^10+35*a*b^4*x^8+84*a^2*b^3*x^6+105*a^3*b^2*x^4+70*a^4*b*x
^2+21*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.25, size = 56, normalized size = 0.84

$$\frac{1}{14}\,b^{5}x^{14}+\frac{5}{12}\,ab^{4}x^{12}+a^{2}b^{3}x^{10}+\frac{5}{4}\,a^{3}b^{2}x^{8}+\frac{5}{6}\,a^{4}bx^{6}+\frac{1}{4}\,a^{5}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/14*b^5*x^14 + 5/12*a*b^4*x^12 + a^2*b^3*x^10 + 5/4*a^3*b^2*x^8 + 5/6*a^4*
b*x^6 + 1/4*a^5*x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**3*((a + b*x**2)**2)**(5/2), x)
```

3.591 $\int x \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b}$$

```
[Out] 1/12*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1107, 609}

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(12*b)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}
\int x \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx &= \frac{1}{2}\text{Subst}\left(\int \left(a^2+2abx+b^2x^2\right)^{5/2} \, dx, x, x^2\right)\\
&= \frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^{5/2}}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^(5/2))/(12*b)
```

fricas [A] time = 1.04, size = 57, normalized size = 1.58

$$\frac{1}{12}\,b^5x^{12}+\frac{1}{2}\,ab^4x^{10}+\frac{5}{4}\,a^2b^3x^8+\frac{5}{3}\,a^3b^2x^6+\frac{5}{4}\,a^4bx^4+\frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*b^5*x^12 + 1/2*a*b^4*x^10 + 5/4*a^2*b^3*x^8 + 5/3*a^3*b^2*x^6 + 5/4*a^
4*b*x^4 + 1/2*a^5*x^2
```

giac [B] time = 0.16, size = 66, normalized size = 1.83

$$\frac{1}{12}\left(3\left(bx^4+2\,ax^2\right)a^4+3\left(bx^4+2\,ax^2\right)^2a^2b+\left(bx^4+2\,ax^2\right)^3b^2\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*(3*(b*x^4 + 2*a*x^2)*a^4 + 3*(b*x^4 + 2*a*x^2)^2*a^2*b + (b*x^4 + 2*a*
x^2)^3*b^2)*sgn(b*x^2 + a)
```

maple [B] time = 0.01, size = 79, normalized size = 2.19

$$\frac{\left(b^5x^{10}+6a\,b^4x^8+15a^2b^3x^6+20a^3b^2x^4+15a^4b\,x^2+6a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x^2}{12\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/12*x^2*(b^5*x^10+6*a*b^4*x^8+15*a^2*b^3*x^6+20*a^3*b^2*x^4+15*a^4*b*x^2+6
*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.31, size = 57, normalized size = 1.58

$$\frac{1}{12}\,b^5x^{12}+\frac{1}{2}\,ab^4x^{10}+\frac{5}{4}\,a^2b^3x^8+\frac{5}{3}\,a^3b^2x^6+\frac{5}{4}\,a^4bx^4+\frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*b^5*x^12 + 1/2*a*b^4*x^10 + 5/4*a^2*b^3*x^8 + 5/3*a^3*b^2*x^6 + 5/4*a^
4*b*x^4 + 1/2*a^5*x^2
```

mupad [B] time = 4.40, size = 36, normalized size = 1.00

$$\frac{\left(b^2\,x^2+a\,b\right)\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] ((a*b + b^2*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2))/(12*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(\left(a + bx^{2}\right)^{2}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x*((a + b*x**2)**2)**(5/2), x)
```

3.592 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x}\,dx$

Optimal. Leaf size=251

$$\frac{b^5x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10(a+bx^2)}+\frac{5ab^4x^8\sqrt{a^2+2abx^2+b^2x^4}}{8(a+bx^2)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}+\frac{a^5\log(x)\sqrt{a^2+2abx^2+}}{a+bx^2}$$

```
[Out] 5/2*a^4*b*x^2*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/2*a^3*b^2*x^4*((b*x^2+a)^2)^(
1/2)/(b*x^2+a)+5/3*a^2*b^3*x^6*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/8*a*b^4*x^8*
((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/10*b^5*x^10*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+a
^5*ln(x)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.07, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$\frac{b^5x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10(a+bx^2)}+\frac{5ab^4x^8\sqrt{a^2+2abx^2+b^2x^4}}{8(a+bx^2)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^2+}}{2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x,x]
```

```
[Out] (5*a^4*b*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a^3*b^2*
x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a^2*b^3*x^6*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (5*a*b^4*x^8*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(8*(a + b*x^2)) + (b^5*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])/(10*(a + b*x^2)) + (a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x
^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x}\,dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \left(5a^4b^6+\frac{a^5b^5}{x}+10a^3b^7x+10a^2b^8x^2+5ab^9x^3+b^1\right.\right.}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{5a^4bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)} + \frac{5a^3b^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)} + \frac{5a^2b^3x^6\sqrt{a^2+2ab}}{3\left(a+bx^2\right.}$$

Mathematica [A] time = 0.02, size = 82, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(120a^5\log(x)+bx^2\left(300a^4+300a^3bx^2+200a^2b^2x^4+75ab^3x^6+12b^4x^8\right)\right)}{120\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(b*x^2*(300*a^4 + 300*a^3*b*x^2 + 200*a^2*b^2*x^4 + 75
*a*b^3*x^6 + 12*b^4*x^8) + 120*a^5*Log[x]))/(120*(a + b*x^2))
```

fricas [A] time = 0.96, size = 55, normalized size = 0.22

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{8}\,ab^4x^8+\frac{5}{3}\,a^2b^3x^6+\frac{5}{2}\,a^3b^2x^4+\frac{5}{2}\,a^4bx^2+a^5\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] 1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^4
*b*x^2 + a^5*log(x)
```

giac [A] time = 0.19, size = 106, normalized size = 0.42

$$\frac{1}{10}\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{8}\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{3}\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{2}\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{2}\,a^4bx^2\mathrm{sgn}\left(bx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 1/10*b^5*x^10*sgn(b*x^2 + a) + 5/8*a*b^4*x^8*sgn(b*x^2 + a) + 5/3*a^2*b^3*x
^6*sgn(b*x^2 + a) + 5/2*a^3*b^2*x^4*sgn(b*x^2 + a) + 5/2*a^4*b*x^2*sgn(b*x^
2 + a) + 1/2*a^5*log(x^2)*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 79, normalized size = 0.31

$$\frac{\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}\left(12b^5x^{10}+75a\,b^4x^8+200a^2b^3x^6+300a^3b^2x^4+300a^4b\,x^2+120a^5\ln(x)\right)}{120\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x,x)
```

```
[Out] 1/120*((b*x^2+a)^2)^(5/2)*(12*b^5*x^10+75*a*b^4*x^8+200*a^2*b^3*x^6+300*a^3
*b^2*x^4+300*a^4*b*x^2+120*a^5*ln(x))/(b*x^2+a)^5
```

maxima [A] time = 1.36, size = 55, normalized size = 0.22

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{8}\,ab^4x^8+\frac{5}{3}\,a^2b^3x^6+\frac{5}{2}\,a^3b^2x^4+\frac{5}{2}\,a^4bx^2+a^5\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] 1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^4
*b*x^2 + a^5*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x, x)
```

3.593 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^3} dx$

Optimal. Leaf size=250

$$\frac{b^5x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}+\frac{5ab^4x^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+$$

```
[Out] -1/2*a^5*((b*x^2+a)^2)^(1/2)/x^2/(b*x^2+a)+5*a^3*b^2*x^2*((b*x^2+a)^2)^(1/2
)/(b*x^2+a)+5/2*a^2*b^3*x^4*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/6*a*b^4*x^6*((b
*x^2+a)^2)^(1/2)/(b*x^2+a)+1/8*b^5*x^8*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5*a^4*
b*ln(x)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.07, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$\frac{b^5x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}+\frac{5ab^4x^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{5a^3b^2x^2\sqrt{a^2+2abx^2+b}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^3,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (5*a^3*b^2*x^2
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (5*a^2*b^3*x^4*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a*b^4*x^6*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(6*(a + b*x^2)) + (b^5*x^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a
 + b*x^2)) + (5*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^3}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^3}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^2}\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(10a^3b^7+\frac{a^5b^5}{x^2}+\frac{5a^4b^6}{x}+10a^2b^8x+5ab^9x^2+b^{10}x^3\right.\right.}{2b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{5a^3b^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^2-}}{2\left(a+bx^2\right)}$$

Mathematica [A] time = 0.03, size = 85, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-12a^5+120a^4bx^2\log(x)+120a^3b^2x^4+60a^2b^3x^6+20ab^4x^8+3b^5x^{10}\right)}{24x^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-12*a^5 + 120*a^3*b^2*x^4 + 60*a^2*b^3*x^6 + 20*a*b^4
*x^8 + 3*b^5*x^10 + 120*a^4*b*x^2*Log[x]))/(24*x^2*(a + b*x^2))
```

fricas [A] time = 0.69, size = 61, normalized size = 0.24

$$\frac{3\,b^5x^{10}+20\,ab^4x^8+60\,a^2b^3x^6+120\,a^3b^2x^4+120\,a^4bx^2\log(x)-12\,a^5}{24\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^3,x, algorithm="fricas")
```

[Out] 1/24*(3*b^5*x^10 + 20*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^4 + 120*a^
4*b*x^2*log(x) - 12*a^5)/x^2

giac [A] time = 0.16, size = 125, normalized size = 0.50

$$\frac{1}{8}\,b^5x^8\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{6}\,ab^4x^6\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{2}\,a^2b^3x^4\mathrm{sgn}\left(bx^2+a\right)+5\,a^3b^2x^2\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{2}\,a^4b\log\left(x^2\right)\mathrm{sgn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^3,x, algorithm="giac")

[Out] 1/8*b^5*x^8*sgn(b*x^2 + a) + 5/6*a*b^4*x^6*sgn(b*x^2 + a) + 5/2*a^2*b^3*x^4
*sgn(b*x^2 + a) + 5*a^3*b^2*x^2*sgn(b*x^2 + a) + 5/2*a^4*b*log(x^2)*sgn(b*x
^2 + a) - 1/2*(5*a^4*b*x^2*sgn(b*x^2 + a) + a^5*sgn(b*x^2 + a))/x^2

maple [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}\left(3b^5x^{10}+20a\,b^4x^8+60a^2b^3x^6+120a^3b^2x^4+120a^4b\,x^2\ln(x)-12a^5\right)}{24\left(b\,x^2+a\right)^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^3,x)

[Out] 1/24*((b*x^2+a)^2)^(5/2)*(3*b^5*x^10+20*a*b^4*x^8+60*a^2*b^3*x^6+120*a^3*b^
2*x^4+120*a^4*b*ln(x)*x^2-12*a^5)/(b*x^2+a)^5/x^2

maxima [A] time = 1.34, size = 56, normalized size = 0.22

$$\frac{1}{8}\,b^5x^8+\frac{5}{6}\,ab^4x^6+\frac{5}{2}\,a^2b^3x^4+5\,a^3b^2x^2+5\,a^4b\log(x)-\frac{a^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^3,x, algorithm="maxima")

[Out] 1/8*b^5*x^8 + 5/6*a*b^4*x^6 + 5/2*a^2*b^3*x^4 + 5*a^3*b^2*x^2 + 5*a^4*b*log
(x) - 1/2*a^5/x^2

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^3,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**3,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**3, x)
```

3.594 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^5}\,dx$

Optimal. Leaf size=250

$$\frac{b^5x^6\sqrt{a^2+2abx^2+b^2x^4}}{6(a+bx^2)}+\frac{5ab^4x^4\sqrt{a^2+2abx^2+b^2x^4}}{4(a+bx^2)}+\frac{5a^2b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{4x^4(a+bx^2)}-$$

```
[Out] -1/4*a^5*((b*x^2+a)^2)^(1/2)/x^4/(b*x^2+a)-5/2*a^4*b*((b*x^2+a)^2)^(1/2)/x^
2/(b*x^2+a)+5*a^2*b^3*x^2*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/4*a*b^4*x^4*((b*x
^2+a)^2)^(1/2)/(b*x^2+a)+1/6*b^5*x^6*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+10*a^3*b
^2*ln(x)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.07, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$\frac{b^5x^6\sqrt{a^2+2abx^2+b^2x^4}}{6(a+bx^2)}+\frac{5ab^4x^4\sqrt{a^2+2abx^2+b^2x^4}}{4(a+bx^2)}+\frac{5a^2b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^5,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (5*a^2*b^3*x^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (5*a*b^4*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(4*(a + b*x^2)) + (b^5*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*(a +
 b*x^2)) + (10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^5}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^5}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^3}\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(10a^2b^8+\frac{a^5b^5}{x^3}+\frac{5a^4b^6}{x^2}+\frac{10a^3b^7}{x}+5ab^9x+b^{10}x^2\right) d$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)} + \frac{5a^2b^3x^2\sqrt{a^2+2abx^2+b^2}}{a+bx^2}$$

Mathematica [A] time = 0.03, size = 85, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-3a^5-30a^4bx^2+120a^3b^2x^4\log(x)+60a^2b^3x^6+15ab^4x^8+2b^5x^{10}\right)}{12x^4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^5,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-3*a^5 - 30*a^4*b*x^2 + 60*a^2*b^3*x^6 + 15*a*b^4*x^8
 + 2*b^5*x^10 + 120*a^3*b^2*x^4*Log[x]))/(12*x^4*(a + b*x^2))
```

fricas [A] time = 1.05, size = 61, normalized size = 0.24

$$\frac{2\,b^5x^{10}+15\,ab^4x^8+60\,a^2b^3x^6+120\,a^3b^2x^4\log(x)-30\,a^4bx^2-3\,a^5}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^5,x, algorithm="fricas")
```

[Out] 1/12*(2*b^5*x^10 + 15*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^4*log(x) - 30*a^4*b*x^2 - 3*a^5)/x^4

giac [A] time = 0.16, size = 127, normalized size = 0.51

$$\frac{1}{6}b^5x^6\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{4}ab^4x^4\mathrm{sgn}\left(bx^2+a\right)+5a^2b^3x^2\mathrm{sgn}\left(bx^2+a\right)+5a^3b^2\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)-\frac{30a^3b^2x^4\mathrm{sg}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^5,x, algorithm="giac")

[Out] 1/6*b^5*x^6*sgn(b*x^2 + a) + 5/4*a*b^4*x^4*sgn(b*x^2 + a) + 5*a^2*b^3*x^2*sgn(b*x^2 + a) + 5*a^3*b^2*log(x^2)*sgn(b*x^2 + a) - 1/4*(30*a^3*b^2*x^4*sgn(b*x^2 + a) + 10*a^4*b*x^2*sgn(b*x^2 + a) + a^5*sgn(b*x^2 + a))/x^4

maple [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}\left(2b^5x^{10}+15ab^4x^8+60a^2b^3x^6+120a^3b^2x^4\ln(x)-30a^4bx^2-3a^5\right)}{12\left(bx^2+a\right)^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^5,x)

[Out] 1/12*((b*x^2+a)^2)^(5/2)*(2*b^5*x^10+15*a*b^4*x^8+60*a^2*b^3*x^6+120*a^3*b^2*ln(x)*x^4-30*a^4*b*x^2-3*a^5)/(b*x^2+a)^5/x^4

maxima [A] time = 1.34, size = 56, normalized size = 0.22

$$\frac{1}{6}b^5x^6+\frac{5}{4}ab^4x^4+5a^2b^3x^2+10a^3b^2\log(x)-\frac{5a^4b}{2x^2}-\frac{a^5}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^5,x, algorithm="maxima")

[Out] 1/6*b^5*x^6 + 5/4*a*b^4*x^4 + 5*a^2*b^3*x^2 + 10*a^3*b^2*log(x) - 5/2*a^4*b/x^2 - 1/4*a^5/x^4

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^5,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**5,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**5, x)
```

3.595 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^7}\,dx$

Optimal. Leaf size=250

$$\frac{b^5x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{5ab^4x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{10a^2b^3\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}$$

```
[Out] -1/6*a^5*((b*x^2+a)^2)^(1/2)/x^6/(b*x^2+a)-5/4*a^4*b*((b*x^2+a)^2)^(1/2)/x^
4/(b*x^2+a)-5*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^2/(b*x^2+a)+5/2*a*b^4*x^2*((b*x
^2+a)^2)^(1/2)/(b*x^2+a)+1/4*b^5*x^4*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+10*a^2*b
^3*ln(x)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.07, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^2\left(a+bx^2\right)}+\frac{5ab^4x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^7,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(x^2*(a + b*x^2)) + (5*a*b^4*x^2*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(2*(a + b*x^2)) + (b^5*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a
 + b*x^2)) + (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2
)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^7}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^7}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\;\mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^4}\,dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\;\mathrm{Subst}\left(\int \left(5ab^9+\frac{a^5b^5}{x^4}+\frac{5a^4b^6}{x^3}+\frac{10a^3b^7}{x^2}+\frac{10a^2b^8}{x}+b^{10}x\right)dx, x\right.}{2b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)} - \frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^2\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 85, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-2a^5-15a^4bx^2-60a^3b^2x^4+120a^2b^3x^6\log(x)+30ab^4x^8+3b^5x^{10}\right)}{12x^6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^7,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-2*a^5 - 15*a^4*b*x^2 - 60*a^3*b^2*x^4 + 30*a*b^4*x^8
 + 3*b^5*x^10 + 120*a^2*b^3*x^6*Log[x]))/(12*x^6*(a + b*x^2))
```

fricas [A] time = 0.88, size = 61, normalized size = 0.24

$$\frac{3\,b^5x^{10}+30\,ab^4x^8+120\,a^2b^3x^6\log(x)-60\,a^3b^2x^4-15\,a^4bx^2-2\,a^5}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^10 + 30*a*b^4*x^8 + 120*a^2*b^3*x^6*log(x) - 60*a^3*b^2*x^4 -
 15*a^4*b*x^2 - 2*a^5)/x^6
```

giac [A] time = 0.16, size = 128, normalized size = 0.51

$$\frac{1}{4}\,b^5x^4\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{2}\,ab^4x^2\mathrm{sgn}\left(bx^2+a\right)+5\,a^2b^3\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)-\frac{110\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+60\,a^3b^2x^4 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/4*b^5*x^4*sgn(b*x^2 + a) + 5/2*a*b^4*x^2*sgn(b*x^2 + a) + 5*a^2*b^3*log(x
^2)*sgn(b*x^2 + a) - 1/12*(110*a^2*b^3*x^6*sgn(b*x^2 + a) + 60*a^3*b^2*x^4*
sgn(b*x^2 + a) + 15*a^4*b*x^2*sgn(b*x^2 + a) + 2*a^5*sgn(b*x^2 + a))/x^6
```

maple [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}\left(3b^5x^{10}+30a\,b^4x^8+120a^2b^3x^6\ln(x)-60a^3b^2x^4-15a^4b\,x^2-2a^5\right)}{12\left(b\,x^2+a\right)^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^7,x)
```

```
[Out] 1/12*((b*x^2+a)^2)^(5/2)*(3*b^5*x^10+30*a*b^4*x^8+120*a^2*b^3*ln(x)*x^6-60*
a^3*b^2*x^4-15*a^4*b*x^2-2*a^5)/(b*x^2+a)^5/x^6
```

maxima [A] time = 1.38, size = 56, normalized size = 0.22

$$\frac{1}{4}\,b^5x^4+\frac{5}{2}\,ab^4x^2+10\,a^2b^3\log(x)-\frac{5\,a^3b^2}{x^2}-\frac{5\,a^4b}{4\,x^4}-\frac{a^5}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*b^5*x^4 + 5/2*a*b^4*x^2 + 10*a^2*b^3*log(x) - 5*a^3*b^2/x^2 - 5/4*a^4*b
/x^4 - 1/6*a^5/x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^7,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^{2}\right)^{2}\right)^{\frac{5}{2}}}{x^{7}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**7,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**7, x)
```

3.596 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^9}\,dx$

Optimal. Leaf size=250

$$\frac{b^5x^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{5ab^4\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x^2(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}$$

```
[Out] -1/8*a^5*((b*x^2+a)^2)^(1/2)/x^8/(b*x^2+a)-5/6*a^4*b*((b*x^2+a)^2)^(1/2)/x^
6/(b*x^2+a)-5/2*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^4/(b*x^2+a)-5*a^2*b^3*((b*x^2
+a)^2)^(1/2)/x^2/(b*x^2+a)+1/2*b^5*x^2*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5*a*b^
4*ln(x)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.07, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6(a+bx^2)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^4(a+bx^2)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x^2(a+bx^2)}+\frac{b^5}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^9,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(2*x^4*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(x^2*(a + b*x^2)) + (b^5*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*
(a + b*x^2)) + (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^9}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^9}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^5}\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(b^{10}+\frac{a^5b^5}{x^5}+\frac{5a^4b^6}{x^4}+\frac{10a^3b^7}{x^3}+\frac{10a^2b^8}{x^2}+\frac{5ab^9}{x}\right)dx,x,x\ldots}{2b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^4\left(a+bx^2\right)}\ldots$$

Mathematica [A] time = 0.02, size = 85, normalized size = 0.34

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(3a^5+20a^4bx^2+60a^3b^2x^4+120a^2b^3x^6-120ab^4x^8\log(x)-12b^5x^{10}\right)}{24x^8\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^9,x]
```

```
[Out] -1/24*(Sqrt[(a + b*x^2)^2]*(3*a^5 + 20*a^4*b*x^2 + 60*a^3*b^2*x^4 + 120*a^2
*b^3*x^6 - 12*b^5*x^10 - 120*a*b^4*x^8*Log[x]))/(x^8*(a + b*x^2))
```

fricas [A] time = 0.68, size = 61, normalized size = 0.24

$$\frac{12\,b^5x^{10}+120\,ab^4x^8\log(x)-120\,a^2b^3x^6-60\,a^3b^2x^4-20\,a^4bx^2-3\,a^5}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(12*b^5*x^10 + 120*a*b^4*x^8*log(x) - 120*a^2*b^3*x^6 - 60*a^3*b^2*x^4
 - 20*a^4*b*x^2 - 3*a^5)/x^8
```

giac [A] time = 0.19, size = 126, normalized size = 0.50

$$\frac{1}{2}\,b^5x^2\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{2}\,ab^4\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)-\frac{125\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+120\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+60\,a^3b}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x^2*sgn(b*x^2 + a) + 5/2*a*b^4*log(x^2)*sgn(b*x^2 + a) - 1/24*(125*
a*b^4*x^8*sgn(b*x^2 + a) + 120*a^2*b^3*x^6*sgn(b*x^2 + a) + 60*a^3*b^2*x^4*
sgn(b*x^2 + a) + 20*a^4*b*x^2*sgn(b*x^2 + a) + 3*a^5*sgn(b*x^2 + a))/x^8
```

maple [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}\left(12b^5x^{10}+120a\,b^4x^8\ln(x)-120a^2b^3x^6-60a^3b^2x^4-20a^4b\,x^2-3a^5\right)}{24\left(b\,x^2+a\right)^5x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^9,x)
```

```
[Out] 1/24*((b*x^2+a)^2)^(5/2)*(12*b^5*x^10+120*a*b^4*ln(x)*x^8-120*a^2*b^3*x^6-6
0*a^3*b^2*x^4-20*a^4*b*x^2-3*a^5)/(b*x^2+a)^5/x^8
```

maxima [A] time = 1.36, size = 56, normalized size = 0.22

$$\frac{1}{2}\,b^5x^2+5\,ab^4\log(x)-\frac{5\,a^2b^3}{x^2}-\frac{5\,a^3b^2}{2\,x^4}-\frac{5\,a^4b}{6\,x^6}-\frac{a^5}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*b^5*x^2 + 5*a*b^4*log(x) - 5*a^2*b^3/x^2 - 5/2*a^3*b^2/x^4 - 5/6*a^4*b/
x^6 - 1/8*a^5/x^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^9,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^9, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**9,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**9, x)
```

3.597 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{11}} \, dx$

Optimal. Leaf size=251

$$\frac{b^5 \log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^4\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-$$

```
[Out] -1/10*a^5*((b*x^2+a)^2)^(1/2)/x^10/(b*x^2+a)-5/8*a^4*b*((b*x^2+a)^2)^(1/2)/
x^8/(b*x^2+a)-5/3*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^6/(b*x^2+a)-5/2*a^2*b^3*((b
*x^2+a)^2)^(1/2)/x^4/(b*x^2+a)-5/2*a*b^4*((b*x^2+a)^2)^(1/2)/x^2/(b*x^2+a)+
b^5*ln(x)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.07, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^6\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^4\left(a+bx^2\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^11,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(3*x^6*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])/(2*x^4*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
/(2*x^2*(a + b*x^2)) + (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*
x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{11}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{11}}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^6}\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(\frac{a^5b^5}{x^6}+\frac{5a^4b^6}{x^5}+\frac{10a^3b^7}{x^4}+\frac{10a^2b^8}{x^3}+\frac{5ab^9}{x^2}+\frac{b^{10}}{x}\right)dx,x,x$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^6\left(a+bx^2\right)}$$

Mathematica [A] time = 0.03, size = 85, normalized size = 0.34

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a\left(12a^4+75a^3bx^2+200a^2b^2x^4+300ab^3x^6+300b^4x^8\right)-120b^5x^{10}\log(x)\right)}{120x^{10}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^11,x]
```

```
[Out] -1/120*(Sqrt[(a + b*x^2)^2]*(a*(12*a^4 + 75*a^3*b*x^2 + 200*a^2*b^2*x^4 + 3
00*a*b^3*x^6 + 300*b^4*x^8) - 120*b^5*x^10*Log[x]))/(x^10*(a + b*x^2))
```

fricas [A] time = 1.09, size = 61, normalized size = 0.24

$$\frac{120\,b^5x^{10}\log(x)-300\,ab^4x^8-300\,a^2b^3x^6-200\,a^3b^2x^4-75\,a^4bx^2-12\,a^5}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/120*(120*b^5*x^10*log(x) - 300*a*b^4*x^8 - 300*a^2*b^3*x^6 - 200*a^3*b^2*
x^4 - 75*a^4*b*x^2 - 12*a^5)/x^10
```

giac [A] time = 0.16, size = 125, normalized size = 0.50

$$\frac{1}{2}\,b^5\log\left(x^2\right)\operatorname{sgn}\left(bx^2+a\right)-\frac{137\,b^5x^{10}\operatorname{sgn}\left(bx^2+a\right)+300\,ab^4x^8\operatorname{sgn}\left(bx^2+a\right)+300\,a^2b^3x^6\operatorname{sgn}\left(bx^2+a\right)+200}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/2*b^5*log(x^2)*sgn(b*x^2 + a) - 1/120*(137*b^5*x^10*sgn(b*x^2 + a) + 300*
a*b^4*x^8*sgn(b*x^2 + a) + 300*a^2*b^3*x^6*sgn(b*x^2 + a) + 200*a^3*b^2*x^4
*sgn(b*x^2 + a) + 75*a^4*b*x^2*sgn(b*x^2 + a) + 12*a^5*sgn(b*x^2 + a))/x^10
```

maple [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}\left(120b^5x^{10}\ln(x)-300a\,b^4x^8-300a^2b^3x^6-200a^3b^2x^4-75a^4b\,x^2-12a^5\right)}{120\left(b\,x^2+a\right)^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^11,x)
```

```
[Out] 1/120*((b*x^2+a)^2)^(5/2)*(120*b^5*ln(x)*x^10-300*a*b^4*x^8-300*a^2*b^3*x^6
-200*a^3*b^2*x^4-75*a^4*b*x^2-12*a^5)/(b*x^2+a)^5/x^10
```

maxima [A] time = 1.32, size = 55, normalized size = 0.22

$$b^5\log(x)-\frac{5\,ab^4}{2\,x^2}-\frac{5\,a^2b^3}{2\,x^4}-\frac{5\,a^3b^2}{3\,x^6}-\frac{5\,a^4b}{8\,x^8}-\frac{a^5}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^11,x, algorithm="maxima")
```

```
[Out] b^5*log(x) - 5/2*a*b^4/x^2 - 5/2*a^2*b^3/x^4 - 5/3*a^3*b^2/x^6 - 5/8*a^4*b/
x^8 - 1/10*a^5/x^10
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^11,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^11, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{11}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**11,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**11, x)
```

3.598 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{13}}\,dx$

Optimal. Leaf size=41

$$-\frac{\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{12ax^{12}}$$

```
[Out] -1/12*(b*x^2+a)^5*((b*x^2+a)^2)^(1/2)/a/x^12
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 37}

$$-\frac{\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^13,x]
```

```
[Out] -((a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*a*x^12)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{13}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^7}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^7}\,dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{12ax^{12}}$$

Mathematica [A] time = 0.02, size = 81, normalized size = 1.98

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a^5+6a^4bx^2+15a^3b^2x^4+20a^2b^3x^6+15ab^4x^8+6b^5x^{10}\right)}{12x^{12}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^13,x]
```

```
[Out] -1/12*(Sqrt[(a + b*x^2)^2]*(a^5 + 6*a^4*b*x^2 + 15*a^3*b^2*x^4 + 20*a^2*b^3
*x^6 + 15*a*b^4*x^8 + 6*b^5*x^10))/(x^12*(a + b*x^2))
```

fricas [B] time = 0.87, size = 57, normalized size = 1.39

$$-\frac{6\,b^5x^{10}+15\,ab^4x^8+20\,a^2b^3x^6+15\,a^3b^2x^4+6\,a^4bx^2+a^5}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^5*x^10 + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^4 + 6*a^4*
b*x^2 + a^5)/x^12
```

giac [B] time = 0.16, size = 106, normalized size = 2.59

$$-\frac{6\,b^5x^{10}\text{sgn}\left(bx^2+a\right)+15\,ab^4x^8\text{sgn}\left(bx^2+a\right)+20\,a^2b^3x^6\text{sgn}\left(bx^2+a\right)+15\,a^3b^2x^4\text{sgn}\left(bx^2+a\right)+6\,a^4bx^2\text{sgn}}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^13,x, algorithm="giac")
```

```
[Out] -1/12*(6*b^5*x^10*sgn(b*x^2 + a) + 15*a*b^4*x^8*sgn(b*x^2 + a) + 20*a^2*b^3
*x^6*sgn(b*x^2 + a) + 15*a^3*b^2*x^4*sgn(b*x^2 + a) + 6*a^4*b*x^2*sgn(b*x^2
 + a) + a^5*sgn(b*x^2 + a))/x^12
```

maple [B] time = 0.01, size = 78, normalized size = 1.90

$$-\frac{\left(6b^{5}x^{10}+15a\,b^{4}x^{8}+20a^{2}b^{3}x^{6}+15a^{3}b^{2}x^{4}+6a^{4}b\,x^{2}+a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{12\left(b\,x^{2}+a\right)^{5}x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^13,x)
```

```
[Out] -1/12*(6*b^5*x^10+15*a*b^4*x^8+20*a^2*b^3*x^6+15*a^3*b^2*x^4+6*a^4*b*x^2+a^
5)*((b*x^2+a)^2)^(5/2)/x^12/(b*x^2+a)^5
```

maxima [B] time = 1.40, size = 57, normalized size = 1.39

$$-\frac{b^{5}}{2\,x^{2}}-\frac{5\,ab^{4}}{4\,x^{4}}-\frac{5\,a^{2}b^{3}}{3\,x^{6}}-\frac{5\,a^{3}b^{2}}{4\,x^{8}}-\frac{a^{4}b}{2\,x^{10}}-\frac{a^{5}}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^13,x, algorithm="maxima")
```

```
[Out] -1/2*b^5/x^2 - 5/4*a*b^4/x^4 - 5/3*a^2*b^3/x^6 - 5/4*a^3*b^2/x^8 - 1/2*a^4*
b/x^10 - 1/12*a^5/x^12
```

mupad [B] time = 4.18, size = 231, normalized size = 5.63

$$-\frac{a^{5}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{12\,x^{12}\,\left(b\,x^{2}+a\right)}-\frac{b^{5}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{2\,x^{2}\,\left(b\,x^{2}+a\right)}-\frac{5\,a\,b^{4}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{4\,x^{4}\,\left(b\,x^{2}+a\right)}-\frac{a^{4}\,b\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{2\,x^{10}\,\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^13,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(12*x^12*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(2*x^2*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(4*x^4*(a + b*x^2)) - (a^4*b*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(2*x^10*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1
/2))/(3*x^6*(a + b*x^2)) - (5*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(4
*x^8*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**13,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**13, x)
```

3.599 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{15}} \, dx$

Optimal. Leaf size=72

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{84a^2x^{14}}-\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12ax^{14}}$$

```
[Out] -1/12*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/a/x^14+1/84*(b^2*x^4+2*a*b*x^
2+a^2)^(7/2)/a^2/x^14
```

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1110}

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{84a^2x^{14}}-\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^15,x]
```

```
[Out] -((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(12*a*x^14) + (a^2 + 2*a*b
*x^2 + b^2*x^4)^(7/2)/(84*a^2*x^14)
```

Rule 1110

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(4*a*d*(p + 1)*(2*p +
1)), x] - Simp[((d*x)^(m + 1)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^p)/(4*a*d*(
2*p + 1)), x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !I
ntegerQ[p] && EqQ[m + 4*p + 5, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{15}} \, dx=-\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12ax^{14}}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{84a^2x^{14}}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 1.15

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(6a^5+35a^4bx^2+84a^3b^2x^4+105a^2b^3x^6+70ab^4x^8+21b^5x^{10}\right)}{84x^{14}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^15,x]
```

```
[Out] -1/84*(Sqrt[(a + b*x^2)^2]*(6*a^5 + 35*a^4*b*x^2 + 84*a^3*b^2*x^4 + 105*a^2
*b^3*x^6 + 70*a*b^4*x^8 + 21*b^5*x^10))/(x^14*(a + b*x^2))
```

fricas [A] time = 0.62, size = 59, normalized size = 0.82

$$-\frac{21\,b^5x^{10}+70\,ab^4x^8+105\,a^2b^3x^6+84\,a^3b^2x^4+35\,a^4bx^2+6\,a^5}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^15,x, algorithm="fricas")
```

```
[Out] -1/84*(21*b^5*x^10 + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*x^4 + 35*a
^4*b*x^2 + 6*a^5)/x^14
```

giac [A] time = 0.16, size = 107, normalized size = 1.49

$$-\frac{21\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+70\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+105\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+84\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+35\,a^4bx^2}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^15,x, algorithm="giac")
```

```
[Out] -1/84*(21*b^5*x^10*sgn(b*x^2 + a) + 70*a*b^4*x^8*sgn(b*x^2 + a) + 105*a^2*b
^3*x^6*sgn(b*x^2 + a) + 84*a^3*b^2*x^4*sgn(b*x^2 + a) + 35*a^4*b*x^2*sgn(b*
x^2 + a) + 6*a^5*sgn(b*x^2 + a))/x^14
```

maple [A] time = 0.01, size = 80, normalized size = 1.11

$$-\frac{\left(21b^5x^{10}+70a\,b^4x^8+105a^2b^3x^6+84a^3b^2x^4+35a^4b\,x^2+6a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{84\left(b\,x^2+a\right)^5x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^15,x)
```

```
[Out] -1/84*(21*b^5*x^10+70*a*b^4*x^8+105*a^2*b^3*x^6+84*a^3*b^2*x^4+35*a^4*b*x^2
+6*a^5)*((b*x^2+a)^2)^(5/2)/x^14/(b*x^2+a)^5
```

maxima [A] time = 1.35, size = 57, normalized size = 0.79

$$-\frac{b^5}{4\,x^4}-\frac{5\,ab^4}{6\,x^6}-\frac{5\,a^2b^3}{4\,x^8}-\frac{a^3b^2}{x^{10}}-\frac{5\,a^4b}{12\,x^{12}}-\frac{a^5}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^15,x, algorithm="maxima")
```

```
[Out] -1/4*b^5/x^4 - 5/6*a*b^4/x^6 - 5/4*a^2*b^3/x^8 - a^3*b^2/x^10 - 5/12*a^4*b/
x^12 - 1/14*a^5/x^14
```

mupad [B] time = 4.22, size = 231, normalized size = 3.21

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{14\,x^{14}\,\left(b\,x^2+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^4\,\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{6\,x^6\,\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x}}{12\,x^{12}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^15,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(14*x^14*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(4*x^4*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(6*x^6*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a*b
*x^2)^(1/2))/(12*x^12*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2)
^(1/2))/(4*x^8*(a + b*x^2)) - (a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(
x^10*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{15}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**15,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**15, x)
```

3.600 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{17}}\,dx$

Optimal. Leaf size=128

$$-\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^5}{16ax^{16}}+\frac{b\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^5}{56a^2x^{14}}-\frac{b^2\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^5}{336a^3x^{12}}$$

```
[Out] -1/16*(b*x^2+a)^5*((b*x^2+a)^2)^(1/2)/a/x^16+1/56*b*(b*x^2+a)^5*((b*x^2+a)^
2)^(1/2)/a^2/x^14-1/336*b^2*(b*x^2+a)^5*((b*x^2+a)^2)^(1/2)/a^3/x^12
```

Rubi [A] time = 0.09, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1111, 646, 45, 37}

$$-\frac{b^2\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^5}{336a^3x^{12}}+\frac{b\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^5}{56a^2x^{14}}-\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^5}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^17,x]
```

```
[Out] -((a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*a*x^16) + (b*(a + b*x^
2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(56*a^2*x^14) - (b^2*(a + b*x^2)^5*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(336*a^3*x^12)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{17}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^9}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^9}\,dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{16ax^{16}} - \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^8}\,dx, x\right.}{8ab^3\left(ab+b^2x^2\right)}$$

$$= -\frac{\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{16ax^{16}} + \frac{b\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{56a^2x^{14}} + \frac{\sqrt{a^2+2}}{}$$

$$= -\frac{\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{16ax^{16}} + \frac{b\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{56a^2x^{14}} - \frac{b^2\left(a+\right.}{}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.65

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(21a^5+120a^4bx^2+280a^3b^2x^4+336a^2b^3x^6+210ab^4x^8+56b^5x^{10}\right)}{336x^{16}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^17,x]
```

```
[Out] -1/336*(Sqrt[(a + b*x^2)^2]*(21*a^5 + 120*a^4*b*x^2 + 280*a^3*b^2*x^4 + 336
*a^2*b^3*x^6 + 210*a*b^4*x^8 + 56*b^5*x^10))/(x^16*(a + b*x^2))
```

fricas [A] time = 1.21, size = 59, normalized size = 0.46

$$-\frac{56\,b^5x^{10}+210\,ab^4x^8+336\,a^2b^3x^6+280\,a^3b^2x^4+120\,a^4bx^2+21\,a^5}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/336*(56*b^5*x^10 + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b^2*x^4 + 1
20*a^4*b*x^2 + 21*a^5)/x^16
```

giac [A] time = 0.21, size = 107, normalized size = 0.84

$$-\frac{56\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+210\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+336\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+280\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+120\,a^4}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^17,x, algorithm="giac")
```

```
[Out] -1/336*(56*b^5*x^10*sgn(b*x^2 + a) + 210*a*b^4*x^8*sgn(b*x^2 + a) + 336*a^2
*b^3*x^6*sgn(b*x^2 + a) + 280*a^3*b^2*x^4*sgn(b*x^2 + a) + 120*a^4*b*x^2*sg
n(b*x^2 + a) + 21*a^5*sgn(b*x^2 + a))/x^16
```

maple [A] time = 0.01, size = 80, normalized size = 0.62

$$-\frac{\left(56b^5x^{10}+210a\,b^4x^8+336a^2b^3x^6+280a^3b^2x^4+120a^4b\,x^2+21a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{336\left(b\,x^2+a\right)^5x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^17,x)
```

```
[Out] -1/336*(56*b^5*x^10+210*a*b^4*x^8+336*a^2*b^3*x^6+280*a^3*b^2*x^4+120*a^4*b
*x^2+21*a^5)*((b*x^2+a)^2)^(5/2)/x^16/(b*x^2+a)^5
```

maxima [A] time = 1.35, size = 57, normalized size = 0.45

$$-\frac{b^5}{6\,x^6}-\frac{5\,ab^4}{8\,x^8}-\frac{a^2b^3}{x^{10}}-\frac{5\,a^3b^2}{6\,x^{12}}-\frac{5\,a^4b}{14\,x^{14}}-\frac{a^5}{16\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^17,x, algorithm="maxima")
```

```
[Out] -1/6*b^5/x^6 - 5/8*a*b^4/x^8 - a^2*b^3/x^10 - 5/6*a^3*b^2/x^12 - 5/14*a^4*b
/x^14 - 1/16*a^5/x^16
```

mupad [B] time = 4.24, size = 231, normalized size = 1.80

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{16\,x^{16}\,\left(b\,x^2+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{6\,x^6\,\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^8\,\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x}}{14\,x^{14}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^17,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(16*x^16*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(6*x^6*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(8*x^8*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a*b
*x^2)^(1/2))/(14*x^14*(a + b*x^2)) - (a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(
1/2))/(x^10*(a + b*x^2)) - (5*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(6
*x^12*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^{2}\right)^{2}\right)^{\frac{5}{2}}}{x^{17}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**17,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**17, x)
```

3.601 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{19}} dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{2x^{10}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^{12}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2}}{16x}$$

```
[Out] -1/18*a^5*((b*x^2+a)^2)^(1/2)/x^18/(b*x^2+a)-5/16*a^4*b*((b*x^2+a)^2)^(1/2)
/x^16/(b*x^2+a)-5/7*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^14/(b*x^2+a)-5/6*a^2*b^3*
((b*x^2+a)^2)^(1/2)/x^12/(b*x^2+a)-1/2*a*b^4*((b*x^2+a)^2)^(1/2)/x^10/(b*x^
2+a)-1/8*b^5*((b*x^2+a)^2)^(1/2)/x^8/(b*x^2+a)
```

Rubi [A] time = 0.16, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^{14}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^{12}\left(a+bx^2\right)}-ab^4$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^19,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(18*x^18*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*x^16*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(7*x^14*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(6*x^12*(a + b*x^2)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])/(2*x^10*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a +
 b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
```

```
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{19}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{10}}\,dx,x,x^2\right)$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\;\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^{10}}\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\;\text{Subst}\left(\int \left(\frac{a^5b^5}{x^{10}}+\frac{5a^4b^6}{x^9}+\frac{10a^3b^7}{x^8}+\frac{10a^2b^8}{x^7}+\frac{5ab^9}{x^6}+\frac{b^{10}}{x^5}\right)dx,x,\right.}{2b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x}}{7x^{14}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(56a^5+315a^4bx^2+720a^3b^2x^4+840a^2b^3x^6+504ab^4x^8+126b^5x^{10}\right)}{1008x^{18}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^19,x]
```

```
[Out] -1/1008*(Sqrt[(a + b*x^2)^2]*(56*a^5 + 315*a^4*b*x^2 + 720*a^3*b^2*x^4 + 84
0*a^2*b^3*x^6 + 504*a*b^4*x^8 + 126*b^5*x^10))/(x^18*(a + b*x^2))
```

fricas [A] time = 1.17, size = 59, normalized size = 0.23

$$-\frac{126\,b^5x^{10}+504\,ab^4x^8+840\,a^2b^3x^6+720\,a^3b^2x^4+315\,a^4bx^2+56\,a^5}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^19,x, algorithm="fricas")
```

```
[Out] -1/1008*(126*b^5*x^10 + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3*b^2*x^4 +
 315*a^4*b*x^2 + 56*a^5)/x^18
```

giac [A] time = 0.17, size = 107, normalized size = 0.42

$$-\frac{126\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+504\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+840\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+720\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+315\,a}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^19,x, algorithm="giac")
```

```
[Out] -1/1008*(126*b^5*x^10*sgn(b*x^2 + a) + 504*a*b^4*x^8*sgn(b*x^2 + a) + 840*a
^2*b^3*x^6*sgn(b*x^2 + a) + 720*a^3*b^2*x^4*sgn(b*x^2 + a) + 315*a^4*b*x^2*
sgn(b*x^2 + a) + 56*a^5*sgn(b*x^2 + a))/x^18
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(126b^5x^{10}+504a\,b^4x^8+840a^2b^3x^6+720a^3b^2x^4+315a^4b\,x^2+56a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{1008\left(b\,x^2+a\right)^5x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^19,x)
```

```
[Out] -1/1008*(126*b^5*x^10+504*a*b^4*x^8+840*a^2*b^3*x^6+720*a^3*b^2*x^4+315*a^4
*b*x^2+56*a^5)*((b*x^2+a)^2)^(5/2)/x^18/(b*x^2+a)^5
```

maxima [A] time = 1.33, size = 57, normalized size = 0.22

$$-\frac{b^5}{8\,x^8}-\frac{ab^4}{2\,x^{10}}-\frac{5\,a^2b^3}{6\,x^{12}}-\frac{5\,a^3b^2}{7\,x^{14}}-\frac{5\,a^4b}{16\,x^{16}}-\frac{a^5}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^19,x, algorithm="maxima")
```

```
[Out] -1/8*b^5/x^8 - 1/2*a*b^4/x^10 - 5/6*a^2*b^3/x^12 - 5/7*a^3*b^2/x^14 - 5/16*
a^4*b/x^16 - 1/18*a^5/x^18
```

mupad [B] time = 4.27, size = 231, normalized size = 0.91

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{18\,x^{18}\,\left(b\,x^2+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^8\,\left(b\,x^2+a\right)}-\frac{a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^{10}\,\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{16\,x^{16}\,\left(b\,x^2+a\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^19,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(18*x^18*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(8*x^8*(a + b*x^2)) - (a*b^4*(a^2 + b^2*x^4
+ 2*a*b*x^2)^(1/2))/(2*x^10*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a*b*
x^2)^(1/2))/(16*x^16*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^
(1/2))/(6*x^12*(a + b*x^2)) - (5*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))
/(7*x^14*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{19}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**19,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**19, x)
```

3.602 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{21}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^{14}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{20x^{20}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{\ldots}}{18\ldots}$$

```
[Out] -1/20*a^5*((b*x^2+a)^2)^(1/2)/x^20/(b*x^2+a)-5/18*a^4*b*((b*x^2+a)^2)^(1/2)
/x^18/(b*x^2+a)-5/8*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^16/(b*x^2+a)-5/7*a^2*b^3*
((b*x^2+a)^2)^(1/2)/x^14/(b*x^2+a)-5/12*a*b^4*((b*x^2+a)^2)^(1/2)/x^12/(b*x
^2+a)-1/10*b^5*((b*x^2+a)^2)^(1/2)/x^10/(b*x^2+a)
```

Rubi [A] time = 0.15, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{20x^{20}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^{16}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^{14}\left(a+bx^2\right)}-5ab\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^21,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(20*x^20*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(18*x^18*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(8*x^16*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(7*x^14*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])/(12*x^12*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10
*(a + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
```

```
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{21}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{11}}\,dx,x,x^2\right)$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^{11}}\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \left(\frac{a^5b^5}{x^{11}}+\frac{5a^4b^6}{x^{10}}+\frac{10a^3b^7}{x^9}+\frac{10a^2b^8}{x^8}+\frac{5ab^9}{x^7}+\frac{b^{10}}{x^6}\right)dx,x,\right.}{2b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{20x^{20}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x}}{8x^{16}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(126a^5+700a^4bx^2+1575a^3b^2x^4+1800a^2b^3x^6+1050ab^4x^8+252b^5x^{10}\right)}{2520x^{20}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^21,x]
```

```
[Out] -1/2520*(Sqrt[(a + b*x^2)^2]*(126*a^5 + 700*a^4*b*x^2 + 1575*a^3*b^2*x^4 +
1800*a^2*b^3*x^6 + 1050*a*b^4*x^8 + 252*b^5*x^10))/(x^20*(a + b*x^2))
```

fricas [A] time = 1.13, size = 59, normalized size = 0.23

$$-\frac{252\,b^5x^{10}+1050\,ab^4x^8+1800\,a^2b^3x^6+1575\,a^3b^2x^4+700\,a^4bx^2+126\,a^5}{2520\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^21,x, algorithm="fricas")

[Out] -1/2520*(252*b^5*x^10 + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*a^3*b^2*x^4 + 700*a^4*b*x^2 + 126*a^5)/x^20

giac [A] time = 0.16, size = 107, normalized size = 0.42

$$-\frac{252\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+1050\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+1800\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+1575\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+7}{2520\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^21,x, algorithm="giac")

[Out] -1/2520*(252*b^5*x^10*sgn(b*x^2 + a) + 1050*a*b^4*x^8*sgn(b*x^2 + a) + 1800*a^2*b^3*x^6*sgn(b*x^2 + a) + 1575*a^3*b^2*x^4*sgn(b*x^2 + a) + 700*a^4*b*x^2*sgn(b*x^2 + a) + 126*a^5*sgn(b*x^2 + a))/x^20

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(252b^5x^{10}+1050a\,b^4x^8+1800a^2b^3x^6+1575a^3b^2x^4+700a^4b\,x^2+126a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{2520\left(b\,x^2+a\right)^5x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^21,x)

[Out] -1/2520*(252*b^5*x^10+1050*a*b^4*x^8+1800*a^2*b^3*x^6+1575*a^3*b^2*x^4+700*a^4*b*x^2+126*a^5)*((b*x^2+a)^2)^(5/2)/x^20/(b*x^2+a)^5

maxima [A] time = 1.34, size = 57, normalized size = 0.22

$$-\frac{b^5}{10\,x^{10}}-\frac{5\,ab^4}{12\,x^{12}}-\frac{5\,a^2b^3}{7\,x^{14}}-\frac{5\,a^3b^2}{8\,x^{16}}-\frac{5\,a^4b}{18\,x^{18}}-\frac{a^5}{20\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^21,x, algorithm="maxima")

[Out] -1/10*b^5/x^10 - 5/12*a*b^4/x^12 - 5/7*a^2*b^3/x^14 - 5/8*a^3*b^2/x^16 - 5/18*a^4*b/x^18 - 1/20*a^5/x^20

mupad [B] time = 4.22, size = 231, normalized size = 0.91

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{20\,x^{20}\left(b\,x^2+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{10\,x^{10}\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{12\,x^{12}\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{18\,x^{18}\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^21,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(20*x^20*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(10*x^10*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(12*x^12*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2
*a*b*x^2)^(1/2))/(18*x^18*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*
x^2)^(1/2))/(7*x^14*(a + b*x^2)) - (5*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(
1/2))/(8*x^16*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{21}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**21,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**21, x)
```

3.603 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{23}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{8x^{16}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}\left(a+bx^2\right)}-\frac{a^4b\sqrt{a^2}}{4x}$$

```
[Out] -1/22*a^5*((b*x^2+a)^2)^(1/2)/x^22/(b*x^2+a)-1/4*a^4*b*((b*x^2+a)^2)^(1/2)/
x^20/(b*x^2+a)-5/9*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^18/(b*x^2+a)-5/8*a^2*b^3*(
(b*x^2+a)^2)^(1/2)/x^16/(b*x^2+a)-5/14*a*b^4*((b*x^2+a)^2)^(1/2)/x^14/(b*x^
2+a)-1/12*b^5*((b*x^2+a)^2)^(1/2)/x^12/(b*x^2+a)
```

Rubi [A] time = 0.15, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}\left(a+bx^2\right)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{20}\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{9x^{18}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{8x^{16}\left(a+bx^2\right)}-\frac{5ab^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^23,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(22*x^22*(a + b*x^2)) - (a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^20*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(9*x^18*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])/(8*x^16*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
)/(14*x^14*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*x^12*(a
 + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
```

```
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{23}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{12}}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^{12}}\,dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\ \text{Subst}\left(\int \left(\frac{a^5b^5}{x^{12}}+\frac{5a^4b^6}{x^{11}}+\frac{10a^3b^7}{x^{10}}+\frac{10a^2b^8}{x^9}+\frac{5ab^9}{x^8}+\frac{b^{10}}{x^7}\right) dx, x,\right.}{2b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}\left(a+bx^2\right)} - \frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{20}\left(a+bx^2\right)} - \frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{9x^{18}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(252a^5+1386a^4bx^2+3080a^3b^2x^4+3465a^2b^3x^6+1980ab^4x^8+462b^5x^{10}\right)}{5544x^{22}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^23,x]
```

```
[Out] -1/5544*(Sqrt[(a + b*x^2)^2]*(252*a^5 + 1386*a^4*b*x^2 + 3080*a^3*b^2*x^4 +
 3465*a^2*b^3*x^6 + 1980*a*b^4*x^8 + 462*b^5*x^10))/(x^22*(a + b*x^2))
```

fricas [A] time = 1.00, size = 59, normalized size = 0.23

$$-\frac{462\,b^5x^{10}+1980\,ab^4x^8+3465\,a^2b^3x^6+3080\,a^3b^2x^4+1386\,a^4bx^2+252\,a^5}{5544\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^23,x, algorithm="fricas")
```

```
[Out] -1/5544*(462*b^5*x^10 + 1980*a*b^4*x^8 + 3465*a^2*b^3*x^6 + 3080*a^3*b^2*x^
4 + 1386*a^4*b*x^2 + 252*a^5)/x^22
```

giac [A] time = 0.19, size = 107, normalized size = 0.42

$$-\frac{462\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+1980\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+3465\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+3080\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+1}{5544\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^23,x, algorithm="giac")
```

```
[Out] -1/5544*(462*b^5*x^10*sgn(b*x^2 + a) + 1980*a*b^4*x^8*sgn(b*x^2 + a) + 3465
*a^2*b^3*x^6*sgn(b*x^2 + a) + 3080*a^3*b^2*x^4*sgn(b*x^2 + a) + 1386*a^4*b*
x^2*sgn(b*x^2 + a) + 252*a^5*sgn(b*x^2 + a))/x^22
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(462b^5x^{10}+1980a\,b^4x^8+3465a^2b^3x^6+3080a^3b^2x^4+1386a^4b\,x^2+252a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{5544\left(b\,x^2+a\right)^5x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^23,x)
```

```
[Out] -1/5544*(462*b^5*x^10+1980*a*b^4*x^8+3465*a^2*b^3*x^6+3080*a^3*b^2*x^4+1386
*a^4*b*x^2+252*a^5)*((b*x^2+a)^2)^(5/2)/x^22/(b*x^2+a)^5
```

maxima [A] time = 1.34, size = 57, normalized size = 0.22

$$-\frac{b^5}{12\,x^{12}}-\frac{5\,ab^4}{14\,x^{14}}-\frac{5\,a^2b^3}{8\,x^{16}}-\frac{5\,a^3b^2}{9\,x^{18}}-\frac{a^4b}{4\,x^{20}}-\frac{a^5}{22\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^23,x, algorithm="maxima")
```

```
[Out] -1/12*b^5/x^12 - 5/14*a*b^4/x^14 - 5/8*a^2*b^3/x^16 - 5/9*a^3*b^2/x^18 - 1/
4*a^4*b/x^20 - 1/22*a^5/x^22
```

mupad [B] time = 4.23, size = 231, normalized size = 0.91

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{22\,x^{22}\left(b\,x^2+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{12\,x^{12}\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{14\,x^{14}\left(b\,x^2+a\right)}-\frac{a^4\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^{20}\left(b\,x^2+a\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^23,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(22*x^22*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(12*x^12*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(14*x^14*(a + b*x^2)) - (a^4*b*(a^2 + b^2*x^4 + 2*a
*b*x^2)^(1/2))/(4*x^20*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2
)^(1/2))/(8*x^16*(a + b*x^2)) - (5*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2
))/(9*x^18*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{23}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**23,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**23, x)
```

3.604 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{25}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^{18}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{24x^{24}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{}}{22}$$

```
[Out] -1/24*a^5*((b*x^2+a)^2)^(1/2)/x^24/(b*x^2+a)-5/22*a^4*b*((b*x^2+a)^2)^(1/2)
/x^22/(b*x^2+a)-1/2*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^20/(b*x^2+a)-5/9*a^2*b^3*
((b*x^2+a)^2)^(1/2)/x^18/(b*x^2+a)-5/16*a*b^4*((b*x^2+a)^2)^(1/2)/x^16/(b*x
^2+a)-1/14*b^5*((b*x^2+a)^2)^(1/2)/x^14/(b*x^2+a)
```

Rubi [A] time = 0.15, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{24x^{24}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}\left(a+bx^2\right)}-\frac{a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^{20}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^{18}\left(a+bx^2\right)}-\frac{5ab^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^25,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(24*x^24*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(22*x^22*(a + b*x^2)) - (a^3*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(2*x^20*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(9*x^18*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])/(16*x^16*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*x^14*(
a + b*x^2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
```

```
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{25}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{13}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^{13}}\,dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int \left(\frac{a^5b^5}{x^{13}}+\frac{5a^4b^6}{x^{12}}+\frac{10a^3b^7}{x^{11}}+\frac{10a^2b^8}{x^{10}}+\frac{5ab^9}{x^9}+\frac{b^{10}}{x^8}\right)dx, x,}{2b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{24x^{24}\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}\left(a+bx^2\right)} - \frac{a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^{20}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(462a^5+2520a^4bx^2+5544a^3b^2x^4+6160a^2b^3x^6+3465ab^4x^8+792b^5x^{10}\right)}{11088x^{24}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^25,x]
```

```
[Out] -1/11088*(Sqrt[(a + b*x^2)^2]*(462*a^5 + 2520*a^4*b*x^2 + 5544*a^3*b^2*x^4
+ 6160*a^2*b^3*x^6 + 3465*a*b^4*x^8 + 792*b^5*x^10))/(x^24*(a + b*x^2))
```

fricas [A] time = 1.16, size = 59, normalized size = 0.23

$$-\frac{792\,b^5x^{10}+3465\,ab^4x^8+6160\,a^2b^3x^6+5544\,a^3b^2x^4+2520\,a^4bx^2+462\,a^5}{11088\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^25,x, algorithm="fricas")
```

```
[Out] -1/11088*(792*b^5*x^10 + 3465*a*b^4*x^8 + 6160*a^2*b^3*x^6 + 5544*a^3*b^2*x
^4 + 2520*a^4*b*x^2 + 462*a^5)/x^24
```

giac [A] time = 0.17, size = 107, normalized size = 0.42

$$-\frac{792\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+3465\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+6160\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+5544\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+2}{11088\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^25,x, algorithm="giac")
```

```
[Out] -1/11088*(792*b^5*x^10*sgn(b*x^2 + a) + 3465*a*b^4*x^8*sgn(b*x^2 + a) + 616
0*a^2*b^3*x^6*sgn(b*x^2 + a) + 5544*a^3*b^2*x^4*sgn(b*x^2 + a) + 2520*a^4*b
*x^2*sgn(b*x^2 + a) + 462*a^5*sgn(b*x^2 + a))/x^24
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(792b^5x^{10}+3465a\,b^4x^8+6160a^2b^3x^6+5544a^3b^2x^4+2520a^4b\,x^2+462a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{11088\left(b\,x^2+a\right)^5x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^25,x)
```

```
[Out] -1/11088*(792*b^5*x^10+3465*a*b^4*x^8+6160*a^2*b^3*x^6+5544*a^3*b^2*x^4+252
0*a^4*b*x^2+462*a^5)*((b*x^2+a)^2)^(5/2)/x^24/(b*x^2+a)^5
```

maxima [A] time = 1.38, size = 57, normalized size = 0.22

$$-\frac{b^5}{14\,x^{14}}-\frac{5\,ab^4}{16\,x^{16}}-\frac{5\,a^2b^3}{9\,x^{18}}-\frac{a^3b^2}{2\,x^{20}}-\frac{5\,a^4b}{22\,x^{22}}-\frac{a^5}{24\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^25,x, algorithm="maxima")
```

```
[Out] -1/14*b^5/x^14 - 5/16*a*b^4/x^16 - 5/9*a^2*b^3/x^18 - 1/2*a^3*b^2/x^20 - 5/
22*a^4*b/x^22 - 1/24*a^5/x^24
```

mupad [B] time = 4.22, size = 231, normalized size = 0.91

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{24\,x^{24}\left(b\,x^2+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{14\,x^{14}\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{16\,x^{16}\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{22\,x^{22}\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^25,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(24*x^24*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(14*x^14*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(16*x^16*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2
*a*b*x^2)^(1/2))/(22*x^22*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*
x^2)^(1/2))/(9*x^18*(a + b*x^2)) - (a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/
2))/(2*x^20*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{25}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**25,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**25, x)
```

3.605 $\int x^{12}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{b^5x^{23}\sqrt{a^2+2abx^2+b^2x^4}}{23\left(a+bx^2\right)}+\frac{5ab^4x^{21}\sqrt{a^2+2abx^2+b^2x^4}}{21\left(a+bx^2\right)}+\frac{10a^2b^3x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19\left(a+bx^2\right)}+\frac{a^5x^{13}\sqrt{a^2+2abx^2+b}}{13\left(a+bx^2\right)}$$

```
[Out] 1/13*a^5*x^13*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/3*a^4*b*x^15*((b*x^2+a)^2)^(1
/2)/(b*x^2+a)+10/17*a^3*b^2*x^17*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+10/19*a^2*b^
3*x^19*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/21*a*b^4*x^21*((b*x^2+a)^2)^(1/2)/(b
*x^2+a)+1/23*b^5*x^23*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^{23}\sqrt{a^2+2abx^2+b^2x^4}}{23\left(a+bx^2\right)}+\frac{5ab^4x^{21}\sqrt{a^2+2abx^2+b^2x^4}}{21\left(a+bx^2\right)}+\frac{10a^2b^3x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19\left(a+bx^2\right)}+\frac{10a^3b^2x^{17}\sqrt{a^2+2abx^2}}{17\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (a^4*b*x^15*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (10*a^3*b^2*x^17*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4])/(17*(a + b*x^2)) + (10*a^2*b^3*x^19*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(19*(a + b*x^2)) + (5*a*b^4*x^21*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(21*(a + b*x^2)) + (b^5*x^23*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23
*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^{12} \left(a^2 + 2abx^2 + b^2x^4\right)^{5/2} dx = \frac{\sqrt{a^2 + 2abx^2 + b^2x^4} \int x^{12} \left(ab + b^2x^2\right)^5 dx}{b^4 \left(ab + b^2x^2\right)}$$
$$= \frac{\sqrt{a^2 + 2abx^2 + b^2x^4} \int \left(a^5b^5x^{12} + 5a^4b^6x^{14} + 10a^3b^7x^{16} + 10a^2b^8x^{18} + 5ab^9\right.}{b^4 \left(ab + b^2x^2\right)}$$
$$= \frac{a^5x^{13}\sqrt{a^2 + 2abx^2 + b^2x^4}}{13 \left(a + bx^2\right)} + \frac{a^4bx^{15}\sqrt{a^2 + 2abx^2 + b^2x^4}}{3 \left(a + bx^2\right)} + \frac{10a^3b^2x^{17}\sqrt{a^2 +}}{17 \left(a +\right.}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^{13}\sqrt{\left(a + bx^2\right)^2} \left(156009a^5 + 676039a^4bx^2 + 1193010a^3b^2x^4 + 1067430a^2b^3x^6 + 482885ab^4x^8 + 88179b^5x^{10}\right)}{2028117 \left(a + bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^13*Sqrt[(a + b*x^2)^2]*(156009*a^5 + 676039*a^4*b*x^2 + 1193010*a^3*b^2*
x^4 + 1067430*a^2*b^3*x^6 + 482885*a*b^4*x^8 + 88179*b^5*x^10))/(2028117*(a
 + b*x^2))
```

fricas [A] time = 1.15, size = 57, normalized size = 0.22

$$\frac{1}{23}\,b^5x^{23} + \frac{5}{21}\,ab^4x^{21} + \frac{10}{19}\,a^2b^3x^{19} + \frac{10}{17}\,a^3b^2x^{17} + \frac{1}{3}\,a^4bx^{15} + \frac{1}{13}\,a^5x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/23*b^5*x^23 + 5/21*a*b^4*x^21 + 10/19*a^2*b^3*x^19 + 10/17*a^3*b^2*x^17 +
 1/3*a^4*b*x^15 + 1/13*a^5*x^13
```

giac [A] time = 0.16, size = 105, normalized size = 0.41

$$\frac{1}{23}\,b^5x^{23}\mathrm{sgn}\left(bx^2 + a\right) + \frac{5}{21}\,ab^4x^{21}\mathrm{sgn}\left(bx^2 + a\right) + \frac{10}{19}\,a^2b^3x^{19}\mathrm{sgn}\left(bx^2 + a\right) + \frac{10}{17}\,a^3b^2x^{17}\mathrm{sgn}\left(bx^2 + a\right) + \frac{1}{3}\,a^4bx^{15}\mathrm{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/23*b^5*x^23*sgn(b*x^2 + a) + 5/21*a*b^4*x^21*sgn(b*x^2 + a) + 10/19*a^2*b
^3*x^19*sgn(b*x^2 + a) + 10/17*a^3*b^2*x^17*sgn(b*x^2 + a) + 1/3*a^4*b*x^15
*sgn(b*x^2 + a) + 1/13*a^5*x^13*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(88179b^5x^{10}+482885a\,b^4x^8+1067430a^2b^3x^6+1193010a^3b^2x^4+676039a^4b\,x^2+156009a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x^{13}}{2028117\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/2028117*x^13*(88179*b^5*x^10+482885*a*b^4*x^8+1067430*a^2*b^3*x^6+1193010
*a^3*b^2*x^4+676039*a^4*b*x^2+156009*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.28, size = 57, normalized size = 0.22

$$\frac{1}{23}\,b^5x^{23}+\frac{5}{21}\,ab^4x^{21}+\frac{10}{19}\,a^2b^3x^{19}+\frac{10}{17}\,a^3b^2x^{17}+\frac{1}{3}\,a^4bx^{15}+\frac{1}{13}\,a^5x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/23*b^5*x^23 + 5/21*a*b^4*x^21 + 10/19*a^2*b^3*x^19 + 10/17*a^3*b^2*x^17 +
 1/3*a^4*b*x^15 + 1/13*a^5*x^13
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{12}\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^12*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{12}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**12*((a + b*x**2)**2)**(5/2), x)
```

3.606 $\int x^{10}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{b^5x^{21}\sqrt{a^2+2abx^2+b^2x^4}}{21\left(a+bx^2\right)}+\frac{5ab^4x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19\left(a+bx^2\right)}+\frac{10a^2b^3x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\left(a+bx^2\right)}+\frac{a^5x^{11}\sqrt{a^2+2abx^2+}}{11\left(a+bx^2\right)}$$

```
[Out] 1/11*a^5*x^11*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/13*a^4*b*x^13*((b*x^2+a)^2)^(
1/2)/(b*x^2+a)+2/3*a^3*b^2*x^15*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+10/17*a^2*b^3
*x^17*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/19*a*b^4*x^19*((b*x^2+a)^2)^(1/2)/(b*
x^2+a)+1/21*b^5*x^21*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^{21}\sqrt{a^2+2abx^2+b^2x^4}}{21\left(a+bx^2\right)}+\frac{5ab^4x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19\left(a+bx^2\right)}+\frac{10a^2b^3x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\left(a+bx^2\right)}+\frac{2a^3b^2x^{15}\sqrt{a^2+2abx}}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^10*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (5*a^4*b*x^13
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (2*a^3*b^2*x^15*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (10*a^2*b^3*x^17*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(17*(a + b*x^2)) + (5*a*b^4*x^19*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(19*(a + b*x^2)) + (b^5*x^21*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2
1*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^{10}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^{10}\left(ab+b^2x^2\right)^5\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(a^5b^5x^{10}+5a^4b^6x^{12}+10a^3b^7x^{14}+10a^2b^8x^{16}+5ab^9x\right.}{b^4\left(ab+b^2x^2\right)}$$
$$=\frac{a^5x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{5a^4bx^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{2a^3b^2x^{15}\sqrt{a^2+2}}{3\left(a+b\right.}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^{11}\sqrt{\left(a+bx^2\right)^2}\left(88179a^5+373065a^4bx^2+646646a^3b^2x^4+570570a^2b^3x^6+255255ab^4x^8+46189b^5x^{10}\right)}{969969\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^11*Sqrt[(a + b*x^2)^2]*(88179*a^5 + 373065*a^4*b*x^2 + 646646*a^3*b^2*x^
4 + 570570*a^2*b^3*x^6 + 255255*a*b^4*x^8 + 46189*b^5*x^10))/(969969*(a + b
*x^2))
```

fricas [A] time = 0.64, size = 57, normalized size = 0.22

$$\frac{1}{21}\,b^5x^{21}+\frac{5}{19}\,ab^4x^{19}+\frac{10}{17}\,a^2b^3x^{17}+\frac{2}{3}\,a^3b^2x^{15}+\frac{5}{13}\,a^4bx^{13}+\frac{1}{11}\,a^5x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/21*b^5*x^21 + 5/19*a*b^4*x^19 + 10/17*a^2*b^3*x^17 + 2/3*a^3*b^2*x^15 + 5
/13*a^4*b*x^13 + 1/11*a^5*x^11
```

giac [A] time = 0.16, size = 105, normalized size = 0.41

$$\frac{1}{21}\,b^5x^{21}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{19}\,ab^4x^{19}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{17}\,a^2b^3x^{17}\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{3}\,a^3b^2x^{15}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{13}\,a^4bx^{13}\mathrm{sg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/21*b^5*x^21*sgn(b*x^2 + a) + 5/19*a*b^4*x^19*sgn(b*x^2 + a) + 10/17*a^2*b
^3*x^17*sgn(b*x^2 + a) + 2/3*a^3*b^2*x^15*sgn(b*x^2 + a) + 5/13*a^4*b*x^13*
sgn(b*x^2 + a) + 1/11*a^5*x^11*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(46189b^{5}x^{10}+255255a\,b^{4}x^{8}+570570a^{2}b^{3}x^{6}+646646a^{3}b^{2}x^{4}+373065a^{4}b\,x^{2}+88179a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}x^{11}}{969969\left(b\,x^{2}+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/969969*x^11*(46189*b^5*x^10+255255*a*b^4*x^8+570570*a^2*b^3*x^6+646646*a^
3*b^2*x^4+373065*a^4*b*x^2+88179*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.35, size = 57, normalized size = 0.22

$$\frac{1}{21}\,b^{5}x^{21}+\frac{5}{19}\,ab^{4}x^{19}+\frac{10}{17}\,a^{2}b^{3}x^{17}+\frac{2}{3}\,a^{3}b^{2}x^{15}+\frac{5}{13}\,a^{4}bx^{13}+\frac{1}{11}\,a^{5}x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/21*b^5*x^21 + 5/19*a*b^4*x^19 + 10/17*a^2*b^3*x^17 + 2/3*a^3*b^2*x^15 + 5
/13*a^4*b*x^13 + 1/11*a^5*x^11
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{10}\left(a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^10*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{10}\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**10*((a + b*x**2)**2)**(5/2), x)
```

3.607 $\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19\left(a+bx^2\right)}+\frac{5ab^4x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\left(a+bx^2\right)}+\frac{2a^2b^3x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{a^5x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}$$

```
[Out] 1/9*a^5*x^9*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/11*a^4*b*x^11*((b*x^2+a)^2)^(1/
2)/(b*x^2+a)+10/13*a^3*b^2*x^13*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+2/3*a^2*b^3*x
^15*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/17*a*b^4*x^17*((b*x^2+a)^2)^(1/2)/(b*x^
2+a)+1/19*b^5*x^19*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19\left(a+bx^2\right)}+\frac{5ab^4x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\left(a+bx^2\right)}+\frac{2a^2b^3x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2abx^2}}{13\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (5*a^4*b*x^11*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (10*a^3*b^2*x^13*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (2*a^2*b^3*x^15*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (5*a*b^4*x^17*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(17*(a + b*x^2)) + (b^5*x^19*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*
(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^8\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^8\left(ab+b^2x^2\right)^5\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(a^5b^5x^8+5a^4b^6x^{10}+10a^3b^7x^{12}+10a^2b^8x^{14}+5ab^9x^{1}\right.}{b^4\left(ab+b^2x^2\right)}$$
$$=\frac{a^5x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{5a^4bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2}}{13\left(a+b\right.}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^9\sqrt{\left(a+bx^2\right)^2}\left(46189a^5+188955a^4bx^2+319770a^3b^2x^4+277134a^2b^3x^6+122265ab^4x^8+21879b^5x^{10}\right)}{415701\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^9*Sqrt[(a + b*x^2)^2]*(46189*a^5 + 188955*a^4*b*x^2 + 319770*a^3*b^2*x^4
 + 277134*a^2*b^3*x^6 + 122265*a*b^4*x^8 + 21879*b^5*x^10))/(415701*(a + b*
x^2))
```

fricas [A] time = 1.14, size = 57, normalized size = 0.22

$$\frac{1}{19}\,b^5x^{19}+\frac{5}{17}\,ab^4x^{17}+\frac{2}{3}\,a^2b^3x^{15}+\frac{10}{13}\,a^3b^2x^{13}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{9}\,a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^2*x^13 + 5
/11*a^4*b*x^11 + 1/9*a^5*x^9
```

giac [A] time = 0.16, size = 105, normalized size = 0.41

$$\frac{1}{19}\,b^5x^{19}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{17}\,ab^4x^{17}\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{3}\,a^2b^3x^{15}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{13}\,a^3b^2x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{11}\,a^4bx^{11}\mathrm{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

[Out] 1/19*b^5*x^19*sgn(b*x^2 + a) + 5/17*a*b^4*x^17*sgn(b*x^2 + a) + 2/3*a^2*b^3*x^15*sgn(b*x^2 + a) + 10/13*a^3*b^2*x^13*sgn(b*x^2 + a) + 5/11*a^4*b*x^11*sgn(b*x^2 + a) + 1/9*a^5*x^9*sgn(b*x^2 + a)

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(21879b^5x^{10}+122265a\,b^4x^8+277134a^2b^3x^6+319770a^3b^2x^4+188955a^4b\,x^2+46189a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x^9}{415701\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)

[Out] 1/415701*x^9*(21879*b^5*x^10+122265*a*b^4*x^8+277134*a^2*b^3*x^6+319770*a^3*b^2*x^4+188955*a^4*b*x^2+46189*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5

maxima [A] time = 1.33, size = 57, normalized size = 0.22

$$\frac{1}{19}\,b^5x^{19}+\frac{5}{17}\,ab^4x^{17}+\frac{2}{3}\,a^2b^3x^{15}+\frac{10}{13}\,a^3b^2x^{13}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{9}\,a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")

[Out] 1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^2*x^13 + 5/11*a^4*b*x^11 + 1/9*a^5*x^9

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^8\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)

[Out] int(x^8*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)

[Out] Integral(x**8*((a + b*x**2)**2)**(5/2), x)

3.608 $\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{b^5x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\left(a+bx^2\right)}+\frac{ab^4x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{a^5x^7\sqrt{a^2+2abx^2+b^2}}{7\left(a+bx^2\right)}$$

```
[Out] 1/7*a^5*x^7*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/9*a^4*b*x^9*((b*x^2+a)^2)^(1/2)
/(b*x^2+a)+10/11*a^3*b^2*x^11*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+10/13*a^2*b^3*x
^13*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/3*a*b^4*x^15*((b*x^2+a)^2)^(1/2)/(b*x^2
+a)+1/17*b^5*x^17*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\left(a+bx^2\right)}+\frac{ab^4x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{10a^3b^2x^{11}\sqrt{a^2+2abx}}{11\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (5*a^4*b*x^9*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (10*a^3*b^2*x^11*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (10*a^2*b^3*x^13*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (a*b^4*x^15*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(3*(a + b*x^2)) + (b^5*x^17*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*(a
+ b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^6 \left(ab+b^2x^2\right)^5 dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a^5b^5x^6+5a^4b^6x^8+10a^3b^7x^{10}+10a^2b^8x^{12}+5ab^9x^{14}\right.}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{a^5x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{5a^4bx^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{10a^3b^2x^{11}\sqrt{a^2+2a}}{11\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^7\sqrt{\left(a+bx^2\right)^2}\left(21879a^5+85085a^4bx^2+139230a^3b^2x^4+117810a^2b^3x^6+51051ab^4x^8+9009b^5x^{10}\right)}{153153\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^7*Sqrt[(a + b*x^2)^2]*(21879*a^5 + 85085*a^4*b*x^2 + 139230*a^3*b^2*x^4
+ 117810*a^2*b^3*x^6 + 51051*a*b^4*x^8 + 9009*b^5*x^10))/(153153*(a + b*x^2
))
```

fricas [A] time = 0.65, size = 57, normalized size = 0.22

$$\frac{1}{17}\,b^5x^{17}+\frac{1}{3}\,ab^4x^{15}+\frac{10}{13}\,a^2b^3x^{13}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{9}\,a^4bx^9+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b^2*x^11 +
5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

giac [A] time = 0.17, size = 105, normalized size = 0.41

$$\frac{1}{17}\,b^5x^{17}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{3}\,ab^4x^{15}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{13}\,a^2b^3x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{11}\,a^3b^2x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{9}\,a^4bx^9\mathrm{sgn}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/17*b^5*x^17*sgn(b*x^2 + a) + 1/3*a*b^4*x^15*sgn(b*x^2 + a) + 10/13*a^2*b^
3*x^13*sgn(b*x^2 + a) + 10/11*a^3*b^2*x^11*sgn(b*x^2 + a) + 5/9*a^4*b*x^9*s
gn(b*x^2 + a) + 1/7*a^5*x^7*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(9009b^{5}x^{10}+51051a\,b^{4}x^{8}+117810a^{2}b^{3}x^{6}+139230a^{3}b^{2}x^{4}+85085a^{4}b\,x^{2}+21879a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}x^{7}}{153153\left(b\,x^{2}+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/153153*x^7*(9009*b^5*x^10+51051*a*b^4*x^8+117810*a^2*b^3*x^6+139230*a^3*b
^2*x^4+85085*a^4*b*x^2+21879*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.29, size = 57, normalized size = 0.22

$$\frac{1}{17}\,b^{5}x^{17}+\frac{1}{3}\,ab^{4}x^{15}+\frac{10}{13}\,a^{2}b^{3}x^{13}+\frac{10}{11}\,a^{3}b^{2}x^{11}+\frac{5}{9}\,a^{4}bx^{9}+\frac{1}{7}\,a^{5}x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b^2*x^11 +
5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{6}\left(a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{6}\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**6*((a + b*x**2)**2)**(5/2), x)
```

3.609 $\int x^4 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{15\left(a+bx^2\right)}+\frac{5ab^4x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{a^5x^5\sqrt{a^2+2abx^2+b^2}}{5\left(a+bx^2\right)}$$

```
[Out] 1/5*a^5*x^5*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/7*a^4*b*x^7*((b*x^2+a)^2)^(1/2)
/(b*x^2+a)+10/9*a^3*b^2*x^9*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+10/11*a^2*b^3*x^1
1*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/13*a*b^4*x^13*((b*x^2+a)^2)^(1/2)/(b*x^2+
a)+1/15*b^5*x^15*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{15\left(a+bx^2\right)}+\frac{5ab^4x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{10a^3b^2x^9\sqrt{a^2+2abx^2}}{9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (5*a^4*b*x^7*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (10*a^3*b^2*x^9*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (10*a^2*b^3*x^11*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(11*(a + b*x^2)) + (5*a*b^4*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(13*(a + b*x^2)) + (b^5*x^15*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*(a
 + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^4 \left(ab+b^2x^2\right)^5 dx}{b^4 \left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^5b^5x^4+5a^4b^6x^6+10a^3b^7x^8+10a^2b^8x^{10}+5ab^9x^{12}\right.}{b^4 \left(ab+b^2x^2\right)}$$
$$= \frac{a^5x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)} + \frac{5a^4bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)} + \frac{10a^3b^2x^9\sqrt{a^2+2a}}{9\left(a+bx\right.}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^5\sqrt{\left(a+bx^2\right)^2}\left(9009a^5+32175a^4bx^2+50050a^3b^2x^4+40950a^2b^3x^6+17325ab^4x^8+3003b^5x^{10}\right)}{45045\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^2)^2]*(9009*a^5 + 32175*a^4*b*x^2 + 50050*a^3*b^2*x^4 +
40950*a^2*b^3*x^6 + 17325*a*b^4*x^8 + 3003*b^5*x^10))/(45045*(a + b*x^2))
```

fricas [A] time = 1.11, size = 57, normalized size = 0.22

$$\frac{1}{15}\,b^5x^{15}+\frac{5}{13}\,ab^4x^{13}+\frac{10}{11}\,a^2b^3x^{11}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{7}\,a^4bx^7+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b^2*x^9 + 5
/7*a^4*b*x^7 + 1/5*a^5*x^5
```

giac [A] time = 0.16, size = 105, normalized size = 0.41

$$\frac{1}{15}\,b^5x^{15}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{13}\,ab^4x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{11}\,a^2b^3x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{9}\,a^3b^2x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{7}\,a^4bx^7\mathrm{sgr}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/15*b^5*x^15*sgn(b*x^2 + a) + 5/13*a*b^4*x^13*sgn(b*x^2 + a) + 10/11*a^2*b
^3*x^11*sgn(b*x^2 + a) + 10/9*a^3*b^2*x^9*sgn(b*x^2 + a) + 5/7*a^4*b*x^7*sg
n(b*x^2 + a) + 1/5*a^5*x^5*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{\left(3003b^5x^{10}+17325a\,b^4x^8+40950a^2b^3x^6+50050a^3b^2x^4+32175a^4b\,x^2+9009a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x^5}{45045\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/45045*x^5*(3003*b^5*x^10+17325*a*b^4*x^8+40950*a^2*b^3*x^6+50050*a^3*b^2*
x^4+32175*a^4*b*x^2+9009*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.33, size = 57, normalized size = 0.22

$$\frac{1}{15}\,b^5x^{15}+\frac{5}{13}\,ab^4x^{13}+\frac{10}{11}\,a^2b^3x^{11}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{7}\,a^4bx^7+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b^2*x^9 + 5
/7*a^4*b*x^7 + 1/5*a^5*x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{4}\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**4*((a + b*x**2)**2)**(5/2), x)
```

3.610 $\int x^2 \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx$

Optimal. Leaf size=252

$$\frac{b^5x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{5ab^4x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{a^5x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

```
[Out] 1/3*a^5*x^3*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+a^4*b*x^5*((b*x^2+a)^2)^(1/2)/(b*
x^2+a)+10/7*a^3*b^2*x^7*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+10/9*a^2*b^3*x^9*((b*
x^2+a)^2)^(1/2)/(b*x^2+a)+5/11*a*b^4*x^11*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/1
3*b^5*x^13*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.06, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{5ab^4x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{10a^3b^2x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (a^4*b*x^5*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (10*a^3*b^2*x^7*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (10*a^2*b^3*x^9*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(9*(a + b*x^2)) + (5*a*b^4*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(
11*(a + b*x^2)) + (b^5*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2
))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^2 \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int x^2\left(ab+b^2x^2\right)^5\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int \left(a^5b^5x^2+5a^4b^6x^4+10a^3b^7x^6+10a^2b^8x^8+5ab^9x^{10}+\right.}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{a^5x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)} + \frac{a^4bx^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{10a^3b^2x^7\sqrt{a^2+2abx}}{7\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^3\sqrt{\left(a+bx^2\right)^2}\left(3003a^5+9009a^4bx^2+12870a^3b^2x^4+10010a^2b^3x^6+4095ab^4x^8+693b^5x^{10}\right)}{9009\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x^2)^2]*(3003*a^5 + 9009*a^4*b*x^2 + 12870*a^3*b^2*x^4 + 1
0010*a^2*b^3*x^6 + 4095*a*b^4*x^8 + 693*b^5*x^10))/(9009*(a + b*x^2))
```

fricas [A] time = 0.93, size = 56, normalized size = 0.22

$$\frac{1}{13}\,b^5x^{13}+\frac{5}{11}\,ab^4x^{11}+\frac{10}{9}\,a^2b^3x^9+\frac{10}{7}\,a^3b^2x^7+a^4bx^5+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2*x^7 + a^4
*b*x^5 + 1/3*a^5*x^3
```

giac [A] time = 0.18, size = 104, normalized size = 0.41

$$\frac{1}{13}\,b^5x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{11}\,ab^4x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{9}\,a^2b^3x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{7}\,a^3b^2x^7\mathrm{sgn}\left(bx^2+a\right)+a^4bx^5\mathrm{sgn}\left(bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/13*b^5*x^13*sgn(b*x^2 + a) + 5/11*a*b^4*x^11*sgn(b*x^2 + a) + 10/9*a^2*b^
3*x^9*sgn(b*x^2 + a) + 10/7*a^3*b^2*x^7*sgn(b*x^2 + a) + a^4*b*x^5*sgn(b*x^
2 + a) + 1/3*a^5*x^3*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$\frac{\left(693b^5x^{10}+4095a\,b^4x^8+10010a^2b^3x^6+12870a^3b^2x^4+9009a^4b\,x^2+3003a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x^3}{9009\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/9009*x^3*(693*b^5*x^10+4095*a*b^4*x^8+10010*a^2*b^3*x^6+12870*a^3*b^2*x^4
+9009*a^4*b*x^2+3003*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.36, size = 56, normalized size = 0.22

$$\frac{1}{13}\,b^5x^{13}+\frac{5}{11}\,ab^4x^{11}+\frac{10}{9}\,a^2b^3x^9+\frac{10}{7}\,a^3b^2x^7+a^4bx^5+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2*x^7 + a^4
*b*x^5 + 1/3*a^5*x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**2*((a + b*x**2)**2)**(5/2), x)
```

3.611 $\int \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$

Optimal. Leaf size=248

$$\frac{b^5x^{11}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{11\left(a+bx^2\right)^5}+\frac{5ab^4x^9\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{9\left(a+bx^2\right)^5}+\frac{10a^2b^3x^7\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{7\left(a+bx^2\right)^5}+\frac{a^5x\left(a^2+2abx^2\right.}{\left(a+bx^2\right.}$$

```
[Out] a^5*x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(b*x^2+a)^5+5/3*a^4*b*x^3*(b^2*x^4+2*a*
b*x^2+a^2)^(5/2)/(b*x^2+a)^5+2*a^3*b^2*x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(b
*x^2+a)^5+10/7*a^2*b^3*x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(b*x^2+a)^5+5/9*a*
b^4*x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(b*x^2+a)^5+1/11*b^5*x^11*(b^2*x^4+2*
a*b*x^2+a^2)^(5/2)/(b*x^2+a)^5
```

Rubi [A] time = 0.05, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1088, 194}

$$\frac{b^5x^{11}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{11\left(a+bx^2\right)^5}+\frac{5ab^4x^9\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{9\left(a+bx^2\right)^5}+\frac{10a^2b^3x^7\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{7\left(a+bx^2\right)^5}+\frac{2a^3b^2x^5\left(a^2+2a\right.}{\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(a + b*x^2)^5 + (5*a^4*b*x^3*(a^2
 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(3*(a + b*x^2)^5) + (2*a^3*b^2*x^5*(a^2 + 2*
a*b*x^2 + b^2*x^4)^(5/2))/(a + b*x^2)^5 + (10*a^2*b^3*x^7*(a^2 + 2*a*b*x^2
+ b^2*x^4)^(5/2))/(7*(a + b*x^2)^5) + (5*a*b^4*x^9*(a^2 + 2*a*b*x^2 + b^2*x
^4)^(5/2))/(9*(a + b*x^2)^5) + (b^5*x^11*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))
/(11*(a + b*x^2)^5)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1088

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx = \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}\int \left(2ab+2b^2x^2\right)^5\,dx}{\left(2ab+2b^2x^2\right)^5}$$
$$= \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}\int \left(32a^5b^5+160a^4b^6x^2+320a^3b^7x^4+320a^2b^8x^6+160a\right.}{\left(2ab+2b^2x^2\right)^5}$$
$$= \frac{a^5x\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\left(a+bx^2\right)^5}+\frac{5a^4bx^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{3\left(a+bx^2\right)^5}+\frac{2a^3b^2x^5\left(a^2+2a\right.}{\left(a+b\right.}$$

Mathematica [A] time = 0.02, size = 81, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(693a^5x+1155a^4bx^3+1386a^3b^2x^5+990a^2b^3x^7+385ab^4x^9+63b^5x^{11}\right)}{693\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(693*a^5*x + 1155*a^4*b*x^3 + 1386*a^3*b^2*x^5 + 990*a
^2*b^3*x^7 + 385*a*b^4*x^9 + 63*b^5*x^11))/(693*(a + b*x^2))
```

fricas [A] time = 0.61, size = 54, normalized size = 0.22

$$\frac{1}{11}\,b^5x^{11}+\frac{5}{9}\,ab^4x^9+\frac{10}{7}\,a^2b^3x^7+2\,a^3b^2x^5+\frac{5}{3}\,a^4bx^3+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4*
b*x^3 + a^5*x
```

giac [A] time = 0.16, size = 102, normalized size = 0.41

$$\frac{1}{11}\,b^5x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{9}\,ab^4x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{7}\,a^2b^3x^7\mathrm{sgn}\left(bx^2+a\right)+2\,a^3b^2x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{3}\,a^4bx^3\mathrm{sgn}\left(bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/11*b^5*x^11*sgn(b*x^2 + a) + 5/9*a*b^4*x^9*sgn(b*x^2 + a) + 10/7*a^2*b^3*
x^7*sgn(b*x^2 + a) + 2*a^3*b^2*x^5*sgn(b*x^2 + a) + 5/3*a^4*b*x^3*sgn(b*x^2
 + a) + a^5*x*sgn(b*x^2 + a)
```

maple [A] time = 0.00, size = 78, normalized size = 0.31

$$\frac{\left(63b^5x^{10}+385a\,b^4x^8+990a^2b^3x^6+1386a^3b^2x^4+1155a^4b\,x^2+693a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x}{693\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/693*x*(63*b^5*x^10+385*a*b^4*x^8+990*a^2*b^3*x^6+1386*a^3*b^2*x^4+1155*a^
4*b*x^2+693*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

maxima [A] time = 1.32, size = 54, normalized size = 0.22

$$\frac{1}{11}\,b^5x^{11}+\frac{5}{9}\,ab^4x^9+\frac{10}{7}\,a^2b^3x^7+2\,a^3b^2x^5+\frac{5}{3}\,a^4bx^3+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4*
b*x^3 + a^5*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2), x)
```

3.612 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=247

$$\frac{b^5x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{5ab^4x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{2a^2b^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+$$

```
[Out] -a^5*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)+5*a^4*b*x*((b*x^2+a)^2)^(1/2)/(b*x^2+a
)+10/3*a^3*b^2*x^3*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+2*a^2*b^3*x^5*((b*x^2+a)^2
)^(1/2)/(b*x^2+a)+5/7*a*b^4*x^7*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/9*b^5*x^9*(
(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.06, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{5ab^4x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{2a^2b^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^2+}}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^2,x]
```

```
[Out] -((a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))) + (5*a^4*b*x*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (10*a^3*b^2*x^3*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(3*(a + b*x^2)) + (2*a^2*b^3*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(a + b*x^2) + (5*a*b^4*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a +
b*x^2)) + (b^5*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^2}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(5a^4b^6+\frac{a^5b^5}{x^2}+10a^3b^7x^2+10a^2b^8x^4+5ab^9x^6+b^{10}x^8\right) d}{b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{5a^4bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^2+}}{3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-63a^5+315a^4bx^2+210a^3b^2x^4+126a^2b^3x^6+45ab^4x^8+7b^5x^{10}\right)}{63x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-63*a^5 + 315*a^4*b*x^2 + 210*a^3*b^2*x^4 + 126*a^2*b
^3*x^6 + 45*a*b^4*x^8 + 7*b^5*x^10))/(63*x*(a + b*x^2))
```

fricas [A] time = 1.07, size = 59, normalized size = 0.24

$$\frac{7\,b^5x^{10}+45\,ab^4x^8+126\,a^2b^3x^6+210\,a^3b^2x^4+315\,a^4bx^2-63\,a^5}{63\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/63*(7*b^5*x^10 + 45*a*b^4*x^8 + 126*a^2*b^3*x^6 + 210*a^3*b^2*x^4 + 315*a
^4*b*x^2 - 63*a^5)/x
```

giac [A] time = 0.16, size = 103, normalized size = 0.42

$$\frac{1}{9}\,b^5x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{7}\,ab^4x^7\mathrm{sgn}\left(bx^2+a\right)+2\,a^2b^3x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{3}\,a^3b^2x^3\mathrm{sgn}\left(bx^2+a\right)+5\,a^4bx\mathrm{sgn}\left(bx^2+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^2,x, algorithm="giac")
```

[Out] 1/9*b^5*x^9*sgn(b*x^2 + a) + 5/7*a*b^4*x^7*sgn(b*x^2 + a) + 2*a^2*b^3*x^5*sgn(b*x^2 + a) + 10/3*a^3*b^2*x^3*sgn(b*x^2 + a) + 5*a^4*b*x*sgn(b*x^2 + a) - a^5*sgn(b*x^2 + a)/x

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-7b^5x^{10}-45a\,b^4x^8-126a^2b^3x^6-210a^3b^2x^4-315a^4b\,x^2+63a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{63\left(b\,x^2+a\right)^5 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^2,x)

[Out] -1/63*(-7*b^5*x^10-45*a*b^4*x^8-126*a^2*b^3*x^6-210*a^3*b^2*x^4-315*a^4*b*x^2+63*a^5)*((b*x^2+a)^2)^(5/2)/x/(b*x^2+a)^5

maxima [A] time = 1.32, size = 55, normalized size = 0.22

$$\frac{1}{9}\,b^5x^9+\frac{5}{7}\,ab^4x^7+2\,a^2b^3x^5+\frac{10}{3}\,a^3b^2x^3+5\,a^4bx-\frac{a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^2,x, algorithm="maxima")

[Out] 1/9*b^5*x^9 + 5/7*a*b^4*x^7 + 2*a^2*b^3*x^5 + 10/3*a^3*b^2*x^3 + 5*a^4*b*x - a^5/x

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^2,x)

[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**2,x)
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**2, x)
```

3.613 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=246

$$\frac{b^5x^7\sqrt{a^2+2abx^2+b^2x^4}}{7(a+bx^2)}+\frac{ab^4x^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^2b^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}-$$

```
[Out] -1/3*a^5*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)-5*a^4*b*((b*x^2+a)^2)^(1/2)/x/(b
*x^2+a)+10*a^3*b^2*x*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+10/3*a^2*b^3*x^3*((b*x^2
+a)^2)^(1/2)/(b*x^2+a)+a*b^4*x^5*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/7*b^5*x^7*
((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.06, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^7\sqrt{a^2+2abx^2+b^2x^4}}{7(a+bx^2)}+\frac{ab^4x^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^2b^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}+\frac{10a^3b^2x\sqrt{a^2+2abx^2+b}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^4,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (10*a^3*b^2*x*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(a + b*x^2) + (10*a^2*b^3*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])/(3*(a + b*x^2)) + (a*b^4*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*
x^2) + (b^5*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^4}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(10a^3b^7+\frac{a^5b^5}{x^4}+\frac{5a^4b^6}{x^2}+10a^2b^8x^2+5ab^9x^4+b^{10}x^6\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)} + \frac{10a^3b^2x\sqrt{a^2+2abx^2+b^2}}{a+bx^2}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-7a^5-105a^4bx^2+210a^3b^2x^4+70a^2b^3x^6+21ab^4x^8+3b^5x^{10}\right)}{21x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^4,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-7*a^5 - 105*a^4*b*x^2 + 210*a^3*b^2*x^4 + 70*a^2*b^3
*x^6 + 21*a*b^4*x^8 + 3*b^5*x^10))/(21*x^3*(a + b*x^2))
```

fricas [A] time = 0.77, size = 59, normalized size = 0.24

$$\frac{3\,b^5x^{10}+21\,ab^4x^8+70\,a^2b^3x^6+210\,a^3b^2x^4-105\,a^4bx^2-7\,a^5}{21\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/21*(3*b^5*x^10 + 21*a*b^4*x^8 + 70*a^2*b^3*x^6 + 210*a^3*b^2*x^4 - 105*a^
4*b*x^2 - 7*a^5)/x^3
```

giac [A] time = 0.16, size = 104, normalized size = 0.42

$$\frac{1}{7}\,b^5x^7\mathrm{sgn}\left(bx^2+a\right)+ab^4x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{3}\,a^2b^3x^3\mathrm{sgn}\left(bx^2+a\right)+10\,a^3b^2x\mathrm{sgn}\left(bx^2+a\right)-\frac{15\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right.}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/7*b^5*x^7*sgn(b*x^2 + a) + a*b^4*x^5*sgn(b*x^2 + a) + 10/3*a^2*b^3*x^3*sg
n(b*x^2 + a) + 10*a^3*b^2*x*sgn(b*x^2 + a) - 1/3*(15*a^4*b*x^2*sgn(b*x^2 +
a) + a^5*sgn(b*x^2 + a))/x^3
```

maple [A] time = 0.01, size = 80, normalized size = 0.33

$$-\frac{\left(-3b^5x^{10}-21a\,b^4x^8-70a^2b^3x^6-210a^3b^2x^4+105a^4b\,x^2+7a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{21\left(b\,x^2+a\right)^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^4,x)
```

```
[Out] -1/21*(-3*b^5*x^10-21*a*b^4*x^8-70*a^2*b^3*x^6-210*a^3*b^2*x^4+105*a^4*b*x^
2+7*a^5)*((b*x^2+a)^2)^(5/2)/x^3/(b*x^2+a)^5
```

maxima [A] time = 1.33, size = 54, normalized size = 0.22

$$\frac{1}{7}\,b^5x^7+ab^4x^5+\frac{10}{3}\,a^2b^3x^3+10\,a^3b^2x-\frac{5\,a^4b}{x}-\frac{a^5}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/7*b^5*x^7 + a*b^4*x^5 + 10/3*a^2*b^3*x^3 + 10*a^3*b^2*x - 5*a^4*b/x - 1/3
*a^5/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^4,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**4,x)
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**4, x)
```

3.614 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^6}\,dx$

Optimal. Leaf size=249

$$\frac{b^5x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{5ab^4x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{10a^2b^3x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-$$

```
[Out] -1/5*a^5*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)-5/3*a^4*b*((b*x^2+a)^2)^(1/2)/x^
3/(b*x^2+a)-10*a^3*b^2*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)+10*a^2*b^3*x*((b*x^2
+a)^2)^(1/2)/(b*x^2+a)+5/3*a*b^4*x^3*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/5*b^5*
x^5*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.06, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{10a^2b^3x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^6,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (10*a^2*b^3*x*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(a + b*x^2) + (5*a*b^4*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a
 + b*x^2)) + (b^5*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^6}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^6}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(10a^2b^8+\frac{a^5b^5}{x^6}+\frac{5a^4b^6}{x^4}+\frac{10a^3b^7}{x^2}+5ab^9x^2+b^{10}x^4\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x}}{x\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-3a^5-25a^4bx^2-150a^3b^2x^4+150a^2b^3x^6+25ab^4x^8+3b^5x^{10}\right)}{15x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^6,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-3*a^5 - 25*a^4*b*x^2 - 150*a^3*b^2*x^4 + 150*a^2*b^3
*x^6 + 25*a*b^4*x^8 + 3*b^5*x^10))/(15*x^5*(a + b*x^2))
```

fricas [A] time = 0.81, size = 59, normalized size = 0.24

$$\frac{3\,b^5x^{10}+25\,ab^4x^8+150\,a^2b^3x^6-150\,a^3b^2x^4-25\,a^4bx^2-3\,a^5}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^5*x^10 + 25*a*b^4*x^8 + 150*a^2*b^3*x^6 - 150*a^3*b^2*x^4 - 25*a^
4*b*x^2 - 3*a^5)/x^5
```

giac [A] time = 0.17, size = 106, normalized size = 0.43

$$\frac{1}{5}\,b^5x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{3}\,ab^4x^3\mathrm{sgn}\left(bx^2+a\right)+10\,a^2b^3x\mathrm{sgn}\left(bx^2+a\right)-\frac{150\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+25\,a^4bx^2\mathrm{sgn}\left(bx^2\right.}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/5*b^5*x^5*sgn(b*x^2 + a) + 5/3*a*b^4*x^3*sgn(b*x^2 + a) + 10*a^2*b^3*x*sg
n(b*x^2 + a) - 1/15*(150*a^3*b^2*x^4*sgn(b*x^2 + a) + 25*a^4*b*x^2*sgn(b*x^
2 + a) + 3*a^5*sgn(b*x^2 + a))/x^5
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-3b^5x^{10}-25a\,b^4x^8-150a^2b^3x^6+150a^3b^2x^4+25a^4b\,x^2+3a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{15\left(b\,x^2+a\right)^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^6,x)
```

```
[Out] -1/15*(-3*b^5*x^10-25*a*b^4*x^8-150*a^2*b^3*x^6+150*a^3*b^2*x^4+25*a^4*b*x^
2+3*a^5)*((b*x^2+a)^2)^(5/2)/x^5/(b*x^2+a)^5
```

maxima [A] time = 1.35, size = 55, normalized size = 0.22

$$\frac{1}{5}\,b^5x^5+\frac{5}{3}\,ab^4x^3+10\,a^2b^3x-\frac{10\,a^3b^2}{x}-\frac{5\,a^4b}{3\,x^3}-\frac{a^5}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*b^5*x^5 + 5/3*a*b^4*x^3 + 10*a^2*b^3*x - 10*a^3*b^2/x - 5/3*a^4*b/x^3 -
 1/5*a^5/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^6,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**6,x)
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**6, x)
```

3.615 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^8} dx$

Optimal. Leaf size=247

$$\frac{b^5x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{5ab^4x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{a^4}{}$$

```
[Out] -1/7*a^5*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)-a^4*b*((b*x^2+a)^2)^(1/2)/x^5/(b
*x^2+a)-10/3*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)-10*a^2*b^3*((b*x^2+a
)^2)^(1/2)/x/(b*x^2+a)+5*a*b^4*x*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/3*b^5*x^3*
((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.06, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{5}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^8,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (a^4*b*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(x^5*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(x*(a + b*x^2)) + (5*a*b^4*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a +
b*x^2) + (b^5*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^8}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^8}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(5ab^9+\frac{a^5b^5}{x^8}+\frac{5a^4b^6}{x^6}+\frac{10a^3b^7}{x^4}+\frac{10a^2b^8}{x^2}+b^{10}x^2\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.34

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(3a^5+21a^4bx^2+70a^3b^2x^4+210a^2b^3x^6-105ab^4x^8-7b^5x^{10}\right)}{21x^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^8,x]
```

```
[Out] -1/21*(Sqrt[(a + b*x^2)^2]*(3*a^5 + 21*a^4*b*x^2 + 70*a^3*b^2*x^4 + 210*a^2
*b^3*x^6 - 105*a*b^4*x^8 - 7*b^5*x^10))/(x^7*(a + b*x^2))
```

fricas [A] time = 0.95, size = 59, normalized size = 0.24

$$\frac{7\,b^5x^{10}+105\,ab^4x^8-210\,a^2b^3x^6-70\,a^3b^2x^4-21\,a^4bx^2-3\,a^5}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/21*(7*b^5*x^10 + 105*a*b^4*x^8 - 210*a^2*b^3*x^6 - 70*a^3*b^2*x^4 - 21*a^
4*b*x^2 - 3*a^5)/x^7
```

giac [A] time = 0.16, size = 106, normalized size = 0.43

$$\frac{1}{3}\,b^5x^3\mathrm{sgn}\left(bx^2+a\right)+5\,ab^4x\mathrm{sgn}\left(bx^2+a\right)-\frac{210\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+70\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+21\,a^4bx^2\mathrm{sgn}\left(bx\right.}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/3*b^5*x^3*sgn(b*x^2 + a) + 5*a*b^4*x*sgn(b*x^2 + a) - 1/21*(210*a^2*b^3*x
^6*sgn(b*x^2 + a) + 70*a^3*b^2*x^4*sgn(b*x^2 + a) + 21*a^4*b*x^2*sgn(b*x^2
+ a) + 3*a^5*sgn(b*x^2 + a))/x^7
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(-7b^{5}x^{10}-105a\,b^{4}x^{8}+210a^{2}b^{3}x^{6}+70a^{3}b^{2}x^{4}+21a^{4}b\,x^{2}+3a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{21\left(b\,x^{2}+a\right)^{5}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^8,x)
```

```
[Out] -1/21*(-7*b^5*x^10-105*a*b^4*x^8+210*a^2*b^3*x^6+70*a^3*b^2*x^4+21*a^4*b*x^
2+3*a^5)*((b*x^2+a)^2)^(5/2)/x^7/(b*x^2+a)^5
```

maxima [A] time = 1.29, size = 55, normalized size = 0.22

$$\frac{1}{3}\,b^{5}x^{3}+5\,ab^{4}x-\frac{10\,a^{2}b^{3}}{x}-\frac{10\,a^{3}b^{2}}{3\,x^{3}}-\frac{a^{4}b}{x^{5}}-\frac{a^{5}}{7\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] 1/3*b^5*x^3 + 5*a*b^4*x - 10*a^2*b^3/x - 10/3*a^3*b^2/x^3 - a^4*b/x^5 - 1/7
*a^5/x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}{x^8} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^8,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^{2}\right)^{2}\right)^{\frac{5}{2}}}{x^{8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**8,x)
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**8, x)
```

3.616 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{10}} dx$

Optimal. Leaf size=246

$$\frac{b^5x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{5a^4b}{}$$

```
[Out] -1/9*a^5*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)-5/7*a^4*b*((b*x^2+a)^2)^(1/2)/x^
7/(b*x^2+a)-2*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)-10/3*a^2*b^3*((b*x^
2+a)^2)^(1/2)/x^3/(b*x^2+a)-5*a*b^4*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)+b^5*x*(
(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.06, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-5$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^10,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (2*a^3*b^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(x^5*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(3*x^3*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x
*(a + b*x^2)) + (b^5*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{10}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{10}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^{10}+\frac{a^5b^5}{x^{10}}+\frac{5a^4b^6}{x^8}+\frac{10a^3b^7}{x^6}+\frac{10a^2b^8}{x^4}+\frac{5ab^9}{x^2}\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)} - \frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.34

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(7a^5+45a^4bx^2+126a^3b^2x^4+210a^2b^3x^6+315ab^4x^8-63b^5x^{10}\right)}{63x^9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^10,x]
```

```
[Out] -1/63*(Sqrt[(a + b*x^2)^2]*(7*a^5 + 45*a^4*b*x^2 + 126*a^3*b^2*x^4 + 210*a^
2*b^3*x^6 + 315*a*b^4*x^8 - 63*b^5*x^10))/(x^9*(a + b*x^2))
```

fricas [A] time = 1.07, size = 59, normalized size = 0.24

$$\frac{63\,b^5x^{10}-315\,ab^4x^8-210\,a^2b^3x^6-126\,a^3b^2x^4-45\,a^4bx^2-7\,a^5}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/63*(63*b^5*x^10 - 315*a*b^4*x^8 - 210*a^2*b^3*x^6 - 126*a^3*b^2*x^4 - 45*
a^4*b*x^2 - 7*a^5)/x^9
```

giac [A] time = 0.17, size = 105, normalized size = 0.43

$$b^5x\mathrm{sgn}\left(bx^2+a\right)-\frac{315\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+210\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+126\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+45\,a^4bx^2\mathrm{sgn}\left(\right.}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^10,x, algorithm="giac")
```

```
[Out] b^5*x*sgn(b*x^2 + a) - 1/63*(315*a*b^4*x^8*sgn(b*x^2 + a) + 210*a^2*b^3*x^6
*sgn(b*x^2 + a) + 126*a^3*b^2*x^4*sgn(b*x^2 + a) + 45*a^4*b*x^2*sgn(b*x^2 +
 a) + 7*a^5*sgn(b*x^2 + a))/x^9
```

maple [A] time = 0.01, size = 80, normalized size = 0.33

$$-\frac{\left(-63b^5x^{10}+315a\,b^4x^8+210a^2b^3x^6+126a^3b^2x^4+45a^4b\,x^2+7a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{63\left(b\,x^2+a\right)^5x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^10,x)
```

```
[Out] -1/63*(-63*b^5*x^10+315*a*b^4*x^8+210*a^2*b^3*x^6+126*a^3*b^2*x^4+45*a^4*b*
x^2+7*a^5)*((b*x^2+a)^2)^(5/2)/x^9/(b*x^2+a)^5
```

maxima [A] time = 1.35, size = 54, normalized size = 0.22

$$b^5x-\frac{5\,ab^4}{x}-\frac{10\,a^2b^3}{3\,x^3}-\frac{2\,a^3b^2}{x^5}-\frac{5\,a^4b}{7\,x^7}-\frac{a^5}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^10,x, algorithm="maxima")
```

```
[Out] b^5*x - 5*a*b^4/x - 10/3*a^2*b^3/x^3 - 2*a^3*b^2/x^5 - 5/7*a^4*b/x^7 - 1/9*
a^5/x^9
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^10,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^10, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**10,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**10, x)
```

3.617 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{12}}\,dx$

Optimal. Leaf size=251

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{}}{}$$

```
[Out] -1/11*a^5*((b*x^2+a)^2)^(1/2)/x^11/(b*x^2+a)-5/9*a^4*b*((b*x^2+a)^2)^(1/2)/
x^9/(b*x^2+a)-10/7*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)-2*a^2*b^3*((b*
x^2+a)^2)^(1/2)/x^5/(b*x^2+a)-5/3*a*b^4*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)-b
^5*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)
```

Rubi [A] time = 0.06, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)}-5$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^12,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (2*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(x^5*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/
(3*x^3*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{12}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{12}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{12}}+\frac{5a^4b^6}{x^{10}}+\frac{10a^3b^7}{x^8}+\frac{10a^2b^8}{x^6}+\frac{5ab^9}{x^4}+\frac{b^{10}}{x^2}\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x}}{7x^7\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(63a^5+385a^4bx^2+990a^3b^2x^4+1386a^2b^3x^6+1155ab^4x^8+693b^5x^{10}\right)}{693x^{11}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^12,x]
```

```
[Out] -1/693*(Sqrt[(a + b*x^2)^2]*(63*a^5 + 385*a^4*b*x^2 + 990*a^3*b^2*x^4 + 138
6*a^2*b^3*x^6 + 1155*a*b^4*x^8 + 693*b^5*x^10))/(x^11*(a + b*x^2))
```

fricas [A] time = 1.06, size = 59, normalized size = 0.24

$$-\frac{693\,b^5x^{10}+1155\,ab^4x^8+1386\,a^2b^3x^6+990\,a^3b^2x^4+385\,a^4bx^2+63\,a^5}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/693*(693*b^5*x^10 + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4
+ 385*a^4*b*x^2 + 63*a^5)/x^11
```

giac [A] time = 0.16, size = 107, normalized size = 0.43

$$-\frac{693\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+1155\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+1386\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+990\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+38}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^12,x, algorithm="giac")
```

```
[Out] -1/693*(693*b^5*x^10*sgn(b*x^2 + a) + 1155*a*b^4*x^8*sgn(b*x^2 + a) + 1386*
a^2*b^3*x^6*sgn(b*x^2 + a) + 990*a^3*b^2*x^4*sgn(b*x^2 + a) + 385*a^4*b*x^2
*sgn(b*x^2 + a) + 63*a^5*sgn(b*x^2 + a))/x^11
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(693b^5x^{10}+1155a\,b^4x^8+1386a^2b^3x^6+990a^3b^2x^4+385a^4b\,x^2+63a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{693\left(b\,x^2+a\right)^5x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^12,x)
```

```
[Out] -1/693*(693*b^5*x^10+1155*a*b^4*x^8+1386*a^2*b^3*x^6+990*a^3*b^2*x^4+385*a^
4*b*x^2+63*a^5)*((b*x^2+a)^2)^(5/2)/x^11/(b*x^2+a)^5
```

maxima [A] time = 1.33, size = 57, normalized size = 0.23

$$-\frac{b^5}{x}-\frac{5\,ab^4}{3\,x^3}-\frac{2\,a^2b^3}{x^5}-\frac{10\,a^3b^2}{7\,x^7}-\frac{5\,a^4b}{9\,x^9}-\frac{a^5}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^12,x, algorithm="maxima")
```

```
[Out] -b^5/x - 5/3*a*b^4/x^3 - 2*a^2*b^3/x^5 - 10/7*a^3*b^2/x^7 - 5/9*a^4*b/x^9 -
 1/11*a^5/x^11
```

mupad [B] time = 4.22, size = 231, normalized size = 0.92

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\,\left(b\,x^2+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x\,\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^3\,\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x}}{9\,x^9\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^12,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(11*x^11*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(x*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*x^4 +
2*a*b*x^2)^(1/2))/(3*x^3*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a*b*x^2
)^(1/2))/(9*x^9*(a + b*x^2)) - (2*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)
)/(x^5*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(7*x^7
*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**12,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**12, x)
```

3.618 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{14}} dx$

Optimal. Leaf size=253

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{}}{1}$$

```
[Out] -1/13*a^5*((b*x^2+a)^2)^(1/2)/x^13/(b*x^2+a)-5/11*a^4*b*((b*x^2+a)^2)^(1/2)
/x^11/(b*x^2+a)-10/9*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)-10/7*a^2*b^3
*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)-a*b^4*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)-
1/3*b^5*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)
```

Rubi [A] time = 0.06, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^14,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])/(x^5*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*
x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{14}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{14}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{14}}+\frac{5a^4b^6}{x^{12}}+\frac{10a^3b^7}{x^{10}}+\frac{10a^2b^8}{x^8}+\frac{5ab^9}{x^6}+\frac{b^{10}}{x^4}\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x}}{9x^9\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(693a^5+4095a^4bx^2+10010a^3b^2x^4+12870a^2b^3x^6+9009ab^4x^8+3003b^5x^{10}\right)}{9009x^{13}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^14,x]
```

```
[Out] -1/9009*(Sqrt[(a + b*x^2)^2]*(693*a^5 + 4095*a^4*b*x^2 + 10010*a^3*b^2*x^4
+ 12870*a^2*b^3*x^6 + 9009*a*b^4*x^8 + 3003*b^5*x^10))/(x^13*(a + b*x^2))
```

fricas [A] time = 0.98, size = 59, normalized size = 0.23

$$-\frac{3003\,b^5x^{10}+9009\,ab^4x^8+12870\,a^2b^3x^6+10010\,a^3b^2x^4+4095\,a^4bx^2+693\,a^5}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^14,x, algorithm="fricas")
```

```
[Out] -1/9009*(3003*b^5*x^10 + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^2
*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^13
```

giac [A] time = 0.18, size = 107, normalized size = 0.42

$$-\frac{3003\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+9009\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+12870\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+10010\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^14,x, algorithm="giac")
```

```
[Out] -1/9009*(3003*b^5*x^10*sgn(b*x^2 + a) + 9009*a*b^4*x^8*sgn(b*x^2 + a) + 128
70*a^2*b^3*x^6*sgn(b*x^2 + a) + 10010*a^3*b^2*x^4*sgn(b*x^2 + a) + 4095*a^4
*b*x^2*sgn(b*x^2 + a) + 693*a^5*sgn(b*x^2 + a))/x^13
```

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$-\frac{\left(3003 b^{5} x^{10}+9009 a\, b^{4} x^{8}+12870 a^{2} b^{3} x^{6}+10010 a^{3} b^{2} x^{4}+4095 a^{4} b\, x^{2}+693 a^{5}\right) \left(\left(b\, x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{9009 \left(b\, x^{2}+a\right)^{5} x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^14,x)
```

```
[Out] -1/9009*(3003*b^5*x^10+9009*a*b^4*x^8+12870*a^2*b^3*x^6+10010*a^3*b^2*x^4+4
095*a^4*b*x^2+693*a^5)*((b*x^2+a)^2)^(5/2)/x^13/(b*x^2+a)^5
```

maxima [A] time = 1.34, size = 57, normalized size = 0.23

$$-\frac{b^{5}}{3\,x^{3}}-\frac{ab^{4}}{x^{5}}-\frac{10\,a^{2}b^{3}}{7\,x^{7}}-\frac{10\,a^{3}b^{2}}{9\,x^{9}}-\frac{5\,a^{4}b}{11\,x^{11}}-\frac{a^{5}}{13\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^14,x, algorithm="maxima")
```

```
[Out] -1/3*b^5/x^3 - a*b^4/x^5 - 10/7*a^2*b^3/x^7 - 10/9*a^3*b^2/x^9 - 5/11*a^4*b
/x^11 - 1/13*a^5/x^13
```

mupad [B] time = 4.35, size = 231, normalized size = 0.91

$$-\frac{a^{5}\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{13\,x^{13}\,\left(b\,x^{2}+a\right)}-\frac{b^{5}\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{3\,x^{3}\,\left(b\,x^{2}+a\right)}-\frac{a\,b^{4}\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{x^{5}\,\left(b\,x^{2}+a\right)}-\frac{5\,a^{4}\,b\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{11\,x^{11}\,\left(b\,x^{2}+a\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^14,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(13*x^13*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(3*x^3*(a + b*x^2)) - (a*b^4*(a^2 + b^2*x^4
+ 2*a*b*x^2)^(1/2))/(x^5*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a*b*x^2
)^(1/2))/(11*x^11*(a + b*x^2)) - (10*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1
/2))/(7*x^7*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(
9*x^9*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**14,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**14, x)
```

3.619 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{16}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)}-\frac{5a^4b}{}$$

```
[Out] -1/15*a^5*((b*x^2+a)^2)^(1/2)/x^15/(b*x^2+a)-5/13*a^4*b*((b*x^2+a)^2)^(1/2)
/x^13/(b*x^2+a)-10/11*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^11/(b*x^2+a)-10/9*a^2*b
^3*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)-5/7*a*b^4*((b*x^2+a)^2)^(1/2)/x^7/(b*x
^2+a)-1/5*b^5*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)
```

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^16,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*x^15*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(7*x^7*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(
a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{16}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{16}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{16}}+\frac{5a^4b^6}{x^{14}}+\frac{10a^3b^7}{x^{12}}+\frac{10a^2b^8}{x^{10}}+\frac{5ab^9}{x^8}+\frac{b^{10}}{x^6}\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x}}{11x^{11}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(3003a^5+17325a^4bx^2+40950a^3b^2x^4+50050a^2b^3x^6+32175ab^4x^8+9009b^5x^{10}\right)}{45045x^{15}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^16,x]
```

```
[Out] -1/45045*(Sqrt[(a + b*x^2)^2]*(3003*a^5 + 17325*a^4*b*x^2 + 40950*a^3*b^2*x
^4 + 50050*a^2*b^3*x^6 + 32175*a*b^4*x^8 + 9009*b^5*x^10))/(x^15*(a + b*x^2
))
```

fricas [A] time = 0.97, size = 59, normalized size = 0.23

$$-\frac{9009\,b^5x^{10}+32175\,ab^4x^8+50050\,a^2b^3x^6+40950\,a^3b^2x^4+17325\,a^4bx^2+3003\,a^5}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(9009*b^5*x^10 + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*b
^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^15
```

giac [A] time = 0.16, size = 107, normalized size = 0.42

$$-\frac{9009\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+32175\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+50050\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+40950\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^16,x, algorithm="giac")
```

```
[Out] -1/45045*(9009*b^5*x^10*sgn(b*x^2 + a) + 32175*a*b^4*x^8*sgn(b*x^2 + a) + 5
0050*a^2*b^3*x^6*sgn(b*x^2 + a) + 40950*a^3*b^2*x^4*sgn(b*x^2 + a) + 17325*
a^4*b*x^2*sgn(b*x^2 + a) + 3003*a^5*sgn(b*x^2 + a))/x^15
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(9009b^5x^{10}+32175a\,b^4x^8+50050a^2b^3x^6+40950a^3b^2x^4+17325a^4b\,x^2+3003a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{45045\left(b\,x^2+a\right)^5x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^16,x)
```

```
[Out] -1/45045*(9009*b^5*x^10+32175*a*b^4*x^8+50050*a^2*b^3*x^6+40950*a^3*b^2*x^4
+17325*a^4*b*x^2+3003*a^5)*((b*x^2+a)^2)^(5/2)/x^15/(b*x^2+a)^5
```

maxima [A] time = 1.35, size = 57, normalized size = 0.22

$$-\frac{b^5}{5\,x^5}-\frac{5\,ab^4}{7\,x^7}-\frac{10\,a^2b^3}{9\,x^9}-\frac{10\,a^3b^2}{11\,x^{11}}-\frac{5\,a^4b}{13\,x^{13}}-\frac{a^5}{15\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^16,x, algorithm="maxima")
```

```
[Out] -1/5*b^5/x^5 - 5/7*a*b^4/x^7 - 10/9*a^2*b^3/x^9 - 10/11*a^3*b^2/x^11 - 5/13
*a^4*b/x^13 - 1/15*a^5/x^15
```

mupad [B] time = 4.21, size = 231, normalized size = 0.91

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{15\,x^{15}\left(b\,x^2+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{5\,x^5\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^7\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x}}{13\,x^{13}\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^16,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(15*x^15*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(5*x^5*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(7*x^7*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a*b
*x^2)^(1/2))/(13*x^13*(a + b*x^2)) - (10*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2
)^(1/2))/(9*x^9*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2
))/(11*x^11*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**16,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**16, x)
```

3.620 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{18}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{a^4b\sqrt{}}{}$$

```
[Out] -1/17*a^5*((b*x^2+a)^2)^(1/2)/x^17/(b*x^2+a)-1/3*a^4*b*((b*x^2+a)^2)^(1/2)/
x^15/(b*x^2+a)-10/13*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^13/(b*x^2+a)-10/11*a^2*b
^3*((b*x^2+a)^2)^(1/2)/x^11/(b*x^2+a)-5/9*a*b^4*((b*x^2+a)^2)^(1/2)/x^9/(b*
x^2+a)-1/7*b^5*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)
```

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-5$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^18,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2)) - (a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^15*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])/(9*x^9*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a
 + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{18}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{18}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{18}}+\frac{5a^4b^6}{x^{16}}+\frac{10a^3b^7}{x^{14}}+\frac{10a^2b^8}{x^{12}}+\frac{5ab^9}{x^{10}}+\frac{b^{10}}{x^8}\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(9009a^5+51051a^4bx^2+117810a^3b^2x^4+139230a^2b^3x^6+85085ab^4x^8+21879b^5x^{10}\right)}{153153x^{17}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^18,x]
```

```
[Out] -1/153153*(Sqrt[(a + b*x^2)^2]*(9009*a^5 + 51051*a^4*b*x^2 + 117810*a^3*b^2
*x^4 + 139230*a^2*b^3*x^6 + 85085*a*b^4*x^8 + 21879*b^5*x^10))/(x^17*(a + b
*x^2))
```

fricas [A] time = 0.93, size = 59, normalized size = 0.23

$$-\frac{21879\,b^5x^{10}+85085\,ab^4x^8+139230\,a^2b^3x^6+117810\,a^3b^2x^4+51051\,a^4bx^2+9009\,a^5}{153153\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^18,x, algorithm="fricas")
```

```
[Out] -1/153153*(21879*b^5*x^10 + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*a
^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^17
```

giac [A] time = 0.17, size = 107, normalized size = 0.42

$$-\frac{21879\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+85085\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+139230\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+117810\,a^3b^2x^4\mathrm{sgn}\left(bx^2\right.}{153153\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^18,x, algorithm="giac")
```

```
[Out] -1/153153*(21879*b^5*x^10*sgn(b*x^2 + a) + 85085*a*b^4*x^8*sgn(b*x^2 + a) +
 139230*a^2*b^3*x^6*sgn(b*x^2 + a) + 117810*a^3*b^2*x^4*sgn(b*x^2 + a) + 51
051*a^4*b*x^2*sgn(b*x^2 + a) + 9009*a^5*sgn(b*x^2 + a))/x^17
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(21879 b^{5} x^{10}+85085 a\,b^{4} x^{8}+139230 a^{2} b^{3} x^{6}+117810 a^{3} b^{2} x^{4}+51051 a^{4} b\,x^{2}+9009 a^{5}\right) \left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{153153 \left(b\,x^{2}+a\right)^{5} x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^18,x)
```

```
[Out] -1/153153*(21879*b^5*x^10+85085*a*b^4*x^8+139230*a^2*b^3*x^6+117810*a^3*b^2
*x^4+51051*a^4*b*x^2+9009*a^5)*((b*x^2+a)^2)^(5/2)/x^17/(b*x^2+a)^5
```

maxima [A] time = 1.35, size = 57, normalized size = 0.22

$$-\frac{b^{5}}{7\,x^{7}}-\frac{5\,a b^{4}}{9\,x^{9}}-\frac{10\,a^{2} b^{3}}{11\,x^{11}}-\frac{10\,a^{3} b^{2}}{13\,x^{13}}-\frac{a^{4} b}{3\,x^{15}}-\frac{a^{5}}{17\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^18,x, algorithm="maxima")
```

```
[Out] -1/7*b^5/x^7 - 5/9*a*b^4/x^9 - 10/11*a^2*b^3/x^11 - 10/13*a^3*b^2/x^13 - 1/
3*a^4*b/x^15 - 1/17*a^5/x^17
```

mupad [B] time = 4.31, size = 231, normalized size = 0.91

$$-\frac{a^{5}\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{17\,x^{17}\,\left(b\,x^{2}+a\right)}-\frac{b^{5}\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{7\,x^{7}\,\left(b\,x^{2}+a\right)}-\frac{5\,a\,b^{4}\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{9\,x^{9}\,\left(b\,x^{2}+a\right)}-\frac{a^{4}\,b\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{3\,x^{15}\,\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^18,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(17*x^17*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(7*x^7*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(9*x^9*(a + b*x^2)) - (a^4*b*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(3*x^15*(a + b*x^2)) - (10*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(
1/2))/(11*x^11*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)
)/(13*x^13*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{18}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**18,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**18, x)
```

3.621 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{20}} \, dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)}-\frac{5a^4b}{}$$

```
[Out] -1/19*a^5*((b*x^2+a)^2)^(1/2)/x^19/(b*x^2+a)-5/17*a^4*b*((b*x^2+a)^2)^(1/2)
/x^17/(b*x^2+a)-2/3*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^15/(b*x^2+a)-10/13*a^2*b^
3*((b*x^2+a)^2)^(1/2)/x^13/(b*x^2+a)-5/11*a*b^4*((b*x^2+a)^2)^(1/2)/x^11/(b
*x^2+a)-1/9*b^5*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)
```

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-5$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^20,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*x^19*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2)) - (2*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(3*x^15*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(11*x^11*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9
*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{20}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{20}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{20}}+\frac{5a^4b^6}{x^{18}}+\frac{10a^3b^7}{x^{16}}+\frac{10a^2b^8}{x^{14}}+\frac{5ab^9}{x^{12}}+\frac{b^{10}}{x^{10}}\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)} - \frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(21879a^5+122265a^4bx^2+277134a^3b^2x^4+319770a^2b^3x^6+188955ab^4x^8+46189b^5x^{10}\right)}{415701x^{19}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^20,x]
```

```
[Out] -1/415701*(Sqrt[(a + b*x^2)^2]*(21879*a^5 + 122265*a^4*b*x^2 + 277134*a^3*b
^2*x^4 + 319770*a^2*b^3*x^6 + 188955*a*b^4*x^8 + 46189*b^5*x^10))/(x^19*(a
+ b*x^2))
```

fricas [A] time = 0.58, size = 59, normalized size = 0.23

$$-\frac{46189\,b^5x^{10}+188955\,ab^4x^8+319770\,a^2b^3x^6+277134\,a^3b^2x^4+122265\,a^4bx^2+21879\,a^5}{415701\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^20,x, algorithm="fricas")
```

```
[Out] -1/415701*(46189*b^5*x^10 + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134*
a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^19
```

giac [A] time = 0.20, size = 107, normalized size = 0.42

$$-\frac{46189\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+188955\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+319770\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+277134\,a^3b^2x^4\mathrm{sgn}\left(bx\right.}{415701\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^20,x, algorithm="giac")
```

```
[Out] -1/415701*(46189*b^5*x^10*sgn(b*x^2 + a) + 188955*a*b^4*x^8*sgn(b*x^2 + a)
+ 319770*a^2*b^3*x^6*sgn(b*x^2 + a) + 277134*a^3*b^2*x^4*sgn(b*x^2 + a) + 1
22265*a^4*b*x^2*sgn(b*x^2 + a) + 21879*a^5*sgn(b*x^2 + a))/x^19
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(46189 b^{5} x^{10}+188955 a\,b^{4} x^{8}+319770 a^{2} b^{3} x^{6}+277134 a^{3} b^{2} x^{4}+122265 a^{4} b\,x^{2}+21879 a^{5}\right) \left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{415701 \left(b\,x^{2}+a\right)^{5} x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^20,x)
```

```
[Out] -1/415701*(46189*b^5*x^10+188955*a*b^4*x^8+319770*a^2*b^3*x^6+277134*a^3*b^
2*x^4+122265*a^4*b*x^2+21879*a^5)*((b*x^2+a)^2)^(5/2)/x^19/(b*x^2+a)^5
```

maxima [A] time = 1.34, size = 57, normalized size = 0.22

$$-\frac{b^{5}}{9\,x^{9}}-\frac{5\,ab^{4}}{11\,x^{11}}-\frac{10\,a^{2}b^{3}}{13\,x^{13}}-\frac{2\,a^{3}b^{2}}{3\,x^{15}}-\frac{5\,a^{4}b}{17\,x^{17}}-\frac{a^{5}}{19\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^20,x, algorithm="maxima")
```

```
[Out] -1/9*b^5/x^9 - 5/11*a*b^4/x^11 - 10/13*a^2*b^3/x^13 - 2/3*a^3*b^2/x^15 - 5/
17*a^4*b/x^17 - 1/19*a^5/x^19
```

mupad [B] time = 4.27, size = 231, normalized size = 0.91

$$-\frac{a^{5}\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{19\,x^{19}\,\left(b\,x^{2}+a\right)}-\frac{b^{5}\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{9\,x^{9}\,\left(b\,x^{2}+a\right)}-\frac{5\,a\,b^{4}\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{11\,x^{11}\,\left(b\,x^{2}+a\right)}-\frac{5\,a^{4}\,b\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x}}{17\,x^{17}\,\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^20,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(19*x^19*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(9*x^9*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(11*x^11*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a
*b*x^2)^(1/2))/(17*x^17*(a + b*x^2)) - (10*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(13*x^13*(a + b*x^2)) - (2*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(
1/2))/(3*x^15*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{20}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**20,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**20, x)
```

3.622 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{22}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{}}{1}$$

```
[Out] -1/21*a^5*((b*x^2+a)^2)^(1/2)/x^21/(b*x^2+a)-5/19*a^4*b*((b*x^2+a)^2)^(1/2)
/x^19/(b*x^2+a)-10/17*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^17/(b*x^2+a)-2/3*a^2*b^
3*((b*x^2+a)^2)^(1/2)/x^15/(b*x^2+a)-5/13*a*b^4*((b*x^2+a)^2)^(1/2)/x^13/(b
*x^2+a)-1/11*b^5*((b*x^2+a)^2)^(1/2)/x^11/(b*x^2+a)
```

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-5$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^22,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*x^21*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*x^19*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2)) - (2*a^2*b^3*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(3*x^15*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(13*x^13*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^
11*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{22}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{22}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{22}}+\frac{5a^4b^6}{x^{20}}+\frac{10a^3b^7}{x^{18}}+\frac{10a^2b^8}{x^{16}}+\frac{5ab^9}{x^{14}}+\frac{b^{10}}{x^{12}}\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(46189a^5+255255a^4bx^2+570570a^3b^2x^4+646646a^2b^3x^6+373065ab^4x^8+88179b^5x^{10}\right)}{969969x^{21}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^22,x]
```

```
[Out] -1/969969*(Sqrt[(a + b*x^2)^2]*(46189*a^5 + 255255*a^4*b*x^2 + 570570*a^3*b
^2*x^4 + 646646*a^2*b^3*x^6 + 373065*a*b^4*x^8 + 88179*b^5*x^10))/(x^21*(a
+ b*x^2))
```

fricas [A] time = 1.00, size = 59, normalized size = 0.23

$$-\frac{88179\,b^5x^{10}+373065\,ab^4x^8+646646\,a^2b^3x^6+570570\,a^3b^2x^4+255255\,a^4bx^2+46189\,a^5}{969969\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^22,x, algorithm="fricas")
```

```
[Out] -1/969969*(88179*b^5*x^10 + 373065*a*b^4*x^8 + 646646*a^2*b^3*x^6 + 570570*
a^3*b^2*x^4 + 255255*a^4*b*x^2 + 46189*a^5)/x^21
```

giac [A] time = 0.16, size = 107, normalized size = 0.42

$$-\frac{88179\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+373065\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+646646\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+570570\,a^3b^2x^4\mathrm{sgn}\left(bx\right.}{969969\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^22,x, algorithm="giac")
```

```
[Out] -1/969969*(88179*b^5*x^10*sgn(b*x^2 + a) + 373065*a*b^4*x^8*sgn(b*x^2 + a)
+ 646646*a^2*b^3*x^6*sgn(b*x^2 + a) + 570570*a^3*b^2*x^4*sgn(b*x^2 + a) + 2
55255*a^4*b*x^2*sgn(b*x^2 + a) + 46189*a^5*sgn(b*x^2 + a))/x^21
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(88179 b^{5} x^{10}+373065 a\,b^{4} x^{8}+646646 a^{2} b^{3} x^{6}+570570 a^{3} b^{2} x^{4}+255255 a^{4} b\,x^{2}+46189 a^{5}\right) \left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{969969 \left(b\,x^{2}+a\right)^{5} x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^22,x)
```

```
[Out] -1/969969*(88179*b^5*x^10+373065*a*b^4*x^8+646646*a^2*b^3*x^6+570570*a^3*b^
2*x^4+255255*a^4*b*x^2+46189*a^5)*((b*x^2+a)^2)^(5/2)/x^21/(b*x^2+a)^5
```

maxima [A] time = 1.34, size = 57, normalized size = 0.22

$$-\frac{b^{5}}{11\,x^{11}}-\frac{5\,ab^{4}}{13\,x^{13}}-\frac{2\,a^{2}b^{3}}{3\,x^{15}}-\frac{10\,a^{3}b^{2}}{17\,x^{17}}-\frac{5\,a^{4}b}{19\,x^{19}}-\frac{a^{5}}{21\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^22,x, algorithm="maxima")
```

```
[Out] -1/11*b^5/x^11 - 5/13*a*b^4/x^13 - 2/3*a^2*b^3/x^15 - 10/17*a^3*b^2/x^17 -
5/19*a^4*b/x^19 - 1/21*a^5/x^21
```

mupad [B] time = 4.34, size = 231, normalized size = 0.91

$$-\frac{a^{5}\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{21\,x^{21}\,\left(b\,x^{2}+a\right)}-\frac{b^{5}\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{11\,x^{11}\,\left(b\,x^{2}+a\right)}-\frac{5\,a\,b^{4}\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{13\,x^{13}\,\left(b\,x^{2}+a\right)}-\frac{5\,a^{4}\,b\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x}}{19\,x^{19}\,\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^22,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(21*x^21*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(11*x^11*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(13*x^13*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2
*a*b*x^2)^(1/2))/(19*x^19*(a + b*x^2)) - (2*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*
x^2)^(1/2))/(3*x^15*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^
(1/2))/(17*x^17*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{22}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**22,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**22, x)
```

3.623 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{24}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{23x^{23}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{}}{2}$$

```
[Out] -1/23*a^5*((b*x^2+a)^2)^(1/2)/x^23/(b*x^2+a)-5/21*a^4*b*((b*x^2+a)^2)^(1/2)
/x^21/(b*x^2+a)-10/19*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^19/(b*x^2+a)-10/17*a^2*
b^3*((b*x^2+a)^2)^(1/2)/x^17/(b*x^2+a)-1/3*a*b^4*((b*x^2+a)^2)^(1/2)/x^15/(
b*x^2+a)-1/13*b^5*((b*x^2+a)^2)^(1/2)/x^13/(b*x^2+a)
```

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{23x^{23}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^24,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23*x^23*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*x^21*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(19*x^19*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(3*x^15*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^1
3*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{24}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{24}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{24}}+\frac{5a^4b^6}{x^{22}}+\frac{10a^3b^7}{x^{20}}+\frac{10a^2b^8}{x^{18}}+\frac{5ab^9}{x^{16}}+\frac{b^{10}}{x^{14}}\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{23x^{23}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x}}{19x^{19}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(88179a^5+482885a^4bx^2+1067430a^3b^2x^4+1193010a^2b^3x^6+676039ab^4x^8+156009b^5x^{10}\right)}{2028117x^{23}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^24,x]
```

```
[Out] -1/2028117*(Sqrt[(a + b*x^2)^2]*(88179*a^5 + 482885*a^4*b*x^2 + 1067430*a^3
*b^2*x^4 + 1193010*a^2*b^3*x^6 + 676039*a*b^4*x^8 + 156009*b^5*x^10))/(x^23
*(a + b*x^2))
```

fricas [A] time = 0.86, size = 59, normalized size = 0.23

$$-\frac{156009\,b^5x^{10}+676039\,ab^4x^8+1193010\,a^2b^3x^6+1067430\,a^3b^2x^4+482885\,a^4bx^2+88179\,a^5}{2028117\,x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^24,x, algorithm="fricas")
```

```
[Out] -1/2028117*(156009*b^5*x^10 + 676039*a*b^4*x^8 + 1193010*a^2*b^3*x^6 + 1067
430*a^3*b^2*x^4 + 482885*a^4*b*x^2 + 88179*a^5)/x^23
```

giac [A] time = 0.16, size = 107, normalized size = 0.42

$$-\frac{156009\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+676039\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+1193010\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+1067430\,a^3b^2x^4\mathrm{sgn}}{2028117\,x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^24,x, algorithm="giac")
```

```
[Out] -1/2028117*(156009*b^5*x^10*sgn(b*x^2 + a) + 676039*a*b^4*x^8*sgn(b*x^2 + a
) + 1193010*a^2*b^3*x^6*sgn(b*x^2 + a) + 1067430*a^3*b^2*x^4*sgn(b*x^2 + a)
 + 482885*a^4*b*x^2*sgn(b*x^2 + a) + 88179*a^5*sgn(b*x^2 + a))/x^23
```

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$-\frac{\left(156009 b^{5} x^{10}+676039 a\,b^{4} x^{8}+1193010 a^{2} b^{3} x^{6}+1067430 a^{3} b^{2} x^{4}+482885 a^{4} b\,x^{2}+88179 a^{5}\right) \left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{2028117 \left(b\,x^{2}+a\right)^{5} x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^24,x)
```

```
[Out] -1/2028117*(156009*b^5*x^10+676039*a*b^4*x^8+1193010*a^2*b^3*x^6+1067430*a^
3*b^2*x^4+482885*a^4*b*x^2+88179*a^5)*((b*x^2+a)^2)^(5/2)/x^23/(b*x^2+a)^5
```

maxima [A] time = 1.38, size = 57, normalized size = 0.22

$$-\frac{b^{5}}{13\,x^{13}}-\frac{ab^{4}}{3\,x^{15}}-\frac{10\,a^{2}b^{3}}{17\,x^{17}}-\frac{10\,a^{3}b^{2}}{19\,x^{19}}-\frac{5\,a^{4}b}{21\,x^{21}}-\frac{a^{5}}{23\,x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^24,x, algorithm="maxima")
```

```
[Out] -1/13*b^5/x^13 - 1/3*a*b^4/x^15 - 10/17*a^2*b^3/x^17 - 10/19*a^3*b^2/x^19 -
 5/21*a^4*b/x^21 - 1/23*a^5/x^23
```

mupad [B] time = 4.31, size = 231, normalized size = 0.91

$$-\frac{a^{5}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{23\,x^{23}\,\left(b\,x^{2}+a\right)}-\frac{b^{5}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{13\,x^{13}\,\left(b\,x^{2}+a\right)}-\frac{a\,b^{4}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{3\,x^{15}\,\left(b\,x^{2}+a\right)}-\frac{5\,a^{4}\,b\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{21\,x^{21}\,\left(b\,x^{2}+a\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^24,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(23*x^23*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(13*x^13*(a + b*x^2)) - (a*b^4*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(3*x^15*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a*
b*x^2)^(1/2))/(21*x^21*(a + b*x^2)) - (10*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^
2)^(1/2))/(17*x^17*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(
1/2))/(19*x^19*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{24}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**24,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**24, x)
```

3.624 $\int \frac{x^5}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=127

$$\frac{x^4\left(a+bx^2\right)}{4b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{ax^2\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^2\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -1/2*a*x^2*(b*x^2+a)/b^2/((b*x^2+a)^2)^(1/2)+1/4*x^4*(b*x^2+a)/b/((b*x^2+a)
^2)^(1/2)+1/2*a^2*(b*x^2+a)*ln(b*x^2+a)/b^3/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{x^4\left(a+bx^2\right)}{4b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{ax^2\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^2\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -(a*x^2*(a + b*x^2))/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (x^4*(a + b*
x^2))/(4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^2*(a + b*x^2)*Log[a + b*x^
2])/(2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2}}\,dx,x,x^2\right)\\
&= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{x^2}{ab+b^2x}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \left(-\frac{a}{b^3}+\frac{x}{b^2}+\frac{a^2}{b^3(a+bx)}\right)dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= -\frac{ax^2\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^4\left(a+bx^2\right)}{4b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^2\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.43

$$\frac{\left(a+bx^2\right)\left(2a^2\log\left(a+bx^2\right)+bx^2\left(bx^2-2a\right)\right)}{4b^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(b*x^2*(-2*a + b*x^2) + 2*a^2*Log[a + b*x^2]))/(4*b^3*Sqrt[(a
+ b*x^2)^2])
```

fricas [A] time = 1.00, size = 33, normalized size = 0.26

$$\frac{b^2x^4-2\,abx^2+2\,a^2\log\left(bx^2+a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^4 - 2*a*b*x^2 + 2*a^2*log(b*x^2 + a))/b^3
```

giac [A] time = 0.18, size = 59, normalized size = 0.46

$$\frac{a^2\log\left(\left|bx^2+a\right|\right)\text{sgn}\left(bx^2+a\right)}{2\,b^3}+\frac{bx^4\text{sgn}\left(bx^2+a\right)-2\,ax^2\text{sgn}\left(bx^2+a\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*a^2*log(abs(b*x^2 + a))*sgn(b*x^2 + a)/b^3 + 1/4*(b*x^4*sgn(b*x^2 + a)
- 2*a*x^2*sgn(b*x^2 + a))/b^2
```

maple [A] time = 0.01, size = 52, normalized size = 0.41

$$\frac{\left(b\,x^{2}+a\right)\left(b^{2}x^{4}-2ab\,x^{2}+2a^{2}\ln\left(b\,x^{2}+a\right)\right)}{4\sqrt{\left(b\,x^{2}+a\right)^{2}}\;b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/4*(b*x^2+a)*(b^2*x^4-2*a*b*x^2+2*a^2*ln(b*x^2+a))/((b*x^2+a)^2)^(1/2)/b^3
```

maxima [A] time = 1.39, size = 34, normalized size = 0.27

$$\frac{a^{2}\log\left(bx^{2}+a\right)}{2\,b^{3}}+\frac{bx^{4}-2\,ax^{2}}{4\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*log(b*x^2 + a)/b^3 + 1/4*(b*x^4 - 2*a*x^2)/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5}}{\sqrt{\left(b\,x^{2}+a\right)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(x^5/((a + b*x^2)^2)^(1/2), x)
```

sympy [A] time = 0.20, size = 32, normalized size = 0.25

$$\frac{a^{2}\log\left(a+bx^{2}\right)}{2b^{3}}-\frac{ax^{2}}{2b^{2}}+\frac{x^{4}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/((b*x**2+a)**2)**(1/2),x)

[Out] a**2*log(a + b*x**2)/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b)
```

3.625 $\int \frac{x^3}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=75

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}-\frac{a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -1/2*a*(b*x^2+a)*ln(b*x^2+a)/b^2/((b*x^2+a)^2)^(1/2)+1/2*((b*x^2+a)^2)^(1/2
)/b^2
```

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1111, 640, 608, 31}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}-\frac{a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(2*b^2) - (a*(a + b*x^2)*Log[a + b*x^2])/(2
*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, x^2\right)\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} - \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, x^2\right)}{2b}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} - \frac{\left(a\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx, x, x^2\right)}{2b\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} - \frac{a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 0.59

$$\frac{\left(a+bx^2\right)\left(bx^2-a\log\left(a+bx^2\right)\right)}{2b^2\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] ((a + b*x^2)*(b*x^2 - a*Log[a + b*x^2]))/(2*b^2*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.73, size = 22, normalized size = 0.29

$$\frac{bx^2-a\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(b*x^2 - a*log(b*x^2 + a))/b^2
```

giac [A] time = 0.17, size = 33, normalized size = 0.44

$$\frac{1}{2}\left(\frac{x^2}{b} - \frac{a\log\left(\left|bx^2 + a\right|\right)}{b^2}\right)\operatorname{sgn}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*(x^2/b - a*log(abs(b*x^2 + a))/b^2)*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 41, normalized size = 0.55

$$-\frac{\left(b\,x^2 + a\right)\left(-b\,x^2 + a\ln\left(b\,x^2 + a\right)\right)}{2\sqrt{\left(b\,x^2 + a\right)^2}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x^2+a)^2)^(1/2),x)
[Out] -1/2*(b*x^2+a)*(-b*x^2+a*ln(b*x^2+a))/((b*x^2+a)^2)^(1/2)/b^2
```

maxima [A] time = 1.31, size = 23, normalized size = 0.31

$$\frac{x^2}{2\,b} - \frac{a\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*x^2/b - 1/2*a*log(b*x^2 + a)/b^2
```

mupad [B] time = 4.52, size = 64, normalized size = 0.85

$$\frac{\sqrt{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}}{2\,b^2} - \frac{a\,b\,\ln\left(a\,b + \sqrt{\left(b\,x^2 + a\right)^2}\,\sqrt{b^2} + b^2\,x^2\right)}{2\left(b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)^2)^(1/2),x)
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/(2*b^2) - (a*b*log(a*b + ((a + b*x^2)^2)^
(1/2)*(b^2)^(1/2) + b^2*x^2))/(2*(b^2)^(3/2))
```

sympy [A] time = 0.18, size = 20, normalized size = 0.27

$$-\frac{a\log\left(a+bx^2\right)}{2b^2}+\frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -a*log(a + b*x**2)/(2*b**2) + x**2/(2*b)
```

3.626 $\int \frac{x}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=44

$$\frac{(a+bx^2)\log(a+bx^2)}{2b\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/2*(b*x^2+a)*ln(b*x^2+a)/b/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1107, 608, 31}

$$\frac{(a+bx^2)\log(a+bx^2)}{2b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*Log[a + b*x^2])/(2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx,x,x^2\right)$$
$$= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.80

$$\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] ((a + b*x^2)*Log[a + b*x^2])/(2*b*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.76, size = 13, normalized size = 0.30

$$\frac{\log\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*log(b*x^2 + a)/b
```

giac [A] time = 0.15, size = 22, normalized size = 0.50

$$\frac{\log\left(\left|bx^2+a\right|\right)\text{sgn}\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*log(abs(b*x^2 + a))*sgn(b*x^2 + a)/b
```

maple [A] time = 0.00, size = 32, normalized size = 0.73

$$\frac{\left(b\,x^{2}+a\right)\ln\left(b\,x^{2}+a\right)}{2\sqrt{\left(b\,x^{2}+a\right)^{2}}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x^2+a)^2)^(1/2),x)
[Out] 1/2*(b*x^2+a)*ln(b*x^2+a)/b/((b*x^2+a)^2)^(1/2)
```

maxima [A] time = 1.35, size = 13, normalized size = 0.30

$$\frac{\log\left(bx^{2}+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*log(b*x^2 + a)/b
```

mupad [B] time = 4.42, size = 33, normalized size = 0.75

$$\frac{\ln\left(b^{2}\,x^{2}+a\,b\right)\,\mathrm{sign}\left(2\,b^{2}\,x^{2}+2\,a\,b\right)}{2\,\sqrt{b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)^2)^(1/2),x)
[Out] (log(a*b + b^2*x^2)*sign(2*a*b + 2*b^2*x^2))/(2*(b^2)^(1/2))
```

sympy [A] time = 0.15, size = 10, normalized size = 0.23

$$\frac{\log\left(a+bx^{2}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x**2+a)**2)**(1/2),x)
[Out] log(a + b*x**2)/(2*b)
```

3.627 $\int \frac{1}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=80

$$\frac{\log(x)\left(a+bx^2\right)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (b*x^2+a)*ln(x)/a/((b*x^2+a)^2)^(1/2)-1/2*(b*x^2+a)*ln(b*x^2+a)/a/((b*x^2+a
)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1112, 266, 36, 29, 31}

$$\frac{\log(x)\left(a+bx^2\right)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((a + b*x^2)*Log
[a + b*x^2])/(2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{\left(ab+b^2x^2\right)\int \frac{1}{x\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}} \\
&= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}} \\
&= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx,x,x^2\right)}{2a\sqrt{a^2+2abx^2+b^2x^4}} \\
&= \frac{\left(a+bx^2\right)\log(x)}{a\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.52

$$\frac{\left(a+bx^2\right)\left(2\log(x)-\log\left(a+bx^2\right)\right)}{2a\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*(2*Log[x] - Log[a + b*x^2]))/(2*a*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.94, size = 18, normalized size = 0.22

$$-\frac{\log\left(bx^2+a\right)-2\,\log(x)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(log(b*x^2 + a) - 2*log(x))/a
```

giac [A] time = 0.15, size = 33, normalized size = 0.41

$$\frac{1}{2}\left(\frac{\log\left(x^2\right)}{a}-\frac{\log\left(\left|bx^2+a\right|\right)}{a}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*(log(x^2)/a - log(abs(b*x^2 + a))/a)*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 39, normalized size = 0.49

$$\frac{\left(b\,x^2+a\right)\left(2\ln(x)-\ln\left(b\,x^2+a\right)\right)}{2\sqrt{\left(b\,x^2+a\right)^2}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x^2+a)^2)^(1/2),x)
[Out] 1/2*(b*x^2+a)*(2*ln(x)-ln(b*x^2+a))/((b*x^2+a)^2)^(1/2)/a
```

maxima [A] time = 1.28, size = 23, normalized size = 0.29

$$-\frac{\log\left(bx^2+a\right)}{2\,a}+\frac{\log\left(x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*log(b*x^2 + a)/a + 1/2*log(x^2)/a
```

mupad [B] time = 4.45, size = 40, normalized size = 0.50

$$-\frac{\ln\left(\sqrt{\left(b\,x^2+a\right)^2}\;\sqrt{a^2}\;+a^2+a\,b\,x^2\right)+\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] -(log(((a + b*x^2)^2)^(1/2)*(a^2)^(1/2) + a^2 + a*b*x^2) + log(1/x^2))/(2*(
a^2)^(1/2))
```

sympy [A] time = 0.26, size = 15, normalized size = 0.19

$$\frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b} + x^2\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] log(x)/a - log(a/b + x**2)/(2*a)
```

3.628 $\int \frac{1}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=125

$$\frac{-a-bx^2}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b\log(x)\left(a+bx^2\right)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/2*(-b*x^2-a)/a/x^2/((b*x^2+a)^2)^(1/2)-b*(b*x^2+a)*ln(x)/a^2/((b*x^2+a)^2
)^(1/2)+1/2*b*(b*x^2+a)*ln(b*x^2+a)/a^2/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 122, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 44}

$$-\frac{a+bx^2}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b\log(x)\left(a+bx^2\right)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -(a + b*x^2)/(2*a*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*(a + b*x^2)*Log
[x])/(a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b*(a + b*x^2)*Log[a + b*x^2])
/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{1}{x^3\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \left(\frac{1}{abx^2}-\frac{1}{a^2x}+\frac{b}{a^2(a+bx)}\right)dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= -\frac{a+bx^2}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b\left(a+bx^2\right)\log(x)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b\left(a+bx^2\right)\log\left(a+bx^2\right.}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.43

$$-\frac{\left(a+bx^2\right)\left(-bx^2\log\left(a+bx^2\right)+a+2bx^2\log(x)\right)}{2a^2x^2\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -1/2*((a + b*x^2)*(a + 2*b*x^2*Log[x] - b*x^2*Log[a + b*x^2]))/(a^2*x^2*Sqr
t[(a + b*x^2)^2])
```

fricas [A] time = 0.98, size = 33, normalized size = 0.26

$$\frac{bx^2\log\left(bx^2+a\right)-2\,bx^2\log(x)-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*log(b*x^2 + a) - 2*b*x^2*log(x) - a)/(a^2*x^2)
```

giac [A] time = 0.16, size = 52, normalized size = 0.42

$$-\frac{1}{2}\left(\frac{b\log\left(x^2\right)}{a^2}-\frac{b\log\left(\left|bx^2+a\right|\right)}{a^2}-\frac{bx^2-a}{a^2x^2}\right)\text{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(b*log(x^2)/a^2 - b*log(abs(b*x^2 + a))/a^2 - (b*x^2 - a)/(a^2*x^2))*s
gn(b*x^2 + a)
```

maple [A] time = 0.01, size = 51, normalized size = 0.41

$$-\frac{\left(b\,x^{2}+a\right)\left(2b\,x^{2}\ln(x)-b\,x^{2}\ln\left(b\,x^{2}+a\right)+a\right)}{2\sqrt{\left(b\,x^{2}+a\right)^{2}}\;a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/2*(b*x^2+a)*(2*b*x^2*ln(x)-b*ln(b*x^2+a)*x^2+a)/((b*x^2+a)^2)^(1/2)/x^2/
a^2
```

maxima [A] time = 1.29, size = 33, normalized size = 0.26

$$\frac{b\log\left(bx^{2}+a\right)}{2\,a^{2}}-\frac{b\log\left(x^{2}\right)}{2\,a^{2}}-\frac{1}{2\,ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 + a)/a^2 - 1/2*b*log(x^2)/a^2 - 1/2/(a*x^2)
```

mupad [B] time = 4.45, size = 75, normalized size = 0.60

$$\frac{a\,b\,\mathrm{atanh}\left(\frac{a^{2}+b\,a\,x^{2}}{\sqrt{a^{2}}\,\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}\right)}{2\left(a^{2}\right)^{3/2}}-\frac{\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{2\,a^{2}\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] (a*b*atanh((a^2 + a*b*x^2)/((a^2)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)))
)/(2*(a^2)^(3/2)) - (a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/(2*a^2*x^2)
```

sympy [A] time = 0.32, size = 31, normalized size = 0.25

$$-\frac{1}{2ax^{2}}-\frac{b\log\left(x\right)}{a^{2}}+\frac{b\log\left(\frac{a}{b}+x^{2}\right)}{2a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/((b*x**2+a)**2)**(1/2),x)
[Out] -1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2)
```

3.629 $\int \frac{x^4}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=129

$$-\frac{ax(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^3(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -a*x*(b*x^2+a)/b^2/((b*x^2+a)^2)^(1/2)+1/3*x^3*(b*x^2+a)/b/((b*x^2+a)^2)^(1
/2)+a^(3/2)*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/b^(5/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 302, 205}

$$\frac{x^3(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{ax(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -((a*x*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) + (x^3*(a + b*x^
2))/(3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(3/2)*(a + b*x^2)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{x^4}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(ab+b^2x^2\right)\int \left(-\frac{a}{b^3}+\frac{x^2}{b^2}+\frac{a^2}{b^2\left(ab+b^2x^2\right)}\right)dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= -\frac{ax\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a^2\left(ab+b^2x^2\right)\right)\int \frac{1}{ab+b^2x^2}\,dx}{b^2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= -\frac{ax\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.51

$$\frac{\left(a+bx^2\right)\left(3a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+\sqrt{b}\,x\left(bx^2-3a\right)\right)}{3b^{5/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(Sqrt[b]*x*(-3*a + b*x^2) + 3*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]]))/(3*b^(5/2)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.71, size = 99, normalized size = 0.77

$$\left[\frac{2\,bx^3+3\,a\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-6\,ax}{6\,b^2},\ \frac{bx^3+3\,a\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-3\,ax}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b*x^3 + 3*a*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 +
a)) - 6*a*x)/b^2, 1/3*(b*x^3 + 3*a*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 3*a*
x)/b^2]
```

giac [A] time = 0.16, size = 64, normalized size = 0.50

$$\frac{a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right) \operatorname{sgn}\left(bx^2+a\right)}{\sqrt{ab}\, b^2} + \frac{b^2 x^3 \operatorname{sgn}\left(bx^2+a\right) - 3\, abx \operatorname{sgn}\left(bx^2+a\right)}{3\, b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))*sgn(b*x^2 + a)/(sqrt(a*b)*b^2) + 1/3*(b^2*x^3*sgn
(b*x^2 + a) - 3*a*b*x*sgn(b*x^2 + a))/b^3
```

maple [A] time = 0.01, size = 63, normalized size = 0.49

$$\frac{\left(b\,x^2+a\right)\left(\sqrt{ab}\;b\,x^3+3a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)-3\sqrt{ab}\;ax\right)}{3\sqrt{\left(b\,x^2+a\right)^2}\;\sqrt{ab}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)*((a*b)^(1/2)*x^3*b-3*(a*b)^(1/2)*x*a+3*a^2*arctan(1/(a*b)^(1/
2)*b*x))/((b*x^2+a)^2)^(1/2)/b^2/(a*b)^(1/2)
```

maxima [A] time = 2.93, size = 37, normalized size = 0.29

$$\frac{a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, b^2} + \frac{bx^3 - 3\, ax}{3\, b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b*x^3 - 3*a*x)/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{\left(b\,x^2+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(x^4/((a + b*x^2)^2)^(1/2), x)
```

sympy [A] time = 0.21, size = 80, normalized size = 0.62

$$-\frac{ax}{b^2}-\frac{\sqrt{-\frac{a^3}{b^5}}\log\left(x-\frac{b^2\sqrt{-\frac{a^3}{b^5}}}{a}\right)}{2}+\frac{\sqrt{-\frac{a^3}{b^5}}\log\left(x+\frac{b^2\sqrt{-\frac{a^3}{b^5}}}{a}\right)}{2}+\frac{x^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -a*x/b**2 - sqrt(-a**3/b**5)*log(x - b**2*sqrt(-a**3/b**5)/a)/2 + sqrt(-a**
3/b**5)*log(x + b**2*sqrt(-a**3/b**5)/a)/2 + x**3/(3*b)
```

3.630 $\int \frac{x^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=89

$$\frac{x(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{a}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] x*(b*x^2+a)/b/((b*x^2+a)^2)^(1/2)-(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/
2)/b^(3/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 321, 205}

$$\frac{x(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{a}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (x*(a + b*x^2))/(b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (Sqrt[a]*(a + b*x^2)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{\left(ab+b^2x^2\right)\int \frac{x^2}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{x\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a\left(ab+b^2x^2\right)\right)\int \frac{1}{ab+b^2x^2}\,dx}{b\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{x\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{a}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 0.61

$$\frac{\left(a+bx^2\right)\left(\sqrt{b}\,x-\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right)}{b^{3/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(Sqrt[b]*x - Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(b^(3/2)*Sq
rt[(a + b*x^2)^2])
```

fricas [A] time = 0.84, size = 82, normalized size = 0.92

$$\left[\frac{\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\,x}{2\,b},\,-\frac{\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-x}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*x)/b,
-(sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - x)/b]
```

giac [A] time = 0.23, size = 42, normalized size = 0.47

$$-\frac{a \arctan\left(\frac{bx}{\sqrt{ab}}\right)\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{ab}\,b}+\frac{x\mathrm{sgn}\left(bx^2+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))*sgn(b*x^2 + a)/(sqrt(a*b)*b) + x*sgn(b*x^2 + a)/b
```

maple [A] time = 0.01, size = 48, normalized size = 0.54

$$\frac{\left(b\,x^2+a\right)\left(-a \arctan\left(\frac{bx}{\sqrt{ab}}\right)+\sqrt{ab}\;x\right)}{\sqrt{\left(b\,x^2+a\right)^2}\;\sqrt{ab}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] (b*x^2+a)*(x*(a*b)^(1/2)-a*arctan(1/(a*b)^(1/2)*b*x))/((b*x^2+a)^2)^(1/2)/b
/(a*b)^(1/2)
```

maxima [A] time = 2.91, size = 26, normalized size = 0.29

$$-\frac{a \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{\left(b\,x^2+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(x^2/((a + b*x^2)^2)^(1/2), x)
```

sympy [A] time = 0.19, size = 56, normalized size = 0.63

$$\frac{\sqrt{-\frac{a}{b^3}}\log\left(-b\sqrt{-\frac{a}{b^3}}+x\right)}{2}-\frac{\sqrt{-\frac{a}{b^3}}\log\left(b\sqrt{-\frac{a}{b^3}}+x\right)}{2}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x)/2 - sqrt(-a/b**3)*log(b*sqrt(-a/b**
3) + x)/2 + x/b
```

3.631 $\int \frac{1}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=53

$$\frac{(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1088, 205}

$$\frac{(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1088

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{\left(2ab+2b^2x^2\right)\int \frac{1}{2ab+2b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 0.83

$$\frac{(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] ((a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[(a + b*x^2)
^2])
```

fricas [A] time = 0.84, size = 67, normalized size = 1.26

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

giac [A] time = 0.18, size = 23, normalized size = 0.43

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)\operatorname{sgn}\left(bx^2+a\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] arctan(b*x/sqrt(a*b))*sgn(b*x^2 + a)/sqrt(a*b)
```

maple [A] time = 0.00, size = 34, normalized size = 0.64

$$\frac{(b\,x^2+a)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{(b\,x^2+a)^2}\,\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/((b*x^2+a)^2)^(1/2)*(b*x^2+a)/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.03, size = 15, normalized size = 0.28

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\left(b\,x^2+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(1/((a + b*x^2)^2)^(1/2), x)
```

sympy [A] time = 0.18, size = 53, normalized size = 1.00

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1
/(a*b)) + x)/2
```

3.632 $\int \frac{1}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=92

$$-\frac{a+bx^2}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{b}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-b*x^2-a)/a/x/((b*x^2+a)^2)^(1/2)-(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))*b^(1
/2)/a^(3/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 325, 205}

$$-\frac{a+bx^2}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{b}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -((a + b*x^2)/(a*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) - (Sqrt[b]*(a + b*x^2)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= -\frac{a+bx^2}{ax\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b\left(ab+b^2x^2\right)\right)\int \frac{1}{ab+b^2x^2}\,dx}{a\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= -\frac{a+bx^2}{ax\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt{b}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 0.61

$$-\frac{\left(a+bx^2\right)\left(\sqrt{b}\,x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+\sqrt{a}\right)}{a^{3/2}x\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -(((a + b*x^2)*(Sqrt[a] + Sqrt[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(a^(3/2)*
x*Sqrt[(a + b*x^2)^2]))
```

fricas [A] time = 0.92, size = 82, normalized size = 0.89

$$\left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2}{2\,ax},\ -\frac{x\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+1}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2)/(a*
x), -(x*sqrt(b/a)*arctan(x*sqrt(b/a)) + 1)/(a*x)]
```

giac [A] time = 0.15, size = 37, normalized size = 0.40

$$-\left(\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{1}{ax}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/(a*x))*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 50, normalized size = 0.54

$$-\frac{\left(b\,x^{2}+a\right)\left(bx\arctan\left(\frac{bx}{\sqrt{ab}}\right)+\sqrt{ab}\right)}{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\sqrt{ab}\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -(b*x^2+a)*(b*arctan(1/(a*b)^(1/2)*b*x)*x+(a*b)^(1/2))/((b*x^2+a)^2)^(1/2)/
a/x/(a*b)^(1/2)
```

maxima [A] time = 2.94, size = 29, normalized size = 0.32

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{2}\,\sqrt{\left(b\,x^{2}+a\right)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] int(1/(x^2*((a + b*x^2)^2)^(1/2)), x)
```

sympy [A] time = 0.23, size = 65, normalized size = 0.71

$$\frac{\sqrt{-\frac{b}{a^{3}}}\,\log\left(-\frac{a^{2}\sqrt{-\frac{b}{a^{3}}}}{b}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^{3}}}\,\log\left(\frac{a^{2}\sqrt{-\frac{b}{a^{3}}}}{b}+x\right)}{2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x)/2 - sqrt(-b/a**3)*log(a**2*sqr
t(-b/a**3)/b + x)/2 - 1/(a*x)
```

3.633 $\int \frac{1}{x^4\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=133

$$\frac{b\left(a+bx^2\right)}{a^2x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{-a-bx^2}{3ax^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/3*(-b*x^2-a)/a/x^3/((b*x^2+a)^2)^(1/2)+b*(b*x^2+a)/a^2/x/((b*x^2+a)^2)^(1
/2)+b^(3/2)*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 130, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 325, 205}

$$\frac{b\left(a+bx^2\right)}{a^2x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+bx^2}{3ax^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -(a + b*x^2)/(3*a*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b*(a + b*x^2))/(a
^2*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(3/2)*(a + b*x^2)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b\left(ab+b^2x^2\right)\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)}\,dx}{a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b\left(a+bx^2\right)}{a^2x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{ab+b^2}}{a^2\sqrt{a^2+2abx^2+b^2x}}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b\left(a+bx^2\right)}{a^2x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.}{a^{5/2}\sqrt{a^2+2abx^2+b^2x}}$$

Mathematica [A] time = 0.02, size = 70, normalized size = 0.53

$$-\frac{\left(a+bx^2\right)\left(\sqrt{a}\left(a-3bx^2\right)-3b^{3/2}x^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right)}{3a^{5/2}x^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -1/3*((a + b*x^2)*(Sqrt[a]*(a - 3*b*x^2) - 3*b^(3/2)*x^3*ArcTan[(Sqrt[b]*x)
/Sqrt[a]]))/(a^(5/2)*x^3*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 1.19, size = 106, normalized size = 0.80

$$\left[\frac{3\,bx^3\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+6\,bx^2-2\,a}{6\,a^2x^3},\ \frac{3\,bx^3\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)+3\,bx^2-a}{3\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*b*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) +
6*b*x^2 - 2*a)/(a^2*x^3), 1/3*(3*b*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*b*
x^2 - a)/(a^2*x^3)]
```

giac [A] time = 0.16, size = 50, normalized size = 0.38

$$\frac{1}{3}\left(\frac{3\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{3\,bx^2-a}{a^2x^3}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + (3*b*x^2 - a)/(a^2*x^3))
*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 69, normalized size = 0.52

$$\frac{\left(b\,x^2+a\right)\left(3b^2x^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)+3\sqrt{ab}\;b\,x^2-\sqrt{ab}\;a\right)}{3\sqrt{\left(b\,x^2+a\right)^2}\;\sqrt{ab}\;a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)*(3*b^2*arctan(1/(a*b)^(1/2)*b*x)*x^3+3*b*x^2*(a*b)^(1/2)-a*(a
*b)^(1/2))/((b*x^2+a)^2)^(1/2)/a^2/x^3/(a*b)^(1/2)
```

maxima [A] time = 3.01, size = 40, normalized size = 0.30

$$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{3\,bx^2-a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{x^4\,\sqrt{\left(b\,x^2+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] int(1/(x^4*((a + b*x^2)^2)^(1/2)), x)
```

sympy [A] time = 0.28, size = 87, normalized size = 0.65

$$-\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{-a+3bx^2}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -sqrt(-b**3/a**5)*log(-a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + sqrt(-b**3/a**5)
*log(a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + (-a + 3*b*x**2)/(3*a**2*x**3)
```

3.634 $\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=158

$$-\frac{3a^2}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^2\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^3}{4b^4\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -3/2*a^2/b^4/((b*x^2+a)^2)^(1/2)+1/4*a^3/b^4/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+
1/2*x^2*(b*x^2+a)/b^3/((b*x^2+a)^2)^(1/2)-3/2*a*(b*x^2+a)*ln(b*x^2+a)/b^4/(
(b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{a^3}{4b^4\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3a^2}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^2\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-3*a^2)/(2*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + a^3/(4*b^4*(a + b*x^2)*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (x^2*(a + b*x^2))/(2*b^3*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]) - (3*a*(a + b*x^2)*Log[a + b*x^2])/(2*b^4*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx,x,x^2\right)\\ &= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\operatorname{Subst}\left(\int \frac{x^3}{\left(ab+b^2x\right)^3}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\operatorname{Subst}\left(\int \left(\frac{1}{b^6}-\frac{a^3}{b^6(a+bx)^3}+\frac{3a^2}{b^6(a+bx)^2}-\frac{3a}{b^6(a+bx)}\right)dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\ &= -\frac{3a^2}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^3}{4b^4\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^2\left(a+b\right.}{2b^3\sqrt{a^2+2abx}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 81, normalized size = 0.51

$$\frac{-5a^3-4a^2bx^2+4ab^2x^4-6a\left(a+bx^2\right)^2\log\left(a+bx^2\right)+2b^3x^6}{4b^4\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-5*a^3 - 4*a^2*b*x^2 + 4*a*b^2*x^4 + 2*b^3*x^6 - 6*a*(a + b*x^2)^2*Log[a +
 b*x^2])/(4*b^4*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 1.17, size = 91, normalized size = 0.58

$$\frac{2\,b^3x^6+4\,ab^2x^4-4\,a^2bx^2-5\,a^3-6\left(ab^2x^4+2\,a^2bx^2+a^3\right)\log\left(bx^2+a\right)}{4\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*x^6 + 4*a*b^2*x^4 - 4*a^2*b*x^2 - 5*a^3 - 6*(a*b^2*x^4 + 2*a^2*b
*x^2 + a^3)*log(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

giac [A] time = 0.25, size = 83, normalized size = 0.53

$$\frac{x^2}{2\,b^3\mathrm{sgn}\left(bx^2+a\right)}-\frac{3\,a\log\left(\left|bx^2+a\right|\right)}{2\,b^4\mathrm{sgn}\left(bx^2+a\right)}-\frac{6\,a^2bx^2+5\,a^3}{4\left(bx^2+a\right)^2b^4\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2/(b^3*sgn(b*x^2 + a)) - 3/2*a*log(abs(b*x^2 + a))/(b^4*sgn(b*x^2 + a
)) - 1/4*(6*a^2*b*x^2 + 5*a^3)/((b*x^2 + a)^2*b^4*sgn(b*x^2 + a))
```

maple [A] time = 0.02, size = 103, normalized size = 0.65

$$-\frac{\left(-2b^3x^6+6a\,b^2x^4\ln\left(b\,x^2+a\right)-4a\,b^2x^4+12a^2b\,x^2\ln\left(b\,x^2+a\right)+4a^2b\,x^2+6a^3\ln\left(b\,x^2+a\right)+5a^3\right)\left(b\,x^2+a\right)}{4\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/4*(-2*b^3*x^6+6*ln(b*x^2+a)*x^4*a*b^2-4*a*b^2*x^4+12*ln(b*x^2+a)*x^2*a^2
*b+4*a^2*b*x^2+6*ln(b*x^2+a)*a^3+5*a^3)*(b*x^2+a)/b^4/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 1.36, size = 66, normalized size = 0.42

$$-\frac{6\,a^2bx^2+5\,a^3}{4\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}+\frac{x^2}{2\,b^3}-\frac{3\,a\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(6*a^2*b*x^2 + 5*a^3)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4) + 1/2*x^2/b^3
- 3/2*a*log(b*x^2 + a)/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**7/((a + b*x**2)**2)**(3/2), x)
```

3.635 $\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{a^2}{4b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a}{b^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] a/b^3/((b*x^2+a)^2)^(1/2)-1/4*a^2/b^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+1/2*(b*
x^2+a)*ln(b*x^2+a)/b^3/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^2}{4b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a}{b^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] a/(b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - a^2/(4*b^3*(a + b*x^2)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]) + ((a + b*x^2)*Log[a + b*x^2])/(2*b^3*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\operatorname{Subst}\left(\int \frac{x^2}{\left(ab+b^2x\right)^3}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\operatorname{Subst}\left(\int \left(\frac{a^2}{b^5(a+bx)^3}-\frac{2a}{b^5(a+bx)^2}+\frac{1}{b^5(a+bx)}\right)dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{a}{b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^2}{4b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log\left(a+\right.}{2b^3\sqrt{a^2+2abx^2+}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.54

$$\frac{a\left(3a+4bx^2\right)+2\left(a+bx^2\right)^2\log\left(a+bx^2\right)}{4b^3\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a*(3*a + 4*b*x^2) + 2*(a + b*x^2)^2*Log[a + b*x^2])/(4*b^3*(a + b*x^2)*Sqr
t[(a + b*x^2)^2])
```

fricas [A] time = 0.96, size = 69, normalized size = 0.61

$$\frac{4\,abx^2+3\,a^2+2\left(b^2x^4+2\,abx^2+a^2\right)\log\left(bx^2+a\right)}{4\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*b*x^2 + 3*a^2 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(b*x^2 + a))/(b^5
*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

giac [A] time = 0.23, size = 64, normalized size = 0.57

$$\frac{\log\left(\left|bx^2+a\right|\right)}{2\,b^3\mathrm{sgn}\left(bx^2+a\right)}+\frac{4\,ax^2+\frac{3\,a^2}{b}}{4\left(bx^2+a\right)^2b^2\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/(b^3*sgn(b*x^2 + a)) + 1/4*(4*a*x^2 + 3*a^2/b)/((b*
x^2 + a)^2*b^2*sgn(b*x^2 + a))
```

maple [A] time = 0.02, size = 81, normalized size = 0.72

$$\frac{\left(2b^2x^4\ln\left(b\,x^2+a\right)+4ab\,x^2\ln\left(b\,x^2+a\right)+4ab\,x^2+2a^2\ln\left(b\,x^2+a\right)+3a^2\right)\left(b\,x^2+a\right)}{4\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/4*(2*ln(b*x^2+a)*x^4*b^2+4*ln(b*x^2+a)*x^2*a*b+4*a*b*x^2+2*a^2*ln(b*x^2+a
)+3*a^2)*(b*x^2+a)/b^3/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 1.37, size = 55, normalized size = 0.49

$$\frac{4\,abx^2+3\,a^2}{4\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}+\frac{\log\left(bx^2+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a*b*x^2 + 3*a^2)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3) + 1/2*log(b*x^2 +
 a)/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)**2)**(3/2), x)
```

3.636 $\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=41

$$\frac{x^4}{4a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/4*x^4/a/(b*x^2+a)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.68, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 640, 607}

$$\frac{a}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -1/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + a/(4*b^2*(a + b*x^2)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx, x, x^2\right)$$
$$= -\frac{1}{2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a\,\text{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx, x, x^2\right)}{2b}$$
$$= -\frac{1}{2b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{a}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.95

$$\frac{-a-2bx^2}{4b^2\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
[Out] (-a - 2*b*x^2)/(4*b^2*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 1.16, size = 36, normalized size = 0.88

$$-\frac{2\,bx^2+a}{4\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
[Out] -1/4*(2*b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

giac [A] time = 0.23, size = 32, normalized size = 0.78

$$-\frac{2\,bx^2+a}{4\left(bx^2+a\right)^2b^2\text{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
[Out] -1/4*(2*b*x^2 + a)/((b*x^2 + a)^2*b^2*sgn(b*x^2 + a))
```

maple [A] time = 0.01, size = 32, normalized size = 0.78

$$-\frac{\left(b\,x^{2}+a\right)\left(2b\,x^{2}+a\right)}{4\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
[Out] -1/4*(b*x^2+a)*(2*b*x^2+a)/b^2/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 1.38, size = 36, normalized size = 0.88

$$-\frac{2\,bx^{2}+a}{4\left(b^{4}x^{4}+2\,ab^{3}x^{2}+a^{2}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
[Out] -1/4*(2*b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

mupad [B] time = 4.24, size = 42, normalized size = 1.02

$$-\frac{\left(2\,b\,x^{2}+a\right)\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{4\,b^{2}\left(b\,x^{2}+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
[Out] -((a + 2*b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(4*b^2*(a + b*x^2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{3}}{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
[Out] Integral(x**3/((a + b*x**2)**2)**(3/2), x)
```

3.637 $\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -1/4/b/(b*x^2+a)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1107, 607}

$$-\frac{1}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -1/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx,x,x^2\right)\\ &= -\frac{1}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.71

$$-\frac{a+bx^2}{4b\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
[Out] -1/4*(a + b*x^2)/(b*((a + b*x^2)^2)^(3/2))
```

fricas [A] time = 0.90, size = 26, normalized size = 0.68

$$-\frac{1}{4\left(b^3x^4+2\,ab^2x^2+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
[Out] -1/4/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

giac [A] time = 0.20, size = 24, normalized size = 0.63

$$-\frac{1}{4\left(bx^2+a\right)^2 b\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
[Out] -1/4/((b*x^2 + a)^2*b*sgn(b*x^2 + a))
```

maple [A] time = 0.00, size = 24, normalized size = 0.63

$$-\frac{b\,x^2+a}{4\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
[Out] -1/4*(b*x^2+a)/b/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 1.32, size = 26, normalized size = 0.68

$$-\frac{1}{4\left(b^3x^4+2\,ab^2x^2+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

mupad [B] time = 4.34, size = 34, normalized size = 0.89

$$-\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,b\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] -(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/(4*b*(a + b*x^2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x/((a + b*x**2)**2)**(3/2), x)
```

3.638 $\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=147

$$\frac{1}{4a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\log(x)\left(a+bx^2\right)}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/2/a^2/((b*x^2+a)^2)^(1/2)+1/4/a/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+(b*x^2+a)*l
n(x)/a^3/((b*x^2+a)^2)^(1/2)-1/2*(b*x^2+a)*ln(b*x^2+a)/a^3/((b*x^2+a)^2)^(1
/2)
```

Rubi [A] time = 0.08, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 44}

$$\frac{1}{4a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\log(x)\left(a+bx^2\right)}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 1/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*(a + b*x^2)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) + ((a + b*x^2)*Log[x])/(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) - ((a + b*x^2)*Log[a + b*x^2])/(2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
```

```
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{x\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^3}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\operatorname{Subst}\left(\int \left(\frac{1}{a^3b^3x}-\frac{1}{ab^2(a+bx)^3}-\frac{1}{a^2b^2(a+bx)^2}-\frac{1}{a^3b^2(a+bx)}\right)dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{1}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log}{a^3\sqrt{a^2+2abx^2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 74, normalized size = 0.50

$$\frac{a\left(3a+2bx^2\right)+4\log(x)\left(a+bx^2\right)^2-2\left(a+bx^2\right)^2\log\left(a+bx^2\right)}{4a^3\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (a*(3*a + 2*b*x^2) + 4*(a + b*x^2)^2*Log[x] - 2*(a + b*x^2)^2*Log[a + b*x^2
])/(4*a^3*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.84, size = 90, normalized size = 0.61

$$\frac{2\,abx^2+3\,a^2-2\left(b^2x^4+2\,abx^2+a^2\right)\log\left(bx^2+a\right)+4\left(b^2x^4+2\,abx^2+a^2\right)\log(x)}{4\left(a^3b^2x^4+2\,a^4bx^2+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

[Out] 1/4*(2*a*b*x^2 + 3*a^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(b*x^2 + a) + 4*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(x))/(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)

giac [A] time = 0.27, size = 79, normalized size = 0.54

$$-\frac{\log\left(\left|bx^2+a\right|\right)}{2\,a^3\mathrm{sgn}\left(bx^2+a\right)}+\frac{\log\left(|x|\right)}{a^3\mathrm{sgn}\left(bx^2+a\right)}+\frac{2\,abx^2+3\,a^2}{4\left(bx^2+a\right)^2a^3\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")

[Out] -1/2*log(abs(b*x^2 + a))/(a^3*sgn(b*x^2 + a)) + log(abs(x))/(a^3*sgn(b*x^2 + a)) + 1/4*(2*a*b*x^2 + 3*a^2)/((b*x^2 + a)^2*a^3*sgn(b*x^2 + a))

maple [A] time = 0.02, size = 107, normalized size = 0.73

$$\frac{\left(4b^2x^4\ln(x)-2b^2x^4\ln\left(b\,x^2+a\right)+8ab\,x^2\ln(x)-4ab\,x^2\ln\left(b\,x^2+a\right)+2ab\,x^2+4a^2\ln(x)-2a^2\ln\left(b\,x^2+a\right)+\right.}{4\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)

[Out] 1/4*(4*ln(x)*x^4*b^2-2*b^2*x^4*ln(b*x^2+a)+8*ln(x)*x^2*a*b-4*a*b*x^2*ln(b*x^2+a)+2*a*b*x^2+4*a^2*ln(x)-2*a^2*ln(b*x^2+a)+3*a^2)*(b*x^2+a)/a^3/((b*x^2+a)^2)^(3/2)

maxima [A] time = 1.44, size = 57, normalized size = 0.39

$$\frac{2\,bx^2+3\,a}{4\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)}-\frac{\log\left(bx^2+a\right)}{2\,a^3}+\frac{\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")

[Out] 1/4*(2*b*x^2 + 3*a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) - 1/2*log(b*x^2 + a)/a^3 + log(x)/a^3

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**2)**2)**(3/2)), x)
```

3.639 $\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=189

$$-\frac{b}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3b\log(x)\left(a+bx^2\right)}{a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3b\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{a^3\sqrt{a^2+2abx^2+b^2x}}$$

```
[Out] -b/a^3/((b*x^2+a)^2)^(1/2)-1/4*b/a^2/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+1/2*(-b*
x^2-a)/a^3/x^2/((b*x^2+a)^2)^(1/2)-3*b*(b*x^2+a)*ln(x)/a^4/((b*x^2+a)^2)^(1
/2)+3/2*b*(b*x^2+a)*ln(b*x^2+a)/a^4/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 44}

$$-\frac{b}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+bx^2}{2a^3x^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3b\log(x)\left(a+bx^2\right)}{a^4\sqrt{a^2+2abx^2+b^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] -(b/(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) - b/(4*a^2*(a + b*x^2)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4]) - (a + b*x^2)/(2*a^3*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) - (3*b*(a + b*x^2)*Log[x])/(a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (
3*b*(a + b*x^2)*Log[a + b*x^2])/(2*a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
```

```
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{x^3\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)^3}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \left(\frac{1}{a^3b^3x^2}-\frac{3}{a^4b^2x}+\frac{1}{a^2b(a+bx)^3}+\frac{2}{a^3b(a+bx)^2}+\frac{3}{a^4b(a+bx)}\right)\,dx\right.}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{b}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+b}{2a^3x^2\sqrt{a^2+2}}$$

Mathematica [A] time = 0.04, size = 97, normalized size = 0.51

$$\frac{-a\left(2a^2+9abx^2+6b^2x^4\right)-12bx^2\log(x)\left(a+bx^2\right)^2+6bx^2\left(a+bx^2\right)^2\log\left(a+bx^2\right)}{4a^4x^2\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (-(a*(2*a^2 + 9*a*b*x^2 + 6*b^2*x^4)) - 12*b*x^2*(a + b*x^2)^2*Log[x] + 6*b
*x^2*(a + b*x^2)^2*Log[a + b*x^2])/(4*a^4*x^2*(a + b*x^2)*Sqrt[(a + b*x^2)^
2])
```

fricas [A] time = 1.00, size = 119, normalized size = 0.63

$$-\frac{6\,ab^2x^4+9\,a^2bx^2+2\,a^3-6\left(b^3x^6+2\,ab^2x^4+a^2bx^2\right)\log\left(bx^2+a\right)+12\left(b^3x^6+2\,ab^2x^4+a^2bx^2\right)\log(x)}{4\left(a^4b^2x^6+2\,a^5bx^4+a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(6*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3 - 6*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*
x^2)*log(b*x^2 + a) + 12*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*log(x))/(a^4*b
^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)
```

giac [A] time = 0.24, size = 96, normalized size = 0.51

$$\frac{3\,b\log\left(\left|bx^2+a\right|\right)}{2\,a^4\mathrm{sgn}\left(bx^2+a\right)}-\frac{3\,b\log\left(|x|\right)}{a^4\mathrm{sgn}\left(bx^2+a\right)}-\frac{6\,ab^2x^4+9\,a^2bx^2+2\,a^3}{4\left(bx^2+a\right)^2a^4x^2\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3/2*b*log(abs(b*x^2 + a))/(a^4*sgn(b*x^2 + a)) - 3*b*log(abs(x))/(a^4*sgn(b
*x^2 + a)) - 1/4*(6*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3)/((b*x^2 + a)^2*a^4*x^2
*sgn(b*x^2 + a))
```

maple [A] time = 0.02, size = 133, normalized size = 0.70

$$-\frac{\left(12b^3x^6\ln(x)-6b^3x^6\ln\left(b\,x^2+a\right)+24a\,b^2x^4\ln(x)-12a\,b^2x^4\ln\left(b\,x^2+a\right)+6a\,b^2x^4+12a^2b\,x^2\ln(x)-6a^2b}{4\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/4*(12*b^3*x^6*ln(x)-6*ln(b*x^2+a)*x^6*b^3+24*a*b^2*x^4*ln(x)-12*a*b^2*x^
4*ln(b*x^2+a)+6*a*b^2*x^4+12*a^2*b*x^2*ln(x)-6*a^2*b*x^2*ln(b*x^2+a)+9*a^2*
b*x^2+2*a^3)*(b*x^2+a)/a^4/x^2/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 1.37, size = 75, normalized size = 0.40

$$-\frac{6\,b^2x^4+9\,abx^2+2\,a^2}{4\left(a^3b^2x^6+2\,a^4bx^4+a^5x^2\right)}+\frac{3\,b\log\left(bx^2+a\right)}{2\,a^4}-\frac{3\,b\log(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(6*b^2*x^4 + 9*a*b*x^2 + 2*a^2)/(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)
+ 3/2*b*log(b*x^2 + a)/a^4 - 3*b*log(x)/a^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x**2)**2)**(3/2)), x)
```

3.640 $\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{3x}{8b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x^3}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -3/8*x/b^2/((b*x^2+a)^2)^(1/2)-1/4*x^3/b/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+3/8*
(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/b^(5/2)/a^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 288, 205}

$$-\frac{x^3}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3x}{8b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-3*x)/(8*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x^3/(4*b*(a + b*x^2)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
8*Sqrt[a]*b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
```

```
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{x^4}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x^3}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3\left(ab+b^2x^2\right)\right)\int \frac{x^2}{\left(ab+b^2x^2\right)^2}\,dx}{4\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{3x}{8b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^3}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3\left(ab+b^2x^2\right)\right)\ldots}{8b^2\sqrt{a^2+2ab\ldots}}$$

$$= -\frac{3x}{8b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^3}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{3\left(a+bx^2\right)\tan\ldots}{8\sqrt{a}\,b^{5/2}\sqrt{a^2+2\ldots}}$$

Mathematica [A] time = 0.03, size = 84, normalized size = 0.66

$$\frac{3\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\sqrt{a}\,\sqrt{b}\,x\left(3a+5bx^2\right)}{8\sqrt{a}\,b^{5/2}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-(Sqrt[a]*Sqrt[b]*x*(3*a + 5*b*x^2)) + 3*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(8*Sqrt[a]*b^(5/2)*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 1.01, size = 188, normalized size = 1.47

$$\left[-\frac{10\,ab^2x^3+6\,a^2bx+3\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{16\left(ab^5x^4+2\,a^2b^4x^2+a^3b^3\right)},-\frac{5\,ab^2x^3+3\,a^2bx-3\left(b^2x^4+2\,abx^2+a\ldots\right.}{8\left(ab^5x^4+2\,a^2b^4x^2+a\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(10*a*b^2*x^3 + 6*a^2*b*x + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)
*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a*b^5*x^4 + 2*a^2*b^4*x^2
+ a^3*b^3), -1/8*(5*a*b^2*x^3 + 3*a^2*b*x - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*s
qrt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 97, normalized size = 0.76

$$-\frac{\left(-3 b^{2} x^{4} \arctan \left(\frac{b x}{\sqrt{a b}}\right)-6 a b\, x^{2} \arctan \left(\frac{b x}{\sqrt{a b}}\right)+5 \sqrt{a b}\, b\, x^{3}-3 a^{2} \arctan \left(\frac{b x}{\sqrt{a b}}\right)+3 \sqrt{a b}\, a x\right) \left(b\, x^{2}+a\right)}{8 \sqrt{a b}\, \left(\left(b\, x^{2}+a\right)^{2}\right)^{\frac{3}{2}} b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/8*(-3*arctan(1/(a*b)^(1/2)*b*x)*x^4*b^2+5*(a*b)^(1/2)*b*x^3-6*arctan(1/(
a*b)^(1/2)*b*x)*x^2*a*b+3*(a*b)^(1/2)*a*x-3*a^2*arctan(1/(a*b)^(1/2)*b*x))*
(b*x^2+a)/(a*b)^(1/2)/b^2/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 3.03, size = 59, normalized size = 0.46

$$-\frac{5 \, b x^{3} + 3 \, a x}{8 \, {\left(b^{4} x^{4} + 2 \, a b^{3} x^{2} + a^{2} b^{2}\right)}} + \frac{3 \, \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{8 \, \sqrt{a b} b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(5*b*x^3 + 3*a*x)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2) + 3/8*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)**2)**(3/2), x)
```

3.641 $\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=129

$$\frac{x}{8ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/8*x/a/b/((b*x^2+a)^2)^(1/2)-1/4*x/b/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+1/8*(b*
x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 288, 199, 205}

$$\frac{x}{8ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] x/(8*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x/(4*b*(a + b*x^2)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) + ((a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)
*b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{x^2}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(ab+b^2x^2\right)\int \frac{1}{\left(ab+b^2x^2\right)^2}\,dx}{4\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{x}{8ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(ab+b^2x^2\right)\int \frac{1}{ab+}}{8ab\sqrt{a^2+2abx^2}}$$

$$= \frac{x}{8ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(a+bx^2\right)\tan^{-}}{8a^{3/2}b^{3/2}\sqrt{a^2+2a}}$$

Mathematica [A] time = 0.03, size = 81, normalized size = 0.63

$$\frac{\sqrt{a}\,\sqrt{b}\,x\left(bx^2-a\right)+\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-a + b*x^2) + (a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(8*a^(3/2)*b^(3/2)*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.94, size = 190, normalized size = 1.47

$$\left[\frac{2\,ab^2x^3 - 2\,a^2bx - \left(b^2x^4 + 2\,abx^2 + a^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2 - 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right)}{16\,\left(a^2b^4x^4 + 2\,a^3b^3x^2 + a^4b^2\right)}, \frac{ab^2x^3 - a^2bx + \left(b^2x^4 + 2\,abx^2 + a^2\right)\sqrt{ab}\,\mathrm{arc}}{8\,\left(a^2b^4x^4 + 2\,a^3b^3x^2 + a^4b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*a*b^2*x^3 - 2*a^2*b*x - (b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)*log
((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 +
a^4*b^2), 1/8*(a*b^2*x^3 - a^2*b*x + (b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(a*b)*
arctan(sqrt(a*b)*x/a))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 97, normalized size = 0.75

$$\frac{\left(b^2x^4\arctan\left(\frac{bx}{\sqrt{ab}}\right) + 2ab\,x^2\arctan\left(\frac{bx}{\sqrt{ab}}\right) + \sqrt{ab}\;b\,x^3 + a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right) - \sqrt{ab}\;ax\right)\left(b\,x^2 + a\right)}{8\sqrt{ab}\;\left(\left(b\,x^2 + a\right)^2\right)^{\frac{3}{2}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/8*(b^2*x^4*arctan(1/(a*b)^(1/2)*b*x)+(a*b)^(1/2)*b*x^3+2*a*b*x^2*arctan(1
/(a*b)^(1/2)*b*x)-(a*b)^(1/2)*a*x+a^2*arctan(1/(a*b)^(1/2)*b*x))*(b*x^2+a)/
(a*b)^(1/2)/b/a/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 2.93, size = 62, normalized size = 0.48

$$\frac{bx^3 - ax}{8\,\left(ab^3x^4 + 2\,a^2b^2x^2 + a^3b\right)} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^3 - a*x)/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b) + 1/8*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)**2)**(3/2), x)
```

3.642 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=135

$$\frac{3x\left(a+bx^2\right)^2}{8a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{x\left(a+bx^2\right)}{4a\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{3\left(a+bx^2\right)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

```
[Out] 1/4*x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)+3/8*x*(b*x^2+a)^2/a^2/(b^2*
x^4+2*a*b*x^2+a^2)^(3/2)+3/8*(b*x^2+a)^3*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/
(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/b^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1088, 199, 205}

$$\frac{3x\left(a+bx^2\right)^2}{8a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{x\left(a+bx^2\right)}{4a\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{3\left(a+bx^2\right)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/2),x]
```

```
[Out] (x*(a + b*x^2))/(4*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + (3*x*(a + b*x^2)^
2)/(8*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + (3*(a + b*x^2)^3*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1088

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(2ab+2b^2x^2\right)^3 \int \frac{1}{\left(2ab+2b^2x^2\right)^3}\,dx}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$
$$= \frac{x\left(a+bx^2\right)}{4a\left(a^2+2abx^2+b^2x^4\right)^{3/2}} + \frac{\left(3\left(2ab+2b^2x^2\right)^3\right)\int \frac{1}{\left(2ab+2b^2x^2\right)^2}\,dx}{8ab\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$
$$= \frac{x\left(a+bx^2\right)}{4a\left(a^2+2abx^2+b^2x^4\right)^{3/2}} + \frac{3x\left(a+bx^2\right)^2}{8a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}} + \frac{\left(3\left(2ab+2b^2x^2\right)^3\right)\int \frac{}{2}}{32a^2b^2\left(a^2+2abx^2+\right.}$$
$$= \frac{x\left(a+bx^2\right)}{4a\left(a^2+2abx^2+b^2x^4\right)^{3/2}} + \frac{3x\left(a+bx^2\right)^2}{8a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}} + \frac{3\left(a+bx^2\right)^3\tan^{-1}\left(\right.}{8a^{5/2}\sqrt{b}\left(a^2+2abx^2+\right.}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.61

$$\frac{\sqrt{a}\sqrt{b}\,x\left(5a+3bx^2\right)+3\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(5*a + 3*b*x^2) + 3*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(8*a^(5/2)*Sqrt[b]*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 1.07, size = 188, normalized size = 1.39

$$\left[\frac{6\,ab^2x^3+10\,a^2bx-3\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{16\left(a^3b^3x^4+2\,a^4b^2x^2+a^5b\right)},\ \frac{3\,ab^2x^3+5\,a^2bx+3\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{}}{8\left(a^3b^3x^4+2\,a^4b^2x^2+a^5b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(6*a*b^2*x^3 + 10*a^2*b*x - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)*
log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2
 + a^5*b), 1/8*(3*a*b^2*x^3 + 5*a^2*b*x + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqr
t(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 97, normalized size = 0.72

$$\frac{\left(3b^2x^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right) + 6ab\,x^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right) + 3\sqrt{ab}\;b\,x^3 + 3a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right) + 5\sqrt{ab}\;ax\right)\left(b\,x^2 + a\right)}{8\sqrt{ab}\;\left(\left(b\,x^2 + a\right)^2\right)^{\frac{3}{2}} a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/8*(3*b^2*x^4*arctan(1/(a*b)^(1/2)*b*x)+3*(a*b)^(1/2)*b*x^3+6*a*b*x^2*arct
an(1/(a*b)^(1/2)*b*x)+5*(a*b)^(1/2)*a*x+3*a^2*arctan(1/(a*b)^(1/2)*b*x))*(b
*x^2+a)/(a*b)^(1/2)/a^2/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 2.99, size = 58, normalized size = 0.43

$$\frac{3\,bx^3 + 5\,ax}{8\left(a^2b^2x^4 + 2\,a^3bx^2 + a^4\right)} + \frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(3*b*x^3 + 5*a*x)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 3/8*arctan(b*x/sq
rt(a*b))/(sqrt(a*b)*a^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2 + 2abx^2 + b^2x^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-3/2), x)
```

3.643 $\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=169

$$\frac{5}{8a^2x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ax\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)}-\frac{15\sqrt{b}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{15\left(a+bx^2\right.}{8a^3x\sqrt{a^2+2abx^2}}$$

```
[Out] 5/8/a^2/x/((b*x^2+a)^2)^(1/2)+1/4/a/x/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-15/8*(b
*x^2+a)/a^3/x/((b*x^2+a)^2)^(1/2)-15/8*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))*
b^(1/2)/a^(7/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 290, 325, 205}

$$-\frac{15\left(a+bx^2\right)}{8a^3x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5}{8a^2x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ax\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)}-\frac{15\sqrt{b}\,(a+bx^2)\tan}{8a^{7/2}\sqrt{a^2+2abx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 5/(8*a^2*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*x*(a + b*x^2)*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4]) - (15*(a + b*x^2))/(8*a^3*x*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (15*Sqrt[b]*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{4ax\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5b\left(ab+b^2x^2\right)\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^2}\,dx}{4a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{5}{8a^2x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ax\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(15\left(ab+b^2\right.\right.}{8a^2\sqrt{a^2}}$$

$$= \frac{5}{8a^2x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ax\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{15\left(a\right.}{8a^3x\sqrt{a^2+2}}$$

$$= \frac{5}{8a^2x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ax\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{15\left(a\right.}{8a^3x\sqrt{a^2+2}}$$

Mathematica [A] time = 0.03, size = 93, normalized size = 0.55

$$\frac{-\sqrt{a}\left(8a^2+25abx^2+15b^2x^4\right)-15\sqrt{b}\,x\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}x\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (-(Sqrt[a]*(8*a^2 + 25*a*b*x^2 + 15*b^2*x^4)) - 15*Sqrt[b]*x*(a + b*x^2)^2*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*x*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.60, size = 202, normalized size = 1.20

$$\left[-\frac{30\,b^2x^4+50\,abx^2-15\left(b^2x^5+2\,abx^3+a^2x\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+16\,a^2}{16\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)},-\frac{15\,b^2x^4+25\,abx^2+15\left(b^2x^5\right.}{8\left(a^3b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(30*b^2*x^4 + 50*a*b*x^2 - 15*(b^2*x^5 + 2*a*b*x^3 + a^2*x)*sqrt(-b/
a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 16*a^2)/(a^3*b^2*x^5 +
 2*a^4*b*x^3 + a^5*x), -1/8*(15*b^2*x^4 + 25*a*b*x^2 + 15*(b^2*x^5 + 2*a*b*
x^3 + a^2*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*
x^3 + a^5*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 119, normalized size = 0.70

$$-\frac{\left(15b^3x^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)+30a\,b^2x^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)+15\sqrt{ab}\;b^2x^4+15a^2bx\arctan\left(\frac{bx}{\sqrt{ab}}\right)+25\sqrt{ab}\;ab\,x^2+8\sqrt{ab}\right.}{8\sqrt{ab}\;\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/8*(15*arctan(1/(a*b)^(1/2)*b*x)*x^5*b^3+15*(a*b)^(1/2)*x^4*b^2+30*arctan
(1/(a*b)^(1/2)*b*x)*x^3*a*b^2+25*(a*b)^(1/2)*x^2*a*b+15*arctan(1/(a*b)^(1/2
)*b*x)*x*a^2*b+8*(a*b)^(1/2)*a^2)*(b*x^2+a)/(a*b)^(1/2)/x/a^3/((b*x^2+a)^2)
^(3/2)
```

maxima [A] time = 2.98, size = 71, normalized size = 0.42

$$-\frac{15\,b^2x^4 + 25\,abx^2 + 8\,a^2}{8\left(a^3b^2x^5 + 2\,a^4bx^3 + a^5x\right)} - \frac{15\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(15*b^2*x^4 + 25*a*b*x^2 + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)
- 15/8*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**2)**2)**(3/2)), x)
```

3.644 $\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=209

$$\frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35b^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35b\left(a+\right.}{8a^4x\sqrt{a^2+2ab}}$$

```
[Out] 7/8/a^2/x^3/((b*x^2+a)^2)^(1/2)+1/4/a/x^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-35/
24*(b*x^2+a)/a^3/x^3/((b*x^2+a)^2)^(1/2)+35/8*b*(b*x^2+a)/a^4/x/((b*x^2+a)^
2)^(1/2)+35/8*b^(3/2)*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^(9/2)/((b*x^2+a
)^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 290, 325, 205}

$$\frac{35b\left(a+bx^2\right)}{8a^4x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35\left(a+bx^2\right)}{24a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7}{8a^2x^3\sqrt{a^2+2abx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 7/(8*a^2*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*x^3*(a + b*x^2)*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*(a + b*x^2))/(24*a^3*x^3*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4]) + (35*b*(a + b*x^2))/(8*a^4*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) + (35*b^(3/2)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2)*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(7b\left(ab+b^2x^2\right)\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^2}\,dx}{4a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(35\left(ab+\right.\right.}{8a^2\sqrt{a}}$$

$$= \frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{35}{24a^3x^3\sqrt{a}}$$

$$= \frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{35}{24a^3x^3\sqrt{a}}$$

$$= \frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{35}{24a^3x^3\sqrt{a}}$$

Mathematica [A] time = 0.04, size = 105, normalized size = 0.50

$$\frac{\sqrt{a}\left(-8a^3+56a^2bx^2+175ab^2x^4+105b^3x^6\right)+105b^{3/2}x^3\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{24a^{9/2}x^3\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[a]*(-8*a^3 + 56*a^2*b*x^2 + 175*a*b^2*x^4 + 105*b^3*x^6) + 105*b^(3/2
)*x^3*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(24*a^(9/2)*x^3*(a + b*x^2
)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.65, size = 238, normalized size = 1.14

$$\left[\frac{210\,b^3x^6+350\,ab^2x^4+112\,a^2bx^2-16\,a^3+105\left(b^3x^7+2\,ab^2x^5+a^2bx^3\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{48\left(a^4b^2x^7+2\,a^5bx^5+a^6x^3\right)},\frac{105\,b^3x^6+\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(210*b^3*x^6 + 350*a*b^2*x^4 + 112*a^2*b*x^2 - 16*a^3 + 105*(b^3*x^7
+ 2*a*b^2*x^5 + a^2*b*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b
*x^2 + a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3), 1/24*(105*b^3*x^6 + 175*
a*b^2*x^4 + 56*a^2*b*x^2 - 8*a^3 + 105*(b^3*x^7 + 2*a*b^2*x^5 + a^2*b*x^3)*
sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 139, normalized size = 0.67

$$\frac{\left(105b^4x^7\arctan\left(\frac{bx}{\sqrt{ab}}\right)+210a\,b^3x^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)+105\sqrt{ab}\;b^3x^6+105a^2b^2x^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)+175\sqrt{ab}\;a\,b^2x^4+\cdots\right.}{24\sqrt{ab}\;\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/24*(105*arctan(1/(a*b)^(1/2)*b*x)*x^7*b^4+105*(a*b)^(1/2)*x^6*b^3+210*arc
tan(1/(a*b)^(1/2)*b*x)*x^5*a*b^3+175*(a*b)^(1/2)*x^4*a*b^2+105*arctan(1/(a*
b)^(1/2)*b*x)*x^3*a^2*b^2+56*(a*b)^(1/2)*x^2*a^2*b-8*(a*b)^(1/2)*a^3)*(b*x^
2+a)/(a*b)^(1/2)/x^3/a^4/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 3.03, size = 86, normalized size = 0.41

$$\frac{105\,b^3x^6 + 175\,ab^2x^4 + 56\,a^2bx^2 - 8\,a^3}{24\left(a^4b^2x^7 + 2\,a^5bx^5 + a^6x^3\right)} + \frac{35\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*(105*b^3*x^6 + 175*a*b^2*x^4 + 56*a^2*b*x^2 - 8*a^3)/(a^4*b^2*x^7 + 2*
a^5*b*x^5 + a^6*x^3) + 35/8*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**4*((a + b*x**2)**2)**(3/2)), x)
```

3.645 $\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=238

$$-\frac{5a^2}{b^6\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^2\left(a+bx^2\right)}{2b^5\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^5}{8b^6\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b}}$$

```
[Out] -5*a^2/b^6/((b*x^2+a)^2)^(1/2)+1/8*a^5/b^6/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)-
5/6*a^4/b^6/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)+5/2*a^3/b^6/(b*x^2+a)/((b*x^2+a
)^2)^(1/2)+1/2*x^2*(b*x^2+a)/b^5/((b*x^2+a)^2)^(1/2)-5/2*a*(b*x^2+a)*ln(b*x
^2+a)/b^6/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{a^5}{8b^6\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5a^4}{6b^6\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5a^3}{2b^6\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-5*a^2)/(b^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + a^5/(8*b^6*(a + b*x^2)^3*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*a^4)/(6*b^6*(a + b*x^2)^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) + (5*a^3)/(2*b^6*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]) + (x^2*(a + b*x^2))/(2*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*a*
(a + b*x^2)*Log[a + b*x^2])/(2*b^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
```

```
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^5}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx,x,x^2\right)$$

$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{x^5}{\left(ab+b^2x\right)^5}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \left(\frac{1}{b^{10}}-\frac{a^5}{b^{10}(a+bx)^5}+\frac{5a^4}{b^{10}(a+bx)^4}-\frac{10a^3}{b^{10}(a+bx)^3}+\frac{10a^2}{b^{10}(a+bx)^2}-\frac{}{b^{10}}\right.\right.}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{5a^2}{b^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^5}{8b^6\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{6b^6\left(a+bx^2\right)^2}$$

Mathematica [A] time = 0.04, size = 103, normalized size = 0.43

$$\frac{-77a^5-248a^4bx^2-252a^3b^2x^4-48a^2b^3x^6+48ab^4x^8-60a\left(a+bx^2\right)^4\log\left(a+bx^2\right)+12b^5x^{10}}{24b^6\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-77*a^5 - 248*a^4*b*x^2 - 252*a^3*b^2*x^4 - 48*a^2*b^3*x^6 + 48*a*b^4*x^8
+ 12*b^5*x^10 - 60*a*(a + b*x^2)^4*Log[a + b*x^2])/(24*b^6*(a + b*x^2)^3*Sq
rt[(a + b*x^2)^2])
```

fricas [A] time = 1.03, size = 157, normalized size = 0.66

$$\frac{12\,b^5x^{10}+48\,ab^4x^8-48\,a^2b^3x^6-252\,a^3b^2x^4-248\,a^4bx^2-77\,a^5-60\left(ab^4x^8+4\,a^2b^3x^6+6\,a^3b^2x^4+4\,a^4bx^2+\right.}{24\left(b^{10}x^8+4\,ab^9x^6+6\,a^2b^8x^4+4\,a^3b^7x^2+a^4b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(12*b^5*x^10 + 48*a*b^4*x^8 - 48*a^2*b^3*x^6 - 252*a^3*b^2*x^4 - 248*a
^4*b*x^2 - 77*a^5 - 60*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*b
*x^2 + a^5)*log(b*x^2 + a))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3
*b^7*x^2 + a^4*b^6)
```

giac [A] time = 0.25, size = 105, normalized size = 0.44

$$\frac{x^2}{2\,b^5\mathrm{sgn}\left(bx^2+a\right)}-\frac{5\,a\log\left(\left|bx^2+a\right|\right)}{2\,b^6\mathrm{sgn}\left(bx^2+a\right)}-\frac{120\,a^2b^3x^6+300\,a^3b^2x^4+260\,a^4bx^2+77\,a^5}{24\left(bx^2+a\right)^4b^6\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2/(b^5*sgn(b*x^2 + a)) - 5/2*a*log(abs(b*x^2 + a))/(b^6*sgn(b*x^2 + a
)) - 1/24*(120*a^2*b^3*x^6 + 300*a^3*b^2*x^4 + 260*a^4*b*x^2 + 77*a^5)/((b*
x^2 + a)^4*b^6*sgn(b*x^2 + a))
```

maple [A] time = 0.02, size = 163, normalized size = 0.68

$$-\frac{\left(-12b^5x^{10}+60a\,b^4x^8\ln\left(b\,x^2+a\right)-48a\,b^4x^8+240a^2b^3x^6\ln\left(b\,x^2+a\right)+48a^2b^3x^6+360a^3b^2x^4\ln\left(b\,x^2+a\right)\right.}{24\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24*(-12*b^5*x^10+60*ln(b*x^2+a)*x^8*a*b^4-48*a*b^4*x^8+240*ln(b*x^2+a)*x
^6*a^2*b^3+48*a^2*b^3*x^6+360*ln(b*x^2+a)*x^4*a^3*b^2+252*a^3*b^2*x^4+240*l
n(b*x^2+a)*x^2*a^4*b+248*a^4*b*x^2+60*ln(b*x^2+a)*a^5+77*a^5)*(b*x^2+a)/b^6
/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 1.38, size = 110, normalized size = 0.46

$$-\frac{120\,a^2b^3x^6+300\,a^3b^2x^4+260\,a^4bx^2+77\,a^5}{24\left(b^{10}x^8+4\,ab^9x^6+6\,a^2b^8x^4+4\,a^3b^7x^2+a^4b^6\right)}+\frac{x^2}{2\,b^5}-\frac{5\,a\log\left(bx^2+a\right)}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/24*(120*a^2*b^3*x^6 + 300*a^3*b^2*x^4 + 260*a^4*b*x^2 + 77*a^5)/(b^10*x^
8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6) + 1/2*x^2/b^5 -
5/2*a*log(b*x^2 + a)/b^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{11}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^11/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**11/((a + b*x**2)**2)**(5/2), x)
```

3.646 $\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^{5/2}} dx$

Optimal. Leaf size=196

$$-\frac{3a^2}{2b^5\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2a}{b^5\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^4}{8b^5\left(a+bx^2\right)^3\sqrt{a^2+2a}}$$

```
[Out] 2*a/b^5/((b*x^2+a)^2)^(1/2)-1/8*a^4/b^5/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)+2/3
*a^3/b^5/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)-3/2*a^2/b^5/(b*x^2+a)/((b*x^2+a)^2
)^(1/2)+1/2*(b*x^2+a)*ln(b*x^2+a)/b^5/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^4}{8b^5\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2a^3}{3b^5\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3a^2}{2b^5\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*a)/(b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - a^4/(8*b^5*(a + b*x^2)^3*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4]) + (2*a^3)/(3*b^5*(a + b*x^2)^2*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4]) - (3*a^2)/(2*b^5*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4]) + ((a + b*x^2)*Log[a + b*x^2])/(2*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{x^9}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, x^2\right)$$
$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{x^4}{\left(ab+b^2x\right)^5}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \left(\frac{a^4}{b^9(a+bx)^5}-\frac{4a^3}{b^9(a+bx)^4}+\frac{6a^2}{b^9(a+bx)^3}-\frac{4a}{b^9(a+bx)^2}+\frac{1}{b^9(a+bx)}\right) d\right.}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{2a}{b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^4}{8b^5\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2}{3b^5\left(a+bx^2\right)^2\sqrt{}}$$

Mathematica [A] time = 0.03, size = 83, normalized size = 0.42

$$\frac{a\left(25a^3+88a^2bx^2+108ab^2x^4+48b^3x^6\right)+12\left(a+bx^2\right)^4\log\left(a+bx^2\right)}{24b^5\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a*(25*a^3 + 88*a^2*b*x^2 + 108*a*b^2*x^4 + 48*b^3*x^6) + 12*(a + b*x^2)^4*
Log[a + b*x^2])/(24*b^5*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 1.07, size = 135, normalized size = 0.69

$$\frac{48\,ab^3x^6+108\,a^2b^2x^4+88\,a^3bx^2+25\,a^4+12\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\log\left(bx^2+a\right)}{24\left(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(48*a*b^3*x^6 + 108*a^2*b^2*x^4 + 88*a^3*b*x^2 + 25*a^4 + 12*(b^4*x^8
+ 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*log(b*x^2 + a))/(b^9*x^8
 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)
```

giac [A] time = 0.21, size = 84, normalized size = 0.43

$$\frac{\log\left(\left|bx^2+a\right|\right)}{2\,b^5\mathrm{sgn}\left(bx^2+a\right)}+\frac{48\,ab^2x^6+108\,a^2bx^4+88\,a^3x^2+\frac{25\,a^4}{b}}{24\left(bx^2+a\right)^4b^4\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/(b^5*sgn(b*x^2 + a)) + 1/24*(48*a*b^2*x^6 + 108*a^2
*b*x^4 + 88*a^3*x^2 + 25*a^4/b)/((b*x^2 + a)^4*b^4*sgn(b*x^2 + a))
```

maple [A] time = 0.02, size = 141, normalized size = 0.72

$$\frac{\left(12b^4x^8\ln\left(b\,x^2+a\right)+48a\,b^3x^6\ln\left(b\,x^2+a\right)+48a\,b^3x^6+72a^2b^2x^4\ln\left(b\,x^2+a\right)+108a^2b^2x^4+48a^3b\,x^2\ln\left(b\,x^2\right.\right.}{24\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24*(12*ln(b*x^2+a)*x^8*b^4+48*ln(b*x^2+a)*x^6*a*b^3+48*a*b^3*x^6+72*ln(b*
x^2+a)*x^4*a^2*b^2+108*a^2*b^2*x^4+48*ln(b*x^2+a)*x^2*a^3*b+88*a^3*b*x^2+12
*ln(b*x^2+a)*a^4+25*a^4)*(b*x^2+a)/b^5/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 1.46, size = 99, normalized size = 0.51

$$\frac{48\,ab^3x^6+108\,a^2b^2x^4+88\,a^3bx^2+25\,a^4}{24\left(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5\right)}+\frac{\log\left(bx^2+a\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*(48*a*b^3*x^6 + 108*a^2*b^2*x^4 + 88*a^3*b*x^2 + 25*a^4)/(b^9*x^8 + 4*
a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5) + 1/2*log(b*x^2 + a)/b
^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int(x^9/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral(x**9/((a + b*x**2)**2)**(5/2), x)
```

3.647 $\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=41

$$\frac{x^8}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/8*x^8/a/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 37}

$$\frac{x^8}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] x^8/(8*a*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, x^2\right)$$
$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{x^3}{\left(ab+b^2x\right)^5}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{x^8}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 1.49

$$\frac{-a^3-4a^2bx^2-6ab^2x^4-4b^3x^6}{8b^4\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-a^3 - 4*a^2*b*x^2 - 6*a*b^2*x^4 - 4*b^3*x^6)/(8*b^4*(a + b*x^2)^3*Sqrt[(a
 + b*x^2)^2])
```

fricas [B] time = 0.92, size = 80, normalized size = 1.95

$$-\frac{4\,b^3x^6+6\,ab^2x^4+4\,a^2bx^2+a^3}{8\left(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/(b^8*x^8 + 4*a*b^7*x^6 +
 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

giac [A] time = 0.22, size = 54, normalized size = 1.32

$$-\frac{4\,b^3x^6+6\,ab^2x^4+4\,a^2bx^2+a^3}{8\left(bx^2+a\right)^4b^4\text{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/((b*x^2 + a)^4*b^4*sgn(b
*x^2 + a))
```

maple [A] time = 0.01, size = 54, normalized size = 1.32

$$-\frac{\left(b\,x^{2}+a\right)\left(4b^{3}x^{6}+6a\,b^{2}x^{4}+4a^{2}b\,x^{2}+a^{3}\right)}{8\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/8*(b*x^2+a)*(4*b^3*x^6+6*a*b^2*x^4+4*a^2*b*x^2+a^3)/b^4/((b*x^2+a)^2)^(5
/2)
```

maxima [B] time = 1.40, size = 80, normalized size = 1.95

$$-\frac{4\,b^{3}x^{6}+6\,ab^{2}x^{4}+4\,a^{2}bx^{2}+a^{3}}{8\left(b^{8}x^{8}+4\,ab^{7}x^{6}+6\,a^{2}b^{6}x^{4}+4\,a^{3}b^{5}x^{2}+a^{4}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/(b^8*x^8 + 4*a*b^7*x^6 +
 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

mupad [B] time = 4.29, size = 144, normalized size = 3.51

$$\frac{a^{3}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{8\,b^{4}\left(b\,x^{2}+a\right)^{5}}-\frac{a^{2}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{2\,b^{4}\left(b\,x^{2}+a\right)^{4}}-\frac{\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{2\,b^{4}\left(b\,x^{2}+a\right)^{2}}+\frac{3\,a\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{4\,b^{4}\left(b\,x^{2}+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(8*b^4*(a + b*x^2)^5) - (a^2*(a^2 +
 b^2*x^4 + 2*a*b*x^2)^(1/2))/(2*b^4*(a + b*x^2)^4) - (a^2 + b^2*x^4 + 2*a*b
*x^2)^(1/2)/(2*b^4*(a + b*x^2)^2) + (3*a*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))
/(4*b^4*(a + b*x^2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral(x**7/((a + b*x**2)**2)**(5/2), x)
```

3.648 $\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=74

$$\frac{x^6}{8a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{x^6}{24a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

```
[Out] 1/24*x^6/a^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)+1/8*x^6/a/(b*x^2+a)/(b^2*x^4+2*a
*b*x^2+a^2)^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1109}

$$\frac{x^6}{8a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{x^6}{24a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] x^6/(24*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + x^6/(8*a*(a + b*x^2)*(a^2
+ 2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rule 1109

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(2*(d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(d*(m + 3)*(2*a + b*
x^2)), x] - Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*d*(m + 3)
*(p + 1)), x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !I
ntegerQ[p] && EqQ[m + 4*p + 5, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\frac{x^6}{24a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{x^6}{8a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.68

$$\frac{-a^2-4abx^2-6b^2x^4}{24b^3\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-a^2 - 4*a*b*x^2 - 6*b^2*x^4)/(24*b^3*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.83, size = 69, normalized size = 0.93

$$-\frac{6\,b^2x^4 + 4\,abx^2 + a^2}{24\,\left(b^7x^8 + 4\,ab^6x^6 + 6\,a^2b^5x^4 + 4\,a^3b^4x^2 + a^4b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/24*(6*b^2*x^4 + 4*a*b*x^2 + a^2)/(b^7*x^8 + 4*a*b^6*x^6 + 6*a^2*b^5*x^4
+ 4*a^3*b^4*x^2 + a^4*b^3)
```

giac [A] time = 0.21, size = 43, normalized size = 0.58

$$-\frac{6\,b^2x^4 + 4\,abx^2 + a^2}{24\,\left(bx^2 + a\right)^4 b^3 \mathrm{sgn}\left(bx^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24*(6*b^2*x^4 + 4*a*b*x^2 + a^2)/((b*x^2 + a)^4*b^3*sgn(b*x^2 + a))
```

maple [A] time = 0.01, size = 43, normalized size = 0.58

$$-\frac{\left(b\,x^2 + a\right)\left(6b^2x^4 + 4ab\,x^2 + a^2\right)}{24\left(\left(b\,x^2 + a\right)^2\right)^{\frac{5}{2}} b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24*(b*x^2+a)*(6*b^2*x^4+4*a*b*x^2+a^2)/b^3/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 1.79, size = 69, normalized size = 0.93

$$-\frac{6\,b^2x^4 + 4\,abx^2 + a^2}{24\,\left(b^7x^8 + 4\,ab^6x^6 + 6\,a^2b^5x^4 + 4\,a^3b^4x^2 + a^4b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/24*(6*b^2*x^4 + 4*a*b*x^2 + a^2)/(b^7*x^8 + 4*a*b^6*x^6 + 6*a^2*b^5*x^4
+ 4*a^3*b^4*x^2 + a^4*b^3)
```

mupad [B] time = 4.23, size = 53, normalized size = 0.72

$$-\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(a^2+4\,a\,b\,x^2+6\,b^2\,x^4\right)}{24\,b^3\,{\left(b\,x^2+a\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] -((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(a^2 + 6*b^2*x^4 + 4*a*b*x^2))/(24*b^3*
(a + b*x^2)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)**2)**(5/2), x)
```

3.649 $\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^{5/2}} dx$

Optimal. Leaf size=69

$$\frac{a}{8b^2(a+bx^2)(a^2+2abx^2+b^2x^4)^{3/2}} - \frac{1}{6b^2(a^2+2abx^2+b^2x^4)^{3/2}}$$

```
[Out] -1/6/b^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)+1/8*a/b^2/(b*x^2+a)/(b^2*x^4+2*a*b*x
^2+a^2)^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 640, 607}

$$\frac{a}{8b^2(a+bx^2)(a^2+2abx^2+b^2x^4)^{3/2}} - \frac{1}{6b^2(a^2+2abx^2+b^2x^4)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] -1/(6*b^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + a/(8*b^2*(a + b*x^2)*(a^2 +
2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{x}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx,x,x^2\right)$$
$$=-\frac{1}{6b^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}-\frac{a\,\text{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx,x,x^2\right)}{2b}$$
$$=-\frac{1}{6b^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{a}{8b^2\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.57

$$\frac{-a-4bx^2}{24b^2\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
[Out] (-a - 4*b*x^2)/(24*b^2*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.85, size = 58, normalized size = 0.84

$$-\frac{4\,bx^2+a}{24\left(b^6x^8+4\,ab^5x^6+6\,a^2b^4x^4+4\,a^3b^3x^2+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
[Out] -1/24*(4*b*x^2 + a)/(b^6*x^8 + 4*a*b^5*x^6 + 6*a^2*b^4*x^4 + 4*a^3*b^3*x^2
+ a^4*b^2)
```

giac [A] time = 0.21, size = 32, normalized size = 0.46

$$-\frac{4\,bx^2+a}{24\left(bx^2+a\right)^4b^2\text{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24*(4*b*x^2 + a)/((b*x^2 + a)^4*b^2*sgn(b*x^2 + a))
```

maple [A] time = 0.01, size = 32, normalized size = 0.46

$$-\frac{\left(b\,x^{2}+a\right)\left(4b\,x^{2}+a\right)}{24\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}} b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24*(b*x^2+a)*(4*b*x^2+a)/b^2/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 1.36, size = 58, normalized size = 0.84

$$-\frac{4\,bx^{2}+a}{24\left(b^{6}x^{8}+4\,ab^{5}x^{6}+6\,a^{2}b^{4}x^{4}+4\,a^{3}b^{3}x^{2}+a^{4}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/24*(4*b*x^2 + a)/(b^6*x^8 + 4*a*b^5*x^6 + 6*a^2*b^4*x^4 + 4*a^3*b^3*x^2
+ a^4*b^2)
```

mupad [B] time = 4.26, size = 42, normalized size = 0.61

$$-\frac{\left(4\,b\,x^{2}+a\right)\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{24\,b^{2}\left(b\,x^{2}+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] -((a + 4*b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(24*b^2*(a + b*x^2)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral(x**3/((a + b*x**2)**2)**(5/2), x)
```

3.650 $\int \frac{x}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{8b\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

```
[Out] -1/8/b/(b*x^2+a)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1107, 607}

$$-\frac{1}{8b\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] -1/(8*b*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx,x,x^2\right)\\ &= -\frac{1}{8b\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.71

$$-\frac{a+bx^2}{8b\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
[Out] -1/8*(a + b*x^2)/(b*((a + b*x^2)^2)^(5/2))
```

fricas [A] time = 0.97, size = 48, normalized size = 1.26

$$-\frac{1}{8\left(b^5x^8+4\,ab^4x^6+6\,a^2b^3x^4+4\,a^3b^2x^2+a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
[Out] -1/8/(b^5*x^8 + 4*a*b^4*x^6 + 6*a^2*b^3*x^4 + 4*a^3*b^2*x^2 + a^4*b)
```

giac [A] time = 0.21, size = 24, normalized size = 0.63

$$-\frac{1}{8\left(bx^2+a\right)^4 b\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
[Out] -1/8/((b*x^2 + a)^4*b*sgn(b*x^2 + a))
```

maple [A] time = 0.01, size = 24, normalized size = 0.63

$$-\frac{b\,x^2+a}{8\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
[Out] -1/8*(b*x^2+a)/b/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 1.32, size = 48, normalized size = 1.26

$$-\frac{1}{8\left(b^5x^8+4\,ab^4x^6+6\,a^2b^3x^4+4\,a^3b^2x^2+a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/8/(b^5*x^8 + 4*a*b^4*x^6 + 6*a^2*b^3*x^4 + 4*a^3*b^2*x^2 + a^4*b)
```

mupad [B] time = 4.27, size = 34, normalized size = 0.89

$$-\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,b\,\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] -(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/(8*b*(a + b*x^2)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x/((a + b*x**2)**2)**(5/2), x)
```

3.651 $\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{1}{6a^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\log(x)\left(a+bx^2\right)}{a^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log}{2a^5\sqrt{a^2+2ab}}$$

```
[Out] 1/2/a^4/((b*x^2+a)^2)^(1/2)+1/8/a/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)+1/6/a^2/(
b*x^2+a)^2/((b*x^2+a)^2)^(1/2)+1/4/a^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+(b*x^2
+a)*ln(x)/a^5/((b*x^2+a)^2)^(1/2)-1/2*(b*x^2+a)*ln(b*x^2+a)/a^5/((b*x^2+a)^
2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 44}

$$\frac{1}{4a^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{6a^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 1/(2*a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*(a + b*x^2)^3*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]) + 1/(6*a^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) + 1/(4*a^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((a + b*x^2
)*Log[x])/(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((a + b*x^2)*Log[a + b*x^
2])/(2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{x\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^5}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \left(\frac{1}{a^5b^5x}-\frac{1}{ab^4(a+bx)^5}-\frac{1}{a^2b^4(a+bx)^4}-\frac{1}{a^3b^4(a+bx)^3}-\frac{1}{a^4b^4(a+bx}\right.\right.}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{6a^2\left(a+bx^2\right)^2}$$

Mathematica [A] time = 0.04, size = 96, normalized size = 0.43

$$\frac{a\left(25a^3+52a^2bx^2+42ab^2x^4+12b^3x^6\right)+24\log(x)\left(a+bx^2\right)^4-12\left(a+bx^2\right)^4\log\left(a+bx^2\right)}{24a^5\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
[Out] (a*(25*a^3 + 52*a^2*b*x^2 + 42*a*b^2*x^4 + 12*b^3*x^6) + 24*(a + b*x^2)^4*L
og[x] - 12*(a + b*x^2)^4*Log[a + b*x^2])/(24*a^5*(a + b*x^2)^3*Sqrt[(a + b*
x^2)^2])
```

fricas [A] time = 0.62, size = 178, normalized size = 0.80

$$\frac{12\,ab^3x^6+42\,a^2b^2x^4+52\,a^3bx^2+25\,a^4-12\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\log\left(bx^2+a\right)+24\left(b^4x^8\right.}{24\left(a^5b^4x^8+4\,a^6b^3x^6+6\,a^7b^2x^4+4\,a^8bx^2+a^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(12*a*b^3*x^6 + 42*a^2*b^2*x^4 + 52*a^3*b*x^2 + 25*a^4 - 12*(b^4*x^8 +
 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*log(b*x^2 + a) + 24*(b^4*
x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*log(x))/(a^5*b^4*x^8
 + 4*a^6*b^3*x^6 + 6*a^7*b^2*x^4 + 4*a^8*b*x^2 + a^9)
```

giac [A] time = 0.27, size = 101, normalized size = 0.45

$$-\frac{\log\left(\left|bx^2+a\right|\right)}{2\,a^5\mathrm{sgn}\left(bx^2+a\right)}+\frac{\log\left(\left|x\right|\right)}{a^5\mathrm{sgn}\left(bx^2+a\right)}+\frac{12\,ab^3x^6+42\,a^2b^2x^4+52\,a^3bx^2+25\,a^4}{24\,\left(bx^2+a\right)^4a^5\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(b*x^2 + a))/(a^5*sgn(b*x^2 + a)) + log(abs(x))/(a^5*sgn(b*x^2
+ a)) + 1/24*(12*a*b^3*x^6 + 42*a^2*b^2*x^4 + 52*a^3*b*x^2 + 25*a^4)/((b*x^
2 + a)^4*a^5*sgn(b*x^2 + a))
```

maple [A] time = 0.02, size = 193, normalized size = 0.87

$$\frac{\left(24b^4x^8\ln(x)-12b^4x^8\ln\left(b\,x^2+a\right)+96a\,b^3x^6\ln(x)-48a\,b^3x^6\ln\left(b\,x^2+a\right)+12a\,b^3x^6+144a^2b^2x^4\ln(x)-72\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24*(24*ln(x)*x^8*b^4-12*b^4*x^8*ln(b*x^2+a)+96*ln(x)*x^6*a*b^3-48*a*b^3*x
^6*ln(b*x^2+a)+12*a*b^3*x^6+144*ln(x)*x^4*a^2*b^2-72*a^2*b^2*x^4*ln(b*x^2+a
)+42*a^2*b^2*x^4+96*ln(x)*x^2*a^3*b-48*a^3*b*x^2*ln(b*x^2+a)+52*a^3*b*x^2+2
4*a^4*ln(x)-12*a^4*ln(b*x^2+a)+25*a^4)*(b*x^2+a)/a^5/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 1.44, size = 101, normalized size = 0.45

$$\frac{12\,b^3x^6+42\,ab^2x^4+52\,a^2bx^2+25\,a^3}{24\,\left(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8\right)}-\frac{\log\left(bx^2+a\right)}{2\,a^5}+\frac{\log\left(x\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*(12*b^3*x^6 + 42*a*b^2*x^4 + 52*a^2*b*x^2 + 25*a^3)/(a^4*b^4*x^8 + 4*a
^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8) - 1/2*log(b*x^2 + a)/a^5 +
log(x)/a^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
[Out] int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral(1/(x*((a + b*x**2)**2)**(5/2)), x)
```

3.652 $\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=267

$$-\frac{b}{8a^2\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5b\log(x)\left(a+bx^2\right)}{a^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5b\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^6\sqrt{a^2+2abx^2+b^2x^4}}-\frac{2b}{a^5\sqrt{a^2+2abx^2+b^2}}$$

```
[Out] -2*b/a^5/((b*x^2+a)^2)^(1/2)-1/8*b/a^2/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)-1/3*
b/a^3/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)-3/4*b/a^4/(b*x^2+a)/((b*x^2+a)^2)^(1/
2)+1/2*(-b*x^2-a)/a^5/x^2/((b*x^2+a)^2)^(1/2)-5*b*(b*x^2+a)*ln(x)/a^6/((b*x
^2+a)^2)^(1/2)+5/2*b*(b*x^2+a)*ln(b*x^2+a)/a^6/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 44}

$$-\frac{3b}{4a^4\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{3a^3\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{8a^2\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-2*b)/(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - b/(8*a^2*(a + b*x^2)^3*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4]) - b/(3*a^3*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (3*b)/(4*a^4*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a
 + b*x^2)/(2*a^5*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*b*(a + b*x^2)*Lo
g[x])/(a^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*b*(a + b*x^2)*Log[a + b*x^
2])/(2*a^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{x^3\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)^5}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int\left(\frac{1}{a^5b^5x^2}-\frac{5}{a^6b^4x}+\frac{1}{a^2b^3(a+bx)^5}+\frac{2}{a^3b^3(a+bx)^4}+\frac{3}{a^4b^3(a+bx)^3}\right.\right.}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2b}{a^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{8a^2\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{3a^3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.05, size = 119, normalized size = 0.45

$$\frac{-a\left(12a^4+125a^3bx^2+260a^2b^2x^4+210ab^3x^6+60b^4x^8\right)-120bx^2\log(x)\left(a+bx^2\right)^4+60bx^2\left(a+bx^2\right)^4\log\left(a+\right.}{24a^6x^2\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-(a*(12*a^4 + 125*a^3*b*x^2 + 260*a^2*b^2*x^4 + 210*a*b^3*x^6 + 60*b^4*x^8
)) - 120*b*x^2*(a + b*x^2)^4*Log[x] + 60*b*x^2*(a + b*x^2)^4*Log[a + b*x^2]
)/(24*a^6*x^2*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.94, size = 207, normalized size = 0.78

$$-\frac{60\,ab^4x^8+210\,a^2b^3x^6+260\,a^3b^2x^4+125\,a^4bx^2+12\,a^5-60\left(b^5x^{10}+4\,ab^4x^8+6\,a^2b^3x^6+4\,a^3b^2x^4+a^4bx^2\right)\mathrm{lo}}{24\left(a^6b^4x^{10}+4\,a^7b^3x^8+6\,a^8b^2x^6+4\,a^9bx^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/24*(60*a*b^4*x^8 + 210*a^2*b^3*x^6 + 260*a^3*b^2*x^4 + 125*a^4*b*x^2 + 1
2*a^5 - 60*(b^5*x^10 + 4*a*b^4*x^8 + 6*a^2*b^3*x^6 + 4*a^3*b^2*x^4 + a^4*b*
x^2)*log(b*x^2 + a) + 120*(b^5*x^10 + 4*a*b^4*x^8 + 6*a^2*b^3*x^6 + 4*a^3*b
^2*x^4 + a^4*b*x^2)*log(x))/(a^6*b^4*x^10 + 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 +
 4*a^9*b*x^4 + a^10*x^2)
```

giac [A] time = 0.22, size = 118, normalized size = 0.44

$$\frac{5\,b\log\left(\left|bx^2+a\right|\right)}{2\,a^6\mathrm{sgn}\left(bx^2+a\right)}-\frac{5\,b\log\left(|x|\right)}{a^6\mathrm{sgn}\left(bx^2+a\right)}-\frac{60\,ab^4x^8+210\,a^2b^3x^6+260\,a^3b^2x^4+125\,a^4bx^2+12\,a^5}{24\left(bx^2+a\right)^4a^6x^2\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 5/2*b*log(abs(b*x^2 + a))/(a^6*sgn(b*x^2 + a)) - 5*b*log(abs(x))/(a^6*sgn(b
*x^2 + a)) - 1/24*(60*a*b^4*x^8 + 210*a^2*b^3*x^6 + 260*a^3*b^2*x^4 + 125*a
^4*b*x^2 + 12*a^5)/((b*x^2 + a)^4*a^6*x^2*sgn(b*x^2 + a))
```

maple [A] time = 0.02, size = 219, normalized size = 0.82

$$-\frac{\left(120b^5x^{10}\ln(x)-60b^5x^{10}\ln\left(b\,x^2+a\right)+480a\,b^4x^8\ln(x)-240a\,b^4x^8\ln\left(b\,x^2+a\right)+60a\,b^4x^8+720a^2b^3x^6\ln(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24*(120*b^5*x^10*ln(x)-60*ln(b*x^2+a)*x^10*b^5+480*a*b^4*x^8*ln(x)-240*a
*b^4*x^8*ln(b*x^2+a)+60*a*b^4*x^8+720*a^2*b^3*x^6*ln(x)-360*a^2*b^3*x^6*ln(
b*x^2+a)+210*a^2*b^3*x^6+480*a^3*b^2*x^4*ln(x)-240*a^3*b^2*x^4*ln(b*x^2+a)+
260*a^3*b^2*x^4+120*a^4*b*x^2*ln(x)-60*a^4*b*x^2*ln(b*x^2+a)+125*a^4*b*x^2+
12*a^5)*(b*x^2+a)/x^2/a^6/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 1.42, size = 119, normalized size = 0.45

$$-\frac{60\,b^4x^8+210\,ab^3x^6+260\,a^2b^2x^4+125\,a^3bx^2+12\,a^4}{24\left(a^5b^4x^{10}+4\,a^6b^3x^8+6\,a^7b^2x^6+4\,a^8bx^4+a^9x^2\right)}+\frac{5\,b\log\left(bx^2+a\right)}{2\,a^6}-\frac{5\,b\log(x)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/24*(60*b^4*x^8 + 210*a*b^3*x^6 + 260*a^2*b^2*x^4 + 125*a^3*b*x^2 + 12*a^
4)/(a^5*b^4*x^10 + 4*a^6*b^3*x^8 + 6*a^7*b^2*x^6 + 4*a^8*b*x^4 + a^9*x^2) +
 5/2*b*log(b*x^2 + a)/a^6 - 5*b*log(x)/a^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3 \left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
```

```
[Out] int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x**2)**2)**(5/2)), x)
```

3.653 $\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=211

$$-\frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5x^3}{48b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5x}{64b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 5/128*x/a/b^3/((b*x^2+a)^2)^(1/2)-1/8*x^5/b/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)
-5/48*x^3/b^2/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)-5/64*x/b^3/(b*x^2+a)/((b*x^2+
a)^2)^(1/2)+5/128*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(7/2)/((b*x
^2+a)^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 288, 199, 205}

$$-\frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5x^3}{48b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5x}{64b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (5*x)/(128*a*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x^5/(8*b*(a + b*x^2)^3*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*x^3)/(48*b^2*(a + b*x^2)^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]) - (5*x)/(64*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4]) + (5*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(3/2)*b^(7/2)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{x^6}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5b^2\left(ab+b^2x^2\right)\right)\int \frac{x^4}{\left(ab+b^2x^2\right)^4}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5x^3}{48b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(5}{}$$

$$= -\frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5x^3}{48b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{64b}$$

$$= \frac{5x}{128ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{48b^2\left(a+bx^2\right.}$$

$$= \frac{5x}{128ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{48b^2\left(a+bx^2\right.}$$

Mathematica [A] time = 0.04, size = 105, normalized size = 0.50

$$\frac{\sqrt{a}\sqrt{b}\,x\left(-15a^3-55a^2bx^2-73ab^2x^4+15b^3x^6\right)+15\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{384a^{3/2}b^{7/2}\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-15*a^3 - 55*a^2*b*x^2 - 73*a*b^2*x^4 + 15*b^3*x^6) + 1
5*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(384*a^(3/2)*b^(7/2)*(a + b*x^
2)^3*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.96, size = 324, normalized size = 1.54

$$\left[\frac{30\,ab^4x^7-146\,a^2b^3x^5-110\,a^3b^2x^3-30\,a^4bx-15\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2}{bx}\right.}{768\left(a^2b^8x^8+4\,a^3b^7x^6+6\,a^4b^6x^4+4\,a^5b^5x^2+a^6b^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(30*a*b^4*x^7 - 146*a^2*b^3*x^5 - 110*a^3*b^2*x^3 - 30*a^4*b*x - 15*
(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*log(
(b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^8*x^8 + 4*a^3*b^7*x^6 + 6
*a^4*b^6*x^4 + 4*a^5*b^5*x^2 + a^6*b^4), 1/384*(15*a*b^4*x^7 - 73*a^2*b^3*x
^5 - 55*a^3*b^2*x^3 - 15*a^4*b*x + 15*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^
4 + 4*a^3*b*x^2 + a^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^2*b^8*x^8 + 4*a^
3*b^7*x^6 + 6*a^4*b^6*x^4 + 4*a^5*b^5*x^2 + a^6*b^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 172, normalized size = 0.82

$$\frac{\left(15b^4x^8\arctan\left(\frac{bx}{\sqrt{ab}}\right)+60a\,b^3x^6\arctan\left(\frac{bx}{\sqrt{ab}}\right)+15\sqrt{ab}\,b^3x^7+90a^2b^2x^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)-73\sqrt{ab}\,a\,b^2x^5+60a^3\right.}{384\sqrt{ab}\,\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}a\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/384*(15*arctan(1/(a*b)^(1/2)*b*x)*x^8*b^4+15*(a*b)^(1/2)*x^7*b^3+60*arcta
n(1/(a*b)^(1/2)*b*x)*x^6*a*b^3-73*(a*b)^(1/2)*x^5*a*b^2+90*arctan(1/(a*b)^(
1/2)*b*x)*x^4*a^2*b^2-55*(a*b)^(1/2)*x^3*a^2*b+60*arctan(1/(a*b)^(1/2)*b*x)
*x^2*a^3*b-15*(a*b)^(1/2)*x*a^3+15*arctan(1/(a*b)^(1/2)*b*x)*a^4)*(b*x^2+a)
/(a*b)^(1/2)/b^3/a/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 3.03, size = 109, normalized size = 0.52

$$\frac{15\,b^{3}x^{7}-73\,ab^{2}x^{5}-55\,a^{2}bx^{3}-15\,a^{3}x}{384\left(ab^{7}x^{8}+4\,a^{2}b^{6}x^{6}+6\,a^{3}b^{5}x^{4}+4\,a^{4}b^{4}x^{2}+a^{5}b^{3}\right)}+\frac{5\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}\,ab^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/384*(15*b^3*x^7 - 73*a*b^2*x^5 - 55*a^2*b*x^3 - 15*a^3*x)/(a*b^7*x^8 + 4*
a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3) + 5/128*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*a*b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^6/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(\left(a + bx^{2}\right)^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**6/((a + b*x**2)**2)**(5/2), x)
```

3.654 $\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=212

$$\frac{x}{64ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3x}{128a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{16b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{8b\left(\right.}$$

[Out] 3/128*x/a^2/b^2/((b*x^2+a)^2)^(1/2)-1/8*x^3/b/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)-1/16*x/b^2/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)+1/64*x/a/b^2/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+3/128*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(5/2)/((b*x^2+a)^2)^(1/2)

Rubi [A] time = 0.09, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 288, 199, 205}

$$-\frac{x^3}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x}{64ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3x}{128a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{16b^2}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]

[Out] (3*x)/(128*a^2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x^3/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x/(16*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + x/(64*a*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(5/2)*b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{x^4}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x^3}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3b^2\left(ab+b^2x^2\right)\right)\int \frac{x^2}{\left(ab+b^2x^2\right)^4}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x^3}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x}{16b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(ab \cdots}{1 \cdots}$$

$$= -\frac{x^3}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x}{16b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{64a \cdots}$$

$$= \frac{3x}{128a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^3}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{16b^2\left(a+bx \cdots\right.}$$

$$= \frac{3x}{128a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^3}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{16b^2\left(a+bx \cdots\right.}$$

Mathematica [A] time = 0.04, size = 105, normalized size = 0.50

$$\frac{\sqrt{a}\sqrt{b}x\left(-3a^3-11a^2bx^2+11ab^2x^4+3b^3x^6\right)+3\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{5/2}b^{5/2}\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-3*a^3 - 11*a^2*b*x^2 + 11*a*b^2*x^4 + 3*b^3*x^6) + 3*(
a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(5/2)*b^(5/2)*(a + b*x^2)^
3*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 1.12, size = 324, normalized size = 1.53

$$\left[\frac{6\,ab^4x^7+22\,a^2b^3x^5-22\,a^3b^2x^3-6\,a^4bx-3\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-}{bx^2+a}\right.}{256\left(a^3b^7x^8+4\,a^4b^6x^6+6\,a^5b^5x^4+4\,a^6b^4x^2+a^7b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/256*(6*a*b^4*x^7 + 22*a^2*b^3*x^5 - 22*a^3*b^2*x^3 - 6*a^4*b*x - 3*(b^4*
x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*log((b*x^
2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^7*x^8 + 4*a^4*b^6*x^6 + 6*a^5*
b^5*x^4 + 4*a^6*b^4*x^2 + a^7*b^3), 1/128*(3*a*b^4*x^7 + 11*a^2*b^3*x^5 - 1
1*a^3*b^2*x^3 - 3*a^4*b*x + 3*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^
3*b*x^2 + a^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^7*x^8 + 4*a^4*b^6*x^
6 + 6*a^5*b^5*x^4 + 4*a^6*b^4*x^2 + a^7*b^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 172, normalized size = 0.81

$$\frac{\left(3b^4x^8\arctan\left(\frac{bx}{\sqrt{ab}}\right)+12a\,b^3x^6\arctan\left(\frac{bx}{\sqrt{ab}}\right)+3\sqrt{ab}\;b^3x^7+18a^2b^2x^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)+11\sqrt{ab}\;a\,b^2x^5+12a^3b\,x\right.}{128\sqrt{ab}\;\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/128*(3*b^4*x^8*arctan(1/(a*b)^(1/2)*b*x)+3*(a*b)^(1/2)*b^3*x^7+12*a*b^3*x
^6*arctan(1/(a*b)^(1/2)*b*x)+11*(a*b)^(1/2)*a*b^2*x^5+18*a^2*b^2*x^4*arctan
(1/(a*b)^(1/2)*b*x)-11*(a*b)^(1/2)*a^2*b*x^3+12*a^3*b*x^2*arctan(1/(a*b)^(1
/2)*b*x)-3*(a*b)^(1/2)*a^3*x+3*a^4*arctan(1/(a*b)^(1/2)*b*x))*(b*x^2+a)/(a*
b)^(1/2)/b^2/a^2/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 2.99, size = 111, normalized size = 0.52

$$\frac{3 b^3 x^7 + 11 ab^2 x^5 - 11 a^2 b x^3 - 3 a^3 x}{128 \left(a^2 b^6 x^8 + 4 a^3 b^5 x^6 + 6 a^4 b^4 x^4 + 4 a^5 b^3 x^2 + a^6 b^2\right)} + \frac{3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128 \sqrt{ab} a^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/128*(3*b^3*x^7 + 11*a*b^2*x^5 - 11*a^2*b*x^3 - 3*a^3*x)/(a^2*b^6*x^8 + 4*
a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2) + 3/128*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*a^2*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(a^2 + 2 a b x^2 + b^2 x^4\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)**2)**(5/2), x)
```

3.655 $\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=213

$$\frac{5x}{192a^2b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x}{48ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 5/128*x/a^3/b/((b*x^2+a)^2)^(1/2)-1/8*x/b/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)+1
/48*x/a/b/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)+5/192*x/a^2/b/(b*x^2+a)/((b*x^2+a
)^2)^(1/2)+5/128*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(3/2)/((b*x^
2+a)^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 288, 199, 205}

$$\frac{5x}{128a^3b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5x}{192a^2b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x}{48ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{8b\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (5*x)/(128*a^3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x/(8*b*(a + b*x^2)^3*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) + x/(48*a*b*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4]) + (5*x)/(192*a^2*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) + (5*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(7/2)*b^(3/2)*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{x^2}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{\left(ab+b^2x^2\right)^4}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{x}{48ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(5}{}$$

$$= -\frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{x}{48ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{19}$$

$$= \frac{5x}{128a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{48ab\left(a+bx^2\right.}$$

$$= \frac{5x}{128a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{48ab\left(a+bx^2\right.}$$

Mathematica [A] time = 0.03, size = 105, normalized size = 0.49

$$\frac{\sqrt{a}\sqrt{b}\,x\left(-15a^3+73a^2bx^2+55ab^2x^4+15b^3x^6\right)+15\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{384a^{7/2}b^{3/2}\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-15*a^3 + 73*a^2*b*x^2 + 55*a*b^2*x^4 + 15*b^3*x^6) + 1
5*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(384*a^(7/2)*b^(3/2)*(a + b*x^
2)^3*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 1.05, size = 324, normalized size = 1.52

$$\left[\frac{30\,ab^4x^7+110\,a^2b^3x^5+146\,a^3b^2x^3-30\,a^4bx-15\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2}{bx}\right.}{768\left(a^4b^6x^8+4\,a^5b^5x^6+6\,a^6b^4x^4+4\,a^7b^3x^2+a^8b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(30*a*b^4*x^7 + 110*a^2*b^3*x^5 + 146*a^3*b^2*x^3 - 30*a^4*b*x - 15*
(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*log(
(b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^6*x^8 + 4*a^5*b^5*x^6 + 6
*a^6*b^4*x^4 + 4*a^7*b^3*x^2 + a^8*b^2), 1/384*(15*a*b^4*x^7 + 55*a^2*b^3*x
^5 + 73*a^3*b^2*x^3 - 15*a^4*b*x + 15*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^
4 + 4*a^3*b*x^2 + a^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b^6*x^8 + 4*a^
5*b^5*x^6 + 6*a^6*b^4*x^4 + 4*a^7*b^3*x^2 + a^8*b^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 172, normalized size = 0.81

$$\frac{\left(15b^4x^8\arctan\left(\frac{bx}{\sqrt{ab}}\right)+60a\,b^3x^6\arctan\left(\frac{bx}{\sqrt{ab}}\right)+15\sqrt{ab}\;b^3x^7+90a^2b^2x^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)+55\sqrt{ab}\;a\,b^2x^5+60a^3\right.}{384\sqrt{ab}\;\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/384*(15*b^4*x^8*arctan(1/(a*b)^(1/2)*b*x)+15*(a*b)^(1/2)*b^3*x^7+60*a*b^3
*x^6*arctan(1/(a*b)^(1/2)*b*x)+55*(a*b)^(1/2)*a*b^2*x^5+90*a^2*b^2*x^4*arct
an(1/(a*b)^(1/2)*b*x)+73*(a*b)^(1/2)*a^2*b*x^3+60*a^3*b*x^2*arctan(1/(a*b)^
(1/2)*b*x)-15*(a*b)^(1/2)*a^3*x+15*a^4*arctan(1/(a*b)^(1/2)*b*x))*(b*x^2+a)
/(a*b)^(1/2)/b/a^3/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 2.90, size = 109, normalized size = 0.51

$$\frac{15\,b^3x^7+55\,ab^2x^5+73\,a^2bx^3-15\,a^3x}{384\left(a^3b^5x^8+4\,a^4b^4x^6+6\,a^5b^3x^4+4\,a^6b^2x^2+a^7b\right)}+\frac{5\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/384*(15*b^3*x^7 + 55*a*b^2*x^5 + 73*a^2*b*x^3 - 15*a^3*x)/(a^3*b^5*x^8 +
4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b) + 5/128*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*a^3*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(\left(a + b x^{2}\right)^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)**2)**(5/2), x)
```

3.656 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=213

$$\frac{7x\left(a+bx^2\right)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{x\left(a+bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{35\left(a+bx^2\right)^5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{9/2}\sqrt{b}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{35x\left(a+bx^2\right)^4}{128a^4\left(a^2+2abx^2+b^2x^4\right)^{5/2}}$$

```
[Out] 1/8*x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(5/2)+7/48*x*(b*x^2+a)^2/a^2/(b^2
*x^4+2*a*b*x^2+a^2)^(5/2)+35/192*x*(b*x^2+a)^3/a^3/(b^2*x^4+2*a*b*x^2+a^2)^
(5/2)+35/128*x*(b*x^2+a)^4/a^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2)+35/128*(b*x^2+
a)^5*arctan(x*b^(1/2)/a^(1/2))/a^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/b^(1/2
)
```

Rubi [A] time = 0.07, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1088, 199, 205}

$$\frac{35x\left(a+bx^2\right)^4}{128a^4\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{35x\left(a+bx^2\right)^3}{192a^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{7x\left(a+bx^2\right)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{x\left(a+bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/2),x]
```

```
[Out] (x*(a + b*x^2))/(8*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)) + (7*x*(a + b*x^2)^
2)/(48*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)) + (35*x*(a + b*x^2)^3)/(192*a
^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)) + (35*x*(a + b*x^2)^4)/(128*a^4*(a^2
+ 2*a*b*x^2 + b^2*x^4)^(5/2)) + (35*(a + b*x^2)^5*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(128*a^(9/2)*Sqrt[b]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1088

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(2ab+2b^2x^2\right)^5 \int \frac{1}{\left(2ab+2b^2x^2\right)^5}\,dx}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}$$

$$= \frac{x\left(a+bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{\left(7\left(2ab+2b^2x^2\right)^5\right)\int \frac{1}{\left(2ab+2b^2x^2\right)^4}\,dx}{16ab\left(a^2+2abx^2+b^2x^4\right)^{5/2}}$$

$$= \frac{x\left(a+bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{7x\left(a+bx^2\right)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{\left(35\left(2ab+2b^2x^2\right)^5\right)\int}{192a^2b^2\left(a^2+2ab\right.}$$

$$= \frac{x\left(a+bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{7x\left(a+bx^2\right)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{35x\left(a+bx^2\right)^3}{192a^3\left(a^2+2abx^2+b\right.}$$

$$= \frac{x\left(a+bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{7x\left(a+bx^2\right)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{35x\left(a+bx^2\right)^3}{192a^3\left(a^2+2abx^2+b\right.}$$

$$= \frac{x\left(a+bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{7x\left(a+bx^2\right)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{35x\left(a+bx^2\right)^3}{192a^3\left(a^2+2abx^2+b\right.}$$

Mathematica [A] time = 0.04, size = 105, normalized size = 0.49

$$\frac{\sqrt{a}\sqrt{b}\,x\left(279a^3+511a^2bx^2+385ab^2x^4+105b^3x^6\right)+105\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{384a^{9/2}\sqrt{b}\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/2),x]
```

[Out] (Sqrt[a]*Sqrt[b]*x*(279*a^3 + 511*a^2*b*x^2 + 385*a*b^2*x^4 + 105*b^3*x^6) + 105*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(384*a^(9/2)*Sqrt[b]*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])

fricas [A] time = 1.12, size = 320, normalized size = 1.50

$$\left[\frac{210\,ab^4x^7 + 770\,a^2b^3x^5 + 1022\,a^3b^2x^3 + 558\,a^4bx - 105\left(b^4x^8 + 4\,ab^3x^6 + 6\,a^2b^2x^4 + 4\,a^3bx^2 + a^4\right)\sqrt{-ab}\,\log\left(\right.}{768\left(a^5b^5x^8 + 4\,a^6b^4x^6 + 6\,a^7b^3x^4 + 4\,a^8b^2x^2 + a^9b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(210*a*b^4*x^7 + 770*a^2*b^3*x^5 + 1022*a^3*b^2*x^3 + 558*a^4*b*x -
105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*
log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^5*b^5*x^8 + 4*a^6*b^4*x^6
 + 6*a^7*b^3*x^4 + 4*a^8*b^2*x^2 + a^9*b), 1/384*(105*a*b^4*x^7 + 385*a^2*b
^3*x^5 + 511*a^3*b^2*x^3 + 279*a^4*b*x + 105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2
*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^5*b^5*x^8
 + 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4 + 4*a^8*b^2*x^2 + a^9*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 169, normalized size = 0.79

$$\frac{\left(105b^4x^8\arctan\left(\frac{bx}{\sqrt{ab}}\right) + 420a\,b^3x^6\arctan\left(\frac{bx}{\sqrt{ab}}\right) + 105\sqrt{ab}\,b^3x^7 + 630a^2b^2x^4\arctan\left(\frac{bx}{\sqrt{ab}}\right) + 385\sqrt{ab}\,a\,b^2x^5 -\right.}{384\sqrt{ab}\,\left(\left(b\,x^2 + a\right)^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/384*(105*b^4*x^8*arctan(1/(a*b)^(1/2)*b*x)+105*(a*b)^(1/2)*b^3*x^7+420*a*
b^3*x^6*arctan(1/(a*b)^(1/2)*b*x)+385*(a*b)^(1/2)*a*b^2*x^5+630*a^2*b^2*x^4
*arctan(1/(a*b)^(1/2)*b*x)+511*(a*b)^(1/2)*a^2*b*x^3+420*a^3*b*x^2*arctan(1
```

```
/(a*b)^(1/2)*b*x)+279*(a*b)^(1/2)*a^3*x+105*a^4*arctan(1/(a*b)^(1/2)*b*x))*
(b*x^2+a)/(a*b)^(1/2)/a^4/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 3.05, size = 102, normalized size = 0.48

$$\frac{105\,b^3x^7+385\,ab^2x^5+511\,a^2bx^3+279\,a^3x}{384\left(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8\right)}+\frac{35\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/384*(105*b^3*x^7 + 385*a*b^2*x^5 + 511*a^2*b*x^3 + 279*a^3*x)/(a^4*b^4*x^
8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8) + 35/128*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*a^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-5/2), x)
```

3.657 $\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=251

$$\frac{3}{16a^2x\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^2}+\frac{1}{8ax\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^3}-\frac{315\sqrt{b}\,\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{128}$$

```
[Out] 105/128/a^4/x/((b*x^2+a)^2)^(1/2)+1/8/a/x/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)+3
/16/a^2/x/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)+21/64/a^3/x/(b*x^2+a)/((b*x^2+a)^
2)^(1/2)-315/128*(b*x^2+a)/a^5/x/((b*x^2+a)^2)^(1/2)-315/128*(b*x^2+a)*arct
an(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(11/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 290, 325, 205}

$$-\frac{315\left(a+bx^2\right)}{128a^5x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{105}{128a^4x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21}{64a^3x\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)}+\frac{}{16a^2x\sqrt{a^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 105/(128*a^4*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*x*(a + b*x^2)^3*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 3/(16*a^2*x*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]) + 21/(64*a^3*x*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) - (315*(a + b*x^2))/(128*a^5*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (315*Sq
rt[b]*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(11/2)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

```
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(9b^3\left(ab+b^2x^2\right)\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^4}\,dx}{8a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{3}{16a^2x\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{3}{16a^2x\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{105}{128a^4x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{16a^2x\left(\right.}$$

$$= \frac{105}{128a^4x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{16a^2x\left(\right.}$$

$$= \frac{105}{128a^4x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{16a^2x\left(\right.}$$

Mathematica [A] time = 0.04, size = 115, normalized size = 0.46

$$\frac{-\sqrt{a}\left(128a^4+837a^3bx^2+1533a^2b^2x^4+1155ab^3x^6+315b^4x^8\right)-315\sqrt{b}\,x\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{11/2}x\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-(Sqrt[a]*(128*a^4 + 837*a^3*b*x^2 + 1533*a^2*b^2*x^4 + 1155*a*b^3*x^6 + 3
15*b^4*x^8)) - 315*Sqrt[b]*x*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(12
8*a^(11/2)*x*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.91, size = 334, normalized size = 1.33

$$\left[-\frac{630\,b^4x^8+2310\,ab^3x^6+3066\,a^2b^2x^4+1674\,a^3bx^2+256\,a^4-315\left(b^4x^9+4\,ab^3x^7+6\,a^2b^2x^5+4\,a^3bx^3+a^4x\right)}{256\left(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4\,a^8bx^3+a^9x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/256*(630*b^4*x^8 + 2310*a*b^3*x^6 + 3066*a^2*b^2*x^4 + 1674*a^3*b*x^2 +
 256*a^4 - 315*(b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*x
)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^5*b^4*x^9
+ 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x), -1/128*(315*b^4*x^8
 + 1155*a*b^3*x^6 + 1533*a^2*b^2*x^4 + 837*a^3*b*x^2 + 128*a^4 + 315*(b^4*x
^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*x)*sqrt(b/a)*arctan(x*
sqrt(b/a)))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^
9*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 191, normalized size = 0.76

$$-\frac{\left(315b^5x^9\arctan\left(\frac{bx}{\sqrt{ab}}\right)+1260a\,b^4x^7\arctan\left(\frac{bx}{\sqrt{ab}}\right)+315\sqrt{ab}\;b^4x^8+1890a^2b^3x^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)+1155\sqrt{ab}\;a\,b^3\right.}{128\sqrt{ab}\;\left(\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/128*(315*arctan(1/(a*b)^(1/2)*b*x)*x^9*b^5+315*(a*b)^(1/2)*x^8*b^4+1260*
arctan(1/(a*b)^(1/2)*b*x)*x^7*a*b^4+1155*(a*b)^(1/2)*x^6*a*b^3+1890*arctan(
1/(a*b)^(1/2)*b*x)*x^5*a^2*b^3+1533*(a*b)^(1/2)*x^4*a^2*b^2+1260*arctan(1/(
a*b)^(1/2)*b*x)*x^3*a^3*b^2+837*(a*b)^(1/2)*x^2*a^3*b+315*arctan(1/(a*b)^(1
```

```
/2)*b*x)*x*a^4*b+128*(a*b)^(1/2)*a^4)*(b*x^2+a)/(a*b)^(1/2)/x/a^5/((b*x^2+a
)^2)^(5/2)
```

maxima [A] time = 3.04, size = 115, normalized size = 0.46

$$-\frac{315\,b^4x^8+1155\,ab^3x^6+1533\,a^2b^2x^4+837\,a^3bx^2+128\,a^4}{128\left(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4\,a^8bx^3+a^9x\right)}-\frac{315\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/128*(315*b^4*x^8 + 1155*a*b^3*x^6 + 1533*a^2*b^2*x^4 + 837*a^3*b*x^2 + 1
28*a^4)/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)
 - 315/128*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
```

```
[Out] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**2)**2)**(5/2)), x)
```

3.658 $\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=291

$$\frac{11}{48a^2x^3\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155b^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}+$$

```
[Out] 231/128/a^4/x^3/((b*x^2+a)^2)^(1/2)+1/8/a/x^3/(b*x^2+a)^3/((b*x^2+a)^2)^(1/
2)+11/48/a^2/x^3/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)+33/64/a^3/x^3/(b*x^2+a)/((
b*x^2+a)^2)^(1/2)-385/128*(b*x^2+a)/a^5/x^3/((b*x^2+a)^2)^(1/2)+1155/128*b*
(b*x^2+a)/a^6/x/((b*x^2+a)^2)^(1/2)+1155/128*b^(3/2)*(b*x^2+a)*arctan(x*b^(
1/2)/a^(1/2))/a^(13/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 290, 325, 205}

$$\frac{1155b\left(a+bx^2\right)}{128a^6x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{385\left(a+bx^2\right)}{128a^5x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{33}{64a^3x^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{48a^2x^3\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 231/(128*a^4*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*x^3*(a + b*x^2)^
3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 11/(48*a^2*x^3*(a + b*x^2)^2*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4]) + 33/(64*a^3*x^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (385*(a + b*x^2))/(128*a^5*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) + (1155*b*(a + b*x^2))/(128*a^6*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (115
5*b^(3/2)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(13/2)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
```

```
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$=\frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(11b^3\left(ab+b^2x^2\right)\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^4}\,dx}{8a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$=\frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{11}{48a^2x^3\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$=\frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{11}{48a^2x^3\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$=\frac{231}{128a^4x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{48a^2x^3}$$

$$=\frac{231}{128a^4x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{48a^2x^3}$$

$$=\frac{231}{128a^4x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{48a^2x^3}$$

$$=\frac{231}{128a^4x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{48a^2x^3}$$

Mathematica [A] time = 0.05, size = 127, normalized size = 0.44

$$\frac{\sqrt{a}\left(-128a^5+1408a^4bx^2+9207a^3b^2x^4+16863a^2b^3x^6+12705ab^4x^8+3465b^5x^{10}\right)+3465b^{3/2}x^3\left(a+bx^2\right)^4\tan}{384a^{13/2}x^3\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (Sqrt[a]*(-128*a^5 + 1408*a^4*b*x^2 + 9207*a^3*b^2*x^4 + 16863*a^2*b^3*x^6
+ 12705*a*b^4*x^8 + 3465*b^5*x^10) + 3465*b^(3/2)*x^3*(a + b*x^2)^4*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(384*a^(13/2)*x^3*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 1.04, size = 370, normalized size = 1.27

$$\left[\frac{6930\,b^5x^{10}+25410\,ab^4x^8+33726\,a^2b^3x^6+18414\,a^3b^2x^4+2816\,a^4bx^2-256\,a^5+3465\left(b^5x^{11}+4\,ab^4x^9+6\,a\right.}{768\left(a^6b^4x^{11}+4\,a^7b^3x^9+6\,a^8b^2x^7+4\,a^9bx^5+a^{10}x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(6930*b^5*x^10 + 25410*a*b^4*x^8 + 33726*a^2*b^3*x^6 + 18414*a^3*b^2
*x^4 + 2816*a^4*b*x^2 - 256*a^5 + 3465*(b^5*x^11 + 4*a*b^4*x^9 + 6*a^2*b^3*
x^7 + 4*a^3*b^2*x^5 + a^4*b*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)))/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x
^5 + a^10*x^3), 1/384*(3465*b^5*x^10 + 12705*a*b^4*x^8 + 16863*a^2*b^3*x^6
+ 9207*a^3*b^2*x^4 + 1408*a^4*b*x^2 - 128*a^5 + 3465*(b^5*x^11 + 4*a*b^4*x^
9 + 6*a^2*b^3*x^7 + 4*a^3*b^2*x^5 + a^4*b*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)
))/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 211, normalized size = 0.73

$$\frac{\left(3465b^6x^{11}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+13860a\,b^5x^9\arctan\left(\frac{bx}{\sqrt{ab}}\right)+3465\sqrt{ab}\;b^5x^{10}+20790a^2b^4x^7\arctan\left(\frac{bx}{\sqrt{ab}}\right)+12705\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/384*(3465*arctan(1/(a*b)^(1/2)*b*x)*x^11*b^6+3465*(a*b)^(1/2)*x^10*b^5+13
860*arctan(1/(a*b)^(1/2)*b*x)*x^9*a*b^5+12705*(a*b)^(1/2)*x^8*a*b^4+20790*a
```

```
rctan(1/(a*b)^(1/2)*b*x)*x^7*a^2*b^4+16863*(a*b)^(1/2)*x^6*a^2*b^3+13860*ar
ctan(1/(a*b)^(1/2)*b*x)*x^5*a^3*b^3+9207*(a*b)^(1/2)*x^4*a^3*b^2+3465*arcta
n(1/(a*b)^(1/2)*b*x)*x^3*a^4*b^2+1408*(a*b)^(1/2)*x^2*a^4*b-128*(a*b)^(1/2)
*a^5)*(b*x^2+a)/(a*b)^(1/2)/x^3/a^6/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 3.13, size = 130, normalized size = 0.45

$$\frac{3465\,b^5x^{10}+12705\,ab^4x^8+16863\,a^2b^3x^6+9207\,a^3b^2x^4+1408\,a^4bx^2-128\,a^5}{384\left(a^6b^4x^{11}+4\,a^7b^3x^9+6\,a^8b^2x^7+4\,a^9bx^5+a^{10}x^3\right)}+\frac{1155\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/384*(3465*b^5*x^10 + 12705*a*b^4*x^8 + 16863*a^2*b^3*x^6 + 9207*a^3*b^2*x
^4 + 1408*a^4*b*x^2 - 128*a^5)/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^
7 + 4*a^9*b*x^5 + a^10*x^3) + 1155/128*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
```

```
[Out] int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**4*((a + b*x**2)**2)**(5/2)), x)
```

3.659 $\int \frac{x^2}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=298

$$\frac{3x\left(a+bx^2\right)}{5b\sqrt[3]{a^2+2abx^2+b^2x^4}}+\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\left(\frac{bx^2}{a}+1\right)^{2/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}\cdots}{-\sqrt[3]{\frac{bx^2}{a}+1}\cdots}\right.\right.\cdots}{5b^2x\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}$$

```
[Out] 3/5*x*(b*x^2+a)/b/(b^2*x^4+2*a*b*x^2+a^2)^(1/3)+3/5*3^(3/4)*a^2*(1+b*x^2/a)
^(2/3)*(1-(1+b*x^2/a)^(1/3))*EllipticF((1-(1+b*x^2/a)^(1/3)+3^(1/2))/(1-(1+
b*x^2/a)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(1+b*x
^2/a)^(1/3)+(1+b*x^2/a)^(2/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^(1/2)/b^2/x
/(b^2*x^4+2*a*b*x^2+a^2)^(1/3)/((-1+(1+b*x^2/a)^(1/3))/(1-(1+b*x^2/a)^(1/3)
-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1113, 321, 236, 219}

$$\frac{3x\left(a+bx^2\right)}{5b\sqrt[3]{a^2+2abx^2+b^2x^4}}+\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\left(\frac{bx^2}{a}+1\right)^{2/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}\cdots}{-\sqrt[3]{\frac{bx^2}{a}+1}\cdots}\right.\right.\cdots}{5b^2x\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2))/(5*b*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3)) + (3*3^(3/4)*Sqrt
[2 - Sqrt[3]]*a^2*(1 + (b*x^2)/a)^(2/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1
 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^
2)/a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 -
 Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(5*b^2*x*(a^2 + 2*a*b*
x^2 + b^2*x^4)^(1/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 +
 (b*x^2)/a)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{2/3}\int \frac{x^2}{\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{3x\left(a+bx^2\right)}{5b\sqrt[3]{a^2+2abx^2+b^2x^4}} - \frac{\left(3a\left(1+\frac{bx^2}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{5b\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{3x\left(a+bx^2\right)}{5b\sqrt[3]{a^2+2abx^2+b^2x^4}} - \frac{\left(9a^2\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right)^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{1+\frac{bx^2}{a}}\right.}{10b^2x\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{3x\left(a+bx^2\right)}{5b\sqrt[3]{a^2+2abx^2+b^2x^4}} + \frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\left(1+\frac{bx^2}{a}\right)^{2/3}\left(1-\sqrt[3]{1+\frac{bx^2}{a}}\right)\sqrt{\frac{1+\sqrt[3]{1+\cdots}}{\left(1-\sqrt{3}\cdots\right.}}}{5b^2x\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

Mathematica [C] time = 0.03, size = 64, normalized size = 0.21

$$\frac{3x\left(-a\left(\frac{bx^2}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};-\frac{bx^2}{a}\right)+a+bx^2\right)}{5b\sqrt[3]{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2,
-((b*x^2)/a)]))/(5*b*((a + b*x^2)^2)^(1/3))
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="fricas")
[Out] integral(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="giac")
[Out] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b^2x^4 + 2ab\,x^2 + a^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
[Out] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="maxima")
[Out] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/3),x)
```

```
[Out] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[3]{\left(a+bx^2\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(1/3),x)
[Out] Integral(x**2/((a + b*x**2)**2)**(1/3), x)
```

3.660 $\int \frac{1}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=256

$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(\frac{bx^2}{a}+1\right)^{2/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{bx\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}$$

```
[Out] -3^(3/4)*a*(1+b*x^2/a)^(2/3)*(1-(1+b*x^2/a)^(1/3))*EllipticF((1-(1+b*x^2/a)
^(1/3)+3^(1/2))/(1-(1+b*x^2/a)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1
/2*2^(1/2))*((1+(1+b*x^2/a)^(1/3)+(1+b*x^2/a)^(2/3))/(1-(1+b*x^2/a)^(1/3)-3
^(1/2))^2)^(1/2)/b/x/(b^2*x^4+2*a*b*x^2+a^2)^(1/3)/((-1+(1+b*x^2/a)^(1/3))/
(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1089, 236, 219}

$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(\frac{bx^2}{a}+1\right)^{2/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{bx\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1/3),x]
```

```
[Out] -((3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(1 + (b*x^2)/a)^(2/3)*(1 - (1 + (b*x^2)/a)^(
1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3]
 - (1 + (b*x^2)/a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a
)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(b*x*(a^2
 + 2*a*b*x^2 + b^2*x^4)^(1/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[
3] - (1 + (b*x^2)/a)^(1/3))^2)]))
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(3a\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{1+\frac{bx^2}{a}}\right)}{2bx\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(1+\frac{bx^2}{a}\right)^{2/3}\left(1-\sqrt[3]{1+\frac{bx^2}{a}}\right)\sqrt{\frac{1+\sqrt[3]{1+\frac{bx^2}{a}}+\left(1+\frac{bx^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}\cdots}{1-\sqrt{3}\cdots}\right.\right.}{bx\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{1+\frac{bx^2}{a}}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.19

$$\frac{x\left(\frac{bx^2}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[3]{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1/3),x]
[Out] (x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, -((b*x^2)/a)])/((
a + b*x^2)^2)^(1/3)
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="fricas")
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="giac")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/3), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b^2x^4+2ab\,x^2+a^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

```
[Out] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="maxima")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/3),x)
[Out] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a^2 + 2abx^2 + b^2x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(1/3),x)
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-1/3), x)
```

3.661 $\int \frac{1}{x^2 \sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=289

$$\frac{\sqrt{2-\sqrt{3}}\left(\frac{bx^2}{a}+1\right)^{2/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,x\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}-\frac{a+}{ax\sqrt[3]{a^2+2}}$$

```
[Out] (-b*x^2-a)/a/x/(b^2*x^4+2*a*b*x^2+a^2)^(1/3)+1/3*(1+b*x^2/a)^(2/3)*(1-(1+b*
x^2/a)^(1/3))*EllipticF((1-(1+b*x^2/a)^(1/3)+3^(1/2))/(1-(1+b*x^2/a)^(1/3)-
3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(1+b*x^2/a)^(1/3)+(1+
b*x^2/a)^(2/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^(1/2)*3^(3/4)/x/(b^2*x^4+2
*a*b*x^2+a^2)^(1/3)/((-1+(1+b*x^2/a)^(1/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2
)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1113, 325, 236, 219}

$$\frac{\sqrt{2-\sqrt{3}}\left(\frac{bx^2}{a}+1\right)^{2/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,x\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}-\frac{a+}{ax\sqrt[3]{a^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3)),x]
```

```
[Out] -((a + b*x^2)/(a*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3))) + (Sqrt[2 - Sqrt[3]]
*(1 + (b*x^2)/a)^(2/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a
)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*E
llipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (
b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^
(1/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/
3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{2/3}\int \frac{1}{x^2\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{a+bx^2}{ax\sqrt[3]{a^2+2abx^2+b^2x^4}} - \frac{\left(b\left(1+\frac{bx^2}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{3a\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{a+bx^2}{ax\sqrt[3]{a^2+2abx^2+b^2x^4}} - \frac{\left(\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{1+\frac{bx^2}{a}}\right.}{2x\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{a+bx^2}{ax\sqrt[3]{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{2-\sqrt{3}}\left(1+\frac{bx^2}{a}\right)^{2/3}\left(1-\sqrt[3]{1+\frac{bx^2}{a}}\right)\sqrt{\frac{1+\sqrt[3]{1+\frac{bx^2}{a}}+\left(1\right.}{\left(1-\sqrt{3}-\sqrt[3]{1-\right.}}}}{\sqrt[4]{3}\,x\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.18

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[3]{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3)),x]
```

```
[Out] -(((1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-1/2, 2/3, 1/2, -((b*x^2)/a)])/(
x*((a + b*x^2)^2)^(1/3)))
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{2}{3}}}{b^2x^6+2\,abx^4+a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3)*x^2), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^4 + 2ab\,x^2 + a^2\right)^{\frac{1}{3}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

```
[Out] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/3)),x)
[Out] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt[3]{\left(a+bx^2\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(1/3),x)
[Out] Integral(1/(x**2*((a + b*x**2)**2)**(1/3)), x)
```

3.662 $\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}\,dx$

Optimal. Leaf size=618

$$-\frac{9ax\left(\frac{bx^2}{a}+1\right)^{4/3}}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)}-\frac{3x\left(a+bx^2\right)}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}}-\frac{3\,3^{3/4}a^2\left(\frac{bx^2}{a}+1\right)^{4/3}\left(1-\sqrt[3]{\frac{bx^2}{a}}\cdots\right.}{\sqrt{2}\,b^2x\left(\cdots\right.}$$

```
[Out] -3/2*x*(b*x^2+a)/b/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)-9/2*a*x*(1+b*x^2/a)^(4/3)/
b/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)/(1-(1+b*x^2/a)^(1/3)-3^(1/2))-3/2*3^(3/4)*a
^2*(1+b*x^2/a)^(4/3)*(1-(1+b*x^2/a)^(1/3))*EllipticF((1-(1+b*x^2/a)^(1/3)+3
^(1/2))/(1-(1+b*x^2/a)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(1+b*x^2/a)^(1/3)+
(1+b*x^2/a)^(2/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^(1/2)/b^2/x/(b^2*x^4+2*
a*b*x^2+a^2)^(2/3)*2^(1/2)/((-1+(1+b*x^2/a)^(1/3))/(1-(1+b*x^2/a)^(1/3)-3^(
1/2))^2)^(1/2)+9/4*3^(1/4)*a^2*(1+b*x^2/a)^(4/3)*(1-(1+b*x^2/a)^(1/3))*Elli
pticE((1-(1+b*x^2/a)^(1/3)+3^(1/2))/(1-(1+b*x^2/a)^(1/3)-3^(1/2)),2*I-I*3^(
1/2))*((1+(1+b*x^2/a)^(1/3)+(1+b*x^2/a)^(2/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2)
)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^2/x/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)/((
-1+(1+b*x^2/a)^(1/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 618, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1113, 288, 235, 304, 219, 1879}

$$-\frac{9ax\left(\frac{bx^2}{a}+1\right)^{4/3}}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)}-\frac{3x\left(a+bx^2\right)}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}}-\frac{3\,3^{3/4}a^2\left(\frac{bx^2}{a}+1\right)^{4/3}\left(1-\sqrt[3]{\frac{bx^2}{a}}\cdots\right.}{\sqrt{2}\,b^2x\left(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3),x]
```

```
[Out] (-3*x*(a + b*x^2))/(2*b*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) - (9*a*x*(1 + (b
*x^2)/a)^(4/3))/(2*b*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*(1 - Sqrt[3] - (1 +
(b*x^2)/a)^(1/3))) + (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^2*(1 + (b*x^2)/a)^(4/3)
*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)
/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticE[ArcSin[(1 + S
qrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7
+ 4*Sqrt[3]])/(4*b^2*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1 + (
b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)]) - (3*3^(3/4)*a^
2*(1 + (b*x^2)/a)^(4/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/
a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*
EllipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 +
(b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[2]*b^2*x*(a^2 + 2*a*b*x^2 + b^2*
x^4)^(2/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a
)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{4/3}\int \frac{x^2}{\left(1+\frac{bx^2}{a}\right)^{4/3}}\,dx}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$$= -\frac{3x\left(a+bx^2\right)}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}} + \frac{\left(3a\left(1+\frac{bx^2}{a}\right)^{4/3}\right)\int \frac{1}{\sqrt[3]{1+\frac{bx^2}{a}}}\,dx}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$$= -\frac{3x\left(a+bx^2\right)}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}} + \frac{\left(9a^2\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right)^{4/3}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{1\right.}{4b^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$$= -\frac{3x\left(a+bx^2\right)}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{\left(9a^2\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right)^{4/3}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{1\right.}{4b^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$$= -\frac{3x\left(a+bx^2\right)}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{9ax\left(1+\frac{bx^2}{a}\right)^{4/3}}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)} + $$

Mathematica [C] time = 0.03, size = 64, normalized size = 0.10

$$\frac{3x\left(a+bx^2\right)\left(\sqrt[3]{\frac{bx^2}{a}+1}\ {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)-1\right)}{2b\left(\left(a+bx^2\right)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3),x]
```

```
[Out] (3*x*(a + b*x^2)*(-1 + (1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/
2, -((b*x^2)/a)]))/(2*b*((a + b*x^2)^2)^(2/3))
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{2}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b^2x^4+2ab\,x^2+a^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

```
[Out] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(2/3),x)
[Out] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\left(a+bx^2\right)^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(2/3),x)
[Out] Integral(x**2/((a + b*x**2)**2)**(2/3), x)
```

3.663 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}\,dx$

Optimal. Leaf size=609

$$\frac{3x\left(\frac{bx^2}{a}+1\right)^{4/3}}{2\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)}+\frac{3x\left(a+bx^2\right)}{2a\left(a^2+2abx^2+b^2x^4\right)^{2/3}}+\frac{3^{3/4}a\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\cdots}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}\cdots\right.}}}{\sqrt{2}\,bx\left(a^2+\cdots\right.}$$

```
[Out] 3/2*x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)+3/2*x*(1+b*x^2/a)^(4/3)/(b^
2*x^4+2*a*b*x^2+a^2)^(2/3)/(1-(1+b*x^2/a)^(1/3)-3^(1/2))+1/2*3^(3/4)*a*(1+b
*x^2/a)^(4/3)*(1-(1+b*x^2/a)^(1/3))*EllipticF((1-(1+b*x^2/a)^(1/3)+3^(1/2))
/(1-(1+b*x^2/a)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(1+b*x^2/a)^(1/3)+(1+b*x^
2/a)^(2/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^(1/2)/b/x/(b^2*x^4+2*a*b*x^2+a
^2)^(2/3)*2^(1/2)/((-1+(1+b*x^2/a)^(1/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^
(1/2)-3/4*3^(1/4)*a*(1+b*x^2/a)^(4/3)*(1-(1+b*x^2/a)^(1/3))*EllipticE((1-(1
+b*x^2/a)^(1/3)+3^(1/2))/(1-(1+b*x^2/a)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(
1+b*x^2/a)^(1/3)+(1+b*x^2/a)^(2/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^(1/2)*
(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)/((-1+(1+b*x^2/a
)^(1/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 609, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1089, 199, 235, 304, 219, 1879}

$$\frac{3x\left(\frac{bx^2}{a}+1\right)^{4/3}}{2\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)}+\frac{3x\left(a+bx^2\right)}{2a\left(a^2+2abx^2+b^2x^4\right)^{2/3}}+\frac{3^{3/4}a\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\cdots}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}\cdots\right.}}}{\sqrt{2}\,bx\left(a^2+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2/3),x]
```

```
[Out] (3*x*(a + b*x^2))/(2*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) + (3*x*(1 + (b*x^
2)/a)^(4/3))/(2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*(1 - Sqrt[3] - (1 + (b*x^
```

```
2)/a)^(1/3))) - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*a*(1 + (b*x^2)/a)^(4/3)*(1 - (
1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/
3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3]
- (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqr
t[3]])/(4*b*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1 + (b*x^2)/a)
^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)]) + (3^(3/4)*a*(1 + (b*x^2
)/a)^(4/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1
 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticF[Arc
Sin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1
/3))], -7 + 4*Sqrt[3]])/(Sqrt[2]*b*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt
[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
```

```
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{4/3}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{4/3}}\,dx}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$$= \frac{3x\left(a+bx^2\right)}{2a\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{\left(1+\frac{bx^2}{a}\right)^{4/3}\int \frac{1}{\sqrt[3]{1+\frac{bx^2}{a}}}\,dx}{2\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$$= \frac{3x\left(a+bx^2\right)}{2a\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{\left(3a\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right)^{4/3}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{1+\right.}{4bx\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$$= \frac{3x\left(a+bx^2\right)}{2a\left(a^2+2abx^2+b^2x^4\right)^{2/3}} + \frac{\left(3a\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right)^{4/3}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{1-\right.}{4bx\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

3

$$= \frac{3x\left(a+bx^2\right)}{2a\left(a^2+2abx^2+b^2x^4\right)^{2/3}} + \frac{3x\left(1+\frac{bx^2}{a}\right)^{4/3}}{2\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)} - -$$

Mathematica [C] time = 0.02, size = 64, normalized size = 0.11

$$-\frac{x\left(a+bx^2\right)\left(\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)-3\right)}{2a\left(\left(a+bx^2\right)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2/3),x]
```

```
[Out] -1/2*(x*(a + b*x^2)*(-3 + (1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2,
 3/2, -((b*x^2)/a)]))/(a*((a + b*x^2)^2)^(2/3))
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{2}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^(-2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-2/3), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^4+2ab\,x^2+a^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

```
[Out] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="maxima")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(2/3),x)
[Out] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2 + 2abx^2 + b^2x^4\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(2/3),x)
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-2/3), x)
```

3.664 $\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^{2/3}}\,dx$

Optimal. Leaf size=649

$$-\frac{5\left(a+bx^2\right)^2}{2a^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}}+\frac{3\left(a+bx^2\right)}{2ax\left(a^2+2abx^2+b^2x^4\right)^{2/3}}-\frac{5bx\left(\frac{bx^2}{a}+1\right)^{4/3}}{2a\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)}-\ 5$$

```
[Out] 3/2*(b*x^2+a)/a/x/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)-5/2*(b*x^2+a)^2/a^2/x/(b^2*
x^4+2*a*b*x^2+a^2)^(2/3)-5/2*b*x*(1+b*x^2/a)^(4/3)/a/(b^2*x^4+2*a*b*x^2+a^2
)^(2/3)/(1-(1+b*x^2/a)^(1/3)-3^(1/2))-5/6*(1+b*x^2/a)^(4/3)*(1-(1+b*x^2/a)^
(1/3))*EllipticF((1-(1+b*x^2/a)^(1/3)+3^(1/2))/(1-(1+b*x^2/a)^(1/3)-3^(1/2)
),2*I-I*3^(1/2))*((1+(1+b*x^2/a)^(1/3)+(1+b*x^2/a)^(2/3))/(1-(1+b*x^2/a)^(1
/3)-3^(1/2))^2)^(1/2)*3^(3/4)/x/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)*2^(1/2)/((-1+
(1+b*x^2/a)^(1/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^(1/2)+5/4*3^(1/4)*(1+b*
x^2/a)^(4/3)*(1-(1+b*x^2/a)^(1/3))*EllipticE((1-(1+b*x^2/a)^(1/3)+3^(1/2))/
(1-(1+b*x^2/a)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(1+b*x^2/a)^(1/3)+(1+b*x^2
/a)^(2/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))
/x/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)/((-1+(1+b*x^2/a)^(1/3))/(1-(1+b*x^2/a)^(1/
3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 649, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1113, 290, 325, 235, 304, 219, 1879}

$$-\frac{5\left(a+bx^2\right)^2}{2a^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}}+\frac{3\left(a+bx^2\right)}{2ax\left(a^2+2abx^2+b^2x^4\right)^{2/3}}-\frac{5bx\left(\frac{bx^2}{a}+1\right)^{4/3}}{2a\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)}-\ 5$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)),x]
```

```
[Out] (3*(a + b*x^2))/(2*a*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) - (5*(a + b*x^2)^
2)/(2*a^2*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) - (5*b*x*(1 + (b*x^2)/a)^(4/
3))/(2*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*(1 - Sqrt[3] - (1 + (b*x^2)/a)^(
1/3))) + (5*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + (b*x^2)/a)^(4/3)*(1 - (1 + (b*x^
2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 -
Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 + (b
*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(4
*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1
- Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)]) - (5*(1 + (b*x^2)/a)^(4/3)*(1 - (1
+ (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3)
)/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
(1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[
3]])/(Sqrt[2]*3^(1/4)*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1 +
(b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^{2/3}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{4/3}\int \frac{1}{x^2\left(1+\frac{bx^2}{a}\right)^{4/3}}\,dx}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$$= \frac{3\left(a+bx^2\right)}{2ax\left(a^2+2abx^2+b^2x^4\right)^{2/3}} + \frac{\left(5\left(1+\frac{bx^2}{a}\right)^{4/3}\right)\int \frac{1}{x^2\sqrt[3]{1+\frac{bx^2}{a}}}\,dx}{2\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$$= \frac{3\left(a+bx^2\right)}{2ax\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{5\left(a+bx^2\right)^2}{2a^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}} + \frac{\left(5b\left(1+\frac{bx^2}{a}\right)^{4/3}\right.}{6a\left(a^2+2abx\right.}$$

$$= \frac{3\left(a+bx^2\right)}{2ax\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{5\left(a+bx^2\right)^2}{2a^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}} + \frac{\left(5\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right.\right.}{4}$$

$$= \frac{3\left(a+bx^2\right)}{2ax\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{5\left(a+bx^2\right)^2}{2a^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{\left(5\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right.\right.}{4}$$

$$= \frac{3\left(a+bx^2\right)}{2ax\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{5\left(a+bx^2\right)^2}{2a^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{5}{2a\left(a^2+2abx^2+\right.}$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.09

$$-\frac{\left(a+bx^2\right)\sqrt[3]{\frac{bx^2}{a}+1}\ {}_2F_1\left(-\frac{1}{2},\frac{4}{3};\frac{1}{2};-\frac{bx^2}{a}\right)}{ax\left(\left(a+bx^2\right)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)),x]
```

[Out] -(((a + b*x^2)*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[-1/2, 4/3, 1/2, -((b*x^2)/a)])/(a*x*((a + b*x^2)^2)^(2/3)))

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(b^{2} x^{4} + 2 \, a b x^{2} + a^{2}\right)^{\frac{1}{3}}}{b^{2} x^{6} + 2 \, a b x^{4} + a^{2} x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="fricas")

[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{{\left(b^{2} x^{4} + 2 \, a b x^{2} + a^{2}\right)}^{\frac{2}{3}} x^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="giac")

[Out] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3)*x^2), x)

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{\frac{2}{3}} x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)

[Out] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{{\left(b^{2} x^{4} + 2 \, a b x^{2} + a^{2}\right)}^{\frac{2}{3}} x^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(2/3)),x)
[Out] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(2/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(\left(a + b x^{2}\right)^{2}\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(2/3),x)
[Out] Integral(1/(x**2*((a + b*x**2)**2)**(2/3)), x)
```

3.665 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right) \, dx$

Optimal. Leaf size=51

$$\frac{2a^2(dx)^{7/2}}{7d}+\frac{4ab(dx)^{11/2}}{11d^3}+\frac{2b^2(dx)^{15/2}}{15d^5}$$

```
[Out] 2/7*a^2*(d*x)^(7/2)/d+4/11*a*b*(d*x)^(11/2)/d^3+2/15*b^2*(d*x)^(15/2)/d^5
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {14}

$$\frac{2a^2(dx)^{7/2}}{7d}+\frac{4ab(dx)^{11/2}}{11d^3}+\frac{2b^2(dx)^{15/2}}{15d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*a^2*(d*x)^(7/2))/(7*d) + (4*a*b*(d*x)^(11/2))/(11*d^3) + (2*b^2*(d*x)^(1
5/2))/(15*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)\,dx&=\int\left(a^2(dx)^{5/2}+\frac{2ab(dx)^{9/2}}{d^2}+\frac{b^2(dx)^{13/2}}{d^4}\right)dx\\&=\frac{2a^2(dx)^{7/2}}{7d}+\frac{4ab(dx)^{11/2}}{11d^3}+\frac{2b^2(dx)^{15/2}}{15d^5}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.65

$$\frac{2x(dx)^{5/2}\left(165a^2+210abx^2+77b^2x^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*x*(d*x)^(5/2)*(165*a^2 + 210*a*b*x^2 + 77*b^2*x^4))/1155
```

fricas [A] time = 0.63, size = 40, normalized size = 0.78

$$\frac{2}{1155}\left(77\,b^2d^2x^7 + 210\,abd^2x^5 + 165\,a^2d^2x^3\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^2*d^2*x^7 + 210*a*b*d^2*x^5 + 165*a^2*d^2*x^3)*sqrt(d*x)
```

giac [A] time = 0.17, size = 48, normalized size = 0.94

$$\frac{2}{15}\sqrt{dx}\,b^2d^2x^7 + \frac{4}{11}\sqrt{dx}\,abd^2x^5 + \frac{2}{7}\sqrt{dx}\,a^2d^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 2/15*sqrt(d*x)*b^2*d^2*x^7 + 4/11*sqrt(d*x)*a*b*d^2*x^5 + 2/7*sqrt(d*x)*a^2
*d^2*x^3
```

maple [A] time = 0.01, size = 30, normalized size = 0.59

$$\frac{2\left(77b^2x^4 + 210ab\,x^2 + 165a^2\right)(dx)^{\frac{5}{2}}\,x}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 2/1155*x*(77*b^2*x^4+210*a*b*x^2+165*a^2)*(d*x)^(5/2)
```

maxima [A] time = 1.30, size = 41, normalized size = 0.80

$$\frac{2\left(77\,(dx)^{\frac{15}{2}}\,b^2 + 210\,(dx)^{\frac{11}{2}}\,abd^2 + 165\,(dx)^{\frac{7}{2}}\,a^2d^4\right)}{1155\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 2/1155*(77*(d*x)^(15/2)*b^2 + 210*(d*x)^(11/2)*a*b*d^2 + 165*(d*x)^(7/2)*a^
2*d^4)/d^5
```

mupad [B] time = 0.07, size = 40, normalized size = 0.78

$$\frac{\frac{2\,b^2\,{\left(d\,x\right)}^{15/2}}{15}+\frac{2\,a^2\,d^4\,{\left(d\,x\right)}^{7/2}}{7}+\frac{4\,a\,b\,d^2\,{\left(d\,x\right)}^{11/2}}{11}}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] ((2*b^2*(d*x)^(15/2))/15 + (2*a^2*d^4*(d*x)^(7/2))/7 + (4*a*b*d^2*(d*x)^(11
/2))/11)/d^5
```

sympy [A] time = 2.67, size = 49, normalized size = 0.96

$$\frac{2a^2d^{\frac{5}{2}}x^{\frac{7}{2}}}{7}+\frac{4abd^{\frac{5}{2}}x^{\frac{11}{2}}}{11}+\frac{2b^2d^{\frac{5}{2}}x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 2*a**2*d**(5/2)*x**(7/2)/7 + 4*a*b*d**(5/2)*x**(11/2)/11 + 2*b**2*d**(5/2)*
x**(15/2)/15
```

3.666 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right) \, dx$

Optimal. Leaf size=51

$$\frac{2a^2(dx)^{5/2}}{5d}+\frac{4ab(dx)^{9/2}}{9d^3}+\frac{2b^2(dx)^{13/2}}{13d^5}$$

```
[Out] 2/5*a^2*(d*x)^(5/2)/d+4/9*a*b*(d*x)^(9/2)/d^3+2/13*b^2*(d*x)^(13/2)/d^5
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {14}

$$\frac{2a^2(dx)^{5/2}}{5d}+\frac{4ab(dx)^{9/2}}{9d^3}+\frac{2b^2(dx)^{13/2}}{13d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*a^2*(d*x)^(5/2))/(5*d) + (4*a*b*(d*x)^(9/2))/(9*d^3) + (2*b^2*(d*x)^(13/
2))/(13*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)\,dx &= \int \left(a^2(dx)^{3/2}+\frac{2ab(dx)^{7/2}}{d^2}+\frac{b^2(dx)^{11/2}}{d^4}\right)dx\\ &= \frac{2a^2(dx)^{5/2}}{5d}+\frac{4ab(dx)^{9/2}}{9d^3}+\frac{2b^2(dx)^{13/2}}{13d^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.65

$$\frac{2}{585}x(dx)^{3/2}\left(117a^2+130abx^2+45b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*x*(d*x)^(3/2)*(117*a^2 + 130*a*b*x^2 + 45*b^2*x^4))/585
```

fricas [A] time = 0.87, size = 34, normalized size = 0.67

$$\frac{2}{585}\left(45\,b^2dx^6+130\,abdx^4+117\,a^2dx^2\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*b^2*d*x^6 + 130*a*b*d*x^4 + 117*a^2*d*x^2)*sqrt(d*x)
```

giac [A] time = 0.15, size = 42, normalized size = 0.82

$$\frac{2}{585}\left(45\,\sqrt{dx}\,b^2x^6+130\,\sqrt{dx}\,abx^4+117\,\sqrt{dx}\,a^2x^2\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 2/585*(45*sqrt(d*x)*b^2*x^6 + 130*sqrt(d*x)*a*b*x^4 + 117*sqrt(d*x)*a^2*x^2
)*d
```

maple [A] time = 0.01, size = 30, normalized size = 0.59

$$\frac{2\left(45b^2x^4+130ab\,x^2+117a^2\right)(dx)^{\frac{3}{2}}\,x}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 2/585*x*(45*b^2*x^4+130*a*b*x^2+117*a^2)*(d*x)^(3/2)
```

maxima [A] time = 1.34, size = 41, normalized size = 0.80

$$\frac{2\left(45\,(dx)^{\frac{13}{2}}\,b^2+130\,(dx)^{\frac{9}{2}}\,abd^2+117\,(dx)^{\frac{5}{2}}\,a^2d^4\right)}{585\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 2/585*(45*(d*x)^(13/2)*b^2 + 130*(d*x)^(9/2)*a*b*d^2 + 117*(d*x)^(5/2)*a^2*
d^4)/d^5
```

mupad [B] time = 4.22, size = 41, normalized size = 0.80

$$\frac{90\,b^2\,(d\,x)^{13/2}+234\,a^2\,d^4\,(d\,x)^{5/2}+260\,a\,b\,d^2\,(d\,x)^{9/2}}{585\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (90*b^2*(d*x)^(13/2) + 234*a^2*d^4*(d*x)^(5/2) + 260*a*b*d^2*(d*x)^(9/2))/(
585*d^5)
```

sympy [A] time = 1.24, size = 49, normalized size = 0.96

$$\frac{2a^2d^{\frac{3}{2}}x^{\frac{5}{2}}}{5}+\frac{4abd^{\frac{3}{2}}x^{\frac{9}{2}}}{9}+\frac{2b^2d^{\frac{3}{2}}x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 2*a**2*d**(3/2)*x**(5/2)/5 + 4*a*b*d**(3/2)*x**(9/2)/9 + 2*b**2*d**(3/2)*x*
*(13/2)/13
```

3.667 $\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)\,dx$

Optimal. Leaf size=51

$$\frac{2a^2(dx)^{3/2}}{3d}+\frac{4ab(dx)^{7/2}}{7d^3}+\frac{2b^2(dx)^{11/2}}{11d^5}$$

```
[Out] 2/3*a^2*(d*x)^(3/2)/d+4/7*a*b*(d*x)^(7/2)/d^3+2/11*b^2*(d*x)^(11/2)/d^5
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {14}

$$\frac{2a^2(dx)^{3/2}}{3d}+\frac{4ab(dx)^{7/2}}{7d^3}+\frac{2b^2(dx)^{11/2}}{11d^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*a^2*(d*x)^(3/2))/(3*d) + (4*a*b*(d*x)^(7/2))/(7*d^3) + (2*b^2*(d*x)^(11/
2))/(11*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)\,dx&=\int\left(a^2\sqrt{dx}+\frac{2ab(dx)^{5/2}}{d^2}+\frac{b^2(dx)^{9/2}}{d^4}\right)dx\\&=\frac{2a^2(dx)^{3/2}}{3d}+\frac{4ab(dx)^{7/2}}{7d^3}+\frac{2b^2(dx)^{11/2}}{11d^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.65

$$\frac{2}{231}x\sqrt{dx}\left(77a^2+66abx^2+21b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*x*Sqrt[d*x]*(77*a^2 + 66*a*b*x^2 + 21*b^2*x^4))/231
```

fricas [A] time = 0.63, size = 29, normalized size = 0.57

$$\frac{2}{231}\left(21\,b^2x^5 + 66\,abx^3 + 77\,a^2x\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*b^2*x^5 + 66*a*b*x^3 + 77*a^2*x)*sqrt(d*x)
```

giac [A] time = 0.15, size = 37, normalized size = 0.73

$$\frac{2}{11}\sqrt{dx}\,b^2x^5 + \frac{4}{7}\sqrt{dx}\,abx^3 + \frac{2}{3}\sqrt{dx}\,a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/11*sqrt(d*x)*b^2*x^5 + 4/7*sqrt(d*x)*a*b*x^3 + 2/3*sqrt(d*x)*a^2*x
```

maple [A] time = 0.01, size = 30, normalized size = 0.59

$$\frac{2\left(21b^2x^4 + 66ab\,x^2 + 77a^2\right)\sqrt{dx}\;x}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)*(d*x)^(1/2),x)
```

```
[Out] 2/231*x*(21*b^2*x^4+66*a*b*x^2+77*a^2)*(d*x)^(1/2)
```

maxima [A] time = 1.36, size = 41, normalized size = 0.80

$$\frac{2\left(21\,(dx)^{\frac{11}{2}}\,b^2 + 66\,(dx)^{\frac{7}{2}}\,abd^2 + 77\,(dx)^{\frac{3}{2}}\,a^2d^4\right)}{231\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/231*(21*(d*x)^(11/2)*b^2 + 66*(d*x)^(7/2)*a*b*d^2 + 77*(d*x)^(3/2)*a^2*d^
4)/d^5
```

mupad [B] time = 0.05, size = 41, normalized size = 0.80

$$\frac{42\,b^2\,{\left(d\,x\right)}^{11/2}+154\,a^2\,d^4\,{\left(d\,x\right)}^{3/2}+132\,a\,b\,d^2\,{\left(d\,x\right)}^{7/2}}{231\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (42*b^2*(d*x)^(11/2) + 154*a^2*d^4*(d*x)^(3/2) + 132*a*b*d^2*(d*x)^(7/2))/(
231*d^5)
```

sympy [A] time = 0.48, size = 49, normalized size = 0.96

$$\frac{2a^2\sqrt{d}\,x^{\frac{3}{2}}}{3}+\frac{4ab\sqrt{d}\,x^{\frac{7}{2}}}{7}+\frac{2b^2\sqrt{d}\,x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)*(d*x)**(1/2),x)
```

```
[Out] 2*a**2*sqrt(d)*x**(3/2)/3 + 4*a*b*sqrt(d)*x**(7/2)/7 + 2*b**2*sqrt(d)*x**(1
1/2)/11
```

3.668 $\int \frac{a^2+2abx^2+b^2x^4}{\sqrt{dx}} dx$

Optimal. Leaf size=49

$$\frac{2a^2\sqrt{dx}}{d}+\frac{4ab(dx)^{5/2}}{5d^3}+\frac{2b^2(dx)^{9/2}}{9d^5}$$

```
[Out] 4/5*a*b*(d*x)^(5/2)/d^3+2/9*b^2*(d*x)^(9/2)/d^5+2*a^2*(d*x)^(1/2)/d
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {14}

$$\frac{2a^2\sqrt{dx}}{d}+\frac{4ab(dx)^{5/2}}{5d^3}+\frac{2b^2(dx)^{9/2}}{9d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/Sqrt[d*x],x]
```

```
[Out] (2*a^2*Sqrt[d*x])/d + (4*a*b*(d*x)^(5/2))/(5*d^3) + (2*b^2*(d*x)^(9/2))/(9*
d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{\sqrt{dx}}\,dx &= \int \left(\frac{a^2}{\sqrt{dx}}+\frac{2ab(dx)^{3/2}}{d^2}+\frac{b^2(dx)^{7/2}}{d^4}\right)dx\\ &= \frac{2a^2\sqrt{dx}}{d}+\frac{4ab(dx)^{5/2}}{5d^3}+\frac{2b^2(dx)^{9/2}}{9d^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.67

$$\frac{2\left(45a^2x+18abx^3+5b^2x^5\right)}{45\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/Sqrt[d*x],x]
[Out] (2*(45*a^2*x + 18*a*b*x^3 + 5*b^2*x^5))/(45*Sqrt[d*x])
```

fricas [A] time = 0.98, size = 31, normalized size = 0.63

$$\frac{2\left(5\,b^2x^4+18\,abx^2+45\,a^2\right)\sqrt{dx}}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="fricas")
[Out] 2/45*(5*b^2*x^4 + 18*a*b*x^2 + 45*a^2)*sqrt(d*x)/d
```

giac [A] time = 0.15, size = 41, normalized size = 0.84

$$\frac{2\left(5\,\sqrt{dx}\,b^2x^4+18\,\sqrt{dx}\,abx^2+45\,\sqrt{dx}\,a^2\right)}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="giac")
[Out] 2/45*(5*sqrt(d*x)*b^2*x^4 + 18*sqrt(d*x)*a*b*x^2 + 45*sqrt(d*x)*a^2)/d
```

maple [A] time = 0.01, size = 30, normalized size = 0.61

$$\frac{2\left(5b^2x^4+18ab\,x^2+45a^2\right)x}{45\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x)
[Out] 2/45*(5*b^2*x^4+18*a*b*x^2+45*a^2)*x/(d*x)^(1/2)
```

maxima [A] time = 1.27, size = 41, normalized size = 0.84

$$\frac{2\left(45\,\sqrt{dx}\,a^2+\frac{5\,(dx)^{\frac{9}{2}}\,b^2}{d^4}+\frac{18\,(dx)^{\frac{5}{2}}\,ab}{d^2}\right)}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="maxima")
```

[Out] 2/45*(45*sqrt(d*x)*a^2 + 5*(d*x)^(9/2)*b^2/d^4 + 18*(d*x)^(5/2)*a*b/d^2)/d

mupad [B] time = 0.05, size = 41, normalized size = 0.84

$$\frac{10\,b^2\,{\left(d\,x\right)}^{9/2}+90\,a^2\,d^4\,\sqrt{d\,x}+36\,a\,b\,d^2\,{\left(d\,x\right)}^{5/2}}{45\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/(d*x)^(1/2),x)
```

```
[Out] (10*b^2*(d*x)^(9/2) + 90*a^2*d^4*(d*x)^(1/2) + 36*a*b*d^2*(d*x)^(5/2))/(45*
d^5)
```

sympy [A] time = 0.63, size = 48, normalized size = 0.98

$$\frac{2a^2\sqrt{x}}{\sqrt{d}}+\frac{4abx^{\frac{5}{2}}}{5\sqrt{d}}+\frac{2b^2x^{\frac{9}{2}}}{9\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(1/2),x)
```

```
[Out] 2*a**2*sqrt(x)/sqrt(d) + 4*a*b*x**(5/2)/(5*sqrt(d)) + 2*b**2*x**(9/2)/(9*sq
rt(d))
```

3.669 $\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{2a^2}{d\sqrt{dx}}+\frac{4ab(dx)^{3/2}}{3d^3}+\frac{2b^2(dx)^{7/2}}{7d^5}$$

```
[Out] 4/3*a*b*(d*x)^(3/2)/d^3+2/7*b^2*(d*x)^(7/2)/d^5-2*a^2/d/(d*x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {14}

$$-\frac{2a^2}{d\sqrt{dx}}+\frac{4ab(dx)^{3/2}}{3d^3}+\frac{2b^2(dx)^{7/2}}{7d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(3/2),x]
```

```
[Out] (-2*a^2)/(d*Sqrt[d*x]) + (4*a*b*(d*x)^(3/2))/(3*d^3) + (2*b^2*(d*x)^(7/2))/
(7*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{3/2}}\,dx &= \int \left(\frac{a^2}{(dx)^{3/2}}+\frac{2ab\sqrt{dx}}{d^2}+\frac{b^2(dx)^{5/2}}{d^4}\right)dx\\ &= -\frac{2a^2}{d\sqrt{dx}}+\frac{4ab(dx)^{3/2}}{3d^3}+\frac{2b^2(dx)^{7/2}}{7d^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.67

$$\frac{2x\left(-21a^2+14abx^2+3b^2x^4\right)}{21(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(3/2),x]
```

```
[Out] (2*x*(-21*a^2 + 14*a*b*x^2 + 3*b^2*x^4))/(21*(d*x)^(3/2))
```

fricas [A] time = 0.73, size = 34, normalized size = 0.69

$$\frac{2\left(3\,b^2x^4+14\,abx^2-21\,a^2\right)\sqrt{dx}}{21\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*b^2*x^4 + 14*a*b*x^2 - 21*a^2)*sqrt(d*x)/(d^2*x)
```

giac [A] time = 0.22, size = 51, normalized size = 1.04

$$-\frac{2\left(\frac{21\,a^2}{\sqrt{dx}}-\frac{3\,\sqrt{dx}\,b^2d^{27}x^3+14\,\sqrt{dx}\,abd^{27}x}{d^{28}}\right)}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/21*(21*a^2/sqrt(d*x) - (3*sqrt(d*x)*b^2*d^27*x^3 + 14*sqrt(d*x)*a*b*d^27
*x)/d^28)/d
```

maple [A] time = 0.01, size = 30, normalized size = 0.61

$$-\frac{2\left(-3b^2x^4-14ab\,x^2+21a^2\right)x}{21\,(dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(3/2),x)
```

```
[Out] -2/21*(-3*b^2*x^4-14*a*b*x^2+21*a^2)*x/(d*x)^(3/2)
```

maxima [A] time = 1.36, size = 44, normalized size = 0.90

$$-\frac{2\left(\frac{21\,a^2}{\sqrt{dx}}-\frac{3\,(dx)^{\frac{7}{2}}b^2+14\,(dx)^{\frac{3}{2}}abd^2}{d^4}\right)}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/21*(21*a^2/sqrt(d*x) - (3*(d*x)^(7/2)*b^2 + 14*(d*x)^(3/2)*a*b*d^2)/d^4)
/d
```

mupad [B] time = 0.05, size = 31, normalized size = 0.63

$$\frac{-42\,a^2+28\,a\,b\,x^2+6\,b^2\,x^4}{21\,d\,\sqrt{d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/(d*x)^(3/2),x)
```

```
[Out] (6*b^2*x^4 - 42*a^2 + 28*a*b*x^2)/(21*d*(d*x)^(1/2))
```

sympy [A] time = 0.66, size = 48, normalized size = 0.98

$$-\frac{2a^2}{d^{\frac{3}{2}}\sqrt{x}}+\frac{4abx^{\frac{3}{2}}}{3d^{\frac{3}{2}}}+\frac{2b^2x^{\frac{7}{2}}}{7d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(3/2),x)
```

```
[Out] -2*a**2/(d**(3/2)*sqrt(x)) + 4*a*b*x**(3/2)/(3*d**(3/2)) + 2*b**2*x**(7/2)/
(7*d**(3/2))
```

3.670 $\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{5/2}} \, dx$

Optimal. Leaf size=49

$$-\frac{2a^2}{3d(dx)^{3/2}}+\frac{4ab\sqrt{dx}}{d^3}+\frac{2b^2(dx)^{5/2}}{5d^5}$$

```
[Out] -2/3*a^2/d/(d*x)^(3/2)+2/5*b^2*(d*x)^(5/2)/d^5+4*a*b*(d*x)^(1/2)/d^3
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {14}

$$-\frac{2a^2}{3d(dx)^{3/2}}+\frac{4ab\sqrt{dx}}{d^3}+\frac{2b^2(dx)^{5/2}}{5d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(5/2),x]
```

```
[Out] (-2*a^2)/(3*d*(d*x)^(3/2)) + (4*a*b*Sqrt[d*x])/d^3 + (2*b^2*(d*x)^(5/2))/(5
*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{5/2}} \, dx &= \int \left(\frac{a^2}{(dx)^{5/2}}+\frac{2ab}{d^2\sqrt{dx}}+\frac{b^2(dx)^{3/2}}{d^4}\right) dx\\ &= -\frac{2a^2}{3d(dx)^{3/2}}+\frac{4ab\sqrt{dx}}{d^3}+\frac{2b^2(dx)^{5/2}}{5d^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.67

$$\frac{x\left(-10a^2+60abx^2+6b^2x^4\right)}{15(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(5/2),x]
```

```
[Out] (x*(-10*a^2 + 60*a*b*x^2 + 6*b^2*x^4))/(15*(d*x)^(5/2))
```

fricas [A] time = 0.73, size = 34, normalized size = 0.69

$$\frac{2\left(3\,b^2x^4+30\,abx^2-5\,a^2\right)\sqrt{dx}}{15\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^4 + 30*a*b*x^2 - 5*a^2)*sqrt(d*x)/(d^3*x^2)
```

giac [A] time = 0.15, size = 53, normalized size = 1.08

$$-\frac{2\left(\frac{5\,a^2d}{\sqrt{dx}\,x}-\frac{3\left(\sqrt{dx}\,b^2d^{10}x^2+10\,\sqrt{dx}\,abd^{10}\right)}{d^{10}}\right)}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/15*(5*a^2*d/(sqrt(d*x)*x) - 3*(sqrt(d*x)*b^2*d^10*x^2 + 10*sqrt(d*x)*a*b
*d^10)/d^10)/d^3
```

maple [A] time = 0.01, size = 30, normalized size = 0.61

$$-\frac{2\left(-3b^2x^4-30ab\,x^2+5a^2\right)x}{15\,(dx)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(5/2),x)
```

```
[Out] -2/15*(-3*b^2*x^4-30*a*b*x^2+5*a^2)*x/(d*x)^(5/2)
```

maxima [A] time = 1.40, size = 43, normalized size = 0.88

$$-\frac{2\left(\frac{5\,a^2}{(dx)^{\frac{3}{2}}}-\frac{3\left((dx)^{\frac{5}{2}}b^2+10\,\sqrt{dx}\,abd^2\right)}{d^4}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/15*(5*a^2/(d*x)^(3/2) - 3*((d*x)^(5/2)*b^2 + 10*sqrt(d*x)*a*b*d^2)/d^4)/
d
```

mupad [B] time = 4.23, size = 34, normalized size = 0.69

$$\frac{-10\,a^2+60\,a\,b\,x^2+6\,b^2\,x^4}{15\,d^2\,x\,\sqrt{d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/(d*x)^(5/2),x)
```

```
[Out] (6*b^2*x^4 - 10*a^2 + 60*a*b*x^2)/(15*d^2*x*(d*x)^(1/2))
```

sympy [A] time = 0.91, size = 48, normalized size = 0.98

$$-\frac{2a^2}{3d^{\frac{5}{2}}x^{\frac{3}{2}}}+\frac{4ab\sqrt{x}}{d^{\frac{5}{2}}}+\frac{2b^2x^{\frac{5}{2}}}{5d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(5/2),x)
```

```
[Out] -2*a**2/(3*d**(5/2)*x**(3/2)) + 4*a*b*sqrt(x)/d**(5/2) + 2*b**2*x**(5/2)/(5
*d**(5/2))
```

3.671 $\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{2a^2}{5d(dx)^{5/2}}-\frac{4ab}{d^3\sqrt{dx}}+\frac{2b^2(dx)^{3/2}}{3d^5}$$

```
[Out] -2/5*a^2/d/(d*x)^(5/2)+2/3*b^2*(d*x)^(3/2)/d^5-4*a*b/d^3/(d*x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {14}

$$-\frac{2a^2}{5d(dx)^{5/2}}-\frac{4ab}{d^3\sqrt{dx}}+\frac{2b^2(dx)^{3/2}}{3d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(7/2),x]
```

```
[Out] (-2*a^2)/(5*d*(d*x)^(5/2)) - (4*a*b)/(d^3*Sqrt[d*x]) + (2*b^2*(d*x)^(3/2))/
(3*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{7/2}}\,dx &= \int \left(\frac{a^2}{(dx)^{7/2}}+\frac{2ab}{d^2(dx)^{3/2}}+\frac{b^2\sqrt{dx}}{d^4}\right)dx\\ &= -\frac{2a^2}{5d(dx)^{5/2}}-\frac{4ab}{d^3\sqrt{dx}}+\frac{2b^2(dx)^{3/2}}{3d^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 0.78

$$\frac{2\sqrt{dx}\left(-3a^2-30abx^2+5b^2x^4\right)}{15d^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(7/2),x]
```

```
[Out] (2*Sqrt[d*x]*(-3*a^2 - 30*a*b*x^2 + 5*b^2*x^4))/(15*d^4*x^3)
```

fricas [A] time = 0.84, size = 34, normalized size = 0.69

$$\frac{2\left(5\,b^{2}x^{4}-30\,abx^{2}-3\,a^{2}\right)\sqrt{dx}}{15\,d^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(5*b^2*x^4 - 30*a*b*x^2 - 3*a^2)*sqrt(d*x)/(d^4*x^3)
```

giac [A] time = 0.15, size = 48, normalized size = 0.98

$$\frac{2\left(5\,\sqrt{dx}\,b^{2}x-\frac{3\left(10\,abd^{3}x^{2}+a^{2}d^{3}\right)}{\sqrt{dx}\,d^{2}x^{2}}\right)}{15\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*(5*sqrt(d*x)*b^2*x - 3*(10*a*b*d^3*x^2 + a^2*d^3)/(sqrt(d*x)*d^2*x^2))
/d^4
```

maple [A] time = 0.01, size = 30, normalized size = 0.61

$$-\frac{2\left(-5b^{2}x^{4}+30ab\,x^{2}+3a^{2}\right)x}{15\left(dx\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(7/2),x)
```

```
[Out] -2/15*(-5*b^2*x^4+30*a*b*x^2+3*a^2)*x/(d*x)^(7/2)
```

maxima [A] time = 1.30, size = 47, normalized size = 0.96

$$\frac{2\left(\frac{5\left(dx\right)^{\frac{3}{2}}b^{2}}{d^{4}}-\frac{3\left(10\,abd^{2}x^{2}+a^{2}d^{2}\right)}{\left(dx\right)^{\frac{5}{2}}d^{2}}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(5*(d*x)^(3/2)*b^2/d^4 - 3*(10*a*b*d^2*x^2 + a^2*d^2)/((d*x)^(5/2)*d^2
))/d
```

mupad [B] time = 0.05, size = 34, normalized size = 0.69

$$-\frac{6\,a^2+60\,a\,b\,x^2-10\,b^2\,x^4}{15\,d^3\,x^2\,\sqrt{d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/(d*x)^(7/2),x)
```

```
[Out] -(6*a^2 - 10*b^2*x^4 + 60*a*b*x^2)/(15*d^3*x^2*(d*x)^(1/2))
```

sympy [A] time = 1.97, size = 48, normalized size = 0.98

$$-\frac{2a^2}{5d^{\frac{7}{2}}x^{\frac{5}{2}}}-\frac{4ab}{d^{\frac{7}{2}}\sqrt{x}}+\frac{2b^2x^{\frac{3}{2}}}{3d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(7/2),x)
```

```
[Out] -2*a**2/(5*d**(7/2)*x**(5/2)) - 4*a*b/(d**(7/2)*sqrt(x)) + 2*b**2*x**(3/2)/
(3*d**(7/2))
```

3.672 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=91

$$\frac{2a^4(dx)^{7/2}}{7d}+\frac{8a^3b(dx)^{11/2}}{11d^3}+\frac{4a^2b^2(dx)^{15/2}}{5d^5}+\frac{8ab^3(dx)^{19/2}}{19d^7}+\frac{2b^4(dx)^{23/2}}{23d^9}$$

```
[Out] 2/7*a^4*(d*x)^(7/2)/d+8/11*a^3*b*(d*x)^(11/2)/d^3+4/5*a^2*b^2*(d*x)^(15/2)/
d^5+8/19*a*b^3*(d*x)^(19/2)/d^7+2/23*b^4*(d*x)^(23/2)/d^9
```

Rubi [A] time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{4a^2b^2(dx)^{15/2}}{5d^5}+\frac{8a^3b(dx)^{11/2}}{11d^3}+\frac{2a^4(dx)^{7/2}}{7d}+\frac{8ab^3(dx)^{19/2}}{19d^7}+\frac{2b^4(dx)^{23/2}}{23d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*a^4*(d*x)^(7/2))/(7*d) + (8*a^3*b*(d*x)^(11/2))/(11*d^3) + (4*a^2*b^2*(d
*x)^(15/2))/(5*d^5) + (8*a*b^3*(d*x)^(19/2))/(19*d^7) + (2*b^4*(d*x)^(23/2)
)/(23*d^9)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{\int (dx)^{5/2}\left(ab+b^2x^2\right)^4\,dx}{b^4}$$
$$=\frac{\int \left(a^4b^4(dx)^{5/2}+\frac{4a^3b^5(dx)^{9/2}}{d^2}+\frac{6a^2b^6(dx)^{13/2}}{d^4}+\frac{4ab^7(dx)^{17/2}}{d^6}+\frac{b^8(dx)^{21/2}}{d^8}\right)dx}{b^4}$$
$$=\frac{2a^4(dx)^{7/2}}{7d}+\frac{8a^3b(dx)^{11/2}}{11d^3}+\frac{4a^2b^2(dx)^{15/2}}{5d^5}+\frac{8ab^3(dx)^{19/2}}{19d^7}+\frac{2b^4(dx)^{23/2}}{23d^9}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.60

$$\frac{2x(dx)^{5/2}\left(24035a^4+61180a^3bx^2+67298a^2b^2x^4+35420ab^3x^6+7315b^4x^8\right)}{168245}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*x*(d*x)^(5/2)*(24035*a^4 + 61180*a^3*b*x^2 + 67298*a^2*b^2*x^4 + 35420*a
*b^3*x^6 + 7315*b^4*x^8))/168245
```

fricas [A] time = 0.85, size = 68, normalized size = 0.75

$$\frac{2}{168245}\left(7315\,b^4d^2x^{11}+35420\,ab^3d^2x^9+67298\,a^2b^2d^2x^7+61180\,a^3bd^2x^5+24035\,a^4d^2x^3\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 2/168245*(7315*b^4*d^2*x^11 + 35420*a*b^3*d^2*x^9 + 67298*a^2*b^2*d^2*x^7 +
 61180*a^3*b*d^2*x^5 + 24035*a^4*d^2*x^3)*sqrt(d*x)
```

giac [A] time = 0.15, size = 86, normalized size = 0.95

$$\frac{2}{23}\sqrt{dx}\,b^4d^2x^{11}+\frac{8}{19}\sqrt{dx}\,ab^3d^2x^9+\frac{4}{5}\sqrt{dx}\,a^2b^2d^2x^7+\frac{8}{11}\sqrt{dx}\,a^3bd^2x^5+\frac{2}{7}\sqrt{dx}\,a^4d^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 2/23*sqrt(d*x)*b^4*d^2*x^11 + 8/19*sqrt(d*x)*a*b^3*d^2*x^9 + 4/5*sqrt(d*x)*
a^2*b^2*d^2*x^7 + 8/11*sqrt(d*x)*a^3*b*d^2*x^5 + 2/7*sqrt(d*x)*a^4*d^2*x^3
```

maple [A] time = 0.01, size = 52, normalized size = 0.57

$$\frac{2\left(7315b^{4}x^{8}+35420a\,b^{3}x^{6}+67298a^{2}b^{2}x^{4}+61180a^{3}b\,x^{2}+24035a^{4}\right)(dx)^{\frac{5}{2}}x}{168245}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 2/168245*x*(7315*b^4*x^8+35420*a*b^3*x^6+67298*a^2*b^2*x^4+61180*a^3*b*x^2+
24035*a^4)*(d*x)^(5/2)
```

maxima [A] time = 1.35, size = 73, normalized size = 0.80

$$\frac{2\left(7315\,(dx)^{\frac{23}{2}}b^{4}+35420\,(dx)^{\frac{19}{2}}ab^{3}d^{2}+67298\,(dx)^{\frac{15}{2}}a^{2}b^{2}d^{4}+61180\,(dx)^{\frac{11}{2}}a^{3}bd^{6}+24035\,(dx)^{\frac{7}{2}}a^{4}d^{8}\right)}{168245\,d^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 2/168245*(7315*(d*x)^(23/2)*b^4 + 35420*(d*x)^(19/2)*a*b^3*d^2 + 67298*(d*x
)^(15/2)*a^2*b^2*d^4 + 61180*(d*x)^(11/2)*a^3*b*d^6 + 24035*(d*x)^(7/2)*a^4
*d^8)/d^9
```

mupad [B] time = 4.20, size = 71, normalized size = 0.78

$$\frac{2\,a^{4}\,(d\,x)^{7/2}}{7\,d}+\frac{2\,b^{4}\,(d\,x)^{23/2}}{23\,d^{9}}+\frac{4\,a^{2}\,b^{2}\,(d\,x)^{15/2}}{5\,d^{5}}+\frac{8\,a^{3}\,b\,(d\,x)^{11/2}}{11\,d^{3}}+\frac{8\,a\,b^{3}\,(d\,x)^{19/2}}{19\,d^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (2*a^4*(d*x)^(7/2))/(7*d) + (2*b^4*(d*x)^(23/2))/(23*d^9) + (4*a^2*b^2*(d*x
)^(15/2))/(5*d^5) + (8*a^3*b*(d*x)^(11/2))/(11*d^3) + (8*a*b^3*(d*x)^(19/2)
)/(19*d^7)
```

sympy [A] time = 5.68, size = 90, normalized size = 0.99

$$\frac{2a^{4}d^{\frac{5}{2}}x^{\frac{7}{2}}}{7}+\frac{8a^{3}bd^{\frac{5}{2}}x^{\frac{11}{2}}}{11}+\frac{4a^{2}b^{2}d^{\frac{5}{2}}x^{\frac{15}{2}}}{5}+\frac{8ab^{3}d^{\frac{5}{2}}x^{\frac{19}{2}}}{19}+\frac{2b^{4}d^{\frac{5}{2}}x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 2*a**4*d**(5/2)*x**(7/2)/7 + 8*a**3*b*d**(5/2)*x**(11/2)/11 + 4*a**2*b**2*d
**(5/2)*x**(15/2)/5 + 8*a*b**3*d**(5/2)*x**(19/2)/19 + 2*b**4*d**(5/2)*x**(
23/2)/23
```

3.673 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=91

$$\frac{2a^4(dx)^{5/2}}{5d}+\frac{8a^3b(dx)^{9/2}}{9d^3}+\frac{12a^2b^2(dx)^{13/2}}{13d^5}+\frac{8ab^3(dx)^{17/2}}{17d^7}+\frac{2b^4(dx)^{21/2}}{21d^9}$$

```
[Out] 2/5*a^4*(d*x)^(5/2)/d+8/9*a^3*b*(d*x)^(9/2)/d^3+12/13*a^2*b^2*(d*x)^(13/2)/
d^5+8/17*a*b^3*(d*x)^(17/2)/d^7+2/21*b^4*(d*x)^(21/2)/d^9
```

Rubi [A] time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{12a^2b^2(dx)^{13/2}}{13d^5}+\frac{8a^3b(dx)^{9/2}}{9d^3}+\frac{2a^4(dx)^{5/2}}{5d}+\frac{8ab^3(dx)^{17/2}}{17d^7}+\frac{2b^4(dx)^{21/2}}{21d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*a^4*(d*x)^(5/2))/(5*d) + (8*a^3*b*(d*x)^(9/2))/(9*d^3) + (12*a^2*b^2*(d*
x)^(13/2))/(13*d^5) + (8*a*b^3*(d*x)^(17/2))/(17*d^7) + (2*b^4*(d*x)^(21/2)
)/(21*d^9)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{\int (dx)^{3/2}\left(ab+b^2x^2\right)^4\,dx}{b^4}$$
$$=\frac{\int \left(a^4b^4(dx)^{3/2}+\frac{4a^3b^5(dx)^{7/2}}{d^2}+\frac{6a^2b^6(dx)^{11/2}}{d^4}+\frac{4ab^7(dx)^{15/2}}{d^6}+\frac{b^8(dx)^{19/2}}{d^8}\right)dx}{b^4}$$
$$=\frac{2a^4(dx)^{5/2}}{5d}+\frac{8a^3b(dx)^{9/2}}{9d^3}+\frac{12a^2b^2(dx)^{13/2}}{13d^5}+\frac{8ab^3(dx)^{17/2}}{17d^7}+\frac{2b^4(dx)^{21/2}}{21d^9}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.60

$$\frac{2x(dx)^{3/2}\left(13923a^4+30940a^3bx^2+32130a^2b^2x^4+16380ab^3x^6+3315b^4x^8\right)}{69615}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*x*(d*x)^(3/2)*(13923*a^4 + 30940*a^3*b*x^2 + 32130*a^2*b^2*x^4 + 16380*a
*b^3*x^6 + 3315*b^4*x^8))/69615
```

fricas [A] time = 0.97, size = 58, normalized size = 0.64

$$\frac{2}{69615}\left(3315\,b^4dx^{10}+16380\,ab^3dx^8+32130\,a^2b^2dx^6+30940\,a^3bdx^4+13923\,a^4dx^2\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*b^4*d*x^10 + 16380*a*b^3*d*x^8 + 32130*a^2*b^2*d*x^6 + 30940*
a^3*b*d*x^4 + 13923*a^4*d*x^2)*sqrt(d*x)
```

giac [A] time = 0.17, size = 74, normalized size = 0.81

$$\frac{2}{69615}\left(3315\sqrt{dx}\,b^4x^{10}+16380\sqrt{dx}\,ab^3x^8+32130\sqrt{dx}\,a^2b^2x^6+30940\sqrt{dx}\,a^3bx^4+13923\sqrt{dx}\,a^4x^2\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 2/69615*(3315*sqrt(d*x)*b^4*x^10 + 16380*sqrt(d*x)*a*b^3*x^8 + 32130*sqrt(d
*x)*a^2*b^2*x^6 + 30940*sqrt(d*x)*a^3*b*x^4 + 13923*sqrt(d*x)*a^4*x^2)*d
```

maple [A] time = 0.01, size = 52, normalized size = 0.57

$$\frac{2\left(3315b^4x^8+16380a\,b^3x^6+32130a^2b^2x^4+30940a^3b\,x^2+13923a^4\right)(dx)^{\frac{3}{2}}\,x}{69615}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 2/69615*x*(3315*b^4*x^8+16380*a*b^3*x^6+32130*a^2*b^2*x^4+30940*a^3*b*x^2+1
3923*a^4)*(d*x)^(3/2)
```

maxima [A] time = 1.35, size = 73, normalized size = 0.80

$$\frac{2\left(3315\,(dx)^{\frac{21}{2}}\,b^4+16380\,(dx)^{\frac{17}{2}}\,ab^3d^2+32130\,(dx)^{\frac{13}{2}}\,a^2b^2d^4+30940\,(dx)^{\frac{9}{2}}\,a^3bd^6+13923\,(dx)^{\frac{5}{2}}\,a^4d^8\right)}{69615\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 2/69615*(3315*(d*x)^(21/2)*b^4 + 16380*(d*x)^(17/2)*a*b^3*d^2 + 32130*(d*x)
^(13/2)*a^2*b^2*d^4 + 30940*(d*x)^(9/2)*a^3*b*d^6 + 13923*(d*x)^(5/2)*a^4*d
^8)/d^9
```

mupad [B] time = 0.03, size = 71, normalized size = 0.78

$$\frac{2\,a^4\,{(d\,x)}^{5/2}}{5\,d}+\frac{2\,b^4\,{(d\,x)}^{21/2}}{21\,d^9}+\frac{12\,a^2\,b^2\,{(d\,x)}^{13/2}}{13\,d^5}+\frac{8\,a^3\,b\,{(d\,x)}^{9/2}}{9\,d^3}+\frac{8\,a\,b^3\,{(d\,x)}^{17/2}}{17\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (2*a^4*(d*x)^(5/2))/(5*d) + (2*b^4*(d*x)^(21/2))/(21*d^9) + (12*a^2*b^2*(d*
x)^(13/2))/(13*d^5) + (8*a^3*b*(d*x)^(9/2))/(9*d^3) + (8*a*b^3*(d*x)^(17/2)
)/(17*d^7)
```

sympy [A] time = 2.69, size = 90, normalized size = 0.99

$$\frac{2a^4d^{\frac{3}{2}}x^{\frac{5}{2}}}{5}+\frac{8a^3bd^{\frac{3}{2}}x^{\frac{9}{2}}}{9}+\frac{12a^2b^2d^{\frac{3}{2}}x^{\frac{13}{2}}}{13}+\frac{8ab^3d^{\frac{3}{2}}x^{\frac{17}{2}}}{17}+\frac{2b^4d^{\frac{3}{2}}x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 2*a**4*d**(3/2)*x**(5/2)/5 + 8*a**3*b*d**(3/2)*x**(9/2)/9 + 12*a**2*b**2*d*
*(3/2)*x**(13/2)/13 + 8*a*b**3*d**(3/2)*x**(17/2)/17 + 2*b**4*d**(3/2)*x**(
21/2)/21
```

3.674 $\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2\,dx$

Optimal. Leaf size=91

$$\frac{2a^4(dx)^{3/2}}{3d}+\frac{8a^3b(dx)^{7/2}}{7d^3}+\frac{12a^2b^2(dx)^{11/2}}{11d^5}+\frac{8ab^3(dx)^{15/2}}{15d^7}+\frac{2b^4(dx)^{19/2}}{19d^9}$$

```
[Out] 2/3*a^4*(d*x)^(3/2)/d+8/7*a^3*b*(d*x)^(7/2)/d^3+12/11*a^2*b^2*(d*x)^(11/2)/
d^5+8/15*a*b^3*(d*x)^(15/2)/d^7+2/19*b^4*(d*x)^(19/2)/d^9
```

Rubi [A] time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{12a^2b^2(dx)^{11/2}}{11d^5}+\frac{8a^3b(dx)^{7/2}}{7d^3}+\frac{2a^4(dx)^{3/2}}{3d}+\frac{8ab^3(dx)^{15/2}}{15d^7}+\frac{2b^4(dx)^{19/2}}{19d^9}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*a^4*(d*x)^(3/2))/(3*d) + (8*a^3*b*(d*x)^(7/2))/(7*d^3) + (12*a^2*b^2*(d*
x)^(11/2))/(11*d^5) + (8*a*b^3*(d*x)^(15/2))/(15*d^7) + (2*b^4*(d*x)^(19/2)
)/(19*d^9)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2 dx = \frac{\int \sqrt{dx}\left(ab+b^2x^2\right)^4 dx}{b^4}$$
$$= \frac{\int \left(a^4b^4\sqrt{dx}+\frac{4a^3b^5(dx)^{5/2}}{d^2}+\frac{6a^2b^6(dx)^{9/2}}{d^4}+\frac{4ab^7(dx)^{13/2}}{d^6}+\frac{b^8(dx)^{17/2}}{d^8}\right)dx}{b^4}$$
$$= \frac{2a^4(dx)^{3/2}}{3d}+\frac{8a^3b(dx)^{7/2}}{7d^3}+\frac{12a^2b^2(dx)^{11/2}}{11d^5}+\frac{8ab^3(dx)^{15/2}}{15d^7}+\frac{2b^4(dx)^{19/2}}{19d^9}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 0.60

$$\frac{2x\sqrt{dx}\left(7315a^4+12540a^3bx^2+11970a^2b^2x^4+5852ab^3x^6+1155b^4x^8\right)}{21945}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*x*Sqrt[d*x]*(7315*a^4 + 12540*a^3*b*x^2 + 11970*a^2*b^2*x^4 + 5852*a*b^3
*x^6 + 1155*b^4*x^8))/21945
```

fricas [A] time = 1.01, size = 51, normalized size = 0.56

$$\frac{2}{21945}\left(1155\,b^4x^9+5852\,ab^3x^7+11970\,a^2b^2x^5+12540\,a^3bx^3+7315\,a^4x\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*b^4*x^9 + 5852*a*b^3*x^7 + 11970*a^2*b^2*x^5 + 12540*a^3*b*x^
3 + 7315*a^4*x)*sqrt(d*x)
```

giac [A] time = 0.16, size = 69, normalized size = 0.76

$$\frac{2}{19}\sqrt{dx}\,b^4x^9+\frac{8}{15}\sqrt{dx}\,ab^3x^7+\frac{12}{11}\sqrt{dx}\,a^2b^2x^5+\frac{8}{7}\sqrt{dx}\,a^3bx^3+\frac{2}{3}\sqrt{dx}\,a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/19*sqrt(d*x)*b^4*x^9 + 8/15*sqrt(d*x)*a*b^3*x^7 + 12/11*sqrt(d*x)*a^2*b^2
*x^5 + 8/7*sqrt(d*x)*a^3*b*x^3 + 2/3*sqrt(d*x)*a^4*x
```

maple [A] time = 0.01, size = 52, normalized size = 0.57

$$\frac{2\left(1155b^4x^8+5852a\,b^3x^6+11970a^2b^2x^4+12540a^3b\,x^2+7315a^4\right)\sqrt{dx}\;x}{21945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2*(d*x)^(1/2),x)
```

```
[Out] 2/21945*x*(1155*b^4*x^8+5852*a*b^3*x^6+11970*a^2*b^2*x^4+12540*a^3*b*x^2+73
15*a^4)*(d*x)^(1/2)
```

maxima [A] time = 1.33, size = 73, normalized size = 0.80

$$\frac{2\left(1155\,(dx)^{\frac{19}{2}}b^4+5852\,(dx)^{\frac{15}{2}}ab^3d^2+11970\,(dx)^{\frac{11}{2}}a^2b^2d^4+12540\,(dx)^{\frac{7}{2}}a^3bd^6+7315\,(dx)^{\frac{3}{2}}a^4d^8\right)}{21945\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21945*(1155*(d*x)^(19/2)*b^4 + 5852*(d*x)^(15/2)*a*b^3*d^2 + 11970*(d*x)^
(11/2)*a^2*b^2*d^4 + 12540*(d*x)^(7/2)*a^3*b*d^6 + 7315*(d*x)^(3/2)*a^4*d^8
)/d^9
```

mupad [B] time = 0.03, size = 71, normalized size = 0.78

$$\frac{2\,a^4\,{\left(d\,x\right)}^{3/2}}{3\,d}+\frac{2\,b^4\,{\left(d\,x\right)}^{19/2}}{19\,d^9}+\frac{12\,a^2\,b^2\,{\left(d\,x\right)}^{11/2}}{11\,d^5}+\frac{8\,a^3\,b\,{\left(d\,x\right)}^{7/2}}{7\,d^3}+\frac{8\,a\,b^3\,{\left(d\,x\right)}^{15/2}}{15\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (2*a^4*(d*x)^(3/2))/(3*d) + (2*b^4*(d*x)^(19/2))/(19*d^9) + (12*a^2*b^2*(d*
x)^(11/2))/(11*d^5) + (8*a^3*b*(d*x)^(7/2))/(7*d^3) + (8*a*b^3*(d*x)^(15/2)
)/(15*d^7)
```

sympy [A] time = 1.27, size = 90, normalized size = 0.99

$$\frac{2a^4\sqrt{d}\,x^{\frac{3}{2}}}{3}+\frac{8a^3b\sqrt{d}\,x^{\frac{7}{2}}}{7}+\frac{12a^2b^2\sqrt{d}\,x^{\frac{11}{2}}}{11}+\frac{8ab^3\sqrt{d}\,x^{\frac{15}{2}}}{15}+\frac{2b^4\sqrt{d}\,x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2*(d*x)**(1/2),x)
```

```
[Out] 2*a**4*sqrt(d)*x**(3/2)/3 + 8*a**3*b*sqrt(d)*x**(7/2)/7 + 12*a**2*b**2*sqrt
(d)*x**(11/2)/11 + 8*a*b**3*sqrt(d)*x**(15/2)/15 + 2*b**4*sqrt(d)*x**(19/2)
/19
```

3.675 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{\sqrt{dx}} dx$

Optimal. Leaf size=89

$$\frac{2a^4\sqrt{dx}}{d} + \frac{8a^3b(dx)^{5/2}}{5d^3} + \frac{4a^2b^2(dx)^{9/2}}{3d^5} + \frac{8ab^3(dx)^{13/2}}{13d^7} + \frac{2b^4(dx)^{17/2}}{17d^9}$$

```
[Out] 8/5*a^3*b*(d*x)^(5/2)/d^3+4/3*a^2*b^2*(d*x)^(9/2)/d^5+8/13*a*b^3*(d*x)^(13/
2)/d^7+2/17*b^4*(d*x)^(17/2)/d^9+2*a^4*(d*x)^(1/2)/d
```

Rubi [A] time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{4a^2b^2(dx)^{9/2}}{3d^5} + \frac{8a^3b(dx)^{5/2}}{5d^3} + \frac{2a^4\sqrt{dx}}{d} + \frac{8ab^3(dx)^{13/2}}{13d^7} + \frac{2b^4(dx)^{17/2}}{17d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/Sqrt[d*x],x]
```

```
[Out] (2*a^4*Sqrt[d*x])/d + (8*a^3*b*(d*x)^(5/2))/(5*d^3) + (4*a^2*b^2*(d*x)^(9/2
))/(3*d^5) + (8*a*b^3*(d*x)^(13/2))/(13*d^7) + (2*b^4*(d*x)^(17/2))/(17*d^9
)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{\sqrt{dx}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{\sqrt{dx}}\,dx}{b^4}$$
$$= \frac{\int \left(\frac{a^4b^4}{\sqrt{dx}}+\frac{4a^3b^5(dx)^{3/2}}{d^2}+\frac{6a^2b^6(dx)^{7/2}}{d^4}+\frac{4ab^7(dx)^{11/2}}{d^6}+\frac{b^8(dx)^{15/2}}{d^8}\right)dx}{b^4}$$
$$= \frac{2a^4\sqrt{dx}}{d}+\frac{8a^3b(dx)^{5/2}}{5d^3}+\frac{4a^2b^2(dx)^{9/2}}{3d^5}+\frac{8ab^3(dx)^{13/2}}{13d^7}+\frac{2b^4(dx)^{17/2}}{17d^9}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.62

$$\frac{2\left(3315a^4x+2652a^3bx^3+2210a^2b^2x^5+1020ab^3x^7+195b^4x^9\right)}{3315\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/Sqrt[d*x],x]
```

```
[Out] (2*(3315*a^4*x + 2652*a^3*b*x^3 + 2210*a^2*b^2*x^5 + 1020*a*b^3*x^7 + 195*b
^4*x^9))/(3315*Sqrt[d*x])
```

fricas [A] time = 0.82, size = 53, normalized size = 0.60

$$\frac{2\left(195\,b^4x^8+1020\,ab^3x^6+2210\,a^2b^2x^4+2652\,a^3bx^2+3315\,a^4\right)\sqrt{dx}}{3315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3315*(195*b^4*x^8 + 1020*a*b^3*x^6 + 2210*a^2*b^2*x^4 + 2652*a^3*b*x^2 +
3315*a^4)*sqrt(d*x)/d
```

giac [A] time = 0.26, size = 73, normalized size = 0.82

$$\frac{2\left(195\,\sqrt{dx}\,b^4x^8+1020\,\sqrt{dx}\,ab^3x^6+2210\,\sqrt{dx}\,a^2b^2x^4+2652\,\sqrt{dx}\,a^3bx^2+3315\,\sqrt{dx}\,a^4\right)}{3315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3315*(195*sqrt(d*x)*b^4*x^8 + 1020*sqrt(d*x)*a*b^3*x^6 + 2210*sqrt(d*x)*a
^2*b^2*x^4 + 2652*sqrt(d*x)*a^3*b*x^2 + 3315*sqrt(d*x)*a^4)/d
```

maple [A] time = 0.01, size = 52, normalized size = 0.58

$$\frac{2\left(195b^{4}x^{8}+1020a\,b^{3}x^{6}+2210a^{2}b^{2}x^{4}+2652a^{3}b\,x^{2}+3315a^{4}\right)x}{3315\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x)
```

```
[Out] 2/3315*(195*b^4*x^8+1020*a*b^3*x^6+2210*a^2*b^2*x^4+2652*a^3*b*x^2+3315*a^4
)*x/(d*x)^(1/2)
```

maxima [A] time = 1.36, size = 90, normalized size = 1.01

$$\frac{2\left(9945\,\sqrt{dx}\,a^{4}+\frac{585\,(dx)^{\frac{17}{2}}\,b^{4}}{d^{8}}+\frac{3060\,(dx)^{\frac{13}{2}}\,ab^{3}}{d^{6}}+\frac{4420\,(dx)^{\frac{9}{2}}\,a^{2}b^{2}}{d^{4}}+442\left(\frac{5\,(dx)^{\frac{9}{2}}\,b^{2}}{d^{4}}+\frac{18\,(dx)^{\frac{5}{2}}\,ab}{d^{2}}\right)a^{2}\right)}{9945\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9945*(9945*sqrt(d*x)*a^4 + 585*(d*x)^(17/2)*b^4/d^8 + 3060*(d*x)^(13/2)*a
*b^3/d^6 + 4420*(d*x)^(9/2)*a^2*b^2/d^4 + 442*(5*(d*x)^(9/2)*b^2/d^4 + 18*(
d*x)^(5/2)*a*b/d^2)*a^2)/d
```

mupad [B] time = 0.03, size = 71, normalized size = 0.80

$$\frac{2\,a^4\,\sqrt{d\,x}}{d}+\frac{2\,b^4\,{\left(d\,x\right)}^{17/2}}{17\,d^9}+\frac{4\,a^2\,b^2\,{\left(d\,x\right)}^{9/2}}{3\,d^5}+\frac{8\,a^3\,b\,{\left(d\,x\right)}^{5/2}}{5\,d^3}+\frac{8\,a\,b^3\,{\left(d\,x\right)}^{13/2}}{13\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/(d*x)^(1/2),x)
```

```
[Out] (2*a^4*(d*x)^(1/2))/d + (2*b^4*(d*x)^(17/2))/(17*d^9) + (4*a^2*b^2*(d*x)^(9
/2))/(3*d^5) + (8*a^3*b*(d*x)^(5/2))/(5*d^3) + (8*a*b^3*(d*x)^(13/2))/(13*d
^7)
```

sympy [A] time = 1.35, size = 88, normalized size = 0.99

$$\frac{2a^{4}\sqrt{x}}{\sqrt{d}}+\frac{8a^{3}bx^{\frac{5}{2}}}{5\sqrt{d}}+\frac{4a^{2}b^{2}x^{\frac{9}{2}}}{3\sqrt{d}}+\frac{8ab^{3}x^{\frac{13}{2}}}{13\sqrt{d}}+\frac{2b^{4}x^{\frac{17}{2}}}{17\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(1/2),x)
```

```
[Out] 2*a**4*sqrt(x)/sqrt(d) + 8*a**3*b*x**(5/2)/(5*sqrt(d)) + 4*a**2*b**2*x**(9/
2)/(3*sqrt(d)) + 8*a*b**3*x**(13/2)/(13*sqrt(d)) + 2*b**4*x**(17/2)/(17*sqr
t(d))
```

3.676 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2a^4}{d\sqrt{dx}}+\frac{8a^3b(dx)^{3/2}}{3d^3}+\frac{12a^2b^2(dx)^{7/2}}{7d^5}+\frac{8ab^3(dx)^{11/2}}{11d^7}+\frac{2b^4(dx)^{15/2}}{15d^9}$$

```
[Out] 8/3*a^3*b*(d*x)^(3/2)/d^3+12/7*a^2*b^2*(d*x)^(7/2)/d^5+8/11*a*b^3*(d*x)^(11
/2)/d^7+2/15*b^4*(d*x)^(15/2)/d^9-2*a^4/d/(d*x)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{12a^2b^2(dx)^{7/2}}{7d^5}+\frac{8a^3b(dx)^{3/2}}{3d^3}-\frac{2a^4}{d\sqrt{dx}}+\frac{8ab^3(dx)^{11/2}}{11d^7}+\frac{2b^4(dx)^{15/2}}{15d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(3/2),x]
```

```
[Out] (-2*a^4)/(d*Sqrt[d*x]) + (8*a^3*b*(d*x)^(3/2))/(3*d^3) + (12*a^2*b^2*(d*x)^
(7/2))/(7*d^5) + (8*a*b^3*(d*x)^(11/2))/(11*d^7) + (2*b^4*(d*x)^(15/2))/(15
*d^9)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{(dx)^{3/2}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{(dx)^{3/2}}\,dx}{b^4}$$
$$= \frac{\int \left(\frac{a^4b^4}{(dx)^{3/2}} + \frac{4a^3b^5\sqrt{dx}}{d^2} + \frac{6a^2b^6(dx)^{5/2}}{d^4} + \frac{4ab^7(dx)^{9/2}}{d^6} + \frac{b^8(dx)^{13/2}}{d^8}\right)dx}{b^4}$$
$$= -\frac{2a^4}{d\sqrt{dx}} + \frac{8a^3b(dx)^{3/2}}{3d^3} + \frac{12a^2b^2(dx)^{7/2}}{7d^5} + \frac{8ab^3(dx)^{11/2}}{11d^7} + \frac{2b^4(dx)^{15/2}}{15d^9}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.62

$$\frac{2x\left(-1155a^4+1540a^3bx^2+990a^2b^2x^4+420ab^3x^6+77b^4x^8\right)}{1155(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(3/2),x]
```

```
[Out] (2*x*(-1155*a^4 + 1540*a^3*b*x^2 + 990*a^2*b^2*x^4 + 420*a*b^3*x^6 + 77*b^4
*x^8))/(1155*(d*x)^(3/2))
```

fricas [A] time = 0.80, size = 56, normalized size = 0.63

$$\frac{2\left(77\,b^4x^8+420\,ab^3x^6+990\,a^2b^2x^4+1540\,a^3bx^2-1155\,a^4\right)\sqrt{dx}}{1155\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^4*x^8 + 420*a*b^3*x^6 + 990*a^2*b^2*x^4 + 1540*a^3*b*x^2 - 115
5*a^4)*sqrt(d*x)/(d^2*x)
```

giac [A] time = 0.18, size = 89, normalized size = 1.00

$$-\frac{2\left(\frac{1155\,a^4}{\sqrt{dx}} - \frac{77\,\sqrt{dx}\,b^4d^{119}x^7+420\,\sqrt{dx}\,ab^3d^{119}x^5+990\,\sqrt{dx}\,a^2b^2d^{119}x^3+1540\,\sqrt{dx}\,a^3bd^{119}x}{d^{120}}\right)}{1155\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/1155*(1155*a^4/sqrt(d*x) - (77*sqrt(d*x)*b^4*d^119*x^7 + 420*sqrt(d*x)*a
*b^3*d^119*x^5 + 990*sqrt(d*x)*a^2*b^2*d^119*x^3 + 1540*sqrt(d*x)*a^3*b*d^1
19*x)/d^120)/d
```

maple [A] time = 0.01, size = 52, normalized size = 0.58

$$-\frac{2\left(-77b^4x^8-420a\,b^3x^6-990a^2b^2x^4-1540a^3b\,x^2+1155a^4\right)x}{1155\,(dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(3/2),x)
```

```
[Out] -2/1155*(-77*b^4*x^8-420*a*b^3*x^6-990*a^2*b^2*x^4-1540*a^3*b*x^2+1155*a^4)
*x/(d*x)^(3/2)
```

maxima [A] time = 1.21, size = 76, normalized size = 0.85

$$-\frac{2\left(\frac{1155\,a^4}{\sqrt{dx}}-\frac{77\,(dx)^{\frac{15}{2}}b^4+420\,(dx)^{\frac{11}{2}}ab^3d^2+990\,(dx)^{\frac{7}{2}}a^2b^2d^4+1540\,(dx)^{\frac{3}{2}}a^3bd^6}{d^8}\right)}{1155\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/1155*(1155*a^4/sqrt(d*x) - (77*(d*x)^(15/2)*b^4 + 420*(d*x)^(11/2)*a*b^3
*d^2 + 990*(d*x)^(7/2)*a^2*b^2*d^4 + 1540*(d*x)^(3/2)*a^3*b*d^6)/d^8)/d
```

mupad [B] time = 0.03, size = 71, normalized size = 0.80

$$\frac{2\,b^4\,{\left(d\,x\right)}^{15/2}}{15\,d^9}-\frac{2\,a^4}{d\,\sqrt{d\,x}}+\frac{12\,a^2\,b^2\,{\left(d\,x\right)}^{7/2}}{7\,d^5}+\frac{8\,a^3\,b\,{\left(d\,x\right)}^{3/2}}{3\,d^3}+\frac{8\,a\,b^3\,{\left(d\,x\right)}^{11/2}}{11\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/(d*x)^(3/2),x)
```

```
[Out] (2*b^4*(d*x)^(15/2))/(15*d^9) - (2*a^4)/(d*(d*x)^(1/2)) + (12*a^2*b^2*(d*x)
^(7/2))/(7*d^5) + (8*a^3*b*(d*x)^(3/2))/(3*d^3) + (8*a*b^3*(d*x)^(11/2))/(1
1*d^7)
```

sympy [A] time = 1.38, size = 88, normalized size = 0.99

$$-\frac{2a^4}{d^{\frac{3}{2}}\sqrt{x}}+\frac{8a^3bx^{\frac{3}{2}}}{3d^{\frac{3}{2}}}+\frac{12a^2b^2x^{\frac{7}{2}}}{7d^{\frac{3}{2}}}+\frac{8ab^3x^{\frac{11}{2}}}{11d^{\frac{3}{2}}}+\frac{2b^4x^{\frac{15}{2}}}{15d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(3/2),x)
```

```
[Out] -2*a**4/(d**(3/2)*sqrt(x)) + 8*a**3*b*x**(3/2)/(3*d**(3/2)) + 12*a**2*b**2*
x**(7/2)/(7*d**(3/2)) + 8*a*b**3*x**(11/2)/(11*d**(3/2)) + 2*b**4*x**(15/2)
/(15*d**(3/2))
```

3.677 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2a^4}{3d(dx)^{3/2}}+\frac{8a^3b\sqrt{dx}}{d^3}+\frac{12a^2b^2(dx)^{5/2}}{5d^5}+\frac{8ab^3(dx)^{9/2}}{9d^7}+\frac{2b^4(dx)^{13/2}}{13d^9}$$

```
[Out] -2/3*a^4/d/(d*x)^(3/2)+12/5*a^2*b^2*(d*x)^(5/2)/d^5+8/9*a*b^3*(d*x)^(9/2)/d
^7+2/13*b^4*(d*x)^(13/2)/d^9+8*a^3*b*(d*x)^(1/2)/d^3
```

Rubi [A] time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{12a^2b^2(dx)^{5/2}}{5d^5}+\frac{8a^3b\sqrt{dx}}{d^3}-\frac{2a^4}{3d(dx)^{3/2}}+\frac{8ab^3(dx)^{9/2}}{9d^7}+\frac{2b^4(dx)^{13/2}}{13d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(5/2),x]
```

```
[Out] (-2*a^4)/(3*d*(d*x)^(3/2)) + (8*a^3*b*Sqrt[d*x])/d^3 + (12*a^2*b^2*(d*x)^(5
/2))/(5*d^5) + (8*a*b^3*(d*x)^(9/2))/(9*d^7) + (2*b^4*(d*x)^(13/2))/(13*d^9
)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{(dx)^{5/2}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{(dx)^{5/2}}\,dx}{b^4}$$
$$= \frac{\int \left(\frac{a^4b^4}{(dx)^{5/2}} + \frac{4a^3b^5}{d^2\sqrt{dx}} + \frac{6a^2b^6(dx)^{3/2}}{d^4} + \frac{4ab^7(dx)^{7/2}}{d^6} + \frac{b^8(dx)^{11/2}}{d^8}\right)dx}{b^4}$$
$$= -\frac{2a^4}{3d(dx)^{3/2}} + \frac{8a^3b\sqrt{dx}}{d^3} + \frac{12a^2b^2(dx)^{5/2}}{5d^5} + \frac{8ab^3(dx)^{9/2}}{9d^7} + \frac{2b^4(dx)^{13/2}}{13d^9}$$

Mathematica [**A**] time = 0.02, size = 55, normalized size = 0.62

$$\frac{x\left(-390a^4+4680a^3bx^2+1404a^2b^2x^4+520ab^3x^6+90b^4x^8\right)}{585(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(5/2),x]
```

```
[Out] (x*(-390*a^4 + 4680*a^3*b*x^2 + 1404*a^2*b^2*x^4 + 520*a*b^3*x^6 + 90*b^4*x
^8))/(585*(d*x)^(5/2))
```

fricas [**A**] time = 0.57, size = 56, normalized size = 0.63

$$\frac{2\left(45\,b^4x^8+260\,ab^3x^6+702\,a^2b^2x^4+2340\,a^3bx^2-195\,a^4\right)\sqrt{dx}}{585\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*b^4*x^8 + 260*a*b^3*x^6 + 702*a^2*b^2*x^4 + 2340*a^3*b*x^2 - 195*
a^4)*sqrt(d*x)/(d^3*x^2)
```

giac [**A**] time = 0.16, size = 92, normalized size = 1.03

$$-\frac{2\left(\frac{195\,a^4d}{\sqrt{dx}\,x} - \frac{45\,\sqrt{dx}\,b^4d^{78}x^6+260\,\sqrt{dx}\,ab^3d^{78}x^4+702\,\sqrt{dx}\,a^2b^2d^{78}x^2+2340\,\sqrt{dx}\,a^3bd^{78}}{d^{78}}\right)}{585\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/585*(195*a^4*d/(sqrt(d*x)*x) - (45*sqrt(d*x)*b^4*d^78*x^6 + 260*sqrt(d*x
)*a*b^3*d^78*x^4 + 702*sqrt(d*x)*a^2*b^2*d^78*x^2 + 2340*sqrt(d*x)*a^3*b*d^
78)/d^78)/d^3
```

maple [A] time = 0.01, size = 52, normalized size = 0.58

$$-\frac{2\left(-45b^4x^8-260a\,b^3x^6-702a^2b^2x^4-2340a^3b\,x^2+195a^4\right)x}{585\,(dx)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(5/2),x)
```

```
[Out] -2/585*(-45*b^4*x^8-260*a*b^3*x^6-702*a^2*b^2*x^4-2340*a^3*b*x^2+195*a^4)*x
/(d*x)^(5/2)
```

maxima [A] time = 1.35, size = 76, normalized size = 0.85

$$-\frac{2\left(\frac{195\,a^4}{(dx)^{\frac{3}{2}}}-\frac{45\,(dx)^{\frac{13}{2}}b^4+260\,(dx)^{\frac{9}{2}}ab^3d^2+702\,(dx)^{\frac{5}{2}}a^2b^2d^4+2340\,\sqrt{dx}\,a^3bd^6}{d^8}\right)}{585\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/585*(195*a^4/(d*x)^(3/2) - (45*(d*x)^(13/2)*b^4 + 260*(d*x)^(9/2)*a*b^3*
d^2 + 702*(d*x)^(5/2)*a^2*b^2*d^4 + 2340*sqrt(d*x)*a^3*b*d^6)/d^8)/d
```

mupad [B] time = 0.03, size = 71, normalized size = 0.80

$$\frac{2\,b^4\,(d\,x)^{13/2}}{13\,d^9}-\frac{2\,a^4}{3\,d\,(d\,x)^{3/2}}+\frac{12\,a^2\,b^2\,(d\,x)^{5/2}}{5\,d^5}+\frac{8\,a^3\,b\,\sqrt{d\,x}}{d^3}+\frac{8\,a\,b^3\,(d\,x)^{9/2}}{9\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/(d*x)^(5/2),x)
```

```
[Out] (2*b^4*(d*x)^(13/2))/(13*d^9) - (2*a^4)/(3*d*(d*x)^(3/2)) + (12*a^2*b^2*(d*
x)^(5/2))/(5*d^5) + (8*a^3*b*(d*x)^(1/2))/d^3 + (8*a*b^3*(d*x)^(9/2))/(9*d^
7)
```

sympy [A] time = 1.74, size = 88, normalized size = 0.99

$$-\frac{2a^4}{3d^{\frac{5}{2}}x^{\frac{3}{2}}}+\frac{8a^3b\sqrt{x}}{d^{\frac{5}{2}}}+\frac{12a^2b^2x^{\frac{5}{2}}}{5d^{\frac{5}{2}}}+\frac{8ab^3x^{\frac{9}{2}}}{9d^{\frac{5}{2}}}+\frac{2b^4x^{\frac{13}{2}}}{13d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(5/2),x)
```

```
[Out] -2*a**4/(3*d**(5/2)*x**(3/2)) + 8*a**3*b*sqrt(x)/d**(5/2) + 12*a**2*b**2*x*
*(5/2)/(5*d**(5/2)) + 8*a*b**3*x**(9/2)/(9*d**(5/2)) + 2*b**4*x**(13/2)/(13
*d**(5/2))
```

3.678 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{7/2}} dx$

Optimal. Leaf size=87

$$-\frac{2a^4}{5d(dx)^{5/2}}-\frac{8a^3b}{d^3\sqrt{dx}}+\frac{4a^2b^2(dx)^{3/2}}{d^5}+\frac{8ab^3(dx)^{7/2}}{7d^7}+\frac{2b^4(dx)^{11/2}}{11d^9}$$

```
[Out] -2/5*a^4/d/(d*x)^(5/2)+4*a^2*b^2*(d*x)^(3/2)/d^5+8/7*a*b^3*(d*x)^(7/2)/d^7+
2/11*b^4*(d*x)^(11/2)/d^9-8*a^3*b/d^3/(d*x)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{4a^2b^2(dx)^{3/2}}{d^5}-\frac{8a^3b}{d^3\sqrt{dx}}-\frac{2a^4}{5d(dx)^{5/2}}+\frac{8ab^3(dx)^{7/2}}{7d^7}+\frac{2b^4(dx)^{11/2}}{11d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(7/2),x]
```

```
[Out] (-2*a^4)/(5*d*(d*x)^(5/2)) - (8*a^3*b)/(d^3*Sqrt[d*x]) + (4*a^2*b^2*(d*x)^(
3/2))/d^5 + (8*a*b^3*(d*x)^(7/2))/(7*d^7) + (2*b^4*(d*x)^(11/2))/(11*d^9)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{(dx)^{7/2}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{(dx)^{7/2}}\,dx}{b^4}$$
$$= \frac{\int \left(\frac{a^4b^4}{(dx)^{7/2}}+\frac{4a^3b^5}{d^2(dx)^{3/2}}+\frac{6a^2b^6\sqrt{dx}}{d^4}+\frac{4ab^7(dx)^{5/2}}{d^6}+\frac{b^8(dx)^{9/2}}{d^8}\right)dx}{b^4}$$
$$= -\frac{2a^4}{5d(dx)^{5/2}}-\frac{8a^3b}{d^3\sqrt{dx}}+\frac{4a^2b^2(dx)^{3/2}}{d^5}+\frac{8ab^3(dx)^{7/2}}{7d^7}+\frac{2b^4(dx)^{11/2}}{11d^9}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 0.69

$$\frac{2\sqrt{dx}\left(-77a^4-1540a^3bx^2+770a^2b^2x^4+220ab^3x^6+35b^4x^8\right)}{385d^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(7/2),x]
```

```
[Out] (2*Sqrt[d*x]*(-77*a^4 - 1540*a^3*b*x^2 + 770*a^2*b^2*x^4 + 220*a*b^3*x^6 +
35*b^4*x^8))/(385*d^4*x^3)
```

fricas [A] time = 0.81, size = 56, normalized size = 0.64

$$\frac{2\left(35\,b^4x^8+220\,ab^3x^6+770\,a^2b^2x^4-1540\,a^3bx^2-77\,a^4\right)\sqrt{dx}}{385\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/385*(35*b^4*x^8 + 220*a*b^3*x^6 + 770*a^2*b^2*x^4 - 1540*a^3*b*x^2 - 77*a
^4)*sqrt(d*x)/(d^4*x^3)
```

giac [A] time = 0.18, size = 95, normalized size = 1.09

$$-\frac{2\left(\frac{77\left(20\,a^3bd^3x^2+a^4d^3\right)}{\sqrt{dx}\,d^2x^2}-\frac{5\left(7\,\sqrt{dx}\,b^4d^{55}x^5+44\,\sqrt{dx}\,ab^3d^{55}x^3+154\,\sqrt{dx}\,a^2b^2d^{55}x\right)}{d^{55}}\right)}{385\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/385*(77*(20*a^3*b*d^3*x^2 + a^4*d^3)/(sqrt(d*x)*d^2*x^2) - 5*(7*sqrt(d*x
)*b^4*d^55*x^5 + 44*sqrt(d*x)*a*b^3*d^55*x^3 + 154*sqrt(d*x)*a^2*b^2*d^55*x
)/d^55)/d^4
```

maple [A] time = 0.01, size = 52, normalized size = 0.60

$$-\frac{2\left(-35b^4x^8-220a\,b^3x^6-770a^2b^2x^4+1540a^3b\,x^2+77a^4\right)x}{385\,(dx)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(7/2),x)
```

```
[Out] -2/385*(-35*b^4*x^8-220*a*b^3*x^6-770*a^2*b^2*x^4+1540*a^3*b*x^2+77*a^4)*x/
(d*x)^(7/2)
```

maxima [A] time = 1.44, size = 82, normalized size = 0.94

$$-\frac{2\left(\frac{77\left(20\,a^3bd^2x^2+a^4d^2\right)}{(dx)^{\frac{5}{2}}d^2}-\frac{5\left(7\,(dx)^{\frac{11}{2}}b^4+44\,(dx)^{\frac{7}{2}}ab^3d^2+154\,(dx)^{\frac{3}{2}}a^2b^2d^4\right)}{d^8}\right)}{385\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/385*(77*(20*a^3*b*d^2*x^2 + a^4*d^2)/((d*x)^(5/2)*d^2) - 5*(7*(d*x)^(11/
2)*b^4 + 44*(d*x)^(7/2)*a*b^3*d^2 + 154*(d*x)^(3/2)*a^2*b^2*d^4)/d^8)/d
```

mupad [B] time = 0.06, size = 75, normalized size = 0.86

$$\frac{2\,b^4\,(d\,x)^{11/2}}{11\,d^9}-\frac{\frac{2\,a^4\,d^2}{5}+8\,b\,a^3\,d^2\,x^2}{d^3\,(d\,x)^{5/2}}+\frac{4\,a^2\,b^2\,(d\,x)^{3/2}}{d^5}+\frac{8\,a\,b^3\,(d\,x)^{7/2}}{7\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/(d*x)^(7/2),x)
```

```
[Out] (2*b^4*(d*x)^(11/2))/(11*d^9) - ((2*a^4*d^2)/5 + 8*a^3*b*d^2*x^2)/(d^3*(d*x
)^(5/2)) + (4*a^2*b^2*(d*x)^(3/2))/d^5 + (8*a*b^3*(d*x)^(7/2))/(7*d^7)
```

sympy [A] time = 2.47, size = 87, normalized size = 1.00

$$-\frac{2a^4}{5d^{\frac{7}{2}}x^{\frac{5}{2}}}-\frac{8a^3b}{d^{\frac{7}{2}}\sqrt{x}}+\frac{4a^2b^2x^{\frac{3}{2}}}{d^{\frac{7}{2}}}+\frac{8ab^3x^{\frac{7}{2}}}{7d^{\frac{7}{2}}}+\frac{2b^4x^{\frac{11}{2}}}{11d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(7/2),x)
```

```
[Out] -2*a**4/(5*d**(7/2)*x**(5/2)) - 8*a**3*b/(d**(7/2)*sqrt(x)) + 4*a**2*b**2*x
**(3/2)/d**(7/2) + 8*a*b**3*x**(7/2)/(7*d**(7/2)) + 2*b**4*x**(11/2)/(11*d*
*(7/2))
```

3.679 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=129

$$\frac{2a^6(dx)^{7/2}}{7d}+\frac{12a^5b(dx)^{11/2}}{11d^3}+\frac{2a^4b^2(dx)^{15/2}}{d^5}+\frac{40a^3b^3(dx)^{19/2}}{19d^7}+\frac{30a^2b^4(dx)^{23/2}}{23d^9}+\frac{4ab^5(dx)^{27/2}}{9d^{11}}+\frac{2b^6(dx)^{31/2}}{31d^{13}}$$

```
[Out] 2/7*a^6*(d*x)^(7/2)/d+12/11*a^5*b*(d*x)^(11/2)/d^3+2*a^4*b^2*(d*x)^(15/2)/d
^5+40/19*a^3*b^3*(d*x)^(19/2)/d^7+30/23*a^2*b^4*(d*x)^(23/2)/d^9+4/9*a*b^5*
(d*x)^(27/2)/d^11+2/31*b^6*(d*x)^(31/2)/d^13
```

Rubi [A] time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{30a^2b^4(dx)^{23/2}}{23d^9}+\frac{40a^3b^3(dx)^{19/2}}{19d^7}+\frac{2a^4b^2(dx)^{15/2}}{d^5}+\frac{12a^5b(dx)^{11/2}}{11d^3}+\frac{2a^6(dx)^{7/2}}{7d}+\frac{4ab^5(dx)^{27/2}}{9d^{11}}+\frac{2b^6(dx)^{31/2}}{31d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*a^6*(d*x)^(7/2))/(7*d) + (12*a^5*b*(d*x)^(11/2))/(11*d^3) + (2*a^4*b^2*(
d*x)^(15/2))/d^5 + (40*a^3*b^3*(d*x)^(19/2))/(19*d^7) + (30*a^2*b^4*(d*x)^(
23/2))/(23*d^9) + (4*a*b^5*(d*x)^(27/2))/(9*d^11) + (2*b^6*(d*x)^(31/2))/(3
1*d^13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (dx)^{5/2} \left(a^2 + 2abx^2 + b^2x^4\right)^3 \, dx = \frac{\int (dx)^{5/2} \left(ab + b^2x^2\right)^6 \, dx}{b^6}$$
$$= \frac{\int \left(a^6b^6(dx)^{5/2} + \frac{6a^5b^7(dx)^{9/2}}{d^2} + \frac{15a^4b^8(dx)^{13/2}}{d^4} + \frac{20a^3b^9(dx)^{17/2}}{d^6} + \frac{15a^2b^{10}(dx)^{21/2}}{d^8} + \frac{6ab}{}\right.}{b^6}$$
$$= \frac{2a^6(dx)^{7/2}}{7d} + \frac{12a^5b(dx)^{11/2}}{11d^3} + \frac{2a^4b^2(dx)^{15/2}}{d^5} + \frac{40a^3b^3(dx)^{19/2}}{19d^7} + \frac{30a^2b^4(dx)^2}{23d^9}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.60

$$\frac{2x(dx)^{5/2} \left(1341153a^6 + 5120766a^5bx^2 + 9388071a^4b^2x^4 + 9882180a^3b^3x^6 + 6122655a^2b^4x^8 + 2086238ab^5x^{10} + \right.}{9388071}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*x*(d*x)^(5/2)*(1341153*a^6 + 5120766*a^5*b*x^2 + 9388071*a^4*b^2*x^4 + 9
882180*a^3*b^3*x^6 + 6122655*a^2*b^4*x^8 + 2086238*a*b^5*x^10 + 302841*b^6*
x^12))/9388071
```

fricas [A] time = 0.96, size = 96, normalized size = 0.74

$$\frac{2}{9388071} \left(302841\, b^6d^2x^{15} + 2086238\, ab^5d^2x^{13} + 6122655\, a^2b^4d^2x^{11} + 9882180\, a^3b^3d^2x^9 + 9388071\, a^4b^2d^2x^7 + 5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 2/9388071*(302841*b^6*d^2*x^15 + 2086238*a*b^5*d^2*x^13 + 6122655*a^2*b^4*d
^2*x^11 + 9882180*a^3*b^3*d^2*x^9 + 9388071*a^4*b^2*d^2*x^7 + 5120766*a^5*b
*d^2*x^5 + 1341153*a^6*d^2*x^3)*sqrt(d*x)
```

giac [A] time = 0.16, size = 124, normalized size = 0.96

$$\frac{2}{31}\sqrt{dx}\, b^6d^2x^{15} + \frac{4}{9}\sqrt{dx}\, ab^5d^2x^{13} + \frac{30}{23}\sqrt{dx}\, a^2b^4d^2x^{11} + \frac{40}{19}\sqrt{dx}\, a^3b^3d^2x^9 + 2\sqrt{dx}\, a^4b^2d^2x^7 + \frac{12}{11}\sqrt{dx}\, a^5bd^2x^5 + \frac{2}{7}\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 2/31*sqrt(d*x)*b^6*d^2*x^15 + 4/9*sqrt(d*x)*a*b^5*d^2*x^13 + 30/23*sqrt(d*x
)*a^2*b^4*d^2*x^11 + 40/19*sqrt(d*x)*a^3*b^3*d^2*x^9 + 2*sqrt(d*x)*a^4*b^2*
d^2*x^7 + 12/11*sqrt(d*x)*a^5*b*d^2*x^5 + 2/7*sqrt(d*x)*a^6*d^2*x^3
```

maple [A] time = 0.01, size = 74, normalized size = 0.57

$$\frac{2\left(302841 b^{6} x^{12}+2086238 a\,b^{5} x^{10}+6122655 a^{2} b^{4} x^{8}+9882180 a^{3} b^{3} x^{6}+9388071 a^{4} b^{2} x^{4}+5120766 a^{5} b\,x^{2}+134\right.}{9388071}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 2/9388071*x*(302841*b^6*x^12+2086238*a*b^5*x^10+6122655*a^2*b^4*x^8+9882180
*a^3*b^3*x^6+9388071*a^4*b^2*x^4+5120766*a^5*b*x^2+1341153*a^6)*(d*x)^(5/2)
```

maxima [A] time = 1.40, size = 105, normalized size = 0.81

$$\frac{2\left(302841\,(dx)^{\frac{31}{2}}\,b^{6}+2086238\,(dx)^{\frac{27}{2}}\,ab^{5}d^{2}+6122655\,(dx)^{\frac{23}{2}}\,a^{2}b^{4}d^{4}+9882180\,(dx)^{\frac{19}{2}}\,a^{3}b^{3}d^{6}+9388071\,(dx\right.}{9388071\,d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 2/9388071*(302841*(d*x)^(31/2)*b^6 + 2086238*(d*x)^(27/2)*a*b^5*d^2 + 61226
55*(d*x)^(23/2)*a^2*b^4*d^4 + 9882180*(d*x)^(19/2)*a^3*b^3*d^6 + 9388071*(d
*x)^(15/2)*a^4*b^2*d^8 + 5120766*(d*x)^(11/2)*a^5*b*d^10 + 1341153*(d*x)^(7
/2)*a^6*d^12)/d^13
```

mupad [B] time = 0.04, size = 103, normalized size = 0.80

$$\frac{2\,a^6\,{\left(d\,x\right)}^{7/2}}{7\,d}+\frac{2\,b^6\,{\left(d\,x\right)}^{31/2}}{31\,d^{13}}+\frac{2\,a^4\,b^2\,{\left(d\,x\right)}^{15/2}}{d^5}+\frac{40\,a^3\,b^3\,{\left(d\,x\right)}^{19/2}}{19\,d^7}+\frac{30\,a^2\,b^4\,{\left(d\,x\right)}^{23/2}}{23\,d^9}+\frac{12\,a^5\,b\,{\left(d\,x\right)}^{11/2}}{11\,d^3}+\frac{4\,a\,b^5\,{\left(d\,x\right)}^{27/2}}{9\,d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (2*a^6*(d*x)^(7/2))/(7*d) + (2*b^6*(d*x)^(31/2))/(31*d^13) + (2*a^4*b^2*(d*
x)^(15/2))/d^5 + (40*a^3*b^3*(d*x)^(19/2))/(19*d^7) + (30*a^2*b^4*(d*x)^(23
/2))/(23*d^9) + (12*a^5*b*(d*x)^(11/2))/(11*d^3) + (4*a*b^5*(d*x)^(27/2))/(
9*d^11)
```

sympy [A] time = 10.80, size = 129, normalized size = 1.00

$$\frac{2a^{6}d^{\frac{5}{2}}x^{\frac{7}{2}}}{7}+\frac{12a^{5}bd^{\frac{5}{2}}x^{\frac{11}{2}}}{11}+2a^{4}b^{2}d^{\frac{5}{2}}x^{\frac{15}{2}}+\frac{40a^{3}b^{3}d^{\frac{5}{2}}x^{\frac{19}{2}}}{19}+\frac{30a^{2}b^{4}d^{\frac{5}{2}}x^{\frac{23}{2}}}{23}+\frac{4ab^{5}d^{\frac{5}{2}}x^{\frac{27}{2}}}{9}+\frac{2b^{6}d^{\frac{5}{2}}x^{\frac{31}{2}}}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 2*a**6*d**(5/2)*x**(7/2)/7 + 12*a**5*b*d**(5/2)*x**(11/2)/11 + 2*a**4*b**2*
d**(5/2)*x**(15/2) + 40*a**3*b**3*d**(5/2)*x**(19/2)/19 + 30*a**2*b**4*d**(
5/2)*x**(23/2)/23 + 4*a*b**5*d**(5/2)*x**(27/2)/9 + 2*b**6*d**(5/2)*x**(31/
2)/31
```

3.680 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=131

$$\frac{2a^6(dx)^{5/2}}{5d}+\frac{4a^5b(dx)^{9/2}}{3d^3}+\frac{30a^4b^2(dx)^{13/2}}{13d^5}+\frac{40a^3b^3(dx)^{17/2}}{17d^7}+\frac{10a^2b^4(dx)^{21/2}}{7d^9}+\frac{12ab^5(dx)^{25/2}}{25d^{11}}+\frac{2b^6(dx)^{29/2}}{29d^{13}}$$

```
[Out] 2/5*a^6*(d*x)^(5/2)/d+4/3*a^5*b*(d*x)^(9/2)/d^3+30/13*a^4*b^2*(d*x)^(13/2)/
d^5+40/17*a^3*b^3*(d*x)^(17/2)/d^7+10/7*a^2*b^4*(d*x)^(21/2)/d^9+12/25*a*b^
5*(d*x)^(25/2)/d^11+2/29*b^6*(d*x)^(29/2)/d^13
```

Rubi [A] time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{10a^2b^4(dx)^{21/2}}{7d^9}+\frac{40a^3b^3(dx)^{17/2}}{17d^7}+\frac{30a^4b^2(dx)^{13/2}}{13d^5}+\frac{4a^5b(dx)^{9/2}}{3d^3}+\frac{2a^6(dx)^{5/2}}{5d}+\frac{12ab^5(dx)^{25/2}}{25d^{11}}+\frac{2b^6(dx)^{29/2}}{29d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*a^6*(d*x)^(5/2))/(5*d) + (4*a^5*b*(d*x)^(9/2))/(3*d^3) + (30*a^4*b^2*(d*
x)^(13/2))/(13*d^5) + (40*a^3*b^3*(d*x)^(17/2))/(17*d^7) + (10*a^2*b^4*(d*x
)^(21/2))/(7*d^9) + (12*a*b^5*(d*x)^(25/2))/(25*d^11) + (2*b^6*(d*x)^(29/2)
)/(29*d^13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{\int (dx)^{3/2}\left(ab+b^2x^2\right)^6\,dx}{b^6}$$
$$=\frac{\int\left(a^6b^6(dx)^{3/2}+\frac{6a^5b^7(dx)^{7/2}}{d^2}+\frac{15a^4b^8(dx)^{11/2}}{d^4}+\frac{20a^3b^9(dx)^{15/2}}{d^6}+\frac{15a^2b^{10}(dx)^{19/2}}{d^8}+\frac{6ab}{}\right.}{b^6}$$
$$=\frac{2a^6(dx)^{5/2}}{5d}+\frac{4a^5b(dx)^{9/2}}{3d^3}+\frac{30a^4b^2(dx)^{13/2}}{13d^5}+\frac{40a^3b^3(dx)^{17/2}}{17d^7}+\frac{10a^2b^4(dx)^{21}}{7d^9}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.59

$$\frac{2x(dx)^{3/2}\left(672945a^6+2243150a^5bx^2+3882375a^4b^2x^4+3958500a^3b^3x^6+2403375a^2b^4x^8+807534ab^5x^{10}+11\right.}{3364725}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*x*(d*x)^(3/2)*(672945*a^6 + 2243150*a^5*b*x^2 + 3882375*a^4*b^2*x^4 + 39
58500*a^3*b^3*x^6 + 2403375*a^2*b^4*x^8 + 807534*a*b^5*x^10 + 116025*b^6*x^
12))/3364725
```

fricas [A] time = 1.05, size = 82, normalized size = 0.63

$$\frac{2}{3364725}\left(116025\,b^6dx^{14}+807534\,ab^5dx^{12}+2403375\,a^2b^4dx^{10}+3958500\,a^3b^3dx^8+3882375\,a^4b^2dx^6+224315\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 2/3364725*(116025*b^6*d*x^14 + 807534*a*b^5*d*x^12 + 2403375*a^2*b^4*d*x^10
 + 3958500*a^3*b^3*d*x^8 + 3882375*a^4*b^2*d*x^6 + 2243150*a^5*b*d*x^4 + 67
2945*a^6*d*x^2)*sqrt(d*x)
```

giac [A] time = 0.19, size = 106, normalized size = 0.81

$$\frac{2}{3364725}\left(116025\,\sqrt{dx}\,b^6x^{14}+807534\,\sqrt{dx}\,ab^5x^{12}+2403375\,\sqrt{dx}\,a^2b^4x^{10}+3958500\,\sqrt{dx}\,a^3b^3x^8+3882375\,\sqrt{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 2/3364725*(116025*sqrt(d*x)*b^6*x^14 + 807534*sqrt(d*x)*a*b^5*x^12 + 240337
5*sqrt(d*x)*a^2*b^4*x^10 + 3958500*sqrt(d*x)*a^3*b^3*x^8 + 3882375*sqrt(d*x
)*a^4*b^2*x^6 + 2243150*sqrt(d*x)*a^5*b*x^4 + 672945*sqrt(d*x)*a^6*x^2)*d
```

maple [A] time = 0.01, size = 74, normalized size = 0.56

$$\frac{2\left(116025b^6x^{12} + 807534a\,b^5x^{10} + 2403375a^2b^4x^8 + 3958500a^3b^3x^6 + 3882375a^4b^2x^4 + 2243150a^5b\,x^2 + 672\right.}{3364725}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 2/3364725*x*(116025*b^6*x^12+807534*a*b^5*x^10+2403375*a^2*b^4*x^8+3958500*
a^3*b^3*x^6+3882375*a^4*b^2*x^4+2243150*a^5*b*x^2+672945*a^6)*(d*x)^(3/2)
```

maxima [A] time = 1.33, size = 105, normalized size = 0.80

$$\frac{2\left(116025\,(dx)^{\frac{29}{2}}\,b^6 + 807534\,(dx)^{\frac{25}{2}}\,ab^5d^2 + 2403375\,(dx)^{\frac{21}{2}}\,a^2b^4d^4 + 3958500\,(dx)^{\frac{17}{2}}\,a^3b^3d^6 + 3882375\,(dx)\right.}{3364725\,d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 2/3364725*(116025*(d*x)^(29/2)*b^6 + 807534*(d*x)^(25/2)*a*b^5*d^2 + 240337
5*(d*x)^(21/2)*a^2*b^4*d^4 + 3958500*(d*x)^(17/2)*a^3*b^3*d^6 + 3882375*(d*
x)^(13/2)*a^4*b^2*d^8 + 2243150*(d*x)^(9/2)*a^5*b*d^10 + 672945*(d*x)^(5/2)
*a^6*d^12)/d^13
```

mupad [B] time = 0.04, size = 103, normalized size = 0.79

$$\frac{2\,a^6\,(d\,x)^{5/2}}{5\,d}+\frac{2\,b^6\,(d\,x)^{29/2}}{29\,d^{13}}+\frac{30\,a^4\,b^2\,(d\,x)^{13/2}}{13\,d^5}+\frac{40\,a^3\,b^3\,(d\,x)^{17/2}}{17\,d^7}+\frac{10\,a^2\,b^4\,(d\,x)^{21/2}}{7\,d^9}+\frac{4\,a^5\,b\,(d\,x)^{9/2}}{3\,d^3}+\frac{12\,a\,b^5\,(d\,x)^{25/}}{25\,d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (2*a^6*(d*x)^(5/2))/(5*d) + (2*b^6*(d*x)^(29/2))/(29*d^13) + (30*a^4*b^2*(d
*x)^(13/2))/(13*d^5) + (40*a^3*b^3*(d*x)^(17/2))/(17*d^7) + (10*a^2*b^4*(d*
x)^(21/2))/(7*d^9) + (4*a^5*b*(d*x)^(9/2))/(3*d^3) + (12*a*b^5*(d*x)^(25/2)
)/(25*d^11)
```

sympy [A] time = 5.34, size = 131, normalized size = 1.00

$$\frac{2a^6d^{\frac{3}{2}}x^{\frac{5}{2}}}{5} + \frac{4a^5bd^{\frac{3}{2}}x^{\frac{9}{2}}}{3} + \frac{30a^4b^2d^{\frac{3}{2}}x^{\frac{13}{2}}}{13} + \frac{40a^3b^3d^{\frac{3}{2}}x^{\frac{17}{2}}}{17} + \frac{10a^2b^4d^{\frac{3}{2}}x^{\frac{21}{2}}}{7} + \frac{12ab^5d^{\frac{3}{2}}x^{\frac{25}{2}}}{25} + \frac{2b^6d^{\frac{3}{2}}x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 2*a**6*d**(3/2)*x**(5/2)/5 + 4*a**5*b*d**(3/2)*x**(9/2)/3 + 30*a**4*b**2*d*
*(3/2)*x**(13/2)/13 + 40*a**3*b**3*d**(3/2)*x**(17/2)/17 + 10*a**2*b**4*d**
(3/2)*x**(21/2)/7 + 12*a*b**5*d**(3/2)*x**(25/2)/25 + 2*b**6*d**(3/2)*x**(2
9/2)/29
```

3.681 $\int \sqrt{dx} \left(a^2 + 2abx^2 + b^2x^4\right)^3 dx$

Optimal. Leaf size=131

$$\frac{2a^6(dx)^{3/2}}{3d}+\frac{12a^5b(dx)^{7/2}}{7d^3}+\frac{30a^4b^2(dx)^{11/2}}{11d^5}+\frac{8a^3b^3(dx)^{15/2}}{3d^7}+\frac{30a^2b^4(dx)^{19/2}}{19d^9}+\frac{12ab^5(dx)^{23/2}}{23d^{11}}+\frac{2b^6(dx)^{27/2}}{27d^{13}}$$

```
[Out] 2/3*a^6*(d*x)^(3/2)/d+12/7*a^5*b*(d*x)^(7/2)/d^3+30/11*a^4*b^2*(d*x)^(11/2)
/d^5+8/3*a^3*b^3*(d*x)^(15/2)/d^7+30/19*a^2*b^4*(d*x)^(19/2)/d^9+12/23*a*b^
5*(d*x)^(23/2)/d^11+2/27*b^6*(d*x)^(27/2)/d^13
```

Rubi [A] time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{30a^2b^4(dx)^{19/2}}{19d^9}+\frac{8a^3b^3(dx)^{15/2}}{3d^7}+\frac{30a^4b^2(dx)^{11/2}}{11d^5}+\frac{12a^5b(dx)^{7/2}}{7d^3}+\frac{2a^6(dx)^{3/2}}{3d}+\frac{12ab^5(dx)^{23/2}}{23d^{11}}+\frac{2b^6(dx)^{27/2}}{27d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*a^6*(d*x)^(3/2))/(3*d) + (12*a^5*b*(d*x)^(7/2))/(7*d^3) + (30*a^4*b^2*(d
*x)^(11/2))/(11*d^5) + (8*a^3*b^3*(d*x)^(15/2))/(3*d^7) + (30*a^2*b^4*(d*x)
^(19/2))/(19*d^9) + (12*a*b^5*(d*x)^(23/2))/(23*d^11) + (2*b^6*(d*x)^(27/2)
)/(27*d^13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3 dx = \frac{\int \sqrt{dx}\left(ab+b^2x^2\right)^6 dx}{b^6}$$
$$= \frac{\int \left(a^6b^6\sqrt{dx} + \frac{6a^5b^7(dx)^{5/2}}{d^2} + \frac{15a^4b^8(dx)^{9/2}}{d^4} + \frac{20a^3b^9(dx)^{13/2}}{d^6} + \frac{15a^2b^{10}(dx)^{17/2}}{d^8} + \frac{6ab^{11}(dx}{d^{10}}\right.}{b^6}$$
$$= \frac{2a^6(dx)^{3/2}}{3d} + \frac{12a^5b(dx)^{7/2}}{7d^3} + \frac{30a^4b^2(dx)^{11/2}}{11d^5} + \frac{8a^3b^3(dx)^{15/2}}{3d^7} + \frac{30a^2b^4(dx)^{19/2}}{19d^9}$$

Mathematica [**A**] time = 0.02, size = 77, normalized size = 0.59

$$\frac{2x\sqrt{dx}\left(302841a^6+778734a^5bx^2+1238895a^4b^2x^4+1211364a^3b^3x^6+717255a^2b^4x^8+237006ab^5x^{10}+33649\right.}{908523}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*x*Sqrt[d*x]*(302841*a^6 + 778734*a^5*b*x^2 + 1238895*a^4*b^2*x^4 + 12113
64*a^3*b^3*x^6 + 717255*a^2*b^4*x^8 + 237006*a*b^5*x^10 + 33649*b^6*x^12))/
908523
```

fricas [**A**] time = 0.89, size = 73, normalized size = 0.56

$$\frac{2}{908523}\left(33649\,b^6x^{13}+237006\,ab^5x^{11}+717255\,a^2b^4x^9+1211364\,a^3b^3x^7+1238895\,a^4b^2x^5+778734\,a^5bx^3+30\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/908523*(33649*b^6*x^13 + 237006*a*b^5*x^11 + 717255*a^2*b^4*x^9 + 1211364
*a^3*b^3*x^7 + 1238895*a^4*b^2*x^5 + 778734*a^5*b*x^3 + 302841*a^6*x)*sqrt(
d*x)
```

giac [**A**] time = 0.18, size = 101, normalized size = 0.77

$$\frac{2}{27}\sqrt{dx}\,b^6x^{13}+\frac{12}{23}\sqrt{dx}\,ab^5x^{11}+\frac{30}{19}\sqrt{dx}\,a^2b^4x^9+\frac{8}{3}\sqrt{dx}\,a^3b^3x^7+\frac{30}{11}\sqrt{dx}\,a^4b^2x^5+\frac{12}{7}\sqrt{dx}\,a^5bx^3+\frac{2}{3}\sqrt{dx}\,a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/27*sqrt(d*x)*b^6*x^13 + 12/23*sqrt(d*x)*a*b^5*x^11 + 30/19*sqrt(d*x)*a^2*
b^4*x^9 + 8/3*sqrt(d*x)*a^3*b^3*x^7 + 30/11*sqrt(d*x)*a^4*b^2*x^5 + 12/7*sq
rt(d*x)*a^5*b*x^3 + 2/3*sqrt(d*x)*a^6*x
```

maple [A] time = 0.01, size = 74, normalized size = 0.56

$$\frac{2\left(33649b^{6}x^{12}+237006a\,b^{5}x^{10}+717255a^{2}b^{4}x^{8}+1211364a^{3}b^{3}x^{6}+1238895a^{4}b^{2}x^{4}+778734a^{5}b\,x^{2}+302841a\right.}{908523}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3*(d*x)^(1/2),x)
```

```
[Out] 2/908523*x*(33649*b^6*x^12+237006*a*b^5*x^10+717255*a^2*b^4*x^8+1211364*a^3
*b^3*x^6+1238895*a^4*b^2*x^4+778734*a^5*b*x^2+302841*a^6)*(d*x)^(1/2)
```

maxima [A] time = 1.39, size = 105, normalized size = 0.80

$$\frac{2\left(33649\,(dx)^{\frac{27}{2}}\,b^{6}+237006\,(dx)^{\frac{23}{2}}\,ab^{5}d^{2}+717255\,(dx)^{\frac{19}{2}}\,a^{2}b^{4}d^{4}+1211364\,(dx)^{\frac{15}{2}}\,a^{3}b^{3}d^{6}+1238895\,(dx)^{\frac{11}{2}}\,a\right.}{908523\,d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/908523*(33649*(d*x)^(27/2)*b^6 + 237006*(d*x)^(23/2)*a*b^5*d^2 + 717255*(
d*x)^(19/2)*a^2*b^4*d^4 + 1211364*(d*x)^(15/2)*a^3*b^3*d^6 + 1238895*(d*x)^
(11/2)*a^4*b^2*d^8 + 778734*(d*x)^(7/2)*a^5*b*d^10 + 302841*(d*x)^(3/2)*a^6
*d^12)/d^13
```

mupad [B] time = 0.04, size = 103, normalized size = 0.79

$$\frac{2\,a^6\,{\left(d\,x\right)}^{3/2}}{3\,d}+\frac{2\,b^6\,{\left(d\,x\right)}^{27/2}}{27\,d^{13}}+\frac{30\,a^4\,b^2\,{\left(d\,x\right)}^{11/2}}{11\,d^5}+\frac{8\,a^3\,b^3\,{\left(d\,x\right)}^{15/2}}{3\,d^7}+\frac{30\,a^2\,b^4\,{\left(d\,x\right)}^{19/2}}{19\,d^9}+\frac{12\,a^5\,b\,{\left(d\,x\right)}^{7/2}}{7\,d^3}+\frac{12\,a\,b^5\,{\left(d\,x\right)}^{23/2}}{23\,d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (2*a^6*(d*x)^(3/2))/(3*d) + (2*b^6*(d*x)^(27/2))/(27*d^13) + (30*a^4*b^2*(d
*x)^(11/2))/(11*d^5) + (8*a^3*b^3*(d*x)^(15/2))/(3*d^7) + (30*a^2*b^4*(d*x)
^(19/2))/(19*d^9) + (12*a^5*b*(d*x)^(7/2))/(7*d^3) + (12*a*b^5*(d*x)^(23/2)
)/(23*d^11)
```

sympy [A] time = 3.00, size = 131, normalized size = 1.00

$$\frac{2a^{6}\sqrt{d}\,x^{\frac{3}{2}}}{3}+\frac{12a^{5}b\sqrt{d}\,x^{\frac{7}{2}}}{7}+\frac{30a^{4}b^{2}\sqrt{d}\,x^{\frac{11}{2}}}{11}+\frac{8a^{3}b^{3}\sqrt{d}\,x^{\frac{15}{2}}}{3}+\frac{30a^{2}b^{4}\sqrt{d}\,x^{\frac{19}{2}}}{19}+\frac{12ab^{5}\sqrt{d}\,x^{\frac{23}{2}}}{23}+\frac{2b^{6}\sqrt{d}\,x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3*(d*x)**(1/2),x)
```

```
[Out] 2*a**6*sqrt(d)*x**(3/2)/3 + 12*a**5*b*sqrt(d)*x**(7/2)/7 + 30*a**4*b**2*sqr
t(d)*x**(11/2)/11 + 8*a**3*b**3*sqrt(d)*x**(15/2)/3 + 30*a**2*b**4*sqrt(d)*
x**(19/2)/19 + 12*a*b**5*sqrt(d)*x**(23/2)/23 + 2*b**6*sqrt(d)*x**(27/2)/27
```

3.682 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{\sqrt{dx}}\,dx$

Optimal. Leaf size=129

$$\frac{2a^6\sqrt{dx}}{d}+\frac{12a^5b(dx)^{5/2}}{5d^3}+\frac{10a^4b^2(dx)^{9/2}}{3d^5}+\frac{40a^3b^3(dx)^{13/2}}{13d^7}+\frac{30a^2b^4(dx)^{17/2}}{17d^9}+\frac{4ab^5(dx)^{21/2}}{7d^{11}}+\frac{2b^6(dx)^{25/2}}{25d^{13}}$$

```
[Out] 12/5*a^5*b*(d*x)^(5/2)/d^3+10/3*a^4*b^2*(d*x)^(9/2)/d^5+40/13*a^3*b^3*(d*x)
^(13/2)/d^7+30/17*a^2*b^4*(d*x)^(17/2)/d^9+4/7*a*b^5*(d*x)^(21/2)/d^11+2/25
*b^6*(d*x)^(25/2)/d^13+2*a^6*(d*x)^(1/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{30a^2b^4(dx)^{17/2}}{17d^9}+\frac{40a^3b^3(dx)^{13/2}}{13d^7}+\frac{10a^4b^2(dx)^{9/2}}{3d^5}+\frac{12a^5b(dx)^{5/2}}{5d^3}+\frac{2a^6\sqrt{dx}}{d}+\frac{4ab^5(dx)^{21/2}}{7d^{11}}+\frac{2b^6(dx)^{25/2}}{25d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/Sqrt[d*x],x]
```

```
[Out] (2*a^6*Sqrt[d*x])/d + (12*a^5*b*(d*x)^(5/2))/(5*d^3) + (10*a^4*b^2*(d*x)^(9
/2))/(3*d^5) + (40*a^3*b^3*(d*x)^(13/2))/(13*d^7) + (30*a^2*b^4*(d*x)^(17/2
))/(17*d^9) + (4*a*b^5*(d*x)^(21/2))/(7*d^11) + (2*b^6*(d*x)^(25/2))/(25*d^
13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{\sqrt{dx}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{\sqrt{dx}}\,dx}{b^6}$$
$$= \frac{\int \left(\frac{a^6b^6}{\sqrt{dx}} + \frac{6a^5b^7(dx)^{3/2}}{d^2} + \frac{15a^4b^8(dx)^{7/2}}{d^4} + \frac{20a^3b^9(dx)^{11/2}}{d^6} + \frac{15a^2b^{10}(dx)^{15/2}}{d^8} + \frac{6ab^{11}(dx)^{19/2}}{d^{10}} + \frac{b^{12}(\ldots}{\ldots}\right.}{b^6}$$
$$= \frac{2a^6\sqrt{dx}}{d} + \frac{12a^5b(dx)^{5/2}}{5d^3} + \frac{10a^4b^2(dx)^{9/2}}{3d^5} + \frac{40a^3b^3(dx)^{13/2}}{13d^7} + \frac{30a^2b^4(dx)^{17/2}}{17d^9} + \frac{4ab\ldots}{\ldots}$$

Mathematica [A] time = 0.02, size = 77, normalized size = 0.60

$$\frac{2\left(116025a^6x+139230a^5bx^3+193375a^4b^2x^5+178500a^3b^3x^7+102375a^2b^4x^9+33150ab^5x^{11}+4641b^6x^{13}\right)}{116025\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/Sqrt[d*x],x]
```

```
[Out] (2*(116025*a^6*x + 139230*a^5*b*x^3 + 193375*a^4*b^2*x^5 + 178500*a^3*b^3*x
^7 + 102375*a^2*b^4*x^9 + 33150*a*b^5*x^11 + 4641*b^6*x^13))/(116025*Sqrt[d
*x])
```

fricas [A] time = 0.87, size = 75, normalized size = 0.58

$$\frac{2\left(4641\,b^6x^{12}+33150\,ab^5x^{10}+102375\,a^2b^4x^8+178500\,a^3b^3x^6+193375\,a^4b^2x^4+139230\,a^5bx^2+116025\,a^6\right)\sqrt{\ldots}}{116025\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/116025*(4641*b^6*x^12 + 33150*a*b^5*x^10 + 102375*a^2*b^4*x^8 + 178500*a^
3*b^3*x^6 + 193375*a^4*b^2*x^4 + 139230*a^5*b*x^2 + 116025*a^6)*sqrt(d*x)/d
```

giac [A] time = 0.18, size = 105, normalized size = 0.81

$$\frac{2\left(4641\,\sqrt{dx}\,b^6x^{12}+33150\,\sqrt{dx}\,ab^5x^{10}+102375\,\sqrt{dx}\,a^2b^4x^8+178500\,\sqrt{dx}\,a^3b^3x^6+193375\,\sqrt{dx}\,a^4b^2x^4+1392\ldots\right.}{116025\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/116025*(4641*sqrt(d*x)*b^6*x^12 + 33150*sqrt(d*x)*a*b^5*x^10 + 102375*sqr
t(d*x)*a^2*b^4*x^8 + 178500*sqrt(d*x)*a^3*b^3*x^6 + 193375*sqrt(d*x)*a^4*b^
2*x^4 + 139230*sqrt(d*x)*a^5*b*x^2 + 116025*sqrt(d*x)*a^6)/d
```

maple [A] time = 0.01, size = 74, normalized size = 0.57

$$\frac{2\left(4641 b^{6} x^{12}+33150 a\,b^{5} x^{10}+102375 a^{2} b^{4} x^{8}+178500 a^{3} b^{3} x^{6}+193375 a^{4} b^{2} x^{4}+139230 a^{5} b\,x^{2}+116025 a^{6}\right) x}{116025 \sqrt{d x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x)
```

```
[Out] 2/116025*(4641*b^6*x^12+33150*a*b^5*x^10+102375*a^2*b^4*x^8+178500*a^3*b^3*
x^6+193375*a^4*b^2*x^4+139230*a^5*b*x^2+116025*a^6)*x/(d*x)^(1/2)
```

maxima [A] time = 1.36, size = 155, normalized size = 1.20

$$\frac{2\left(116025\,\sqrt{d x}\,a^{6}+\frac{4641\,(d x)^{\frac{25}{2}}\,b^{6}}{d^{12}}+\frac{33150\,(d x)^{\frac{21}{2}}\,a b^{5}}{d^{10}}+\frac{81900\,(d x)^{\frac{17}{2}}\,a^{2} b^{4}}{d^{8}}+\frac{71400\,(d x)^{\frac{13}{2}}\,a^{3} b^{3}}{d^{6}}+7735\left(\frac{5\,(d x)^{\frac{9}{2}}\,b^{2}}{d^{4}}+\frac{18\,(d x)^{\frac{5}{2}}\,a b}{d^{2}}\right)a\right.}{116025\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/116025*(116025*sqrt(d*x)*a^6 + 4641*(d*x)^(25/2)*b^6/d^12 + 33150*(d*x)^(
21/2)*a*b^5/d^10 + 81900*(d*x)^(17/2)*a^2*b^4/d^8 + 71400*(d*x)^(13/2)*a^3*
b^3/d^6 + 7735*(5*(d*x)^(9/2)*b^2/d^4 + 18*(d*x)^(5/2)*a*b/d^2)*a^4 + 175*(
117*(d*x)^(17/2)*b^4/d^8 + 612*(d*x)^(13/2)*a*b^3/d^6 + 884*(d*x)^(9/2)*a^2
*b^2/d^4)*a^2)/d
```

mupad [B] time = 0.04, size = 103, normalized size = 0.80

$$\frac{2\,a^6\,\sqrt{d\,x}}{d}+\frac{2\,b^6\,{\left(d\,x\right)}^{25/2}}{25\,d^{13}}+\frac{10\,a^4\,b^2\,{\left(d\,x\right)}^{9/2}}{3\,d^5}+\frac{40\,a^3\,b^3\,{\left(d\,x\right)}^{13/2}}{13\,d^7}+\frac{30\,a^2\,b^4\,{\left(d\,x\right)}^{17/2}}{17\,d^9}+\frac{12\,a^5\,b\,{\left(d\,x\right)}^{5/2}}{5\,d^3}+\frac{4\,a\,b^5\,{\left(d\,x\right)}^{21/2}}{7\,d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/(d*x)^(1/2),x)
```

```
[Out] (2*a^6*(d*x)^(1/2))/d + (2*b^6*(d*x)^(25/2))/(25*d^13) + (10*a^4*b^2*(d*x)^
(9/2))/(3*d^5) + (40*a^3*b^3*(d*x)^(13/2))/(13*d^7) + (30*a^2*b^4*(d*x)^(17
/2))/(17*d^9) + (12*a^5*b*(d*x)^(5/2))/(5*d^3) + (4*a*b^5*(d*x)^(21/2))/(7*
d^11)
```

sympy [A] time = 2.94, size = 129, normalized size = 1.00

$$\frac{2a^6\sqrt{x}}{\sqrt{d}}+\frac{12a^5bx^{\frac{5}{2}}}{5\sqrt{d}}+\frac{10a^4b^2x^{\frac{9}{2}}}{3\sqrt{d}}+\frac{40a^3b^3x^{\frac{13}{2}}}{13\sqrt{d}}+\frac{30a^2b^4x^{\frac{17}{2}}}{17\sqrt{d}}+\frac{4ab^5x^{\frac{21}{2}}}{7\sqrt{d}}+\frac{2b^6x^{\frac{25}{2}}}{25\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(1/2),x)
```

```
[Out] 2*a**6*sqrt(x)/sqrt(d) + 12*a**5*b*x**(5/2)/(5*sqrt(d)) + 10*a**4*b**2*x**(
9/2)/(3*sqrt(d)) + 40*a**3*b**3*x**(13/2)/(13*sqrt(d)) + 30*a**2*b**4*x**(1
7/2)/(17*sqrt(d)) + 4*a*b**5*x**(21/2)/(7*sqrt(d)) + 2*b**6*x**(25/2)/(25*s
qrt(d))
```

3.683 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{2a^6}{d\sqrt{dx}}+\frac{4a^5b(dx)^{3/2}}{d^3}+\frac{30a^4b^2(dx)^{7/2}}{7d^5}+\frac{40a^3b^3(dx)^{11/2}}{11d^7}+\frac{2a^2b^4(dx)^{15/2}}{d^9}+\frac{12ab^5(dx)^{19/2}}{19d^{11}}+\frac{2b^6(dx)^{23/2}}{23d^{13}}$$

```
[Out] 4*a^5*b*(d*x)^(3/2)/d^3+30/7*a^4*b^2*(d*x)^(7/2)/d^5+40/11*a^3*b^3*(d*x)^(1
1/2)/d^7+2*a^2*b^4*(d*x)^(15/2)/d^9+12/19*a*b^5*(d*x)^(19/2)/d^11+2/23*b^6*
(d*x)^(23/2)/d^13-2*a^6/d/(d*x)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{2a^2b^4(dx)^{15/2}}{d^9}+\frac{40a^3b^3(dx)^{11/2}}{11d^7}+\frac{30a^4b^2(dx)^{7/2}}{7d^5}+\frac{4a^5b(dx)^{3/2}}{d^3}-\frac{2a^6}{d\sqrt{dx}}+\frac{12ab^5(dx)^{19/2}}{19d^{11}}+\frac{2b^6(dx)^{23/2}}{23d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(3/2),x]
```

```
[Out] (-2*a^6)/(d*Sqrt[d*x]) + (4*a^5*b*(d*x)^(3/2))/d^3 + (30*a^4*b^2*(d*x)^(7/2
))/(7*d^5) + (40*a^3*b^3*(d*x)^(11/2))/(11*d^7) + (2*a^2*b^4*(d*x)^(15/2))/
d^9 + (12*a*b^5*(d*x)^(19/2))/(19*d^11) + (2*b^6*(d*x)^(23/2))/(23*d^13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{(dx)^{3/2}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{(dx)^{3/2}}\,dx}{b^6}$$

$$= \frac{\int \left(\frac{a^6b^6}{(dx)^{3/2}} + \frac{6a^5b^7\sqrt{dx}}{d^2} + \frac{15a^4b^8(dx)^{5/2}}{d^4} + \frac{20a^3b^9(dx)^{9/2}}{d^6} + \frac{15a^2b^{10}(dx)^{13/2}}{d^8} + \frac{6ab^{11}(dx)^{17/2}}{d^{10}} + \frac{b^{12}(d}{d}\right.}{b^6}$$

$$= -\frac{2a^6}{d\sqrt{dx}} + \frac{4a^5b(dx)^{3/2}}{d^3} + \frac{30a^4b^2(dx)^{7/2}}{7d^5} + \frac{40a^3b^3(dx)^{11/2}}{11d^7} + \frac{2a^2b^4(dx)^{15/2}}{d^9} + \frac{12ab^5(}{19}$$

Mathematica [A] time = 0.02, size = 77, normalized size = 0.62

$$\frac{2x\left(-33649a^6+67298a^5bx^2+72105a^4b^2x^4+61180a^3b^3x^6+33649a^2b^4x^8+10626ab^5x^{10}+1463b^6x^{12}\right)}{33649(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(3/2),x]
```

```
[Out] (2*x*(-33649*a^6 + 67298*a^5*b*x^2 + 72105*a^4*b^2*x^4 + 61180*a^3*b^3*x^6
+ 33649*a^2*b^4*x^8 + 10626*a*b^5*x^10 + 1463*b^6*x^12))/(33649*(d*x)^(3/2)
)
```

fricas [A] time = 1.10, size = 78, normalized size = 0.62

$$\frac{2\left(1463\,b^6x^{12}+10626\,ab^5x^{10}+33649\,a^2b^4x^8+61180\,a^3b^3x^6+72105\,a^4b^2x^4+67298\,a^5bx^2-33649\,a^6\right)\sqrt{dx}}{33649\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/33649*(1463*b^6*x^12 + 10626*a*b^5*x^10 + 33649*a^2*b^4*x^8 + 61180*a^3*b
^3*x^6 + 72105*a^4*b^2*x^4 + 67298*a^5*b*x^2 - 33649*a^6)*sqrt(d*x)/(d^2*x)
```

giac [A] time = 0.20, size = 127, normalized size = 1.02

$$-\frac{2\left(\frac{33649\,a^6}{\sqrt{dx}} - \frac{1463\,\sqrt{dx}\,b^6d^{275}x^{11}+10626\,\sqrt{dx}\,ab^5d^{275}x^9+33649\,\sqrt{dx}\,a^2b^4d^{275}x^7+61180\,\sqrt{dx}\,a^3b^3d^{275}x^5+72105\,\sqrt{dx}\,a^4b^2d^{275}x^3+67298\,\sqrt{dx}\,a^5b}{d^{276}}\right.}{33649\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/33649*(33649*a^6/sqrt(d*x) - (1463*sqrt(d*x)*b^6*d^275*x^11 + 10626*sqrt
(d*x)*a*b^5*d^275*x^9 + 33649*sqrt(d*x)*a^2*b^4*d^275*x^7 + 61180*sqrt(d*x)
*a^3*b^3*d^275*x^5 + 72105*sqrt(d*x)*a^4*b^2*d^275*x^3 + 67298*sqrt(d*x)*a^
5*b*d^275*x)/d^276)/d
```

maple [A] time = 0.01, size = 74, normalized size = 0.59

$$-\frac{2\left(-1463b^{6}x^{12}-10626a\,b^{5}x^{10}-33649a^{2}b^{4}x^{8}-61180a^{3}b^{3}x^{6}-72105a^{4}b^{2}x^{4}-67298a^{5}b\,x^{2}+33649a^{6}\right)x}{33649\left(dx\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(3/2),x)
```

```
[Out] -2/33649*(-1463*b^6*x^12-10626*a*b^5*x^10-33649*a^2*b^4*x^8-61180*a^3*b^3*x
^6-72105*a^4*b^2*x^4-67298*a^5*b*x^2+33649*a^6)*x/(d*x)^(3/2)
```

maxima [A] time = 1.39, size = 108, normalized size = 0.86

$$-\frac{2\left(\frac{33649\,a^{6}}{\sqrt{dx}}-\frac{1463\,(dx)^{\frac{23}{2}}b^{6}+10626\,(dx)^{\frac{19}{2}}ab^{5}d^{2}+33649\,(dx)^{\frac{15}{2}}a^{2}b^{4}d^{4}+61180\,(dx)^{\frac{11}{2}}a^{3}b^{3}d^{6}+72105\,(dx)^{\frac{7}{2}}a^{4}b^{2}d^{8}+67298\,(dx)^{\frac{3}{2}}a^{5}bd^{10}}{d^{12}}\right)}{33649\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/33649*(33649*a^6/sqrt(d*x) - (1463*(d*x)^(23/2)*b^6 + 10626*(d*x)^(19/2)
*a*b^5*d^2 + 33649*(d*x)^(15/2)*a^2*b^4*d^4 + 61180*(d*x)^(11/2)*a^3*b^3*d^
6 + 72105*(d*x)^(7/2)*a^4*b^2*d^8 + 67298*(d*x)^(3/2)*a^5*b*d^10)/d^12)/d
```

mupad [B] time = 0.04, size = 103, normalized size = 0.82

$$\frac{2\,b^6\,{\left(d\,x\right)}^{23/2}}{23\,d^{13}}-\frac{2\,a^6}{d\,\sqrt{d\,x}}+\frac{30\,a^4\,b^2\,{\left(d\,x\right)}^{7/2}}{7\,d^5}+\frac{40\,a^3\,b^3\,{\left(d\,x\right)}^{11/2}}{11\,d^7}+\frac{2\,a^2\,b^4\,{\left(d\,x\right)}^{15/2}}{d^9}+\frac{4\,a^5\,b\,{\left(d\,x\right)}^{3/2}}{d^3}+\frac{12\,a\,b^5\,{\left(d\,x\right)}^{19/2}}{19\,d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/(d*x)^(3/2),x)
```

```
[Out] (2*b^6*(d*x)^(23/2))/(23*d^13) - (2*a^6)/(d*(d*x)^(1/2)) + (30*a^4*b^2*(d*x
)^(7/2))/(7*d^5) + (40*a^3*b^3*(d*x)^(11/2))/(11*d^7) + (2*a^2*b^4*(d*x)^(1
5/2))/d^9 + (4*a^5*b*(d*x)^(3/2))/d^3 + (12*a*b^5*(d*x)^(19/2))/(19*d^11)
```

sympy [A] time = 3.01, size = 126, normalized size = 1.01

$$-\frac{2a^{6}}{d^{\frac{3}{2}}\sqrt{x}}+\frac{4a^{5}bx^{\frac{3}{2}}}{d^{\frac{3}{2}}}+\frac{30a^{4}b^{2}x^{\frac{7}{2}}}{7d^{\frac{3}{2}}}+\frac{40a^{3}b^{3}x^{\frac{11}{2}}}{11d^{\frac{3}{2}}}+\frac{2a^{2}b^{4}x^{\frac{15}{2}}}{d^{\frac{3}{2}}}+\frac{12ab^{5}x^{\frac{19}{2}}}{19d^{\frac{3}{2}}}+\frac{2b^{6}x^{\frac{23}{2}}}{23d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(3/2),x)
```

```
[Out] -2*a**6/(d**(3/2)*sqrt(x)) + 4*a**5*b*x**(3/2)/d**(3/2) + 30*a**4*b**2*x**(
7/2)/(7*d**(3/2)) + 40*a**3*b**3*x**(11/2)/(11*d**(3/2)) + 2*a**2*b**4*x**(
15/2)/d**(3/2) + 12*a*b**5*x**(19/2)/(19*d**(3/2)) + 2*b**6*x**(23/2)/(23*d
**(3/2))
```

3.684 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{2a^6}{3d(dx)^{3/2}}+\frac{12a^5b\sqrt{dx}}{d^3}+\frac{6a^4b^2(dx)^{5/2}}{d^5}+\frac{40a^3b^3(dx)^{9/2}}{9d^7}+\frac{30a^2b^4(dx)^{13/2}}{13d^9}+\frac{12ab^5(dx)^{17/2}}{17d^{11}}+\frac{2b^6(dx)^{21/2}}{21d^{13}}$$

```
[Out] -2/3*a^6/d/(d*x)^(3/2)+6*a^4*b^2*(d*x)^(5/2)/d^5+40/9*a^3*b^3*(d*x)^(9/2)/d
^7+30/13*a^2*b^4*(d*x)^(13/2)/d^9+12/17*a*b^5*(d*x)^(17/2)/d^11+2/21*b^6*(d
*x)^(21/2)/d^13+12*a^5*b*(d*x)^(1/2)/d^3
```

Rubi [A] time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{30a^2b^4(dx)^{13/2}}{13d^9}+\frac{40a^3b^3(dx)^{9/2}}{9d^7}+\frac{6a^4b^2(dx)^{5/2}}{d^5}+\frac{12a^5b\sqrt{dx}}{d^3}-\frac{2a^6}{3d(dx)^{3/2}}+\frac{12ab^5(dx)^{17/2}}{17d^{11}}+\frac{2b^6(dx)^{21/2}}{21d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(5/2),x]
```

```
[Out] (-2*a^6)/(3*d*(d*x)^(3/2)) + (12*a^5*b*Sqrt[d*x])/d^3 + (6*a^4*b^2*(d*x)^(5
/2))/d^5 + (40*a^3*b^3*(d*x)^(9/2))/(9*d^7) + (30*a^2*b^4*(d*x)^(13/2))/(13
*d^9) + (12*a*b^5*(d*x)^(17/2))/(17*d^11) + (2*b^6*(d*x)^(21/2))/(21*d^13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{(dx)^{5/2}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{(dx)^{5/2}}\,dx}{b^6}$$
$$= \frac{\int \left(\frac{a^6b^6}{(dx)^{5/2}} + \frac{6a^5b^7}{d^2\sqrt{dx}} + \frac{15a^4b^8(dx)^{3/2}}{d^4} + \frac{20a^3b^9(dx)^{7/2}}{d^6} + \frac{15a^2b^{10}(dx)^{11/2}}{d^8} + \frac{6ab^{11}(dx)^{15/2}}{d^{10}} + \frac{b^{12}(dx)^{19}}{d^{12}}\right.}{b^6}$$
$$= -\frac{2a^6}{3d(dx)^{3/2}} + \frac{12a^5b\sqrt{dx}}{d^3} + \frac{6a^4b^2(dx)^{5/2}}{d^5} + \frac{40a^3b^3(dx)^{9/2}}{9d^7} + \frac{30a^2b^4(dx)^{13/2}}{13d^9} + \frac{12ab}{1}$$

Mathematica [A] time = 0.02, size = 77, normalized size = 0.61

$$\frac{2x\left(-4641a^6+83538a^5bx^2+41769a^4b^2x^4+30940a^3b^3x^6+16065a^2b^4x^8+4914ab^5x^{10}+663b^6x^{12}\right)}{13923(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(5/2),x]
```

```
[Out] (2*x*(-4641*a^6 + 83538*a^5*b*x^2 + 41769*a^4*b^2*x^4 + 30940*a^3*b^3*x^6 +
 16065*a^2*b^4*x^8 + 4914*a*b^5*x^10 + 663*b^6*x^12))/(13923*(d*x)^(5/2))
```

fricas [A] time = 0.69, size = 78, normalized size = 0.61

$$\frac{2\left(663\,b^6x^{12}+4914\,ab^5x^{10}+16065\,a^2b^4x^8+30940\,a^3b^3x^6+41769\,a^4b^2x^4+83538\,a^5bx^2-4641\,a^6\right)\sqrt{dx}}{13923\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*b^6*x^12 + 4914*a*b^5*x^10 + 16065*a^2*b^4*x^8 + 30940*a^3*b^3
*x^6 + 41769*a^4*b^2*x^4 + 83538*a^5*b*x^2 - 4641*a^6)*sqrt(d*x)/(d^3*x^2)
```

giac [A] time = 0.17, size = 130, normalized size = 1.02

$$-\frac{2\left(\frac{4641\,a^6d}{\sqrt{dx}\,x} - \frac{663\,\sqrt{dx}\,b^6d^{210}x^{10}+4914\,\sqrt{dx}\,ab^5d^{210}x^8+16065\,\sqrt{dx}\,a^2b^4d^{210}x^6+30940\,\sqrt{dx}\,a^3b^3d^{210}x^4+41769\,\sqrt{dx}\,a^4b^2d^{210}x^2+83538\,\sqrt{dx}\,a^5bd^{2}}{d^{210}}\right.}{13923\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/13923*(4641*a^6*d/(sqrt(d*x)*x) - (663*sqrt(d*x)*b^6*d^210*x^10 + 4914*s
qrt(d*x)*a*b^5*d^210*x^8 + 16065*sqrt(d*x)*a^2*b^4*d^210*x^6 + 30940*sqrt(d
*x)*a^3*b^3*d^210*x^4 + 41769*sqrt(d*x)*a^4*b^2*d^210*x^2 + 83538*sqrt(d*x)
*a^5*b*d^210)/d^210)/d^3
```

maple [A] time = 0.01, size = 74, normalized size = 0.58

$$-\frac{2\left(-663b^6x^{12}-4914a\,b^5x^{10}-16065a^2b^4x^8-30940a^3b^3x^6-41769a^4b^2x^4-83538a^5b\,x^2+4641a^6\right)x}{13923\,(dx)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(5/2),x)
```

```
[Out] -2/13923*(-663*b^6*x^12-4914*a*b^5*x^10-16065*a^2*b^4*x^8-30940*a^3*b^3*x^6
-41769*a^4*b^2*x^4-83538*a^5*b*x^2+4641*a^6)*x/(d*x)^(5/2)
```

maxima [A] time = 1.39, size = 108, normalized size = 0.85

$$-\frac{2\left(\frac{4641\,a^6}{(dx)^{\frac{3}{2}}}-\frac{663\,(dx)^{\frac{21}{2}}b^6+4914\,(dx)^{\frac{17}{2}}ab^5d^2+16065\,(dx)^{\frac{13}{2}}a^2b^4d^4+30940\,(dx)^{\frac{9}{2}}a^3b^3d^6+41769\,(dx)^{\frac{5}{2}}a^4b^2d^8+83538\,\sqrt{dx}\,a^5bd^{10}}{d^{12}}\right)}{13923\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/13923*(4641*a^6/(d*x)^(3/2) - (663*(d*x)^(21/2)*b^6 + 4914*(d*x)^(17/2)*
a*b^5*d^2 + 16065*(d*x)^(13/2)*a^2*b^4*d^4 + 30940*(d*x)^(9/2)*a^3*b^3*d^6
+ 41769*(d*x)^(5/2)*a^4*b^2*d^8 + 83538*sqrt(d*x)*a^5*b*d^10)/d^12)/d
```

mupad [B] time = 0.04, size = 103, normalized size = 0.81

$$\frac{2\,b^6\,{\left(d\,x\right)}^{21/2}}{21\,d^{13}}-\frac{2\,a^6}{3\,d\,{\left(d\,x\right)}^{3/2}}+\frac{6\,a^4\,b^2\,{\left(d\,x\right)}^{5/2}}{d^5}+\frac{40\,a^3\,b^3\,{\left(d\,x\right)}^{9/2}}{9\,d^7}+\frac{30\,a^2\,b^4\,{\left(d\,x\right)}^{13/2}}{13\,d^9}+\frac{12\,a^5\,b\,\sqrt{d\,x}}{d^3}+\frac{12\,a\,b^5\,{\left(d\,x\right)}^{17/2}}{17\,d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/(d*x)^(5/2),x)
```

```
[Out] (2*b^6*(d*x)^(21/2))/(21*d^13) - (2*a^6)/(3*d*(d*x)^(3/2)) + (6*a^4*b^2*(d*
x)^(5/2))/d^5 + (40*a^3*b^3*(d*x)^(9/2))/(9*d^7) + (30*a^2*b^4*(d*x)^(13/2)
)/(13*d^9) + (12*a^5*b*(d*x)^(1/2))/d^3 + (12*a*b^5*(d*x)^(17/2))/(17*d^11)
```

sympy [A] time = 3.57, size = 128, normalized size = 1.01

$$-\frac{2a^6}{3d^{\frac{5}{2}}x^{\frac{3}{2}}}+\frac{12a^5b\sqrt{x}}{d^{\frac{5}{2}}}+\frac{6a^4b^2x^{\frac{5}{2}}}{d^{\frac{5}{2}}}+\frac{40a^3b^3x^{\frac{9}{2}}}{9d^{\frac{5}{2}}}+\frac{30a^2b^4x^{\frac{13}{2}}}{13d^{\frac{5}{2}}}+\frac{12ab^5x^{\frac{17}{2}}}{17d^{\frac{5}{2}}}+\frac{2b^6x^{\frac{21}{2}}}{21d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(5/2),x)
```

```
[Out] -2*a**6/(3*d**(5/2)*x**(3/2)) + 12*a**5*b*sqrt(x)/d**(5/2) + 6*a**4*b**2*x*
*(5/2)/d**(5/2) + 40*a**3*b**3*x**(9/2)/(9*d**(5/2)) + 30*a**2*b**4*x**(13/
2)/(13*d**(5/2)) + 12*a*b**5*x**(17/2)/(17*d**(5/2)) + 2*b**6*x**(21/2)/(21
*d**(5/2))
```

3.685 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{2a^6}{5d(dx)^{5/2}}-\frac{12a^5b}{d^3\sqrt{dx}}+\frac{10a^4b^2(dx)^{3/2}}{d^5}+\frac{40a^3b^3(dx)^{7/2}}{7d^7}+\frac{30a^2b^4(dx)^{11/2}}{11d^9}+\frac{4ab^5(dx)^{15/2}}{5d^{11}}+\frac{2b^6(dx)^{19/2}}{19d^{13}}$$

```
[Out] -2/5*a^6/d/(d*x)^(5/2)+10*a^4*b^2*(d*x)^(3/2)/d^5+40/7*a^3*b^3*(d*x)^(7/2)/
d^7+30/11*a^2*b^4*(d*x)^(11/2)/d^9+4/5*a*b^5*(d*x)^(15/2)/d^11+2/19*b^6*(d*
x)^(19/2)/d^13-12*a^5*b/d^3/(d*x)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{30a^2b^4(dx)^{11/2}}{11d^9}+\frac{40a^3b^3(dx)^{7/2}}{7d^7}+\frac{10a^4b^2(dx)^{3/2}}{d^5}-\frac{12a^5b}{d^3\sqrt{dx}}-\frac{2a^6}{5d(dx)^{5/2}}+\frac{4ab^5(dx)^{15/2}}{5d^{11}}+\frac{2b^6(dx)^{19/2}}{19d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(7/2),x]
```

```
[Out] (-2*a^6)/(5*d*(d*x)^(5/2)) - (12*a^5*b)/(d^3*Sqrt[d*x]) + (10*a^4*b^2*(d*x)
^(3/2))/d^5 + (40*a^3*b^3*(d*x)^(7/2))/(7*d^7) + (30*a^2*b^4*(d*x)^(11/2))/
(11*d^9) + (4*a*b^5*(d*x)^(15/2))/(5*d^11) + (2*b^6*(d*x)^(19/2))/(19*d^13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{(dx)^{7/2}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{(dx)^{7/2}}\,dx}{b^6}$$
$$= \frac{\int \left(\frac{a^6b^6}{(dx)^{7/2}} + \frac{6a^5b^7}{d^2(dx)^{3/2}} + \frac{15a^4b^8\sqrt{dx}}{d^4} + \frac{20a^3b^9(dx)^{5/2}}{d^6} + \frac{15a^2b^{10}(dx)^{9/2}}{d^8} + \frac{6ab^{11}(dx)^{13/2}}{d^{10}} + \frac{b^{12}(dx)^{17/}}{d^{12}}\right.}{b^6}$$
$$= -\frac{2a^6}{5d(dx)^{5/2}} - \frac{12a^5b}{d^3\sqrt{dx}} + \frac{10a^4b^2(dx)^{3/2}}{d^5} + \frac{40a^3b^3(dx)^{7/2}}{7d^7} + \frac{30a^2b^4(dx)^{11/2}}{11d^9} + \frac{4ab^5(d}{5d^1}$$

Mathematica [**A**] time = 0.03, size = 82, normalized size = 0.65

$$\frac{2\sqrt{dx}\left(-1463a^6-43890a^5bx^2+36575a^4b^2x^4+20900a^3b^3x^6+9975a^2b^4x^8+2926ab^5x^{10}+385b^6x^{12}\right)}{7315d^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(7/2),x]
```

```
[Out] (2*Sqrt[d*x]*(-1463*a^6 - 43890*a^5*b*x^2 + 36575*a^4*b^2*x^4 + 20900*a^3*b
^3*x^6 + 9975*a^2*b^4*x^8 + 2926*a*b^5*x^10 + 385*b^6*x^12))/(7315*d^4*x^3)
```

fricas [**A**] time = 0.77, size = 78, normalized size = 0.61

$$\frac{2\left(385\,b^6x^{12}+2926\,ab^5x^{10}+9975\,a^2b^4x^8+20900\,a^3b^3x^6+36575\,a^4b^2x^4-43890\,a^5bx^2-1463\,a^6\right)\sqrt{dx}}{7315\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/7315*(385*b^6*x^12 + 2926*a*b^5*x^10 + 9975*a^2*b^4*x^8 + 20900*a^3*b^3*x
^6 + 36575*a^4*b^2*x^4 - 43890*a^5*b*x^2 - 1463*a^6)*sqrt(d*x)/(d^4*x^3)
```

giac [**A**] time = 0.17, size = 133, normalized size = 1.05

$$-\frac{2\left(\frac{1463\left(30\,a^5bd^3x^2+a^6d^3\right)}{\sqrt{dx}\,d^2x^2}-\frac{385\,\sqrt{dx}\,b^6d^{171}x^9+2926\,\sqrt{dx}\,ab^5d^{171}x^7+9975\,\sqrt{dx}\,a^2b^4d^{171}x^5+20900\,\sqrt{dx}\,a^3b^3d^{171}x^3+36575\,\sqrt{dx}\,a^4b^2d^{171}x}{d^{171}}\right)}{7315\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/7315*(1463*(30*a^5*b*d^3*x^2 + a^6*d^3)/(sqrt(d*x)*d^2*x^2) - (385*sqrt(
d*x)*b^6*d^171*x^9 + 2926*sqrt(d*x)*a*b^5*d^171*x^7 + 9975*sqrt(d*x)*a^2*b^
4*d^171*x^5 + 20900*sqrt(d*x)*a^3*b^3*d^171*x^3 + 36575*sqrt(d*x)*a^4*b^2*d
^171*x)/d^171)/d^4
```

maple [A] time = 0.01, size = 74, normalized size = 0.58

$$-\frac{2\left(-385b^{6}x^{12}-2926a\,b^{5}x^{10}-9975a^{2}b^{4}x^{8}-20900a^{3}b^{3}x^{6}-36575a^{4}b^{2}x^{4}+43890a^{5}b\,x^{2}+1463a^{6}\right)x}{7315\left(dx\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(7/2),x)
```

```
[Out] -2/7315*(-385*b^6*x^12-2926*a*b^5*x^10-9975*a^2*b^4*x^8-20900*a^3*b^3*x^6-3
6575*a^4*b^2*x^4+43890*a^5*b*x^2+1463*a^6)*x/(d*x)^(7/2)
```

maxima [A] time = 1.35, size = 114, normalized size = 0.90

$$-\frac{2\left(\frac{1463\left(30\,a^{5}bd^{2}x^{2}+a^{6}d^{2}\right)}{\left(dx\right)^{\frac{5}{2}}d^{2}}-\frac{385\left(dx\right)^{\frac{19}{2}}b^{6}+2926\left(dx\right)^{\frac{15}{2}}ab^{5}d^{2}+9975\left(dx\right)^{\frac{11}{2}}a^{2}b^{4}d^{4}+20900\left(dx\right)^{\frac{7}{2}}a^{3}b^{3}d^{6}+36575\left(dx\right)^{\frac{3}{2}}a^{4}b^{2}d^{8}}{d^{12}}\right)}{7315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/7315*(1463*(30*a^5*b*d^2*x^2 + a^6*d^2)/((d*x)^(5/2)*d^2) - (385*(d*x)^(
19/2)*b^6 + 2926*(d*x)^(15/2)*a*b^5*d^2 + 9975*(d*x)^(11/2)*a^2*b^4*d^4 + 2
0900*(d*x)^(7/2)*a^3*b^3*d^6 + 36575*(d*x)^(3/2)*a^4*b^2*d^8)/d^12)/d
```

mupad [B] time = 0.04, size = 107, normalized size = 0.84

$$\frac{2\,b^{6}\,\left(d\,x\right)^{19/2}}{19\,d^{13}}-\frac{\frac{2\,a^{6}\,d^{2}}{5}+12\,b\,a^{5}\,d^{2}\,x^{2}}{d^{3}\,\left(d\,x\right)^{5/2}}+\frac{10\,a^{4}\,b^{2}\,\left(d\,x\right)^{3/2}}{d^{5}}+\frac{40\,a^{3}\,b^{3}\,\left(d\,x\right)^{7/2}}{7\,d^{7}}+\frac{30\,a^{2}\,b^{4}\,\left(d\,x\right)^{11/2}}{11\,d^{9}}+\frac{4\,a\,b^{5}\,\left(d\,x\right)^{15/2}}{5\,d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/(d*x)^(7/2),x)
```

```
[Out] (2*b^6*(d*x)^(19/2))/(19*d^13) - ((2*a^6*d^2)/5 + 12*a^5*b*d^2*x^2)/(d^3*(d
*x)^(5/2)) + (10*a^4*b^2*(d*x)^(3/2))/d^5 + (40*a^3*b^3*(d*x)^(7/2))/(7*d^7
) + (30*a^2*b^4*(d*x)^(11/2))/(11*d^9) + (4*a*b^5*(d*x)^(15/2))/(5*d^11)
```

sympy [A] time = 4.56, size = 128, normalized size = 1.01

$$-\frac{2a^{6}}{5d^{\frac{7}{2}}x^{\frac{5}{2}}}-\frac{12a^{5}b}{d^{\frac{7}{2}}\sqrt{x}}+\frac{10a^{4}b^{2}x^{\frac{3}{2}}}{d^{\frac{7}{2}}}+\frac{40a^{3}b^{3}x^{\frac{7}{2}}}{7d^{\frac{7}{2}}}+\frac{30a^{2}b^{4}x^{\frac{11}{2}}}{11d^{\frac{7}{2}}}+\frac{4ab^{5}x^{\frac{15}{2}}}{5d^{\frac{7}{2}}}+\frac{2b^{6}x^{\frac{19}{2}}}{19d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(7/2),x)
```

```
[Out] -2*a**6/(5*d**(7/2)*x**(5/2)) - 12*a**5*b/(d**(7/2)*sqrt(x)) + 10*a**4*b**2
*x**(3/2)/d**(7/2) + 40*a**3*b**3*x**(7/2)/(7*d**(7/2)) + 30*a**2*b**4*x**(
11/2)/(11*d**(7/2)) + 4*a*b**5*x**(15/2)/(5*d**(7/2)) + 2*b**6*x**(19/2)/(1
9*d**(7/2))
```

3.686 $\int \frac{(dx)^{11/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=316

$$-\frac{9a^{5/4}d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{13/4}}+\frac{9a^{5/4}d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{13/4}}-$$

```
[Out] 9/10*d^3*(d*x)^(5/2)/b^2-1/2*d*(d*x)^(9/2)/b/(b*x^2+a)-9/8*a^(5/4)*d^(11/2)
*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(13/4)*2^(1/2)+9/8
*a^(5/4)*d^(11/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(
13/4)*2^(1/2)-9/16*a^(5/4)*d^(11/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^
(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(13/4)*2^(1/2)+9/16*a^(5/4)*d^(11/2)*l
n(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^
(13/4)*2^(1/2)-9/2*a*d^5*(d*x)^(1/2)/b^3
```

Rubi [A] time = 0.38, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{9a^{5/4}d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{13/4}}+\frac{9a^{5/4}d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{13/4}}-$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-9*a*d^5*Sqrt[d*x])/(2*b^3) + (9*d^3*(d*x)^(5/2))/(10*b^2) - (d*(d*x)^(9/2
))/(2*b*(a + b*x^2)) - (9*a^(5/4)*d^(11/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(13/4)) + (9*a^(5/4)*d^(11/2)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(13/4))
- (9*a^(5/4)*d^(11/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(13/4)) + (9*a^(5/4)*d^(11/2)*Log[Sqr
t[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*S
qrt[2]*b^(13/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{11/2}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)} + \frac{1}{4}\left(9d^2\right)\int \frac{(dx)^{7/2}}{ab+b^2x^2}\,dx$$

$$= \frac{9d^3(dx)^{5/2}}{10b^2} - \frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)} - \frac{\left(9ad^4\right)\int \frac{(dx)^{3/2}}{ab+b^2x^2}\,dx}{4b}$$

$$= -\frac{9ad^5\sqrt{dx}}{2b^3} + \frac{9d^3(dx)^{5/2}}{10b^2} - \frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)} + \frac{\left(9a^2d^6\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{4b^2}$$

$$= -\frac{9ad^5\sqrt{dx}}{2b^3} + \frac{9d^3(dx)^{5/2}}{10b^2} - \frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)} + \frac{\left(9a^2d^5\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{2b^2}$$

$$= -\frac{9ad^5\sqrt{dx}}{2b^3} + \frac{9d^3(dx)^{5/2}}{10b^2} - \frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)} + \frac{\left(9a^{3/2}d^4\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4b^2}$$

$$= -\frac{9ad^5\sqrt{dx}}{2b^3} + \frac{9d^3(dx)^{5/2}}{10b^2} - \frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)} - \frac{\left(9a^{5/4}d^{11/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx,\right.}{8\sqrt{2}\,b^{13/4}}$$

$$= -\frac{9ad^5\sqrt{dx}}{2b^3} + \frac{9d^3(dx)^{5/2}}{10b^2} - \frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)} - \frac{9a^{5/4}d^{11/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\right.}{8\sqrt{2}\,b^{13/4}}$$

$$= -\frac{9ad^5\sqrt{dx}}{2b^3} + \frac{9d^3(dx)^{5/2}}{10b^2} - \frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)} - \frac{9a^{5/4}d^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,b^{13/4}} + \frac{9a^{5/4}d^{11}}{}$$

Mathematica [A] time = 0.34, size = 235, normalized size = 0.74

$$\frac{d^5\sqrt{dx}\left(-45\sqrt{2}\,a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+45\sqrt{2}\,a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-90\sqrt{2}\,a^{5}\right.}{80b^{13/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (d^5*Sqrt[d*x]*((8*b^(1/4)*Sqrt[x]*(-45*a^2 - 36*a*b*x^2 + 4*b^2*x^4))/(a +
 b*x^2) - 90*Sqrt[2]*a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
+ 90*Sqrt[2]*a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 45*Sqr
t[2]*a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 4
5*Sqrt[2]*a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
]))/(80*b^(13/4)*Sqrt[x])
```

fricas [A] time = 1.01, size = 283, normalized size = 0.90

$$\frac{180\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{1}{4}}\left(b^4x^2+ab^3\right)\arctan\left(-\frac{\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{3}{4}}\sqrt{dx}\,ab^{10}d^5-\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{3}{4}}\sqrt{a^2d^{11}x+\sqrt{-\frac{a^5d^{22}}{b^{13}}}\,b^6}\,b^{10}}{a^5d^{22}}\right)+45\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{1}{4}}\left(b^4x^2+a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/40*(180*(-a^5*d^22/b^13)^(1/4)*(b^4*x^2 + a*b^3)*arctan(-((-a^5*d^22/b^13
)^(3/4)*sqrt(d*x)*a*b^10*d^5 - (-a^5*d^22/b^13)^(3/4)*sqrt(a^2*d^11*x + sqr
t(-a^5*d^22/b^13)*b^6)*b^10)/(a^5*d^22)) + 45*(-a^5*d^22/b^13)^(1/4)*(b^4*x
^2 + a*b^3)*log(9*sqrt(d*x)*a*d^5 + 9*(-a^5*d^22/b^13)^(1/4)*b^3) - 45*(-a^
5*d^22/b^13)^(1/4)*(b^4*x^2 + a*b^3)*log(9*sqrt(d*x)*a*d^5 - 9*(-a^5*d^22/b
^13)^(1/4)*b^3) + 4*(4*b^2*d^5*x^4 - 36*a*b*d^5*x^2 - 45*a^2*d^5)*sqrt(d*x)
)/(b^4*x^2 + a*b^3)
```

giac [A] time = 0.21, size = 297, normalized size = 0.94

$$-\frac{1}{80}d^5\left(\frac{40\sqrt{dx}\,a^2d^2}{\left(bd^2x^2+ad^2\right)b^3}-\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4}-\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\right.}{}\right.}{b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/80*d^5*(40*sqrt(d*x)*a^2*d^2/((b*d^2*x^2 + a*d^2)*b^3) - 90*sqrt(2)*(a*b
^3*d^2)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/
(a*d^2/b)^(1/4))/b^4 - 90*sqrt(2)*(a*b^3*d^2)^(1/4)*a*arctan(-1/2*sqrt(2)*(
sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^4 - 45*sqrt(2)*(a
*b^3*d^2)^(1/4)*a*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/
b))/b^4 + 45*sqrt(2)*(a*b^3*d^2)^(1/4)*a*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*
sqrt(d*x) + sqrt(a*d^2/b))/b^4 - 32*(sqrt(d*x)*b^8*d^10*x^2 - 10*sqrt(d*x)*
a*b^7*d^10)/(b^10*d^10))
```

maple [A] time = 0.02, size = 242, normalized size = 0.77

$$-\frac{\sqrt{dx}\, a^2 d^7}{2\left(b\, d^2 x^2+d^2 a\right) b^3}+\frac{9\left(\frac{a\, d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\, a\, d^5 \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\, d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{8b^3}+\frac{9\left(\frac{a\, d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\, a\, d^5 \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\, d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{8b^3}+\frac{9\left(\frac{a\, d^2}{b}\right)^{\frac{1}{4}}\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 2/5*d^3*(d*x)^(5/2)/b^2-4*a*d^5*(d*x)^(1/2)/b^3-1/2*d^7/b^3*a^2*(d*x)^(1/2)
/(b*d^2*x^2+a*d^2)+9/16*d^5/b^3*a*(d^2*a/b)^(1/4)*2^(1/2)*ln((d*x+(d^2*a/b)
^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2*a/b)^(1/2))/(d*x-(d^2*a/b)^(1/4)*(d*x)^(1/2
)*2^(1/2)+(d^2*a/b)^(1/2)))+9/8*d^5/b^3*a*(d^2*a/b)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(d^2*a/b)^(1/4)*(d*x)^(1/2)+1)+9/8*d^5/b^3*a*(d^2*a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(d^2*a/b)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.04, size = 300, normalized size = 0.95

$$-\frac{\frac{40\sqrt{dx}\, a^2 d^8}{b^4 d^2 x^2+a b^3 d^2}-\frac{45\left(\frac{\sqrt{2}\, d^8 \log\left(\sqrt{b}\, dx+\sqrt{2}\left(a d^2\right)^{\frac{1}{4}}\sqrt{dx}\, b^{\frac{1}{4}}+\sqrt{a}\, d\right)}{\left(a d^2\right)^{\frac{3}{4}} b^{\frac{1}{4}}}-\frac{\sqrt{2}\, d^8 \log\left(\sqrt{b}\, dx-\sqrt{2}\left(a d^2\right)^{\frac{1}{4}}\sqrt{dx}\, b^{\frac{1}{4}}+\sqrt{a}\, d\right)}{\left(a d^2\right)^{\frac{3}{4}} b^{\frac{1}{4}}}+\frac{2\sqrt{2}\, d^7 \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(a d^2\right)^{\frac{1}{4}} b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\, d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\, d}\,\sqrt{a}}+\cdots\right)}{b^3}}{80\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/80*(40*sqrt(d*x)*a^2*d^8/(b^4*d^2*x^2 + a*b^3*d^2) - 45*(sqrt(2)*d^8*log
(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2
```

```
)^(3/4)*b^(1/4)) - sqrt(2)*d^8*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt
(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^7*arctan(1
/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(
a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^7*arctan(-1/
2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a
)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))*a^2/b^3 - 32*((d*x)^(5/2)*
b*d^4 - 10*sqrt(d*x)*a*d^6)/b^3)/d
```

mupad [B] time = 4.27, size = 129, normalized size = 0.41

$$\frac{2\,d^3\,{\left(d\,x\right)}^{5/2}}{5\,b^2}-\frac{9\,{\left(-a\right)}^{5/4}\,d^{11/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{4\,b^{13/4}}-\frac{a^2\,d^7\,\sqrt{d\,x}}{2\,\left(b^4\,d^2\,x^2+a\,b^3\,d^2\right)}-\frac{4\,a\,d^5\,\sqrt{d\,x}}{b^3}+\frac{{\left(-a\right)}^{5/4}\,d^{11/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{4\,b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (2*d^3*(d*x)^(5/2))/(5*b^2) - (9*(-a)^(5/4)*d^(11/2)*atan((b^(1/4)*(d*x)^(1
/2))/((-a)^(1/4)*d^(1/2))))/(4*b^(13/4)) + ((-a)^(5/4)*d^(11/2)*atan((b^(1/
4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*9i)/(4*b^(13/4)) - (a^2*d^7*(d*x)^
(1/2))/(2*(a*b^3*d^2 + b^4*d^2*x^2)) - (4*a*d^5*(d*x)^(1/2))/b^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{11}{2}}}{\left(a + bx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(11/2)/(a + b*x**2)**2, x)
```

3.687 $\int \frac{(dx)^{9/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=298

$$-\frac{7a^{3/4}d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{11/4}}+\frac{7a^{3/4}d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{11/4}}+\frac{7a}{}$$

```
[Out] 7/6*d^3*(d*x)^(3/2)/b^2-1/2*d*(d*x)^(7/2)/b/(b*x^2+a)+7/8*a^(3/4)*d^(9/2)*a
rctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(11/4)*2^(1/2)-7/8*a
^(3/4)*d^(9/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(11/
4)*2^(1/2)-7/16*a^(3/4)*d^(9/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4
)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(11/4)*2^(1/2)+7/16*a^(3/4)*d^(9/2)*ln(a^(
1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(11/4
)*2^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{7a^{3/4}d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{11/4}}+\frac{7a^{3/4}d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{11/4}}+\frac{7a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (7*d^3*(d*x)^(3/2))/(6*b^2) - (d*(d*x)^(7/2))/(2*b*(a + b*x^2)) + (7*a^(3/4
)*d^(9/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqr
t[2]*b^(11/4)) - (7*a^(3/4)*d^(9/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/
(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(11/4)) - (7*a^(3/4)*d^(9/2)*Log[Sqrt[a]*S
qrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]
*b^(11/4)) + (7*a^(3/4)*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(11/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{9/2}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)} + \frac{1}{4}\left(7d^2\right)\int \frac{(dx)^{5/2}}{ab+b^2x^2}\,dx$$

$$= \frac{7d^3(dx)^{3/2}}{6b^2} - \frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)} - \frac{\left(7ad^4\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{4b}$$

$$= \frac{7d^3(dx)^{3/2}}{6b^2} - \frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)} - \frac{\left(7ad^3\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{2b}$$

$$= \frac{7d^3(dx)^{3/2}}{6b^2} - \frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)} + \frac{\left(7ad^3\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4b^{3/2}} - \frac{\left(7ad^3\right)\text{Subst}}{}$$

$$= \frac{7d^3(dx)^{3/2}}{6b^2} - \frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)} - \frac{\left(7a^{3/4}d^{9/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{dx}\right)}{8\sqrt{2}\,b^{11/4}} - \ (7$$

$$= \frac{7d^3(dx)^{3/2}}{6b^2} - \frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)} - \frac{7a^{3/4}d^{9/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{8\sqrt{2}\,b^{11/4}} + \ 7$$

$$= \frac{7d^3(dx)^{3/2}}{6b^2} - \frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)} + \frac{7a^{3/4}d^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,b^{11/4}} - \frac{7a^{3/4}d^{9/2}\tan^{-1}\left(1+\right.}{4\sqrt{2}\,b^{11/4}}$$

Mathematica [C] time = 0.02, size = 63, normalized size = 0.21

$$-\frac{2d^4x\sqrt{dx}\left(7\left(a+bx^2\right){}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)-7a-bx^2\right)}{3b^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-2*d^4*x*Sqrt[d*x]*(-7*a - b*x^2 + 7*(a + b*x^2)*Hypergeometric2F1[3/4, 2,
 7/4, -((b*x^2)/a)]))/(3*b^2*(a + b*x^2))
```

fricas [A] time = 0.86, size = 283, normalized size = 0.95

$$84\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{1}{4}}\left(b^3x^2+ab^2\right)\arctan\left(-\frac{\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{1}{4}}\sqrt{dx}\,a^2b^3d^{13}-\sqrt{a^4d^{27}x-\sqrt{-\frac{a^3d^{18}}{b^{11}}}\,a^3b^5d^{18}}\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{1}{4}}b^3}{a^3d^{18}}\right)-21\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{1}{4}}\left(b^3x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/24*(84*(-a^3*d^18/b^11)^(1/4)*(b^3*x^2 + a*b^2)*arctan(-((-a^3*d^18/b^11)
^(1/4)*sqrt(d*x)*a^2*b^3*d^13 - sqrt(a^4*d^27*x - sqrt(-a^3*d^18/b^11)*a^3*
b^5*d^18)*(-a^3*d^18/b^11)^(1/4)*b^3)/(a^3*d^18)) - 21*(-a^3*d^18/b^11)^(1/
4)*(b^3*x^2 + a*b^2)*log(343*sqrt(d*x)*a^2*d^13 + 343*(-a^3*d^18/b^11)^(3/4
)*b^8) + 21*(-a^3*d^18/b^11)^(1/4)*(b^3*x^2 + a*b^2)*log(343*sqrt(d*x)*a^2*
d^13 - 343*(-a^3*d^18/b^11)^(3/4)*b^8) + 4*(4*b*d^4*x^3 + 7*a*d^4*x)*sqrt(d
*x))/(b^3*x^2 + a*b^2)
```

giac [A] time = 0.19, size = 277, normalized size = 0.93

$$\frac{1}{48}\left(\frac{24\sqrt{dx}\,ad^2x}{\left(bd^2x^2+ad^2\right)b^2}+\frac{32\sqrt{dx}\,x}{b^2}-\frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^5d}-\frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\cdots}{\cdots}\right)}{b^5d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/48*(24*sqrt(d*x)*a*d^2*x/((b*d^2*x^2 + a*d^2)*b^2) + 32*sqrt(d*x)*x/b^2 -
 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) +
 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^5*d) - 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^
5*d) + 21*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(
```

```
d*x) + sqrt(a*d^2/b))/(b^5*d) - 21*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt
(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^5*d))*d^4
```

maple [A] time = 0.02, size = 226, normalized size = 0.76

$$\frac{(dx)^{\frac{3}{2}} a d^5}{2\left(b d^2 x^2 + d^2 a\right) b^2} - \frac{7\sqrt{2}\, a d^5 \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}} - 1\right)}{8\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} b^3} - \frac{7\sqrt{2}\, a d^5 \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}} + 1\right)}{8\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} b^3} - \frac{7\sqrt{2}\, a d^5 \ln\left(\frac{dx - \left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{dx} \cdots}{dx + \left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{dx} \cdots}\right)}{16\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 2/3*d^3*(d*x)^(3/2)/b^2+1/2*d^5*a/b^2*(d*x)^(3/2)/(b*d^2*x^2+a*d^2)-7/16*d^
5*a/b^3/(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))
)-7/8*d^5*a/b^3/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x
)^(1/2)+1)-7/8*d^5*a/b^3/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(
1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.12, size = 273, normalized size = 0.92

$$\frac{\frac{24\,(dx)^{\frac{3}{2}} a d^6}{b^3 d^2 x^2 + a b^2 d^2} - \frac{21\, a d^6 \left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(a d^2\right)^{\frac{1}{4}} b^{\frac{1}{4}} + 2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b} d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b} d}\,\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(a d^2\right)^{\frac{1}{4}} b^{\frac{1}{4}} - 2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b} d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b} d}\,\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{b} dx + \sqrt{2}\left(a d^2\right)^{\frac{1}{4}} \sqrt{dx}\, b^{\frac{1}{4}} + \sqrt{a} d\right)}{\left(a d^2\right)^{\frac{1}{4}} b^{\frac{3}{4}}} + \cdots\right)}{b^2}}{48\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/48*(24*(d*x)^(3/2)*a*d^6/(b^3*d^2*x^2 + a*b^2*d^2) - 21*a*d^6*(2*sqrt(2)*
arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sq
rt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sq
rt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d
*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^
```

```
(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
+ sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b^2 + 32*(d*x)^(3/2)*d^4/b^2)/d
```

mupad [B] time = 0.12, size = 112, normalized size = 0.38

$$\frac{2\,d^3\,{\left(d\,x\right)}^{3/2}}{3\,b^2}+\frac{7\,{\left(-a\right)}^{3/4}\,d^{9/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{4\,b^{11/4}}+\frac{a\,d^5\,{\left(d\,x\right)}^{3/2}}{2\,\left(b^3\,d^2\,x^2+a\,b^2\,d^2\right)}+\frac{{\left(-a\right)}^{3/4}\,d^{9/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)\,7{}\mathrm{i}}{4\,b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (2*d^3*(d*x)^(3/2))/(3*b^2) + (7*(-a)^(3/4)*d^(9/2)*atan((b^(1/4)*(d*x)^(1/
2))/((-a)^(1/4)*d^(1/2))))/(4*b^(11/4)) + ((-a)^(3/4)*d^(9/2)*atan((b^(1/4)
*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*7i)/(4*b^(11/4)) + (a*d^5*(d*x)^(3/2
))/(2*(a*b^2*d^2 + b^3*d^2*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{9}{2}}}{\left(a + bx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(9/2)/(a + b*x**2)**2, x)
```

3.688 $\int \frac{(dx)^{7/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=298

$$\frac{5\sqrt[4]{a}\,d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{a}\,d^{7/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{9/4}}+\frac{5\sqrt[4]{}}{}$$

```
[Out] -1/2*d*(d*x)^(5/2)/b/(b*x^2+a)+5/8*a^(1/4)*d^(7/2)*arctan(1-b^(1/4)*2^(1/2)
*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(9/4)*2^(1/2)-5/8*a^(1/4)*d^(7/2)*arctan(1+
b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(9/4)*2^(1/2)+5/16*a^(1/4)*d
^(7/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(
1/2))/b^(9/4)*2^(1/2)-5/16*a^(1/4)*d^(7/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(
1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(9/4)*2^(1/2)+5/2*d^3*(d*x)^(1/
2)/b^2
```

Rubi [A] time = 0.29, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{5\sqrt[4]{a}\,d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{a}\,d^{7/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{9/4}}+\frac{5\sqrt[4]{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (5*d^3*Sqrt[d*x])/(2*b^2) - (d*(d*x)^(5/2))/(2*b*(a + b*x^2)) + (5*a^(1/4)*
d^(7/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[
2]*b^(9/4)) - (5*a^(1/4)*d^(7/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(9/4)) + (5*a^(1/4)*d^(7/2)*Log[Sqrt[a]*Sqrt[
d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(
9/4)) - (5*a^(1/4)*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(9/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{7/2}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}+\frac{1}{4}\left(5d^2\right)\int \frac{(dx)^{3/2}}{ab+b^2x^2}\,dx$$

$$= \frac{5d^3\sqrt{dx}}{2b^2}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}-\frac{\left(5ad^4\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{4b}$$

$$= \frac{5d^3\sqrt{dx}}{2b^2}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}-\frac{\left(5ad^3\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{2b}$$

$$= \frac{5d^3\sqrt{dx}}{2b^2}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}-\frac{\left(5\sqrt{a}\,d^2\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{4b}-\frac{\left(5\sqrt{a}\,d^2\right)\text{Subs}}{}$$

$$= \frac{5d^3\sqrt{dx}}{2b^2}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}+\frac{\left(5\sqrt[4]{a}\,d^{7/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx,x,\sqrt{dx}\right)}{8\sqrt{2}\,b^{9/4}}+\frac{\left(5\sqrt[4]{a}\right.}{}$$

$$= \frac{5d^3\sqrt{dx}}{2b^2}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}+\frac{5\sqrt[4]{a}\,d^{7/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{8\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{a}}{}$$

$$= \frac{5d^3\sqrt{dx}}{2b^2}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}+\frac{5\sqrt[4]{a}\,d^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{a}\,d^{7/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{a}}\right.}{4\sqrt{2}\,b^{9/4}}$$

Mathematica [A] time = 0.16, size = 244, normalized size = 0.82

$$\frac{d^3\sqrt{dx}\left(\frac{32b^{5/4}x^2}{a+bx^2}+\frac{40a\sqrt[4]{b}}{a+bx^2}+\frac{5\sqrt{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{\sqrt{x}}-\frac{5\sqrt{2}\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{\sqrt{x}}+\frac{10\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{\sqrt{x}}\right.}{16b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (d^3*Sqrt[d*x]*((40*a*b^(1/4))/(a + b*x^2) + (32*b^(5/4)*x^2)/(a + b*x^2) +
 (10*Sqrt[2]*a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/Sqrt[x]
 - (10*Sqrt[2]*a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/Sqrt[
x] + (5*Sqrt[2]*a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/Sqrt[x] - (5*Sqrt[2]*a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/Sqrt[x]))/(16*b^(9/4))
```

fricas [A] time = 0.89, size = 247, normalized size = 0.83

$$-\frac{20\left(-\frac{ad^{14}}{b^{9}}\right)^{\frac{1}{4}}\left(b^{3}x^{2}+ab^{2}\right)\arctan\left(-\frac{\left(-\frac{ad^{14}}{b^{9}}\right)^{\frac{3}{4}}\sqrt{dx}\,b^{7}d^{3}-\sqrt{d^{7}x+\sqrt{-\frac{ad^{14}}{b^{9}}}\,b^{4}}\left(-\frac{ad^{14}}{b^{9}}\right)^{\frac{3}{4}}b^{7}}{ad^{14}}\right)+5\left(-\frac{ad^{14}}{b^{9}}\right)^{\frac{1}{4}}\left(b^{3}x^{2}+ab^{2}\right)\log\left(5\right.}{8\left(b^{3}x^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/8*(20*(-a*d^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*arctan(-((-a*d^14/b^9)^(3/4)
*sqrt(d*x)*b^7*d^3 - sqrt(d^7*x + sqrt(-a*d^14/b^9)*b^4)*(-a*d^14/b^9)^(3/4
)*b^7)/(a*d^14)) + 5*(-a*d^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*log(5*sqrt(d*x)*
d^3 + 5*(-a*d^14/b^9)^(1/4)*b^2) - 5*(-a*d^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*
log(5*sqrt(d*x)*d^3 - 5*(-a*d^14/b^9)^(1/4)*b^2) - 4*(4*b*d^3*x^2 + 5*a*d^3
)*sqrt(d*x))/(b^3*x^2 + a*b^2)
```

giac [A] time = 0.19, size = 263, normalized size = 0.88

$$\frac{1}{16}d^{3}\left(\frac{8\sqrt{dx}\,ad^{2}}{\left(bd^{2}x^{2}+ad^{2}\right)b^{2}}-\frac{10\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{b^{3}}-\frac{10\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\right.}{2\left(\frac{ad^{2}}{b}\right)}\right.}{b^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/16*d^3*(8*sqrt(d*x)*a*d^2/((b*d^2*x^2 + a*d^2)*b^2) - 10*sqrt(2)*(a*b^3*d
^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2
```

```
/b)^(1/4))/b^3 - 10*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*
(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^3 - 5*sqrt(2)*(a*b^3*d^2)
^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^3 + 5
*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sq
rt(a*d^2/b))/b^3 + 32*sqrt(d*x)/b^2)
```

maple [A] time = 0.02, size = 223, normalized size = 0.75

$$\frac{\sqrt{dx}\, a\, d^5}{2\left(b\, d^2x^2 + d^2a\right) b^2} - \frac{5\left(\frac{a\, d^2}{b}\right)^{\frac{1}{4}} \sqrt{2}\, d^3 \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\, d^2}{b}\right)^{\frac{1}{4}}} - 1\right)}{8b^2} - \frac{5\left(\frac{a\, d^2}{b}\right)^{\frac{1}{4}} \sqrt{2}\, d^3 \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\, d^2}{b}\right)^{\frac{1}{4}}} + 1\right)}{8b^2} - \frac{5\left(\frac{a\, d^2}{b}\right)^{\frac{1}{4}} \sqrt{2}\, d^3 \mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 2*d^3*(d*x)^(1/2)/b^2+1/2*d^5/b^2*a*(d*x)^(1/2)/(b*d^2*x^2+a*d^2)-5/16*d^3/
b^2*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/
b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))-5/
8*d^3/b^2*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2
)+1)-5/8*d^3/b^2*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*
x)^(1/2)-1)
```

maxima [A] time = 3.03, size = 282, normalized size = 0.95

$$\frac{\frac{8\sqrt{dx}\, ad^6}{b^3d^2x^2+ab^2d^2} + \frac{32\sqrt{dx}\, d^4}{b^2} - \frac{5\left(\frac{\sqrt{2}\, d^6 \log\left(\sqrt{b}\, dx + \sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\, b^{\frac{1}{4}} + \sqrt{a}\, d\right)}{\left(ad^2\right)^{\frac{3}{4}} b^{\frac{1}{4}}} - \frac{\sqrt{2}\, d^6 \log\left(\sqrt{b}\, dx - \sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\, b^{\frac{1}{4}} + \sqrt{a}\, d\right)}{\left(ad^2\right)^{\frac{3}{4}} b^{\frac{1}{4}}} + \frac{2\sqrt{2}\, d^5 \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}} b^{\frac{1}{4}} + 2\sqrt{}\right.}{2\sqrt{\sqrt{a}\,\sqrt{b}\, d}}\right.}{\sqrt{\sqrt{a}\,\sqrt{b}\, d}\,\sqrt{a}}\right.}{b^2}}{16\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/16*(8*sqrt(d*x)*a*d^6/(b^3*d^2*x^2 + a*b^2*d^2) + 32*sqrt(d*x)*d^4/b^2 -
5*(sqrt(2)*d^6*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) +
sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^6*log(sqrt(b)*d*x - sqrt(2)*
(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sq
rt(2)*d^5*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*s
```

```
qrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt
(2)*d^5*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sq
rt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))*a/b^2)/d
```

mupad [B] time = 0.12, size = 112, normalized size = 0.38

$$\frac{2\,d^3\,\sqrt{d\,x}}{b^2}-\frac{5\,{\left(-a\right)}^{1/4}\,d^{7/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{4\,b^{9/4}}+\frac{a\,d^5\,\sqrt{d\,x}}{2\,\left(b^3\,d^2\,x^2+a\,b^2\,d^2\right)}+\frac{{\left(-a\right)}^{1/4}\,d^{7/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)\,5{}\mathrm{i}}{4\,b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (2*d^3*(d*x)^(1/2))/b^2 - (5*(-a)^(1/4)*d^(7/2)*atan((b^(1/4)*(d*x)^(1/2))/
((-a)^(1/4)*d^(1/2))))/(4*b^(9/4)) + ((-a)^(1/4)*d^(7/2)*atan((b^(1/4)*(d*x
)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*5i)/(4*b^(9/4)) + (a*d^5*(d*x)^(1/2))/(2*
(a*b^2*d^2 + b^3*d^2*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{7}{2}}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(7/2)/(a + b*x**2)**2, x)
```

3.689 $\int \frac{(dx)^{5/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=281

$$\frac{3d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}}-\frac{3d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}}-\frac{3d^{5/2}\tan^{-1}\left(\right.}{4\sqrt{2}}$$

```
[Out] -1/2*d*(d*x)^(3/2)/b/(b*x^2+a)-3/8*d^(5/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(
1/2)/a^(1/4)/d^(1/2))/a^(1/4)/b^(7/4)*2^(1/2)+3/8*d^(5/2)*arctan(1+b^(1/4)*
2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(1/4)/b^(7/4)*2^(1/2)+3/16*d^(5/2)*l
n(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^
(1/4)/b^(7/4)*2^(1/2)-3/16*d^(5/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(
1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(1/4)/b^(7/4)*2^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}}-\frac{3d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}}-\frac{3d^{5/2}\tan^{-1}\left(\right.}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -(d*(d*x)^(3/2))/(2*b*(a + b*x^2)) - (3*d^(5/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(1/4)*b^(7/4)) + (3*d^(5/2)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(1/4)
*b^(7/4)) + (3*d^(5/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(1/4)*b^(7/4)) - (3*d^(5/2)*Log[Sqrt
[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sq
rt[2]*a^(1/4)*b^(7/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)} + \frac{1}{4}\left(3d^2\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx$$

$$= -\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)} + \frac{1}{2}(3d)\,\mathrm{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)$$

$$= -\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)} - \frac{(3d)\,\mathrm{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4\sqrt{b}} + \frac{(3d)\,\mathrm{Subst}\left(\int \frac{\sqrt{a}\,d+\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x\right.}{4\sqrt{b}}$$

$$= -\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)} + \frac{\left(3d^{5/2}\right)\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{dx}\right)}{8\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}} + \frac{\left(3d^{5/2}\right)\mathrm{Subst}\left(\int -\right.}{8}$$

$$= -\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)} + \frac{3d^{5/2}\log\left(\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}\right)}{8\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}} - \frac{3d^{5/2}\log\left(\sqrt{a}\,\sqrt{d}\right.}{}$$

$$= -\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)} - \frac{3d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}} + \frac{3d^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}} + \frac{3d^{5/2}\log}{}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.19

$$\frac{2d(dx)^{3/2}\left(\left(a+bx^2\right)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)-a\right)}{ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*d*(d*x)^(3/2)*(-a + (a + b*x^2)*Hypergeometric2F1[3/4, 2, 7/4, -((b*x^2)
/a)]))/(a*b*(a + b*x^2))
```

fricas [A] time = 0.74, size = 247, normalized size = 0.88

$$-\frac{4\sqrt{dx}\,d^2x+12\left(b^2x^2+ab\right)\left(-\frac{d^{10}}{ab^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{10}}{ab^7}\right)^{\frac{1}{4}}\sqrt{dx}\,b^2d^7-\sqrt{d^{15}x-\sqrt{-\frac{d^{10}}{ab^7}}\,ab^3d^{10}}\left(-\frac{d^{10}}{ab^7}\right)^{\frac{1}{4}}b^2}{d^{10}}\right)-3\left(b^2x^2+ab\right)\left(-\frac{d^{10}}{ab^7}\right.}{8\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*sqrt(d*x)*d^2*x + 12*(b^2*x^2 + a*b)*(-d^10/(a*b^7))^(1/4)*arctan(-
((-d^10/(a*b^7))^(1/4)*sqrt(d*x)*b^2*d^7 - sqrt(d^15*x - sqrt(-d^10/(a*b^7)
)*a*b^3*d^10)*(-d^10/(a*b^7))^(1/4)*b^2)/d^10) - 3*(b^2*x^2 + a*b)*(-d^10/(
a*b^7))^(1/4)*log(27*sqrt(d*x)*d^7 + 27*(-d^10/(a*b^7))^(3/4)*a*b^5) + 3*(b
^2*x^2 + a*b)*(-d^10/(a*b^7))^(1/4)*log(27*sqrt(d*x)*d^7 - 27*(-d^10/(a*b^7
))^(3/4)*a*b^5))/(b^2*x^2 + a*b)
```

giac [A] time = 0.23, size = 277, normalized size = 0.99

$$-\frac{1}{16}\left(\frac{8\sqrt{dx}\,d^2x}{\left(bd^2x^2+ad^2\right)b}-\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^4d}-\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{\right.}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right.}{ab^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/16*(8*sqrt(d*x)*d^2*x/((b*d^2*x^2 + a*d^2)*b) - 6*sqrt(2)*(a*b^3*d^2)^(3
/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1
/4))/(a*b^4*d) - 6*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^4*d) + 3*sqrt(2)*(a*b^3
*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a
```

```
*b^4*d) - 3*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqr
t(d*x) + sqrt(a*d^2/b))/(a*b^4*d))*d^2
```

maple [A] time = 0.02, size = 209, normalized size = 0.74

$$-\frac{(dx)^{\frac{3}{2}} d^3}{2\left(b d^2x^2 + d^2a\right) b}+\frac{3\sqrt{2}\, d^3 \arctan\left(\frac{\sqrt{2}\, \sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} b^2}+\frac{3\sqrt{2}\, d^3 \arctan\left(\frac{\sqrt{2}\, \sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} b^2}+\frac{3\sqrt{2}\, d^3 \ln\left(\frac{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\cdots}}{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\cdots}}\right)}{16\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2*d^3/b*(d*x)^(3/2)/(b*d^2*x^2+a*d^2)+3/16*d^3/b^2/(a/b*d^2)^(1/4)*2^(1/
2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d
^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+3/8*d^3/b^2/(a/b*d^2)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+3/8*d^3/b^2/(a/b*d^2
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.10, size = 256, normalized size = 0.91

$$-\frac{\frac{8\,(dx)^{\frac{3}{2}} d^4}{b^2d^2x^2+abd^2}-\frac{3\,d^4\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\cdots\right)}{b}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/16*(8*(d*x)^(3/2)*d^4/(b^2*d^2*x^2 + a*b*d^2) - 3*d^4*(2*sqrt(2)*arctan(
1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt
(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*s
qrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*s
qrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sq
rt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4))
+ sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(
a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b)/d
```

mupad [B] time = 4.25, size = 92, normalized size = 0.33

$$\frac{3\,d^{5/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{1/4}\,b^{7/4}}-\frac{3\,d^{5/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{1/4}\,b^{7/4}}-\frac{d^3\,(d\,x)^{3/2}}{2\,b\,\left(b\,d^2\,x^2+a\,d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (3*d^(5/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(-a)^(1/4)*
b^(7/4)) - (3*d^(5/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4
*(-a)^(1/4)*b^(7/4)) - (d^3*(d*x)^(3/2))/(2*b*(a*d^2 + b*d^2*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{5}{2}}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(5/2)/(a + b*x**2)**2, x)
```

3.690 $\int \frac{(dx)^{3/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=281

$$-\frac{d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{d^{3/2}\tan^{-1}\left(1\right.}{4\sqrt{2}\,a}$$

```
[Out] -1/8*d^(3/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(3/4)/
b^(5/4)*2^(1/2)+1/8*d^(3/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^
(1/2))/a^(3/4)/b^(5/4)*2^(1/2)-1/16*d^(3/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^
(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(3/4)/b^(5/4)*2^(1/2)+1/16*d^(
3/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/
2))/a^(3/4)/b^(5/4)*2^(1/2)-1/2*d*(d*x)^(1/2)/b/(b*x^2+a)
```

Rubi [A] time = 0.26, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {28, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{d^{3/2}\tan^{-1}\left(1\right.}{4\sqrt{2}\,a}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -(d*Sqrt[d*x])/(2*b*(a + b*x^2)) - (d^(3/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + (d^(3/2)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(3/4)*b^(5/
4)) - (d^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(3/4)*b^(5/4)) + (d^(3/2)*Log[Sqrt[a]*Sqrt[d
] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(3
/4)*b^(5/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)} + \frac{1}{4}d^2\int \frac{1}{\sqrt{dx}\,\left(ab+b^2x^2\right)}\,dx$$

$$= -\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)} + \frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)$$

$$= -\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4\sqrt{a}} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}\,d+\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4\sqrt{a}}$$

$$= -\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)} - \frac{d^{3/2}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{dx}\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}} - \frac{d^{3/2}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{b}}}{-\frac{\sqrt{a}\,d}{\sqrt{b}}+\frac{\sqrt{2}}{}}\right)}{8\sqrt{2}\,a}$$

$$= -\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)} - \frac{d^{3/2}\log\left(\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}} + \frac{d^{3/2}\log\left(\sqrt{a}\,\sqrt{d}+\right.}{8}$$

$$= -\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)} - \frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}} + \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}} - \frac{d^{3/2}\log\left(\right.}{}$$

Mathematica [A] time = 0.17, size = 210, normalized size = 0.75

$$\frac{(dx)^{3/2}\left(-\frac{\sqrt{2}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{3/4}} + \frac{\sqrt{2}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{3/4}} - \frac{2\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}} + \frac{2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{3/4}}\right.}{16b^{5/4}x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] ((d*x)^(3/2)*((-8*b^(1/4)*Sqrt[x])/(a + b*x^2) - (2*Sqrt[2]*ArcTan[1 - (Sqr
t[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) + (2*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^
(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) - (Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4) + (Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4)))/(16*b^(5/4)*x^(3/2))
```

fricas [A] time = 0.85, size = 234, normalized size = 0.83

$$\frac{4\left(b^2x^2+ab\right)\left(-\frac{d^6}{a^3b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^2b^4d\left(-\frac{d^6}{a^3b^5}\right)^{\frac{3}{4}}-\sqrt{a^2b^2\sqrt{-\frac{d^6}{a^3b^5}}+d^3x}\,a^2b^4\left(-\frac{d^6}{a^3b^5}\right)^{\frac{3}{4}}}{d^6}\right)+\left(b^2x^2+ab\right)\left(-\frac{d^6}{a^3b^5}\right)^{\frac{1}{4}}\log\left(ab\left(\cdots\right.\right.}{8\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b^2*x^2 + a*b)*(-d^6/(a^3*b^5))^(1/4)*arctan(-(sqrt(d*x)*a^2*b^4*d*
(-d^6/(a^3*b^5))^(3/4) - sqrt(a^2*b^2*sqrt(-d^6/(a^3*b^5)) + d^3*x)*a^2*b^4
*(-d^6/(a^3*b^5))^(3/4))/d^6) + (b^2*x^2 + a*b)*(-d^6/(a^3*b^5))^(1/4)*log(
a*b*(-d^6/(a^3*b^5))^(1/4) + sqrt(d*x)*d) - (b^2*x^2 + a*b)*(-d^6/(a^3*b^5)
)^(1/4)*log(-a*b*(-d^6/(a^3*b^5))^(1/4) + sqrt(d*x)*d) - 4*sqrt(d*x)*d)/(b^
2*x^2 + a*b)
```

giac [A] time = 0.23, size = 261, normalized size = 0.93

$$-\frac{1}{16}\,d\left(\frac{8\sqrt{dx}\,d^2}{\left(bd^2x^2+ad^2\right)b}-\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^2}-\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\cdots\right.}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right.}{ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/16*d*(8*sqrt(d*x)*d^2/((b*d^2*x^2 + a*d^2)*b) - 2*sqrt(2)*(a*b^3*d^2)^(1
/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1
/4))/(a*b^2) - 2*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^2) - sqrt(2)*(a*b^3*d^2)^
(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^2)
+ sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + s
qrt(a*d^2/b))/(a*b^2))
```

maple [A] time = 0.01, size = 212, normalized size = 0.75

$$-\frac{\sqrt{dx}\, d^{3}}{2\left(b\, d^{2} x^{2}+d^{2} a\right) b}+\frac{\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{2}\, d \arctan\left(\frac{\sqrt{2}\, \sqrt{dx}}{\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}}}-1\right)}{8ab}+\frac{\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{2}\, d \arctan\left(\frac{\sqrt{2}\, \sqrt{dx}}{\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)}{8ab}+\frac{\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{2}\, d \ln\left(\frac{dx+(\cdots}{dx-(\cdots}\right.}{16\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2*d^3/b*(d*x)^(1/2)/(b*d^2*x^2+a*d^2)+1/16*d/b*(a/b*d^2)^(1/4)/a*2^(1/2)
*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+1/8*d/b*(a/b*d^2)^(1/4)/a*2^(
1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+1/8*d/b*(a/b*d^2)^(1/4)/
a*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.06, size = 265, normalized size = 0.94

$$-\frac{\frac{8\sqrt{dx}\, d^{4}}{b^{2} d^{2} x^{2}+a b d^{2}}-\frac{\frac{\sqrt{2}\, d^{4} \log\left(\sqrt{b}\, dx+\sqrt{2} \left(a d^{2}\right)^{\frac{1}{4}} \sqrt{dx}\, b^{\frac{1}{4}}+\sqrt{a}\, d\right)}{\left(a d^{2}\right)^{\frac{3}{4}} b^{\frac{1}{4}}}-\frac{\sqrt{2}\, d^{4} \log\left(\sqrt{b}\, dx-\sqrt{2} \left(a d^{2}\right)^{\frac{1}{4}} \sqrt{dx}\, b^{\frac{1}{4}}+\sqrt{a}\, d\right)}{\left(a d^{2}\right)^{\frac{3}{4}} b^{\frac{1}{4}}}+\frac{2\sqrt{2}\, d^{3} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2} \left(a d^{2}\right)^{\frac{1}{4}} b^{\frac{1}{4}}+2\sqrt{dx}\, \sqrt{b}\right)}{2\sqrt{\sqrt{a}\, \sqrt{b}\, d}}\right)}{\sqrt{\sqrt{a}\, \sqrt{b}\, d}\, \sqrt{a}}+\frac{2\sqrt{2}\, d^{3} \cdots}{\cdots}}{b}}{16\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/16*(8*sqrt(d*x)*d^4/(b^2*d^2*x^2 + a*b*d^2) - (sqrt(2)*d^4*log(sqrt(b)*d
*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^
(1/4)) - sqrt(2)*d^4*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1
/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^3*arctan(1/2*sqrt(2)
*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)
*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^3*arctan(-1/2*sqrt(2)*
```

```
(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*
d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/b)/d
```

mupad [B] time = 4.35, size = 92, normalized size = 0.33

$$-\frac{d^{3/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{3/4}\,b^{5/4}}-\frac{d^{3/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{3/4}\,b^{5/4}}-\frac{d^3\,\sqrt{d\,x}}{2\,b\,\left(b\,d^2\,x^2+a\,d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] - (d^(3/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(-a)^(3/4)*
b^(5/4)) - (d^(3/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(
-a)^(3/4)*b^(5/4)) - (d^3*(d*x)^(1/2))/(2*b*(a*d^2 + b*d^2*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(dx)^{\frac{3}{2}}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(3/2)/(a + b*x**2)**2, x)
```

3.691 $\int \frac{\sqrt{dx}}{a^2+2abx^2+b^2x^4} \, dx$

Optimal. Leaf size=283

$$\frac{\sqrt{d}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{\sqrt{d}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{\sqrt{d}\,\tan^{-1}\Big(1-}{4\sqrt{2}\,a^{5/4}}$$

```
[Out] 1/2*(d*x)^(3/2)/a/d/(b*x^2+a)-1/8*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1
/4)/d^(1/2))*d^(1/2)/a^(5/4)/b^(3/4)*2^(1/2)+1/8*arctan(1+b^(1/4)*2^(1/2)*(
d*x)^(1/2)/a^(1/4)/d^(1/2))*d^(1/2)/a^(5/4)/b^(3/4)*2^(1/2)+1/16*ln(a^(1/2)
*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))*d^(1/2)/a^(
5/4)/b^(3/4)*2^(1/2)-1/16*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1
/4)*2^(1/2)*(d*x)^(1/2))*d^(1/2)/a^(5/4)/b^(3/4)*2^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt{d}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{\sqrt{d}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{\sqrt{d}\,\tan^{-1}\Big(1-}{4\sqrt{2}\,a^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (d*x)^(3/2)/(2*a*d*(a + b*x^2)) - (Sqrt[d]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(5/4)*b^(3/4)) + (Sqrt[d]*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(5/4)*b^(3/4
)) + (Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(5/4)*b^(3/4)) - (Sqrt[d]*Log[Sqrt[a]*Sqrt[d]
 + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(5/
4)*b^(3/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)} + \frac{b\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{4a}$$

$$= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)} + \frac{b\,\mathrm{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{2ad}$$

$$= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)} - \frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4ad} + \frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{\sqrt{a}\,d+\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x,\right.}{4ad}$$

$$= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)} + \frac{\sqrt{d}\,\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{dx}\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}} + \frac{\sqrt{d}\,\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}}{-\frac{\sqrt{a}\,d}{\sqrt{b}}+\sqrt{2}}\right.}{8\sqrt{2}\,a}$$

$$= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)} + \frac{\sqrt{d}\,\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}} - \frac{\sqrt{d}\,\log\left(\sqrt{a}\sqrt{d}+\right.}{8}$$

$$= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)} - \frac{\sqrt{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}} + \frac{\sqrt{d}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}} + \frac{\sqrt{d}\,\log\left(\sqrt{\right.}{}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.11

$$\frac{2x\sqrt{dx}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*x*Sqrt[d*x]*Hypergeometric2F1[3/4, 2, 7/4, -((b*x^2)/a)])/(3*a^2)
```

fricas [A] time = 1.13, size = 232, normalized size = 0.82

$$-\frac{4\left(abx^2+a^2\right)\left(-\frac{d^2}{a^5b^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,abd\left(-\frac{d^2}{a^5b^3}\right)^{\frac{1}{4}}-\sqrt{-a^3bd^2\sqrt{-\frac{d^2}{a^5b^3}}+d^3x}\,ab\left(-\frac{d^2}{a^5b^3}\right)^{\frac{1}{4}}}{d^2}\right)-\left(abx^2+a^2\right)\left(-\frac{d^2}{a^5b^3}\right)^{\frac{1}{4}}\log\left(a^4b^2\cdots\right.}{8\left(abx^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*(a*b*x^2 + a^2)*(-d^2/(a^5*b^3))^(1/4)*arctan(-(sqrt(d*x)*a*b*d*(-d
^2/(a^5*b^3))^(1/4) - sqrt(-a^3*b*d^2*sqrt(-d^2/(a^5*b^3)) + d^3*x)*a*b*(-d
^2/(a^5*b^3))^(1/4))/d^2) - (a*b*x^2 + a^2)*(-d^2/(a^5*b^3))^(1/4)*log(a^4*
b^2*(-d^2/(a^5*b^3))^(3/4) + sqrt(d*x)*d) + (a*b*x^2 + a^2)*(-d^2/(a^5*b^3)
)^(1/4)*log(-a^4*b^2*(-d^2/(a^5*b^3))^(3/4) + sqrt(d*x)*d) - 4*sqrt(d*x)*x)
/(a*b*x^2 + a^2)
```

giac [A] time = 0.19, size = 264, normalized size = 0.93

$$\frac{\frac{8\sqrt{dx}\,d^3x}{\left(bd^2x^2+ad^2\right)a}+\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^3}+\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^3}-\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\cdots\right.\right.}{a^2b^3}\cdots}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/16*(8*sqrt(d*x)*d^3*x/((b*d^2*x^2 + a*d^2)*a) + 2*sqrt(2)*(a*b^3*d^2)^(3/
4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/
4))/(a^2*b^3) + 2*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^3) - sqrt(2)*(a*b^3*d^
2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*
b^3) + sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x
) + sqrt(a*d^2/b))/(a^2*b^3))/d
```

maple [A] time = 0.01, size = 210, normalized size = 0.74

$$\frac{\sqrt{2}\, d \arctan\left(\frac{\sqrt{2}\, \sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}} ab}+\frac{\sqrt{2}\, d \arctan\left(\frac{\sqrt{2}\, \sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}} ab}+\frac{\sqrt{2}\, d \ln\left(\frac{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)}{16\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}} ab}+\frac{(dx)^{\frac{3}{2}}\, d}{2\left(b\,d^{2}x^{2}+d^{2}a\right) a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2*d*(d*x)^(3/2)/a/(b*d^2*x^2+a*d^2)+1/16*d/a/b/(a/b*d^2)^(1/4)*2^(1/2)*ln
((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+1/8*d/a/b/(a/b*d^2)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+1/8*d/a/b/(a/b*d^2)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.06, size = 255, normalized size = 0.90

$$\frac{\frac{8\,(dx)^{\frac{3}{2}} d^{2}}{abd^{2}x^{2}+a^{2}d^{2}}+\frac{d^{2}\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}} b^{\frac{1}{4}}+2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}} b^{\frac{1}{4}}-2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^{2}\right)^{\frac{1}{4}} b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log \cdots}{\cdots}\right)}{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/16*(8*(d*x)^(3/2)*d^2/(a*b*d^2*x^2 + a^2*d^2) + d^2*(2*sqrt(2)*arctan(1/2
*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)
*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt
(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt
(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(
2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + s
qrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*
d)/((a*d^2)^(1/4)*b^(3/4)))/a)/d
```

mupad [B] time = 0.11, size = 90, normalized size = 0.32

$$\frac{\sqrt{d}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{5/4}\,b^{3/4}}-\frac{\sqrt{d}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{5/4}\,b^{3/4}}+\frac{d\,(d\,x)^{3/2}}{2\,a\,\left(b\,d^2\,x^2+a\,d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (d^(1/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(-a)^(5/4)*b
^(3/4)) - (d^(1/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(-a
)^(5/4)*b^(3/4)) + (d*(d*x)^(3/2))/(2*a*(a*d^2 + b*d^2*x^2))
```

sympy [A] time = 6.67, size = 78, normalized size = 0.28

$$\frac{2d^3\,(dx)^{\frac{3}{2}}}{4a^2d^4+4abd^4x^2}+2d^3\,\mathrm{RootSum}\left(65536t^4a^5b^3d^{10}+1,\left(t\mapsto t\log\left(4096t^3a^4b^2d^8+\sqrt{dx}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 2*d**3*(d*x)**(3/2)/(4*a**2*d**4 + 4*a*b*d**4*x**2) + 2*d**3*RootSum(65536*
_t**4*a**5*b**3*d**10 + 1, Lambda(_t, _t*log(4096*_t**3*a**4*b**2*d**8 + sq
rt(d*x))))
```

3.692 $\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)}\,dx$

Optimal. Leaf size=283

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}\sqrt{d}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}\sqrt{d}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}\sqrt{\ }}$$

```
[Out] -3/8*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(7/4)/b^(1/4)*
2^(1/2)/d^(1/2)+3/8*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a
^(7/4)/b^(1/4)*2^(1/2)/d^(1/2)-3/16*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^
(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(7/4)/b^(1/4)*2^(1/2)/d^(1/2)+3/16*ln(
a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(7
/4)/b^(1/4)*2^(1/2)/d^(1/2)+1/2*(d*x)^(1/2)/a/d/(b*x^2+a)
```

Rubi [A] time = 0.26, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}\sqrt{d}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}\sqrt{d}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}\sqrt{\ }}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] Sqrt[d*x]/(2*a*d*(a + b*x^2)) - (3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(
a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(7/4)*b^(1/4)*Sqrt[d]) + (3*ArcTan[1 + (Sqr
t[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(7/4)*b^(1/4)*Sqrt
[d]) - (3*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(8*Sqrt[2]*a^(7/4)*b^(1/4)*Sqrt[d]) + (3*Log[Sqrt[a]*Sqrt[d] +
 Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(7/4)
*b^(1/4)*Sqrt[d])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)}\,dx = b^2\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^2}\,dx$$

$$= \frac{\sqrt{dx}}{2ad\left(a+bx^2\right)} + \frac{(3b)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{4a}$$

$$= \frac{\sqrt{dx}}{2ad\left(a+bx^2\right)} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{2ad}$$

$$= \frac{\sqrt{dx}}{2ad\left(a+bx^2\right)} + \frac{(3b)\,\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4a^{3/2}d^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{\sqrt{a}\,d+\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4a^{3/2}d^2}$$

$$= \frac{\sqrt{dx}}{2ad\left(a+bx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt{d}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{dx}\right)}{8a^{3/2}\sqrt{b}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}\,d}{\sqrt{b}}+\dots}\,dx, \dots\right)}{8a\dots}$$

$$= \frac{\sqrt{dx}}{2ad\left(a+bx^2\right)} - \frac{3\log\left(\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}\,\sqrt{d}} + \frac{3\log\left(\sqrt{a}\,\sqrt{d}\dots\right.}{\dots}$$

$$= \frac{\sqrt{dx}}{2ad\left(a+bx^2\right)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}\,\sqrt{d}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}\,\sqrt{d}} - \frac{3\log\left(\sqrt{\dots}\right.}{\dots}$$

Mathematica [A] time = 0.17, size = 211, normalized size = 0.75

$$\frac{\sqrt{x}\left(\frac{8a^{3/4}\sqrt{x}}{a+bx^2}-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}+\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}-\frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}+\frac{6\sqrt{2}\tan^{-1}\left(\sqrt{\right.}}{\sqrt[4]{b}}\right.}{16a^{7/4}\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (Sqrt[x]*((8*a^(3/4)*Sqrt[x])/(a + b*x^2) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/b^(1/4) - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4)))/(16*a^(7/4)*Sqrt[d*x])
```

fricas [A] time = 1.00, size = 232, normalized size = 0.82

$$\frac{12\left(abdx^2+a^2d\right)\left(-\frac{1}{a^7bd^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^4d^2\sqrt{-\frac{1}{a^7bd^2}}+dx}\,a^5bd\left(-\frac{1}{a^7bd^2}\right)^{\frac{3}{4}}-\sqrt{dx}\,a^5bd\left(-\frac{1}{a^7bd^2}\right)^{\frac{3}{4}}\right)+3\left(abdx^2+a^2\right.}{8\left(abdx^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(12*(a*b*d*x^2 + a^2*d)*(-1/(a^7*b*d^2))^(1/4)*arctan(sqrt(a^4*d^2*sqrt
(-1/(a^7*b*d^2)) + d*x)*a^5*b*d*(-1/(a^7*b*d^2))^(3/4) - sqrt(d*x)*a^5*b*d*
(-1/(a^7*b*d^2))^(3/4)) + 3*(a*b*d*x^2 + a^2*d)*(-1/(a^7*b*d^2))^(1/4)*log(
a^2*d*(-1/(a^7*b*d^2))^(1/4) + sqrt(d*x)) - 3*(a*b*d*x^2 + a^2*d)*(-1/(a^7*
b*d^2))^(1/4)*log(-a^2*d*(-1/(a^7*b*d^2))^(1/4) + sqrt(d*x)) + 4*sqrt(d*x))
/(a*b*d*x^2 + a^2*d)
```

giac [A] time = 0.19, size = 269, normalized size = 0.95

$$\frac{\sqrt{dx}\,d}{2\left(bd^2x^2+ad^2\right)a}+\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8a^2bd}+\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8a^2bd}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x)*d/((b*d^2*x^2 + a*d^2)*a) + 3/8*sqrt(2)*(a*b^3*d^2)^(1/4)*arc
tan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a
^2*b*d) + 3/8*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b*d) + 3/16*sqrt(2)*(a*b^3*d
^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2
*b*d) - 3/16*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sq
rt(d*x) + sqrt(a*d^2/b))/(a^2*b*d)
```

maple [A] time = 0.01, size = 207, normalized size = 0.73

$$\frac{\sqrt{dx}\, d}{2\left(b d^2 x^2 + d^2 a\right) a} + \frac{3\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}} - 1\right)}{8a^2 d} + \frac{3\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}} + 1\right)}{8a^2 d} + \frac{3\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \ln\left(\frac{dx+\left(\cdots\right.}{dx-\left(\cdots\right.}\right.}{16a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x)
```

```
[Out] 1/2*d*(d*x)^(1/2)/a/(b*d^2*x^2+a*d^2)+3/16/d/a^2*(a/b*d^2)^(1/4)*2^(1/2)*ln
((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+3/8/d/a^2*(a/b*d^2)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+3/8/d/a^2*(a/b*d^2)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.00, size = 261, normalized size = 0.92

$$\frac{\frac{8\sqrt{dx}\, d^2}{abd^2x^2+a^2d^2} + \frac{3\left(\frac{\sqrt{2}\, d^2 \log\left(\sqrt{b}\, dx + \sqrt{2}\left(ad^2\right)^{\frac{1}{4}} \sqrt{dx}\, b^{\frac{1}{4}} + \sqrt{a}\, d\right)}{\left(ad^2\right)^{\frac{3}{4}} b^{\frac{1}{4}}} - \frac{\sqrt{2}\, d^2 \log\left(\sqrt{b}\, dx - \sqrt{2}\left(ad^2\right)^{\frac{1}{4}} \sqrt{dx}\, b^{\frac{1}{4}} + \sqrt{a}\, d\right)}{\left(ad^2\right)^{\frac{3}{4}} b^{\frac{1}{4}}} + \frac{2\sqrt{2}\, d \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}} b^{\frac{1}{4}} + 2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\, d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\, d}\,\sqrt{a}} + \frac{2\sqrt{2}\, d\cdots}{\cdots}\right.}{a}}{16\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(8*sqrt(d*x)*d^2/(a*b*d^2*x^2 + a^2*d^2) + 3*(sqrt(2)*d^2*log(sqrt(b)*
d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b
```

```
^(1/4)) - sqrt(2)*d^2*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(
1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d*arctan(1/2*sqrt(2)*
(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*
d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d*arctan(-1/2*sqrt(2)*(sq
rt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))
/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/a)/d
```

mupad [B] time = 0.10, size = 90, normalized size = 0.32

$$\frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{7/4}\,b^{1/4}\,\sqrt{d}}+\frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{7/4}\,b^{1/4}\,\sqrt{d}}+\frac{d\,\sqrt{d\,x}}{2\,a\,\left(b\,d^2\,x^2+a\,d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] (3*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(-a)^(7/4)*b^(1/4)*
d^(1/2)) + (3*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(-a)^(7
/4)*b^(1/4)*d^(1/2)) + (d*(d*x)^(1/2))/(2*a*(a*d^2 + b*d^2*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\,\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*(a + b*x**2)**2), x)
```

3.693 $\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx$

Optimal. Leaf size=300

$$-\frac{5\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{9/4}d^{3/2}}+\frac{5\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{9/4}d^{3/2}}+\frac{5\sqrt[4]{b}\tan^{-1}}{4\sqrt{}}$$

```
[Out] 5/8*b^(1/4)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(9/4)/d
^(3/2)*2^(1/2)-5/8*b^(1/4)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(
1/2))/a^(9/4)/d^(3/2)*2^(1/2)-5/16*b^(1/4)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(
1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(9/4)/d^(3/2)*2^(1/2)+5/16*b^(1
/4)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2
))/a^(9/4)/d^(3/2)*2^(1/2)-5/2/a^2/d/(d*x)^(1/2)+1/2/a/d/(b*x^2+a)/(d*x)^(1
/2)
```

Rubi [A] time = 0.32, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{5\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{9/4}d^{3/2}}+\frac{5\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{9/4}d^{3/2}}+\frac{5\sqrt[4]{b}\tan^{-1}}{4\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -5/(2*a^2*d*Sqrt[d*x]) + 1/(2*a*d*Sqrt[d*x]*(a + b*x^2)) + (5*b^(1/4)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(9/4)*d^
(3/2)) - (5*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
)])/(4*Sqrt[2]*a^(9/4)*d^(3/2)) - (5*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*
Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(9/4)*d^(3/2))
 + (5*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(9/4)*d^(3/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx = b^2\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^2}\,dx$$

$$= \frac{1}{2ad\sqrt{dx}\,\left(a+bx^2\right)} + \frac{(5b)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)}\,dx}{4a}$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\,\left(a+bx^2\right)} - \frac{\left(5b^2\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{4a^2d^2}$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\,\left(a+bx^2\right)} - \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{2a^2d^3}$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\,\left(a+bx^2\right)} + \frac{\left(5b^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4a^2d^3}$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\,\left(a+bx^2\right)} - \frac{\left(5\sqrt[4]{b}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx, x\right.}{8\sqrt{2}\,a^{9/4}d^{3/2}}$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\,\left(a+bx^2\right)} - \frac{5\sqrt[4]{b}\,\log\left(\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x-\sqrt{2}\,\sqrt[4]{a}\right.}{8\sqrt{2}\,a^{9/4}d^{3/2}}$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\,\left(a+bx^2\right)} + \frac{5\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,a^{9/4}d^{3/2}} - \frac{5\sqrt[4]{b}\,\tan}{4}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.10

$$-\frac{2x\,{}_2F_1\left(-\frac{1}{4},2;\frac{3}{4};-\frac{bx^2}{a}\right)}{a^2(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (-2*x*Hypergeometric2F1[-1/4, 2, 3/4, -((b*x^2)/a)])/(a^2*(d*x)^(3/2))
```

fricas [A] time = 1.03, size = 276, normalized size = 0.92

$$\frac{20\left(a^{2}bd^{2}x^{3}+a^{3}d^{2}x\right)\left(-\frac{b}{a^{9}d^{6}}\right)^{\frac{1}{4}}\arctan\left(-\frac{125\sqrt{dx}\,a^{2}bd\left(-\frac{b}{a^{9}d^{6}}\right)^{\frac{1}{4}}-\sqrt{-15625\,a^{5}bd^{4}\sqrt{-\frac{b}{a^{9}d^{6}}}+15625\,b^{2}dx}\,a^{2}d\left(-\frac{b}{a^{9}d^{6}}\right)^{\frac{1}{4}}}{125\,b}\right)-5\left(a^{2}bd^{2}x^{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/8*(20*(a^2*b*d^2*x^3 + a^3*d^2*x)*(-b/(a^9*d^6))^(1/4)*arctan(-1/125*(125
*sqrt(d*x)*a^2*b*d*(-b/(a^9*d^6))^(1/4) - sqrt(-15625*a^5*b*d^4*sqrt(-b/(a^
9*d^6)) + 15625*b^2*d*x)*a^2*d*(-b/(a^9*d^6))^(1/4))/b) - 5*(a^2*b*d^2*x^3
+ a^3*d^2*x)*(-b/(a^9*d^6))^(1/4)*log(125*a^7*d^5*(-b/(a^9*d^6))^(3/4) + 12
5*sqrt(d*x)*b) + 5*(a^2*b*d^2*x^3 + a^3*d^2*x)*(-b/(a^9*d^6))^(1/4)*log(-12
5*a^7*d^5*(-b/(a^9*d^6))^(3/4) + 125*sqrt(d*x)*b) - 4*(5*b*x^2 + 4*a)*sqrt(
d*x))/(a^2*b*d^2*x^3 + a^3*d^2*x)
```

giac [A] time = 0.18, size = 294, normalized size = 0.98

$$-\frac{\frac{8\left(5\,bd^{2}x^{2}+4\,ad^{2}\right)}{\left(\sqrt{dx}\,bd^{2}x^{2}+\sqrt{dx}\,ad^{2}\right)a^{2}}+\frac{10\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{3}b^{2}d^{2}}+\frac{10\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{3}b^{2}d^{2}}-\frac{5\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/16*(8*(5*b*d^2*x^2 + 4*a*d^2)/((sqrt(d*x)*b*d^2*x^2 + sqrt(d*x)*a*d^2)*a
^2) + 10*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^2*d^2) + 10*sqrt(2)*(a*b^3*d^2)^
(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)
^(1/4))/(a^3*b^2*d^2) - 5*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^
2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^2*d^2) + 5*sqrt(2)*(a*b^3*d^2)
^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^
2*d^2))/d
```

maple [A] time = 0.02, size = 223, normalized size = 0.74

$$-\frac{(dx)^{\frac{3}{2}} b}{2\left(b\,d^2x^2+d^2a\right)a^2d}-\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^2d}-\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^2d}-\frac{5\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{16\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2/d*b/a^2*(d*x)^(3/2)/(b*d^2*x^2+a*d^2)-5/16/d/a^2/(a/b*d^2)^(1/4)*2^(1/
2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d
^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))-5/8/d/a^2/(a/b*d^2)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)-5/8/d/a^2/(a/b*d^2)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)-2/a^2/d/(d*x)^(1/
2)
```

maxima [A] time = 3.12, size = 268, normalized size = 0.89

$$-\frac{\frac{8\left(5\,bd^2x^2+4\,ad^2\right)}{(dx)^{\frac{5}{2}}a^2b+\sqrt{dx}\,a^3d^2}+\frac{5\,b\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\cdots\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\cdots\right)}{a^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/16*(8*(5*b*d^2*x^2 + 4*a*d^2)/((d*x)^(5/2)*a^2*b + sqrt(d*x)*a^3*d^2) +
5*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d
*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2
*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*s
qrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2
)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((
a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqr
t(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/a^2)/d
```

mupad [B] time = 0.12, size = 102, normalized size = 0.34

$$\frac{5\,{\left(-b\right)}^{1/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{4\,a^{9/4}\,d^{3/2}}-\frac{5\,{\left(-b\right)}^{1/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{4\,a^{9/4}\,d^{3/2}}-\frac{\frac{2\,d}{a}+\frac{5\,b\,d\,x^2}{2\,a^2}}{b\,{\left(d\,x\right)}^{5/2}+a\,d^2\,\sqrt{d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] (5*(-b)^(1/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(4*a^(9/4)
*d^(3/2)) - (5*(-b)^(1/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))
/(4*a^(9/4)*d^(3/2)) - ((2*d)/a + (5*b*d*x^2)/(2*a^2))/(b*(d*x)^(5/2) + a*d
^2*(d*x)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral(1/((d*x)**(3/2)*(a + b*x**2)**2), x)
```

3.694 $\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx$

Optimal. Leaf size=300

$$\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{11/4}d^{5/2}}-\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{11/4}d^{5/2}}+\frac{7b^{3/4}\tan^{-1}}{4\sqrt{}}$$

```
[Out] -7/6/a^2/d/(d*x)^(3/2)+1/2/a/d/(d*x)^(3/2)/(b*x^2+a)+7/8*b^(3/4)*arctan(1-b
^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(11/4)/d^(5/2)*2^(1/2)-7/8*b^
(3/4)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(11/4)/d^(5/2
)*2^(1/2)+7/16*b^(3/4)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)
*2^(1/2)*(d*x)^(1/2))/a^(11/4)/d^(5/2)*2^(1/2)-7/16*b^(3/4)*ln(a^(1/2)*d^(1
/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(11/4)/d^(5/2)
*2^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{11/4}d^{5/2}}-\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{11/4}d^{5/2}}+\frac{7b^{3/4}\tan^{-1}}{4\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -7/(6*a^2*d*(d*x)^(3/2)) + 1/(2*a*d*(d*x)^(3/2)*(a + b*x^2)) + (7*b^(3/4)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(11/
4)*d^(5/2)) - (7*b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sq
rt[d])])/(4*Sqrt[2]*a^(11/4)*d^(5/2)) + (7*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sq
rt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(11/4)*d
^(5/2)) - (7*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(11/4)*d^(5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx = b^2\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^2}\,dx$$

$$= \frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)} + \frac{(7b)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)}\,dx}{4a}$$

$$= -\frac{7}{6a^2d(dx)^{3/2}} + \frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)} - \frac{\left(7b^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{4a^2d^2}$$

$$= -\frac{7}{6a^2d(dx)^{3/2}} + \frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)} - \frac{\left(7b^2\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{2a^2d^3}$$

$$= -\frac{7}{6a^2d(dx)^{3/2}} + \frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)} - \frac{\left(7b^2\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4a^{5/2}d^4}$$

$$= -\frac{7}{6a^2d(dx)^{3/2}} + \frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)} + \frac{\left(7b^{3/4}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx\right.}{8\sqrt{2}\,a^{11/4}d^{5/2}}$$

$$= -\frac{7}{6a^2d(dx)^{3/2}} + \frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)} + \frac{7b^{3/4}\log\left(\sqrt{a}\,\sqrt{d} + \sqrt{b}\,\sqrt{d}\,x - \sqrt{2}\,\sqrt[4]{a}\right.}{8\sqrt{2}\,a^{11/4}d^{5/2}}$$

$$= -\frac{7}{6a^2d(dx)^{3/2}} + \frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)} + \frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,a^{11/4}d^{5/2}} - \frac{7b^{3/4}\,\text{ta}}{}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.11

$$-\frac{2x\,{}_2F_1\left(-\frac{3}{4},2;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^2(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (-2*x*Hypergeometric2F1[-3/4, 2, 1/4, -((b*x^2)/a)])/(3*a^2*(d*x)^(5/2))
```

fricas [A] time = 1.09, size = 300, normalized size = 1.00

$$-\frac{84\left(a^2bd^3x^4+a^3d^3x^2\right)\left(-\frac{b^3}{a^{11}d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^8bd^7\left(-\frac{b^3}{a^{11}d^{10}}\right)^{\frac{3}{4}}-\sqrt{a^6d^6\sqrt{-\frac{b^3}{a^{11}d^{10}}}+b^2dx}\,a^8d^7\left(-\frac{b^3}{a^{11}d^{10}}\right)^{\frac{3}{4}}}{b^3}\right)+21\left(a^2bd^3x^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/24*(84*(a^2*b*d^3*x^4 + a^3*d^3*x^2)*(-b^3/(a^11*d^10))^(1/4)*arctan(-(s
qrt(d*x)*a^8*b*d^7*(-b^3/(a^11*d^10))^(3/4) - sqrt(a^6*d^6*sqrt(-b^3/(a^11*
d^10)) + b^2*d*x)*a^8*d^7*(-b^3/(a^11*d^10))^(3/4))/b^3) + 21*(a^2*b*d^3*x^
4 + a^3*d^3*x^2)*(-b^3/(a^11*d^10))^(1/4)*log(7*a^3*d^3*(-b^3/(a^11*d^10))^
(1/4) + 7*sqrt(d*x)*b) - 21*(a^2*b*d^3*x^4 + a^3*d^3*x^2)*(-b^3/(a^11*d^10)
)^(1/4)*log(-7*a^3*d^3*(-b^3/(a^11*d^10))^(1/4) + 7*sqrt(d*x)*b) + 4*(7*b*x
^2 + 4*a)*sqrt(d*x))/(a^2*b*d^3*x^4 + a^3*d^3*x^2)
```

giac [A] time = 0.19, size = 276, normalized size = 0.92

$$-\frac{\sqrt{dx}\,b}{2\left(bd^2x^2+ad^2\right)a^2d}-\frac{7\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8a^3d^3}-\frac{7\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8a^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(d*x)*b/((b*d^2*x^2 + a*d^2)*a^2*d) - 7/8*sqrt(2)*(a*b^3*d^2)^(1/4
)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4
))/(a^3*d^3) - 7/8*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*d^3) - 7/16*sqrt(2)*(a*
b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))
/(a^3*d^3) + 7/16*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/
4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*d^3) - 2/3/(sqrt(d*x)*a^2*d^2*x)
```

maple [A] time = 0.02, size = 226, normalized size = 0.75

$$-\frac{\sqrt{dx}\,b}{2\left(b\,d^2x^2+d^2a\right)a^2d}-\frac{2}{3\,(dx)^{\frac{3}{2}}\,a^2d}-\frac{7\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{8a^3d^3}-\frac{7\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)}{8a^3d^3}-\frac{7(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2/d/a^2*b*(d*x)^(1/2)/(b*d^2*x^2+a*d^2)-7/16/d^3/a^3*b*(a/b*d^2)^(1/4)*2
^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a
/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))-7/8/d^3/a^3*b*(a/b*d^2)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)-7/8/d^3/a^3*b*
(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)-2/3/a
^2/d/(d*x)^(3/2)
```

maxima [A] time = 3.08, size = 275, normalized size = 0.92

$$-\frac{\frac{8\left(7\,bd^2x^2+4\,ad^2\right)}{(dx)^{\frac{7}{2}}a^2b+(dx)^{\frac{3}{2}}a^3d^2}+\frac{21\left(\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}}+\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\cdots\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{a}\,d}\cdots\right)}{a^2}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/48*(8*(7*b*d^2*x^2 + 4*a*d^2)/((d*x)^(7/2)*a^2*b + (d*x)^(3/2)*a^3*d^2)
+ 21*(sqrt(2)*b^(3/4)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(
1/4) + sqrt(a)*d)/(a*d^2)^(3/4) - sqrt(2)*b^(3/4)*log(sqrt(b)*d*x - sqrt(2)
*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/(a*d^2)^(3/4) + 2*sqrt(2)*b*a
rctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqr
t(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)*d) + 2*sqrt(2)*b*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt
(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)*d))/a^2)/d
```

mupad [B] time = 4.40, size = 102, normalized size = 0.34

$$\frac{7\,{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{4\,a^{11/4}\,d^{5/2}}-\frac{\frac{2\,d}{3\,a}+\frac{7\,b\,d\,x^2}{6\,a^2}}{b\,{\left(d\,x\right)}^{7/2}+a\,d^2\,{\left(d\,x\right)}^{3/2}}+\frac{7\,{\left(-b\right)}^{3/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{4\,a^{11/4}\,d^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] (7*(-b)^(3/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(4*a^(11/4)
*d^(5/2)) - ((2*d)/(3*a) + (7*b*d*x^2)/(6*a^2))/(b*(d*x)^(7/2) + a*d^2*(d*x
)^(3/2)) + (7*(-b)^(3/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))
/(4*a^(11/4)*d^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{5}{2}}\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral(1/((d*x)**(5/2)*(a + b*x**2)**2), x)
```

3.695 $\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx$

Optimal. Leaf size=318

$$\frac{9b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{13/4}d^{7/2}}-\frac{9b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{13/4}d^{7/2}}-\frac{9b^{5/4}\tan^{-1}\left(\right.}{4\sqrt{2}}$$

```
[Out] -9/10/a^2/d/(d*x)^(5/2)+1/2/a/d/(d*x)^(5/2)/(b*x^2+a)-9/8*b^(5/4)*arctan(1-
b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(13/4)/d^(7/2)*2^(1/2)+9/8*b
^(5/4)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(13/4)/d^(7/
2)*2^(1/2)+9/16*b^(5/4)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4
)*2^(1/2)*(d*x)^(1/2))/a^(13/4)/d^(7/2)*2^(1/2)-9/16*b^(5/4)*ln(a^(1/2)*d^(
1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(13/4)/d^(7/2
)*2^(1/2)+9/2*b/a^3/d^3/(d*x)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{9b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{13/4}d^{7/2}}-\frac{9b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{13/4}d^{7/2}}-\frac{9b^{5/4}\tan^{-1}\left(\right.}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -9/(10*a^2*d*(d*x)^(5/2)) + (9*b)/(2*a^3*d^3*Sqrt[d*x]) + 1/(2*a*d*(d*x)^(5
/2)*(a + b*x^2)) - (9*b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/
4)*Sqrt[d])])/(4*Sqrt[2]*a^(13/4)*d^(7/2)) + (9*b^(5/4)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(13/4)*d^(7/2)) + (9*b
^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(8*Sqrt[2]*a^(13/4)*d^(7/2)) - (9*b^(5/4)*Log[Sqrt[a]*Sqrt[d] + S
qrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(13/4)*
d^(7/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx = b^2\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^2}\,dx$$

$$= \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} + \frac{(9b)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)}\,dx}{4a}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} - \frac{\left(9b^2\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)}\,dx}{4a^2d^2}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} + \frac{\left(9b^3\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{4a^3d^4}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} + \frac{\left(9b^3\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2}{d}\cdots}\cdots\right.}{2a^3d^5\cdots}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} - \frac{\left(9b^{5/2}\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d\cdots}{ab\cdots}\cdots\right.}{4a^3d\cdots}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} + \frac{\left(9b^{5/4}\right)\text{Subst}\left(\int \frac{\cdots}{-\frac{\sqrt{a}\cdots}{\sqrt{\cdots}}}\cdots\right.}{8\sqrt{2}\cdots}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} + \frac{9b^{5/4}\log\left(\sqrt{a}\sqrt{d}+\cdots\right.}{8\sqrt{\cdots}}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} - \frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{a}\cdots}\right.}{4\sqrt{2}\,a^{13/4}d^{7/2}\cdots}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.12

$$-\frac{2\sqrt{dx}\,{}_2F_1\left(-\frac{5}{4},2;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^2d^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (-2*Sqrt[d*x]*Hypergeometric2F1[-5/4, 2, -1/4, -((b*x^2)/a)])/(5*a^2*d^4*x^
3)
```

fricas [A] time = 1.06, size = 323, normalized size = 1.02

$$-\frac{180\left(a^{3}bd^{4}x^{5}+a^{4}d^{4}x^{3}\right)\left(-\frac{b^{5}}{a^{13}d^{14}}\right)^{\frac{1}{4}}\arctan\left(-\frac{729\sqrt{dx}\,a^{3}b^{4}d^{3}\left(-\frac{b^{5}}{a^{13}d^{14}}\right)^{\frac{1}{4}}-\sqrt{-531441\,a^{7}b^{5}d^{8}\sqrt{-\frac{b^{5}}{a^{13}d^{14}}}+531441\,b^{8}dx}\,a^{3}d^{3}\left(-\frac{b^{5}}{a^{13}d^{14}}\right)}{729\,b^{5}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/40*(180*(a^3*b*d^4*x^5 + a^4*d^4*x^3)*(-b^5/(a^13*d^14))^(1/4)*arctan(-1
/729*(729*sqrt(d*x)*a^3*b^4*d^3*(-b^5/(a^13*d^14))^(1/4) - sqrt(-531441*a^7
*b^5*d^8*sqrt(-b^5/(a^13*d^14)) + 531441*b^8*d*x)*a^3*d^3*(-b^5/(a^13*d^14)
)^(1/4))/b^5) - 45*(a^3*b*d^4*x^5 + a^4*d^4*x^3)*(-b^5/(a^13*d^14))^(1/4)*l
og(729*a^10*d^11*(-b^5/(a^13*d^14))^(3/4) + 729*sqrt(d*x)*b^4) + 45*(a^3*b*
d^4*x^5 + a^4*d^4*x^3)*(-b^5/(a^13*d^14))^(1/4)*log(-729*a^10*d^11*(-b^5/(a
^13*d^14))^(3/4) + 729*sqrt(d*x)*b^4) - 4*(45*b^2*x^4 + 36*a*b*x^2 - 4*a^2)
*sqrt(d*x))/(a^3*b*d^4*x^5 + a^4*d^4*x^3)
```

giac [A] time = 0.18, size = 307, normalized size = 0.97

$$\frac{\sqrt{dx}\,b^{2}x}{2\left(bd^{2}x^{2}+ad^{2}\right)a^{3}d^{2}}+\frac{9\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{4}bd^{5}}+\frac{9\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{4}bd^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x)*b^2*x/((b*d^2*x^2 + a*d^2)*a^3*d^2) + 9/8*sqrt(2)*(a*b^3*d^2)
^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)
^(1/4))/(a^4*b*d^5) + 9/8*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b*d^5) - 9/16*sq
rt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(a^4*b*d^5) + 9/16*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a
```

```
*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b*d^5) + 2/5*(10*b*d^2*x^2 -
a*d^2)/(sqrt(d*x)*a^3*d^5*x^2)
```

maple [A] time = 0.02, size = 242, normalized size = 0.76

$$-\frac{2}{5\,(dx)^{\frac{5}{2}}\,a^{2}d}+\frac{(dx)^{\frac{3}{2}}\,b^{2}}{2\left(b\,d^{2}x^{2}+d^{2}a\right)a^{3}d^{3}}+\frac{9\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}a^{3}d^{3}}+\frac{9\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}a^{3}d^{3}}+\frac{9\sqrt{2}\,b\ln\left(\frac{dx-(\cdots}{dx+(\cdots}\right.}{16\,(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2/d^3*b^2/a^3*(d*x)^(3/2)/(b*d^2*x^2+a*d^2)+9/16/d^3*b/a^3/(a/b*d^2)^(1/4
)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x
+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+9/8/d^3*b/a^3/(a/b*d
^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+9/8/d^3*b/a
^3/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)-2/
5/a^2/d/(d*x)^(5/2)+4*b/a^3/d^3/(d*x)^(1/2)
```

maxima [A] time = 3.01, size = 290, normalized size = 0.91

$$\frac{\frac{8\left(45\,b^{2}d^{4}x^{4}+36\,abd^{4}x^{2}-4\,a^{2}d^{4}\right)}{(dx)^{\frac{9}{2}}a^{3}bd^{2}+(dx)^{\frac{5}{2}}a^{4}d^{4}}+\frac{45\,b^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^{2}\cdots\right.}{\left(ad^{2}\right)^{\frac{1}{4}}\cdots}\cdots\right)}{a^{3}d^{2}}}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/80*(8*(45*b^2*d^4*x^4 + 36*a*b*d^4*x^2 - 4*a^2*d^4)/((d*x)^(9/2)*a^3*b*d^
2 + (d*x)^(5/2)*a^4*d^4) + 45*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(s
qrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)
^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a
)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqr
```

```
t(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d
*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^
(3/4)))/(a^3*d^2))/d
```

mupad [B] time = 4.35, size = 113, normalized size = 0.36

$$\frac{\frac{9\,b^2\,d\,x^4}{2\,a^3}-\frac{2\,d}{5\,a}+\frac{18\,b\,d\,x^2}{5\,a^2}}{b\,{\left(d\,x\right)}^{9/2}+a\,d^2\,{\left(d\,x\right)}^{5/2}}-\frac{9\,{\left(-b\right)}^{5/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{4\,a^{13/4}\,d^{7/2}}+\frac{9\,{\left(-b\right)}^{5/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{4\,a^{13/4}\,d^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] ((9*b^2*d*x^4)/(2*a^3) - (2*d)/(5*a) + (18*b*d*x^2)/(5*a^2))/(b*(d*x)^(9/2)
 + a*d^2*(d*x)^(5/2)) - (9*(-b)^(5/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4
)*d^(1/2))))/(4*a^(13/4)*d^(7/2)) + (9*(-b)^(5/4)*atanh(((-b)^(1/4)*(d*x)^(
1/2))/(a^(1/4)*d^(1/2))))/(4*a^(13/4)*d^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{7}{2}}\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral(1/((d*x)**(7/2)*(a + b*x**2)**2), x)
```

3.696 $\int \frac{(dx)^{19/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=368

$$-\frac{663a^{5/4}d^{19/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,b^{21/4}}+\frac{663a^{5/4}d^{19/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\right.}{256\sqrt{2}\,b^{21/4}}$$

```
[Out] 663/320*d^7*(d*x)^(5/2)/b^4-1/6*d*(d*x)^(17/2)/b/(b*x^2+a)^3-17/48*d^3*(d*x
)^(13/2)/b^2/(b*x^2+a)^2-221/192*d^5*(d*x)^(9/2)/b^3/(b*x^2+a)-663/256*a^(5
/4)*d^(19/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(21/4)
*2^(1/2)+663/256*a^(5/4)*d^(19/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1
/4)/d^(1/2))/b^(21/4)*2^(1/2)-663/512*a^(5/4)*d^(19/2)*ln(a^(1/2)*d^(1/2)+x
*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(21/4)*2^(1/2)+663/
512*a^(5/4)*d^(19/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2
^(1/2)*(d*x)^(1/2))/b^(21/4)*2^(1/2)-663/64*a*d^9*(d*x)^(1/2)/b^5
```

Rubi [A] time = 0.42, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{663a^{5/4}d^{19/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,b^{21/4}}+\frac{663a^{5/4}d^{19/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\right.}{256\sqrt{2}\,b^{21/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-663*a*d^9*Sqrt[d*x])/(64*b^5) + (663*d^7*(d*x)^(5/2))/(320*b^4) - (d*(d*x
)^(17/2))/(6*b*(a + b*x^2)^3) - (17*d^3*(d*x)^(13/2))/(48*b^2*(a + b*x^2)^2
) - (221*d^5*(d*x)^(9/2))/(192*b^3*(a + b*x^2)) - (663*a^(5/4)*d^(19/2)*Arc
Tan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(21/
4)) + (663*a^(5/4)*d^(19/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)
*Sqrt[d])])/(128*Sqrt[2]*b^(21/4)) - (663*a^(5/4)*d^(19/2)*Log[Sqrt[a]*Sqrt
[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*
b^(21/4)) + (663*a^(5/4)*d^(19/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*b^(21/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{19/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{19/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} + \frac{1}{12}\left(17b^2d^2\right)\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} + \frac{1}{96}\left(221d^4\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right)} + \frac{\left(663d^6\right)\int \frac{(dx)^{7/2}}{ab+b^2x^2}\,dx}{128b^2}$$

$$= \frac{663d^7(dx)^{5/2}}{320b^4} - \frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right)} - \frac{\left(663ad^8\right)\int \cdots}{128b^3}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5} + \frac{663d^7(dx)^{5/2}}{320b^4} - \frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right)}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5} + \frac{663d^7(dx)^{5/2}}{320b^4} - \frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right)}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5} + \frac{663d^7(dx)^{5/2}}{320b^4} - \frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right)}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5} + \frac{663d^7(dx)^{5/2}}{320b^4} - \frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right)}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5} + \frac{663d^7(dx)^{5/2}}{320b^4} - \frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right)}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5} + \frac{663d^7(dx)^{5/2}}{320b^4} - \frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.23, size = 347, normalized size = 0.94

$$d^9\sqrt{dx}\left(\frac{-69615\sqrt{2}a^{5/4}\left(a+bx^2\right)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+69615\sqrt{2}a^{5/4}\left(a+bx^2\right)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+139230\sqrt{2}a^{5/4}\left(a+bx\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^9*Sqrt[d*x]*(-139230*Sqrt[2]*a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)] + (-848640*a^4*b^(1/4)*Sqrt[x] - 2036736*a^3*b^(5/4)*x^(5/2) - 1
584128*a^2*b^(9/4)*x^(9/2) - 365568*a*b^(13/4)*x^(13/2) + 21504*b^(17/4)*x^
(17/2) + 106080*a^3*b^(1/4)*Sqrt[x]*(a + b*x^2) + 185640*a^2*b^(1/4)*Sqrt[x
]*(a + b*x^2)^2 + 139230*Sqrt[2]*a^(5/4)*(a + b*x^2)^3*ArcTan[1 + (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)] - 69615*Sqrt[2]*a^(5/4)*(a + b*x^2)^3*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 69615*Sqrt[2]*a^(5/4)*(a
+ b*x^2)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a +
 b*x^2)^3))/(53760*b^(21/4)*Sqrt[x])
```

fricas [A] time = 0.75, size = 399, normalized size = 1.08

$$39780\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{1}{4}}\left(b^8x^6+3ab^7x^4+3a^2b^6x^2+a^3b^5\right)\arctan\left(-\frac{\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{3}{4}}\sqrt{dx}\,ab^{16}d^9-\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{3}{4}}\sqrt{a^2d^{19}x+\sqrt{-\frac{a^5d^{38}}{b^{21}}}\,b^{10}}\,b^{16}}{a^5d^{38}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3840*(39780*(-a^5*d^38/b^21)^(1/4)*(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2
 + a^3*b^5)*arctan(-((-a^5*d^38/b^21)^(3/4)*sqrt(d*x)*a*b^16*d^9 - (-a^5*d^
38/b^21)^(3/4)*sqrt(a^2*d^19*x + sqrt(-a^5*d^38/b^21)*b^10)*b^16)/(a^5*d^38
)) + 9945*(-a^5*d^38/b^21)^(1/4)*(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2 + a
^3*b^5)*log(663*sqrt(d*x)*a*d^9 + 663*(-a^5*d^38/b^21)^(1/4)*b^5) - 9945*(-
a^5*d^38/b^21)^(1/4)*(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2 + a^3*b^5)*log(
663*sqrt(d*x)*a*d^9 - 663*(-a^5*d^38/b^21)^(1/4)*b^5) + 4*(384*b^4*d^9*x^8
- 6528*a*b^3*d^9*x^6 - 24973*a^2*b^2*d^9*x^4 - 27846*a^3*b*d^9*x^2 - 9945*a
^4*d^9)*sqrt(d*x))/(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2 + a^3*b^5)
```

giac [A] time = 0.22, size = 336, normalized size = 0.91

$$\frac{1}{7680}d^9\left(\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6}+\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/7680*d^9*(19890*sqrt(2)*(a*b^3*d^2)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^6 + 19890*sqrt(2)*(a*b^3*d
^2)^(1/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*
d^2/b)^(1/4))/b^6 + 9945*sqrt(2)*(a*b^3*d^2)^(1/4)*a*log(d*x + sqrt(2)*(a*d
^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^6 - 9945*sqrt(2)*(a*b^3*d^2)^(1/4)
*a*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^6 - 40*(6
17*sqrt(d*x)*a^2*b^2*d^6*x^4 + 1038*sqrt(d*x)*a^3*b*d^6*x^2 + 453*sqrt(d*x)
*a^4*d^6)/((b*d^2*x^2 + a*d^2)^3*b^5) + 3072*(sqrt(d*x)*b^16*d^10*x^2 - 20*
sqrt(d*x)*a*b^15*d^10)/(b^20*d^10))
```

maple [A] time = 0.02, size = 306, normalized size = 0.83

$$-\frac{151\sqrt{dx}\,a^4d^{15}}{64\left(b\,d^2x^2+d^2a\right)^3b^5}-\frac{173\,(dx)^{\frac{5}{2}}\,a^3d^{13}}{32\left(b\,d^2x^2+d^2a\right)^3b^4}-\frac{617\,(dx)^{\frac{9}{2}}\,a^2d^{11}}{192\left(b\,d^2x^2+d^2a\right)^3b^3}+\frac{663\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^9\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{256b^5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 2/5*d^7*(d*x)^(5/2)/b^4-8*a*d^9*(d*x)^(1/2)/b^5-617/192*d^11/b^3*a^2/(b*d^2
*x^2+a*d^2)^3*(d*x)^(9/2)-173/32*d^13/b^4*a^3/(b*d^2*x^2+a*d^2)^3*(d*x)^(5/
2)-151/64*d^15/b^5*a^4/(b*d^2*x^2+a*d^2)^3*(d*x)^(1/2)+663/512*d^9/b^5*a*(a
/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)
^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+663/256*
```

```
d^9/b^5*a*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2
)+1)+663/256*d^9/b^5*a*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/
4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.10, size = 361, normalized size = 0.98

$$-\frac{\frac{40\left(617\,(dx)^{\frac{9}{2}}a^2b^2d^{12}+1038\,(dx)^{\frac{5}{2}}a^3bd^{14}+453\,\sqrt{dx}\,a^4d^{16}\right)}{b^8d^6x^6+3\,ab^7d^6x^4+3\,a^2b^6d^6x^2+a^3b^5d^6}-\frac{9945\left(\frac{\sqrt{2}\,d^{12}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^{12}\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{\cdots}\right.}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}\cdots\right.}{\cdots}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/7680*(40*(617*(d*x)^(9/2)*a^2*b^2*d^12 + 1038*(d*x)^(5/2)*a^3*b*d^14 + 4
53*sqrt(d*x)*a^4*d^16)/(b^8*d^6*x^6 + 3*a*b^7*d^6*x^4 + 3*a^2*b^6*d^6*x^2 +
 a^3*b^5*d^6) - 9945*(sqrt(2)*d^12*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*
sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^12*log(s
qrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^
(3/4)*b^(1/4)) + 2*sqrt(2)*d^11*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b
^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b
)*d)*sqrt(a)) + 2*sqrt(2)*d^11*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b
^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b
)*d)*sqrt(a)))*a^2/b^5 - 3072*((d*x)^(5/2)*b*d^8 - 20*sqrt(d*x)*a*d^10)/b^5
)/d
```

mupad [B] time = 0.13, size = 188, normalized size = 0.51

$$\frac{2\,d^7\,{\left(d\,x\right)}^{5/2}}{5\,b^4}-\frac{\frac{151\,a^4\,d^{15}\,\sqrt{d\,x}}{64}+\frac{617\,a^2\,b^2\,d^{11}\,{\left(d\,x\right)}^{9/2}}{192}+\frac{173\,a^3\,b\,d^{13}\,{\left(d\,x\right)}^{5/2}}{32}}{a^3\,b^5\,d^6+3\,a^2\,b^6\,d^6\,x^2+3\,a\,b^7\,d^6\,x^4+b^8\,d^6\,x^6}-\frac{663\,{\left(-a\right)}^{5/4}\,d^{19/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,b^{21/4}}-\frac{8\,a\,d^9\,\sqrt{d\,x}}{b^5}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(19/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (2*d^7*(d*x)^(5/2))/(5*b^4) - ((151*a^4*d^15*(d*x)^(1/2))/64 + (617*a^2*b^2
*d^11*(d*x)^(9/2))/192 + (173*a^3*b*d^13*(d*x)^(5/2))/32)/(a^3*b^5*d^6 + b^
8*d^6*x^6 + 3*a*b^7*d^6*x^4 + 3*a^2*b^6*d^6*x^2) - (663*(-a)^(5/4)*d^(19/2)
*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*b^(21/4)) + ((-a)^(
```

```
5/4)*d^(19/2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*663i)/(12
8*b^(21/4)) - (8*a*d^9*(d*x)^(1/2))/b^5
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(19/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)

[Out] Timed out
```

3.697 $\int \frac{(dx)^{17/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=350

$$-\frac{385a^{3/4}d^{17/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,b^{19/4}}+\frac{385a^{3/4}d^{17/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\right.}{256\sqrt{2}\,b^{19/4}}$$

```
[Out] 385/192*d^7*(d*x)^(3/2)/b^4-1/6*d*(d*x)^(15/2)/b/(b*x^2+a)^3-5/16*d^3*(d*x)
^(11/2)/b^2/(b*x^2+a)^2-55/64*d^5*(d*x)^(7/2)/b^3/(b*x^2+a)+385/256*a^(3/4)
*d^(17/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(19/4)*2^
(1/2)-385/256*a^(3/4)*d^(17/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)
/d^(1/2))/b^(19/4)*2^(1/2)-385/512*a^(3/4)*d^(17/2)*ln(a^(1/2)*d^(1/2)+x*b^
(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(19/4)*2^(1/2)+385/512
*a^(3/4)*d^(17/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1
/2)*(d*x)^(1/2))/b^(19/4)*2^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{385a^{3/4}d^{17/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,b^{19/4}}+\frac{385a^{3/4}d^{17/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\right.}{256\sqrt{2}\,b^{19/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (385*d^7*(d*x)^(3/2))/(192*b^4) - (d*(d*x)^(15/2))/(6*b*(a + b*x^2)^3) - (5
*d^3*(d*x)^(11/2))/(16*b^2*(a + b*x^2)^2) - (55*d^5*(d*x)^(7/2))/(64*b^3*(a
 + b*x^2)) + (385*a^(3/4)*d^(17/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(
a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(19/4)) - (385*a^(3/4)*d^(17/2)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(19/4)) -
(385*a^(3/4)*d^(17/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*b^(19/4)) + (385*a^(3/4)*d^(17/2)*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/
(256*Sqrt[2]*b^(19/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{17/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{17/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3} + \frac{1}{4}\left(5b^2d^2\right)\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2} + \frac{1}{32}\left(55d^4\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2} - \frac{55d^5(dx)^{7/2}}{64b^3\left(a+bx^2\right)} + \frac{\left(385d^6\right)\int \frac{(dx)^{5/2}}{ab+b^2x^2}\,dx}{128b^2}$$

$$= \frac{385d^7(dx)^{3/2}}{192b^4} - \frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2} - \frac{55d^5(dx)^{7/2}}{64b^3\left(a+bx^2\right)} - \frac{\left(385ad^8\right)\int \frac{\sqrt{}}{ab\text{-}}}{128b^3}$$

$$= \frac{385d^7(dx)^{3/2}}{192b^4} - \frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2} - \frac{55d^5(dx)^{7/2}}{64b^3\left(a+bx^2\right)} - \frac{\left(385ad^7\right)\text{Subs}}{}$$

$$= \frac{385d^7(dx)^{3/2}}{192b^4} - \frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2} - \frac{55d^5(dx)^{7/2}}{64b^3\left(a+bx^2\right)} + \frac{\left(385ad^7\right)\text{Subs}}{}$$

$$= \frac{385d^7(dx)^{3/2}}{192b^4} - \frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2} - \frac{55d^5(dx)^{7/2}}{64b^3\left(a+bx^2\right)} - \frac{\left(385a^{3/4}d^{17/2}\right)}{}$$

$$= \frac{385d^7(dx)^{3/2}}{192b^4} - \frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2} - \frac{55d^5(dx)^{7/2}}{64b^3\left(a+bx^2\right)} - \frac{385a^{3/4}d^{17/2}\,\text{lo}}{}$$

$$= \frac{385d^7(dx)^{3/2}}{192b^4} - \frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2} - \frac{55d^5(dx)^{7/2}}{64b^3\left(a+bx^2\right)} + \frac{385a^{3/4}d^{17/2}\,\text{ta}}{128}$$

Mathematica [C] time = 0.03, size = 87, normalized size = 0.25

$$-\frac{2d^8x\sqrt{dx}\left(-77a^3-99a^2bx^2-45ab^2x^4+77\left(a+bx^2\right)^3\,{}_2F_1\left(\frac{3}{4},4;\frac{7}{4};-\frac{bx^2}{a}\right)-3b^3x^6\right)}{9b^4\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-2*d^8*x*Sqrt[d*x]*(-77*a^3 - 99*a^2*b*x^2 - 45*a*b^2*x^4 - 3*b^3*x^6 + 77
*(a + b*x^2)^3*Hypergeometric2F1[3/4, 4, 7/4, -((b*x^2)/a)]))/(9*b^4*(a + b
*x^2)^3)
```

fricas [A] time = 1.12, size = 399, normalized size = 1.14

$$4620\left(-\frac{a^3d^{34}}{b^{19}}\right)^{\frac{1}{4}}\left(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4\right)\arctan\left(-\frac{\left(-\frac{a^3d^{34}}{b^{19}}\right)^{\frac{1}{4}}\sqrt{dx}\,a^2b^5d^{25}-\sqrt{a^4d^{51}x-\sqrt{-\frac{a^3d^{34}}{b^{19}}}\,a^3b^9d^{34}}\left(-\frac{a^3d^{34}}{b^{19}}\right)^{\frac{1}{4}}b}{a^3d^{34}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(4620*(-a^3*d^34/b^19)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 +
 a^3*b^4)*arctan(-((-a^3*d^34/b^19)^(1/4)*sqrt(d*x)*a^2*b^5*d^25 - sqrt(a^4
*d^51*x - sqrt(-a^3*d^34/b^19)*a^3*b^9*d^34)*(-a^3*d^34/b^19)^(1/4)*b^5)/(a
^3*d^34)) - 1155*(-a^3*d^34/b^19)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*
x^2 + a^3*b^4)*log(57066625*sqrt(d*x)*a^2*d^25 + 57066625*(-a^3*d^34/b^19)^
(3/4)*b^14) + 1155*(-a^3*d^34/b^19)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^
5*x^2 + a^3*b^4)*log(57066625*sqrt(d*x)*a^2*d^25 - 57066625*(-a^3*d^34/b^19
)^(3/4)*b^14) + 4*(128*b^3*d^8*x^7 + 765*a*b^2*d^8*x^5 + 990*a^2*b*d^8*x^3
+ 385*a^3*d^8*x)*sqrt(d*x))/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^
4)
```

giac [A] time = 0.24, size = 316, normalized size = 0.90

$$\frac{1}{1536}d^{8}\left(\frac{1024\sqrt{dx}\,x}{b^{4}}-\frac{2310\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{b^{7}d}-\frac{2310\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right.\right.}{2\left(\frac{ad^{2}}{b}\right.}\right.}{b^{7}d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^8*(1024*sqrt(d*x)*x/b^4 - 2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/
2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^7*d)
- 2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^7*d) + 1155*sqrt(2)*(a*b^3*d^2)^(3/4)
*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^7*d) - 115
5*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + s
qrt(a*d^2/b))/(b^7*d) + 8*(381*sqrt(d*x)*a*b^2*d^6*x^5 + 606*sqrt(d*x)*a^2*
b*d^6*x^3 + 257*sqrt(d*x)*a^3*d^6*x)/((b*d^2*x^2 + a*d^2)^3*b^4))
```

maple [A] time = 0.02, size = 290, normalized size = 0.83

$$\frac{257\left(dx\right)^{\frac{3}{2}}a^{3}d^{13}}{192\left(b\,d^{2}x^{2}+d^{2}a\right)^{3}b^{4}}+\frac{101\left(dx\right)^{\frac{7}{2}}a^{2}d^{11}}{32\left(b\,d^{2}x^{2}+d^{2}a\right)^{3}b^{3}}+\frac{127\left(dx\right)^{\frac{11}{2}}a\,d^{9}}{64\left(b\,d^{2}x^{2}+d^{2}a\right)^{3}b^{2}}-\frac{385\sqrt{2}\,a\,d^{9}\arctan\left(\frac{\sqrt{2}\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}-1\right)}{256\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}b^{5}}-\frac{385\sqrt{2}\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 2/3*d^7*(d*x)^(3/2)/b^4+127/64*d^9*a/b^2/(b*d^2*x^2+a*d^2)^3*(d*x)^(11/2)+1
01/32*d^11*a^2/b^3/(b*d^2*x^2+a*d^2)^3*(d*x)^(7/2)+257/192*d^13*a^3/b^4/(b*
d^2*x^2+a*d^2)^3*(d*x)^(3/2)-385/512*d^9*a/b^5/(a/b*d^2)^(1/4)*2^(1/2)*ln((
d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/
4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))-385/256*d^9*a/b^5/(a/b*d^2)^(1/4)*
```

```
2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)-385/256*d^9*a/b^5/(a/
b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.08, size = 334, normalized size = 0.95

$$-\frac{1155\,ad^{10}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx\right.}{}\right.}{b^4}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/1536*(1155*a*d^10*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b
^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b
)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4
) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*
sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4
) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*
(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b^4 -
 1024*(d*x)^(3/2)*d^8/b^4 - 8*(381*(d*x)^(11/2)*a*b^2*d^10 + 606*(d*x)^(7/2
)*a^2*b*d^12 + 257*(d*x)^(3/2)*a^3*d^14)/(b^7*d^6*x^6 + 3*a*b^6*d^6*x^4 + 3
*a^2*b^5*d^6*x^2 + a^3*b^4*d^6))/d
```

mupad [B] time = 4.33, size = 171, normalized size = 0.49

$$\frac{\frac{257\,a^3\,d^{13}\,(d\,x)^{3/2}}{192}+\frac{101\,a^2\,b\,d^{11}\,(d\,x)^{7/2}}{32}+\frac{127\,a\,b^2\,d^9\,(d\,x)^{11/2}}{64}}{a^3\,b^4\,d^6+3\,a^2\,b^5\,d^6\,x^2+3\,a\,b^6\,d^6\,x^4+b^7\,d^6\,x^6}+\frac{2\,d^7\,(d\,x)^{3/2}}{3\,b^4}+\frac{385\,(-a)^{3/4}\,d^{17/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,b^{19/4}}+\frac{(-a)^{3/4}\,d^{17/2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(17/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((257*a^3*d^13*(d*x)^(3/2))/192 + (101*a^2*b*d^11*(d*x)^(7/2))/32 + (127*a*
b^2*d^9*(d*x)^(11/2))/64)/(a^3*b^4*d^6 + b^7*d^6*x^6 + 3*a*b^6*d^6*x^4 + 3*
a^2*b^5*d^6*x^2) + (2*d^7*(d*x)^(3/2))/(3*b^4) + (385*(-a)^(3/4)*d^(17/2)*a
tan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*b^(19/4)) + ((-a)^(3/
4)*d^(17/2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*385i)/(128*
b^(19/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(17/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Timed out
```

3.698 $\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=350

$$\frac{195\sqrt[4]{a}\,d^{15/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,b^{17/4}}-\frac{195\sqrt[4]{a}\,d^{15/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right.}{256\sqrt{2}\,b^{17/4}}$$

```
[Out] -1/6*d*(d*x)^(13/2)/b/(b*x^2+a)^3-13/48*d^3*(d*x)^(9/2)/b^2/(b*x^2+a)^2-39/
64*d^5*(d*x)^(5/2)/b^3/(b*x^2+a)+195/256*a^(1/4)*d^(15/2)*arctan(1-b^(1/4)*
2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(17/4)*2^(1/2)-195/256*a^(1/4)*d^(15
/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(17/4)*2^(1/2)+
195/512*a^(1/4)*d^(15/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/
4)*2^(1/2)*(d*x)^(1/2))/b^(17/4)*2^(1/2)-195/512*a^(1/4)*d^(15/2)*ln(a^(1/2
)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(17/4)*2
^(1/2)+195/64*d^7*(d*x)^(1/2)/b^4
```

Rubi [A] time = 0.38, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)}-\frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2}+\frac{195\sqrt[4]{a}\,d^{15/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,b^{17/4}}-\frac{195\sqrt[4]{a}\,d^{15/2}\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (195*d^7*Sqrt[d*x])/(64*b^4) - (d*(d*x)^(13/2))/(6*b*(a + b*x^2)^3) - (13*d
^3*(d*x)^(9/2))/(48*b^2*(a + b*x^2)^2) - (39*d^5*(d*x)^(5/2))/(64*b^3*(a +
b*x^2)) + (195*a^(1/4)*d^(15/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(
1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(17/4)) - (195*a^(1/4)*d^(15/2)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(17/4)) + (19
5*a^(1/4)*d^(15/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*b^(17/4)) - (195*a^(1/4)*d^(15/2)*Log[Sq
rt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(25
6*Sqrt[2]*b^(17/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3} + \frac{1}{12}\left(13b^2d^2\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3} - \frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2} + \frac{1}{32}\left(39d^4\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3} - \frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2} - \frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)} + \frac{\left(195d^6\right)\int \frac{(dx)^{3/2}}{ab+b^2x^2}\,dx}{128b^2}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4} - \frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3} - \frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2} - \frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)} - \frac{\left(195ad^8\right)\int \frac{}{\sqrt{dx}(}}{128b^3}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4} - \frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3} - \frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2} - \frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)} - \frac{\left(195ad^7\right)\text{Subst}}{}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4} - \frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3} - \frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2} - \frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)} - \frac{\left(195\sqrt{a}\,d^6\right)\text{Sub}}{}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4} - \frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3} - \frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2} - \frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)} + \frac{\left(195\sqrt[4]{a}\,d^{15/2}\right)\text{S}}{}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4} - \frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3} - \frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2} - \frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)} + \frac{195\sqrt[4]{a}\,d^{15/2}\log}{}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4} - \frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3} - \frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2} - \frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)} + \frac{195\sqrt[4]{a}\,d^{15/2}\tan}{128\sqrt{}}$$

Mathematica [A] time = 0.14, size = 324, normalized size = 0.93

$$\frac{d^7\sqrt{dx}\left(\frac{49920a^3\sqrt[4]{b}}{(a+bx^2)^3}+\frac{119808a^2b^{5/4}x^2}{(a+bx^2)^3}-\frac{6240a^2\sqrt[4]{b}}{(a+bx^2)^2}+\frac{21504b^{13/4}x^6}{(a+bx^2)^3}+\frac{93184ab^{9/4}x^4}{(a+bx^2)^3}-\frac{10920a\sqrt[4]{b}}{a+bx^2}+\frac{4095\sqrt{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{\sqrt{x}}\right.}{10752b^{17/}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^7*Sqrt[d*x]*((49920*a^3*b^(1/4))/(a + b*x^2)^3 + (119808*a^2*b^(5/4)*x^2
)/(a + b*x^2)^3 + (93184*a*b^(9/4)*x^4)/(a + b*x^2)^3 + (21504*b^(13/4)*x^6
)/(a + b*x^2)^3 - (6240*a^2*b^(1/4))/(a + b*x^2)^2 - (10920*a*b^(1/4))/(a +
 b*x^2) + (8190*Sqrt[2]*a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4
)])/Sqrt[x] - (8190*Sqrt[2]*a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)])/Sqrt[x] + (4095*Sqrt[2]*a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/Sqrt[x] - (4095*Sqrt[2]*a^(1/4)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/Sqrt[x]))/(10752*b^(17/4))
```

fricas [A] time = 0.88, size = 363, normalized size = 1.04

$$-\frac{2340\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{1}{4}}\left(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4\right)\arctan\left(-\frac{\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{3}{4}}\sqrt{dx}\,b^{13}d^7-\sqrt{d^{15}x+\sqrt{-\frac{ad^{30}}{b^{17}}}\,b^8}\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{3}{4}}b^{13}}{ad^{30}}\right)+585}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(2340*(-a*d^30/b^17)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 +
a^3*b^4)*arctan(-((-a*d^30/b^17)^(3/4)*sqrt(d*x)*b^13*d^7 - sqrt(d^15*x + s
qrt(-a*d^30/b^17)*b^8)*(-a*d^30/b^17)^(3/4)*b^13)/(a*d^30)) + 585*(-a*d^30/
b^17)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4)*log(195*sqrt(
d*x)*d^7 + 195*(-a*d^30/b^17)^(1/4)*b^4) - 585*(-a*d^30/b^17)^(1/4)*(b^7*x^
6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4)*log(195*sqrt(d*x)*d^7 - 195*(-a*
d^30/b^17)^(1/4)*b^4) - 4*(384*b^3*d^7*x^6 + 1469*a*b^2*d^7*x^4 + 1638*a^2*
b*d^7*x^2 + 585*a^3*d^7)*sqrt(d*x))/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2
+ a^3*b^4)
```

giac [A] time = 0.20, size = 302, normalized size = 0.86

$$-\frac{1}{1536}d^7\left(\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^5}+\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^5}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/1536*d^7*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^5 + 1170*sqrt(2)*(a*b^3*d^2)
^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b
)^(1/4))/b^5 + 585*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1
/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^5 - 585*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x
- sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^5 - 3072*sqrt(d*x)/b
^4 - 8*(317*sqrt(d*x)*a*b^2*d^6*x^4 + 486*sqrt(d*x)*a^2*b*d^6*x^2 + 201*sqr
t(d*x)*a^3*d^6)/((b*d^2*x^2 + a*d^2)^3*b^4))
```

maple [A] time = 0.02, size = 287, normalized size = 0.82

$$\frac{67\sqrt{dx}\,a^3d^{13}}{64\left(b\,d^2x^2+d^2a\right)^3b^4}+\frac{81\,(dx)^{\frac{5}{2}}\,a^2d^{11}}{32\left(b\,d^2x^2+d^2a\right)^3b^3}+\frac{317\,(dx)^{\frac{9}{2}}\,a\,d^9}{192\left(b\,d^2x^2+d^2a\right)^3b^2}-\frac{195\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^7\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{256b^4}-\frac{19}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 2*d^7*(d*x)^(1/2)/b^4+317/192*d^9/b^2*a/(b*d^2*x^2+a*d^2)^3*(d*x)^(9/2)+81/
32*d^11/b^3*a^2/(b*d^2*x^2+a*d^2)^3*(d*x)^(5/2)+67/64*d^13/b^4*a^3/(b*d^2*x
^2+a*d^2)^3*(d*x)^(1/2)-195/512*d^7/b^4*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/
b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x
)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))-195/256*d^7/b^4*(a/b*d^2)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)-195/256*d^7/b^4*(a/b*d^2)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.14, size = 343, normalized size = 0.98

$$\frac{\frac{3072\sqrt{dx}\,d^8}{b^4}+\frac{8\left(317\,(dx)^{\frac{9}{2}}ab^2d^{10}+486\,(dx)^{\frac{5}{2}}a^2bd^{12}+201\sqrt{dx}\,a^3d^{14}\right)}{b^7d^6x^6+3\,ab^6d^6x^4+3\,a^2b^5d^6x^2+a^3b^4d^6}-585\left(\frac{\sqrt{2}\,d^{10}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^{10}\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\right.}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}\right.}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*(3072*sqrt(d*x)*d^8/b^4 + 8*(317*(d*x)^(9/2)*a*b^2*d^10 + 486*(d*x)^
(5/2)*a^2*b*d^12 + 201*sqrt(d*x)*a^3*d^14)/(b^7*d^6*x^6 + 3*a*b^6*d^6*x^4 +
 3*a^2*b^5*d^6*x^2 + a^3*b^4*d^6) - 585*(sqrt(2)*d^10*log(sqrt(b)*d*x + sqr
t(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) -
 sqrt(2)*d^10*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + s
qrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^9*arctan(1/2*sqrt(2)*(sqrt(
2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(s
qrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^9*arctan(-1/2*sqrt(2)*(sqrt(2
)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sq
rt(sqrt(a)*sqrt(b)*d)*sqrt(a)))*a/b^4)/d
```

mupad [B] time = 4.30, size = 171, normalized size = 0.49

$$\frac{\frac{67\,a^3\,d^{13}\,\sqrt{d\,x}}{64}+\frac{81\,a^2\,b\,d^{11}\,{\left(d\,x\right)}^{5/2}}{32}+\frac{317\,a\,b^2\,d^9\,{\left(d\,x\right)}^{9/2}}{192}}{a^3\,b^4\,d^6+3\,a^2\,b^5\,d^6\,x^2+3\,a\,b^6\,d^6\,x^4+b^7\,d^6\,x^6}+\frac{2\,d^7\,\sqrt{d\,x}}{b^4}-\frac{195\,{\left(-a\right)}^{1/4}\,d^{15/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,b^{17/4}}+\frac{{\left(-a\right)}^{1/4}\,d^{15/2}\,\mathrm{atan}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((67*a^3*d^13*(d*x)^(1/2))/64 + (81*a^2*b*d^11*(d*x)^(5/2))/32 + (317*a*b^2
*d^9*(d*x)^(9/2))/192)/(a^3*b^4*d^6 + b^7*d^6*x^6 + 3*a*b^6*d^6*x^4 + 3*a^2
*b^5*d^6*x^2) + (2*d^7*(d*x)^(1/2))/b^4 - (195*(-a)^(1/4)*d^(15/2)*atan((b^
(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*b^(17/4)) + ((-a)^(1/4)*d^(1
5/2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*195i)/(128*b^(17/4
))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Timed out
```

3.699 $\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=333

$$\frac{77d^{13/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\sqrt[4]{a}\,b^{15/4}}-\frac{77d^{13/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\sqrt[4]{a}\,b^{15/4}}-\frac{77d^{13/}}{}$$

```
[Out] -1/6*d*(d*x)^(11/2)/b/(b*x^2+a)^3-11/48*d^3*(d*x)^(7/2)/b^2/(b*x^2+a)^2-77/
192*d^5*(d*x)^(3/2)/b^3/(b*x^2+a)-77/256*d^(13/2)*arctan(1-b^(1/4)*2^(1/2)*
(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(1/4)/b^(15/4)*2^(1/2)+77/256*d^(13/2)*arcta
n(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(1/4)/b^(15/4)*2^(1/2)+7
7/512*d^(13/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)
*(d*x)^(1/2))/a^(1/4)/b^(15/4)*2^(1/2)-77/512*d^(13/2)*ln(a^(1/2)*d^(1/2)+x
*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(1/4)/b^(15/4)*2^(1
/2)
```

Rubi [A] time = 0.35, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)}-\frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2}+\frac{77d^{13/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\sqrt[4]{a}\,b^{15/4}}-\frac{77d^{13/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{}\right.}{256}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(d*(d*x)^(11/2))/(6*b*(a + b*x^2)^3) - (11*d^3*(d*x)^(7/2))/(48*b^2*(a + b
*x^2)^2) - (77*d^5*(d*x)^(3/2))/(192*b^3*(a + b*x^2)) - (77*d^(13/2)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(1/4)*b
^(15/4)) + (77*d^(13/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])])/(128*Sqrt[2]*a^(1/4)*b^(15/4)) + (77*d^(13/2)*Log[Sqrt[a]*Sqrt[d] +
 Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(1/
4)*b^(15/4)) - (77*d^(13/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(1/4)*b^(15/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3} + \frac{1}{12}\left(11b^2d^2\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3} - \frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2} + \frac{1}{96}\left(77d^4\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3} - \frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2} - \frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)} + \frac{\left(77d^6\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{128b^2}$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3} - \frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2} - \frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)} + \frac{\left(77d^5\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,\ldots\right.}{64b^2}$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3} - \frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2} - \frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)} - \frac{\left(77d^5\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\ldots\right.}{128b^{5/2}}$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3} - \frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2} - \frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)} + \frac{\left(77d^{13/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{\ldots}}{\sqrt[4]{\ldots}}}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\ldots}\ldots\right.}{256\sqrt{2}\sqrt[4]{a}\ldots}$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3} - \frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2} - \frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)} + \frac{77d^{13/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\ldots\right.}{256\sqrt{2}\sqrt[4]{a}\ldots}$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3} - \frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2} - \frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)} - \frac{77d^{13/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\ldots\right.}{128\sqrt{2}\sqrt[4]{a}\,b^{15/4}}$$

Mathematica [C] time = 0.03, size = 83, normalized size = 0.25

$$\frac{2d^6x\sqrt{dx}\left(77\left(a+bx^2\right)^3\,{}_2F_1\left(\frac{3}{4},4;\frac{7}{4};-\frac{bx^2}{a}\right)-a\left(77a^2+99abx^2+45b^2x^4\right)\right)}{45ab^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*d^6*x*Sqrt[d*x]*(-(a*(77*a^2 + 99*a*b*x^2 + 45*b^2*x^4)) + 77*(a + b*x^2
)^3*Hypergeometric2F1[3/4, 4, 7/4, -((b*x^2)/a)]))/(45*a*b^3*(a + b*x^2)^3)
```

fricas [A] time = 0.80, size = 370, normalized size = 1.11

$$-\frac{924\left(b^{6}x^{6}+3\,ab^{5}x^{4}+3\,a^{2}b^{4}x^{2}+a^{3}b^{3}\right)\left(-\frac{d^{26}}{ab^{15}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{26}}{ab^{15}}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{4}d^{19}-\sqrt{d^{39}x-\sqrt{-\frac{d^{26}}{ab^{15}}}\,ab^{7}d^{26}}\left(-\frac{d^{26}}{ab^{15}}\right)^{\frac{1}{4}}b^{4}}{d^{26}}\right)-231\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(924*(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^26/(a*b^1
5))^(1/4)*arctan(-((-d^26/(a*b^15))^(1/4)*sqrt(d*x)*b^4*d^19 - sqrt(d^39*x
- sqrt(-d^26/(a*b^15))*a*b^7*d^26)*(-d^26/(a*b^15))^(1/4)*b^4)/d^26) - 231*
(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^26/(a*b^15))^(1/4)*lo
g(456533*sqrt(d*x)*d^19 + 456533*(-d^26/(a*b^15))^(3/4)*a*b^11) + 231*(b^6*
x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^26/(a*b^15))^(1/4)*log(456
533*sqrt(d*x)*d^19 - 456533*(-d^26/(a*b^15))^(3/4)*a*b^11) + 4*(153*b^2*d^6
*x^5 + 198*a*b*d^6*x^3 + 77*a^2*d^6*x)*sqrt(d*x))/(b^6*x^6 + 3*a*b^5*x^4 +
3*a^2*b^4*x^2 + a^3*b^3)
```

giac [A] time = 0.20, size = 314, normalized size = 0.94

$$-\frac{1}{1536}\,d^{6}\left(\frac{8\left(153\sqrt{dx}\,b^{2}d^{6}x^{5}+198\sqrt{dx}\,abd^{6}x^{3}+77\sqrt{dx}\,a^{2}d^{6}x\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{3}b^{3}}-\frac{462\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\cdots\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{ab^{6}d}\cdots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/1536*d^6*(8*(153*sqrt(d*x)*b^2*d^6*x^5 + 198*sqrt(d*x)*a*b*d^6*x^3 + 77*
sqrt(d*x)*a^2*d^6*x)/((b*d^2*x^2 + a*d^2)^3*b^3) - 462*sqrt(2)*(a*b^3*d^2)^
(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^
(1/4))/(a*b^6*d) - 462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(
2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^6*d) + 231*sqrt(2)*
(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/
b))/(a*b^6*d) - 231*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(
1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^6*d))
```

maple [A] time = 0.02, size = 271, normalized size = 0.81

$$-\frac{77\,(dx)^{\frac{3}{2}}\,a^2 d^{11}}{192\left(b\,d^2x^2+d^2a\right)^3 b^3}-\frac{33\,(dx)^{\frac{7}{2}}\,a\,d^9}{32\left(b\,d^2x^2+d^2a\right)^3 b^2}-\frac{51\,(dx)^{\frac{11}{2}}\,d^7}{64\left(b\,d^2x^2+d^2a\right)^3 b}+\frac{77\sqrt{2}\,d^7\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{256\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}} b^4}+\frac{77\sqrt{2}\,d^7\,\mathrm{ar}}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -51/64*d^7/(b*d^2*x^2+a*d^2)^3/b*(d*x)^(11/2)-33/32*d^9/(b*d^2*x^2+a*d^2)^3
/b^2*a*(d*x)^(7/2)-77/192*d^11/(b*d^2*x^2+a*d^2)^3/b^3*a^2*(d*x)^(3/2)+77/5
12*d^7/b^4/(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/
2)))+77/256*d^7/b^4/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*
(d*x)^(1/2)+1)+77/256*d^7/b^4/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d
^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.44, size = 317, normalized size = 0.95

$$\frac{\frac{231\,d^8\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad\right.\right.}{\left(ad^2\right)^{\frac{1}{4}}}\right.}{b^3}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*(231*d^8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4
) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*
sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2
*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(
b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + s
qrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^
2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b^3 - 8*(1
53*(d*x)^(11/2)*b^2*d^8 + 198*(d*x)^(7/2)*a*b*d^10 + 77*(d*x)^(3/2)*a^2*d^1
2)/(b^6*d^6*x^6 + 3*a*b^5*d^6*x^4 + 3*a^2*b^4*d^6*x^2 + a^3*b^3*d^6))/d
```

mupad [B] time = 0.11, size = 153, normalized size = 0.46

$$\frac{77\,d^{13/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{1/4}\,b^{15/4}}-\frac{\frac{51\,d^7\,(d\,x)^{11/2}}{64\,b}+\frac{77\,a^2\,d^{11}\,(d\,x)^{3/2}}{192\,b^3}+\frac{33\,a\,d^9\,(d\,x)^{7/2}}{32\,b^2}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}-\frac{77\,d^{13/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{1/4}\,b^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (77*d^(13/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-a)^(1
/4)*b^(15/4)) - ((51*d^7*(d*x)^(11/2))/(64*b) + (77*a^2*d^11*(d*x)^(3/2))/(
192*b^3) + (33*a*d^9*(d*x)^(7/2))/(32*b^2))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*
b*d^6*x^2 + 3*a*b^2*d^6*x^4) - (77*d^(13/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-
a)^(1/4)*d^(1/2))))/(128*(-a)^(1/4)*b^(15/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{13}{2}}}{\left(a+bx^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(13/2)/(a + b*x**2)**4, x)
```

3.700 $\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=333

$$-\frac{15d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{15d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{3/4}b^{13/4}}-\frac{15d^{11/2}}{}$$

```
[Out] -1/6*d*(d*x)^(9/2)/b/(b*x^2+a)^3-3/16*d^3*(d*x)^(5/2)/b^2/(b*x^2+a)^2-15/25
6*d^(11/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(3/4)/b^
(13/4)*2^(1/2)+15/256*d^(11/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)
/d^(1/2))/a^(3/4)/b^(13/4)*2^(1/2)-15/512*d^(11/2)*ln(a^(1/2)*d^(1/2)+x*b^(
1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(3/4)/b^(13/4)*2^(1/2)+
15/512*d^(11/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2
)*(d*x)^(1/2))/a^(3/4)/b^(13/4)*2^(1/2)-15/64*d^5*(d*x)^(1/2)/b^3/(b*x^2+a)
```

Rubi [A] time = 0.34, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {28, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{15d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{15d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{3/4}b^{13/4}}-\frac{15d^{11/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(d*(d*x)^(9/2))/(6*b*(a + b*x^2)^3) - (3*d^3*(d*x)^(5/2))/(16*b^2*(a + b*x
^2)^2) - (15*d^5*Sqrt[d*x])/(64*b^3*(a + b*x^2)) - (15*d^(11/2)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(3/4)*b^(13/
4)) + (15*d^(11/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(128*Sqrt[2]*a^(3/4)*b^(13/4)) - (15*d^(11/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt
[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(3/4)*b^
(13/4)) + (15*d^(11/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(3/4)*b^(13/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}+\frac{1}{4}\left(3b^2d^2\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}+\frac{1}{32}\left(15d^4\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}-\frac{15d^5\sqrt{dx}}{64b^3\left(a+bx^2\right)}+\frac{\left(15d^6\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{128b^2}$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}-\frac{15d^5\sqrt{dx}}{64b^3\left(a+bx^2\right)}+\frac{\left(15d^5\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx\right.}{64b^2}$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}-\frac{15d^5\sqrt{dx}}{64b^3\left(a+bx^2\right)}+\frac{\left(15d^4\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\right.}{128\sqrt{a}\,b^2}$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}-\frac{15d^5\sqrt{dx}}{64b^3\left(a+bx^2\right)}-\frac{\left(15d^{11/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{}}{\sqrt[4]{}}}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{}}{}}\right.}{256\sqrt{2}\,a^{3/}}$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}-\frac{15d^5\sqrt{dx}}{64b^3\left(a+bx^2\right)}-\frac{15d^{11/2}\log\left(\sqrt{a}\,\sqrt{d}+\sqrt{b}\right.}{256\sqrt{2}\,a^{3/}}$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}-\frac{15d^5\sqrt{dx}}{64b^3\left(a+bx^2\right)}-\frac{15d^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{d}}{\sqrt[4]{a}\,\sqrt{d}}\right.}{128\sqrt{2}\,a^{3/4}b^{13/4}}$$

Mathematica [A] time = 0.13, size = 299, normalized size = 0.90

$$\frac{d^5\sqrt{dx}\left(-\frac{315\sqrt{2}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{3/4}\sqrt{x}}+\frac{315\sqrt{2}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{3/4}\sqrt{x}}-\frac{630\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}\sqrt{x}}+\frac{630\sqrt{2}\tan^{-1}}{a^{3/}}\right.}{10752b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^5*Sqrt[d*x]*((-3840*a^2*b^(1/4))/(a + b*x^2)^3 - (9216*a*b^(5/4)*x^2)/(a
 + b*x^2)^3 - (7168*b^(9/4)*x^4)/(a + b*x^2)^3 + (480*a*b^(1/4))/(a + b*x^2
)^2 + (840*b^(1/4))/(a + b*x^2) - (630*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(a^(3/4)*Sqrt[x]) + (630*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)])/(a^(3/4)*Sqrt[x]) - (315*Sqrt[2]*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(3/4)*Sqrt[x]) + (315*Sqrt[2]*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(3/4)*Sqrt[x
])))/(10752*b^(13/4))
```

fricas [A] time = 1.35, size = 373, normalized size = 1.12

$$180\left(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3\right)\left(-\frac{d^{22}}{a^3b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{22}}{a^3b^{13}}\right)^{\frac{3}{4}}\sqrt{dx}\,a^2b^{10}d^5-\sqrt{d^{11}x+\sqrt{-\frac{d^{22}}{a^3b^{13}}}\,a^2b^6}\left(-\frac{d^{22}}{a^3b^{13}}\right)^{\frac{3}{4}}a^2b^{10}}{d^{22}}\right)+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(180*(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^22/(a^3*b^
13))^(1/4)*arctan(-((-d^22/(a^3*b^13))^(3/4)*sqrt(d*x)*a^2*b^10*d^5 - sqrt(
d^11*x + sqrt(-d^22/(a^3*b^13))*a^2*b^6)*(-d^22/(a^3*b^13))^(3/4)*a^2*b^10)
/d^22) + 45*(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^22/(a^3*b
^13))^(1/4)*log(15*sqrt(d*x)*d^5 + 15*(-d^22/(a^3*b^13))^(1/4)*a*b^3) - 45*
(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^22/(a^3*b^13))^(1/4)*
log(15*sqrt(d*x)*d^5 - 15*(-d^22/(a^3*b^13))^(1/4)*a*b^3) - 4*(113*b^2*d^5*
x^4 + 126*a*b*d^5*x^2 + 45*a^2*d^5)*sqrt(d*x))/(b^6*x^6 + 3*a*b^5*x^4 + 3*a
^2*b^4*x^2 + a^3*b^3)
```

giac [A] time = 0.21, size = 301, normalized size = 0.90

$$\frac{1}{1536}d^5\left(\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^4}+\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^4}+\frac{45\sqrt{2}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^5*(90*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^4) + 90*sqrt(2)*(a*b^3*d^2)^
(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)
^(1/4))/(a*b^4) + 45*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^
(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^4) - 45*sqrt(2)*(a*b^3*d^2)^(1/4)*log
(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^4) - 8*(113*
sqrt(d*x)*b^2*d^6*x^4 + 126*sqrt(d*x)*a*b*d^6*x^2 + 45*sqrt(d*x)*a^2*d^6)/(
(b*d^2*x^2 + a*d^2)^3*b^3))
```

maple [A] time = 0.02, size = 280, normalized size = 0.84

$$-\frac{15\sqrt{dx}\,a^2d^{11}}{64\left(b\,d^2x^2+d^2a\right)^3 b^3}-\frac{21\,(dx)^{\frac{5}{2}}\,a\,d^9}{32\left(b\,d^2x^2+d^2a\right)^3 b^2}-\frac{113\,(dx)^{\frac{9}{2}}\,d^7}{192\left(b\,d^2x^2+d^2a\right)^3 b}+\frac{15\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^5\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{256a\,b^3}+\frac{15}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -113/192*d^7/(b*d^2*x^2+a*d^2)^3/b*(d*x)^(9/2)-21/32*d^9/(b*d^2*x^2+a*d^2)^
3/b^2*a*(d*x)^(5/2)-15/64*d^11/(b*d^2*x^2+a*d^2)^3/b^3*a^2*(d*x)^(1/2)+15/5
12*d^5/b^3*(a/b*d^2)^(1/4)/a*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^
(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(
1/2)))+15/256*d^5/b^3*(a/b*d^2)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1
/4)*(d*x)^(1/2)+1)+15/256*d^5/b^3*(a/b*d^2)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/
(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.13, size = 326, normalized size = 0.98

$$-\frac{\frac{8\left(113\,(dx)^{\frac{9}{2}}b^2d^8+126\,(dx)^{\frac{5}{2}}abd^{10}+45\,\sqrt{dx}\,a^2d^{12}\right)}{b^6d^6x^6+3\,ab^5d^6x^4+3\,a^2b^4d^6x^2+a^3b^3d^6}-\frac{45\left(\frac{\sqrt{2}\,d^8\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^8\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d^7}{}\right.}{b^3}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/1536*(8*(113*(d*x)^(9/2)*b^2*d^8 + 126*(d*x)^(5/2)*a*b*d^10 + 45*sqrt(d*
x)*a^2*d^12)/(b^6*d^6*x^6 + 3*a*b^5*d^6*x^4 + 3*a^2*b^4*d^6*x^2 + a^3*b^3*d
^6) - 45*(sqrt(2)*d^8*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(
1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^8*log(sqrt(b)*d*x - s
qrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4))
 + 2*sqrt(2)*d^7*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt
(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) +
 2*sqrt(2)*d^7*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(
d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/b
^3)/d
```

mupad [B] time = 4.29, size = 153, normalized size = 0.46

$$-\frac{\frac{113\,d^7\,{\left(d\,x\right)}^{9/2}}{192\,b}+\frac{15\,a^2\,d^{11}\,\sqrt{d\,x}}{64\,b^3}+\frac{21\,a\,d^9\,{\left(d\,x\right)}^{5/2}}{32\,b^2}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}-\frac{15\,d^{11/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{3/4}\,b^{13/4}}-\frac{15\,d^{11/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{3/4}\,b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] - ((113*d^7*(d*x)^(9/2))/(192*b) + (15*a^2*d^11*(d*x)^(1/2))/(64*b^3) + (21
*a*d^9*(d*x)^(5/2))/(32*b^2))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 + 3*
a*b^2*d^6*x^4) - (15*d^(11/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2
))))/(128*(-a)^(3/4)*b^(13/4)) - (15*d^(11/2)*atanh((b^(1/4)*(d*x)^(1/2))/(
(-a)^(1/4)*d^(1/2))))/(128*(-a)^(3/4)*b^(13/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx\right)^{\frac{11}{2}}}{\left(a + bx^{2}\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(11/2)/(a + b*x**2)**4, x)
```

3.701 $\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=336

$$\frac{7d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{5/4}b^{11/4}}-\frac{7d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{5/4}b^{11/4}}-\frac{7d^{9/2}\tan^{-1}}{128}$$

```
[Out] -1/6*d*(d*x)^(7/2)/b/(b*x^2+a)^3-7/48*d^3*(d*x)^(3/2)/b^2/(b*x^2+a)^2+7/64*
d^3*(d*x)^(3/2)/a/b^2/(b*x^2+a)-7/256*d^(9/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x
)^(1/2)/a^(1/4)/d^(1/2))/a^(5/4)/b^(11/4)*2^(1/2)+7/256*d^(9/2)*arctan(1+b^
(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(5/4)/b^(11/4)*2^(1/2)+7/512*d
^(9/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(
1/2))/a^(5/4)/b^(11/4)*2^(1/2)-7/512*d^(9/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d
^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(5/4)/b^(11/4)*2^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{7d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{5/4}b^{11/4}}-\frac{7d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{5/4}b^{11/4}}-\frac{7d^{9/2}\tan^{-1}}{128}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(d*(d*x)^(7/2))/(6*b*(a + b*x^2)^3) - (7*d^3*(d*x)^(3/2))/(48*b^2*(a + b*x
^2)^2) + (7*d^3*(d*x)^(3/2))/(64*a*b^2*(a + b*x^2)) - (7*d^(9/2)*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(5/4)*b^(11
/4)) + (7*d^(9/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(128*Sqrt[2]*a^(5/4)*b^(11/4)) + (7*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]
*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(5/4)*b^(11
/4)) - (7*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(5/4)*b^(11/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}+\frac{1}{12}\left(7b^2d^2\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}-\frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2}+\frac{1}{32}\left(7d^4\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}-\frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2}+\frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)}+\frac{\left(7d^4\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{128ab}$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}-\frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2}+\frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)}+\frac{\left(7d^3\right)\mathrm{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,\ldots\right.}{64ab}$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}-\frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2}+\frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)}-\frac{\left(7d^3\right)\mathrm{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,d\ldots\right.}{128ab^{3/2}}$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}-\frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2}+\frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)}+\frac{\left(7d^{9/2}\right)\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt{\ldots}}{\sqrt[4]{b}}}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{\ldots}}{\ldots}}\ldots\right.}{256\sqrt{2}\,a^{5/4}\ldots}$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}-\frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2}+\frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)}+\frac{7d^{9/2}\log\left(\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\ldots\right.}{256\sqrt{2}\,a^{5/4}\ldots}$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}-\frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2}+\frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)}-\frac{7d^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{128\sqrt{2}\,a^{5/4}b^{11/4}}$$

Mathematica [C] time = 0.02, size = 74, normalized size = 0.22

$$\frac{2d^4x\sqrt{dx}\left(7\left(a+bx^2\right)^3\,{}_2F_1\left(\frac{3}{4},4;\frac{7}{4};-\frac{bx^2}{a}\right)-a^2\left(7a+9bx^2\right)\right)}{45a^2b^2\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*d^4*x*Sqrt[d*x]*(-(a^2*(7*a + 9*b*x^2)) + 7*(a + b*x^2)^3*Hypergeometric
2F1[3/4, 4, 7/4, -((b*x^2)/a)]))/(45*a^2*b^2*(a + b*x^2)^3)
```

fricas [A] time = 0.96, size = 390, normalized size = 1.16

$$-\frac{84\left(ab^5x^6+3a^2b^4x^4+3a^3b^3x^2+a^4b^2\right)\left(-\frac{d^{18}}{a^5b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{18}}{a^5b^{11}}\right)^{\frac{1}{4}}\sqrt{dx}\,ab^3d^{13}-\sqrt{d^{27}x-\sqrt{-\frac{d^{18}}{a^5b^{11}}}\,a^3b^5d^{18}}\left(-\frac{d^{18}}{a^5b^{11}}\right)^{\frac{1}{4}}ab^3}{d^{18}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(84*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^18/(a^
5*b^11))^(1/4)*arctan(-((-d^18/(a^5*b^11))^(1/4)*sqrt(d*x)*a*b^3*d^13 - sqr
t(d^27*x - sqrt(-d^18/(a^5*b^11))*a^3*b^5*d^18)*(-d^18/(a^5*b^11))^(1/4)*a*
b^3)/d^18) - 21*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^1
8/(a^5*b^11))^(1/4)*log(343*sqrt(d*x)*d^13 + 343*(-d^18/(a^5*b^11))^(3/4)*a
^4*b^8) + 21*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^18/(
a^5*b^11))^(1/4)*log(343*sqrt(d*x)*d^13 - 343*(-d^18/(a^5*b^11))^(3/4)*a^4*
b^8) - 4*(21*b^2*d^4*x^5 - 18*a*b*d^4*x^3 - 7*a^2*d^4*x)*sqrt(d*x))/(a*b^5*
x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)
```

giac [A] time = 0.21, size = 317, normalized size = 0.94

$$\frac{1}{1536}d^4\left(\frac{8\left(21\sqrt{dx}\,b^2d^6x^5-18\sqrt{dx}\,abd^6x^3-7\sqrt{dx}\,a^2d^6x\right)}{\left(bd^2x^2+ad^2\right)^3ab^2}+\frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^5d}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^4*(8*(21*sqrt(d*x)*b^2*d^6*x^5 - 18*sqrt(d*x)*a*b*d^6*x^3 - 7*sqrt
(d*x)*a^2*d^6*x)/((b*d^2*x^2 + a*d^2)^3*a*b^2) + 42*sqrt(2)*(a*b^3*d^2)^(3/
4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/
4))/(a^2*b^5*d) + 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^5*d) - 21*sqrt(2)*(
a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b
))/(a^2*b^5*d) + 21*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(
1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^5*d))
```

maple [A] time = 0.02, size = 277, normalized size = 0.82

$$-\frac{7 (dx)^{\frac{3}{2}} a d^9}{192 \left(b d^2x^2 + d^2a\right)^3 b^2} - \frac{3 (dx)^{\frac{7}{2}} d^7}{32 \left(b d^2x^2 + d^2a\right)^3 b} + \frac{7 (dx)^{\frac{11}{2}} d^5}{64 \left(b d^2x^2 + d^2a\right)^3 a} + \frac{7\sqrt{2}\, d^5 \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}} - 1\right)}{256 \left(\frac{a d^2}{b}\right)^{\frac{1}{4}} a b^3} + \frac{7\sqrt{2}\, d^5 \operatorname{arcta}}{256 \left(\frac{a}{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 7/64*d^5/(b*d^2*x^2+a*d^2)^3/a*(d*x)^(11/2)-3/32*d^7/(b*d^2*x^2+a*d^2)^3/b*
(d*x)^(7/2)-7/192*d^9/(b*d^2*x^2+a*d^2)^3/b^2*a*(d*x)^(3/2)+7/512*d^5/a/b^3
/(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d
^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+7/256
*d^5/a/b^3/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/
2)+1)+7/256*d^5/a/b^3/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4
)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.02, size = 323, normalized size = 0.96

$$\frac{\frac{21 d^6 \left(\frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}} b^{\frac{1}{4}} + 2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}} + \frac{2\sqrt{2} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}} b^{\frac{1}{4}} - 2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{b}dx + \sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\, b^{\frac{1}{4}} + \sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}} b^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(\sqrt{b}dx - \sqrt{2}\left(ad^2\right.\right.}{\left(ad^2\right)^{\frac{1}{4}} b}\right.}{ab^2}}{1536 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*(21*d^6*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4)
 + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*s
qrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*
sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b
)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sq
rt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2
)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a*b^2) + 8
*(21*(d*x)^(11/2)*b^2*d^6 - 18*(d*x)^(7/2)*a*b*d^8 - 7*(d*x)^(3/2)*a^2*d^10
)/(a*b^5*d^6*x^6 + 3*a^2*b^4*d^6*x^4 + 3*a^3*b^3*d^6*x^2 + a^4*b^2*d^6))/d
```

mupad [B] time = 4.26, size = 150, normalized size = 0.45

$$\frac{7\,d^{9/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{5/4}\,b^{11/4}}-\frac{7\,d^{9/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{5/4}\,b^{11/4}}-\frac{\frac{3\,d^7\,{\left(d\,x\right)}^{7/2}}{32\,b}-\frac{7\,d^5\,{\left(d\,x\right)}^{11/2}}{64\,a}+\frac{7\,a\,d^9\,{\left(d\,x\right)}^{3/2}}{192\,b^2}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (7*d^(9/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-a)^(5/
4)*b^(11/4)) - (7*d^(9/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))
/(128*(-a)^(5/4)*b^(11/4)) - ((3*d^7*(d*x)^(7/2))/(32*b) - (7*d^5*(d*x)^(11
/2))/(64*a) + (7*a*d^9*(d*x)^(3/2))/(192*b^2))/(a^3*d^6 + b^3*d^6*x^6 + 3*a
^2*b*d^6*x^2 + 3*a*b^2*d^6*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{9}{2}}}{\left(a + bx^2\right)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(9/2)/(a + b*x**2)**4, x)
```

3.702 $\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=336

$$-\frac{5d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{5d^{7/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{7/4}b^{9/4}}-\frac{5d^{7/2}\tan^{-}}{128}$$

```
[Out] -1/6*d*(d*x)^(5/2)/b/(b*x^2+a)^3-5/256*d^(7/2)*arctan(1-b^(1/4)*2^(1/2)*(d*
x)^(1/2)/a^(1/4)/d^(1/2))/a^(7/4)/b^(9/4)*2^(1/2)+5/256*d^(7/2)*arctan(1+b^
(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(7/4)/b^(9/4)*2^(1/2)-5/512*d^
(7/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1
/2))/a^(7/4)/b^(9/4)*2^(1/2)+5/512*d^(7/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(
1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(7/4)/b^(9/4)*2^(1/2)-5/48*d^3*
(d*x)^(1/2)/b^2/(b*x^2+a)^2+5/192*d^3*(d*x)^(1/2)/a/b^2/(b*x^2+a)
```

Rubi [A] time = 0.34, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{5d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{5d^{7/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{7/4}b^{9/4}}-\frac{5d^{7/2}\tan^{-}}{128}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(d*(d*x)^(5/2))/(6*b*(a + b*x^2)^3) - (5*d^3*Sqrt[d*x])/(48*b^2*(a + b*x^2
)^2) + (5*d^3*Sqrt[d*x])/(192*a*b^2*(a + b*x^2)) - (5*d^(7/2)*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(7/4)*b^(9/4))
 + (5*d^(7/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(1
28*Sqrt[2]*a^(7/4)*b^(9/4)) - (5*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt
[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(7/4)*b^(9/4)) +
 (5*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(7/4)*b^(9/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3} + \frac{1}{12}\left(5b^2d^2\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2} + \frac{1}{96}\left(5d^4\right)\int \frac{1}{\sqrt{dx}\,\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2} + \frac{5d^3\sqrt{dx}}{192ab^2\left(a+bx^2\right)} + \frac{\left(5d^4\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{128ab}$$

$$= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2} + \frac{5d^3\sqrt{dx}}{192ab^2\left(a+bx^2\right)} + \frac{\left(5d^3\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,d \cdots\right.}{64ab}$$

$$= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2} + \frac{5d^3\sqrt{dx}}{192ab^2\left(a+bx^2\right)} + \frac{\left(5d^2\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x \cdots}{ab+\frac{b^2x^4}{d^2}} \cdots\right.}{128a^{3/2}b}$$

$$= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2} + \frac{5d^3\sqrt{dx}}{192ab^2\left(a+bx^2\right)} - \frac{\left(5d^{7/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\cdots} \cdots\right.}{256\sqrt{2}\,a \cdots}$$

$$= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2} + \frac{5d^3\sqrt{dx}}{192ab^2\left(a+bx^2\right)} - \frac{5d^{7/2}\log\left(\sqrt{a}\,\sqrt{d}\,+\sqrt{b}\cdots\right.}{256\sqrt{2}\,a\cdots}$$

$$= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2} + \frac{5d^3\sqrt{dx}}{192ab^2\left(a+bx^2\right)} - \frac{5d^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{d\cdots}}{\sqrt[4]{a}\,\sqrt{d}}\cdots\right.}{128\sqrt{2}\,a^{7/4}b^{9/4}}$$

Mathematica [A] time = 0.17, size = 279, normalized size = 0.83

$$\frac{d^3\sqrt{dx}\left(-\frac{105\sqrt{2}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{7/4}\sqrt{x}} + \frac{105\sqrt{2}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{7/4}\sqrt{x}} - \frac{210\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{7/4}\sqrt{x}} + \frac{210\sqrt{2}\,\tan^{-1}\cdots}{a^{7/\cdots}}\cdots\right.}{10752b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^3*Sqrt[d*x]*((-1280*a*b^(1/4))/(a + b*x^2)^3 - (3072*b^(5/4)*x^2)/(a + b
*x^2)^3 + (160*b^(1/4))/(a + b*x^2)^2 + (280*b^(1/4))/(a^2 + a*b*x^2) - (21
0*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(7/4)*Sqrt[x])
+ (210*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(7/4)*Sqrt
[x]) - (105*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x])/(a^(7/4)*Sqrt[x]) + (105*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(a^(7/4)*Sqrt[x])))/(10752*b^(9/4))
```

fricas [A] time = 0.94, size = 389, normalized size = 1.16

$$\frac{60\left(ab^5x^6+3\,a^2b^4x^4+3\,a^3b^3x^2+a^4b^2\right)\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^5b^7d^3\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{3}{4}}-\sqrt{a^4b^4\sqrt{-\frac{d^{14}}{a^7b^9}}+d^7x}\,a^5b^7\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{3}{4}}}{d^{14}}\right)+15\,(a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(60*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^14/(a^7
*b^9))^(1/4)*arctan(-(sqrt(d*x)*a^5*b^7*d^3*(-d^14/(a^7*b^9))^(3/4) - sqrt(
a^4*b^4*sqrt(-d^14/(a^7*b^9)) + d^7*x)*a^5*b^7*(-d^14/(a^7*b^9))^(3/4))/d^1
4) + 15*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^14/(a^7*b
^9))^(1/4)*log(5*a^2*b^2*(-d^14/(a^7*b^9))^(1/4) + 5*sqrt(d*x)*d^3) - 15*(a
*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^14/(a^7*b^9))^(1/4)
*log(-5*a^2*b^2*(-d^14/(a^7*b^9))^(1/4) + 5*sqrt(d*x)*d^3) + 4*(5*b^2*d^3*x
^4 - 42*a*b*d^3*x^2 - 15*a^2*d^3)*sqrt(d*x))/(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3
*a^3*b^3*x^2 + a^4*b^2)
```

giac [A] time = 0.24, size = 304, normalized size = 0.90

$$\frac{1}{1536}\,d^3\left(\frac{30\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^3}+\frac{30\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^3}+\frac{15\sqrt{2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^3*(30*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^3) + 30*sqrt(2)*(a*b^3*d^2
)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/
b)^(1/4))/(a^2*b^3) + 15*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2
/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^3) - 15*sqrt(2)*(a*b^3*d^2)^(1/
4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^3) +
 8*(5*sqrt(d*x)*b^2*d^6*x^4 - 42*sqrt(d*x)*a*b*d^6*x^2 - 15*sqrt(d*x)*a^2*d
^6)/((b*d^2*x^2 + a*d^2)^3*a*b^2))
```

maple [A] time = 0.02, size = 277, normalized size = 0.82

$$-\frac{5\sqrt{dx}\,a\,d^9}{64\left(b\,d^2x^2+d^2a\right)^3 b^2}-\frac{7\,(dx)^{\frac{5}{2}}\,d^7}{32\left(b\,d^2x^2+d^2a\right)^3 b}+\frac{5\,(dx)^{\frac{9}{2}}\,d^5}{192\left(b\,d^2x^2+d^2a\right)^3 a}+\frac{5\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^3\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{256a^2b^2}+\frac{5\left(\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 5/192*d^5/(b*d^2*x^2+a*d^2)^3/a*(d*x)^(9/2)-7/32*d^7/(b*d^2*x^2+a*d^2)^3/b*
(d*x)^(5/2)-5/64*d^9/(b*d^2*x^2+a*d^2)^3/b^2*a*(d*x)^(1/2)+5/512*d^3/a^2/b^
2*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*
d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+5/25
6*d^3/a^2/b^2*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^
(1/2)+1)+5/256*d^3/a^2/b^2*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)
^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 2.96, size = 332, normalized size = 0.99

$$\frac{\frac{8\left(5\,(dx)^{\frac{9}{2}}b^2d^6-42\,(dx)^{\frac{5}{2}}abd^8-15\,\sqrt{dx}\,a^2d^{10}\right)}{ab^5d^6x^6+3\,a^2b^4d^6x^4+3\,a^3b^3d^6x^2+a^4b^2d^6}+\frac{15\left(\frac{\sqrt{2}\,d^6\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^6\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d^5\arctan\left(\cdots\right.}{}\right.}{ab^2}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*(8*(5*(d*x)^(9/2)*b^2*d^6 - 42*(d*x)^(5/2)*a*b*d^8 - 15*sqrt(d*x)*a^
2*d^10)/(a*b^5*d^6*x^6 + 3*a^2*b^4*d^6*x^4 + 3*a^3*b^3*d^6*x^2 + a^4*b^2*d^
6) + 15*(sqrt(2)*d^6*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1
/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^6*log(sqrt(b)*d*x - sq
rt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4))
+ 2*sqrt(2)*d^5*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(
d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) +
2*sqrt(2)*d^5*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d
*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/(a
*b^2))/d
```

mupad [B] time = 4.26, size = 150, normalized size = 0.45

$$\frac{5\,d^{7/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{7/4}\,b^{9/4}}-\frac{\frac{7\,d^7\,{\left(d\,x\right)}^{5/2}}{32\,b}-\frac{5\,d^5\,{\left(d\,x\right)}^{9/2}}{192\,a}+\frac{5\,a\,d^9\,\sqrt{d\,x}}{64\,b^2}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}+\frac{5\,d^{7/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{7/4}\,b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (5*d^(7/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-a)^(7/4
)*b^(9/4)) - ((7*d^7*(d*x)^(5/2))/(32*b) - (5*d^5*(d*x)^(9/2))/(192*a) + (5
*a*d^9*(d*x)^(1/2))/(64*b^2))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 + 3*
a*b^2*d^6*x^4) + (5*d^(7/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2)
)))/(128*(-a)^(7/4)*b^(9/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{7}{2}}}{\left(a + bx^2\right)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(7/2)/(a + b*x**2)**4, x)
```

3.703 $\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=335

$$\frac{5d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{9/4}b^{7/4}}-\frac{5d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{9/4}b^{7/4}}-\frac{5d^{5/2}\tan^{-1}}{128}$$

```
[Out] -1/6*d*(d*x)^(3/2)/b/(b*x^2+a)^3+1/16*d*(d*x)^(3/2)/a/b/(b*x^2+a)^2+5/64*d*
(d*x)^(3/2)/a^2/b/(b*x^2+a)-5/256*d^(5/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1
/2)/a^(1/4)/d^(1/2))/a^(9/4)/b^(7/4)*2^(1/2)+5/256*d^(5/2)*arctan(1+b^(1/4)
*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(9/4)/b^(7/4)*2^(1/2)+5/512*d^(5/2)
*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/
a^(9/4)/b^(7/4)*2^(1/2)-5/512*d^(5/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+
a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(9/4)/b^(7/4)*2^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{5d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{9/4}b^{7/4}}-\frac{5d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{9/4}b^{7/4}}-\frac{5d^{5/2}\tan^{-1}}{128}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(d*(d*x)^(3/2))/(6*b*(a + b*x^2)^3) + (d*(d*x)^(3/2))/(16*a*b*(a + b*x^2)^
2) + (5*d*(d*x)^(3/2))/(64*a^2*b*(a + b*x^2)) - (5*d^(5/2)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(9/4)*b^(7/4)) +
(5*d^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*
Sqrt[2]*a^(9/4)*b^(7/4)) + (5*d^(5/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(9/4)*b^(7/4)) - (5
*d^(5/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d*x]])/(256*Sqrt[2]*a^(9/4)*b^(7/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}+\frac{1}{4}\left(b^2d^2\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}+\frac{d(dx)^{3/2}}{16ab\left(a+bx^2\right)^2}+\frac{\left(5bd^2\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^2}\,dx}{32a}$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}+\frac{d(dx)^{3/2}}{16ab\left(a+bx^2\right)^2}+\frac{5d(dx)^{3/2}}{64a^2b\left(a+bx^2\right)}+\frac{\left(5d^2\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{128a^2}$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}+\frac{d(dx)^{3/2}}{16ab\left(a+bx^2\right)^2}+\frac{5d(dx)^{3/2}}{64a^2b\left(a+bx^2\right)}+\frac{(5d)\,\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,x\right.}{64a^2}$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}+\frac{d(dx)^{3/2}}{16ab\left(a+bx^2\right)^2}+\frac{5d(dx)^{3/2}}{64a^2b\left(a+bx^2\right)}-\frac{(5d)\,\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx\right.}{128a^2\sqrt{b}}$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}+\frac{d(dx)^{3/2}}{16ab\left(a+bx^2\right)^2}+\frac{5d(dx)^{3/2}}{64a^2b\left(a+bx^2\right)}+\frac{\left(5d^{5/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt{}}{\sqrt[4]{b}}}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{}}{}}\right.}{256\sqrt{2}\,a^{9/4}}$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}+\frac{d(dx)^{3/2}}{16ab\left(a+bx^2\right)^2}+\frac{5d(dx)^{3/2}}{64a^2b\left(a+bx^2\right)}+\frac{5d^{5/2}\log\left(\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\right.}{256\sqrt{2}\,a^{9/}}$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}+\frac{d(dx)^{3/2}}{16ab\left(a+bx^2\right)^2}+\frac{5d(dx)^{3/2}}{64a^2b\left(a+bx^2\right)}-\frac{5d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{128\sqrt{2}\,a^{9/4}b^{7/4}}$$

Mathematica [C] time = 0.02, size = 60, normalized size = 0.18

$$\frac{2d(dx)^{3/2}\left(\left(a+bx^2\right)^3\,{}_2F_1\left(\frac{3}{4},4;\frac{7}{4};-\frac{bx^2}{a}\right)-a^3\right)}{9a^3b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*d*(d*x)^(3/2)*(-a^3 + (a + b*x^2)^3*Hypergeometric2F1[3/4, 4, 7/4, -((b*
x^2)/a)]))/(9*a^3*b*(a + b*x^2)^3)
```

fricas [A] time = 0.80, size = 396, normalized size = 1.18

$$-\frac{60\left(a^2b^4x^6+3a^3b^3x^4+3a^4b^2x^2+a^5b\right)\left(-\frac{d^{10}}{a^9b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{125\sqrt{dx}\,a^2b^2d^7\left(-\frac{d^{10}}{a^9b^7}\right)^{\frac{1}{4}}-\sqrt{-15625a^5b^3d^{10}\sqrt{-\frac{d^{10}}{a^9b^7}}+15625d^{15}x}}{125d^{10}}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(60*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*(-d^10/(a^
9*b^7))^(1/4)*arctan(-1/125*(125*sqrt(d*x)*a^2*b^2*d^7*(-d^10/(a^9*b^7))^(1
/4) - sqrt(-15625*a^5*b^3*d^10*sqrt(-d^10/(a^9*b^7)) + 15625*d^15*x)*a^2*b^
2*(-d^10/(a^9*b^7))^(1/4))/d^10) - 15*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*
b^2*x^2 + a^5*b)*(-d^10/(a^9*b^7))^(1/4)*log(125*a^7*b^5*(-d^10/(a^9*b^7))^
(3/4) + 125*sqrt(d*x)*d^7) + 15*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^
2 + a^5*b)*(-d^10/(a^9*b^7))^(1/4)*log(-125*a^7*b^5*(-d^10/(a^9*b^7))^(3/4)
 + 125*sqrt(d*x)*d^7) - 4*(15*b^2*d^2*x^5 + 42*a*b*d^2*x^3 - 5*a^2*d^2*x)*s
qrt(d*x))/(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)
```

giac [A] time = 0.22, size = 317, normalized size = 0.95

$$\frac{1}{1536}d^2\left(\frac{8\left(15\sqrt{dx}\,b^2d^6x^5+42\sqrt{dx}\,abd^6x^3-5\sqrt{dx}\,a^2d^6x\right)}{\left(bd^2x^2+ad^2\right)^3a^2b}+\frac{30\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^4d}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^2*(8*(15*sqrt(d*x)*b^2*d^6*x^5 + 42*sqrt(d*x)*a*b*d^6*x^3 - 5*sqrt
(d*x)*a^2*d^6*x)/((b*d^2*x^2 + a*d^2)^3*a^2*b) + 30*sqrt(2)*(a*b^3*d^2)^(3/
4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/
4))/(a^3*b^4*d) + 30*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^4*d) - 15*sqrt(2)*(
a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b
))/(a^3*b^4*d) + 15*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(
1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^4*d))
```

maple [A] time = 0.02, size = 277, normalized size = 0.83

$$-\frac{5\,(dx)^{\frac{3}{2}}\,d^{7}}{192\left(b\,d^{2}x^{2}+d^{2}a\right)^{3}b}+\frac{7\,(dx)^{\frac{7}{2}}\,d^{5}}{32\left(b\,d^{2}x^{2}+d^{2}a\right)^{3}a}+\frac{5\,(dx)^{\frac{11}{2}}\,b\,d^{3}}{64\left(b\,d^{2}x^{2}+d^{2}a\right)^{3}a^{2}}+\frac{5\sqrt{2}\,d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}-1\right)}{256\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}a^{2}b^{2}}+\frac{5\sqrt{2}\,d^{3}\,\mathrm{arcta}}{256\left(\frac{a}{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 5/64*d^3/(b*d^2*x^2+a*d^2)^3/a^2*b*(d*x)^(11/2)+7/32*d^5/(b*d^2*x^2+a*d^2)^
3/a*(d*x)^(7/2)-5/192*d^7/(b*d^2*x^2+a*d^2)^3/b*(d*x)^(3/2)+5/512*d^3/a^2/b
^2/(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b
*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+5/2
56*d^3/a^2/b^2/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)
^(1/2)+1)+5/256*d^3/a^2/b^2/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2
)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.02, size = 323, normalized size = 0.96

$$\frac{\frac{15\,d^{4}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^{2}\right.\right.}{\left(ad^{2}\right)^{\frac{1}{4}}b}\right.}{a^{2}b}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*(15*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4)
 + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*s
qrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*
sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b
)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sq
rt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2
)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^2*b) + 8
*(15*(d*x)^(11/2)*b^2*d^4 + 42*(d*x)^(7/2)*a*b*d^6 - 5*(d*x)^(3/2)*a^2*d^8)
/(a^2*b^4*d^6*x^6 + 3*a^3*b^3*d^6*x^4 + 3*a^4*b^2*d^6*x^2 + a^5*b*d^6))/d
```

mupad [B] time = 4.23, size = 149, normalized size = 0.44

$$\frac{\frac{7\,d^5\,(d\,x)^{7/2}}{32\,a}-\frac{5\,d^7\,(d\,x)^{3/2}}{192\,b}+\frac{5\,b\,d^3\,(d\,x)^{11/2}}{64\,a^2}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}+\frac{5\,d^{5/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{9/4}\,b^{7/4}}-\frac{5\,d^{5/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{9/4}\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((7*d^5*(d*x)^(7/2))/(32*a) - (5*d^7*(d*x)^(3/2))/(192*b) + (5*b*d^3*(d*x)^
(11/2))/(64*a^2))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 + 3*a*b^2*d^6*x^
4) + (5*d^(5/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-a)
^(9/4)*b^(7/4)) - (5*d^(5/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2
))))/(128*(-a)^(9/4)*b^(7/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{5}{2}}}{\left(a + bx^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(5/2)/(a + b*x**2)**4, x)
```

3.704 $\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=335

$$-\frac{7d^{3/2}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{7d^{3/2}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{11/4}b^{5/4}}-\frac{7d^{3/2}\tan^{-}}{128}$$

```
[Out] -7/256*d^(3/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(11/
4)/b^(5/4)*2^(1/2)+7/256*d^(3/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/
4)/d^(1/2))/a^(11/4)/b^(5/4)*2^(1/2)-7/512*d^(3/2)*ln(a^(1/2)*d^(1/2)+x*b^(
1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(11/4)/b^(5/4)*2^(1/2)+
7/512*d^(3/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*
(d*x)^(1/2))/a^(11/4)/b^(5/4)*2^(1/2)-1/6*d*(d*x)^(1/2)/b/(b*x^2+a)^3+1/48*
d*(d*x)^(1/2)/a/b/(b*x^2+a)^2+7/192*d*(d*x)^(1/2)/a^2/b/(b*x^2+a)
```

Rubi [A] time = 0.35, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{7d^{3/2}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{7d^{3/2}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{11/4}b^{5/4}}-\frac{7d^{3/2}\tan^{-}}{128}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(d*Sqrt[d*x])/(6*b*(a + b*x^2)^3) + (d*Sqrt[d*x])/(48*a*b*(a + b*x^2)^2) +
 (7*d*Sqrt[d*x])/(192*a^2*b*(a + b*x^2)) - (7*d^(3/2)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(11/4)*b^(5/4)) + (7*d
^(3/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt
[2]*a^(11/4)*b^(5/4)) - (7*d^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x
- Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(11/4)*b^(5/4)) + (7*d
^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(256*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{1}{12}\left(b^2d^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2} + \frac{\left(7bd^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^2}\,dx}{96a}$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2} + \frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)} + \frac{\left(7d^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{128a^2}$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2} + \frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)} + \frac{(7d)\,\mathrm{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx \ldots\right.}{64a^2}$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2} + \frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)} + \frac{7\,\mathrm{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx \ldots\right.}{128a^{5/2}}$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2} + \frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)} - \frac{\left(7d^{3/2}\right)\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{\ldots}}{\sqrt[4]{\ldots}}}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\ldots}\right.}{256\sqrt{2}\,a\ldots}$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2} + \frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)} - \frac{7d^{3/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\ldots\right.}{256\sqrt{2}\,a\ldots}$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2} + \frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)} - \frac{7d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{\ldots}}{\sqrt[4]{a}\,\sqrt{d}}\right.}{128\sqrt{2}\,a^{11/4}b^{5/4}}$$

Mathematica [A] time = 0.13, size = 260, normalized size = 0.78

$$\frac{d\sqrt{dx}\left(-\frac{21\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{11/4}\sqrt{x}}+\frac{21\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{11/4}\sqrt{x}}-\frac{42\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{11/4}\sqrt{x}}+\frac{42\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{a^{11/4}\sqrt{x}}\right.}{1536b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d*Sqrt[d*x]*((-256*b^(1/4))/(a + b*x^2)^3 + (32*b^(1/4))/(a*(a + b*x^2)^2)
 + (56*b^(1/4))/(a^2*(a + b*x^2)) - (42*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/(a^(11/4)*Sqrt[x]) + (42*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^
(1/4)*Sqrt[x])/a^(1/4)])/(a^(11/4)*Sqrt[x]) - (21*Sqrt[2]*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(11/4)*Sqrt[x]) + (21*Sqrt[2]
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(11/4)*Sqrt
[x])))/(1536*b^(5/4))
```

fricas [A] time = 1.05, size = 373, normalized size = 1.11

$$\frac{84\left(a^2b^4x^6+3a^3b^3x^4+3a^4b^2x^2+a^5b\right)\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^8b^4d\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac{3}{4}}-\sqrt{a^6b^2\sqrt{-\frac{d^6}{a^{11}b^5}}+d^3x}\,a^8b^4\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac{3}{4}}}{d^6}\right)+21}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(84*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*(-d^6/(a^11
*b^5))^(1/4)*arctan(-(sqrt(d*x)*a^8*b^4*d*(-d^6/(a^11*b^5))^(3/4) - sqrt(a^
6*b^2*sqrt(-d^6/(a^11*b^5)) + d^3*x)*a^8*b^4*(-d^6/(a^11*b^5))^(3/4))/d^6)
+ 21*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*(-d^6/(a^11*b^5)
)^(1/4)*log(7*a^3*b*(-d^6/(a^11*b^5))^(1/4) + 7*sqrt(d*x)*d) - 21*(a^2*b^4*
x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*(-d^6/(a^11*b^5))^(1/4)*log(-7
*a^3*b*(-d^6/(a^11*b^5))^(1/4) + 7*sqrt(d*x)*d) + 4*(7*b^2*d*x^4 + 18*a*b*d
*x^2 - 21*a^2*d)*sqrt(d*x))/(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 +
a^5*b)
```

giac [A] time = 0.20, size = 302, normalized size = 0.90

$$\frac{1}{1536}d\left(\frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^2}+\frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^2}+\frac{21\sqrt{2}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d*(42*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b
)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^2) + 42*sqrt(2)*(a*b^3*d^2)^
(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)
^(1/4))/(a^3*b^2) + 21*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b
)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^2) - 21*sqrt(2)*(a*b^3*d^2)^(1/4)
*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^2) + 8
*(7*sqrt(d*x)*b^2*d^6*x^4 + 18*sqrt(d*x)*a*b*d^6*x^2 - 21*sqrt(d*x)*a^2*d^6
)/((b*d^2*x^2 + a*d^2)^3*a^2*b))
```

maple [A] time = 0.02, size = 271, normalized size = 0.81

$$-\frac{7\sqrt{dx}\,d^7}{64\left(b\,d^2x^2+d^2a\right)^3b}+\frac{3\,(dx)^{\frac{5}{2}}\,d^5}{32\left(b\,d^2x^2+d^2a\right)^3a}+\frac{7\,(dx)^{\frac{9}{2}}\,b\,d^3}{192\left(b\,d^2x^2+d^2a\right)^3a^2}+\frac{7\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{256a^3b}+\frac{7\left(\frac{a\,d^2}{b}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 7/192*d^3/(b*d^2*x^2+a*d^2)^3/a^2*b*(d*x)^(9/2)+3/32*d^5/(b*d^2*x^2+a*d^2)^
3/a*(d*x)^(5/2)-7/64*d^7/(b*d^2*x^2+a*d^2)^3/b*(d*x)^(1/2)+7/512*d/a^3/b*(a
/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)
^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+7/256*d/
a^3/b*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)
+7/256*d/a^3/b*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)
^(1/2)-1)
```

maxima [A] time = 3.02, size = 332, normalized size = 0.99

$$\frac{\frac{8\left(7\,(dx)^{\frac{9}{2}}b^2d^4+18\,(dx)^{\frac{5}{2}}abd^6-21\,\sqrt{dx}\,a^2d^8\right)}{a^2b^4d^6x^6+3\,a^3b^3d^6x^4+3\,a^4b^2d^6x^2+a^5bd^6}+\frac{21\left(\frac{\sqrt{2}\,d^4\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^4\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\,\sqrt{2}\,d^3\arctan\left(\frac{\sqrt{2}\cdots}{\cdots}\right)}{\sqrt{\cdots}}\cdots\right)}{a^2b}\cdots}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*(8*(7*(d*x)^(9/2)*b^2*d^4 + 18*(d*x)^(5/2)*a*b*d^6 - 21*sqrt(d*x)*a^
2*d^8)/(a^2*b^4*d^6*x^6 + 3*a^3*b^3*d^6*x^4 + 3*a^4*b^2*d^6*x^2 + a^5*b*d^6
) + 21*(sqrt(2)*d^4*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/
4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^4*log(sqrt(b)*d*x - sqr
t(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) +
 2*sqrt(2)*d^3*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d
*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2
*sqrt(2)*d^3*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*
x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/(a^
2*b))/d
```

mupad [B] time = 4.27, size = 149, normalized size = 0.44

$$\frac{\frac{3\,d^5\,{\left(d\,x\right)}^{5/2}}{32\,a}-\frac{7\,d^7\,\sqrt{d\,x}}{64\,b}+\frac{7\,b\,d^3\,{\left(d\,x\right)}^{9/2}}{192\,a^2}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}-\frac{7\,d^{3/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{11/4}\,b^{5/4}}-\frac{7\,d^{3/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{11/4}\,b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((3*d^5*(d*x)^(5/2))/(32*a) - (7*d^7*(d*x)^(1/2))/(64*b) + (7*b*d^3*(d*x)^(
9/2))/(192*a^2))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 + 3*a*b^2*d^6*x^4
) - (7*d^(3/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-a)^
(11/4)*b^(5/4)) - (7*d^(3/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2
))))/(128*(-a)^(11/4)*b^(5/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{\left(a+bx^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(3/2)/(a + b*x**2)**4, x)
```

3.705 $\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=335

$$\frac{15\sqrt{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{13/4}b^{3/4}}-\frac{15\sqrt{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{13/4}b^{3/4}}-\frac{15\sqrt{d}\tan}{128}$$

```
[Out] 1/6*(d*x)^(3/2)/a/d/(b*x^2+a)^3+3/16*(d*x)^(3/2)/a^2/d/(b*x^2+a)^2+15/64*(d
*x)^(3/2)/a^3/d/(b*x^2+a)-15/256*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/
4)/d^(1/2))*d^(1/2)/a^(13/4)/b^(3/4)*2^(1/2)+15/256*arctan(1+b^(1/4)*2^(1/2
)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*d^(1/2)/a^(13/4)/b^(3/4)*2^(1/2)+15/512*ln(a
^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))*d^(1/
2)/a^(13/4)/b^(3/4)*2^(1/2)-15/512*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(
1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))*d^(1/2)/a^(13/4)/b^(3/4)*2^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {28, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{15\sqrt{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{13/4}b^{3/4}}-\frac{15\sqrt{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{13/4}b^{3/4}}-\frac{15\sqrt{d}\tan}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d*x)^(3/2)/(6*a*d*(a + b*x^2)^3) + (3*(d*x)^(3/2))/(16*a^2*d*(a + b*x^2)^2
) + (15*(d*x)^(3/2))/(64*a^3*d*(a + b*x^2)) - (15*Sqrt[d]*ArcTan[1 - (Sqrt[
2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(13/4)*b^(3/4)) +
(15*Sqrt[d]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128
*Sqrt[2]*a^(13/4)*b^(3/4)) + (15*Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt
[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(13/4)*b^(3/4))
- (15*Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(13/4)*b^(3/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{\left(3b^3\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^3}\,dx}{4a}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{\left(15b^2\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^2}\,dx}{32a^2}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} + \frac{(15b)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{128a^3}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} + \frac{(15b)\,\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,d \ldots\right.}{64a^3d}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} - \frac{\left(15\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{\ldots}}{ab+\frac{b^2 \ldots}{d \ldots}} \ldots\right.}{128a^3d}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} + \frac{\left(15\sqrt{d}\right)\text{Subst}\left(\int \frac{\sqrt{2} \ldots}{-\frac{\sqrt{a}\,d}{\sqrt{b}}- \ldots} \ldots\right.}{256\sqrt{2}\,a \ldots}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} + \frac{15\sqrt{d}\,\log\left(\sqrt{a}\sqrt{d}+\sqrt{b} \ldots\right.}{256\sqrt{2}\,a \ldots}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} - \frac{15\sqrt{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{\ldots}}{\sqrt[4]{a}\sqrt{\ldots}}\right.}{128\sqrt{2}\,a^{13/4}b^{3/4}}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.10

$$\frac{2x\sqrt{dx}\;{}_2F_1\left(\frac{3}{4},4;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*x*Sqrt[d*x]*Hypergeometric2F1[3/4, 4, 7/4, -((b*x^2)/a)])/(3*a^4)
```

fricas [A] time = 1.01, size = 359, normalized size = 1.07

$$-\frac{180\left(a^{3}b^{3}x^{6}+3\,a^{4}b^{2}x^{4}+3\,a^{5}bx^{2}+a^{6}\right)\left(-\frac{d^{2}}{a^{13}b^{3}}\right)^{\frac{1}{4}}\arctan\left(-\frac{3375\,\sqrt{dx}\,a^{3}bd\left(-\frac{d^{2}}{a^{13}b^{3}}\right)^{\frac{1}{4}}-\sqrt{-11390625\,a^{7}bd^{2}\sqrt{-\frac{d^{2}}{a^{13}b^{3}}}+11390625\cdots}}{3375\,d^{2}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(180*(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)*(-d^2/(a^13*b
^3))^(1/4)*arctan(-1/3375*(3375*sqrt(d*x)*a^3*b*d*(-d^2/(a^13*b^3))^(1/4) -
 sqrt(-11390625*a^7*b*d^2*sqrt(-d^2/(a^13*b^3)) + 11390625*d^3*x)*a^3*b*(-d
^2/(a^13*b^3))^(1/4))/d^2) - 45*(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2
+ a^6)*(-d^2/(a^13*b^3))^(1/4)*log(3375*a^10*b^2*(-d^2/(a^13*b^3))^(3/4) +
3375*sqrt(d*x)*d) + 45*(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)*(-
d^2/(a^13*b^3))^(1/4)*log(-3375*a^10*b^2*(-d^2/(a^13*b^3))^(3/4) + 3375*sqr
t(d*x)*d) - 4*(45*b^2*x^5 + 126*a*b*x^3 + 113*a^2*x)*sqrt(d*x))/(a^3*b^3*x^
6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)
```

giac [A] time = 0.22, size = 302, normalized size = 0.90

$$\frac{\frac{90\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{4}b^{3}}+\frac{90\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{4}b^{3}}-\frac{45\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\cdots}\right.}{a^{4}b^{3}}\cdots}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*(90*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^3) + 90*sqrt(2)*(a*b^3*d^2)^(3
/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(
1/4))/(a^4*b^3) - 45*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^
```

```
(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^3) + 45*sqrt(2)*(a*b^3*d^2)^(3/4)*l
og(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^3) + 8*(
45*sqrt(d*x)*b^2*d^7*x^5 + 126*sqrt(d*x)*a*b*d^7*x^3 + 113*sqrt(d*x)*a^2*d^
7*x)/((b*d^2*x^2 + a*d^2)^3*a^3))/d
```

maple [A] time = 0.02, size = 272, normalized size = 0.81

$$\frac{113 (dx)^{\frac{3}{2}} d^5}{192 \left(b d^2x^2 + d^2a\right)^3 a} + \frac{21 (dx)^{\frac{7}{2}} b d^3}{32 \left(b d^2x^2 + d^2a\right)^3 a^2} + \frac{15 (dx)^{\frac{11}{2}} b^2 d}{64 \left(b d^2x^2 + d^2a\right)^3 a^3} + \frac{15\sqrt{2}\, d \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}} - 1\right)}{256 \left(\frac{a d^2}{b}\right)^{\frac{1}{4}} a^3 b} + \frac{15\sqrt{2}\, d \operatorname{arc}\cdots}{256 \left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 15/64*d/(b*d^2*x^2+a*d^2)^3/a^3*b^2*(d*x)^(11/2)+21/32*d^3/(b*d^2*x^2+a*d^2
)^3/a^2*b*(d*x)^(7/2)+113/192*d^5/(b*d^2*x^2+a*d^2)^3/a*(d*x)^(3/2)+15/512*
d/a^3/b/(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))
)+15/256*d/a^3/b/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*
x)^(1/2)+1)+15/256*d/a^3/b/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)
^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 2.98, size = 317, normalized size = 0.95

$$\frac{\frac{8\left(45 (dx)^{\frac{11}{2}} b^2d^2 + 126 (dx)^{\frac{7}{2}} abd^4 + 113 (dx)^{\frac{3}{2}} a^2d^6\right)}{a^3b^3d^6x^6 + 3a^4b^2d^6x^4 + 3a^5bd^6x^2 + a^6d^6} + \frac{45 d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}} b^{\frac{1}{4}} + 2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}} b^{\frac{1}{4}} - 2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}} - \cdots\right.}{a^3 \cdots}}{1536\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*(8*(45*(d*x)^(11/2)*b^2*d^2 + 126*(d*x)^(7/2)*a*b*d^4 + 113*(d*x)^(3
/2)*a^2*d^6)/(a^3*b^3*d^6*x^6 + 3*a^4*b^2*d^6*x^4 + 3*a^5*b*d^6*x^2 + a^6*d
^6) + 45*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) +
 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqr
```

```
t(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sq
rt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b))
 - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt
(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^
(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/a^3)/d
```

mupad [B] time = 0.10, size = 150, normalized size = 0.45

$$\frac{\frac{113\,d^5\,(d\,x)^{3/2}}{192\,a}+\frac{21\,b\,d^3\,(d\,x)^{7/2}}{32\,a^2}+\frac{15\,b^2\,d\,(d\,x)^{11/2}}{64\,a^3}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}-\frac{15\,\sqrt{d}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{13/4}\,b^{3/4}}+\frac{15\,\sqrt{d}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{13/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((113*d^5*(d*x)^(3/2))/(192*a) + (21*b*d^3*(d*x)^(7/2))/(32*a^2) + (15*b^2*
d*(d*x)^(11/2))/(64*a^3))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 + 3*a*b^
2*d^6*x^4) - (15*d^(1/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/
(128*(-a)^(13/4)*b^(3/4)) + (15*d^(1/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(
1/4)*d^(1/2))))/(128*(-a)^(13/4)*b^(3/4))
```

sympy [A] time = 28.99, size = 252, normalized size = 0.75

$$\frac{226a^2d^{11}(dx)^{\frac{3}{2}}}{384a^6d^{12}+1152a^5bd^{12}x^2+1152a^4b^2d^{12}x^4+384a^3b^3d^{12}x^6}+\frac{252abd^9(dx)^{\frac{7}{2}}}{384a^6d^{12}+1152a^5bd^{12}x^2+1152a^4b^2d^{12}x^4+384a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 226*a**2*d**11*(d*x)**(3/2)/(384*a**6*d**12 + 1152*a**5*b*d**12*x**2 + 1152
*a**4*b**2*d**12*x**4 + 384*a**3*b**3*d**12*x**6) + 252*a*b*d**9*(d*x)**(7/
2)/(384*a**6*d**12 + 1152*a**5*b*d**12*x**2 + 1152*a**4*b**2*d**12*x**4 + 3
84*a**3*b**3*d**12*x**6) + 90*b**2*d**7*(d*x)**(11/2)/(384*a**6*d**12 + 115
2*a**5*b*d**12*x**2 + 1152*a**4*b**2*d**12*x**4 + 384*a**3*b**3*d**12*x**6)
 + 2*d**7*RootSum(68719476736*_t**4*a**13*b**3*d**26 + 50625, Lambda(_t, _t
*log(134217728*_t**3*a**10*b**2*d**20/3375 + sqrt(d*x))))
```

3.706 $\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=335

$$-\frac{77\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{15/4}\sqrt[4]{b}\sqrt{d}}+\frac{77\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{15/4}\sqrt[4]{b}\sqrt{d}}-\frac{77\tan^{-1}\left(1-\right.}{128\sqrt{2}\,a^{15/}}$$

```
[Out] -77/256*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(15/4)/b^(1
/4)*2^(1/2)/d^(1/2)+77/256*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(
1/2))/a^(15/4)/b^(1/4)*2^(1/2)/d^(1/2)-77/512*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*
d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(15/4)/b^(1/4)*2^(1/2)/d^(1/
2)+77/512*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x
)^(1/2))/a^(15/4)/b^(1/4)*2^(1/2)/d^(1/2)+1/6*(d*x)^(1/2)/a/d/(b*x^2+a)^3+1
1/48*(d*x)^(1/2)/a^2/d/(b*x^2+a)^2+77/192*(d*x)^(1/2)/a^3/d/(b*x^2+a)
```

Rubi [A] time = 0.35, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)}+\frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2}-\frac{77\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{15/4}\sqrt[4]{b}\sqrt{d}}+\frac{77\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right.}{256\sqrt{2}\,a^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] Sqrt[d*x]/(6*a*d*(a + b*x^2)^3) + (11*Sqrt[d*x])/(48*a^2*d*(a + b*x^2)^2) +
 (77*Sqrt[d*x])/(192*a^3*d*(a + b*x^2)) - (77*ArcTan[1 - (Sqrt[2]*b^(1/4)*S
qrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(15/4)*b^(1/4)*Sqrt[d]) + (77*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(
15/4)*b^(1/4)*Sqrt[d]) - (77*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(15/4)*b^(1/4)*Sqrt[d]) + (7
7*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(256*Sqrt[2]*a^(15/4)*b^(1/4)*Sqrt[d])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\,\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{\sqrt{dx}\,\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{\left(11b^3\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^3}\,dx}{12a}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{\left(77b^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^2}\,dx}{96a^2}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} + \frac{(77b)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right.}}{128a^3}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} + \frac{(77b)\,\mathrm{Subst}\left(\int \frac{1}{ab+}\right.}{64a^3}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} + \frac{(77b)\,\mathrm{Subst}\left(\int \frac{\sqrt{a}\,d}{ab-}\right.}{128a^7}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} + \frac{77\,\mathrm{Subst}\left(\int \frac{}{\frac{\sqrt{a}\,d}{\sqrt{b}}-}\right.}{256}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} - \frac{77\log\left(\sqrt{a}\,\sqrt{d}\,+\right.}{256\sqrt{2}}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} - \frac{77\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{128\sqrt{2}\,a^{15/4}\sqrt[4]{b}}$$

Mathematica [A] time = 0.11, size = 253, normalized size = 0.76

$$\frac{\sqrt{x}\left(\frac{256a^{11/4}\sqrt{x}}{\left(a+bx^2\right)^3}+\frac{352a^{7/4}\sqrt{x}}{\left(a+bx^2\right)^2}+\frac{616a^{3/4}\sqrt{x}}{a+bx^2}-\frac{231\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}+\frac{231\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}-\frac{462\sqrt{\cdots}}{\cdots}\right.}{1536a^{15/4}\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (Sqrt[x]*((256*a^(11/4)*Sqrt[x])/(a + b*x^2)^3 + (352*a^(7/4)*Sqrt[x])/(a +
 b*x^2)^2 + (616*a^(3/4)*Sqrt[x])/(a + b*x^2) - (462*Sqrt[2]*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) + (462*Sqrt[2]*ArcTan[1 + (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) - (231*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4) + (231*Sqrt[2]*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4)))/(1536*a^(15/4)*Sqrt[d
*x])
```

fricas [A] time = 1.09, size = 357, normalized size = 1.07

$$\frac{924\left(a^3b^3dx^6+3\,a^4b^2dx^4+3\,a^5bdx^2+a^6d\right)\left(-\frac{1}{a^{15}bd^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^8d^2\sqrt{-\frac{1}{a^{15}bd^2}}+dx}\,a^{11}bd\left(-\frac{1}{a^{15}bd^2}\right)^{\frac{3}{4}}-\sqrt{dx}\,a\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(924*(a^3*b^3*d*x^6 + 3*a^4*b^2*d*x^4 + 3*a^5*b*d*x^2 + a^6*d)*(-1/(a
^15*b*d^2))^(1/4)*arctan(sqrt(a^8*d^2*sqrt(-1/(a^15*b*d^2)) + d*x)*a^11*b*d
*(-1/(a^15*b*d^2))^(3/4) - sqrt(d*x)*a^11*b*d*(-1/(a^15*b*d^2))^(3/4)) + 23
1*(a^3*b^3*d*x^6 + 3*a^4*b^2*d*x^4 + 3*a^5*b*d*x^2 + a^6*d)*(-1/(a^15*b*d^2
))^(1/4)*log(a^4*d*(-1/(a^15*b*d^2))^(1/4) + sqrt(d*x)) - 231*(a^3*b^3*d*x^
6 + 3*a^4*b^2*d*x^4 + 3*a^5*b*d*x^2 + a^6*d)*(-1/(a^15*b*d^2))^(1/4)*log(-a
^4*d*(-1/(a^15*b*d^2))^(1/4) + sqrt(d*x)) + 4*(77*b^2*x^4 + 198*a*b*x^2 + 1
53*a^2)*sqrt(d*x))/(a^3*b^3*d*x^6 + 3*a^4*b^2*d*x^4 + 3*a^5*b*d*x^2 + a^6*d
)
```

giac [A] time = 0.27, size = 308, normalized size = 0.92

$$\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^4bd}+\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^4bd}+\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 77/256*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4
) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b*d) + 77/256*sqrt(2)*(a*b^3*d^2)^(1
/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(
1/4))/(a^4*b*d) + 77/512*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2
/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b*d) - 77/512*sqrt(2)*(a*b^3*d^2)
^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b*
d) + 1/192*(77*sqrt(d*x)*b^2*d^5*x^4 + 198*sqrt(d*x)*a*b*d^5*x^2 + 153*sqrt
(d*x)*a^2*d^5)/((b*d^2*x^2 + a*d^2)^3*a^3)
```

maple [A] time = 0.02, size = 269, normalized size = 0.80

$$\frac{51\sqrt{dx}\;d^5}{64\left(b\,d^2x^2+d^2a\right)^3a}+\frac{33\,(dx)^{\frac{5}{2}}\,b\,d^3}{32\left(b\,d^2x^2+d^2a\right)^3a^2}+\frac{77\,(dx)^{\frac{9}{2}}\,b^2d}{192\left(b\,d^2x^2+d^2a\right)^3a^3}+\frac{77\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\arctan\left(\frac{\sqrt{2}\;\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{256a^4d}+\frac{77\left(\frac{a\,d}{b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x)
```

```
[Out] 77/192*d/(b*d^2*x^2+a*d^2)^3/a^3*b^2*(d*x)^(9/2)+33/32*d^3/(b*d^2*x^2+a*d^2
)^3/a^2*b*(d*x)^(5/2)+51/64*d^5/(b*d^2*x^2+a*d^2)^3/a*(d*x)^(1/2)+77/512/d/
a^4*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/
b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+77
/256/d/a^4*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/
2)+1)+77/256/d/a^4*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(
d*x)^(1/2)-1)
```

maxima [A] time = 3.09, size = 322, normalized size = 0.96

$$\frac{\frac{8\left(77\,(dx)^{\frac{9}{2}}b^2d^2+198\,(dx)^{\frac{5}{2}}abd^4+153\,\sqrt{dx}\,a^2d^6\right)}{a^3b^3d^6x^6+3\,a^4b^2d^6x^4+3\,a^5bd^6x^2+a^6d^6}+\frac{231\left(\frac{\sqrt{2}\,d^2\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^2\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d\,\mathrm{arcta}\cdots}{\cdots}\right)}{a^3}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/1536*(8*(77*(d*x)^(9/2)*b^2*d^2 + 198*(d*x)^(5/2)*a*b*d^4 + 153*sqrt(d*x)
*a^2*d^6)/(a^3*b^3*d^6*x^6 + 3*a^4*b^2*d^6*x^4 + 3*a^5*b*d^6*x^2 + a^6*d^6)
 + 231*(sqrt(2)*d^2*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/
4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^2*log(sqrt(b)*d*x - sqr
t(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) +
 2*sqrt(2)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x
)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*s
qrt(2)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*s
qrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/a^3)/d
```

mupad [B] time = 4.28, size = 150, normalized size = 0.45

$$\frac{\frac{51\,d^5\,\sqrt{d\,x}}{64\,a}+\frac{33\,b\,d^3\,{(d\,x)}^{5/2}}{32\,a^2}+\frac{77\,b^2\,d\,{(d\,x)}^{9/2}}{192\,a^3}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}+\frac{77\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{128\,{(-a)}^{15/4}\,b^{1/4}\,\sqrt{d}}+\frac{77\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{128\,{(-a)}^{15/4}\,b^{1/4}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] ((51*d^5*(d*x)^(1/2))/(64*a) + (33*b*d^3*(d*x)^(5/2))/(32*a^2) + (77*b^2*d*
(d*x)^(9/2))/(192*a^3))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 + 3*a*b^2*
d^6*x^4) + (77*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-a)^
(15/4)*b^(1/4)*d^(1/2)) + (77*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/
2))))/(128*(-a)^(15/4)*b^(1/4)*d^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\,\left(a+bx^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*(a + b*x**2)**4), x)
```

3.707 $\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=352

$$-\frac{195\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{17/4}d^{3/2}}+\frac{195\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{17/4}d^{3/2}}+\frac{195}{}$$

```
[Out] 195/256*b^(1/4)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(17
/4)/d^(3/2)*2^(1/2)-195/256*b^(1/4)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^
(1/4)/d^(1/2))/a^(17/4)/d^(3/2)*2^(1/2)-195/512*b^(1/4)*ln(a^(1/2)*d^(1/2)+
x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(17/4)/d^(3/2)*2^(
1/2)+195/512*b^(1/4)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2
^(1/2)*(d*x)^(1/2))/a^(17/4)/d^(3/2)*2^(1/2)-195/64/a^4/d/(d*x)^(1/2)+1/6/a
/d/(b*x^2+a)^3/(d*x)^(1/2)+13/48/a^2/d/(b*x^2+a)^2/(d*x)^(1/2)+39/64/a^3/d/
(b*x^2+a)/(d*x)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{195\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{17/4}d^{3/2}}+\frac{195\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{17/4}d^{3/2}}+\frac{195}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -195/(64*a^4*d*Sqrt[d*x]) + 1/(6*a*d*Sqrt[d*x]*(a + b*x^2)^3) + 13/(48*a^2*
d*Sqrt[d*x]*(a + b*x^2)^2) + 39/(64*a^3*d*Sqrt[d*x]*(a + b*x^2)) + (195*b^(
1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2
]*a^(17/4)*d^(3/2)) - (195*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(
a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(17/4)*d^(3/2)) - (195*b^(1/4)*Log[Sqrt[a
]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sq
rt[2]*a^(17/4)*d^(3/2)) + (195*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d
]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(17/4)*d^(3/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{\left(13b^3\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^3}\,dx}{12a}$$

$$= \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{\left(39b^2\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^2}\,dx}{32a^2}$$

$$= \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{39}{64a^3d\sqrt{dx}\left(a+bx^2\right)} + \frac{(195}{}$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{39}{64a^3d\sqrt{dx}\,(}$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{39}{64a^3d\sqrt{dx}\,(}$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{39}{64a^3d\sqrt{dx}\,(}$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{39}{64a^3d\sqrt{dx}\,(}$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{39}{64a^3d\sqrt{dx}\,(}$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{39}{64a^3d\sqrt{dx}\,(}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.09

$$-\frac{2x\,{}_2F_1\left(-\frac{1}{4},4;\frac{3}{4};-\frac{bx^2}{a}\right)}{a^4(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (-2*x*Hypergeometric2F1[-1/4, 4, 3/4, -((b*x^2)/a)])/(a^4*(d*x)^(3/2))
```

fricas [A] time = 1.23, size = 410, normalized size = 1.16

$$\frac{2340\left(a^4b^3d^2x^7+3\,a^5b^2d^2x^5+3\,a^6bd^2x^3+a^7d^2x\right)\left(-\frac{b}{a^{17}d^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{7414875\,\sqrt{dx}\,a^4bd\left(-\frac{b}{a^{17}d^6}\right)^{\frac{1}{4}}-\sqrt{-54980371265625\,a \cdots}}{7414 \cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(2340*(a^4*b^3*d^2*x^7 + 3*a^5*b^2*d^2*x^5 + 3*a^6*b*d^2*x^3 + a^7*d^
2*x)*(-b/(a^17*d^6))^(1/4)*arctan(-1/7414875*(7414875*sqrt(d*x)*a^4*b*d*(-b
/(a^17*d^6))^(1/4) - sqrt(-54980371265625*a^9*b*d^4*sqrt(-b/(a^17*d^6)) + 5
4980371265625*b^2*d*x)*a^4*d*(-b/(a^17*d^6))^(1/4))/b) - 585*(a^4*b^3*d^2*x
^7 + 3*a^5*b^2*d^2*x^5 + 3*a^6*b*d^2*x^3 + a^7*d^2*x)*(-b/(a^17*d^6))^(1/4)
*log(7414875*a^13*d^5*(-b/(a^17*d^6))^(3/4) + 7414875*sqrt(d*x)*b) + 585*(a
^4*b^3*d^2*x^7 + 3*a^5*b^2*d^2*x^5 + 3*a^6*b*d^2*x^3 + a^7*d^2*x)*(-b/(a^17
*d^6))^(1/4)*log(-7414875*a^13*d^5*(-b/(a^17*d^6))^(3/4) + 7414875*sqrt(d*x
)*b) - 4*(585*b^3*x^6 + 1638*a*b^2*x^4 + 1469*a^2*b*x^2 + 384*a^3)*sqrt(d*x
))/(a^4*b^3*d^2*x^7 + 3*a^5*b^2*d^2*x^5 + 3*a^6*b*d^2*x^3 + a^7*d^2*x)
```

giac [A] time = 0.20, size = 327, normalized size = 0.93

$$-\frac{\frac{3072}{\sqrt{dx}\,a^4}+\frac{8\left(201\,\sqrt{dx}\,b^3d^5x^5+486\,\sqrt{dx}\,ab^2d^5x^3+317\,\sqrt{dx}\,a^2bd^5x\right)}{\left(bd^2x^2+ad^2\right)^3a^4}+\frac{1170\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^2d^2}+\frac{1170\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\,\text{ar}\cdots}{\cdots}}{1536\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/1536*(3072/(sqrt(d*x)*a^4) + 8*(201*sqrt(d*x)*b^3*d^5*x^5 + 486*sqrt(d*x
)*a*b^2*d^5*x^3 + 317*sqrt(d*x)*a^2*b*d^5*x)/((b*d^2*x^2 + a*d^2)^3*a^4) +
1170*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4)
+ 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^2*d^2) + 1170*sqrt(2)*(a*b^3*d^2)^(3
/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(
1/4))/(a^5*b^2*d^2) - 585*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^
2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b^2*d^2) + 585*sqrt(2)*(a*b^3*d^
2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*
b^2*d^2))/d
```

maple [A] time = 0.02, size = 285, normalized size = 0.81

$$-\frac{317\,(dx)^{\frac{3}{2}}\,b\,d^3}{192\left(b\,d^2x^2+d^2a\right)^3 a^2}-\frac{81\,(dx)^{\frac{7}{2}}\,b^2d}{32\left(b\,d^2x^2+d^2a\right)^3 a^3}-\frac{67\,(dx)^{\frac{11}{2}}\,b^3}{64\left(b\,d^2x^2+d^2a\right)^3 a^4d}-\frac{195\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)}{256\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}} a^4d}-\frac{195\sqrt{2}\,\mathrm{ar}}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -67/64/d*b^3/a^4/(b*d^2*x^2+a*d^2)^3*(d*x)^(11/2)-81/32*d*b^2/a^3/(b*d^2*x^
2+a*d^2)^3*(d*x)^(7/2)-317/192*d^3*b/a^2/(b*d^2*x^2+a*d^2)^3*(d*x)^(3/2)-19
5/512/d/a^4/(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(
1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1
/2)))-195/256/d/a^4/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*
(d*x)^(1/2)+1)-195/256/d/a^4/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^
2)^(1/4)*(d*x)^(1/2)-1)-2/a^4/d/(d*x)^(1/2)
```

maxima [A] time = 3.07, size = 328, normalized size = 0.93

$$-\frac{\frac{8\left(585\,b^3d^6x^6+1638\,ab^2d^6x^4+1469\,a^2bd^6x^2+384\,a^3d^6\right)}{(dx)^{\frac{13}{2}}a^4b^3+3\,(dx)^{\frac{9}{2}}a^5b^2d^2+3\,(dx)^{\frac{5}{2}}a^6bd^4+\sqrt{dx}\,a^7d^6}+\frac{585\,b\left(\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}\right.}{a^4}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/1536*(8*(585*b^3*d^6*x^6 + 1638*a*b^2*d^6*x^4 + 1469*a^2*b*d^6*x^2 + 384
*a^3*d^6)/((d*x)^(13/2)*a^4*b^3 + 3*(d*x)^(9/2)*a^5*b^2*d^2 + 3*(d*x)^(5/2)
*a^6*b*d^4 + sqrt(d*x)*a^7*d^6) + 585*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt
(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(
sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(
sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/
4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqr
t(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1
/4)*b^(3/4)))/a^4)/d
```

mupad [B] time = 0.14, size = 166, normalized size = 0.47

$$\frac{195\,(-b)^{1/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{128\,a^{17/4}\,d^{3/2}}-\frac{195\,(-b)^{1/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{128\,a^{17/4}\,d^{3/2}}-\frac{\frac{2\,d^5}{a}+\frac{1469\,b\,d^5\,x^2}{192\,a^2}+\frac{273\,b^2\,d^5\,x^4}{32\,a^3}+\frac{195\,b^3\,d^5\,x}{64\,a^4}}{b^3\,(d\,x)^{13/2}+a^3\,d^6\,\sqrt{d\,x}+3\,a^2\,b\,d^4\,(d\,x)^{5/2}+3\,a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] (195*(-b)^(1/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(128*a^(
17/4)*d^(3/2)) - (195*(-b)^(1/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(
1/2))))/(128*a^(17/4)*d^(3/2)) - ((2*d^5)/a + (1469*b*d^5*x^2)/(192*a^2) +
(273*b^2*d^5*x^4)/(32*a^3) + (195*b^3*d^5*x^6)/(64*a^4))/(b^3*(d*x)^(13/2)
+ a^3*d^6*(d*x)^(1/2) + 3*a^2*b*d^4*(d*x)^(5/2) + 3*a*b^2*d^2*(d*x)^(9/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(a+bx^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral(1/((d*x)**(3/2)*(a + b*x**2)**4), x)
```

3.708 $\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=352

$$\frac{385b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{19/4}d^{5/2}}-\frac{385b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{19/4}d^{5/2}}+\frac{385b^{3/4}}{}$$

```
[Out] -385/192/a^4/d/(d*x)^(3/2)+1/6/a/d/(d*x)^(3/2)/(b*x^2+a)^3+5/16/a^2/d/(d*x)
^(3/2)/(b*x^2+a)^2+55/64/a^3/d/(d*x)^(3/2)/(b*x^2+a)+385/256*b^(3/4)*arctan
(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(19/4)/d^(5/2)*2^(1/2)-38
5/256*b^(3/4)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(19/4
)/d^(5/2)*2^(1/2)+385/512*b^(3/4)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1
/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(19/4)/d^(5/2)*2^(1/2)-385/512*b^(3/4)*l
n(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^
(19/4)/d^(5/2)*2^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{385b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{19/4}d^{5/2}}-\frac{385b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{19/4}d^{5/2}}+\frac{385b^{3/4}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -385/(192*a^4*d*(d*x)^(3/2)) + 1/(6*a*d*(d*x)^(3/2)*(a + b*x^2)^3) + 5/(16*
a^2*d*(d*x)^(3/2)*(a + b*x^2)^2) + 55/(64*a^3*d*(d*x)^(3/2)*(a + b*x^2)) +
(385*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(12
8*Sqrt[2]*a^(19/4)*d^(5/2)) - (385*b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(19/4)*d^(5/2)) + (385*b^(3/4)*Lo
g[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])
/(256*Sqrt[2]*a^(19/4)*d^(5/2)) - (385*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b
]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(19/4)*d^(
5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{\left(5b^3\right)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^3}\,dx}{4a}$$

$$= \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{\left(55b^2\right)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^2}}{32a^2}$$

$$= \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{55}{64a^3d(dx)^{3/2}\left(a+bx^2\right)}$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{}{64a^3}$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{}{64a^3}$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{}{64a^3}$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{}{64a^3}$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{}{64a^3}$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{}{64a^3}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.09

$$-\frac{2x\,{}_2F_1\left(-\frac{3}{4},4;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^4(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (-2*x*Hypergeometric2F1[-3/4, 4, 1/4, -((b*x^2)/a)])/(3*a^4*(d*x)^(5/2))
```

fricas [A] time = 1.12, size = 434, normalized size = 1.23

$$-\frac{4620\left(a^4b^3d^3x^8+3a^5b^2d^3x^6+3a^6bd^3x^4+a^7d^3x^2\right)\left(-\frac{b^3}{a^{19}d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^{14}bd^7\left(-\frac{b^3}{a^{19}d^{10}}\right)^{\frac{3}{4}}-\sqrt{a^{10}d^6\sqrt{-\frac{b^3}{a^{19}d^{10}}}+b^2dx}\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(4620*(a^4*b^3*d^3*x^8 + 3*a^5*b^2*d^3*x^6 + 3*a^6*b*d^3*x^4 + a^7*d
^3*x^2)*(-b^3/(a^19*d^10))^(1/4)*arctan(-(sqrt(d*x)*a^14*b*d^7*(-b^3/(a^19*
d^10))^(3/4) - sqrt(a^10*d^6*sqrt(-b^3/(a^19*d^10)) + b^2*d*x)*a^14*d^7*(-b
^3/(a^19*d^10))^(3/4))/b^3) + 1155*(a^4*b^3*d^3*x^8 + 3*a^5*b^2*d^3*x^6 + 3
*a^6*b*d^3*x^4 + a^7*d^3*x^2)*(-b^3/(a^19*d^10))^(1/4)*log(385*a^5*d^3*(-b^
3/(a^19*d^10))^(1/4) + 385*sqrt(d*x)*b) - 1155*(a^4*b^3*d^3*x^8 + 3*a^5*b^2
*d^3*x^6 + 3*a^6*b*d^3*x^4 + a^7*d^3*x^2)*(-b^3/(a^19*d^10))^(1/4)*log(-385
*a^5*d^3*(-b^3/(a^19*d^10))^(1/4) + 385*sqrt(d*x)*b) + 4*(385*b^3*x^6 + 990
*a*b^2*x^4 + 765*a^2*b*x^2 + 128*a^3)*sqrt(d*x))/(a^4*b^3*d^3*x^8 + 3*a^5*b
^2*d^3*x^6 + 3*a^6*b*d^3*x^4 + a^7*d^3*x^2)
```

giac [A] time = 0.22, size = 308, normalized size = 0.88

$$-\frac{385\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256a^5d^3}-\frac{385\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256a^5d^3}-\frac{385\sqrt{2}\left(ab^3d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -385/256*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*d^3) - 385/256*sqrt(2)*(a*b^3*d^2)
^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b
)^(1/4))/(a^5*d^3) - 385/512*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a
*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*d^3) + 385/512*sqrt(2)*(a*b^3
*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a
^5*d^3) - 1/192*(385*b^3*d^6*x^6 + 990*a*b^2*d^6*x^4 + 765*a^2*b*d^6*x^2 +
128*a^3*d^6)/((sqrt(d*x)*b*d^2*x^2 + sqrt(d*x)*a*d^2)^3*a^4*d)
```

maple [A] time = 0.02, size = 288, normalized size = 0.82

$$-\frac{127\sqrt{dx}\,b\,d^3}{64\left(b\,d^2x^2+d^2a\right)^3a^2}-\frac{101\,(dx)^{\frac{5}{2}}\,b^2d}{32\left(b\,d^2x^2+d^2a\right)^3a^3}-\frac{257\,(dx)^{\frac{9}{2}}\,b^3}{192\left(b\,d^2x^2+d^2a\right)^3a^4d}-\frac{2}{3\,(dx)^{\frac{3}{2}}\,a^4d}-\frac{385\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\right.}{256a^5d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -257/192/d/a^4*b^3/(b*d^2*x^2+a*d^2)^3*(d*x)^(9/2)-101/32*d/a^3*b^2/(b*d^2*
x^2+a*d^2)^3*(d*x)^(5/2)-127/64*d^3/a^2*b/(b*d^2*x^2+a*d^2)^3*(d*x)^(1/2)-3
85/512/d^3/a^5*b*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2
)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^
2)^(1/2)))-385/256/d^3/a^5*b*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^
2)^(1/4)*(d*x)^(1/2)+1)-385/256/d^3/a^5*b*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)-2/3/a^4/d/(d*x)^(3/2)
```

maxima [A] time = 3.07, size = 335, normalized size = 0.95

$$-\frac{\frac{8\left(385\,b^3d^6x^6+990\,ab^2d^6x^4+765\,a^2bd^6x^2+128\,a^3d^6\right)}{(dx)^{\frac{15}{2}}a^4b^3+3\,(dx)^{\frac{11}{2}}a^5b^2d^2+3\,(dx)^{\frac{7}{2}}a^6bd^4+(dx)^{\frac{3}{2}}a^7d^6}+\frac{1155\left(\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}}\right.}{a}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/1536*(8*(385*b^3*d^6*x^6 + 990*a*b^2*d^6*x^4 + 765*a^2*b*d^6*x^2 + 128*a
^3*d^6)/((d*x)^(15/2)*a^4*b^3 + 3*(d*x)^(11/2)*a^5*b^2*d^2 + 3*(d*x)^(7/2)*
a^6*b*d^4 + (d*x)^(3/2)*a^7*d^6) + 1155*(sqrt(2)*b^(3/4)*log(sqrt(b)*d*x +
sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/(a*d^2)^(3/4) - sqrt(2
)*b^(3/4)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(
a)*d)/(a*d^2)^(3/4) + 2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)
*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt
(b)*d)*sqrt(a)*d) + 2*sqrt(2)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*
b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(
b)*d)*sqrt(a)*d))/a^4)/d
```

mupad [B] time = 4.25, size = 166, normalized size = 0.47

$$\frac{385\,{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{128\,a^{19/4}\,d^{5/2}}-\frac{\frac{2\,d^5}{3\,a}+\frac{255\,b\,d^5\,x^2}{64\,a^2}+\frac{165\,b^2\,d^5\,x^4}{32\,a^3}+\frac{385\,b^3\,d^5\,x^6}{192\,a^4}}{b^3\,{\left(d\,x\right)}^{15/2}+a^3\,d^6\,{\left(d\,x\right)}^{3/2}+3\,a^2\,b\,d^4\,{\left(d\,x\right)}^{7/2}+3\,a\,b^2\,d^2\,{\left(d\,x\right)}^{11/2}}+\frac{385\,{\left(-b\right)}^{3/4}\,\mathrm{atanh}}{128\,a^{19/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] (385*(-b)^(3/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(128*a^(1
9/4)*d^(5/2)) - ((2*d^5)/(3*a) + (255*b*d^5*x^2)/(64*a^2) + (165*b^2*d^5*x^
4)/(32*a^3) + (385*b^3*d^5*x^6)/(192*a^4))/(b^3*(d*x)^(15/2) + a^3*d^6*(d*x
)^(3/2) + 3*a^2*b*d^4*(d*x)^(7/2) + 3*a*b^2*d^2*(d*x)^(11/2)) + (385*(-b)^(
3/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(128*a^(19/4)*d^(5/
2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{5}{2}}\left(a+bx^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral(1/((d*x)**(5/2)*(a + b*x**2)**4), x)
```

3.709 $\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=370

$$\frac{663b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{21/4}d^{7/2}}-\frac{663b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{21/4}d^{7/2}}-\frac{663b^5}{}$$

```
[Out] -663/320/a^4/d/(d*x)^(5/2)+1/6/a/d/(d*x)^(5/2)/(b*x^2+a)^3+17/48/a^2/d/(d*x
)^(5/2)/(b*x^2+a)^2+221/192/a^3/d/(d*x)^(5/2)/(b*x^2+a)-663/256*b^(5/4)*arc
tan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(21/4)/d^(7/2)*2^(1/2)
+663/256*b^(5/4)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(2
1/4)/d^(7/2)*2^(1/2)+663/512*b^(5/4)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a
^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(21/4)/d^(7/2)*2^(1/2)-663/512*b^(5/4
)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))
/a^(21/4)/d^(7/2)*2^(1/2)+663/64*b/a^5/d^3/(d*x)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{663b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{21/4}d^{7/2}}-\frac{663b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{21/4}d^{7/2}}-\frac{663b^5}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -663/(320*a^4*d*(d*x)^(5/2)) + (663*b)/(64*a^5*d^3*Sqrt[d*x]) + 1/(6*a*d*(d
*x)^(5/2)*(a + b*x^2)^3) + 17/(48*a^2*d*(d*x)^(5/2)*(a + b*x^2)^2) + 221/(1
92*a^3*d*(d*x)^(5/2)*(a + b*x^2)) - (663*b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4
)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(21/4)*d^(7/2)) + (663*b^(5
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]
*a^(21/4)*d^(7/2)) + (663*b^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(21/4)*d^(7/2)) - (663*
b^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[d*x]])/(256*Sqrt[2]*a^(21/4)*d^(7/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{\left(17b^3\right)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^3}\,dx}{12a}$$

$$= \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+bx^2\right)^2} + \frac{\left(221b^2\right)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^2}}{96a^2}$$

$$= \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+bx^2\right)^2} + \frac{221}{192a^3d(dx)^{5/2}\left(a+bx^2\right)}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+bx^2\right)^2} + \frac{}{192a^3}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+\right.}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+\right.}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+\right.}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+\right.}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+\right.}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+\right.}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.10

$$-\frac{2\sqrt{dx}\ {}_2F_1\left(-\frac{5}{4},4;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^4d^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (-2*Sqrt[d*x]*Hypergeometric2F1[-5/4, 4, -1/4, -((b*x^2)/a)])/(5*a^4*d^4*x^
3)
```

fricas [A] time = 0.99, size = 457, normalized size = 1.24

$$-\frac{39780\left(a^5b^3d^4x^9+3\,a^6b^2d^4x^7+3\,a^7bd^4x^5+a^8d^4x^3\right)\left(-\frac{b^5}{a^{21}d^{14}}\right)^{\frac{1}{4}}\arctan\left(-\frac{291434247\,\sqrt{dx}\,a^5b^4d^3\left(-\frac{b^5}{a^{21}d^{14}}\right)^{\frac{1}{4}}-\sqrt{-849339\cdots}}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3840*(39780*(a^5*b^3*d^4*x^9 + 3*a^6*b^2*d^4*x^7 + 3*a^7*b*d^4*x^5 + a^8
*d^4*x^3)*(-b^5/(a^21*d^14))^(1/4)*arctan(-1/291434247*(291434247*sqrt(d*x)
*a^5*b^4*d^3*(-b^5/(a^21*d^14))^(1/4) - sqrt(-84933920324457009*a^11*b^5*d^
8*sqrt(-b^5/(a^21*d^14)) + 84933920324457009*b^8*d*x)*a^5*d^3*(-b^5/(a^21*d
^14))^(1/4))/b^5) - 9945*(a^5*b^3*d^4*x^9 + 3*a^6*b^2*d^4*x^7 + 3*a^7*b*d^4
*x^5 + a^8*d^4*x^3)*(-b^5/(a^21*d^14))^(1/4)*log(291434247*a^16*d^11*(-b^5/
(a^21*d^14))^(3/4) + 291434247*sqrt(d*x)*b^4) + 9945*(a^5*b^3*d^4*x^9 + 3*a
^6*b^2*d^4*x^7 + 3*a^7*b*d^4*x^5 + a^8*d^4*x^3)*(-b^5/(a^21*d^14))^(1/4)*lo
g(-291434247*a^16*d^11*(-b^5/(a^21*d^14))^(3/4) + 291434247*sqrt(d*x)*b^4)
- 4*(9945*b^4*x^8 + 27846*a*b^3*x^6 + 24973*a^2*b^2*x^4 + 6528*a^3*b*x^2 -
384*a^4)*sqrt(d*x))/(a^5*b^3*d^4*x^9 + 3*a^6*b^2*d^4*x^7 + 3*a^7*b*d^4*x^5
+ a^8*d^4*x^3)
```

giac [A] time = 0.22, size = 349, normalized size = 0.94

$$\frac{663\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^6bd^5}+\frac{663\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^6bd^5}-\frac{663\sqrt{2}\left(ab^3d\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 663/256*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*b*d^5) + 663/256*sqrt(2)*(a*b^3*d^2
)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/
b)^(1/4))/(a^6*b*d^5) - 663/512*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)
*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b*d^5) + 663/512*sqrt(2)*(
a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b
))/(a^6*b*d^5) + 1/192*(453*sqrt(d*x)*b^4*d^5*x^5 + 1038*sqrt(d*x)*a*b^3*d^
5*x^3 + 617*sqrt(d*x)*a^2*b^2*d^5*x)/((b*d^2*x^2 + a*d^2)^3*a^5*d^3) + 2/5*
(20*b*d^2*x^2 - a*d^2)/(sqrt(d*x)*a^5*d^5*x^2)
```

maple [A] time = 0.03, size = 304, normalized size = 0.82

$$\frac{617 (dx)^{\frac{3}{2}} b^2 d}{192 \left(b d^2 x^2 + d^2 a\right)^3 a^3} + \frac{173 (dx)^{\frac{7}{2}} b^3}{32 \left(b d^2 x^2 + d^2 a\right)^3 a^4 d} + \frac{151 (dx)^{\frac{11}{2}} b^4}{64 \left(b d^2 x^2 + d^2 a\right)^3 a^5 d^3} - \frac{2}{5 (dx)^{\frac{5}{2}} a^4 d} + \frac{663 \sqrt{2}\, b \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}} - \right.}{256 \left(\frac{a d^2}{b}\right)^{\frac{1}{4}} a^5 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 151/64/d^3*b^4/a^5/(b*d^2*x^2+a*d^2)^3*(d*x)^(11/2)+173/32/d*b^3/a^4/(b*d^2
*x^2+a*d^2)^3*(d*x)^(7/2)+617/192*d*b^2/a^3/(b*d^2*x^2+a*d^2)^3*(d*x)^(3/2)
+663/512/d^3*b/a^5/(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1
/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*
d^2)^(1/2)))+663/256/d^3*b/a^5/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*
d^2)^(1/4)*(d*x)^(1/2)+1)+663/256/d^3*b/a^5/(a/b*d^2)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)-2/5/a^4/d/(d*x)^(5/2)+8*b/a^5/d^3/(d
*x)^(1/2)
```

maxima [A] time = 3.17, size = 350, normalized size = 0.95

$$\frac{\frac{8\left(9945\,b^4d^8x^8+27846\,ab^3d^8x^6+24973\,a^2b^2d^8x^4+6528\,a^3bd^8x^2-384\,a^4d^8\right)}{(dx)^{\frac{17}{2}}a^5b^3d^2+3\,(dx)^{\frac{13}{2}}a^6b^2d^4+3\,(dx)^{\frac{9}{2}}a^7bd^6+(dx)^{\frac{5}{2}}a^8d^8}+\frac{9945\,b^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\ldots\right)}{\ldots}\ldots\right)}{\ldots}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/7680*(8*(9945*b^4*d^8*x^8 + 27846*a*b^3*d^8*x^6 + 24973*a^2*b^2*d^8*x^4 +
 6528*a^3*b*d^8*x^2 - 384*a^4*d^8)/((d*x)^(17/2)*a^5*b^3*d^2 + 3*(d*x)^(13/
2)*a^6*b^2*d^4 + 3*(d*x)^(9/2)*a^7*b*d^6 + (d*x)^(5/2)*a^8*d^8) + 9945*b^2*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*
sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqr
t(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(
b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*lo
g(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^
2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*
x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^5*d^2))/d
```

mupad [B] time = 4.33, size = 179, normalized size = 0.48

$$\frac{\frac{34\,b\,d^5\,x^2}{5\,a^2}-\frac{2\,d^5}{5\,a}+\frac{24973\,b^2\,d^5\,x^4}{960\,a^3}+\frac{4641\,b^3\,d^5\,x^6}{160\,a^4}+\frac{663\,b^4\,d^5\,x^8}{64\,a^5}}{b^3\,(d\,x)^{17/2}+a^3\,d^6\,(d\,x)^{5/2}+3\,a^2\,b\,d^4\,(d\,x)^{9/2}+3\,a\,b^2\,d^2\,(d\,x)^{13/2}}-\frac{663\,(-b)^{5/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{128\,a^{21/4}\,d^{7/2}}+\frac{663\,(-b)^{5/4}\,\mathrm{atan}\ldots}{128\,a^{21\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] ((34*b*d^5*x^2)/(5*a^2) - (2*d^5)/(5*a) + (24973*b^2*d^5*x^4)/(960*a^3) + (
4641*b^3*d^5*x^6)/(160*a^4) + (663*b^4*d^5*x^8)/(64*a^5))/(b^3*(d*x)^(17/2)
 + a^3*d^6*(d*x)^(5/2) + 3*a^2*b*d^4*(d*x)^(9/2) + 3*a*b^2*d^2*(d*x)^(13/2)
) - (663*(-b)^(5/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(128*
a^(21/4)*d^(7/2)) + (663*(-b)^(5/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)
*d^(1/2))))/(128*a^(21/4)*d^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{7}{2}} \left(a + bx^2\right)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral(1/((d*x)**(7/2)*(a + b*x**2)**4), x)
```

3.710 $\int \frac{(dx)^{27/2}}{\left(a^2+2abx^2+b^2x^4\right)^3} dx$

Optimal. Leaf size=420

$$-\frac{69615a^{5/4}d^{27/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}b^{29/4}}+\frac{69615a^{5/4}d^{27/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{\right.}}{16384\sqrt{2}b^{29/4}}$$

```
[Out] 13923/4096*d^11*(d*x)^(5/2)/b^6-1/10*d*(d*x)^(25/2)/b/(b*x^2+a)^5-5/32*d^3*
(d*x)^(21/2)/b^2/(b*x^2+a)^4-35/128*d^5*(d*x)^(17/2)/b^3/(b*x^2+a)^3-595/10
24*d^7*(d*x)^(13/2)/b^4/(b*x^2+a)^2-7735/4096*d^9*(d*x)^(9/2)/b^5/(b*x^2+a)
-69615/16384*a^(5/4)*d^(27/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/
d^(1/2))/b^(29/4)*2^(1/2)+69615/16384*a^(5/4)*d^(27/2)*arctan(1+b^(1/4)*2^(
1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(29/4)*2^(1/2)-69615/32768*a^(5/4)*d^(2
7/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/
2))/b^(29/4)*2^(1/2)+69615/32768*a^(5/4)*d^(27/2)*ln(a^(1/2)*d^(1/2)+x*b^(1
/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(29/4)*2^(1/2)-69615/409
6*a*d^13*(d*x)^(1/2)/b^7
```

Rubi [A] time = 0.53, antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{69615a^{5/4}d^{27/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}b^{29/4}}+\frac{69615a^{5/4}d^{27/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{\right.}}{16384\sqrt{2}b^{29/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(27/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-69615*a*d^13*Sqrt[d*x])/(4096*b^7) + (13923*d^11*(d*x)^(5/2))/(4096*b^6)
- (d*(d*x)^(25/2))/(10*b*(a + b*x^2)^5) - (5*d^3*(d*x)^(21/2))/(32*b^2*(a +
 b*x^2)^4) - (35*d^5*(d*x)^(17/2))/(128*b^3*(a + b*x^2)^3) - (595*d^7*(d*x)
^(13/2))/(1024*b^4*(a + b*x^2)^2) - (7735*d^9*(d*x)^(9/2))/(4096*b^5*(a + b
*x^2)) - (69615*a^(5/4)*d^(27/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(29/4)) + (69615*a^(5/4)*d^(27/2)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(29/4))
- (69615*a^(5/4)*d^(27/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*b^(29/4)) + (69615*a^(5/4)*d^(2
7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d*x]])/(16384*Sqrt[2]*b^(29/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{27/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{27/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{4}\left(5b^4d^2\right)\int \frac{(dx)^{23/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} + \frac{1}{64}\left(105b^2d^4\right)\int \frac{(dx)^{19/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5(dx)^{17/2}}{128b^3\left(a+bx^2\right)^3} + \frac{1}{256}\left(595d^6\right)\int \frac{(dx)^{15/\cdots}}{\left(ab+b^2x\cdots\right.}$$

$$= -\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5(dx)^{17/2}}{128b^3\left(a+bx^2\right)^3} - \frac{595d^7(dx)^{13/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{(77\cdots}{\cdots}$$

$$= -\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5(dx)^{17/2}}{128b^3\left(a+bx^2\right)^3} - \frac{595d^7(dx)^{13/2}}{1024b^4\left(a+bx^2\right)^2} - \frac{77\cdots}{409\cdots}$$

$$= \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5(dx)^{17/2}}{128b^3\left(a+bx^2\right)^3} - \frac{595d^7\cdots}{1024b^4\cdots}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5\cdots}{128b^3\cdots}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5\cdots}{128b^3\cdots}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5\cdots}{128b^3\cdots}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5\cdots}{128b^3\cdots}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5\cdots}{128b^3\cdots}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5\cdots}{128b^3\cdots}$$

Mathematica [A] time = 0.18, size = 432, normalized size = 1.03

$$d^{13}\sqrt{dx}\left(-3828825\sqrt{2}\,a^{5/4}\left(a+bx^2\right)^5\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+3828825\sqrt{2}\,a^{5/4}\left(a+bx^2\right)^5\log\left(\sqrt{2}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(27/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^13*Sqrt[d*x]*(-54312960*a^6*b^(1/4)*Sqrt[x] - 217251840*a^5*b^(5/4)*x^(5
/2) - 362086400*a^4*b^(9/4)*x^(9/2) - 306380800*a^3*b^(13/4)*x^(13/2) - 126
156800*a^2*b^(17/4)*x^(17/2) - 18022400*a*b^(21/4)*x^(21/2) + 720896*b^(25/
4)*x^(25/2) + 3394560*a^5*b^(1/4)*Sqrt[x]*(a + b*x^2) + 4243200*a^4*b^(1/4)
*Sqrt[x]*(a + b*x^2)^2 + 5834400*a^3*b^(1/4)*Sqrt[x]*(a + b*x^2)^3 + 102102
00*a^2*b^(1/4)*Sqrt[x]*(a + b*x^2)^4 - 7657650*Sqrt[2]*a^(5/4)*(a + b*x^2)^
5*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 7657650*Sqrt[2]*a^(5/4)*(
a + b*x^2)^5*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 3828825*Sqrt[2
]*a^(5/4)*(a + b*x^2)^5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x] + 3828825*Sqrt[2]*a^(5/4)*(a + b*x^2)^5*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(1802240*b^(29/4)*Sqrt[x]*(a + b*x^2)^5)
```

fricas [A] time = 0.91, size = 515, normalized size = 1.23

$$1392300\left(-\frac{a^5d^{54}}{b^{29}}\right)^{\frac{1}{4}}\left(b^{12}x^{10}+5\,ab^{11}x^8+10\,a^2b^{10}x^6+10\,a^3b^9x^4+5\,a^4b^8x^2+a^5b^7\right)\arctan\left(-\frac{\left(-\frac{a^5d^{54}}{b^{29}}\right)^{\frac{3}{4}}\sqrt{dx}\,ab^{22}d^{13}-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(27/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(1392300*(-a^5*d^54/b^29)^(1/4)*(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*
b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7)*arctan(-((-a^5*d^54/b^
29)^(3/4)*sqrt(d*x)*a*b^22*d^13 - (-a^5*d^54/b^29)^(3/4)*sqrt(a^2*d^27*x +
sqrt(-a^5*d^54/b^29)*b^14)*b^22)/(a^5*d^54)) + 348075*(-a^5*d^54/b^29)^(1/4
)*(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*
x^2 + a^5*b^7)*log(69615*sqrt(d*x)*a*d^13 + 69615*(-a^5*d^54/b^29)^(1/4)*b^
7) - 348075*(-a^5*d^54/b^29)^(1/4)*(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*
x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7)*log(69615*sqrt(d*x)*a*d^13
- 69615*(-a^5*d^54/b^29)^(1/4)*b^7) + 4*(8192*b^6*d^13*x^12 - 204800*a*b^5*
d^13*x^10 - 1317575*a^2*b^4*d^13*x^8 - 2951200*a^3*b^3*d^13*x^6 - 3171350*a
^4*b^2*d^13*x^4 - 1670760*a^5*b*d^13*x^2 - 348075*a^6*d^13)*sqrt(d*x))/(b^1
```

```
2*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 +
a^5*b^7)
```

giac [A] time = 0.24, size = 374, normalized size = 0.89

$$\frac{1}{163840}d^{13}\left(\frac{696150\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^8}+\frac{696150\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-\right.}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right.}{b^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(27/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^13*(696150*sqrt(2)*(a*b^3*d^2)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(
2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^8 + 696150*sqrt(2)*(a*
b^3*d^2)^(1/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x)
)/(a*d^2/b)^(1/4))/b^8 + 348075*sqrt(2)*(a*b^3*d^2)^(1/4)*a*log(d*x + sqrt(
2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^8 - 348075*sqrt(2)*(a*b^3*d
^2)^(1/4)*a*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^
8 - 8*(170695*sqrt(d*x)*a^2*b^4*d^10*x^8 + 575520*sqrt(d*x)*a^3*b^3*d^10*x^
6 + 754710*sqrt(d*x)*a^4*b^2*d^10*x^4 + 450152*sqrt(d*x)*a^5*b*d^10*x^2 + 1
02315*sqrt(d*x)*a^6*d^10)/((b*d^2*x^2 + a*d^2)^5*b^7) + 65536*(sqrt(d*x)*b^
24*d^10*x^2 - 30*sqrt(d*x)*a*b^23*d^10)/(b^30*d^10))
```

maple [A] time = 0.03, size = 370, normalized size = 0.88

$$-\frac{20463\sqrt{dx}\,a^6d^{23}}{4096\left(b\,d^2x^2+d^2a\right)^5b^7}-\frac{56269\,(dx)^{\frac{5}{2}}\,a^5d^{21}}{2560\left(b\,d^2x^2+d^2a\right)^5b^6}-\frac{75471\,(dx)^{\frac{9}{2}}\,a^4d^{19}}{2048\left(b\,d^2x^2+d^2a\right)^5b^5}-\frac{3597\,(dx)^{\frac{13}{2}}\,a^3d^{17}}{128\left(b\,d^2x^2+d^2a\right)^5b^4}-\frac{34139\,(d}{4096\left(b\,d^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(27/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 2/5*d^11*(d*x)^(5/2)/b^6-12*a*d^13*(d*x)^(1/2)/b^7-20463/4096*d^23/b^7*a^6/
(b*d^2*x^2+a*d^2)^5*(d*x)^(1/2)-56269/2560*d^21/b^6*a^5/(b*d^2*x^2+a*d^2)^5
```

```
*(d*x)^(5/2)-75471/2048*d^19/b^5*a^4/(b*d^2*x^2+a*d^2)^5*(d*x)^(9/2)-3597/1
28*d^17/b^4*a^3/(b*d^2*x^2+a*d^2)^5*(d*x)^(13/2)-34139/4096*d^15/b^3*a^2/(b
*d^2*x^2+a*d^2)^5*(d*x)^(17/2)+69615/32768*d^13/b^7*a*(a/b*d^2)^(1/4)*2^(1/
2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d
^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+69615/16384*d^13/b^7*a*(a/b
*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+69615/163
84*d^13/b^7*a*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^
(1/2)-1)
```

maxima [A] time = 3.18, size = 421, normalized size = 1.00

$$-\frac{\frac{8\left(170695\,(dx)^{\frac{17}{2}}a^{2}b^{4}d^{16}+575520\,(dx)^{\frac{13}{2}}a^{3}b^{3}d^{18}+754710\,(dx)^{\frac{9}{2}}a^{4}b^{2}d^{20}+450152\,(dx)^{\frac{5}{2}}a^{5}bd^{22}+102315\,\sqrt{dx}\,a^{6}d^{24}\right)}{b^{12}d^{10}x^{10}+5\,ab^{11}d^{10}x^{8}+10\,a^{2}b^{10}d^{10}x^{6}+10\,a^{3}b^{9}d^{10}x^{4}+5\,a^{4}b^{8}d^{10}x^{2}+a^{5}b^{7}d^{10}}-\frac{348075\left(\frac{\sqrt{2}d^{16}\log\left(\sqrt{b}\,dx+\sqrt{\cdots}\right.}{(ad^{2}\cdots}\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(27/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/163840*(8*(170695*(d*x)^(17/2)*a^2*b^4*d^16 + 575520*(d*x)^(13/2)*a^3*b^
3*d^18 + 754710*(d*x)^(9/2)*a^4*b^2*d^20 + 450152*(d*x)^(5/2)*a^5*b*d^22 +
102315*sqrt(d*x)*a^6*d^24)/(b^12*d^10*x^10 + 5*a*b^11*d^10*x^8 + 10*a^2*b^1
0*d^10*x^6 + 10*a^3*b^9*d^10*x^4 + 5*a^4*b^8*d^10*x^2 + a^5*b^7*d^10) - 348
075*(sqrt(2)*d^16*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
 + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^16*log(sqrt(b)*d*x - sqrt
(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) +
2*sqrt(2)*d^15*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d
*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2
*sqrt(2)*d^15*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d
*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))*a^
2/b^7 - 65536*((d*x)^(5/2)*b*d^12 - 30*sqrt(d*x)*a*d^14)/b^7)/d
```

mupad [B] time = 4.40, size = 248, normalized size = 0.59

$$\frac{2\,d^{11}\,(d\,x)^{5/2}}{5\,b^{6}}-\frac{\frac{20463\,a^{6}\,d^{23}\,\sqrt{d\,x}}{4096}+\frac{75471\,a^{4}\,b^{2}\,d^{19}\,(d\,x)^{9/2}}{2048}+\frac{3597\,a^{3}\,b^{3}\,d^{17}\,(d\,x)^{13/2}}{128}+\frac{34139\,a^{2}\,b^{4}\,d^{15}\,(d\,x)^{17/2}}{4096}+\frac{56269\,a^{5}\,b\,d^{21}\,(d\,x)^{5/2}}{2560}\cdots}{a^{5}\,b^{7}\,d^{10}+5\,a^{4}\,b^{8}\,d^{10}\,x^{2}+10\,a^{3}\,b^{9}\,d^{10}\,x^{4}+10\,a^{2}\,b^{10}\,d^{10}\,x^{6}+5\,a\,b^{11}\,d^{10}\,x^{8}+b^{12}\,d^{10}\,x^{10}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(27/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (2*d^11*(d*x)^(5/2))/(5*b^6) - ((20463*a^6*d^23*(d*x)^(1/2))/4096 + (75471*
a^4*b^2*d^19*(d*x)^(9/2))/2048 + (3597*a^3*b^3*d^17*(d*x)^(13/2))/128 + (34
139*a^2*b^4*d^15*(d*x)^(17/2))/4096 + (56269*a^5*b*d^21*(d*x)^(5/2))/2560)/
(a^5*b^7*d^10 + b^12*d^10*x^10 + 5*a*b^11*d^10*x^8 + 5*a^4*b^8*d^10*x^2 + 1
0*a^3*b^9*d^10*x^4 + 10*a^2*b^10*d^10*x^6) - (69615*(-a)^(5/4)*d^(27/2)*ata
n((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*b^(29/4)) + ((-a)^(5/4
)*d^(27/2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*69615i)/(819
2*b^(29/4)) - (12*a*d^13*(d*x)^(1/2))/b^7
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(27/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

3.711 $\int \frac{(dx)^{25/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=402

$$-\frac{33649a^{3/4}d^{25/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,b^{27/4}}+\frac{33649a^{3/4}d^{25/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\cdots\right.}{16384\sqrt{2}\,b^{27/4}}$$

```
[Out] 33649/12288*d^11*(d*x)^(3/2)/b^6-1/10*d*(d*x)^(23/2)/b/(b*x^2+a)^5-23/160*d
^3*(d*x)^(19/2)/b^2/(b*x^2+a)^4-437/1920*d^5*(d*x)^(15/2)/b^3/(b*x^2+a)^3-4
37/1024*d^7*(d*x)^(11/2)/b^4/(b*x^2+a)^2-4807/4096*d^9*(d*x)^(7/2)/b^5/(b*x
^2+a)+33649/16384*a^(3/4)*d^(25/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(
1/4)/d^(1/2))/b^(27/4)*2^(1/2)-33649/16384*a^(3/4)*d^(25/2)*arctan(1+b^(1/4
)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(27/4)*2^(1/2)-33649/32768*a^(3/4)
*d^(25/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x
)^(1/2))/b^(27/4)*2^(1/2)+33649/32768*a^(3/4)*d^(25/2)*ln(a^(1/2)*d^(1/2)+x
*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(27/4)*2^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 288, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{33649a^{3/4}d^{25/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,b^{27/4}}+\frac{33649a^{3/4}d^{25/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\cdots\right.}{16384\sqrt{2}\,b^{27/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(25/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (33649*d^11*(d*x)^(3/2))/(12288*b^6) - (d*(d*x)^(23/2))/(10*b*(a + b*x^2)^5
) - (23*d^3*(d*x)^(19/2))/(160*b^2*(a + b*x^2)^4) - (437*d^5*(d*x)^(15/2))/
(1920*b^3*(a + b*x^2)^3) - (437*d^7*(d*x)^(11/2))/(1024*b^4*(a + b*x^2)^2)
- (4807*d^9*(d*x)^(7/2))/(4096*b^5*(a + b*x^2)) + (33649*a^(3/4)*d^(25/2)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(
27/4)) - (33649*a^(3/4)*d^(25/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(27/4)) - (33649*a^(3/4)*d^(25/2)*Log[Sqrt
[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(1638
4*Sqrt[2]*b^(27/4)) + (33649*a^(3/4)*d^(25/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]
*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*b^(27/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{25/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{25/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(23b^4d^2\right)\int \frac{(dx)^{21/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{320}\left(437b^2d^4\right)\int \frac{(dx)^{17/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{1}{256}\left(437d^6\right)\int \frac{(dx)}{(ab+}$$

$$= -\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{437d^7(dx)^{11/2}}{1024b^4\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{437d^7(dx)^{11/2}}{1024b^4\left(a+bx^2\right)^2} -$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6} - \frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{437}{1024}$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6} - \frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{437}{1024}$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6} - \frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{437}{1024}$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6} - \frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{437}{1024}$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6} - \frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{437}{1024}$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6} - \frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{437}{1024}$$

Mathematica [C] time = 0.03, size = 109, normalized size = 0.27

$$-\frac{2d^{12}x\sqrt{dx}\left(-168245a^5-408595a^4bx^2-482885a^3b^2x^4-289731a^2b^3x^6-76245ab^4x^8+168245\left(a+bx^2\right)^5{}_2\right.}{9945b^6\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(25/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-2*d^12*x*Sqrt[d*x]*(-168245*a^5 - 408595*a^4*b*x^2 - 482885*a^3*b^2*x^4 -
 289731*a^2*b^3*x^6 - 76245*a*b^4*x^8 - 3315*b^5*x^10 + 168245*(a + b*x^2)^
5*Hypergeometric2F1[3/4, 6, 7/4, -((b*x^2)/a)]))/(9945*b^6*(a + b*x^2)^5)
```

fricas [A] time = 1.13, size = 515, normalized size = 1.28

$$\frac{2018940\left(-\frac{a^3d^{50}}{b^{27}}\right)^{\frac{1}{4}}\left(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6\right)\arctan\left(-\frac{\left(-\frac{a^3d^{50}}{b^{27}}\right)^{\frac{1}{4}}\sqrt{dx}\,a^2b^7d^{37}-\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(25/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/245760*(2018940*(-a^3*d^50/b^27)^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2
*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)*arctan(-((-a^3*d^50/b^
27)^(1/4)*sqrt(d*x)*a^2*b^7*d^37 - sqrt(a^4*d^75*x - sqrt(-a^3*d^50/b^27)*a
^3*b^13*d^50)*(-a^3*d^50/b^27)^(1/4)*b^7)/(a^3*d^50)) - 504735*(-a^3*d^50/b
^27)^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*
a^4*b^7*x^2 + a^5*b^6)*log(38099255258449*sqrt(d*x)*a^2*d^37 + 380992552584
49*(-a^3*d^50/b^27)^(3/4)*b^20) + 504735*(-a^3*d^50/b^27)^(1/4)*(b^11*x^10
+ 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)
*log(38099255258449*sqrt(d*x)*a^2*d^37 - 38099255258449*(-a^3*d^50/b^27)^(3
/4)*b^20) + 4*(40960*b^5*d^12*x^11 + 437345*a*b^4*d^12*x^9 + 1157176*a^2*b^
3*d^12*x^7 + 1367810*a^3*b^2*d^12*x^5 + 769120*a^4*b*d^12*x^3 + 168245*a^5*
d^12*x)*sqrt(d*x))/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*
x^4 + 5*a^4*b^7*x^2 + a^5*b^6)
```

giac [A] time = 0.22, size = 354, normalized size = 0.88

$$\frac{1}{491520}d^{12}\left(\frac{327680\sqrt{dx}\,x}{b^6}-\frac{1009470\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^9d}-\frac{1009470\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan}{b^9d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(25/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/491520*d^12*(327680*sqrt(d*x)*x/b^6 - 1009470*sqrt(2)*(a*b^3*d^2)^(3/4)*a
rctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/
(b^9*d) - 1009470*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^9*d) + 504735*sqrt(2)*(a*b
^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/
(b^9*d) - 504735*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4
)*sqrt(d*x) + sqrt(a*d^2/b))/(b^9*d) + 8*(232545*sqrt(d*x)*a*b^4*d^10*x^9 +
 747576*sqrt(d*x)*a^2*b^3*d^10*x^7 + 958210*sqrt(d*x)*a^3*b^2*d^10*x^5 + 56
4320*sqrt(d*x)*a^4*b*d^10*x^3 + 127285*sqrt(d*x)*a^5*d^10*x)/((b*d^2*x^2 +
a*d^2)^5*b^6))
```

maple [A] time = 0.03, size = 354, normalized size = 0.88

$$\frac{25457\,(dx)^{\frac{3}{2}}\,a^5d^{21}}{12288\left(b\,d^2x^2+d^2a\right)^5b^6}+\frac{3527\,(dx)^{\frac{7}{2}}\,a^4d^{19}}{384\left(b\,d^2x^2+d^2a\right)^5b^5}+\frac{95821\,(dx)^{\frac{11}{2}}\,a^3d^{17}}{6144\left(b\,d^2x^2+d^2a\right)^5b^4}+\frac{31149\,(dx)^{\frac{15}{2}}\,a^2d^{15}}{2560\left(b\,d^2x^2+d^2a\right)^5b^3}+\frac{15503\,(d}{4096\left(b\,d^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(25/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 2/3*d^11*(d*x)^(3/2)/b^6+25457/12288*d^21*a^5/b^6/(b*d^2*x^2+a*d^2)^5*(d*x)
^(3/2)+3527/384*d^19*a^4/b^5/(b*d^2*x^2+a*d^2)^5*(d*x)^(7/2)+95821/6144*d^1
7*a^3/b^4/(b*d^2*x^2+a*d^2)^5*(d*x)^(11/2)+31149/2560*d^15*a^2/b^3/(b*d^2*x
^2+a*d^2)^5*(d*x)^(15/2)+15503/4096*d^13*a/b^2/(b*d^2*x^2+a*d^2)^5*(d*x)^(1
```

```
9/2)-33649/32768*d^13*a/b^7/(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)
*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a/b*d^2)^(1/2)))-33649/16384*d^13*a/b^7/(a/b*d^2)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)-33649/16384*d^13*a/b^7/(a/b*d^2)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.20, size = 394, normalized size = 0.98

$$-\frac{504735\,ad^{14}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\cdots\right.}{\cdots}\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(25/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/491520*(504735*a*d^14*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/
4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sq
rt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^
(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)
*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^
(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt
(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b
^6 - 327680*(d*x)^(3/2)*d^12/b^6 - 8*(232545*(d*x)^(19/2)*a*b^4*d^14 + 7475
76*(d*x)^(15/2)*a^2*b^3*d^16 + 958210*(d*x)^(11/2)*a^3*b^2*d^18 + 564320*(d
*x)^(7/2)*a^4*b*d^20 + 127285*(d*x)^(3/2)*a^5*d^22)/(b^11*d^10*x^10 + 5*a*b
^10*d^10*x^8 + 10*a^2*b^9*d^10*x^6 + 10*a^3*b^8*d^10*x^4 + 5*a^4*b^7*d^10*x
^2 + a^5*b^6*d^10))/d
```

mupad [B] time = 0.24, size = 231, normalized size = 0.57

$$\frac{\frac{25457\,a^5\,d^{21}\,(d\,x)^{3/2}}{12288}+\frac{95821\,a^3\,b^2\,d^{17}\,(d\,x)^{11/2}}{6144}+\frac{31149\,a^2\,b^3\,d^{15}\,(d\,x)^{15/2}}{2560}+\frac{3527\,a^4\,b\,d^{19}\,(d\,x)^{7/2}}{384}+\frac{15503\,a\,b^4\,d^{13}\,(d\,x)^{19/2}}{4096}}{a^5\,b^6\,d^{10}+5\,a^4\,b^7\,d^{10}\,x^2+10\,a^3\,b^8\,d^{10}\,x^4+10\,a^2\,b^9\,d^{10}\,x^6+5\,a\,b^{10}\,d^{10}\,x^8+b^{11}\,d^{10}\,x^{10}}+\frac{2\,d^{11}\,(d\,x)^{3/2}}{3\,b^6}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(25/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((25457*a^5*d^21*(d*x)^(3/2))/12288 + (95821*a^3*b^2*d^17*(d*x)^(11/2))/614
4 + (31149*a^2*b^3*d^15*(d*x)^(15/2))/2560 + (3527*a^4*b*d^19*(d*x)^(7/2))/
```

```
384 + (15503*a*b^4*d^13*(d*x)^(19/2))/4096)/(a^5*b^6*d^10 + b^11*d^10*x^10
+ 5*a*b^10*d^10*x^8 + 5*a^4*b^7*d^10*x^2 + 10*a^3*b^8*d^10*x^4 + 10*a^2*b^9
*d^10*x^6) + (2*d^11*(d*x)^(3/2))/(3*b^6) + (33649*(-a)^(3/4)*d^(25/2)*atan
((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*b^(27/4)) + ((-a)^(3/4)
*d^(25/2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*33649i)/(8192
*b^(27/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(25/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

3.712 $\int \frac{(dx)^{23/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=402

$$\frac{13923\sqrt[4]{a}\,d^{23/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,b^{25/4}}-\frac{13923\sqrt[4]{a}\,d^{23/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\right.}{16384\sqrt{2}\,b^{25/4}}$$

```
[Out] -1/10*d*(d*x)^(21/2)/b/(b*x^2+a)^5-21/160*d^3*(d*x)^(17/2)/b^2/(b*x^2+a)^4-
119/640*d^5*(d*x)^(13/2)/b^3/(b*x^2+a)^3-1547/5120*d^7*(d*x)^(9/2)/b^4/(b*x
^2+a)^2-13923/20480*d^9*(d*x)^(5/2)/b^5/(b*x^2+a)+13923/16384*a^(1/4)*d^(23
/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(25/4)*2^(1/2)-
13923/16384*a^(1/4)*d^(23/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d
^(1/2))/b^(25/4)*2^(1/2)+13923/32768*a^(1/4)*d^(23/2)*ln(a^(1/2)*d^(1/2)+x*
b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(25/4)*2^(1/2)-13923
/32768*a^(1/4)*d^(23/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4
)*2^(1/2)*(d*x)^(1/2))/b^(25/4)*2^(1/2)+13923/4096*d^11*(d*x)^(1/2)/b^6
```

Rubi [A] time = 0.49, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{13923d^9(dx)^{5/2}}{20480b^5\left(a+bx^2\right)}-\frac{1547d^7(dx)^{9/2}}{5120b^4\left(a+bx^2\right)^2}-\frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3}-\frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4}+\frac{13923\sqrt[4]{a}\,d^{23/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{16384\sqrt{2}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (13923*d^11*Sqrt[d*x])/(4096*b^6) - (d*(d*x)^(21/2))/(10*b*(a + b*x^2)^5) -
 (21*d^3*(d*x)^(17/2))/(160*b^2*(a + b*x^2)^4) - (119*d^5*(d*x)^(13/2))/(64
0*b^3*(a + b*x^2)^3) - (1547*d^7*(d*x)^(9/2))/(5120*b^4*(a + b*x^2)^2) - (1
3923*d^9*(d*x)^(5/2))/(20480*b^5*(a + b*x^2)) + (13923*a^(1/4)*d^(23/2)*Arc
Tan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(25
/4)) - (13923*a^(1/4)*d^(23/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1
/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(25/4)) + (13923*a^(1/4)*d^(23/2)*Log[Sqrt[a
]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*
Sqrt[2]*b^(25/4)) - (13923*a^(1/4)*d^(23/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*S
qrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*b^(25/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{23/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{23/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(21b^4d^2\right)\int \frac{(dx)^{19/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{320}\left(357b^2d^4\right)\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} + \frac{\left(1547d^6\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^3}\,dx}{1280}$$

$$= -\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} - \frac{1547d^7(dx)^{9/2}}{5120b^4\left(a+bx^2\right)^2} + \frac{(13}{}$$

$$= -\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} - \frac{1547d^7(dx)^{9/2}}{5120b^4\left(a+bx^2\right)^2} - \frac{1}{204}$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} - \frac{1547d}{5120b^4(}$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} - \frac{1547d}{5120b^4(}$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} - \frac{1547d}{5120b^4(}$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} - \frac{1547d}{5120b^4(}$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} - \frac{1547d}{5120b^4(}$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} - \frac{1547d}{5120b^4(}$$

Mathematica [A] time = 0.30, size = 408, normalized size = 1.01

$$\frac{d^{11}\sqrt{dx}\left(\frac{10862592a^5\sqrt[4]{b}\sqrt{x}+43450368a^4b^{5/4}x^{5/2}-678912a^4\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)+72417280a^3b^{9/4}x^{9/2}-848640a^3\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^2+61276160a^2b^{13/4}x}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^11*Sqrt[d*x]*(1531530*Sqrt[2]*a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)] + (10862592*a^5*b^(1/4)*Sqrt[x] + 43450368*a^4*b^(5/4)*x^(5/2)
+ 72417280*a^3*b^(9/4)*x^(9/2) + 61276160*a^2*b^(13/4)*x^(13/2) + 25231360*
a*b^(17/4)*x^(17/2) + 3604480*b^(21/4)*x^(21/2) - 678912*a^4*b^(1/4)*Sqrt[x
]*(a + b*x^2) - 848640*a^3*b^(1/4)*Sqrt[x]*(a + b*x^2)^2 - 1166880*a^2*b^(1
/4)*Sqrt[x]*(a + b*x^2)^3 - 2042040*a*b^(1/4)*Sqrt[x]*(a + b*x^2)^4 - 15315
30*Sqrt[2]*a^(1/4)*(a + b*x^2)^5*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/
4)] + 765765*Sqrt[2]*a^(1/4)*(a + b*x^2)^5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x] - 765765*Sqrt[2]*a^(1/4)*(a + b*x^2)^5*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a + b*x^2)^5))/(1802240
*b^(25/4)*Sqrt[x])
```

fricas [A] time = 0.80, size = 479, normalized size = 1.19

$$-\frac{278460\left(-\frac{ad^{46}}{b^{25}}\right)^{\frac{1}{4}}\left(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6\right)\arctan\left(-\frac{\left(-\frac{ad^{46}}{b^{25}}\right)^{\frac{3}{4}}\sqrt{dx}\,b^{19}d^{11}-\sqrt{a}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(278460*(-a*d^46/b^25)^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^
9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)*arctan(-((-a*d^46/b^25)^(
3/4)*sqrt(d*x)*b^19*d^11 - sqrt(d^23*x + sqrt(-a*d^46/b^25)*b^12)*(-a*d^46/
b^25)^(3/4)*b^19)/(a*d^46)) + 69615*(-a*d^46/b^25)^(1/4)*(b^11*x^10 + 5*a*b
^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)*log(13
923*sqrt(d*x)*d^11 + 13923*(-a*d^46/b^25)^(1/4)*b^6) - 69615*(-a*d^46/b^25)
^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*
b^7*x^2 + a^5*b^6)*log(13923*sqrt(d*x)*d^11 - 13923*(-a*d^46/b^25)^(1/4)*b^
6) - 4*(40960*b^5*d^11*x^10 + 263515*a*b^4*d^11*x^8 + 590240*a^2*b^3*d^11*x
^6 + 634270*a^3*b^2*d^11*x^4 + 334152*a^4*b*d^11*x^2 + 69615*a^5*d^11)*sqrt
```

```
(d*x))/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*
b^7*x^2 + a^5*b^6)
```

giac [A] time = 0.25, size = 340, normalized size = 0.85

$$-\frac{1}{163840}\,d^{11}\left(\frac{139230\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{b^{7}}+\frac{139230\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\right.}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right.}{b^{7}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/163840*d^11*(139230*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2
)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^7 + 139230*sqrt(2)*(a*b
^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(
a*d^2/b)^(1/4))/b^7 + 69615*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*
d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^7 - 69615*sqrt(2)*(a*b^3*d^2)^(1/
4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^7 - 32768
0*sqrt(d*x)/b^6 - 8*(58715*sqrt(d*x)*a*b^4*d^10*x^8 + 180640*sqrt(d*x)*a^2*
b^3*d^10*x^6 + 224670*sqrt(d*x)*a^3*b^2*d^10*x^4 + 129352*sqrt(d*x)*a^4*b*d
^10*x^2 + 28655*sqrt(d*x)*a^5*d^10)/((b*d^2*x^2 + a*d^2)^5*b^6))
```

maple [A] time = 0.03, size = 351, normalized size = 0.87

$$\frac{5731\sqrt{dx}\;a^{5}d^{21}}{4096\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}b^{6}}+\frac{16169\left(dx\right)^{\frac{5}{2}}a^{4}d^{19}}{2560\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}b^{5}}+\frac{22467\left(dx\right)^{\frac{9}{2}}a^{3}d^{17}}{2048\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}b^{4}}+\frac{1129\left(dx\right)^{\frac{13}{2}}a^{2}d^{15}}{128\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}b^{3}}+\frac{11743\left(dx\right.}{4096\left(b\,d^{2}x^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 2*d^11*(d*x)^(1/2)/b^6+5731/4096*d^21/b^6*a^5/(b*d^2*x^2+a*d^2)^5*(d*x)^(1/
2)+16169/2560*d^19/b^5*a^4/(b*d^2*x^2+a*d^2)^5*(d*x)^(5/2)+22467/2048*d^17/
b^4*a^3/(b*d^2*x^2+a*d^2)^5*(d*x)^(9/2)+1129/128*d^15/b^3*a^2/(b*d^2*x^2+a*
```

```
d^2)^5*(d*x)^(13/2)+11743/4096*d^13/b^2*a/(b*d^2*x^2+a*d^2)^5*(d*x)^(17/2)-
13923/32768*d^11/b^6*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^
(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/
b*d^2)^(1/2)))-13923/16384*d^11/b^6*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)-13923/16384*d^11/b^6*(a/b*d^2)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.23, size = 403, normalized size = 1.00

$$\frac{\frac{327680\sqrt{dx}\,d^{12}}{b^6}+\frac{8\left(58715\,(dx)^{\frac{17}{2}}ab^4d^{14}+180640\,(dx)^{\frac{13}{2}}a^2b^3d^{16}+224670\,(dx)^{\frac{9}{2}}a^3b^2d^{18}+129352\,(dx)^{\frac{5}{2}}a^4bd^{20}+28655\sqrt{dx}\,a^5d^{22}\right)}{b^{11}d^{10}x^{10}+5\,ab^{10}d^{10}x^8+10\,a^2b^9d^{10}x^6+10\,a^3b^8d^{10}x^4+5\,a^4b^7d^{10}x^2+a^5b^6d^{10}}-\frac{69615\left(\frac{\sqrt{2}\,d^{1}\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/163840*(327680*sqrt(d*x)*d^12/b^6 + 8*(58715*(d*x)^(17/2)*a*b^4*d^14 + 18
0640*(d*x)^(13/2)*a^2*b^3*d^16 + 224670*(d*x)^(9/2)*a^3*b^2*d^18 + 129352*(
d*x)^(5/2)*a^4*b*d^20 + 28655*sqrt(d*x)*a^5*d^22)/(b^11*d^10*x^10 + 5*a*b^1
0*d^10*x^8 + 10*a^2*b^9*d^10*x^6 + 10*a^3*b^8*d^10*x^4 + 5*a^4*b^7*d^10*x^2
 + a^5*b^6*d^10) - 69615*(sqrt(2)*d^14*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1
/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^14*l
og(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d
^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^13*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/
4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sq
rt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^13*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/
4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sq
rt(b)*d)*sqrt(a)))*a/b^6)/d
```

mupad [B] time = 4.36, size = 231, normalized size = 0.57

$$\frac{\frac{5731\,a^5\,d^{21}\,\sqrt{d\,x}}{4096}+\frac{22467\,a^3\,b^2\,d^{17}\,{\left(d\,x\right)}^{9/2}}{2048}+\frac{1129\,a^2\,b^3\,d^{15}\,{\left(d\,x\right)}^{13/2}}{128}+\frac{16169\,a^4\,b\,d^{19}\,{\left(d\,x\right)}^{5/2}}{2560}+\frac{11743\,a\,b^4\,d^{13}\,{\left(d\,x\right)}^{17/2}}{4096}}{a^5\,b^6\,d^{10}+5\,a^4\,b^7\,d^{10}\,x^2+10\,a^3\,b^8\,d^{10}\,x^4+10\,a^2\,b^9\,d^{10}\,x^6+5\,a\,b^{10}\,d^{10}\,x^8+b^{11}\,d^{10}\,x^{10}}+\frac{2\,d^{11}\,\sqrt{d\,x}}{b^6}-\frac{139\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(23/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((5731*a^5*d^21*(d*x)^(1/2))/4096 + (22467*a^3*b^2*d^17*(d*x)^(9/2))/2048 +
 (1129*a^2*b^3*d^15*(d*x)^(13/2))/128 + (16169*a^4*b*d^19*(d*x)^(5/2))/2560
 + (11743*a*b^4*d^13*(d*x)^(17/2))/4096)/(a^5*b^6*d^10 + b^11*d^10*x^10 + 5
*a*b^10*d^10*x^8 + 5*a^4*b^7*d^10*x^2 + 10*a^3*b^8*d^10*x^4 + 10*a^2*b^9*d^
10*x^6) + (2*d^11*(d*x)^(1/2))/b^6 - (13923*(-a)^(1/4)*d^(23/2)*atan((b^(1/
4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*b^(25/4)) + ((-a)^(1/4)*d^(23/
2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*13923i)/(8192*b^(25/
4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(23/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

3.713 $\int \frac{(dx)^{21/2}}{\left(a^2+2abx^2+b^2x^4\right)^3} dx$

Optimal. Leaf size=385

$$\frac{4389d^{21/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\sqrt[4]{a}\,b^{23/4}}-\frac{4389d^{21/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\sqrt[4]{a}\,b^{23/4}}-$$

```
[Out] -1/10*d*(d*x)^(19/2)/b/(b*x^2+a)^5-19/160*d^3*(d*x)^(15/2)/b^2/(b*x^2+a)^4-
19/128*d^5*(d*x)^(11/2)/b^3/(b*x^2+a)^3-209/1024*d^7*(d*x)^(7/2)/b^4/(b*x^2
+a)^2-1463/4096*d^9*(d*x)^(3/2)/b^5/(b*x^2+a)-4389/16384*d^(21/2)*arctan(1-
b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(1/4)/b^(23/4)*2^(1/2)+4389/
16384*d^(21/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(1/4
)/b^(23/4)*2^(1/2)+4389/32768*d^(21/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)
-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(1/4)/b^(23/4)*2^(1/2)-4389/32768*d
^(21/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^
(1/2))/a^(1/4)/b^(23/4)*2^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{1463d^9(dx)^{3/2}}{4096b^5\left(a+bx^2\right)}-\frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2}-\frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3}-\frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4}+\frac{4389d^{21/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\right.}{16384\sqrt{2}\sqrt[4]{a}\,b^{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(19/2))/(10*b*(a + b*x^2)^5) - (19*d^3*(d*x)^(15/2))/(160*b^2*(a
+ b*x^2)^4) - (19*d^5*(d*x)^(11/2))/(128*b^3*(a + b*x^2)^3) - (209*d^7*(d*x
)^(7/2))/(1024*b^4*(a + b*x^2)^2) - (1463*d^9*(d*x)^(3/2))/(4096*b^5*(a + b
*x^2)) - (4389*d^(21/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])])/(8192*Sqrt[2]*a^(1/4)*b^(23/4)) + (4389*d^(21/2)*ArcTan[1 + (Sqrt[2
]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(1/4)*b^(23/4)) +
(4389*d^(21/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(1/4)*b^(23/4)) - (4389*d^(21/2)*Log[Sqr
t[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(163
84*Sqrt[2]*a^(1/4)*b^(23/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{21/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{21/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(19b^4d^2\right)\int \frac{(dx)^{17/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{64}\left(57b^2d^4\right)\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} + \frac{1}{256}\left(209d^6\right)\int \frac{(dx)^{9}}{\left(ab+b^{2}\right.}$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{(14}{}$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2} - \frac{14}{409}$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2} - \frac{14}{409}$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2} - \frac{14}{409}$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2} - \frac{14}{409}$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2} - \frac{14}{409}$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2} - \frac{14}{409}$$

Mathematica [C] time = 0.04, size = 104, normalized size = 0.27

$$\frac{2d^9(dx)^{3/2}\left(7315\left(a+bx^2\right)^5 {}_2F_1\left(\frac{3}{4},6;\frac{7}{4};-\frac{bx^2}{a}\right)-a\left(7315a^4+17765a^3bx^2+20995a^2b^2x^4+12597ab^3x^6+3315b\right.\right.}{3315ab^5\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*d^9*(d*x)^(3/2)*(-(a*(7315*a^4 + 17765*a^3*b*x^2 + 20995*a^2*b^2*x^4 + 1
2597*a*b^3*x^6 + 3315*b^4*x^8)) + 7315*(a + b*x^2)^5*Hypergeometric2F1[3/4,
 6, 7/4, -((b*x^2)/a)]))/(3315*a*b^5*(a + b*x^2)^5)
```

fricas [A] time = 1.09, size = 486, normalized size = 1.26

$$-\frac{87780\left(b^{10}x^{10}+5\,ab^9x^8+10\,a^2b^8x^6+10\,a^3b^7x^4+5\,a^4b^6x^2+a^5b^5\right)\left(-\frac{d^{42}}{ab^{23}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{42}}{ab^{23}}\right)^{\frac{1}{4}}\sqrt{dx}\,b^6d^{31}-\sqrt{d^{63}x-}}{d^4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(87780*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4
+ 5*a^4*b^6*x^2 + a^5*b^5)*(-d^42/(a*b^23))^(1/4)*arctan(-((-d^42/(a*b^23))
^(1/4)*sqrt(d*x)*b^6*d^31 - sqrt(d^63*x - sqrt(-d^42/(a*b^23))*a*b^11*d^42)
*(-d^42/(a*b^23))^(1/4)*b^6)/d^42) - 21945*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^
2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)*(-d^42/(a*b^23))^(1/4
)*log(84546715869*sqrt(d*x)*d^31 + 84546715869*(-d^42/(a*b^23))^(3/4)*a*b^1
7) + 21945*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a
^4*b^6*x^2 + a^5*b^5)*(-d^42/(a*b^23))^(1/4)*log(84546715869*sqrt(d*x)*d^31
 - 84546715869*(-d^42/(a*b^23))^(3/4)*a*b^17) + 4*(19015*b^4*d^10*x^9 + 503
12*a*b^3*d^10*x^7 + 59470*a^2*b^2*d^10*x^5 + 33440*a^3*b*d^10*x^3 + 7315*a^
4*d^10*x)*sqrt(d*x))/(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7
*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)
```

giac [A] time = 0.21, size = 352, normalized size = 0.91

$$\frac{1}{163840}d^{10}\left(\frac{43890\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^8d}+\frac{43890\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^8d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^10*(43890*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*
(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^8*d) + 43890*sqrt(2)*(
a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x)
)/(a*d^2/b)^(1/4))/(a*b^8*d) - 21945*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sq
rt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^8*d) + 21945*sqrt(2)*
(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/
b))/(a*b^8*d) - 8*(19015*sqrt(d*x)*b^4*d^10*x^9 + 50312*sqrt(d*x)*a*b^3*d^1
0*x^7 + 59470*sqrt(d*x)*a^2*b^2*d^10*x^5 + 33440*sqrt(d*x)*a^3*b*d^10*x^3 +
 7315*sqrt(d*x)*a^4*d^10*x)/((b*d^2*x^2 + a*d^2)^5*b^5))
```

maple [A] time = 0.02, size = 335, normalized size = 0.87

$$-\frac{1463\left(dx\right)^{\frac{3}{2}}a^4d^{19}}{4096\left(b\,d^2x^2+d^2a\right)^5b^5}-\frac{209\left(dx\right)^{\frac{7}{2}}a^3d^{17}}{128\left(b\,d^2x^2+d^2a\right)^5b^4}-\frac{5947\left(dx\right)^{\frac{11}{2}}a^2d^{15}}{2048\left(b\,d^2x^2+d^2a\right)^5b^3}-\frac{6289\left(dx\right)^{\frac{15}{2}}a\,d^{13}}{2560\left(b\,d^2x^2+d^2a\right)^5b^2}-\frac{3803\left(d\right.}{4096\left(b\,d^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1463/4096*d^19/(b*d^2*x^2+a*d^2)^5/b^5*a^4*(d*x)^(3/2)-209/128*d^17/(b*d^2
*x^2+a*d^2)^5/b^4*a^3*(d*x)^(7/2)-5947/2048*d^15/(b*d^2*x^2+a*d^2)^5/b^3*a^
2*(d*x)^(11/2)-6289/2560*d^13/(b*d^2*x^2+a*d^2)^5/b^2*a*(d*x)^(15/2)-3803/4
096*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(19/2)+4389/32768*d^11/b^6/(a/b*d^2)^(
1/4)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(
```

```
d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+4389/16384*d^11/b
^6/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+43
89/16384*d^11/b^6/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d
*x)^(1/2)-1)
```

maxima [A] time = 3.16, size = 377, normalized size = 0.98

$$\frac{21945\,d^{12}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx-\cdots\right)}{(\cdots}\right)}{b^5}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/163840*(21945*d^12*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b
^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b
)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4
) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*
sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4
) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*
(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b^5 -
 8*(19015*(d*x)^(19/2)*b^4*d^12 + 50312*(d*x)^(15/2)*a*b^3*d^14 + 59470*(d*
x)^(11/2)*a^2*b^2*d^16 + 33440*(d*x)^(7/2)*a^3*b*d^18 + 7315*(d*x)^(3/2)*a^
4*d^20)/(b^10*d^10*x^10 + 5*a*b^9*d^10*x^8 + 10*a^2*b^8*d^10*x^6 + 10*a^3*b
^7*d^10*x^4 + 5*a^4*b^6*d^10*x^2 + a^5*b^5*d^10))/d
```

mupad [B] time = 0.21, size = 213, normalized size = 0.55

$$\frac{4389\,d^{21/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{1/4}\,b^{23/4}}-\frac{\frac{3803\,d^{11}\,(d\,x)^{19/2}}{4096\,b}+\frac{5947\,a^2\,d^{15}\,(d\,x)^{11/2}}{2048\,b^3}+\frac{209\,a^3\,d^{17}\,(d\,x)^{7/2}}{128\,b^4}+\frac{1463\,a^4\,d^{19}\,(d\,x)^{3/2}}{4096\,b^5}+\frac{6289\,a\,d^{13}\,(d\,x)^{15/2}}{2560\,b^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(21/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (4389*d^(21/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)
^(1/4)*b^(23/4)) - ((3803*d^11*(d*x)^(19/2))/(4096*b) + (5947*a^2*d^15*(d*x
)^(11/2))/(2048*b^3) + (209*a^3*d^17*(d*x)^(7/2))/(128*b^4) + (1463*a^4*d^1
```

```
9*(d*x)^(3/2))/(4096*b^5) + (6289*a*d^13*(d*x)^(15/2))/(2560*b^2))/(a^5*d^1
0 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x
^4 + 10*a^2*b^3*d^10*x^6) - (4389*d^(21/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a
)^(1/4)*d^(1/2))))/(8192*(-a)^(1/4)*b^(23/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(21/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

3.714 $\int \frac{(dx)^{19/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=385

$$-\frac{663d^{19/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{3/4}b^{21/4}}+\frac{663d^{19/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{3/4}b^{21/4}}-\frac{66}{}$$

```
[Out] -1/10*d*(d*x)^(17/2)/b/(b*x^2+a)^5-17/160*d^3*(d*x)^(13/2)/b^2/(b*x^2+a)^4-
221/1920*d^5*(d*x)^(9/2)/b^3/(b*x^2+a)^3-663/5120*d^7*(d*x)^(5/2)/b^4/(b*x^
2+a)^2-663/16384*d^(19/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1
/2))/a^(3/4)/b^(21/4)*2^(1/2)+663/16384*d^(19/2)*arctan(1+b^(1/4)*2^(1/2)*(
d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(3/4)/b^(21/4)*2^(1/2)-663/32768*d^(19/2)*ln(
a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(3
/4)/b^(21/4)*2^(1/2)+663/32768*d^(19/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2
)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(3/4)/b^(21/4)*2^(1/2)-663/4096*d^
9*(d*x)^(1/2)/b^5/(b*x^2+a)
```

Rubi [A] time = 0.45, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{663d^{19/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{3/4}b^{21/4}}+\frac{663d^{19/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{3/4}b^{21/4}}-\frac{66}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(17/2))/(10*b*(a + b*x^2)^5) - (17*d^3*(d*x)^(13/2))/(160*b^2*(a
+ b*x^2)^4) - (221*d^5*(d*x)^(9/2))/(1920*b^3*(a + b*x^2)^3) - (663*d^7*(d*
x)^(5/2))/(5120*b^4*(a + b*x^2)^2) - (663*d^9*Sqrt[d*x])/(4096*b^5*(a + b*x
^2)) - (663*d^(19/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d
]])/(8192*Sqrt[2]*a^(3/4)*b^(21/4)) + (663*d^(19/2)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(3/4)*b^(21/4)) - (663
*d^(19/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(16384*Sqrt[2]*a^(3/4)*b^(21/4)) + (663*d^(19/2)*Log[Sqrt[a]*S
qrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqr
t[2]*a^(3/4)*b^(21/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{19/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{19/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(17b^4d^2\right)\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{320}\left(221b^2d^4\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{\left(663d^6\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^3}\,d}{1280}$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\left(a+bx^2\right)^2} -$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\left(a+bx^2\right)^2} -$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\left(a+bx^2\right)^2} -$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\left(a+bx^2\right)^2} -$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\left(a+bx^2\right)^2} -$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\left(a+bx^2\right)^2} -$$

Mathematica [A] time = 0.20, size = 381, normalized size = 0.99

$$d^9\sqrt{dx}\left(-\frac{765765\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}\sqrt{x}}+\frac{765765\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}\sqrt{x}}-\frac{1531530\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}\sqrt{x}}+\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^9*Sqrt[d*x]*((-10862592*a^4*b^(1/4))/(a + b*x^2)^5 - (43450368*a^3*b^(5/
4)*x^2)/(a + b*x^2)^5 - (72417280*a^2*b^(9/4)*x^4)/(a + b*x^2)^5 - (6127616
0*a*b^(13/4)*x^6)/(a + b*x^2)^5 - (25231360*b^(17/4)*x^8)/(a + b*x^2)^5 + (
678912*a^3*b^(1/4))/(a + b*x^2)^4 + (848640*a^2*b^(1/4))/(a + b*x^2)^3 + (1
166880*a*b^(1/4))/(a + b*x^2)^2 + (2042040*b^(1/4))/(a + b*x^2) - (1531530*
Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(3/4)*Sqrt[x]) +
(1531530*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(3/4)*Sq
rt[x]) - (765765*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(a^(3/4)*Sqrt[x]) + (765765*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(3/4)*Sqrt[x])))/(37847040*b^(21/4))
```

fricas [A] time = 1.17, size = 489, normalized size = 1.27

$$39780\left(b^{10}x^{10}+5\,ab^{9}x^{8}+10\,a^{2}b^{8}x^{6}+10\,a^{3}b^{7}x^{4}+5\,a^{4}b^{6}x^{2}+a^{5}b^{5}\right)\left(-\frac{d^{38}}{a^{3}b^{21}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{38}}{a^{3}b^{21}}\right)^{\frac{3}{4}}\sqrt{dx}\,a^{2}b^{16}d^{9}-\sqrt{d^{19}\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/245760*(39780*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4
+ 5*a^4*b^6*x^2 + a^5*b^5)*(-d^38/(a^3*b^21))^(1/4)*arctan(-((-d^38/(a^3*b^
21))^(3/4)*sqrt(d*x)*a^2*b^16*d^9 - sqrt(d^19*x + sqrt(-d^38/(a^3*b^21))*a^
2*b^10)*(-d^38/(a^3*b^21))^(3/4)*a^2*b^16)/d^38) + 9945*(b^10*x^10 + 5*a*b^
9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)*(-d^38/(
a^3*b^21))^(1/4)*log(663*sqrt(d*x)*d^9 + 663*(-d^38/(a^3*b^21))^(1/4)*a*b^5
) - 9945*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4
*b^6*x^2 + a^5*b^5)*(-d^38/(a^3*b^21))^(1/4)*log(663*sqrt(d*x)*d^9 - 663*(-
d^38/(a^3*b^21))^(1/4)*a*b^5) - 4*(37645*b^4*d^9*x^8 + 84320*a*b^3*d^9*x^6
+ 90610*a^2*b^2*d^9*x^4 + 47736*a^3*b*d^9*x^2 + 9945*a^4*d^9)*sqrt(d*x))/(b
^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 +
a^5*b^5)
```

giac [A] time = 0.24, size = 339, normalized size = 0.88

$$\frac{1}{491520}d^9\left(\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^6}+\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/491520*d^9*(19890*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^6) + 19890*sqrt(2)*(a*b
^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(
a*d^2/b)^(1/4))/(a*b^6) + 9945*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*
(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^6) - 9945*sqrt(2)*(a*b^3*d^
2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^
6) - 8*(37645*sqrt(d*x)*b^4*d^10*x^8 + 84320*sqrt(d*x)*a*b^3*d^10*x^6 + 906
10*sqrt(d*x)*a^2*b^2*d^10*x^4 + 47736*sqrt(d*x)*a^3*b*d^10*x^2 + 9945*sqrt(
d*x)*a^4*d^10)/((b*d^2*x^2 + a*d^2)^5*b^5))
```

maple [A] time = 0.03, size = 344, normalized size = 0.89

$$-\frac{663\sqrt{dx}\,a^4d^{19}}{4096\left(b\,d^2x^2+d^2a\right)^5b^5}-\frac{1989\,(dx)^{\frac{5}{2}}\,a^3d^{17}}{2560\left(b\,d^2x^2+d^2a\right)^5b^4}-\frac{9061\,(dx)^{\frac{9}{2}}\,a^2d^{15}}{6144\left(b\,d^2x^2+d^2a\right)^5b^3}-\frac{527\,(dx)^{\frac{13}{2}}\,a\,d^{13}}{384\left(b\,d^2x^2+d^2a\right)^5b^2}-\frac{7529\,(d}{12288\left(b\,d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -663/4096*d^19/(b*d^2*x^2+a*d^2)^5/b^5*a^4*(d*x)^(1/2)-1989/2560*d^17/(b*d^
2*x^2+a*d^2)^5/b^4*a^3*(d*x)^(5/2)-9061/6144*d^15/(b*d^2*x^2+a*d^2)^5/b^3*a
^2*(d*x)^(9/2)-527/384*d^13/(b*d^2*x^2+a*d^2)^5/b^2*a*(d*x)^(13/2)-7529/122
88*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(17/2)+663/32768*d^9/b^5*(a/b*d^2)^(1/4
)/a*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d
```

```
*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+663/16384*d^9/b^5*
(a/b*d^2)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+663
/16384*d^9/b^5*(a/b*d^2)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*
x)^(1/2)-1)
```

maxima [A] time = 3.11, size = 386, normalized size = 1.00

$$-\frac{\frac{8\left(37645\,(dx)^{\frac{17}{2}}b^4d^{12}+84320\,(dx)^{\frac{13}{2}}ab^3d^{14}+90610\,(dx)^{\frac{9}{2}}a^2b^2d^{16}+47736\,(dx)^{\frac{5}{2}}a^3bd^{18}+9945\,\sqrt{dx}\,a^4d^{20}\right)}{b^{10}d^{10}x^{10}+5\,ab^9d^{10}x^8+10\,a^2b^8d^{10}x^6+10\,a^3b^7d^{10}x^4+5\,a^4b^6d^{10}x^2+a^5b^5d^{10}}-\frac{9945\left(\frac{\sqrt{2}\,d^{12}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b\right.}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}\right.}{}}{491520}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/491520*(8*(37645*(d*x)^(17/2)*b^4*d^12 + 84320*(d*x)^(13/2)*a*b^3*d^14 +
 90610*(d*x)^(9/2)*a^2*b^2*d^16 + 47736*(d*x)^(5/2)*a^3*b*d^18 + 9945*sqrt(
d*x)*a^4*d^20)/(b^10*d^10*x^10 + 5*a*b^9*d^10*x^8 + 10*a^2*b^8*d^10*x^6 + 1
0*a^3*b^7*d^10*x^4 + 5*a^4*b^6*d^10*x^2 + a^5*b^5*d^10) - 9945*(sqrt(2)*d^1
2*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((
a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^12*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4
)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^11*a
rctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqr
t(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^11*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqr
t(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/b^5)/d
```

mupad [B] time = 4.27, size = 213, normalized size = 0.55

$$-\frac{\frac{7529\,d^{11}\,(d\,x)^{17/2}}{12288\,b}+\frac{9061\,a^2\,d^{15}\,(d\,x)^{9/2}}{6144\,b^3}+\frac{1989\,a^3\,d^{17}\,(d\,x)^{5/2}}{2560\,b^4}+\frac{663\,a^4\,d^{19}\,\sqrt{d\,x}}{4096\,b^5}+\frac{527\,a\,d^{13}\,(d\,x)^{13/2}}{384\,b^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}-\frac{663\,d^{19/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{3/4}\,b^{21/4}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(19/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] - ((7529*d^11*(d*x)^(17/2))/(12288*b) + (9061*a^2*d^15*(d*x)^(9/2))/(6144*b
^3) + (1989*a^3*d^17*(d*x)^(5/2))/(2560*b^4) + (663*a^4*d^19*(d*x)^(1/2))/(
4096*b^5) + (527*a*d^13*(d*x)^(13/2))/(384*b^2))/(a^5*d^10 + b^5*d^10*x^10
```

```
+ 5*a^4*b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^
10*x^6) - (663*d^(19/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(
8192*(-a)^(3/4)*b^(21/4)) - (663*d^(19/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)
^(1/4)*d^(1/2))))/(8192*(-a)^(3/4)*b^(21/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(19/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

3.715 $\int \frac{(dx)^{17/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=388

$$\frac{231d^{17/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{5/4}b^{19/4}}-\frac{231d^{17/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{5/4}b^{19/4}}-\frac{231}{}$$

```
[Out] -1/10*d*(d*x)^(15/2)/b/(b*x^2+a)^5-3/32*d^3*(d*x)^(11/2)/b^2/(b*x^2+a)^4-11
/128*d^5*(d*x)^(7/2)/b^3/(b*x^2+a)^3-77/1024*d^7*(d*x)^(3/2)/b^4/(b*x^2+a)^
2+231/4096*d^7*(d*x)^(3/2)/a/b^4/(b*x^2+a)-231/16384*d^(17/2)*arctan(1-b^(1
/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(5/4)/b^(19/4)*2^(1/2)+231/16384
*d^(17/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(5/4)/b^(
19/4)*2^(1/2)+231/32768*d^(17/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/
4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(5/4)/b^(19/4)*2^(1/2)-231/32768*d^(17/2)
*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/
a^(5/4)/b^(19/4)*2^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{231d^{17/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{5/4}b^{19/4}}-\frac{231d^{17/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{5/4}b^{19/4}}-\frac{231}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(15/2))/(10*b*(a + b*x^2)^5) - (3*d^3*(d*x)^(11/2))/(32*b^2*(a +
b*x^2)^4) - (11*d^5*(d*x)^(7/2))/(128*b^3*(a + b*x^2)^3) - (77*d^7*(d*x)^(3
/2))/(1024*b^4*(a + b*x^2)^2) + (231*d^7*(d*x)^(3/2))/(4096*a*b^4*(a + b*x^
2)) - (231*d^(17/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
)])/(8192*Sqrt[2]*a^(5/4)*b^(19/4)) + (231*d^(17/2)*ArcTan[1 + (Sqrt[2]*b^(
1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(5/4)*b^(19/4)) + (231*
d^(17/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d*x]])/(16384*Sqrt[2]*a^(5/4)*b^(19/4)) - (231*d^(17/2)*Log[Sqrt[a]*Sq
rt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt
[2]*a^(5/4)*b^(19/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{17/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{17/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{4}\left(3b^4d^2\right)\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} + \frac{1}{64}\left(33b^2d^4\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} + \frac{1}{256}\left(77d^6\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2 \right.}$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{(231}{}$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{2}{409}$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{2}{409}$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{2}{409}$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{2}{409}$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{2}{409}$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{2}{409}$$

Mathematica [C] time = 0.03, size = 96, normalized size = 0.25

$$\frac{2d^8x\sqrt{dx}\left(385\left(a+bx^2\right)^5{}_2F_1\left(\frac{3}{4},6;\frac{7}{4};-\frac{bx^2}{a}\right)-a^2\left(385a^3+935a^2bx^2+1105ab^2x^4+663b^3x^6\right)\right)}{3315a^2b^4\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*d^8*x*Sqrt[d*x]*(-(a^2*(385*a^3 + 935*a^2*b*x^2 + 1105*a*b^2*x^4 + 663*b
^3*x^6)) + 385*(a + b*x^2)^5*Hypergeometric2F1[3/4, 6, 7/4, -((b*x^2)/a)]))
/(3315*a^2*b^4*(a + b*x^2)^5)
```

fricas [A] time = 1.08, size = 506, normalized size = 1.30

$$-\frac{4620\left(ab^9x^{10}+5\,a^2b^8x^8+10\,a^3b^7x^6+10\,a^4b^6x^4+5\,a^5b^5x^2+a^6b^4\right)\left(-\frac{d^{34}}{a^5b^{19}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{34}}{a^5b^{19}}\right)^{\frac{1}{4}}\sqrt{dx}\,ab^5d^{25}-\sqrt{d^{5}}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(4620*(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^
4 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^34/(a^5*b^19))^(1/4)*arctan(-((-d^34/(a^5*
b^19))^(1/4)*sqrt(d*x)*a*b^5*d^25 - sqrt(d^51*x - sqrt(-d^34/(a^5*b^19))*a^
3*b^9*d^34)*(-d^34/(a^5*b^19))^(1/4)*a*b^5)/d^34) - 1155*(a*b^9*x^10 + 5*a^
2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^
34/(a^5*b^19))^(1/4)*log(12326391*sqrt(d*x)*d^25 + 12326391*(-d^34/(a^5*b^1
9))^(3/4)*a^4*b^14) + 1155*(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 1
0*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^34/(a^5*b^19))^(1/4)*log(12326
391*sqrt(d*x)*d^25 - 12326391*(-d^34/(a^5*b^19))^(3/4)*a^4*b^14) - 4*(1155*
b^4*d^8*x^9 - 2648*a*b^3*d^8*x^7 - 3130*a^2*b^2*d^8*x^5 - 1760*a^3*b*d^8*x^
3 - 385*a^4*d^8*x)*sqrt(d*x))/(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6
+ 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)
```

giac [A] time = 0.21, size = 355, normalized size = 0.91

$$\frac{1}{163840}d^8\left(\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^7d}+\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^7d}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^8*(2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^7*d) + 2310*sqrt(2)*(a
*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))
/(a*d^2/b)^(1/4))/(a^2*b^7*d) - 1155*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sq
rt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^7*d) + 1155*sqrt(2)
*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2
/b))/(a^2*b^7*d) + 8*(1155*sqrt(d*x)*b^4*d^10*x^9 - 2648*sqrt(d*x)*a*b^3*d^
10*x^7 - 3130*sqrt(d*x)*a^2*b^2*d^10*x^5 - 1760*sqrt(d*x)*a^3*b*d^10*x^3 -
385*sqrt(d*x)*a^4*d^10*x)/((b*d^2*x^2 + a*d^2)^5*a*b^4))
```

maple [A] time = 0.03, size = 341, normalized size = 0.88

$$-\frac{77\left(dx\right)^{\frac{3}{2}}a^3d^{17}}{4096\left(b\,d^2x^2+d^2a\right)^5b^4}-\frac{11\left(dx\right)^{\frac{7}{2}}a^2d^{15}}{128\left(b\,d^2x^2+d^2a\right)^5b^3}-\frac{313\left(dx\right)^{\frac{11}{2}}a\,d^{13}}{2048\left(b\,d^2x^2+d^2a\right)^5b^2}-\frac{331\left(dx\right)^{\frac{15}{2}}d^{11}}{2560\left(b\,d^2x^2+d^2a\right)^5b}+\frac{231\left(dx\right.}{4096\left(b\,d^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -77/4096*d^17/(b*d^2*x^2+a*d^2)^5/b^4*a^3*(d*x)^(3/2)-11/128*d^15/(b*d^2*x^
2+a*d^2)^5/b^3*a^2*(d*x)^(7/2)-313/2048*d^13/(b*d^2*x^2+a*d^2)^5/b^2*a*(d*x
)^(11/2)-331/2560*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(15/2)+231/4096*d^9/(b*d
^2*x^2+a*d^2)^5/a*(d*x)^(19/2)+231/32768*d^9/a/b^5/(a/b*d^2)^(1/4)*2^(1/2)*
ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)
```

```
^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+231/16384*d^9/a/b^5/(a/b*d^2)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+231/16384*d^9/a
/b^5/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.04, size = 383, normalized size = 0.99

$$\frac{\frac{1155\,d^{10}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx-\sqrt{\cdots}\right)}{\left(a\cdots\right.}\right.}{ab^4}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/163840*(1155*d^10*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^
(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)
*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4)
 - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*s
qrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
 + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(
a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a*b^4
) + 8*(1155*(d*x)^(19/2)*b^4*d^10 - 2648*(d*x)^(15/2)*a*b^3*d^12 - 3130*(d*
x)^(11/2)*a^2*b^2*d^14 - 1760*(d*x)^(7/2)*a^3*b*d^16 - 385*(d*x)^(3/2)*a^4*
d^18)/(a*b^9*d^10*x^10 + 5*a^2*b^8*d^10*x^8 + 10*a^3*b^7*d^10*x^6 + 10*a^4*
b^6*d^10*x^4 + 5*a^5*b^5*d^10*x^2 + a^6*b^4*d^10))/d
```

mupad [B] time = 4.29, size = 210, normalized size = 0.54

$$\frac{231\,d^{17/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{5/4}\,b^{19/4}}-\frac{231\,d^{17/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{5/4}\,b^{19/4}}-\frac{\frac{331\,d^{11}\,(d\,x)^{15/2}}{2560\,b}-\frac{231\,d^9\,(d\,x)^{19/2}}{4096\,a}+\frac{11\,a^2\,d^{15}\,(d\,x)^{7/2}}{128\,b^3}+\frac{77\,a^3\,d^{17}\,(\cdots}{4096\cdots}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(17/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (231*d^(17/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)
^(5/4)*b^(19/4)) - (231*d^(17/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(
1/2))))/(8192*(-a)^(5/4)*b^(19/4)) - ((331*d^11*(d*x)^(15/2))/(2560*b) - (2
31*d^9*(d*x)^(19/2))/(4096*a) + (11*a^2*d^15*(d*x)^(7/2))/(128*b^3) + (77*a
```

```
^3*d^17*(d*x)^(3/2))/(4096*b^4) + (313*a*d^13*(d*x)^(11/2))/(2048*b^2))/(a^
5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d
^10*x^4 + 10*a^2*b^3*d^10*x^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(17/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
[Out] Timed out
```

3.716 $\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=388

$$-\frac{117d^{15/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{7/4}b^{17/4}}+\frac{117d^{15/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{7/4}b^{17/4}}-\frac{117}{}$$

```
[Out] -1/10*d*(d*x)^(13/2)/b/(b*x^2+a)^5-13/160*d^3*(d*x)^(9/2)/b^2/(b*x^2+a)^4-3
9/640*d^5*(d*x)^(5/2)/b^3/(b*x^2+a)^3-117/16384*d^(15/2)*arctan(1-b^(1/4)*2
^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(7/4)/b^(17/4)*2^(1/2)+117/16384*d^(1
5/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(7/4)/b^(17/4)
*2^(1/2)-117/32768*d^(15/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^
(1/4)*2^(1/2)*(d*x)^(1/2))/a^(7/4)/b^(17/4)*2^(1/2)+117/32768*d^(15/2)*ln(a
^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(7/
4)/b^(17/4)*2^(1/2)-39/1024*d^7*(d*x)^(1/2)/b^4/(b*x^2+a)^2+39/4096*d^7*(d*
x)^(1/2)/a/b^4/(b*x^2+a)
```

Rubi [A] time = 0.45, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{117d^{15/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{7/4}b^{17/4}}+\frac{117d^{15/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{7/4}b^{17/4}}-\frac{117}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(13/2))/(10*b*(a + b*x^2)^5) - (13*d^3*(d*x)^(9/2))/(160*b^2*(a +
 b*x^2)^4) - (39*d^5*(d*x)^(5/2))/(640*b^3*(a + b*x^2)^3) - (39*d^7*Sqrt[d*
x])/(1024*b^4*(a + b*x^2)^2) + (39*d^7*Sqrt[d*x])/(4096*a*b^4*(a + b*x^2))
- (117*d^(15/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/
(8192*Sqrt[2]*a^(7/4)*b^(17/4)) + (117*d^(15/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(7/4)*b^(17/4)) - (117*d^(1
5/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d*x]])/(16384*Sqrt[2]*a^(7/4)*b^(17/4)) + (117*d^(15/2)*Log[Sqrt[a]*Sqrt[d
] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*
a^(7/4)*b^(17/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(13b^4d^2\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{320}\left(117b^2d^4\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} + \frac{1}{256}\left(39d^6\right)\int \cdots$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2} + \cdots$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2} + \cdots$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2} + \cdots$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2} + \cdots$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2} + \cdots$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2} + \cdots$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2} + \cdots$$

Mathematica [A] time = 0.26, size = 359, normalized size = 0.93

$$\frac{d^7\sqrt{dx}\left(-\frac{45045\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}\sqrt{x}}+\frac{45045\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}\sqrt{x}}-\frac{90090\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{7/4}\sqrt{x}}+\frac{90090 \cdots}{\cdots}\right.}{1\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^7*Sqrt[d*x]*((-638976*a^3*b^(1/4))/(a + b*x^2)^5 - (2555904*a^2*b^(5/4)*
x^2)/(a + b*x^2)^5 - (4259840*a*b^(9/4)*x^4)/(a + b*x^2)^5 - (3604480*b^(13
/4)*x^6)/(a + b*x^2)^5 + (39936*a^2*b^(1/4))/(a + b*x^2)^4 + (49920*a*b^(1/
4))/(a + b*x^2)^3 + (68640*b^(1/4))/(a + b*x^2)^2 + (120120*b^(1/4))/(a^2 +
 a*b*x^2) - (90090*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
a^(7/4)*Sqrt[x]) + (90090*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(a^(7/4)*Sqrt[x]) - (45045*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(a^(7/4)*Sqrt[x]) + (45045*Sqrt[2]*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(7/4)*Sqrt[x])))/(12615680
*b^(17/4))
```

fricas [A] time = 1.14, size = 505, normalized size = 1.30

$$\frac{2340\left(ab^9x^{10}+5a^2b^8x^8+10a^3b^7x^6+10a^4b^6x^4+5a^5b^5x^2+a^6b^4\right)\left(-\frac{d^{30}}{a^7b^{17}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{30}}{a^7b^{17}}\right)^{\frac{3}{4}}\sqrt{dx}\,a^5b^{13}d^7-\sqrt{d^{15}\cdots}}{\cdots}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(2340*(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4
 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^30/(a^7*b^17))^(1/4)*arctan(-((-d^30/(a^7*b
^17))^(3/4)*sqrt(d*x)*a^5*b^13*d^7 - sqrt(d^15*x + sqrt(-d^30/(a^7*b^17))*a
^4*b^8)*(-d^30/(a^7*b^17))^(3/4)*a^5*b^13)/d^30) + 585*(a*b^9*x^10 + 5*a^2*
b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^30
/(a^7*b^17))^(1/4)*log(117*sqrt(d*x)*d^7 + 117*(-d^30/(a^7*b^17))^(1/4)*a^2
*b^4) - 585*(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 +
 5*a^5*b^5*x^2 + a^6*b^4)*(-d^30/(a^7*b^17))^(1/4)*log(117*sqrt(d*x)*d^7 -
117*(-d^30/(a^7*b^17))^(1/4)*a^2*b^4) + 4*(195*b^4*d^7*x^8 - 4960*a*b^3*d^7
*x^6 - 5330*a^2*b^2*d^7*x^4 - 2808*a^3*b*d^7*x^2 - 585*a^4*d^7)*sqrt(d*x))/
(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x
^2 + a^6*b^4)
```

giac [A] time = 0.22, size = 342, normalized size = 0.88

$$\frac{1}{163840}d^7\left(\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^5}+\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^5}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^7*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^5) + 1170*sqrt(2)*(a*b
^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(
a*d^2/b)^(1/4))/(a^2*b^5) + 585*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)
*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^5) - 585*sqrt(2)*(a*b^3*
d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^
2*b^5) + 8*(195*sqrt(d*x)*b^4*d^10*x^8 - 4960*sqrt(d*x)*a*b^3*d^10*x^6 - 53
30*sqrt(d*x)*a^2*b^2*d^10*x^4 - 2808*sqrt(d*x)*a^3*b*d^10*x^2 - 585*sqrt(d*
x)*a^4*d^10)/((b*d^2*x^2 + a*d^2)^5*a*b^4))
```

maple [A] time = 0.02, size = 341, normalized size = 0.88

$$-\frac{117\sqrt{dx}\,a^3d^{17}}{4096\left(b\,d^2x^2+d^2a\right)^5b^4}-\frac{351\,(dx)^{\frac{5}{2}}\,a^2d^{15}}{2560\left(b\,d^2x^2+d^2a\right)^5b^3}-\frac{533\,(dx)^{\frac{9}{2}}\,a\,d^{13}}{2048\left(b\,d^2x^2+d^2a\right)^5b^2}-\frac{31\,(dx)^{\frac{13}{2}}\,d^{11}}{128\left(b\,d^2x^2+d^2a\right)^5b}+\frac{39\,(dx)}{4096\left(b\,d^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -117/4096*d^17/(b*d^2*x^2+a*d^2)^5/b^4*a^3*(d*x)^(1/2)-351/2560*d^15/(b*d^2
*x^2+a*d^2)^5/b^3*a^2*(d*x)^(5/2)-533/2048*d^13/(b*d^2*x^2+a*d^2)^5/b^2*a*(
d*x)^(9/2)-31/128*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(13/2)+39/4096*d^9/(b*d^
2*x^2+a*d^2)^5/a*(d*x)^(17/2)+117/32768*d^7/a^2/b^4*(a/b*d^2)^(1/4)*2^(1/2)
*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2
```

```
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+117/16384*d^7/a^2/b^4*(a/b*d^
2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+117/16384*d^
7/a^2/b^4*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2
)-1)
```

maxima [A] time = 3.22, size = 392, normalized size = 1.01

$$\frac{\frac{8\left(195\,(dx)^{\frac{17}{2}}b^4d^{10}-4960\,(dx)^{\frac{13}{2}}ab^3d^{12}-5330\,(dx)^{\frac{9}{2}}a^2b^2d^{14}-2808\,(dx)^{\frac{5}{2}}a^3bd^{16}-585\,\sqrt{dx}\,a^4d^{18}\right)}{ab^9d^{10}x^{10}+5\,a^2b^8d^{10}x^8+10\,a^3b^7d^{10}x^6+10\,a^4b^6d^{10}x^4+5\,a^5b^5d^{10}x^2+a^6b^4d^{10}}+\frac{585\left(\frac{\sqrt{2}\,d^{10}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\cdots\right.}{\cdots}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/163840*(8*(195*(d*x)^(17/2)*b^4*d^10 - 4960*(d*x)^(13/2)*a*b^3*d^12 - 533
0*(d*x)^(9/2)*a^2*b^2*d^14 - 2808*(d*x)^(5/2)*a^3*b*d^16 - 585*sqrt(d*x)*a^
4*d^18)/(a*b^9*d^10*x^10 + 5*a^2*b^8*d^10*x^8 + 10*a^3*b^7*d^10*x^6 + 10*a^
4*b^6*d^10*x^4 + 5*a^5*b^5*d^10*x^2 + a^6*b^4*d^10) + 585*(sqrt(2)*d^10*log
(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2
)^(3/4)*b^(1/4)) - sqrt(2)*d^10*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqr
t(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^9*arctan(
1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt
(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^9*arctan(-1
/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(
a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/(a*b^4))/d
```

mupad [B] time = 0.13, size = 210, normalized size = 0.54

$$\frac{117\,d^{15/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{7/4}\,b^{17/4}}-\frac{\frac{31\,d^{11}\,(d\,x)^{13/2}}{128\,b}-\frac{39\,d^9\,(d\,x)^{17/2}}{4096\,a}+\frac{351\,a^2\,d^{15}\,(d\,x)^{5/2}}{2560\,b^3}+\frac{117\,a^3\,d^{17}\,\sqrt{d\,x}}{4096\,b^4}+\frac{533\,a\,d^{13}\,(d\,x)^{9/2}}{2048\,b^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (117*d^(15/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^
(7/4)*b^(17/4)) - ((31*d^11*(d*x)^(13/2))/(128*b) - (39*d^9*(d*x)^(17/2))/(
4096*a) + (351*a^2*d^15*(d*x)^(5/2))/(2560*b^3) + (117*a^3*d^17*(d*x)^(1/2)
```

```
)/(4096*b^4) + (533*a*d^13*(d*x)^(9/2))/(2048*b^2))/(a^5*d^10 + b^5*d^10*x^
10 + 5*a^4*b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3
*d^10*x^6) + (117*d^(15/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))
))/(8192*(-a)^(7/4)*b^(17/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

3.717 $\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=391

$$\frac{77d^{13/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{9/4}b^{15/4}}-\frac{77d^{13/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{9/4}b^{15/4}}-\frac{77d^{13/}}{}$$

```
[Out] -1/10*d*(d*x)^(11/2)/b/(b*x^2+a)^5-11/160*d^3*(d*x)^(7/2)/b^2/(b*x^2+a)^4-7
7/1920*d^5*(d*x)^(3/2)/b^3/(b*x^2+a)^3+77/5120*d^5*(d*x)^(3/2)/a/b^3/(b*x^2
+a)^2+77/4096*d^5*(d*x)^(3/2)/a^2/b^3/(b*x^2+a)-77/16384*d^(13/2)*arctan(1-
b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(9/4)/b^(15/4)*2^(1/2)+77/16
384*d^(13/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(9/4)/
b^(15/4)*2^(1/2)+77/32768*d^(13/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(
1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(9/4)/b^(15/4)*2^(1/2)-77/32768*d^(13/2
)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))
/a^(9/4)/b^(15/4)*2^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{77d^5(dx)^{3/2}}{4096a^2b^3\left(a+bx^2\right)}+\frac{77d^{13/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{9/4}b^{15/4}}-\frac{77d^{13/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{a}\right.}{16384\sqrt{2}\,a^{9/4}b^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(11/2))/(10*b*(a + b*x^2)^5) - (11*d^3*(d*x)^(7/2))/(160*b^2*(a +
 b*x^2)^4) - (77*d^5*(d*x)^(3/2))/(1920*b^3*(a + b*x^2)^3) + (77*d^5*(d*x)^
(3/2))/(5120*a*b^3*(a + b*x^2)^2) + (77*d^5*(d*x)^(3/2))/(4096*a^2*b^3*(a +
 b*x^2)) - (77*d^(13/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])])/(8192*Sqrt[2]*a^(9/4)*b^(15/4)) + (77*d^(13/2)*ArcTan[1 + (Sqrt[2]*
b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(9/4)*b^(15/4)) + (7
7*d^(13/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(9/4)*b^(15/4)) - (77*d^(13/2)*Log[Sqrt[a]*S
qrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqr
t[2]*a^(9/4)*b^(15/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(11b^4d^2\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{320}\left(77b^2d^4\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{\left(77d^6\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^3}\,dx}{1280}$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\left(a+bx^2\right)^2} +$$

Mathematica [C] time = 0.03, size = 85, normalized size = 0.22

$$\frac{2d^6x\sqrt{dx}\left(77\left(a+bx^2\right)^5\ {}_2F_1\left(\frac{3}{4},6;\frac{7}{4};-\frac{bx^2}{a}\right)-a^3\left(77a^2+187abx^2+221b^2x^4\right)\right)}{1989a^3b^3\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*d^6*x*Sqrt[d*x]*(-(a^3*(77*a^2 + 187*a*b*x^2 + 221*b^2*x^4)) + 77*(a + b
*x^2)^5*Hypergeometric2F1[3/4, 6, 7/4, -((b*x^2)/a)]))/(1989*a^3*b^3*(a + b
*x^2)^5)
```

fricas [A] time = 1.01, size = 518, normalized size = 1.32

$$-\frac{4620\left(a^2b^8x^{10}+5\,a^3b^7x^8+10\,a^4b^6x^6+10\,a^5b^5x^4+5\,a^6b^4x^2+a^7b^3\right)\left(-\frac{d^{26}}{a^9b^{15}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{26}}{a^9b^{15}}\right)^{\frac{1}{4}}\sqrt{dx}\,a^2b^4d^{19}-\sqrt{\cdots}}{\cdots}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/245760*(4620*(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5
*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)*(-d^26/(a^9*b^15))^(1/4)*arctan(-((-d^26/(a
^9*b^15))^(1/4)*sqrt(d*x)*a^2*b^4*d^19 - sqrt(d^39*x - sqrt(-d^26/(a^9*b^15
))*a^5*b^7*d^26)*(-d^26/(a^9*b^15))^(1/4)*a^2*b^4)/d^26) - 1155*(a^2*b^8*x^
10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*
b^3)*(-d^26/(a^9*b^15))^(1/4)*log(456533*sqrt(d*x)*d^19 + 456533*(-d^26/(a^
9*b^15))^(3/4)*a^7*b^11) + 1155*(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*
x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)*(-d^26/(a^9*b^15))^(1/4)*lo
g(456533*sqrt(d*x)*d^19 - 456533*(-d^26/(a^9*b^15))^(3/4)*a^7*b^11) - 4*(11
55*b^4*d^6*x^9 + 5544*a*b^3*d^6*x^7 - 3130*a^2*b^2*d^6*x^5 - 1760*a^3*b*d^6
*x^3 - 385*a^4*d^6*x)*sqrt(d*x))/(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6
*x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)
```

giac [A] time = 0.23, size = 355, normalized size = 0.91

$$\frac{1}{491520}\,d^{6}\left(\frac{2310\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{3}b^{6}d}+\frac{2310\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{3}b^{6}d}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/491520*d^6*(2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^6*d) + 2310*sqrt(2)*(a
*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))
/(a*d^2/b)^(1/4))/(a^3*b^6*d) - 1155*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sq
rt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^6*d) + 1155*sqrt(2)
*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2
/b))/(a^3*b^6*d) + 8*(1155*sqrt(d*x)*b^4*d^10*x^9 + 5544*sqrt(d*x)*a*b^3*d^
10*x^7 - 3130*sqrt(d*x)*a^2*b^2*d^10*x^5 - 1760*sqrt(d*x)*a^3*b*d^10*x^3 -
385*sqrt(d*x)*a^4*d^10*x)/((b*d^2*x^2 + a*d^2)^5*a^2*b^3))
```

maple [A] time = 0.02, size = 339, normalized size = 0.87

$$-\frac{77\,(dx)^{\frac{3}{2}}\,a^{2}d^{15}}{12288\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}b^{3}}-\frac{11\,(dx)^{\frac{7}{2}}\,a\,d^{13}}{384\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}b^{2}}-\frac{313\,(dx)^{\frac{11}{2}}\,d^{11}}{6144\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}b}+\frac{231\,(dx)^{\frac{15}{2}}\,d^{9}}{2560\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a}+\frac{77\,(dx)}{4096\left(b\,d^{2}x^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -77/12288*d^15/(b*d^2*x^2+a*d^2)^5/b^3*a^2*(d*x)^(3/2)-11/384*d^13/(b*d^2*x
^2+a*d^2)^5/b^2*a*(d*x)^(7/2)-313/6144*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(11
/2)+231/2560*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^(15/2)+77/4096*d^7/(b*d^2*x^2+
a*d^2)^5/a^2*b*(d*x)^(19/2)+77/32768*d^7/a^2/b^4/(a/b*d^2)^(1/4)*2^(1/2)*ln
((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(
```

```
1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+77/16384*d^7/a^2/b^4/(a/b*d^2)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+77/16384*d^7/a^2
/b^4/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.00, size = 385, normalized size = 0.98

$$\frac{\frac{1155\,d^8\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx-\sqrt{2}\cdots\right.}{\left(ad\cdots\right.}\right.}{a^2b^3}}{491520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/491520*(1155*d^8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(
1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*
d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4)
- 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sq
rt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
+ sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a
*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^2*b^
3) + 8*(1155*(d*x)^(19/2)*b^4*d^8 + 5544*(d*x)^(15/2)*a*b^3*d^10 - 3130*(d*
x)^(11/2)*a^2*b^2*d^12 - 1760*(d*x)^(7/2)*a^3*b*d^14 - 385*(d*x)^(3/2)*a^4*
d^16)/(a^2*b^8*d^10*x^10 + 5*a^3*b^7*d^10*x^8 + 10*a^4*b^6*d^10*x^6 + 10*a^
5*b^5*d^10*x^4 + 5*a^6*b^4*d^10*x^2 + a^7*b^3*d^10))/d
```

mupad [B] time = 4.32, size = 208, normalized size = 0.53

$$\frac{77\,d^{13/2}\,\mathrm{atan}\left(\frac{b^{1/4}\sqrt{d\,x}}{(-a)^{1/4}\sqrt{d}}\right)}{8192\,(-a)^{9/4}\,b^{15/4}}-\frac{\frac{313\,d^{11}\,(d\,x)^{11/2}}{6144\,b}-\frac{231\,d^9\,(d\,x)^{15/2}}{2560\,a}+\frac{77\,a^2\,d^{15}\,(d\,x)^{3/2}}{12288\,b^3}+\frac{11\,a\,d^{13}\,(d\,x)^{7/2}}{384\,b^2}-\frac{77\,b\,d^7\,(d\,x)^{19/2}}{4096\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}-\frac{77\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (77*d^(13/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(
9/4)*b^(15/4)) - ((313*d^11*(d*x)^(11/2))/(6144*b) - (231*d^9*(d*x)^(15/2))
/(2560*a) + (77*a^2*d^15*(d*x)^(3/2))/(12288*b^3) + (11*a*d^13*(d*x)^(7/2))
/(384*b^2) - (77*b*d^7*(d*x)^(19/2))/(4096*a^2))/(a^5*d^10 + b^5*d^10*x^10
```

```
+ 5*a^4*b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^
10*x^6) - (77*d^(13/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(
8192*(-a)^(9/4)*b^(15/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

3.718 $\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=391

$$-\frac{63d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{11/4}b^{13/4}}+\frac{63d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{11/4}b^{13/4}}-\frac{63d^{1}}{}$$

```
[Out] -1/10*d*(d*x)^(9/2)/b/(b*x^2+a)^5-9/160*d^3*(d*x)^(5/2)/b^2/(b*x^2+a)^4-63/
16384*d^(11/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(11/
4)/b^(13/4)*2^(1/2)+63/16384*d^(11/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/
a^(1/4)/d^(1/2))/a^(11/4)/b^(13/4)*2^(1/2)-63/32768*d^(11/2)*ln(a^(1/2)*d^(
1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(11/4)/b^(13/
4)*2^(1/2)+63/32768*d^(11/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b
^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(11/4)/b^(13/4)*2^(1/2)-3/128*d^5*(d*x)^(1/2)
/b^3/(b*x^2+a)^3+3/1024*d^5*(d*x)^(1/2)/a/b^3/(b*x^2+a)^2+21/4096*d^5*(d*x)
^(1/2)/a^2/b^3/(b*x^2+a)
```

Rubi [A] time = 0.47, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{21d^5\sqrt{dx}}{4096a^2b^3\left(a+bx^2\right)}-\frac{63d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{11/4}b^{13/4}}+\frac{63d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}\right.}{16384\sqrt{2}\,a^{11/4}b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(9/2))/(10*b*(a + b*x^2)^5) - (9*d^3*(d*x)^(5/2))/(160*b^2*(a + b
*x^2)^4) - (3*d^5*Sqrt[d*x])/(128*b^3*(a + b*x^2)^3) + (3*d^5*Sqrt[d*x])/(1
024*a*b^3*(a + b*x^2)^2) + (21*d^5*Sqrt[d*x])/(4096*a^2*b^3*(a + b*x^2)) -
(63*d^(11/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(81
92*Sqrt[2]*a^(11/4)*b^(13/4)) + (63*d^(11/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(11/4)*b^(13/4)) - (63*d^(11/2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(16384*Sqrt[2]*a^(11/4)*b^(13/4)) + (63*d^(11/2)*Log[Sqrt[a]*Sqrt[d] +
 Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(
11/4)*b^(13/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(9b^4d^2\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{64}\left(9b^2d^4\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{1}{256}\left(3d^6\right)\int \frac{1}{\sqrt{dx}\,(ab+ \cdots}$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2} + \cdots$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2} + \cdots$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2} + \cdots$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2} + \cdots$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2} + \cdots$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2} + \cdots$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2} + \cdots$$

Mathematica [A] time = 0.18, size = 337, normalized size = 0.86

$$\frac{d^5\sqrt{dx}\left(-\frac{3465\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{11/4}\sqrt{x}}+\frac{3465\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{11/4}\sqrt{x}}-\frac{6930\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{11/4}\sqrt{x}}+\frac{6930\sqrt{2}\ t}{}\right.}{1802240b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^5*Sqrt[d*x]*((-49152*a^2*b^(1/4))/(a + b*x^2)^5 - (196608*a*b^(5/4)*x^2)
/(a + b*x^2)^5 - (327680*b^(9/4)*x^4)/(a + b*x^2)^5 + (3072*a*b^(1/4))/(a +
 b*x^2)^4 + (3840*b^(1/4))/(a + b*x^2)^3 + (5280*b^(1/4))/(a*(a + b*x^2)^2)
 + (9240*b^(1/4))/(a^2*(a + b*x^2)) - (6930*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)])/(a^(11/4)*Sqrt[x]) + (6930*Sqrt[2]*ArcTan[1 + (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(11/4)*Sqrt[x]) - (3465*Sqrt[2]*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(11/4)*Sqrt[x]) + (34
65*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(
11/4)*Sqrt[x])))/(1802240*b^(13/4))
```

fricas [A] time = 1.09, size = 513, normalized size = 1.31

$$\frac{1260\left(a^2b^8x^{10}+5a^3b^7x^8+10a^4b^6x^6+10a^5b^5x^4+5a^6b^4x^2+a^7b^3\right)\left(-\frac{d^{22}}{a^{11}b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^8b^{10}d^5\left(-\frac{d^{22}}{a^{11}b^{13}}\right)^{\frac{3}{4}}-\sqrt{}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(1260*(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x
^4 + 5*a^6*b^4*x^2 + a^7*b^3)*(-d^22/(a^11*b^13))^(1/4)*arctan(-(sqrt(d*x)*
a^8*b^10*d^5*(-d^22/(a^11*b^13))^(3/4) - sqrt(a^6*b^6*sqrt(-d^22/(a^11*b^13
)) + d^11*x)*a^8*b^10*(-d^22/(a^11*b^13))^(3/4))/d^22) + 315*(a^2*b^8*x^10
+ 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3
)*(-d^22/(a^11*b^13))^(1/4)*log(63*a^3*b^3*(-d^22/(a^11*b^13))^(1/4) + 63*s
qrt(d*x)*d^5) - 315*(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5
*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)*(-d^22/(a^11*b^13))^(1/4)*log(-63*a^3*b
^3*(-d^22/(a^11*b^13))^(1/4) + 63*sqrt(d*x)*d^5) + 4*(105*b^4*d^5*x^8 + 480
*a*b^3*d^5*x^6 - 2870*a^2*b^2*d^5*x^4 - 1512*a^3*b*d^5*x^2 - 315*a^4*d^5)*s
qrt(d*x))/(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x^4 +
 5*a^6*b^4*x^2 + a^7*b^3)
```

giac [A] time = 0.21, size = 342, normalized size = 0.87

$$\frac{1}{163840}d^5\left(\frac{630\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^4}+\frac{630\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^4}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^5*(630*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^4) + 630*sqrt(2)*(a*b^3
*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*
d^2/b)^(1/4))/(a^3*b^4) + 315*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(
a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^4) - 315*sqrt(2)*(a*b^3*d^
2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*
b^4) + 8*(105*sqrt(d*x)*b^4*d^10*x^8 + 480*sqrt(d*x)*a*b^3*d^10*x^6 - 2870*
sqrt(d*x)*a^2*b^2*d^10*x^4 - 1512*sqrt(d*x)*a^3*b*d^10*x^2 - 315*sqrt(d*x)*
a^4*d^10)/((b*d^2*x^2 + a*d^2)^5*a^2*b^3))
```

maple [A] time = 0.02, size = 339, normalized size = 0.87

$$-\frac{63\sqrt{dx}\,a^2d^{15}}{4096\left(b\,d^2x^2+d^2a\right)^5b^3}-\frac{189\,(dx)^{\frac{5}{2}}\,a\,d^{13}}{2560\left(b\,d^2x^2+d^2a\right)^5b^2}-\frac{287\,(dx)^{\frac{9}{2}}\,d^{11}}{2048\left(b\,d^2x^2+d^2a\right)^5b}+\frac{3\,(dx)^{\frac{13}{2}}\,d^9}{128\left(b\,d^2x^2+d^2a\right)^5a}+\frac{21\,(dx)^{\frac{17}{2}}}{4096\left(b\,d^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -63/4096*d^15/(b*d^2*x^2+a*d^2)^5/b^3*a^2*(d*x)^(1/2)-189/2560*d^13/(b*d^2*
x^2+a*d^2)^5/b^2*a*(d*x)^(5/2)-287/2048*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(9
/2)+3/128*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^(13/2)+21/4096*d^7/(b*d^2*x^2+a*d
^2)^5/a^2*b*(d*x)^(17/2)+63/32768*d^5/a^3/b^3*(a/b*d^2)^(1/4)*2^(1/2)*ln((d
*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4
```

```
)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+63/16384*d^5/a^3/b^3*(a/b*d^2)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+63/16384*d^5/a^3/b^
3*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.16, size = 394, normalized size = 1.01

$$\frac{\frac{8\left(105\,(dx)^{\frac{17}{2}}b^4d^8+480\,(dx)^{\frac{13}{2}}ab^3d^{10}-2870\,(dx)^{\frac{9}{2}}a^2b^2d^{12}-1512\,(dx)^{\frac{5}{2}}a^3bd^{14}-315\,\sqrt{dx}\,a^4d^{16}\right)}{a^2b^8d^{10}x^{10}+5\,a^3b^7d^{10}x^8+10\,a^4b^6d^{10}x^6+10\,a^5b^5d^{10}x^4+5\,a^6b^4d^{10}x^2+a^7b^3d^{10}}+\frac{315\left(\frac{\sqrt{2}d^8\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d\cdots}{\cdots}\right.}{\cdots}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/163840*(8*(105*(d*x)^(17/2)*b^4*d^8 + 480*(d*x)^(13/2)*a*b^3*d^10 - 2870*
(d*x)^(9/2)*a^2*b^2*d^12 - 1512*(d*x)^(5/2)*a^3*b*d^14 - 315*sqrt(d*x)*a^4*
d^16)/(a^2*b^8*d^10*x^10 + 5*a^3*b^7*d^10*x^8 + 10*a^4*b^6*d^10*x^6 + 10*a^
5*b^5*d^10*x^4 + 5*a^6*b^4*d^10*x^2 + a^7*b^3*d^10) + 315*(sqrt(2)*d^8*log(
sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)
^(3/4)*b^(1/4)) - sqrt(2)*d^8*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(
d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^7*arctan(1/
2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a
)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^7*arctan(-1/2
*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)
*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/(a^2*b^3))/d
```

mupad [B] time = 4.23, size = 208, normalized size = 0.53

$$-\frac{\frac{287\,d^{11}\,(d\,x)^{9/2}}{2048\,b}-\frac{3\,d^9\,(d\,x)^{13/2}}{128\,a}+\frac{63\,a^2\,d^{15}\,\sqrt{d\,x}}{4096\,b^3}+\frac{189\,a\,d^{13}\,(d\,x)^{5/2}}{2560\,b^2}-\frac{21\,b\,d^7\,(d\,x)^{17/2}}{4096\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}-\frac{63\,d^{11/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{11/4}\,b^{13/4}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] - ((287*d^11*(d*x)^(9/2))/(2048*b) - (3*d^9*(d*x)^(13/2))/(128*a) + (63*a^2
*d^15*(d*x)^(1/2))/(4096*b^3) + (189*a*d^13*(d*x)^(5/2))/(2560*b^2) - (21*b
*d^7*(d*x)^(17/2))/(4096*a^2))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^2
 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6) - (63*d^(1
```

```
1/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(11/4)*b^
(13/4)) - (63*d^(11/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(
8192*(-a)^(11/4)*b^(13/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

3.719 $\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=394

$$\frac{63d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{13/4}b^{11/4}}-\frac{63d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{13/4}b^{11/4}}-\frac{63d^{9/2}\,\mathrm{ta}}{81}$$

```
[Out] -1/10*d*(d*x)^(7/2)/b/(b*x^2+a)^5-7/160*d^3*(d*x)^(3/2)/b^2/(b*x^2+a)^4+7/6
40*d^3*(d*x)^(3/2)/a/b^2/(b*x^2+a)^3+63/5120*d^3*(d*x)^(3/2)/a^2/b^2/(b*x^2
+a)^2+63/4096*d^3*(d*x)^(3/2)/a^3/b^2/(b*x^2+a)-63/16384*d^(9/2)*arctan(1-b
^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(13/4)/b^(11/4)*2^(1/2)+63/16
384*d^(9/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(13/4)/
b^(11/4)*2^(1/2)+63/32768*d^(9/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1
/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(13/4)/b^(11/4)*2^(1/2)-63/32768*d^(9/2)
*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/
a^(13/4)/b^(11/4)*2^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{63d^3(dx)^{3/2}}{4096a^3b^2\left(a+bx^2\right)}+\frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2}+\frac{63d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{13/4}b^{11/4}}-\frac{63d^{9/2}\log\left(\sqrt{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(7/2))/(10*b*(a + b*x^2)^5) - (7*d^3*(d*x)^(3/2))/(160*b^2*(a + b
*x^2)^4) + (7*d^3*(d*x)^(3/2))/(640*a*b^2*(a + b*x^2)^3) + (63*d^3*(d*x)^(3
/2))/(5120*a^2*b^2*(a + b*x^2)^2) + (63*d^3*(d*x)^(3/2))/(4096*a^3*b^2*(a +
 b*x^2)) - (63*d^(9/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(8192*Sqrt[2]*a^(13/4)*b^(11/4)) + (63*d^(9/2)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(13/4)*b^(11/4)) + (6
3*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(16384*Sqrt[2]*a^(13/4)*b^(11/4)) - (63*d^(9/2)*Log[Sqrt[a]*Sq
rt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt
[2]*a^(13/4)*b^(11/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(7b^4d^2\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{320}\left(21b^2d^4\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{\left(63bd^4\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^3}\,dx}{1280a}$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2} +$$

Mathematica [C] time = 0.03, size = 61, normalized size = 0.15

$$\frac{2d^4x\sqrt{dx}\left(\frac{7\,{}_2F_1\left(\frac{3}{4},6;\frac{7}{4};-\frac{bx^2}{a}\right)}{a^4}+\frac{-7a-17bx^2}{\left(a+bx^2\right)^5}\right)}{221b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*d^4*x*Sqrt[d*x]*((-7*a - 17*b*x^2)/(a + b*x^2)^5 + (7*Hypergeometric2F1[
3/4, 6, 7/4, -((b*x^2)/a)])/a^4))/(221*b^2)
```

fricas [A] time = 0.90, size = 520, normalized size = 1.32

$$-\frac{1260\left(a^3b^7x^{10}+5\,a^4b^6x^8+10\,a^5b^5x^6+10\,a^6b^4x^4+5\,a^7b^3x^2+a^8b^2\right)\left(-\frac{d^{18}}{a^{13}b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{250047\,\sqrt{dx}\,a^3b^3d^{13}\left(-\frac{d^{1}}{a^{13}}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(1260*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*
x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d^18/(a^13*b^11))^(1/4)*arctan(-1/250047*(
250047*sqrt(d*x)*a^3*b^3*d^13*(-d^18/(a^13*b^11))^(1/4) - sqrt(-62523502209
*a^7*b^5*d^18*sqrt(-d^18/(a^13*b^11)) + 62523502209*d^27*x)*a^3*b^3*(-d^18/
(a^13*b^11))^(1/4))/d^18) - 315*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*
x^6 + 10*a^6*b^4*x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d^18/(a^13*b^11))^(1/4)*l
og(250047*a^10*b^8*(-d^18/(a^13*b^11))^(3/4) + 250047*sqrt(d*x)*d^13) + 315
*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a^7*b^
3*x^2 + a^8*b^2)*(-d^18/(a^13*b^11))^(1/4)*log(-250047*a^10*b^8*(-d^18/(a^1
3*b^11))^(3/4) + 250047*sqrt(d*x)*d^13) - 4*(315*b^4*d^4*x^9 + 1512*a*b^3*d
^4*x^7 + 2870*a^2*b^2*d^4*x^5 - 480*a^3*b*d^4*x^3 - 105*a^4*d^4*x)*sqrt(d*x
))/(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a^7*
b^3*x^2 + a^8*b^2)
```

giac [A] time = 0.26, size = 355, normalized size = 0.90

$$\frac{1}{163840}d^4\left(\frac{630\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^5d}+\frac{630\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^5d}-\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^4*(630*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^5*d) + 630*sqrt(2)*(a*b
^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(
a*d^2/b)^(1/4))/(a^4*b^5*d) - 315*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(
2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^5*d) + 315*sqrt(2)*(a*
b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))
/(a^4*b^5*d) + 8*(315*sqrt(d*x)*b^4*d^10*x^9 + 1512*sqrt(d*x)*a*b^3*d^10*x^
7 + 2870*sqrt(d*x)*a^2*b^2*d^10*x^5 - 480*sqrt(d*x)*a^3*b*d^10*x^3 - 105*sq
rt(d*x)*a^4*d^10*x)/((b*d^2*x^2 + a*d^2)^5*a^3*b^2))
```

maple [A] time = 0.03, size = 339, normalized size = 0.86

$$-\frac{21\left(dx\right)^{\frac{3}{2}}a\,d^{13}}{4096\left(b\,d^2x^2+d^2a\right)^5b^2}-\frac{3\left(dx\right)^{\frac{7}{2}}d^{11}}{128\left(b\,d^2x^2+d^2a\right)^5b}+\frac{287\left(dx\right)^{\frac{11}{2}}d^9}{2048\left(b\,d^2x^2+d^2a\right)^5a}+\frac{189\left(dx\right)^{\frac{15}{2}}b\,d^7}{2560\left(b\,d^2x^2+d^2a\right)^5a^2}+\frac{63\left(dx\right)^{\frac{1}{2}}\cdots}{4096\left(b\,d^2x^2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -21/4096*d^13/(b*d^2*x^2+a*d^2)^5/b^2*a*(d*x)^(3/2)-3/128*d^11/(b*d^2*x^2+a
*d^2)^5/b*(d*x)^(7/2)+287/2048*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^(11/2)+189/2
560*d^7/(b*d^2*x^2+a*d^2)^5/a^2*b*(d*x)^(15/2)+63/4096*d^5/(b*d^2*x^2+a*d^2
)^5/a^3*b^2*(d*x)^(19/2)+63/32768*d^5/a^3/b^3/(a/b*d^2)^(1/4)*2^(1/2)*ln((d
*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4
```

```
)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+63/16384*d^5/a^3/b^3/(a/b*d^2)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+63/16384*d^5/a^3/b^
3/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.17, size = 385, normalized size = 0.98

$$\frac{\frac{315\,d^6\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx-\sqrt{2}\left(\cdots\right.\right.}{\left(ad^2\cdots\right.}\right.}{a^3b^2}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/163840*(315*d^6*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1
/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d
)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) -
 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqr
t(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) +
 sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*
d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^3*b^2
) + 8*(315*(d*x)^(19/2)*b^4*d^6 + 1512*(d*x)^(15/2)*a*b^3*d^8 + 2870*(d*x)^
(11/2)*a^2*b^2*d^10 - 480*(d*x)^(7/2)*a^3*b*d^12 - 105*(d*x)^(3/2)*a^4*d^14
)/(a^3*b^7*d^10*x^10 + 5*a^4*b^6*d^10*x^8 + 10*a^5*b^5*d^10*x^6 + 10*a^6*b^
4*d^10*x^4 + 5*a^7*b^3*d^10*x^2 + a^8*b^2*d^10))/d
```

mupad [B] time = 0.12, size = 207, normalized size = 0.53

$$\frac{\frac{287\,d^9\,(d\,x)^{11/2}}{2048\,a}-\frac{3\,d^{11}\,(d\,x)^{7/2}}{128\,b}+\frac{63\,b^2\,d^5\,(d\,x)^{19/2}}{4096\,a^3}-\frac{21\,a\,d^{13}\,(d\,x)^{3/2}}{4096\,b^2}+\frac{189\,b\,d^7\,(d\,x)^{15/2}}{2560\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}-\frac{63\,d^{9/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{13/4}\,b^{11/4}}+\frac{63\,d\cdots}{8\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((287*d^9*(d*x)^(11/2))/(2048*a) - (3*d^11*(d*x)^(7/2))/(128*b) + (63*b^2*d
^5*(d*x)^(19/2))/(4096*a^3) - (21*a*d^13*(d*x)^(3/2))/(4096*b^2) + (189*b*d
^7*(d*x)^(15/2))/(2560*a^2))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^2 +
 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6) - (63*d^(9/2
```

```
)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(13/4)*b^(11
/4)) + (63*d^(9/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192
*(-a)^(13/4)*b^(11/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

3.720 $\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=394

$$-\frac{77d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{15/4}b^{9/4}}+\frac{77d^{7/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{15/4}b^{9/4}}-\frac{77d^{7/2}}{8}$$

```
[Out] -1/10*d*(d*x)^(5/2)/b/(b*x^2+a)^5-77/16384*d^(7/2)*arctan(1-b^(1/4)*2^(1/2)
*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(15/4)/b^(9/4)*2^(1/2)+77/16384*d^(7/2)*arc
tan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(15/4)/b^(9/4)*2^(1/2)
-77/32768*d^(7/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1
/2)*(d*x)^(1/2))/a^(15/4)/b^(9/4)*2^(1/2)+77/32768*d^(7/2)*ln(a^(1/2)*d^(1/
2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(15/4)/b^(9/4)*
2^(1/2)-1/32*d^3*(d*x)^(1/2)/b^2/(b*x^2+a)^4+1/384*d^3*(d*x)^(1/2)/a/b^2/(b
*x^2+a)^3+11/3072*d^3*(d*x)^(1/2)/a^2/b^2/(b*x^2+a)^2+77/12288*d^3*(d*x)^(1
/2)/a^3/b^2/(b*x^2+a)
```

Rubi [A] time = 0.45, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{77d^3\sqrt{dx}}{12288a^3b^2\left(a+bx^2\right)}+\frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2}-\frac{77d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{15/4}b^{9/4}}+\frac{77d^{7/2}\log\left(\sqrt{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(5/2))/(10*b*(a + b*x^2)^5) - (d^3*Sqrt[d*x])/(32*b^2*(a + b*x^2)
^4) + (d^3*Sqrt[d*x])/(384*a*b^2*(a + b*x^2)^3) + (11*d^3*Sqrt[d*x])/(3072*
a^2*b^2*(a + b*x^2)^2) + (77*d^3*Sqrt[d*x])/(12288*a^3*b^2*(a + b*x^2)) - (
77*d^(7/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192
*Sqrt[2]*a^(15/4)*b^(9/4)) + (77*d^(7/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d
*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(15/4)*b^(9/4)) - (77*d^(7/2)*Log[
Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(
16384*Sqrt[2]*a^(15/4)*b^(9/4)) + (77*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]
*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(15/4)*b^
(9/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{4}\left(b^4d^2\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{1}{64}\left(b^2d^4\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{\left(11bd^4\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^3}}{768a}$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2} +$$

Mathematica [A] time = 0.17, size = 317, normalized size = 0.80

$$\frac{d^3\sqrt{dx}\left(-\frac{1155\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{15/4}\sqrt{x}}+\frac{1155\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{15/4}\sqrt{x}}-\frac{2310\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{15/4}\sqrt{x}}+\frac{2310\sqrt{2}\,t}{}\right.}{491520b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^3*Sqrt[d*x]*((-16384*a*b^(1/4))/(a + b*x^2)^5 - (65536*b^(5/4)*x^2)/(a +
 b*x^2)^5 + (1024*b^(1/4))/(a + b*x^2)^4 + (1280*b^(1/4))/(a*(a + b*x^2)^3)
 + (1760*b^(1/4))/(a^2*(a + b*x^2)^2) + (3080*b^(1/4))/(a^3*(a + b*x^2)) -
(2310*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(15/4)*Sqrt
[x]) + (2310*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(15/
4)*Sqrt[x]) - (1155*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(a^(15/4)*Sqrt[x]) + (1155*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(15/4)*Sqrt[x])))/(491520*b^(9/4))
```

fricas [A] time = 1.05, size = 513, normalized size = 1.30

$$\frac{4620\left(a^3b^7x^{10}+5a^4b^6x^8+10a^5b^5x^6+10a^6b^4x^4+5a^7b^3x^2+a^8b^2\right)\left(-\frac{d^{14}}{a^{15}b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^{11}b^7d^3\left(-\frac{d^{14}}{a^{15}b^9}\right)^{\frac{3}{4}}-\sqrt{a^8}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/245760*(4620*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*
x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d^14/(a^15*b^9))^(1/4)*arctan(-(sqrt(d*x)*
a^11*b^7*d^3*(-d^14/(a^15*b^9))^(3/4) - sqrt(a^8*b^4*sqrt(-d^14/(a^15*b^9))
 + d^7*x)*a^11*b^7*(-d^14/(a^15*b^9))^(3/4))/d^14) + 1155*(a^3*b^7*x^10 + 5
*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(
-d^14/(a^15*b^9))^(1/4)*log(77*a^4*b^2*(-d^14/(a^15*b^9))^(1/4) + 77*sqrt(d
*x)*d^3) - 1155*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4
*x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d^14/(a^15*b^9))^(1/4)*log(-77*a^4*b^2*(-
d^14/(a^15*b^9))^(1/4) + 77*sqrt(d*x)*d^3) + 4*(385*b^4*d^3*x^8 + 1760*a*b^
3*d^3*x^6 + 3130*a^2*b^2*d^3*x^4 - 5544*a^3*b*d^3*x^2 - 1155*a^4*d^3)*sqrt(
d*x))/(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a
^7*b^3*x^2 + a^8*b^2)
```

giac [A] time = 0.22, size = 342, normalized size = 0.87

$$\frac{1}{491520}d^3\left(\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^3}+\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^3}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/491520*d^3*(2310*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^3) + 2310*sqrt(2)*(a*b
^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(
a*d^2/b)^(1/4))/(a^4*b^3) + 1155*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2
)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^3) - 1155*sqrt(2)*(a*b^
3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(
a^4*b^3) + 8*(385*sqrt(d*x)*b^4*d^10*x^8 + 1760*sqrt(d*x)*a*b^3*d^10*x^6 +
3130*sqrt(d*x)*a^2*b^2*d^10*x^4 - 5544*sqrt(d*x)*a^3*b*d^10*x^2 - 1155*sqrt
(d*x)*a^4*d^10)/((b*d^2*x^2 + a*d^2)^5*a^3*b^2))
```

maple [A] time = 0.02, size = 339, normalized size = 0.86

$$-\frac{77\sqrt{dx}\,a\,d^{13}}{4096\left(b\,d^2x^2+d^2a\right)^5b^2}-\frac{231\,(dx)^{\frac{5}{2}}\,d^{11}}{2560\left(b\,d^2x^2+d^2a\right)^5b}+\frac{313\,(dx)^{\frac{9}{2}}\,d^9}{6144\left(b\,d^2x^2+d^2a\right)^5a}+\frac{11\,(dx)^{\frac{13}{2}}\,b\,d^7}{384\left(b\,d^2x^2+d^2a\right)^5a^2}+\frac{77\,(dx)}{12288\left(b\,d^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -77/4096*d^13/(b*d^2*x^2+a*d^2)^5/b^2*a*(d*x)^(1/2)-231/2560*d^11/(b*d^2*x^
2+a*d^2)^5/b*(d*x)^(5/2)+313/6144*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^(9/2)+11/
384*d^7/(b*d^2*x^2+a*d^2)^5/a^2*b*(d*x)^(13/2)+77/12288*d^5/(b*d^2*x^2+a*d^
2)^5/a^3*b^2*(d*x)^(17/2)+77/32768*d^3/a^4/b^2*(a/b*d^2)^(1/4)*2^(1/2)*ln((
d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/
```

```
4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+77/16384*d^3/a^4/b^2*(a/b*d^2)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+77/16384*d^3/a^4/b
^2*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.15, size = 394, normalized size = 1.00

$$\frac{\frac{8\left(385\,(dx)^{\frac{17}{2}}b^4d^6+1760\,(dx)^{\frac{13}{2}}ab^3d^8+3130\,(dx)^{\frac{9}{2}}a^2b^2d^{10}-5544\,(dx)^{\frac{5}{2}}a^3bd^{12}-1155\,\sqrt{dx}\,a^4d^{14}\right)}{a^3b^7d^{10}x^{10}+5\,a^4b^6d^{10}x^8+10\,a^5b^5d^{10}x^6+10\,a^6b^4d^{10}x^4+5\,a^7b^3d^{10}x^2+a^8b^2d^{10}}+\frac{1155\left(\frac{\sqrt{2}\,d^6\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\cdots\right)}{\cdots}}{491520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/491520*(8*(385*(d*x)^(17/2)*b^4*d^6 + 1760*(d*x)^(13/2)*a*b^3*d^8 + 3130*
(d*x)^(9/2)*a^2*b^2*d^10 - 5544*(d*x)^(5/2)*a^3*b*d^12 - 1155*sqrt(d*x)*a^4
*d^14)/(a^3*b^7*d^10*x^10 + 5*a^4*b^6*d^10*x^8 + 10*a^5*b^5*d^10*x^6 + 10*a
^6*b^4*d^10*x^4 + 5*a^7*b^3*d^10*x^2 + a^8*b^2*d^10) + 1155*(sqrt(2)*d^6*lo
g(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^
2)^(3/4)*b^(1/4)) - sqrt(2)*d^6*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqr
t(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^5*arctan(
1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt
(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^5*arctan(-1
/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(
a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/(a^3*b^2))/d
```

mupad [B] time = 4.27, size = 207, normalized size = 0.53

$$\frac{\frac{313\,d^9\,(d\,x)^{9/2}}{6144\,a}-\frac{231\,d^{11}\,(d\,x)^{5/2}}{2560\,b}+\frac{77\,b^2\,d^5\,(d\,x)^{17/2}}{12288\,a^3}-\frac{77\,a\,d^{13}\,\sqrt{d\,x}}{4096\,b^2}+\frac{11\,b\,d^7\,(d\,x)^{13/2}}{384\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}+\frac{77\,d^{7/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{15/4}\,b^{9/4}}+\frac{77\,a\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((313*d^9*(d*x)^(9/2))/(6144*a) - (231*d^11*(d*x)^(5/2))/(2560*b) + (77*b^2
*d^5*(d*x)^(17/2))/(12288*a^3) - (77*a*d^13*(d*x)^(1/2))/(4096*b^2) + (11*b
*d^7*(d*x)^(13/2))/(384*a^2))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^2
+ 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6) + (77*d^(7/
```

```
2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(15/4)*b^(9
/4)) + (77*d^(7/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192
*(-a)^(15/4)*b^(9/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{7}{2}}}{(a+bx^2)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Integral((d*x)**(7/2)/(a + b*x**2)**6, x)
```

3.721 $\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=389

$$\frac{117d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{17/4}b^{7/4}}-\frac{117d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{17/4}b^{7/4}}-\frac{117d^{5/}}{}$$

```
[Out] -1/10*d*(d*x)^(3/2)/b/(b*x^2+a)^5+3/160*d*(d*x)^(3/2)/a/b/(b*x^2+a)^4+13/64
0*d*(d*x)^(3/2)/a^2/b/(b*x^2+a)^3+117/5120*d*(d*x)^(3/2)/a^3/b/(b*x^2+a)^2+
117/4096*d*(d*x)^(3/2)/a^4/b/(b*x^2+a)-117/16384*d^(5/2)*arctan(1-b^(1/4)*2
^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(17/4)/b^(7/4)*2^(1/2)+117/16384*d^(5
/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(17/4)/b^(7/4)*
2^(1/2)+117/32768*d^(5/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1
/4)*2^(1/2)*(d*x)^(1/2))/a^(17/4)/b^(7/4)*2^(1/2)-117/32768*d^(5/2)*ln(a^(1
/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(17/4)
/b^(7/4)*2^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{117d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{17/4}b^{7/4}}-\frac{117d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{17/4}b^{7/4}}-\frac{117d^{5/}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(3/2))/(10*b*(a + b*x^2)^5) + (3*d*(d*x)^(3/2))/(160*a*b*(a + b*x
^2)^4) + (13*d*(d*x)^(3/2))/(640*a^2*b*(a + b*x^2)^3) + (117*d*(d*x)^(3/2))
/(5120*a^3*b*(a + b*x^2)^2) + (117*d*(d*x)^(3/2))/(4096*a^4*b*(a + b*x^2))
- (117*d^(5/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(
8192*Sqrt[2]*a^(17/4)*b^(7/4)) + (117*d^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*S
qrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(17/4)*b^(7/4)) + (117*d^(5/2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(16384*Sqrt[2]*a^(17/4)*b^(7/4)) - (117*d^(5/2)*Log[Sqrt[a]*Sqrt[d] +
Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(1
7/4)*b^(7/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(3b^4d^2\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{\left(39b^3d^2\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^4}\,dx}{320a}$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{\left(117b^2d^2\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^3}}{1280a^2}$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2} +$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2} +$$

Mathematica [C] time = 0.02, size = 48, normalized size = 0.12

$$\frac{2d(dx)^{3/2}\left(\frac{{}_2F_1\left(\frac{3}{4},6;\frac{7}{4};-\frac{bx^2}{a}\right)}{a^5}-\frac{1}{(a+bx^2)^5}\right)}{17b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*d*(d*x)^(3/2)*(-(a + b*x^2)^(-5) + Hypergeometric2F1[3/4, 6, 7/4, -((b*x
^2)/a)]/a^5))/(17*b)
```

fricas [A] time = 1.17, size = 512, normalized size = 1.32

$$-\frac{2340\left(a^4b^6x^{10}+5\,a^5b^5x^8+10\,a^6b^4x^6+10\,a^7b^3x^4+5\,a^8b^2x^2+a^9b\right)\left(-\frac{d^{10}}{a^{17}b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{1601613\,\sqrt{dx}\,a^4b^2d^7\left(-\frac{d^{10}}{a^{17}b^7}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(2340*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*
x^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^10/(a^17*b^7))^(1/4)*arctan(-1/1601613*(16
01613*sqrt(d*x)*a^4*b^2*d^7*(-d^10/(a^17*b^7))^(1/4) - sqrt(-2565164201769*
a^9*b^3*d^10*sqrt(-d^10/(a^17*b^7)) + 2565164201769*d^15*x)*a^4*b^2*(-d^10/
(a^17*b^7))^(1/4))/d^10) - 585*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x
^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^10/(a^17*b^7))^(1/4)*log(1
601613*a^13*b^5*(-d^10/(a^17*b^7))^(3/4) + 1601613*sqrt(d*x)*d^7) + 585*(a^
4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^
2 + a^9*b)*(-d^10/(a^17*b^7))^(1/4)*log(-1601613*a^13*b^5*(-d^10/(a^17*b^7)
)^(3/4) + 1601613*sqrt(d*x)*d^7) - 4*(585*b^4*d^2*x^9 + 2808*a*b^3*d^2*x^7
+ 5330*a^2*b^2*d^2*x^5 + 4960*a^3*b*d^2*x^3 - 195*a^4*d^2*x)*sqrt(d*x))/(a^
4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^
2 + a^9*b)
```

giac [A] time = 0.22, size = 355, normalized size = 0.91

$$\frac{1}{163840}\,d^2\left(\frac{1170\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^4d}+\frac{1170\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^4d}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^2*(1170*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^4*d) + 1170*sqrt(2)*(a
*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))
/(a*d^2/b)^(1/4))/(a^5*b^4*d) - 585*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqr
t(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b^4*d) + 585*sqrt(2)*(
a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b
))/(a^5*b^4*d) + 8*(585*sqrt(d*x)*b^4*d^10*x^9 + 2808*sqrt(d*x)*a*b^3*d^10*
x^7 + 5330*sqrt(d*x)*a^2*b^2*d^10*x^5 + 4960*sqrt(d*x)*a^3*b*d^10*x^3 - 195
*sqrt(d*x)*a^4*d^10*x)/((b*d^2*x^2 + a*d^2)^5*a^4*b))
```

maple [A] time = 0.02, size = 341, normalized size = 0.88

$$-\frac{39\,(dx)^{\frac{3}{2}}\,d^{11}}{4096\left(b\,d^2x^2+d^2a\right)^5 b}+\frac{31\,(dx)^{\frac{7}{2}}\,d^9}{128\left(b\,d^2x^2+d^2a\right)^5 a}+\frac{533\,(dx)^{\frac{11}{2}}\,b\,d^7}{2048\left(b\,d^2x^2+d^2a\right)^5 a^2}+\frac{351\,(dx)^{\frac{15}{2}}\,b^2d^5}{2560\left(b\,d^2x^2+d^2a\right)^5 a^3}+\frac{117\,(dx)}{4096\left(b\,d^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -39/4096*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(3/2)+31/128*d^9/(b*d^2*x^2+a*d^2
)^5/a*(d*x)^(7/2)+533/2048*d^7/(b*d^2*x^2+a*d^2)^5/a^2*b*(d*x)^(11/2)+351/2
560*d^5/(b*d^2*x^2+a*d^2)^5/a^3*b^2*(d*x)^(15/2)+117/4096*d^3/(b*d^2*x^2+a*
d^2)^5/a^4*b^3*(d*x)^(19/2)+117/32768*d^3/a^4/b^2/(a/b*d^2)^(1/4)*2^(1/2)*l
n((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^
```

```
(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+117/16384*d^3/a^4/b^2/(a/b*d^2)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+117/16384*d^3/
a^4/b^2/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-
1)
```

maxima [A] time = 3.16, size = 383, normalized size = 0.98

$$\frac{\frac{8\left(585\,(dx)^{\frac{19}{2}}b^4d^4+2808\,(dx)^{\frac{15}{2}}ab^3d^6+5330\,(dx)^{\frac{11}{2}}a^2b^2d^8+4960\,(dx)^{\frac{7}{2}}a^3bd^{10}-195\,(dx)^{\frac{3}{2}}a^4d^{12}\right)}{a^4b^6d^{10}x^{10}+5\,a^5b^5d^{10}x^8+10\,a^6b^4d^{10}x^6+10\,a^7b^3d^{10}x^4+5\,a^8b^2d^{10}x^2+a^9bd^{10}}+\frac{585\,d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}\cdots}{\cdots}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/163840*(8*(585*(d*x)^(19/2)*b^4*d^4 + 2808*(d*x)^(15/2)*a*b^3*d^6 + 5330*
(d*x)^(11/2)*a^2*b^2*d^8 + 4960*(d*x)^(7/2)*a^3*b*d^10 - 195*(d*x)^(3/2)*a^
4*d^12)/(a^4*b^6*d^10*x^10 + 5*a^5*b^5*d^10*x^8 + 10*a^6*b^4*d^10*x^6 + 10*
a^7*b^3*d^10*x^4 + 5*a^8*b^2*d^10*x^2 + a^9*b*d^10) + 585*d^4*(2*sqrt(2)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt
(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt
(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x
 + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3
/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) +
sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^4*b))/d
```

mupad [B] time = 4.28, size = 209, normalized size = 0.54

$$\frac{\frac{31\,d^9\,(d\,x)^{7/2}}{128\,a}-\frac{39\,d^{11}\,(d\,x)^{3/2}}{4096\,b}+\frac{351\,b^2\,d^5\,(d\,x)^{15/2}}{2560\,a^3}+\frac{117\,b^3\,d^3\,(d\,x)^{19/2}}{4096\,a^4}+\frac{533\,b\,d^7\,(d\,x)^{11/2}}{2048\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}+\frac{117\,d^{5/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{17/4}\,b^{7/4}}-\frac{117\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((31*d^9*(d*x)^(7/2))/(128*a) - (39*d^11*(d*x)^(3/2))/(4096*b) + (351*b^2*d
^5*(d*x)^(15/2))/(2560*a^3) + (117*b^3*d^3*(d*x)^(19/2))/(4096*a^4) + (533*
b*d^7*(d*x)^(11/2))/(2048*a^2))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^
```

```
2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6) + (117*d^
(5/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(17/4)*b
^(7/4)) - (117*d^(5/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(
8192*(-a)^(17/4)*b^(7/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{5}{2}}}{\left(a+bx^2\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Integral((d*x)**(5/2)/(a + b*x**2)**6, x)
```

3.722 $\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=389

$$-\frac{231d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{19/4}b^{5/4}}+\frac{231d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{19/4}b^{5/4}}-\frac{231d}{}$$

```
[Out] -231/16384*d^(3/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^
(19/4)/b^(5/4)*2^(1/2)+231/16384*d^(3/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/
2)/a^(1/4)/d^(1/2))/a^(19/4)/b^(5/4)*2^(1/2)-231/32768*d^(3/2)*ln(a^(1/2)*d
^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(19/4)/b^(5
/4)*2^(1/2)+231/32768*d^(3/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*
b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(19/4)/b^(5/4)*2^(1/2)-1/10*d*(d*x)^(1/2)/b/
(b*x^2+a)^5+1/160*d*(d*x)^(1/2)/a/b/(b*x^2+a)^4+1/128*d*(d*x)^(1/2)/a^2/b/(
b*x^2+a)^3+11/1024*d*(d*x)^(1/2)/a^3/b/(b*x^2+a)^2+77/4096*d*(d*x)^(1/2)/a^
4/b/(b*x^2+a)
```

Rubi [A] time = 0.50, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{231d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{19/4}b^{5/4}}+\frac{231d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{19/4}b^{5/4}}-\frac{231d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*Sqrt[d*x])/(10*b*(a + b*x^2)^5) + (d*Sqrt[d*x])/(160*a*b*(a + b*x^2)^4)
 + (d*Sqrt[d*x])/(128*a^2*b*(a + b*x^2)^3) + (11*d*Sqrt[d*x])/(1024*a^3*b*(
a + b*x^2)^2) + (77*d*Sqrt[d*x])/(4096*a^4*b*(a + b*x^2)) - (231*d^(3/2)*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(1
9/4)*b^(5/4)) + (231*d^(3/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4
)*Sqrt[d])])/(8192*Sqrt[2]*a^(19/4)*b^(5/4)) - (231*d^(3/2)*Log[Sqrt[a]*Sqr
t[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[
2]*a^(19/4)*b^(5/4)) + (231*d^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x
 + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(19/4)*b^(5/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(b^4d^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{\left(3b^3d^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^4}\,dx}{64a}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{\left(11b^2d^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right.}}{256a^2}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2} +$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2} +$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2} +$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2} +$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2} +$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2} +$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2} +$$

Mathematica [A] time = 0.17, size = 298, normalized size = 0.77

$$\frac{d\sqrt{dx}\left(-\frac{1155\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{19/4}\sqrt{x}}+\frac{1155\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{19/4}\sqrt{x}}-\frac{2310\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{19/4}\sqrt{x}}+\frac{2310\sqrt{2}\ta\ldots}{\ldots}\right.}{163840b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d*Sqrt[d*x]*((-16384*b^(1/4))/(a + b*x^2)^5 + (1024*b^(1/4))/(a*(a + b*x^2
)^4) + (1280*b^(1/4))/(a^2*(a + b*x^2)^3) + (1760*b^(1/4))/(a^3*(a + b*x^2)
^2) + (3080*b^(1/4))/(a^4*(a + b*x^2)) - (2310*Sqrt[2]*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(19/4)*Sqrt[x]) + (2310*Sqrt[2]*ArcTan[1 + (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(19/4)*Sqrt[x]) - (1155*Sqrt[2]*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(19/4)*Sqrt[x]) +
(1155*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
a^(19/4)*Sqrt[x])))/(163840*b^(5/4))
```

fricas [A] time = 1.20, size = 485, normalized size = 1.25

$$4620\left(a^4b^6x^{10}+5a^5b^5x^8+10a^6b^4x^6+10a^7b^3x^4+5a^8b^2x^2+a^9b\right)\left(-\frac{d^6}{a^{19}b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^{14}b^4d\left(-\frac{d^6}{a^{19}b^5}\right)^{\frac{3}{4}}-\sqrt{a^{10}b\ldots}}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(4620*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x
^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^6/(a^19*b^5))^(1/4)*arctan(-(sqrt(d*x)*a^14
*b^4*d*(-d^6/(a^19*b^5))^(3/4) - sqrt(a^10*b^2*sqrt(-d^6/(a^19*b^5)) + d^3*
x)*a^14*b^4*(-d^6/(a^19*b^5))^(3/4))/d^6) + 1155*(a^4*b^6*x^10 + 5*a^5*b^5*
x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^6/(a^19*
b^5))^(1/4)*log(231*a^5*b*(-d^6/(a^19*b^5))^(1/4) + 231*sqrt(d*x)*d) - 1155
*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^
2*x^2 + a^9*b)*(-d^6/(a^19*b^5))^(1/4)*log(-231*a^5*b*(-d^6/(a^19*b^5))^(1/
4) + 231*sqrt(d*x)*d) + 4*(385*b^4*d*x^8 + 1760*a*b^3*d*x^6 + 3130*a^2*b^2*
d*x^4 + 2648*a^3*b*d*x^2 - 1155*a^4*d)*sqrt(d*x))/(a^4*b^6*x^10 + 5*a^5*b^5
*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)
```

giac [A] time = 0.21, size = 340, normalized size = 0.87

$$\frac{1}{163840}d\left(\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^2}+\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d*(2310*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^2) + 2310*sqrt(2)*(a*b^3
*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*
d^2/b)^(1/4))/(a^5*b^2) + 1155*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*
(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b^2) - 1155*sqrt(2)*(a*b^3*
d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^
5*b^2) + 8*(385*sqrt(d*x)*b^4*d^10*x^8 + 1760*sqrt(d*x)*a*b^3*d^10*x^6 + 31
30*sqrt(d*x)*a^2*b^2*d^10*x^4 + 2648*sqrt(d*x)*a^3*b*d^10*x^2 - 1155*sqrt(d
*x)*a^4*d^10)/((b*d^2*x^2 + a*d^2)^5*a^4*b))
```

maple [A] time = 0.02, size = 335, normalized size = 0.86

$$-\frac{231\sqrt{dx}\,d^{11}}{4096\left(b\,d^2x^2+d^2a\right)^5b}+\frac{331\,(dx)^{\frac{5}{2}}\,d^9}{2560\left(b\,d^2x^2+d^2a\right)^5a}+\frac{313\,(dx)^{\frac{9}{2}}\,b\,d^7}{2048\left(b\,d^2x^2+d^2a\right)^5a^2}+\frac{11\,(dx)^{\frac{13}{2}}\,b^2d^5}{128\left(b\,d^2x^2+d^2a\right)^5a^3}+\frac{77\,(dx)^{\frac{1}{}}}{4096\left(b\,d^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -231/4096*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(1/2)+331/2560*d^9/(b*d^2*x^2+a*
d^2)^5/a*(d*x)^(5/2)+313/2048*d^7/(b*d^2*x^2+a*d^2)^5/a^2*b*(d*x)^(9/2)+11/
128*d^5/(b*d^2*x^2+a*d^2)^5/a^3*b^2*(d*x)^(13/2)+77/4096*d^3/(b*d^2*x^2+a*d
^2)^5/a^4*b^3*(d*x)^(17/2)+231/32768*d/a^5/b*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*
x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)
```

```
*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+231/16384*d/a^5/b*(a/b*d^2)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+231/16384*d/a^5/b*(a/b*
d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.23, size = 392, normalized size = 1.01

$$\frac{\frac{8\left(385\,(dx)^{\frac{17}{2}}b^4d^4+1760\,(dx)^{\frac{13}{2}}ab^3d^6+3130\,(dx)^{\frac{9}{2}}a^2b^2d^8+2648\,(dx)^{\frac{5}{2}}a^3bd^{10}-1155\,\sqrt{dx}\,a^4d^{12}\right)}{a^4b^6d^{10}x^{10}+5\,a^5b^5d^{10}x^8+10\,a^6b^4d^{10}x^6+10\,a^7b^3d^{10}x^4+5\,a^8b^2d^{10}x^2+a^9bd^{10}}+\frac{1155\left(\frac{\sqrt{2}d^4\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\cdots\right.}{\cdots}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/163840*(8*(385*(d*x)^(17/2)*b^4*d^4 + 1760*(d*x)^(13/2)*a*b^3*d^6 + 3130*
(d*x)^(9/2)*a^2*b^2*d^8 + 2648*(d*x)^(5/2)*a^3*b*d^10 - 1155*sqrt(d*x)*a^4*
d^12)/(a^4*b^6*d^10*x^10 + 5*a^5*b^5*d^10*x^8 + 10*a^6*b^4*d^10*x^6 + 10*a^
7*b^3*d^10*x^4 + 5*a^8*b^2*d^10*x^2 + a^9*b*d^10) + 1155*(sqrt(2)*d^4*log(s
qrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^
(3/4)*b^(1/4)) - sqrt(2)*d^4*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d
*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^3*arctan(1/2
*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)
*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^3*arctan(-1/2*
sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*
sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/(a^4*b))/d
```

mupad [B] time = 0.13, size = 209, normalized size = 0.54

$$\frac{\frac{331\,d^9\,(d\,x)^{5/2}}{2560\,a}-\frac{231\,d^{11}\,\sqrt{d\,x}}{4096\,b}+\frac{11\,b^2\,d^5\,(d\,x)^{13/2}}{128\,a^3}+\frac{77\,b^3\,d^3\,(d\,x)^{17/2}}{4096\,a^4}+\frac{313\,b\,d^7\,(d\,x)^{9/2}}{2048\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}-\frac{231\,d^{3/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{19/4}\,b^{5/4}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((331*d^9*(d*x)^(5/2))/(2560*a) - (231*d^11*(d*x)^(1/2))/(4096*b) + (11*b^2
*d^5*(d*x)^(13/2))/(128*a^3) + (77*b^3*d^3*(d*x)^(17/2))/(4096*a^4) + (313*
b*d^7*(d*x)^(9/2))/(2048*a^2))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^2
 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6) - (231*d^(
```

```
3/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(19/4)*b^
(5/4)) - (231*d^(3/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8
192*(-a)^(19/4)*b^(5/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{\left(a+bx^2\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Integral((d*x)**(3/2)/(a + b*x**2)**6, x)
```

3.723 $\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^3} dx$

Optimal. Leaf size=387

$$\frac{663\sqrt{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{21/4}b^{3/4}}-\frac{663\sqrt{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{21/4}b^{3/4}}-\frac{663\sqrt{}}{}$$

```
[Out] 1/10*(d*x)^(3/2)/a/d/(b*x^2+a)^5+17/160*(d*x)^(3/2)/a^2/d/(b*x^2+a)^4+221/1
920*(d*x)^(3/2)/a^3/d/(b*x^2+a)^3+663/5120*(d*x)^(3/2)/a^4/d/(b*x^2+a)^2+66
3/4096*(d*x)^(3/2)/a^5/d/(b*x^2+a)-663/16384*arctan(1-b^(1/4)*2^(1/2)*(d*x)
^(1/2)/a^(1/4)/d^(1/2))*d^(1/2)/a^(21/4)/b^(3/4)*2^(1/2)+663/16384*arctan(1
+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*d^(1/2)/a^(21/4)/b^(3/4)*2^(1
/2)+663/32768*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*
(d*x)^(1/2))*d^(1/2)/a^(21/4)/b^(3/4)*2^(1/2)-663/32768*ln(a^(1/2)*d^(1/2)+
x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))*d^(1/2)/a^(21/4)/b^(
3/4)*2^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {28, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{663\sqrt{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{21/4}b^{3/4}}-\frac{663\sqrt{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{21/4}b^{3/4}}-\frac{663\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d*x)^(3/2)/(10*a*d*(a + b*x^2)^5) + (17*(d*x)^(3/2))/(160*a^2*d*(a + b*x^2
)^4) + (221*(d*x)^(3/2))/(1920*a^3*d*(a + b*x^2)^3) + (663*(d*x)^(3/2))/(51
20*a^4*d*(a + b*x^2)^2) + (663*(d*x)^(3/2))/(4096*a^5*d*(a + b*x^2)) - (663
*Sqrt[d]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*S
qrt[2]*a^(21/4)*b^(3/4)) + (663*Sqrt[d]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*
x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(21/4)*b^(3/4)) + (663*Sqrt[d]*Log[
Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(
16384*Sqrt[2]*a^(21/4)*b^(3/4)) - (663*Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b
]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(21/4)*b
^(3/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{\left(17b^5\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^5}\,dx}{20a}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{\left(221b^4\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^4}\,dx}{320a^2}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{\left(663b^3\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^3}}{1280a^3}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)^2}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)^2}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)^2}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)^2}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)^2}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)^2}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)^2}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.08

$$\frac{2x\sqrt{dx}\,{}_2F_1\left(\frac{3}{4},6;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*x*Sqrt[d*x]*Hypergeometric2F1[3/4, 6, 7/4, -((b*x^2)/a)])/(3*a^6)
```

fricas [A] time = 0.99, size = 469, normalized size = 1.21

$$-\frac{39780\left(a^5b^5x^{10}+5\,a^6b^4x^8+10\,a^7b^3x^6+10\,a^8b^2x^4+5\,a^9bx^2+a^{10}\right)\left(-\frac{d^2}{a^{21}b^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{291434247\,\sqrt{dx}\,a^5bd\left(-\frac{d^2}{a^{21}b^3}\right.}{\quad}\right.}{\quad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/245760*(39780*(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^
2*x^4 + 5*a^9*b*x^2 + a^10)*(-d^2/(a^21*b^3))^(1/4)*arctan(-1/291434247*(29
1434247*sqrt(d*x)*a^5*b*d*(-d^2/(a^21*b^3))^(1/4) - sqrt(-84933920324457009
*a^11*b*d^2*sqrt(-d^2/(a^21*b^3)) + 84933920324457009*d^3*x)*a^5*b*(-d^2/(a
^21*b^3))^(1/4))/d^2) - 9945*(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6
 + 10*a^8*b^2*x^4 + 5*a^9*b*x^2 + a^10)*(-d^2/(a^21*b^3))^(1/4)*log(2914342
47*a^16*b^2*(-d^2/(a^21*b^3))^(3/4) + 291434247*sqrt(d*x)*d) + 9945*(a^5*b^
5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^2*x^4 + 5*a^9*b*x^2 + a^
10)*(-d^2/(a^21*b^3))^(1/4)*log(-291434247*a^16*b^2*(-d^2/(a^21*b^3))^(3/4)
 + 291434247*sqrt(d*x)*d) - 4*(9945*b^4*x^9 + 47736*a*b^3*x^7 + 90610*a^2*b
^2*x^5 + 84320*a^3*b*x^3 + 37645*a^4*x)*sqrt(d*x))/(a^5*b^5*x^10 + 5*a^6*b^
4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^2*x^4 + 5*a^9*b*x^2 + a^10)
```

giac [A] time = 0.22, size = 340, normalized size = 0.88

$$\frac{\frac{19890\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^6b^3}+\frac{19890\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^6b^3}-\frac{9945\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right.\right.}{a^6b^3}}{\quad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/491520*(19890*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^
2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*b^3) + 19890*sqrt(2)*(a*b^3
*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*
d^2/b)^(1/4))/(a^6*b^3) - 9945*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*
(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b^3) + 9945*sqrt(2)*(a*b^3*
d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^
6*b^3) + 8*(9945*sqrt(d*x)*b^4*d^11*x^9 + 47736*sqrt(d*x)*a*b^3*d^11*x^7 +
90610*sqrt(d*x)*a^2*b^2*d^11*x^5 + 84320*sqrt(d*x)*a^3*b*d^11*x^3 + 37645*s
qrt(d*x)*a^4*d^11*x)/((b*d^2*x^2 + a*d^2)^5*a^5))/d
```

maple [A] time = 0.03, size = 336, normalized size = 0.87

$$\frac{7529\,(dx)^{\frac{3}{2}}\,d^9}{12288\left(b\,d^2x^2+d^2a\right)^5 a}+\frac{527\,(dx)^{\frac{7}{2}}\,b\,d^7}{384\left(b\,d^2x^2+d^2a\right)^5 a^2}+\frac{9061\,(dx)^{\frac{11}{2}}\,b^2d^5}{6144\left(b\,d^2x^2+d^2a\right)^5 a^3}+\frac{1989\,(dx)^{\frac{15}{2}}\,b^3d^3}{2560\left(b\,d^2x^2+d^2a\right)^5 a^4}+\frac{663\,(dx}{4096\left(b\,d^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 7529/12288*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^(3/2)+527/384*d^7/(b*d^2*x^2+a*d
^2)^5/a^2*b*(d*x)^(7/2)+9061/6144*d^5/(b*d^2*x^2+a*d^2)^5/a^3*b^2*(d*x)^(11
/2)+1989/2560*d^3/(b*d^2*x^2+a*d^2)^5/a^4*b^3*(d*x)^(15/2)+663/4096*d/(b*d^
2*x^2+a*d^2)^5/a^5*b^4*(d*x)^(19/2)+663/32768*d/a^5/b/(a/b*d^2)^(1/4)*2^(1/
2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x+(a/b*d
^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+663/16384*d/a^5/b/(a/b*d^2)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+663/16384*d/a^
5/b/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.19, size = 377, normalized size = 0.97

$$\frac{\frac{8\left(9945\,(dx)^{\frac{19}{2}}b^4d^2+47736\,(dx)^{\frac{15}{2}}ab^3d^4+90610\,(dx)^{\frac{11}{2}}a^2b^2d^6+84320\,(dx)^{\frac{7}{2}}a^3bd^8+37645\,(dx)^{\frac{3}{2}}a^4d^{10}\right)}{a^5b^5d^{10}x^{10}+5\,a^6b^4d^{10}x^8+10\,a^7b^3d^{10}x^6+10\,a^8b^2d^{10}x^4+5\,a^9bd^{10}x^2+a^{10}d^{10}}+\frac{9945\,d^2\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\right.}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right.}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}\right.}{}}{491520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/491520*(8*(9945*(d*x)^(19/2)*b^4*d^2 + 47736*(d*x)^(15/2)*a*b^3*d^4 + 906
10*(d*x)^(11/2)*a^2*b^2*d^6 + 84320*(d*x)^(7/2)*a^3*b*d^8 + 37645*(d*x)^(3/
2)*a^4*d^10)/(a^5*b^5*d^10*x^10 + 5*a^6*b^4*d^10*x^8 + 10*a^7*b^3*d^10*x^6
+ 10*a^8*b^2*d^10*x^4 + 5*a^9*b*d^10*x^2 + a^10*d^10) + 9945*d^2*(2*sqrt(2)
*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/s
qrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(s
qrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*
d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b
^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
 + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/a^5)/d
```

mupad [B] time = 4.25, size = 210, normalized size = 0.54

$$\frac{\frac{7529\,d^9\,{\left(d\,x\right)}^{3/2}}{12288\,a}+\frac{9061\,b^2\,d^5\,{\left(d\,x\right)}^{11/2}}{6144\,a^3}+\frac{1989\,b^3\,d^3\,{\left(d\,x\right)}^{15/2}}{2560\,a^4}+\frac{527\,b\,d^7\,{\left(d\,x\right)}^{7/2}}{384\,a^2}+\frac{663\,b^4\,d\,{\left(d\,x\right)}^{19/2}}{4096\,a^5}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}-\frac{663\,\sqrt{d}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,{\left(-a\right)}^{21/4}\,b^{3/4}}+\frac{66\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((7529*d^9*(d*x)^(3/2))/(12288*a) + (9061*b^2*d^5*(d*x)^(11/2))/(6144*a^3)
+ (1989*b^3*d^3*(d*x)^(15/2))/(2560*a^4) + (527*b*d^7*(d*x)^(7/2))/(384*a^2
) + (663*b^4*d*(d*x)^(19/2))/(4096*a^5))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*
b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6)
- (663*d^(1/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)
^(21/4)*b^(3/4)) + (663*d^(1/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(
1/2))))/(8192*(-a)^(21/4)*b^(3/4))
```

sympy [A] time = 89.24, size = 547, normalized size = 1.41

$$\frac{75290a^4d^{19}\left(dx\right)^{\frac{3}{2}}}{122880a^{10}d^{20}+614400a^9bd^{20}x^2+1228800a^8b^2d^{20}x^4+1228800a^7b^3d^{20}x^6+614400a^6b^4d^{20}x^8+122880a^5b^5d^{2\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 75290*a**4*d**19*(d*x)**(3/2)/(122880*a**10*d**20 + 614400*a**9*b*d**20*x**
2 + 1228800*a**8*b**2*d**20*x**4 + 1228800*a**7*b**3*d**20*x**6 + 614400*a*
*6*b**4*d**20*x**8 + 122880*a**5*b**5*d**20*x**10) + 168640*a**3*b*d**17*(d
```

```
*x)**(7/2)/(122880*a**10*d**20 + 614400*a**9*b*d**20*x**2 + 1228800*a**8*b*
*2*d**20*x**4 + 1228800*a**7*b**3*d**20*x**6 + 614400*a**6*b**4*d**20*x**8
+ 122880*a**5*b**5*d**20*x**10) + 181220*a**2*b**2*d**15*(d*x)**(11/2)/(122
880*a**10*d**20 + 614400*a**9*b*d**20*x**2 + 1228800*a**8*b**2*d**20*x**4 +
 1228800*a**7*b**3*d**20*x**6 + 614400*a**6*b**4*d**20*x**8 + 122880*a**5*b
**5*d**20*x**10) + 95472*a*b**3*d**13*(d*x)**(15/2)/(122880*a**10*d**20 + 6
14400*a**9*b*d**20*x**2 + 1228800*a**8*b**2*d**20*x**4 + 1228800*a**7*b**3*
d**20*x**6 + 614400*a**6*b**4*d**20*x**8 + 122880*a**5*b**5*d**20*x**10) +
19890*b**4*d**11*(d*x)**(19/2)/(122880*a**10*d**20 + 614400*a**9*b*d**20*x*
*2 + 1228800*a**8*b**2*d**20*x**4 + 1228800*a**7*b**3*d**20*x**6 + 614400*a
**6*b**4*d**20*x**8 + 122880*a**5*b**5*d**20*x**10) + 2*d**11*RootSum(11529
21504606846976*_t**4*a**21*b**3*d**42 + 193220905761, Lambda(_t, _t*log(351
84372088832*_t**3*a**16*b**2*d**32/291434247 + sqrt(d*x)))
```

3.724 $\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=387

$$-\frac{4389\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{23/4}\sqrt[4]{b}\sqrt{d}}+\frac{4389\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{23/4}\sqrt[4]{b}\sqrt{d}}-\frac{4389\tan}{8192}$$

```
[Out] -4389/16384*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(23/4)/
b^(1/4)*2^(1/2)/d^(1/2)+4389/16384*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(
1/4)/d^(1/2))/a^(23/4)/b^(1/4)*2^(1/2)/d^(1/2)-4389/32768*ln(a^(1/2)*d^(1/2
)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(23/4)/b^(1/4)*2
^(1/2)/d^(1/2)+4389/32768*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1
/4)*2^(1/2)*(d*x)^(1/2))/a^(23/4)/b^(1/4)*2^(1/2)/d^(1/2)+1/10*(d*x)^(1/2)/
a/d/(b*x^2+a)^5+19/160*(d*x)^(1/2)/a^2/d/(b*x^2+a)^4+19/128*(d*x)^(1/2)/a^3
/d/(b*x^2+a)^3+209/1024*(d*x)^(1/2)/a^4/d/(b*x^2+a)^2+1463/4096*(d*x)^(1/2)
/a^5/d/(b*x^2+a)
```

Rubi [A] time = 0.50, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {28, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{1463\sqrt{dx}}{4096a^5d\left(a+bx^2\right)}+\frac{209\sqrt{dx}}{1024a^4d\left(a+bx^2\right)^2}+\frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3}+\frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4}-\frac{4389\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\right.}{16384\sqrt{2}a^{23/4}\sqrt[4]{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] Sqrt[d*x]/(10*a*d*(a + b*x^2)^5) + (19*Sqrt[d*x])/(160*a^2*d*(a + b*x^2)^4)
 + (19*Sqrt[d*x])/(128*a^3*d*(a + b*x^2)^3) + (209*Sqrt[d*x])/(1024*a^4*d*(
a + b*x^2)^2) + (1463*Sqrt[d*x])/(4096*a^5*d*(a + b*x^2)) - (4389*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(23/4)*b^
(1/4)*Sqrt[d]) + (4389*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(8192*Sqrt[2]*a^(23/4)*b^(1/4)*Sqrt[d]) - (4389*Log[Sqrt[a]*Sqrt[d]
+ Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^
(23/4)*b^(1/4)*Sqrt[d]) + (4389*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(23/4)*b^(1/4)*Sqrt[d])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\,\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{\sqrt{dx}\,\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{\left(19b^5\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^5}\,dx}{20a}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{\left(57b^4\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^4}\,dx}{64a^2}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{\left(209b^3\right)\int \frac{1}{\sqrt{dx}\left(a\right.}}{256a^3}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{dx}}{1024a^4d\left(a+bx\right.}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{dx}}{1024a^4d\left(a+bx\right.}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{dx}}{1024a^4d\left(a+bx\right.}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{dx}}{1024a^4d\left(a+bx\right.}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{dx}}{1024a^4d\left(a+bx\right.}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{dx}}{1024a^4d\left(a+bx\right.}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{dx}}{1024a^4d\left(a+bx\right.}$$

Mathematica [A] time = 0.16, size = 295, normalized size = 0.76

$$\frac{\sqrt{x}\left(\frac{16384a^{19/4}\sqrt{x}}{(a+bx^2)^5}+\frac{19456a^{15/4}\sqrt{x}}{(a+bx^2)^4}+\frac{24320a^{11/4}\sqrt{x}}{(a+bx^2)^3}+\frac{33440a^{7/4}\sqrt{x}}{(a+bx^2)^2}+\frac{58520a^{3/4}\sqrt{x}}{a+bx^2}-\frac{21945\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}+\ldots\right.}{163840a^{23/4}\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] (Sqrt[x]*((16384*a^(19/4)*Sqrt[x])/(a + b*x^2)^5 + (19456*a^(15/4)*Sqrt[x])
/(a + b*x^2)^4 + (24320*a^(11/4)*Sqrt[x])/(a + b*x^2)^3 + (33440*a^(7/4)*Sq
rt[x])/(a + b*x^2)^2 + (58520*a^(3/4)*Sqrt[x])/(a + b*x^2) - (43890*Sqrt[2]
*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) + (43890*Sqrt[2]*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) - (21945*Sqrt[2]*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4) + (21945*Sqr
t[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4)))/
(163840*a^(23/4)*Sqrt[d*x])
```

fricas [A] time = 0.80, size = 475, normalized size = 1.23

$$\frac{87780\left(a^5b^5dx^{10}+5a^6b^4dx^8+10a^7b^3dx^6+10a^8b^2dx^4+5a^9bdx^2+a^{10}d\right)\left(-\frac{1}{a^{23}bd^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^{12}d^2\sqrt{-\frac{1}{a^{23}bd^2}}\ldots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/81920*(87780*(a^5*b^5*d*x^10 + 5*a^6*b^4*d*x^8 + 10*a^7*b^3*d*x^6 + 10*a^
8*b^2*d*x^4 + 5*a^9*b*d*x^2 + a^10*d)*(-1/(a^23*b*d^2))^(1/4)*arctan(sqrt(a
^12*d^2*sqrt(-1/(a^23*b*d^2)) + d*x)*a^17*b*d*(-1/(a^23*b*d^2))^(3/4) - sqr
t(d*x)*a^17*b*d*(-1/(a^23*b*d^2))^(3/4)) + 21945*(a^5*b^5*d*x^10 + 5*a^6*b^
4*d*x^8 + 10*a^7*b^3*d*x^6 + 10*a^8*b^2*d*x^4 + 5*a^9*b*d*x^2 + a^10*d)*(-1
/(a^23*b*d^2))^(1/4)*log(a^6*d*(-1/(a^23*b*d^2))^(1/4) + sqrt(d*x)) - 21945
*(a^5*b^5*d*x^10 + 5*a^6*b^4*d*x^8 + 10*a^7*b^3*d*x^6 + 10*a^8*b^2*d*x^4 +
5*a^9*b*d*x^2 + a^10*d)*(-1/(a^23*b*d^2))^(1/4)*log(-a^6*d*(-1/(a^23*b*d^2)
)^(1/4) + sqrt(d*x)) + 4*(7315*b^4*x^8 + 33440*a*b^3*x^6 + 59470*a^2*b^2*x^
4 + 50312*a^3*b*x^2 + 19015*a^4)*sqrt(d*x))/(a^5*b^5*d*x^10 + 5*a^6*b^4*d*x
^8 + 10*a^7*b^3*d*x^6 + 10*a^8*b^2*d*x^4 + 5*a^9*b*d*x^2 + a^10*d)
```

giac [A] time = 0.19, size = 346, normalized size = 0.89

$$\frac{4389\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^6bd}+\frac{4389\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^6bd}+\frac{4389\,\sqrt{2}\,(ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4389/16384*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*b*d) + 4389/16384*sqrt(2)*(a*b^3
*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*
d^2/b)^(1/4))/(a^6*b*d) + 4389/32768*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sq
rt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b*d) - 4389/32768*sqr
t(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a
*d^2/b))/(a^6*b*d) + 1/20480*(7315*sqrt(d*x)*b^4*d^9*x^8 + 33440*sqrt(d*x)*
a*b^3*d^9*x^6 + 59470*sqrt(d*x)*a^2*b^2*d^9*x^4 + 50312*sqrt(d*x)*a^3*b*d^9
*x^2 + 19015*sqrt(d*x)*a^4*d^9)/((b*d^2*x^2 + a*d^2)^5*a^5)
```

maple [A] time = 0.03, size = 333, normalized size = 0.86

$$\frac{3803\sqrt{dx}\,d^9}{4096\left(b\,d^2x^2+d^2a\right)^5a}+\frac{6289\,(dx)^{\frac{5}{2}}\,b\,d^7}{2560\left(b\,d^2x^2+d^2a\right)^5a^2}+\frac{5947\,(dx)^{\frac{9}{2}}\,b^2d^5}{2048\left(b\,d^2x^2+d^2a\right)^5a^3}+\frac{209\,(dx)^{\frac{13}{2}}\,b^3d^3}{128\left(b\,d^2x^2+d^2a\right)^5a^4}+\frac{1463\,(dx}{4096\left(b\,d^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x)
```

```
[Out] 3803/4096*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^(1/2)+6289/2560*d^7/(b*d^2*x^2+a*
d^2)^5/a^2*b*(d*x)^(5/2)+5947/2048*d^5/(b*d^2*x^2+a*d^2)^5/a^3*b^2*(d*x)^(9
/2)+209/128*d^3/(b*d^2*x^2+a*d^2)^5/a^4*b^3*(d*x)^(13/2)+1463/4096*d/(b*d^2
*x^2+a*d^2)^5/a^5*b^4*(d*x)^(17/2)+4389/32768/d/a^6*(a/b*d^2)^(1/4)*2^(1/2)
*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+4389/16384/d/a^6*(a/b*d^2)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)+4389/16384/d/a^6*
(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 3.11, size = 382, normalized size = 0.99

$$\frac{\frac{8\left(7315\,(dx)^{\frac{17}{2}}b^4d^2+33440\,(dx)^{\frac{13}{2}}ab^3d^4+59470\,(dx)^{\frac{9}{2}}a^2b^2d^6+50312\,(dx)^{\frac{5}{2}}a^3bd^8+19015\,\sqrt{dx}\,a^4d^{10}\right)}{a^5b^5d^{10}x^{10}+5\,a^6b^4d^{10}x^8+10\,a^7b^3d^{10}x^6+10\,a^8b^2d^{10}x^4+5\,a^9bd^{10}x^2+a^{10}d^{10}}+\frac{21945\left(\frac{\sqrt{2}\,d^2\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\right.}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}\right.}{}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/163840*(8*(7315*(d*x)^(17/2)*b^4*d^2 + 33440*(d*x)^(13/2)*a*b^3*d^4 + 594
70*(d*x)^(9/2)*a^2*b^2*d^6 + 50312*(d*x)^(5/2)*a^3*b*d^8 + 19015*sqrt(d*x)*
a^4*d^10)/(a^5*b^5*d^10*x^10 + 5*a^6*b^4*d^10*x^8 + 10*a^7*b^3*d^10*x^6 + 1
0*a^8*b^2*d^10*x^4 + 5*a^9*b*d^10*x^2 + a^10*d^10) + 21945*(sqrt(2)*d^2*log
(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2
)^(3/4)*b^(1/4)) - sqrt(2)*d^2*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt
(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d*arctan(1/2
*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)
*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d*arctan(-1/2*sq
rt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sq
rt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/a^5)/d
```

mupad [B] time = 4.29, size = 210, normalized size = 0.54

$$\frac{\frac{3803\,d^9\,\sqrt{d\,x}}{4096\,a}+\frac{5947\,b^2\,d^5\,(d\,x)^{9/2}}{2048\,a^3}+\frac{209\,b^3\,d^3\,(d\,x)^{13/2}}{128\,a^4}+\frac{6289\,b\,d^7\,(d\,x)^{5/2}}{2560\,a^2}+\frac{1463\,b^4\,d\,(d\,x)^{17/2}}{4096\,a^5}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}+\frac{4389\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{23/4}\,b^{1/4}\,\sqrt{d}}+\frac{4389}{8192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] ((3803*d^9*(d*x)^(1/2))/(4096*a) + (5947*b^2*d^5*(d*x)^(9/2))/(2048*a^3) +
(209*b^3*d^3*(d*x)^(13/2))/(128*a^4) + (6289*b*d^7*(d*x)^(5/2))/(2560*a^2)
+ (1463*b^4*d*(d*x)^(17/2))/(4096*a^5))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b
*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6) +
 (4389*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(23/4)*
b^(1/4)*d^(1/2)) + (4389*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))
/(8192*(-a)^(23/4)*b^(1/4)*d^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\,\left(a+bx^2\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(1/2),x)
[Out] Integral(1/(sqrt(d*x)*(a + b*x**2)**6), x)
```

3.725 $\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=404

$$-\frac{13923\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{25/4}d^{3/2}}+\frac{13923\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{25/4}d^{3/2}}+$$

```
[Out] 13923/16384*b^(1/4)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a
^(25/4)/d^(3/2)*2^(1/2)-13923/16384*b^(1/4)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^
(1/2)/a^(1/4)/d^(1/2))/a^(25/4)/d^(3/2)*2^(1/2)-13923/32768*b^(1/4)*ln(a^(1
/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(25/4)
/d^(3/2)*2^(1/2)+13923/32768*b^(1/4)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a
^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(25/4)/d^(3/2)*2^(1/2)-13923/4096/a^6
/d/(d*x)^(1/2)+1/10/a/d/(b*x^2+a)^5/(d*x)^(1/2)+21/160/a^2/d/(b*x^2+a)^4/(d
*x)^(1/2)+119/640/a^3/d/(b*x^2+a)^3/(d*x)^(1/2)+1547/5120/a^4/d/(b*x^2+a)^2
/(d*x)^(1/2)+13923/20480/a^5/d/(b*x^2+a)/(d*x)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{13923\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{25/4}d^{3/2}}+\frac{13923\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{25/4}d^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -13923/(4096*a^6*d*Sqrt[d*x]) + 1/(10*a*d*Sqrt[d*x]*(a + b*x^2)^5) + 21/(16
0*a^2*d*Sqrt[d*x]*(a + b*x^2)^4) + 119/(640*a^3*d*Sqrt[d*x]*(a + b*x^2)^3)
+ 1547/(5120*a^4*d*Sqrt[d*x]*(a + b*x^2)^2) + 13923/(20480*a^5*d*Sqrt[d*x]*
(a + b*x^2)) + (13923*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/
4)*Sqrt[d])])/(8192*Sqrt[2]*a^(25/4)*d^(3/2)) - (13923*b^(1/4)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(25/4)*d^(3/
2)) - (13923*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(25/4)*d^(3/2)) + (13923*b^(1/4)*L
og[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]]
)/(16384*Sqrt[2]*a^(25/4)*d^(3/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{\left(21b^5\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^5}\,dx}{20a}$$

$$= \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{\left(357b^4\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^4}\,d}{320a^2}$$

$$= \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{119}{640a^3d\sqrt{dx}\left(a+bx^2\right)^3} +$$

$$= \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{119}{640a^3d\sqrt{dx}\left(a+bx^2\right)^3} +$$

$$= \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{119}{640a^3d\sqrt{dx}\left(a+bx^2\right)^3} +$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{}{640a^3d}$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{}{640a^3d}$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{}{640a^3d}$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{}{640a^3d}$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{}{640a^3d}$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{}{640a^3d}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.07

$$-\frac{2x\,{}_2F_1\left(-\frac{1}{4},6;\frac{3}{4};-\frac{bx^2}{a}\right)}{a^6(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] (-2*x*Hypergeometric2F1[-1/4, 6, 3/4, -((b*x^2)/a)])/(a^6*(d*x)^(3/2))
```

fricas [A] time = 1.14, size = 544, normalized size = 1.35

$$\frac{278460\left(a^6b^5d^2x^{11}+5\,a^7b^4d^2x^9+10\,a^8b^3d^2x^7+10\,a^9b^2d^2x^5+5\,a^{10}bd^2x^3+a^{11}d^2x\right)\left(-\frac{b}{a^{25}d^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{2698972\cdots}{\cdots}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(278460*(a^6*b^5*d^2*x^11 + 5*a^7*b^4*d^2*x^9 + 10*a^8*b^3*d^2*x^7
+ 10*a^9*b^2*d^2*x^5 + 5*a^10*b*d^2*x^3 + a^11*d^2*x)*(-b/(a^25*d^6))^(1/4)
*arctan(-1/2698972561467*(2698972561467*sqrt(d*x)*a^6*b*d*(-b/(a^25*d^6))^(
1/4) - sqrt(-7284452887551739093192089*a^13*b*d^4*sqrt(-b/(a^25*d^6)) + 728
4452887551739093192089*b^2*d*x)*a^6*d*(-b/(a^25*d^6))^(1/4))/b) - 69615*(a^
6*b^5*d^2*x^11 + 5*a^7*b^4*d^2*x^9 + 10*a^8*b^3*d^2*x^7 + 10*a^9*b^2*d^2*x^
5 + 5*a^10*b*d^2*x^3 + a^11*d^2*x)*(-b/(a^25*d^6))^(1/4)*log(2698972561467*
a^19*d^5*(-b/(a^25*d^6))^(3/4) + 2698972561467*sqrt(d*x)*b) + 69615*(a^6*b^
5*d^2*x^11 + 5*a^7*b^4*d^2*x^9 + 10*a^8*b^3*d^2*x^7 + 10*a^9*b^2*d^2*x^5 +
5*a^10*b*d^2*x^3 + a^11*d^2*x)*(-b/(a^25*d^6))^(1/4)*log(-2698972561467*a^1
9*d^5*(-b/(a^25*d^6))^(3/4) + 2698972561467*sqrt(d*x)*b) - 4*(69615*b^5*x^1
0 + 334152*a*b^4*x^8 + 634270*a^2*b^3*x^6 + 590240*a^3*b^2*x^4 + 263515*a^4
*b*x^2 + 40960*a^5)*sqrt(d*x))/(a^6*b^5*d^2*x^11 + 5*a^7*b^4*d^2*x^9 + 10*a
^8*b^3*d^2*x^7 + 10*a^9*b^2*d^2*x^5 + 5*a^10*b*d^2*x^3 + a^11*d^2*x)
```

giac [A] time = 0.20, size = 365, normalized size = 0.90

$$-\frac{\frac{327680}{\sqrt{dx}\,a^6}+\frac{139230\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^7b^2d^2}+\frac{139230\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^7b^2d^2}-\frac{69615\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/163840*(327680/(sqrt(d*x)*a^6) + 139230*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan
(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*
b^2*d^2) + 139230*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*b^2*d^2) - 69615*sqrt(2)
*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2
/b))/(a^7*b^2*d^2) + 69615*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d
^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^7*b^2*d^2) + 8*(28655*sqrt(d*x)*b
^5*d^9*x^9 + 129352*sqrt(d*x)*a*b^4*d^9*x^7 + 224670*sqrt(d*x)*a^2*b^3*d^9*
x^5 + 180640*sqrt(d*x)*a^3*b^2*d^9*x^3 + 58715*sqrt(d*x)*a^4*b*d^9*x)/((b*d
^2*x^2 + a*d^2)^5*a^6))/d
```

maple [A] time = 0.03, size = 349, normalized size = 0.86

$$-\frac{11743\,(dx)^{\frac{3}{2}}\,b\,d^{7}}{4096\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a^{2}}-\frac{1129\,(dx)^{\frac{7}{2}}\,b^{2}d^{5}}{128\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a^{3}}-\frac{22467\,(dx)^{\frac{11}{2}}\,b^{3}d^{3}}{2048\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a^{4}}-\frac{16169\,(dx)^{\frac{15}{2}}\,b^{4}d}{2560\left(b\,d^{2}x^{2}+d^{2}a\right)^{5}a^{5}}-\frac{5731\,(}{4096\left(b\,d^{2}x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -11743/4096*d^7*b/a^2/(b*d^2*x^2+a*d^2)^5*(d*x)^(3/2)-1129/128*d^5*b^2/a^3/
(b*d^2*x^2+a*d^2)^5*(d*x)^(7/2)-22467/2048*d^3*b^3/a^4/(b*d^2*x^2+a*d^2)^5*
(d*x)^(11/2)-16169/2560*d*b^4/a^5/(b*d^2*x^2+a*d^2)^5*(d*x)^(15/2)-5731/409
6/d*b^5/a^6/(b*d^2*x^2+a*d^2)^5*(d*x)^(19/2)-13923/32768/d/a^6/(a/b*d^2)^(1
/4)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d
*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))-13923/16384/d/a^6/
(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)+1)-13923
/16384/d/a^6/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(
1/2)-1)-2/a^6/d/(d*x)^(1/2)
```

maxima [A] time = 3.23, size = 388, normalized size = 0.96

$$-\frac{\frac{8\left(69615\,b^{5}d^{10}x^{10}+334152\,ab^{4}d^{10}x^{8}+634270\,a^{2}b^{3}d^{10}x^{6}+590240\,a^{3}b^{2}d^{10}x^{4}+263515\,a^{4}bd^{10}x^{2}+40960\,a^{5}d^{10}\right)}{(dx)^{\frac{21}{2}}a^{6}b^{5}+5\,(dx)^{\frac{17}{2}}a^{7}b^{4}d^{2}+10\,(dx)^{\frac{13}{2}}a^{8}b^{3}d^{4}+10\,(dx)^{\frac{9}{2}}a^{9}b^{2}d^{6}+5\,(dx)^{\frac{5}{2}}a^{10}bd^{8}+\sqrt{dx}\,a^{11}d^{10}}+69615\,b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\cdots\right)}{2\sqrt{\sqrt{\cdots}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{\cdots}}\cdots\right)}{163840\,a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/163840*(8*(69615*b^5*d^10*x^10 + 334152*a*b^4*d^10*x^8 + 634270*a^2*b^3*
d^10*x^6 + 590240*a^3*b^2*d^10*x^4 + 263515*a^4*b*d^10*x^2 + 40960*a^5*d^10
)/((d*x)^(21/2)*a^6*b^5 + 5*(d*x)^(17/2)*a^7*b^4*d^2 + 10*(d*x)^(13/2)*a^8*
b^3*d^4 + 10*(d*x)^(9/2)*a^9*b^2*d^6 + 5*(d*x)^(5/2)*a^10*b*d^8 + sqrt(d*x)
*a^11*d^10) + 69615*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*
b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(
b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/
4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)
*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/
4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)
*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/a^6)
/d
```

mupad [B] time = 0.21, size = 226, normalized size = 0.56

$$\frac{13923\,(-b)^{1/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\sqrt{d\,x}}{a^{1/4}\sqrt{d}}\right)}{8192\,a^{25/4}\,d^{3/2}}-\frac{13923\,(-b)^{1/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\sqrt{d\,x}}{a^{1/4}\sqrt{d}}\right)}{8192\,a^{25/4}\,d^{3/2}}-\frac{\frac{2\,d^{9}}{a}+\frac{52703\,b\,d^{9}\,x^{2}}{4096\,a^{2}}+\frac{3689\,b^{2}\,d^{9}\cdots}{128\,a^{3}}\cdots}{b^{5}\,(d\,x)^{21/2}+a^{5}\,d^{10}\sqrt{d\,x}+10\,a^{3}\,b^{2}\,d^{6}\,(d\,x)^{9/\cdots}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] (13923*(-b)^(1/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(8192*
a^(25/4)*d^(3/2)) - (13923*(-b)^(1/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4
)*d^(1/2))))/(8192*a^(25/4)*d^(3/2)) - ((2*d^9)/a + (52703*b*d^9*x^2)/(4096
*a^2) + (3689*b^2*d^9*x^4)/(128*a^3) + (63427*b^3*d^9*x^6)/(2048*a^4) + (41
769*b^4*d^9*x^8)/(2560*a^5) + (13923*b^5*d^9*x^10)/(4096*a^6))/(b^5*(d*x)^(
21/2) + a^5*d^10*(d*x)^(1/2) + 10*a^3*b^2*d^6*(d*x)^(9/2) + 10*a^2*b^3*d^4*
(d*x)^(13/2) + 5*a^4*b*d^8*(d*x)^(5/2) + 5*a*b^4*d^2*(d*x)^(17/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}} \left(a + bx^2\right)^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Integral(1/((d*x)**(3/2)*(a + b*x**2)**6), x)
```

3.726 $\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=404

$$\frac{33649b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{27/4}d^{5/2}}-\frac{33649b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{27/4}d^{5/2}}+\cdots$$

```
[Out] -33649/12288/a^6/d/(d*x)^(3/2)+1/10/a/d/(d*x)^(3/2)/(b*x^2+a)^5+23/160/a^2/
d/(d*x)^(3/2)/(b*x^2+a)^4+437/1920/a^3/d/(d*x)^(3/2)/(b*x^2+a)^3+437/1024/a
^4/d/(d*x)^(3/2)/(b*x^2+a)^2+4807/4096/a^5/d/(d*x)^(3/2)/(b*x^2+a)+33649/16
384*b^(3/4)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(27/4)/
d^(5/2)*2^(1/2)-33649/16384*b^(3/4)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^
(1/4)/d^(1/2))/a^(27/4)/d^(5/2)*2^(1/2)+33649/32768*b^(3/4)*ln(a^(1/2)*d^(1
/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(27/4)/d^(5/2)
*2^(1/2)-33649/32768*b^(3/4)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b
^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(27/4)/d^(5/2)*2^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{33649b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{27/4}d^{5/2}}-\frac{33649b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{27/4}d^{5/2}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -33649/(12288*a^6*d*(d*x)^(3/2)) + 1/(10*a*d*(d*x)^(3/2)*(a + b*x^2)^5) + 2
3/(160*a^2*d*(d*x)^(3/2)*(a + b*x^2)^4) + 437/(1920*a^3*d*(d*x)^(3/2)*(a +
b*x^2)^3) + 437/(1024*a^4*d*(d*x)^(3/2)*(a + b*x^2)^2) + 4807/(4096*a^5*d*(
d*x)^(3/2)*(a + b*x^2)) + (33649*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d
*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(27/4)*d^(5/2)) - (33649*b^(3/4)*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(
27/4)*d^(5/2)) + (33649*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(27/4)*d^(5/2)) - (3364
9*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(16384*Sqrt[2]*a^(27/4)*d^(5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{\left(23b^5\right)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^5}\,dx}{20a}$$

$$= \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{\left(437b^4\right)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x\right.}}{320a^2}$$

$$= \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{437}{1920a^3d(dx)^{3/2}\left(a+bx\right.}$$

$$= \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{437}{1920a^3d(dx)^{3/2}\left(a+bx\right.}$$

$$= \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{437}{1920a^3d(dx)^{3/2}\left(a+bx\right.}$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{}{1}$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{}{1}$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{}{1}$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{}{1}$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{}{1}$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{}{1}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.08

$$-\frac{2x\,{}_2F_1\left(-\frac{3}{4},6;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^6(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
[Out] (-2*x*Hypergeometric2F1[-3/4, 6, 1/4, -((b*x^2)/a)])/(3*a^6*(d*x)^(5/2))
```

fricas [A] time = 1.12, size = 568, normalized size = 1.41

$$-\frac{2018940\left(a^6b^5d^3x^{12}+5\,a^7b^4d^3x^{10}+10\,a^8b^3d^3x^8+10\,a^9b^2d^3x^6+5\,a^{10}bd^3x^4+a^{11}d^3x^2\right)\left(-\frac{b^3}{a^{27}d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
[Out] -1/245760*(2018940*(a^6*b^5*d^3*x^12 + 5*a^7*b^4*d^3*x^10 + 10*a^8*b^3*d^3*
x^8 + 10*a^9*b^2*d^3*x^6 + 5*a^10*b*d^3*x^4 + a^11*d^3*x^2)*(-b^3/(a^27*d^1
0))^(1/4)*arctan(-(sqrt(d*x)*a^20*b*d^7*(-b^3/(a^27*d^10))^(3/4) - sqrt(a^1
4*d^6*sqrt(-b^3/(a^27*d^10)) + b^2*d*x)*a^20*d^7*(-b^3/(a^27*d^10))^(3/4))/
b^3) + 504735*(a^6*b^5*d^3*x^12 + 5*a^7*b^4*d^3*x^10 + 10*a^8*b^3*d^3*x^8 +
 10*a^9*b^2*d^3*x^6 + 5*a^10*b*d^3*x^4 + a^11*d^3*x^2)*(-b^3/(a^27*d^10))^(
1/4)*log(33649*a^7*d^3*(-b^3/(a^27*d^10))^(1/4) + 33649*sqrt(d*x)*b) - 5047
35*(a^6*b^5*d^3*x^12 + 5*a^7*b^4*d^3*x^10 + 10*a^8*b^3*d^3*x^8 + 10*a^9*b^2
*d^3*x^6 + 5*a^10*b*d^3*x^4 + a^11*d^3*x^2)*(-b^3/(a^27*d^10))^(1/4)*log(-3
3649*a^7*d^3*(-b^3/(a^27*d^10))^(1/4) + 33649*sqrt(d*x)*b) + 4*(168245*b^5*
x^10 + 769120*a*b^4*x^8 + 1367810*a^2*b^3*x^6 + 1157176*a^3*b^2*x^4 + 43734
5*a^4*b*x^2 + 40960*a^5)*sqrt(d*x))/(a^6*b^5*d^3*x^12 + 5*a^7*b^4*d^3*x^10
+ 10*a^8*b^3*d^3*x^8 + 10*a^9*b^2*d^3*x^6 + 5*a^10*b*d^3*x^4 + a^11*d^3*x^2
)
```

giac [A] time = 0.20, size = 356, normalized size = 0.88

$$-\frac{33649\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^7d^3}-\frac{33649\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^7d^3}-\frac{33649\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -33649/16384*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b
)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*d^3) - 33649/16384*sqrt(2)*(a*
b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/
(a*d^2/b)^(1/4))/(a^7*d^3) - 33649/32768*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x
+ sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^7*d^3) + 33649/3276
8*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + s
qrt(a*d^2/b))/(a^7*d^3) - 2/3/(sqrt(d*x)*a^6*d^2*x) - 1/61440*(127285*sqrt(
d*x)*b^5*d^8*x^8 + 564320*sqrt(d*x)*a*b^4*d^8*x^6 + 958210*sqrt(d*x)*a^2*b^
3*d^8*x^4 + 747576*sqrt(d*x)*a^3*b^2*d^8*x^2 + 232545*sqrt(d*x)*a^4*b*d^8)/
((b*d^2*x^2 + a*d^2)^5*a^6*d)
```

maple [A] time = 0.03, size = 352, normalized size = 0.87

$$-\frac{15503\sqrt{dx}\,b\,d^7}{4096\left(b\,d^2x^2+d^2a\right)^5a^2}-\frac{31149\,(dx)^{\frac{5}{2}}\,b^2d^5}{2560\left(b\,d^2x^2+d^2a\right)^5a^3}-\frac{95821\,(dx)^{\frac{9}{2}}\,b^3d^3}{6144\left(b\,d^2x^2+d^2a\right)^5a^4}-\frac{3527\,(dx)^{\frac{13}{2}}\,b^4d}{384\left(b\,d^2x^2+d^2a\right)^5a^5}-\frac{25457}{12288\left(b\,d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -15503/4096*d^7/a^2*b/(b*d^2*x^2+a*d^2)^5*(d*x)^(1/2)-31149/2560*d^5/a^3*b^
2/(b*d^2*x^2+a*d^2)^5*(d*x)^(5/2)-95821/6144*d^3/a^4*b^3/(b*d^2*x^2+a*d^2)^
5*(d*x)^(9/2)-3527/384*d/a^5*b^4/(b*d^2*x^2+a*d^2)^5*(d*x)^(13/2)-25457/122
88/d/a^6*b^5/(b*d^2*x^2+a*d^2)^5*(d*x)^(17/2)-33649/32768/d^3/a^7*b*(a/b*d^
2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2
))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))-33649/16384/d
^3/a^7*b*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2)
+1)-33649/16384/d^3/a^7*b*(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^
(1/4)*(d*x)^(1/2)-1)-2/3/a^6/d/(d*x)^(3/2)
```

maxima [A] time = 3.23, size = 395, normalized size = 0.98

$$-\frac{\frac{8\left(168245\,b^5d^{10}x^{10}+769120\,ab^4d^{10}x^8+1367810\,a^2b^3d^{10}x^6+1157176\,a^3b^2d^{10}x^4+437345\,a^4bd^{10}x^2+40960\,a^5d^{10}\right)}{(dx)^{\frac{23}{2}}a^6b^5+5\,(dx)^{\frac{19}{2}}a^7b^4d^2+10\,(dx)^{\frac{15}{2}}a^8b^3d^4+10\,(dx)^{\frac{11}{2}}a^9b^2d^6+5\,(dx)^{\frac{7}{2}}a^{10}bd^8+(dx)^{\frac{3}{2}}a^{11}d^{10}}+\frac{504735\left(\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(a \cdots\right.}{(ad^2)\cdots}\cdots\right.}{4915\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/491520*(8*(168245*b^5*d^10*x^10 + 769120*a*b^4*d^10*x^8 + 1367810*a^2*b^
3*d^10*x^6 + 1157176*a^3*b^2*d^10*x^4 + 437345*a^4*b*d^10*x^2 + 40960*a^5*d
^10)/((d*x)^(23/2)*a^6*b^5 + 5*(d*x)^(19/2)*a^7*b^4*d^2 + 10*(d*x)^(15/2)*a
^8*b^3*d^4 + 10*(d*x)^(11/2)*a^9*b^2*d^6 + 5*(d*x)^(7/2)*a^10*b*d^8 + (d*x)
^(3/2)*a^11*d^10) + 504735*(sqrt(2)*b^(3/4)*log(sqrt(b)*d*x + sqrt(2)*(a*d^
2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/(a*d^2)^(3/4) - sqrt(2)*b^(3/4)*log
(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/(a*d^2)
^(3/4) + 2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*
sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a
)*d) + 2*sqrt(2)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*s
qrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)
*d))/a^6)/d
```

mupad [B] time = 4.46, size = 226, normalized size = 0.56

$$\frac{33649\,{(-b)}^{3/4}\,\mathrm{atan}\left(\frac{{(-b)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{8192\,a^{27/4}\,d^{5/2}}-\frac{\frac{2\,d^9}{3\,a}+\frac{87469\,b\,d^9\,x^2}{12288\,a^2}+\frac{144647\,b^2\,d^9\,x^4}{7680\,a^3}+\frac{136781\,b^3\,d^9\,x^6}{6144\,a^4}+\frac{4807\,b^4\,d^9\,x^8}{384\,a^5}+\cdots}{b^5\,{(d\,x)}^{23/2}+a^5\,d^{10}\,{(d\,x)}^{3/2}+10\,a^3\,b^2\,d^6\,{(d\,x)}^{11/2}+10\,a^2\,b^3\,d^4\,{(d\,x)}^{15/2}+5\,a^4\,b\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] (33649*(-b)^(3/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(8192*a
^(27/4)*d^(5/2)) - ((2*d^9)/(3*a) + (87469*b*d^9*x^2)/(12288*a^2) + (144647
*b^2*d^9*x^4)/(7680*a^3) + (136781*b^3*d^9*x^6)/(6144*a^4) + (4807*b^4*d^9*
x^8)/(384*a^5) + (33649*b^5*d^9*x^10)/(12288*a^6))/(b^5*(d*x)^(23/2) + a^5*
d^10*(d*x)^(3/2) + 10*a^3*b^2*d^6*(d*x)^(11/2) + 10*a^2*b^3*d^4*(d*x)^(15/2
) + 5*a^4*b*d^8*(d*x)^(7/2) + 5*a*b^4*d^2*(d*x)^(19/2)) + (33649*(-b)^(3/4)
*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(8192*a^(27/4)*d^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

3.727 $\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=422

$$\frac{69615b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{29/4}d^{7/2}}-\frac{69615b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{29/4}d^{7/2}}-\cdots$$

```
[Out] -13923/4096/a^6/d/(d*x)^(5/2)+1/10/a/d/(d*x)^(5/2)/(b*x^2+a)^5+5/32/a^2/d/(
d*x)^(5/2)/(b*x^2+a)^4+35/128/a^3/d/(d*x)^(5/2)/(b*x^2+a)^3+595/1024/a^4/d/
(d*x)^(5/2)/(b*x^2+a)^2+7735/4096/a^5/d/(d*x)^(5/2)/(b*x^2+a)-69615/16384*b
^(5/4)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(29/4)/d^(7/
2)*2^(1/2)+69615/16384*b^(5/4)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)
/d^(1/2))/a^(29/4)/d^(7/2)*2^(1/2)+69615/32768*b^(5/4)*ln(a^(1/2)*d^(1/2)+x
*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(29/4)/d^(7/2)*2^(1
/2)-69615/32768*b^(5/4)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4
)*2^(1/2)*(d*x)^(1/2))/a^(29/4)/d^(7/2)*2^(1/2)+69615/4096*b/a^7/d^3/(d*x)^
(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{69615b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{29/4}d^{7/2}}-\frac{69615b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{29/4}d^{7/2}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -13923/(4096*a^6*d*(d*x)^(5/2)) + (69615*b)/(4096*a^7*d^3*Sqrt[d*x]) + 1/(1
0*a*d*(d*x)^(5/2)*(a + b*x^2)^5) + 5/(32*a^2*d*(d*x)^(5/2)*(a + b*x^2)^4) +
 35/(128*a^3*d*(d*x)^(5/2)*(a + b*x^2)^3) + 595/(1024*a^4*d*(d*x)^(5/2)*(a
+ b*x^2)^2) + 7735/(4096*a^5*d*(d*x)^(5/2)*(a + b*x^2)) - (69615*b^(5/4)*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(2
9/4)*d^(7/2)) + (69615*b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1
/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(29/4)*d^(7/2)) + (69615*b^(5/4)*Log[Sqrt[a]
*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*S
qrt[2]*a^(29/4)*d^(7/2)) - (69615*b^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqr
t[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(29/4)*d^(7/2
))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{\left(5b^5\right)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^5}\,dx}{4a}$$

$$= \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4} + \frac{\left(105b^4\right)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right.}}{64a^2}$$

$$= \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4} + \frac{35}{128a^3d(dx)^{5/2}\left(a+bx^2\right.}$$

$$= \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4} + \frac{35}{128a^3d(dx)^{5/2}\left(a+bx^2\right.}$$

$$= \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4} + \frac{35}{128a^3d(dx)^{5/2}\left(a+bx^2\right.}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4} + \frac{}{128}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.09

$$-\frac{2\sqrt{dx}\ {}_2F_1\left(-\frac{5}{4},6;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^6d^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] (-2*Sqrt[d*x]*Hypergeometric2F1[-5/4, 6, -1/4, -((b*x^2)/a)])/(5*a^6*d^4*x^
3)
```

fricas [A] time = 1.10, size = 591, normalized size = 1.40

$$-\frac{1392300\left(a^7b^5d^4x^{13}+5\,a^8b^4d^4x^{11}+10\,a^9b^3d^4x^9+10\,a^{10}b^2d^4x^7+5\,a^{11}bd^4x^5+a^{12}d^4x^3\right)\left(-\frac{b^5}{a^{29}d^{14}}\right)^{\frac{1}{4}}\arctan\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(1392300*(a^7*b^5*d^4*x^13 + 5*a^8*b^4*d^4*x^11 + 10*a^9*b^3*d^4*x
^9 + 10*a^10*b^2*d^4*x^7 + 5*a^11*b*d^4*x^5 + a^12*d^4*x^3)*(-b^5/(a^29*d^1
4))^(1/4)*arctan(-1/337371570183375*(337371570183375*sqrt(d*x)*a^7*b^4*d^3*
(-b^5/(a^29*d^14))^(1/4) - sqrt(-113819576367995923331126390625*a^15*b^5*d^
8*sqrt(-b^5/(a^29*d^14)) + 113819576367995923331126390625*b^8*d*x)*a^7*d^3*
(-b^5/(a^29*d^14))^(1/4))/b^5) - 348075*(a^7*b^5*d^4*x^13 + 5*a^8*b^4*d^4*x
^11 + 10*a^9*b^3*d^4*x^9 + 10*a^10*b^2*d^4*x^7 + 5*a^11*b*d^4*x^5 + a^12*d^
4*x^3)*(-b^5/(a^29*d^14))^(1/4)*log(337371570183375*a^22*d^11*(-b^5/(a^29*d
^14))^(3/4) + 337371570183375*sqrt(d*x)*b^4) + 348075*(a^7*b^5*d^4*x^13 + 5
*a^8*b^4*d^4*x^11 + 10*a^9*b^3*d^4*x^9 + 10*a^10*b^2*d^4*x^7 + 5*a^11*b*d^4
*x^5 + a^12*d^4*x^3)*(-b^5/(a^29*d^14))^(1/4)*log(-337371570183375*a^22*d^1
1*(-b^5/(a^29*d^14))^(3/4) + 337371570183375*sqrt(d*x)*b^4) - 4*(348075*b^6
*x^12 + 1670760*a*b^5*x^10 + 3171350*a^2*b^4*x^8 + 2951200*a^3*b^3*x^6 + 13
17575*a^4*b^2*x^4 + 204800*a^5*b*x^2 - 8192*a^6)*sqrt(d*x))/(a^7*b^5*d^4*x^
13 + 5*a^8*b^4*d^4*x^11 + 10*a^9*b^3*d^4*x^9 + 10*a^10*b^2*d^4*x^7 + 5*a^11
*b*d^4*x^5 + a^12*d^4*x^3)
```

giac [A] time = 0.20, size = 362, normalized size = 0.86

$$\frac{69615\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^8bd^5}+\frac{69615\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^8bd^5}-\frac{69615\,\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 69615/16384*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)
^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^8*b*d^5) + 69615/16384*sqrt(2)*(a
*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))
/(a*d^2/b)^(1/4))/(a^8*b*d^5) - 69615/32768*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d
*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^8*b*d^5) + 69615
/32768*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x
) + sqrt(a*d^2/b))/(a^8*b*d^5) + 1/20480*(348075*b^6*d^12*x^12 + 1670760*a*
b^5*d^12*x^10 + 3171350*a^2*b^4*d^12*x^8 + 2951200*a^3*b^3*d^12*x^6 + 13175
75*a^4*b^2*d^12*x^4 + 204800*a^5*b*d^12*x^2 - 8192*a^6*d^12)/((sqrt(d*x)*b*
d^2*x^2 + sqrt(d*x)*a*d^2)^5*a^7*d^3)
```

maple [A] time = 0.04, size = 368, normalized size = 0.87

$$\frac{34139\,(dx)^{\frac{3}{2}}\,b^2d^5}{4096\left(b\,d^2x^2+d^2a\right)^5a^3}+\frac{3597\,(dx)^{\frac{7}{2}}\,b^3d^3}{128\left(b\,d^2x^2+d^2a\right)^5a^4}+\frac{75471\,(dx)^{\frac{11}{2}}\,b^4d}{2048\left(b\,d^2x^2+d^2a\right)^5a^5}+\frac{56269\,(dx)^{\frac{15}{2}}\,b^5}{2560\left(b\,d^2x^2+d^2a\right)^5a^6d}+\frac{2046}{4096\left(b\,d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 34139/4096*d^5*b^2/a^3/(b*d^2*x^2+a*d^2)^5*(d*x)^(3/2)+3597/128*d^3*b^3/a^4
/(b*d^2*x^2+a*d^2)^5*(d*x)^(7/2)+75471/2048*d*b^4/a^5/(b*d^2*x^2+a*d^2)^5*(
d*x)^(11/2)+56269/2560/d*b^5/a^6/(b*d^2*x^2+a*d^2)^5*(d*x)^(15/2)+20463/409
6/d^3*b^6/a^7/(b*d^2*x^2+a*d^2)^5*(d*x)^(19/2)+69615/32768/d^3*b/a^7/(a/b*d
^2)^(1/4)*2^(1/2)*ln((d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/
2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+69615/16384/
d^3*b/a^7/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)^(1/4)*(d*x)^(1/2
```

```
)+1)+69615/16384/d^3*b/a^7/(a/b*d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b*d^2)
^(1/4)*(d*x)^(1/2)-1)-2/5/a^6/d/(d*x)^(5/2)+12*b/a^7/d^3/(d*x)^(1/2)
```

maxima [A] time = 3.32, size = 410, normalized size = 0.97

$$\frac{\frac{8\left(348075\,b^6d^{12}x^{12}+1670760\,ab^5d^{12}x^{10}+3171350\,a^2b^4d^{12}x^8+2951200\,a^3b^3d^{12}x^6+1317575\,a^4b^2d^{12}x^4+204800\,a^5bd^{12}x^2-8192\,a^6d^{12}\right)}{(dx)^{\frac{25}{2}}a^7b^5d^2+5\,(dx)^{\frac{21}{2}}a^8b^4d^4+10\,(dx)^{\frac{17}{2}}a^9b^3d^6+10\,(dx)^{\frac{13}{2}}a^{10}b^2d^8+5\,(dx)^{\frac{9}{2}}a^{11}bd^{10}+(dx)^{\frac{5}{2}}a^{12}d^{12}}+\frac{348075\,b^2 \cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/163840*(8*(348075*b^6*d^12*x^12 + 1670760*a*b^5*d^12*x^10 + 3171350*a^2*b
^4*d^12*x^8 + 2951200*a^3*b^3*d^12*x^6 + 1317575*a^4*b^2*d^12*x^4 + 204800*
a^5*b*d^12*x^2 - 8192*a^6*d^12)/((d*x)^(25/2)*a^7*b^5*d^2 + 5*(d*x)^(21/2)*
a^8*b^4*d^4 + 10*(d*x)^(17/2)*a^9*b^3*d^6 + 10*(d*x)^(13/2)*a^10*b^2*d^8 +
5*(d*x)^(9/2)*a^11*b*d^10 + (d*x)^(5/2)*a^12*d^12) + 348075*b^2*(2*sqrt(2)*
arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sq
rt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sq
rt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d
*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^
(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
+ sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^7*d^2))/d
```

mupad [B] time = 0.27, size = 239, normalized size = 0.57

$$\frac{\frac{10\,b\,d^9\,x^2}{a^2}-\frac{2\,d^9}{5\,a}+\frac{263515\,b^2\,d^9\,x^4}{4096\,a^3}+\frac{18445\,b^3\,d^9\,x^6}{128\,a^4}+\frac{317135\,b^4\,d^9\,x^8}{2048\,a^5}+\frac{41769\,b^5\,d^9\,x^{10}}{512\,a^6}+\frac{69615\,b^6\,d^9\,x^{12}}{4096\,a^7}}{b^5\,(d\,x)^{25/2}+a^5\,d^{10}\,(d\,x)^{5/2}+10\,a^3\,b^2\,d^6\,(d\,x)^{13/2}+10\,a^2\,b^3\,d^4\,(d\,x)^{17/2}+5\,a^4\,b\,d^8\,(d\,x)^{9/2}+5\,a\,b^4\,d^2\,(d\,x)^{21/2}}-\frac{69\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] ((10*b*d^9*x^2)/a^2 - (2*d^9)/(5*a) + (263515*b^2*d^9*x^4)/(4096*a^3) + (18
445*b^3*d^9*x^6)/(128*a^4) + (317135*b^4*d^9*x^8)/(2048*a^5) + (41769*b^5*d
^9*x^10)/(512*a^6) + (69615*b^6*d^9*x^12)/(4096*a^7))/(b^5*(d*x)^(25/2) + a
^5*d^10*(d*x)^(5/2) + 10*a^3*b^2*d^6*(d*x)^(13/2) + 10*a^2*b^3*d^4*(d*x)^(1
```

```
7/2) + 5*a^4*b*d^8*(d*x)^(9/2) + 5*a*b^4*d^2*(d*x)^(21/2)) - (69615*(-b)^(5
/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(8192*a^(29/4)*d^(7/2
)) + (69615*(-b)^(5/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(
8192*a^(29/4)*d^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

3.728 $\int (dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=93

$$\frac{2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{2a(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)}$$

```
[Out] 2/7*a*(d*x)^(7/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+2/11*b*(d*x)^(11/2)*((b*x
^2+a)^2)^(1/2)/d^3/(b*x^2+a)
```

Rubi [A] time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 14}

$$\frac{2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{2a(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*a*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d*(a + b*x^2)) + (2*b*
(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^3*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^{5/2}\left(ab+b^2x^2\right)\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(ab(dx)^{5/2}+\frac{b^2(dx)^{9/2}}{d^2}\right)\,dx}{ab+b^2x^2}$$
$$= \frac{2a(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)} + \frac{2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}$$

Mathematica [**A**] time = 0.02, size = 44, normalized size = 0.47

$$\frac{2x(dx)^{5/2}\sqrt{\left(a+bx^2\right)^2}\left(11a+7bx^2\right)}{77\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (2*x*(d*x)^(5/2)*Sqrt[(a + b*x^2)^2]*(11*a + 7*b*x^2))/(77*(a + b*x^2))
```

fricas [**A**] time = 0.83, size = 26, normalized size = 0.28

$$\frac{2}{77}\left(7\,bd^2x^5+11\,ad^2x^3\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 2/77*(7*b*d^2*x^5 + 11*a*d^2*x^3)*sqrt(d*x)
```

giac [**A**] time = 0.18, size = 45, normalized size = 0.48

$$\frac{2}{11}\sqrt{dx}\,bd^2x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{7}\sqrt{dx}\,ad^2x^3\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 2/11*sqrt(d*x)*b*d^2*x^5*sgn(b*x^2 + a) + 2/7*sqrt(d*x)*a*d^2*x^3*sgn(b*x^2
 + a)
```

maple [A] time = 0.00, size = 39, normalized size = 0.42

$$\frac{2\left(7b\,x^{2}+11a\right)(dx)^{\frac{5}{2}}\sqrt{\left(b\,x^{2}+a\right)^{2}}\;x}{77\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 2/77*x*(7*b*x^2+11*a)*(d*x)^(5/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

maxima [A] time = 1.32, size = 25, normalized size = 0.27

$$\frac{2\left(7\,(dx)^{\frac{11}{2}}\,b+11\,(dx)^{\frac{7}{2}}\,ad^{2}\right)}{77\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/77*(7*(d*x)^(11/2)*b + 11*(d*x)^(7/2)*a*d^2)/d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^{5/2}\,\sqrt{\left(b\,x^{2}+a\right)^{2}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((d*x)^(5/2)*((a + b*x^2)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.729 $\int (dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=93

$$\frac{2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3\left(a+bx^2\right)}+\frac{2a(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)}$$

```
[Out] 2/5*a*(d*x)^(5/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+2/9*b*(d*x)^(9/2)*((b*x^2
+a)^2)^(1/2)/d^3/(b*x^2+a)
```

Rubi [A] time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 14}

$$\frac{2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3\left(a+bx^2\right)}+\frac{2a(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*a*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x^2)) + (2*b*
(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^3*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^{3/2}\left(ab+b^2x^2\right)dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(ab(dx)^{3/2}+\frac{b^2(dx)^{7/2}}{d^2}\right)dx}{ab+b^2x^2}$$
$$= \frac{2a(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)}+\frac{2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 0.47

$$\frac{2x(dx)^{3/2}\sqrt{\left(a+bx^2\right)^2}\left(9a+5bx^2\right)}{45\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (2*x*(d*x)^(3/2)*Sqrt[(a + b*x^2)^2]*(9*a + 5*b*x^2))/(45*(a + b*x^2))
```

fricas [A] time = 0.64, size = 22, normalized size = 0.24

$$\frac{2}{45}\left(5\,bdx^4+9\,adx^2\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 2/45*(5*b*d*x^4 + 9*a*d*x^2)*sqrt(d*x)
```

giac [A] time = 0.16, size = 42, normalized size = 0.45

$$\frac{2}{45}\left(5\,\sqrt{dx}\,bx^4\mathrm{sgn}\left(bx^2+a\right)+9\,\sqrt{dx}\,ax^2\mathrm{sgn}\left(bx^2+a\right)\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 2/45*(5*sqrt(d*x)*b*x^4*sgn(b*x^2 + a) + 9*sqrt(d*x)*a*x^2*sgn(b*x^2 + a))*
d
```

maple [A] time = 0.00, size = 39, normalized size = 0.42

$$\frac{2\left(5b\,x^{2}+9a\right)(dx)^{\frac{3}{2}}\sqrt{\left(b\,x^{2}+a\right)^{2}}\;x}{45\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*((b*x^2+a)^2)^(1/2),x)
[Out] 2/45*x*(5*b*x^2+9*a)*(d*x)^(3/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

maxima [A] time = 1.25, size = 25, normalized size = 0.27

$$\frac{2\left(5\,(dx)^{\frac{9}{2}}\,b+9\,(dx)^{\frac{5}{2}}\,ad^{2}\right)}{45\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 2/45*(5*(d*x)^(9/2)*b + 9*(d*x)^(5/2)*a*d^2)/d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^{3/2}\,\sqrt{\left(b\,x^{2}+a\right)^{2}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*((a + b*x^2)^2)^(1/2),x)
[Out] int((d*x)^(3/2)*((a + b*x^2)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*((b*x**2+a)**2)**(1/2),x)
[Out] Timed out
```

3.730 $\int \sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=93

$$\frac{2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}+\frac{2a(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)}$$

```
[Out] 2/3*a*(d*x)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+2/7*b*(d*x)^(7/2)*((b*x^2
+a)^2)^(1/2)/d^3/(b*x^2+a)
```

Rubi [A] time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 14}

$$\frac{2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}+\frac{2a(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*a*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) + (2*b*
(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^3*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \sqrt{dx}\left(ab+b^2x^2\right)dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(ab\sqrt{dx}+\frac{b^2(dx)^{5/2}}{d^2}\right)dx}{ab+b^2x^2}$$
$$= \frac{2a(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)} + \frac{2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}$$

Mathematica [**A**] time = 0.01, size = 44, normalized size = 0.47

$$\frac{2\sqrt{dx}\sqrt{\left(a+bx^2\right)^2}\left(7ax+3bx^3\right)}{21\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (2*Sqrt[d*x]*Sqrt[(a + b*x^2)^2]*(7*a*x + 3*b*x^3))/(21*(a + b*x^2))
```

fricas [**A**] time = 0.91, size = 18, normalized size = 0.19

$$\frac{2}{21}\left(3\,bx^3+7\,ax\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 2/21*(3*b*x^3 + 7*a*x)*sqrt(d*x)
```

giac [**A**] time = 0.17, size = 37, normalized size = 0.40

$$\frac{2}{7}\sqrt{dx}\,bx^3\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{3}\sqrt{dx}\,ax\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 2/7*sqrt(d*x)*b*x^3*sgn(b*x^2 + a) + 2/3*sqrt(d*x)*a*x*sgn(b*x^2 + a)
```

maple [A] time = 0.00, size = 39, normalized size = 0.42

$$\frac{2\left(3b\,x^{2}+7a\right)\sqrt{dx}\,\sqrt{\left(b\,x^{2}+a\right)^{2}}\,x}{21\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*((b*x^2+a)^2)^(1/2),x)
[Out] 2/21*x*(3*b*x^2+7*a)*(d*x)^(1/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

maxima [A] time = 1.40, size = 25, normalized size = 0.27

$$\frac{2\left(3\,(dx)^{\frac{7}{2}}\,b+7\,(dx)^{\frac{3}{2}}\,ad^{2}\right)}{21\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 2/21*(3*(d*x)^(7/2)*b + 7*(d*x)^(3/2)*a*d^2)/d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d\,x}\,\sqrt{\left(b\,x^{2}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*((a + b*x^2)^2)^(1/2),x)
[Out] int((d*x)^(1/2)*((a + b*x^2)^2)^(1/2), x)
```

sympy [A] time = 133.05, size = 27, normalized size = 0.29

$$\frac{2a\,(dx)^{\frac{3}{2}}}{3d}+\frac{2b\,(dx)^{\frac{7}{2}}}{7d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*((b*x**2+a)**2)**(1/2),x)
[Out] 2*a*(d*x)**(3/2)/(3*d) + 2*b*(d*x)**(7/2)/(7*d**3)
```

3.731 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{\sqrt{dx}}\,dx$

Optimal. Leaf size=91

$$\frac{2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\left(a+bx^2\right)}+\frac{2a\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d\left(a+bx^2\right)}$$

```
[Out] 2/5*b*(d*x)^(5/2)*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+2*a*(d*x)^(1/2)*((b*x^2
+a)^2)^(1/2)/d/(b*x^2+a)
```

Rubi [A] time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 14}

$$\frac{2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\left(a+bx^2\right)}+\frac{2a\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/Sqrt[d*x],x]
```

```
[Out] (2*a*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(a + b*x^2)) + (2*b*(d*x
)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^3*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{\sqrt{dx}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{\sqrt{dx}}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{\sqrt{dx}}+\frac{b^2(dx)^{3/2}}{d^2}\right)dx}{ab+b^2x^2}$$
$$= \frac{2a\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d\left(a+bx^2\right)} + \frac{2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.47

$$\frac{2\sqrt{\left(a+bx^2\right)^2}\left(5ax+bx^3\right)}{5\sqrt{dx}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/Sqrt[d*x],x]
[Out] (2*Sqrt[(a + b*x^2)^2]*(5*a*x + b*x^3))/(5*Sqrt[d*x]*(a + b*x^2))
```

fricas [A] time = 0.73, size = 19, normalized size = 0.21

$$\frac{2\left(bx^2+5a\right)\sqrt{dx}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(1/2),x, algorithm="fricas")
[Out] 2/5*(b*x^2 + 5*a)*sqrt(d*x)/d
```

giac [A] time = 0.17, size = 40, normalized size = 0.44

$$\frac{2\left(\sqrt{dx}\,bx^2\mathrm{sgn}\left(bx^2+a\right)+5\sqrt{dx}\,a\mathrm{sgn}\left(bx^2+a\right)\right)}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(1/2),x, algorithm="giac")
[Out] 2/5*(sqrt(d*x)*b*x^2*sgn(b*x^2 + a) + 5*sqrt(d*x)*a*sgn(b*x^2 + a))/d
```

maple [A] time = 0.00, size = 38, normalized size = 0.42

$$\frac{2\left(b\,x^{2}+5a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}\;x}{5\left(b\,x^{2}+a\right)\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/(d*x)^(1/2),x)
[Out] 2/5*x*(b*x^2+5*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(d*x)^(1/2)
```

maxima [A] time = 1.37, size = 24, normalized size = 0.26

$$\frac{2\left(5\sqrt{dx}\,a+\frac{(dx)^{\frac{5}{2}}b}{d^{2}}\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(1/2),x, algorithm="maxima")
[Out] 2/5*(5*sqrt(d*x)*a + (d*x)^(5/2)*b/d^2)/d
```

mupad [B] time = 4.36, size = 47, normalized size = 0.52

$$\frac{\left(\frac{2\,x^{3}}{5}+\frac{2\,a\,x}{b}\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{x^{2}\sqrt{d\,x}+\frac{a\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/(d*x)^(1/2),x)
[Out] (((2*x^3)/5 + (2*a*x)/b)*((a + b*x^2)^2)^(1/2))/(x^2*(d*x)^(1/2) + (a*(d*x)
^(1/2))/b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\left(a+bx^{2}\right)^{2}}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(1/2),x)
[Out] Integral(sqrt((a + b*x**2)**2)/sqrt(d*x), x)
```

3.732 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{3/2}} dx$

Optimal. Leaf size=91

$$\frac{2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\left(a+bx^2\right)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\left(a+bx^2\right)}$$

```
[Out] 2/3*b*(d*x)^(3/2)*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)-2*a*((b*x^2+a)^2)^(1/2)
/d/(b*x^2+a)/(d*x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 14}

$$\frac{2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\left(a+bx^2\right)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(3/2),x]
```

```
[Out] (-2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*Sqrt[d*x]*(a + b*x^2)) + (2*b*(d*
x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^3*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{3/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{(dx)^{3/2}}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{(dx)^{3/2}}+\frac{b^2\sqrt{dx}}{d^2}\right)dx}{ab+b^2x^2}$$
$$= -\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\left(a+bx^2\right)}+\frac{2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.47

$$\frac{2x\left(bx^2-3a\right)\sqrt{\left(a+bx^2\right)^2}}{3(dx)^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(3/2),x]
```

```
[Out] (2*x*(-3*a + b*x^2)*Sqrt[(a + b*x^2)^2])/(3*(d*x)^(3/2)*(a + b*x^2))
```

fricas [A] time = 0.52, size = 22, normalized size = 0.24

$$\frac{2\left(bx^2-3\,a\right)\sqrt{dx}}{3\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x^2 - 3*a)*sqrt(d*x)/(d^2*x)
```

giac [A] time = 0.16, size = 41, normalized size = 0.45

$$\frac{2\left(\frac{\sqrt{dx}\,bx\mathrm{sgn}\left(bx^2+a\right)}{d}-\frac{3\,a\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{dx}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(sqrt(d*x)*b*x*sgn(b*x^2 + a)/d - 3*a*sgn(b*x^2 + a)/sqrt(d*x))/d
```

maple [**A**] time = 0.00, size = 39, normalized size = 0.43

$$-\frac{2\left(-b\,x^{2}+3a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}\,x}{3\left(b\,x^{2}+a\right)\left(dx\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/(d*x)^(3/2),x)
```

```
[Out] -2/3*x*(-b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(d*x)^(3/2)
```

maxima [**A**] time = 1.36, size = 25, normalized size = 0.27

$$-\frac{2\left(\frac{3\,a}{\sqrt{dx}}-\frac{\left(dx\right)^{\frac{3}{2}}b}{d^{2}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3*a/sqrt(d*x) - (d*x)^(3/2)*b/d^2)/d
```

mupad [**B**] time = 4.35, size = 52, normalized size = 0.57

$$\frac{\left(\frac{2\,x^{2}}{3\,d}-\frac{2\,a}{b\,d}\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{x^{2}\,\sqrt{d\,x}+\frac{a\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/(d*x)^(3/2),x)
```

```
[Out] (((2*x^2)/(3*d) - (2*a)/(b*d))*((a + b*x^2)^2)^(1/2))/(x^2*(d*x)^(1/2) + (a
*(d*x)^(1/2))/b)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(3/2),x)
```

```
[Out] Timed out
```

3.733 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=91

$$\frac{2b\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)}$$

```
[Out] -2/3*a*((b*x^2+a)^2)^(1/2)/d/(d*x)^(3/2)/(b*x^2+a)+2*b*(d*x)^(1/2)*((b*x^2+
a)^2)^(1/2)/d^3/(b*x^2+a)
```

Rubi [A] time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 14}

$$\frac{2b\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(5/2),x]
```

```
[Out] (-2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(d*x)^(3/2)*(a + b*x^2)) + (2*b
*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{5/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{(dx)^{5/2}}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{(dx)^{5/2}}+\frac{b^2}{d^2\sqrt{dx}}\right)dx}{ab+b^2x^2}$$
$$= -\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)} + \frac{2b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.46

$$-\frac{2x\left(a-3bx^2\right)\sqrt{\left(a+bx^2\right)^2}}{3(dx)^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(5/2),x]
[Out] (-2*x*(a - 3*b*x^2)*Sqrt[(a + b*x^2)^2])/(3*(d*x)^(5/2)*(a + b*x^2))
```

fricas [A] time = 0.80, size = 23, normalized size = 0.25

$$\frac{2\left(3\,bx^2-a\right)\sqrt{dx}}{3\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(5/2),x, algorithm="fricas")
[Out] 2/3*(3*b*x^2 - a)*sqrt(d*x)/(d^3*x^2)
```

giac [A] time = 0.16, size = 42, normalized size = 0.46

$$\frac{2\left(3\sqrt{dx}\,b\mathrm{sgn}\left(bx^2+a\right)-\frac{ad\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{dx}\,x}\right)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(5/2),x, algorithm="giac")
[Out] 2/3*(3*sqrt(d*x)*b*sgn(b*x^2 + a) - a*d*sgn(b*x^2 + a)/(sqrt(d*x)*x))/d^3
```

maple [A] time = 0.00, size = 37, normalized size = 0.41

$$-\frac{2\left(-3b\,x^{2}+a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}\,x}{3\left(b\,x^{2}+a\right)\left(dx\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/(d*x)^(5/2),x)
[Out] -2/3*x*(-3*b*x^2+a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(d*x)^(5/2)
```

maxima [A] time = 1.40, size = 24, normalized size = 0.26

$$-\frac{2\left(\frac{a}{\left(dx\right)^{\frac{3}{2}}}-\frac{3\sqrt{dx}\,b}{d^{2}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(5/2),x, algorithm="maxima")
[Out] -2/3*(a/(d*x)^(3/2) - 3*sqrt(d*x)*b/d^2)/d
```

mupad [B] time = 4.38, size = 53, normalized size = 0.58

$$\frac{\left(\frac{2\,x^{2}}{d^{2}}-\frac{2\,a}{3\,b\,d^{2}}\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{x^{3}\,\sqrt{d\,x}+\frac{a\,x\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/(d*x)^(5/2),x)
[Out] (((2*x^2)/d^2 - (2*a)/(3*b*d^2))*((a + b*x^2)^2)^(1/2))/(x^3*(d*x)^(1/2) +
(a*x*(d*x)^(1/2))/b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(5/2),x)
[Out] Timed out
```

3.734 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\,(a+bx^2)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\,(a+bx^2)}$$

```
[Out] -2/5*a*((b*x^2+a)^2)^(1/2)/d/(d*x)^(5/2)/(b*x^2+a)-2*b*((b*x^2+a)^2)^(1/2)/
d^3/(b*x^2+a)/(d*x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 14}

$$-\frac{2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\,(a+bx^2)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(7/2),x]
```

```
[Out] (-2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(d*x)^(5/2)*(a + b*x^2)) - (2*b
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*Sqrt[d*x]*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{7/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{(dx)^{7/2}}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{(dx)^{7/2}}+\frac{b^2}{d^2(dx)^{3/2}}\right)dx}{ab+b^2x^2}$$
$$= -\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\left(a+bx^2\right)} - \frac{2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.46

$$-\frac{2x\sqrt{\left(a+bx^2\right)^2}\left(a+5bx^2\right)}{5(dx)^{7/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(7/2),x]
[Out] (-2*x*Sqrt[(a + b*x^2)^2]*(a + 5*b*x^2))/(5*(d*x)^(7/2)*(a + b*x^2))
```

fricas [A] time = 0.77, size = 21, normalized size = 0.23

$$-\frac{2\left(5\,bx^2+a\right)\sqrt{dx}}{5\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(7/2),x, algorithm="fricas")
[Out] -2/5*(5*b*x^2 + a)*sqrt(d*x)/(d^4*x^3)
```

giac [A] time = 0.18, size = 44, normalized size = 0.48

$$-\frac{2\left(5\,bd^3x^2\mathrm{sgn}\left(bx^2+a\right)+ad^3\mathrm{sgn}\left(bx^2+a\right)\right)}{5\,\sqrt{dx}\,d^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(7/2),x, algorithm="giac")
[Out] -2/5*(5*b*d^3*x^2*sgn(b*x^2 + a) + a*d^3*sgn(b*x^2 + a))/(sqrt(d*x)*d^6*x^2
)
```

maple [A] time = 0.00, size = 37, normalized size = 0.41

$$-\frac{2\left(5b\,x^{2}+a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}\;x}{5\left(b\,x^{2}+a\right)(dx)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/(d*x)^(7/2),x)
```

```
[Out] -2/5*x*(5*b*x^2+a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(d*x)^(7/2)
```

maxima [A] time = 1.27, size = 25, normalized size = 0.27

$$-\frac{2\left(5\,bd^{2}x^{2}+ad^{2}\right)}{5\,(dx)^{\frac{5}{2}}\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*(5*b*d^2*x^2 + a*d^2)/((d*x)^(5/2)*d^3)
```

mupad [B] time = 4.32, size = 56, normalized size = 0.62

$$-\frac{\left(\frac{2\,x^{2}}{d^{3}}+\frac{2\,a}{5\,b\,d^{3}}\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{x^{4}\,\sqrt{d\,x}+\frac{a\,x^{2}\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/(d*x)^(7/2),x)
```

```
[Out] -((((2*x^2)/d^3 + (2*a)/(5*b*d^3))*((a + b*x^2)^2)^(1/2))/(x^4*(d*x)^(1/2) +
 (a*x^2*(d*x)^(1/2))/b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(7/2),x)
```

```
[Out] Timed out
```

3.735 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx$

Optimal. Leaf size=195

$$\frac{2ab^2(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^5\left(a+bx^2\right)}+\frac{6a^2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{2b^3(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^7\left(a+bx^2\right)}+\frac{2a^3(dx)^{7/2}}{7}$$

```
[Out] 2/7*a^3*(d*x)^(7/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+6/11*a^2*b*(d*x)^(11/2)
*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+2/5*a*b^2*(d*x)^(15/2)*((b*x^2+a)^2)^(1/
2)/d^5/(b*x^2+a)+2/19*b^3*(d*x)^(19/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*x^2+a)
```

Rubi [A] time = 0.06, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^3(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^7\left(a+bx^2\right)}+\frac{2ab^2(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^5\left(a+bx^2\right)}+\frac{6a^2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{2a^3(dx)^{7/2}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*a^3*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d*(a + b*x^2)) + (6*
a^2*b*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^3*(a + b*x^2)) +
(2*a*b^2*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^5*(a + b*x^2))
+ (2*b^3*(d*x)^(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^7*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^{5/2}\left(ab+b^2x^2\right)^3\,dx}{b^2\left(ab+b^2x^2\right)}$$

$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(a^3b^3(dx)^{5/2}+\frac{3a^2b^4(dx)^{9/2}}{d^2}+\frac{3ab^5(dx)^{13/2}}{d^4}+\frac{b^6(dx)^{17/2}}{d^6}\right)}{b^2\left(ab+b^2x^2\right)}$$

$$=\frac{2a^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)}+\frac{6a^2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{2ab}{}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.34

$$\frac{2x(dx)^{5/2}\sqrt{\left(a+bx^2\right)^2}\left(1045a^3+1995a^2bx^2+1463ab^2x^4+385b^3x^6\right)}{7315\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*x*(d*x)^(5/2)*Sqrt[(a + b*x^2)^2]*(1045*a^3 + 1995*a^2*b*x^2 + 1463*a*b^
2*x^4 + 385*b^3*x^6))/(7315*(a + b*x^2))
```

fricas [A] time = 0.85, size = 54, normalized size = 0.28

$$\frac{2}{7315}\left(385\,b^3d^2x^9+1463\,ab^2d^2x^7+1995\,a^2bd^2x^5+1045\,a^3d^2x^3\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/7315*(385*b^3*d^2*x^9 + 1463*a*b^2*d^2*x^7 + 1995*a^2*b*d^2*x^5 + 1045*a^
3*d^2*x^3)*sqrt(d*x)
```

giac [A] time = 0.16, size = 99, normalized size = 0.51

$$\frac{2}{19}\sqrt{dx}\,b^3d^2x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{5}\sqrt{dx}\,ab^2d^2x^7\mathrm{sgn}\left(bx^2+a\right)+\frac{6}{11}\sqrt{dx}\,a^2bd^2x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{7}\sqrt{dx}\,a^3d^2x^3\mathrm{sgn}\left(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/19*sqrt(d*x)*b^3*d^2*x^9*sgn(b*x^2 + a) + 2/5*sqrt(d*x)*a*b^2*d^2*x^7*sgn
(b*x^2 + a) + 6/11*sqrt(d*x)*a^2*b*d^2*x^5*sgn(b*x^2 + a) + 2/7*sqrt(d*x)*a
^3*d^2*x^3*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 61, normalized size = 0.31

$$\frac{2\left(385b^{3}x^{6}+1463a\,b^{2}x^{4}+1995a^{2}b\,x^{2}+1045a^{3}\right)(dx)^{\frac{5}{2}}\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}x}{7315\left(b\,x^{2}+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 2/7315*x*(385*b^3*x^6+1463*a*b^2*x^4+1995*a^2*b*x^2+1045*a^3)*(d*x)^(5/2)*(
(b*x^2+a)^2)^(3/2)/(b*x^2+a)^3
```

maxima [A] time = 1.46, size = 83, normalized size = 0.43

$$\frac{2}{285}\left(15\,b^{3}d^{\frac{5}{2}}x^{3}+19\,ab^{2}d^{\frac{5}{2}}x\right)x^{\frac{13}{2}}+\frac{4}{165}\left(11\,ab^{2}d^{\frac{5}{2}}x^{3}+15\,a^{2}bd^{\frac{5}{2}}x\right)x^{\frac{9}{2}}+\frac{2}{77}\left(7\,a^{2}bd^{\frac{5}{2}}x^{3}+11\,a^{3}d^{\frac{5}{2}}x\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/285*(15*b^3*d^(5/2)*x^3 + 19*a*b^2*d^(5/2)*x)*x^(13/2) + 4/165*(11*a*b^2*
d^(5/2)*x^3 + 15*a^2*b*d^(5/2)*x)*x^(9/2) + 2/77*(7*a^2*b*d^(5/2)*x^3 + 11*
a^3*d^(5/2)*x)*x^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^{5/2}\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{5}{2}}\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**(5/2)*((a + b*x**2)**2)**(3/2), x)
```

3.736 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=195

$$\frac{6ab^2(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^5\left(a+bx^2\right)}+\frac{2a^2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\left(a+bx^2\right)}+\frac{2b^3(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^7\left(a+bx^2\right)}+\frac{2a^3(dx)^{5/2}}{5}$$

```
[Out] 2/5*a^3*(d*x)^(5/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+2/3*a^2*b*(d*x)^(9/2)*(
(b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+6/13*a*b^2*(d*x)^(13/2)*((b*x^2+a)^2)^(1/2
)/d^5/(b*x^2+a)+2/17*b^3*(d*x)^(17/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*x^2+a)
```

Rubi [A] time = 0.06, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^3(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^7\left(a+bx^2\right)}+\frac{6ab^2(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^5\left(a+bx^2\right)}+\frac{2a^2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\left(a+bx^2\right)}+\frac{2a^3(dx)^{5/2}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*a^3*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x^2)) + (2*
a^2*b*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^3*(a + b*x^2)) + (6
*a*b^2*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^5*(a + b*x^2)) +
 (2*b^3*(d*x)^(17/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^7*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^{3/2}\left(ab+b^2x^2\right)^3\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a^3b^3(dx)^{3/2}+\frac{3a^2b^4(dx)^{7/2}}{d^2}+\frac{3ab^5(dx)^{11/2}}{d^4}+\frac{b^6(dx)^{15/2}}{d^6}\right)d}{b^2\left(ab+b^2x^2\right)}$$
$$=\frac{2a^3(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)}+\frac{2a^2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\left(a+bx^2\right)}+\frac{6ab^2(}{}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 0.34

$$\frac{2x(dx)^{3/2}\sqrt{\left(a+bx^2\right)^2}\left(663a^3+1105a^2bx^2+765ab^2x^4+195b^3x^6\right)}{3315\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*x*(d*x)^(3/2)*Sqrt[(a + b*x^2)^2]*(663*a^3 + 1105*a^2*b*x^2 + 765*a*b^2*
x^4 + 195*b^3*x^6))/(3315*(a + b*x^2))
```

fricas [A] time = 0.69, size = 46, normalized size = 0.24

$$\frac{2}{3315}\left(195\,b^3dx^8+765\,ab^2dx^6+1105\,a^2bdx^4+663\,a^3dx^2\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3315*(195*b^3*d*x^8 + 765*a*b^2*d*x^6 + 1105*a^2*b*d*x^4 + 663*a^3*d*x^2)
*sqrt(d*x)
```

giac [A] time = 0.21, size = 90, normalized size = 0.46

$$\frac{2}{3315}\left(195\,\sqrt{dx}\,b^3x^8\mathrm{sgn}\left(bx^2+a\right)+765\,\sqrt{dx}\,ab^2x^6\mathrm{sgn}\left(bx^2+a\right)+1105\,\sqrt{dx}\,a^2bx^4\mathrm{sgn}\left(bx^2+a\right)+663\,\sqrt{dx}\,a^3x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3315*(195*sqrt(d*x)*b^3*x^8*sgn(b*x^2 + a) + 765*sqrt(d*x)*a*b^2*x^6*sgn(
b*x^2 + a) + 1105*sqrt(d*x)*a^2*b*x^4*sgn(b*x^2 + a) + 663*sqrt(d*x)*a^3*x^
2*sgn(b*x^2 + a))*d
```

maple [A] time = 0.01, size = 61, normalized size = 0.31

$$\frac{2\left(195b^{3}x^{6}+765a\,b^{2}x^{4}+1105a^{2}b\,x^{2}+663a^{3}\right)(dx)^{\frac{3}{2}}\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}x}{3315\left(b\,x^{2}+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 2/3315*x*(195*b^3*x^6+765*a*b^2*x^4+1105*a^2*b*x^2+663*a^3)*(d*x)^(3/2)*((b
*x^2+a)^2)^(3/2)/(b*x^2+a)^3
```

maxima [A] time = 1.41, size = 83, normalized size = 0.43

$$\frac{2}{221}\left(13\,b^{3}d^{\frac{3}{2}}x^{3}+17\,ab^{2}d^{\frac{3}{2}}x\right)x^{\frac{11}{2}}+\frac{4}{117}\left(9\,ab^{2}d^{\frac{3}{2}}x^{3}+13\,a^{2}bd^{\frac{3}{2}}x\right)x^{\frac{7}{2}}+\frac{2}{45}\left(5\,a^{2}bd^{\frac{3}{2}}x^{3}+9\,a^{3}d^{\frac{3}{2}}x\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/221*(13*b^3*d^(3/2)*x^3 + 17*a*b^2*d^(3/2)*x)*x^(11/2) + 4/117*(9*a*b^2*d
^(3/2)*x^3 + 13*a^2*b*d^(3/2)*x)*x^(7/2) + 2/45*(5*a^2*b*d^(3/2)*x^3 + 9*a^
3*d^(3/2)*x)*x^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^{3/2}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}}\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**(3/2)*((a + b*x**2)**2)**(3/2), x)
```

3.737 $\int \sqrt{dx} \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=195

$$\frac{6ab^2(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^5\left(a+bx^2\right)}+\frac{6a^2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}+\frac{2b^3(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{15d^7\left(a+bx^2\right)}+\frac{2a^3(dx)^{3/2}\sqrt{}}{3a}$$

```
[Out] 2/3*a^3*(d*x)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+6/7*a^2*b*(d*x)^(7/2)*(
(b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+6/11*a*b^2*(d*x)^(11/2)*((b*x^2+a)^2)^(1/2
)/d^5/(b*x^2+a)+2/15*b^3*(d*x)^(15/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*x^2+a)
```

Rubi [A] time = 0.05, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^3(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{15d^7\left(a+bx^2\right)}+\frac{6ab^2(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^5\left(a+bx^2\right)}+\frac{6a^2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}+\frac{2a^3(dx)^{3/2}\sqrt{}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*a^3*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) + (6*
a^2*b*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^3*(a + b*x^2)) + (6
*a*b^2*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^5*(a + b*x^2)) +
 (2*b^3*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*d^7*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \sqrt{dx}\left(ab+b^2x^2\right)^3 dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a^3b^3\sqrt{dx}+\frac{3a^2b^4(dx)^{5/2}}{d^2}+\frac{3ab^5(dx)^{9/2}}{d^4}+\frac{b^6(dx)^{13/2}}{d^6}\right) dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{2a^3(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)}+\frac{6a^2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}+\frac{6ab^2(}{}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 0.34

$$\frac{2\sqrt{dx}\sqrt{\left(a+bx^2\right)^2}\left(385a^3x+495a^2bx^3+315ab^2x^5+77b^3x^7\right)}{1155\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[d*x]*Sqrt[(a + b*x^2)^2]*(385*a^3*x + 495*a^2*b*x^3 + 315*a*b^2*x^5
 + 77*b^3*x^7))/(1155*(a + b*x^2))
```

fricas [A] time = 0.96, size = 40, normalized size = 0.21

$$\frac{2}{1155}\left(77\,b^3x^7+315\,ab^2x^5+495\,a^2bx^3+385\,a^3x\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^3*x^7 + 315*a*b^2*x^5 + 495*a^2*b*x^3 + 385*a^3*x)*sqrt(d*x)
```

giac [A] time = 0.16, size = 85, normalized size = 0.44

$$\frac{2}{15}\sqrt{dx}\,b^3x^7\mathrm{sgn}\left(bx^2+a\right)+\frac{6}{11}\sqrt{dx}\,ab^2x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{6}{7}\sqrt{dx}\,a^2bx^3\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{3}\sqrt{dx}\,a^3x\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*sqrt(d*x)*b^3*x^7*sgn(b*x^2 + a) + 6/11*sqrt(d*x)*a*b^2*x^5*sgn(b*x^2
+ a) + 6/7*sqrt(d*x)*a^2*b*x^3*sgn(b*x^2 + a) + 2/3*sqrt(d*x)*a^3*x*sgn(b*x
^2 + a)
```

maple [A] time = 0.01, size = 61, normalized size = 0.31

$$\frac{2\left(77b^3x^6+315a\,b^2x^4+495a^2b\,x^2+385a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}\sqrt{dx}\;x}{1155\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)*(d*x)^(1/2),x)
```

```
[Out] 2/1155*x*(77*b^3*x^6+315*a*b^2*x^4+495*a^2*b*x^2+385*a^3)*((b*x^2+a)^2)^(3/
2)*(d*x)^(1/2)/(b*x^2+a)^3
```

maxima [A] time = 1.44, size = 83, normalized size = 0.43

$$\frac{2}{165}\left(11\,b^3\sqrt{d}\,x^3+15\,ab^2\sqrt{d}\,x\right)x^{\frac{9}{2}}+\frac{4}{77}\left(7\,ab^2\sqrt{d}\,x^3+11\,a^2b\sqrt{d}\,x\right)x^{\frac{5}{2}}+\frac{2}{21}\left(3\,a^2b\sqrt{d}\,x^3+7\,a^3\sqrt{d}\,x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/165*(11*b^3*sqrt(d)*x^3 + 15*a*b^2*sqrt(d)*x)*x^(9/2) + 4/77*(7*a*b^2*sqr
t(d)*x^3 + 11*a^2*b*sqrt(d)*x)*x^(5/2) + 2/21*(3*a^2*b*sqrt(d)*x^3 + 7*a^3*
sqrt(d)*x)*sqrt(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d\,x}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)*(d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(d*x)*((a + b*x**2)**2)**(3/2), x)
```

3.738 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\sqrt{dx}}\,dx$

Optimal. Leaf size=193

$$\frac{2ab^2(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^5\left(a+bx^2\right)}+\frac{6a^2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\left(a+bx^2\right)}+\frac{2b^3(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^7\left(a+bx^2\right)}+\frac{2a^3\sqrt{dx}\,\sqrt{}}{d}$$

```
[Out] 6/5*a^2*b*(d*x)^(5/2)*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+2/3*a*b^2*(d*x)^(9/
2)*((b*x^2+a)^2)^(1/2)/d^5/(b*x^2+a)+2/13*b^3*(d*x)^(13/2)*((b*x^2+a)^2)^(1
/2)/d^7/(b*x^2+a)+2*a^3*(d*x)^(1/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)
```

Rubi [A] time = 0.05, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^3(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^7\left(a+bx^2\right)}+\frac{2ab^2(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^5\left(a+bx^2\right)}+\frac{6a^2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\left(a+bx^2\right)}+\frac{2a^3\sqrt{dx}\,\sqrt{}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/Sqrt[d*x],x]
```

```
[Out] (2*a^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(a + b*x^2)) + (6*a^2*
b*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^3*(a + b*x^2)) + (2*a*b
^2*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^5*(a + b*x^2)) + (2*b^
3*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^7*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\sqrt{dx}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{\sqrt{dx}}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{\sqrt{dx}}+\frac{3a^2b^4(dx)^{3/2}}{d^2}+\frac{3ab^5(dx)^{7/2}}{d^4}+\frac{b^6(dx)^{11/2}}{d^6}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{2a^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d\left(a+bx^2\right)}+\frac{6a^2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\left(a+bx^2\right)}+\frac{2ab^2(dx)^{9/2}\sqrt{a}}{3d^5\left(\right.}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 0.34

$$\frac{2\sqrt{\left(a+bx^2\right)^2}\left(195a^3x+117a^2bx^3+65ab^2x^5+15b^3x^7\right)}{195\sqrt{dx}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/Sqrt[d*x],x]
```

```
[Out] (2*Sqrt[(a + b*x^2)^2]*(195*a^3*x + 117*a^2*b*x^3 + 65*a*b^2*x^5 + 15*b^3*x
^7))/(195*Sqrt[d*x]*(a + b*x^2))
```

fricas [A] time = 0.94, size = 42, normalized size = 0.22

$$\frac{2\left(15\,b^3x^6+65\,ab^2x^4+117\,a^2bx^2+195\,a^3\right)\sqrt{dx}}{195\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/195*(15*b^3*x^6 + 65*a*b^2*x^4 + 117*a^2*b*x^2 + 195*a^3)*sqrt(d*x)/d
```

giac [A] time = 0.17, size = 89, normalized size = 0.46

$$\frac{2\left(15\sqrt{dx}\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+65\sqrt{dx}\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+117\sqrt{dx}\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+195\sqrt{dx}\,a^3\mathrm{sgn}\left(bx^2\right.}{195\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/195*(15*sqrt(d*x)*b^3*x^6*sgn(b*x^2 + a) + 65*sqrt(d*x)*a*b^2*x^4*sgn(b*x
^2 + a) + 117*sqrt(d*x)*a^2*b*x^2*sgn(b*x^2 + a) + 195*sqrt(d*x)*a^3*sgn(b*
x^2 + a))/d
```

maple [A] time = 0.01, size = 61, normalized size = 0.32

$$\frac{2\left(15b^{3}x^{6}+65a\,b^{2}x^{4}+117a^{2}b\,x^{2}+195a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}x}{195\left(b\,x^{2}+a\right)^{3}\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x)
```

```
[Out] 2/195*x*(15*b^3*x^6+65*a*b^2*x^4+117*a^2*b*x^2+195*a^3)*((b*x^2+a)^2)^(3/2)
/(b*x^2+a)^3/(d*x)^(1/2)
```

maxima [A] time = 1.46, size = 87, normalized size = 0.45

$$\frac{2\left(5\left(9\,b^{3}\sqrt{d}\,x^{3}+13\,ab^{2}\sqrt{d}\,x\right)x^{\frac{7}{2}}+26\left(5\,ab^{2}\sqrt{d}\,x^{3}+9\,a^{2}b\sqrt{d}\,x\right)x^{\frac{3}{2}}+\frac{117\left(a^{2}b\sqrt{d}\,x^{3}+5\,a^{3}\sqrt{d}\,x\right)}{\sqrt{x}}\right)}{585\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/585*(5*(9*b^3*sqrt(d)*x^3 + 13*a*b^2*sqrt(d)*x)*x^(7/2) + 26*(5*a*b^2*sqr
t(d)*x^3 + 9*a^2*b*sqrt(d)*x)*x^(3/2) + 117*(a^2*b*sqrt(d)*x^3 + 5*a^3*sqrt
(d)*x)/sqrt(x))/d
```

mupad [B] time = 4.50, size = 76, normalized size = 0.39

$$\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(\frac{6\,a^2\,x^3}{5}+\frac{2\,b^2\,x^7}{13}+\frac{2\,a^3\,x}{b}+\frac{2\,a\,b\,x^5}{3}\right)}{x^2\,\sqrt{d\,x}+\frac{a\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/(d*x)^(1/2),x)
```

```
[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((6*a^2*x^3)/5 + (2*b^2*x^7)/13 + (2*a^3
*x)/b + (2*a*b*x^5)/3))/(x^2*(d*x)^(1/2) + (a*(d*x)^(1/2))/b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(1/2),x)
[Out] Integral(((a + b*x**2)**2)**(3/2)/sqrt(d*x), x)
```

3.739 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=191

$$\frac{6ab^2(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^5\left(a+bx^2\right)}+\frac{2a^2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{2b^3(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^7\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+}}{d\sqrt{dx}}$$

```
[Out] 2*a^2*b*(d*x)^(3/2)*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+6/7*a*b^2*(d*x)^(7/2)
*((b*x^2+a)^2)^(1/2)/d^5/(b*x^2+a)+2/11*b^3*(d*x)^(11/2)*((b*x^2+a)^2)^(1/2
)/d^7/(b*x^2+a)-2*a^3*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)/(d*x)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^3(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^7\left(a+bx^2\right)}+\frac{6ab^2(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^5\left(a+bx^2\right)}+\frac{2a^2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+}}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(3/2),x]
```

```
[Out] (-2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*Sqrt[d*x]*(a + b*x^2)) + (2*a^2
*b*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2)) + (6*a*b^
2*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^5*(a + b*x^2)) + (2*b^3
*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^7*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{3/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{(dx)^{3/2}}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{(dx)^{3/2}}+\frac{3a^2b^4\sqrt{dx}}{d^2}+\frac{3ab^5(dx)^{5/2}}{d^4}+\frac{b^6(dx)^{9/2}}{d^6}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\left(a+bx^2\right)}+\frac{2a^2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{6ab^2(dx)^{7/2}\sqrt{a^2+}}{7d^5\left(a+\right.}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 0.35

$$\frac{2x\sqrt{\left(a+bx^2\right)^2}\left(-77a^3+77a^2bx^2+33ab^2x^4+7b^3x^6\right)}{77(dx)^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(3/2),x]
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-77*a^3 + 77*a^2*b*x^2 + 33*a*b^2*x^4 + 7*b^3*x^6
))/(77*(d*x)^(3/2)*(a + b*x^2))
```

fricas [A] time = 1.06, size = 45, normalized size = 0.24

$$\frac{2\left(7\,b^3x^6+33\,ab^2x^4+77\,a^2bx^2-77\,a^3\right)\sqrt{dx}}{77\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(3/2),x, algorithm="fricas")
[Out] 2/77*(7*b^3*x^6 + 33*a*b^2*x^4 + 77*a^2*b*x^2 - 77*a^3)*sqrt(d*x)/(d^2*x)
```

giac [A] time = 0.17, size = 102, normalized size = 0.53

$$-\frac{2\left(\frac{77\,a^3\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{dx}}-\frac{7\,\sqrt{dx}\,b^3d^{65}x^5\mathrm{sgn}\left(bx^2+a\right)+33\,\sqrt{dx}\,ab^2d^{65}x^3\mathrm{sgn}\left(bx^2+a\right)+77\,\sqrt{dx}\,a^2bd^{65}x\mathrm{sgn}\left(bx^2+a\right)}{d^{66}}\right)}{77\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/77*(77*a^3*sgn(b*x^2 + a)/sqrt(d*x) - (7*sqrt(d*x)*b^3*d^65*x^5*sgn(b*x^
2 + a) + 33*sqrt(d*x)*a*b^2*d^65*x^3*sgn(b*x^2 + a) + 77*sqrt(d*x)*a^2*b*d^
65*x*sgn(b*x^2 + a))/d^66)/d
```

maple [A] time = 0.01, size = 61, normalized size = 0.32

$$-\frac{2\left(-7b^{3}x^{6}-33a\,b^{2}x^{4}-77a^{2}b\,x^{2}+77a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}x}{77\left(b\,x^{2}+a\right)^{3}(dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(3/2),x)
```

```
[Out] -2/77*x*(-7*b^3*x^6-33*a*b^2*x^4-77*a^2*b*x^2+77*a^3)*((b*x^2+a)^2)^(3/2)/(
b*x^2+a)^3/(d*x)^(3/2)
```

maxima [A] time = 1.42, size = 87, normalized size = 0.46

$$\frac{2\left(3\left(7\,b^{3}\sqrt{d}\,x^{3}+11\,ab^{2}\sqrt{d}\,x\right)x^{\frac{5}{2}}+22\left(3\,ab^{2}\sqrt{d}\,x^{3}+7\,a^{2}b\sqrt{d}\,x\right)\sqrt{x}+\frac{77\left(a^{2}b\sqrt{d}\,x^{3}-3\,a^{3}\sqrt{d}\,x\right)}{x^{\frac{3}{2}}}\right)}{231\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/231*(3*(7*b^3*sqrt(d)*x^3 + 11*a*b^2*sqrt(d)*x)*x^(5/2) + 22*(3*a*b^2*sqr
t(d)*x^3 + 7*a^2*b*sqrt(d)*x)*sqrt(x) + 77*(a^2*b*sqrt(d)*x^3 - 3*a^3*sqrt(
d)*x)/x^(3/2))/d^2
```

mupad [B] time = 4.53, size = 87, normalized size = 0.46

$$\frac{\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}\,\left(\frac{2\,a^{2}\,x^{2}}{d}-\frac{2\,a^{3}}{b\,d}+\frac{2\,b^{2}\,x^{6}}{11\,d}+\frac{6\,a\,b\,x^{4}}{7\,d}\right)}{x^{2}\,\sqrt{d\,x}+\frac{a\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/(d*x)^(3/2),x)
```

```
[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((2*a^2*x^2)/d - (2*a^3)/(b*d) + (2*b^2*
x^6)/(11*d) + (6*a*b*x^4)/(7*d)))/(x^2*(d*x)^(1/2) + (a*(d*x)^(1/2))/b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(3/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/(d*x)**(3/2), x)
```

3.740 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=193

$$\frac{6ab^2(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^5\left(a+bx^2\right)}+\frac{6a^2b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{2b^3(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^7\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+2a\ldots}}{3d(dx)^{3/2}\left(\ldots\right.}$$

```
[Out] -2/3*a^3*((b*x^2+a)^2)^(1/2)/d/(d*x)^(3/2)/(b*x^2+a)+6/5*a*b^2*(d*x)^(5/2)*
((b*x^2+a)^2)^(1/2)/d^5/(b*x^2+a)+2/9*b^3*(d*x)^(9/2)*((b*x^2+a)^2)^(1/2)/d
^7/(b*x^2+a)+6*a^2*b*(d*x)^(1/2)*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)
```

Rubi [A] time = 0.05, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^3(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^7\left(a+bx^2\right)}+\frac{6ab^2(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^5\left(a+bx^2\right)}+\frac{6a^2b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+2a\ldots}}{3d(dx)^{3/2}\left(\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(5/2),x]
```

```
[Out] (-2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(d*x)^(3/2)*(a + b*x^2)) + (6
*a^2*b*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2)) + (6*a*
b^2*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^5*(a + b*x^2)) + (2*b
^3*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^7*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{5/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{(dx)^{5/2}}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{(dx)^{5/2}}+\frac{3a^2b^4}{d^2\sqrt{dx}}+\frac{3ab^5(dx)^{3/2}}{d^4}+\frac{b^6(dx)^{7/2}}{d^6}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)}+\frac{6a^2b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{6ab^2(dx)^{5/2}\sqrt{a^2+2}}{5d^5\left(a+b\right.}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.34

$$\frac{2x\sqrt{\left(a+bx^2\right)^2}\left(-15a^3+135a^2bx^2+27ab^2x^4+5b^3x^6\right)}{45(dx)^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(5/2),x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-15*a^3 + 135*a^2*b*x^2 + 27*a*b^2*x^4 + 5*b^3*x^
6))/(45*(d*x)^(5/2)*(a + b*x^2))
```

fricas [A] time = 0.78, size = 45, normalized size = 0.23

$$\frac{2\left(5\,b^3x^6+27\,ab^2x^4+135\,a^2bx^2-15\,a^3\right)\sqrt{dx}}{45\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*b^3*x^6 + 27*a*b^2*x^4 + 135*a^2*b*x^2 - 15*a^3)*sqrt(d*x)/(d^3*x^2
)
```

giac [A] time = 0.17, size = 105, normalized size = 0.54

$$-\frac{2\left(\frac{15\,a^3d\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{dx}\,x}-\frac{5\,\sqrt{dx}\,b^3d^{36}x^4\mathrm{sgn}\left(bx^2+a\right)+27\,\sqrt{dx}\,ab^2d^{36}x^2\mathrm{sgn}\left(bx^2+a\right)+135\,\sqrt{dx}\,a^2bd^{36}\mathrm{sgn}\left(bx^2+a\right)}{d^{36}}\right)}{45\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/45*(15*a^3*d*sgn(b*x^2 + a)/(sqrt(d*x)*x) - (5*sqrt(d*x)*b^3*d^36*x^4*sg
n(b*x^2 + a) + 27*sqrt(d*x)*a*b^2*d^36*x^2*sgn(b*x^2 + a) + 135*sqrt(d*x)*a
^2*b*d^36*sgn(b*x^2 + a))/d^36)/d^3
```

maple [A] time = 0.01, size = 61, normalized size = 0.32

$$-\frac{2\left(-5b^3x^6-27a\,b^2x^4-135a^2b\,x^2+15a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}x}{45\left(b\,x^2+a\right)^3(dx)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(5/2),x)
```

```
[Out] -2/45*x*(-5*b^3*x^6-27*a*b^2*x^4-135*a^2*b*x^2+15*a^3)*((b*x^2+a)^2)^(3/2)/
(b*x^2+a)^3/(d*x)^(5/2)
```

maxima [A] time = 1.47, size = 86, normalized size = 0.45

$$\frac{2\left(\left(5\,b^3\sqrt{d}\,x^3+9\,ab^2\sqrt{d}\,x\right)x^{\frac{3}{2}}+\frac{18\left(ab^2\sqrt{d}\,x^3+5\,a^2b\sqrt{d}\,x\right)}{\sqrt{x}}+\frac{15\left(3\,a^2b\sqrt{d}\,x^3-a^3\sqrt{d}\,x\right)}{x^{\frac{5}{2}}}\right)}{45\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/45*((5*b^3*sqrt(d)*x^3 + 9*a*b^2*sqrt(d)*x)*x^(3/2) + 18*(a*b^2*sqrt(d)*x
^3 + 5*a^2*b*sqrt(d)*x)/sqrt(x) + 15*(3*a^2*b*sqrt(d)*x^3 - a^3*sqrt(d)*x)/
x^(5/2))/d^3
```

mupad [B] time = 4.49, size = 88, normalized size = 0.46

$$\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(\frac{6\,a^2\,x^2}{d^2}-\frac{2\,a^3}{3\,b\,d^2}+\frac{2\,b^2\,x^6}{9\,d^2}+\frac{6\,a\,b\,x^4}{5\,d^2}\right)}{x^3\,\sqrt{d\,x}+\frac{a\,x\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/(d*x)^(5/2),x)
```

```
[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((6*a^2*x^2)/d^2 - (2*a^3)/(3*b*d^2) + (
2*b^2*x^6)/(9*d^2) + (6*a*b*x^4)/(5*d^2)))/(x^3*(d*x)^(1/2) + (a*x*(d*x)^(1
/2))/b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^{2}\right)^{2}\right)^{\frac{3}{2}}}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(5/2),x)
[Out] Integral(((a + b*x**2)**2)**(3/2)/(d*x)**(5/2), x)
```

3.741 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{7/2}} \, dx$

Optimal. Leaf size=191

$$\frac{2ab^2(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^5\left(a+bx^2\right)}-\frac{6a^2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\left(a+bx^2\right)}+\frac{2b^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^7\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+2abx^2+}}{5d(dx)^{5/2}\left(a+b\right.}$$

```
[Out] -2/5*a^3*((b*x^2+a)^2)^(1/2)/d/(d*x)^(5/2)/(b*x^2+a)+2*a*b^2*(d*x)^(3/2)*((
b*x^2+a)^2)^(1/2)/d^5/(b*x^2+a)+2/7*b^3*(d*x)^(7/2)*((b*x^2+a)^2)^(1/2)/d^7
/(b*x^2+a)-6*a^2*b*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)/(d*x)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^7\left(a+bx^2\right)}+\frac{2ab^2(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^5\left(a+bx^2\right)}-\frac{6a^2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+2abx^2+}}{5d(dx)^{5/2}\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(7/2),x]
```

```
[Out] (-2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(d*x)^(5/2)*(a + b*x^2)) - (6
*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*Sqrt[d*x]*(a + b*x^2)) + (2*a*
b^2*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^5*(a + b*x^2)) + (2*b^3
*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^7*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{7/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{(dx)^{7/2}}\,dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{(dx)^{7/2}}+\frac{3a^2b^4}{d^2(dx)^{3/2}}+\frac{3ab^5\sqrt{dx}}{d^4}+\frac{b^6(dx)^{5/2}}{d^6}\right)dx}{b^2\left(ab+b^2x^2\right)}$$

$$= -\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\left(a+bx^2\right)} - \frac{6a^2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\left(a+bx^2\right)} + \frac{2ab^2(dx)^{3/2}\sqrt{a^2+2abx^2}}{d^5\left(a+bx^2\right)}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.35

$$\frac{2x\sqrt{\left(a+bx^2\right)^2}\left(-7a^3-105a^2bx^2+35ab^2x^4+5b^3x^6\right)}{35(dx)^{7/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(7/2),x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-7*a^3 - 105*a^2*b*x^2 + 35*a*b^2*x^4 + 5*b^3*x^6
))/(35*(d*x)^(7/2)*(a + b*x^2))
```

fricas [A] time = 1.07, size = 45, normalized size = 0.24

$$\frac{2\left(5\,b^3x^6+35\,ab^2x^4-105\,a^2bx^2-7\,a^3\right)\sqrt{dx}}{35\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^6 + 35*a*b^2*x^4 - 105*a^2*b*x^2 - 7*a^3)*sqrt(d*x)/(d^4*x^3)
```

giac [A] time = 0.22, size = 107, normalized size = 0.56

$$-\frac{2\left(\frac{7\left(15\,a^2bd^3x^2\mathrm{sgn}\left(bx^2+a\right)+a^3d^3\mathrm{sgn}\left(bx^2+a\right)\right)}{\sqrt{dx}\,d^2x^2}-\frac{5\left(\sqrt{dx}\,b^3d^{21}x^3\mathrm{sgn}\left(bx^2+a\right)+7\,\sqrt{dx}\,ab^2d^{21}x\mathrm{sgn}\left(bx^2+a\right)\right)}{d^{21}}\right)}{35\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/35*(7*(15*a^2*b*d^3*x^2*sgn(b*x^2 + a) + a^3*d^3*sgn(b*x^2 + a))/(sqrt(d
*x)*d^2*x^2) - 5*(sqrt(d*x)*b^3*d^21*x^3*sgn(b*x^2 + a) + 7*sqrt(d*x)*a*b^2
*d^21*x*sgn(b*x^2 + a))/d^21)/d^4
```

maple [A] time = 0.01, size = 61, normalized size = 0.32

$$-\frac{2\left(-5b^{3}x^{6}-35a\,b^{2}x^{4}+105a^{2}b\,x^{2}+7a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}x}{35\left(b\,x^{2}+a\right)^{3}\left(dx\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(7/2),x)
```

```
[Out] -2/35*x*(-5*b^3*x^6-35*a*b^2*x^4+105*a^2*b*x^2+7*a^3)*((b*x^2+a)^2)^(3/2)/(
b*x^2+a)^3/(d*x)^(7/2)
```

maxima [A] time = 1.45, size = 86, normalized size = 0.45

$$\frac{2\left(5\left(3\,b^{3}\sqrt{d}\,x^{3}+7\,ab^{2}\sqrt{d}\,x\right)\sqrt{x}+\frac{70\left(ab^{2}\sqrt{d}\,x^{3}-3\,a^{2}b\sqrt{d}\,x\right)}{x^{\frac{3}{2}}}-\frac{21\left(5\,a^{2}b\sqrt{d}\,x^{3}+a^{3}\sqrt{d}\,x\right)}{x^{\frac{7}{2}}}\right)}{105\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/105*(5*(3*b^3*sqrt(d)*x^3 + 7*a*b^2*sqrt(d)*x)*sqrt(x) + 70*(a*b^2*sqrt(d
)*x^3 - 3*a^2*b*sqrt(d)*x)/x^(3/2) - 21*(5*a^2*b*sqrt(d)*x^3 + a^3*sqrt(d)*
x)/x^(7/2))/d^4
```

mupad [B] time = 4.53, size = 91, normalized size = 0.48

$$-\frac{\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}\,\left(\frac{2\,a^{3}}{5\,b\,d^{3}}+\frac{6\,a^{2}\,x^{2}}{d^{3}}-\frac{2\,b^{2}\,x^{6}}{7\,d^{3}}-\frac{2\,a\,b\,x^{4}}{d^{3}}\right)}{x^{4}\,\sqrt{d\,x}+\frac{a\,x^{2}\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/(d*x)^(7/2),x)
```

```
[Out] -((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((2*a^3)/(5*b*d^3) + (6*a^2*x^2)/d^3 -
(2*b^2*x^6)/(7*d^3) - (2*a*b*x^4)/d^3))/(x^4*(d*x)^(1/2) + (a*x^2*(d*x)^(1/
2))/b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{(dx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(7/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/(d*x)**(7/2), x)
```

3.742 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=297

$$\frac{2b^5(dx)^{27/2}\sqrt{a^2+2abx^2+b^2x^4}}{27d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{23/2}\sqrt{a^2+2abx^2+b^2x^4}}{23d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^7\left(a+bx^2\right)}+\frac{2a^5(d$$

```
[Out] 2/7*a^5*(d*x)^(7/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+10/11*a^4*b*(d*x)^(11/2
)*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+4/3*a^3*b^2*(d*x)^(15/2)*((b*x^2+a)^2)^
(1/2)/d^5/(b*x^2+a)+20/19*a^2*b^3*(d*x)^(19/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*x
^2+a)+10/23*a*b^4*(d*x)^(23/2)*((b*x^2+a)^2)^(1/2)/d^9/(b*x^2+a)+2/27*b^5*(
d*x)^(27/2)*((b*x^2+a)^2)^(1/2)/d^11/(b*x^2+a)
```

Rubi [A] time = 0.08, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^5(dx)^{27/2}\sqrt{a^2+2abx^2+b^2x^4}}{27d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{23/2}\sqrt{a^2+2abx^2+b^2x^4}}{23d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^7\left(a+bx^2\right)}+\frac{4a^3b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*a^5*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d*(a + b*x^2)) + (10
*a^4*b*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^3*(a + b*x^2)) +
 (4*a^3*b^2*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^5*(a + b*x^2
)) + (20*a^2*b^3*(d*x)^(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^7*(a +
 b*x^2)) + (10*a*b^4*(d*x)^(23/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23*d^9*
(a + b*x^2)) + (2*b^5*(d*x)^(27/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(27*d^1
1*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^{5/2}\left(ab+b^2x^2\right)^5\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(a^5b^5(dx)^{5/2}+\frac{5a^4b^6(dx)^{9/2}}{d^2}+\frac{10a^3b^7(dx)^{13/2}}{d^4}+\frac{10a^2b^8(dx)^{17}}{d^6}\right.}{b^4\left(ab+b^2x^2\right)}$$
$$=\frac{2a^5(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)}+\frac{10a^4b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{4a^3}{}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 0.30

$$\frac{2x(dx)^{5/2}\sqrt{\left(a+bx^2\right)^2}\left(129789a^5+412965a^4bx^2+605682a^3b^2x^4+478170a^2b^3x^6+197505ab^4x^8+33649b^5x^{10}\right.}{908523\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*x*(d*x)^(5/2)*Sqrt[(a + b*x^2)^2]*(129789*a^5 + 412965*a^4*b*x^2 + 60568
2*a^3*b^2*x^4 + 478170*a^2*b^3*x^6 + 197505*a*b^4*x^8 + 33649*b^5*x^10))/(9
08523*(a + b*x^2))
```

fricas [A] time = 0.64, size = 82, normalized size = 0.28

$$\frac{2}{908523}\left(33649\,b^5d^2x^{13}+197505\,ab^4d^2x^{11}+478170\,a^2b^3d^2x^9+605682\,a^3b^2d^2x^7+412965\,a^4bd^2x^5+129789\,a^5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/908523*(33649*b^5*d^2*x^13 + 197505*a*b^4*d^2*x^11 + 478170*a^2*b^3*d^2*x
^9 + 605682*a^3*b^2*d^2*x^7 + 412965*a^4*b*d^2*x^5 + 129789*a^5*d^2*x^3)*sq
rt(d*x)
```

giac [A] time = 0.17, size = 153, normalized size = 0.52

$$\frac{2}{27}\sqrt{dx}\,b^5d^2x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{23}\sqrt{dx}\,ab^4d^2x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{20}{19}\sqrt{dx}\,a^2b^3d^2x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{4}{3}\sqrt{dx}\,a^3b^2d^2x^7\mathrm{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 2/27*sqrt(d*x)*b^5*d^2*x^13*sgn(b*x^2 + a) + 10/23*sqrt(d*x)*a*b^4*d^2*x^11
*sgn(b*x^2 + a) + 20/19*sqrt(d*x)*a^2*b^3*d^2*x^9*sgn(b*x^2 + a) + 4/3*sqrt
(d*x)*a^3*b^2*d^2*x^7*sgn(b*x^2 + a) + 10/11*sqrt(d*x)*a^4*b*d^2*x^5*sgn(b*
x^2 + a) + 2/7*sqrt(d*x)*a^5*d^2*x^3*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 83, normalized size = 0.28

$$\frac{2\left(33649b^5x^{10}+197505a\,b^4x^8+478170a^2b^3x^6+605682a^3b^2x^4+412965a^4b\,x^2+129789a^5\right)(dx)^{\frac{5}{2}}\left(\left(b\,x^2+a\right)^2\right.}{908523\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 2/908523*x*(33649*b^5*x^10+197505*a*b^4*x^8+478170*a^2*b^3*x^6+605682*a^3*b
^2*x^4+412965*a^4*b*x^2+129789*a^5)*(d*x)^(5/2)*((b*x^2+a)^2)^(5/2)/(b*x^2+
a)^5
```

maxima [A] time = 1.47, size = 147, normalized size = 0.49

$$\frac{2}{621}\left(23\,b^5d^{\frac{5}{2}}x^3+27\,ab^4d^{\frac{5}{2}}x\right)x^{\frac{21}{2}}+\frac{8}{437}\left(19\,ab^4d^{\frac{5}{2}}x^3+23\,a^2b^3d^{\frac{5}{2}}x\right)x^{\frac{17}{2}}+\frac{4}{95}\left(15\,a^2b^3d^{\frac{5}{2}}x^3+19\,a^3b^2d^{\frac{5}{2}}x\right)x^{\frac{13}{2}}+\frac{8}{165}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/621*(23*b^5*d^(5/2)*x^3 + 27*a*b^4*d^(5/2)*x)*x^(21/2) + 8/437*(19*a*b^4*
d^(5/2)*x^3 + 23*a^2*b^3*d^(5/2)*x)*x^(17/2) + 4/95*(15*a^2*b^3*d^(5/2)*x^3
 + 19*a^3*b^2*d^(5/2)*x)*x^(13/2) + 8/165*(11*a^3*b^2*d^(5/2)*x^3 + 15*a^4*
b*d^(5/2)*x)*x^(9/2) + 2/77*(7*a^4*b*d^(5/2)*x^3 + 11*a^5*d^(5/2)*x)*x^(5/2
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^{5/2}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

3.743 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=297

$$\frac{2b^5(dx)^{25/2}\sqrt{a^2+2abx^2+b^2x^4}}{25d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{21/2}\sqrt{a^2+2abx^2+b^2x^4}}{21d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^7\left(a+bx^2\right)}+\frac{2a^5(d$$

```
[Out] 2/5*a^5*(d*x)^(5/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+10/9*a^4*b*(d*x)^(9/2)*
((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+20/13*a^3*b^2*(d*x)^(13/2)*((b*x^2+a)^2)^
(1/2)/d^5/(b*x^2+a)+20/17*a^2*b^3*(d*x)^(17/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*x
^2+a)+10/21*a*b^4*(d*x)^(21/2)*((b*x^2+a)^2)^(1/2)/d^9/(b*x^2+a)+2/25*b^5*(
d*x)^(25/2)*((b*x^2+a)^2)^(1/2)/d^11/(b*x^2+a)
```

Rubi [A] time = 0.08, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^5(dx)^{25/2}\sqrt{a^2+2abx^2+b^2x^4}}{25d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{21/2}\sqrt{a^2+2abx^2+b^2x^4}}{21d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^7\left(a+bx^2\right)}+\frac{20a^3b$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*a^5*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x^2)) + (10
*a^4*b*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^3*(a + b*x^2)) + (
20*a^3*b^2*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^5*(a + b*x^2
)) + (20*a^2*b^3*(d*x)^(17/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^7*(a +
 b*x^2)) + (10*a*b^4*(d*x)^(21/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*d^9*
(a + b*x^2)) + (2*b^5*(d*x)^(25/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(25*d^1
1*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int (dx)^{3/2}\left(ab+b^2x^2\right)^5\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int \left(a^5b^5(dx)^{3/2}+\frac{5a^4b^6(dx)^{7/2}}{d^2}+\frac{10a^3b^7(dx)^{11/2}}{d^4}+\frac{10a^2b^8(dx)^{15}}{d^6}\right.}{b^4\left(ab+b^2x^2\right)}$$

$$=\frac{2a^5(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)}+\frac{10a^4b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3\left(a+bx^2\right)}+\frac{20a^3}{}$$

Mathematica [A] time = 0.03, size = 88, normalized size = 0.30

$$\frac{2x(dx)^{3/2}\sqrt{\left(a+bx^2\right)^2}\left(69615a^5+193375a^4bx^2+267750a^3b^2x^4+204750a^2b^3x^6+82875ab^4x^8+13923b^5x^{10}\right)}{348075\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*x*(d*x)^(3/2)*Sqrt[(a + b*x^2)^2]*(69615*a^5 + 193375*a^4*b*x^2 + 267750
*a^3*b^2*x^4 + 204750*a^2*b^3*x^6 + 82875*a*b^4*x^8 + 13923*b^5*x^10))/(348
075*(a + b*x^2))
```

fricas [A] time = 0.64, size = 70, normalized size = 0.24

$$\frac{2}{348075}\left(13923\,b^5dx^{12}+82875\,ab^4dx^{10}+204750\,a^2b^3dx^8+267750\,a^3b^2dx^6+193375\,a^4bdx^4+69615\,a^5dx^2\right)\sqrt{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/348075*(13923*b^5*d*x^12 + 82875*a*b^4*d*x^10 + 204750*a^2*b^3*d*x^8 + 26
7750*a^3*b^2*d*x^6 + 193375*a^4*b*d*x^4 + 69615*a^5*d*x^2)*sqrt(d*x)
```

giac [A] time = 0.20, size = 138, normalized size = 0.46

$$\frac{2}{348075}\left(13923\sqrt{dx}\,b^5x^{12}\mathrm{sgn}\left(bx^2+a\right)+82875\sqrt{dx}\,ab^4x^{10}\mathrm{sgn}\left(bx^2+a\right)+204750\sqrt{dx}\,a^2b^3x^8\mathrm{sgn}\left(bx^2+a\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 2/348075*(13923*sqrt(d*x)*b^5*x^12*sgn(b*x^2 + a) + 82875*sqrt(d*x)*a*b^4*x
^10*sgn(b*x^2 + a) + 204750*sqrt(d*x)*a^2*b^3*x^8*sgn(b*x^2 + a) + 267750*s
qrt(d*x)*a^3*b^2*x^6*sgn(b*x^2 + a) + 193375*sqrt(d*x)*a^4*b*x^4*sgn(b*x^2
+ a) + 69615*sqrt(d*x)*a^5*x^2*sgn(b*x^2 + a))*d
```

maple [A] time = 0.01, size = 83, normalized size = 0.28

$$\frac{2\left(13923b^5x^{10}+82875a\,b^4x^8+204750a^2b^3x^6+267750a^3b^2x^4+193375a^4b\,x^2+69615a^5\right)(dx)^{\frac{3}{2}}\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{348075\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 2/348075*x*(13923*b^5*x^10+82875*a*b^4*x^8+204750*a^2*b^3*x^6+267750*a^3*b^
2*x^4+193375*a^4*b*x^2+69615*a^5)*(d*x)^(3/2)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)
^5
```

maxima [A] time = 1.42, size = 147, normalized size = 0.49

$$\frac{2}{525}\left(21\,b^5d^{\frac{3}{2}}x^3+25\,ab^4d^{\frac{3}{2}}x\right)x^{\frac{19}{2}}+\frac{8}{357}\left(17\,ab^4d^{\frac{3}{2}}x^3+21\,a^2b^3d^{\frac{3}{2}}x\right)x^{\frac{15}{2}}+\frac{12}{221}\left(13\,a^2b^3d^{\frac{3}{2}}x^3+17\,a^3b^2d^{\frac{3}{2}}x\right)x^{\frac{11}{2}}+\frac{8}{117}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/525*(21*b^5*d^(3/2)*x^3 + 25*a*b^4*d^(3/2)*x)*x^(19/2) + 8/357*(17*a*b^4*
d^(3/2)*x^3 + 21*a^2*b^3*d^(3/2)*x)*x^(15/2) + 12/221*(13*a^2*b^3*d^(3/2)*x
^3 + 17*a^3*b^2*d^(3/2)*x)*x^(11/2) + 8/117*(9*a^3*b^2*d^(3/2)*x^3 + 13*a^4
*b*d^(3/2)*x)*x^(7/2) + 2/45*(5*a^4*b*d^(3/2)*x^3 + 9*a^5*d^(3/2)*x)*x^(3/2
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^{3/2}\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}} \left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**(3/2)*((a + b*x**2)**2)**(5/2), x)
```

3.744 $\int \sqrt{dx} \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=297

$$\frac{2b^5(dx)^{23/2}\sqrt{a^2+2abx^2+b^2x^4}}{23d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^9\left(a+bx^2\right)}+\frac{4a^2b^3(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^7\left(a+bx^2\right)}+\frac{2a^5(dx}{}$$

```
[Out] 2/3*a^5*(d*x)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+10/7*a^4*b*(d*x)^(7/2)*
((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+20/11*a^3*b^2*(d*x)^(11/2)*((b*x^2+a)^2)^
(1/2)/d^5/(b*x^2+a)+4/3*a^2*b^3*(d*x)^(15/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*x^2
+a)+10/19*a*b^4*(d*x)^(19/2)*((b*x^2+a)^2)^(1/2)/d^9/(b*x^2+a)+2/23*b^5*(d*
x)^(23/2)*((b*x^2+a)^2)^(1/2)/d^11/(b*x^2+a)
```

Rubi [A] time = 0.08, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^5(dx)^{23/2}\sqrt{a^2+2abx^2+b^2x^4}}{23d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^9\left(a+bx^2\right)}+\frac{4a^2b^3(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^7\left(a+bx^2\right)}+\frac{20a^3b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*a^5*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) + (10
*a^4*b*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^3*(a + b*x^2)) + (
20*a^3*b^2*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^5*(a + b*x^2
)) + (4*a^2*b^3*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^7*(a + b
*x^2)) + (10*a*b^4*(d*x)^(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^9*(a
 + b*x^2)) + (2*b^5*(d*x)^(23/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23*d^11*
(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \sqrt{dx}\,\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int \sqrt{dx}\,\left(ab+b^2x^2\right)^5\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int \left(a^5b^5\sqrt{dx}+\frac{5a^4b^6(dx)^{5/2}}{d^2}+\frac{10a^3b^7(dx)^{9/2}}{d^4}+\frac{10a^2b^8(dx)^{13/2}}{d^6}+\right.}{b^4\left(ab+b^2x^2\right)}$$

$$=\frac{2a^5(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)}+\frac{10a^4b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}+\frac{20a^3b}{}$$

Mathematica [A] time = 0.03, size = 88, normalized size = 0.30

$$\frac{2\sqrt{dx}\sqrt{\left(a+bx^2\right)^2}\left(33649a^5x+72105a^4bx^3+91770a^3b^2x^5+67298a^2b^3x^7+26565ab^4x^9+4389b^5x^{11}\right)}{100947\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*Sqrt[d*x]*Sqrt[(a + b*x^2)^2]*(33649*a^5*x + 72105*a^4*b*x^3 + 91770*a^3
*b^2*x^5 + 67298*a^2*b^3*x^7 + 26565*a*b^4*x^9 + 4389*b^5*x^11))/(100947*(a
 + b*x^2))
```

fricas [A] time = 1.01, size = 62, normalized size = 0.21

$$\frac{2}{100947}\left(4389\,b^5x^{11}+26565\,ab^4x^9+67298\,a^2b^3x^7+91770\,a^3b^2x^5+72105\,a^4bx^3+33649\,a^5x\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/100947*(4389*b^5*x^11 + 26565*a*b^4*x^9 + 67298*a^2*b^3*x^7 + 91770*a^3*b
^2*x^5 + 72105*a^4*b*x^3 + 33649*a^5*x)*sqrt(d*x)
```

giac [A] time = 0.20, size = 133, normalized size = 0.45

$$\frac{2}{23}\sqrt{dx}\,b^5x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{19}\sqrt{dx}\,ab^4x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{4}{3}\sqrt{dx}\,a^2b^3x^7\mathrm{sgn}\left(bx^2+a\right)+\frac{20}{11}\sqrt{dx}\,a^3b^2x^5\mathrm{sgn}\left(bx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/23*sqrt(d*x)*b^5*x^11*sgn(b*x^2 + a) + 10/19*sqrt(d*x)*a*b^4*x^9*sgn(b*x^
2 + a) + 4/3*sqrt(d*x)*a^2*b^3*x^7*sgn(b*x^2 + a) + 20/11*sqrt(d*x)*a^3*b^2
*x^5*sgn(b*x^2 + a) + 10/7*sqrt(d*x)*a^4*b*x^3*sgn(b*x^2 + a) + 2/3*sqrt(d*
x)*a^5*x*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 83, normalized size = 0.28

$$\frac{2\left(4389b^5x^{10}+26565a\,b^4x^8+67298a^2b^3x^6+91770a^3b^2x^4+72105a^4b\,x^2+33649a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}\sqrt{dx}\;x}{100947\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)*(d*x)^(1/2),x)
```

```
[Out] 2/100947*x*(4389*b^5*x^10+26565*a*b^4*x^8+67298*a^2*b^3*x^6+91770*a^3*b^2*x
^4+72105*a^4*b*x^2+33649*a^5)*((b*x^2+a)^2)^(5/2)*(d*x)^(1/2)/(b*x^2+a)^5
```

maxima [A] time = 1.53, size = 147, normalized size = 0.49

$$\frac{2}{437}\left(19\,b^5\sqrt{d}\,x^3+23\,ab^4\sqrt{d}\,x\right)x^{\frac{17}{2}}+\frac{8}{285}\left(15\,ab^4\sqrt{d}\,x^3+19\,a^2b^3\sqrt{d}\,x\right)x^{\frac{13}{2}}+\frac{4}{55}\left(11\,a^2b^3\sqrt{d}\,x^3+15\,a^3b^2\sqrt{d}\,x\right)x^{\frac{9}{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/437*(19*b^5*sqrt(d)*x^3 + 23*a*b^4*sqrt(d)*x)*x^(17/2) + 8/285*(15*a*b^4*
sqrt(d)*x^3 + 19*a^2*b^3*sqrt(d)*x)*x^(13/2) + 4/55*(11*a^2*b^3*sqrt(d)*x^3
 + 15*a^3*b^2*sqrt(d)*x)*x^(9/2) + 8/77*(7*a^3*b^2*sqrt(d)*x^3 + 11*a^4*b*s
qrt(d)*x)*x^(5/2) + 2/21*(3*a^4*b*sqrt(d)*x^3 + 7*a^5*sqrt(d)*x)*sqrt(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{d\,x}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)*(d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(d*x)*((a + b*x**2)**2)**(5/2), x)
```

3.745 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\sqrt{dx}} \, dx$

Optimal. Leaf size=293

$$\frac{2b^5(dx)^{21/2}\sqrt{a^2+2abx^2+b^2x^4}}{21d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^7\left(a+bx^2\right)}+\frac{2a^5\sqrt{}}{}$$

```
[Out] 2*a^4*b*(d*x)^(5/2)*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+20/9*a^3*b^2*(d*x)^(9
/2)*((b*x^2+a)^2)^(1/2)/d^5/(b*x^2+a)+20/13*a^2*b^3*(d*x)^(13/2)*((b*x^2+a)
^2)^(1/2)/d^7/(b*x^2+a)+10/17*a*b^4*(d*x)^(17/2)*((b*x^2+a)^2)^(1/2)/d^9/(b
*x^2+a)+2/21*b^5*(d*x)^(21/2)*((b*x^2+a)^2)^(1/2)/d^11/(b*x^2+a)+2*a^5*(d*x
)^(1/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)
```

Rubi [A] time = 0.08, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^5(dx)^{21/2}\sqrt{a^2+2abx^2+b^2x^4}}{21d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^7\left(a+bx^2\right)}+\frac{20a^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/Sqrt[d*x],x]
```

```
[Out] (2*a^5*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(a + b*x^2)) + (2*a^4*
b*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2)) + (20*a^3*
b^2*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^5*(a + b*x^2)) + (20*
a^2*b^3*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^7*(a + b*x^2))
+ (10*a*b^4*(d*x)^(17/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^9*(a + b*x^
2)) + (2*b^5*(d*x)^(21/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*d^11*(a + b*
x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\sqrt{dx}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{\sqrt{dx}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{\sqrt{dx}}+\frac{5a^4b^6(dx)^{3/2}}{d^2}+\frac{10a^3b^7(dx)^{7/2}}{d^4}+\frac{10a^2b^8(dx)^{11/2}}{d^6}+\frac{5ab^9(dx)^{15/2}}{d^8}+\right.}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{2a^5\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d\left(a+bx^2\right)}+\frac{2a^4b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{9/2}}{9d^5}$$

Mathematica [A] time = 0.03, size = 88, normalized size = 0.30

$$\frac{2\sqrt{\left(a+bx^2\right)^2}\left(13923a^5x+13923a^4bx^3+15470a^3b^2x^5+10710a^2b^3x^7+4095ab^4x^9+663b^5x^{11}\right)}{13923\sqrt{dx}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/Sqrt[d*x],x]
```

```
[Out] (2*Sqrt[(a + b*x^2)^2]*(13923*a^5*x + 13923*a^4*b*x^3 + 15470*a^3*b^2*x^5 +
 10710*a^2*b^3*x^7 + 4095*a*b^4*x^9 + 663*b^5*x^11))/(13923*Sqrt[d*x]*(a +
b*x^2))
```

fricas [A] time = 0.91, size = 64, normalized size = 0.22

$$\frac{2\left(663\,b^5x^{10}+4095\,ab^4x^8+10710\,a^2b^3x^6+15470\,a^3b^2x^4+13923\,a^4bx^2+13923\,a^5\right)\sqrt{dx}}{13923\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*b^5*x^10 + 4095*a*b^4*x^8 + 10710*a^2*b^3*x^6 + 15470*a^3*b^2*
x^4 + 13923*a^4*b*x^2 + 13923*a^5)*sqrt(d*x)/d
```

giac [A] time = 0.17, size = 137, normalized size = 0.47

$$\frac{2\left(663\sqrt{dx}\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+4095\sqrt{dx}\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+10710\sqrt{dx}\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+15470\sqrt{dx}\,a\right.}{13923\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/13923*(663*sqrt(d*x)*b^5*x^10*sgn(b*x^2 + a) + 4095*sqrt(d*x)*a*b^4*x^8*s
gn(b*x^2 + a) + 10710*sqrt(d*x)*a^2*b^3*x^6*sgn(b*x^2 + a) + 15470*sqrt(d*x
)*a^3*b^2*x^4*sgn(b*x^2 + a) + 13923*sqrt(d*x)*a^4*b*x^2*sgn(b*x^2 + a) + 1
3923*sqrt(d*x)*a^5*sgn(b*x^2 + a))/d
```

maple [A] time = 0.01, size = 83, normalized size = 0.28

$$\frac{2\left(663b^5x^{10}+4095a\,b^4x^8+10710a^2b^3x^6+15470a^3b^2x^4+13923a^4b\,x^2+13923a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x}{13923\left(b\,x^2+a\right)^5\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x)
```

```
[Out] 2/13923*x*(663*b^5*x^10+4095*a*b^4*x^8+10710*a^2*b^3*x^6+15470*a^3*b^2*x^4+
13923*a^4*b*x^2+13923*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5/(d*x)^(1/2)
```

maxima [A] time = 1.45, size = 151, normalized size = 0.52

$$\frac{2\left(195\left(17\,b^5\sqrt{d}\,x^3+21\,ab^4\sqrt{d}\,x\right)x^{\frac{15}{2}}+1260\left(13\,ab^4\sqrt{d}\,x^3+17\,a^2b^3\sqrt{d}\,x\right)x^{\frac{11}{2}}+3570\left(9\,a^2b^3\sqrt{d}\,x^3+13\,a^3b^2\sqrt{d}\right.\right.}{69615\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/69615*(195*(17*b^5*sqrt(d)*x^3 + 21*a*b^4*sqrt(d)*x)*x^(15/2) + 1260*(13*
a*b^4*sqrt(d)*x^3 + 17*a^2*b^3*sqrt(d)*x)*x^(11/2) + 3570*(9*a^2*b^3*sqrt(d
)*x^3 + 13*a^3*b^2*sqrt(d)*x)*x^(7/2) + 6188*(5*a^3*b^2*sqrt(d)*x^3 + 9*a^4
*b*sqrt(d)*x)*x^(3/2) + 13923*(a^4*b*sqrt(d)*x^3 + 5*a^5*sqrt(d)*x)/sqrt(x)
)/d
```

mupad [B] time = 4.57, size = 112, normalized size = 0.38

$$\frac{2\,x\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(5731\,a^4+8192\,a^3\,b\,x^2+7278\,a^2\,b^2\,x^4+3432\,a\,b^3\,x^6+663\,b^4\,x^8\right)}{13923\,\sqrt{d\,x}}+\frac{16384\,a^5\,x\,\sqrt{a^2+}}{13923\,\sqrt{d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/(d*x)^(1/2),x)
```

```
[Out] (2*x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(5731*a^4 + 663*b^4*x^8 + 8192*a^3*b
*x^2 + 3432*a*b^3*x^6 + 7278*a^2*b^2*x^4))/(13923*(d*x)^(1/2)) + (16384*a^5
*x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(13923*(d*x)^(1/2)*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{\sqrt{dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(1/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/sqrt(d*x), x)
```

3.746 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=295

$$\frac{2b^5(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^{11}\left(a+bx^2\right)}+\frac{2ab^4(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^7\left(a+bx^2\right)}-\frac{2a^5\sqrt{a}}{d\sqrt{}}$$

```
[Out] 10/3*a^4*b*(d*x)^(3/2)*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+20/7*a^3*b^2*(d*x)
^(7/2)*((b*x^2+a)^2)^(1/2)/d^5/(b*x^2+a)+20/11*a^2*b^3*(d*x)^(11/2)*((b*x^2
+a)^2)^(1/2)/d^7/(b*x^2+a)+2/3*a*b^4*(d*x)^(15/2)*((b*x^2+a)^2)^(1/2)/d^9/(
b*x^2+a)+2/19*b^5*(d*x)^(19/2)*((b*x^2+a)^2)^(1/2)/d^11/(b*x^2+a)-2*a^5*((b
*x^2+a)^2)^(1/2)/d/(b*x^2+a)/(d*x)^(1/2)
```

Rubi [**A**] time = 0.08, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^5(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^{11}\left(a+bx^2\right)}+\frac{2ab^4(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^7\left(a+bx^2\right)}+\frac{20a^3b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(3/2),x]
```

```
[Out] (-2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*Sqrt[d*x]*(a + b*x^2)) + (10*a^
4*b*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^3*(a + b*x^2)) + (20*
a^3*b^2*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^5*(a + b*x^2)) +
(20*a^2*b^3*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^7*(a + b*x^
2)) + (2*a*b^4*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^9*(a + b*
x^2)) + (2*b^5*(d*x)^(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^11*(a +
b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{(dx)^{3/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{(dx)^{3/2}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{(dx)^{3/2}}+\frac{5a^4b^6\sqrt{dx}}{d^2}+\frac{10a^3b^7(dx)^{5/2}}{d^4}+\frac{10a^2b^8(dx)^{9/2}}{d^6}+\frac{5ab^9(dx)^{13/2}}{d^8}+\right.}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\left(a+bx^2\right)}+\frac{10a^4b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{7/2}\sqrt{a}}{7d^5\left(a\right.}$$

Mathematica [A] time = 0.03, size = 88, normalized size = 0.30

$$\frac{2x\sqrt{\left(a+bx^2\right)^2}\left(-4389a^5+7315a^4bx^2+6270a^3b^2x^4+3990a^2b^3x^6+1463ab^4x^8+231b^5x^{10}\right)}{4389(dx)^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(3/2),x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-4389*a^5 + 7315*a^4*b*x^2 + 6270*a^3*b^2*x^4 + 3
990*a^2*b^3*x^6 + 1463*a*b^4*x^8 + 231*b^5*x^10))/(4389*(d*x)^(3/2)*(a + b*
x^2))
```

fricas [A] time = 0.82, size = 67, normalized size = 0.23

$$\frac{2\left(231\,b^5x^{10}+1463\,ab^4x^8+3990\,a^2b^3x^6+6270\,a^3b^2x^4+7315\,a^4bx^2-4389\,a^5\right)\sqrt{dx}}{4389\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/4389*(231*b^5*x^10 + 1463*a*b^4*x^8 + 3990*a^2*b^3*x^6 + 6270*a^3*b^2*x^4
 + 7315*a^4*b*x^2 - 4389*a^5)*sqrt(d*x)/(d^2*x)
```

giac [A] time = 0.19, size = 156, normalized size = 0.53

$$-\frac{2\left(\frac{4389\,a^5\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{dx}}-\frac{231\,\sqrt{dx}\,b^5d^{189}x^9\mathrm{sgn}\left(bx^2+a\right)+1463\,\sqrt{dx}\,ab^4d^{189}x^7\mathrm{sgn}\left(bx^2+a\right)+3990\,\sqrt{dx}\,a^2b^3d^{189}x^5\mathrm{sgn}\left(bx^2+a\right)+6270\,\sqrt{dx}\,a^3b^2d^{189}x^3}{d^{190}}\right.}{4389\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/4389*(4389*a^5*sgn(b*x^2 + a)/sqrt(d*x) - (231*sqrt(d*x)*b^5*d^189*x^9*s
gn(b*x^2 + a) + 1463*sqrt(d*x)*a*b^4*d^189*x^7*sgn(b*x^2 + a) + 3990*sqrt(d
*x)*a^2*b^3*d^189*x^5*sgn(b*x^2 + a) + 6270*sqrt(d*x)*a^3*b^2*d^189*x^3*sgn
(b*x^2 + a) + 7315*sqrt(d*x)*a^4*b*d^189*x*sgn(b*x^2 + a))/d^190)/d
```

maple [A] time = 0.00, size = 83, normalized size = 0.28

$$-\frac{2\left(-231b^5x^{10}-1463a\,b^4x^8-3990a^2b^3x^6-6270a^3b^2x^4-7315a^4b\,x^2+4389a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x}{4389\left(b\,x^2+a\right)^5(dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(3/2),x)
```

```
[Out] -2/4389*x*(-231*b^5*x^10-1463*a*b^4*x^8-3990*a^2*b^3*x^6-6270*a^3*b^2*x^4-7
315*a^4*b*x^2+4389*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5/(d*x)^(3/2)
```

maxima [A] time = 1.50, size = 151, normalized size = 0.51

$$\frac{2\left(77\left(15\,b^5\sqrt{d}\,x^3+19\,ab^4\sqrt{d}\,x\right)x^{\frac{13}{2}}+532\left(11\,ab^4\sqrt{d}\,x^3+15\,a^2b^3\sqrt{d}\,x\right)x^{\frac{9}{2}}+1710\left(7\,a^2b^3\sqrt{d}\,x^3+11\,a^3b^2\sqrt{d}\,x\right)x}{21945\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/21945*(77*(15*b^5*sqrt(d)*x^3 + 19*a*b^4*sqrt(d)*x)*x^(13/2) + 532*(11*a*
b^4*sqrt(d)*x^3 + 15*a^2*b^3*sqrt(d)*x)*x^(9/2) + 1710*(7*a^2*b^3*sqrt(d)*x
^3 + 11*a^3*b^2*sqrt(d)*x)*x^(5/2) + 4180*(3*a^3*b^2*sqrt(d)*x^3 + 7*a^4*b*
sqrt(d)*x)*sqrt(x) + 7315*(a^4*b*sqrt(d)*x^3 - 3*a^5*sqrt(d)*x)/x^(3/2))/d^
2
```

mupad [B] time = 4.54, size = 116, normalized size = 0.39

$$\frac{2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(3803\,a^4+3512\,a^3\,b\,x^2+2758\,a^2\,b^2\,x^4+1232\,a\,b^3\,x^6+231\,b^4\,x^8\right)}{4389\,d\,\sqrt{d\,x}}-\frac{16384\,a^5\,\sqrt{a^2+2\,a}}{4389\,d\,\sqrt{d\,x}\,\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/(d*x)^(3/2),x)
```

```
[Out] (2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(3803*a^4 + 231*b^4*x^8 + 3512*a^3*b*x
^2 + 1232*a*b^3*x^6 + 2758*a^2*b^2*x^4))/(4389*d*(d*x)^(1/2)) - (16384*a^5*
(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(4389*d*(d*x)^(1/2)*(a + b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(3/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/(d*x)**(3/2), x)
```

3.747 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=293

$$\frac{2b^5(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^7\left(a+bx^2\right)}-\frac{2a^5\sqrt{a}}{3d(a}$$

```
[Out] -2/3*a^5*((b*x^2+a)^2)^(1/2)/d/(d*x)^(3/2)/(b*x^2+a)+4*a^3*b^2*(d*x)^(5/2)*
((b*x^2+a)^2)^(1/2)/d^5/(b*x^2+a)+20/9*a^2*b^3*(d*x)^(9/2)*((b*x^2+a)^2)^(1
/2)/d^7/(b*x^2+a)+10/13*a*b^4*(d*x)^(13/2)*((b*x^2+a)^2)^(1/2)/d^9/(b*x^2+a
)+2/17*b^5*(d*x)^(17/2)*((b*x^2+a)^2)^(1/2)/d^11/(b*x^2+a)+10*a^4*b*(d*x)^(
1/2)*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)
```

Rubi [A] time = 0.08, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^5(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^7\left(a+bx^2\right)}+\frac{4a^3b^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(5/2),x]
```

```
[Out] (-2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(d*x)^(3/2)*(a + b*x^2)) + (1
0*a^4*b*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2)) + (4*a
^3*b^2*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^5*(a + b*x^2)) + (20
*a^2*b^3*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^7*(a + b*x^2)) +
 (10*a*b^4*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^9*(a + b*x^2
)) + (2*b^5*(d*x)^(17/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^11*(a + b*x
^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{(dx)^{5/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{(dx)^{5/2}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{(dx)^{5/2}}+\frac{5a^4b^6}{d^2\sqrt{dx}}+\frac{10a^3b^7(dx)^{3/2}}{d^4}+\frac{10a^2b^8(dx)^{7/2}}{d^6}+\frac{5ab^9(dx)^{11/2}}{d^8}+\cdots\right.}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)}+\frac{10a^4b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{4a^3b^2(dx)^{5/2}\sqrt{a^2+\cdots}}{d^5\left(a+\cdots\right.}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 0.30

$$\frac{2x\sqrt{\left(a+bx^2\right)^2}\left(-663a^5+9945a^4bx^2+3978a^3b^2x^4+2210a^2b^3x^6+765ab^4x^8+117b^5x^{10}\right)}{1989(dx)^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(5/2),x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-663*a^5 + 9945*a^4*b*x^2 + 3978*a^3*b^2*x^4 + 22
10*a^2*b^3*x^6 + 765*a*b^4*x^8 + 117*b^5*x^10))/(1989*(d*x)^(5/2)*(a + b*x^
2))
```

fricas [A] time = 0.94, size = 67, normalized size = 0.23

$$\frac{2\left(117\,b^5x^{10}+765\,ab^4x^8+2210\,a^2b^3x^6+3978\,a^3b^2x^4+9945\,a^4bx^2-663\,a^5\right)\sqrt{dx}}{1989\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/1989*(117*b^5*x^10 + 765*a*b^4*x^8 + 2210*a^2*b^3*x^6 + 3978*a^3*b^2*x^4
+ 9945*a^4*b*x^2 - 663*a^5)*sqrt(d*x)/(d^3*x^2)
```

giac [A] time = 0.19, size = 159, normalized size = 0.54

$$-\frac{2\left(\frac{663\,a^5d\,\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{dx}\,x}-\frac{117\,\sqrt{dx}\,b^5d^{136}x^8\mathrm{sgn}\left(bx^2+a\right)+765\,\sqrt{dx}\,ab^4d^{136}x^6\mathrm{sgn}\left(bx^2+a\right)+2210\,\sqrt{dx}\,a^2b^3d^{136}x^4\mathrm{sgn}\left(bx^2+a\right)+3978\,\sqrt{dx}\,a^3b^2d^{136}x^2\mathrm{s}\cdots}{d^{136}}\right.}{1989\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/1989*(663*a^5*d*sgn(b*x^2 + a)/(sqrt(d*x)*x) - (117*sqrt(d*x)*b^5*d^136*
x^8*sgn(b*x^2 + a) + 765*sqrt(d*x)*a*b^4*d^136*x^6*sgn(b*x^2 + a) + 2210*sq
rt(d*x)*a^2*b^3*d^136*x^4*sgn(b*x^2 + a) + 3978*sqrt(d*x)*a^3*b^2*d^136*x^2
*sgn(b*x^2 + a) + 9945*sqrt(d*x)*a^4*b*d^136*sgn(b*x^2 + a))/d^136)/d^3
```

maple [A] time = 0.01, size = 83, normalized size = 0.28

$$-\frac{2\left(-117b^5x^{10}-765a\,b^4x^8-2210a^2b^3x^6-3978a^3b^2x^4-9945a^4b\,x^2+663a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x}{1989\left(b\,x^2+a\right)^5(dx)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(5/2),x)
```

```
[Out] -2/1989*x*(-117*b^5*x^10-765*a*b^4*x^8-2210*a^2*b^3*x^6-3978*a^3*b^2*x^4-99
45*a^4*b*x^2+663*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5/(d*x)^(5/2)
```

maxima [A] time = 1.54, size = 151, normalized size = 0.52

$$\frac{2\left(45\left(13\,b^5\sqrt{d}\,x^3+17\,ab^4\sqrt{d}\,x\right)x^{\frac{11}{2}}+340\left(9\,ab^4\sqrt{d}\,x^3+13\,a^2b^3\sqrt{d}\,x\right)x^{\frac{7}{2}}+1326\left(5\,a^2b^3\sqrt{d}\,x^3+9\,a^3b^2\sqrt{d}\,x\right)x^{\frac{3}{2}}}{9945\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9945*(45*(13*b^5*sqrt(d)*x^3 + 17*a*b^4*sqrt(d)*x)*x^(11/2) + 340*(9*a*b^
4*sqrt(d)*x^3 + 13*a^2*b^3*sqrt(d)*x)*x^(7/2) + 1326*(5*a^2*b^3*sqrt(d)*x^3
 + 9*a^3*b^2*sqrt(d)*x)*x^(3/2) + 7956*(a^3*b^2*sqrt(d)*x^3 + 5*a^4*b*sqrt(
d)*x)/sqrt(x) + 3315*(3*a^4*b*sqrt(d)*x^3 - a^5*sqrt(d)*x)/x^(5/2))/d^3
```

mupad [B] time = 4.56, size = 116, normalized size = 0.40

$$\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(\frac{10\,a^4\,x^2}{d^2}-\frac{2\,a^5}{3\,b\,d^2}+\frac{2\,b^4\,x^{10}}{17\,d^2}+\frac{4\,a^3\,b\,x^4}{d^2}+\frac{10\,a\,b^3\,x^8}{13\,d^2}+\frac{20\,a^2\,b^2\,x^6}{9\,d^2}\right)}{x^3\,\sqrt{d\,x}+\frac{a\,x\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/(d*x)^(5/2),x)
```

```
[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((10*a^4*x^2)/d^2 - (2*a^5)/(3*b*d^2) +
(2*b^4*x^10)/(17*d^2) + (4*a^3*b*x^4)/d^2 + (10*a*b^3*x^8)/(13*d^2) + (20*a
^2*b^2*x^6)/(9*d^2)))/(x^3*(d*x)^(1/2) + (a*x*(d*x)^(1/2))/b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{(dx)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(5/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/(d*x)**(5/2), x)
```

3.748 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=295

$$\frac{2b^5(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{15d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^7\left(a+bx^2\right)}-\frac{2a^5\sqrt{a}}{5d(a}$$

```
[Out] -2/5*a^5*((b*x^2+a)^2)^(1/2)/d/(d*x)^(5/2)/(b*x^2+a)+20/3*a^3*b^2*(d*x)^(3/
2)*((b*x^2+a)^2)^(1/2)/d^5/(b*x^2+a)+20/7*a^2*b^3*(d*x)^(7/2)*((b*x^2+a)^2)
^(1/2)/d^7/(b*x^2+a)+10/11*a*b^4*(d*x)^(11/2)*((b*x^2+a)^2)^(1/2)/d^9/(b*x^
2+a)+2/15*b^5*(d*x)^(15/2)*((b*x^2+a)^2)^(1/2)/d^11/(b*x^2+a)-10*a^4*b*((b*
x^2+a)^2)^(1/2)/d^3/(b*x^2+a)/(d*x)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^5(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{15d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^7\left(a+bx^2\right)}+\frac{20a^3b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(7/2),x]
```

```
[Out] (-2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(d*x)^(5/2)*(a + b*x^2)) - (1
0*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*Sqrt[d*x]*(a + b*x^2)) + (20*
a^3*b^2*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^5*(a + b*x^2)) +
(20*a^2*b^3*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^7*(a + b*x^2)
) + (10*a*b^4*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^9*(a + b*
x^2)) + (2*b^5*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*d^11*(a +
b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{(dx)^{7/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{(dx)^{7/2}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{(dx)^{7/2}}+\frac{5a^4b^6}{d^2(dx)^{3/2}}+\frac{10a^3b^7\sqrt{dx}}{d^4}+\frac{10a^2b^8(dx)^{5/2}}{d^6}+\frac{5ab^9(dx)^{9/2}}{d^8}+\frac{b}{\ldots}\right)}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\left(a+bx^2\right)}-\frac{10a^4b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{3/2}\sqrt{a^2+2a\ldots}}{3d^5\left(a+bx^2\right)}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 0.30

$$\frac{2x\sqrt{\left(a+bx^2\right)^2}\left(-231a^5-5775a^4bx^2+3850a^3b^2x^4+1650a^2b^3x^6+525ab^4x^8+77b^5x^{10}\right)}{1155(dx)^{7/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(7/2),x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-231*a^5 - 5775*a^4*b*x^2 + 3850*a^3*b^2*x^4 + 16
50*a^2*b^3*x^6 + 525*a*b^4*x^8 + 77*b^5*x^10))/(1155*(d*x)^(7/2)*(a + b*x^2
))
```

fricas [A] time = 1.20, size = 67, normalized size = 0.23

$$\frac{2\left(77\,b^5x^{10}+525\,ab^4x^8+1650\,a^2b^3x^6+3850\,a^3b^2x^4-5775\,a^4bx^2-231\,a^5\right)\sqrt{dx}}{1155\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^5*x^10 + 525*a*b^4*x^8 + 1650*a^2*b^3*x^6 + 3850*a^3*b^2*x^4 -
 5775*a^4*b*x^2 - 231*a^5)*sqrt(d*x)/(d^4*x^3)
```

giac [A] time = 0.20, size = 162, normalized size = 0.55

$$-\frac{2\left(\frac{231\left(25\,a^4bd^3x^2\mathrm{sgn}\left(bx^2+a\right)+a^5d^3\mathrm{sgn}\left(bx^2+a\right)\right)}{\sqrt{dx}\,d^2x^2}-\frac{77\,\sqrt{dx}\,b^5d^{105}x^7\mathrm{sgn}\left(bx^2+a\right)+525\,\sqrt{dx}\,ab^4d^{105}x^5\mathrm{sgn}\left(bx^2+a\right)+1650\,\sqrt{dx}\,a^2b^3d^{105}x^3\mathrm{sgn}\left(bx^2+\ldots\right.}{d^{105}}\right.}{1155\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/1155*(231*(25*a^4*b*d^3*x^2*sgn(b*x^2 + a) + a^5*d^3*sgn(b*x^2 + a))/(sq
rt(d*x)*d^2*x^2) - (77*sqrt(d*x)*b^5*d^105*x^7*sgn(b*x^2 + a) + 525*sqrt(d*
x)*a*b^4*d^105*x^5*sgn(b*x^2 + a) + 1650*sqrt(d*x)*a^2*b^3*d^105*x^3*sgn(b*
x^2 + a) + 3850*sqrt(d*x)*a^3*b^2*d^105*x*sgn(b*x^2 + a))/d^105)/d^4
```

maple [A] time = 0.01, size = 83, normalized size = 0.28

$$-\frac{2\left(-77b^5x^{10}-525a\,b^4x^8-1650a^2b^3x^6-3850a^3b^2x^4+5775a^4b\,x^2+231a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}x}{1155\left(b\,x^2+a\right)^5(dx)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(7/2),x)
```

```
[Out] -2/1155*x*(-77*b^5*x^10-525*a*b^4*x^8-1650*a^2*b^3*x^6-3850*a^3*b^2*x^4+577
5*a^4*b*x^2+231*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5/(d*x)^(7/2)
```

maxima [A] time = 1.55, size = 150, normalized size = 0.51

$$\frac{2\left(7\left(11\,b^5\sqrt{d}\,x^3+15\,ab^4\sqrt{d}\,x\right)x^{\frac{9}{2}}+60\left(7\,ab^4\sqrt{d}\,x^3+11\,a^2b^3\sqrt{d}\,x\right)x^{\frac{5}{2}}+330\left(3\,a^2b^3\sqrt{d}\,x^3+7\,a^3b^2\sqrt{d}\,x\right)\sqrt{x}+\cdots\right.}{1155\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(7*(11*b^5*sqrt(d)*x^3 + 15*a*b^4*sqrt(d)*x)*x^(9/2) + 60*(7*a*b^4*s
qrt(d)*x^3 + 11*a^2*b^3*sqrt(d)*x)*x^(5/2) + 330*(3*a^2*b^3*sqrt(d)*x^3 + 7
*a^3*b^2*sqrt(d)*x)*sqrt(x) + 1540*(a^3*b^2*sqrt(d)*x^3 - 3*a^4*b*sqrt(d)*x
)/x^(3/2) - 231*(5*a^4*b*sqrt(d)*x^3 + a^5*sqrt(d)*x)/x^(7/2))/d^4
```

mupad [B] time = 4.72, size = 118, normalized size = 0.40

$$\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(\frac{2\,b^4\,x^{10}}{15\,d^3}-\frac{10\,a^4\,x^2}{d^3}-\frac{2\,a^5}{5\,b\,d^3}+\frac{20\,a^3\,b\,x^4}{3\,d^3}+\frac{10\,a\,b^3\,x^8}{11\,d^3}+\frac{20\,a^2\,b^2\,x^6}{7\,d^3}\right)}{x^4\,\sqrt{d\,x}+\frac{a\,x^2\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/(d*x)^(7/2),x)
```

```
[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((2*b^4*x^10)/(15*d^3) - (10*a^4*x^2)/d^
3 - (2*a^5)/(5*b*d^3) + (20*a^3*b*x^4)/(3*d^3) + (10*a*b^3*x^8)/(11*d^3) +
(20*a^2*b^2*x^6)/(7*d^3)))/(x^4*(d*x)^(1/2) + (a*x^2*(d*x)^(1/2))/b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{(dx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(7/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/(d*x)**(7/2), x)
```

3.749 $\int \frac{(dx)^{7/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=457

$$-\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{5/4}d^{7/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right.}{2\sqrt{2}\,b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 2/5*d*(d*x)^(5/2)*(b*x^2+a)/b/((b*x^2+a)^2)^(1/2)-1/2*a^(5/4)*d^(7/2)*(b*x^
2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(9/4)*2^(1/2)/
((b*x^2+a)^2)^(1/2)+1/2*a^(5/4)*d^(7/2)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*
(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(9/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-1/4*a^(5/4
)*d^(7/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^
(1/2)*(d*x)^(1/2))/b^(9/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+1/4*a^(5/4)*d^(7/2)*
(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x
)^(1/2))/b^(9/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-2*a*d^3*(b*x^2+a)*(d*x)^(1/2)/
b^2/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{5/4}d^{7/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{5/4}d^{7/2}\left(a+bx^2\right)\log}{2\sqrt{2}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (-2*a*d^3*Sqrt[d*x]*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (2
*d*(d*x)^(5/2)*(a + b*x^2))/(5*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(5/4
)*d^(7/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[
d])])/(Sqrt[2]*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(5/4)*d^(7/2)*
(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqr
t[2]*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(5/4)*d^(7/2)*(a + b*x^2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(2*Sqrt[2]*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(5/4)*d^(7/2)
*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[d*x]])/(2*Sqrt[2]*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{7/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{(dx)^{7/2}}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(ad^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{3/2}}{ab+b^2x^2}\,dx}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(a^2d^4\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b\right.}}{b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(2a^2d^3\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int\right.}{b^2\sqrt{a^2+2abx^2+}}$$

$$= -\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(a^{3/2}d^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int\right.}{b^2\sqrt{a^2+2abx^2}}$$

$$= -\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(a^{5/4}d^{7/2}\left(ab+b^2x^2\right)\right)\text{Subst}\left(\right.}{2\sqrt{2}\,b^{13/4}\sqrt{a^2}}$$

$$= -\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a^{5/4}d^{7/2}\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}\right.}{2\sqrt{2}\,b^{9/4}\sqrt{a^2+}}$$

$$= -\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a^{5/4}d^{7/2}\left(a+bx^2\right)\tan^{-1}\left(1-\right.}{\sqrt{2}\,b^{9/4}\sqrt{a^2+2abx^2+b}}$$

Mathematica [A] time = 0.09, size = 238, normalized size = 0.52

$$\frac{d^3\sqrt{dx}\left(a+bx^2\right)\left(-5\sqrt{2}\,a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+5\sqrt{2}\,a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-\right.}{20b^{9/4}\sqrt{x}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (d^3*Sqrt[d*x]*(a + b*x^2)*(-40*a*b^(1/4)*Sqrt[x] + 8*b^(5/4)*x^(5/2) - 10*
Sqrt[2]*a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 10*Sqrt[2]*
a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 5*Sqrt[2]*a^(5/4)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 5*Sqrt[2]*a^(5/
4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(20*b^(9/4)
*Sqrt[x]*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 1.09, size = 223, normalized size = 0.49

$$\frac{20 \left(-\frac{a^5 d^{14}}{b^9}\right)^{\frac{1}{4}} b^2 \arctan\left(-\frac{\left(-\frac{a^5 d^{14}}{b^9}\right)^{\frac{3}{4}} \sqrt{dx}\, a b^7 d^3 - \left(-\frac{a^5 d^{14}}{b^9}\right)^{\frac{3}{4}} \sqrt{a^2 d^7 x + \sqrt{-\frac{a^5 d^{14}}{b^9}}\, b^4}\, b^7}{a^5 d^{14}}\right) + 5 \left(-\frac{a^5 d^{14}}{b^9}\right)^{\frac{1}{4}} b^2 \log\left(\sqrt{dx}\, a d^3 + \left(-\frac{a^5 d^{1}}{b^9}\right.\right.}{10 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*(20*(-a^5*d^14/b^9)^(1/4)*b^2*arctan(-((-a^5*d^14/b^9)^(3/4)*sqrt(d*x)
*a*b^7*d^3 - (-a^5*d^14/b^9)^(3/4)*sqrt(a^2*d^7*x + sqrt(-a^5*d^14/b^9)*b^4
)*b^7)/(a^5*d^14)) + 5*(-a^5*d^14/b^9)^(1/4)*b^2*log(sqrt(d*x)*a*d^3 + (-a^
5*d^14/b^9)^(1/4)*b^2) - 5*(-a^5*d^14/b^9)^(1/4)*b^2*log(sqrt(d*x)*a*d^3 -
(-a^5*d^14/b^9)^(1/4)*b^2) + 4*(b*d^3*x^2 - 5*a*d^3)*sqrt(d*x))/b^2
```

giac [A] time = 0.20, size = 273, normalized size = 0.60

$$\frac{1}{20} d^3 \left(\frac{10 \sqrt{2} \left(a b^3 d^2\right)^{\frac{1}{4}} a \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} + 2 \sqrt{dx}\right)}{2 \left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)}{b^3} + \frac{10 \sqrt{2} \left(a b^3 d^2\right)^{\frac{1}{4}} a \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} - 2 \sqrt{dx}\right)}{2 \left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)}{b^3} + \frac{5 \sqrt{}}{} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/20*d^3*(10*sqrt(2)*(a*b^3*d^2)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^3 + 10*sqrt(2)*(a*b^3*d^2)^(1/4
)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(
1/4))/b^3 + 5*sqrt(2)*(a*b^3*d^2)^(1/4)*a*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)
```

```
*sqrt(d*x) + sqrt(a*d^2/b))/b^3 - 5*sqrt(2)*(a*b^3*d^2)^(1/4)*a*log(d*x - s
qrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^3 + 8*(sqrt(d*x)*b^4*d^
10*x^2 - 5*sqrt(d*x)*a*b^3*d^10)/(b^5*d^10))*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 239, normalized size = 0.52

$$\frac{\left(b\,x^{2}+a\right)\left(10\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\;a\,d^{2}\arctan\left(\frac{\sqrt{2}\;\sqrt{dx}-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)+10\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\;a\,d^{2}\arctan\left(\frac{\sqrt{2}\;\sqrt{dx}+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)+5\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}}{20\sqrt{\left(b\,x^{2}+a\right)^{2}}\;b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/20*(b*x^2+a)*d*(5*a*d^2*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(
d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2
)+(a/b*d^2)^(1/2)))+10*a*d^2*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^
(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+10*a*d^2*(a/b*d^2)^(1/4)*2^(1/2)*ar
ctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+8*(d*x)^(5/2)*b
-40*d^2*a*(d*x)^(1/2))/((b*x^2+a)^2)^(1/2)/b^2
```

maxima [A] time = 2.94, size = 266, normalized size = 0.58

$$\frac{\frac{5\left(\frac{\sqrt{2}\,d^{6}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^{2}\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^{6}\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^{2}\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d^{5}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\;\sqrt{a}}+\frac{2\sqrt{2}\,d^{5}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\cdots\right.\right.}{}\right)}{\sqrt{\sqrt{a}\sqrt{b}\cdots}}\right.}{b^{2}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/20*(5*(sqrt(2)*d^6*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1
/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^6*log(sqrt(b)*d*x - sq
rt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4))
+ 2*sqrt(2)*d^5*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(
d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) +
2*sqrt(2)*d^5*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d
```

```
*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))*a^
2/b^2 + 8*((d*x)^(5/2)*b*d^2 - 5*sqrt(d*x)*a*d^4)/b^2)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{7/2}}{\sqrt{\left(b\,x^2+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/((a + b*x^2)^2)^(1/2),x)
[Out] int((d*x)^(7/2)/((a + b*x^2)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/((b*x**2+a)**2)**(1/2),x)
[Out] Timed out
```

3.750 $\int \frac{(dx)^{5/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=412

$$\frac{2d(dx)^{3/2}\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{\right.}{2\sqrt{2}\,b^{7/4}}$$

```
[Out] 2/3*d*(d*x)^(3/2)*(b*x^2+a)/b/((b*x^2+a)^2)^(1/2)+1/2*a^(3/4)*d^(5/2)*(b*x^
2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(7/4)*2^(1/2)/
((b*x^2+a)^2)^(1/2)-1/2*a^(3/4)*d^(5/2)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*
(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(7/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-1/4*a^(3/4
)*d^(5/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^
(1/2)*(d*x)^(1/2))/b^(7/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+1/4*a^(3/4)*d^(5/2)*
(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x
)^(1/2))/b^(7/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}\right.}{2\sqrt{2}\,b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*d*(d*x)^(3/2)*(a + b*x^2))/(3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(3
/4)*d^(5/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])])/(Sqrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(3/4)*d^(5/2
)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(S
qrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(3/4)*d^(5/2)*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(2*Sqrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(3/4)*d^(5/
2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{(dx)^{5/2}}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d(dx)^{3/2}\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(ad^2\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d(dx)^{3/2}\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(2ad\left(ab+b^2x^2\right)\right)\mathrm{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d(dx)^{3/2}\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(ad\left(ab+b^2x^2\right)\right)\mathrm{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(ad\left(a\right.\right.}{}$$

$$= \frac{2d(dx)^{3/2}\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(a^{3/4}d^{5/2}\left(ab+b^2x^2\right)\right)\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{\right.}{2\sqrt{2}\,b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d(dx)^{3/2}\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{d}\right.}{2\sqrt{2}\,b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d(dx)^{3/2}\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{\sqrt{2}\,b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a^{3/4}d^{5/2}\left(a+bx\right.}{\sqrt{2}\,b^{7/4}\sqrt{}}$$

Mathematica [A] time = 0.05, size = 110, normalized size = 0.27

$$\frac{(dx)^{5/2}\left(a+bx^2\right)\left(3(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)-3(-a)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)+2b^{3/4}x^{3/2}\right)}{3b^{7/4}x^{5/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((d*x)^(5/2)*(a + b*x^2)*(2*b^(3/4)*x^(3/2) + 3*(-a)^(3/4)*ArcTan[(b^(1/4)*
Sqrt[x])/(-a)^(1/4)] - 3*(-a)^(3/4)*ArcTanh[(b^(1/4)*Sqrt[x])/(-a)^(1/4)]))
/(3*b^(7/4)*x^(5/2)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 1.09, size = 219, normalized size = 0.53

$$\frac{4\sqrt{dx}\,d^2x+12\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{1}{4}}b\arctan\left(-\frac{\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{1}{4}}\sqrt{dx}\,a^2b^2d^7-\sqrt{a^4d^{15}x-\sqrt{-\frac{a^3d^{10}}{b^7}}\,a^3b^3d^{10}}\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{1}{4}}b^2}{a^3d^{10}}\right)-3\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{1}{4}}b\log\Bigl(\ldots}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(d*x)*d^2*x + 12*(-a^3*d^10/b^7)^(1/4)*b*arctan(-((-a^3*d^10/b^7
)^(1/4)*sqrt(d*x)*a^2*b^2*d^7 - sqrt(a^4*d^15*x - sqrt(-a^3*d^10/b^7)*a^3*b
^3*d^10)*(-a^3*d^10/b^7)^(1/4)*b^2)/(a^3*d^10)) - 3*(-a^3*d^10/b^7)^(1/4)*b
*log(sqrt(d*x)*a^2*d^7 + (-a^3*d^10/b^7)^(3/4)*b^5) + 3*(-a^3*d^10/b^7)^(1/
4)*b*log(sqrt(d*x)*a^2*d^7 - (-a^3*d^10/b^7)^(3/4)*b^5))/b
```

giac [A] time = 0.20, size = 254, normalized size = 0.62

$$\frac{1}{12}d^2\left(\frac{8\sqrt{dx}\,x}{b}-\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4d}-\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4d}+\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*d^2*(8*sqrt(d*x)*x/b - 6*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*
(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^4*d) - 6*sqrt(2
)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d
*x))/(a*d^2/b)^(1/4))/(b^4*d) + 3*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(
2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^4*d) - 3*sqrt(2)*(a*b^3*d^
2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^4*
d))*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 221, normalized size = 0.54

$$-\frac{\left(b x^2+a\right)\left(6\sqrt{2}\,a d^2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a d^2}{b}\right)^{\frac14}}{\left(\frac{a d^2}{b}\right)^{\frac14}}\right)+6\sqrt{2}\,a d^2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a d^2}{b}\right)^{\frac14}}{\left(\frac{a d^2}{b}\right)^{\frac14}}\right)+3\sqrt{2}\,a d^2\ln\left(-\frac{-dx+\left(\frac{a d^2}{b}\right)^{\frac14}\sqrt{dx}}{dx+\left(\frac{a d^2}{b}\right)^{\frac14}\sqrt{dx}}\right.\right.}{12\sqrt{\left(b x^2+a\right)^2}\,\left(\frac{a d^2}{b}\right)^{\frac14} b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/12*(b*x^2+a)*d*(3*a*d^2*2^(1/2)*ln(-((a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)
-d*x-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1
/2)))+6*a*d^2*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2
)^(1/4))+6*a*d^2*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*
d^2)^(1/4))-8*(d*x)^(3/2)*b*(a/b*d^2)^(1/4))/((b*x^2+a)^2)^(1/2)/b^2/(a/b*d
^2)^(1/4)
```

maxima [A] time = 2.97, size = 241, normalized size = 0.58

$$-\frac{\frac{3\,a d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(a d^2\right)^{\frac14} b^{\frac14}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(a d^2\right)^{\frac14} b^{\frac14}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(a d^2\right)^{\frac14}\sqrt{dx}\,b^{\frac14}+\sqrt{a}\,d\right)}{\left(a d^2\right)^{\frac14} b^{\frac34}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\left(a d^2\right.\right.}{\left(a d^2\right)}\right.}{b}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4)
 + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*s
qrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*
sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b
)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sq
rt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2
)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b - 8*(d*x)
^(3/2)*d^2/b)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{5/2}}{\sqrt{\left(b\,x^2+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((d*x)^(5/2)/((a + b*x^2)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.751 $\int \frac{(dx)^{3/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=410

$$\frac{2d\sqrt{dx}\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{a}\,d^{3/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{a}\,d^{3/2}\left(a+bx^2\right)\log\left(\sqrt{2}\right.}{2\sqrt{2}\,b^{5/4}\sqrt{}}$$

```
[Out] 1/2*a^(1/4)*d^(3/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/
d^(1/2))/b^(5/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-1/2*a^(1/4)*d^(3/2)*(b*x^2+a)*
arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(5/4)*2^(1/2)/((b*x
^2+a)^2)^(1/2)+1/4*a^(1/4)*d^(3/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d
^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(5/4)*2^(1/2)/((b*x^2+a)^2)^(
1/2)-1/4*a^(1/4)*d^(3/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(
1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(5/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+2*d*(
b*x^2+a)*(d*x)^(1/2)/b/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{a}\,d^{3/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{a}\,d^{3/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\right.}{2\sqrt{2}\,b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*d*Sqrt[d*x]*(a + b*x^2))/(b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(1/4)*
d^(3/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
)])/(Sqrt[2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(1/4)*d^(3/2)*(a
 + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[
2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(1/4)*d^(3/2)*(a + b*x^2)*
Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]
])/(2*Sqrt[2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(1/4)*d^(3/2)*(
a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(2*Sqrt[2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{(dx)^{3/2}}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d\sqrt{dx}\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(ad^2\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d\sqrt{dx}\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(2ad\left(ab+b^2x^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d\sqrt{dx}\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(\sqrt{a}\left(ab+b^2x^2\right)\right)\mathrm{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(\sqrt{a}\left(\right.\right.}{}$$

$$= \frac{2d\sqrt{dx}\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(\sqrt[4]{a}\,d^{3/2}\left(ab+b^2x^2\right)\right)\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{d\right.}{2\sqrt{2}\,b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d\sqrt{dx}\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt[4]{a}\,d^{3/2}\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right.}{2\sqrt{2}\,b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d\sqrt{dx}\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt[4]{a}\,d^{3/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\,b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt[4]{a}\,d^{3/2}\left(a+bx^2\right.}{\sqrt{2}\,b^{5/4}\sqrt{a}}$$

Mathematica [A] time = 0.08, size = 221, normalized size = 0.54

$$\frac{(dx)^{3/2}\left(a+bx^2\right)\left(\sqrt{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-\sqrt{2}\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+2\sqrt{2}\cdots\right.}{4b^{5/4}x^{3/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((d*x)^(3/2)*(a + b*x^2)*(8*b^(1/4)*Sqrt[x] + 2*Sqrt[2]*a^(1/4)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*Sqrt[2]*a^(1/4)*ArcTan[1 + (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)] + Sqrt[2]*a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x] - Sqrt[2]*a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(4*b^(5/4)*x^(3/2)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.95, size = 170, normalized size = 0.41

$$-\frac{4\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b\arctan\left(-\frac{\left(-\frac{ad^6}{b^5}\right)^{\frac{3}{4}}\sqrt{dx}\,b^4d-\sqrt{d^3x+\sqrt{-\frac{ad^6}{b^5}}\,b^2}\left(-\frac{ad^6}{b^5}\right)^{\frac{3}{4}}b^4}{ad^6}\right)+\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b\log\left(\sqrt{dx}\,d+\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b\right)-\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b\cdots}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(-a*d^6/b^5)^(1/4)*b*arctan(-((-a*d^6/b^5)^(3/4)*sqrt(d*x)*b^4*d -
sqrt(d^3*x + sqrt(-a*d^6/b^5)*b^2)*(-a*d^6/b^5)^(3/4)*b^4)/(a*d^6)) + (-a*d
^6/b^5)^(1/4)*b*log(sqrt(d*x)*d + (-a*d^6/b^5)^(1/4)*b) - (-a*d^6/b^5)^(1/4
)*b*log(sqrt(d*x)*d - (-a*d^6/b^5)^(1/4)*b) - 4*sqrt(d*x)*d)/b
```

giac [A] time = 0.24, size = 238, normalized size = 0.58

$$-\frac{1}{4}d\left(\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{\sqrt{2}\left(ab^3d^2\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*d*(2*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(
1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^2 + 2*sqrt(2)*(a*b^3*d^2)^(1/4)*arct
an(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^
2 + sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) +
 sqrt(a*d^2/b))/b^2 - sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)
^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^2 - 8*sqrt(d*x)/b)*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 214, normalized size = 0.52

$$-\frac{\left(b x^2+a\right)\left(2\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+2\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{a}{\cdots}\right.}{dx-\left(\frac{a}{\cdots}\right.}\right.\cdots}{4\sqrt{\left(b x^2+a\right)^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/4*(b*x^2+a)*d*((a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/
2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d
^2)^(1/2)))+2*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)
^(1/4))/(a/b*d^2)^(1/4))+2*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1
/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))-8*(d*x)^(1/2))/((b*x^2+a)^2)^(1/2)/b
```

maxima [A] time = 3.10, size = 250, normalized size = 0.61

$$\frac{\frac{8\sqrt{dx}\,d^2}{b}-\frac{\left(\frac{\sqrt{2}\,d^4\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^4\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d^3\arctan\cdots}{\cdots}\right.}{b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*sqrt(d*x)*d^2/b - (sqrt(2)*d^4*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/
4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^4*log
(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2
)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^3*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*
b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(
b)*d)*sqrt(a)) + 2*sqrt(2)*d^3*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b
^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b
)*d)*sqrt(a)))*a/b)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{3/2}}{\sqrt{\left(b\,x^2+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((d*x)^(3/2)/((a + b*x^2)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.752 $\int \frac{\sqrt{dx}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=368

$$\frac{\sqrt{d}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{d}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{a}\right.}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -1/2*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*d^(1/2
)/a^(1/4)/b^(3/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+1/2*(b*x^2+a)*arctan(1+b^(1/4
)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*d^(1/2)/a^(1/4)/b^(3/4)*2^(1/2)/((b*
x^2+a)^2)^(1/2)+1/4*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*
b^(1/4)*2^(1/2)*(d*x)^(1/2))*d^(1/2)/a^(1/4)/b^(3/4)*2^(1/2)/((b*x^2+a)^2)^
(1/2)-1/4*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^
(1/2)*(d*x)^(1/2))*d^(1/2)/a^(1/4)/b^(3/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1112, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt{d}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{d}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{a}\right.}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -((Sqrt[d]*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(Sqrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) + (Sqrt[d
]*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(S
qrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (Sqrt[d]*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(2*Sqrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (Sqrt[
d]*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{d\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{\sqrt{b}\,d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d+\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,\right.}{\sqrt{b}\,d\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(\sqrt{d}\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{dx}\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(\sqrt{d}\left(ab+b^2x^2\right)\right)\text{Subs}}{2\sqrt{2}\sqrt[4]{a}\,b^{7}}$$

$$= \frac{\sqrt{d}\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt{d}\left(a+bx^2\right)\log\left(\sqrt{a}\,\sqrt{}\right.}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}}$$

$$= -\frac{\sqrt{d}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{d}\left(a+bx^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{d}}{}$$

Mathematica [A] time = 0.04, size = 85, normalized size = 0.23

$$\frac{\sqrt{dx}\left(a+bx^2\right)\left(\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)+\tanh^{-1}\left(\frac{a\sqrt[4]{b}\sqrt{x}}{(-a)^{5/4}}\right)\right)}{\sqrt[4]{-a}\,b^{3/4}\sqrt{x}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

[Out] (Sqrt[d*x]*(a + b*x^2)*(ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/4)] + ArcTanh[(a*b
^(1/4)*Sqrt[x])/(-a)^(5/4)]))/((-a)^(1/4)*b^(3/4)*Sqrt[x]*Sqrt[(a + b*x^2)^
2])

fricas [A] time = 0.83, size = 173, normalized size = 0.47

$$-2\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,bd\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}-\sqrt{-abd^2\sqrt{-\frac{d^2}{ab^3}}+d^3x}\,b\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}}{d^2}\right)+\frac{1}{2}\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}\log\left(ab^2\left(-\frac{d^2}{ab^3}\right)^{\frac{3}{4}}+\sqrt{d\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")

[Out] -2*(-d^2/(a*b^3))^(1/4)*arctan(-(sqrt(d*x)*b*d*(-d^2/(a*b^3))^(1/4) - sqrt(
-a*b*d^2*sqrt(-d^2/(a*b^3)) + d^3*x)*b*(-d^2/(a*b^3))^(1/4))/d^2) + 1/2*(-d
^2/(a*b^3))^(1/4)*log(a*b^2*(-d^2/(a*b^3))^(3/4) + sqrt(d*x)*d) - 1/2*(-d^2
/(a*b^3))^(1/4)*log(-a*b^2*(-d^2/(a*b^3))^(3/4) + sqrt(d*x)*d)

giac [A] time = 0.19, size = 242, normalized size = 0.66

$$\frac{\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^3}+\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^3}-\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\ldots}\right.}{ab^3}\ldots}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")

[Out] 1/4*(2*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4
) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^3) + 2*sqrt(2)*(a*b^3*d^2)^(3/4)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(
a*b^3) - sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d
*x) + sqrt(a*d^2/b))/(a*b^3) + sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*
(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^3))*sgn(b*x^2 + a)/d

maple [A] time = 0.01, size = 183, normalized size = 0.50

$$\frac{\left(b x^{2}+a\right) \sqrt{2}\, \left(2 \arctan\left(\frac{\sqrt{2}\, \sqrt{d x}-\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}}\right)+2 \arctan\left(\frac{\sqrt{2}\, \sqrt{d x}+\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}}\right)+\ln\left(-\frac{-d x+\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{d x}\, \sqrt{2}-\sqrt{\frac{a d^{2}}{b}}}{d x+\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{d x}\, \sqrt{2}+\sqrt{\frac{a d^{2}}{b}}}\right)\right) d}{4 \sqrt{\left(b x^{2}+a\right)^{2}}\, \left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/4/((b*x^2+a)^2)^(1/2)*(b*x^2+a)*d/b/(a/b*d^2)^(1/4)*2^(1/2)*(ln(-(-d*x+(a
/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^
(1/4))/(a/b*d^2)^(1/4))+2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b
*d^2)^(1/4)))
```

maxima [A] time = 3.00, size = 216, normalized size = 0.59

$$\frac{1}{4} d\left(\frac{2 \sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(a d^{2}\right)^{\frac{1}{4}} b^{\frac{1}{4}}+2 \sqrt{d x} \sqrt{b}\right)}{2 \sqrt{\sqrt{a} \sqrt{b} d}}\right)}{\sqrt{\sqrt{a} \sqrt{b} d} \sqrt{b}}+\frac{2 \sqrt{2} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(a d^{2}\right)^{\frac{1}{4}} b^{\frac{1}{4}}-2 \sqrt{d x} \sqrt{b}\right)}{2 \sqrt{\sqrt{a} \sqrt{b} d}}\right)}{\sqrt{\sqrt{a} \sqrt{b} d} \sqrt{b}}-\frac{\sqrt{2} \log\left(\sqrt{b} d x+\sqrt{2}\,(\right.}{\left(a d^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*d*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt
(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) +
 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)
*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt
(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/
((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*s
qrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x}}{\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((d*x)^(1/2)/((a + b*x^2)^2)^(1/2), x)
```

sympy [A] time = 57.26, size = 41, normalized size = 0.11

$$2d\,\mathrm{RootSum}\left(256t^4ab^3d^2+1,\left(t\mapsto t\log\left(64t^3ab^2d^2+\sqrt{dx}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] 2*d*RootSum(256*_t**4*a*b**3*d**2 + 1, Lambda(_t, _t*log(64*_t**3*a*b**2*d*
*2 + sqrt(d*x))))
```

3.753 $\int \frac{1}{\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=368

$$-\frac{(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}-$$

```
[Out] -1/2*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(3/4
)/b^(1/4)*2^(1/2)/d^(1/2)/((b*x^2+a)^2)^(1/2)+1/2*(b*x^2+a)*arctan(1+b^(1/4
)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(3/4)/b^(1/4)*2^(1/2)/d^(1/2)/((b*
x^2+a)^2)^(1/2)-1/4*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*
b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(3/4)/b^(1/4)*2^(1/2)/d^(1/2)/((b*x^2+a)^2)^
(1/2)+1/4*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^
(1/2)*(d*x)^(1/2))/a^(3/4)/b^(1/4)*2^(1/2)/d^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1112, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -(((a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(
Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) + ((a + b
*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a
^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((a + b*x^2)*Log[
Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(
2*Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((a +
b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(2*Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{d\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{\sqrt{a}\,d^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d+\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\right.}{\sqrt{a}\,d^2\sqrt{a^2+2abx^2+b^{2}}}$$

$$= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{dx}\right)}{2\sqrt{a}\,b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}}\right.}{2\sqrt{a}\,b^{3/2}\sqrt{a^2}}$$

$$= -\frac{\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}\right.}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}}$$

$$= -\frac{\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(a+bx^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}} - $$

Mathematica [A] time = 0.05, size = 178, normalized size = 0.48

$$-\frac{\sqrt{x}\left(a+bx^2\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.\right.}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{dx}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -1/2*(Sqrt[x]*(a + b*x^2)*(2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
- 2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x]))/(Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d*x]*Sqrt[(a + b*x^2)
^2])
```

fricas [A] time = 0.70, size = 165, normalized size = 0.45

$$2\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^2d^2\sqrt{-\frac{1}{a^3bd^2}}+dx}\,a^2bd\left(-\frac{1}{a^3bd^2}\right)^{\frac{3}{4}}-\sqrt{dx}\,a^2bd\left(-\frac{1}{a^3bd^2}\right)^{\frac{3}{4}}\right)+\frac{1}{2}\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}\log\left(ad\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(-1/(a^3*b*d^2))^(1/4)*arctan(sqrt(a^2*d^2*sqrt(-1/(a^3*b*d^2)) + d*x)*a^
2*b*d*(-1/(a^3*b*d^2))^(3/4) - sqrt(d*x)*a^2*b*d*(-1/(a^3*b*d^2))^(3/4)) +
1/2*(-1/(a^3*b*d^2))^(1/4)*log(a*d*(-1/(a^3*b*d^2))^(1/4) + sqrt(d*x)) - 1/
2*(-1/(a^3*b*d^2))^(1/4)*log(-a*d*(-1/(a^3*b*d^2))^(1/4) + sqrt(d*x))
```

giac [A] time = 0.26, size = 251, normalized size = 0.68

$$\frac{1}{4}\left(\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{abd}+\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{abd}+\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4
) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b*d) + 2*sqrt(2)*(a*b^3*d^2)^(1/4)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(
a*b*d) + sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d
*x) + sqrt(a*d^2/b))/(a*b*d) - sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*
(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b*d))*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 182, normalized size = 0.49

$$\frac{\left(b x^2+a\right)\left(\frac{a d^2}{b}\right)^{\frac14}\sqrt{2}\left(2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a d^2}{b}\right)^{\frac14}}{\left(\frac{a d^2}{b}\right)^{\frac14}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a d^2}{b}\right)^{\frac14}}{\left(\frac{a d^2}{b}\right)^{\frac14}}\right)+\ln\left(\frac{dx+\left(\frac{a d^2}{b}\right)^{\frac14}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx-\left(\frac{a d^2}{b}\right)^{\frac14}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)\right)}{4\sqrt{\left(b x^2+a\right)^2}\, a d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/4/((b*x^2+a)^2)^(1/2)*(b*x^2+a)/d*(a/b*d^2)^(1/4)/a*2^(1/2)*(ln((d*x+(a/b
*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)
^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1
/4))/(a/b*d^2)^(1/4))+2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d
^2)^(1/4)))
```

maxima [A] time = 3.06, size = 226, normalized size = 0.61

$$\frac{\frac{\sqrt{2}\,d^2\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac14}\sqrt{dx}\,b^{\frac14}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac34}b^{\frac14}}-\frac{\sqrt{2}\,d^2\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac14}\sqrt{dx}\,b^{\frac14}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac34}b^{\frac14}}+\frac{2\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac14}b^{\frac14}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\cdots}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(sqrt(2)*d^2*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
+ sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^2*log(sqrt(b)*d*x - sqrt(2
)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*
sqrt(2)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*s
qrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt
(2)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt
(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{d\,x}\,\sqrt{\left(b\,x^2+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(1/2)*((a + b*x^2)^2)^(1/2)),x)
[Out] int(1/((d*x)^(1/2)*((a + b*x^2)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\,\sqrt{\left(a+bx^2\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(1/2)/((b*x**2+a)**2)**(1/2),x)
[Out] Integral(1/(sqrt(d*x)*sqrt((a + b*x**2)**2)), x)
```

3.754 $\int \frac{1}{(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=412

$$-\frac{2\left(a+bx^2\right)}{ad\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{b}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{b}\left(a+bx^2\right)\log\left(\sqrt{2}\right.}{2\sqrt{2}\,a^{5/4}d^{3}}$$

```
[Out] 1/2*b^(1/4)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))
/a^(5/4)/d^(3/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)-1/2*b^(1/4)*(b*x^2+a)*arctan(1
+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(5/4)/d^(3/2)*2^(1/2)/((b*x
^2+a)^2)^(1/2)-1/4*b^(1/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a
^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(5/4)/d^(3/2)*2^(1/2)/((b*x^2+a)^2)^(
1/2)+1/4*b^(1/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(
1/4)*2^(1/2)*(d*x)^(1/2))/a^(5/4)/d^(3/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)-2*(b*
x^2+a)/a/d/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{b}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{b}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\right.}{2\sqrt{2}\,a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-2*(a + b*x^2))/(a*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(1/4)
*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sq
rt[2]*a^(5/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(1/4)*(a + b*x^
2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(5
/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(1/4)*(a + b*x^2)*Log[Sqr
t[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*S
qrt[2]*a^(5/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(1/4)*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(2*Sqrt[2]*a^(5/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{ad\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{ad^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{ad\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(2b\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right.}{ad^3\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{ad\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(\sqrt{b}\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x,\right.}{ad^3\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{ad\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx, x\right.}{2\sqrt{2}\,a^{5/4}b^{3/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{ad\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt[4]{b}\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\right.}{2\sqrt{2}\,a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{ad\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt[4]{b}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\,a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt[4]{b}\,(}{\sqrt{2}}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.12

$$-\frac{2x\left(a+bx^2\right){}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{bx^2}{a}\right)}{a(dx)^{3/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-2*x*(a + b*x^2)*Hypergeometric2F1[-1/4, 1, 3/4, -((b*x^2)/a)])/(a*(d*x)^(
3/2)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.98, size = 198, normalized size = 0.48

$$\frac{4\,ad^2x\left(-\frac{b}{a^5d^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,abd\left(-\frac{b}{a^5d^6}\right)^{\frac{1}{4}}-\sqrt{-a^3bd^4\sqrt{-\frac{b}{a^5d^6}}+b^2dx}\,ad\left(-\frac{b}{a^5d^6}\right)^{\frac{1}{4}}}{b}\right)-ad^2x\left(-\frac{b}{a^5d^6}\right)^{\frac{1}{4}}\log\left(a^4d^5\left(-\frac{b}{a^5d^6}\right)^{\frac{3}{4}}+\sqrt{\cdots}\right.}{2\,ad^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*d^2*x*(-b/(a^5*d^6))^(1/4)*arctan(-(sqrt(d*x)*a*b*d*(-b/(a^5*d^6))
^(1/4) - sqrt(-a^3*b*d^4*sqrt(-b/(a^5*d^6)) + b^2*d*x)*a*d*(-b/(a^5*d^6))^(
1/4))/b) - a*d^2*x*(-b/(a^5*d^6))^(1/4)*log(a^4*d^5*(-b/(a^5*d^6))^(3/4) +
sqrt(d*x)*b) + a*d^2*x*(-b/(a^5*d^6))^(1/4)*log(-a^4*d^5*(-b/(a^5*d^6))^(3/
4) + sqrt(d*x)*b) - 4*sqrt(d*x))/(a*d^2*x)
```

giac [A] time = 0.23, size = 264, normalized size = 0.64

$$-\frac{\left(\frac{8}{\sqrt{dx}\,a}+\frac{2\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^2d^2}+\frac{2\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^2d^2}-\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)\cdots\right.}{a^2b^2d^2}\cdots\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(8/(sqrt(d*x)*a) + 2*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqr
t(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^2*d^2) + 2*sqrt
(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt
(d*x))/(a*d^2/b)^(1/4))/(a^2*b^2*d^2) - sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x +
 sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^2*d^2) + sqrt(2)
*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2
/b))/(a^2*b^2*d^2))*sgn(b*x^2 + a)/d
```

maple [A] time = 0.01, size = 224, normalized size = 0.54

$$-\frac{\left(b x^2+a\right)\left(2\sqrt{2}\,\sqrt{dx}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+2\sqrt{2}\,\sqrt{dx}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+\sqrt{2}\,\sqrt{dx}\,\ln\left(-\frac{-dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\cdots}{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\cdots}\right.\cdots\right.}{4\sqrt{\left(b x^2+a\right)^2}\,\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/4*(b*x^2+a)/d*(2^(1/2)*(d*x)^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)
*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2
)^(1/2)))+2*2^(1/2)*(d*x)^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4)
)/(a/b*d^2)^(1/4))+2*2^(1/2)*(d*x)^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d
^2)^(1/4))/(a/b*d^2)^(1/4))+8*(a/b*d^2)^(1/4))/((b*x^2+a)^2)^(1/2)/a/(a/b*d
^2)^(1/4)/(d*x)^(1/2)
```

maxima [A] time = 3.26, size = 234, normalized size = 0.57

$$-\frac{\frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\cdots\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\cdots\right)}{a}\cdots}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sq
rt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b))
 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*
x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sq
rt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d
)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)
*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/a + 8/(sqrt(d*x)*a
))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d\,x)^{3/2}\,\sqrt{\left(b\,x^2+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(3/2)*((a + b*x^2)^2)^(1/2)),x)
[Out] int(1/((d*x)^(3/2)*((a + b*x^2)^2)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/((b*x**2+a)**2)**(1/2),x)
[Out] Timed out
```

3.755 $\int \frac{1}{(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=414

$$-\frac{2\left(a+bx^2\right)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{3/4}\left(a+bx^2\right)\log\left(\right.}{2\sqrt{2}\,a^{7}}$$

```
[Out] -2/3*(b*x^2+a)/a/d/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2)+1/2*b^(3/4)*(b*x^2+a)*ar
ctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(7/4)/d^(5/2)*2^(1/2)
/((b*x^2+a)^2)^(1/2)-1/2*b^(3/4)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(
1/2)/a^(1/4)/d^(1/2))/a^(7/4)/d^(5/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)+1/4*b^(3/
4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(
d*x)^(1/2))/a^(7/4)/d^(5/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)-1/4*b^(3/4)*(b*x^2+
a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2)
)/a^(7/4)/d^(5/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{3/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{3/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\right.}{2\sqrt{2}\,a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-2*(a + b*x^2))/(3*a*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(
3/4)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])
/(Sqrt[2]*a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(3/4)*(a +
b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*
a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(3/4)*(a + b*x^2)*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/
(2*Sqrt[2]*a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(3/4)*(a +
 b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[d*x]])/(2*Sqrt[2]*a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{ad^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(2b\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{\ldots}\right)}{ad^3\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \ldots\right)}{a^{3/2}d^4\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,d\ldots\right)}{2\sqrt{2}\,a^{7/4}\sqrt[4]{b}\,d^{5/2}\sqrt{a^2+2abx^2+b^2\ldots}}$$

$$= -\frac{2\left(a+bx^2\right)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b^{3/4}\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\ldots\right)}{2\sqrt{2}\,a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2\ldots}}$$

$$= -\frac{2\left(a+bx^2\right)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b^{3/4}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\,a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{b^{3/\ldots}}{\ldots}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.13

$$-\frac{2x\left(a+bx^2\right)\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a(dx)^{5/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-2*x*(a + b*x^2)*Hypergeometric2F1[-3/4, 1, 1/4, -((b*x^2)/a)])/(3*a*(d*x)
^(5/2)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.81, size = 227, normalized size = 0.55

$$-\frac{12\,ad^3x^2\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^5bd^7\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{3}{4}}-\sqrt{a^4d^6\sqrt{-\frac{b^3}{a^7d^{10}}}+b^2dx}\,a^5d^7\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{3}{4}}}{b^3}\right)+3\,ad^3x^2\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{1}{4}}\log\left(a^2d^3\left(-\right.\right.}{6\,ad^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(12*a*d^3*x^2*(-b^3/(a^7*d^10))^(1/4)*arctan(-(sqrt(d*x)*a^5*b*d^7*(-b
^3/(a^7*d^10))^(3/4) - sqrt(a^4*d^6*sqrt(-b^3/(a^7*d^10)) + b^2*d*x)*a^5*d^
7*(-b^3/(a^7*d^10))^(3/4))/b^3) + 3*a*d^3*x^2*(-b^3/(a^7*d^10))^(1/4)*log(a
^2*d^3*(-b^3/(a^7*d^10))^(1/4) + sqrt(d*x)*b) - 3*a*d^3*x^2*(-b^3/(a^7*d^10
))^(1/4)*log(-a^2*d^3*(-b^3/(a^7*d^10))^(1/4) + sqrt(d*x)*b) + 4*sqrt(d*x))
/(a*d^3*x^2)
```

giac [A] time = 0.24, size = 256, normalized size = 0.62

$$-\frac{1}{12}\left(\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2d^3}+\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2d^3}+\frac{3\sqrt{2}\left(ab^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*(6*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*d^3) + 6*sqrt(2)*(a*b^3*d^2)^(1/4)
*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4
))/(a^2*d^3) + 3*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4
)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*d^3) - 3*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*
x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*d^3) + 8/(sqrt(
d*x)*a*d^2*x))*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 239, normalized size = 0.58

$$-\frac{\left(b\,x^{2}+a\right)\left(8a\,d^{2}+6\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(dx\right)^{\frac{3}{2}}b\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)+6\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(dx\right)^{\frac{3}{2}}b\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)\right.}{12\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(dx\right)^{\frac{3}{2}}a^{2}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/12*(b*x^2+a)/d^3*(3*b*(a/b*d^2)^(1/4)*2^(1/2)*(d*x)^(3/2)*ln((d*x+(a/b*d
^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(
1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+6*b*(a/b*d^2)^(1/4)*2^(1/2)*(d*x)^(3/2)*arct
an((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+6*b*(a/b*d^2)^(1/
4)*2^(1/2)*(d*x)^(3/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^
2)^(1/4))+8*d^2*a)/((b*x^2+a)^2)^(1/2)/a^2/(d*x)^(3/2)
```

maxima [A] time = 3.08, size = 242, normalized size = 0.58

$$-\frac{\frac{3\left(\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^{2}\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^{2}\right)^{\frac{3}{4}}}+\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}\,d}+\frac{2\sqrt{2}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\right.}{}\right.}{\sqrt{\sqrt{a}\,\sqrt{}}}\right.}{a}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*(sqrt(2)*b^(3/4)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)
*b^(1/4) + sqrt(a)*d)/(a*d^2)^(3/4) - sqrt(2)*b^(3/4)*log(sqrt(b)*d*x - sqr
t(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/(a*d^2)^(3/4) + 2*sqrt(2)
*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))
/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)*d) + 2*sqrt(2)*b
*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/
sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)*d))/a + 8/((d*x)^
(3/2)*a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d\,x)^{5/2}\,\sqrt{\left(b\,x^2+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(5/2)*((a + b*x^2)^2)^(1/2)),x)
[Out] int(1/((d*x)^(5/2)*((a + b*x^2)^2)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/((b*x**2+a)**2)**(1/2),x)
[Out] Timed out
```

3.756 $\int \frac{1}{(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=459

$$\frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{5/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\right.}{2\sqrt{2}a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -2/5*(b*x^2+a)/a/d/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2)-1/2*b^(5/4)*(b*x^2+a)*ar
ctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(9/4)/d^(7/2)*2^(1/2)
/((b*x^2+a)^2)^(1/2)+1/2*b^(5/4)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(
1/2)/a^(1/4)/d^(1/2))/a^(9/4)/d^(7/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)+1/4*b^(5/
4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(
d*x)^(1/2))/a^(9/4)/d^(7/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)-1/4*b^(5/4)*(b*x^2+
a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2)
)/a^(9/4)/d^(7/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)+2*b*(b*x^2+a)/a^2/d^3/(d*x)^(
1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{5/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{2\sqrt{2}a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{5/4}\left(a+bx^2\right)\log\left(\sqrt{2}\right.}{2\sqrt{2}a^{9/4}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-2*(a + b*x^2))/(5*a*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (2*b
*(a + b*x^2))/(a^2*d^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(5/4
)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(S
qrt[2]*a^(9/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(5/4)*(a + b*x
^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(
9/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(5/4)*(a + b*x^2)*Log[Sq
rt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*
Sqrt[2]*a^(9/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(5/4)*(a + b*
x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d*x]])/(2*Sqrt[2]*a^(9/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)}\,dx}{ad^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(b^2\left(ab\right.\right.}{a^2d^4}$$

$$= -\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(2b^2\left(a\right.\right.}{}$$

$$= -\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b^{3/2}\left(a\right.\right.}{}$$

$$= -\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(\sqrt[4]{b}\left(a\right.\right.}{}$$

$$= -\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b^{5/4}\left(a\right.}{}$$

$$= -\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} - \frac{b^{5/4}\left(a\right.}{\sqrt{2}\,a}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.11

$$-\frac{2x\left(a+bx^2\right)\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a(dx)^{7/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-2*x*(a + b*x^2)*Hypergeometric2F1[-5/4, 1, -1/4, -((b*x^2)/a)])/(5*a*(d*x
)^(7/2)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 1.12, size = 253, normalized size = 0.55

$$-\frac{20\,a^{2}d^{4}x^{3}\left(-\frac{b^{5}}{a^{9}d^{14}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^{2}b^{4}d^{3}\left(-\frac{b^{5}}{a^{9}d^{14}}\right)^{\frac{1}{4}}-\sqrt{-a^{5}b^{5}d^{8}\sqrt{-\frac{b^{5}}{a^{9}d^{14}}}+b^{8}dx}\,a^{2}d^{3}\left(-\frac{b^{5}}{a^{9}d^{14}}\right)^{\frac{1}{4}}}{b^{5}}\right)-5\,a^{2}d^{4}x^{3}\left(-\frac{b^{5}}{a^{9}d^{14}}\right)^{\frac{1}{4}}\log\left(a^{7}\cdots\right.}{10\,a^{2}d^{4}x^{3}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/10*(20*a^2*d^4*x^3*(-b^5/(a^9*d^14))^(1/4)*arctan(-(sqrt(d*x)*a^2*b^4*d^
3*(-b^5/(a^9*d^14))^(1/4) - sqrt(-a^5*b^5*d^8*sqrt(-b^5/(a^9*d^14)) + b^8*d
*x)*a^2*d^3*(-b^5/(a^9*d^14))^(1/4))/b^5) - 5*a^2*d^4*x^3*(-b^5/(a^9*d^14))
^(1/4)*log(a^7*d^11*(-b^5/(a^9*d^14))^(3/4) + sqrt(d*x)*b^4) + 5*a^2*d^4*x^
3*(-b^5/(a^9*d^14))^(1/4)*log(-a^7*d^11*(-b^5/(a^9*d^14))^(3/4) + sqrt(d*x)
*b^4) - 4*(5*b*x^2 - a)*sqrt(d*x))/(a^2*d^4*x^3)
```

giac [A] time = 0.32, size = 284, normalized size = 0.62

$$\frac{1}{20}\left(\frac{10\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{3}bd^{5}}+\frac{10\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{3}bd^{5}}-\frac{5\sqrt{2}\left(ab^{3}d\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/20*(10*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b*d^5) + 10*sqrt(2)*(a*b^3*d^2)^(3
/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(
1/4))/(a^3*b*d^5) - 5*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)
^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b*d^5) + 5*sqrt(2)*(a*b^3*d^2)^(3/4)
*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b*d^5) +
 8*(5*b*d^2*x^2 - a*d^2)/(sqrt(d*x)*a^2*d^5*x^2))*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 251, normalized size = 0.55

$$\frac{\left(b\,x^{2}+a\right)\left(40\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}} b\,d^{2}x^{2}-8\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}} a\,d^{2}+10\sqrt{2}\;(dx)^{\frac{5}{2}}\,b\arctan\left(\frac{\sqrt{2}\;\sqrt{dx}-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)+10\sqrt{2}\;(dx)^{\frac{5}{2}}\,b\arctan\left(\frac{\sqrt{2}\cdots}{\cdots}\right.\right.}{20\sqrt{\left(b\,x^{2}+a\right)^{2}}\;\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}(dx)^{\frac{5}{2}}\,a^{2}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/20*(b*x^2+a)/d^3*(5*b*2^(1/2)*(d*x)^(5/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)
^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a
/b*d^2)^(1/2)))+10*b*2^(1/2)*(d*x)^(5/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d
^2)^(1/4))/(a/b*d^2)^(1/4))+10*b*2^(1/2)*(d*x)^(5/2)*arctan((2^(1/2)*(d*x)^
(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+40*b*(a/b*d^2)^(1/4)*d^2*x^2-8*d^2*
a*(a/b*d^2)^(1/4))/((b*x^2+a)^2)^(1/2)/a^2/(a/b*d^2)^(1/4)/(d*x)^(5/2)
```

maxima [A] time = 3.03, size = 259, normalized size = 0.56

$$\frac{\frac{5\,b^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^{2}\cdots\right.\right.}{\left(ad^{2}\right)^{\frac{1}{4}}\cdots}\cdots\right.}{a^{2}d^{2}}\cdots}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/20*(5*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) +
2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt
(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqr
t(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b))
- sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(
a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(
1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^2*d^2) + 8*
(5*b*d^2*x^2 - a*d^2)/((d*x)^(5/2)*a^2*d^2))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d\,x)^{7/2}\,\sqrt{\left(b\,x^2+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(7/2)*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] int(1/((d*x)^(7/2)*((a + b*x^2)^2)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.757 $\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=551

$$-\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117ad^7\sqrt{dx}\left(a+bx^2\right)}{16b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117d^5(dx)^{5/2}\left(a+b\right.}{80b^3\sqrt{a^2+2abx^2+b}}$$

```
[Out] -13/16*d^3*(d*x)^(9/2)/b^2/((b*x^2+a)^2)^(1/2)-1/4*d*(d*x)^(13/2)/b/(b*x^2+
a)/((b*x^2+a)^2)^(1/2)+117/80*d^5*(d*x)^(5/2)*(b*x^2+a)/b^3/((b*x^2+a)^2)^(
1/2)-117/64*a^(5/4)*d^(15/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)
/a^(1/4)/d^(1/2))/b^(17/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+117/64*a^(5/4)*d^(15
/2)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(17/4
)*2^(1/2)/((b*x^2+a)^2)^(1/2)-117/128*a^(5/4)*d^(15/2)*(b*x^2+a)*ln(a^(1/2)
*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(17/4)*2^
(1/2)/((b*x^2+a)^2)^(1/2)+117/128*a^(5/4)*d^(15/2)*(b*x^2+a)*ln(a^(1/2)*d^(
1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(17/4)*2^(1/2
)/((b*x^2+a)^2)^(1/2)-117/16*a*d^7*(b*x^2+a)*(d*x)^(1/2)/b^4/((b*x^2+a)^2)^
(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{117ad^7\sqrt{dx}\left(a+bx^2\right)}{16b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117d^5(dx)^{5/2}\left(a+bx^2\right)}{80b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117a^{5/4}d^{15/2}\left(a+bx^2\right)\log\left(-\right.}{64\sqrt{2}\,b^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-13*d^3*(d*x)^(9/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(
13/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*a*d^7*Sqrt[
d*x]*(a + b*x^2))/(16*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*d^5*(d*x)
^(5/2)*(a + b*x^2))/(80*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*a^(5/4)
*d^(15/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[
d])])/(32*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*a^(5/4)*
d^(15/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d
])])/(32*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*a^(5/4)*d
^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

```
) + (117*a^(5/4)*d^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(13d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(117d^4\left(ab+b^2\right.\right.}{32b^2\sqrt{a^2+}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{117d^5(dx)^{5/2}\left(\right.}{80b^3\sqrt{a^2+2ab}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\sqrt{dx}\left(\right.}{16b^4\sqrt{a^2+2ab}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\sqrt{dx}\left(\right.}{16b^4\sqrt{a^2+2ab}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\sqrt{dx}\left(\right.}{16b^4\sqrt{a^2+2ab}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\sqrt{dx}\left(\right.}{16b^4\sqrt{a^2+2ab}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\sqrt{dx}\left(\right.}{16b^4\sqrt{a^2+2ab}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\sqrt{dx}\left(\right.}{16b^4\sqrt{a^2+2ab}}$$

Mathematica [A] time = 0.16, size = 498, normalized size = 0.90

$$d^7\sqrt{dx}\left(-585\sqrt{2}\,a^{5/4}b^2x^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+585\sqrt{2}\,a^{5/4}b^2x^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (d^7*Sqrt[d*x]*(-4680*a^3*b^(1/4)*Sqrt[x] - 8424*a^2*b^(5/4)*x^(5/2) - 3328
*a*b^(9/4)*x^(9/2) + 256*b^(13/4)*x^(13/2) - 1170*Sqrt[2]*a^(5/4)*(a + b*x^
2)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 1170*Sqrt[2]*a^(5/4)*(
a + b*x^2)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 585*Sqrt[2]*a^
(13/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 1170*Sq
rt[2]*a^(9/4)*b*x^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x] - 585*Sqrt[2]*a^(5/4)*b^2*x^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x] + Sqrt[b]*x] + 585*Sqrt[2]*a^(13/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x] + 1170*Sqrt[2]*a^(9/4)*b*x^2*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 585*Sqrt[2]*a^(5/4)*b^2*x^4*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(640*b^(17/4)*Sqrt[x]
*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.95, size = 341, normalized size = 0.62

$$2340\left(-\frac{a^5d^{30}}{b^{17}}\right)^{\frac{1}{4}}\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)\arctan\left(-\frac{\left(-\frac{a^5d^{30}}{b^{17}}\right)^{\frac{3}{4}}\sqrt{dx}\,ab^{13}d^7-\left(-\frac{a^5d^{30}}{b^{17}}\right)^{\frac{3}{4}}\sqrt{a^2d^{15}x+\sqrt{-\frac{a^5d^{30}}{b^{17}}}\,b^8}\,b^{13}}{a^5d^{30}}\right)+585\left(-\frac{a^5d^{3}}{b^{17}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/320*(2340*(-a^5*d^30/b^17)^(1/4)*(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)*arctan
(-((-a^5*d^30/b^17)^(3/4)*sqrt(d*x)*a*b^13*d^7 - (-a^5*d^30/b^17)^(3/4)*sqr
t(a^2*d^15*x + sqrt(-a^5*d^30/b^17)*b^8)*b^13)/(a^5*d^30)) + 585*(-a^5*d^30
/b^17)^(1/4)*(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)*log(117*sqrt(d*x)*a*d^7 + 11
7*(-a^5*d^30/b^17)^(1/4)*b^4) - 585*(-a^5*d^30/b^17)^(1/4)*(b^6*x^4 + 2*a*b
^5*x^2 + a^2*b^4)*log(117*sqrt(d*x)*a*d^7 - 117*(-a^5*d^30/b^17)^(1/4)*b^4)
 + 4*(32*b^3*d^7*x^6 - 416*a*b^2*d^7*x^4 - 1053*a^2*b*d^7*x^2 - 585*a^3*d^7
)*sqrt(d*x))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

giac [A] time = 0.35, size = 419, normalized size = 0.76

$$\frac{1}{640}\,d^7\left(\frac{1170\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\,a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{1170\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\,a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/640*d^7*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^5*sgn(b*d^4*x^2 + a*d^4)) +
 1170*sqrt(2)*(a*b^3*d^2)^(1/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^5*sgn(b*d^4*x^2 + a*d^4)) + 585*sqrt
(2)*(a*b^3*d^2)^(1/4)*a*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(b^5*sgn(b*d^4*x^2 + a*d^4)) - 585*sqrt(2)*(a*b^3*d^2)^(1/4)*a*lo
g(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^5*sgn(b*d^4*x
^2 + a*d^4)) - 40*(25*sqrt(d*x)*a^2*b*d^4*x^2 + 21*sqrt(d*x)*a^3*d^4)/((b*d
^2*x^2 + a*d^2)^2*b^4*sgn(b*d^4*x^2 + a*d^4)) + 256*(sqrt(d*x)*b^12*d^10*x^
2 - 15*sqrt(d*x)*a*b^11*d^10)/(b^15*d^10*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 737, normalized size = 1.34

$$\frac{\left(1170\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\;a\,b^2d^2x^4\arctan\left(\frac{\sqrt{2}\;\sqrt{dx}-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)+1170\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\;a\,b^2d^2x^4\arctan\left(\frac{\sqrt{2}\;\sqrt{dx}+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)+585\left(\frac{a\,d^2}{b}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/640*(1170*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*(
a/b*d^2)^(1/4)*2^(1/2)*x^4*a*b^2*d^2+585*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a
/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^4*a*b^2*d^2+1170*(a/b*d^2)^(1/4)*2^(1/
```

```
2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a*b^2*
d^2+256*(d*x)^(5/2)*x^4*b^3+2340*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4
))/(a/b*d^2)^(1/4))*(a/b*d^2)^(1/4)*2^(1/2)*x^2*a^2*b*d^2+1170*(a/b*d^2)^(1
/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d
*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a^2*b*d^2+2340
*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*
d^2)^(1/4))*x^2*a^2*b*d^2+512*(d*x)^(5/2)*x^2*a*b^2-3840*(d*x)^(1/2)*x^4*a*
b^2*d^2+1170*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*
(a/b*d^2)^(1/4)*2^(1/2)*a^3*d^2+585*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^
2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1
/2)*2^(1/2)+(a/b*d^2)^(1/2)))*a^3*d^2+1170*(a/b*d^2)^(1/4)*2^(1/2)*arctan((
2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^3*d^2-744*(d*x)^(5/
2)*a^2*b-7680*(d*x)^(1/2)*x^2*a^2*b*d^2-4680*(d*x)^(1/2)*a^3*d^2)*d^5*(b*x^
2+a)/b^4/((b*x^2+a)^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^{2} d^{\frac{15}{2}} x^{\frac{5}{2}}}{2\left(a b^{4} x^{2}+a^{2} b^{3}+\left(b^{5} x^{2}+a b^{4}\right) x^{2}\right)}-2\, a d^{\frac{15}{2}} \int \frac{x^{\frac{3}{2}}}{b^{4} x^{2}+a b^{3}}\, dx+d^{\frac{15}{2}} \int \frac{x^{\frac{7}{2}}}{b^{3} x^{2}+a b^{2}}\, dx+\frac{21\left(\frac{2\sqrt{2}\, a^{\frac{3}{2}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\, a^{\frac{1}{4}} b^{\frac{1}{4}} \cdots\right.}{2\sqrt{\sqrt{a} \cdots}}\right.}{\sqrt{\sqrt{a}\,\sqrt{b}}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*d^(15/2)*x^(5/2)/(a*b^4*x^2 + a^2*b^3 + (b^5*x^2 + a*b^4)*x^2) - 2
*a*d^(15/2)*integrate(x^(3/2)/(b^4*x^2 + a*b^3), x) + d^(15/2)*integrate(x^
(7/2)/(b^3*x^2 + a*b^2), x) + 21/128*(2*sqrt(2)*a^(3/2)*arctan(1/2*sqrt(2)*
(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(s
qrt(a)*sqrt(b)) + 2*sqrt(2)*a^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^
(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + s
qrt(2)*a^(5/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b
^(1/4) - sqrt(2)*a^(5/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x +
 sqrt(a))/b^(1/4))*d^(15/2)/b^4 - 1/16*(17*a^2*b*d^(15/2)*x^(5/2) + 21*a^3*
d^(15/2)*sqrt(x))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{15/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Timed out
```

3.758 $\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=504

$$-\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^5(dx)^{3/2}\left(a+bx^2\right)}{48b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77a^{3/4}d^{13/2}\left(a+bx^2\right)}{64\sqrt{}}$$

```
[Out] -11/16*d^3*(d*x)^(7/2)/b^2/((b*x^2+a)^2)^(1/2)-1/4*d*(d*x)^(11/2)/b/(b*x^2+
a)/((b*x^2+a)^2)^(1/2)+77/48*d^5*(d*x)^(3/2)*(b*x^2+a)/b^3/((b*x^2+a)^2)^(1
/2)+77/64*a^(3/4)*d^(13/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a
^(1/4)/d^(1/2))/b^(15/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-77/64*a^(3/4)*d^(13/2)
*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(15/4)*2
^(1/2)/((b*x^2+a)^2)^(1/2)-77/128*a^(3/4)*d^(13/2)*(b*x^2+a)*ln(a^(1/2)*d^(
1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(15/4)*2^(1/2
)/((b*x^2+a)^2)^(1/2)+77/128*a^(3/4)*d^(13/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+
x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(15/4)*2^(1/2)/((b
*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 504, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{77d^5(dx)^{3/2}\left(a+bx^2\right)}{48b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77a^{3/4}d^{13/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{}\right.}{64\sqrt{2}\,b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-11*d^3*(d*x)^(7/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(
11/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^5*(d*x)^(3
/2)*(a + b*x^2))/(48*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*a^(3/4)*d^(
13/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(32*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*a^(3/4)*d^(13
/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/
(32*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*a^(3/4)*d^(13/2
)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7
7*a^(3/4)*d^(13/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(11d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(77d^4\left(ab+b^2x\right.\right.}{32b^2\sqrt{a^2+2}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(dx)^{3/2}\left(\right.}{48b^3\sqrt{a^2+2ab}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(dx)^{3/2}\left(\right.}{48b^3\sqrt{a^2+2ab}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(dx)^{3/2}\left(\right.}{48b^3\sqrt{a^2+2ab}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(dx)^{3/2}\left(\right.}{48b^3\sqrt{a^2+2ab}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(dx)^{3/2}\left(\right.}{48b^3\sqrt{a^2+2ab}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(dx)^{3/2}\left(\right.}{48b^3\sqrt{a^2+2ab}}$$

Mathematica [C] time = 0.03, size = 88, normalized size = 0.17

$$-\frac{2d^5(dx)^{3/2}\left(-77a^2+77\left(a+bx^2\right)^2\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)-55abx^2-5b^2x^4\right)}{15b^3\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-2*d^5*(d*x)^(3/2)*(-77*a^2 - 55*a*b*x^2 - 5*b^2*x^4 + 77*(a + b*x^2)^2*Hy
pergeometric2F1[3/4, 3, 7/4, -((b*x^2)/a)]))/(15*b^3*(a + b*x^2)*Sqrt[(a +
b*x^2)^2])
```

fricas [A] time = 1.17, size = 341, normalized size = 0.68

$$\frac{924 \left(-\frac{a^3 d^{26}}{b^{15}}\right)^{\frac{1}{4}} \left(b^5 x^4 + 2\, a b^4 x^2 + a^2 b^3\right) \arctan\left(-\frac{\left(-\frac{a^3 d^{26}}{b^{15}}\right)^{\frac{1}{4}} \sqrt{dx}\, a^2 b^4 d^{19} - \sqrt{a^4 d^{39} x - \sqrt{-\frac{a^3 d^{26}}{b^{15}}}\, a^3 b^7 d^{26}} \left(-\frac{a^3 d^{26}}{b^{15}}\right)^{\frac{1}{4}} b^4}{a^3 d^{26}}\right) - 231 \left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/192*(924*(-a^3*d^26/b^15)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*arctan(
-((-a^3*d^26/b^15)^(1/4)*sqrt(d*x)*a^2*b^4*d^19 - sqrt(a^4*d^39*x - sqrt(-a
^3*d^26/b^15)*a^3*b^7*d^26)*(-a^3*d^26/b^15)^(1/4)*b^4)/(a^3*d^26)) - 231*(
-a^3*d^26/b^15)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*log(456533*sqrt(d*x
)*a^2*d^19 + 456533*(-a^3*d^26/b^15)^(3/4)*b^11) + 231*(-a^3*d^26/b^15)^(1/
4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*log(456533*sqrt(d*x)*a^2*d^19 - 456533
*(-a^3*d^26/b^15)^(3/4)*b^11) + 4*(32*b^2*d^6*x^5 + 121*a*b*d^6*x^3 + 77*a^
2*d^6*x)*sqrt(d*x))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

giac [A] time = 0.41, size = 399, normalized size = 0.79

$$\frac{1}{384}\, d^6 \left(\frac{256\, \sqrt{dx}\, x}{b^3 \mathrm{sgn}\left(b d^4 x^2 + a d^4\right)} + \frac{24 \left(19 \sqrt{dx}\, a b d^4 x^3 + 15 \sqrt{dx}\, a^2 d^4 x\right)}{\left(b d^2 x^2 + a d^2\right)^2 b^3 \mathrm{sgn}\left(b d^4 x^2 + a d^4\right)} - \frac{462 \sqrt{2} \left(a b^3 d^2\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} + 2\right.}{2\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6 d\, \mathrm{sgn}\left(b d^4 x^2 + a d^4\right)} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/384*d^6*(256*sqrt(d*x)*x/(b^3*sgn(b*d^4*x^2 + a*d^4)) + 24*(19*sqrt(d*x)*
a*b*d^4*x^3 + 15*sqrt(d*x)*a^2*d^4*x)/((b*d^2*x^2 + a*d^2)^2*b^3*sgn(b*d^4*
x^2 + a*d^4)) - 462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^6*d*sgn(b*d^4*x^2 + a*d^4
)) - 462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(
1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^6*d*sgn(b*d^4*x^2 + a*d^4)) + 231*s
qrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt
(a*d^2/b))/(b^6*d*sgn(b*d^4*x^2 + a*d^4)) - 231*sqrt(2)*(a*b^3*d^2)^(3/4)*l
og(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^6*d*sgn(b*d^
4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 679, normalized size = 1.35

$$\left(-462\sqrt{2}\, a\, b^{2} d^{4} x^{4} \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)-462\sqrt{2}\, a\, b^{2} d^{4} x^{4} \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)-231\sqrt{2}\, a\, b^{2} d^{4} x^{4} \ln\left(-\frac{-dx \cdots}{dx+\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/384*(256*(a/b*d^2)^(1/4)*(d*x)^(3/2)*x^4*b^3*d^2-231*2^(1/2)*ln(-(-d*x+(a
/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^4*a*b^2*d^4-462*2^(1/2)*arctan((2^(1/2
)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a*b^2*d^4-462*2^(1/2)*a
rctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a*b^2*d^4+
456*(a/b*d^2)^(1/4)*(d*x)^(7/2)*a*b^2+512*(a/b*d^2)^(1/4)*(d*x)^(3/2)*x^2*a
*b^2*d^2-462*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2
)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a^2
*b*d^4-924*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(
1/4))*x^2*a^2*b*d^4-924*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4)
)/(a/b*d^2)^(1/4))*x^2*a^2*b*d^4+616*(a/b*d^2)^(1/4)*(d*x)^(3/2)*a^2*b*d^2-
231*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/
(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*a^3*d^4-462*2^(1
/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^3*d^4-4
62*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^
3*d^4)*d^3*(b*x^2+a)/(a/b*d^2)^(1/4)/b^4/((b*x^2+a)^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^{2} d^{\frac{13}{2}} x^{\frac{3}{2}}}{2\left(ab^{4}x^{2}+a^{2}b^{3}+\left(b^{5}x^{2}+ab^{4}\right)x^{2}\right)}-2\,ad^{\frac{13}{2}}\int\frac{\sqrt{x}}{b^{4}x^{2}+ab^{3}}\,dx+d^{\frac{13}{2}}\int\frac{x^{\frac{5}{2}}}{b^{3}x^{2}+ab^{2}}\,dx+\frac{19\,ad^{\frac{13}{2}}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}\cdots\right)}{2\sqrt{\cdots}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{\cdots}}\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*d^(13/2)*x^(3/2)/(a*b^4*x^2 + a^2*b^3 + (b^5*x^2 + a*b^4)*x^2) - 2
*a*d^(13/2)*integrate(sqrt(x)/(b^4*x^2 + a*b^3), x) + d^(13/2)*integrate(x^
(5/2)/(b^3*x^2 + a*b^2), x) + 19/128*a*d^(13/2)*(2*sqrt(2)*arctan(1/2*sqrt(
2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sq
rt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/
4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b
))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqr
t(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sq
rt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^3 + 1/16*(19*a*b*d^(13/2)*x^(7/2) +
 23*a^2*d^(13/2)*x^(3/2))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{13/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Timed out
```

3.759 $\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=504

$$-\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45\sqrt[4]{a}\,d^{11/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\right.}{64\sqrt{2}\,b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -9/16*d^3*(d*x)^(5/2)/b^2/((b*x^2+a)^2)^(1/2)-1/4*d*(d*x)^(9/2)/b/(b*x^2+a)
/((b*x^2+a)^2)^(1/2)+45/64*a^(1/4)*d^(11/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1
/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(13/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-45/64
*a^(1/4)*d^(11/2)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^
(1/2))/b^(13/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+45/128*a^(1/4)*d^(11/2)*(b*x^2+
a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2)
)/b^(13/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-45/128*a^(1/4)*d^(11/2)*(b*x^2+a)*ln
(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(
13/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+45/16*d^5*(b*x^2+a)*(d*x)^(1/2)/b^3/((b*x
^2+a)^2)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 504, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{45d^5\sqrt{dx}\left(a+bx^2\right)}{16b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45\sqrt[4]{a}\,d^{11/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\right.}{64\sqrt{2}\,b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-9*d^3*(d*x)^(5/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(9
/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^5*Sqrt[d*x]*
(a + b*x^2))/(16*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*a^(1/4)*d^(11/2
)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(3
2*Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*a^(1/4)*d^(11/2)*
(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*
Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*a^(1/4)*d^(11/2)*(a
 + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(64*Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*a^
(1/4)*d^(11/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(9d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(45d^4\left(ab+b\right.\right.}{32b^2\sqrt{a^2+}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{dx}\,\left(\right.}{16b^3\sqrt{a^2+2a}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{dx}\,\left(\right.}{16b^3\sqrt{a^2+2a}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{dx}\,\left(\right.}{16b^3\sqrt{a^2+2a}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{dx}\,\left(\right.}{16b^3\sqrt{a^2+2a}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{dx}\,\left(\right.}{16b^3\sqrt{a^2+2a}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{dx}\,\left(\right.}{16b^3\sqrt{a^2+2a}}$$

Mathematica [A] time = 0.17, size = 484, normalized size = 0.96

$$\frac{15a^2(dx)^{11/2}\left(a+bx^2\right)}{4b^3x^5\left(\left(a+bx^2\right)^2\right)^{3/2}}+\frac{45\sqrt[4]{a}\,(dx)^{11/2}\left(a+bx^2\right)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,b^{13/4}x^{11/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}-\frac{45\sqrt[4]{a}\,(dx)^{11/2}\left(a+bx^2\right)^3\mathrm{lo}}{64\sqrt{2}\,b^{13/4}x^{1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (15*a^2*(d*x)^(11/2)*(a + b*x^2))/(4*b^3*x^5*((a + b*x^2)^2)^(3/2)) + (6*a*
(d*x)^(11/2)*(a + b*x^2))/(b^2*x^3*((a + b*x^2)^2)^(3/2)) + (2*(d*x)^(11/2)
*(a + b*x^2))/(b*x*((a + b*x^2)^2)^(3/2)) - (15*a*(d*x)^(11/2)*(a + b*x^2)^
2)/(16*b^3*x^5*((a + b*x^2)^2)^(3/2)) + (45*a^(1/4)*(d*x)^(11/2)*(a + b*x^2
)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*b^(13/4)*x^(
11/2)*((a + b*x^2)^2)^(3/2)) - (45*a^(1/4)*(d*x)^(11/2)*(a + b*x^2)^3*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*b^(13/4)*x^(11/2)*((a
 + b*x^2)^2)^(3/2)) + (45*a^(1/4)*(d*x)^(11/2)*(a + b*x^2)^3*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*b^(13/4)*x^(11/2)
*((a + b*x^2)^2)^(3/2)) - (45*a^(1/4)*(d*x)^(11/2)*(a + b*x^2)^3*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*b^(13/4)*x^(1
1/2)*((a + b*x^2)^2)^(3/2))
```

fricas [A] time = 1.01, size = 305, normalized size = 0.61

$$-\frac{180\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{1}{4}}\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)\arctan\left(-\frac{\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{3}{4}}\sqrt{dx}\,b^{10}d^5-\sqrt{d^{11}x+\sqrt{-\frac{ad^{22}}{b^{13}}}\,b^6}\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{3}{4}}b^{10}}{ad^{22}}\right)+45\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{1}{4}}\left(b^5x^{4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(180*(-a*d^22/b^13)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*arctan(-(
(-a*d^22/b^13)^(3/4)*sqrt(d*x)*b^10*d^5 - sqrt(d^11*x + sqrt(-a*d^22/b^13)*
b^6)*(-a*d^22/b^13)^(3/4)*b^10)/(a*d^22)) + 45*(-a*d^22/b^13)^(1/4)*(b^5*x^
4 + 2*a*b^4*x^2 + a^2*b^3)*log(45*sqrt(d*x)*d^5 + 45*(-a*d^22/b^13)^(1/4)*b
^3) - 45*(-a*d^22/b^13)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*log(45*sqrt
(d*x)*d^5 - 45*(-a*d^22/b^13)^(1/4)*b^3) - 4*(32*b^2*d^5*x^4 + 81*a*b*d^5*x
^2 + 45*a^2*d^5)*sqrt(d*x))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

giac [A] time = 0.31, size = 385, normalized size = 0.76

$$-\frac{1}{128}d^5\left(\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{45\sqrt{\ldots}}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/128*d^5*(90*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^4*sgn(b*d^4*x^2 + a*d^4)) + 90
*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2
*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^4*sgn(b*d^4*x^2 + a*d^4)) + 45*sqrt(2)*(a*b
^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/
(b^4*sgn(b*d^4*x^2 + a*d^4)) - 45*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(
2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^4*sgn(b*d^4*x^2 + a*d^4))
- 256*sqrt(d*x)/(b^3*sgn(b*d^4*x^2 + a*d^4)) - 8*(17*sqrt(d*x)*a*b*d^4*x^2
+ 13*sqrt(d*x)*a^2*d^4)/((b*d^2*x^2 + a*d^2)^2*b^3*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 696, normalized size = 1.38

$$-\frac{\left(90\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^2d^2x^4\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)+90\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^2d^2x^4\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)+45\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/128*(45*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/
2)))*x^4*b^2*d^2+90*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/
b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*b^2*d^2+90*(a/b*d^2)^(1/4)*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*b^2*d^2+90*(a/
```

```
b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^
(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a*b*d
^2+180*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))
/(a/b*d^2)^(1/4))*x^2*a*b*d^2+180*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(
d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a*b*d^2-256*(d*x)^(1/2)*x^
4*b^2*d^2+45*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^
(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(
1/2)))*a^2*d^2+90*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*
d^2)^(1/4))/(a/b*d^2)^(1/4))*a^2*d^2+90*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(
1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^2*d^2-136*(d*x)^(5/2)*
a*b-512*(d*x)^(1/2)*x^2*a*b*d^2-360*(d*x)^(1/2)*a^2*d^2)*d^3*(b*x^2+a)/b^3/
((b*x^2+a)^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ad^{\frac{11}{2}}x^{\frac{5}{2}}}{2\left(ab^3x^2+a^2b^2+\left(b^4x^2+ab^3\right)x^2\right)}+d^{\frac{11}{2}}\int\frac{x^{\frac{3}{2}}}{b^3x^2+ab^2}\,dx-\frac{13\left(\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{a}\arctan\left(-\frac{\sqrt{2}\cdots}{\cdots}\right)}{\sqrt{\sqrt{a}\cdots}}\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*d^(11/2)*x^(5/2)/(a*b^3*x^2 + a^2*b^2 + (b^4*x^2 + a*b^3)*x^2) + d^(1
1/2)*integrate(x^(3/2)/(b^3*x^2 + a*b^2), x) - 13/128*(2*sqrt(2)*sqrt(a)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)
*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt(a)*arctan(-1/2*sqrt(2)*(s
qrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqr
t(a)*sqrt(b)) + sqrt(2)*a^(1/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(
b)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(1/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt
(x) + sqrt(b)*x + sqrt(a))/b^(1/4))*d^(11/2)/b^3 + 1/16*(9*a*b*d^(11/2)*x^(
5/2) + 13*a^2*d^(11/2)*sqrt(x))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{(d\,x)^{11/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Timed out
```

3.760 $\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=458

$$-\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21d^{9/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}\right.}{64\sqrt{2}\sqrt[4]{a}\,b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -7/16*d^3*(d*x)^(3/2)/b^2/((b*x^2+a)^2)^(1/2)-1/4*d*(d*x)^(7/2)/b/(b*x^2+a)
/((b*x^2+a)^2)^(1/2)-21/64*d^(9/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)
^(1/2)/a^(1/4)/d^(1/2))/a^(1/4)/b^(11/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+21/64*
d^(9/2)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(
1/4)/b^(11/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+21/128*d^(9/2)*(b*x^2+a)*ln(a^(1/
2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(1/4)/b
^(11/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-21/128*d^(9/2)*(b*x^2+a)*ln(a^(1/2)*d^(
1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(1/4)/b^(11/4
)*2^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 458, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21d^{9/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\sqrt[4]{a}\,b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21d^{9/2}\left(a+bx^2\right)\log\left(\right.}{64\sqrt{2}\sqrt[4]{a}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-7*d^3*(d*x)^(3/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(7
/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (21*d^(9/2)*(a + b
*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2
]*a^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (21*d^(9/2)*(a + b*x^
2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a
^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (21*d^(9/2)*(a + b*x^2)*
Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]
])/(64*Sqrt[2]*a^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (21*d^(9
/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(7d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21d^4\left(ab+b \cdots\right.\right.}{32b^2\sqrt{a^2+\cdots}}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21d^3\left(ab+b \cdots\right.\right.}{16 \cdots}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(21d^3\left(ab+b \cdots\right.\right.}{32 \cdots}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21d^{9/2}\left(ab+ \cdots\right.\right.}{6 \cdots}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{21d^{9/2}\left(a+bx \cdots\right.}{64 \cdots}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{21d^{9/2}\left(a+bx \cdots\right.}{32\sqrt{2}\sqrt[4]{a}\,b^{11 \cdots}}$$

Mathematica [C] time = 0.03, size = 84, normalized size = 0.18

$$\frac{2d^3(dx)^{3/2}\left(7\left(a+bx^2\right)^2 {}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)-a\left(7a+5bx^2\right)\right)}{5ab^2\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*d^3*(d*x)^(3/2)*(-(a*(7*a + 5*b*x^2)) + 7*(a + b*x^2)^2*Hypergeometric2F
1[3/4, 3, 7/4, -((b*x^2)/a)]))/(5*a*b^2*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.96, size = 312, normalized size = 0.68

$$-\frac{84\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac{1}{4}}\sqrt{dx}\,b^3d^{13}-\sqrt{d^{27}x-\sqrt{-\frac{d^{18}}{ab^{11}}}\,ab^5d^{18}}\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac{1}{4}}b^3}{d^{18}}\right)-21\left(b^4x^4+2\,ab^3x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(84*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-d^18/(a*b^11))^(1/4)*arctan(-
((-d^18/(a*b^11))^(1/4)*sqrt(d*x)*b^3*d^13 - sqrt(d^27*x - sqrt(-d^18/(a*b^
11))*a*b^5*d^18)*(-d^18/(a*b^11))^(1/4)*b^3)/d^18) - 21*(b^4*x^4 + 2*a*b^3*
x^2 + a^2*b^2)*(-d^18/(a*b^11))^(1/4)*log(9261*sqrt(d*x)*d^13 + 9261*(-d^18
/(a*b^11))^(3/4)*a*b^8) + 21*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-d^18/(a*b^
11))^(1/4)*log(9261*sqrt(d*x)*d^13 - 9261*(-d^18/(a*b^11))^(3/4)*a*b^8) + 4
*(11*b*d^4*x^3 + 7*a*d^4*x)*sqrt(d*x))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

giac [A] time = 0.32, size = 380, normalized size = 0.83

$$-\frac{1}{128}\,d^4\left(\frac{8\left(11\,\sqrt{dx}\,bd^4x^3+7\,\sqrt{dx}\,ad^4x\right)}{\left(bd^2x^2+ad^2\right)^2b^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{42\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^5d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{42\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\,a}{ab^5ds}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/128*d^4*(8*(11*sqrt(d*x)*b*d^4*x^3 + 7*sqrt(d*x)*a*d^4*x)/((b*d^2*x^2 +
a*d^2)^2*b^2*sgn(b*d^4*x^2 + a*d^4)) - 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(
1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^5
*d*sgn(b*d^4*x^2 + a*d^4)) - 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(
```

```
2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^5*d*sgn(b*
d^4*x^2 + a*d^4)) + 21*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b
)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^5*d*sgn(b*d^4*x^2 + a*d^4)) - 21*sq
rt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(a*b^5*d*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 612, normalized size = 1.34

$$-\frac{\left(-42\sqrt{2}\,b^2d^4x^4\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)-42\sqrt{2}\,b^2d^4x^4\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)-21\sqrt{2}\,b^2d^4x^4\ln\left(-\frac{-dx+\left(\frac{a\,d^2}{b}\right)}{dx+\left(\frac{a\,d^2}{b}\right)}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/128*(-21*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)
^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^4*b^2*
d^4-42*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4)
)*x^4*b^2*d^4-42*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*
d^2)^(1/4))*x^4*b^2*d^4+88*(a/b*d^2)^(1/4)*(d*x)^(7/2)*b^2-42*2^(1/2)*ln(-(
-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a*b*d^4-84*2^(1/2)*arctan((2^
(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a*b*d^4-84*2^(1/2)*
arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a*b*d^4+5
6*(a/b*d^2)^(1/4)*(d*x)^(3/2)*a*b*d^2-21*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*
(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/
2)+(a/b*d^2)^(1/2)))*a^2*d^4-42*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^
2)^(1/4))/(a/b*d^2)^(1/4))*a^2*d^4-42*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(
a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^2*d^4)*d*(b*x^2+a)/(a/b*d^2)^(1/4)/b^3/(
(b*x^2+a)^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ad^{\frac{9}{2}}x^{\frac{3}{2}}}{2\left(ab^3x^2+a^2b^2+\left(b^4x^2+ab^3\right)x^2\right)}+d^{\frac{9}{2}}\int\frac{\sqrt{x}}{b^3x^2+ab^2}\,dx-\frac{11\,d^{\frac{9}{2}}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{}\right.}{}\right.}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*d^(9/2)*x^(3/2)/(a*b^3*x^2 + a^2*b^2 + (b^4*x^2 + a*b^3)*x^2) + d^(9/
2)*integrate(sqrt(x)/(b^3*x^2 + a*b^2), x) - 11/128*d^(9/2)*(2*sqrt(2)*arct
an(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(s
qrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sq
rt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqr
t(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*
sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^2 - 1/16*(11*b*d^(9/2)*
x^(7/2) + 15*a*d^(9/2)*x^(3/2))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{9/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{9}{2}}}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**(9/2)/((a + b*x**2)**2)**(3/2), x)
```

3.761 $\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=458

$$-\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^{7/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}\right.}{64\sqrt{2}\,a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -1/4*d*(d*x)^(5/2)/b/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-5/64*d^(7/2)*(b*x^2+a)*a
rctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(3/4)/b^(9/4)*2^(1/2
)/((b*x^2+a)^2)^(1/2)+5/64*d^(7/2)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)
^(1/2)/a^(1/4)/d^(1/2))/a^(3/4)/b^(9/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-5/128*d
^(7/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/
2)*(d*x)^(1/2))/a^(3/4)/b^(9/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+5/128*d^(7/2)*(
b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)
^(1/2))/a^(3/4)/b^(9/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-5/16*d^3*(d*x)^(1/2)/b^
2/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 458, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^{7/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5d^{7/2}\left(a+bx^2\right)\log\left(\sqrt{\right.}}{64\sqrt{2}\,a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-5*d^3*Sqrt[d*x])/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(5/2
))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*d^(7/2)*(a + b*x^
2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a
^(3/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*d^(7/2)*(a + b*x^2)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(3/4
)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*d^(7/2)*(a + b*x^2)*Log[Sqr
t[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*
Sqrt[2]*a^(3/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*d^(7/2)*(a +
b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(64*Sqrt[2]*a^(3/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5d^4\left(ab+b^2x^2\right.\right.}{32b^2\sqrt{a^2 -}}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5d^3\left(ab+b^2x^2\right.\right.}{16b^2}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5d^2\left(ab+b^2x^2\right.\right.}{32\sqrt{}}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(5d^{7/2}\left(ab+b^2\right.\right.}{64}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^{7/2}\left(a+bx^2\right)}{64}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^{7/2}\left(a+bx^2\right)}{32\sqrt{2}\,a^{3/4}b^{9/4}}$$

Mathematica [A] time = 0.14, size = 447, normalized size = 0.98

$$-\frac{5(dx)^{7/2}\left(a+bx^2\right)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{3/4}b^{9/4}x^{7/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}+\frac{5(dx)^{7/2}\left(a+bx^2\right)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{3/4}b^{9/4}x^{7/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-5*a*(d*x)^(7/2)*(a + b*x^2))/(12*b^2*x^3*((a + b*x^2)^2)^(3/2)) - (2*(d*x
)^(7/2)*(a + b*x^2))/(3*b*x*((a + b*x^2)^2)^(3/2)) + (5*(d*x)^(7/2)*(a + b*
x^2)^2)/(48*b^2*x^3*((a + b*x^2)^2)^(3/2)) - (5*(d*x)^(7/2)*(a + b*x^2)^3*A
rcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(3/4)*b^(9/4)*x
^(7/2)*((a + b*x^2)^2)^(3/2)) + (5*(d*x)^(7/2)*(a + b*x^2)^3*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(3/4)*b^(9/4)*x^(7/2)*((a +
b*x^2)^2)^(3/2)) - (5*(d*x)^(7/2)*(a + b*x^2)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(3/4)*b^(9/4)*x^(7/2)*((a +
b*x^2)^2)^(3/2)) + (5*(d*x)^(7/2)*(a + b*x^2)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(3/4)*b^(9/4)*x^(7/2)*((a +
b*x^2)^2)^(3/2))
```

fricas [A] time = 0.89, size = 315, normalized size = 0.69

$$\frac{20\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{3}{4}}\sqrt{dx}\,a^2b^7d^3-\sqrt{d^7x+\sqrt{-\frac{d^{14}}{a^3b^9}}\,a^2b^4}\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{3}{4}}a^2b^7}{d^{14}}\right)+5\left(b^4x^4+2\,ab^3x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/64*(20*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-d^14/(a^3*b^9))^(1/4)*arctan(-
((-d^14/(a^3*b^9))^(3/4)*sqrt(d*x)*a^2*b^7*d^3 - sqrt(d^7*x + sqrt(-d^14/(a
^3*b^9))*a^2*b^4)*(-d^14/(a^3*b^9))^(3/4)*a^2*b^7)/d^14) + 5*(b^4*x^4 + 2*a
*b^3*x^2 + a^2*b^2)*(-d^14/(a^3*b^9))^(1/4)*log(5*sqrt(d*x)*d^3 + 5*(-d^14/
(a^3*b^9))^(1/4)*a*b^2) - 5*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-d^14/(a^3*b
^9))^(1/4)*log(5*sqrt(d*x)*d^3 - 5*(-d^14/(a^3*b^9))^(1/4)*a*b^2) - 4*(9*b*
d^3*x^2 + 5*a*d^3)*sqrt(d*x))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

giac [A] time = 0.31, size = 367, normalized size = 0.80

$$\frac{1}{128}d^3\left(\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{5\sqrt{2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*d^3*(10*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/
b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^3*sgn(b*d^4*x^2 + a*d^4)) + 1
0*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) -
2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^3*sgn(b*d^4*x^2 + a*d^4)) + 5*sqrt(2)*(a
*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b)
)/(a*b^3*sgn(b*d^4*x^2 + a*d^4)) - 5*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sq
rt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^3*sgn(b*d^4*x^2 + a*d
^4)) - 8*(9*sqrt(d*x)*b*d^4*x^2 + 5*sqrt(d*x)*a*d^4)/((b*d^2*x^2 + a*d^2)^2
*b^2*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 666, normalized size = 1.45

$$\frac{\left(10\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\, b^2 d^2 x^4 \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+10\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\, b^2 d^2 x^4 \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+5\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\, b^2 d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/128*(5*(a/b*d^2)^(1/4)*2^(1/2)*b^2*d^2*x^4*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^
(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/
b*d^2)^(1/2)))+10*(a/b*d^2)^(1/4)*2^(1/2)*b^2*d^2*x^4*arctan((2^(1/2)*(d*x)
^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+10*(a/b*d^2)^(1/4)*2^(1/2)*b^2*d^2
*x^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+10*(a/b*
d^2)^(1/4)*2^(1/2)*a*b*d^2*x^2*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+
(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))
+20*(a/b*d^2)^(1/4)*2^(1/2)*a*b*d^2*x^2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^
2)^(1/4))/(a/b*d^2)^(1/4))+20*(a/b*d^2)^(1/4)*2^(1/2)*a*b*d^2*x^2*arctan((2
^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+5*(a/b*d^2)^(1/4)*2^(1
/2)*a^2*d^2*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d
*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+10*(a/b*d^2)^(1/4)
*2^(1/2)*a^2*d^2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/
4))+10*(a/b*d^2)^(1/4)*2^(1/2)*a^2*d^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2
)^(1/4))/(a/b*d^2)^(1/4))-72*(d*x)^(5/2)*a*b-40*(d*x)^(1/2)*a^2*d^2)*d*(b*x
^2+a)/a/b^2/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 3.23, size = 279, normalized size = 0.61

$$-\frac{d^{\frac{7}{2}}x^{\frac{5}{2}}}{2\left(ab^2x^2+a^2b+\left(b^3x^2+ab^2\right)x^2\right)}+\frac{5\,d^3\left(\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\cdots\right.}{128\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*d^(7/2)*x^(5/2)/(a*b^2*x^2 + a^2*b + (b^3*x^2 + a*b^2)*x^2) + 5/128*d^
3*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b
)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(
2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*sqrt(d)*
log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)
) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt
(a))/(a^(3/4)*b^(1/4)))/b^2 - 1/16*(b*d^(7/2)*x^(5/2) + 5*a*d^(7/2)*sqrt(x)
)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{7/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{7}{2}}}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**(7/2)/((a + b*x**2)**2)**(3/2), x)
```

3.762 $\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=459

$$\frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3d^{5/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\cdots\right.}{64\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 3/16*d*(d*x)^(3/2)/a/b/((b*x^2+a)^2)^(1/2)-1/4*d*(d*x)^(3/2)/b/(b*x^2+a)/((
b*x^2+a)^2)^(1/2)-3/64*d^(5/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/
2)/a^(1/4)/d^(1/2))/a^(5/4)/b^(7/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+3/64*d^(5/2
)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(5/4)/b
^(7/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+3/128*d^(5/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/
2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(5/4)/b^(7/4)*2
^(1/2)/((b*x^2+a)^2)^(1/2)-3/128*d^(5/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(
1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(5/4)/b^(7/4)*2^(1/2)/(
(b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3d^{5/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\cdots\right.}{64\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (3*d*(d*x)^(3/2))/(16*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(3/2)
)/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^(5/2)*(a + b*x^2
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^
(5/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(5/2)*(a + b*x^2)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(5/4)
*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(5/2)*(a + b*x^2)*Log[Sqrt
[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*S
qrt[2]*a^(5/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^(5/2)*(a + b
*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[d*x]])/(64*Sqrt[2]*a^(5/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3d^2\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3d^2\left(ab+b^2x^{2}\right.\right.}{32ab\sqrt{a^2+2}}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3d\left(ab+b^2x^2\right)\right.}{16ab}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(3d\left(ab+b^2x^2\right)\right.}{32ab}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3d^{5/2}\left(ab+b^2\right.\right.}{64}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{3d^{5/2}\left(a+bx^2\right)}{64}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^{5/2}\left(a+bx^2\right)}{32\sqrt{2}\,a^{5/4}b^{7/4}}$$

Mathematica [C] time = 0.03, size = 73, normalized size = 0.16

$$\frac{2d(dx)^{3/2}\left(\left(a+bx^2\right)^2 {}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)-a^2\right)}{5a^2b\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*d*(d*x)^(3/2)*(-a^2 + (a + b*x^2)^2*Hypergeometric2F1[3/4, 3, 7/4, -((b*
x^2)/a)]))/(5*a^2*b*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.92, size = 326, normalized size = 0.71

$$-\frac{12\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{27\,\sqrt{dx}\,ab^2d^7\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{1}{4}}-\sqrt{-729\,a^3b^3d^{10}\sqrt{-\frac{d^{10}}{a^5b^7}}+729\,d^{15}x}\,ab^2\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{1}{4}}}{27\,d^{10}}\right)-3\left(ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(12*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^10/(a^5*b^7))^(1/4)*arcta
n(-1/27*(27*sqrt(d*x)*a*b^2*d^7*(-d^10/(a^5*b^7))^(1/4) - sqrt(-729*a^3*b^3
*d^10*sqrt(-d^10/(a^5*b^7)) + 729*d^15*x)*a*b^2*(-d^10/(a^5*b^7))^(1/4))/d^
10) - 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^10/(a^5*b^7))^(1/4)*log(27*
a^4*b^5*(-d^10/(a^5*b^7))^(3/4) + 27*sqrt(d*x)*d^7) + 3*(a*b^3*x^4 + 2*a^2*
b^2*x^2 + a^3*b)*(-d^10/(a^5*b^7))^(1/4)*log(-27*a^4*b^5*(-d^10/(a^5*b^7))^
(3/4) + 27*sqrt(d*x)*d^7) - 4*(3*b*d^2*x^3 - a*d^2*x)*sqrt(d*x))/(a*b^3*x^4
 + 2*a^2*b^2*x^2 + a^3*b)
```

giac [A] time = 0.32, size = 383, normalized size = 0.83

$$\frac{1}{128}\,d^2\left(\frac{8\left(3\,\sqrt{dx}\,bd^4x^3-\sqrt{dx}\,ad^4x\right)}{\left(bd^2x^2+ad^2\right)^2ab\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{6\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^4d\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{6\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\mathrm{arct}}{a^2b^4d\mathrm{sg}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*d^2*(8*(3*sqrt(d*x)*b*d^4*x^3 - sqrt(d*x)*a*d^4*x)/((b*d^2*x^2 + a*d^
2)^2*a*b*sgn(b*d^4*x^2 + a*d^4)) + 6*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*s
qrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^4*d*
```

```
sgn(b*d^4*x^2 + a*d^4)) + 6*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^4*d*sgn(b*d^
4*x^2 + a*d^4)) - 3*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(
1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^4*d*sgn(b*d^4*x^2 + a*d^4)) + 3*sqrt
(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*
d^2/b))/(a^2*b^4*d*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 617, normalized size = 1.34

$$\frac{\left(6\sqrt{2}\, b^2 d^4 x^4 \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+6\sqrt{2}\, b^2 d^4 x^4 \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+3\sqrt{2}\, b^2 d^4 x^4 \ln\left(-\frac{-dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{}}{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/128*(3*2^(1/2)*b^2*d^4*x^4*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-
(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))
+6*2^(1/2)*b^2*d^4*x^4*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^
2)^(1/4))+6*2^(1/2)*b^2*d^4*x^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4)
)/(a/b*d^2)^(1/4))+24*(a/b*d^2)^(1/4)*(d*x)^(7/2)*b^2+6*2^(1/2)*a*b*d^4*x^2
*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d
^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+12*2^(1/2)*a*b*d^4*x^2*arct
an((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+12*2^(1/2)*a*b*d^
4*x^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))-8*(a/b*
d^2)^(1/4)*(d*x)^(3/2)*a*b*d^2+3*2^(1/2)*a^2*d^4*ln(-(-d*x+(a/b*d^2)^(1/4)*
(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/
2)+(a/b*d^2)^(1/2)))+6*2^(1/2)*a^2*d^4*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2
)^(1/4))/(a/b*d^2)^(1/4))+6*2^(1/2)*a^2*d^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/
b*d^2)^(1/4))/(a/b*d^2)^(1/4)))/d*(b*x^2+a)/(a/b*d^2)^(1/4)/b^2/a/((b*x^2+a
)^2)^(3/2)
```

maxima [A] time = 3.19, size = 272, normalized size = 0.59

$$-\frac{d^{\frac{5}{2}} x^{\frac{3}{2}}}{2\left(ab^2x^2+a^2b+\left(b^3x^2+ab^2\right)x^2\right)}+\frac{3\,d^{\frac{5}{2}}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log}{}\right.}{128\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*d^(5/2)*x^(3/2)/(a*b^2*x^2 + a^2*b + (b^3*x^2 + a*b^2)*x^2) + 3/128*d^
(5/2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sq
rt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(
a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*
b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqr
t(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b
) + 1/16*(3*b*d^(5/2)*x^(7/2) + 7*a*d^(5/2)*x^(3/2))/(a*b^3*x^4 + 2*a^2*b^2
*x^2 + a^3*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{5/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{5}{2}}}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**(5/2)/((a + b*x**2)**2)**(3/2), x)
```

3.763 $\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}} dx$

Optimal. Leaf size=459

$$\frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{3/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\right.}{64\sqrt{2}a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -3/64*d^(3/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2
))/a^(7/4)/b^(5/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+3/64*d^(3/2)*(b*x^2+a)*arcta
n(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)/((
b*x^2+a)^2)^(1/2)-3/128*d^(3/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1
/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)/((b*x^2+a)
^2)^(1/2)+3/128*d^(3/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1
/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)/((b*x^2+a)^2)^(1/2
)+1/16*d*(d*x)^(1/2)/a/b/((b*x^2+a)^2)^(1/2)-1/4*d*(d*x)^(1/2)/b/(b*x^2+a)/
((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3d^{3/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3d^{3/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\right.}{64\sqrt{2}a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (d*Sqrt[d*x])/(16*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*Sqrt[d*x])/(4*b
*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^(3/2)*(a + b*x^2)*ArcT
an[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(7/4)*
b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(3/2)*(a + b*x^2)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(7/4)*b^(5/
4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^(3/2)*(a + b*x^2)*Log[Sqrt[a]*Sq
rt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]
*a^(7/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(3/2)*(a + b*x^2)*
Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]
])/(64*Sqrt[2]*a^(7/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(d^2\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3d^2\left(ab+b^2x^2\right)\right.}{32ab\sqrt{a^2+}}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3d\left(ab+b^2x^2\right)\right)}{16ab\sqrt{}}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3\left(ab+b^2x^2\right)\right)S}{32a^{3/2}b}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(3d^{3/2}\left(ab+b^2x^2\right.\right.}{64\sqrt{}}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^{3/2}\left(a+bx^2\right)\text{lo}}{64\sqrt{}}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^{3/2}\left(a+bx^2\right)\text{ta}}{32\sqrt{2}\,a^{7/4}b^{5/4}\sqrt{}}$$

<u>**Mathematica**</u> **[A]** time = 0.21, size = 272, normalized size = 0.59

$$\frac{(dx)^{3/2}\left(a+bx^2\right)\left(8a^{3/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)-32a^{7/4}\sqrt[4]{b}\sqrt{x}-3\sqrt{2}\left(a+bx^2\right)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+\right.}{128a^{7/}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((d*x)^(3/2)*(a + b*x^2)*(-32*a^(7/4)*b^(1/4)*Sqrt[x] + 8*a^(3/4)*b^(1/4)*S
qrt[x]*(a + b*x^2) - 6*Sqrt[2]*(a + b*x^2)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)] + 6*Sqrt[2]*(a + b*x^2)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)] - 3*Sqrt[2]*(a + b*x^2)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x] + 3*Sqrt[2]*(a + b*x^2)^2*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(128*a^(7/4)*b^(5/4)*x^(3/2)*((a + b*x^2
)^2)^(3/2))
```

fricas [A] time = 1.05, size = 308, normalized size = 0.67

$$12\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)\left(-\frac{d^6}{a^7b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^5b^4d\left(-\frac{d^6}{a^7b^5}\right)^{\frac{3}{4}}-\sqrt{a^4b^2\sqrt{-\frac{d^6}{a^7b^5}}+d^3x}\,a^5b^4\left(-\frac{d^6}{a^7b^5}\right)^{\frac{3}{4}}}{d^6}\right)+3\left(ab^3x^4+2\,a^2b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/64*(12*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^6/(a^7*b^5))^(1/4)*arctan(
-(sqrt(d*x)*a^5*b^4*d*(-d^6/(a^7*b^5))^(3/4) - sqrt(a^4*b^2*sqrt(-d^6/(a^7*
b^5)) + d^3*x)*a^5*b^4*(-d^6/(a^7*b^5))^(3/4))/d^6) + 3*(a*b^3*x^4 + 2*a^2*
b^2*x^2 + a^3*b)*(-d^6/(a^7*b^5))^(1/4)*log(3*a^2*b*(-d^6/(a^7*b^5))^(1/4)
+ 3*sqrt(d*x)*d) - 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^6/(a^7*b^5))^(
1/4)*log(-3*a^2*b*(-d^6/(a^7*b^5))^(1/4) + 3*sqrt(d*x)*d) + 4*(b*d*x^2 - 3*
a*d)*sqrt(d*x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
```

giac [A] time = 0.34, size = 367, normalized size = 0.80

$$\frac{1}{128}d\left(\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{3\sqrt{2}\left(ab\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*d*(6*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^2*sgn(b*d^4*x^2 + a*d^4)) + 6*
sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*
sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^2*sgn(b*d^4*x^2 + a*d^4)) + 3*sqrt(2)*(a
*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b)
)/(a^2*b^2*sgn(b*d^4*x^2 + a*d^4)) - 3*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x -
sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^2*sgn(b*d^4*x^2 +
 a*d^4)) + 8*(sqrt(d*x)*b*d^4*x^2 - 3*sqrt(d*x)*a*d^4)/((b*d^2*x^2 + a*d^2)
^2*a*b*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 668, normalized size = 1.46

$$\left(6\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^2 d^2 x^4 \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)+6\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^2 d^2 x^4 \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)+3\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^2 d^2 x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/128*(3*(a/b*d^2)^(1/4)*2^(1/2)*b^2*d^2*x^4*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^
(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/
b*d^2)^(1/2)))+6*(a/b*d^2)^(1/4)*2^(1/2)*b^2*d^2*x^4*arctan((2^(1/2)*(d*x)^
(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+6*(a/b*d^2)^(1/4)*2^(1/2)*b^2*d^2*x
^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+6*(a/b*d^2
)^(1/4)*2^(1/2)*a*b*d^2*x^2*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/
b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+12
*(a/b*d^2)^(1/4)*2^(1/2)*a*b*d^2*x^2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^
(1/4))/(a/b*d^2)^(1/4))+12*(a/b*d^2)^(1/4)*2^(1/2)*a*b*d^2*x^2*arctan((2^(1
/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+3*(a/b*d^2)^(1/4)*2^(1/2)
*a^2*d^2*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-
(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+6*(a/b*d^2)^(1/4)*2^(
1/2)*a^2*d^2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+
6*(a/b*d^2)^(1/4)*2^(1/2)*a^2*d^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/
4))/(a/b*d^2)^(1/4))+8*(d*x)^(5/2)*a*b-24*(d*x)^(1/2)*a^2*d^2)/d*(b*x^2+a)/
b/a^2/((b*x^2+a)^2)^(3/2)
```

maxima [A] time = 3.22, size = 281, normalized size = 0.61

$$\frac{d^{\frac{3}{2}}x^{\frac{5}{2}}}{2\left(a^{2}bx^{2}+a^{3}+\left(ab^{2}x^{2}+a^{2}b\right)x^{2}\right)}-\frac{7\,bd^{\frac{3}{2}}x^{\frac{5}{2}}+3\,ad^{\frac{3}{2}}\sqrt{x}}{16\left(ab^{3}x^{4}+2\,a^{2}b^{2}x^{2}+a^{3}b\right)}+\frac{3\,d\left(\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{d}\,\mathrm{ar}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*d^(3/2)*x^(5/2)/(a^2*b*x^2 + a^3 + (a*b^2*x^2 + a^2*b)*x^2) - 1/16*(7*b
*d^(3/2)*x^(5/2) + 3*a*d^(3/2)*sqrt(x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
 + 3/128*d*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) +
 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
+ 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b
)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)
*sqrt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4
)*b^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)
*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{3/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**(3/2)/((a + b*x**2)**2)**(3/2), x)
```

3.764 $\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=460

$$\frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\sqrt{d}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}\right.}{64\sqrt{2}a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 5/16*(d*x)^(3/2)/a^2/d/((b*x^2+a)^2)^(1/2)+1/4*(d*x)^(3/2)/a/d/(b*x^2+a)/((
b*x^2+a)^2)^(1/2)-5/64*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/
4)/d^(1/2))*d^(1/2)/a^(9/4)/b^(3/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+5/64*(b*x^2
+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*d^(1/2)/a^(9/4)/b
^(3/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+5/128*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(
1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))*d^(1/2)/a^(9/4)/b^(3/4)*2
^(1/2)/((b*x^2+a)^2)^(1/2)-5/128*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(
1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))*d^(1/2)/a^(9/4)/b^(3/4)*2^(1/2)/(
(b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1112, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\sqrt{d}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}\right.}{64\sqrt{2}a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (5*(d*x)^(3/2))/(16*a^2*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*x)^(3/2)/(4
*a*d*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*Sqrt[d]*(a + b*x^2)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(9
/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*Sqrt[d]*(a + b*x^2)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(9/4)*b
^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*Sqrt[d]*(a + b*x^2)*Log[Sqrt[a
]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqr
t[2]*a^(9/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*Sqrt[d]*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(64*Sqrt[2]*a^(9/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5b\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^2}\,dx}{8a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5\left(ab+b^2x\right.\right.}{32a^2\sqrt{a^2+}}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5\left(ab+b^2x\right.\right.}{16a}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(5\left(ab+b^2x\right.\right.}{32a}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5\sqrt{d}\left(ab+\right.\right.}{}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{5\sqrt{d}\left(a+bx\right.}{6}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5\sqrt{d}\left(a+bx\right.}{32\sqrt{2}\,a^{9/4}b}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.12

$$\frac{2x\sqrt{dx}\left(a+bx^2\right)^3 {}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^3\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*x*Sqrt[d*x]*(a + b*x^2)^3*Hypergeometric2F1[3/4, 3, 7/4, -((b*x^2)/a)])/
(3*a^3*((a + b*x^2)^2)^(3/2))
```

fricas [A] time = 0.97, size = 304, normalized size = 0.66

$$-\frac{20\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)\left(-\frac{d^2}{a^9b^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{125\,\sqrt{dx}\,a^2bd\left(-\frac{d^2}{a^9b^3}\right)^{\frac{1}{4}}-\sqrt{-15625\,a^5bd^2\sqrt{-\frac{d^2}{a^9b^3}}+15625\,d^3x}\,a^2b\left(-\frac{d^2}{a^9b^3}\right)^{\frac{1}{4}}}{125\,d^2}\right)-5\left(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(20*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-d^2/(a^9*b^3))^(1/4)*arctan(-
1/125*(125*sqrt(d*x)*a^2*b*d*(-d^2/(a^9*b^3))^(1/4) - sqrt(-15625*a^5*b*d^2
*sqrt(-d^2/(a^9*b^3)) + 15625*d^3*x)*a^2*b*(-d^2/(a^9*b^3))^(1/4))/d^2) - 5
*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-d^2/(a^9*b^3))^(1/4)*log(125*a^7*b^2*(
-d^2/(a^9*b^3))^(3/4) + 125*sqrt(d*x)*d) + 5*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a
^4)*(-d^2/(a^9*b^3))^(1/4)*log(-125*a^7*b^2*(-d^2/(a^9*b^3))^(3/4) + 125*sq
rt(d*x)*d) - 4*(5*b*x^3 + 9*a*x)*sqrt(d*x))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^
4)
```

giac [A] time = 0.34, size = 368, normalized size = 0.80

$$\frac{\frac{8\left(5\,\sqrt{dx}\,bd^5x^3+9\,\sqrt{dx}\,ad^5x\right)}{\left(bd^2x^2+ad^2\right)^2a^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{10\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{10\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{5\,\sqrt{2}\left(a\right.}{}}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*(8*(5*sqrt(d*x)*b*d^5*x^3 + 9*sqrt(d*x)*a*d^5*x)/((b*d^2*x^2 + a*d^2)
^2*a^2*sgn(b*d^4*x^2 + a*d^4)) + 10*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sq
rt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^3*sgn
(b*d^4*x^2 + a*d^4)) + 10*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^3*sgn(b*d^4*x^
2 + a*d^4)) - 5*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)
```

```
*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^3*sgn(b*d^4*x^2 + a*d^4)) + 5*sqrt(2)*(a
*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b)
)/(a^3*b^3*sgn(b*d^4*x^2 + a*d^4)))/d
```

maple [B] time = 0.01, size = 617, normalized size = 1.34

$$\frac{\left(10\sqrt{2}\,b^2d^2x^4\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+10\sqrt{2}\,b^2d^2x^4\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+5\sqrt{2}\,b^2d^2x^4\ln\left(-\frac{-dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{d\ldots}}{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx\ldots}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/128*(5*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1
/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^4*b^2*d^2
+10*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x
^4*b^2*d^2+10*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2
)^(1/4))*x^4*b^2*d^2+40*(a/b*d^2)^(1/4)*(d*x)^(3/2)*x^2*b^2+10*2^(1/2)*ln(-
(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a*b*d^2+20*2^(1/2)*arctan((2
^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a*b*d^2+20*2^(1/2)
*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a*b*d^2+
72*(d*x)^(3/2)*a*b*(a/b*d^2)^(1/4)+5*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x
)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(
a/b*d^2)^(1/2)))*a^2*d^2+10*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(
1/4))/(a/b*d^2)^(1/4))*a^2*d^2+10*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*
d^2)^(1/4))/(a/b*d^2)^(1/4))*a^2*d^2)/d*(b*x^2+a)/(a/b*d^2)^(1/4)/b/a^2/((b
*x^2+a)^2)^(3/2)
```

maxima [A] time = 3.21, size = 265, normalized size = 0.58

$$\frac{\sqrt{d}\,x^{\frac{3}{2}}}{2\left(a^2bx^2+a^3+\left(ab^2x^2+a^2b\right)x^2\right)}+\frac{5\,b\sqrt{d}\,x^{\frac{7}{2}}+a\sqrt{d}\,x^{\frac{3}{2}}}{16\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)}+\frac{5\,\sqrt{d}\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\ldots}{\ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(d)*x^(3/2)/(a^2*b*x^2 + a^3 + (a*b^2*x^2 + a^2*b)*x^2) + 1/16*(5*b
*sqrt(d)*x^(7/2) + a*sqrt(d)*x^(3/2))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 5
/128*sqrt(d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqr
t(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqr
t(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*l
og(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)
))/a^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{\left(\left(a + bx^{2}\right)^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(sqrt(d*x)/((a + b*x**2)**2)**(3/2), x)
```

3.765 $\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=460

$$\frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\right.}{64\sqrt{2}a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -21/64*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(1
1/4)/b^(1/4)*2^(1/2)/d^(1/2)/((b*x^2+a)^2)^(1/2)+21/64*(b*x^2+a)*arctan(1+b
^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(11/4)/b^(1/4)*2^(1/2)/d^(1/2
)/((b*x^2+a)^2)^(1/2)-21/128*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)
-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(11/4)/b^(1/4)*2^(1/2)/d^(1/2)/((b*
x^2+a)^2)^(1/2)+21/128*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/
4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(11/4)/b^(1/4)*2^(1/2)/d^(1/2)/((b*x^2+a)
^2)^(1/2)+7/16*(d*x)^(1/2)/a^2/d/((b*x^2+a)^2)^(1/2)+1/4*(d*x)^(1/2)/a/d/(b
*x^2+a)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\right.}{64\sqrt{2}a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (7*Sqrt[d*x])/(16*a^2*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + Sqrt[d*x]/(4*a*d
*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (21*(a + b*x^2)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(11/4)*b^(1/4
)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (21*(a + b*x^2)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(11/4)*b^(1/4)*S
qrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (21*(a + b*x^2)*Log[Sqrt[a]*Sqrt[
d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^
(11/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (21*(a + b*x^2)*L
og[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]]
)/(64*Sqrt[2]*a^(11/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$=\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(7b\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^2}\,dx}{8a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$=\frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(21\left(ab+\right.\right.}{32a^2}$$

$$=\frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(21\left(ab+\right.\right.}{}$$

$$=\frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(21\left(ab+\right.\right.}{}$$

$$=\frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(21\left(ab+\right.\right.}{}$$

$$=\frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21\left(a+b\right.}{6}$$

$$=\frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21\left(a+\right.}{32\sqrt{2}\,a^{11}}$$

Mathematica [A] time = 0.19, size = 272, normalized size = 0.59

$$\frac{\sqrt{x}\left(a+bx^2\right)\left(56a^{3/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)+32a^{7/4}\sqrt[4]{b}\sqrt{x}-21\sqrt{2}\left(a+bx^2\right)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+2\right.}{128a^{1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[x]*(a + b*x^2)*(32*a^(7/4)*b^(1/4)*Sqrt[x] + 56*a^(3/4)*b^(1/4)*Sqrt[
x]*(a + b*x^2) - 42*Sqrt[2]*(a + b*x^2)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)] + 42*Sqrt[2]*(a + b*x^2)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)] - 21*Sqrt[2]*(a + b*x^2)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x] + 21*Sqrt[2]*(a + b*x^2)^2*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(128*a^(11/4)*b^(1/4)*Sqrt[d*x]*((a + b*
x^2)^2)^(3/2))
```

fricas [A] time = 1.08, size = 298, normalized size = 0.65

$$\frac{84\left(a^2b^2dx^4+2\,a^3bdx^2+a^4d\right)\left(-\frac{1}{a^{11}bd^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^6d^2\sqrt{-\frac{1}{a^{11}bd^2}}+dx}\,a^8bd\left(-\frac{1}{a^{11}bd^2}\right)^{\frac{3}{4}}-\sqrt{dx}\,a^8bd\left(-\frac{1}{a^{11}bd^2}\right)^{\frac{3}{4}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/64*(84*(a^2*b^2*d*x^4 + 2*a^3*b*d*x^2 + a^4*d)*(-1/(a^11*b*d^2))^(1/4)*ar
ctan(sqrt(a^6*d^2*sqrt(-1/(a^11*b*d^2)) + d*x)*a^8*b*d*(-1/(a^11*b*d^2))^(3
/4) - sqrt(d*x)*a^8*b*d*(-1/(a^11*b*d^2))^(3/4)) + 21*(a^2*b^2*d*x^4 + 2*a^
3*b*d*x^2 + a^4*d)*(-1/(a^11*b*d^2))^(1/4)*log(a^3*d*(-1/(a^11*b*d^2))^(1/4
) + sqrt(d*x)) - 21*(a^2*b^2*d*x^4 + 2*a^3*b*d*x^2 + a^4*d)*(-1/(a^11*b*d^2
))^(1/4)*log(-a^3*d*(-1/(a^11*b*d^2))^(1/4) + sqrt(d*x)) + 4*(7*b*x^2 + 11*
a)*sqrt(d*x))/(a^2*b^2*d*x^4 + 2*a^3*b*d*x^2 + a^4*d)
```

giac [A] time = 0.29, size = 374, normalized size = 0.81

$$\frac{7\sqrt{dx}\,bd^3x^2+11\sqrt{dx}\,ad^3}{16\left(bd^2x^2+ad^2\right)^2a^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3bd\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\right.}{64\,a^3bd\mathrm{sgn}\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(7*sqrt(d*x)*b*d^3*x^2 + 11*sqrt(d*x)*a*d^3)/((b*d^2*x^2 + a*d^2)^2*a^
2*sgn(b*d^4*x^2 + a*d^4)) + 21/64*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt
(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b*d*sgn(b
```

```
*d^4*x^2 + a*d^4)) + 21/64*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b*d*sgn(b*d^4*x
^2 + a*d^4)) + 21/128*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)
^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b*d*sgn(b*d^4*x^2 + a*d^4)) - 21/128
*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sq
rt(a*d^2/b))/(a^3*b*d*sgn(b*d^4*x^2 + a*d^4))
```

maple [B] time = 0.01, size = 638, normalized size = 1.39

$$\frac{\left(42\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\, b^2x^4\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+42\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\, b^2x^4\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+21\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\, b^2x^4 \ln\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x)
```

```
[Out] 1/128*(42*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*(a/
b*d^2)^(1/4)*2^(1/2)*x^4*b^2+42*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4)
)/(a/b*d^2)^(1/4))*(a/b*d^2)^(1/4)*2^(1/2)*x^4*b^2+21*(a/b*d^2)^(1/4)*2^(1/
2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d
^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^4*b^2+84*arctan((2^(1/2)*
(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*(a/b*d^2)^(1/4)*2^(1/2)*x^2*a
*b+84*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*(a/b*d^
2)^(1/4)*2^(1/2)*x^2*a*b+42*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)
*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a/b*d^2)^(1/2)))*x^2*a*b+42*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4
))/(a/b*d^2)^(1/4))*(a/b*d^2)^(1/4)*2^(1/2)*a^2+42*arctan((2^(1/2)*(d*x)^(1
/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*(a/b*d^2)^(1/4)*2^(1/2)*a^2+21*(a/b*d
^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/
2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*a^2+56*(d*x)
^(1/2)*x^2*a*b+88*(d*x)^(1/2)*a^2)/d*(b*x^2+a)/a^3/((b*x^2+a)^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b\sqrt{d}\,x^{\frac{5}{2}}}{2\left(a^3bdx^2+a^4d+\left(a^2b^2dx^2+a^3bd\right)x^2\right)}+\frac{15\,bx^{\frac{5}{2}}+11\,a\sqrt{x}}{16\left(a^2b^2\sqrt{d}\,x^4+2\,a^3b\sqrt{d}\,x^2+a^4\sqrt{d}\right)}-\frac{11\left(\frac{2\sqrt{2}\,\sqrt{d}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\cdots\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}\cdots}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\cdots}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/2*b*sqrt(d)*x^(5/2)/(a^3*b*d*x^2 + a^4*d + (a^2*b^2*d*x^2 + a^3*b*d)*x^2
) + 1/16*(15*b*x^(5/2) + 11*a*sqrt(x))/(a^2*b^2*sqrt(d)*x^4 + 2*a^3*b*sqrt(
d)*x^2 + a^4*sqrt(d)) - 11/128*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(
2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqr
t(sqrt(a)*sqrt(b))) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4
)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)
*sqrt(b))) + sqrt(2)*sqrt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*
x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/
4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a^2*d) + integrate(1/
((a^2*b*sqrt(d)*x^2 + a^3*sqrt(d))*sqrt(x)), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{d\,x}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*((a + b*x**2)**2)**(3/2)), x)
```

3.766 $\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=506

$$\frac{9}{16a^2d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)}-\frac{45\sqrt[4]{b}\,\left(a+bx^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}\right.}{64\sqrt{2}\,a^{13/4}d^{3/2}\sqrt{a^2+2abx}}$$

```
[Out] 45/64*b^(1/4)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2
))/a^(13/4)/d^(3/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)-45/64*b^(1/4)*(b*x^2+a)*arc
tan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(13/4)/d^(3/2)*2^(1/2)
/((b*x^2+a)^2)^(1/2)-45/128*b^(1/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*
d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(13/4)/d^(3/2)*2^(1/2)/((b*x
^2+a)^2)^(1/2)+45/128*b^(1/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2
)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(13/4)/d^(3/2)*2^(1/2)/((b*x^2+a)^
2)^(1/2)+9/16/a^2/d/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)+1/4/a/d/(b*x^2+a)/(d*x)
^(1/2)/((b*x^2+a)^2)^(1/2)-45/16*(b*x^2+a)/a^3/d/(d*x)^(1/2)/((b*x^2+a)^2)^
(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 506, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{45\sqrt[4]{b}\,\left(a+bx^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45\sqrt[4]{b}\,\left(a+bx^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\right.}{64\sqrt{2}\,a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 9/(16*a^2*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*d*Sqrt[d*x]
*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*(a + b*x^2))/(16*a^3*d*
Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*b^(1/4)*(a + b*x^2)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(13/4)*d
^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*b^(1/4)*(a + b*x^2)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(13/4)*d^(3
/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*b^(1/4)*(a + b*x^2)*Log[Sqrt[a]*
Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[
2]*a^(13/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*b^(1/4)*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(64*Sqrt[2]*a^(13/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(9b\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x\right.}}{8a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.10

$$-\frac{2x\left(a+bx^2\right)^3 {}_2F_1\left(-\frac{1}{4},3;\frac{3}{4};-\frac{bx^2}{a}\right)}{a^3(dx)^{3/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (-2*x*(a + b*x^2)^3*Hypergeometric2F1[-1/4, 3, 3/4, -((b*x^2)/a)])/(a^3*(d*
x)^(3/2)*((a + b*x^2)^2)^(3/2))
```

fricas [A] time = 0.95, size = 343, normalized size = 0.68

$$\frac{180\left(a^3b^2d^2x^5+2\,a^4bd^2x^3+a^5d^2x\right)\left(-\frac{b}{a^{13}d^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{91125\,\sqrt{dx}\,a^3bd\left(-\frac{b}{a^{13}d^6}\right)^{\frac{1}{4}}-\sqrt{-8303765625\,a^7bd^4\sqrt{-\frac{b}{a^{13}d^6}}+8303765625}}{91125\,b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/64*(180*(a^3*b^2*d^2*x^5 + 2*a^4*b*d^2*x^3 + a^5*d^2*x)*(-b/(a^13*d^6))^(
1/4)*arctan(-1/91125*(91125*sqrt(d*x)*a^3*b*d*(-b/(a^13*d^6))^(1/4) - sqrt(
-8303765625*a^7*b*d^4*sqrt(-b/(a^13*d^6)) + 8303765625*b^2*d*x)*a^3*d*(-b/(
a^13*d^6))^(1/4))/b) - 45*(a^3*b^2*d^2*x^5 + 2*a^4*b*d^2*x^3 + a^5*d^2*x)*(
-b/(a^13*d^6))^(1/4)*log(91125*a^10*d^5*(-b/(a^13*d^6))^(3/4) + 91125*sqrt(
d*x)*b) + 45*(a^3*b^2*d^2*x^5 + 2*a^4*b*d^2*x^3 + a^5*d^2*x)*(-b/(a^13*d^6)
)^(1/4)*log(-91125*a^10*d^5*(-b/(a^13*d^6))^(3/4) + 91125*sqrt(d*x)*b) - 4*
(45*b^2*x^4 + 81*a*b*x^2 + 32*a^2)*sqrt(d*x))/(a^3*b^2*d^2*x^5 + 2*a^4*b*d^
2*x^3 + a^5*d^2*x)
```

giac [A] time = 0.31, size = 410, normalized size = 0.81

$$-\frac{\frac{256}{\sqrt{dx}\,a^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{8\left(13\,\sqrt{dx}\,b^2d^3x^3+17\,\sqrt{dx}\,abd^3x\right)}{\left(bd^2x^2+ad^2\right)^2a^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{90\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^2d^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{90\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\ldots\right)}{a^4b^2d^2\mathrm{sgn}\left(bd^4x^2\ldots\right.}}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/128*(256/(sqrt(d*x)*a^3*sgn(b*d^4*x^2 + a*d^4)) + 8*(13*sqrt(d*x)*b^2*d^
3*x^3 + 17*sqrt(d*x)*a*b*d^3*x)/((b*d^2*x^2 + a*d^2)^2*a^3*sgn(b*d^4*x^2 +
```

```
a*d^4)) + 90*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b
)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^2*d^2*sgn(b*d^4*x^2 + a*d^4)
) + 90*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^2*d^2*sgn(b*d^4*x^2 + a*d^4)) - 4
5*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + s
qrt(a*d^2/b))/(a^4*b^2*d^2*sgn(b*d^4*x^2 + a*d^4)) + 45*sqrt(2)*(a*b^3*d^2)
^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^
2*d^2*sgn(b*d^4*x^2 + a*d^4)))/d
```

maple [A] time = 0.02, size = 645, normalized size = 1.27

$$-\frac{\left(90\sqrt{2}\,\sqrt{dx}\,b^2x^4\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)+90\sqrt{2}\,\sqrt{dx}\,b^2x^4\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)+45\sqrt{2}\,\sqrt{dx}\,b^2x^4\ln\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/128/d*(45*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2
)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*(d*x)^(
1/2)*x^4*b^2+90*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d
^2)^(1/4))*(d*x)^(1/2)*x^4*b^2+90*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*
d^2)^(1/4))/(a/b*d^2)^(1/4))*(d*x)^(1/2)*x^4*b^2+360*(a/b*d^2)^(1/4)*x^4*b^
2+90*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))
/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*(d*x)^(1/2)*x^2
*a*b+180*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/
4))*(d*x)^(1/2)*x^2*a*b+180*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(
1/4))/(a/b*d^2)^(1/4))*(d*x)^(1/2)*x^2*a*b+648*(a/b*d^2)^(1/4)*x^2*a*b+45*2
^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+
(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*(d*x)^(1/2)*a^2+90*2^
(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*(d*x)^(
1/2)*a^2+90*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^
(1/4))*(d*x)^(1/2)*a^2+256*(a/b*d^2)^(1/4)*a^2)*(b*x^2+a)/(d*x)^(1/2)/(a/b*
d^2)^(1/4)/a^3/((b*x^2+a)^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bx^{\frac{3}{2}}}{2\left(a^3bd^{\frac{3}{2}}x^2+a^4d^{\frac{3}{2}}+\left(a^2b^2d^{\frac{3}{2}}x^2+a^3bd^{\frac{3}{2}}\right)x^2\right)}-\frac{13\,b^2x^{\frac{7}{2}}+9\,abx^{\frac{3}{2}}}{16\left(a^3b^2d^{\frac{3}{2}}x^4+2\,a^4bd^{\frac{3}{2}}x^2+a^5d^{\frac{3}{2}}\right)}-\frac{13\,b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{\cdots}\right.}{2\sqrt{\sqrt{a}\sqrt{b}}}\right.}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/2*b*x^(3/2)/(a^3*b*d^(3/2)*x^2 + a^4*d^(3/2) + (a^2*b^2*d^(3/2)*x^2 + a^
3*b*d^(3/2))*x^2) - 1/16*(13*b^2*x^(7/2) + 9*a*b*x^(3/2))/(a^3*b^2*d^(3/2)*
x^4 + 2*a^4*b*d^(3/2)*x^2 + a^5*d^(3/2)) - 13/128*b*(2*sqrt(2)*arctan(1/2*s
qrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))
/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a
^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sq
rt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x +
 sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
+ sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^3*d^(3/2)) + integrate(1/((a^2
*b*d^(3/2)*x^2 + a^3*d^(3/2))*x^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{(d\,x)}^{3/2}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
[Out] Integral(1/((d*x)**(3/2)*((a + b*x**2)**2)**(3/2)), x)
```

3.767 $\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=506

$$\frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)}+\frac{77b^{3/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{\right.}{64\sqrt{2}\,a^{15/4}d^{5/2}\sqrt{a^2+2}$$

```
[Out] 11/16/a^2/d/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2)+1/4/a/d/(d*x)^(3/2)/(b*x^2+a)/(
(b*x^2+a)^2)^(1/2)-77/48*(b*x^2+a)/a^3/d/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2)+77
/64*b^(3/4)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))
/a^(15/4)/d^(5/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)-77/64*b^(3/4)*(b*x^2+a)*arcta
n(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(15/4)/d^(5/2)*2^(1/2)/(
(b*x^2+a)^2)^(1/2)+77/128*b^(3/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^
(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(15/4)/d^(5/2)*2^(1/2)/((b*x^2
+a)^2)^(1/2)-77/128*b^(3/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+
a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(15/4)/d^(5/2)*2^(1/2)/((b*x^2+a)^2)
^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 506, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{77b^{3/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77b^{3/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\right.}{64\sqrt{2}\,a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 11/(16*a^2*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*d*(d*x)^
(3/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*(a + b*x^2))/(48*a
^3*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*b^(3/4)*(a + b*x^2)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(
15/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*b^(3/4)*(a + b*x^2)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(15/
4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*b^(3/4)*(a + b*x^2)*Log[S
qrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(6
4*Sqrt[2]*a^(15/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*b^(3/4)*(
a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(64*Sqrt[2]*a^(15/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(11b\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{5/2}\left(ab\cdots\right.}}{8a\sqrt{a^2+2abx^2+b^2x\cdots}}$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+\cdots}}$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+\cdots}}$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+\cdots}}$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+\cdots}}$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+\cdots}}$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+\cdots}}$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+\cdots}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.11

$$-\frac{2x\left(a+bx^2\right)^3 {}_2F_1\left(-\frac{3}{4},3;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^3(dx)^{5/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (-2*x*(a + b*x^2)^3*Hypergeometric2F1[-3/4, 3, 1/4, -((b*x^2)/a)])/(3*a^3*(
d*x)^(5/2)*((a + b*x^2)^2)^(3/2))
```

fricas [A] time = 0.82, size = 367, normalized size = 0.73

$$-\frac{924\left(a^3b^2d^3x^6+2\,a^4bd^3x^4+a^5d^3x^2\right)\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^{11}bd^7\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{3}{4}}-\sqrt{a^8d^6\sqrt{-\frac{b^3}{a^{15}d^{10}}}+b^2dx}\,a^{11}d^7\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{3}{4}}}{b^3}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/192*(924*(a^3*b^2*d^3*x^6 + 2*a^4*b*d^3*x^4 + a^5*d^3*x^2)*(-b^3/(a^15*d
^10))^(1/4)*arctan(-(sqrt(d*x)*a^11*b*d^7*(-b^3/(a^15*d^10))^(3/4) - sqrt(a
^8*d^6*sqrt(-b^3/(a^15*d^10)) + b^2*d*x)*a^11*d^7*(-b^3/(a^15*d^10))^(3/4))
/b^3) + 231*(a^3*b^2*d^3*x^6 + 2*a^4*b*d^3*x^4 + a^5*d^3*x^2)*(-b^3/(a^15*d
^10))^(1/4)*log(77*a^4*d^3*(-b^3/(a^15*d^10))^(1/4) + 77*sqrt(d*x)*b) - 231
*(a^3*b^2*d^3*x^6 + 2*a^4*b*d^3*x^4 + a^5*d^3*x^2)*(-b^3/(a^15*d^10))^(1/4)
*log(-77*a^4*d^3*(-b^3/(a^15*d^10))^(1/4) + 77*sqrt(d*x)*b) + 4*(77*b^2*x^4
 + 121*a*b*x^2 + 32*a^2)*sqrt(d*x))/(a^3*b^2*d^3*x^6 + 2*a^4*b*d^3*x^4 + a^
5*d^3*x^2)
```

giac [A] time = 0.35, size = 401, normalized size = 0.79

$$-\frac{15\sqrt{dx}\,b^2d^2x^2+19\sqrt{dx}\,abd^2}{16\left(bd^2x^2+ad^2\right)^2a^3d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4d^3\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan}{64\,a^4d^3\,\mathrm{sgn}\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/16*(15*sqrt(d*x)*b^2*d^2*x^2 + 19*sqrt(d*x)*a*b*d^2)/((b*d^2*x^2 + a*d^2
)^2*a^3*d*sgn(b*d^4*x^2 + a*d^4)) - 77/64*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(
1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*d
^3*sgn(b*d^4*x^2 + a*d^4)) - 77/64*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sq
rt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*d^3*sgn
(b*d^4*x^2 + a*d^4)) - 77/128*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(
a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*d^3*sgn(b*d^4*x^2 + a*d^4))
+ 77/128*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d
*x) + sqrt(a*d^2/b))/(a^4*d^3*sgn(b*d^4*x^2 + a*d^4)) - 2/3/(sqrt(d*x)*a^3*
d^2*x*sgn(b*d^4*x^2 + a*d^4))
```

maple [B] time = 0.02, size = 707, normalized size = 1.40

$$-\frac{\left(616 a b^{2} d^{2} x^{4}+462 (dx)^{\frac{3}{2}} \left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{2}\, b^{3} x^{4} \arctan\left(\frac{\sqrt{2}\, \sqrt{dx}-\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}}\right)+462 (dx)^{\frac{3}{2}} \left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{2}\, b^{3} x^{4} \arctan\left(\frac{\sqrt{2}\, \sqrt{dx} \cdots}{\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/384/d^3*(231*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)
*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a/b*d^2)^(1/2)))*x^4*b^3+462*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*b^3+462*(d*x)^
(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/
(a/b*d^2)^(1/4))*x^4*b^3+462*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a
/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a*b^2+924*(d*x)^(3/2)*(a/b*d^2)^(1/4
)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2
*a*b^2+924*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-
(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a*b^2+231*(d*x)^(3/2)*(a/b*d^2)^(1/4)
*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-
(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*a^2*b+462*(d*x)^(3/2)
*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*
d^2)^(1/4))*a^2*b+462*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(
d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^2*b+616*a*d^2*b^2*x^4+968*x^
2*a^2*b*d^2+256*a^3*d^2)*(b*x^2+a)/(d*x)^(3/2)/a^4/((b*x^2+a)^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b^2x^{\frac{5}{2}}}{2\left(a^4bd^{\frac{5}{2}}x^2+a^5d^{\frac{5}{2}}+\left(a^3b^2d^{\frac{5}{2}}x^2+a^4bd^{\frac{5}{2}}\right)x^2\right)}-2\,b\int\frac{1}{\left(a^3bd^{\frac{5}{2}}x^2+a^4d^{\frac{5}{2}}\right)\sqrt{x}}\,dx-\frac{23\,b^2x^{\frac{5}{2}}+19\,ab\sqrt{x}}{16\left(a^3b^2d^{\frac{5}{2}}x^4+2\,a^4bd^{\frac{5}{2}}x^2+a^5d^{\frac{5}{2}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] 1/2*b^2*x^(5/2)/(a^4*b*d^(5/2)*x^2 + a^5*d^(5/2) + (a^3*b^2*d^(5/2)*x^2 + a
^4*b*d^(5/2))*x^2) - 2*b*integrate(1/((a^3*b*d^(5/2)*x^2 + a^4*d^(5/2))*sqr
t(x)), x) - 1/16*(23*b^2*x^(5/2) + 19*a*b*sqrt(x))/(a^3*b^2*d^(5/2)*x^4 + 2
*a^4*b*d^(5/2)*x^2 + a^5*d^(5/2)) + 19/128*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*
(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(
a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4
)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)
*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*
x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
 + sqrt(b)*x + sqrt(a))/a^(3/4))/(a^3*d^(5/2)) + integrate(1/((a^2*b*d^(5/2
)*x^2 + a^3*d^(5/2))*x^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{1}{{\left(d\,x\right)}^{5/2}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(dx)^{\frac{5}{2}}\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/((d*x)**(5/2)*((a + b*x**2)**2)**(3/2)), x)
```

3.768 $\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=553

$$\frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117b^{5/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{64\sqrt{2}\,a^{17/4}d^{7/2}\sqrt{a^2+}}$$

```
[Out] 13/16/a^2/d/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2)+1/4/a/d/(d*x)^(5/2)/(b*x^2+a)/(
(b*x^2+a)^2)^(1/2)-117/80*(b*x^2+a)/a^3/d/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2)-1
17/64*b^(5/4)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2
))/a^(17/4)/d^(7/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)+117/64*b^(5/4)*(b*x^2+a)*ar
ctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(17/4)/d^(7/2)*2^(1/2
)/((b*x^2+a)^2)^(1/2)+117/128*b^(5/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2
)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(17/4)/d^(7/2)*2^(1/2)/((b
*x^2+a)^2)^(1/2)-117/128*b^(5/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(
1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(17/4)/d^(7/2)*2^(1/2)/((b*x^2+
a)^2)^(1/2)+117/16*b*(b*x^2+a)/a^4/d^3/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1112, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{117b\left(a+bx^2\right)}{16a^4d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117b^{5/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{17/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117b^{5/4}\left(a+bx^2\right.}{64}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 13/(16*a^2*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*d*(d*x)^
(5/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*(a + b*x^2))/(80*
a^3*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*b*(a + b*x^2))/(1
6*a^4*d^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*b^(5/4)*(a + b*
x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]
*a^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*b^(5/4)*(a + b*x^
2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a
^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*b^(5/4)*(a + b*x^2)
*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x
]])/(64*Sqrt[2]*a^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*b^
(5/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)
```

```
*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(13b\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{7/2}\left(ab\cdots\right.}}{8a\sqrt{a^2+2abx^2+b^2x\cdots}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+\cdots}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+\cdots}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+\cdots}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+\cdots}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+\cdots}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+\cdots}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+\cdots}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+\cdots}}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.10

$$-\frac{2x\left(a+bx^2\right)^3 {}_2F_1\left(-\frac{5}{4},3;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^3(dx)^{7/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (-2*x*(a + b*x^2)^3*Hypergeometric2F1[-5/4, 3, -1/4, -((b*x^2)/a)])/(5*a^3*
(d*x)^(7/2)*((a + b*x^2)^2)^(3/2))
```

fricas [A] time = 1.19, size = 390, normalized size = 0.71

$$-\frac{2340\left(a^4b^2d^4x^7+2\,a^5bd^4x^5+a^6d^4x^3\right)\left(-\frac{b^5}{a^{17}d^{14}}\right)^{\frac{1}{4}}\arctan\left(-\frac{1601613\,\sqrt{dx}\,a^4b^4d^3\left(-\frac{b^5}{a^{17}d^{14}}\right)^{\frac{1}{4}}-\sqrt{-2565164201769\,a^9b^5d^8\sqrt{-\frac{b^5}{a^{17}d^{14}}}}}{1601613\,b^5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/320*(2340*(a^4*b^2*d^4*x^7 + 2*a^5*b*d^4*x^5 + a^6*d^4*x^3)*(-b^5/(a^17*
d^14))^(1/4)*arctan(-1/1601613*(1601613*sqrt(d*x)*a^4*b^4*d^3*(-b^5/(a^17*d
^14))^(1/4) - sqrt(-2565164201769*a^9*b^5*d^8*sqrt(-b^5/(a^17*d^14)) + 2565
164201769*b^8*d*x)*a^4*d^3*(-b^5/(a^17*d^14))^(1/4))/b^5) - 585*(a^4*b^2*d^
4*x^7 + 2*a^5*b*d^4*x^5 + a^6*d^4*x^3)*(-b^5/(a^17*d^14))^(1/4)*log(1601613
*a^13*d^11*(-b^5/(a^17*d^14))^(3/4) + 1601613*sqrt(d*x)*b^4) + 585*(a^4*b^2
*d^4*x^7 + 2*a^5*b*d^4*x^5 + a^6*d^4*x^3)*(-b^5/(a^17*d^14))^(1/4)*log(-160
1613*a^13*d^11*(-b^5/(a^17*d^14))^(3/4) + 1601613*sqrt(d*x)*b^4) - 4*(585*b
^3*x^6 + 1053*a*b^2*x^4 + 416*a^2*b*x^2 - 32*a^3)*sqrt(d*x))/(a^4*b^2*d^4*x
^7 + 2*a^5*b*d^4*x^5 + a^6*d^4*x^3)
```

giac [A] time = 0.34, size = 432, normalized size = 0.78

$$\frac{21\,\sqrt{dx}\,b^3d^3x^3+25\,\sqrt{dx}\,ab^2d^3x}{16\left(bd^2x^2+ad^2\right)^2a^4d^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{117\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^5bd^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{117\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\mathrm{arcta}}{64\,a^5bd^5\mathrm{sgr}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*(21*sqrt(d*x)*b^3*d^3*x^3 + 25*sqrt(d*x)*a*b^2*d^3*x)/((b*d^2*x^2 + a*
d^2)^2*a^4*d^3*sgn(b*d^4*x^2 + a*d^4)) + 117/64*sqrt(2)*(a*b^3*d^2)^(3/4)*a
rctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/
(a^5*b*d^5*sgn(b*d^4*x^2 + a*d^4)) + 117/64*sqrt(2)*(a*b^3*d^2)^(3/4)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^
5*b*d^5*sgn(b*d^4*x^2 + a*d^4)) - 117/128*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x
 + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b*d^5*sgn(b*d^4*
x^2 + a*d^4)) + 117/128*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/
b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b*d^5*sgn(b*d^4*x^2 + a*d^4)) + 2/
5*(15*b*d^2*x^2 - a*d^2)/(sqrt(d*x)*a^4*d^5*x^2*sgn(b*d^4*x^2 + a*d^4))
```

maple [A] time = 0.03, size = 687, normalized size = 1.24

$$\left(4680\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} b^3 d^2 x^6 + 8424\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} a b^2 d^2 x^4 + 1170\sqrt{2}\ (dx)^{\frac{5}{2}} b^3 x^4 \arctan\left(\frac{\sqrt{2}\ \sqrt{dx}-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right) + 1170\sqrt{2}\ (dx)^{\frac{5}{2}} b^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/640/d^3*(585*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d
^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*(d*x)
^(5/2)*x^4*b^3+1170*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a
/b*d^2)^(1/4))*(d*x)^(5/2)*x^4*b^3+1170*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)
-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*(d*x)^(5/2)*x^4*b^3+4680*(a/b*d^2)^(1/4)
*x^6*b^3*d^2+1170*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/
b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*(d
*x)^(5/2)*x^2*a*b^2+2340*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4
))/(a/b*d^2)^(1/4))*(d*x)^(5/2)*x^2*a*b^2+2340*2^(1/2)*arctan((2^(1/2)*(d*x
)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*(d*x)^(5/2)*x^2*a*b^2+8424*(a/b*d
^2)^(1/4)*x^4*a*b^2*d^2+585*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2
^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^
(1/2)))*(d*x)^(5/2)*a^2*b+1170*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2
)^(1/4))/(a/b*d^2)^(1/4))*(d*x)^(5/2)*a^2*b+1170*2^(1/2)*arctan((2^(1/2)*(d
*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*(d*x)^(5/2)*a^2*b+3328*(a/b*d^2
)^(1/4)*x^2*a^2*b*d^2-256*(a/b*d^2)^(1/4)*a^3*d^2)*(b*x^2+a)/(d*x)^(5/2)/(a
/b*d^2)^(1/4)/a^4/((b*x^2+a)^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b^2x^{\frac{3}{2}}}{2\left(a^4bd^{\frac{7}{2}}x^2+a^5d^{\frac{7}{2}}+\left(a^3b^2d^{\frac{7}{2}}x^2+a^4bd^{\frac{7}{2}}\right)x^2\right)}-2\,b\int\frac{1}{\left(a^3bd^{\frac{7}{2}}x^2+a^4d^{\frac{7}{2}}\right)x^{\frac{3}{2}}}\,dx+\frac{21\,b^3x^{\frac{7}{2}}+17\,ab^2x^{\frac{3}{2}}}{16\left(a^4b^2d^{\frac{7}{2}}x^4+2\,a^5bd^{\frac{7}{2}}x^2+a^6d^{\frac{7}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] 1/2*b^2*x^(3/2)/(a^4*b*d^(7/2)*x^2 + a^5*d^(7/2) + (a^3*b^2*d^(7/2)*x^2 + a
^4*b*d^(7/2))*x^2) - 2*b*integrate(1/((a^3*b*d^(7/2)*x^2 + a^4*d^(7/2))*x^(
3/2)), x) + 1/16*(21*b^3*x^(7/2) + 17*a*b^2*x^(3/2))/(a^4*b^2*d^(7/2)*x^4 +
 2*a^5*b*d^(7/2)*x^2 + a^6*d^(7/2)) + 21/128*b^2*(2*sqrt(2)*arctan(1/2*sqrt
(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(s
qrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1
/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(
b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sq
rt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + s
qrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^4*d^(7/2)) + integrate(1/((a^2*b*
d^(7/2)*x^2 + a^3*d^(7/2))*x^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(d\,x\right)}^{7/2}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx\right)^{\frac{7}{2}} \left(\left(a + bx^{2}\right)^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/((d*x)**(7/2)*((a + b*x**2)**2)**(3/2)), x)
```

3.769 $\int \frac{(dx)^{23/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=647

$$-\frac{7d^3(dx)^{17/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923ad^{11}\sqrt{dx}\left(a+bx^2\right)}{1024b^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923}{5120b^5}$$

```
[Out] -1547/1024*d^7*(d*x)^(9/2)/b^4/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(21/2)/b/(b*
x^2+a)^3/((b*x^2+a)^2)^(1/2)-7/32*d^3*(d*x)^(17/2)/b^2/(b*x^2+a)^2/((b*x^2+
a)^2)^(1/2)-119/256*d^5*(d*x)^(13/2)/b^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+1392
3/5120*d^9*(d*x)^(5/2)*(b*x^2+a)/b^5/((b*x^2+a)^2)^(1/2)-13923/4096*a^(5/4)
*d^(23/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b
^(25/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+13923/4096*a^(5/4)*d^(23/2)*(b*x^2+a)*a
rctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(25/4)*2^(1/2)/((b*x
^2+a)^2)^(1/2)-13923/8192*a^(5/4)*d^(23/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b
^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(25/4)*2^(1/2)/((b*x^
2+a)^2)^(1/2)+13923/8192*a^(5/4)*d^(23/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^
(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(25/4)*2^(1/2)/((b*x^2
+a)^2)^(1/2)-13923/1024*a*d^11*(b*x^2+a)*(d*x)^(1/2)/b^6/((b*x^2+a)^2)^(1/2
)
```

Rubi [A] time = 0.51, antiderivative size = 647, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{13923ad^{11}\sqrt{dx}\left(a+bx^2\right)}{1024b^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923d^9(dx)^{5/2}\left(a+bx^2\right)}{5120b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{119d^5(dx)^{13/2}}{256b^3\left(a+bx^2\right)\sqrt{a^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-1547*d^7*(d*x)^(9/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*
x)^(21/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7*d^3*(d*
x)^(17/2))/(32*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (119*d^
5*(d*x)^(13/2))/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13
923*a*d^11*Sqrt[d*x]*(a + b*x^2))/(1024*b^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) + (13923*d^9*(d*x)^(5/2)*(a + b*x^2))/(5120*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]) - (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*S
qrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(25/4)*Sqrt[a^2 + 2*a*b*x^2 +
```

```
b^2*x^4]) + (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(25/4)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4]) - (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + S
qrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*b^(25/
4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*L
og[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]]
)/(4096*Sqrt[2]*b^(25/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{23/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{23/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{19/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx)^{17/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \cdots$$

$$= -\frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx)^{17/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \cdots$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{32b^2\left(a+\cdots\right.}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{32b^2\left(a+\cdots\right.}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{32b^2\left(a+\cdots\right.}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{32b^2\left(a+\cdots\right.}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{32b^2\left(a+\cdots\right.}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{32b^2\left(a+\cdots\right.}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{32b^2\left(a+\cdots\right.}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \cdots$$

Mathematica [A] time = 0.30, size = 401, normalized size = 0.62

$$(dx)^{23/2}\left(a+bx^2\right)\left(-765765\sqrt{2}\,a^{5/4}\left(a+bx^2\right)^4\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+765765\sqrt{2}\,a^{5/4}\left(a+bx^2\right)^4\log\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(23/2)*(a + b*x^2)*(-10183680*a^5*b^(1/4)*Sqrt[x] - 32587776*a^4*b^(
5/4)*x^(5/2) - 39829504*a^3*b^(9/4)*x^(9/2) - 21446656*a^2*b^(13/4)*x^(13/2
) - 3784704*a*b^(17/4)*x^(17/2) + 180224*b^(21/4)*x^(21/2) + 848640*a^4*b^(
1/4)*Sqrt[x]*(a + b*x^2) + 1166880*a^3*b^(1/4)*Sqrt[x]*(a + b*x^2)^2 + 2042
040*a^2*b^(1/4)*Sqrt[x]*(a + b*x^2)^3 - 1531530*Sqrt[2]*a^(5/4)*(a + b*x^2)
^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 1531530*Sqrt[2]*a^(5/4)*
(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 765765*Sqrt[2
]*a^(5/4)*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x] + 765765*Sqrt[2]*a^(5/4)*(a + b*x^2)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(450560*b^(25/4)*x^(23/2)*((a + b*x^2)^2)^
(5/2))
```

fricas [A] time = 1.12, size = 457, normalized size = 0.71

$$278460\left(-\frac{a^5d^{46}}{b^{25}}\right)^{\frac{1}{4}}\left(b^{10}x^8+4\,ab^9x^6+6\,a^2b^8x^4+4\,a^3b^7x^2+a^4b^6\right)\arctan\left(-\frac{\left(-\frac{a^5d^{46}}{b^{25}}\right)^{\frac{3}{4}}\sqrt{dx}\,ab^{19}d^{11}-\left(-\frac{a^5d^{46}}{b^{25}}\right)^{\frac{3}{4}}\sqrt{a^2d^{23}x+\sqrt{}}}{a^5d^{46}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20480*(278460*(-a^5*d^46/b^25)^(1/4)*(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*
x^4 + 4*a^3*b^7*x^2 + a^4*b^6)*arctan(-((-a^5*d^46/b^25)^(3/4)*sqrt(d*x)*a*
b^19*d^11 - (-a^5*d^46/b^25)^(3/4)*sqrt(a^2*d^23*x + sqrt(-a^5*d^46/b^25)*b
^12)*b^19)/(a^5*d^46)) + 69615*(-a^5*d^46/b^25)^(1/4)*(b^10*x^8 + 4*a*b^9*x
^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)*log(13923*sqrt(d*x)*a*d^11 +
13923*(-a^5*d^46/b^25)^(1/4)*b^6) - 69615*(-a^5*d^46/b^25)^(1/4)*(b^10*x^8
+ 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)*log(13923*sqrt(d*x
)*a*d^11 - 13923*(-a^5*d^46/b^25)^(1/4)*b^6) + 4*(2048*b^5*d^11*x^10 - 4300
8*a*b^4*d^11*x^8 - 220507*a^2*b^3*d^11*x^6 - 369733*a^3*b^2*d^11*x^4 - 2645
37*a^4*b*d^11*x^2 - 69615*a^5*d^11)*sqrt(d*x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*
a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)
```

giac [A] time = 0.45, size = 457, normalized size = 0.71

$$\frac{1}{40960}\, d^{11} \left(\frac{139230\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}} a \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7 \operatorname{sgn}\left(bd^4x^2+ad^4\right)} + \frac{139230\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}} a \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}} \cdots\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7 \operatorname{sgn}\left(bd^4x^2+ad^4\right)} \cdots \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/40960*d^11*(139230*sqrt(2)*(a*b^3*d^2)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2
)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^7*sgn(b*d^4*x^2 + a*d^
4)) + 139230*sqrt(2)*(a*b^3*d^2)^(1/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^
2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^7*sgn(b*d^4*x^2 + a*d^4)) + 6
9615*sqrt(2)*(a*b^3*d^2)^(1/4)*a*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x
) + sqrt(a*d^2/b))/(b^7*sgn(b*d^4*x^2 + a*d^4)) - 69615*sqrt(2)*(a*b^3*d^2)
^(1/4)*a*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^7*
sgn(b*d^4*x^2 + a*d^4)) - 40*(5599*sqrt(d*x)*a^2*b^3*d^8*x^6 + 14145*sqrt(d
*x)*a^3*b^2*d^8*x^4 + 12357*sqrt(d*x)*a^4*b*d^8*x^2 + 3683*sqrt(d*x)*a^5*d^
8)/((b*d^2*x^2 + a*d^2)^4*b^6*sgn(b*d^4*x^2 + a*d^4)) + 16384*(sqrt(d*x)*b^
20*d^10*x^2 - 25*sqrt(d*x)*a*b^19*d^10)/(b^25*d^10*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.03, size = 1287, normalized size = 1.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/40960*(477896*(d*x)^(5/2)*a^4*b*d^4+565800*(d*x)^(9/2)*a^3*b^2*d^2-16384
*(d*x)^(5/2)*x^8*b^5*d^4-65536*(d*x)^(5/2)*x^6*a*b^4*d^4+409600*(d*x)^(1/2)
*x^8*a*b^4*d^6-98304*(d*x)^(5/2)*x^4*a^2*b^3*d^4+1638400*(d*x)^(1/2)*x^6*a^
2*b^3*d^6-65536*(d*x)^(5/2)*x^2*a^3*b^2*d^4+2457600*(d*x)^(1/2)*x^4*a^3*b^2
*d^6+1638400*(d*x)^(1/2)*x^2*a^4*b*d^6-69615*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*
x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)
*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*a^5*d^6-139230*(a/b*d^2)^(1/4)*2^(1/
2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^5*d^6-13
9230*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(
```

```
a/b*d^2)^(1/4))*a^5*d^6-556920*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x
)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a^4*b*d^6-69615*(a/b*d^2)^(1/
4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*
x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^8*a*b^4*d^6-13923
0*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b
*d^2)^(1/4))*x^8*a*b^4*d^6-139230*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(
d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^8*a*b^4*d^6-278460*(a/b*d^2)
^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))
/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^6*a^2*b^3*d^6
-556920*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4)
)/(a/b*d^2)^(1/4))*x^6*a^2*b^3*d^6-556920*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2
^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^6*a^2*b^3*d^6-417690
*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d
^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^4*a
^3*b^2*d^6-835380*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*
d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a^3*b^2*d^6-278460*(a/b*d^2)^(1/4)*2^(1/2)
*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a^4*b*d^6-556920*(a/b*d^2
)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4
))*x^2*a^4*b*d^6-835380*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)
+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a^3*b^2*d^6+223960*(d*x)^(13/2)*a^2*
b^3+556920*(d*x)^(1/2)*a^5*d^6)*d^5*(b*x^2+a)/b^6/((b*x^2+a)^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-4\,ad^{\frac{23}{2}}\int\frac{x^{\frac{3}{2}}}{b^6x^2+ab^5}\,dx+d^{\frac{23}{2}}\int\frac{x^{\frac{7}{2}}}{b^5x^2+ab^4}\,dx+\frac{3683\left(\frac{2\sqrt{2}a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}a^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-\cdots\right)}{2\sqrt{\sqrt{a}\cdots}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -4*a*d^(23/2)*integrate(x^(3/2)/(b^6*x^2 + a*b^5), x) + d^(23/2)*integrate(
x^(7/2)/(b^5*x^2 + a*b^4), x) + 3683/8192*(2*sqrt(2)*a^(3/2)*arctan(1/2*sqr
t(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/s
qrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*a^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/
4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)
) + sqrt(2)*a^(5/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(
a))/b^(1/4) - sqrt(2)*a^(5/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b
```

```
)*x + sqrt(a))/b^(1/4))*d^(23/2)/b^6 - 1/3072*(6925*a^2*b^3*d^(23/2)*x^(13/
2) + 23395*a^3*b^2*d^(23/2)*x^(9/2) + 27135*a^4*b*d^(23/2)*x^(5/2) + 11049*
a^5*d^(23/2)*sqrt(x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x
^2 + a^4*b^6) - 1/192*((617*a^2*b^4*d^(23/2)*x^5 + 1386*a^3*b^3*d^(23/2)*x^
3 + 801*a^4*b^2*d^(23/2)*x)*x^(11/2) + 2*(519*a^3*b^3*d^(23/2)*x^5 + 1182*a
^4*b^2*d^(23/2)*x^3 + 695*a^5*b*d^(23/2)*x)*x^(7/2) + (453*a^4*b^2*d^(23/2)
*x^5 + 1042*a^5*b*d^(23/2)*x^3 + 621*a^6*d^(23/2)*x)*x^(3/2))/(a^3*b^8*x^6
+ 3*a^4*b^7*x^4 + 3*a^5*b^6*x^2 + a^6*b^5 + (b^11*x^6 + 3*a*b^10*x^4 + 3*a^
2*b^9*x^2 + a^3*b^8)*x^6 + 3*(a*b^10*x^6 + 3*a^2*b^9*x^4 + 3*a^3*b^8*x^2 +
a^4*b^7)*x^4 + 3*(a^2*b^9*x^6 + 3*a^3*b^8*x^4 + 3*a^4*b^7*x^2 + a^5*b^6)*x^
2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{23/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(23/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((d*x)^(23/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(23/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Timed out
```

3.770 $\int \frac{(dx)^{21/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=600

$$-\frac{19d^3(dx)^{15/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7315d^9(dx)^{3/2}\left(a+bx^2\right)}{3072b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1}{1024b^4}$$

```
[Out] -1045/1024*d^7*(d*x)^(7/2)/b^4/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(19/2)/b/(b*
x^2+a)^3/((b*x^2+a)^2)^(1/2)-19/96*d^3*(d*x)^(15/2)/b^2/(b*x^2+a)^2/((b*x^2
+a)^2)^(1/2)-95/256*d^5*(d*x)^(11/2)/b^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+7315
/3072*d^9*(d*x)^(3/2)*(b*x^2+a)/b^5/((b*x^2+a)^2)^(1/2)+7315/4096*a^(3/4)*d
^(21/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(
23/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-7315/4096*a^(3/4)*d^(21/2)*(b*x^2+a)*arct
an(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(23/4)*2^(1/2)/((b*x^2+
a)^2)^(1/2)-7315/8192*a^(3/4)*d^(21/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/
2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(23/4)*2^(1/2)/((b*x^2+a)
^2)^(1/2)+7315/8192*a^(3/4)*d^(21/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)
*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(23/4)*2^(1/2)/((b*x^2+a)^2
)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 600, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{7315d^9(dx)^{3/2}\left(a+bx^2\right)}{3072b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{95d^5(dx)^{11/2}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{19d}{96b^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-1045*d^7*(d*x)^(7/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*
x)^(19/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (19*d^3*(d
*x)^(15/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (95*d^
5*(d*x)^(11/2))/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (73
15*d^9*(d*x)^(3/2)*(a + b*x^2))/(3072*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
+ (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])
/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(23/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) - (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x
])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(23/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
```

```
4]) - (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt
[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*b^(23/4)*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4]) + (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*Log[Sqrt[a]*S
qrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt
[2]*b^(23/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{21/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{21/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(19b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{17/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{19d^3(dx)^{15/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \cdots$$

$$= -\frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{19d^3(dx)^{15/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{2\cdots}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

Mathematica [C] time = 0.05, size = 110, normalized size = 0.18

$$-\frac{2d^9(dx)^{3/2}\left(-1463a^4-2717a^3bx^2-2223a^2b^2x^4-741ab^3x^6+1463\left(a+bx^2\right)^4\,{}_2F_1\left(\frac{3}{4},5;\frac{7}{4};-\frac{bx^2}{a}\right)-39b^4x^8\right)}{117b^5\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-2*d^9*(d*x)^(3/2)*(-1463*a^4 - 2717*a^3*b*x^2 - 2223*a^2*b^2*x^4 - 741*a*
b^3*x^6 - 39*b^4*x^8 + 1463*(a + b*x^2)^4*Hypergeometric2F1[3/4, 5, 7/4, -(
(b*x^2)/a)]))/(117*b^5*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 0.72, size = 457, normalized size = 0.76

$$87780\left(-\frac{a^3d^{42}}{b^{23}}\right)^{\frac{1}{4}}\left(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5\right)\arctan\left(-\frac{\left(-\frac{a^3d^{42}}{b^{23}}\right)^{\frac{1}{4}}\sqrt{dx}\,a^2b^6d^{31}-\sqrt{a^4d^{63}x-\sqrt{-\frac{a^3d^{42}}{b^{23}}}\,a^3b^{11}d}}{a^3d^{42}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12288*(87780*(-a^3*d^42/b^23)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^
4 + 4*a^3*b^6*x^2 + a^4*b^5)*arctan(-((-a^3*d^42/b^23)^(1/4)*sqrt(d*x)*a^2*
b^6*d^31 - sqrt(a^4*d^63*x - sqrt(-a^3*d^42/b^23)*a^3*b^11*d^42)*(-a^3*d^42
/b^23)^(1/4)*b^6)/(a^3*d^42)) - 21945*(-a^3*d^42/b^23)^(1/4)*(b^9*x^8 + 4*a
*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)*log(391419980875*sqrt(d
*x)*a^2*d^31 + 391419980875*(-a^3*d^42/b^23)^(3/4)*b^17) + 21945*(-a^3*d^42
/b^23)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b
^5)*log(391419980875*sqrt(d*x)*a^2*d^31 - 391419980875*(-a^3*d^42/b^23)^(3/
4)*b^17) + 4*(2048*b^4*d^10*x^9 + 16967*a*b^3*d^10*x^7 + 33345*a^2*b^2*d^10
*x^5 + 26125*a^3*b*d^10*x^3 + 7315*a^4*d^10*x)*sqrt(d*x))/(b^9*x^8 + 4*a*b^
8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)
```

giac [A] time = 0.42, size = 437, normalized size = 0.73

$$\frac{1}{24576}d^{10}\left(\frac{16384\sqrt{dx}\,x}{b^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{43890\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^8d\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{43890\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\mathrm{arcta}}{b^8d\mathrm{sgn}\left(bd\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^10*(16384*sqrt(d*x)*x/(b^5*sgn(b*d^4*x^2 + a*d^4)) - 43890*sqrt(2
)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*
x))/(a*d^2/b)^(1/4))/(b^8*d*sgn(b*d^4*x^2 + a*d^4)) - 43890*sqrt(2)*(a*b^3*
d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d
^2/b)^(1/4))/(b^8*d*sgn(b*d^4*x^2 + a*d^4)) + 21945*sqrt(2)*(a*b^3*d^2)^(3/
4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^8*d*sgn(
b*d^4*x^2 + a*d^4)) - 21945*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*
d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^8*d*sgn(b*d^4*x^2 + a*d^4)) + 8*
(8775*sqrt(d*x)*a*b^3*d^8*x^7 + 21057*sqrt(d*x)*a^2*b^2*d^8*x^5 + 17933*sqr
t(d*x)*a^3*b*d^8*x^3 + 5267*sqrt(d*x)*a^4*d^8*x)/((b*d^2*x^2 + a*d^2)^4*b^5
*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.03, size = 1171, normalized size = 1.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(16384*(d*x)^(3/2)*(a/b*d^2)^(1/4)*x^8*b^5*d^6-21945*2^(1/2)*ln(-(-
d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/
4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^8*a*b^4*d^8-43890*2^(1/2)*arctan
((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^8*a*b^4*d^8-43890
*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^8*
a*b^4*d^8+70200*(d*x)^(15/2)*(a/b*d^2)^(1/4)*a*b^4+65536*(d*x)^(3/2)*(a/b*d
^2)^(1/4)*x^6*a*b^4*d^6-87780*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)
*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2
)^(1/2)))*x^6*a^2*b^3*d^8-175560*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d
```

```
^2)^(1/4))/(a/b*d^2)^(1/4))*x^6*a^2*b^3*d^8-175560*2^(1/2)*arctan((2^(1/2)*
(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^6*a^2*b^3*d^8+168456*(d*x)^
(11/2)*(a/b*d^2)^(1/4)*a^2*b^3*d^2+98304*(d*x)^(3/2)*(a/b*d^2)^(1/4)*x^4*a^
2*b^3*d^6-131670*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b
*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^4
*a^3*b^2*d^8-263340*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a
/b*d^2)^(1/4))*x^4*a^3*b^2*d^8-263340*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(
a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a^3*b^2*d^8+143464*(d*x)^(7/2)*(a/b*d^
2)^(1/4)*a^3*b^2*d^4+65536*(d*x)^(3/2)*(a/b*d^2)^(1/4)*x^2*a^3*b^2*d^6-8778
0*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d
*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a^4*b*d^8-1755
60*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^
2*a^4*b*d^8-175560*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/
b*d^2)^(1/4))*x^2*a^4*b*d^8+58520*(d*x)^(3/2)*(a/b*d^2)^(1/4)*a^4*b*d^6-219
45*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(
d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*a^5*d^8-43890*2^(
1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^5*d^8-
43890*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))
*a^5*d^8)*d^3*(b*x^2+a)/(a/b*d^2)^(1/4)/b^6/((b*x^2+a)^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-4\,ad^{\frac{21}{2}}\int\frac{\sqrt{x}}{b^6x^2+ab^5}\,dx+d^{\frac{21}{2}}\int\frac{x^{\frac{5}{2}}}{b^5x^2+ab^4}\,dx+\frac{2925\,ad^{\frac{21}{2}}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-\cdots\right)}{2\sqrt{\sqrt{a}\cdots}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}\cdots\right)}{81\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -4*a*d^(21/2)*integrate(sqrt(x)/(b^6*x^2 + a*b^5), x) + d^(21/2)*integrate(
x^(5/2)/(b^5*x^2 + a*b^4), x) + 2925/8192*a*d^(21/2)*(2*sqrt(2)*arctan(1/2*
sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*
a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*s
qrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
+ sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
 + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^5 + 1/3072*(8775*a*b^3*d^(21/2
)*x^(15/2) + 29649*a^2*b^2*d^(21/2)*x^(11/2) + 34285*a^3*b*d^(21/2)*x^(7/2)
 + 13795*a^4*d^(21/2)*x^(3/2))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a
```

```
^3*b^6*x^2 + a^4*b^5) - 1/192*((537*a^2*b^4*d^(21/2)*x^5 + 1210*a^3*b^3*d^(
21/2)*x^3 + 705*a^4*b^2*d^(21/2)*x)*x^(9/2) + 2*(443*a^3*b^3*d^(21/2)*x^5 +
 1014*a^4*b^2*d^(21/2)*x^3 + 603*a^5*b*d^(21/2)*x)*x^(5/2) + (381*a^4*b^2*d
^(21/2)*x^5 + 882*a^5*b*d^(21/2)*x^3 + 533*a^6*d^(21/2)*x)*sqrt(x))/(a^3*b^
8*x^6 + 3*a^4*b^7*x^4 + 3*a^5*b^6*x^2 + a^6*b^5 + (b^11*x^6 + 3*a*b^10*x^4
+ 3*a^2*b^9*x^2 + a^3*b^8)*x^6 + 3*(a*b^10*x^6 + 3*a^2*b^9*x^4 + 3*a^3*b^8*
x^2 + a^4*b^7)*x^4 + 3*(a^2*b^9*x^6 + 3*a^3*b^8*x^4 + 3*a^4*b^7*x^2 + a^5*b
^6)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{21/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(21/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((d*x)^(21/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(21/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Timed out
```

3.771 $\int \frac{(dx)^{19/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}} dx$

Optimal. Leaf size=600

$$-\frac{17d^3(dx)^{13/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3315\sqrt[4]{a}\,d^{19/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\right.}{4096\sqrt{2}\,b^{21/4}\sqrt{a^2+}}$$

```
[Out] -663/1024*d^7*(d*x)^(5/2)/b^4/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(17/2)/b/(b*x
^2+a)^3/((b*x^2+a)^2)^(1/2)-17/96*d^3*(d*x)^(13/2)/b^2/(b*x^2+a)^2/((b*x^2+
a)^2)^(1/2)-221/768*d^5*(d*x)^(9/2)/b^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+3315/
4096*a^(1/4)*d^(19/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4
)/d^(1/2))/b^(21/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-3315/4096*a^(1/4)*d^(19/2)*
(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(21/4)*2^
(1/2)/((b*x^2+a)^2)^(1/2)+3315/8192*a^(1/4)*d^(19/2)*(b*x^2+a)*ln(a^(1/2)*d
^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(21/4)*2^(1
/2)/((b*x^2+a)^2)^(1/2)-3315/8192*a^(1/4)*d^(19/2)*(b*x^2+a)*ln(a^(1/2)*d^(
1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(21/4)*2^(1/2
)/((b*x^2+a)^2)^(1/2)+3315/1024*d^9*(b*x^2+a)*(d*x)^(1/2)/b^5/((b*x^2+a)^2)
^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 600, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1112, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{3315d^9\sqrt{dx}\left(a+bx^2\right)}{1024b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{221d^5(dx)^{9/2}}{768b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{17d}{96b^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-663*d^7*(d*x)^(5/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x
)^(17/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (17*d^3*(d*
x)^(13/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (221*d^
5*(d*x)^(9/2))/(768*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (331
5*d^9*Sqrt[d*x]*(a + b*x^2))/(1024*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (
3315*a^(1/4)*d^(19/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a
^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(21/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
 (3315*a^(1/4)*d^(19/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/
(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(21/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

```
 + (3315*a^(1/4)*d^(19/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*b^(21/4)*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4]) - (3315*a^(1/4)*d^(19/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt
[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]
*b^(21/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{19/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{19/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(17b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{17d^3(dx)^{13/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \cdots$$

$$= -\frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{17d^3(dx)^{13/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{7\cdots}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

Mathematica [A] time = 0.28, size = 384, normalized size = 0.64

$$\frac{(dx)^{19/2}\left(a+bx^2\right)\left(10183680a^4\sqrt[4]{b}\sqrt{x}+32587776a^3b^{5/4}x^{5/2}-848640a^3\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)+39829504a^2b^{9/4}x^{9/2}-\dots\right.}{\dots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(19/2)*(a + b*x^2)*(10183680*a^4*b^(1/4)*Sqrt[x] + 32587776*a^3*b^(5
/4)*x^(5/2) + 39829504*a^2*b^(9/4)*x^(9/2) + 21446656*a*b^(13/4)*x^(13/2) +
 3784704*b^(17/4)*x^(17/2) - 848640*a^3*b^(1/4)*Sqrt[x]*(a + b*x^2) - 11668
80*a^2*b^(1/4)*Sqrt[x]*(a + b*x^2)^2 - 2042040*a*b^(1/4)*Sqrt[x]*(a + b*x^2
)^3 + 1531530*Sqrt[2]*a^(1/4)*(a + b*x^2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)] - 1531530*Sqrt[2]*a^(1/4)*(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)] + 765765*Sqrt[2]*a^(1/4)*(a + b*x^2)^4*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 765765*Sqrt[2]*a^(1/4)*(
a + b*x^2)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(
1892352*b^(21/4)*x^(19/2)*((a + b*x^2)^2)^(5/2))
```

fricas [A] time = 0.84, size = 421, normalized size = 0.70

$$-\frac{39780\left(-\frac{ad^{38}}{b^{21}}\right)^{\frac{1}{4}}\left(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5\right)\arctan\left(-\frac{\left(-\frac{ad^{38}}{b^{21}}\right)^{\frac{3}{4}}\sqrt{dx}\,b^{16}d^9-\sqrt{d^{19}x+\sqrt{-\frac{ad^{38}}{b^{21}}}\,b^{10}}\left(-\frac{ad^{38}}{b^{21}}\right)\dots}{ad^{38}}\right.}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12288*(39780*(-a*d^38/b^21)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4
 + 4*a^3*b^6*x^2 + a^4*b^5)*arctan(-((-a*d^38/b^21)^(3/4)*sqrt(d*x)*b^16*d^
9 - sqrt(d^19*x + sqrt(-a*d^38/b^21)*b^10)*(-a*d^38/b^21)^(3/4)*b^16)/(a*d^
38)) + 9945*(-a*d^38/b^21)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4
*a^3*b^6*x^2 + a^4*b^5)*log(3315*sqrt(d*x)*d^9 + 3315*(-a*d^38/b^21)^(1/4)*
b^5) - 9945*(-a*d^38/b^21)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4
*a^3*b^6*x^2 + a^4*b^5)*log(3315*sqrt(d*x)*d^9 - 3315*(-a*d^38/b^21)^(1/4)*
b^5) - 4*(6144*b^4*d^9*x^8 + 31501*a*b^3*d^9*x^6 + 52819*a^2*b^2*d^9*x^4 +
37791*a^3*b*d^9*x^2 + 9945*a^4*d^9)*sqrt(d*x))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a
^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)
```

giac [A] time = 0.42, size = 423, normalized size = 0.70

$$-\frac{1}{24576}\,d^9\left(\frac{19890\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{19890\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6\mathrm{sgn}\left(bd^4x^2+ad^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24576*d^9*(19890*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^6*sgn(b*d^4*x^2 + a*d^4))
 + 19890*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(
1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^6*sgn(b*d^4*x^2 + a*d^4)) + 9945*sq
rt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(b^6*sgn(b*d^4*x^2 + a*d^4)) - 9945*sqrt(2)*(a*b^3*d^2)^(1/4)*log
(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^6*sgn(b*d^4*x^
2 + a*d^4)) - 49152*sqrt(d*x)/(b^5*sgn(b*d^4*x^2 + a*d^4)) - 8*(6925*sqrt(d
*x)*a*b^3*d^8*x^6 + 15955*sqrt(d*x)*a^2*b^2*d^8*x^4 + 13215*sqrt(d*x)*a^3*b
*d^8*x^2 + 3801*sqrt(d*x)*a^4*d^8)/((b*d^2*x^2 + a*d^2)^4*b^5*sgn(b*d^4*x^2
 + a*d^4)))
```

maple [B] time = 0.03, size = 1202, normalized size = 2.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24576*(9945*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*
2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)
^(1/2)))*x^8*b^4*d^6+19890*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1
/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^8*b^4*d^6+19890*(a/b*d^2)^(1/4)*2^(
1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^8*b^4*
d^6+39780*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/
2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2
)))*x^6*a*b^3*d^6+79560*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)
+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^6*a*b^3*d^6+79560*(a/b*d^2)^(1/4)*2^(1
```

```
/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^6*a*b^3
*d^6-49152*(d*x)^(1/2)*x^8*b^4*d^6+59670*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a
/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^4*a^2*b^2*d^6+119340*(a/b*d^2)^(1/4)*2
^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a^
2*b^2*d^6+119340*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d
^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a^2*b^2*d^6-55400*(d*x)^(13/2)*a*b^3-196608
*(d*x)^(1/2)*x^6*a*b^3*d^6+39780*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^
(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)
*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a^3*b*d^6+79560*(a/b*d^2)^(1/4)*2^(1/2)*arct
an((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a^3*b*d^6+795
60*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/
b*d^2)^(1/4))*x^2*a^3*b*d^6-127640*(d*x)^(9/2)*a^2*b^2*d^2-294912*(d*x)^(1/
2)*x^4*a^2*b^2*d^6+9945*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+
(a/b*d^2)^(1/2)))*a^4*d^6+19890*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*
x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^4*d^6+19890*(a/b*d^2)^(1/4)*2^
(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^4*d^6
-105720*(d*x)^(5/2)*a^3*b*d^4-196608*(d*x)^(1/2)*x^2*a^3*b*d^6-79560*(d*x)^
(1/2)*a^4*d^6)*d^3*(b*x^2+a)/b^5/((b*x^2+a)^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$d^{\frac{19}{2}}\int \frac{x^{\frac{3}{2}}}{b^5x^2+ab^4}\,dx - \frac{1267\left(\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}\sqrt{a}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right.}{b^{\frac{1}{4}}}\right.}{8192b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] d^(19/2)*integrate(x^(3/2)/(b^5*x^2 + a*b^4), x) - 1267/8192*(2*sqrt(2)*sqr
t(a)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt(a)*arctan(-1/2*sqr
t(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/s
qrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(1/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
+ sqrt(b)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(1/4)*log(-sqrt(2)*a^(1/4)*b^(1/
4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4))*d^(19/2)/b^5 + 1/3072*(1853*a*b^
3*d^(19/2)*x^(13/2) + 6515*a^2*b^2*d^(19/2)*x^(9/2) + 8079*a^3*b*d^(19/2)*x
^(5/2) + 3801*a^4*d^(19/2)*sqrt(x))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4
```

```
+ 4*a^3*b^6*x^2 + a^4*b^5) + 1/192*((317*a*b^4*d^(19/2)*x^5 + 738*a^2*b^3*d
^(19/2)*x^3 + 453*a^3*b^2*d^(19/2)*x)*x^(11/2) + 2*(243*a^2*b^3*d^(19/2)*x^
5 + 582*a^3*b^2*d^(19/2)*x^3 + 371*a^4*b*d^(19/2)*x)*x^(7/2) + (201*a^3*b^2
*d^(19/2)*x^5 + 490*a^4*b*d^(19/2)*x^3 + 321*a^5*d^(19/2)*x)*x^(3/2))/(a^3*
b^7*x^6 + 3*a^4*b^6*x^4 + 3*a^5*b^5*x^2 + a^6*b^4 + (b^10*x^6 + 3*a*b^9*x^4
 + 3*a^2*b^8*x^2 + a^3*b^7)*x^6 + 3*(a*b^9*x^6 + 3*a^2*b^8*x^4 + 3*a^3*b^7*
x^2 + a^4*b^6)*x^4 + 3*(a^2*b^8*x^6 + 3*a^3*b^7*x^4 + 3*a^4*b^6*x^2 + a^5*b
^5)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{19/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(19/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(19/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(19/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

3.772 $\int \frac{(dx)^{17/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=554

$$-\frac{5d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155d^{17/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{4096\sqrt{2}\sqrt[4]{a}\,b^{19/4}\sqrt{a^2+}}$$

```
[Out] -385/1024*d^7*(d*x)^(3/2)/b^4/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(15/2)/b/(b*x
^2+a)^3/((b*x^2+a)^2)^(1/2)-5/32*d^3*(d*x)^(11/2)/b^2/(b*x^2+a)^2/((b*x^2+a
)^2)^(1/2)-55/256*d^5*(d*x)^(7/2)/b^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-1155/40
96*d^(17/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))
/a^(1/4)/b^(19/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+1155/4096*d^(17/2)*(b*x^2+a)*
arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(1/4)/b^(19/4)*2^(1
/2)/((b*x^2+a)^2)^(1/2)+1155/8192*d^(17/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b
^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(1/4)/b^(19/4)*2^(1/2
)/((b*x^2+a)^2)^(1/2)-1155/8192*d^(17/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(
1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(1/4)/b^(19/4)*2^(1/2)/
((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 554, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{55d^5(dx)^{7/2}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-385*d^7*(d*x)^(3/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x
)^(15/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*d^3*(d*x
)^(11/2))/(32*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (55*d^5*
(d*x)^(7/2))/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (1155*
d^(17/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d
])])/(2048*Sqrt[2]*a^(1/4)*b^(19/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (115
5*d^(17/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(2048*Sqrt[2]*a^(1/4)*b^(19/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1
155*d^(17/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(1/4)*b^(19/4)*Sqrt[a^2 + 2*a*b
```

```
*x^2 + b^2*x^4]) - (1155*d^(17/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]
*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(1/4)*b^(1
9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{17/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{17/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(15b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \cdots$$

$$= -\frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \cdots$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{32b^2\left(a+\cdots\right.}$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{32b^2\left(a+\cdots\right.}$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{32b^2\left(a+\cdots\right.}$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{32b^2\left(a+\cdots\right.}$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{32b^2\left(a+\cdots\right.}$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{32b^2\left(a+\cdots\right.}$$

Mathematica [C] time = 0.05, size = 106, normalized size = 0.19

$$\frac{2d^7(dx)^{3/2}\left(77\left(a+bx^2\right)^4\ {}_2F_1\left(\frac{3}{4},5;\frac{7}{4};-\frac{bx^2}{a}\right)-a\left(77a^3+143a^2bx^2+117ab^2x^4+39b^3x^6\right)\right)}{39ab^4\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*d^7*(d*x)^(3/2)*(-(a*(77*a^3 + 143*a^2*b*x^2 + 117*a*b^2*x^4 + 39*b^3*x^
6)) + 77*(a + b*x^2)^4*Hypergeometric2F1[3/4, 5, 7/4, -((b*x^2)/a)]))/(39*a
*b^4*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 1.06, size = 428, normalized size = 0.77

$$-\frac{4620\left(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4\right)\left(-\frac{d^{34}}{ab^{19}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{34}}{ab^{19}}\right)^{\frac{1}{4}}\sqrt{dx}\,b^5d^{25}-\sqrt{d^{51}x-\sqrt{-\frac{d^{34}}{ab^{19}}}\,ab^9d^{34}}\left(-\frac{d^{34}}{ab^{19}}\right.}{d^{34}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4096*(4620*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*
b^4)*(-d^34/(a*b^19))^(1/4)*arctan(-((-d^34/(a*b^19))^(1/4)*sqrt(d*x)*b^5*d
^25 - sqrt(d^51*x - sqrt(-d^34/(a*b^19))*a*b^9*d^34)*(-d^34/(a*b^19))^(1/4)
*b^5)/d^34) - 1155*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 +
 a^4*b^4)*(-d^34/(a*b^19))^(1/4)*log(1540798875*sqrt(d*x)*d^25 + 1540798875
*(-d^34/(a*b^19))^(3/4)*a*b^14) + 1155*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x
^4 + 4*a^3*b^5*x^2 + a^4*b^4)*(-d^34/(a*b^19))^(1/4)*log(1540798875*sqrt(d*
x)*d^25 - 1540798875*(-d^34/(a*b^19))^(3/4)*a*b^14) + 4*(893*b^3*d^8*x^7 +
1755*a*b^2*d^8*x^5 + 1375*a^2*b*d^8*x^3 + 385*a^3*d^8*x)*sqrt(d*x))/(b^8*x^
8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

giac [A] time = 0.37, size = 418, normalized size = 0.75

$$\frac{1}{8192}d^8\left(\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^7d\operatorname{sgn}\left(bd^4x^2+ad^4\right)}+\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^7d\operatorname{sgn}\left(bd^4x^2+ad^4\right)}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8192*d^8*(2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^7*d*sgn(b*d^4*x^2 + a*d^4)
) + 2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(
1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^7*d*sgn(b*d^4*x^2 + a*d^4)) - 115
5*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + s
qrt(a*d^2/b))/(a*b^7*d*sgn(b*d^4*x^2 + a*d^4)) + 1155*sqrt(2)*(a*b^3*d^2)^(
3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^7*d*
sgn(b*d^4*x^2 + a*d^4)) - 8*(893*sqrt(d*x)*b^3*d^8*x^7 + 1755*sqrt(d*x)*a*b
^2*d^8*x^5 + 1375*sqrt(d*x)*a^2*b*d^8*x^3 + 385*sqrt(d*x)*a^3*d^8*x)/((b*d^
2*x^2 + a*d^2)^4*b^4*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 1046, normalized size = 1.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/8192*(-1155*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d
^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^8*b
^4*d^8-2310*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^
(1/4))*x^8*b^4*d^8-2310*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4)
)/(a/b*d^2)^(1/4))*x^8*b^4*d^8+7144*(a/b*d^2)^(1/4)*(d*x)^(15/2)*b^4-4620*2
^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+
(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^6*a*b^3*d^8-9240*2^
(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^6*a*b
^3*d^8-9240*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^
(1/4))*x^6*a*b^3*d^8+14040*(a/b*d^2)^(1/4)*(d*x)^(11/2)*a*b^3*d^2-6930*2^(1
```

```
/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/
b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^4*a^2*b^2*d^8-13860*2^
(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a^2
*b^2*d^8-13860*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^
2)^(1/4))*x^4*a^2*b^2*d^8+11000*(a/b*d^2)^(1/4)*(d*x)^(7/2)*a^2*b^2*d^4-462
0*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d
*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a^3*b*d^8-9240
*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*
a^3*b*d^8-9240*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^
2)^(1/4))*x^2*a^3*b*d^8+3080*(a/b*d^2)^(1/4)*(d*x)^(3/2)*a^3*b*d^6-1155*2^(
1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a
/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*a^4*d^8-2310*2^(1/2)*ar
ctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^4*d^8-2310*2^
(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^4*d^8
)*d*(b*x^2+a)/(a/b*d^2)^(1/4)/b^5/((b*x^2+a)^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$d^{\frac{17}{2}}\int\frac{\sqrt{x}}{b^5x^2+ab^4}\,dx-\frac{893\,d^{\frac{17}{2}}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{}\right.}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{8192\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] d^(17/2)*integrate(sqrt(x)/(b^5*x^2 + a*b^4), x) - 893/8192*d^(17/2)*(2*sqr
t(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*s
qrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))
/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt
(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)
*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^4 - 1/3072*(26
79*b^3*d^(17/2)*x^(15/2) + 9441*a*b^2*d^(17/2)*x^(11/2) + 11645*a^2*b*d^(17
/2)*x^(7/2) + 5267*a^3*d^(17/2)*x^(3/2))/(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6
*x^4 + 4*a^3*b^5*x^2 + a^4*b^4) + 1/192*((261*a*b^4*d^(17/2)*x^5 + 610*a^2*
b^3*d^(17/2)*x^3 + 381*a^3*b^2*d^(17/2)*x)*x^(9/2) + 2*(191*a^2*b^3*d^(17/2
)*x^5 + 462*a^3*b^2*d^(17/2)*x^3 + 303*a^4*b*d^(17/2)*x)*x^(5/2) + (153*a^3
*b^2*d^(17/2)*x^5 + 378*a^4*b*d^(17/2)*x^3 + 257*a^5*d^(17/2)*x)*sqrt(x))/(
a^3*b^7*x^6 + 3*a^4*b^6*x^4 + 3*a^5*b^5*x^2 + a^6*b^4 + (b^10*x^6 + 3*a*b^9
```

```
*x^4 + 3*a^2*b^8*x^2 + a^3*b^7)*x^6 + 3*(a*b^9*x^6 + 3*a^2*b^8*x^4 + 3*a^3*
b^7*x^2 + a^4*b^6)*x^4 + 3*(a^2*b^8*x^6 + 3*a^3*b^7*x^4 + 3*a^4*b^6*x^2 + a
^5*b^5)*x^2)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{17/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(17/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((d*x)^(17/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(17/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Timed out
```

3.773 $\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=554

$$-\frac{13d^3(dx)^{9/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{256b^3\left(\right.}$$

```
[Out] -1/8*d*(d*x)^(13/2)/b/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)-13/96*d^3*(d*x)^(9/2)
/b^2/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)-39/256*d^5*(d*x)^(5/2)/b^3/(b*x^2+a)/(
(b*x^2+a)^2)^(1/2)-195/4096*d^(15/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*
x)^(1/2)/a^(1/4)/d^(1/2))/a^(3/4)/b^(17/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+195/
4096*d^(15/2)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2
))/a^(3/4)/b^(17/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-195/8192*d^(15/2)*(b*x^2+a)
*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/
a^(3/4)/b^(17/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+195/8192*d^(15/2)*(b*x^2+a)*ln
(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(
3/4)/b^(17/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-195/1024*d^7*(d*x)^(1/2)/b^4/((b*
x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 554, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{39d^5(dx)^{5/2}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13d^3(dx)^{9/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-195*d^7*Sqrt[d*x])/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^
(13/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13*d^3*(d*x)
^(9/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (39*d^5*(d
*x)^(5/2))/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (195*d^(
15/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(2048*Sqrt[2]*a^(3/4)*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (195*d^
(15/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(2048*Sqrt[2]*a^(3/4)*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (195*d
^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(3/4)*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2
```

```
+ b^2*x^4]) + (195*d^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[
d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(3/4)*b^(17/4)*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(13b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{13d^3(dx)^{9/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \cdots$$

$$= -\frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{13d^3(dx)^{9/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{2\cdots}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

Mathematica [A] time = 0.26, size = 366, normalized size = 0.66

$$\frac{(dx)^{15/2}\left(a+bx^2\right)\left(-\frac{45045\sqrt{2}\left(a+bx^2\right)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}+\frac{45045\sqrt{2}\left(a+bx^2\right)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}-\frac{90090\sqrt{2}(\cdots}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(15/2)*(a + b*x^2)*(-599040*a^3*b^(1/4)*Sqrt[x] - 1916928*a^2*b^(5/4
)*x^(5/2) - 2342912*a*b^(9/4)*x^(9/2) - 1261568*b^(13/4)*x^(13/2) + 49920*a
^2*b^(1/4)*Sqrt[x]*(a + b*x^2) + 68640*a*b^(1/4)*Sqrt[x]*(a + b*x^2)^2 + 12
0120*b^(1/4)*Sqrt[x]*(a + b*x^2)^3 - (90090*Sqrt[2]*(a + b*x^2)^4*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) + (90090*Sqrt[2]*(a + b*x^2)^
4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) - (45045*Sqrt[2]*(
a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^
(3/4) + (45045*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/a^(3/4)))/(1892352*b^(17/4)*x^(15/2)*((a + b*x^2)^2)^
(5/2))
```

fricas [A] time = 1.07, size = 431, normalized size = 0.78

$$\frac{2340\left(b^8x^8+4ab^7x^6+6a^2b^6x^4+4a^3b^5x^2+a^4b^4\right)\left(-\frac{d^{30}}{a^3b^{17}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{30}}{a^3b^{17}}\right)^{\frac{3}{4}}\sqrt{dx}\,a^2b^{13}d^7-\sqrt{d^{15}x+\sqrt{-\frac{d^{30}}{a^3b^{17}}}\,a^2b^8}\left(-\frac{d^{3}\cdots}{a^3b\cdots}\right.}{d^{30}}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12288*(2340*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*
b^4)*(-d^30/(a^3*b^17))^(1/4)*arctan(-((-d^30/(a^3*b^17))^(3/4)*sqrt(d*x)*a
^2*b^13*d^7 - sqrt(d^15*x + sqrt(-d^30/(a^3*b^17))*a^2*b^8)*(-d^30/(a^3*b^1
7))^(3/4)*a^2*b^13)/d^30) + 585*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*
a^3*b^5*x^2 + a^4*b^4)*(-d^30/(a^3*b^17))^(1/4)*log(195*sqrt(d*x)*d^7 + 195
*(-d^30/(a^3*b^17))^(1/4)*a*b^4) - 585*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x
^4 + 4*a^3*b^5*x^2 + a^4*b^4)*(-d^30/(a^3*b^17))^(1/4)*log(195*sqrt(d*x)*d^
7 - 195*(-d^30/(a^3*b^17))^(1/4)*a*b^4) - 4*(1853*b^3*d^7*x^6 + 3107*a*b^2*
d^7*x^4 + 2223*a^2*b*d^7*x^2 + 585*a^3*d^7)*sqrt(d*x))/(b^8*x^8 + 4*a*b^7*x
^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

giac [A] time = 0.35, size = 405, normalized size = 0.73

$$\frac{1}{24576}d^7\left(\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^7*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^5*sgn(b*d^4*x^2 + a*d^4))
 + 1170*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^5*sgn(b*d^4*x^2 + a*d^4)) + 585*sq
rt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(a*b^5*sgn(b*d^4*x^2 + a*d^4)) - 585*sqrt(2)*(a*b^3*d^2)^(1/4)*lo
g(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^5*sgn(b*d^4
*x^2 + a*d^4)) - 8*(1853*sqrt(d*x)*b^3*d^8*x^6 + 3107*sqrt(d*x)*a*b^2*d^8*x
^4 + 2223*sqrt(d*x)*a^2*b*d^8*x^2 + 585*sqrt(d*x)*a^3*d^8)/((b*d^2*x^2 + a*
d^2)^4*b^4*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 1134, normalized size = 2.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(585*(a/b*d^2)^(1/4)*2^(1/2)*b^4*d^6*x^8*ln((d*x+(a/b*d^2)^(1/4)*(d
*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a/b*d^2)^(1/2)))+1170*(a/b*d^2)^(1/4)*2^(1/2)*b^4*d^6*x^8*arctan((2^(1/2)
*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+1170*(a/b*d^2)^(1/4)*2^(1/2)
*b^4*d^6*x^8*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+
2340*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^6*x^6*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1
/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*
d^2)^(1/2)))+4680*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^6*x^6*arctan((2^(1/2)*(d*
x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+4680*(a/b*d^2)^(1/4)*2^(1/2)*a*b
^3*d^6*x^6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+35
```

```
10*(a/b*d^2)^(1/4)*2^(1/2)*a^2*b^2*d^6*x^4*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1
/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*
d^2)^(1/2)))+7020*(a/b*d^2)^(1/4)*2^(1/2)*a^2*b^2*d^6*x^4*arctan((2^(1/2)*(
d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+7020*(a/b*d^2)^(1/4)*2^(1/2)*a
^2*b^2*d^6*x^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4)
)-14824*(d*x)^(13/2)*a*b^3+2340*(a/b*d^2)^(1/4)*2^(1/2)*a^3*b*d^6*x^2*ln((d
*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4
)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+4680*(a/b*d^2)^(1/4)*2^(1/2)*a^3*b*
d^6*x^2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+4680*
(a/b*d^2)^(1/4)*2^(1/2)*a^3*b*d^6*x^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)
^(1/4))/(a/b*d^2)^(1/4))-24856*(d*x)^(9/2)*a^2*b^2*d^2+585*(a/b*d^2)^(1/4)*
2^(1/2)*a^4*d^6*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)
)/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+1170*(a/b*d^2)
^(1/4)*2^(1/2)*a^4*d^6*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^
2)^(1/4))+1170*(a/b*d^2)^(1/4)*2^(1/2)*a^4*d^6*arctan((2^(1/2)*(d*x)^(1/2)-
(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))-17784*(d*x)^(5/2)*a^3*b*d^4-4680*(d*x)^(1
/2)*a^4*d^6)*d*(b*x^2+a)/a/b^4/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 3.76, size = 583, normalized size = 1.05

$$\frac{195\,d^{7}\left(\frac{2\,\sqrt{2}\,\sqrt{d}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,\sqrt{d}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,\sqrt{d}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\,\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,\sqrt{d}\cdots}{}\right.}{8192\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 195/8192*d^7*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
 + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))
) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt
(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(
2)*sqrt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3
/4)*b^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(
b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^4 - 1/1024*(15*b^3*d^(15/2)*x^(13/2) +
 65*a*b^2*d^(15/2)*x^(9/2) + 117*a^2*b*d^(15/2)*x^(5/2) + 195*a^3*d^(15/2)*
sqrt(x))/(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
- 1/192*((113*b^4*d^(15/2)*x^5 + 282*a*b^3*d^(15/2)*x^3 + 201*a^2*b^2*d^(15
/2)*x)*x^(11/2) + 2*(63*a*b^3*d^(15/2)*x^5 + 174*a^2*b^2*d^(15/2)*x^3 + 143
*a^3*b*d^(15/2)*x)*x^(7/2) + (45*a^2*b^2*d^(15/2)*x^5 + 130*a^3*b*d^(15/2)*
```

```
x^3 + 117*a^4*d^(15/2)*x)*x^(3/2))/(a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4
*x^2 + a^6*b^3 + (b^9*x^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3*b^6)*x^6 + 3*
(a*b^8*x^6 + 3*a^2*b^7*x^4 + 3*a^3*b^6*x^2 + a^4*b^5)*x^4 + 3*(a^2*b^7*x^6
+ 3*a^3*b^6*x^4 + 3*a^4*b^5*x^2 + a^5*b^4)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{15/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Timed out
```

3.774 $\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=557

$$-\frac{11d^3(dx)^{7/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{768b^3}$$

```
[Out] 77/1024*d^5*(d*x)^(3/2)/a/b^3/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(11/2)/b/(b*x
^2+a)^3/((b*x^2+a)^2)^(1/2)-11/96*d^3*(d*x)^(7/2)/b^2/(b*x^2+a)^2/((b*x^2+a
)^2)^(1/2)-77/768*d^5*(d*x)^(3/2)/b^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-77/4096
*d^(13/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a
^(5/4)/b^(15/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+77/4096*d^(13/2)*(b*x^2+a)*arct
an(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(5/4)/b^(15/4)*2^(1/2)/
((b*x^2+a)^2)^(1/2)+77/8192*d^(13/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)
*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(5/4)/b^(15/4)*2^(1/2)/((b*
x^2+a)^2)^(1/2)-77/8192*d^(13/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(
1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(5/4)/b^(15/4)*2^(1/2)/((b*x^2+
a)^2)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77d^5(dx)^{3/2}}{768b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{11d^3(dx)^{7/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{13}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (77*d^5*(d*x)^(3/2))/(1024*a*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x
)^(11/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (11*d^3*(d*
x)^(7/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^5*
(d*x)^(3/2))/(768*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^
(13/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(2048*Sqrt[2]*a^(5/4)*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^
(13/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(2048*Sqrt[2]*a^(5/4)*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^
(13/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(5/4)*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 +
```

```
b^2*x^4]) - (77*d^(13/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(5/4)*b^(15/4)*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(11b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{11d^3(dx)^{7/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \cdots$$

$$= -\frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{11d^3(dx)^{7/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{7\cdots}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

Mathematica [C] time = 0.04, size = 97, normalized size = 0.17

$$\frac{2d^5(dx)^{3/2}\left(77\left(a+bx^2\right)^4 {}_2F_1\left(\frac{3}{4},5;\frac{7}{4};-\frac{bx^2}{a}\right)-a^2\left(77a^2+143abx^2+117b^2x^4\right)\right)}{585a^2b^3\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*d^5*(d*x)^(3/2)*(-(a^2*(77*a^2 + 143*a*b*x^2 + 117*b^2*x^4)) + 77*(a + b
*x^2)^4*Hypergeometric2F1[3/4, 5, 7/4, -((b*x^2)/a)]))/(585*a^2*b^3*(a + b*
x^2)^3*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 1.16, size = 448, normalized size = 0.80

$$-\frac{924\left(ab^7x^8+4\,a^2b^6x^6+6\,a^3b^5x^4+4\,a^4b^4x^2+a^5b^3\right)\left(-\frac{d^{26}}{a^5b^{15}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{26}}{a^5b^{15}}\right)^{\frac{1}{4}}\sqrt{dx}\,ab^4d^{19}-\sqrt{d^{39}x-\sqrt{-\frac{d^{26}}{a^5b^{15}}}\,a^3b^7d^{26}}\,\left(\cdots\right.}{d^{26}}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12288*(924*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 +
a^5*b^3)*(-d^26/(a^5*b^15))^(1/4)*arctan(-((-d^26/(a^5*b^15))^(1/4)*sqrt(d*
x)*a*b^4*d^19 - sqrt(d^39*x - sqrt(-d^26/(a^5*b^15))*a^3*b^7*d^26)*(-d^26/(
a^5*b^15))^(1/4)*a*b^4)/d^26) - 231*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*
x^4 + 4*a^4*b^4*x^2 + a^5*b^3)*(-d^26/(a^5*b^15))^(1/4)*log(456533*sqrt(d*x
)*d^19 + 456533*(-d^26/(a^5*b^15))^(3/4)*a^4*b^11) + 231*(a*b^7*x^8 + 4*a^2
*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3)*(-d^26/(a^5*b^15))^(1/4
)*log(456533*sqrt(d*x)*d^19 - 456533*(-d^26/(a^5*b^15))^(3/4)*a^4*b^11) - 4
*(231*b^3*d^6*x^7 - 351*a*b^2*d^6*x^5 - 275*a^2*b*d^6*x^3 - 77*a^3*d^6*x)*s
qrt(d*x))/(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*
b^3)
```

giac [A] time = 0.36, size = 421, normalized size = 0.76

$$\frac{1}{24576}d^6\left(\frac{462\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^6d\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{462\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^6d\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^6*(462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^6*d*sgn(b*d^4*x^2 + a*d^
4)) + 462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^6*d*sgn(b*d^4*x^2 + a*d^4)) -
231*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) +
 sqrt(a*d^2/b))/(a^2*b^6*d*sgn(b*d^4*x^2 + a*d^4)) + 231*sqrt(2)*(a*b^3*d^2
)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b
^6*d*sgn(b*d^4*x^2 + a*d^4)) + 8*(231*sqrt(d*x)*b^3*d^8*x^7 - 351*sqrt(d*x)
*a*b^2*d^8*x^5 - 275*sqrt(d*x)*a^2*b*d^8*x^3 - 77*sqrt(d*x)*a^3*d^8*x)/((b*
d^2*x^2 + a*d^2)^4*a*b^3*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 1051, normalized size = 1.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(231*2^(1/2)*b^4*d^8*x^8*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1
/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/
2)))+462*2^(1/2)*b^4*d^8*x^8*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(
a/b*d^2)^(1/4))+462*2^(1/2)*b^4*d^8*x^8*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^
2)^(1/4))/(a/b*d^2)^(1/4))+1848*(a/b*d^2)^(1/4)*(d*x)^(15/2)*b^4+924*2^(1/2
)*a*b^3*d^8*x^6*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/
2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+1848*2^(1/2)
*a*b^3*d^8*x^6*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4)
)+1848*2^(1/2)*a*b^3*d^8*x^6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(
a/b*d^2)^(1/4))-2808*(a/b*d^2)^(1/4)*(d*x)^(11/2)*a*b^3*d^2+1386*2^(1/2)*a^
```

```
2*b^2*d^8*x^4*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2)
)/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+2772*2^(1/2)*a
^2*b^2*d^8*x^4*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4)
)+2772*2^(1/2)*a^2*b^2*d^8*x^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))
/(a/b*d^2)^(1/4))-2200*(a/b*d^2)^(1/4)*(d*x)^(7/2)*a^2*b^2*d^4+924*2^(1/2)*
a^3*b*d^8*x^2*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2)
)/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+1848*2^(1/2)*a
^3*b*d^8*x^2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+
1848*2^(1/2)*a^3*b*d^8*x^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/
b*d^2)^(1/4))-616*(a/b*d^2)^(1/4)*(d*x)^(3/2)*a^3*b*d^6+231*2^(1/2)*a^4*d^8
*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d
^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+462*2^(1/2)*a^4*d^8*arctan(
(2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+462*2^(1/2)*a^4*d^8*
arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4)))/d*(b*x^2+a)/
(a/b*d^2)^(1/4)/b^4/a/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 3.75, size = 577, normalized size = 1.04

$$\frac{77\, d^{\frac{13}{2}} \left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\,\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\,\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b\cdots\right.}{a^{\frac{1}{4}}\cdots} \right.}{8192\, ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 77/8192*d^(13/2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2
*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) +
2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(
2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(
2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(
3/4)))/(a*b^3) + 1/1024*(77*b^3*d^(13/2)*x^(15/2) + 315*a*b^2*d^(13/2)*x^(1
1/2) + 495*a^2*b*d^(13/2)*x^(7/2) + 385*a^3*d^(13/2)*x^(3/2))/(a*b^7*x^8 +
4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3) - 1/192*((81*b^4*d
^(13/2)*x^5 + 202*a*b^3*d^(13/2)*x^3 + 153*a^2*b^2*d^(13/2)*x)*x^(9/2) + 2*
(35*a*b^3*d^(13/2)*x^5 + 102*a^2*b^2*d^(13/2)*x^3 + 99*a^3*b*d^(13/2)*x)*x^
(5/2) + (21*a^2*b^2*d^(13/2)*x^5 + 66*a^3*b*d^(13/2)*x^3 + 77*a^4*d^(13/2)*
x)*sqrt(x))/(a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*x^2 + a^6*b^3 + (b^9*x
^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3*b^6)*x^6 + 3*(a*b^8*x^6 + 3*a^2*b^7*
```

```
x^4 + 3*a^3*b^6*x^2 + a^4*b^5)*x^4 + 3*(a^2*b^7*x^6 + 3*a^3*b^6*x^4 + 3*a^4
*b^5*x^2 + a^5*b^4)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{13/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Timed out
```

3.775 $\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=557

$$-\frac{3d^3(dx)^{5/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{256b^3}$$

```
[Out] -1/8*d*(d*x)^(9/2)/b/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)-3/32*d^3*(d*x)^(5/2)/b
^2/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)-45/4096*d^(11/2)*(b*x^2+a)*arctan(1-b^(1
/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(7/4)/b^(13/4)*2^(1/2)/((b*x^2+a
)^2)^(1/2)+45/4096*d^(11/2)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/
a^(1/4)/d^(1/2))/a^(7/4)/b^(13/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-45/8192*d^(11
/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*
(d*x)^(1/2))/a^(7/4)/b^(13/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+45/8192*d^(11/2)*
(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x
)^(1/2))/a^(7/4)/b^(13/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+15/1024*d^5*(d*x)^(1/
2)/a/b^3/((b*x^2+a)^2)^(1/2)-15/256*d^5*(d*x)^(1/2)/b^3/(b*x^2+a)/((b*x^2+a
)^2)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{15d^5\sqrt{dx}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^3(dx)^{5/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45d^{11}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (15*d^5*Sqrt[d*x])/(1024*a*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^
(9/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^3*(d*x)^(
5/2))/(32*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (15*d^5*Sqrt
[d*x])/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*d^(11/2)
*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(20
48*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^(11/2)
*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(20
48*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*d^(11/2)
*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
```

```
^4]) + (45*d^(11/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(9b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^3(dx)^{5/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \cdots$$

$$= -\frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^3(dx)^{5/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \cdots$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{32b^2\left(a+\cdots\right.}$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{32b^2\left(a+\cdots\right.}$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{32b^2\left(a+\cdots\right.}$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{32b^2\left(a+\cdots\right.}$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{32b^2\left(a+\cdots\right.}$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{32b^2\left(a+\cdots\right.}$$

Mathematica [A]   time = 0.29, size = 352, normalized size = 0.63

$$d(dx)^{9/2}\left(a+bx^2\right)\left(-\frac{3465\sqrt{2}\left(a+bx^2\right)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}}+\frac{3465\sqrt{2}\left(a+bx^2\right)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}}-\frac{6930\sqrt{2}\left(a+\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (d*(d*x)^(9/2)*(a + b*x^2)*(-46080*a^2*b^(1/4)*Sqrt[x] - 147456*a*b^(5/4)*x
^(5/2) - 180224*b^(9/4)*x^(9/2) + 3840*a*b^(1/4)*Sqrt[x]*(a + b*x^2) + 5280
*b^(1/4)*Sqrt[x]*(a + b*x^2)^2 + (9240*b^(1/4)*Sqrt[x]*(a + b*x^2)^3)/a - (
6930*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a
^(7/4) + (6930*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/a^(7/4) - (3465*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4) + (3465*Sqrt[2]*(a + b*x^2)^4*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4)))/(630784*b^
(13/4)*x^(9/2)*((a + b*x^2)^2)^(5/2))
```

fricas [A]   time = 1.05, size = 447, normalized size = 0.80

$$180\left(ab^7x^8+4a^2b^6x^6+6a^3b^5x^4+4a^4b^4x^2+a^5b^3\right)\left(-\frac{d^{22}}{a^7b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{22}}{a^7b^{13}}\right)^{\frac{3}{4}}\sqrt{dx}\,a^5b^{10}d^5-\sqrt{d^{11}x+\sqrt{-\frac{d^{22}}{a^7b^{13}}}\,a^4b^6}\left(-\frac{d}{a^7}\right.}{d^{22}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4096*(180*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^
5*b^3)*(-d^22/(a^7*b^13))^(1/4)*arctan(-((-d^22/(a^7*b^13))^(3/4)*sqrt(d*x)
*a^5*b^10*d^5 - sqrt(d^11*x + sqrt(-d^22/(a^7*b^13))*a^4*b^6)*(-d^22/(a^7*b
^13))^(3/4)*a^5*b^10)/d^22) + 45*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4
 + 4*a^4*b^4*x^2 + a^5*b^3)*(-d^22/(a^7*b^13))^(1/4)*log(45*sqrt(d*x)*d^5 +
 45*(-d^22/(a^7*b^13))^(1/4)*a^2*b^3) - 45*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a
^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3)*(-d^22/(a^7*b^13))^(1/4)*log(45*sqrt(
d*x)*d^5 - 45*(-d^22/(a^7*b^13))^(1/4)*a^2*b^3) + 4*(15*b^3*d^5*x^6 - 239*a
*b^2*d^5*x^4 - 171*a^2*b*d^5*x^2 - 45*a^3*d^5)*sqrt(d*x))/(a*b^7*x^8 + 4*a^
2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3)
```

giac [A] time = 0.35, size = 408, normalized size = 0.73

$$\frac{1}{8192}\,d^5\left(\frac{90\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^4\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{90\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^4\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{45\,\sqrt{}\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8192*d^5*(90*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^4*sgn(b*d^4*x^2 + a*d^4))
+ 90*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4)
 - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^4*sgn(b*d^4*x^2 + a*d^4)) + 45*sqrt
(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*
d^2/b))/(a^2*b^4*sgn(b*d^4*x^2 + a*d^4)) - 45*sqrt(2)*(a*b^3*d^2)^(1/4)*log
(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^4*sgn(b*d^
4*x^2 + a*d^4)) + 8*(15*sqrt(d*x)*b^3*d^8*x^6 - 239*sqrt(d*x)*a*b^2*d^8*x^4
 - 171*sqrt(d*x)*a^2*b*d^8*x^2 - 45*sqrt(d*x)*a^3*d^8)/((b*d^2*x^2 + a*d^2)
^4*a*b^3*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 1136, normalized size = 2.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/8192*(45*(a/b*d^2)^(1/4)*2^(1/2)*b^4*d^6*x^8*ln((d*x+(a/b*d^2)^(1/4)*(d*x
)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(
a/b*d^2)^(1/2)))+90*(a/b*d^2)^(1/4)*2^(1/2)*b^4*d^6*x^8*arctan((2^(1/2)*(d*
x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+90*(a/b*d^2)^(1/4)*2^(1/2)*b^4*d
^6*x^8*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+180*(a
/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^6*x^6*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(
1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1
/2)))+360*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^6*x^6*arctan((2^(1/2)*(d*x)^(1/2)
+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+360*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^6*x^
6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+270*(a/b*d^
```

2)^(1/4)*2^(1/2)*a^2*b^2*d^6*x^4*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2
)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)
))+540*(a/b*d^2)^(1/4)*2^(1/2)*a^2*b^2*d^6*x^4*arctan((2^(1/2)*(d*x)^(1/2)+
(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+540*(a/b*d^2)^(1/4)*2^(1/2)*a^2*b^2*d^6*x
^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+120*(d*x)^
(13/2)*a*b^3+180*(a/b*d^2)^(1/4)*2^(1/2)*a^3*b*d^6*x^2*ln((d*x+(a/b*d^2)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2
^(1/2)+(a/b*d^2)^(1/2)))+360*(a/b*d^2)^(1/4)*2^(1/2)*a^3*b*d^6*x^2*arctan((
2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+360*(a/b*d^2)^(1/4)*2
^(1/2)*a^3*b*d^6*x^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)
^(1/4))-1912*(d*x)^(9/2)*a^2*b^2*d^2+45*(a/b*d^2)^(1/4)*2^(1/2)*a^4*d^6*ln(
(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+90*(a/b*d^2)^(1/4)*2^(1/2)*a^4*d^
6*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+90*(a/b*d^2
)^(1/4)*2^(1/2)*a^4*d^6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d
^2)^(1/4))-1368*(d*x)^(5/2)*a^3*b*d^4-360*(d*x)^(1/2)*a^4*d^6)/d*(b*x^2+a)/
b^3/a^2/((b*x^2+a)^2)^(5/2)

maxima [A] time = 3.71, size = 595, normalized size = 1.07

$$\frac{45\,d^5\left(\frac{2\,\sqrt{2}\,\sqrt{d}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,\sqrt{d}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,\sqrt{d}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,\sqrt{d}\,\cdots}{\cdots}\right.}{8192\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")

[Out] 45/8192*d^5*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
+ 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
+ 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(
b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2
)*sqrt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/
4)*b^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b
)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^3) - 1/3072*(35*b^3*d^(11/2)*x^(13/2
) + 173*a*b^2*d^(11/2)*x^(9/2) + 657*a^2*b*d^(11/2)*x^(5/2) + 135*a^3*d^(11
/2)*sqrt(x))/(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a
^5*b^3) + 1/192*((5*b^4*d^(11/2)*x^5 + 18*a*b^3*d^(11/2)*x^3 + 45*a^2*b^2*d
^(11/2)*x)*x^(11/2) - 2*(21*a*b^3*d^(11/2)*x^5 + 42*a^2*b^2*d^(11/2)*x^3 -
11*a^3*b*d^(11/2)*x)*x^(7/2) - (15*a^2*b^2*d^(11/2)*x^5 + 38*a^3*b*d^(11/2)

```
*x^3 - 9*a^4*d^(11/2)*x)*x^(3/2))/(a^4*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*
x^2 + a^7*b^2 + (a*b^8*x^6 + 3*a^2*b^7*x^4 + 3*a^3*b^6*x^2 + a^4*b^5)*x^6 +
 3*(a^2*b^7*x^6 + 3*a^3*b^6*x^4 + 3*a^4*b^5*x^2 + a^5*b^4)*x^4 + 3*(a^3*b^6
*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*x^2 + a^6*b^3)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{11/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

3.776 $\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=560

$$\frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7d^3(dx)^{3/2}}{256ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7d^3(dx)^{3/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{8b}$$

```
[Out] 35/1024*d^3*(d*x)^(3/2)/a^2/b^2/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(7/2)/b/(b*
x^2+a)^3/((b*x^2+a)^2)^(1/2)-7/96*d^3*(d*x)^(3/2)/b^2/(b*x^2+a)^2/((b*x^2+a
)^2)^(1/2)+7/256*d^3*(d*x)^(3/2)/a/b^2/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-35/409
6*d^(9/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a
^(9/4)/b^(11/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+35/4096*d^(9/2)*(b*x^2+a)*arcta
n(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(9/4)/b^(11/4)*2^(1/2)/(
(b*x^2+a)^2)^(1/2)+35/8192*d^(9/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d
^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(9/4)/b^(11/4)*2^(1/2)/((b*x^
2+a)^2)^(1/2)-35/8192*d^(9/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2
)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(9/4)/b^(11/4)*2^(1/2)/((b*x^2+a)^
2)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7d^3(dx)^{3/2}}{256ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7d^3(dx)^{3/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (35*d^3*(d*x)^(3/2))/(1024*a^2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d
*x)^(7/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7*d^3*(d*
x)^(3/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7*d^3*(
d*x)^(3/2))/(256*a*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*d
^(9/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(2048*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*d^
(9/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(2048*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*d^(
9/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b
```

```
^2*x^4]) - (35*d^(9/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(7b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx)^{3/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + -$$

$$= -\frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx)^{3/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + -$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\left(a+\right.}$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\left(a+\right.}$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\left(a+\right.}$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\left(a+\right.}$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\left(a+\right.}$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\left(a+\right.}$$

Mathematica [C] time = 0.04, size = 86, normalized size = 0.15

$$\frac{2d^3(dx)^{3/2}\left(7\left(a+bx^2\right)^4\ {}_2F_1\left(\frac{3}{4},5;\frac{7}{4};-\frac{bx^2}{a}\right)-a^3\left(7a+13bx^2\right)\right)}{117a^3b^2\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*d^3*(d*x)^(3/2)*(-(a^3*(7*a + 13*b*x^2)) + 7*(a + b*x^2)^4*Hypergeometri
c2F1[3/4, 5, 7/4, -((b*x^2)/a)]))/(117*a^3*b^2*(a + b*x^2)^3*Sqrt[(a + b*x^
2)^2])
```

fricas [A] time = 0.94, size = 462, normalized size = 0.82

$$-\frac{420\left(a^2b^6x^8+4\,a^3b^5x^6+6\,a^4b^4x^4+4\,a^5b^3x^2+a^6b^2\right)\left(-\frac{d^{18}}{a^9b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{42875\,\sqrt{dx}\,a^2b^3d^{13}\left(-\frac{d^{18}}{a^9b^{11}}\right)^{\frac{1}{4}}-\sqrt{-1838265625\,a^5\cdots}}{4287\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12288*(420*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2
+ a^6*b^2)*(-d^18/(a^9*b^11))^(1/4)*arctan(-1/42875*(42875*sqrt(d*x)*a^2*b^
3*d^13*(-d^18/(a^9*b^11))^(1/4) - sqrt(-1838265625*a^5*b^5*d^18*sqrt(-d^18/
(a^9*b^11)) + 1838265625*d^27*x)*a^2*b^3*(-d^18/(a^9*b^11))^(1/4))/d^18) -
105*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2)
*(-d^18/(a^9*b^11))^(1/4)*log(42875*a^7*b^8*(-d^18/(a^9*b^11))^(3/4) + 4287
5*sqrt(d*x)*d^13) + 105*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^
5*b^3*x^2 + a^6*b^2)*(-d^18/(a^9*b^11))^(1/4)*log(-42875*a^7*b^8*(-d^18/(a^
9*b^11))^(3/4) + 42875*sqrt(d*x)*d^13) - 4*(105*b^3*d^4*x^7 + 399*a*b^2*d^4
*x^5 - 125*a^2*b*d^4*x^3 - 35*a^3*d^4*x)*sqrt(d*x))/(a^2*b^6*x^8 + 4*a^3*b^
5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2)
```

giac [A] time = 0.36, size = 421, normalized size = 0.75

$$\frac{1}{24576}\,d^4\left(\frac{210\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^5d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{210\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^5d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^4*(210*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^5*d*sgn(b*d^4*x^2 + a*d^
4)) + 210*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^5*d*sgn(b*d^4*x^2 + a*d^4)) -
105*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) +
 sqrt(a*d^2/b))/(a^3*b^5*d*sgn(b*d^4*x^2 + a*d^4)) + 105*sqrt(2)*(a*b^3*d^2
)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b
^5*d*sgn(b*d^4*x^2 + a*d^4)) + 8*(105*sqrt(d*x)*b^3*d^8*x^7 + 399*sqrt(d*x)
*a*b^2*d^8*x^5 - 125*sqrt(d*x)*a^2*b*d^8*x^3 - 35*sqrt(d*x)*a^3*d^8*x)/((b*
d^2*x^2 + a*d^2)^4*a^2*b^2*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.03, size = 1051, normalized size = 1.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(105*2^(1/2)*b^4*d^8*x^8*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1
/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/
2)))+210*2^(1/2)*b^4*d^8*x^8*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(
a/b*d^2)^(1/4))+210*2^(1/2)*b^4*d^8*x^8*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^
2)^(1/4))/(a/b*d^2)^(1/4))+840*(a/b*d^2)^(1/4)*(d*x)^(15/2)*b^4+420*2^(1/2)
*a*b^3*d^8*x^6*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2
))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+840*2^(1/2)*a
*b^3*d^8*x^6*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+
840*2^(1/2)*a*b^3*d^8*x^6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b
*d^2)^(1/4))+3192*(a/b*d^2)^(1/4)*(d*x)^(11/2)*a*b^3*d^2+630*2^(1/2)*a^2*b^
```

```
2*d^8*x^4*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d
*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+1260*2^(1/2)*a^2*b
^2*d^8*x^4*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+12
60*2^(1/2)*a^2*b^2*d^8*x^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/
b*d^2)^(1/4))-1000*(a/b*d^2)^(1/4)*(d*x)^(7/2)*a^2*b^2*d^4+420*2^(1/2)*a^3*
b*d^8*x^2*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d
*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+840*2^(1/2)*a^3*b*
d^8*x^2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+840*2
^(1/2)*a^3*b*d^8*x^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)
^(1/4))-280*(a/b*d^2)^(1/4)*(d*x)^(3/2)*a^3*b*d^6+105*2^(1/2)*a^4*d^8*ln(-(
-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+210*2^(1/2)*a^4*d^8*arctan((2^(1/
2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+210*2^(1/2)*a^4*d^8*arctan
((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4)))/d^3*(b*x^2+a)/(a/b
*d^2)^(1/4)/b^3/a^2/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 3.69, size = 584, normalized size = 1.04

$$\frac{35\, d^{\frac{9}{2}} \left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\cdots}\right.}{a^{\frac{1}{4}}\cdots} \right.}{8192\, a^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 35/8192*d^(9/2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*
sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2
*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/
sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2
)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2
)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3
/4)))/(a^2*b^2) + 1/3072*(105*b^3*d^(9/2)*x^(15/2) + 447*a*b^2*d^(9/2)*x^(1
1/2) + 803*a^2*b*d^(9/2)*x^(7/2) + 77*a^3*d^(9/2)*x^(3/2))/(a^2*b^6*x^8 + 4
*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2) - 1/192*((3*b^4*d^(
9/2)*x^5 + 14*a*b^3*d^(9/2)*x^3 - 21*a^2*b^2*d^(9/2)*x)*x^(9/2) + 2*(25*a*b
^3*d^(9/2)*x^5 + 66*a^2*b^2*d^(9/2)*x^3 + 9*a^3*b*d^(9/2)*x)*x^(5/2) + (15*
a^2*b^2*d^(9/2)*x^5 + 54*a^3*b*d^(9/2)*x^3 + 7*a^4*d^(9/2)*x)*sqrt(x))/(a^4
*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b^2 + (a*b^8*x^6 + 3*a^2*b^7
*x^4 + 3*a^3*b^6*x^2 + a^4*b^5)*x^6 + 3*(a^2*b^7*x^6 + 3*a^3*b^6*x^4 + 3*a^
```

```
4*b^5*x^2 + a^5*b^4)*x^4 + 3*(a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*x^2 +
 a^6*b^3)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{9/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{9}{2}}}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**(9/2)/((a + b*x**2)**2)**(5/2), x)
```

3.777 $\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=560

$$\frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5d^3\sqrt{dx}}{768ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^3\sqrt{dx}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{8b}$$

```
[Out] -1/8*d*(d*x)^(5/2)/b/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)-35/4096*d^(7/2)*(b*x^2
+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(11/4)/b^(9/4)*
2^(1/2)/((b*x^2+a)^2)^(1/2)+35/4096*d^(7/2)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1
/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(11/4)/b^(9/4)*2^(1/2)/((b*x^2+a)^2)^(1/
2)-35/8192*d^(7/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b
^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(11/4)/b^(9/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+35
/8192*d^(7/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4
)*2^(1/2)*(d*x)^(1/2))/a^(11/4)/b^(9/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+35/3072
*d^3*(d*x)^(1/2)/a^2/b^2/((b*x^2+a)^2)^(1/2)-5/96*d^3*(d*x)^(1/2)/b^2/(b*x^
2+a)^2/((b*x^2+a)^2)^(1/2)+5/768*d^3*(d*x)^(1/2)/a/b^2/(b*x^2+a)/((b*x^2+a)
^2)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5d^3\sqrt{dx}}{768ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^3\sqrt{dx}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (35*d^3*Sqrt[d*x])/(3072*a^2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x
)^(5/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*d^3*Sqrt[
d*x])/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*d^3*Sqrt[
d*x])/(768*a*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*d^(7/2)
*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(20
48*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*d^(7/2)*
(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(204
8*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*d^(7/2)*(
a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
```

```
]) + (35*d^(7/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3\sqrt{dx}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \cdots$$

$$= -\frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3\sqrt{dx}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \cdots$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a+\cdots\right.}$$

Mathematica [A] time = 0.31, size = 341, normalized size = 0.61

$$\frac{(dx)^{7/2}\left(a+bx^2\right)\left(-49152a^{11/4}b^{5/4}x^{5/2}+3080a^{3/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^3+1760a^{7/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^2+1280a^{11/4}\sqrt[4]{b}\sqrt{\cdots}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(7/2)*(a + b*x^2)*(-15360*a^(15/4)*b^(1/4)*Sqrt[x] - 49152*a^(11/4)*
b^(5/4)*x^(5/2) + 1280*a^(11/4)*b^(1/4)*Sqrt[x]*(a + b*x^2) + 1760*a^(7/4)*
b^(1/4)*Sqrt[x]*(a + b*x^2)^2 + 3080*a^(3/4)*b^(1/4)*Sqrt[x]*(a + b*x^2)^3
- 2310*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
+ 2310*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
- 1155*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x] + 1155*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x]))/(270336*a^(11/4)*b^(9/4)*x^(7/2)*((a + b*x^2)^2
)^(5/2))
```

fricas [A] time = 1.09, size = 455, normalized size = 0.81

$$\frac{420\left(a^2b^6x^8+4\,a^3b^5x^6+6\,a^4b^4x^4+4\,a^5b^3x^2+a^6b^2\right)\left(-\frac{d^{14}}{a^{11}b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^8b^7d^3\left(-\frac{d^{14}}{a^{11}b^9}\right)^{\frac{3}{4}}-\sqrt{a^6b^4\sqrt{-\frac{d^{14}}{a^{11}b^9}}+d^7x}\,a^8b^7\left(\cdots\right.}{d^{14}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12288*(420*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 +
 a^6*b^2)*(-d^14/(a^11*b^9))^(1/4)*arctan(-(sqrt(d*x)*a^8*b^7*d^3*(-d^14/(a
^11*b^9))^(3/4) - sqrt(a^6*b^4*sqrt(-d^14/(a^11*b^9)) + d^7*x)*a^8*b^7*(-d^
14/(a^11*b^9))^(3/4))/d^14) + 105*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*
x^4 + 4*a^5*b^3*x^2 + a^6*b^2)*(-d^14/(a^11*b^9))^(1/4)*log(35*a^3*b^2*(-d^
14/(a^11*b^9))^(1/4) + 35*sqrt(d*x)*d^3) - 105*(a^2*b^6*x^8 + 4*a^3*b^5*x^6
 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2)*(-d^14/(a^11*b^9))^(1/4)*log(-3
5*a^3*b^2*(-d^14/(a^11*b^9))^(1/4) + 35*sqrt(d*x)*d^3) + 4*(35*b^3*d^3*x^6
+ 125*a*b^2*d^3*x^4 - 399*a^2*b*d^3*x^2 - 105*a^3*d^3)*sqrt(d*x))/(a^2*b^6*
x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2)
```

giac [A] time = 0.35, size = 408, normalized size = 0.73

$$\frac{1}{24576}d^3\left(\frac{210\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{210\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^3*(210*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^3*sgn(b*d^4*x^2 + a*d^4)
) + 210*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^3*sgn(b*d^4*x^2 + a*d^4)) + 105*
sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqr
t(a*d^2/b))/(a^3*b^3*sgn(b*d^4*x^2 + a*d^4)) - 105*sqrt(2)*(a*b^3*d^2)^(1/4
)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^3*sgn
(b*d^4*x^2 + a*d^4)) + 8*(35*sqrt(d*x)*b^3*d^8*x^6 + 125*sqrt(d*x)*a*b^2*d^
8*x^4 - 399*sqrt(d*x)*a^2*b*d^8*x^2 - 105*sqrt(d*x)*a^3*d^8)/((b*d^2*x^2 +
a*d^2)^4*a^2*b^2*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 1136, normalized size = 2.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(105*(a/b*d^2)^(1/4)*2^(1/2)*b^4*d^6*x^8*ln((d*x+(a/b*d^2)^(1/4)*(d
*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a/b*d^2)^(1/2)))+210*(a/b*d^2)^(1/4)*2^(1/2)*b^4*d^6*x^8*arctan((2^(1/2)*
(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+210*(a/b*d^2)^(1/4)*2^(1/2)*b
^4*d^6*x^8*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+42
0*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^6*x^6*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)
*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2
)^(1/2)))+840*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^6*x^6*arctan((2^(1/2)*(d*x)^(
1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+840*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^
6*x^6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+630*(a/
```

```
b*d^2)^(1/4)*2^(1/2)*a^2*b^2*d^6*x^4*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^
(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(
1/2)))+1260*(a/b*d^2)^(1/4)*2^(1/2)*a^2*b^2*d^6*x^4*arctan((2^(1/2)*(d*x)^(
1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+1260*(a/b*d^2)^(1/4)*2^(1/2)*a^2*b^2
*d^6*x^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+280*
(d*x)^(13/2)*a*b^3+420*(a/b*d^2)^(1/4)*2^(1/2)*a^3*b*d^6*x^2*ln((d*x+(a/b*d
^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(
1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+840*(a/b*d^2)^(1/4)*2^(1/2)*a^3*b*d^6*x^2*ar
ctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+840*(a/b*d^2)^(
1/4)*2^(1/2)*a^3*b*d^6*x^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/
b*d^2)^(1/4))+1000*(d*x)^(9/2)*a^2*b^2*d^2+105*(a/b*d^2)^(1/4)*2^(1/2)*a^4*
d^6*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*
d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+210*(a/b*d^2)^(1/4)*2^(1/2
)*a^4*d^6*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+210
*(a/b*d^2)^(1/4)*2^(1/2)*a^4*d^6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4
))/(a/b*d^2)^(1/4))-3192*(d*x)^(5/2)*a^3*b*d^4-840*(d*x)^(1/2)*a^4*d^6)/d^3
*(b*x^2+a)/b^2/a^3/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 3.81, size = 597, normalized size = 1.07

$$-\frac{77\,b^3 d^{\frac{7}{2}} x^{\frac{13}{2}} + 803\,a b^2 d^{\frac{7}{2}} x^{\frac{9}{2}} + 447\,a^2 b d^{\frac{7}{2}} x^{\frac{5}{2}} + 105\,a^3 d^{\frac{7}{2}} \sqrt{x}}{3072\left(a^2 b^6 x^8 + 4\,a^3 b^5 x^6 + 6\,a^4 b^4 x^4 + 4\,a^5 b^3 x^2 + a^6 b^2\right)} + \frac{\left(7\,b^4 d^{\frac{7}{2}} x^5 + 54\,a b^3 d^{\frac{7}{2}} x^3 + 15\,a^2 \cdots\right.}{192\left(a^5 b^4 x^6 + 3\,a^6 b^3 x^4 + 3\,a^7 b^2 x^2 + a^8 b + \left(a^2 b^7 x^6 + \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3072*(77*b^3*d^(7/2)*x^(13/2) + 803*a*b^2*d^(7/2)*x^(9/2) + 447*a^2*b*d^
(7/2)*x^(5/2) + 105*a^3*d^(7/2)*sqrt(x))/(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a
^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2) + 1/192*((7*b^4*d^(7/2)*x^5 + 54*a*b^
3*d^(7/2)*x^3 + 15*a^2*b^2*d^(7/2)*x)*x^(11/2) + 2*(9*a*b^3*d^(7/2)*x^5 + 6
6*a^2*b^2*d^(7/2)*x^3 + 25*a^3*b*d^(7/2)*x)*x^(7/2) - (21*a^2*b^2*d^(7/2)*x
^5 - 14*a^3*b*d^(7/2)*x^3 - 3*a^4*d^(7/2)*x)*x^(3/2))/(a^5*b^4*x^6 + 3*a^6*
b^3*x^4 + 3*a^7*b^2*x^2 + a^8*b + (a^2*b^7*x^6 + 3*a^3*b^6*x^4 + 3*a^4*b^5*
x^2 + a^5*b^4)*x^6 + 3*(a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*x^2 + a^6*b
^3)*x^4 + 3*(a^4*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b^2)*x^2) +
35/8192*d^3*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
+ 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
 + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(
```

```
b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2
)*sqrt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/
4)*b^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b
)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a^2*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{7/2}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{7}{2}}}{\left(\left(a + bx^{2}\right)^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**(7/2)/((a + b*x**2)**2)**(5/2), x)
```

3.778 $\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=557

$$\frac{9d(dx)^{3/2}}{256a^2b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d(dx)^{3/2}}{32ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 45/1024*d*(d*x)^(3/2)/a^3/b/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(3/2)/b/(b*x^2+
a)^3/((b*x^2+a)^2)^(1/2)+1/32*d*(d*x)^(3/2)/a/b/(b*x^2+a)^2/((b*x^2+a)^2)^(
1/2)+9/256*d*(d*x)^(3/2)/a^2/b/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-45/4096*d^(5/2
)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(13/4)/
b^(7/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+45/4096*d^(5/2)*(b*x^2+a)*arctan(1+b^(1
/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(13/4)/b^(7/4)*2^(1/2)/((b*x^2+a
)^2)^(1/2)+45/8192*d^(5/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a
^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(13/4)/b^(7/4)*2^(1/2)/((b*x^2+a)^2)^
(1/2)-45/8192*d^(5/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4
)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(13/4)/b^(7/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{45d^{5/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{4096\sqrt{2}\,a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{}\right.}{4096\sqrt{2}\,a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (45*d*(d*x)^(3/2))/(1024*a^3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^
(3/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*(d*x)^(3/2)
)/(32*a*b*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (9*d*(d*x)^(3/2)
)/(256*a^2*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*d^(5/2)*(a
+ b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*S
qrt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^(5/2)*(a +
 b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sq
rt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^(5/2)*(a +
b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(4096*Sqrt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
 (45*d^(5/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*
```

```
a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d(dx)^{3/2}}{32ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \cdots$$

$$= -\frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d(dx)^{3/2}}{32ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \cdots$$

$$= \frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{32ab\left(a+\cdots\right.}$$

$$= \frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{32ab\left(a+\cdots\right.}$$

$$= \frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{32ab\left(a+\cdots\right.}$$

$$= \frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{32ab\left(a+\cdots\right.}$$

$$= \frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{32ab\left(a+\cdots\right.}$$

$$= \frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{32ab\left(a+\cdots\right.}$$

Mathematica [C] time = 0.03, size = 73, normalized size = 0.13

$$\frac{2d(dx)^{3/2}\left(\left(a+bx^2\right)^4 {}_2F_1\left(\frac{3}{4},5;\frac{7}{4};-\frac{bx^2}{a}\right)-a^4\right)}{13a^4b\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*d*(d*x)^(3/2)*(-a^4 + (a + b*x^2)^4*Hypergeometric2F1[3/4, 5, 7/4, -((b*
x^2)/a)]))/(13*a^4*b*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

fricas [A] time = 1.09, size = 454, normalized size = 0.82

$$-\frac{180\left(a^3b^5x^8+4\,a^4b^4x^6+6\,a^5b^3x^4+4\,a^6b^2x^2+a^7b\right)\left(-\frac{d^{10}}{a^{13}b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{91125\,\sqrt{dx}\,a^3b^2d^7\left(-\frac{d^{10}}{a^{13}b^7}\right)^{\frac{1}{4}}-\sqrt{-8303765625\,a^7b^3}}{91125}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4096*(180*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 +
 a^7*b)*(-d^10/(a^13*b^7))^(1/4)*arctan(-1/91125*(91125*sqrt(d*x)*a^3*b^2*d
^7*(-d^10/(a^13*b^7))^(1/4) - sqrt(-8303765625*a^7*b^3*d^10*sqrt(-d^10/(a^1
3*b^7)) + 8303765625*d^15*x)*a^3*b^2*(-d^10/(a^13*b^7))^(1/4))/d^10) - 45*(
a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b)*(-d^10
/(a^13*b^7))^(1/4)*log(91125*a^10*b^5*(-d^10/(a^13*b^7))^(3/4) + 91125*sqrt
(d*x)*d^7) + 45*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^
2 + a^7*b)*(-d^10/(a^13*b^7))^(1/4)*log(-91125*a^10*b^5*(-d^10/(a^13*b^7))^
(3/4) + 91125*sqrt(d*x)*d^7) - 4*(45*b^3*d^2*x^7 + 171*a*b^2*d^2*x^5 + 239*
a^2*b*d^2*x^3 - 15*a^3*d^2*x)*sqrt(d*x))/(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a
^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b)
```

giac [A] time = 0.37, size = 421, normalized size = 0.76

$$\frac{1}{8192}d^2\left(\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^4d\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^4d\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{45\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8192*d^2*(90*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^4*d*sgn(b*d^4*x^2 + a*d^4)
) + 90*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^4*d*sgn(b*d^4*x^2 + a*d^4)) - 45*
sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqr
t(a*d^2/b))/(a^4*b^4*d*sgn(b*d^4*x^2 + a*d^4)) + 45*sqrt(2)*(a*b^3*d^2)^(3/
4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^4*d*
sgn(b*d^4*x^2 + a*d^4)) + 8*(45*sqrt(d*x)*b^3*d^8*x^7 + 171*sqrt(d*x)*a*b^2
*d^8*x^5 + 239*sqrt(d*x)*a^2*b*d^8*x^3 - 15*sqrt(d*x)*a^3*d^8*x)/((b*d^2*x^
2 + a*d^2)^4*a^3*b*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 1051, normalized size = 1.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/8192*(45*2^(1/2)*b^4*d^8*x^8*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2
)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)
))+90*2^(1/2)*b^4*d^8*x^8*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b
*d^2)^(1/4))+90*2^(1/2)*b^4*d^8*x^8*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(
1/4))/(a/b*d^2)^(1/4))+360*(a/b*d^2)^(1/4)*(d*x)^(15/2)*b^4+180*2^(1/2)*a*b
^3*d^8*x^6*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(
d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+360*2^(1/2)*a*b^3
*d^8*x^6*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+360*
2^(1/2)*a*b^3*d^8*x^6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2
)^(1/4))+1368*(a/b*d^2)^(1/4)*(d*x)^(11/2)*a*b^3*d^2+270*2^(1/2)*a^2*b^2*d^
```

```
8*x^4*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(
a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+540*2^(1/2)*a^2*b^2*d^
8*x^4*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+540*2^(
1/2)*a^2*b^2*d^8*x^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)
^(1/4))+1912*(a/b*d^2)^(1/4)*(d*x)^(7/2)*a^2*b^2*d^4+180*2^(1/2)*a^3*b*d^8*
x^2*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/
b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+360*2^(1/2)*a^3*b*d^8*x^
2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+360*2^(1/2)
*a^3*b*d^8*x^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4)
)-120*(a/b*d^2)^(1/4)*(d*x)^(3/2)*a^3*b*d^6+45*2^(1/2)*a^4*d^8*ln(-(-d*x+(a
/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+90*2^(1/2)*a^4*d^8*arctan((2^(1/2)*(d*x)
^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+90*2^(1/2)*a^4*d^8*arctan((2^(1/2)
*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4)))/d^5*(b*x^2+a)/(a/b*d^2)^(1/
4)/b^2/a^3/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 3.71, size = 582, normalized size = 1.04

$$\frac{135\,b^3d^{\frac{5}{2}}x^{\frac{15}{2}} + 657\,ab^2d^{\frac{5}{2}}x^{\frac{11}{2}} + 173\,a^2bd^{\frac{5}{2}}x^{\frac{7}{2}} + 35\,a^3d^{\frac{5}{2}}x^{\frac{3}{2}}}{3072\left(a^3b^5x^8 + 4\,a^4b^4x^6 + 6\,a^5b^3x^4 + 4\,a^6b^2x^2 + a^7b\right)} - \frac{\left(9\,b^4d^{\frac{5}{2}}x^5 - 38\,ab^3d^{\frac{5}{2}}x^3 - 15\,a^2b^2 \cdots\right.}{192\left(a^5b^4x^6 + 3\,a^6b^3x^4 + 3\,a^7b^2x^2 + a^8b + \left(a^2b^7x^6 + 3 \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3072*(135*b^3*d^(5/2)*x^(15/2) + 657*a*b^2*d^(5/2)*x^(11/2) + 173*a^2*b*d
^(5/2)*x^(7/2) + 35*a^3*d^(5/2)*x^(3/2))/(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a
^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b) - 1/192*((9*b^4*d^(5/2)*x^5 - 38*a*b^3*
d^(5/2)*x^3 - 15*a^2*b^2*d^(5/2)*x)*x^(9/2) + 2*(11*a*b^3*d^(5/2)*x^5 - 42*
a^2*b^2*d^(5/2)*x^3 - 21*a^3*b*d^(5/2)*x)*x^(5/2) + (45*a^2*b^2*d^(5/2)*x^5
 + 18*a^3*b*d^(5/2)*x^3 + 5*a^4*d^(5/2)*x)*sqrt(x))/(a^5*b^4*x^6 + 3*a^6*b^
3*x^4 + 3*a^7*b^2*x^2 + a^8*b + (a^2*b^7*x^6 + 3*a^3*b^6*x^4 + 3*a^4*b^5*x^
2 + a^5*b^4)*x^6 + 3*(a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*x^2 + a^6*b^3
)*x^4 + 3*(a^4*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b^2)*x^2) + 45
/8192*d^(5/2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sq
rt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sq
rt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*
a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*
```

```
log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4
)))/(a^3*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{5/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{5}{2}}}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral((d*x)**(5/2)/((a + b*x**2)**2)**(5/2), x)
```

3.779 $\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}} dx$

Optimal. Leaf size=557

$$\frac{11d\sqrt{dx}}{768a^2b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d\sqrt{dx}}{96ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -77/4096*d^(3/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(
1/2))/a^(15/4)/b^(5/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+77/4096*d^(3/2)*(b*x^2+a
)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(15/4)/b^(5/4)*2^
(1/2)/((b*x^2+a)^2)^(1/2)-77/8192*d^(3/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^
(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(15/4)/b^(5/4)*2^(1/2)
/((b*x^2+a)^2)^(1/2)+77/8192*d^(3/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)
*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(15/4)/b^(5/4)*2^(1/2)/((b*
x^2+a)^2)^(1/2)+77/3072*d*(d*x)^(1/2)/a^3/b/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)
^(1/2)/b/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)+1/96*d*(d*x)^(1/2)/a/b/(b*x^2+a)^2
/((b*x^2+a)^2)^(1/2)+11/768*d*(d*x)^(1/2)/a^2/b/(b*x^2+a)/((b*x^2+a)^2)^(1/
2)
```

Rubi [A] time = 0.43, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{77d^{3/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{3/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\right.}{4096\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (77*d*Sqrt[d*x])/(3072*a^3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*Sqrt[d*x
])/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*Sqrt[d*x])/(96*
a*b*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (11*d*Sqrt[d*x])/(768*
a^2*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^(3/2)*(a + b*x^2
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*
a^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^(3/2)*(a + b*x^2)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a
^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^(3/2)*(a + b*x^2)*
Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]
])/(4096*Sqrt[2]*a^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^
```

```
(3/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d\sqrt{dx}}{96ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \cdots$$

$$= -\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d\sqrt{dx}}{96ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \cdots$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96ab\left(a+\cdots\right.}$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96ab\left(a+\cdots\right.}$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96ab\left(a+\cdots\right.}$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96ab\left(a+\cdots\right.}$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96ab\left(a+\cdots\right.}$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96ab\left(a+\cdots\right.}$$

Mathematica [A] time = 0.30, size = 324, normalized size = 0.58

$$(dx)^{3/2}\left(a+bx^2\right)\left(616a^{3/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^3+352a^{7/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^2+256a^{11/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)-3072a^{15/4}\sqrt[4]{\vphantom{b}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(3/2)*(a + b*x^2)*(-3072*a^(15/4)*b^(1/4)*Sqrt[x] + 256*a^(11/4)*b^(
1/4)*Sqrt[x]*(a + b*x^2) + 352*a^(7/4)*b^(1/4)*Sqrt[x]*(a + b*x^2)^2 + 616*
a^(3/4)*b^(1/4)*Sqrt[x]*(a + b*x^2)^3 - 462*Sqrt[2]*(a + b*x^2)^4*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 462*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 +
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 231*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 231*Sqrt[2]*(a + b*x^2)^4
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(24576*a^(15/
4)*b^(5/4)*x^(3/2)*((a + b*x^2)^2)^(5/2))
```

fricas [A] time = 1.14, size = 429, normalized size = 0.77

$$924\left(a^3b^5x^8+4\,a^4b^4x^6+6\,a^5b^3x^4+4\,a^6b^2x^2+a^7b\right)\left(-\frac{d^6}{a^{15}b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^{11}b^4d\left(-\frac{d^6}{a^{15}b^5}\right)^{\frac{3}{4}}-\sqrt{a^8b^2\sqrt{-\frac{d^6}{a^{15}b^5}}+d^3x}\,a^{11}b^4\left(\right.}{d^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12288*(924*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 +
 a^7*b)*(-d^6/(a^15*b^5))^(1/4)*arctan(-(sqrt(d*x)*a^11*b^4*d*(-d^6/(a^15*b
^5))^(3/4) - sqrt(a^8*b^2*sqrt(-d^6/(a^15*b^5)) + d^3*x)*a^11*b^4*(-d^6/(a^
15*b^5))^(3/4))/d^6) + 231*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4
*a^6*b^2*x^2 + a^7*b)*(-d^6/(a^15*b^5))^(1/4)*log(77*a^4*b*(-d^6/(a^15*b^5)
)^(1/4) + 77*sqrt(d*x)*d) - 231*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^
4 + 4*a^6*b^2*x^2 + a^7*b)*(-d^6/(a^15*b^5))^(1/4)*log(-77*a^4*b*(-d^6/(a^1
5*b^5))^(1/4) + 77*sqrt(d*x)*d) + 4*(77*b^3*d*x^6 + 275*a*b^2*d*x^4 + 351*a
^2*b*d*x^2 - 231*a^3*d)*sqrt(d*x))/(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3
*x^4 + 4*a^6*b^2*x^2 + a^7*b)
```

giac [A] time = 0.35, size = 406, normalized size = 0.73

$$\frac{1}{24576}d\left(\frac{462\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{462\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{23}{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d*(462*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^2*sgn(b*d^4*x^2 + a*d^4))
+ 462*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4
) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^2*sgn(b*d^4*x^2 + a*d^4)) + 231*sq
rt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(a^4*b^2*sgn(b*d^4*x^2 + a*d^4)) - 231*sqrt(2)*(a*b^3*d^2)^(1/4)*
log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^2*sgn(b
*d^4*x^2 + a*d^4)) + 8*(77*sqrt(d*x)*b^3*d^8*x^6 + 275*sqrt(d*x)*a*b^2*d^8*
x^4 + 351*sqrt(d*x)*a^2*b*d^8*x^2 - 231*sqrt(d*x)*a^3*d^8)/((b*d^2*x^2 + a*
d^2)^4*a^3*b*sgn(b*d^4*x^2 + a*d^4)))
```

maple [B] time = 0.02, size = 1136, normalized size = 2.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(231*(a/b*d^2)^(1/4)*2^(1/2)*b^4*d^6*x^8*ln((d*x+(a/b*d^2)^(1/4)*(d
*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a/b*d^2)^(1/2)))+462*(a/b*d^2)^(1/4)*2^(1/2)*b^4*d^6*x^8*arctan((2^(1/2)*
(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+462*(a/b*d^2)^(1/4)*2^(1/2)*b
^4*d^6*x^8*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+92
4*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^6*x^6*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)
*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2
)^(1/2)))+1848*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^6*x^6*arctan((2^(1/2)*(d*x)^
(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+1848*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*
d^6*x^6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+1386*
```

```
(a/b*d^2)^(1/4)*2^(1/2)*a^2*b^2*d^6*x^4*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)
*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2
)^(1/2)))+2772*(a/b*d^2)^(1/4)*2^(1/2)*a^2*b^2*d^6*x^4*arctan((2^(1/2)*(d*x
)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+2772*(a/b*d^2)^(1/4)*2^(1/2)*a^2*
b^2*d^6*x^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+6
16*(d*x)^(13/2)*a*b^3+924*(a/b*d^2)^(1/4)*2^(1/2)*a^3*b*d^6*x^2*ln((d*x+(a/
b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x
)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+1848*(a/b*d^2)^(1/4)*2^(1/2)*a^3*b*d^6*x^
2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+1848*(a/b*d
^2)^(1/4)*2^(1/2)*a^3*b*d^6*x^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4)
)/(a/b*d^2)^(1/4))+2200*(d*x)^(9/2)*a^2*b^2*d^2+231*(a/b*d^2)^(1/4)*2^(1/2)
*a^4*d^6*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-
(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+462*(a/b*d^2)^(1/4)*2
^(1/2)*a^4*d^6*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4)
)+462*(a/b*d^2)^(1/4)*2^(1/2)*a^4*d^6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)
^(1/4))/(a/b*d^2)^(1/4))+2808*(d*x)^(5/2)*a^3*b*d^4-1848*(d*x)^(1/2)*a^4*d^
6)/d^5*(b*x^2+a)/b/a^4/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 3.64, size = 586, normalized size = 1.05

$$-\frac{385\, b^3 d^{\frac{3}{2}} x^{\frac{13}{2}} + 495\, a b^2 d^{\frac{3}{2}} x^{\frac{9}{2}} + 315\, a^2 b d^{\frac{3}{2}} x^{\frac{5}{2}} + 77\, a^3 d^{\frac{3}{2}} \sqrt{x}}{1024 \left(a^3 b^5 x^8 + 4\, a^4 b^4 x^6 + 6\, a^5 b^3 x^4 + 4\, a^6 b^2 x^2 + a^7 b\right)} + \frac{\left(77\, b^4 d^{\frac{3}{2}} x^5 + 66\, a b^3 d^{\frac{3}{2}} x^3 + 21\, a^2 b^2 d^{\frac{3}{2}} \cdots\right.}{192 \left(a^6 b^3 x^6 + 3\, a^7 b^2 x^4 + 3\, a^8 b x^2 + a^9 + \left(a^3 b^6 x^6 + 3 \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/1024*(385*b^3*d^(3/2)*x^(13/2) + 495*a*b^2*d^(3/2)*x^(9/2) + 315*a^2*b*d
^(3/2)*x^(5/2) + 77*a^3*d^(3/2)*sqrt(x))/(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a
^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b) + 1/192*((77*b^4*d^(3/2)*x^5 + 66*a*b^3
*d^(3/2)*x^3 + 21*a^2*b^2*d^(3/2)*x)*x^(11/2) + 2*(99*a*b^3*d^(3/2)*x^5 + 1
02*a^2*b^2*d^(3/2)*x^3 + 35*a^3*b*d^(3/2)*x)*x^(7/2) + (153*a^2*b^2*d^(3/2)
*x^5 + 202*a^3*b*d^(3/2)*x^3 + 81*a^4*d^(3/2)*x)*x^(3/2))/(a^6*b^3*x^6 + 3*
a^7*b^2*x^4 + 3*a^8*b*x^2 + a^9 + (a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*
x^2 + a^6*b^3)*x^6 + 3*(a^4*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b
^2)*x^4 + 3*(a^5*b^4*x^6 + 3*a^6*b^3*x^4 + 3*a^7*b^2*x^2 + a^8*b)*x^2) + 77
/8192*d*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*
sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2
*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*s
```

```
qrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*sq
rt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b
^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
+ sqrt(a))/(a^(3/4)*b^(1/4)))/(a^3*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{3/2}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral((d*x)**(3/2)/((a + b*x**2)**2)**(5/2), x)
```

3.780 $\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=556

$$\frac{13(dx)^{3/2}}{96a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{195\sqrt{d}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{4096\sqrt{2}a^{17/4}b^{3/4}\sqrt{a^2+}}$$

```
[Out] 195/1024*(d*x)^(3/2)/a^4/d/((b*x^2+a)^2)^(1/2)+1/8*(d*x)^(3/2)/a/d/(b*x^2+a
)^3/((b*x^2+a)^2)^(1/2)+13/96*(d*x)^(3/2)/a^2/d/(b*x^2+a)^2/((b*x^2+a)^2)^(
1/2)+39/256*(d*x)^(3/2)/a^3/d/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-195/4096*(b*x^2
+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*d^(1/2)/a^(17/4)/
b^(3/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+195/4096*(b*x^2+a)*arctan(1+b^(1/4)*2^(
1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*d^(1/2)/a^(17/4)/b^(3/4)*2^(1/2)/((b*x^2+
a)^2)^(1/2)+195/8192*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)
*b^(1/4)*2^(1/2)*(d*x)^(1/2))*d^(1/2)/a^(17/4)/b^(3/4)*2^(1/2)/((b*x^2+a)^2
)^(1/2)-195/8192*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(
1/4)*2^(1/2)*(d*x)^(1/2))*d^(1/2)/a^(17/4)/b^(3/4)*2^(1/2)/((b*x^2+a)^2)^(1
/2)
```

Rubi [A] time = 0.43, antiderivative size = 556, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{39(dx)^{3/2}}{256a^3d\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13(dx)^{3/2}}{96a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (195*(d*x)^(3/2))/(1024*a^4*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*x)^(3/2
)/(8*a*d*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13*(d*x)^(3/2))/
(96*a^2*d*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (39*(d*x)^(3/2))
/(256*a^3*d*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (195*Sqrt[d]*(a
+ b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*S
qrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (195*Sqrt[d]*(a
+ b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*S
qrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (195*Sqrt[d]*(a
+ b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d*x]])/(4096*Sqrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

```
 - (195*Sqrt[d]*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(13b^3\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^4}\,dx}{16a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{13(dx)^{3/2}}{96a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \cdots$$

$$= \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{13(dx)^{3/2}}{96a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \cdots$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96a^2d\left(a\cdots\right.}$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96a^2d\left(a\cdots\right.}$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96a^2d\left(a\cdots\right.}$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96a^2d\left(a\cdots\right.}$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96a^2d\left(a\cdots\right.}$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{96a^2d\left(a\cdots\right.}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.10

$$\frac{2x\sqrt{dx}\,\left(a+bx^2\right)^5\,{}_2F_1\left(\frac{3}{4},5;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^5\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*x*Sqrt[d*x]*(a + b*x^2)^5*Hypergeometric2F1[3/4, 5, 7/4, -((b*x^2)/a)])/
(3*a^5*((a + b*x^2)^2)^(5/2))
```

fricas [A] time = 1.32, size = 414, normalized size = 0.74

$$-\frac{2340\left(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8\right)\left(-\frac{d^2}{a^{17}b^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{7414875\,\sqrt{dx}\,a^4bd\left(-\frac{d^2}{a^{17}b^3}\right)^{\frac{1}{4}}-\sqrt{-54980371265625\,\ldots}}{7414\ldots}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12288*(2340*(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 +
 a^8)*(-d^2/(a^17*b^3))^(1/4)*arctan(-1/7414875*(7414875*sqrt(d*x)*a^4*b*d*
(-d^2/(a^17*b^3))^(1/4) - sqrt(-54980371265625*a^9*b*d^2*sqrt(-d^2/(a^17*b^
3)) + 54980371265625*d^3*x)*a^4*b*(-d^2/(a^17*b^3))^(1/4))/d^2) - 585*(a^4*
b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)*(-d^2/(a^17*b^
3))^(1/4)*log(7414875*a^13*b^2*(-d^2/(a^17*b^3))^(3/4) + 7414875*sqrt(d*x)*
d) + 585*(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)*
(-d^2/(a^17*b^3))^(1/4)*log(-7414875*a^13*b^2*(-d^2/(a^17*b^3))^(3/4) + 741
4875*sqrt(d*x)*d) - 4*(585*b^3*x^7 + 2223*a*b^2*x^5 + 3107*a^2*b*x^3 + 1853
*a^3*x)*sqrt(d*x))/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x
^2 + a^8)
```

giac [A] time = 0.38, size = 406, normalized size = 0.73

$$\frac{\frac{1170\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{1170\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{585\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{\ldots}\right.}{a^5b^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")

[Out] 1/24576*(1170*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^3*sgn(b*d^4*x^2 + a*d^4)) + 1170*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^3*sgn(b*d^4*x^2 + a*d^4)) - 585*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b^3*sgn(b*d^4*x^2 + a*d^4)) + 585*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b^3*sgn(b*d^4*x^2 + a*d^4)) + 8*(585*sqrt(d*x)*b^3*d^9*x^7 + 2223*sqrt(d*x)*a*b^2*d^9*x^5 + 3107*sqrt(d*x)*a^2*b*d^9*x^3 + 1853*sqrt(d*x)*a^3*d^9*x)/((b*d^2*x^2 + a*d^2)^4*a^4*sgn(b*d^4*x^2 + a*d^4)))/d

maple [B] time = 0.02, size = 1051, normalized size = 1.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)

[Out] 1/24576*(585*2^(1/2)*b^4*d^8*x^8*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+1170*2^(1/2)*b^4*d^8*x^8*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+1170*2^(1/2)*b^4*d^8*x^8*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+4680*(a/b*d^2)^(1/4)*(d*x)^(15/2)*b^4+2340*2^(1/2)*a*b^3*d^8*x^6*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+4680*2^(1/2)*a*b^3*d^8*x^6*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+4680*2^(1/2)*a*b^3*d^8*x^6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+17784*(a/b*d^2)^(1/4)*(d*x)^(11/2)*a*b^3*d^2+3510*2^(1/2)*a^2*b^2*d^8*x^4*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+7020*2^(1/2)*a^2*b^2*d^8*x^4*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+7020*2^(1/2)*a^2*b^2*d^8*x^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+24856*(a/b*d^2)^(1/4)*(d*x)^(7/2)*a^2*b^2*d^4+2340*2^(1/2)*a^3*b*d^8*x^2*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+4680*2^(1/2)*a^3*b*d^8*x^2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+4680*2^(1/2)*a^3*b*d^8*x^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+14824*(a/b*d^2)^(1/4)*(d*x)^(3/2)*a^3*b*d^6+585*2^(1/2)*a^4*d^8*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+1170*2^(1/2)*a^4*d^8*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+1170*2^(1/2

```
)*a^4*d^8*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4)))/d^
7*(b*x^2+a)/(a/b*d^2)^(1/4)/b/a^4/((b*x^2+a)^2)^(5/2)
```

maxima [A] time = 3.79, size = 569, normalized size = 1.02

$$\frac{195\,b^3\sqrt{d}\,x^{\frac{15}{2}}+117\,ab^2\sqrt{d}\,x^{\frac{11}{2}}+65\,a^2b\sqrt{d}\,x^{\frac{7}{2}}+15\,a^3\sqrt{d}\,x^{\frac{3}{2}}}{1024\left(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8\right)}+\frac{\left(117\,b^4\sqrt{d}\,x^5+130\,ab^3\sqrt{d}\,x^3+45\,a^2b^2\sqrt{d}\right.}{192\left(a^6b^3x^6+3\,a^7b^2x^4+3\,a^8bx^2+a^9+\left(a^3b^6x^6+3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/1024*(195*b^3*sqrt(d)*x^(15/2) + 117*a*b^2*sqrt(d)*x^(11/2) + 65*a^2*b*sq
rt(d)*x^(7/2) + 15*a^3*sqrt(d)*x^(3/2))/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^
6*b^2*x^4 + 4*a^7*b*x^2 + a^8) + 1/192*((117*b^4*sqrt(d)*x^5 + 130*a*b^3*sq
rt(d)*x^3 + 45*a^2*b^2*sqrt(d)*x)*x^(9/2) + 2*(143*a*b^3*sqrt(d)*x^5 + 174*
a^2*b^2*sqrt(d)*x^3 + 63*a^3*b*sqrt(d)*x)*x^(5/2) + (201*a^2*b^2*sqrt(d)*x^
5 + 282*a^3*b*sqrt(d)*x^3 + 113*a^4*sqrt(d)*x)*sqrt(x))/(a^6*b^3*x^6 + 3*a^
7*b^2*x^4 + 3*a^8*b*x^2 + a^9 + (a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*x^
2 + a^6*b^3)*x^6 + 3*(a^4*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b^2
)*x^4 + 3*(a^5*b^4*x^6 + 3*a^6*b^3*x^4 + 3*a^7*b^2*x^2 + a^8*b)*x^2) + 195/
8192*sqrt(d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqr
t(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqr
t(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*l
og(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)
))/a^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

3.781 $\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=556

$$\frac{5\sqrt{dx}}{32a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1155\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right.}{4096\sqrt{2}\,a^{19/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+}}$$

```
[Out] -1155/4096*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/
a^(19/4)/b^(1/4)*2^(1/2)/d^(1/2)/((b*x^2+a)^2)^(1/2)+1155/4096*(b*x^2+a)*ar
ctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(19/4)/b^(1/4)*2^(1/2
)/d^(1/2)/((b*x^2+a)^2)^(1/2)-1155/8192*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1
/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(19/4)/b^(1/4)*2^(1/2)/d
^(1/2)/((b*x^2+a)^2)^(1/2)+1155/8192*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)
*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(19/4)/b^(1/4)*2^(1/2)/d^(1
/2)/((b*x^2+a)^2)^(1/2)+385/1024*(d*x)^(1/2)/a^4/d/((b*x^2+a)^2)^(1/2)+1/8*
(d*x)^(1/2)/a/d/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)+5/32*(d*x)^(1/2)/a^2/d/(b*x
^2+a)^2/((b*x^2+a)^2)^(1/2)+55/256*(d*x)^(1/2)/a^3/d/(b*x^2+a)/((b*x^2+a)^2
)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 556, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1112, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{55\sqrt{dx}}{256a^3d\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\sqrt{dx}}{32a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (385*Sqrt[d*x])/(1024*a^4*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + Sqrt[d*x]/(8
*a*d*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*Sqrt[d*x])/(32*a^2
*d*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (55*Sqrt[d*x])/(256*a^3
*d*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (1155*(a + b*x^2)*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(19/4)*
b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1155*(a + b*x^2)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(19/4)
*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (1155*(a + b*x^2)*Log[S
qrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4
096*Sqrt[2]*a^(19/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (11
```

```
55*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(19/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\,\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(15b^3\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^4}\,d}{16a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{5\sqrt{dx}}{32a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x}}$$

$$= \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{5\sqrt{dx}}{32a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x}}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{32a}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{32a}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{32a}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{32a}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{32a}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{32a}$$

Mathematica [A] time = 0.13, size = 319, normalized size = 0.57

$$\frac{\sqrt{x}\left(a+bx^2\right)\left(3080a^{3/4}\sqrt{x}\left(a+bx^2\right)^3+1760a^{7/4}\sqrt{x}\left(a+bx^2\right)^2+1280a^{11/4}\sqrt{x}\left(a+bx^2\right)+1024a^{15/4}\sqrt{x}-\frac{1155\sqrt{\cdots}}{\cdots}\cdots\right.}{8\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (Sqrt[x]*(a + b*x^2)*(1024*a^(15/4)*Sqrt[x] + 1280*a^(11/4)*Sqrt[x]*(a + b*
x^2) + 1760*a^(7/4)*Sqrt[x]*(a + b*x^2)^2 + 3080*a^(3/4)*Sqrt[x]*(a + b*x^2
)^3 - (2310*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/b^(1/4) + (2310*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/b^(1/4) - (1155*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4) + (1155*Sqrt[2]*(a + b*x^2)^
4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4)))/(81
92*a^(19/4)*Sqrt[d*x]*((a + b*x^2)^2)^(5/2))
```

fricas [A] time = 0.94, size = 416, normalized size = 0.75

$$\frac{4620\left(a^4b^4dx^8+4\,a^5b^3dx^6+6\,a^6b^2dx^4+4\,a^7bdx^2+a^8d\right)\left(-\frac{1}{a^{19}bd^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^{10}d^2\sqrt{-\frac{1}{a^{19}bd^2}}+dx}\,a^{14}bd\left(-\frac{1}{a^{19}b\cdots}\right.\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/4096*(4620*(a^4*b^4*d*x^8 + 4*a^5*b^3*d*x^6 + 6*a^6*b^2*d*x^4 + 4*a^7*b*d
*x^2 + a^8*d)*(-1/(a^19*b*d^2))^(1/4)*arctan(sqrt(a^10*d^2*sqrt(-1/(a^19*b*
d^2)) + d*x)*a^14*b*d*(-1/(a^19*b*d^2))^(3/4) - sqrt(d*x)*a^14*b*d*(-1/(a^1
9*b*d^2))^(3/4)) + 1155*(a^4*b^4*d*x^8 + 4*a^5*b^3*d*x^6 + 6*a^6*b^2*d*x^4
+ 4*a^7*b*d*x^2 + a^8*d)*(-1/(a^19*b*d^2))^(1/4)*log(a^5*d*(-1/(a^19*b*d^2)
)^(1/4) + sqrt(d*x)) - 1155*(a^4*b^4*d*x^8 + 4*a^5*b^3*d*x^6 + 6*a^6*b^2*d*
x^4 + 4*a^7*b*d*x^2 + a^8*d)*(-1/(a^19*b*d^2))^(1/4)*log(-a^5*d*(-1/(a^19*b
*d^2))^(1/4) + sqrt(d*x)) + 4*(385*b^3*x^6 + 1375*a*b^2*x^4 + 1755*a^2*b*x^
2 + 893*a^3)*sqrt(d*x))/(a^4*b^4*d*x^8 + 4*a^5*b^3*d*x^6 + 6*a^6*b^2*d*x^4
+ 4*a^7*b*d*x^2 + a^8*d)
```

giac [A] time = 0.33, size = 412, normalized size = 0.74

$$\frac{1155\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^5bd\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{1155\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^5bd\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{1155\sqrt{2}\,(a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1155/4096*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(
1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b*d*sgn(b*d^4*x^2 + a*d^4)) + 115
5/4096*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b*d*sgn(b*d^4*x^2 + a*d^4)) + 1155/
8192*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
+ sqrt(a*d^2/b))/(a^5*b*d*sgn(b*d^4*x^2 + a*d^4)) - 1155/8192*sqrt(2)*(a*b^
3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(
a^5*b*d*sgn(b*d^4*x^2 + a*d^4)) + 1/1024*(385*sqrt(d*x)*b^3*d^7*x^6 + 1375*
sqrt(d*x)*a*b^2*d^7*x^4 + 1755*sqrt(d*x)*a^2*b*d^7*x^2 + 893*sqrt(d*x)*a^3*
d^7)/((b*d^2*x^2 + a*d^2)^4*a^4*sgn(b*d^4*x^2 + a*d^4))
```

maple [B] time = 0.02, size = 1133, normalized size = 2.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x)
```

```
[Out] 1/8192*(1155*(a/b*d^2)^(1/4)*2^(1/2)*b^4*d^6*x^8*ln((d*x+(a/b*d^2)^(1/4)*(d
*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a/b*d^2)^(1/2)))+2310*(a/b*d^2)^(1/4)*2^(1/2)*b^4*d^6*x^8*arctan((2^(1/2)
*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+2310*(a/b*d^2)^(1/4)*2^(1/2)
*b^4*d^6*x^8*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+
4620*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^6*x^6*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1
/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*
d^2)^(1/2)))+9240*(a/b*d^2)^(1/4)*2^(1/2)*a*b^3*d^6*x^6*arctan((2^(1/2)*(d*
x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+9240*(a/b*d^2)^(1/4)*2^(1/2)*a*b
^3*d^6*x^6*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+69
30*(a/b*d^2)^(1/4)*2^(1/2)*a^2*b^2*d^6*x^4*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1
/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*
d^2)^(1/2)))+13860*(a/b*d^2)^(1/4)*2^(1/2)*a^2*b^2*d^6*x^4*arctan((2^(1/2)*
```

```
(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+13860*(a/b*d^2)^(1/4)*2^(1/2)
*a^2*b^2*d^6*x^4*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/
4))+3080*(d*x)^(13/2)*a*b^3+4620*(a/b*d^2)^(1/4)*2^(1/2)*a^3*b*d^6*x^2*ln((
d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/
4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+9240*(a/b*d^2)^(1/4)*2^(1/2)*a^3*b
*d^6*x^2*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+9240
*(a/b*d^2)^(1/4)*2^(1/2)*a^3*b*d^6*x^2*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2
)^(1/4))/(a/b*d^2)^(1/4))+11000*(d*x)^(9/2)*a^2*b^2*d^2+1155*(a/b*d^2)^(1/4
)*2^(1/2)*a^4*d^6*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/
2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))+2310*(a/b*d^
2)^(1/4)*2^(1/2)*a^4*d^6*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*
d^2)^(1/4))+2310*(a/b*d^2)^(1/4)*2^(1/2)*a^4*d^6*arctan((2^(1/2)*(d*x)^(1/2
)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))+14040*(d*x)^(5/2)*a^3*b*d^4+7144*(d*x)^
(1/2)*a^4*d^6)/d^7*(b*x^2+a)/a^5/((b*x^2+a)^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{5267\,b^3x^{\frac{13}{2}} + 11645\,ab^2x^{\frac{9}{2}} + 9441\,a^2bx^{\frac{5}{2}} + 2679\,a^3\sqrt{x}}{3072\left(a^4b^4\sqrt{d}\,x^8 + 4\,a^5b^3\sqrt{d}\,x^6 + 6\,a^6b^2\sqrt{d}\,x^4 + 4\,a^7b\sqrt{d}\,x^2 + a^8\sqrt{d}\right)} - \frac{\left(257\,b^5\sqrt{d}\,x^5 + 378\,ab^4\sqrt{\cdots}\right.}{192\left(a^7b^3dx^6 + 3\,a^8b^2dx^4 + 3\,a^9bdx^2 + a^{10}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 1/3072*(5267*b^3*x^(13/2) + 11645*a*b^2*x^(9/2) + 9441*a^2*b*x^(5/2) + 2679
*a^3*sqrt(x))/(a^4*b^4*sqrt(d)*x^8 + 4*a^5*b^3*sqrt(d)*x^6 + 6*a^6*b^2*sqrt
(d)*x^4 + 4*a^7*b*sqrt(d)*x^2 + a^8*sqrt(d)) - 1/192*((257*b^5*sqrt(d)*x^5
+ 378*a*b^4*sqrt(d)*x^3 + 153*a^2*b^3*sqrt(d)*x)*x^(11/2) + 2*(303*a*b^4*sq
rt(d)*x^5 + 462*a^2*b^3*sqrt(d)*x^3 + 191*a^3*b^2*sqrt(d)*x)*x^(7/2) + (381
*a^2*b^3*sqrt(d)*x^5 + 610*a^3*b^2*sqrt(d)*x^3 + 261*a^4*b*sqrt(d)*x)*x^(3/
2))/(a^7*b^3*d*x^6 + 3*a^8*b^2*d*x^4 + 3*a^9*b*d*x^2 + a^10*d + (a^4*b^6*d*
x^6 + 3*a^5*b^5*d*x^4 + 3*a^6*b^4*d*x^2 + a^7*b^3*d)*x^6 + 3*(a^5*b^5*d*x^6
 + 3*a^6*b^4*d*x^4 + 3*a^7*b^3*d*x^2 + a^8*b^2*d)*x^4 + 3*(a^6*b^4*d*x^6 +
3*a^7*b^3*d*x^4 + 3*a^8*b^2*d*x^2 + a^9*b*d)*x^2) - 893/8192*(2*sqrt(2)*sqr
t(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*sqrt(d)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*sqrt(d)*log(sqrt(2)*a^(1
```

```
/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*sqrt
(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^
(1/4)))/(a^4*d) + integrate(1/((a^4*b*sqrt(d)*x^2 + a^5*sqrt(d))*sqrt(x)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{d\,x}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
[Out] int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(1/2),x)
[Out] Integral(1/(sqrt(d*x)*((a + b*x**2)**2)**(5/2)), x)
```

3.782 $\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=602

$$\frac{17}{96a^2d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^2}+\frac{1}{8ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^3}-\frac{3315\sqrt[4]{b}\,\left(a+bx^2\right)\log\left(-\sqrt{\right.}}{4096\sqrt{2}\,a^{21/4}d}$$

[Out] 3315/4096*b^(1/4)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(21/4)/d^(3/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)-3315/4096*b^(1/4)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(21/4)/d^(3/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)-3315/8192*b^(1/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(21/4)/d^(3/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)+3315/8192*b^(1/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(21/4)/d^(3/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)+663/1024/a^4/d/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)+1/8/a/d/(b*x^2+a)^3/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)+17/96/a^2/d/(b*x^2+a)^2/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)+221/768/a^3/d/(b*x^2+a)/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)-3315/1024*(b*x^2+a)/a^5/d/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)

Rubi [A] time = 0.49, antiderivative size = 602, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3315\sqrt[4]{b}\,\left(a+bx^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{4096\sqrt{2}\,a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3315\sqrt[4]{b}\,\left(a+bx^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{\right.}}{4096\sqrt{2}\,a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2}}$$

Antiderivative was successfully verified.

[In] Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]

[Out] 663/(1024*a^4*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*d*Sqrt[d*x]*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 17/(96*a^2*d*Sqrt[d*x]*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 221/(768*a^3*d*Sqrt[d*x]*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3315*(a + b*x^2))/(1024*a^5*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3315*b^(1/4)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3315*b^(1/4)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3315*b^(1/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*

```
x]])/(4096*Sqrt[2]*a^(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (331
5*b^(1/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(17b^3\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{3/2}\left(ab\right.}}{16a\sqrt{a^2+2abx^2+b^2x}}$$

$$= \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{17}{96a^2d\sqrt{dx}\left(a+bx^2\right)^2\sqrt{a^2+}}$$

$$= \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{17}{96a^2d\sqrt{dx}\left(a+bx^2\right)^2\sqrt{a^2+}}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+}}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+}}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+}}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+}}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+}}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+}}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.09

$$-\frac{2x\left(a+bx^2\right)^5 {}_2F_1\left(-\frac{1}{4},5;\frac{3}{4};-\frac{bx^2}{a}\right)}{a^5(dx)^{3/2}\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-2*x*(a + b*x^2)^5*Hypergeometric2F1[-1/4, 5, 3/4, -((b*x^2)/a)])/(a^5*(d*
x)^(3/2)*((a + b*x^2)^2)^(5/2))
```

fricas [A] time = 1.11, size = 477, normalized size = 0.79

$$\frac{39780\left(a^5b^4d^2x^9+4\,a^6b^3d^2x^7+6\,a^7b^2d^2x^5+4\,a^8bd^2x^3+a^9d^2x\right)\left(-\frac{b}{a^{21}d^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{36429280875\,\sqrt{dx}\,a^5bd\left(-\frac{b}{a^{21}d^6}\right)^{\frac{1}{4}}-}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/12288*(39780*(a^5*b^4*d^2*x^9 + 4*a^6*b^3*d^2*x^7 + 6*a^7*b^2*d^2*x^5 + 4
*a^8*b*d^2*x^3 + a^9*d^2*x)*(-b/(a^21*d^6))^(1/4)*arctan(-1/36429280875*(36
429280875*sqrt(d*x)*a^5*b*d*(-b/(a^21*d^6))^(1/4) - sqrt(-13270925050696407
65625*a^11*b*d^4*sqrt(-b/(a^21*d^6)) + 1327092505069640765625*b^2*d*x)*a^5*
d*(-b/(a^21*d^6))^(1/4))/b) - 9945*(a^5*b^4*d^2*x^9 + 4*a^6*b^3*d^2*x^7 + 6
*a^7*b^2*d^2*x^5 + 4*a^8*b*d^2*x^3 + a^9*d^2*x)*(-b/(a^21*d^6))^(1/4)*log(3
6429280875*a^16*d^5*(-b/(a^21*d^6))^(3/4) + 36429280875*sqrt(d*x)*b) + 9945
*(a^5*b^4*d^2*x^9 + 4*a^6*b^3*d^2*x^7 + 6*a^7*b^2*d^2*x^5 + 4*a^8*b*d^2*x^3
 + a^9*d^2*x)*(-b/(a^21*d^6))^(1/4)*log(-36429280875*a^16*d^5*(-b/(a^21*d^6
))^(3/4) + 36429280875*sqrt(d*x)*b) - 4*(9945*b^4*x^8 + 37791*a*b^3*x^6 + 5
2819*a^2*b^2*x^4 + 31501*a^3*b*x^2 + 6144*a^4)*sqrt(d*x))/(a^5*b^4*d^2*x^9
+ 4*a^6*b^3*d^2*x^7 + 6*a^7*b^2*d^2*x^5 + 4*a^8*b*d^2*x^3 + a^9*d^2*x)
```

giac [A] time = 0.37, size = 448, normalized size = 0.74

$$-\frac{\frac{49152}{\sqrt{dx}\,a^{5}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{19890\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{6}b^{2}d^{2}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{19890\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{6}b^{2}d^{2}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}-\frac{9945}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24576*(49152/(sqrt(d*x)*a^5*sgn(b*d^4*x^2 + a*d^4)) + 19890*sqrt(2)*(a*b
^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a
*d^2/b)^(1/4))/(a^6*b^2*d^2*sgn(b*d^4*x^2 + a*d^4)) + 19890*sqrt(2)*(a*b^3*
d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d
^2/b)^(1/4))/(a^6*b^2*d^2*sgn(b*d^4*x^2 + a*d^4)) - 9945*sqrt(2)*(a*b^3*d^2
)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b
^2*d^2*sgn(b*d^4*x^2 + a*d^4)) + 9945*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - s
qrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b^2*d^2*sgn(b*d^4*x^
2 + a*d^4)) + 8*(3801*sqrt(d*x)*b^4*d^7*x^7 + 13215*sqrt(d*x)*a*b^3*d^7*x^5
 + 15955*sqrt(d*x)*a^2*b^2*d^7*x^3 + 6925*sqrt(d*x)*a^3*b*d^7*x)/((b*d^2*x^
2 + a*d^2)^4*a^5*sgn(b*d^4*x^2 + a*d^4)))/d
```

maple [B] time = 0.03, size = 1081, normalized size = 1.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24576/d*(9945*(d*x)^(1/2)*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*
2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)
^(1/2)))*x^8*b^4+19890*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b
*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^8*b^4+19890*(d*x)^(1/2)*2^(1/2)*arctan((2^(
1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^8*b^4+39780*(d*x)^(1/2
)*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d
*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^6*a*b^3+79560*(d
*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1
/4))*x^6*a*b^3+79560*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d
^2)^(1/4))/(a/b*d^2)^(1/4))*x^6*a*b^3+79560*(a/b*d^2)^(1/4)*x^8*b^4+59670*(
d*x)^(1/2)*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^
(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^4*a^2*b
```

```
^2+119340*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/
(a/b*d^2)^(1/4))*x^4*a^2*b^2+119340*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*
x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a^2*b^2+302328*(a/b*d^2)^(1/
4)*x^6*a*b^3+39780*(d*x)^(1/2)*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2
)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^
2)^(1/2)))*x^2*a^3*b+79560*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+
(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a^3*b+79560*(d*x)^(1/2)*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a^3*b+422552*(
a/b*d^2)^(1/4)*x^4*a^2*b^2+9945*(d*x)^(1/2)*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/
4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^
(1/2)+(a/b*d^2)^(1/2)))*a^4+19890*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)
^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^4+19890*(d*x)^(1/2)*2^(1/2)*arct
an((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^4+252008*(a/b*d
^2)^(1/4)*x^2*a^3*b+49152*(a/b*d^2)^(1/4)*a^4)*(b*x^2+a)/(d*x)^(1/2)/(a/b*d
^2)^(1/4)/a^5/((b*x^2+a)^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3801\,b^4x^{\frac{15}{2}} + 8079\,ab^3x^{\frac{11}{2}} + 6515\,a^2b^2x^{\frac{7}{2}} + 1853\,a^3bx^{\frac{3}{2}}}{3072\left(a^5b^4d^{\frac{3}{2}}x^8 + 4\,a^6b^3d^{\frac{3}{2}}x^6 + 6\,a^7b^2d^{\frac{3}{2}}x^4 + 4\,a^8bd^{\frac{3}{2}}x^2 + a^9d^{\frac{3}{2}}\right)} - \frac{\left(321\,b^5\sqrt{d}\,x^5 + 490\,a\right.}{192\left(a^7b^3d^2x^6 + 3\,a^8b^2d^2x^4 + 3\,a^9bd^2x^2 + a^{10}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] -1/3072*(3801*b^4*x^(15/2) + 8079*a*b^3*x^(11/2) + 6515*a^2*b^2*x^(7/2) + 1
853*a^3*b*x^(3/2))/(a^5*b^4*d^(3/2)*x^8 + 4*a^6*b^3*d^(3/2)*x^6 + 6*a^7*b^2
*d^(3/2)*x^4 + 4*a^8*b*d^(3/2)*x^2 + a^9*d^(3/2)) - 1/192*((321*b^5*sqrt(d)
*x^5 + 490*a*b^4*sqrt(d)*x^3 + 201*a^2*b^3*sqrt(d)*x)*x^(9/2) + 2*(371*a*b^
4*sqrt(d)*x^5 + 582*a^2*b^3*sqrt(d)*x^3 + 243*a^3*b^2*sqrt(d)*x)*x^(5/2) +
(453*a^2*b^3*sqrt(d)*x^5 + 738*a^3*b^2*sqrt(d)*x^3 + 317*a^4*b*sqrt(d)*x)*s
qrt(x))/(a^7*b^3*d^2*x^6 + 3*a^8*b^2*d^2*x^4 + 3*a^9*b*d^2*x^2 + a^10*d^2 +
 (a^4*b^6*d^2*x^6 + 3*a^5*b^5*d^2*x^4 + 3*a^6*b^4*d^2*x^2 + a^7*b^3*d^2)*x^
6 + 3*(a^5*b^5*d^2*x^6 + 3*a^6*b^4*d^2*x^4 + 3*a^7*b^3*d^2*x^2 + a^8*b^2*d^
2)*x^4 + 3*(a^6*b^4*d^2*x^6 + 3*a^7*b^3*d^2*x^4 + 3*a^8*b^2*d^2*x^2 + a^9*b
*d^2)*x^2) - 1267/8192*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(
1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqr
t(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*
```

```
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*l
og(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4))
 + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(
1/4)*b^(3/4)))/(a^5*d^(3/2)) + integrate(1/((a^4*b*d^(3/2)*x^2 + a^5*d^(3/2
))*x^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d\,x)^{3/2}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
[Out] int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral(1/((d*x)**(3/2)*((a + b*x**2)**2)**(5/2)), x)
```

3.783 $\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=602

$$\frac{19}{96a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^2}+\frac{1}{8ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^3}+\frac{7315b^{3/4}\left(a+bx^2\right)\log\left(\right.}{4096\sqrt{2}\,a^2}$$

```
[Out] 1045/1024/a^4/d/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2)+1/8/a/d/(d*x)^(3/2)/(b*x^2+
a)^3/((b*x^2+a)^2)^(1/2)+19/96/a^2/d/(d*x)^(3/2)/(b*x^2+a)^2/((b*x^2+a)^2)^
(1/2)+95/256/a^3/d/(d*x)^(3/2)/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-7315/3072*(b*x
^2+a)/a^5/d/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2)+7315/4096*b^(3/4)*(b*x^2+a)*arc
tan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(23/4)/d^(5/2)*2^(1/2)
/((b*x^2+a)^2)^(1/2)-7315/4096*b^(3/4)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(
d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(23/4)/d^(5/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)+73
15/8192*b^(3/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1
/4)*2^(1/2)*(d*x)^(1/2))/a^(23/4)/d^(5/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)-7315/
8192*b^(3/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)
*2^(1/2)*(d*x)^(1/2))/a^(23/4)/d^(5/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 602, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7315b^{3/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{4096\sqrt{2}\,a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7315b^{3/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}\right.}{4096\sqrt{2}\,a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 1045/(1024*a^4*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*d*(d
*x)^(3/2)*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 19/(96*a^2*d*(d*
x)^(3/2)*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 95/(256*a^3*d*(d*
x)^(3/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7315*(a + b*x^2))/
(3072*a^5*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7315*b^(3/4)*(a
 + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*
Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7315*b^(3/4)*(
a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048
*Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7315*b^(3/4)*
(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/
```

```
4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) - (7315*b^(3/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(19b^3\left(ab+b^2x^2\right)\right)\int \frac{}{(dx)^{5/2}(}}{16a\sqrt{a^2+2abx^2+b}}$$

$$= \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{19}{96a^2d(dx)^{3/2}\left(a+bx^2\right)^2\sqrt{a^2}}$$

$$= \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{19}{96a^2d(dx)^{3/2}\left(a+bx^2\right)^2\sqrt{a^2}}$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2}}$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2}}$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2}}$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2}}$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2}}$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2}}$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2}}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.09

$$-\frac{2x\left(a+bx^2\right)^5 {}_2F_1\left(-\frac{3}{4},5;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^5(dx)^{5/2}\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]

[Out] (-2*x*(a + b*x^2)^5*Hypergeometric2F1[-3/4, 5, 1/4, -((b*x^2)/a)])/(3*a^5*(d*x)^(5/2)*((a + b*x^2)^2)^(5/2))

fricas [A] time = 1.13, size = 501, normalized size = 0.83

$$-\frac{87780\left(a^5b^4d^3x^{10}+4\,a^6b^3d^3x^8+6\,a^7b^2d^3x^6+4\,a^8bd^3x^4+a^9d^3x^2\right)\left(-\frac{b^3}{a^{23}d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^{17}bd^7\left(-\frac{b^3}{a^{23}d^{10}}\right)^{\frac{3}{4}}-\sqrt{a^{12}\cdots}}{\cdots}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")

[Out] -1/12288*(87780*(a^5*b^4*d^3*x^10 + 4*a^6*b^3*d^3*x^8 + 6*a^7*b^2*d^3*x^6 + 4*a^8*b*d^3*x^4 + a^9*d^3*x^2)*(-b^3/(a^23*d^10))^(1/4)*arctan(-(sqrt(d*x)*a^17*b*d^7*(-b^3/(a^23*d^10))^(3/4) - sqrt(a^12*d^6*sqrt(-b^3/(a^23*d^10)) + b^2*d*x)*a^17*d^7*(-b^3/(a^23*d^10))^(3/4))/b^3) + 21945*(a^5*b^4*d^3*x^10 + 4*a^6*b^3*d^3*x^8 + 6*a^7*b^2*d^3*x^6 + 4*a^8*b*d^3*x^4 + a^9*d^3*x^2)*(-b^3/(a^23*d^10))^(1/4)*log(7315*a^6*d^3*(-b^3/(a^23*d^10))^(1/4) + 7315*sqrt(d*x)*b) - 21945*(a^5*b^4*d^3*x^10 + 4*a^6*b^3*d^3*x^8 + 6*a^7*b^2*d^3*x^6 + 4*a^8*b*d^3*x^4 + a^9*d^3*x^2)*(-b^3/(a^23*d^10))^(1/4)*log(-7315*a^6*d^3*(-b^3/(a^23*d^10))^(1/4) + 7315*sqrt(d*x)*b) + 4*(7315*b^4*x^8 + 26125*a*b^3*x^6 + 33345*a^2*b^2*x^4 + 16967*a^3*b*x^2 + 2048*a^4)*sqrt(d*x))/(a^5*b^4*d^3*x^10 + 4*a^6*b^3*d^3*x^8 + 6*a^7*b^2*d^3*x^6 + 4*a^8*b*d^3*x^4 + a^9*d^3*x^2)

giac [A] time = 0.74, size = 439, normalized size = 0.73

$$-\frac{7315\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^{6}d^{3}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}-\frac{7315\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^{6}d^{3}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}-\frac{7315\,\sqrt{2}\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -7315/4096*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*d^3*sgn(b*d^4*x^2 + a*d^4)) - 73
15/4096*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*d^3*sgn(b*d^4*x^2 + a*d^4)) - 7315
/8192*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
 + sqrt(a*d^2/b))/(a^6*d^3*sgn(b*d^4*x^2 + a*d^4)) + 7315/8192*sqrt(2)*(a*b
^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/
(a^6*d^3*sgn(b*d^4*x^2 + a*d^4)) - 2/3/(sqrt(d*x)*a^5*d^2*x*sgn(b*d^4*x^2 +
 a*d^4)) - 1/3072*(5267*sqrt(d*x)*b^4*d^6*x^6 + 17933*sqrt(d*x)*a*b^3*d^6*x
^4 + 21057*sqrt(d*x)*a^2*b^2*d^6*x^2 + 8775*sqrt(d*x)*a^3*b*d^6)/((b*d^2*x^
2 + a*d^2)^4*a^5*d*sgn(b*d^4*x^2 + a*d^4))
```

maple [B] time = 0.03, size = 1183, normalized size = 1.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24576/d^3*(21945*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*
2^(1/2)+(a/b*d^2)^(1/2)))*x^8*b^5+43890*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)
*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^8*b^5+4389
0*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)
^(1/4))/(a/b*d^2)^(1/4))*x^8*b^5+87780*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*
ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)
^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^6*a*b^4+175560*(d*x)^(3/2)*(
a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^
2)^(1/4))*x^6*a*b^4+175560*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1
/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^6*a*b^4+131670*(d*x)^(3
/2)*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/
```

```
b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^
4*a^2*b^3+263340*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^
(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a^2*b^3+263340*(d*x)^(3/2)*(a/b
*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^
(1/4))*x^4*a^2*b^3+58520*x^8*a*b^4*d^2+87780*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^
(1/2)*ln((d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/
b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a^3*b^2+175560*(d*x)
^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))
/(a/b*d^2)^(1/4))*x^2*a^3*b^2+175560*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*ar
ctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a^3*b^2+209
000*x^6*a^2*b^3*d^2+21945*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*ln((d*x+(a/b*
d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2))/(d*x-(a/b*d^2)^(1/4)*(d*x)^
(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*a^4*b+43890*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(
1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*a^4*b+43
890*(d*x)^(3/2)*(a/b*d^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^
2)^(1/4))/(a/b*d^2)^(1/4))*a^4*b+266760*x^4*a^3*b^2*d^2+135736*x^2*a^4*b*d^
2+16384*a^5*d^2)*(b*x^2+a)/(d*x)^(3/2)/a^6/((b*x^2+a)^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-4\,b\int \frac{1}{\left(a^5bd^{\frac{5}{2}}x^2+a^6d^{\frac{5}{2}}\right)\sqrt{x}}\,dx-\frac{13795\,b^4x^{\frac{13}{2}}+34285\,ab^3x^{\frac{9}{2}}+29649\,a^2b^2x^{\frac{5}{2}}+8775\,a^3b\sqrt{x}}{3072\left(a^5b^4d^{\frac{5}{2}}x^8+4\,a^6b^3d^{\frac{5}{2}}x^6+6\,a^7b^2d^{\frac{5}{2}}x^4+4\,a^8bd^{\frac{5}{2}}x^2+a^9d^{\frac{5}{2}}\right)}+\frac{}{192\left(a^8b^3d^{\frac{5}{2}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] -4*b*integrate(1/((a^5*b*d^(5/2)*x^2 + a^6*d^(5/2))*sqrt(x)), x) - 1/3072*(
13795*b^4*x^(13/2) + 34285*a*b^3*x^(9/2) + 29649*a^2*b^2*x^(5/2) + 8775*a^3
*b*sqrt(x))/(a^5*b^4*d^(5/2)*x^8 + 4*a^6*b^3*d^(5/2)*x^6 + 6*a^7*b^2*d^(5/2
)*x^4 + 4*a^8*b*d^(5/2)*x^2 + a^9*d^(5/2)) + 1/192*((533*b^6*x^5 + 882*a*b^
5*x^3 + 381*a^2*b^4*x)*x^(11/2) + 2*(603*a*b^5*x^5 + 1014*a^2*b^4*x^3 + 443
*a^3*b^3*x)*x^(7/2) + (705*a^2*b^4*x^5 + 1210*a^3*b^3*x^3 + 537*a^4*b^2*x)*
x^(3/2))/(a^8*b^3*d^(5/2)*x^6 + 3*a^9*b^2*d^(5/2)*x^4 + 3*a^10*b*d^(5/2)*x^
2 + a^11*d^(5/2) + (a^5*b^6*d^(5/2)*x^6 + 3*a^6*b^5*d^(5/2)*x^4 + 3*a^7*b^4
*d^(5/2)*x^2 + a^8*b^3*d^(5/2))*x^6 + 3*(a^6*b^5*d^(5/2)*x^6 + 3*a^7*b^4*d^
(5/2)*x^4 + 3*a^8*b^3*d^(5/2)*x^2 + a^9*b^2*d^(5/2))*x^4 + 3*(a^7*b^4*d^(5/
2)*x^6 + 3*a^8*b^3*d^(5/2)*x^4 + 3*a^9*b^2*d^(5/2)*x^2 + a^10*b*d^(5/2))*x^
```

```
2) + 2925/8192*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2
*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) +
2*sqrt(2)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)
*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(
2)*b^(3/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3
/4))/(a^5*d^(5/2)) + integrate(1/((a^4*b*d^(5/2)*x^2 + a^5*d^(5/2))*x^(5/2)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d\,x)^{5/2}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
[Out] int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx\right)^{\frac{5}{2}} \left(\left(a + bx^{2}\right)^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral(1/((d*x)**(5/2)*((a + b*x**2)**2)**(5/2)), x)
```

3.784 $\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=649

$$\frac{7}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923b^{5/4}\left(a+bx^2\right)\log}{4096\sqrt{2}\,a}$$

```
[Out] 1547/1024/a^4/d/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2)+1/8/a/d/(d*x)^(5/2)/(b*x^2+
a)^3/((b*x^2+a)^2)^(1/2)+7/32/a^2/d/(d*x)^(5/2)/(b*x^2+a)^2/((b*x^2+a)^2)^(
1/2)+119/256/a^3/d/(d*x)^(5/2)/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-13923/5120*(b*
x^2+a)/a^5/d/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2)-13923/4096*b^(5/4)*(b*x^2+a)*a
rctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(25/4)/d^(7/2)*2^(1/
2)/((b*x^2+a)^2)^(1/2)+13923/4096*b^(5/4)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2
)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(25/4)/d^(7/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)
+13923/8192*b^(5/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*
b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(25/4)/d^(7/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)-1
3923/8192*b^(5/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^
(1/4)*2^(1/2)*(d*x)^(1/2))/a^(25/4)/d^(7/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)+139
23/1024*b*(b*x^2+a)/a^6/d^3/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 649, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{13923b\left(a+bx^2\right)}{1024a^6d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923b^{5/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{4096\sqrt{2}\,a^{25/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923b^{5/4}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 1547/(1024*a^4*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*d*(d
*x)^(5/2)*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 7/(32*a^2*d*(d*x
)^(5/2)*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 119/(256*a^3*d*(d*
x)^(5/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13923*(a + b*x^2))
/(5120*a^5*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13923*b*(a + b
*x^2))/(1024*a^6*d^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13923*b^
(5/4)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(2048*Sqrt[2]*a^(25/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13923*
b^(5/4)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
```

```
)])/(2048*Sqrt[2]*a^(25/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1392
3*b^(5/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(25/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4]) - (13923*b^(5/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sq
rt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(25/4)*d^(7/2
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21b^3\left(ab+b^2x^2\right)\right)\int \frac{}{(dx)^{7/2}(\cdots}}{16a\sqrt{a^2+2abx^2+b\cdots}}$$

$$= \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{7}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^2\sqrt{a\cdots}}$$

$$= \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{7}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^2\sqrt{a\cdots}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx\cdots}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx\cdots}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx\cdots}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx\cdots}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx\cdots}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx\cdots}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx\cdots}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx\cdots}}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.08

$$-\frac{2x\left(a+bx^2\right)^5 {}_2F_1\left(-\frac{5}{4},5;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^5(dx)^{7/2}\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-2*x*(a + b*x^2)^5*Hypergeometric2F1[-5/4, 5, -1/4, -((b*x^2)/a)])/(5*a^5*
(d*x)^(7/2)*((a + b*x^2)^2)^(5/2))
```

fricas [A] time = 0.77, size = 524, normalized size = 0.81

$$-\frac{278460\left(a^6b^4d^4x^{11}+4\,a^7b^3d^4x^9+6\,a^8b^2d^4x^7+4\,a^9bd^4x^5+a^{10}d^4x^3\right)\left(-\frac{b^5}{a^{25}d^{14}}\right)^{\frac{1}{4}}\arctan\left(-\frac{2698972561467\,\sqrt{dx}\,a^6b^4d^3}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] -1/20480*(278460*(a^6*b^4*d^4*x^11 + 4*a^7*b^3*d^4*x^9 + 6*a^8*b^2*d^4*x^7
+ 4*a^9*b*d^4*x^5 + a^10*d^4*x^3)*(-b^5/(a^25*d^14))^(1/4)*arctan(-1/269897
2561467*(2698972561467*sqrt(d*x)*a^6*b^4*d^3*(-b^5/(a^25*d^14))^(1/4) - sqr
t(-7284452887551739093192089*a^13*b^5*d^8*sqrt(-b^5/(a^25*d^14)) + 72844528
87551739093192089*b^8*d*x)*a^6*d^3*(-b^5/(a^25*d^14))^(1/4))/b^5) - 69615*(
a^6*b^4*d^4*x^11 + 4*a^7*b^3*d^4*x^9 + 6*a^8*b^2*d^4*x^7 + 4*a^9*b*d^4*x^5
+ a^10*d^4*x^3)*(-b^5/(a^25*d^14))^(1/4)*log(2698972561467*a^19*d^11*(-b^5/
(a^25*d^14))^(3/4) + 2698972561467*sqrt(d*x)*b^4) + 69615*(a^6*b^4*d^4*x^11
 + 4*a^7*b^3*d^4*x^9 + 6*a^8*b^2*d^4*x^7 + 4*a^9*b*d^4*x^5 + a^10*d^4*x^3)*
(-b^5/(a^25*d^14))^(1/4)*log(-2698972561467*a^19*d^11*(-b^5/(a^25*d^14))^(3
/4) + 2698972561467*sqrt(d*x)*b^4) - 4*(69615*b^5*x^10 + 264537*a*b^4*x^8 +
 369733*a^2*b^3*x^6 + 220507*a^3*b^2*x^4 + 43008*a^4*b*x^2 - 2048*a^5)*sqrt
(d*x))/(a^6*b^4*d^4*x^11 + 4*a^7*b^3*d^4*x^9 + 6*a^8*b^2*d^4*x^7 + 4*a^9*b*
d^4*x^5 + a^10*d^4*x^3)
```

giac [A] time = 0.36, size = 470, normalized size = 0.72

$$\frac{13923\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^7bd^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{13923\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^7bd^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{13923\,\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 13923/4096*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*b*d^5*sgn(b*d^4*x^2 + a*d^4)) +
13923/4096*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)
^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*b*d^5*sgn(b*d^4*x^2 + a*d^4)) -
 13923/8192*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqr
t(d*x) + sqrt(a*d^2/b))/(a^7*b*d^5*sgn(b*d^4*x^2 + a*d^4)) + 13923/8192*sqr
t(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a
*d^2/b))/(a^7*b*d^5*sgn(b*d^4*x^2 + a*d^4)) + 1/1024*(3683*sqrt(d*x)*b^5*d^
7*x^7 + 12357*sqrt(d*x)*a*b^4*d^7*x^5 + 14145*sqrt(d*x)*a^2*b^3*d^7*x^3 + 5
599*sqrt(d*x)*a^3*b^2*d^7*x)/((b*d^2*x^2 + a*d^2)^4*a^6*d^3*sgn(b*d^4*x^2 +
 a*d^4)) + 2/5*(25*b*d^2*x^2 - a*d^2)/(sqrt(d*x)*a^6*d^5*x^2*sgn(b*d^4*x^2
+ a*d^4))
```

maple [B] time = 0.03, size = 1129, normalized size = 1.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/40960/d^3*(69615*(d*x)^(5/2)*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2
)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^
2)^(1/2)))*x^8*b^5+139230*(d*x)^(5/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(
a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^8*b^5+139230*(d*x)^(5/2)*2^(1/2)*arctan(
(2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^8*b^5+278460*(d*x)
^(5/2)*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2
))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^6*a*b^4+556
920*(d*x)^(5/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d
^2)^(1/4))*x^6*a*b^4+556920*(d*x)^(5/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)
-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^6*a*b^4+556920*(a/b*d^2)^(1/4)*x^10*b^
5*d^2+417690*(d*x)^(5/2)*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1
```

```
/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/
2)))*x^4*a^2*b^3+835380*(d*x)^(5/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/
b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a^2*b^3+835380*(d*x)^(5/2)*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^4*a^2*b^3+211629
6*(a/b*d^2)^(1/4)*x^8*a*b^4*d^2+278460*(d*x)^(5/2)*2^(1/2)*ln(-(-d*x+(a/b*d
^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(d*x+(a/b*d^2)^(1/4)*(d*x)^(
1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*x^2*a^3*b^2+556920*(d*x)^(5/2)*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4))*x^2*a^3*b^2+556920
*(d*x)^(5/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(1/4))/(a/b*d^2)
^(1/4))*x^2*a^3*b^2+2957864*(a/b*d^2)^(1/4)*x^6*a^2*b^3*d^2+69615*(d*x)^(5/
2)*2^(1/2)*ln(-(-d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)-(a/b*d^2)^(1/2))/(
d*x+(a/b*d^2)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a/b*d^2)^(1/2)))*a^4*b+139230*(d*x
)^(5/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a/b*d^2)^(1/4))/(a/b*d^2)^(1/4
))*a^4*b+139230*(d*x)^(5/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a/b*d^2)^(
1/4))/(a/b*d^2)^(1/4))*a^4*b+1764056*(a/b*d^2)^(1/4)*x^4*a^3*b^2*d^2+344064
*(a/b*d^2)^(1/4)*x^2*a^4*b*d^2-16384*(a/b*d^2)^(1/4)*a^5*d^2)*(b*x^2+a)/(d*
x)^(5/2)/(a/b*d^2)^(1/4)/a^6/((b*x^2+a)^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-4\,b\int\frac{1}{\left(a^5bd^{\frac{7}{2}}x^2+a^6d^{\frac{7}{2}}\right)x^{\frac{3}{2}}}\,dx+\frac{11049\,b^5x^{\frac{15}{2}}+27135\,ab^4x^{\frac{11}{2}}+23395\,a^2b^3x^{\frac{7}{2}}+6925\,a^3b^2x^{\frac{3}{2}}}{3072\left(a^6b^4d^{\frac{7}{2}}x^8+4\,a^7b^3d^{\frac{7}{2}}x^6+6\,a^8b^2d^{\frac{7}{2}}x^4+4\,a^9bd^{\frac{7}{2}}x^2+a^{10}d^{\frac{7}{2}}\right)}+\frac{}{192\left(a^8b^3d^{\frac{7}{2}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] -4*b*integrate(1/((a^5*b*d^(7/2)*x^2 + a^6*d^(7/2))*x^(3/2)), x) + 1/3072*(
11049*b^5*x^(15/2) + 27135*a*b^4*x^(11/2) + 23395*a^2*b^3*x^(7/2) + 6925*a^
3*b^2*x^(3/2))/(a^6*b^4*d^(7/2)*x^8 + 4*a^7*b^3*d^(7/2)*x^6 + 6*a^8*b^2*d^(
7/2)*x^4 + 4*a^9*b*d^(7/2)*x^2 + a^10*d^(7/2)) + 1/192*((621*b^6*x^5 + 1042
*a*b^5*x^3 + 453*a^2*b^4*x)*x^(9/2) + 2*(695*a*b^5*x^5 + 1182*a^2*b^4*x^3 +
 519*a^3*b^3*x)*x^(5/2) + (801*a^2*b^4*x^5 + 1386*a^3*b^3*x^3 + 617*a^4*b^2
*x)*sqrt(x))/(a^8*b^3*d^(7/2)*x^6 + 3*a^9*b^2*d^(7/2)*x^4 + 3*a^10*b*d^(7/2
)*x^2 + a^11*d^(7/2) + (a^5*b^6*d^(7/2)*x^6 + 3*a^6*b^5*d^(7/2)*x^4 + 3*a^7
*b^4*d^(7/2)*x^2 + a^8*b^3*d^(7/2))*x^6 + 3*(a^6*b^5*d^(7/2)*x^6 + 3*a^7*b^
4*d^(7/2)*x^4 + 3*a^8*b^3*d^(7/2)*x^2 + a^9*b^2*d^(7/2))*x^4 + 3*(a^7*b^4*d
^(7/2)*x^6 + 3*a^8*b^3*d^(7/2)*x^4 + 3*a^9*b^2*d^(7/2)*x^2 + a^10*b*d^(7/2)
```

```
)*x^2) + 3683/8192*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/
4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(
b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sq
rt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log
(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) +
 sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/
4)*b^(3/4)))/(a^6*d^(7/2)) + integrate(1/((a^4*b*d^(7/2)*x^2 + a^5*d^(7/2))
*x^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d\,x)^{7/2}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
[Out] int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{7}{2}}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral(1/((d*x)**(7/2)*((a + b*x**2)**2)**(5/2)), x)
```

3.785 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^3\,dx$

Optimal. Leaf size=150

$$\frac{a^6(dx)^{m+1}}{d(m+1)}+\frac{6a^5b(dx)^{m+3}}{d^3(m+3)}+\frac{15a^4b^2(dx)^{m+5}}{d^5(m+5)}+\frac{20a^3b^3(dx)^{m+7}}{d^7(m+7)}+\frac{15a^2b^4(dx)^{m+9}}{d^9(m+9)}+\frac{6ab^5(dx)^{m+11}}{d^{11}(m+11)}+\frac{b^6(dx)^{m+13}}{d^{13}(m+13)}$$

```
[Out] a^6*(d*x)^(1+m)/d/(1+m)+6*a^5*b*(d*x)^(3+m)/d^3/(3+m)+15*a^4*b^2*(d*x)^(5+m
)/d^5/(5+m)+20*a^3*b^3*(d*x)^(7+m)/d^7/(7+m)+15*a^2*b^4*(d*x)^(9+m)/d^9/(9+
m)+6*a*b^5*(d*x)^(11+m)/d^11/(11+m)+b^6*(d*x)^(13+m)/d^13/(13+m)
```

Rubi [A] time = 0.12, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {28, 270}

$$\frac{15a^4b^2(dx)^{m+5}}{d^5(m+5)}+\frac{20a^3b^3(dx)^{m+7}}{d^7(m+7)}+\frac{15a^2b^4(dx)^{m+9}}{d^9(m+9)}+\frac{6a^5b(dx)^{m+3}}{d^3(m+3)}+\frac{a^6(dx)^{m+1}}{d(m+1)}+\frac{6ab^5(dx)^{m+11}}{d^{11}(m+11)}+\frac{b^6(dx)^{m+13}}{d^{13}(m+13)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*(d*x)^(1 + m))/(d*(1 + m)) + (6*a^5*b*(d*x)^(3 + m))/(d^3*(3 + m)) + (
15*a^4*b^2*(d*x)^(5 + m))/(d^5*(5 + m)) + (20*a^3*b^3*(d*x)^(7 + m))/(d^7*(
7 + m)) + (15*a^2*b^4*(d*x)^(9 + m))/(d^9*(9 + m)) + (6*a*b^5*(d*x)^(11 + m
))/(d^11*(11 + m)) + (b^6*(d*x)^(13 + m))/(d^13*(13 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^3\,dx = \frac{\int (dx)^m \left(ab+b^2x^2\right)^6\,dx}{b^6}$$
$$= \frac{\int \left(a^6b^6(dx)^m + \frac{6a^5b^7(dx)^{2+m}}{d^2} + \frac{15a^4b^8(dx)^{4+m}}{d^4} + \frac{20a^3b^9(dx)^{6+m}}{d^6} + \frac{15a^2b^{10}(dx)^{8+m}}{d^8} + \frac{6}{}\right.}{b^6}$$
$$= \frac{a^6(dx)^{1+m}}{d(1+m)} + \frac{6a^5b(dx)^{3+m}}{d^3(3+m)} + \frac{15a^4b^2(dx)^{5+m}}{d^5(5+m)} + \frac{20a^3b^3(dx)^{7+m}}{d^7(7+m)} + \frac{15a^2b^4(dx)}{d^9(9+}$$

Mathematica [A] time = 0.07, size = 105, normalized size = 0.70

$$x(dx)^m\left(\frac{a^6}{m+1}+\frac{6a^5bx^2}{m+3}+\frac{15a^4b^2x^4}{m+5}+\frac{20a^3b^3x^6}{m+7}+\frac{15a^2b^4x^8}{m+9}+\frac{6ab^5x^{10}}{m+11}+\frac{b^6x^{12}}{m+13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] x*(d*x)^m*(a^6/(1 + m) + (6*a^5*b*x^2)/(3 + m) + (15*a^4*b^2*x^4)/(5 + m) +
 (20*a^3*b^3*x^6)/(7 + m) + (15*a^2*b^4*x^8)/(9 + m) + (6*a*b^5*x^10)/(11 +
 m) + (b^6*x^12)/(13 + m))
```

fricas [B] time = 0.91, size = 507, normalized size = 3.38

$$\frac{\left(\left(b^6m^6+36\,b^6m^5+505\,b^6m^4+3480\,b^6m^3+12139\,b^6m^2+19524\,b^6m+10395\,b^6\right)x^{13}+6\left(ab^5m^6+38\,ab^5m^5\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] ((b^6*m^6 + 36*b^6*m^5 + 505*b^6*m^4 + 3480*b^6*m^3 + 12139*b^6*m^2 + 19524
*b^6*m + 10395*b^6)*x^13 + 6*(a*b^5*m^6 + 38*a*b^5*m^5 + 555*a*b^5*m^4 + 39
40*a*b^5*m^3 + 14039*a*b^5*m^2 + 22902*a*b^5*m + 12285*a*b^5)*x^11 + 15*(a^
2*b^4*m^6 + 40*a^2*b^4*m^5 + 613*a^2*b^4*m^4 + 4528*a^2*b^4*m^3 + 16627*a^2
*b^4*m^2 + 27688*a^2*b^4*m + 15015*a^2*b^4)*x^9 + 20*(a^3*b^3*m^6 + 42*a^3*
b^3*m^5 + 679*a^3*b^3*m^4 + 5292*a^3*b^3*m^3 + 20335*a^3*b^3*m^2 + 34986*a^
3*b^3*m + 19305*a^3*b^3)*x^7 + 15*(a^4*b^2*m^6 + 44*a^4*b^2*m^5 + 753*a^4*b
^2*m^4 + 6280*a^4*b^2*m^3 + 25979*a^4*b^2*m^2 + 47436*a^4*b^2*m + 27027*a^4
*b^2)*x^5 + 6*(a^5*b*m^6 + 46*a^5*b*m^5 + 835*a^5*b*m^4 + 7540*a^5*b*m^3 +
34759*a^5*b*m^2 + 73054*a^5*b*m + 45045*a^5*b)*x^3 + (a^6*m^6 + 48*a^6*m^5
+ 925*a^6*m^4 + 9120*a^6*m^3 + 48259*a^6*m^2 + 129072*a^6*m + 135135*a^6)*x
)*(d*x)^m/(m^7 + 49*m^6 + 973*m^5 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 26
4207*m + 135135)
```

giac [B] time = 0.25, size = 847, normalized size = 5.65

$$\frac{(dx)^m b^6m^6x^{13} + 36\,(dx)^m b^6m^5x^{13} + 6\,(dx)^m ab^5m^6x^{11} + 505\,(dx)^m b^6m^4x^{13} + 228\,(dx)^m ab^5m^5x^{11} + 3480\,(dx)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^6*m^6*x^13 + 36*(d*x)^m*b^6*m^5*x^13 + 6*(d*x)^m*a*b^5*m^6*x^11
+ 505*(d*x)^m*b^6*m^4*x^13 + 228*(d*x)^m*a*b^5*m^5*x^11 + 3480*(d*x)^m*b^6*
m^3*x^13 + 15*(d*x)^m*a^2*b^4*m^6*x^9 + 3330*(d*x)^m*a*b^5*m^4*x^11 + 12139
*(d*x)^m*b^6*m^2*x^13 + 600*(d*x)^m*a^2*b^4*m^5*x^9 + 23640*(d*x)^m*a*b^5*m
^3*x^11 + 19524*(d*x)^m*b^6*m*x^13 + 20*(d*x)^m*a^3*b^3*m^6*x^7 + 9195*(d*x
)^m*a^2*b^4*m^4*x^9 + 84234*(d*x)^m*a*b^5*m^2*x^11 + 10395*(d*x)^m*b^6*x^13
 + 840*(d*x)^m*a^3*b^3*m^5*x^7 + 67920*(d*x)^m*a^2*b^4*m^3*x^9 + 137412*(d*
x)^m*a*b^5*m*x^11 + 15*(d*x)^m*a^4*b^2*m^6*x^5 + 13580*(d*x)^m*a^3*b^3*m^4*
x^7 + 249405*(d*x)^m*a^2*b^4*m^2*x^9 + 73710*(d*x)^m*a*b^5*x^11 + 660*(d*x)
^m*a^4*b^2*m^5*x^5 + 105840*(d*x)^m*a^3*b^3*m^3*x^7 + 415320*(d*x)^m*a^2*b^
4*m*x^9 + 6*(d*x)^m*a^5*b*m^6*x^3 + 11295*(d*x)^m*a^4*b^2*m^4*x^5 + 406700*
(d*x)^m*a^3*b^3*m^2*x^7 + 225225*(d*x)^m*a^2*b^4*x^9 + 276*(d*x)^m*a^5*b*m^
5*x^3 + 94200*(d*x)^m*a^4*b^2*m^3*x^5 + 699720*(d*x)^m*a^3*b^3*m*x^7 + (d*x
)^m*a^6*m^6*x + 5010*(d*x)^m*a^5*b*m^4*x^3 + 389685*(d*x)^m*a^4*b^2*m^2*x^5
 + 386100*(d*x)^m*a^3*b^3*x^7 + 48*(d*x)^m*a^6*m^5*x + 45240*(d*x)^m*a^5*b*
m^3*x^3 + 711540*(d*x)^m*a^4*b^2*m*x^5 + 925*(d*x)^m*a^6*m^4*x + 208554*(d*
x)^m*a^5*b*m^2*x^3 + 405405*(d*x)^m*a^4*b^2*x^5 + 9120*(d*x)^m*a^6*m^3*x +
438324*(d*x)^m*a^5*b*m*x^3 + 48259*(d*x)^m*a^6*m^2*x + 270270*(d*x)^m*a^5*b
*x^3 + 129072*(d*x)^m*a^6*m*x + 135135*(d*x)^m*a^6*x)/(m^7 + 49*m^6 + 973*m
^5 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135)
```

maple [B] time = 0.01, size = 602, normalized size = 4.01

$$\frac{\left(b^6m^6x^{12} + 36b^6m^5x^{12} + 6a\,b^5m^6x^{10} + 505b^6m^4x^{12} + 228a\,b^5m^5x^{10} + 3480b^6m^3x^{12} + 15a^2b^4m^6x^8 + 3330a\,b^5m^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] (d*x)^m*(b^6*m^6*x^12+36*b^6*m^5*x^12+6*a*b^5*m^6*x^10+505*b^6*m^4*x^12+228
*a*b^5*m^5*x^10+3480*b^6*m^3*x^12+15*a^2*b^4*m^6*x^8+3330*a*b^5*m^4*x^10+12
139*b^6*m^2*x^12+600*a^2*b^4*m^5*x^8+23640*a*b^5*m^3*x^10+19524*b^6*m*x^12+
20*a^3*b^3*m^6*x^6+9195*a^2*b^4*m^4*x^8+84234*a*b^5*m^2*x^10+10395*b^6*x^12
+840*a^3*b^3*m^5*x^6+67920*a^2*b^4*m^3*x^8+137412*a*b^5*m*x^10+15*a^4*b^2*m
^6*x^4+13580*a^3*b^3*m^4*x^6+249405*a^2*b^4*m^2*x^8+73710*a*b^5*x^10+660*a^
4*b^2*m^5*x^4+105840*a^3*b^3*m^3*x^6+415320*a^2*b^4*m*x^8+6*a^5*b*m^6*x^2+1
```

```
1295*a^4*b^2*m^4*x^4+406700*a^3*b^3*m^2*x^6+225225*a^2*b^4*x^8+276*a^5*b*m^
5*x^2+94200*a^4*b^2*m^3*x^4+699720*a^3*b^3*m*x^6+a^6*m^6+5010*a^5*b*m^4*x^2
+389685*a^4*b^2*m^2*x^4+386100*a^3*b^3*x^6+48*a^6*m^5+45240*a^5*b*m^3*x^2+7
11540*a^4*b^2*m*x^4+925*a^6*m^4+208554*a^5*b*m^2*x^2+405405*a^4*b^2*x^4+912
0*a^6*m^3+438324*a^5*b*m*x^2+48259*a^6*m^2+270270*a^5*b*x^2+129072*a^6*m+13
5135*a^6)*x/(m+13)/(m+11)/(m+9)/(m+7)/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.54, size = 144, normalized size = 0.96

$$\frac{b^6d^mx^{13}x^m}{m+13}+\frac{6\,ab^5d^mx^{11}x^m}{m+11}+\frac{15\,a^2b^4d^mx^9x^m}{m+9}+\frac{20\,a^3b^3d^mx^7x^m}{m+7}+\frac{15\,a^4b^2d^mx^5x^m}{m+5}+\frac{6\,a^5bd^mx^3x^m}{m+3}+\frac{(dx)^{m+1}\,a^6}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] b^6*d^m*x^13*x^m/(m + 13) + 6*a*b^5*d^m*x^11*x^m/(m + 11) + 15*a^2*b^4*d^m*
x^9*x^m/(m + 9) + 20*a^3*b^3*d^m*x^7*x^m/(m + 7) + 15*a^4*b^2*d^m*x^5*x^m/(
m + 5) + 6*a^5*b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*a^6/(d*(m + 1))
```

mupad [B] time = 4.58, size = 540, normalized size = 3.60

$$\frac{a^6\,x\,{\left(d\,x\right)}^m\,\left(m^6+48\,m^5+925\,m^4+9120\,m^3+48259\,m^2+129072\,m+135135\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{b^6\,x^{13}\,{\left(d\,x\right)}^m\,\left(m^6+36\,m^5\right.}{m^7+49\,m^6+973\,m^5+100}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x*(d*x)^m*(129072*m + 48259*m^2 + 9120*m^3 + 925*m^4 + 48*m^5 + m^6 +
135135))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6
+ m^7 + 135135) + (b^6*x^13*(d*x)^m*(19524*m + 12139*m^2 + 3480*m^3 + 505*m
^4 + 36*m^5 + m^6 + 10395))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4
+ 973*m^5 + 49*m^6 + m^7 + 135135) + (6*a*b^5*x^11*(d*x)^m*(22902*m + 14039
*m^2 + 3940*m^3 + 555*m^4 + 38*m^5 + m^6 + 12285))/(264207*m + 177331*m^2 +
 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (6*a^5*b*x^3*(d
*x)^m*(73054*m + 34759*m^2 + 7540*m^3 + 835*m^4 + 46*m^5 + m^6 + 45045))/(2
64207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135
135) + (15*a^2*b^4*x^9*(d*x)^m*(27688*m + 16627*m^2 + 4528*m^3 + 613*m^4 +
40*m^5 + m^6 + 15015))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973
*m^5 + 49*m^6 + m^7 + 135135) + (20*a^3*b^3*x^7*(d*x)^m*(34986*m + 20335*m^
2 + 5292*m^3 + 679*m^4 + 42*m^5 + m^6 + 19305))/(264207*m + 177331*m^2 + 57
379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (15*a^4*b^2*x^5*(d
*x)^m*(47436*m + 25979*m^2 + 6280*m^3 + 753*m^4 + 44*m^5 + m^6 + 27027))/(2
64207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135
135)
```

sympy [A] time = 7.61, size = 3188, normalized size = 21.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Piecewise(((-a**6/(12*x**12) - 3*a**5*b/(5*x**10) - 15*a**4*b**2/(8*x**8) -
 10*a**3*b**3/(3*x**6) - 15*a**2*b**4/(4*x**4) - 3*a*b**5/x**2 + b**6*log(x
))/d**13, Eq(m, -13)), ((-a**6/(10*x**10) - 3*a**5*b/(4*x**8) - 5*a**4*b**2
/(2*x**6) - 5*a**3*b**3/x**4 - 15*a**2*b**4/(2*x**2) + 6*a*b**5*log(x) + b*
*6*x**2/2)/d**11, Eq(m, -11)), ((-a**6/(8*x**8) - a**5*b/x**6 - 15*a**4*b**
2/(4*x**4) - 10*a**3*b**3/x**2 + 15*a**2*b**4*log(x) + 3*a*b**5*x**2 + b**6
*x**4/4)/d**9, Eq(m, -9)), ((-a**6/(6*x**6) - 3*a**5*b/(2*x**4) - 15*a**4*b
**2/(2*x**2) + 20*a**3*b**3*log(x) + 15*a**2*b**4*x**2/2 + 3*a*b**5*x**4/2
+ b**6*x**6/6)/d**7, Eq(m, -7)), ((-a**6/(4*x**4) - 3*a**5*b/x**2 + 15*a**4
*b**2*log(x) + 10*a**3*b**3*x**2 + 15*a**2*b**4*x**4/4 + a*b**5*x**6 + b**6
*x**8/8)/d**5, Eq(m, -5)), ((-a**6/(2*x**2) + 6*a**5*b*log(x) + 15*a**4*b**
2*x**2/2 + 5*a**3*b**3*x**4 + 5*a**2*b**4*x**6/2 + 3*a*b**5*x**8/4 + b**6*x
**10/10)/d**3, Eq(m, -3)), ((a**6*log(x) + 3*a**5*b*x**2 + 15*a**4*b**2*x**
4/4 + 10*a**3*b**3*x**6/3 + 15*a**2*b**4*x**8/8 + 3*a*b**5*x**10/5 + b**6*x
**12/12)/d, Eq(m, -1)), (a**6*d**m*m**6*x*x**m/(m**7 + 49*m**6 + 973*m**5 +
 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 48*a**6*d**m*
m**5*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m
**2 + 264207*m + 135135) + 925*a**6*d**m*m**4*x*x**m/(m**7 + 49*m**6 + 973*
m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 9120*a*
*6*d**m*m**3*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 +
177331*m**2 + 264207*m + 135135) + 48259*a**6*d**m*m**2*x*x**m/(m**7 + 49*m
**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
 + 129072*a**6*d**m*m*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 5737
9*m**3 + 177331*m**2 + 264207*m + 135135) + 135135*a**6*d**m*x*x**m/(m**7 +
 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 13
5135) + 6*a**5*b*d**m*m**6*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**
4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 276*a**5*b*d**m*m**5*x*
*3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2
+ 264207*m + 135135) + 5010*a**5*b*d**m*m**4*x**3*x**m/(m**7 + 49*m**6 + 97
3*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 45240
*a**5*b*d**m*m**3*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379
*m**3 + 177331*m**2 + 264207*m + 135135) + 208554*a**5*b*d**m*m**2*x**3*x**
m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 2642
07*m + 135135) + 438324*a**5*b*d**m*m*x**3*x**m/(m**7 + 49*m**6 + 973*m**5
+ 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 270270*a**5*
b*d**m*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177
331*m**2 + 264207*m + 135135) + 15*a**4*b**2*d**m*m**6*x**5*x**m/(m**7 + 49
```

*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 13513
5) + 660*a**4*b**2*d**m*m**5*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m
4 + 57379*m3 + 177331*m**2 + 264207*m + 135135) + 11295*a**4*b**2*d**m*
m**4*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 17733
1*m**2 + 264207*m + 135135) + 94200*a**4*b**2*d**m*m**3*x**5*x**m/(m**7 + 4
9*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 1351
35) + 389685*a**4*b**2*d**m*m**2*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 100
45*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 711540*a**4*b**2*
d**m*m*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177
331*m**2 + 264207*m + 135135) + 405405*a**4*b**2*d**m*x**5*x**m/(m**7 + 49*
m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135
) + 20*a**3*b**3*d**m*m**6*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**
4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 840*a**3*b**3*d**m*m**5
*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m*
*2 + 264207*m + 135135) + 13580*a**3*b**3*d**m*m**4*x**7*x**m/(m**7 + 49*m*
*6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
+ 105840*a**3*b**3*d**m*m**3*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m
4 + 57379*m3 + 177331*m**2 + 264207*m + 135135) + 406700*a**3*b**3*d**m
*m**2*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 1773
31*m**2 + 264207*m + 135135) + 699720*a**3*b**3*d**m*m*x**7*x**m/(m**7 + 49
*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 13513
5) + 386100*a**3*b**3*d**m*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**
4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 15*a**2*b**4*d**m*m**6*
x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**
2 + 264207*m + 135135) + 600*a**2*b**4*d**m*m**5*x**9*x**m/(m**7 + 49*m**6
+ 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 9
195*a**2*b**4*d**m*m**4*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 +
 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 67920*a**2*b**4*d**m*m**3*
x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**
2 + 264207*m + 135135) + 249405*a**2*b**4*d**m*m**2*x**9*x**m/(m**7 + 49*m*
*6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
+ 415320*a**2*b**4*d**m*m*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4
 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 225225*a**2*b**4*d**m*x*
*9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2
+ 264207*m + 135135) + 6*a*b**5*d**m*m**6*x**11*x**m/(m**7 + 49*m**6 + 973*
m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 228*a*b
5*dm*m**5*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m*
*3 + 177331*m**2 + 264207*m + 135135) + 3330*a*b**5*d**m*m**4*x**11*x**m/(m
7 + 49*m6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m
 + 135135) + 23640*a*b**5*d**m*m**3*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 +
 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 84234*a*b**5*
d**m*m**2*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 +
 177331*m**2 + 264207*m + 135135) + 137412*a*b**5*d**m*m*x**11*x**m/(m**7 +
 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 13
5135) + 73710*a*b**5*d**m*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**

```
4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + b**6*d**m*m**6*x**13*x*
*m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264
207*m + 135135) + 36*b**6*d**m*m**5*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 +
 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 505*b**6*d**m
*m**4*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177
331*m**2 + 264207*m + 135135) + 3480*b**6*d**m*m**3*x**13*x**m/(m**7 + 49*m
**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
 + 12139*b**6*d**m*m**2*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4
+ 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 19524*b**6*d**m*m*x**13*x
**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 26
4207*m + 135135) + 10395*b**6*d**m*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 +
10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135), True))
```

3.786 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=104

$$\frac{a^4(dx)^{m+1}}{d(m+1)}+\frac{4a^3b(dx)^{m+3}}{d^3(m+3)}+\frac{6a^2b^2(dx)^{m+5}}{d^5(m+5)}+\frac{4ab^3(dx)^{m+7}}{d^7(m+7)}+\frac{b^4(dx)^{m+9}}{d^9(m+9)}$$

```
[Out] a^4*(d*x)^(1+m)/d/(1+m)+4*a^3*b*(d*x)^(3+m)/d^3/(3+m)+6*a^2*b^2*(d*x)^(5+m)
/d^5/(5+m)+4*a*b^3*(d*x)^(7+m)/d^7/(7+m)+b^4*(d*x)^(9+m)/d^9/(9+m)
```

Rubi [A] time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {28, 270}

$$\frac{6a^2b^2(dx)^{m+5}}{d^5(m+5)}+\frac{4a^3b(dx)^{m+3}}{d^3(m+3)}+\frac{a^4(dx)^{m+1}}{d(m+1)}+\frac{4ab^3(dx)^{m+7}}{d^7(m+7)}+\frac{b^4(dx)^{m+9}}{d^9(m+9)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*(d*x)^(1 + m))/(d*(1 + m)) + (4*a^3*b*(d*x)^(3 + m))/(d^3*(3 + m)) + (
6*a^2*b^2*(d*x)^(5 + m))/(d^5*(5 + m)) + (4*a*b^3*(d*x)^(7 + m))/(d^7*(7 +
m)) + (b^4*(d*x)^(9 + m))/(d^9*(9 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^2 dx = \frac{\int (dx)^m \left(ab+b^2x^2\right)^4 dx}{b^4}$$
$$= \frac{\int \left(a^4b^4(dx)^m + \frac{4a^3b^5(dx)^{2+m}}{d^2} + \frac{6a^2b^6(dx)^{4+m}}{d^4} + \frac{4ab^7(dx)^{6+m}}{d^6} + \frac{b^8(dx)^{8+m}}{d^8}\right) dx}{b^4}$$
$$= \frac{a^4(dx)^{1+m}}{d(1+m)} + \frac{4a^3b(dx)^{3+m}}{d^3(3+m)} + \frac{6a^2b^2(dx)^{5+m}}{d^5(5+m)} + \frac{4ab^3(dx)^{7+m}}{d^7(7+m)} + \frac{b^4(dx)^{9+m}}{d^9(9+m)}$$

Mathematica [A] time = 0.04, size = 73, normalized size = 0.70

$$x(dx)^m\left(\frac{a^4}{m+1}+\frac{4a^3bx^2}{m+3}+\frac{6a^2b^2x^4}{m+5}+\frac{4ab^3x^6}{m+7}+\frac{b^4x^8}{m+9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] x*(d*x)^m*(a^4/(1 + m) + (4*a^3*b*x^2)/(3 + m) + (6*a^2*b^2*x^4)/(5 + m) +
(4*a*b^3*x^6)/(7 + m) + (b^4*x^8)/(9 + m))
```

fricas [B] time = 1.01, size = 253, normalized size = 2.43

$$\left(\left(b^4m^4+16\,b^4m^3+86\,b^4m^2+176\,b^4m+105\,b^4\right)x^9+4\left(ab^3m^4+18\,ab^3m^3+104\,ab^3m^2+222\,ab^3m+135\,ab^3\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] ((b^4*m^4 + 16*b^4*m^3 + 86*b^4*m^2 + 176*b^4*m + 105*b^4)*x^9 + 4*(a*b^3*m
^4 + 18*a*b^3*m^3 + 104*a*b^3*m^2 + 222*a*b^3*m + 135*a*b^3)*x^7 + 6*(a^2*b
^2*m^4 + 20*a^2*b^2*m^3 + 130*a^2*b^2*m^2 + 300*a^2*b^2*m + 189*a^2*b^2)*x^
5 + 4*(a^3*b*m^4 + 22*a^3*b*m^3 + 164*a^3*b*m^2 + 458*a^3*b*m + 315*a^3*b)*
x^3 + (a^4*m^4 + 24*a^4*m^3 + 206*a^4*m^2 + 744*a^4*m + 945*a^4)*x)*(d*x)^m
/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

giac [B] time = 0.18, size = 415, normalized size = 3.99

$$(dx)^m\,b^4m^4x^9+16\,(dx)^m\,b^4m^3x^9+4\,(dx)^m\,ab^3m^4x^7+86\,(dx)^m\,b^4m^2x^9+72\,(dx)^m\,ab^3m^3x^7+176\,(dx)^m\,b^4mx^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^4*m^4*x^9 + 16*(d*x)^m*b^4*m^3*x^9 + 4*(d*x)^m*a*b^3*m^4*x^7 + 8
6*(d*x)^m*b^4*m^2*x^9 + 72*(d*x)^m*a*b^3*m^3*x^7 + 176*(d*x)^m*b^4*m*x^9 +
6*(d*x)^m*a^2*b^2*m^4*x^5 + 416*(d*x)^m*a*b^3*m^2*x^7 + 105*(d*x)^m*b^4*x^9
 + 120*(d*x)^m*a^2*b^2*m^3*x^5 + 888*(d*x)^m*a*b^3*m*x^7 + 4*(d*x)^m*a^3*b*
m^4*x^3 + 780*(d*x)^m*a^2*b^2*m^2*x^5 + 540*(d*x)^m*a*b^3*x^7 + 88*(d*x)^m*
a^3*b*m^3*x^3 + 1800*(d*x)^m*a^2*b^2*m*x^5 + (d*x)^m*a^4*m^4*x + 656*(d*x)^
m*a^3*b*m^2*x^3 + 1134*(d*x)^m*a^2*b^2*x^5 + 24*(d*x)^m*a^4*m^3*x + 1832*(d
*x)^m*a^3*b*m*x^3 + 206*(d*x)^m*a^4*m^2*x + 1260*(d*x)^m*a^3*b*x^3 + 744*(d
*x)^m*a^4*m*x + 945*(d*x)^m*a^4*x)/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689
*m + 945)
```

maple [B] time = 0.01, size = 292, normalized size = 2.81

$$\frac{\left(b^4m^4x^8+16b^4m^3x^8+4a\,b^3m^4x^6+86b^4m^2x^8+72a\,b^3m^3x^6+176b^4m\,x^8+6a^2b^2m^4x^4+416a\,b^3m^2x^6+105b^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] (d*x)^m*(b^4*m^4*x^8+16*b^4*m^3*x^8+4*a*b^3*m^4*x^6+86*b^4*m^2*x^8+72*a*b^3
*m^3*x^6+176*b^4*m*x^8+6*a^2*b^2*m^4*x^4+416*a*b^3*m^2*x^6+105*b^4*x^8+120*
a^2*b^2*m^3*x^4+888*a*b^3*m*x^6+4*a^3*b*m^4*x^2+780*a^2*b^2*m^2*x^4+540*a*b
^3*x^6+88*a^3*b*m^3*x^2+1800*a^2*b^2*m*x^4+a^4*m^4+656*a^3*b*m^2*x^2+1134*a
^2*b^2*x^4+24*a^4*m^3+1832*a^3*b*m*x^2+206*a^4*m^2+1260*a^3*b*x^2+744*a^4*m
+945*a^4)*x/(m+9)/(m+7)/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.49, size = 100, normalized size = 0.96

$$\frac{b^4d^mx^9x^m}{m+9}+\frac{4\,ab^3d^mx^7x^m}{m+7}+\frac{6\,a^2b^2d^mx^5x^m}{m+5}+\frac{4\,a^3bd^mx^3x^m}{m+3}+\frac{(dx)^{m+1}\,a^4}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] b^4*d^m*x^9*x^m/(m + 9) + 4*a*b^3*d^m*x^7*x^m/(m + 7) + 6*a^2*b^2*d^m*x^5*x
^m/(m + 5) + 4*a^3*b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*a^4/(d*(m + 1))
```

mupad [B] time = 4.51, size = 263, normalized size = 2.53

$$(d\,x)^m\,\left(\frac{b^4\,x^9\,\left(m^4+16\,m^3+86\,m^2+176\,m+105\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{a^4\,x\,\left(m^4+24\,m^3+206\,m^2+744\,m+945\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (d*x)^m*((b^4*x^9*(176*m + 86*m^2 + 16*m^3 + m^4 + 105))/(1689*m + 950*m^2
+ 230*m^3 + 25*m^4 + m^5 + 945) + (a^4*x*(744*m + 206*m^2 + 24*m^3 + m^4 +
945))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (4*a*b^3*x^7*(222
*m + 104*m^2 + 18*m^3 + m^4 + 135))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 +
m^5 + 945) + (4*a^3*b*x^3*(458*m + 164*m^2 + 22*m^3 + m^4 + 315))/(1689*m +
 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (6*a^2*b^2*x^5*(300*m + 130*m^2
+ 20*m^3 + m^4 + 189))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945))
```

sympy [A] time = 3.20, size = 1321, normalized size = 12.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Piecewise(((-a**4/(8*x**8) - 2*a**3*b/(3*x**6) - 3*a**2*b**2/(2*x**4) - 2*a
*b**3/x**2 + b**4*log(x))/d**9, Eq(m, -9)), ((-a**4/(6*x**6) - a**3*b/x**4
- 3*a**2*b**2/x**2 + 4*a*b**3*log(x) + b**4*x**2/2)/d**7, Eq(m, -7)), ((-a*
*4/(4*x**4) - 2*a**3*b/x**2 + 6*a**2*b**2*log(x) + 2*a*b**3*x**2 + b**4*x**
4/4)/d**5, Eq(m, -5)), ((-a**4/(2*x**2) + 4*a**3*b*log(x) + 3*a**2*b**2*x**
2 + a*b**3*x**4 + b**4*x**6/6)/d**3, Eq(m, -3)), ((a**4*log(x) + 2*a**3*b*x
**2 + 3*a**2*b**2*x**4/2 + 2*a*b**3*x**6/3 + b**4*x**8/8)/d, Eq(m, -1)), (a
**4*d**m*m**4*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945)
+ 24*a**4*d**m*m**3*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m +
 945) + 206*a**4*d**m*m**2*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1
689*m + 945) + 744*a**4*d**m*m*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2
 + 1689*m + 945) + 945*a**4*d**m*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m*
*2 + 1689*m + 945) + 4*a**3*b*d**m*m**4*x**3*x**m/(m**5 + 25*m**4 + 230*m**
3 + 950*m**2 + 1689*m + 945) + 88*a**3*b*d**m*m**3*x**3*x**m/(m**5 + 25*m**
4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 656*a**3*b*d**m*m**2*x**3*x**m/(m
**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1832*a**3*b*d**m*m*x*
*3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1260*a**3*b
*d**m*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 6*a
**2*b**2*d**m*m**4*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
 + 945) + 120*a**2*b**2*d**m*m**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 95
0*m**2 + 1689*m + 945) + 780*a**2*b**2*d**m*m**2*x**5*x**m/(m**5 + 25*m**4
+ 230*m**3 + 950*m**2 + 1689*m + 945) + 1800*a**2*b**2*d**m*m*x**5*x**m/(m*
*5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1134*a**2*b**2*d**m*x*
*5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 4*a*b**3*d*
*m*m**4*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 7
2*a*b**3*d**m*m**3*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
 + 945) + 416*a*b**3*d**m*m**2*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m
**2 + 1689*m + 945) + 888*a*b**3*d**m*m*x**7*x**m/(m**5 + 25*m**4 + 230*m**
```

```
3 + 950*m**2 + 1689*m + 945) + 540*a*b**3*d**m*x**7*x**m/(m**5 + 25*m**4 +
230*m**3 + 950*m**2 + 1689*m + 945) + b**4*d**m*m**4*x**9*x**m/(m**5 + 25*m
**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 16*b**4*d**m*m**3*x**9*x**m/(m*
*5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 86*b**4*d**m*m**2*x**9
*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 176*b**4*d**m
*m*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 105*b*
*4*d**m*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945), Tr
ue))
```

3.787 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right) dx$

Optimal. Leaf size=58

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{2ab(dx)^{m+3}}{d^3(m+3)}+\frac{b^2(dx)^{m+5}}{d^5(m+5)}$$

```
[Out] a^2*(d*x)^(1+m)/d/(1+m)+2*a*b*(d*x)^(3+m)/d^3/(3+m)+b^2*(d*x)^(5+m)/d^5/(5+
m)
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {14}

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{2ab(dx)^{m+3}}{d^3(m+3)}+\frac{b^2(dx)^{m+5}}{d^5(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(3 + m))/(d^3*(3 + m)) + (b^
2*(d*x)^(5 + m))/(d^5*(5 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a^2+2abx^2+b^2x^4\right) dx &= \int \left(a^2(dx)^m+\frac{2ab(dx)^{2+m}}{d^2}+\frac{b^2(dx)^{4+m}}{d^4}\right) dx\\ &= \frac{a^2(dx)^{1+m}}{d(1+m)}+\frac{2ab(dx)^{3+m}}{d^3(3+m)}+\frac{b^2(dx)^{5+m}}{d^5(5+m)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.71

$$x(dx)^m\left(\frac{a^2}{m+1}+\frac{2abx^2}{m+3}+\frac{b^2x^4}{m+5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^2)/(3 + m) + (b^2*x^4)/(5 + m))
```

fricas [A] time = 1.02, size = 87, normalized size = 1.50

$$\frac{\left(\left(b^{2} m^{2}+4 \, b^{2} m+3 \, b^{2}\right) x^{5}+2 \, \left(a b m^{2}+6 \, a b m+5 \, a b\right) x^{3}+\left(a^{2} m^{2}+8 \, a^{2} m+15 \, a^{2}\right) x\right) \left(d x\right)^{m}}{m^{3}+9 \, m^{2}+23 \, m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 4*b^2*m + 3*b^2)*x^5 + 2*(a*b*m^2 + 6*a*b*m + 5*a*b)*x^3 + (a^2
*m^2 + 8*a^2*m + 15*a^2)*x)*(d*x)^m/(m^3 + 9*m^2 + 23*m + 15)
```

giac [B] time = 0.16, size = 135, normalized size = 2.33

$$\frac{\left(d x\right)^{m} b^{2} m^{2} x^{5}+4 \, \left(d x\right)^{m} b^{2} m x^{5}+2 \, \left(d x\right)^{m} a b m^{2} x^{3}+3 \, \left(d x\right)^{m} b^{2} x^{5}+12 \, \left(d x\right)^{m} a b m x^{3}+\left(d x\right)^{m} a^{2} m^{2} x+10 \, \left(d x\right)^{m} a}{m^{3}+9 \, m^{2}+23 \, m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^2*m^2*x^5 + 4*(d*x)^m*b^2*m*x^5 + 2*(d*x)^m*a*b*m^2*x^3 + 3*(d*x
)^m*b^2*x^5 + 12*(d*x)^m*a*b*m*x^3 + (d*x)^m*a^2*m^2*x + 10*(d*x)^m*a*b*x^3
 + 8*(d*x)^m*a^2*m*x + 15*(d*x)^m*a^2*x)/(m^3 + 9*m^2 + 23*m + 15)
```

maple [A] time = 0.01, size = 94, normalized size = 1.62

$$\frac{\left(b^{2} m^{2} x^{4}+4 b^{2} m \,x^{4}+2 a b \,m^{2} x^{2}+3 b^{2} x^{4}+12 a b m \,x^{2}+a^{2} m^{2}+10 a b \,x^{2}+8 a^{2} m +15 a^{2}\right) x \left(d x \right)^{m}}{\left(m +5\right) \left(m +3\right) \left(m +1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] (d*x)^m*(b^2*m^2*x^4+4*b^2*m*x^4+2*a*b*m^2*x^2+3*b^2*x^4+12*a*b*m*x^2+a^2*m
^2+10*a*b*x^2+8*a^2*m+15*a^2)*x/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.40, size = 56, normalized size = 0.97

$$\frac{b^{2} d^{m} x^{5} x^{m}}{m+5}+\frac{2 \, a b d^{m} x^{3} x^{m}}{m+3}+\frac{\left(d x\right)^{m+1} a^{2}}{d {\left(m+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] b^2*d^m*x^5*x^m/(m + 5) + 2*a*b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*a^2/(d*
(m + 1))
```

mupad [B] time = 4.27, size = 95, normalized size = 1.64

$$(d\,x)^m\,\left(\frac{a^2\,x\,\left(m^2+8\,m+15\right)}{m^3+9\,m^2+23\,m+15}+\frac{b^2\,x^5\,\left(m^2+4\,m+3\right)}{m^3+9\,m^2+23\,m+15}+\frac{2\,a\,b\,x^3\,\left(m^2+6\,m+5\right)}{m^3+9\,m^2+23\,m+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (d*x)^m*((a^2*x*(8*m + m^2 + 15))/(23*m + 9*m^2 + m^3 + 15) + (b^2*x^5*(4*m
 + m^2 + 3))/(23*m + 9*m^2 + m^3 + 15) + (2*a*b*x^3*(6*m + m^2 + 5))/(23*m
+ 9*m^2 + m^3 + 15))
```

sympy [A] time = 1.01, size = 345, normalized size = 5.95

$$\begin{cases} \frac{-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)}{d^5} \\ \frac{-\frac{a^2}{2x^2}+2ab\log(x)+\frac{b^2x^2}{2}}{d^3} \\ \frac{a^2\log(x)+abx^2+\frac{b^2x^4}{4}}{d} \\ \frac{a^2d^mm^2xx^m}{m^3+9m^2+23m+15}+\frac{8a^2d^mmxx^m}{m^3+9m^2+23m+15}+\frac{15a^2d^mxx^m}{m^3+9m^2+23m+15}+\frac{2abd^mm^2x^3x^m}{m^3+9m^2+23m+15}+\frac{12abd^mmx^3x^m}{m^3+9m^2+23m+15}+\frac{10abd^mx^3x^m}{m^3+9m^2+23m+15}+\frac{b^2d^m}{m^3+9m^2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Piecewise((((-a**2/(4*x**4) - a*b/x**2 + b**2*log(x))/d**5, Eq(m, -5)), ((-a
**2/(2*x**2) + 2*a*b*log(x) + b**2*x**2/2)/d**3, Eq(m, -3)), ((a**2*log(x)
+ a*b*x**2 + b**2*x**4/4)/d, Eq(m, -1)), (a**2*d**m*m**2*x*x**m/(m**3 + 9*m
**2 + 23*m + 15) + 8*a**2*d**m*m*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 15*a*
*2*d**m*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 2*a*b*d**m*m**2*x**3*x**m/(m**
3 + 9*m**2 + 23*m + 15) + 12*a*b*d**m*m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 1
5) + 10*a*b*d**m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + b**2*d**m*m**2*x**
5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 4*b**2*d**m*m*x**5*x**m/(m**3 + 9*m**2
 + 23*m + 15) + 3*b**2*d**m*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15), True))
```

3.788 $\int \frac{(dx)^m}{a^2+2abx^2+b^2x^4} dx$

Optimal. Leaf size=44

$$\frac{(dx)^{m+1} {}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^2d(m+1)}$$

```
[Out] (d*x)^(1+m)*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/d/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {28, 364}

$$\frac{(dx)^{m+1} {}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^2d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] ((d*x)^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a
^2*d*(1 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{a^2 + 2abx^2 + b^2x^4}\,dx = b^2 \int \frac{(dx)^m}{\left(ab + b^2x^2\right)^2}\,dx$$
$$= \frac{(dx)^{1+m}\,{}_2F_1\left(2, \frac{1+m}{2}; \frac{3+m}{2}; -\frac{bx^2}{a}\right)}{a^2d(1+m)}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.95

$$\frac{x(dx)^m\,{}_2F_1\left(2, \frac{m+1}{2}; \frac{m+1}{2} + 1; -\frac{bx^2}{a}\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] (x*(d*x)^m*Hypergeometric2F1[2, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(a
^2*(1 + m))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{b^2x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
[Out] integral((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b^2x^4 + 2\,abx^2 + a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b^2x^4 + 2ab\,x^2 + a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b^2x^4 + 2\,abx^2 + a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(d\,x)^m}{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a + bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**m/(a + b*x**2)**2, x)
```

3.789 $\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=44

$$\frac{(dx)^{m+1}\,{}_2F_1\left(4,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^4d(m+1)}$$

```
[Out] (d*x)^(1+m)*hypergeom([4, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^4/d/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {28, 364}

$$\frac{(dx)^{m+1}\,{}_2F_1\left(4,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^4d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] ((d*x)^(1 + m)*Hypergeometric2F1[4, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a
^4*d*(1 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^m}{\left(ab+b^2x^2\right)^4}\,dx$$
$$= \frac{(dx)^{1+m}\,{}_2F_1\left(4,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^4d(1+m)}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.95

$$\frac{x(dx)^m\,{}_2F_1\left(4,\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a^4(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (x*(d*x)^m*Hypergeometric2F1[4, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(a
^4*(1 + m))
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b^2x^4+2\,abx^2+a^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^2, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b^2x^4 + 2ab\,x^2 + a^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(d\,x)^m}{\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
[Out] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a + bx^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
[Out] Integral((d*x)**m/(a + b*x**2)**4, x)
```

3.790 $\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^3} dx$

Optimal. Leaf size=44

$$\frac{(dx)^{m+1} {}_2F_1\left(6,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^6 d(m+1)}$$

```
[Out] (d*x)^(1+m)*hypergeom([6, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^6/d/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {28, 364}

$$\frac{(dx)^{m+1} {}_2F_1\left(6,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^6 d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((d*x)^(1 + m)*Hypergeometric2F1[6, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a
^6*d*(1 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^m}{\left(ab+b^2x^2\right)^6}\,dx$$
$$= \frac{(dx)^{1+m}\,{}_2F_1\left(6,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^6d(1+m)}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.95

$$\frac{x(dx)^m\,{}_2F_1\left(6,\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a^6(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (x*(d*x)^m*Hypergeometric2F1[6, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(a
^6*(1 + m))
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(dx)^m}{b^6x^{12}+6\,ab^5x^{10}+15\,a^2b^4x^8+20\,a^3b^3x^6+15\,a^4b^2x^4+6\,a^5bx^2+a^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^6*x^12 + 6*a*b^5*x^10 + 15*a^2*b^4*x^8 + 20*a^3*b^3*x^6
 + 15*a^4*b^2*x^4 + 6*a^5*b*x^2 + a^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b^2x^4+2\,abx^2+a^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^3, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b^2x^4 + 2ab\,x^2 + a^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(d\,x)^m}{\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
[Out] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
[Out] Timed out
```

3.791 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=313

$$\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+11}}{d^{11}(m+11)\left(a+bx^2\right)}+\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+9}}{d^9(m+9)\left(a+bx^2\right)}+\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+7}}{d^7(m+7)\left(a+bx^2\right)}+\frac{a^5\sqrt{a^2+\cdots}}{d\cdots}$$

```
[Out] a^5*(d*x)^(1+m)*((b*x^2+a)^2)^(1/2)/d/(1+m)/(b*x^2+a)+5*a^4*b*(d*x)^(3+m)*(
(b*x^2+a)^2)^(1/2)/d^3/(3+m)/(b*x^2+a)+10*a^3*b^2*(d*x)^(5+m)*((b*x^2+a)^2)
^(1/2)/d^5/(5+m)/(b*x^2+a)+10*a^2*b^3*(d*x)^(7+m)*((b*x^2+a)^2)^(1/2)/d^7/(
7+m)/(b*x^2+a)+5*a*b^4*(d*x)^(9+m)*((b*x^2+a)^2)^(1/2)/d^9/(9+m)/(b*x^2+a)+
b^5*(d*x)^(11+m)*((b*x^2+a)^2)^(1/2)/d^11/(11+m)/(b*x^2+a)
```

Rubi [A] time = 0.12, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1112, 270}

$$\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+3}}{d^3(m+3)\left(a+bx^2\right)}+\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+5}}{d^5(m+5)\left(a+bx^2\right)}+\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+7}}{d^7(m+7)\left(a+bx^2\right)}+\frac{5ab\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(1 + m)*(a + b*x^2))
 + (5*a^4*b*(d*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(3 + m)*(a
+ b*x^2)) + (10*a^3*b^2*(d*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^5
*(5 + m)*(a + b*x^2)) + (10*a^2*b^3*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(d^7*(7 + m)*(a + b*x^2)) + (5*a*b^4*(d*x)^(9 + m)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(d^9*(9 + m)*(a + b*x^2)) + (b^5*(d*x)^(11 + m)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(d^11*(11 + m)*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^m \left(ab+b^2x^2\right)^5\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a^5b^5(dx)^m + \frac{5a^4b^6(dx)^{2+m}}{d^2} + \frac{10a^3b^7(dx)^{4+m}}{d^4} + \frac{10a^2b^8(dx)^{6}}{d^6}\right.}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{a^5(dx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{d(1+m)\left(a+bx^2\right)} + \frac{5a^4b(dx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(3+m)\left(a+bx^2\right)} + \frac{10a^3}{}$$

Mathematica [A] time = 0.09, size = 111, normalized size = 0.35

$$\frac{x\left(\left(a+bx^2\right)^2\right)^{5/2}(dx)^m\left(\frac{a^5}{m+1}+\frac{5a^4bx^2}{m+3}+\frac{10a^3b^2x^4}{m+5}+\frac{10a^2b^3x^6}{m+7}+\frac{5ab^4x^8}{m+9}+\frac{b^5x^{10}}{m+11}\right)}{\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x*(d*x)^m*((a + b*x^2)^2)^(5/2)*(a^5/(1 + m) + (5*a^4*b*x^2)/(3 + m) + (10
*a^3*b^2*x^4)/(5 + m) + (10*a^2*b^3*x^6)/(7 + m) + (5*a*b^4*x^8)/(9 + m) +
(b^5*x^10)/(11 + m)))/(a + b*x^2)^5
```

fricas [A] time = 0.86, size = 369, normalized size = 1.18

$$\frac{\left(\left(b^5m^5+25\,b^5m^4+230\,b^5m^3+950\,b^5m^2+1689\,b^5m+945\,b^5\right)x^{11}+5\left(ab^4m^5+27\,ab^4m^4+262\,ab^4m^3+1122\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] ((b^5*m^5 + 25*b^5*m^4 + 230*b^5*m^3 + 950*b^5*m^2 + 1689*b^5*m + 945*b^5)*
x^11 + 5*(a*b^4*m^5 + 27*a*b^4*m^4 + 262*a*b^4*m^3 + 1122*a*b^4*m^2 + 2041*
a*b^4*m + 1155*a*b^4)*x^9 + 10*(a^2*b^3*m^5 + 29*a^2*b^3*m^4 + 302*a^2*b^3*
m^3 + 1366*a^2*b^3*m^2 + 2577*a^2*b^3*m + 1485*a^2*b^3)*x^7 + 10*(a^3*b^2*m
^5 + 31*a^3*b^2*m^4 + 350*a^3*b^2*m^3 + 1730*a^3*b^2*m^2 + 3489*a^3*b^2*m +
 2079*a^3*b^2)*x^5 + 5*(a^4*b*m^5 + 33*a^4*b*m^4 + 406*a^4*b*m^3 + 2262*a^4
*b*m^2 + 5353*a^4*b*m + 3465*a^4*b)*x^3 + (a^5*m^5 + 35*a^5*m^4 + 470*a^5*m
```

```
^3 + 3010*a^5*m^2 + 9129*a^5*m + 10395*a^5)*x)*(d*x)^m/(m^6 + 36*m^5 + 505*
m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

giac [B] time = 0.28, size = 900, normalized size = 2.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^5*m^5*x^11*sgn(b*x^2 + a) + 25*(d*x)^m*b^5*m^4*x^11*sgn(b*x^2 +
a) + 5*(d*x)^m*a*b^4*m^5*x^9*sgn(b*x^2 + a) + 230*(d*x)^m*b^5*m^3*x^11*sgn(
b*x^2 + a) + 135*(d*x)^m*a*b^4*m^4*x^9*sgn(b*x^2 + a) + 950*(d*x)^m*b^5*m^2
*x^11*sgn(b*x^2 + a) + 10*(d*x)^m*a^2*b^3*m^5*x^7*sgn(b*x^2 + a) + 1310*(d*
x)^m*a*b^4*m^3*x^9*sgn(b*x^2 + a) + 1689*(d*x)^m*b^5*m*x^11*sgn(b*x^2 + a)
+ 290*(d*x)^m*a^2*b^3*m^4*x^7*sgn(b*x^2 + a) + 5610*(d*x)^m*a*b^4*m^2*x^9*s
gn(b*x^2 + a) + 945*(d*x)^m*b^5*x^11*sgn(b*x^2 + a) + 10*(d*x)^m*a^3*b^2*m^
5*x^5*sgn(b*x^2 + a) + 3020*(d*x)^m*a^2*b^3*m^3*x^7*sgn(b*x^2 + a) + 10205*
(d*x)^m*a*b^4*m*x^9*sgn(b*x^2 + a) + 310*(d*x)^m*a^3*b^2*m^4*x^5*sgn(b*x^2
+ a) + 13660*(d*x)^m*a^2*b^3*m^2*x^7*sgn(b*x^2 + a) + 5775*(d*x)^m*a*b^4*x^
9*sgn(b*x^2 + a) + 5*(d*x)^m*a^4*b*m^5*x^3*sgn(b*x^2 + a) + 3500*(d*x)^m*a^
3*b^2*m^3*x^5*sgn(b*x^2 + a) + 25770*(d*x)^m*a^2*b^3*m*x^7*sgn(b*x^2 + a) +
 165*(d*x)^m*a^4*b*m^4*x^3*sgn(b*x^2 + a) + 17300*(d*x)^m*a^3*b^2*m^2*x^5*s
gn(b*x^2 + a) + 14850*(d*x)^m*a^2*b^3*x^7*sgn(b*x^2 + a) + (d*x)^m*a^5*m^5*
x*sgn(b*x^2 + a) + 2030*(d*x)^m*a^4*b*m^3*x^3*sgn(b*x^2 + a) + 34890*(d*x)^
m*a^3*b^2*m*x^5*sgn(b*x^2 + a) + 35*(d*x)^m*a^5*m^4*x*sgn(b*x^2 + a) + 1131
0*(d*x)^m*a^4*b*m^2*x^3*sgn(b*x^2 + a) + 20790*(d*x)^m*a^3*b^2*x^5*sgn(b*x^
2 + a) + 470*(d*x)^m*a^5*m^3*x*sgn(b*x^2 + a) + 26765*(d*x)^m*a^4*b*m*x^3*s
gn(b*x^2 + a) + 3010*(d*x)^m*a^5*m^2*x*sgn(b*x^2 + a) + 17325*(d*x)^m*a^4*b
*x^3*sgn(b*x^2 + a) + 9129*(d*x)^m*a^5*m*x*sgn(b*x^2 + a) + 10395*(d*x)^m*a
^5*x*sgn(b*x^2 + a))/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524
*m + 10395)
```

maple [A] time = 0.01, size = 453, normalized size = 1.45

$$\frac{\left(b^5m^5x^{10}+25b^5m^4x^{10}+5a\,b^4m^5x^8+230b^5m^3x^{10}+135a\,b^4m^4x^8+950b^5m^2x^{10}+10a^2b^3m^5x^6+1310a\,b^4m^3x^8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] x*(b^5*m^5*x^10+25*b^5*m^4*x^10+5*a*b^4*m^5*x^8+230*b^5*m^3*x^10+135*a*b^4*
m^4*x^8+950*b^5*m^2*x^10+10*a^2*b^3*m^5*x^6+1310*a*b^4*m^3*x^8+1689*b^5*m*x
```

```
^10+290*a^2*b^3*m^4*x^6+5610*a*b^4*m^2*x^8+945*b^5*x^10+10*a^3*b^2*m^5*x^4+
3020*a^2*b^3*m^3*x^6+10205*a*b^4*m*x^8+310*a^3*b^2*m^4*x^4+13660*a^2*b^3*m^
2*x^6+5775*a*b^4*x^8+5*a^4*b*m^5*x^2+3500*a^3*b^2*m^3*x^4+25770*a^2*b^3*m*x
^6+165*a^4*b*m^4*x^2+17300*a^3*b^2*m^2*x^4+14850*a^2*b^3*x^6+a^5*m^5+2030*a
^4*b*m^3*x^2+34890*a^3*b^2*m*x^4+35*a^5*m^4+11310*a^4*b*m^2*x^2+20790*a^3*b
^2*x^4+470*a^5*m^3+26765*a^4*b*m*x^2+3010*a^5*m^2+17325*a^4*b*x^2+9129*a^5*
m+10395*a^5)*(d*x)^m*((b*x^2+a)^2)^(5/2)/(m+11)/(m+9)/(m+7)/(m+5)/(m+3)/(m+
1)/(b*x^2+a)^5
```

maxima [A] time = 1.41, size = 243, normalized size = 0.78

$$\frac{\left(\left(m^5 + 25\,m^4 + 230\,m^3 + 950\,m^2 + 1689\,m + 945\right)b^5 d^m x^{11} + 5\left(m^5 + 27\,m^4 + 262\,m^3 + 1122\,m^2 + 2041\,m + 11\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] ((m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)*b^5*d^m*x^11 + 5*(m^5 +
27*m^4 + 262*m^3 + 1122*m^2 + 2041*m + 1155)*a*b^4*d^m*x^9 + 10*(m^5 + 29*m
^4 + 302*m^3 + 1366*m^2 + 2577*m + 1485)*a^2*b^3*d^m*x^7 + 10*(m^5 + 31*m^4
 + 350*m^3 + 1730*m^2 + 3489*m + 2079)*a^3*b^2*d^m*x^5 + 5*(m^5 + 33*m^4 +
406*m^3 + 2262*m^2 + 5353*m + 3465)*a^4*b*d^m*x^3 + (m^5 + 35*m^4 + 470*m^3
 + 3010*m^2 + 9129*m + 10395)*a^5*d^m*x)*x^m/(m^6 + 36*m^5 + 505*m^4 + 3480
*m^3 + 12139*m^2 + 19524*m + 10395)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^m\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**m*((a + b*x**2)**2)**(5/2), x)
```

3.792 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx$

Optimal. Leaf size=205

$$\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+5}}{d^5(m+5)\left(a+bx^2\right)}+\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+3}}{d^3(m+3)\left(a+bx^2\right)}+\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+7}}{d^7(m+7)\left(a+bx^2\right)}+\frac{a^3\sqrt{a^2+2}}{d(m}$$

```
[Out] a^3*(d*x)^(1+m)*((b*x^2+a)^2)^(1/2)/d/(1+m)/(b*x^2+a)+3*a^2*b*(d*x)^(3+m)*(
(b*x^2+a)^2)^(1/2)/d^3/(3+m)/(b*x^2+a)+3*a*b^2*(d*x)^(5+m)*((b*x^2+a)^2)^(1
/2)/d^5/(5+m)/(b*x^2+a)+b^3*(d*x)^(7+m)*((b*x^2+a)^2)^(1/2)/d^7/(7+m)/(b*x^
2+a)
```

Rubi [A] time = 0.08, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1112, 270}

$$\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+3}}{d^3(m+3)\left(a+bx^2\right)}+\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+5}}{d^5(m+5)\left(a+bx^2\right)}+\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+7}}{d^7(m+7)\left(a+bx^2\right)}+\frac{a^3\sqrt{a^2+2}}{d(m}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(1 + m)*(a + b*x^2))
 + (3*a^2*b*(d*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(3 + m)*(a
+ b*x^2)) + (3*a*b^2*(d*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^5*(5
 + m)*(a + b*x^2)) + (b^3*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d
^7*(7 + m)*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int (dx)^m \left(ab+b^2x^2\right)^3 dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^3b^3(dx)^m + \frac{3a^2b^4(dx)^{2+m}}{d^2} + \frac{3ab^5(dx)^{4+m}}{d^4} + \frac{b^6(dx)^{6+m}}{d^6}\right) d}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{d(1+m)\left(a+bx^2\right)} + \frac{3a^2b(dx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(3+m)\left(a+bx^2\right)} + \frac{3ab^2}{}$$

Mathematica [A] time = 0.07, size = 131, normalized size = 0.64

$$\frac{x\sqrt{\left(a+bx^2\right)^2}(dx)^m\left(a^3\left(m^3+15m^2+71m+105\right)+3a^2b\left(m^3+13m^2+47m+35\right)x^2+3ab^2\left(m^3+11m^2+31\right.\right.}{(m+1)(m+3)(m+5)(m+7)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x*(d*x)^m*Sqrt[(a + b*x^2)^2]*(a^3*(105 + 71*m + 15*m^2 + m^3) + 3*a^2*b*(
35 + 47*m + 13*m^2 + m^3)*x^2 + 3*a*b^2*(21 + 31*m + 11*m^2 + m^3)*x^4 + b^
3*(15 + 23*m + 9*m^2 + m^3)*x^6))/((1 + m)*(3 + m)*(5 + m)*(7 + m)*(a + b*x
^2))
```

fricas [A] time = 1.13, size = 159, normalized size = 0.78

$$\frac{\left(\left(b^3m^3+9\,b^3m^2+23\,b^3m+15\,b^3\right)x^7+3\left(ab^2m^3+11\,ab^2m^2+31\,ab^2m+21\,ab^2\right)x^5+3\left(a^2bm^3+13\,a^2bm^2+4\right.\right.}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 9*b^3*m^2 + 23*b^3*m + 15*b^3)*x^7 + 3*(a*b^2*m^3 + 11*a*b^2*m^
2 + 31*a*b^2*m + 21*a*b^2)*x^5 + 3*(a^2*b*m^3 + 13*a^2*b*m^2 + 47*a^2*b*m +
 35*a^2*b)*x^3 + (a^3*m^3 + 15*a^3*m^2 + 71*a^3*m + 105*a^3)*x)*(d*x)^m/(m^
4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

giac [B] time = 0.21, size = 384, normalized size = 1.87

$$\frac{(dx)^m\,b^3m^3x^7\mathrm{sgn}\left(bx^2+a\right)+9\,(dx)^m\,b^3m^2x^7\mathrm{sgn}\left(bx^2+a\right)+3\,(dx)^m\,ab^2m^3x^5\mathrm{sgn}\left(bx^2+a\right)+23\,(dx)^m\,b^3mx^7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^3*m^3*x^7*sgn(b*x^2 + a) + 9*(d*x)^m*b^3*m^2*x^7*sgn(b*x^2 + a)
+ 3*(d*x)^m*a*b^2*m^3*x^5*sgn(b*x^2 + a) + 23*(d*x)^m*b^3*m*x^7*sgn(b*x^2 +
 a) + 33*(d*x)^m*a*b^2*m^2*x^5*sgn(b*x^2 + a) + 15*(d*x)^m*b^3*x^7*sgn(b*x^
2 + a) + 3*(d*x)^m*a^2*b*m^3*x^3*sgn(b*x^2 + a) + 93*(d*x)^m*a*b^2*m*x^5*sg
n(b*x^2 + a) + 39*(d*x)^m*a^2*b*m^2*x^3*sgn(b*x^2 + a) + 63*(d*x)^m*a*b^2*x
^5*sgn(b*x^2 + a) + (d*x)^m*a^3*m^3*x*sgn(b*x^2 + a) + 141*(d*x)^m*a^2*b*m*
x^3*sgn(b*x^2 + a) + 15*(d*x)^m*a^3*m^2*x*sgn(b*x^2 + a) + 105*(d*x)^m*a^2*
b*x^3*sgn(b*x^2 + a) + 71*(d*x)^m*a^3*m*x*sgn(b*x^2 + a) + 105*(d*x)^m*a^3*
x*sgn(b*x^2 + a))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

maple [A] time = 0.01, size = 199, normalized size = 0.97

$$\frac{\left(b^3m^3x^6 + 9b^3m^2x^6 + 3a\,b^2m^3x^4 + 23b^3m\,x^6 + 33a\,b^2m^2x^4 + 15b^3x^6 + 3a^2b\,m^3x^2 + 93a\,b^2m\,x^4 + 39a^2b\,m^2x^2 + \cdots}{(m+7)\,(m+5)\,(m+3)\,(m+1\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] x*(b^3*m^3*x^6+9*b^3*m^2*x^6+3*a*b^2*m^3*x^4+23*b^3*m*x^6+33*a*b^2*m^2*x^4+
15*b^3*x^6+3*a^2*b*m^3*x^2+93*a*b^2*m*x^4+39*a^2*b*m^2*x^2+63*a*b^2*x^4+a^3
*m^3+141*a^2*b*m*x^2+15*a^3*m^2+105*a^2*b*x^2+71*a^3*m+105*a^3)*(d*x)^m*((b
*x^2+a)^2)^(3/2)/(m+7)/(m+5)/(m+3)/(m+1)/(b*x^2+a)^3
```

maxima [A] time = 1.44, size = 119, normalized size = 0.58

$$\frac{\left(\left(m^3 + 9\,m^2 + 23\,m + 15\right)b^3d^mx^7 + 3\left(m^3 + 11\,m^2 + 31\,m + 21\right)ab^2d^mx^5 + 3\left(m^3 + 13\,m^2 + 47\,m + 35\right)a^2bd^mx^3 + \cdots}{m^4 + 16\,m^3 + 86\,m^2 + 176\,m + 105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((m^3 + 9*m^2 + 23*m + 15)*b^3*d^m*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*a*b^2
*d^m*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*a^2*b*d^m*x^3 + (m^3 + 15*m^2 + 71*
m + 105)*a^3*d^m*x)*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^m\,\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**m*((a + b*x**2)**2)**(3/2), x)
```

3.793 $\int (dx)^m \sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=97

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+3}}{d^3(m+3)\left(a+bx^2\right)}+\frac{a\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+1}}{d(m+1)\left(a+bx^2\right)}$$

```
[Out] a*(d*x)^(1+m)*((b*x^2+a)^2)^(1/2)/d/(1+m)/(b*x^2+a)+b*(d*x)^(3+m)*((b*x^2+a
)^2)^(1/2)/d^3/(3+m)/(b*x^2+a)
```

Rubi [A] time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1112, 14}

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+3}}{d^3(m+3)\left(a+bx^2\right)}+\frac{a\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+1}}{d(m+1)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(1 + m)*(a + b*x^2)) +
 (b*(d*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(3 + m)*(a + b*x^2)
)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^m\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^m\left(ab+b^2x^2\right)dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(ab(dx)^m+\frac{b^2(dx)^{2+m}}{d^2}\right)dx}{ab+b^2x^2}$$
$$= \frac{a(dx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{d(1+m)\left(a+bx^2\right)}+\frac{b(dx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(3+m)\left(a+bx^2\right)}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.55

$$\frac{x\sqrt{\left(a+bx^2\right)^2}(dx)^m\left(a(m+3)+b(m+1)x^2\right)}{(m+1)(m+3)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(a + b*x^2)^2]*(a*(3 + m) + b*(1 + m)*x^2))/((1 + m)*(3 + m
)*(a + b*x^2))
```

fricas [A] time = 1.08, size = 35, normalized size = 0.36

$$\frac{\left((bm+b)x^3+(am+3\,a)x\right)(dx)^m}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^3 + (a*m + 3*a)*x)*(d*x)^m/(m^2 + 4*m + 3)
```

giac [A] time = 0.16, size = 83, normalized size = 0.86

$$\frac{(dx)^m\,bmx^3\mathrm{sgn}\left(bx^2+a\right)+(dx)^m\,bx^3\mathrm{sgn}\left(bx^2+a\right)+(dx)^m\,amx\mathrm{sgn}\left(bx^2+a\right)+3\,(dx)^m\,ax\mathrm{sgn}\left(bx^2+a\right)}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b*m*x^3*sgn(b*x^2 + a) + (d*x)^m*b*x^3*sgn(b*x^2 + a) + (d*x)^m*a*
m*x*sgn(b*x^2 + a) + 3*(d*x)^m*a*x*sgn(b*x^2 + a))/(m^2 + 4*m + 3)
```

maple [A] time = 0.00, size = 56, normalized size = 0.58

$$\frac{\left(bm\,x^2+b\,x^2+am+3a\right)\sqrt{\left(b\,x^2+a\right)^2}\;x\,(dx)^m}{(m+3)\,(m+1)\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
[Out] x*(b*m*x^2+b*x^2+a*m+3*a)*(d*x)^m*((b*x^2+a)^2)^(1/2)/(m+3)/(m+1)/(b*x^2+a)
```

maxima [A] time = 1.40, size = 35, normalized size = 0.36

$$\frac{\left(bd^m(m+1)x^3+ad^m(m+3)x\right)x^m}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="maxima")
[Out] (b*d^m*(m + 1)*x^3 + a*d^m*(m + 3)*x)*x^m/(m^2 + 4*m + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2),x)
[Out] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m\sqrt{\left(a+bx^2\right)^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
[Out] Integral((d*x)**m*sqrt((a + b*x**2)**2), x)
```

3.794 $\int \frac{(dx)^m}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=73

$$\frac{(a+bx^2)(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (d*x)^(1+m)*(b*x^2+a)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/d/(1
+m)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1112, 364}

$$\frac{(a+bx^2)(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)])/(a*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{(dx)^m}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{(dx)^{1+m}\left(a+bx^2\right)\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ad(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.85

$$\frac{x\left(a+bx^2\right)(dx)^m\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a(m+1)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d*x)^m/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]

[Out] (x*(d*x)^m*(a + b*x^2)*Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(a*(1 + m)*Sqrt[(a + b*x^2)^2])

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{\sqrt{b^2x^4+2\,abx^2+a^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="fricas")

[Out] integral((d*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{b^2x^4+2\,abx^2+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="giac")

[Out] integrate((d*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{b^2x^4 + 2ab\,x^2 + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{b^2x^4 + 2\,abx^2 + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="maxima")
[Out] integrate((d*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{\sqrt{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2),x)
[Out] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{\left(a + bx^2\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
[Out] Integral((d*x)**m/sqrt((a + b*x**2)**2), x)
```

3.795 $\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{\left(a+bx^2\right)(dx)^{m+1}\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (d*x)^(1+m)*(b*x^2+a)*hypergeom([3, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^3/d/
(1+m)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1112, 364}

$$\frac{\left(a+bx^2\right)(dx)^{m+1}\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)])/(a^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^m}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{(dx)^{1+m}\left(a+bx^2\right)\,{}_2F_1\left(3,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^3d(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 0.82

$$\frac{x\left(a+bx^2\right)(dx)^m\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^3(m+1)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x*(d*x)^m*(a + b*x^2)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*x^2)
/a)])/(a^3*(1 + m)*Sqrt[(a + b*x^2)^2])
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b^2x^4+2\,abx^2+a^2}\,(dx)^m}{b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^m/(b^4*x^8 + 4*a*b^3*x^6 + 6
*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b^2x^4 + 2ab\,x^2 + a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
[Out] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
[Out] Integral((d*x)**m/((a + b*x**2)**2)**(3/2), x)
```

3.796 $\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=73

$$\frac{\left(a+bx^2\right)(dx)^{m+1}\,{}_2F_1\left(5,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^5d(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (d*x)^(1+m)*(b*x^2+a)*hypergeom([5, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^5/d/
(1+m)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1112, 364}

$$\frac{\left(a+bx^2\right)(dx)^{m+1}\,{}_2F_1\left(5,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^5d(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[5, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)])/(a^5*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^m}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{(dx)^{1+m}\left(a+bx^2\right)\,{}_2F_1\left(5,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^5d(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 0.82

$$\frac{x\left(a+bx^2\right)(dx)^m\,{}_2F_1\left(5,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^5(m+1)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x*(d*x)^m*(a + b*x^2)*Hypergeometric2F1[5, (1 + m)/2, (3 + m)/2, -((b*x^2)
/a)])/(a^5*(1 + m)*Sqrt[(a + b*x^2)^2])
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b^2x^4+2\,abx^2+a^2}\,(dx)^m}{b^6x^{12}+6\,ab^5x^{10}+15\,a^2b^4x^8+20\,a^3b^3x^6+15\,a^4b^2x^4+6\,a^5bx^2+a^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^m/(b^6*x^12 + 6*a*b^5*x^10 +
 15*a^2*b^4*x^8 + 20*a^3*b^3*x^6 + 15*a^4*b^2*x^4 + 6*a^5*b*x^2 + a^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b^2x^4 + 2ab\,x^2 + a^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral((d*x)**m/((a + b*x**2)**2)**(5/2), x)
```

3.797 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=74

$$\frac{\left(a+bx^2\right)(dx)^{m+1}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(1,\frac{1}{2}(m+4p+3);\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ad(m+1)}$$

```
[Out] (d*x)^(1+m)*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([1, 3/2+1/2*m+2*p
],[3/2+1/2*m],-b*x^2/a)/a/d/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1113, 364}

$$\frac{(dx)^{m+1}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{m+1}{2},-2p;\frac{m+3}{2};-\frac{bx^2}{a}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[(1 + m)/2, -
2*p, (3 + m)/2, -((b*x^2)/a)])/(d*(1 + m)*(1 + (b*x^2)/a)^(2*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[2*p]
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^p\,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int (dx)^m\left(1+\frac{bx^2}{a}\right)^{2p}dx$$
$$= \frac{(dx)^{1+m}\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{1+m}{2},-2p;\frac{3+m}{2};-\frac{bx^2}{a}\right)}{d(1+m)}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 0.89

$$\frac{x(dx)^m\left(\left(a+bx^2\right)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}{}_2F_1\left(\frac{m+1}{2},-2p;\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
[Out] (x*(d*x)^m*((a + b*x^2)^2)^p*Hypergeometric2F1[(1 + m)/2, -2*p, 1 + (1 + m)
/2, -((b*x^2)/a)])/((1 + m)*(1 + (b*x^2)/a)^(2*p))
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2x^4+2\,abx^2+a^2\right)^p(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p*(d*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^4+2\,abx^2+a^2\right)^p(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*(d*x)^m, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int (dx)^m\left(b^2x^4+2ab\,x^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^4 + 2\,abx^2 + a^2\right)^p (dx)^m\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*(d*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m \left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(\left(a + bx^2\right)^2\right)^p\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Integral((d*x)**m*((a + b*x**2)**2)**p, x)
```

3.798 $\int x^7 \left(a^2+2abx^2+b^2x^4\right)^p\,dx$

Optimal. Leaf size=174

$$\frac{\left(a+bx^2\right)^4\left(a^2+2abx^2+b^2x^4\right)^p}{4b^4(p+2)}-\frac{3a\left(a+bx^2\right)^3\left(a^2+2abx^2+b^2x^4\right)^p}{2b^4(2p+3)}+\frac{3a^2\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^4(p+1)}-\frac{a^3}{}$$

```
[Out] -1/2*a^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p/b^4/(1+2*p)+3/4*a^2*(b*x^2+a)^
2*(b^2*x^4+2*a*b*x^2+a^2)^p/b^4/(1+p)-3/2*a*(b*x^2+a)^3*(b^2*x^4+2*a*b*x^2+
a^2)^p/b^4/(3+2*p)+1/4*(b*x^2+a)^4*(b^2*x^4+2*a*b*x^2+a^2)^p/b^4/(2+p)
```

Rubi [A] time = 0.11, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1113, 266, 43}

$$\frac{\left(a+bx^2\right)^4\left(a^2+2abx^2+b^2x^4\right)^p}{4b^4(p+2)}-\frac{3a\left(a+bx^2\right)^3\left(a^2+2abx^2+b^2x^4\right)^p}{2b^4(2p+3)}+\frac{3a^2\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^4(p+1)}-\frac{a^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] -(a^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^4*(1 + 2*p)) + (3*a^2
*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^4*(1 + p)) - (3*a*(a + b
*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^4*(3 + 2*p)) + ((a + b*x^2)^4*(
a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^4*(2 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
```

```
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^p\,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int x^7\left(1+\frac{bx^2}{a}\right)^{2p}dx$$

$$=\frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x^3\left(1+\frac{bx}{a}\right)^{2p}dx,x,x^2\right)$$

$$=\frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int\left(-\frac{a^3\left(1+\frac{bx}{a}\right)^{2p}}{b^3}+\frac{3a^3\left(1+\frac{bx}{a}\right.}{b^3}\right.\right.$$

$$=-\frac{a^3\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^4(1+2p)}+\frac{3a^2\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^4(1+p)}-\frac{3a}{}$$

Mathematica [A] time = 0.06, size = 110, normalized size = 0.63

$$\frac{\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^p\left(-3a^3+3a^2b(2p+1)x^2-3ab^2\left(2p^2+3p+1\right)x^4+b^3\left(4p^3+12p^2+11p+3\right)x^6\right)}{4b^4(p+1)(p+2)(2p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^p*(-3*a^3 + 3*a^2*b*(1 + 2*p)*x^2 - 3*a*b^2*(1
 + 3*p + 2*p^2)*x^4 + b^3*(3 + 11*p + 12*p^2 + 4*p^3)*x^6))/(4*b^4*(1 + p)*
(2 + p)*(1 + 2*p)*(3 + 2*p))
```

fricas [A] time = 0.95, size = 163, normalized size = 0.94

$$\frac{\left(\left(4\,b^4p^3+12\,b^4p^2+11\,b^4p+3\,b^4\right)x^8+6\,a^3bpx^2+2\left(2\,ab^3p^3+3\,ab^3p^2+ab^3p\right)x^6-3\left(2\,a^2b^2p^2+a^2b^2p\right)x^4-3\,a\right.}{4\left(4\,b^4p^4+20\,b^4p^3+35\,b^4p^2+25\,b^4p+6\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/4*((4*b^4*p^3 + 12*b^4*p^2 + 11*b^4*p + 3*b^4)*x^8 + 6*a^3*b*p*x^2 + 2*(2
*a*b^3*p^3 + 3*a*b^3*p^2 + a*b^3*p)*x^6 - 3*(2*a^2*b^2*p^2 + a^2*b^2*p)*x^4
```

```
 - 3*a^4)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(4*b^4*p^4 + 20*b^4*p^3 + 35*b^4*p^
2 + 25*b^4*p + 6*b^4)
```

giac [B] time = 0.19, size = 375, normalized size = 2.16

$$\frac{4\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{p}b^{4}p^{3}x^{8}+12\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{p}b^{4}p^{2}x^{8}+4\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{p}ab^{3}p^{3}x^{6}+11\left(b^{2}x^{4}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/4*(4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*p^3*x^8 + 12*(b^2*x^4 + 2*a*b*x^2
+ a^2)^p*b^4*p^2*x^8 + 4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^3*p^3*x^6 + 11*(
b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*p*x^8 + 6*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*
b^3*p^2*x^6 + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*x^8 + 2*(b^2*x^4 + 2*a*b*
x^2 + a^2)^p*a*b^3*p*x^6 - 6*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p^2*x^4
- 3*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p*x^4 + 6*(b^2*x^4 + 2*a*b*x^2 +
a^2)^p*a^3*b*p*x^2 - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^4)/(4*b^4*p^4 + 20*b
^4*p^3 + 35*b^4*p^2 + 25*b^4*p + 6*b^4)
```

maple [A] time = 0.01, size = 150, normalized size = 0.86

$$-\frac{\left(-4b^{3}p^{3}x^{6}-12b^{3}p^{2}x^{6}-11b^{3}p\,x^{6}+6a\,b^{2}p^{2}x^{4}-3b^{3}x^{6}+9a\,b^{2}p\,x^{4}+3a\,b^{2}x^{4}-6a^{2}bp\,x^{2}-3a^{2}b\,x^{2}+3a^{3}\right)\left(b\,x^{2}\right.}{4\left(4p^{4}+20p^{3}+35p^{2}+25p+6\right)b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] -1/4*(b^2*x^4+2*a*b*x^2+a^2)^p*(-4*b^3*p^3*x^6-12*b^3*p^2*x^6-11*b^3*p*x^6+
6*a*b^2*p^2*x^4-3*b^3*x^6+9*a*b^2*p*x^4+3*a*b^2*x^4-6*a^2*b*p*x^2-3*a^2*b*x
^2+3*a^3)*(b*x^2+a)/b^4/(4*p^4+20*p^3+35*p^2+25*p+6)
```

maxima [A] time = 1.46, size = 115, normalized size = 0.66

$$\frac{\left(\left(4\,p^{3}+12\,p^{2}+11\,p+3\right)b^{4}x^{8}+2\left(2\,p^{3}+3\,p^{2}+p\right)ab^{3}x^{6}-3\left(2\,p^{2}+p\right)a^{2}b^{2}x^{4}+6\,a^{3}bpx^{2}-3\,a^{4}\right)\left(bx^{2}+a\right)^{2\,p}}{4\left(4\,p^{4}+20\,p^{3}+35\,p^{2}+25\,p+6\right)b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/4*((4*p^3 + 12*p^2 + 11*p + 3)*b^4*x^8 + 2*(2*p^3 + 3*p^2 + p)*a*b^3*x^6
- 3*(2*p^2 + p)*a^2*b^2*x^4 + 6*a^3*b*p*x^2 - 3*a^4)*(b*x^2 + a)^(2*p)/((4*
p^4 + 20*p^3 + 35*p^2 + 25*p + 6)*b^4)
```

mupad [B] time = 4.40, size = 206, normalized size = 1.18

$$\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p\,\left(\frac{x^8\,\left(p^3+3\,p^2+\frac{11\,p}{4}+\frac{3}{4}\right)}{4\,p^4+20\,p^3+35\,p^2+25\,p+6}-\frac{3\,a^4}{4\,b^4\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}+\frac{\ldots}{2\,b^3\,\left(4\,p^4+2\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^p*((x^8*((11*p)/4 + 3*p^2 + p^3 + 3/4))/(25*p +
 35*p^2 + 20*p^3 + 4*p^4 + 6) - (3*a^4)/(4*b^4*(25*p + 35*p^2 + 20*p^3 + 4*
p^4 + 6)) + (3*a^3*p*x^2)/(2*b^3*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)) + (a
*p*x^6*(3*p + 2*p^2 + 1))/(2*b*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)) - (3*a
^2*p*x^4*(2*p + 1))/(4*b^2*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((x**8*(a**2)**p/8, Eq(b, 0)), (6*a**3*log(-I*sqrt(a)*sqrt(1/b) +
x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*a
**3*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b
**6*x**4 + 12*b**7*x**6) + 11*a**3/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a
*b**6*x**4 + 12*b**7*x**6) + 18*a**2*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(
12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a**2
*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36
*a*b**6*x**4 + 12*b**7*x**6) + 27*a**2*b*x**2/(12*a**3*b**4 + 36*a**2*b**5*
x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(-I*sqrt(a)*sqrt(
1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6
) + 18*a*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**
5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4/(12*a**3*b**4 + 36
*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b**3*x**6*log(-I*sqrt(
a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b
**7*x**6) + 6*b**3*x**6*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**
2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6), Eq(p, -2)), (Integral(x**7/((
a + b*x**2)**2)**(3/2), x), Eq(p, -3/2)), (6*a**3*log(-I*sqrt(a)*sqrt(1/b)
+ x)/(4*a*b**4 + 4*b**5*x**2) + 6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a*b*
*4 + 4*b**5*x**2) + 6*a**3/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(-I*
sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(I*sqrt(
a)*sqrt(1/b) + x)/(4*a*b**4 + 4*b**5*x**2) - 3*a*b**2*x**4/(4*a*b**4 + 4*b*
```

```
*5*x**2) + b**3*x**6/(4*a*b**4 + 4*b**5*x**2), Eq(p, -1)), (Integral(x**7/s
qrt((a + b*x**2)**2), x), Eq(p, -1/2)), (-3*a**4*(a**2 + 2*a*b*x**2 + b**2*
x**4)**p/(16*b**4*p**4 + 80*b**4*p**3 + 140*b**4*p**2 + 100*b**4*p + 24*b**
4) + 6*a**3*b*p*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(16*b**4*p**4 + 80*
b**4*p**3 + 140*b**4*p**2 + 100*b**4*p + 24*b**4) - 6*a**2*b**2*p**2*x**4*(
a**2 + 2*a*b*x**2 + b**2*x**4)**p/(16*b**4*p**4 + 80*b**4*p**3 + 140*b**4*p
**2 + 100*b**4*p + 24*b**4) - 3*a**2*b**2*p*x**4*(a**2 + 2*a*b*x**2 + b**2*
x**4)**p/(16*b**4*p**4 + 80*b**4*p**3 + 140*b**4*p**2 + 100*b**4*p + 24*b**
4) + 4*a*b**3*p**3*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(16*b**4*p**4 +
80*b**4*p**3 + 140*b**4*p**2 + 100*b**4*p + 24*b**4) + 6*a*b**3*p**2*x**6*(
a**2 + 2*a*b*x**2 + b**2*x**4)**p/(16*b**4*p**4 + 80*b**4*p**3 + 140*b**4*p
**2 + 100*b**4*p + 24*b**4) + 2*a*b**3*p*x**6*(a**2 + 2*a*b*x**2 + b**2*x**
4)**p/(16*b**4*p**4 + 80*b**4*p**3 + 140*b**4*p**2 + 100*b**4*p + 24*b**4)
+ 4*b**4*p**3*x**8*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(16*b**4*p**4 + 80*b*
*4*p**3 + 140*b**4*p**2 + 100*b**4*p + 24*b**4) + 12*b**4*p**2*x**8*(a**2 +
 2*a*b*x**2 + b**2*x**4)**p/(16*b**4*p**4 + 80*b**4*p**3 + 140*b**4*p**2 +
100*b**4*p + 24*b**4) + 11*b**4*p*x**8*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(
16*b**4*p**4 + 80*b**4*p**3 + 140*b**4*p**2 + 100*b**4*p + 24*b**4) + 3*b**
4*x**8*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(16*b**4*p**4 + 80*b**4*p**3 + 14
0*b**4*p**2 + 100*b**4*p + 24*b**4), True))
```

3.799 $\int x^5 \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=130

$$\frac{\left(a+bx^2\right)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(2p+3)}-\frac{a\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(p+1)}+\frac{a^2\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(2p+1)}$$

```
[Out] 1/2*a^2*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(1+2*p)-1/2*a*(b*x^2+a)^2*(
b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(1+p)+1/2*(b*x^2+a)^3*(b^2*x^4+2*a*b*x^2+a^2)^
p/b^3/(3+2*p)
```

Rubi [A] time = 0.08, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1113, 266, 43}

$$\frac{\left(a+bx^2\right)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(2p+3)}-\frac{a\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(p+1)}+\frac{a^2\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (a^2*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^3*(1 + 2*p)) - (a*(a +
 b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^3*(1 + p)) + ((a + b*x^2)^3*(
a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^3*(3 + 2*p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
```

```
c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^p \, dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p} \left(a^2+2abx^2+b^2x^4\right)^p\right) \int x^5 \left(1+\frac{bx^2}{a}\right)^{2p} \, dx$$
$$= \frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p} \left(a^2+2abx^2+b^2x^4\right)^p\right) \text{Subst}\left(\int x^2 \left(1+\frac{bx}{a}\right)^{2p} \, dx, x, x^2\right)$$
$$= \frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p} \left(a^2+2abx^2+b^2x^4\right)^p\right) \text{Subst}\left(\int \left(\frac{a^2\left(1+\frac{bx}{a}\right)^{2p}}{b^2} - \frac{2a^2\left(1+\frac{bx}{a}\right.}{b^2}\right.\right.$$
$$= \frac{a^2\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(1+2p)} - \frac{a\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(1+p)} + \frac{\left(a+\right.}{}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.59

$$\frac{\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^p \left(a^2-ab(2p+1)x^2+b^2\left(2p^2+3p+1\right)x^4\right)}{2b^3(p+1)(2p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^p*(a^2 - a*b*(1 + 2*p)*x^2 + b^2*(1 + 3*p + 2*
p^2)*x^4))/(2*b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))
```

fricas [A] time = 1.12, size = 108, normalized size = 0.83

$$\frac{\left(\left(2\,b^3p^2+3\,b^3p+b^3\right)x^6-2\,a^2bpx^2+\left(2\,ab^2p^2+ab^2p\right)x^4+a^3\right)\left(b^2x^4+2\,abx^2+a^2\right)^p}{2\left(4\,b^3p^3+12\,b^3p^2+11\,b^3p+3\,b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/2*((2*b^3*p^2 + 3*b^3*p + b^3)*x^6 - 2*a^2*b*p*x^2 + (2*a*b^2*p^2 + a*b^2
*p)*x^4 + a^3)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(4*b^3*p^3 + 12*b^3*p^2 + 11*b
^3*p + 3*b^3)
```

giac [A] time = 0.19, size = 235, normalized size = 1.81

$$\frac{2\left(b^2x^4+2\,abx^2+a^2\right)^pb^3p^2x^6+3\left(b^2x^4+2\,abx^2+a^2\right)^pb^3px^6+2\left(b^2x^4+2\,abx^2+a^2\right)^pab^2p^2x^4+\left(b^2x^4+2\,abx^2\right.}{2\left(4\,b^3p^3+12\,b^3p^2+11\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/2*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^3*p^2*x^6 + 3*(b^2*x^4 + 2*a*b*x^2 +
 a^2)^p*b^3*p*x^6 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^2*p^2*x^4 + (b^2*x^
4 + 2*a*b*x^2 + a^2)^p*b^3*x^6 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^2*p*x^4
- 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b*p*x^2 + (b^2*x^4 + 2*a*b*x^2 + a^2)
^p*a^3)/(4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

maple [A] time = 0.01, size = 96, normalized size = 0.74

$$\frac{\left(b\,x^2+a\right)\left(2b^2p^2x^4+3b^2p\,x^4+b^2x^4-2abp\,x^2-ab\,x^2+a^2\right)\left(b^2x^4+2ab\,x^2+a^2\right)^p}{2\left(4p^3+12p^2+11p+3\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] 1/2*(b*x^2+a)*(2*b^2*p^2*x^4+3*b^2*p*x^4+b^2*x^4-2*a*b*p*x^2-a*b*x^2+a^2)*(
b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(4*p^3+12*p^2+11*p+3)
```

maxima [A] time = 1.44, size = 79, normalized size = 0.61

$$\frac{\left(\left(2\,p^2+3\,p+1\right)b^3x^6+\left(2\,p^2+p\right)ab^2x^4-2\,a^2bpx^2+a^3\right)\left(bx^2+a\right)^{2\,p}}{2\left(4\,p^3+12\,p^2+11\,p+3\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((2*p^2 + 3*p + 1)*b^3*x^6 + (2*p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + a^
3)*(b*x^2 + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

mupad [B] time = 4.27, size = 137, normalized size = 1.05

$$\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p\left(\frac{x^6\,\left(p^2+\frac{3\,p}{2}+\frac{1}{2}\right)}{4\,p^3+12\,p^2+11\,p+3}+\frac{a^3}{2\,b^3\,\left(4\,p^3+12\,p^2+11\,p+3\right)}-\frac{a^2\,p\,x^2}{b^2\,\left(4\,p^3+12\,p^2+11\,p+3\right)}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^p*((x^6*((3*p)/2 + p^2 + 1/2))/(11*p + 12*p^2 +
 4*p^3 + 3) + a^3/(2*b^3*(11*p + 12*p^2 + 4*p^3 + 3)) - (a^2*p*x^2)/(b^2*(1
1*p + 12*p^2 + 4*p^3 + 3)) + (a*p*x^4*(2*p + 1))/(2*b*(11*p + 12*p^2 + 4*p^
3 + 3)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^6(a^2)^p}{6} \\ \int \frac{x^5}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx \\ -\frac{2a^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^3+2b^4x^2}-\frac{2a^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^3+2b^4x^2}-\frac{2a^2}{2ab^3+2b^4x^2}-\frac{2abx^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^3+2b^4x^2}-\frac{2abx^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^3+2b^4x^2}+\frac{b^2x^4}{2ab^3+2b^4x^2} \\ \int \frac{x^5}{\sqrt{(a+bx^2)^2}}\,dx \\ \frac{a^3(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+24b^3p^2+22b^3p+6b^3}-\frac{2a^2bpx^2(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+24b^3p^2+22b^3p+6b^3}+\frac{2ab^2p^2x^4(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+24b^3p^2+22b^3p+6b^3}+\frac{ab^2px^4(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+24b^3p^2+22b^3p+6b^3}+\frac{2b^3p^2x^6(a^2+2abx^2+}{8b^3p^3+24b^3p^2+22b^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((x**6*(a**2)**p/6, Eq(b, 0)), (Integral(x**5/((a + b*x**2)**2)**(
3/2), x), Eq(p, -3/2)), (-2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 +
2*b**4*x**2) - 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2)
 - 2*a**2/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) +
x)/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*
b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -1)), (Inte
gral(x**5/sqrt((a + b*x**2)**2), x), Eq(p, -1/2)), (a**3*(a**2 + 2*a*b*x**2
 + b**2*x**4)**p/(8*b**3*p**3 + 24*b**3*p**2 + 22*b**3*p + 6*b**3) - 2*a**2
*b*p*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 24*b**3*p**2 +
22*b**3*p + 6*b**3) + 2*a*b**2*p**2*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p
/(8*b**3*p**3 + 24*b**3*p**2 + 22*b**3*p + 6*b**3) + a*b**2*p*x**4*(a**2 +
2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 24*b**3*p**2 + 22*b**3*p + 6*b**3
) + 2*b**3*p**2*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 24*b
**3*p**2 + 22*b**3*p + 6*b**3) + 3*b**3*p*x**6*(a**2 + 2*a*b*x**2 + b**2*x*
*4)**p/(8*b**3*p**3 + 24*b**3*p**2 + 22*b**3*p + 6*b**3) + b**3*x**6*(a**2
```

```
+ 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 24*b**3*p**2 + 22*b**3*p + 6*b*
*3), True))
```

3.800 $\int x^3 \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=84

$$\frac{\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(p+1)}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(2p+1)}$$

```
[Out] -1/2*a*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p/b^2/(1+2*p)+1/4*(b*x^2+a)^2*(b^2
*x^4+2*a*b*x^2+a^2)^p/b^2/(1+p)
```

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1113, 266, 43}

$$\frac{\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(p+1)}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] -(a*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^2*(1 + 2*p)) + ((a + b*
x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^2*(1 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a^2+2abx^2+b^2x^4\right)^p\,dx &= \left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int x^3\left(1+\frac{bx^2}{a}\right)^{2p}\,dx\\
&= \frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\mathrm{Subst}\left(\int x\left(1+\frac{bx}{a}\right)^{2p}\,dx,x,x^2\right)\\
&= \frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\mathrm{Subst}\left(\int\left(-\frac{a\left(1+\frac{bx}{a}\right)^{2p}}{b}+\frac{a\left(1+\frac{bx}{a}\right)^{1+2}}{b}\right.\right.\\
&= -\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(1+2p)}+\frac{\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(1+p)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.61

$$\frac{\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^p\left(b(2p+1)x^2-a\right)}{4b^2(p+1)(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^p*(-a + b*(1 + 2*p)*x^2))/(4*b^2*(1 + p)*(1 +
2*p))
```

fricas [A] time = 1.11, size = 70, normalized size = 0.83

$$\frac{\left(2\,abpx^2+\left(2\,b^2p+b^2\right)x^4-a^2\right)\left(b^2x^4+2\,abx^2+a^2\right)^p}{4\left(2\,b^2p^2+3\,b^2p+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b*p*x^2 + (2*b^2*p + b^2)*x^4 - a^2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p
/(2*b^2*p^2 + 3*b^2*p + b^2)
```

giac [A] time = 0.22, size = 132, normalized size = 1.57

$$\frac{2\left(b^2x^4+2\,abx^2+a^2\right)^p b^2px^4+\left(b^2x^4+2\,abx^2+a^2\right)^p b^2x^4+2\left(b^2x^4+2\,abx^2+a^2\right)^p abpx^2-\left(b^2x^4+2\,abx^2+a^2\right)^{p}}{4\left(2\,b^2p^2+3\,b^2p+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/4*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^2*p*x^4 + (b^2*x^4 + 2*a*b*x^2 + a^2
)^p*b^2*x^4 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b*p*x^2 - (b^2*x^4 + 2*a*b*
x^2 + a^2)^p*a^2)/(2*b^2*p^2 + 3*b^2*p + b^2)
```

maple [A] time = 0.01, size = 60, normalized size = 0.71

$$-\frac{\left(-2x^{2}pb-b\,x^{2}+a\right)\left(b\,x^{2}+a\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{p}}{4\left(2p^{2}+3p+1\right)b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] -1/4*(b^2*x^4+2*a*b*x^2+a^2)^p*(-2*b*p*x^2-b*x^2+a)*(b*x^2+a)/b^2/(2*p^2+3*
p+1)
```

maxima [A] time = 1.43, size = 54, normalized size = 0.64

$$\frac{\left(b^{2}\left(2\,p+1\right)x^{4}+2\,abpx^{2}-a^{2}\right)\left(bx^{2}+a\right)^{2\,p}}{4\left(2\,p^{2}+3\,p+1\right)b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*(2*p + 1)*x^4 + 2*a*b*p*x^2 - a^2)*(b*x^2 + a)^(2*p)/((2*p^2 + 3*p
 + 1)*b^2)
```

mupad [B] time = 4.26, size = 85, normalized size = 1.01

$$\left(a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}\right)^{p}\left(\frac{x^{4}\,\left(2\,p+1\right)}{4\,\left(2\,p^{2}+3\,p+1\right)}-\frac{a^{2}}{4\,b^{2}\,\left(2\,p^{2}+3\,p+1\right)}+\frac{a\,p\,x^{2}}{2\,b\,\left(2\,p^{2}+3\,p+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^p*((x^4*(2*p + 1))/(4*(3*p + 2*p^2 + 1)) - a^2/
(4*b^2*(3*p + 2*p^2 + 1)) + (a*p*x^2)/(2*b*(3*p + 2*p^2 + 1)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^4(a^2)^p}{4} & \text{for } b = 0 \\ \frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2} + \frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2} + \frac{a}{2ab^2+2b^3x^2} + \frac{bx^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2} + \frac{bx^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2} & \text{for } p = -1 \\ \int \frac{x^3}{\sqrt{\left(a+bx^2\right)^2}}\,dx & \text{for } p = -\frac{1}{2} \\ -\frac{a^2(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+12b^2p+4b^2} + \frac{2abpx^2(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+12b^2p+4b^2} + \frac{2b^2px^4(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+12b^2p+4b^2} + \frac{b^2x^4(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+12b^2p+4b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((x**4*(a**2)**p/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2
*a*b**2 + 2*b**3*x**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*
x**2) + a/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(
2*a*b**2 + 2*b**3*x**2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2
*b**3*x**2), Eq(p, -1)), (Integral(x**3/sqrt((a + b*x**2)**2), x), Eq(p, -1
/2)), (-a**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 12*b**2*p +
4*b**2) + 2*a*b*p*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 12
*b**2*p + 4*b**2) + 2*b**2*p*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**
2*p**2 + 12*b**2*p + 4*b**2) + b**2*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p
/(8*b**2*p**2 + 12*b**2*p + 4*b**2), True))
```

3.801 $\int x\left(a^2+2abx^2+b^2x^4\right)^p\,dx$

Optimal. Leaf size=41

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b(2p+1)}$$

```
[Out] 1/2*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p/b/(1+2*p)
```

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1107, 609}

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b*(1 + 2*p))
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^2+b^2x^4\right)^p\,dx&=\frac{1}{2}\text{Subst}\left(\int\left(a^2+2abx+b^2x^2\right)^p\,dx,x,x^2\right)\\&=\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b(1+2p)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 29, normalized size = 0.71

$$\frac{(a+bx^2)\left((a+bx^2)^2\right)^p}{4bp+2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^p)/(2*b + 4*b*p)
```

fricas [A] time = 0.91, size = 37, normalized size = 0.90

$$\frac{(bx^2+a)(b^2x^4+2\,abx^2+a^2)^p}{2\,(2\,bp+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2 + a)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(2*b*p + b)
```

giac [A] time = 0.26, size = 58, normalized size = 1.41

$$\frac{(b^2x^4+2\,abx^2+a^2)^p bx^2+(b^2x^4+2\,abx^2+a^2)^p a}{2\,(2\,bp+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/2*((b^2*x^4 + 2*a*b*x^2 + a^2)^p*b*x^2 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a)
/(2*b*p + b)
```

maple [A] time = 0.00, size = 40, normalized size = 0.98

$$\frac{(b\,x^2+a)\,(b^2x^4+2ab\,x^2+a^2)^p}{2\,(2p+1)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] 1/2*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p/b/(1+2*p)
```

maxima [A] time = 1.32, size = 30, normalized size = 0.73

$$\frac{\left(bx^2+a\right)\left(bx^2+a\right)^{2p}}{2b\left(2p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + a)*(b*x^2 + a)^(2*p)/(b*(2*p + 1))
```

mupad [B] time = 4.67, size = 46, normalized size = 1.12

$$\left(\frac{x^2}{2\left(2p+1\right)}+\frac{a}{2b\left(2p+1\right)}\right)\left(a^2+2abx^2+b^2x^4\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] (x^2/(2*(2*p + 1)) + a/(2*b*(2*p + 1)))*(a^2 + b^2*x^4 + 2*a*b*x^2)^p
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^2}{2\sqrt{a^2}} & \text{for}\: b=0\wedge p=-\frac{1}{2} \\ \frac{x^2\left(a^2\right)^p}{2} & \text{for}\: b=0 \\ \int \frac{x}{\sqrt{\left(a+bx^2\right)^2}}\, dx & \text{for}\: p=-\frac{1}{2} \\ \frac{a\left(a^2+2abx^2+b^2x^4\right)^p}{4bp+2b}+\frac{bx^2\left(a^2+2abx^2+b^2x^4\right)^p}{4bp+2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((x**2/(2*sqrt(a**2)), Eq(b, 0) & Eq(p, -1/2)), (x**2*(a**2)**p/2,
 Eq(b, 0)), (Integral(x/sqrt((a + b*x**2)**2), x), Eq(p, -1/2)), (a*(a**2 +
 2*a*b*x**2 + b**2*x**4)**p/(4*b*p + 2*b) + b*x**2*(a**2 + 2*a*b*x**2 + b**
2*x**4)**p/(4*b*p + 2*b), True))
```

3.802 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{x} \, dx$

Optimal. Leaf size=63

$$-\frac{(a+bx^2)(a^2+2abx^2+b^2x^4)^p \, {}_2F_1\left(1,2p+1;2(p+1);\frac{bx^2}{a}+1\right)}{2a(2p+1)}$$

```
[Out] -1/2*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([1, 1+2*p],[2+2*p],1+b*x
^2/a)/a/(1+2*p)
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1113, 266, 65}

$$-\frac{(a+bx^2)(a^2+2abx^2+b^2x^4)^p \, {}_2F_1\left(1,2p+1;2(p+1);\frac{bx^2}{a}+1\right)}{2a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x,x]
```

```
[Out] -((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[1, 1 + 2*p, 2
*(1 + p), 1 + (b*x^2)/a])/(2*a*(1 + 2*p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^p}{x}\,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{2p}}{x}\,dx$$
$$= \frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx}{a}\right)^{2p}}{x}\,dx, x, x^2\right)$$
$$= -\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\ {}_2F_1\left(1,1+2p;2(1+p);1+\frac{bx^2}{a}\right)}{2a(1+2p)}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 0.86

$$-\frac{\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^p\ {}_2F_1\left(1,2p+1;2p+2;\frac{bx^2}{a}+1\right)}{2a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x,x]
```

```
[Out] -1/2*((a + b*x^2)*((a + b*x^2)^2)^p*Hypergeometric2F1[1, 1 + 2*p, 2 + 2*p,
1 + (b*x^2)/a])/(a*(1 + 2*p))
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(b^{2} x^{4}+2 a b\,x^{2}+a^{2}\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^p/x,x)
```

```
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^{2} x^{4}+2\,a b x^{2}+a^{2}\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+b x^{2}\right)^{2}\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x,x)
```

```
[Out] Integral(((a + b*x**2)**2)**p/x, x)
```

3.803 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^p}{x^3} \, dx$

Optimal. Leaf size=64

$$\frac{b\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\ {}_2F_1\left(2,2p+1;2(p+1);\frac{bx^2}{a}+1\right)}{2a^2(2p+1)}$$

```
[Out] 1/2*b*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([2, 1+2*p],[2+2*p],1+b*
x^2/a)/a^2/(1+2*p)
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1113, 266, 65}

$$\frac{b\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\ {}_2F_1\left(2,2p+1;2(p+1);\frac{bx^2}{a}+1\right)}{2a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^3,x]
```

```
[Out] (b*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[2, 1 + 2*p,
2*(1 + p), 1 + (b*x^2)/a])/(2*a^2*(1 + 2*p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^p}{x^3}\,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{2p}}{x^3}\,dx$$
$$= \frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx}{a}\right)^{2p}}{x^2}\,dx,x,x^2\right)$$
$$= \frac{b\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(2,1+2p;2(1+p);1+\frac{bx^2}{a}\right)}{2a^2(1+2p)}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 0.86

$$\frac{b\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^p\,{}_2F_1\left(2,2p+1;2p+2;\frac{bx^2}{a}+1\right)}{2a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^3,x]
```

```
[Out] (b*(a + b*x^2)*((a + b*x^2)^2)^p*Hypergeometric2F1[2, 1 + 2*p, 2 + 2*p, 1 +
 (b*x^2)/a])/(2*a^2*(1 + 2*p))
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^3,x, algorithm="giac")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^3, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^4 + 2ab\,x^2 + a^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^p/x^3,x)
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^3,x, algorithm="maxima")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x^3,x)
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x**3,x)
[Out] Integral(((a + b*x**2)**2)**p/x**3, x)
```

3.804 $\int x^4 \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=60

$$\frac{1}{5}x^5\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{5}{2},-2p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

```
[Out] 1/5*x^5*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([5/2, -2*p],[7/2],-b*x^2/a)/((1
+b*x^2/a)^(2*p))
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1113, 364}

$$\frac{1}{5}x^5\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{5}{2},-2p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[5/2, -2*p, 7/2, -((b*x
^2)/a)])/(5*(1 + (b*x^2)/a)^(2*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^p dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int x^4\left(1+\frac{bx^2}{a}\right)^{2p} dx$$
$$= \frac{1}{5}x^5\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{5}{2},-2p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Mathematica [A] time = 0.01, size = 51, normalized size = 0.85

$$\frac{1}{5}x^5\left(\left(a+bx^2\right)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p} {}_2F_1\left(\frac{5}{2},-2p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (x^5*((a + b*x^2)^2)^p*Hypergeometric2F1[5/2, -2*p, 7/2, -((b*x^2)/a)])/(5*
(1 + (b*x^2)/a)^(2*p))
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2x^4+2\,abx^2+a^2\right)^p x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^4+2\,abx^2+a^2\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^4, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x^4\left(b^2x^4+2ab\,x^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^4 + 2\,abx^2 + a^2\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^4 \left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(\left(a + bx^2\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Integral(x**4*((a + b*x**2)**2)**p, x)
```

3.805 $\int x^2 \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=60

$$\frac{1}{3}x^3\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{3}{2},-2p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

```
[Out] 1/3*x^3*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([3/2, -2*p],[5/2],-b*x^2/a)/((1
+b*x^2/a)^(2*p))
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1113, 364}

$$\frac{1}{3}x^3\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{3}{2},-2p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[3/2, -2*p, 5/2, -((b*x
^2)/a)])/(3*(1 + (b*x^2)/a)^(2*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^2\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int x^2\left(1+\frac{bx^2}{a}\right)^{2p}\,dx$$
$$=\frac{1}{3}x^3\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{3}{2},-2p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Mathematica [**A**] time = 0.01, size = 51, normalized size = 0.85

$$\frac{1}{3}x^3\left(\left(a+bx^2\right)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}\,{}_2F_1\left(\frac{3}{2},-2p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
[Out] (x^3*((a + b*x^2)^2)^p*Hypergeometric2F1[3/2, -2*p, 5/2, -((b*x^2)/a)])/(3*
(1 + (b*x^2)/a)^(2*p))
```

fricas [**F**] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2x^4+2\,abx^2+a^2\right)^p x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^2, x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b^2x^4+2\,abx^2+a^2\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^2, x)
```

maple [**F**] time = 0.04, size = 0, normalized size = 0.00

$$\int x^2\left(b^2x^4+2ab\,x^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^4 + 2\,abx^2 + a^2\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(\left(a + bx^2\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Integral(x**2*((a + b*x**2)**2)**p, x)
```

3.806 $\int \left(a^2+2abx^2+b^2x^4\right)^p\,dx$

Optimal. Leaf size=55

$$x\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{1}{2},-2p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

```
[Out] x*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([1/2, -2*p],[3/2],-b*x^2/a)/((1+b*x^2
/a)^(2*p))
```

Rubi [A] time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1089, 245}

$$x\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{1}{2},-2p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (x*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[1/2, -2*p, 3/2, -((b*x^2
)/a)])/(1 + (b*x^2)/a)^(2*p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \left(a^2+2abx^2+b^2x^4\right)^p\,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int \left(1+\frac{bx^2}{a}\right)^{2p}\,dx$$
$$= x\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{1}{2},-2p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.84

$$x\left(\left(a+bx^2\right)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}\,{}_2F_1\left(\frac{1}{2},-2p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
[Out] (x*((a + b*x^2)^2)^p*Hypergeometric2F1[1/2, -2*p, 3/2, -((b*x^2)/a)])/(1 +
(b*x^2)/a)^(2*p)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2x^4+2\,abx^2+a^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^4+2\,abx^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \left(b^2x^4+2ab\,x^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^4 + 2\,abx^2 + a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2abx^2 + b^2x^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**p, x)
```

3.807 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^p}{x^2}\,dx$

Optimal. Leaf size=58

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(-\frac{1}{2},-2p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

```
[Out] -(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([-1/2, -2*p],[1/2],-b*x^2/a)/x/((1+b*x
^2/a)^(2*p))
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1113, 364}

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(-\frac{1}{2},-2p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^2,x]
```

```
[Out] -(((a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-1/2, -2*p, 1/2, -((b*x^
2)/a)])/(x*(1 + (b*x^2)/a)^(2*p)))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^2}\,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{2p}}{x^2}\,dx$$
$$= -\frac{\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\,{}_2F_1\left(-\frac{1}{2},-2p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 0.84

$$-\frac{\left((a+bx^2)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}\,{}_2F_1\left(-\frac{1}{2},-2p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^2,x]
```

```
[Out] -((((a + b*x^2)^2)^p*Hypergeometric2F1[-1/2, -2*p, 1/2, -((b*x^2)/a)])/(x*(
1 + (b*x^2)/a)^(2*p)))
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(b^2x^4+2\,abx^2+a^2)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b^2x^4+2\,abx^2+a^2)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^2, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^4 + 2ab\,x^2 + a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^p/x^2,x)
```

```
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x^2,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x**2,x)
```

```
[Out] Integral(((a + b*x**2)**2)**p/x**2, x)
```

3.808 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^p}{x^4}\,dx$

Optimal. Leaf size=60

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(-\frac{3}{2},-2p;-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3}$$

```
[Out] -1/3*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([-3/2, -2*p],[-1/2],-b*x^2/a)/x^3/
((1+b*x^2/a)^(2*p))
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1113, 364}

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(-\frac{3}{2},-2p;-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^4,x]
```

```
[Out] -((a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-3/2, -2*p, -1/2, -((b*x^
2)/a)])/(3*x^3*(1 + (b*x^2)/a)^(2*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^4}\,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{2p}}{x^4}\,dx$$
$$= -\frac{\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\ {}_2F_1\left(-\frac{3}{2},-2p;-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 0.85

$$-\frac{\left((a+bx^2)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}{}_2F_1\left(-\frac{3}{2},-2p;-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^4,x]
```

```
[Out] -1/3*(((a + b*x^2)^2)^p*Hypergeometric2F1[-3/2, -2*p, -1/2, -((b*x^2)/a)])/
(x^3*(1 + (b*x^2)/a)^(2*p))
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(b^2x^4+2\,abx^2+a^2)^p}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b^2x^4+2\,abx^2+a^2)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^4,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^4, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{p}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^p/x^4,x)
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^{2} x^{4}+2 \, a b x^{2}+a^{2}\right)^{p}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^4,x, algorithm="maxima")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^p}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x^4,x)
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + b x^{2}\right)^{2}\right)^{p}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x**4,x)
[Out] Integral(((a + b*x**2)**2)**p/x**4, x)
```

3.809 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=67

$$\frac{2(dx)^{5/2}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{5}{4},-2p;\frac{9}{4};-\frac{bx^2}{a}\right)}{5d}$$

```
[Out] 2/5*(d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([5/4, -2*p],[9/4],-b*x^
2/a)/d/((1+b*x^2/a)^(2*p))
```

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1113, 364}

$$\frac{2(dx)^{5/2}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{5}{4},-2p;\frac{9}{4};-\frac{bx^2}{a}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (2*(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[5/4, -2*p, 9
/4, -((b*x^2)/a)])/(5*d*(1 + (b*x^2)/a)^(2*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int (dx)^{3/2}\left(1+\frac{bx^2}{a}\right)^{2p}\,dx$$
$$=\frac{2(dx)^{5/2}\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{5}{4},-2p;\frac{9}{4};-\frac{bx^2}{a}\right)}{5d}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 0.84

$$\frac{2}{5}x(dx)^{3/2}\left(\left(a+bx^2\right)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}\,{}_2F_1\left(\frac{5}{4},-2p;\frac{9}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (2*x*(d*x)^(3/2)*((a + b*x^2)^2)^p*Hypergeometric2F1[5/4, -2*p, 9/4, -((b*x
^2)/a)])/(5*(1 + (b*x^2)/a)^(2*p))
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{dx}\left(b^2x^4+2\,abx^2+a^2\right)^p dx,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*d*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}}\left(b^2x^4+2\,abx^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}}\left(b^2x^4+2ab\,x^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}} \left(b^2x^4 + 2\,abx^2 + a^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^(3/2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^{3/2}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}} \left(\left(a + bx^2\right)^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Integral((d*x)**(3/2)*((a + b*x**2)**2)**p, x)
```

3.810 $\int \sqrt{dx} \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=67

$$\frac{2(dx)^{3/2}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{3}{4},-2p;\frac{7}{4};-\frac{bx^2}{a}\right)}{3d}$$

```
[Out] 2/3*(d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([3/4, -2*p],[7/4],-b*x^
2/a)/d/((1+b*x^2/a)^(2*p))
```

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1113, 364}

$$\frac{2(dx)^{3/2}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{3}{4},-2p;\frac{7}{4};-\frac{bx^2}{a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (2*(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[3/4, -2*p, 7
/4, -((b*x^2)/a)])/(3*d*(1 + (b*x^2)/a)^(2*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[2*p]
```

Rubi steps

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int \sqrt{dx}\left(1+\frac{bx^2}{a}\right)^{2p}dx$$

$$=\frac{2(dx)^{3/2}\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{3}{4},-2p;\frac{7}{4};-\frac{bx^2}{a}\right)}{3d}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 0.84

$$\frac{2}{3}x\sqrt{dx}\left(\left(a+bx^2\right)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}{}_2F_1\left(\frac{3}{4},-2p;\frac{7}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (2*x*Sqrt[d*x]*((a + b*x^2)^2)^p*Hypergeometric2F1[3/4, -2*p, 7/4, -((b*x^2
)/a)])/(3*(1 + (b*x^2)/a)^(2*p))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{dx}\left(b^2x^4+2\,abx^2+a^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\left(b^2x^4+2\,abx^2+a^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\,\left(b^2x^4+2ab\,x^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] int((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx} \left(b^2x^4 + 2\,abx^2 + a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d\,x}\,\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\,\left(\left(a + bx^2\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Integral(sqrt(d*x)*((a + b*x**2)**2)**p, x)
```

3.811 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^p}{\sqrt{dx}}\,dx$

Optimal. Leaf size=65

$$\frac{2\sqrt{dx}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{1}{4},-2p;\frac{5}{4};-\frac{bx^2}{a}\right)}{d}$$

```
[Out] 2*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([1/4, -2*p],[5/4],-b*x^2/a)*(d*x)^(1/
2)/d/((1+b*x^2/a)^(2*p))
```

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1113, 364}

$$\frac{2\sqrt{dx}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{1}{4},-2p;\frac{5}{4};-\frac{bx^2}{a}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/Sqrt[d*x],x]
```

```
[Out] (2*Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[1/4, -2*p, 5/4
, -((b*x^2)/a)])/(d*(1 + (b*x^2)/a)^(2*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^p}{\sqrt{dx}}\,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{2p}}{\sqrt{dx}}\,dx$$
$$= \frac{2\sqrt{dx}\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{1}{4},-2p;\frac{5}{4};-\frac{bx^2}{a}\right)}{d}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 0.83

$$\frac{2x\left(\left(a+bx^2\right)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p} {}_2F_1\left(\frac{1}{4},-2p;\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/Sqrt[d*x],x]
```

```
[Out] (2*x*((a + b*x^2)^2)^p*Hypergeometric2F1[1/4, -2*p, 5/4, -((b*x^2)/a)])/(Sq
rt[d*x]*(1 + (b*x^2)/a)^(2*p))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{dx}\left(b^2x^4+2\,abx^2+a^2\right)^p}{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/sqrt(d*x), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{p}}{\sqrt{d x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(1/2),x)
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^{2} x^{4}+2 \, a b x^{2}+a^{2}\right)^{p}}{\sqrt{d x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(1/2),x, algorithm="maxima")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/sqrt(d*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^p}{\sqrt{d\,x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/(d*x)^(1/2),x)
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/(d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + b x^{2}\right)^{2}\right)^{p}}{\sqrt{d x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p/(d*x)**(1/2),x)
[Out] Integral(((a + b*x**2)**2)**p/sqrt(d*x), x)
```

3.812 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(-\frac{1}{4},-2p;\frac{3}{4};-\frac{bx^2}{a}\right)}{d\sqrt{dx}}$$

```
[Out] -2*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([-1/4, -2*p],[3/4],-b*x^2/a)/d/((1+b
*x^2/a)^(2*p))/(d*x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1113, 364}

$$-\frac{2\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(-\frac{1}{4},-2p;\frac{3}{4};-\frac{bx^2}{a}\right)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/(d*x)^(3/2),x]
```

```
[Out] (-2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-1/4, -2*p, 3/4, -((b*x
^2)/a)])/(d*Sqrt[d*x]*(1 + (b*x^2)/a)^(2*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{(dx)^{3/2}}\,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{2p}}{(dx)^{3/2}}\,dx$$
$$= -\frac{2\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\,{}_2F_1\left(-\frac{1}{4},-2p;\frac{3}{4};-\frac{bx^2}{a}\right)}{d\sqrt{dx}}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 0.83

$$-\frac{2x\left((a+bx^2)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}\,{}_2F_1\left(-\frac{1}{4},-2p;\frac{3}{4};-\frac{bx^2}{a}\right)}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/(d*x)^(3/2),x]
```

```
[Out] (-2*x*((a + b*x^2)^2)^p*Hypergeometric2F1[-1/4, -2*p, 3/4, -((b*x^2)/a)])/(
(d*x)^(3/2)*(1 + (b*x^2)/a)^(2*p))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx}\,(b^2x^4+2\,abx^2+a^2)^p}{d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b^2x^4+2\,abx^2+a^2)^p}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(3/2), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{p}}{\left(d x\right)^{\frac{3}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(3/2),x)
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(b^{2} x^{4} + 2 \, a b x^{2} + a^{2}\right)}^{p}}{\left(d x\right)^{\frac{3}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(3/2),x, algorithm="maxima")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^p}{{\left(d\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/(d*x)^(3/2),x)
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/(d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + b x^{2}\right)^{2}\right)^{p}}{\left(d x\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p/(d*x)**(3/2),x)
[Out] Integral(((a + b*x**2)**2)**p/(d*x)**(3/2), x)
```

3.813 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{2\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(-\frac{3}{4},-2p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3d(dx)^{3/2}}$$

```
[Out] -2/3*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([-3/4, -2*p],[1/4],-b*x^2/a)/d/(d*
x)^(3/2)/((1+b*x^2/a)^(2*p))
```

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1113, 364}

$$-\frac{2\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(-\frac{3}{4},-2p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3d(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/(d*x)^(5/2),x]
```

```
[Out] (-2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-3/4, -2*p, 1/4, -((b*x
^2)/a)])/(3*d*(d*x)^(3/2)*(1 + (b*x^2)/a)^(2*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^p}{(dx)^{5/2}}\,dx=\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{2p}}{(dx)^{5/2}}\,dx$$
$$=-\frac{2\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(-\frac{3}{4},-2p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3d(dx)^{3/2}}$$

Mathematica [**A**] time = 0.01, size = 56, normalized size = 0.84

$$-\frac{2x\left(\left(a+bx^2\right)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}\,{}_2F_1\left(-\frac{3}{4},-2p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/(d*x)^(5/2),x]
[Out] (-2*x*((a + b*x^2)^2)^p*Hypergeometric2F1[-3/4, -2*p, 1/4, -((b*x^2)/a)])/(
3*(d*x)^(5/2)*(1 + (b*x^2)/a)^(2*p))
```

fricas [**F**] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx}\left(b^2x^4+2\,abx^2+a^2\right)^p}{d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d^3*x^3), x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(5/2),x, algorithm="giac")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(5/2), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{p}}{\left(d x \right)^{\frac{5}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(5/2),x)
```

```
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^{2} x^{4}+2 \, a b x^{2}+a^{2}\right)^{p}}{\left(d x\right)^{\frac{5}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^p}{{\left(d\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/(d*x)^(5/2),x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/(d*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + b x^{2}\right)^{2}\right)^{p}}{\left(d x\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p/(d*x)**(5/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**p/(d*x)**(5/2), x)
```

3.814 $\int x^2 \left(a + bx^2 + cx^4\right) dx$

Optimal. Leaf size=25

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

```
[Out] 1/3*a*x^3+1/5*b*x^5+1/7*c*x^7
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x^2 \left(a + bx^2 + cx^4\right) dx &= \int \left(ax^2 + bx^4 + cx^6\right) dx \\ &= \frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7} \end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

fricas [A] time = 0.76, size = 19, normalized size = 0.76

$$\frac{1}{7}x^7c + \frac{1}{5}x^5b + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] 1/7*x^7*c + 1/5*x^5*b + 1/3*x^3*a
```

giac [A] time = 0.15, size = 19, normalized size = 0.76

$$\frac{1}{7}cx^7 + \frac{1}{5}bx^5 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{1}{7}c\,x^7 + \frac{1}{5}b\,x^5 + \frac{1}{3}a\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a),x)
[Out] 1/3*a*x^3+1/5*b*x^5+1/7*c*x^7
```

maxima [A] time = 1.33, size = 19, normalized size = 0.76

$$\frac{1}{7}cx^7 + \frac{1}{5}bx^5 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$\frac{c\,x^7}{7} + \frac{b\,x^5}{5} + \frac{a\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2 + c*x^4),x)
```

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

sympy [A] time = 0.07, size = 19, normalized size = 0.76

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*x**3/3 + b*x**5/5 + c*x**7/7
```

3.815 $\int x\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=25

$$\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}$$

```
[Out] 1/2*a*x^2+1/4*b*x^4+1/6*c*x^6
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {14}

$$\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2+cx^4\right)\,dx&=\int\left(ax+bx^3+cx^5\right)\,dx\\&=\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

fricas [A] time = 0.87, size = 19, normalized size = 0.76

$$\frac{1}{6}x^6c + \frac{1}{4}x^4b + \frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] 1/6*x^6*c + 1/4*x^4*b + 1/2*x^2*a
```

giac [A] time = 0.15, size = 19, normalized size = 0.76

$$\frac{1}{6}\,cx^6 + \frac{1}{4}\,bx^4 + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{1}{6}c\,x^6 + \frac{1}{4}b\,x^4 + \frac{1}{2}a\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a),x)
[Out] 1/2*a*x^2+1/4*b*x^4+1/6*c*x^6
```

maxima [A] time = 1.39, size = 19, normalized size = 0.76

$$\frac{1}{6}\,cx^6 + \frac{1}{4}\,bx^4 + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$\frac{c\,x^6}{6} + \frac{b\,x^4}{4} + \frac{a\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2 + c*x^4),x)
```

```
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

sympy [A] time = 0.07, size = 19, normalized size = 0.76

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a),x)
```

```
[Out] a*x**2/2 + b*x**4/4 + c*x**6/6
```

3.816 $\int (a + bx^2 + cx^4) \, dx$

Optimal. Leaf size=20

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

```
[Out] a*x+1/3*b*x^3+1/5*c*x^5
```

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x^2 + c*x^4,x]
```

```
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

Rubi steps

$$\int (a + bx^2 + cx^4) \, dx = ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x^2 + c*x^4,x]
```

```
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

fricas [A] time = 0.88, size = 16, normalized size = 0.80

$$\frac{1}{5}x^5c + \frac{1}{3}x^3b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2+a,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*c + 1/3*x^3*b + x*a
```

giac [A] time = 0.15, size = 16, normalized size = 0.80

$$\frac{1}{5}\,cx^5 + \frac{1}{3}\,bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2+a,x, algorithm="giac")
[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$\frac{1}{5}c\,x^5 + \frac{1}{3}b\,x^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^4+b*x^2+a,x)
[Out] a*x+1/3*b*x^3+1/5*c*x^5
```

maxima [A] time = 1.35, size = 16, normalized size = 0.80

$$\frac{1}{5}\,cx^5 + \frac{1}{3}\,bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2+a,x, algorithm="maxima")
[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

mupad [B] time = 0.02, size = 16, normalized size = 0.80

$$\frac{c\,x^5}{5} + \frac{b\,x^3}{3} + a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*x^2 + c*x^4,x)
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

sympy [A] time = 0.06, size = 15, normalized size = 0.75

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**4+b*x**2+a,x)
```

```
[Out] a*x + b*x**3/3 + c*x**5/5
```

3.817 $\int \frac{a+bx^2+cx^4}{x} dx$

Optimal. Leaf size=21

$$a\log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}$$

```
[Out] 1/2*b*x^2+1/4*c*x^4+a*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$a\log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x,x]
```

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2+cx^4}{x} dx &= \int \left(\frac{a}{x} + bx + cx^3\right) dx \\ &= \frac{bx^2}{2} + \frac{cx^4}{4} + a\log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$a\log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x,x]
```

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

fricas [A] time = 0.80, size = 17, normalized size = 0.81

$$\frac{1}{4}\,cx^4+\frac{1}{2}\,bx^2+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x,x, algorithm="fricas")
[Out] 1/4*c*x^4 + 1/2*b*x^2 + a*log(x)
```

giac [A] time = 0.15, size = 20, normalized size = 0.95

$$\frac{1}{4}\,cx^4+\frac{1}{2}\,bx^2+\frac{1}{2}\,a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x,x, algorithm="giac")
[Out] 1/4*c*x^4 + 1/2*b*x^2 + 1/2*a*log(x^2)
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{c\,x^4}{4}+\frac{b\,x^2}{2}+a\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x,x)
[Out] 1/2*b*x^2+1/4*c*x^4+a*ln(x)
```

maxima [A] time = 1.36, size = 20, normalized size = 0.95

$$\frac{1}{4}\,cx^4+\frac{1}{2}\,bx^2+\frac{1}{2}\,a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x,x, algorithm="maxima")
[Out] 1/4*c*x^4 + 1/2*b*x^2 + 1/2*a*log(x^2)
```

mupad [B] time = 0.02, size = 17, normalized size = 0.81

$$\frac{b\,x^2}{2}+\frac{c\,x^4}{4}+a\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x,x)
```

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*log(x)
```

sympy [A] time = 0.10, size = 17, normalized size = 0.81

$$a\log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x,x)
```

```
[Out] a*log(x) + b*x**2/2 + c*x**4/4
```

3.818 $\int \frac{a+bx^2+cx^4}{x^2} \, dx$

Optimal. Leaf size=18

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

```
[Out] -a/x+b*x+1/3*c*x^3
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^2,x]
[Out] -(a/x) + b*x + (c*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2+cx^4}{x^2} \, dx &= \int \left(b+\frac{a}{x^2}+cx^2\right) \, dx \\ &= -\frac{a}{x}+bx+\frac{cx^3}{3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^2,x]
[Out] -(a/x) + b*x + (c*x^3)/3
```

fricas [A] time = 0.73, size = 20, normalized size = 1.11

$$\frac{cx^4 + 3\,bx^2 - 3\,a}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2,x, algorithm="fricas")
[Out] 1/3*(c*x^4 + 3*b*x^2 - 3*a)/x
```

giac [A] time = 0.15, size = 16, normalized size = 0.89

$$\frac{1}{3}\,cx^3 + bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2,x, algorithm="giac")
[Out] 1/3*c*x^3 + b*x - a/x
```

maple [A] time = 0.00, size = 17, normalized size = 0.94

$$\frac{c\,x^3}{3} + bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^2,x)
[Out] -a/x+b*x+1/3*c*x^3
```

maxima [A] time = 1.39, size = 16, normalized size = 0.89

$$\frac{1}{3}\,cx^3 + bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2,x, algorithm="maxima")
[Out] 1/3*c*x^3 + b*x - a/x
```

mupad [B] time = 0.03, size = 16, normalized size = 0.89

$$b\,x - \frac{a}{x} + \frac{c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^2,x)
```

```
[Out] b*x - a/x + (c*x^3)/3
```

sympy [A] time = 0.10, size = 12, normalized size = 0.67

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**2,x)
```

```
[Out] -a/x + b*x + c*x**3/3
```

3.819 $\int \frac{a+bx^2+cx^4}{x^3} \, dx$

Optimal. Leaf size=21

$$-\frac{a}{2x^2}+b\log(x)+\frac{cx^2}{2}$$

[Out] -1/2*a/x^2+1/2*c*x^2+b*ln(x)

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$-\frac{a}{2x^2}+b\log(x)+\frac{cx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/x^3,x]

[Out] -a/(2*x^2) + (c*x^2)/2 + b*Log[x]

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^3} \, dx &= \int \left(\frac{a}{x^3}+\frac{b}{x}+cx\right) \, dx\\ &= -\frac{a}{2x^2}+\frac{cx^2}{2}+b\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$-\frac{a}{2x^2}+b\log(x)+\frac{cx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/x^3,x]

[Out] -1/2*a/x^2 + (c*x^2)/2 + b*Log[x]

fricas [A] time = 0.98, size = 22, normalized size = 1.05

$$\frac{cx^4 + 2\,bx^2\log(x) - a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(c*x^4 + 2*b*x^2*log(x) - a)/x^2
```

giac [A] time = 0.15, size = 26, normalized size = 1.24

$$\frac{1}{2}\,cx^2 + \frac{1}{2}\,b\log\left(x^2\right) - \frac{bx^2 + a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*c*x^2 + 1/2*b*log(x^2) - 1/2*(b*x^2 + a)/x^2
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{c\,x^2}{2} + b\ln(x) - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^3,x)
```

```
[Out] -1/2*a/x^2+1/2*c*x^2+b*ln(x)
```

maxima [A] time = 1.34, size = 20, normalized size = 0.95

$$\frac{1}{2}\,cx^2 + \frac{1}{2}\,b\log\left(x^2\right) - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*c*x^2 + 1/2*b*log(x^2) - 1/2*a/x^2
```

mupad [B] time = 0.03, size = 17, normalized size = 0.81

$$\frac{c\,x^2}{2} - \frac{a}{2\,x^2} + b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^3,x)
```

```
[Out] (c*x^2)/2 - a/(2*x^2) + b*log(x)
```

sympy [A] time = 0.13, size = 17, normalized size = 0.81

$$-\frac{a}{2x^2} + b\log\left(x\right) + \frac{cx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**3,x)
```

```
[Out] -a/(2*x**2) + b*log(x) + c*x**2/2
```

3.820 $\int \frac{a+bx^2+cx^4}{x^4} dx$

Optimal. Leaf size=18

$$-\frac{a}{3x^3} - \frac{b}{x} + cx$$

```
[Out] -1/3*a/x^3-b/x+c*x
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$-\frac{a}{3x^3} - \frac{b}{x} + cx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^4,x]
```

```
[Out] -a/(3*x^3) - b/x + c*x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2+cx^4}{x^4} dx &= \int \left(c+\frac{a}{x^4}+\frac{b}{x^2}\right) dx \\ &= -\frac{a}{3x^3} - \frac{b}{x} + cx \end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{a}{3x^3} - \frac{b}{x} + cx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^4,x]
```

```
[Out] -1/3*a/x^3 - b/x + c*x
```

fricas [A] time = 0.63, size = 21, normalized size = 1.17

$$\frac{3\,cx^4-3\,bx^2-a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4,x, algorithm="fricas")
[Out] 1/3*(3*c*x^4 - 3*b*x^2 - a)/x^3
```

giac [A] time = 0.15, size = 17, normalized size = 0.94

$$cx-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4,x, algorithm="giac")
[Out] c*x - 1/3*(3*b*x^2 + a)/x^3
```

maple [A] time = 0.01, size = 17, normalized size = 0.94

$$cx-\frac{b}{x}-\frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^4,x)
[Out] -1/3*a/x^3-b/x+c*x
```

maxima [A] time = 1.37, size = 17, normalized size = 0.94

$$cx-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4,x, algorithm="maxima")
[Out] c*x - 1/3*(3*b*x^2 + a)/x^3
```

mupad [B] time = 0.02, size = 18, normalized size = 1.00

$$c\,x-\frac{b\,x^2+\frac{a}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^4,x)
```

```
[Out] c*x - (a/3 + b*x^2)/x^3
```

sympy [**A**] time = 0.13, size = 17, normalized size = 0.94

$$cx + \frac{-a - 3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**4,x)
```

```
[Out] c*x + (-a - 3*b*x**2)/(3*x**3)
```

3.821 $\int \frac{a+bx^2+cx^4}{x^5} dx$

Optimal. Leaf size=21

$$-\frac{a}{4x^4} - \frac{b}{2x^2} + c\log(x)$$

```
[Out] -1/4*a/x^4-1/2*b/x^2+c*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$-\frac{a}{4x^4} - \frac{b}{2x^2} + c\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^5,x]
```

```
[Out] -a/(4*x^4) - b/(2*x^2) + c*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^5} dx &= \int \left(\frac{a}{x^5} + \frac{b}{x^3} + \frac{c}{x}\right) dx \\ &= -\frac{a}{4x^4} - \frac{b}{2x^2} + c\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$-\frac{a}{4x^4} - \frac{b}{2x^2} + c\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^5,x]
```

```
[Out] -1/4*a/x^4 - b/(2*x^2) + c*Log[x]
```

fricas [A] time = 1.01, size = 23, normalized size = 1.10

$$\frac{4\,cx^4\log(x) - 2\,bx^2 - a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^5,x, algorithm="fricas")
[Out] 1/4*(4*c*x^4*log(x) - 2*b*x^2 - a)/x^4
```

giac [A] time = 0.16, size = 27, normalized size = 1.29

$$\frac{1}{2}\,c\log\left(x^2\right) - \frac{3\,cx^4 + 2\,bx^2 + a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^5,x, algorithm="giac")
[Out] 1/2*c*log(x^2) - 1/4*(3*c*x^4 + 2*b*x^2 + a)/x^4
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$c\ln(x) - \frac{b}{2x^2} - \frac{a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^5,x)
[Out] -1/4*a/x^4-1/2*b/x^2+c*ln(x)
```

maxima [A] time = 1.31, size = 21, normalized size = 1.00

$$\frac{1}{2}\,c\log\left(x^2\right) - \frac{2\,bx^2 + a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^5,x, algorithm="maxima")
[Out] 1/2*c*log(x^2) - 1/4*(2*b*x^2 + a)/x^4
```

mupad [B] time = 0.04, size = 20, normalized size = 0.95

$$c\,\ln(x) - \frac{\frac{b\,x^2}{2} + \frac{a}{4}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^5,x)
```

```
[Out] c*log(x) - (a/4 + (b*x^2)/2)/x^4
```

sympy [A]   time = 0.24, size = 19, normalized size = 0.90

$$c\log(x) + \frac{-a - 2bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**5,x)
```

```
[Out] c*log(x) + (-a - 2*b*x**2)/(4*x**4)
```

3.822 $\int \frac{a+bx^2+cx^4}{x^6} \, dx$

Optimal. Leaf size=23

$$-\frac{a}{5x^5}-\frac{b}{3x^3}-\frac{c}{x}$$

```
[Out] -1/5*a/x^5-1/3*b/x^3-c/x
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$-\frac{a}{5x^5}-\frac{b}{3x^3}-\frac{c}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^6,x]
```

```
[Out] -a/(5*x^5) - b/(3*x^3) - c/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^6} \, dx &= \int \left(\frac{a}{x^6}+\frac{b}{x^4}+\frac{c}{x^2}\right) \, dx\\ &= -\frac{a}{5x^5}-\frac{b}{3x^3}-\frac{c}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{a}{5x^5}-\frac{b}{3x^3}-\frac{c}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^6,x]
```

```
[Out] -1/5*a/x^5 - b/(3*x^3) - c/x
```

fricas [A] time = 0.91, size = 21, normalized size = 0.91

$$-\frac{15\,cx^4+5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6,x, algorithm="fricas")
[Out] -1/15*(15*c*x^4 + 5*b*x^2 + 3*a)/x^5
```

giac [A] time = 0.15, size = 21, normalized size = 0.91

$$-\frac{15\,cx^4+5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6,x, algorithm="giac")
[Out] -1/15*(15*c*x^4 + 5*b*x^2 + 3*a)/x^5
```

maple [A] time = 0.00, size = 20, normalized size = 0.87

$$-\frac{c}{x}-\frac{b}{3x^3}-\frac{a}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^6,x)
[Out] -1/5*a/x^5-1/3*b/x^3-c/x
```

maxima [A] time = 1.36, size = 21, normalized size = 0.91

$$-\frac{15\,cx^4+5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6,x, algorithm="maxima")
[Out] -1/15*(15*c*x^4 + 5*b*x^2 + 3*a)/x^5
```

mupad [B] time = 0.03, size = 20, normalized size = 0.87

$$-\frac{c\,x^4+\frac{b\,x^2}{3}+\frac{a}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^6,x)
```

```
[Out] -(a/5 + (b*x^2)/3 + c*x^4)/x^5
```

sympy [A] time = 0.26, size = 22, normalized size = 0.96

$$\frac{-3a - 5bx^2 - 15cx^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**6,x)
```

```
[Out] (-3*a - 5*b*x**2 - 15*c*x**4)/(15*x**5)
```

3.823 $\int \frac{a+bx^2+cx^4}{x^7}\,dx$

Optimal. Leaf size=25

$$-\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}$$

```
[Out] -1/6*a/x^6-1/4*b/x^4-1/2*c/x^2
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$-\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^7,x]
```

```
[Out] -a/(6*x^6) - b/(4*x^4) - c/(2*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^7}\,dx &= \int \left(\frac{a}{x^7}+\frac{b}{x^5}+\frac{c}{x^3}\right)dx\\ &= -\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$-\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^7,x]
```

```
[Out] -1/6*a/x^6 - b/(4*x^4) - c/(2*x^2)
```

fricas [A] time = 0.67, size = 21, normalized size = 0.84

$$-\frac{6\,cx^4 + 3\,bx^2 + 2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^7,x, algorithm="fricas")
```

```
[Out] -1/12*(6*c*x^4 + 3*b*x^2 + 2*a)/x^6
```

giac [A] time = 0.15, size = 21, normalized size = 0.84

$$-\frac{6\,cx^4 + 3\,bx^2 + 2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^7,x, algorithm="giac")
```

```
[Out] -1/12*(6*c*x^4 + 3*b*x^2 + 2*a)/x^6
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$-\frac{c}{2x^2} - \frac{b}{4x^4} - \frac{a}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^7,x)
```

```
[Out] -1/6*a/x^6-1/4*b/x^4-1/2*c/x^2
```

maxima [A] time = 1.30, size = 21, normalized size = 0.84

$$-\frac{6\,cx^4 + 3\,bx^2 + 2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*(6*c*x^4 + 3*b*x^2 + 2*a)/x^6
```

mupad [B] time = 0.03, size = 21, normalized size = 0.84

$$-\frac{\frac{c\,x^4}{2} + \frac{b\,x^2}{4} + \frac{a}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^7,x)
```

```
[Out] -(a/6 + (b*x^2)/4 + (c*x^4)/2)/x^6
```

sympy [A] time = 0.34, size = 22, normalized size = 0.88

$$\frac{-2a - 3bx^2 - 6cx^4}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**7,x)
```

```
[Out] (-2*a - 3*b*x**2 - 6*c*x**4)/(12*x**6)
```

3.824 $\int \frac{a+bx^2+cx^4}{x^8} \, dx$

Optimal. Leaf size=25

$$-\frac{a}{7x^7}-\frac{b}{5x^5}-\frac{c}{3x^3}$$

```
[Out] -1/7*a/x^7-1/5*b/x^5-1/3*c/x^3
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$-\frac{a}{7x^7}-\frac{b}{5x^5}-\frac{c}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^8,x]
```

```
[Out] -a/(7*x^7) - b/(5*x^5) - c/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^8} \, dx &= \int \left(\frac{a}{x^8}+\frac{b}{x^6}+\frac{c}{x^4}\right) dx\\ &= -\frac{a}{7x^7}-\frac{b}{5x^5}-\frac{c}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$-\frac{a}{7x^7}-\frac{b}{5x^5}-\frac{c}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^8,x]
```

```
[Out] -1/7*a/x^7 - b/(5*x^5) - c/(3*x^3)
```

fricas [A] time = 1.40, size = 21, normalized size = 0.84

$$-\frac{35\,cx^{4}+21\,bx^{2}+15\,a}{105\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8,x, algorithm="fricas")
[Out] -1/105*(35*c*x^4 + 21*b*x^2 + 15*a)/x^7
```

giac [A] time = 0.17, size = 21, normalized size = 0.84

$$-\frac{35\,cx^{4}+21\,bx^{2}+15\,a}{105\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8,x, algorithm="giac")
[Out] -1/105*(35*c*x^4 + 21*b*x^2 + 15*a)/x^7
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$-\frac{c}{3x^{3}}-\frac{b}{5x^{5}}-\frac{a}{7x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^8,x)
[Out] -1/7*a/x^7-1/5*b/x^5-1/3*c/x^3
```

maxima [A] time = 1.34, size = 21, normalized size = 0.84

$$-\frac{35\,cx^{4}+21\,bx^{2}+15\,a}{105\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8,x, algorithm="maxima")
[Out] -1/105*(35*c*x^4 + 21*b*x^2 + 15*a)/x^7
```

mupad [B] time = 0.03, size = 21, normalized size = 0.84

$$-\frac{\frac{c\,x^{4}}{3}+\frac{b\,x^{2}}{5}+\frac{a}{7}}{x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^8,x)
```

```
[Out] -(a/7 + (b*x^2)/5 + (c*x^4)/3)/x^7
```

sympy [A] time = 0.32, size = 22, normalized size = 0.88

$$\frac{-15a - 21bx^2 - 35cx^4}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**8,x)
```

```
[Out] (-15*a - 21*b*x**2 - 35*c*x**4)/(105*x**7)
```

3.825 $\int x^2 \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=54

$$\frac{a^2x^3}{3}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*(2*a*c+b^2)*x^7+2/9*b*c*x^9+1/11*c^2*x^11
```

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$\frac{a^2x^3}{3}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2*
x^11)/11
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2+cx^4\right)^2\,dx&=\int\left(a^2x^2+2abx^4+\left(b^2+2ac\right)x^6+2bcx^8+c^2x^{10}\right)dx\\&=\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{a^2x^3}{3}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2*
x^11)/11
```

fricas [A] time = 0.82, size = 46, normalized size = 0.85

$$\frac{1}{11}x^{11}c^2 + \frac{2}{9}x^9cb + \frac{1}{7}x^7b^2 + \frac{2}{7}x^7ca + \frac{2}{5}x^5ba + \frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*c^2 + 2/9*x^9*c*b + 1/7*x^7*b^2 + 2/7*x^7*c*a + 2/5*x^5*b*a + 1/3
*x^3*a^2
```

giac [A] time = 0.15, size = 46, normalized size = 0.85

$$\frac{1}{11}\,c^2x^{11} + \frac{2}{9}\,bcx^9 + \frac{1}{7}\,b^2x^7 + \frac{2}{7}\,acx^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*b^2*x^7 + 2/7*a*c*x^7 + 2/5*a*b*x^5 + 1/3
*a^2*x^3
```

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{c^2x^{11}}{11} + \frac{2bc\,x^9}{9} + \frac{2ab\,x^5}{5} + \frac{\left(2ac + b^2\right)x^7}{7} + \frac{a^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*(2*a*c+b^2)*x^7+2/9*b*c*x^9+1/11*c^2*x^11
```

maxima [A] time = 1.31, size = 44, normalized size = 0.81

$$\frac{1}{11}\,c^2x^{11} + \frac{2}{9}\,bcx^9 + \frac{1}{7}\left(b^2 + 2\,ac\right)x^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*(b^2 + 2*a*c)*x^7 + 2/5*a*b*x^5 + 1/3*a^2
*x^3
```

mupad [B] time = 0.03, size = 45, normalized size = 0.83

$$x^7 \left(\frac{b^2}{7} + \frac{2\,a\,c}{7}\right) + \frac{a^2\,x^3}{3} + \frac{c^2\,x^{11}}{11} + \frac{2\,a\,b\,x^5}{5} + \frac{2\,b\,c\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^7*((2*a*c)/7 + b^2/7) + (a^2*x^3)/3 + (c^2*x^11)/11 + (2*a*b*x^5)/5 + (2*
b*c*x^9)/9
```

sympy [A] time = 0.08, size = 51, normalized size = 0.94

$$\frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{2bcx^9}{9} + \frac{c^2x^{11}}{11} + x^7\left(\frac{2ac}{7} + \frac{b^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + 2*b*c*x**9/9 + c**2*x**11/11 + x**7*(2*a*c/7 +
 b**2/7)
```

3.826 $\int x\left(a+bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=54

$$\frac{a^2x^2}{2}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{1}{2}abx^4+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}$$

```
[Out] 1/2*a^2*x^2+1/2*a*b*x^4+1/6*(2*a*c+b^2)*x^6+1/4*b*c*x^8+1/10*c^2*x^10
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1107, 611}

$$\frac{a^2x^2}{2}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{1}{2}abx^4+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^6)/6 + (b*c*x^8)/4 + (c^2*x^10
)/10
```

Rule 611

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2+cx^4\right)^2\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int\left(a+bx+cx^2\right)^2\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(a^2+2abx+b^2\left(1+\frac{2ac}{b^2}\right)x^2+2bcx^3+c^2x^4\right)dx,x,x^2\right)\\&=\frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{1}{6}\left(b^2+2ac\right)x^6+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 0.89

$$\frac{1}{60}x^2\left(30a^2+10x^4\left(2ac+b^2\right)+30abx^2+15bcx^6+6c^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^2*(30*a^2 + 30*a*b*x^2 + 10*(b^2 + 2*a*c)*x^4 + 15*b*c*x^6 + 6*c^2*x^8))
/60
```

fricas [A] time = 0.75, size = 46, normalized size = 0.85

$$\frac{1}{10}x^{10}c^2+\frac{1}{4}x^8cb+\frac{1}{6}x^6b^2+\frac{1}{3}x^6ca+\frac{1}{2}x^4ba+\frac{1}{2}x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*c^2 + 1/4*x^8*c*b + 1/6*x^6*b^2 + 1/3*x^6*c*a + 1/2*x^4*b*a + 1/2
*x^2*a^2
```

giac [A] time = 0.16, size = 46, normalized size = 0.85

$$\frac{1}{10}c^2x^{10}+\frac{1}{4}bcx^8+\frac{1}{6}b^2x^6+\frac{1}{3}acx^6+\frac{1}{2}abx^4+\frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*b^2*x^6 + 1/3*a*c*x^6 + 1/2*a*b*x^4 + 1/2
*a^2*x^2
```

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{c^2x^{10}}{10}+\frac{bc\,x^8}{4}+\frac{ab\,x^4}{2}+\frac{\left(2ac+b^2\right)x^6}{6}+\frac{a^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*a^2*x^2+1/2*a*b*x^4+1/6*(2*a*c+b^2)*x^6+1/4*b*c*x^8+1/10*c^2*x^10
```

maxima [A] time = 1.38, size = 44, normalized size = 0.81

$$\frac{1}{10}c^2x^{10}+\frac{1}{4}bcx^8+\frac{1}{6}\left(b^2+2\,ac\right)x^6+\frac{1}{2}abx^4+\frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4 + 1/2*a^2
*x^2
```

mupad [B] time = 0.02, size = 45, normalized size = 0.83

$$x^6\left(\frac{b^2}{6}+\frac{a\,c}{3}\right)+\frac{a^2\,x^2}{2}+\frac{c^2\,x^{10}}{10}+\frac{a\,b\,x^4}{2}+\frac{b\,c\,x^8}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^6*((a*c)/3 + b^2/6) + (a^2*x^2)/2 + (c^2*x^10)/10 + (a*b*x^4)/2 + (b*c*x^
8)/4
```

sympy [A] time = 0.08, size = 46, normalized size = 0.85

$$\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{bcx^8}{4}+\frac{c^2x^{10}}{10}+x^6\left(\frac{ac}{3}+\frac{b^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*x**2/2 + a*b*x**4/2 + b*c*x**8/4 + c**2*x**10/10 + x**6*(a*c/3 + b**2/
6)
```

3.827 $\int \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=49

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

```
[Out] a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1090}

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Rule 1090

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2+cx^4\right)^2 \, dx &= \int \left(a^2+2abx^2+b^2\left(1+\frac{2ac}{b^2}\right)x^4+2bcx^6+c^2x^8\right) dx\\ &= a^2x+\frac{2}{3}abx^3+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 49, normalized size = 1.00

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

fricas [A] time = 0.78, size = 43, normalized size = 0.88

$$\frac{1}{9}x^9c^2 + \frac{2}{7}x^7cb + \frac{1}{5}x^5b^2 + \frac{2}{5}x^5ca + \frac{2}{3}x^3ba + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="fricas")
[Out] 1/9*x^9*c^2 + 2/7*x^7*c*b + 1/5*x^5*b^2 + 2/5*x^5*c*a + 2/3*x^3*b*a + x*a^2
```

giac [A] time = 0.15, size = 43, normalized size = 0.88

$$\frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}b^2x^5 + \frac{2}{5}acx^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="giac")
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 2/3*a*b*x^3 + a^2*x
```

maple [A] time = 0.00, size = 42, normalized size = 0.86

$$\frac{c^2x^9}{9} + \frac{2bc\,x^7}{7} + \frac{2ab\,x^3}{3} + \frac{\left(2ac + b^2\right)x^5}{5} + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2,x)
[Out] a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

maxima [A] time = 1.33, size = 45, normalized size = 0.92

$$\frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}b^2x^5 + a^2x + \frac{2}{15}\left(3\,cx^5 + 5\,bx^3\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="maxima")
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + a^2*x + 2/15*(3*c*x^5 + 5*b*x^3)*
a
```

mupad [B] time = 0.02, size = 42, normalized size = 0.86

$$a^2\,x + x^5\,\left(\frac{b^2}{5} + \frac{2\,a\,c}{5}\right) + \frac{c^2\,x^9}{9} + \frac{2\,a\,b\,x^3}{3} + \frac{2\,b\,c\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2,x)
```

```
[Out] a^2*x + x^5*((2*a*c)/5 + b^2/5) + (c^2*x^9)/9 + (2*a*b*x^3)/3 + (2*b*c*x^7)
/7
```

sympy [A] time = 0.08, size = 48, normalized size = 0.98

$$a^2x + \frac{2abx^3}{3} + \frac{2bcx^7}{7} + \frac{c^2x^9}{9} + x^5\left(\frac{2ac}{5} + \frac{b^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/5 + b**2/5
)
```

3.828 $\int \frac{(a+bx^2+cx^4)^2}{x} \, dx$

Optimal. Leaf size=47

$$a^2\log(x) + \frac{1}{4}x^4\left(2ac+b^2\right) + abx^2 + \frac{1}{3}bcx^6 + \frac{c^2x^8}{8}$$

```
[Out] a*b*x^2+1/4*(2*a*c+b^2)*x^4+1/3*b*c*x^6+1/8*c^2*x^8+a^2*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$a^2\log(x) + \frac{1}{4}x^4\left(2ac+b^2\right) + abx^2 + \frac{1}{3}bcx^6 + \frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x,x]
```

```
[Out] a*b*x^2 + ((b^2 + 2*a*c)*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8 + a^2*Log[x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^2}{x}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx+cx^2)^2}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(2ab+\frac{a^2}{x}+(b^2+2ac)\,x+2bcx^2+c^2x^3\right)dx, x, x^2\right)$$
$$= abx^2+\frac{1}{4}(b^2+2ac)\,x^4+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}+a^2\log(x)$$

Mathematica [A] time = 0.01, size = 47, normalized size = 1.00

$$a^2\log(x)+\frac{1}{4}x^4(2ac+b^2)+abx^2+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x,x]
[Out] a*b*x^2 + ((b^2 + 2*a*c)*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8 + a^2*Log[x]
```

fricas [A] time = 0.55, size = 41, normalized size = 0.87

$$\frac{1}{8}c^2x^8+\frac{1}{3}bcx^6+\frac{1}{4}(b^2+2\,ac)x^4+abx^2+a^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x,x, algorithm="fricas")
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*(b^2 + 2*a*c)*x^4 + a*b*x^2 + a^2*log(x)
```

giac [A] time = 0.15, size = 46, normalized size = 0.98

$$\frac{1}{8}c^2x^8+\frac{1}{3}bcx^6+\frac{1}{4}b^2x^4+\frac{1}{2}acx^4+abx^2+\frac{1}{2}a^2\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x,x, algorithm="giac")
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4 + 1/2*a*c*x^4 + a*b*x^2 + 1/2*a^2*l
og(x^2)
```

maple [A] time = 0.00, size = 44, normalized size = 0.94

$$\frac{c^2x^8}{8}+\frac{bc\,x^6}{3}+\frac{ac\,x^4}{2}+\frac{b^2x^4}{4}+ab\,x^2+a^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x,x)
```

```
[Out] 1/8*c^2*x^8+1/3*b*c*x^6+1/2*x^4*a*c+1/4*b^2*x^4+a*b*x^2+a^2*ln(x)
```

maxima [A] time = 1.34, size = 44, normalized size = 0.94

$$\frac{1}{8}c^2x^8 + \frac{1}{3}bcx^6 + \frac{1}{4}\left(b^2 + 2\,ac\right)x^4 + abx^2 + \frac{1}{2}a^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*(b^2 + 2*a*c)*x^4 + a*b*x^2 + 1/2*a^2*log(x
^2)
```

mupad [B] time = 0.02, size = 42, normalized size = 0.89

$$a^2\,\ln(x) + x^4\,\left(\frac{b^2}{4} + \frac{a\,c}{2}\right) + \frac{c^2\,x^8}{8} + a\,b\,x^2 + \frac{b\,c\,x^6}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x,x)
```

```
[Out] a^2*log(x) + x^4*((a*c)/2 + b^2/4) + (c^2*x^8)/8 + a*b*x^2 + (b*c*x^6)/3
```

sympy [A] time = 0.14, size = 42, normalized size = 0.89

$$a^2\log\left(x\right) + abx^2 + \frac{bcx^6}{3} + \frac{c^2x^8}{8} + x^4\left(\frac{ac}{2} + \frac{b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x,x)
```

```
[Out] a**2*log(x) + a*b*x**2 + b*c*x**6/3 + c**2*x**8/8 + x**4*(a*c/2 + b**2/4)
```

3.829 $\int \frac{(a+bx^2+cx^4)^2}{x^2} dx$

Optimal. Leaf size=48

$$-\frac{a^2}{x}+\frac{1}{3}x^3\left(2ac+b^2\right)+2abx+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

```
[Out] -a^2/x+2*a*b*x+1/3*(2*a*c+b^2)*x^3+2/5*b*c*x^5+1/7*c^2*x^7
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$-\frac{a^2}{x}+\frac{1}{3}x^3\left(2ac+b^2\right)+2abx+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + ((b^2 + 2*a*c)*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^2}\,dx &= \int \left(2ab+\frac{a^2}{x^2}+\left(b^2+2ac\right)x^2+2bcx^4+c^2x^6\right)dx\\ &= -\frac{a^2}{x}+2abx+\frac{1}{3}\left(b^2+2ac\right)x^3+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 1.00

$$-\frac{a^2}{x}+\frac{1}{3}x^3\left(2ac+b^2\right)+2abx+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + ((b^2 + 2*a*c)*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

fricas [A] time = 0.89, size = 46, normalized size = 0.96

$$\frac{15\,c^2x^8+42\,bcx^6+35\left(b^2+2\,ac\right)x^4+210\,abx^2-105\,a^2}{105\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*c^2*x^8 + 42*b*c*x^6 + 35*(b^2 + 2*a*c)*x^4 + 210*a*b*x^2 - 105*a
^2)/x
```

giac [A] time = 0.18, size = 44, normalized size = 0.92

$$\frac{1}{7}\,c^2x^7+\frac{2}{5}\,bcx^5+\frac{1}{3}\,b^2x^3+\frac{2}{3}\,acx^3+2\,abx-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3 + 2/3*a*c*x^3 + 2*a*b*x - a^2/x
```

maple [A] time = 0.00, size = 45, normalized size = 0.94

$$\frac{c^2x^7}{7}+\frac{2bc\,x^5}{5}+\frac{2ac\,x^3}{3}+\frac{b^2x^3}{3}+2abx-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^2,x)
```

```
[Out] 1/7*c^2*x^7+2/5*b*c*x^5+2/3*x^3*a*c+1/3*b^2*x^3+2*a*b*x-a^2/x
```

maxima [A] time = 1.22, size = 42, normalized size = 0.88

$$\frac{1}{7}\,c^2x^7+\frac{2}{5}\,bcx^5+\frac{1}{3}\left(b^2+2\,ac\right)x^3+2\,abx-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*(b^2 + 2*a*c)*x^3 + 2*a*b*x - a^2/x
```

mupad [B] time = 0.02, size = 43, normalized size = 0.90

$$x^3\left(\frac{b^2}{3}+\frac{2\,a\,c}{3}\right)-\frac{a^2}{x}+\frac{c^2\,x^7}{7}+2\,a\,b\,x+\frac{2\,b\,c\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^2,x)
```

```
[Out] x^3*((2*a*c)/3 + b^2/3) - a^2/x + (c^2*x^7)/7 + 2*a*b*x + (2*b*c*x^5)/5
```

sympy [A] time = 0.14, size = 44, normalized size = 0.92

$$-\frac{a^2}{x}+2abx+\frac{2bcx^5}{5}+\frac{c^2x^7}{7}+x^3\left(\frac{2ac}{3}+\frac{b^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**2,x)
```

```
[Out] -a**2/x + 2*a*b*x + 2*b*c*x**5/5 + c**2*x**7/7 + x**3*(2*a*c/3 + b**2/3)
```

3.830 $\int \frac{(a+bx^2+cx^4)^2}{x^3} dx$

Optimal. Leaf size=51

$$-\frac{a^2}{2x^2}+\frac{1}{2}x^2\left(2ac+b^2\right)+2ab\log(x)+\frac{1}{2}bcx^4+\frac{c^2x^6}{6}$$

```
[Out] -1/2*a^2/x^2+1/2*x^2*(2*a*c+b^2)+1/2*b*c*x^4+1/6*c^2*x^6+2*a*b*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$-\frac{a^2}{2x^2}+\frac{1}{2}x^2\left(2ac+b^2\right)+2ab\log(x)+\frac{1}{2}bcx^4+\frac{c^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^3,x]
```

```
[Out] -a^2/(2*x^2) + ((b^2 + 2*a*c)*x^2)/2 + (b*c*x^4)/2 + (c^2*x^6)/6 + 2*a*b*Lo
g[x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{x^3}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^2}{x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(b^2\left(1+\frac{2ac}{b^2}\right)+\frac{a^2}{x^2}+\frac{2ab}{x}+2bcx+c^2x^2\right)dx, x, x^2\right)$$
$$= -\frac{a^2}{2x^2}+\frac{1}{2}\left(b^2+2ac\right)x^2+\frac{1}{2}bcx^4+\frac{c^2x^6}{6}+2ab\log(x)$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.90

$$\frac{1}{6}\left(-\frac{3a^2}{x^2}+3x^2\left(2ac+b^2\right)+12ab\log(x)+3bcx^4+c^2x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^3,x]
```

```
[Out] ((-3*a^2)/x^2 + 3*(b^2 + 2*a*c)*x^2 + 3*b*c*x^4 + c^2*x^6 + 12*a*b*Log[x])/
6
```

fricas [A] time = 0.72, size = 47, normalized size = 0.92

$$\frac{c^2x^8+3\,bcx^6+3\left(b^2+2\,ac\right)x^4+12\,abx^2\log(x)-3\,a^2}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(c^2*x^8 + 3*b*c*x^6 + 3*(b^2 + 2*a*c)*x^4 + 12*a*b*x^2*log(x) - 3*a^2)
/x^2
```

giac [A] time = 0.15, size = 53, normalized size = 1.04

$$\frac{1}{6}\,c^2x^6+\frac{1}{2}\,bcx^4+\frac{1}{2}\,b^2x^2+acx^2+ab\log\left(x^2\right)-\frac{2\,abx^2+a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2 + a*c*x^2 + a*b*log(x^2) - 1/2*(2*a
*b*x^2 + a^2)/x^2
```

maple [A] time = 0.01, size = 45, normalized size = 0.88

$$\frac{c^2x^6}{6}+\frac{bc\,x^4}{2}+ac\,x^2+\frac{b^2x^2}{2}+2ab\ln(x)-\frac{a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^3,x)
```

```
[Out] 1/6*c^2*x^6+1/2*b*c*x^4+x^2*a*c+1/2*b^2*x^2-1/2*a^2/x^2+2*a*b*ln(x)
```

maxima [A] time = 1.37, size = 44, normalized size = 0.86

$$\frac{1}{6}\,c^2x^6+\frac{1}{2}\,bcx^4+\frac{1}{2}\left(b^2+2\,ac\right)x^2+ab\log\left(x^2\right)-\frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*(b^2 + 2*a*c)*x^2 + a*b*log(x^2) - 1/2*a^2/
x^2
```

mupad [B] time = 0.03, size = 43, normalized size = 0.84

$$x^2\left(\frac{b^2}{2}+a\,c\right)-\frac{a^2}{2\,x^2}+\frac{c^2\,x^6}{6}+2\,a\,b\,\ln(x)+\frac{b\,c\,x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^3,x)
```

```
[Out] x^2*(a*c + b^2/2) - a^2/(2*x^2) + (c^2*x^6)/6 + 2*a*b*log(x) + (b*c*x^4)/2
```

sympy [A] time = 0.17, size = 44, normalized size = 0.86

$$-\frac{a^2}{2x^2}+2ab\log\left(x\right)+\frac{bcx^4}{2}+\frac{c^2x^6}{6}+x^2\left(ac+\frac{b^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**3,x)
```

```
[Out] -a**2/(2*x**2) + 2*a*b*log(x) + b*c*x**4/2 + c**2*x**6/6 + x**2*(a*c + b**2
/2)
```

3.831 $\int \frac{(a+bx^2+cx^4)^2}{x^4} \, dx$

Optimal. Leaf size=47

$$-\frac{a^2}{3x^3}+x\left(2ac+b^2\right)-\frac{2ab}{x}+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

```
[Out] -1/3*a^2/x^3-2*a*b/x+(2*a*c+b^2)*x+2/3*b*c*x^3+1/5*c^2*x^5
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$-\frac{a^2}{3x^3}+x\left(2ac+b^2\right)-\frac{2ab}{x}+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^4,x]
```

```
[Out] -a^2/(3*x^3) - (2*a*b)/x + (b^2 + 2*a*c)*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] && !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^4} \, dx &= \int \left(b^2\left(1+\frac{2ac}{b^2}\right)+\frac{a^2}{x^4}+\frac{2ab}{x^2}+2bcx^2+c^2x^4\right) dx\\ &= -\frac{a^2}{3x^3}-\frac{2ab}{x}+\left(b^2+2ac\right)x+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.00

$$-\frac{a^2}{3x^3}+x\left(2ac+b^2\right)-\frac{2ab}{x}+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^4,x]
```

```
[Out] -1/3*a^2/x^3 - (2*a*b)/x + (b^2 + 2*a*c)*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

fricas [A] time = 0.93, size = 46, normalized size = 0.98

$$\frac{3\,c^{2}x^{8}+10\,bcx^{6}+15\left(b^{2}+2\,ac\right)x^{4}-30\,abx^{2}-5\,a^{2}}{15\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/15*(3*c^2*x^8 + 10*b*c*x^6 + 15*(b^2 + 2*a*c)*x^4 - 30*a*b*x^2 - 5*a^2)/x
^3
```

giac [A] time = 0.15, size = 42, normalized size = 0.89

$$\frac{1}{5}\,c^{2}x^{5}+\frac{2}{3}\,bcx^{3}+b^{2}x+2\,acx-\frac{6\,abx^{2}+a^{2}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x + 2*a*c*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

maple [A] time = 0.01, size = 42, normalized size = 0.89

$$\frac{c^{2}x^{5}}{5}+\frac{2bc\,x^{3}}{3}+2acx+b^{2}x-\frac{2ab}{x}-\frac{a^{2}}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^4,x)
```

```
[Out] 1/5*c^2*x^5+2/3*b*c*x^3+2*a*c*x+b^2*x-2*a*b/x-1/3*a^2/x^3
```

maxima [A] time = 1.37, size = 42, normalized size = 0.89

$$\frac{1}{5}\,c^{2}x^{5}+\frac{2}{3}\,bcx^{3}+\left(b^{2}+2\,ac\right)x-\frac{6\,abx^{2}+a^{2}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + (b^2 + 2*a*c)*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

mupad [B] time = 0.04, size = 44, normalized size = 0.94

$$x\,\left(b^2+2\,a\,c\right)-\frac{\frac{a^2}{3}+2\,b\,a\,x^2}{x^3}+\frac{c^2\,x^5}{5}+\frac{2\,b\,c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^4,x)
```

```
[Out] x*(2*a*c + b^2) - (a^2/3 + 2*a*b*x^2)/x^3 + (c^2*x^5)/5 + (2*b*c*x^3)/3
```

sympy [A] time = 0.18, size = 46, normalized size = 0.98

$$\frac{2bcx^3}{3}+\frac{c^2x^5}{5}+x\left(2ac+b^2\right)+\frac{-a^2-6abx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**4,x)
```

```
[Out] 2*b*c*x**3/3 + c**2*x**5/5 + x*(2*a*c + b**2) + (-a**2 - 6*a*b*x**2)/(3*x**
3)
```

3.832 $\int \frac{(a+bx^2+cx^4)^2}{x^5} dx$

Optimal. Leaf size=45

$$-\frac{a^2}{4x^4} + \log(x)\left(2ac + b^2\right) - \frac{ab}{x^2} + bcx^2 + \frac{c^2x^4}{4}$$

```
[Out] -1/4*a^2/x^4-a*b/x^2+b*c*x^2+1/4*c^2*x^4+(2*a*c+b^2)*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$-\frac{a^2}{4x^4} + \log(x)\left(2ac + b^2\right) - \frac{ab}{x^2} + bcx^2 + \frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^5,x]
```

```
[Out] -a^2/(4*x^4) - (a*b)/x^2 + b*c*x^2 + (c^2*x^4)/4 + (b^2 + 2*a*c)*Log[x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^2}{x^5}\,dx = \frac{1}{2}\mathrm{Subst}\left(\int \frac{(a+bx+cx^2)^2}{x^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\mathrm{Subst}\left(\int \left(2bc+\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2+2ac}{x}+c^2x\right)dx, x, x^2\right)$$
$$= -\frac{a^2}{4x^4}-\frac{ab}{x^2}+bcx^2+\frac{c^2x^4}{4}+(b^2+2ac)\log(x)$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.91

$$\log(x)\left(2ac+b^2\right)+\frac{(cx^4-a)\left(a+4bx^2+cx^4\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^5,x]
```

```
[Out] ((-a + c*x^4)*(a + 4*b*x^2 + c*x^4))/(4*x^4) + (b^2 + 2*a*c)*Log[x]
```

fricas [A] time = 0.86, size = 47, normalized size = 1.04

$$\frac{c^2x^8+4\,bcx^6+4\left(b^2+2\,ac\right)x^4\log(x)-4\,abx^2-a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(c^2*x^8 + 4*b*c*x^6 + 4*(b^2 + 2*a*c)*x^4*log(x) - 4*a*b*x^2 - a^2)/x^
4
```

giac [A] time = 0.18, size = 60, normalized size = 1.33

$$\frac{1}{4}\,c^2x^4+bcx^2+\frac{1}{2}\left(b^2+2\,ac\right)\log\left(x^2\right)-\frac{3\,b^2x^4+6\,acx^4+4\,abx^2+a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^2 + 1/2*(b^2 + 2*a*c)*log(x^2) - 1/4*(3*b^2*x^4 + 6*a*c
*x^4 + 4*a*b*x^2 + a^2)/x^4
```

maple [A]   time = 0.01, size = 43, normalized size = 0.96

$$\frac{c^2x^4}{4} + bc\,x^2 + 2ac\ln(x) + b^2\ln(x) - \frac{ab}{x^2} - \frac{a^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^5,x)
```

```
[Out] 1/4*c^2*x^4+b*c*x^2-a*b/x^2-1/4*a^2/x^4+2*ln(x)*a*c+b^2*ln(x)
```

maxima [A]   time = 1.34, size = 45, normalized size = 1.00

$$\frac{1}{4}\,c^2x^4 + bcx^2 + \frac{1}{2}\left(b^2 + 2\,ac\right)\log\left(x^2\right) - \frac{4\,abx^2 + a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^2 + 1/2*(b^2 + 2*a*c)*log(x^2) - 1/4*(4*a*b*x^2 + a^2)/
x^4
```

mupad [B]   time = 0.04, size = 43, normalized size = 0.96

$$\ln(x)\left(b^2 + 2\,a\,c\right) - \frac{\frac{a^2}{4} + b\,a\,x^2}{x^4} + \frac{c^2\,x^4}{4} + b\,c\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^5,x)
```

```
[Out] log(x)*(2*a*c + b^2) - (a^2/4 + a*b*x^2)/x^4 + (c^2*x^4)/4 + b*c*x^2
```

sympy [A]   time = 0.37, size = 44, normalized size = 0.98

$$bcx^2 + \frac{c^2x^4}{4} + \left(2ac + b^2\right)\log\left(x\right) + \frac{-a^2 - 4abx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**5,x)
```

```
[Out] b*c*x**2 + c**2*x**4/4 + (2*a*c + b**2)*log(x) + (-a**2 - 4*a*b*x**2)/(4*x*
*4)
```

3.833 $\int \frac{(a+bx^2+cx^4)^2}{x^6} \, dx$

Optimal. Leaf size=48

$$-\frac{a^2}{5x^5}-\frac{2ac+b^2}{x}-\frac{2ab}{3x^3}+2bcx+\frac{c^2x^3}{3}$$

```
[Out] -1/5*a^2/x^5-2/3*a*b/x^3+(-2*a*c-b^2)/x+2*b*c*x+1/3*c^2*x^3
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$-\frac{a^2}{5x^5}-\frac{2ac+b^2}{x}-\frac{2ab}{3x^3}+2bcx+\frac{c^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^6,x]
```

```
[Out] -a^2/(5*x^5) - (2*a*b)/(3*x^3) - (b^2 + 2*a*c)/x + 2*b*c*x + (c^2*x^3)/3
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^2+cx^4)^2}{x^6} \, dx &= \int \left(2bc+\frac{a^2}{x^6}+\frac{2ab}{x^4}+\frac{b^2+2ac}{x^2}+c^2x^2\right) dx \\ &= -\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2+2ac}{x}+2bcx+\frac{c^2x^3}{3} \end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.02

$$-\frac{a^2}{5x^5}+\frac{-2ac-b^2}{x}-\frac{2ab}{3x^3}+2bcx+\frac{c^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^6,x]
```

```
[Out] -1/5*a^2/x^5 - (2*a*b)/(3*x^3) + (-b^2 - 2*a*c)/x + 2*b*c*x + (c^2*x^3)/3
```

fricas [A] time = 0.87, size = 46, normalized size = 0.96

$$\frac{5\,c^2x^8 + 30\,bcx^6 - 15\,\left(b^2 + 2\,ac\right)x^4 - 10\,abx^2 - 3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(5*c^2*x^8 + 30*b*c*x^6 - 15*(b^2 + 2*a*c)*x^4 - 10*a*b*x^2 - 3*a^2)/x
^5
```

giac [A] time = 0.15, size = 47, normalized size = 0.98

$$\frac{1}{3}\,c^2x^3 + 2\,bcx - \frac{15\,b^2x^4 + 30\,acx^4 + 10\,abx^2 + 3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/3*c^2*x^3 + 2*b*c*x - 1/15*(15*b^2*x^4 + 30*a*c*x^4 + 10*a*b*x^2 + 3*a^2)
/x^5
```

maple [A] time = 0.01, size = 43, normalized size = 0.90

$$\frac{c^2x^3}{3} + 2bcx - \frac{2ab}{3x^3} - \frac{2ac + b^2}{x} - \frac{a^2}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^6,x)
```

```
[Out] 1/3*c^2*x^3+2*b*c*x-1/5*a^2/x^5-(2*a*c+b^2)/x-2/3*a*b/x^3
```

maxima [A] time = 1.32, size = 45, normalized size = 0.94

$$\frac{1}{3}\,c^2x^3 + 2\,bcx - \frac{15\,\left(b^2 + 2\,ac\right)x^4 + 10\,abx^2 + 3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^6,x, algorithm="maxima")
```

```
[Out] 1/3*c^2*x^3 + 2*b*c*x - 1/15*(15*(b^2 + 2*a*c)*x^4 + 10*a*b*x^2 + 3*a^2)/x^
5
```

mupad [B] time = 0.04, size = 44, normalized size = 0.92

$$\frac{c^2\,x^3}{3}-\frac{x^4\,\left(b^2+2\,a\,c\right)+\frac{a^2}{5}+\frac{2\,a\,b\,x^2}{3}}{x^5}+2\,b\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^6,x)
```

```
[Out] (c^2*x^3)/3 - (x^4*(2*a*c + b^2) + a^2/5 + (2*a*b*x^2)/3)/x^5 + 2*b*c*x
```

sympy [A] time = 0.43, size = 48, normalized size = 1.00

$$2bcx+\frac{c^2x^3}{3}+\frac{-3a^2-10abx^2+x^4\left(-30ac-15b^2\right)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**6,x)
```

```
[Out] 2*b*c*x + c**2*x**3/3 + (-3*a**2 - 10*a*b*x**2 + x**4*(-30*a*c - 15*b**2))/
(15*x**5)
```

3.834 $\int \frac{(a+bx^2+cx^4)^2}{x^7} \, dx$

Optimal. Leaf size=51

$$-\frac{a^2}{6x^6}-\frac{2ac+b^2}{2x^2}-\frac{ab}{2x^4}+2bc\log(x)+\frac{c^2x^2}{2}$$

```
[Out] -1/6*a^2/x^6-1/2*a*b/x^4+1/2*(-2*a*c-b^2)/x^2+1/2*c^2*x^2+2*b*c*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$-\frac{a^2}{6x^6}-\frac{2ac+b^2}{2x^2}-\frac{ab}{2x^4}+2bc\log(x)+\frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^7,x]
```

```
[Out] -a^2/(6*x^6) - (a*b)/(2*x^4) - (b^2 + 2*a*c)/(2*x^2) + (c^2*x^2)/2 + 2*b*c*
Log[x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{x^7}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^2}{x^4}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(c^2+\frac{a^2}{x^4}+\frac{2ab}{x^3}+\frac{b^2+2ac}{x^2}+\frac{2bc}{x}\right)dx,x,x^2\right)$$
$$= -\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2+2ac}{2x^2}+\frac{c^2x^2}{2}+2bc\log(x)$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.98

$$-\frac{a^2+3abx^2+6acx^4+3b^2x^4-12bcx^6\log(x)-3c^2x^8}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^7,x]
```

```
[Out] -1/6*(a^2 + 3*a*b*x^2 + 3*b^2*x^4 + 6*a*c*x^4 - 3*c^2*x^8 - 12*b*c*x^6*Log[
x])/x^6
```

fricas [A] time = 1.04, size = 48, normalized size = 0.94

$$\frac{3\,c^2x^8+12\,bcx^6\log(x)-3\left(b^2+2\,ac\right)x^4-3\,abx^2-a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(3*c^2*x^8 + 12*b*c*x^6*log(x) - 3*(b^2 + 2*a*c)*x^4 - 3*a*b*x^2 - a^2)
/x^6
```

giac [A] time = 0.16, size = 54, normalized size = 1.06

$$\frac{1}{2}\,c^2x^2+bc\log\left(x^2\right)-\frac{11\,bcx^6+3\,b^2x^4+6\,acx^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/2*c^2*x^2 + b*c*log(x^2) - 1/6*(11*b*c*x^6 + 3*b^2*x^4 + 6*a*c*x^4 + 3*a*
b*x^2 + a^2)/x^6
```

maple [A] time = 0.01, size = 46, normalized size = 0.90

$$\frac{c^2x^2}{2} + 2bc\ln(x) - \frac{ac}{x^2} - \frac{b^2}{2x^2} - \frac{ab}{2x^4} - \frac{a^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^7,x)
```

```
[Out] 1/2*c^2*x^2-1/x^2*a*c-1/2*b^2/x^2-1/6*a^2/x^6-1/2*a*b/x^4+2*b*c*ln(x)
```

maxima [A] time = 1.34, size = 45, normalized size = 0.88

$$\frac{1}{2}\,c^2x^2 + bc\log\left(x^2\right) - \frac{3\left(b^2 + 2\,ac\right)x^4 + 3\,abx^2 + a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*x^2 + b*c*log(x^2) - 1/6*(3*(b^2 + 2*a*c)*x^4 + 3*a*b*x^2 + a^2)/x^
6
```

mupad [B] time = 4.14, size = 46, normalized size = 0.90

$$\frac{c^2\,x^2}{2} - \frac{\frac{a^2}{6} + x^4\,\left(\frac{b^2}{2} + a\,c\right) + \frac{a\,b\,x^2}{2}}{x^6} + 2\,b\,c\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^7,x)
```

```
[Out] (c^2*x^2)/2 - (a^2/6 + x^4*(a*c + b^2/2) + (a*b*x^2)/2)/x^6 + 2*b*c*log(x)
```

sympy [A] time = 0.78, size = 48, normalized size = 0.94

$$2bc\log\left(x\right) + \frac{c^2x^2}{2} + \frac{-a^2 - 3abx^2 + x^4\left(-6ac - 3b^2\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**7,x)
```

```
[Out] 2*b*c*log(x) + c**2*x**2/2 + (-a**2 - 3*a*b*x**2 + x**4*(-6*a*c - 3*b**2))/
(6*x**6)
```

3.835 $\int \frac{\left(a+bx^2+cx^4\right)^2}{x^8}\,dx$

Optimal. Leaf size=47

$$-\frac{a^2}{7x^7}-\frac{2ac+b^2}{3x^3}-\frac{2ab}{5x^5}-\frac{2bc}{x}+c^2x$$

```
[Out] -1/7*a^2/x^7-2/5*a*b/x^5+1/3*(-2*a*c-b^2)/x^3-2*b*c/x+c^2*x
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$-\frac{a^2}{7x^7}-\frac{2ac+b^2}{3x^3}-\frac{2ab}{5x^5}-\frac{2bc}{x}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^8,x]
```

```
[Out] -a^2/(7*x^7) - (2*a*b)/(5*x^5) - (b^2 + 2*a*c)/(3*x^3) - (2*b*c)/x + c^2*x
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] && !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^8}\,dx&=\int \left(c^2+\frac{a^2}{x^8}+\frac{2ab}{x^6}+\frac{b^2+2ac}{x^4}+\frac{2bc}{x^2}\right)dx\\&=-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2+2ac}{3x^3}-\frac{2bc}{x}+c^2x\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.04

$$-\frac{a^2}{7x^7}+\frac{-2ac-b^2}{3x^3}-\frac{2ab}{5x^5}-\frac{2bc}{x}+c^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^8,x]
```

[Out] -1/7*a^2/x^7 - (2*a*b)/(5*x^5) + (-b^2 - 2*a*c)/(3*x^3) - (2*b*c)/x + c^2*x

fricas [A] time = 0.93, size = 46, normalized size = 0.98

$$\frac{105\,c^2x^8 - 210\,bcx^6 - 35\left(b^2 + 2\,ac\right)x^4 - 42\,abx^2 - 15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^2/x^8,x, algorithm="fricas")

[Out] 1/105*(105*c^2*x^8 - 210*b*c*x^6 - 35*(b^2 + 2*a*c)*x^4 - 42*a*b*x^2 - 15*a^2)/x^7

giac [A] time = 0.17, size = 46, normalized size = 0.98

$$c^2x - \frac{210\,bcx^6 + 35\,b^2x^4 + 70\,acx^4 + 42\,abx^2 + 15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^2/x^8,x, algorithm="giac")

[Out] c^2*x - 1/105*(210*b*c*x^6 + 35*b^2*x^4 + 70*a*c*x^4 + 42*a*b*x^2 + 15*a^2)/x^7

maple [A] time = 0.01, size = 42, normalized size = 0.89

$$c^2x - \frac{2bc}{x} - \frac{2ab}{5x^5} - \frac{2ac + b^2}{3x^3} - \frac{a^2}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)^2/x^8,x)

[Out] c^2*x-1/7*a^2/x^7-2/5*a*b/x^5-2*b*c/x-1/3*(2*a*c+b^2)/x^3

maxima [A] time = 1.34, size = 44, normalized size = 0.94

$$c^2x - \frac{210\,bcx^6 + 35\left(b^2 + 2\,ac\right)x^4 + 42\,abx^2 + 15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^2/x^8,x, algorithm="maxima")

[Out] c^2*x - 1/105*(210*b*c*x^6 + 35*(b^2 + 2*a*c)*x^4 + 42*a*b*x^2 + 15*a^2)/x^7

mupad [B] time = 4.17, size = 45, normalized size = 0.96

$$c^2\,x-\frac{\frac{a^2}{7}+x^4\,\left(\frac{b^2}{3}+\frac{2\,a\,c}{3}\right)+\frac{2\,a\,b\,x^2}{5}+2\,b\,c\,x^6}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^8,x)
```

```
[Out] c^2*x - (a^2/7 + x^4*((2*a*c)/3 + b^2/3) + (2*a*b*x^2)/5 + 2*b*c*x^6)/x^7
```

sympy [A] time = 0.76, size = 46, normalized size = 0.98

$$c^2x+\frac{-15a^2-42abx^2-210bcx^6+x^4\left(-70ac-35b^2\right)}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**8,x)
```

```
[Out] c**2*x + (-15*a**2 - 42*a*b*x**2 - 210*b*c*x**6 + x**4*(-70*a*c - 35*b**2))
/(105*x**7)
```

3.836 $\int \frac{(a+bx^2+cx^4)^2}{x^9}\,dx$

Optimal. Leaf size=48

$$-\frac{a^2}{8x^8}-\frac{2ac+b^2}{4x^4}-\frac{ab}{3x^6}-\frac{bc}{x^2}+c^2\log(x)$$

```
[Out] -1/8*a^2/x^8-1/3*a*b/x^6+1/4*(-2*a*c-b^2)/x^4-b*c/x^2+c^2*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$-\frac{a^2}{8x^8}-\frac{2ac+b^2}{4x^4}-\frac{ab}{3x^6}-\frac{bc}{x^2}+c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^9,x]
```

```
[Out] -a^2/(8*x^8) - (a*b)/(3*x^6) - (b^2 + 2*a*c)/(4*x^4) - (b*c)/x^2 + c^2*Log[
x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{x^9}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^2}{x^5}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2+2ac}{x^3}+\frac{2bc}{x^2}+\frac{c^2}{x}\right)dx,x,x^2\right)$$
$$= -\frac{a^2}{8x^8}-\frac{ab}{3x^6}-\frac{b^2+2ac}{4x^4}-\frac{bc}{x^2}+c^2\log(x)$$

Mathematica [A] time = 0.03, size = 50, normalized size = 1.04

$$-\frac{a^2}{8x^8}+\frac{-2ac-b^2}{4x^4}-\frac{ab}{3x^6}-\frac{bc}{x^2}+c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^9,x]
```

```
[Out] -1/8*a^2/x^8 - (a*b)/(3*x^6) + (-b^2 - 2*a*c)/(4*x^4) - (b*c)/x^2 + c^2*Log
[x]
```

fricas [A] time = 0.88, size = 48, normalized size = 1.00

$$\frac{24\,c^2x^8\log(x)-24\,bcx^6-6\left(b^2+2\,ac\right)x^4-8\,abx^2-3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(24*c^2*x^8*log(x) - 24*b*c*x^6 - 6*(b^2 + 2*a*c)*x^4 - 8*a*b*x^2 - 3*
a^2)/x^8
```

giac [A] time = 0.15, size = 58, normalized size = 1.21

$$\frac{1}{2}\,c^2\log\left(x^2\right)-\frac{25\,c^2x^8+24\,bcx^6+6\,b^2x^4+12\,acx^4+8\,abx^2+3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^9,x, algorithm="giac")
```

```
[Out] 1/2*c^2*log(x^2) - 1/24*(25*c^2*x^8 + 24*b*c*x^6 + 6*b^2*x^4 + 12*a*c*x^4 +
 8*a*b*x^2 + 3*a^2)/x^8
```

maple [A] time = 0.01, size = 45, normalized size = 0.94

$$c^2\ln(x)-\frac{bc}{x^2}-\frac{ac}{2x^4}-\frac{b^2}{4x^4}-\frac{ab}{3x^6}-\frac{a^2}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^9,x)
```

```
[Out] -b*c/x^2-1/8*a^2/x^8-1/3*a*b/x^6-1/2/x^4*a*c-1/4*b^2/x^4+c^2*ln(x)
```

maxima [A] time = 1.36, size = 48, normalized size = 1.00

$$\frac{1}{2}\,c^2\log\left(x^2\right)-\frac{24\,bcx^6+6\left(b^2+2\,ac\right)x^4+8\,abx^2+3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log(x^2) - 1/24*(24*b*c*x^6 + 6*(b^2 + 2*a*c)*x^4 + 8*a*b*x^2 + 3*a
^2)/x^8
```

mupad [B] time = 4.18, size = 45, normalized size = 0.94

$$c^2\,\ln(x)-\frac{\frac{a^2}{8}+x^4\,\left(\frac{b^2}{4}+\frac{a\,c}{2}\right)+\frac{a\,b\,x^2}{3}+b\,c\,x^6}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^9,x)
```

```
[Out] c^2*log(x) - (a^2/8 + x^4*((a*c)/2 + b^2/4) + (a*b*x^2)/3 + b*c*x^6)/x^8
```

sympy [A] time = 1.31, size = 48, normalized size = 1.00

$$c^2\log\left(x\right)+\frac{-3a^2-8abx^2-24bcx^6+x^4\left(-12ac-6b^2\right)}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**9,x)
```

```
[Out] c**2*log(x) + (-3*a**2 - 8*a*b*x**2 - 24*b*c*x**6 + x**4*(-12*a*c - 6*b**2)
)/(24*x**8)
```

3.837 $\int \frac{(a+bx^2+cx^4)^2}{x^{10}} dx$

Optimal. Leaf size=52

$$-\frac{a^2}{9x^9}-\frac{2ac+b^2}{5x^5}-\frac{2ab}{7x^7}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

```
[Out] -1/9*a^2/x^9-2/7*a*b/x^7+1/5*(-2*a*c-b^2)/x^5-2/3*b*c/x^3-c^2/x
```

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$-\frac{a^2}{9x^9}-\frac{2ac+b^2}{5x^5}-\frac{2ab}{7x^7}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^10,x]
```

```
[Out] -a^2/(9*x^9) - (2*a*b)/(7*x^7) - (b^2 + 2*a*c)/(5*x^5) - (2*b*c)/(3*x^3) -
c^2/x
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^2}{x^{10}}\,dx &= \int \left(\frac{a^2}{x^{10}}+\frac{2ab}{x^8}+\frac{b^2+2ac}{x^6}+\frac{2bc}{x^4}+\frac{c^2}{x^2}\right)dx\\ &= -\frac{a^2}{9x^9}-\frac{2ab}{7x^7}-\frac{b^2+2ac}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.96

$$-\frac{35a^2+90abx^2+126acx^4+63b^2x^4+210bcx^6+315c^2x^8}{315x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^10,x]
```

```
[Out] -1/315*(35*a^2 + 90*a*b*x^2 + 63*b^2*x^4 + 126*a*c*x^4 + 210*b*c*x^6 + 315*
c^2*x^8)/x^9
```

fricas [A] time = 0.89, size = 46, normalized size = 0.88

$$-\frac{315\,c^2x^8+210\,bcx^6+63\left(b^2+2\,ac\right)x^4+90\,abx^2+35\,a^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(315*c^2*x^8 + 210*b*c*x^6 + 63*(b^2 + 2*a*c)*x^4 + 90*a*b*x^2 + 35*
a^2)/x^9
```

giac [A] time = 0.15, size = 48, normalized size = 0.92

$$-\frac{315\,c^2x^8+210\,bcx^6+63\,b^2x^4+126\,acx^4+90\,abx^2+35\,a^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^10,x, algorithm="giac")
```

```
[Out] -1/315*(315*c^2*x^8 + 210*b*c*x^6 + 63*b^2*x^4 + 126*a*c*x^4 + 90*a*b*x^2 +
 35*a^2)/x^9
```

maple [A] time = 0.00, size = 45, normalized size = 0.87

$$-\frac{c^2}{x}-\frac{2bc}{3x^3}-\frac{2ab}{7x^7}-\frac{2ac+b^2}{5x^5}-\frac{a^2}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^10,x)
```

```
[Out] -2/7*a*b/x^7-1/9*a^2/x^9-1/5*(2*a*c+b^2)/x^5-c^2/x-2/3*b*c/x^3
```

maxima [A] time = 1.33, size = 46, normalized size = 0.88

$$-\frac{315\,c^2x^8+210\,bcx^6+63\left(b^2+2\,ac\right)x^4+90\,abx^2+35\,a^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^10,x, algorithm="maxima")
```

```
[Out] -1/315*(315*c^2*x^8 + 210*b*c*x^6 + 63*(b^2 + 2*a*c)*x^4 + 90*a*b*x^2 + 35*
a^2)/x^9
```

mupad [**B**] time = 0.03, size = 46, normalized size = 0.88

$$-\frac{\frac{a^2}{9}+x^4\left(\frac{b^2}{5}+\frac{2\,a\,c}{5}\right)+c^2\,x^8+\frac{2\,a\,b\,x^2}{7}+\frac{2\,b\,c\,x^6}{3}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^10,x)
```

```
[Out] -(a^2/9 + x^4*((2*a*c)/5 + b^2/5) + c^2*x^8 + (2*a*b*x^2)/7 + (2*b*c*x^6)/3
)/x^9
```

sympy [**A**] time = 1.47, size = 49, normalized size = 0.94

$$\frac{-35a^2-90abx^2-210bcx^6-315c^2x^8+x^4\left(-126ac-63b^2\right)}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**10,x)
```

```
[Out] (-35*a**2 - 90*a*b*x**2 - 210*b*c*x**6 - 315*c**2*x**8 + x**4*(-126*a*c - 6
3*b**2))/(315*x**9)
```

3.838 $\int \frac{(a+bx^2+cx^4)^2}{x^{11}} dx$

Optimal. Leaf size=54

$$-\frac{a^2}{10x^{10}}-\frac{2ac+b^2}{6x^6}-\frac{ab}{4x^8}-\frac{bc}{2x^4}-\frac{c^2}{2x^2}$$

```
[Out] -1/10*a^2/x^10-1/4*a*b/x^8+1/6*(-2*a*c-b^2)/x^6-1/2*b*c/x^4-1/2*c^2/x^2
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$-\frac{a^2}{10x^{10}}-\frac{2ac+b^2}{6x^6}-\frac{ab}{4x^8}-\frac{bc}{2x^4}-\frac{c^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^11,x]
```

```
[Out] -a^2/(10*x^10) - (a*b)/(4*x^8) - (b^2 + 2*a*c)/(6*x^6) - (b*c)/(2*x^4) - c^
2/(2*x^2)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{11}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^2}{x^6}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{x^6}+\frac{2ab}{x^5}+\frac{b^2+2ac}{x^4}+\frac{2bc}{x^3}+\frac{c^2}{x^2}\right)dx,x,x^2\right)\\ &= -\frac{a^2}{10x^{10}}-\frac{ab}{4x^8}-\frac{b^2+2ac}{6x^6}-\frac{bc}{2x^4}-\frac{c^2}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.98

$$-\frac{6a^2+5a\left(3bx^2+4cx^4\right)+10x^4\left(b^2+3bcx^2+3c^2x^4\right)}{60x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^11,x]
```

```
[Out] -1/60*(6*a^2 + 5*a*(3*b*x^2 + 4*c*x^4) + 10*x^4*(b^2 + 3*b*c*x^2 + 3*c^2*x^
4))/x^10
```

fricas [A] time = 0.91, size = 46, normalized size = 0.85

$$-\frac{30\,c^2x^8+30\,bcx^6+10\left(b^2+2\,ac\right)x^4+15\,abx^2+6\,a^2}{60\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^11,x, algorithm="fricas")
```

```
[Out] -1/60*(30*c^2*x^8 + 30*b*c*x^6 + 10*(b^2 + 2*a*c)*x^4 + 15*a*b*x^2 + 6*a^2)
/x^10
```

giac [A] time = 0.22, size = 48, normalized size = 0.89

$$-\frac{30\,c^2x^8+30\,bcx^6+10\,b^2x^4+20\,acx^4+15\,abx^2+6\,a^2}{60\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^11,x, algorithm="giac")
```

```
[Out] -1/60*(30*c^2*x^8 + 30*b*c*x^6 + 10*b^2*x^4 + 20*a*c*x^4 + 15*a*b*x^2 + 6*a
^2)/x^10
```

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$-\frac{c^2}{2x^2}-\frac{bc}{2x^4}-\frac{ab}{4x^8}-\frac{2ac+b^2}{6x^6}-\frac{a^2}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^11,x)
```

```
[Out] -1/2*c^2/x^2-1/6*(2*a*c+b^2)/x^6-1/10*a^2/x^10-1/2*b*c/x^4-1/4*a*b/x^8
```

maxima [A] time = 1.28, size = 46, normalized size = 0.85

$$-\frac{30\,c^2x^8+30\,bcx^6+10\left(b^2+2\,ac\right)x^4+15\,abx^2+6\,a^2}{60\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^11,x, algorithm="maxima")
```

```
[Out] -1/60*(30*c^2*x^8 + 30*b*c*x^6 + 10*(b^2 + 2*a*c)*x^4 + 15*a*b*x^2 + 6*a^2)
/x^10
```

mupad [B] time = 4.12, size = 47, normalized size = 0.87

$$-\frac{\frac{a^2}{10}+x^4\left(\frac{b^2}{6}+\frac{a\,c}{3}\right)+\frac{c^2\,x^8}{2}+\frac{a\,b\,x^2}{4}+\frac{b\,c\,x^6}{2}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^11,x)
```

```
[Out] -(a^2/10 + x^4*((a*c)/3 + b^2/6) + (c^2*x^8)/2 + (a*b*x^2)/4 + (b*c*x^6)/2)
/x^10
```

sympy [A] time = 2.08, size = 49, normalized size = 0.91

$$\frac{-6a^2-15abx^2-30bcx^6-30c^2x^8+x^4\left(-20ac-10b^2\right)}{60x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**11,x)
```

```
[Out] (-6*a**2 - 15*a*b*x**2 - 30*b*c*x**6 - 30*c**2*x**8 + x**4*(-20*a*c - 10*b*
*2))/(60*x**10)
```

3.839 $\int \frac{(a+bx^2+cx^4)^2}{x^{12}} dx$

Optimal. Leaf size=54

$$-\frac{a^2}{11x^{11}}-\frac{2ac+b^2}{7x^7}-\frac{2ab}{9x^9}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

```
[Out] -1/11*a^2/x^11-2/9*a*b/x^9+1/7*(-2*a*c-b^2)/x^7-2/5*b*c/x^5-1/3*c^2/x^3
```

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$-\frac{a^2}{11x^{11}}-\frac{2ac+b^2}{7x^7}-\frac{2ab}{9x^9}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^12,x]
```

```
[Out] -a^2/(11*x^11) - (2*a*b)/(9*x^9) - (b^2 + 2*a*c)/(7*x^7) - (2*b*c)/(5*x^5)
- c^2/(3*x^3)
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^2}{x^{12}} dx &= \int \left(\frac{a^2}{x^{12}}+\frac{2ab}{x^{10}}+\frac{b^2+2ac}{x^8}+\frac{2bc}{x^6}+\frac{c^2}{x^4}\right) dx\\ &= -\frac{a^2}{11x^{11}}-\frac{2ab}{9x^9}-\frac{b^2+2ac}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 56, normalized size = 1.04

$$-\frac{a^2}{11x^{11}}+\frac{-2ac-b^2}{7x^7}-\frac{2ab}{9x^9}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^12,x]
```

```
[Out] -1/11*a^2/x^11 - (2*a*b)/(9*x^9) + (-b^2 - 2*a*c)/(7*x^7) - (2*b*c)/(5*x^5)
 - c^2/(3*x^3)
```

fricas [A] time = 0.80, size = 46, normalized size = 0.85

$$-\frac{1155\,c^2x^8 + 1386\,bcx^6 + 495\,\left(b^2 + 2\,ac\right)x^4 + 770\,abx^2 + 315\,a^2}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^12,x, algorithm="fricas")
```

```
[Out] -1/3465*(1155*c^2*x^8 + 1386*b*c*x^6 + 495*(b^2 + 2*a*c)*x^4 + 770*a*b*x^2
+ 315*a^2)/x^11
```

giac [A] time = 0.19, size = 48, normalized size = 0.89

$$-\frac{1155\,c^2x^8 + 1386\,bcx^6 + 495\,b^2x^4 + 990\,acx^4 + 770\,abx^2 + 315\,a^2}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^12,x, algorithm="giac")
```

```
[Out] -1/3465*(1155*c^2*x^8 + 1386*b*c*x^6 + 495*b^2*x^4 + 990*a*c*x^4 + 770*a*b*
x^2 + 315*a^2)/x^11
```

maple [A] time = 0.01, size = 45, normalized size = 0.83

$$-\frac{c^2}{3x^3} - \frac{2bc}{5x^5} - \frac{2ab}{9x^9} - \frac{2ac + b^2}{7x^7} - \frac{a^2}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^12,x)
```

```
[Out] -1/7*(2*a*c+b^2)/x^7-1/11*a^2/x^11-2/5*b*c/x^5-1/3*c^2/x^3-2/9*a*b/x^9
```

maxima [A] time = 1.38, size = 46, normalized size = 0.85

$$-\frac{1155\,c^2x^8 + 1386\,bcx^6 + 495\,\left(b^2 + 2\,ac\right)x^4 + 770\,abx^2 + 315\,a^2}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^12,x, algorithm="maxima")
```

```
[Out] -1/3465*(1155*c^2*x^8 + 1386*b*c*x^6 + 495*(b^2 + 2*a*c)*x^4 + 770*a*b*x^2
+ 315*a^2)/x^11
```

mupad [B] time = 4.16, size = 47, normalized size = 0.87

$$-\frac{\frac{a^2}{11}+x^4\left(\frac{b^2}{7}+\frac{2\,a\,c}{7}\right)+\frac{c^2\,x^8}{3}+\frac{2\,a\,b\,x^2}{9}+\frac{2\,b\,c\,x^6}{5}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^12,x)
```

```
[Out] -(a^2/11 + x^4*((2*a*c)/7 + b^2/7) + (c^2*x^8)/3 + (2*a*b*x^2)/9 + (2*b*c*x
^6)/5)/x^11
```

sympy [A] time = 1.96, size = 49, normalized size = 0.91

$$\frac{-315a^2-770abx^2-1386bcx^6-1155c^2x^8+x^4\left(-990ac-495b^2\right)}{3465x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**12,x)
```

```
[Out] (-315*a**2 - 770*a*b*x**2 - 1386*b*c*x**6 - 1155*c**2*x**8 + x**4*(-990*a*c
 - 495*b**2))/(3465*x**11)
```

3.840 $\int \frac{(a+bx^2+cx^4)^2}{x^{13}} \, dx$

Optimal. Leaf size=54

$$-\frac{a^2}{12x^{12}}-\frac{2ac+b^2}{8x^8}-\frac{ab}{5x^{10}}-\frac{bc}{3x^6}-\frac{c^2}{4x^4}$$

```
[Out] -1/12*a^2/x^12-1/5*a*b/x^10+1/8*(-2*a*c-b^2)/x^8-1/3*b*c/x^6-1/4*c^2/x^4
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$-\frac{a^2}{12x^{12}}-\frac{2ac+b^2}{8x^8}-\frac{ab}{5x^{10}}-\frac{bc}{3x^6}-\frac{c^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^13,x]
```

```
[Out] -a^2/(12*x^12) - (a*b)/(5*x^10) - (b^2 + 2*a*c)/(8*x^8) - (b*c)/(3*x^6) - c
^2/(4*x^4)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^2}{x^{13}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx+cx^2)^2}{x^7}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{x^7}+\frac{2ab}{x^6}+\frac{b^2+2ac}{x^5}+\frac{2bc}{x^4}+\frac{c^2}{x^3}\right)dx,x,x^2\right)$$
$$= -\frac{a^2}{12x^{12}}-\frac{ab}{5x^{10}}-\frac{b^2+2ac}{8x^8}-\frac{bc}{3x^6}-\frac{c^2}{4x^4}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.93

$$-\frac{10a^2+24abx^2+30acx^4+15b^2x^4+40bcx^6+30c^2x^8}{120x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^13,x]
```

```
[Out] -1/120*(10*a^2 + 24*a*b*x^2 + 15*b^2*x^4 + 30*a*c*x^4 + 40*b*c*x^6 + 30*c^2
*x^8)/x^12
```

fricas [A] time = 0.89, size = 46, normalized size = 0.85

$$-\frac{30\,c^2x^8+40\,bcx^6+15\,(b^2+2\,ac)x^4+24\,abx^2+10\,a^2}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^13,x, algorithm="fricas")
```

```
[Out] -1/120*(30*c^2*x^8 + 40*b*c*x^6 + 15*(b^2 + 2*a*c)*x^4 + 24*a*b*x^2 + 10*a^
2)/x^12
```

giac [A] time = 0.15, size = 48, normalized size = 0.89

$$-\frac{30\,c^2x^8+40\,bcx^6+15\,b^2x^4+30\,acx^4+24\,abx^2+10\,a^2}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^13,x, algorithm="giac")
```

```
[Out] -1/120*(30*c^2*x^8 + 40*b*c*x^6 + 15*b^2*x^4 + 30*a*c*x^4 + 24*a*b*x^2 + 10
*a^2)/x^12
```

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$-\frac{c^2}{4x^4}-\frac{bc}{3x^6}-\frac{ab}{5x^{10}}-\frac{2ac+b^2}{8x^8}-\frac{a^2}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^13,x)
```

```
[Out] -1/3*b*c/x^6-1/12*a^2/x^12-1/5*a*b/x^10-1/8*(2*a*c+b^2)/x^8-1/4*c^2/x^4
```

maxima [A] time = 1.34, size = 46, normalized size = 0.85

$$-\frac{30\,c^2x^8+40\,bcx^6+15\left(b^2+2\,ac\right)x^4+24\,abx^2+10\,a^2}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^13,x, algorithm="maxima")
```

```
[Out] -1/120*(30*c^2*x^8 + 40*b*c*x^6 + 15*(b^2 + 2*a*c)*x^4 + 24*a*b*x^2 + 10*a^
2)/x^12
```

mupad [B] time = 4.16, size = 47, normalized size = 0.87

$$-\frac{\frac{a^2}{12}+x^4\left(\frac{b^2}{8}+\frac{a\,c}{4}\right)+\frac{c^2\,x^8}{4}+\frac{a\,b\,x^2}{5}+\frac{b\,c\,x^6}{3}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^13,x)
```

```
[Out] -(a^2/12 + x^4*((a*c)/4 + b^2/8) + (c^2*x^8)/4 + (a*b*x^2)/5 + (b*c*x^6)/3)
/x^12
```

sympy [A] time = 2.70, size = 49, normalized size = 0.91

$$\frac{-10a^2-24abx^2-40bcx^6-30c^2x^8+x^4\left(-30ac-15b^2\right)}{120x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**13,x)
```

```
[Out] (-10*a**2 - 24*a*b*x**2 - 40*b*c*x**6 - 30*c**2*x**8 + x**4*(-30*a*c - 15*b
**2))/(120*x**12)
```

3.841 $\int x^2 \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=89

$$\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{11}cx^{11}\left(ac+b^2\right)+\frac{1}{9}bx^9\left(6ac+b^2\right)+\frac{3}{7}ax^7\left(ac+b^2\right)+\frac{3}{13}bc^2x^{13}+\frac{c^3x^{15}}{15}$$

```
[Out] 1/3*a^3*x^3+3/5*a^2*b*x^5+3/7*a*(a*c+b^2)*x^7+1/9*b*(6*a*c+b^2)*x^9+3/11*c*
(a*c+b^2)*x^11+3/13*b*c^2*x^13+1/15*c^3*x^15
```

Rubi [A] time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$\frac{3}{5}a^2bx^5+\frac{a^3x^3}{3}+\frac{3}{11}cx^{11}\left(ac+b^2\right)+\frac{1}{9}bx^9\left(6ac+b^2\right)+\frac{3}{7}ax^7\left(ac+b^2\right)+\frac{3}{13}bc^2x^{13}+\frac{c^3x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*(b^2 + a*c)*x^7)/7 + (b*(b^2 + 6*a*c)*
x^9)/9 + (3*c*(b^2 + a*c)*x^11)/11 + (3*b*c^2*x^13)/13 + (c^3*x^15)/15
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2+cx^4\right)^3\,dx &= \int \left(a^3x^2+3a^2bx^4+3a\left(b^2+ac\right)x^6+b\left(b^2+6ac\right)x^8+3c\left(b^2+ac\right)x^{10}+3bc^2x^{12}+\right. \\ &= \frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}a\left(b^2+ac\right)x^7+\frac{1}{9}b\left(b^2+6ac\right)x^9+\frac{3}{11}c\left(b^2+ac\right)x^{11}+\frac{3}{13}bc^2x^{1}\end{aligned}$$

Mathematica [A] time = 0.01, size = 89, normalized size = 1.00

$$\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{11}cx^{11}\left(ac+b^2\right)+\frac{1}{9}bx^9\left(6ac+b^2\right)+\frac{3}{7}ax^7\left(ac+b^2\right)+\frac{3}{13}bc^2x^{13}+\frac{c^3x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*(b^2 + a*c)*x^7)/7 + (b*(b^2 + 6*a*c)*
x^9)/9 + (3*c*(b^2 + a*c)*x^11)/11 + (3*b*c^2*x^13)/13 + (c^3*x^15)/15
```

fricas [A] time = 0.66, size = 87, normalized size = 0.98

$$\frac{1}{15}x^{15}c^3+\frac{3}{13}x^{13}c^2b+\frac{3}{11}x^{11}cb^2+\frac{3}{11}x^{11}c^2a+\frac{1}{9}x^9b^3+\frac{2}{3}x^9cba+\frac{3}{7}x^7b^2a+\frac{3}{7}x^7ca^2+\frac{3}{5}x^5ba^2+\frac{1}{3}x^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*c^3 + 3/13*x^13*c^2*b + 3/11*x^11*c*b^2 + 3/11*x^11*c^2*a + 1/9*x
^9*b^3 + 2/3*x^9*c*b*a + 3/7*x^7*b^2*a + 3/7*x^7*c*a^2 + 3/5*x^5*b*a^2 + 1/
3*x^3*a^3
```

giac [A] time = 0.19, size = 87, normalized size = 0.98

$$\frac{1}{15}\,c^3x^{15}+\frac{3}{13}\,bc^2x^{13}+\frac{3}{11}\,b^2cx^{11}+\frac{3}{11}\,ac^2x^{11}+\frac{1}{9}\,b^3x^9+\frac{2}{3}\,abcx^9+\frac{3}{7}\,ab^2x^7+\frac{3}{7}\,a^2cx^7+\frac{3}{5}\,a^2bx^5+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/15*c^3*x^15 + 3/13*b*c^2*x^13 + 3/11*b^2*c*x^11 + 3/11*a*c^2*x^11 + 1/9*b
^3*x^9 + 2/3*a*b*c*x^9 + 3/7*a*b^2*x^7 + 3/7*a^2*c*x^7 + 3/5*a^2*b*x^5 + 1/
3*a^3*x^3
```

maple [A] time = 0.00, size = 111, normalized size = 1.25

$$\frac{c^3x^{15}}{15}+\frac{3b\,c^2x^{13}}{13}+\frac{\left(a\,c^2+2b^2c+\left(2ac+b^2\right)c\right)x^{11}}{11}+\frac{\left(4abc+\left(2ac+b^2\right)b\right)x^9}{9}+\frac{3a^2b\,x^5}{5}+\frac{\left(a^2c+2a\,b^2+\left(2ac+b^2\right)a\right.}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/15*c^3*x^15+3/13*b*c^2*x^13+1/11*(a*c^2+2*b^2*c+(2*a*c+b^2)*c)*x^11+1/9*(
4*a*b*c+(2*a*c+b^2)*b)*x^9+1/7*(a^2*c+2*a*b^2+(2*a*c+b^2)*a)*x^7+3/5*a^2*b*
x^5+1/3*a^3*x^3
```

maxima [A] time = 1.39, size = 81, normalized size = 0.91

$$\frac{1}{15}\,c^3x^{15}+\frac{3}{13}\,bc^2x^{13}+\frac{3}{11}\left(b^2c+ac^2\right)x^{11}+\frac{1}{9}\left(b^3+6\,abc\right)x^9+\frac{3}{5}\,a^2bx^5+\frac{3}{7}\left(ab^2+a^2c\right)x^7+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/15*c^3*x^15 + 3/13*b*c^2*x^13 + 3/11*(b^2*c + a*c^2)*x^11 + 1/9*(b^3 + 6*
a*b*c)*x^9 + 3/5*a^2*b*x^5 + 3/7*(a*b^2 + a^2*c)*x^7 + 1/3*a^3*x^3
```

mupad [B] time = 0.03, size = 76, normalized size = 0.85

$$x^9\left(\frac{b^3}{9}+\frac{2\,a\,c\,b}{3}\right)+\frac{a^3\,x^3}{3}+\frac{c^3\,x^{15}}{15}+\frac{3\,a^2\,b\,x^5}{5}+\frac{3\,b\,c^2\,x^{13}}{13}+\frac{3\,a\,x^7\,\left(b^2+a\,c\right)}{7}+\frac{3\,c\,x^{11}\,\left(b^2+a\,c\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^9*(b^3/9 + (2*a*b*c)/3) + (a^3*x^3)/3 + (c^3*x^15)/15 + (3*a^2*b*x^5)/5 +
 (3*b*c^2*x^13)/13 + (3*a*x^7*(a*c + b^2))/7 + (3*c*x^11*(a*c + b^2))/11
```

sympy [A] time = 0.09, size = 97, normalized size = 1.09

$$\frac{a^3x^3}{3}+\frac{3a^2bx^5}{5}+\frac{3bc^2x^{13}}{13}+\frac{c^3x^{15}}{15}+x^{11}\left(\frac{3ac^2}{11}+\frac{3b^2c}{11}\right)+x^9\left(\frac{2abc}{3}+\frac{b^3}{9}\right)+x^7\left(\frac{3a^2c}{7}+\frac{3ab^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] a**3*x**3/3 + 3*a**2*b*x**5/5 + 3*b*c**2*x**13/13 + c**3*x**15/15 + x**11*(
3*a*c**2/11 + 3*b**2*c/11) + x**9*(2*a*b*c/3 + b**3/9) + x**7*(3*a**2*c/7 +
 3*a*b**2/7)
```

3.842 $\int x\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=89

$$\frac{a^3x^2}{2}+\frac{3}{4}a^2bx^4+\frac{3}{10}cx^{10}\left(ac+b^2\right)+\frac{1}{8}bx^8\left(6ac+b^2\right)+\frac{1}{2}ax^6\left(ac+b^2\right)+\frac{1}{4}bc^2x^{12}+\frac{c^3x^{14}}{14}$$

```
[Out] 1/2*a^3*x^2+3/4*a^2*b*x^4+1/2*a*(a*c+b^2)*x^6+1/8*b*(6*a*c+b^2)*x^8+3/10*c*
(a*c+b^2)*x^10+1/4*b*c^2*x^12+1/14*c^3*x^14
```

Rubi [A] time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1107, 611}

$$\frac{3}{4}a^2bx^4+\frac{a^3x^2}{2}+\frac{3}{10}cx^{10}\left(ac+b^2\right)+\frac{1}{8}bx^8\left(6ac+b^2\right)+\frac{1}{2}ax^6\left(ac+b^2\right)+\frac{1}{4}bc^2x^{12}+\frac{c^3x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*x^2)/2 + (3*a^2*b*x^4)/4 + (a*(b^2 + a*c)*x^6)/2 + (b*(b^2 + 6*a*c)*x^
8)/8 + (3*c*(b^2 + a*c)*x^10)/10 + (b*c^2*x^12)/4 + (c^3*x^14)/14
```

Rule 611

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2+cx^4\right)^3\,dx&=\frac{1}{2}\text{Subst}\left(\int\left(a+bx+cx^2\right)^3\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(a^3+3a^2bx+3ab^2\left(1+\frac{ac}{b^2}\right)x^2+b^3\left(1+\frac{6ac}{b^2}\right)x^3+3b^2c\left(1+\frac{ac}{b^2}\right)x^4+3b\right.\right.\\&=\frac{a^3x^2}{2}+\frac{3}{4}a^2bx^4+\frac{1}{2}a\left(b^2+ac\right)x^6+\frac{1}{8}b\left(b^2+6ac\right)x^8+\frac{3}{10}c\left(b^2+ac\right)x^{10}+\frac{1}{4}bc^2x^{12}+\end{aligned}$$

Mathematica [A] time = 0.02, size = 79, normalized size = 0.89

$$\frac{1}{280}x^2\left(140a^3+210a^2bx^2+84cx^8\left(ac+b^2\right)+35bx^6\left(6ac+b^2\right)+140ax^4\left(ac+b^2\right)+70bc^2x^{10}+20c^3x^{12}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^2*(140*a^3 + 210*a^2*b*x^2 + 140*a*(b^2 + a*c)*x^4 + 35*b*(b^2 + 6*a*c)*
x^6 + 84*c*(b^2 + a*c)*x^8 + 70*b*c^2*x^10 + 20*c^3*x^12))/280
```

fricas [A] time = 0.97, size = 87, normalized size = 0.98

$$\frac{1}{14}x^{14}c^3+\frac{1}{4}x^{12}c^2b+\frac{3}{10}x^{10}cb^2+\frac{3}{10}x^{10}c^2a+\frac{1}{8}x^8b^3+\frac{3}{4}x^8cba+\frac{1}{2}x^6b^2a+\frac{1}{2}x^6ca^2+\frac{3}{4}x^4ba^2+\frac{1}{2}x^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*c^3 + 1/4*x^12*c^2*b + 3/10*x^10*c*b^2 + 3/10*x^10*c^2*a + 1/8*x^
8*b^3 + 3/4*x^8*c*b*a + 1/2*x^6*b^2*a + 1/2*x^6*c*a^2 + 3/4*x^4*b*a^2 + 1/2
*x^2*a^3
```

giac [A] time = 0.17, size = 87, normalized size = 0.98

$$\frac{1}{14}c^3x^{14}+\frac{1}{4}bc^2x^{12}+\frac{3}{10}b^2cx^{10}+\frac{3}{10}ac^2x^{10}+\frac{1}{8}b^3x^8+\frac{3}{4}abcx^8+\frac{1}{2}ab^2x^6+\frac{1}{2}a^2cx^6+\frac{3}{4}a^2bx^4+\frac{1}{2}a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/14*c^3*x^14 + 1/4*b*c^2*x^12 + 3/10*b^2*c*x^10 + 3/10*a*c^2*x^10 + 1/8*b^
3*x^8 + 3/4*a*b*c*x^8 + 1/2*a*b^2*x^6 + 1/2*a^2*c*x^6 + 3/4*a^2*b*x^4 + 1/2
*a^3*x^2
```

maple [A] time = 0.00, size = 111, normalized size = 1.25

$$\frac{c^3x^{14}}{14}+\frac{b\,c^2x^{12}}{4}+\frac{\left(a\,c^2+2b^2c+\left(2ac+b^2\right)c\right)x^{10}}{10}+\frac{\left(4abc+\left(2ac+b^2\right)b\right)x^8}{8}+\frac{3a^2b\,x^4}{4}+\frac{\left(a^2c+2a\,b^2+\left(2ac+b^2\right)a\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/14*c^3*x^14+1/4*b*c^2*x^12+1/10*(a*c^2+2*b^2*c+(2*a*c+b^2)*c)*x^10+1/8*(4
*a*b*c+(2*a*c+b^2)*b)*x^8+1/6*(a^2*c+2*a*b^2+(2*a*c+b^2)*a)*x^6+3/4*a^2*b*x
^4+1/2*a^3*x^2
```

maxima [A] time = 1.37, size = 81, normalized size = 0.91

$$\frac{1}{14}c^3x^{14} + \frac{1}{4}bc^2x^{12} + \frac{3}{10}\left(b^2c + ac^2\right)x^{10} + \frac{1}{8}\left(b^3 + 6\,abc\right)x^8 + \frac{3}{4}a^2bx^4 + \frac{1}{2}\left(ab^2 + a^2c\right)x^6 + \frac{1}{2}a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/14*c^3*x^14 + 1/4*b*c^2*x^12 + 3/10*(b^2*c + a*c^2)*x^10 + 1/8*(b^3 + 6*a
*b*c)*x^8 + 3/4*a^2*b*x^4 + 1/2*(a*b^2 + a^2*c)*x^6 + 1/2*a^3*x^2
```

mupad [B] time = 0.03, size = 76, normalized size = 0.85

$$x^8\left(\frac{b^3}{8} + \frac{3\,a\,c\,b}{4}\right) + \frac{a^3\,x^2}{2} + \frac{c^3\,x^{14}}{14} + \frac{3\,a^2\,b\,x^4}{4} + \frac{b\,c^2\,x^{12}}{4} + \frac{a\,x^6\,\left(b^2 + a\,c\right)}{2} + \frac{3\,c\,x^{10}\,\left(b^2 + a\,c\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^8*(b^3/8 + (3*a*b*c)/4) + (a^3*x^2)/2 + (c^3*x^14)/14 + (3*a^2*b*x^4)/4 +
 (b*c^2*x^12)/4 + (a*x^6*(a*c + b^2))/2 + (3*c*x^10*(a*c + b^2))/10
```

sympy [A] time = 0.10, size = 92, normalized size = 1.03

$$\frac{a^3x^2}{2} + \frac{3a^2bx^4}{4} + \frac{bc^2x^{12}}{4} + \frac{c^3x^{14}}{14} + x^{10}\left(\frac{3ac^2}{10} + \frac{3b^2c}{10}\right) + x^8\left(\frac{3abc}{4} + \frac{b^3}{8}\right) + x^6\left(\frac{a^2c}{2} + \frac{ab^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] a**3*x**2/2 + 3*a**2*b*x**4/4 + b*c**2*x**12/4 + c**3*x**14/14 + x**10*(3*a
*c**2/10 + 3*b**2*c/10) + x**8*(3*a*b*c/4 + b**3/8) + x**6*(a**2*c/2 + a*b*
*2/2)
```

3.843 $\int \left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=81

$$a^3x+a^2bx^3+\frac{1}{3}cx^9\left(ac+b^2\right)+\frac{1}{7}bx^7\left(6ac+b^2\right)+\frac{3}{5}ax^5\left(ac+b^2\right)+\frac{3}{11}bc^2x^{11}+\frac{c^3x^{13}}{13}$$

```
[Out] a^3*x+a^2*b*x^3+3/5*a*(a*c+b^2)*x^5+1/7*b*(6*a*c+b^2)*x^7+1/3*c*(a*c+b^2)*x
^9+3/11*b*c^2*x^11+1/13*c^3*x^13
```

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1090}

$$a^2bx^3+a^3x+\frac{1}{3}cx^9\left(ac+b^2\right)+\frac{1}{7}bx^7\left(6ac+b^2\right)+\frac{3}{5}ax^5\left(ac+b^2\right)+\frac{3}{11}bc^2x^{11}+\frac{c^3x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] a^3*x + a^2*b*x^3 + (3*a*(b^2 + a*c)*x^5)/5 + (b*(b^2 + 6*a*c)*x^7)/7 + (c*
(b^2 + a*c)*x^9)/3 + (3*b*c^2*x^11)/11 + (c^3*x^13)/13
```

Rule 1090

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2+cx^4\right)^3\,dx &= \int \left(a^3+3a^2bx^2+3ab^2\left(1+\frac{ac}{b^2}\right)x^4+b^3\left(1+\frac{6ac}{b^2}\right)x^6+3b^2c\left(1+\frac{ac}{b^2}\right)x^8+3bc^2x^{10}+c\right.\\ &= a^3x+a^2bx^3+\frac{3}{5}a\left(b^2+ac\right)x^5+\frac{1}{7}b\left(b^2+6ac\right)x^7+\frac{1}{3}c\left(b^2+ac\right)x^9+\frac{3}{11}bc^2x^{11}+\frac{c^3x^{1}}{13}\end{aligned}$$

Mathematica [A] time = 0.01, size = 81, normalized size = 1.00

$$a^3x+a^2bx^3+\frac{1}{3}cx^9\left(ac+b^2\right)+\frac{1}{7}bx^7\left(6ac+b^2\right)+\frac{3}{5}ax^5\left(ac+b^2\right)+\frac{3}{11}bc^2x^{11}+\frac{c^3x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] a^3*x + a^2*b*x^3 + (3*a*(b^2 + a*c)*x^5)/5 + (b*(b^2 + 6*a*c)*x^7)/7 + (c*
(b^2 + a*c)*x^9)/3 + (3*b*c^2*x^11)/11 + (c^3*x^13)/13
```

fricas [A] time = 0.49, size = 83, normalized size = 1.02

$$\frac{1}{13}x^{13}c^3+\frac{3}{11}x^{11}c^2b+\frac{1}{3}x^9cb^2+\frac{1}{3}x^9c^2a+\frac{1}{7}x^7b^3+\frac{6}{7}x^7cba+\frac{3}{5}x^5b^2a+\frac{3}{5}x^5ca^2+x^3ba^2+xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*c^3 + 3/11*x^11*c^2*b + 1/3*x^9*c*b^2 + 1/3*x^9*c^2*a + 1/7*x^7*b
^3 + 6/7*x^7*c*b*a + 3/5*x^5*b^2*a + 3/5*x^5*c*a^2 + x^3*b*a^2 + x*a^3
```

giac [A] time = 0.15, size = 83, normalized size = 1.02

$$\frac{1}{13}c^3x^{13}+\frac{3}{11}bc^2x^{11}+\frac{1}{3}b^2cx^9+\frac{1}{3}ac^2x^9+\frac{1}{7}b^3x^7+\frac{6}{7}abcx^7+\frac{3}{5}ab^2x^5+\frac{3}{5}a^2cx^5+a^2bx^3+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/13*c^3*x^13 + 3/11*b*c^2*x^11 + 1/3*b^2*c*x^9 + 1/3*a*c^2*x^9 + 1/7*b^3*x
^7 + 6/7*a*b*c*x^7 + 3/5*a*b^2*x^5 + 3/5*a^2*c*x^5 + a^2*b*x^3 + a^3*x
```

maple [A] time = 0.00, size = 107, normalized size = 1.32

$$\frac{c^3x^{13}}{13}+\frac{3bc^2x^{11}}{11}+\frac{\left(ac^2+2b^2c+\left(2ac+b^2\right)c\right)x^9}{9}+\frac{\left(4abc+\left(2ac+b^2\right)b\right)x^7}{7}+a^2bx^3+\frac{\left(a^2c+2ab^2+\left(2ac+b^2\right)a\right)x}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/13*c^3*x^13+3/11*b*c^2*x^11+1/9*(a*c^2+2*b^2*c+(2*a*c+b^2)*c)*x^9+1/7*(4*
a*b*c+(2*a*c+b^2)*b)*x^7+1/5*(a^2*c+2*a*b^2+(2*a*c+b^2)*a)*x^5+a^2*b*x^3+a^
3*x
```

maxima [A] time = 1.35, size = 85, normalized size = 1.05

$$\frac{1}{13}c^3x^{13}+\frac{3}{11}bc^2x^{11}+\frac{1}{3}b^2cx^9+\frac{1}{7}b^3x^7+a^3x+\frac{1}{5}\left(3cx^5+5bx^3\right)a^2+\frac{1}{105}\left(35c^2x^9+90bcx^7+63b^2x^5\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/13*c^3*x^13 + 3/11*b*c^2*x^11 + 1/3*b^2*c*x^9 + 1/7*b^3*x^7 + a^3*x + 1/5
*(3*c*x^5 + 5*b*x^3)*a^2 + 1/105*(35*c^2*x^9 + 90*b*c*x^7 + 63*b^2*x^5)*a
```

mupad [B] time = 0.03, size = 72, normalized size = 0.89

$$x^7\left(\frac{b^3}{7}+\frac{6\,a\,c\,b}{7}\right)+a^3\,x+\frac{c^3\,x^{13}}{13}+a^2\,b\,x^3+\frac{3\,b\,c^2\,x^{11}}{11}+\frac{3\,a\,x^5\,\left(b^2+a\,c\right)}{5}+\frac{c\,x^9\,\left(b^2+a\,c\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^7*(b^3/7 + (6*a*b*c)/7) + a^3*x + (c^3*x^13)/13 + a^2*b*x^3 + (3*b*c^2*x^
11)/11 + (3*a*x^5*(a*c + b^2))/5 + (c*x^9*(a*c + b^2))/3
```

sympy [A] time = 0.09, size = 87, normalized size = 1.07

$$a^3x+a^2bx^3+\frac{3bc^2x^{11}}{11}+\frac{c^3x^{13}}{13}+x^9\left(\frac{ac^2}{3}+\frac{b^2c}{3}\right)+x^7\left(\frac{6abc}{7}+\frac{b^3}{7}\right)+x^5\left(\frac{3a^2c}{5}+\frac{3ab^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3,x)
```

```
[Out] a**3*x + a**2*b*x**3 + 3*b*c**2*x**11/11 + c**3*x**13/13 + x**9*(a*c**2/3 +
 b**2*c/3) + x**7*(6*a*b*c/7 + b**3/7) + x**5*(3*a**2*c/5 + 3*a*b**2/5)
```

3.844 $\int \frac{\left(a+bx^2+cx^4\right)^3}{x} \, dx$

Optimal. Leaf size=85

$$a^3 \log(x) + \frac{3}{2}a^2bx^2 + \frac{3}{8}cx^8\left(ac+b^2\right) + \frac{1}{6}bx^6\left(6ac+b^2\right) + \frac{3}{4}ax^4\left(ac+b^2\right) + \frac{3}{10}bc^2x^{10} + \frac{c^3x^{12}}{12}$$

```
[Out] 3/2*a^2*b*x^2+3/4*a*(a*c+b^2)*x^4+1/6*b*(6*a*c+b^2)*x^6+3/8*c*(a*c+b^2)*x^8
+3/10*b*c^2*x^10+1/12*c^3*x^12+a^3*ln(x)
```

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$\frac{3}{2}a^2bx^2 + a^3\log(x) + \frac{3}{8}cx^8\left(ac+b^2\right) + \frac{1}{6}bx^6\left(6ac+b^2\right) + \frac{3}{4}ax^4\left(ac+b^2\right) + \frac{3}{10}bc^2x^{10} + \frac{c^3x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x,x]
```

```
[Out] (3*a^2*b*x^2)/2 + (3*a*(b^2 + a*c)*x^4)/4 + (b*(b^2 + 6*a*c)*x^6)/6 + (3*c*
(b^2 + a*c)*x^8)/8 + (3*b*c^2*x^10)/10 + (c^3*x^12)/12 + a^3*Log[x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^3}{x}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^3}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(3a^2b+\frac{a^3}{x}+3a\left(b^2+ac\right)x+b\left(b^2+6ac\right)x^2+3c\left(b^2+ac\right)x^3+3bc^2x^4+\right.\right.$$
$$= \frac{3}{2}a^2bx^2+\frac{3}{4}a\left(b^2+ac\right)x^4+\frac{1}{6}b\left(b^2+6ac\right)x^6+\frac{3}{8}c\left(b^2+ac\right)x^8+\frac{3}{10}bc^2x^{10}+\frac{c^3x^{12}}{12}+$$

Mathematica [A] time = 0.02, size = 85, normalized size = 1.00

$$a^3\log(x)+\frac{3}{2}a^2bx^2+\frac{3}{8}cx^8\left(ac+b^2\right)+\frac{1}{6}bx^6\left(6ac+b^2\right)+\frac{3}{4}ax^4\left(ac+b^2\right)+\frac{3}{10}bc^2x^{10}+\frac{c^3x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x,x]
```

```
[Out] (3*a^2*b*x^2)/2 + (3*a*(b^2 + a*c)*x^4)/4 + (b*(b^2 + 6*a*c)*x^6)/6 + (3*c*
(b^2 + a*c)*x^8)/8 + (3*b*c^2*x^10)/10 + (c^3*x^12)/12 + a^3*Log[x]
```

fricas [A] time = 0.89, size = 79, normalized size = 0.93

$$\frac{1}{12}c^3x^{12}+\frac{3}{10}bc^2x^{10}+\frac{3}{8}\left(b^2c+ac^2\right)x^8+\frac{1}{6}\left(b^3+6\,abc\right)x^6+\frac{3}{2}a^2bx^2+\frac{3}{4}\left(ab^2+a^2c\right)x^4+a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/12*c^3*x^12 + 3/10*b*c^2*x^10 + 3/8*(b^2*c + a*c^2)*x^8 + 1/6*(b^3 + 6*a*
b*c)*x^6 + 3/2*a^2*b*x^2 + 3/4*(a*b^2 + a^2*c)*x^4 + a^3*log(x)
```

giac [A] time = 0.16, size = 87, normalized size = 1.02

$$\frac{1}{12}c^3x^{12}+\frac{3}{10}bc^2x^{10}+\frac{3}{8}b^2cx^8+\frac{3}{8}ac^2x^8+\frac{1}{6}b^3x^6+abcx^6+\frac{3}{4}ab^2x^4+\frac{3}{4}a^2cx^4+\frac{3}{2}a^2bx^2+\frac{1}{2}a^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/12*c^3*x^12 + 3/10*b*c^2*x^10 + 3/8*b^2*c*x^8 + 3/8*a*c^2*x^8 + 1/6*b^3*x
^6 + a*b*c*x^6 + 3/4*a*b^2*x^4 + 3/4*a^2*c*x^4 + 3/2*a^2*b*x^2 + 1/2*a^3*lo
g(x^2)
```

maple [A] time = 0.00, size = 85, normalized size = 1.00

$$\frac{c^3x^{12}}{12}+\frac{3b\,c^2x^{10}}{10}+\frac{3a\,c^2x^8}{8}+\frac{3b^2c\,x^8}{8}+abc\,x^6+\frac{b^3x^6}{6}+\frac{3a^2c\,x^4}{4}+\frac{3a\,b^2x^4}{4}+\frac{3a^2b\,x^2}{2}+a^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x,x)
```

```
[Out] 1/12*c^3*x^12+3/10*b*c^2*x^10+3/8*x^8*a*c^2+3/8*x^8*b^2*c+x^6*a*b*c+1/6*b^3
*x^6+3/4*x^4*a^2*c+3/4*a*b^2*x^4+3/2*a^2*b*x^2+a^3*ln(x)
```

maxima [A] time = 1.39, size = 82, normalized size = 0.96

$$\frac{1}{12}\,c^3x^{12}+\frac{3}{10}\,bc^2x^{10}+\frac{3}{8}\left(b^2c+ac^2\right)x^8+\frac{1}{6}\left(b^3+6\,abc\right)x^6+\frac{3}{2}\,a^2bx^2+\frac{3}{4}\left(ab^2+a^2c\right)x^4+\frac{1}{2}\,a^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/12*c^3*x^12 + 3/10*b*c^2*x^10 + 3/8*(b^2*c + a*c^2)*x^8 + 1/6*(b^3 + 6*a*
b*c)*x^6 + 3/2*a^2*b*x^2 + 3/4*(a*b^2 + a^2*c)*x^4 + 1/2*a^3*log(x^2)
```

mupad [B] time = 0.03, size = 73, normalized size = 0.86

$$a^3\,\ln(x)+x^6\,\left(\frac{b^3}{6}+a\,c\,b\right)+\frac{c^3\,x^{12}}{12}+\frac{3\,a^2\,b\,x^2}{2}+\frac{3\,b\,c^2\,x^{10}}{10}+\frac{3\,a\,x^4\,\left(b^2+a\,c\right)}{4}+\frac{3\,c\,x^8\,\left(b^2+a\,c\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x,x)
```

```
[Out] a^3*log(x) + x^6*(b^3/6 + a*b*c) + (c^3*x^12)/12 + (3*a^2*b*x^2)/2 + (3*b*c
^2*x^10)/10 + (3*a*x^4*(a*c + b^2))/4 + (3*c*x^8*(a*c + b^2))/8
```

sympy [A] time = 0.22, size = 92, normalized size = 1.08

$$a^3\log(x)+\frac{3a^2bx^2}{2}+\frac{3bc^2x^{10}}{10}+\frac{c^3x^{12}}{12}+x^8\left(\frac{3ac^2}{8}+\frac{3b^2c}{8}\right)+x^6\left(abc+\frac{b^3}{6}\right)+x^4\left(\frac{3a^2c}{4}+\frac{3ab^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x,x)
```

```
[Out] a**3*log(x) + 3*a**2*b*x**2/2 + 3*b*c**2*x**10/10 + c**3*x**12/12 + x**8*(3
*a*c**2/8 + 3*b**2*c/8) + x**6*(a*b*c + b**3/6) + x**4*(3*a**2*c/4 + 3*a*b*
*2/4)
```

3.845 $\int \frac{(a+bx^2+cx^4)^3}{x^2}\,dx$

Optimal. Leaf size=80

$$-\frac{a^3}{x}+3a^2bx+\frac{3}{7}cx^7\left(ac+b^2\right)+\frac{1}{5}bx^5\left(6ac+b^2\right)+ax^3\left(ac+b^2\right)+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

```
[Out] -a^3/x+3*a^2*b*x+a*(a*c+b^2)*x^3+1/5*b*(6*a*c+b^2)*x^5+3/7*c*(a*c+b^2)*x^7+
1/3*b*c^2*x^9+1/11*c^3*x^11
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$3a^2bx-\frac{a^3}{x}+\frac{3}{7}cx^7\left(ac+b^2\right)+\frac{1}{5}bx^5\left(6ac+b^2\right)+ax^3\left(ac+b^2\right)+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a^2*b*x + a*(b^2 + a*c)*x^3 + (b*(b^2 + 6*a*c)*x^5)/5 + (3*c*(
b^2 + a*c)*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^3}{x^2}\,dx &= \int \left(3a^2b+\frac{a^3}{x^2}+3a\left(b^2+ac\right)x^2+b\left(b^2+6ac\right)x^4+3c\left(b^2+ac\right)x^6+3bc^2x^8+c^3x^{10}\right) \\ &= -\frac{a^3}{x}+3a^2bx+a\left(b^2+ac\right)x^3+\frac{1}{5}b\left(b^2+6ac\right)x^5+\frac{3}{7}c\left(b^2+ac\right)x^7+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.02, size = 80, normalized size = 1.00

$$-\frac{a^3}{x}+3a^2bx+\frac{3}{7}cx^7\left(ac+b^2\right)+\frac{1}{5}bx^5\left(6ac+b^2\right)+ax^3\left(ac+b^2\right)+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a^2*b*x + a*(b^2 + a*c)*x^3 + (b*(b^2 + 6*a*c)*x^5)/5 + (3*c*(
b^2 + a*c)*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

fricas [A] time = 0.99, size = 83, normalized size = 1.04

$$\frac{105\,c^3x^{12} + 385\,bc^2x^{10} + 495\,\left(b^2c + ac^2\right)x^8 + 231\,\left(b^3 + 6\,abc\right)x^6 + 3465\,a^2bx^2 + 1155\,\left(ab^2 + a^2c\right)x^4 - 1155\,a^3}{1155\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/1155*(105*c^3*x^12 + 385*b*c^2*x^10 + 495*(b^2*c + a*c^2)*x^8 + 231*(b^3
+ 6*a*b*c)*x^6 + 3465*a^2*b*x^2 + 1155*(a*b^2 + a^2*c)*x^4 - 1155*a^3)/x
```

giac [A] time = 0.15, size = 83, normalized size = 1.04

$$\frac{1}{11}\,c^3x^{11} + \frac{1}{3}\,bc^2x^9 + \frac{3}{7}\,b^2cx^7 + \frac{3}{7}\,ac^2x^7 + \frac{1}{5}\,b^3x^5 + \frac{6}{5}\,abcx^5 + ab^2x^3 + a^2cx^3 + 3\,a^2bx - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 3/7*a*c^2*x^7 + 1/5*b^3*x^5
 + 6/5*a*b*c*x^5 + a*b^2*x^3 + a^2*c*x^3 + 3*a^2*b*x - a^3/x
```

maple [A] time = 0.00, size = 84, normalized size = 1.05

$$\frac{c^3x^{11}}{11} + \frac{b\,c^2x^9}{3} + \frac{3a\,c^2x^7}{7} + \frac{3b^2c\,x^7}{7} + \frac{6abc\,x^5}{5} + \frac{b^3x^5}{5} + a^2c\,x^3 + a\,b^2x^3 + 3a^2bx - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^2,x)
```

```
[Out] 1/11*c^3*x^11+1/3*b*c^2*x^9+3/7*x^7*a*c^2+3/7*b^2*c*x^7+6/5*x^5*a*b*c+1/5*b
^3*x^5+x^3*a^2*c+a*b^2*x^3+3*a^2*b*x-a^3/x
```

maxima [A] time = 1.36, size = 78, normalized size = 0.98

$$\frac{1}{11}\,c^3x^{11} + \frac{1}{3}\,bc^2x^9 + \frac{3}{7}\,\left(b^2c + ac^2\right)x^7 + \frac{1}{5}\,\left(b^3 + 6\,abc\right)x^5 + 3\,a^2bx + \left(ab^2 + a^2c\right)x^3 - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*(b^2*c + a*c^2)*x^7 + 1/5*(b^3 + 6*a*b*
c)*x^5 + 3*a^2*b*x + (a*b^2 + a^2*c)*x^3 - a^3/x
```

mupad [B] time = 0.03, size = 73, normalized size = 0.91

$$x^5\left(\frac{b^3}{5}+\frac{6\,a\,c\,b}{5}\right)-\frac{a^3}{x}+\frac{c^3\,x^{11}}{11}+\frac{b\,c^2\,x^9}{3}+a\,x^3\,\left(b^2+a\,c\right)+\frac{3\,c\,x^7\,\left(b^2+a\,c\right)}{7}+3\,a^2\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x^2,x)
```

```
[Out] x^5*(b^3/5 + (6*a*b*c)/5) - a^3/x + (c^3*x^11)/11 + (b*c^2*x^9)/3 + a*x^3*(
a*c + b^2) + (3*c*x^7*(a*c + b^2))/7 + 3*a^2*b*x
```

sympy [A] time = 0.22, size = 82, normalized size = 1.02

$$-\frac{a^3}{x}+3a^2bx+\frac{bc^2x^9}{3}+\frac{c^3x^{11}}{11}+x^7\left(\frac{3ac^2}{7}+\frac{3b^2c}{7}\right)+x^5\left(\frac{6abc}{5}+\frac{b^3}{5}\right)+x^3\left(a^2c+ab^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**2,x)
```

```
[Out] -a**3/x + 3*a**2*b*x + b*c**2*x**9/3 + c**3*x**11/11 + x**7*(3*a*c**2/7 + 3
*b**2*c/7) + x**5*(6*a*b*c/5 + b**3/5) + x**3*(a**2*c + a*b**2)
```

3.846 $\int \frac{(a+bx^2+cx^4)^3}{x^3}\,dx$

Optimal. Leaf size=86

$$-\frac{a^3}{2x^2}+3a^2b\log(x)+\frac{1}{2}cx^6\left(ac+b^2\right)+\frac{1}{4}bx^4\left(6ac+b^2\right)+\frac{3}{2}ax^2\left(ac+b^2\right)+\frac{3}{8}bc^2x^8+\frac{c^3x^{10}}{10}$$

```
[Out] -1/2*a^3/x^2+3/2*a*(a*c+b^2)*x^2+1/4*b*(6*a*c+b^2)*x^4+1/2*c*(a*c+b^2)*x^6+
3/8*b*c^2*x^8+1/10*c^3*x^10+3*a^2*b*ln(x)
```

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$3a^2b\log(x)-\frac{a^3}{2x^2}+\frac{1}{2}cx^6\left(ac+b^2\right)+\frac{1}{4}bx^4\left(6ac+b^2\right)+\frac{3}{2}ax^2\left(ac+b^2\right)+\frac{3}{8}bc^2x^8+\frac{c^3x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^3,x]
```

```
[Out] -a^3/(2*x^2) + (3*a*(b^2 + a*c)*x^2)/2 + (b*(b^2 + 6*a*c)*x^4)/4 + (c*(b^2
+ a*c)*x^6)/2 + (3*b*c^2*x^8)/8 + (c^3*x^10)/10 + 3*a^2*b*Log[x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^3}{x^3}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx+cx^2)^3}{x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(3a(b^2+ac)+\frac{a^3}{x^2}+\frac{3a^2b}{x}+b(b^2+6ac)x+3c(b^2+ac)x^2+3bc^2x^3+c\right.\right.$$
$$= -\frac{a^3}{2x^2}+\frac{3}{2}a(b^2+ac)x^2+\frac{1}{4}b(b^2+6ac)x^4+\frac{1}{2}c(b^2+ac)x^6+\frac{3}{8}bc^2x^8+\frac{c^3x^{10}}{10}+3a^2b$$

Mathematica [A] time = 0.03, size = 78, normalized size = 0.91

$$\frac{1}{40}\left(-\frac{20a^3}{x^2}+120a^2b\log(x)+20cx^6(ac+b^2)+10bx^4(6ac+b^2)+60ax^2(ac+b^2)+15bc^2x^8+4c^3x^{10}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x^3,x]
```

```
[Out] ((-20*a^3)/x^2 + 60*a*(b^2 + a*c)*x^2 + 10*b*(b^2 + 6*a*c)*x^4 + 20*c*(b^2
+ a*c)*x^6 + 15*b*c^2*x^8 + 4*c^3*x^10 + 120*a^2*b*Log[x])/40
```

fricas [A] time = 0.98, size = 85, normalized size = 0.99

$$\frac{4c^3x^{12}+15bc^2x^{10}+20(b^2c+ac^2)x^8+10(b^3+6abc)x^6+120a^2bx^2\log(x)+60(ab^2+a^2c)x^4-20a^3}{40x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/40*(4*c^3*x^12 + 15*b*c^2*x^10 + 20*(b^2*c + a*c^2)*x^8 + 10*(b^3 + 6*a*b
*c)*x^6 + 120*a^2*b*x^2*log(x) + 60*(a*b^2 + a^2*c)*x^4 - 20*a^3)/x^2
```

giac [A] time = 0.16, size = 98, normalized size = 1.14

$$\frac{1}{10}c^3x^{10}+\frac{3}{8}bc^2x^8+\frac{1}{2}b^2cx^6+\frac{1}{2}ac^2x^6+\frac{1}{4}b^3x^4+\frac{3}{2}abcx^4+\frac{3}{2}ab^2x^2+\frac{3}{2}a^2cx^2+\frac{3}{2}a^2b\log(x^2)-\frac{3a^2bx^2+a^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/2*a*c^2*x^6 + 1/4*b^3*x^4
 + 3/2*a*b*c*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*c*x^2 + 3/2*a^2*b*log(x^2) - 1/2
*(3*a^2*b*x^2 + a^3)/x^2
```

maple [A] time = 0.01, size = 87, normalized size = 1.01

$$\frac{c^3x^{10}}{10}+\frac{3b\,c^2x^8}{8}+\frac{a\,c^2x^6}{2}+\frac{b^2c\,x^6}{2}+\frac{3abc\,x^4}{2}+\frac{b^3x^4}{4}+\frac{3a^2c\,x^2}{2}+\frac{3a\,b^2x^2}{2}+3a^2b\ln(x)-\frac{a^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^3,x)
```

```
[Out] 1/10*c^3*x^10+3/8*b*c^2*x^8+1/2*x^6*a*c^2+1/2*x^6*b^2*c+3/2*x^4*a*b*c+1/4*b
^3*x^4+3/2*x^2*a^2*c+3/2*a*b^2*x^2-1/2*a^3/x^2+3*a^2*b*ln(x)
```

maxima [A] time = 1.37, size = 82, normalized size = 0.95

$$\frac{1}{10}\,c^3x^{10}+\frac{3}{8}\,bc^2x^8+\frac{1}{2}\left(b^2c+ac^2\right)x^6+\frac{1}{4}\left(b^3+6\,abc\right)x^4+\frac{3}{2}\,a^2b\log\left(x^2\right)+\frac{3}{2}\left(ab^2+a^2c\right)x^2-\frac{a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*(b^2*c + a*c^2)*x^6 + 1/4*(b^3 + 6*a*b*
c)*x^4 + 3/2*a^2*b*log(x^2) + 3/2*(a*b^2 + a^2*c)*x^2 - 1/2*a^3/x^2
```

mupad [B] time = 0.04, size = 75, normalized size = 0.87

$$x^4\left(\frac{b^3}{4}+\frac{3\,a\,c\,b}{2}\right)-\frac{a^3}{2\,x^2}+\frac{c^3\,x^{10}}{10}+\frac{3\,b\,c^2\,x^8}{8}+3\,a^2\,b\,\ln(x)+\frac{3\,a\,x^2\,\left(b^2+a\,c\right)}{2}+\frac{c\,x^6\,\left(b^2+a\,c\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x^3,x)
```

```
[Out] x^4*(b^3/4 + (3*a*b*c)/2) - a^3/(2*x^2) + (c^3*x^10)/10 + (3*b*c^2*x^8)/8 +
 3*a^2*b*log(x) + (3*a*x^2*(a*c + b^2))/2 + (c*x^6*(a*c + b^2))/2
```

sympy [A] time = 0.27, size = 92, normalized size = 1.07

$$-\frac{a^3}{2x^2}+3a^2b\log\left(x\right)+\frac{3bc^2x^8}{8}+\frac{c^3x^{10}}{10}+x^6\left(\frac{ac^2}{2}+\frac{b^2c}{2}\right)+x^4\left(\frac{3abc}{2}+\frac{b^3}{4}\right)+x^2\left(\frac{3a^2c}{2}+\frac{3ab^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**3,x)
```

```
[Out] -a**3/(2*x**2) + 3*a**2*b*log(x) + 3*b*c**2*x**8/8 + c**3*x**10/10 + x**6*(
a*c**2/2 + b**2*c/2) + x**4*(3*a*b*c/2 + b**3/4) + x**2*(3*a**2*c/2 + 3*a*b
**2/2)
```

3.847 $\int \frac{(a+bx^2+cx^4)^3}{x^4} \, dx$

Optimal. Leaf size=83

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{x}+\frac{3}{5}cx^5\left(ac+b^2\right)+\frac{1}{3}bx^3\left(6ac+b^2\right)+3ax\left(ac+b^2\right)+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

```
[Out] -1/3*a^3/x^3-3*a^2*b/x+3*a*(a*c+b^2)*x+1/3*b*(6*a*c+b^2)*x^3+3/5*c*(a*c+b^2
)*x^5+3/7*b*c^2*x^7+1/9*c^3*x^9
```

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$-\frac{3a^2b}{x}-\frac{a^3}{3x^3}+\frac{3}{5}cx^5\left(ac+b^2\right)+\frac{1}{3}bx^3\left(6ac+b^2\right)+3ax\left(ac+b^2\right)+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^4,x]
```

```
[Out] -a^3/(3*x^3) - (3*a^2*b)/x + 3*a*(b^2 + a*c)*x + (b*(b^2 + 6*a*c)*x^3)/3 +
(3*c*(b^2 + a*c)*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^3}{x^4}\,dx &= \int \left(3a\left(b^2+ac\right)+\frac{a^3}{x^4}+\frac{3a^2b}{x^2}+b\left(b^2+6ac\right)x^2+3c\left(b^2+ac\right)x^4+3bc^2x^6+c^3x^8\right)dx\\ &= -\frac{a^3}{3x^3}-\frac{3a^2b}{x}+3a\left(b^2+ac\right)x+\frac{1}{3}b\left(b^2+6ac\right)x^3+\frac{3}{5}c\left(b^2+ac\right)x^5+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.03, size = 83, normalized size = 1.00

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{x}+\frac{3}{5}cx^5\left(ac+b^2\right)+\frac{1}{3}bx^3\left(6ac+b^2\right)+3ax\left(ac+b^2\right)+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x^4,x]
```

```
[Out] -1/3*a^3/x^3 - (3*a^2*b)/x + 3*a*(b^2 + a*c)*x + (b*(b^2 + 6*a*c)*x^3)/3 +
(3*c*(b^2 + a*c)*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

fricas [A] time = 0.88, size = 83, normalized size = 1.00

$$\frac{35\,c^3x^{12} + 135\,bc^2x^{10} + 189\,\left(b^2c + ac^2\right)x^8 + 105\,\left(b^3 + 6\,abc\right)x^6 - 945\,a^2bx^2 + 945\,\left(ab^2 + a^2c\right)x^4 - 105\,a^3}{315\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/315*(35*c^3*x^12 + 135*b*c^2*x^10 + 189*(b^2*c + a*c^2)*x^8 + 105*(b^3 +
6*a*b*c)*x^6 - 945*a^2*b*x^2 + 945*(a*b^2 + a^2*c)*x^4 - 105*a^3)/x^3
```

giac [A] time = 0.18, size = 84, normalized size = 1.01

$$\frac{1}{9}\,c^3x^9 + \frac{3}{7}\,bc^2x^7 + \frac{3}{5}\,b^2cx^5 + \frac{3}{5}\,ac^2x^5 + \frac{1}{3}\,b^3x^3 + 2\,abcx^3 + 3\,ab^2x + 3\,a^2cx - \frac{9\,a^2bx^2 + a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 3/5*a*c^2*x^5 + 1/3*b^3*x^3 +
 2*a*b*c*x^3 + 3*a*b^2*x + 3*a^2*c*x - 1/3*(9*a^2*b*x^2 + a^3)/x^3
```

maple [A] time = 0.01, size = 84, normalized size = 1.01

$$\frac{c^3x^9}{9} + \frac{3b\,c^2x^7}{7} + \frac{3a\,c^2x^5}{5} + \frac{3b^2c\,x^5}{5} + 2abc\,x^3 + \frac{b^3x^3}{3} + 3a^2cx + 3a\,b^2x - \frac{3a^2b}{x} - \frac{a^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^4,x)
```

```
[Out] 1/9*c^3*x^9+3/7*b*c^2*x^7+3/5*x^5*a*c^2+3/5*b^2*c*x^5+2*x^3*a*b*c+1/3*b^3*x
^3+3*a^2*c*x+3*a*b^2*x-3*a^2*b/x-1/3*a^3/x^3
```

maxima [A] time = 1.35, size = 80, normalized size = 0.96

$$\frac{1}{9}\,c^3x^9 + \frac{3}{7}\,bc^2x^7 + \frac{3}{5}\,\left(b^2c + ac^2\right)x^5 + \frac{1}{3}\,\left(b^3 + 6\,abc\right)x^3 + 3\,\left(ab^2 + a^2c\right)x - \frac{9\,a^2bx^2 + a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*(b^2*c + a*c^2)*x^5 + 1/3*(b^3 + 6*a*b*c)
*x^3 + 3*(a*b^2 + a^2*c)*x - 1/3*(9*a^2*b*x^2 + a^3)/x^3
```

mupad [B] time = 0.03, size = 77, normalized size = 0.93

$$x^3\left(\frac{b^3}{3}+2\,a\,c\,b\right)-\frac{\frac{a^3}{3}+3\,b\,a^2\,x^2}{x^3}+\frac{c^3\,x^9}{9}+\frac{3\,b\,c^2\,x^7}{7}+3\,a\,x\,\left(b^2+a\,c\right)+\frac{3\,c\,x^5\,\left(b^2+a\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x^4,x)
```

```
[Out] x^3*(b^3/3 + 2*a*b*c) - (a^3/3 + 3*a^2*b*x^2)/x^3 + (c^3*x^9)/9 + (3*b*c^2*
x^7)/7 + 3*a*x*(a*c + b^2) + (3*c*x^5*(a*c + b^2))/5
```

sympy [A] time = 0.24, size = 90, normalized size = 1.08

$$\frac{3bc^2x^7}{7}+\frac{c^3x^9}{9}+x^5\left(\frac{3ac^2}{5}+\frac{3b^2c}{5}\right)+x^3\left(2abc+\frac{b^3}{3}\right)+x\left(3a^2c+3ab^2\right)+\frac{-a^3-9a^2bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**4,x)
```

```
[Out] 3*b*c**2*x**7/7 + c**3*x**9/9 + x**5*(3*a*c**2/5 + 3*b**2*c/5) + x**3*(2*a*
b*c + b**3/3) + x*(3*a**2*c + 3*a*b**2) + (-a**3 - 9*a**2*b*x**2)/(3*x**3)
```

3.848 $\int \frac{x^7}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=100

$$\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)}{4c^3}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

```
[Out] -1/2*b*x^2/c^2+1/4*x^4/c+1/4*(-a*c+b^2)*ln(c*x^4+b*x^2+a)/c^3+1/2*b*(-3*a*c
+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1114, 701, 634, 618, 206, 628}

$$\frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)}{4c^3}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(b*x^2)/(2*c^2) + x^4/(4*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[
b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x^2 + c*x
^4])/(4*c^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^3}{a+bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{ab+\left(b^2-ac\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx,x,x^2\right)\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{\mathrm{Subst}\left(\int \frac{ab+\left(b^2-ac\right)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2c^2}\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}-\frac{\left(b\left(b^2-3ac\right)\right)\mathrm{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^3}+\frac{\left(b^2-ac\right)\mathrm{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx\right.}{4c^3}\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)}{4c^3}+\frac{\left(b\left(b^2-3ac\right)\right)\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+\right.}{2c^3}\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)}{4c^3}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 93, normalized size = 0.93

$$\frac{-\frac{2b\left(b^2-3ac\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)+cx^2\left(cx^2-2b\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2 + c*x^4),x]
```

```
[Out] (c*x^2*(-2*b + c*x^2) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 +
 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

fricas [A] time = 0.64, size = 313, normalized size = 3.13

$$\left[\frac{\left(b^2c^2-4\,ac^3\right)x^4-2\left(b^3c-4\,abc^2\right)x^2-\left(b^3-3\,abc\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\left(b^4-5\right.}{4\left(b^2c^3-4\,ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*c^2 - 4*a*c^3)*x^4 - 2*(b^3*c - 4*a*b*c^2)*x^2 - (b^3 - 3*a*b*c)
*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)
*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*lo
g(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4), 1/4*((b^2*c^2 - 4*a*c^3)*x^4 - 2
*(b^3*c - 4*a*b*c^2)*x^2 + 2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*
c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2
)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]
```

giac [A] time = 0.56, size = 92, normalized size = 0.92

$$\frac{cx^4-2\,bx^2}{4\,c^2}+\frac{\left(b^2-ac\right)\log\left(cx^4+bx^2+a\right)}{4\,c^3}-\frac{\left(b^3-3\,abc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(c*x^4 - 2*b*x^2)/c^2 + 1/4*(b^2 - a*c)*log(c*x^4 + b*x^2 + a)/c^3 - 1/
2*(b^3 - 3*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a
*c)*c^3)
```

maple [A] time = 0.01, size = 142, normalized size = 1.42

$$\frac{x^4}{4c}+\frac{3ab\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\,c^2}-\frac{b^3\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\,c^3}-\frac{b\,x^2}{2c^2}-\frac{a\ln\left(c\,x^4+b\,x^2+a\right)}{4c^2}+\frac{b^2\ln\left(c\,x^4+b\,x^2+a\right)}{4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4/c*x^4-1/2*b/c^2*x^2-1/4/c^2*ln(c*x^4+b*x^2+a)*a+1/4/c^3*ln(c*x^4+b*x^2+
a)*b^2+3/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b-
1/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.40, size = 842, normalized size = 8.42

$$\frac{x^4}{4\,c}-\frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(8\,a^2\,c^2-10\,a\,b^2\,c+2\,b^4\right)}{2\,\left(16\,a\,c^4-4\,b^2\,c^3\right)}-\frac{b\,x^2}{2\,c^2}+\frac{b\,\mathrm{atan}\left(\frac{2\,c^4\,\left(4\,a\,c-b^2\right)\left(\frac{\frac{b\,\left(3\,a\,c-b^2\right)\left(\frac{8\,a^2\,c^4-8\,a\,b^2\,c^3}{c^4}-\frac{8\,a\,c^2\,\left(8\,a^2\,c^2-10\,a\,b^2 \cdots\right)}{16\,a\,c^4-4\,b^2\,c^3}\cdots\right)}{8\,c^3\,\sqrt{4\,a\,c-b^2}}\cdots}{a\cdots}\cdots\right)}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^4/(4*c) - (log(a + b*x^2 + c*x^4)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(1
6*a*c^4 - 4*b^2*c^3)) - (b*x^2)/(2*c^2) + (b*atan((2*c^4*(4*a*c - b^2)*(((b
*(3*a*c - b^2)*((8*a^2*c^4 - 8*a*b^2*c^3)/c^4 - (8*a*c^2*(2*b^4 + 8*a^2*c^2
 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3)))/(8*c^3*(4*a*c - b^2)^(1/2)) - (a*b
*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(c*(4*a*c - b^2)^(1/2)*(16
*a*c^4 - 4*b^2*c^3)))/a - x^2*(((b*((6*b^3*c^3 - 10*a*b*c^4)/c^4 + (4*b*c^2
*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(3*a*c - b^2))/(
8*c^3*(4*a*c - b^2)^(1/2)) + (b^2*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b
^2*c))/(2*c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a + (b*((b^5 + 2*a
^2*b*c^2 - 3*a*b^3*c)/c^4 + (((6*b^3*c^3 - 10*a*b*c^4)/c^4 + (4*b*c^2*(2*b^
4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(2*b^4 + 8*a^2*c^2 - 1
0*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (b^3*(3*a*c - b^2)^2)/(2*c^4*(4*a*
c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) + (b*((((8*a^2*c^4 - 8*a*b^2*c^3)/c^
4 - (8*a*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(2*b
^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (a*b^4 + a^3*c^2
 - 2*a^2*b^2*c)/c^4 + (a*b^2*(3*a*c - b^2)^2)/(c^4*(4*a*c - b^2))))/(2*a*(4
*a*c - b^2)^(1/2)))))/(b^6 + 9*a^2*b^2*c^2 - 6*a*b^4*c))*(3*a*c - b^2))/(2*c
^3*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 2.91, size = 391, normalized size = 3.91

$$-\frac{bx^2}{2c^2}+\left(-\frac{b\sqrt{-4ac+b^2}\left(3ac-b^2\right)}{4c^3\left(4ac-b^2\right)}-\frac{ac-b^2}{4c^3}\right)\log\left(x^2+\frac{2a^2c-ab^2+8ac^3\left(-\frac{b\sqrt{-4ac+b^2}\left(3ac-b^2\right)}{4c^3\left(4ac-b^2\right)}-\frac{ac-b^2}{4c^3}\right)-2b^2c^2\left(-\cdots\right.}{3abc-b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2+a),x)
```

```
[Out] -b*x**2/(2*c**2) + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c -
b**2)) - (a*c - b**2)/(4*c**3))*log(x**2 + (2*a**2*c - a*b**2 + 8*a*c**3*(-
b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)
/(4*c**3)) - 2*b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*
a*c - b**2)) - (a*c - b**2)/(4*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c +
 b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3))*log(
x**2 + (2*a**2*c - a*b**2 + 8*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/
(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)) - 2*b**2*c**2*(b*sqrt(-4*a
*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)))
/(3*a*b*c - b**3)) + x**4/(4*c)
```

3.849 $\int \frac{x^5}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=81

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

```
[Out] 1/2*x^2/c-1/4*b*ln(c*x^4+b*x^2+a)/c^2-1/2*(-2*a*c+b^2)*arctanh((2*c*x^2+b)/
(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1114, 703, 634, 618, 206, 628}

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2 + c*x^4),x]
```

```
[Out] x^2/(2*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= \frac{x^2}{2c} + \frac{\operatorname{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx, x, x^2\right)}{2c}\\
&= \frac{x^2}{2c} - \frac{b\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2} + \frac{\left(b^2-2ac\right)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2}\\
&= \frac{x^2}{2c} - \frac{b\log\left(a+bx^2+cx^4\right)}{4c^2} - \frac{\left(b^2-2ac\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c^2}\\
&= \frac{x^2}{2c} - \frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}} - \frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 78, normalized size = 0.96

$$\frac{\frac{2\left(b^2-2ac\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - b\log\left(a+bx^2+cx^4\right) + 2cx^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*x^2 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - b*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

fricas [A] time = 0.87, size = 254, normalized size = 3.14

$$\left[\frac{2\left(b^{2}c-4\,ac^{2}\right)x^{2}-\left(b^{2}-2\,ac\right)\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,ac+\left(2\,cx^{2}+b\right)\sqrt{b^{2}-4\,ac}}{cx^{4}+bx^{2}+a}\right)-\left(b^{3}-4\,abc\right)\log\left(cx^{4}+bx^{2}\right.}{4\left(b^{2}c^{2}-4\,ac^{3}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c - 4*a*c^2)*x^2 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b
*x^2 + a)) - (b^3 - 4*a*b*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1
/4*(2*(b^2*c - 4*a*c^2)*x^2 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*x^4 +
 b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

giac [A] time = 0.62, size = 75, normalized size = 0.93

$$\frac{x^{2}}{2\,c}-\frac{b\log\left(cx^{4}+bx^{2}+a\right)}{4\,c^{2}}+\frac{\left(b^{2}-2\,ac\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\,\sqrt{-b^{2}+4\,ac}\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c - 1/4*b*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(b^2 - 2*a*c)*arctan((2*
c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

maple [A] time = 0.00, size = 111, normalized size = 1.37

$$-\frac{a\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}\;c}+\frac{b^{2}\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{2\sqrt{4ac-b^{2}}\;c^{2}}+\frac{x^{2}}{2c}-\frac{b\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{4c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/c*x^2-1/4*b*ln(c*x^4+b*x^2+a)/c^2-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
+b)/(4*a*c-b^2)^(1/2))*a+1/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*
c-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.75, size = 655, normalized size = 8.09

$$\frac{x^2}{2c}+\frac{\ln\left(cx^4+bx^2+a\right)\left(2b^3-8abc\right)}{2\left(16ac^3-4b^2c^2\right)}+\frac{\operatorname{atan}\left(\frac{2c^2\left(4ac-b^2\right)\left(\frac{\frac{\left(8ab+\frac{8ac^2\left(2b^3-8abc\right)}{16ac^3-4b^2c^2}\right)\left(2ac-b^2\right)}{8c^2\sqrt{4ac-b^2}}+\frac{a\left(2b^3-8abc\right)\left(2ac-b^2\right)}{\sqrt{4ac-b^2}\left(16ac^3-4b^2c^2\right)}}{a}-x^2\left(\frac{\left(2ac-b^2\right)\cdots}{\cdots}\right.\cdots\right)}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^2/(2*c) + (log(a + b*x^2 + c*x^4)*(2*b^3 - 8*a*b*c))/(2*(16*a*c^3 - 4*b^2
*c^2)) + (atan((2*c^2*(4*a*c - b^2)*((((8*a*b + (8*a*c^2*(2*b^3 - 8*a*b*c))
/(16*a*c^3 - 4*b^2*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (a*(2
*b^3 - 8*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2))
)/a - x^2*((((2*a*c - b^2)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b^3 - 8
*a*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(8*c^2*(4*a*c - b^2)^(1/2)) - (b*(2*b^3 -
 8*a*b*c)*(2*a*c - b^2))/(2*(4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2)))/a
+ (b*(((2*b^3 - 8*a*b*c)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b^3 - 8*a
```

```
*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(2*(16*a*c^3 - 4*b^2*c^2)) - (b^3 - a*b*c)/
c^2 + (b*(2*a*c - b^2)^2)/(2*c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))
) + (b*((a*b^2)/c^2 + ((2*b^3 - 8*a*b*c)*(8*a*b + (8*a*c^2*(2*b^3 - 8*a*b*c
))/(16*a*c^3 - 4*b^2*c^2)))/(2*(16*a*c^3 - 4*b^2*c^2)) - (a*(2*a*c - b^2)^2
)/(c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(b^4 + 4*a^2*c^2 - 4*a*
b^2*c))*(2*a*c - b^2))/(2*c^2*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 2.14, size = 316, normalized size = 3.90

$$\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4c^2\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{-ab-8ac^2\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4c^2\left(4ac-b^2\right)}\right)+2b^2c\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4c^2\left(4ac-b^2\right)}\right)}{2ac-b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] (-b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*
log(x**2 + (-a*b - 8*a*c**2*(-b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**
2)/(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-b/(4*c**2) - sqrt(-4*a*c + b**2)*(
2*a*c - b**2)/(4*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(4*c**2) + sq
rt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (-a*b
- 8*a*c**2*(-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c
 - b**2))) + 2*b**2*c*(-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*
c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**2/(2*c)
```

3.850 $\int \frac{x^3}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=63

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}$$

```
[Out] 1/4*ln(c*x^4+b*x^2+a)/c+1/2*b*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c/(-4
*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1114, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2 + c*x^4),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^2 + c*x^4]/(4*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx,x,x^2\right)\\ &= \frac{\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c}-\frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4c}\\ &= \frac{\log\left(a+bx^2+cx^4\right)}{4c}+\frac{b\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2c}\\ &= \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}\end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.98

$$\frac{\log\left(a+bx^2+cx^4\right)-\frac{2b\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 + b*x^2 + c*x^4])/(4*c)
```

fricas [A] time = 0.59, size = 197, normalized size = 3.13

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\left(b^2-4\,ac\right)\log\left(cx^4+bx^2+a\right)}{4\left(b^2c-4\,ac^2\right)},\frac{2\sqrt{-b^2+4\,ac}\,b\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x
^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^2 - 4*a*c)*log(c*x^4 +
 b*x^2 + a))/(b^2*c - 4*a*c^2), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^
2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^4 + b*x^2
+ a))/(b^2*c - 4*a*c^2)]
```

giac [A] time = 0.57, size = 59, normalized size = 0.94

$$-\frac{b \arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,c}+\frac{\log\left(cx^4+bx^2+a\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/
4*log(c*x^4 + b*x^2 + a)/c
```

maple [A] time = 0.00, size = 60, normalized size = 0.95

$$-\frac{b \arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;c}+\frac{\ln\left(c\,x^4+b\,x^2+a\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*ln(c*x^4+b*x^2+a)/c-1/2*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c
-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.26, size = 118, normalized size = 1.87

$$\frac{4\,a\,c\,\ln\left(c\,x^4+b\,x^2+a\right)}{16\,a\,c^2-4\,b^2\,c}-\frac{b^2\,\ln\left(c\,x^4+b\,x^2+a\right)}{16\,a\,c^2-4\,b^2\,c}-\frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x^2}{\sqrt{4\,a\,c-b^2}}\right)}{2\,c\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2 + c*x^4),x)
```

```
[Out] (4*a*c*log(a + b*x^2 + c*x^4))/(16*a*c^2 - 4*b^2*c) - (b^2*log(a + b*x^2 +
c*x^4))/(16*a*c^2 - 4*b^2*c) - (b*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x^2)/(4
*a*c - b^2)^(1/2)))/(2*c*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 1.03, size = 223, normalized size = 3.54

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)\log\left(x^2+\frac{-8ac\left(-\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)+2a+2b^2\left(-\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (-8*a*c*
(-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) + 2*a + 2*b**2*(-b*
sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)))/b) + (b*sqrt(-4*a*c +
b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (-8*a*c*(b*sqrt(-4*a*c + b
**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) + 2*a + 2*b**2*(b*sqrt(-4*a*c + b**2)/
(4*c*(4*a*c - b**2)) + 1/(4*c)))/b)
```

3.851 $\int \frac{x}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=36

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] -arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1107, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)\\ &= -\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)\\ &= -\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.08

$$\frac{\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2 + c*x^4),x]
```

```
[Out] ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]/Sqrt[-b^2 + 4*a*c]
```

fricas [A] time = 1.19, size = 129, normalized size = 3.58

$$\left[\frac{\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{2\,\sqrt{b^2-4\,ac}}, -\frac{\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4 + b*x^2 + a))/sqrt(b^2 - 4*a*c), -sqrt(-b^2 + 4*a*c)*arctan(-(
2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

giac [A] time = 0.57, size = 35, normalized size = 0.97

$$\frac{\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

maple [A] time = 0.00, size = 36, normalized size = 1.00

$$\frac{\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.27, size = 41, normalized size = 1.14

$$\frac{\operatorname{atan}\left(\frac{2\,a\,c\,x^{2}+a\,b}{a\,\sqrt{4\,a\,c-b^{2}}}\right)}{\sqrt{4\,a\,c-b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2 + c*x^4),x)
```

```
[Out] atan((a*b + 2*a*c*x^2)/(a*(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)^(1/2)
```

sympy [B] time = 0.59, size = 131, normalized size = 3.64

$$-\frac{\sqrt{-\frac{1}{4ac-b^{2}}}\log\left(x^{2}+\frac{-4ac\sqrt{-\frac{1}{4ac-b^{2}}}+b^{2}\sqrt{-\frac{1}{4ac-b^{2}}}+b}{2c}\right)}{2}+\frac{\sqrt{-\frac{1}{4ac-b^{2}}}\log\left(x^{2}+\frac{4ac\sqrt{-\frac{1}{4ac-b^{2}}}-b^{2}\sqrt{-\frac{1}{4ac-b^{2}}}+b}{2c}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2+a),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*
sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2 + sqrt(-1/(4*a*c - b**2))*log(x**2 +
(4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2
```

3.852 $\int \frac{1}{x(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=69

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

```
[Out] ln(x)/a-1/4*ln(c*x^4+b*x^2+a)/a+1/2*b*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2
))/a/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1114, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a + b*x^2 + c*x^4]/(4*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx,x,x^2\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2a}+\frac{\operatorname{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx,x,x^2\right)}{2a}\\
&= \frac{\log(x)}{a}-\frac{\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a}-\frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4a}\\
&= \frac{\log(x)}{a}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}+\frac{b\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2a}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 113, normalized size = 1.64

$$\frac{-\left(\sqrt{b^2-4ac}+b\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)+\left(b-\sqrt{b^2-4ac}\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)+4\log(x)\sqrt{b^2-}}{4a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*
a*c] + 2*c*x^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

fricas [A] time = 0.97, size = 223, normalized size = 3.23

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(b^2-4\,ac\right)\log\left(cx^4+bx^2+a\right)+4\left(b^2-4\,ac\right)\log(x)}{4\left(ab^2-4\,a^2c\right)},\ 2\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x
^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^2 - 4*a*c)*log(c*x^4 +
 b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/4*(2*sqrt(-b^2 +
 4*a*c)*b*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 -
4*a*c)*log(c*x^4 + b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

giac [A] time = 0.57, size = 68, normalized size = 0.99

$$-\frac{b\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,a}-\frac{\log\left(cx^4+bx^2+a\right)}{4\,a}+\frac{\log\left(x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/
4*log(c*x^4 + b*x^2 + a)/a + 1/2*log(x^2)/a
```

maple [A] time = 0.01, size = 66, normalized size = 0.96

$$-\frac{b\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;a}+\frac{\ln(x)}{a}-\frac{\ln\left(c\,x^4+b\,x^2+a\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4*ln(c*x^4+b*x^2+a)/a-1/2/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*
c-b^2)^(1/2))+ln(x)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.94, size = 1014, normalized size = 14.70

$$\frac{\ln(x)}{a}+\frac{\ln\left(c\,x^4+b\,x^2+a\right)\left(8\,a\,c-2\,b^2\right)}{2\left(4\,a\,b^2-16\,a^2\,c\right)}+\frac{b\,\mathrm{atan}\left(\frac{16\,a^3\,x^2\left(\frac{\left(3\,b^3-8\,a\,b\,c\right)\left(\frac{\left(8\,a\,c-2\,b^2\right)^2\left(10\,b\,c^3-\frac{\left(12\,b^3\,c^2-40\,a\,b\,c^3\right)\left(8\,a\,c-2\,b^2\right)}{2\left(4\,a\,b^2-16\,a^2\,c\right)}\right)}{4\left(4\,a\,b^2-16\,a^2\,c\right)^2}-\frac{b^2\left(10\,b\,c^3-\cdots\right.}{\cdots}\cdots\right)}{8\,a^3\,c^2\left(25\,a\,c-6\,b^2\cdots\right.}\cdots\right)}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] log(x)/a + (log(a + b*x^2 + c*x^4)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c)
) + (b*atan((16*a^3*x^2*(((3*b^3 - 8*a*b*c)*(((8*a*c - 2*b^2)^2*(10*b*c^3 -
 ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(4*
(4*a*b^2 - 16*a^2*c)^2) - (b^2*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*
c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(16*a^2*(4*a*c - b^2)) + (b^2*(12*b^
```

```
3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(16*a^2*(4*a*b^2 - 16*a^2*c)*(4*a*c -
b^2))))/(8*a^3*c^2*(25*a*c - 6*b^2)) - ((3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*(
(b^3*(12*b^3*c^2 - 40*a*b*c^3))/(64*a^3*(4*a*c - b^2)^(3/2)) - (b*(12*b^3*c
^2 - 40*a*b*c^3)*(8*a*c - 2*b^2)^2)/(16*a*(4*a*b^2 - 16*a^2*c)^2*(4*a*c - b
^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a
*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c -
b^2)^(1/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2)))*(4*a*c - b^
2)^(3/2))/(b^2*c^2) + (2*(3*b^3 - 8*a*b*c)*(4*a*c - b^2)^(3/2)*(((8*a*c - 2
*b^2)^2*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))/(4*a*b^2 - 16*a^2*c)))/(
4*(4*a*b^2 - 16*a^2*c)^2) - (b^2*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))
/(4*a*b^2 - 16*a^2*c)))/(16*a^2*(4*a*c - b^2)) + (b^4*c^2*(8*a*c - 2*b^2))/
(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2))))/(b^2*c^4*(25*a*c - 6*b^2)) - (2*
(4*a*c - b^2)*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b^5*c^2)/(16*a^2*(4*a*c -
 b^2)^(3/2)) - (b^3*c^2*(8*a*c - 2*b^2)^2)/(4*(4*a*b^2 - 16*a^2*c)^2*(4*a*c
 - b^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^
2))/(4*a*b^2 - 16*a^2*c)))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2))))
/(b^2*c^4*(25*a*c - 6*b^2))))/(2*a*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 4.67, size = 253, normalized size = 3.67

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)\log\left(x^2+\frac{-8a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)+2ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)-2ac+b^2}{bc}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (-8*a**2
*c*(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*b**2*(-b*s
qrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*c + b**2)/(b*c)) +
 (b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (-8*a**2
*c*(b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*b**2*(b*sqr
t(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*c + b**2)/(b*c)) + l
og(x)/a
```

3.853 $\int \frac{1}{x^3(a+bx^2+cx^4)} dx$

Optimal. Leaf size=89

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

```
[Out] -1/2/a/x^2-b*ln(x)/a^2+1/4*b*ln(c*x^4+b*x^2+a)/a^2-1/2*(-2*a*c+b^2)*arctanh
((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1114, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(2*a*x^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*
a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^2 + c*x^4])/(4*a^2
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx,x,x^2\right)$$
$$= -\frac{1}{2ax^2}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x\left(a+bx+cx^2\right)}\,dx,x,x^2\right)}{2a}$$
$$= -\frac{1}{2ax^2}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a\left(a+bx+cx^2\right)}\right)dx,x,x^2\right)}{2a}$$
$$= -\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx,x,x^2\right)}{2a^2}$$
$$= -\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a^2}+\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx\right.}{4a^2}$$
$$= -\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+\right.}{2a^2}$$
$$= -\frac{1}{2ax^2}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}$$

Mathematica [A] time = 0.13, size = 135, normalized size = 1.52

$$\frac{\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}-\frac{2a}{x^2}-4b\log(x)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a)/x^2 - 4*b*Log[x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b - Sqr
t[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((-b^2 + 2*a*c + b*Sqrt[b^2
- 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

fricas [A] time = 1.13, size = 293, normalized size = 3.29

$$\left[-\frac{\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\,x^2\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(b^3-4\,abc\right)x^2\log\left(cx^4+bx^2+a\right)+4\left(b^3\right.}{4\left(a^2b^2-4\,a^3c\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 + 2*b*c*x^2 + b^2
 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^3 - 4
*a*b*c)*x^2*log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2), -1/4*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*
a*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4
*a*b*c)*x^2*log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2)]
```

giac [A] time = 0.58, size = 94, normalized size = 1.06

$$\frac{b\log\left(cx^4+bx^2+a\right)}{4\,a^2}-\frac{b\log\left(x^2\right)}{2\,a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,a^2}+\frac{bx^2-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*b*log(c*x^4 + b*x^2 + a)/a^2 - 1/2*b*log(x^2)/a^2 + 1/2*(b^2 - 2*a*c)*a
rctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/2*(b*x
^2 - a)/(a^2*x^2)
```

maple [A] time = 0.01, size = 119, normalized size = 1.34

$$-\frac{c\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;a}+\frac{b^2\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;a^2}-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(c\,x^4+b\,x^2+a\right)}{4a^2}-\frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*b*ln(c*x^4+b*x^2+a)/a^2-1/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c
-b^2)^(1/2))*c+1/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/
2))*b^2-1/2/a/x^2-b*ln(x)/a^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 5.89, size = 2033, normalized size = 22.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (atan((16*a^6*x^2*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(c^5/a^3 + ((2*b^3 - 8*a*
b*c)*((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3
+ ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*
a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) - ((((2
*a*c - b^2)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*
b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b
^2)^(1/2)) + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^
2))/(8*a^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(2*a*c - b^2))/(4*a
^2*(4*a*c - b^2)^(1/2)) - ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2
)*(2*a*c - b^2)^2)/(32*a^7*(4*a*c - b^2)*(16*a^3*c - 4*a^2*b^2))))/(8*a^3*c
^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + ((((2*b^3 - 8*a*b*c)*(((2*a*c - b^2)*(
(20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^
3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)) +
((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2))/(8*a^5*(4
*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) - ((
40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2)^3)/(64*a^9*(4*a*c - b^2)^(3/2)
) + (((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3
+ ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*
a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*a*c - b^2))/(4*a^2*(4*a*c - b^2)
^(1/2)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2
)*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)))*(4*a*c - b^2)^(3/2))/(4*a^2*c^4 + b^4*c^
2 - 4*a*b^2*c^3) - (2*a^3*(4*a*c - b^2)*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c)
*(((2*b^3 - 8*a*b*c)*((((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a^3 + (2*a*b^2*c^2*(2
*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*a*c - b^2))/(4*a^2*(4*a*c - b^2
)^(1/2)) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2))/(2*a*(4*a*c - b^2)^(1/
2)*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) + ((2*a*c - b^2)*((
a^2*c^4 - 4*a*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*((4*a^3*b*c^3 - 4*a^2*b^3*c
^2)/a^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2)))/(2*(16*a
^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)) - (b^2*c^2*(2*a*c - b^2)^3
)/(16*a^5*(4*a*c - b^2)^(3/2))))/(c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)*(4*a^2
*c^4 + b^4*c^2 - 4*a*b^2*c^3)) + (2*a^3*(4*a*c - b^2)^(3/2)*(3*b^4 + a^2*c^
2 - 9*a*b^2*c)*((b*c^4)/a^3 - ((2*b^3 - 8*a*b*c)*((a^2*c^4 - 4*a*b^2*c^3)/a
^3 + ((2*b^3 - 8*a*b*c)*((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a^3 + (2*a*b^2*c^2*(
```

```
2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2)))/(2*(16*a^3*c - 4*a^2*b^2))))/(2*
(16*a^3*c - 4*a^2*b^2)) + ((2*a*c - b^2)*((((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a
^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*a*c - b^2))
/(4*a^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2))/(2
*a*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)
) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2)^2)/(8*a^3*(4*a*c - b^2)*(16*a^
3*c - 4*a^2*b^2))))/(c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)*(4*a^2*c^4 + b^4*c^
2 - 4*a*b^2*c^3)))*(2*a*c - b^2))/(2*a^2*(4*a*c - b^2)^(1/2)) - (b*log(x))/
a^2 - (log(a + b*x^2 + c*x^4)*(2*b^3 - 8*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2))
 - 1/(2*a*x^2)
```

sympy [B] time = 137.80, size = 345, normalized size = 3.88

$$\left(\frac{b}{4a^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2\left(4ac-b^2\right)}\right)\log\left(x^2 + \frac{-8a^3c\left(\frac{b}{4a^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2\left(4ac-b^2\right)}\right) + 2a^2b^2\left(\frac{b}{4a^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2\left(4ac-b^2\right)}\right) + 3abc}{2ac^2 - b^2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] (b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)))*l
og(x**2 + (-8*a**3*c*(b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a*
*2*(4*a*c - b**2))) + 2*a**2*b**2*(b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c
- b**2)/(4*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) + (
b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)))*lo
g(x**2 + (-8*a**3*c*(b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**
2*(4*a*c - b**2))) + 2*a**2*b**2*(b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c -
 b**2)/(4*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) - 1/
(2*a*x**2) - b*log(x)/a**2
```

3.854 $\int \frac{1}{x^5(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=114

$$\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}}-\frac{(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{\log(x)\left(b^2-ac\right)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

```
[Out] -1/4/a/x^4+1/2*b/a^2/x^2+(-a*c+b^2)*ln(x)/a^3-1/4*(-a*c+b^2)*ln(c*x^4+b*x^2
+a)/a^3+1/2*b*(-3*a*c+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*
a*c+b^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1114, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}}+\frac{\log(x)\left(b^2-ac\right)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(4*a*x^4) + b/(2*a^2*x^2) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[
b^2 - 4*a*c]])/(2*a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])/a^3 - ((b^2
 - a*c)*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\left(a+bx+cx^2\right)}\,dx,x,x^2\right)$$
$$= -\frac{1}{4ax^4} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x^2\left(a+bx+cx^2\right)}\,dx,x,x^2\right)}{2a}$$
$$= -\frac{1}{4ax^4} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax^2}+\frac{b^2-ac}{a^2x}+\frac{-b\left(b^2-2ac\right)-c\left(b^2-ac\right)x}{a^2\left(a+bx+cx^2\right)}\right)\,dx,x,x^2\right)}{2a}$$
$$= -\frac{1}{4ax^4} + \frac{b}{2a^2x^2} + \frac{\left(b^2-ac\right)\log(x)}{a^3} + \frac{\text{Subst}\left(\int \frac{-b\left(b^2-2ac\right)-c\left(b^2-ac\right)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2a^3}$$
$$= -\frac{1}{4ax^4} + \frac{b}{2a^2x^2} + \frac{\left(b^2-ac\right)\log(x)}{a^3} - \frac{\left(b\left(b^2-3ac\right)\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4a^3} - \frac{(b}{}$$
$$= -\frac{1}{4ax^4} + \frac{b}{2a^2x^2} + \frac{\left(b^2-ac\right)\log(x)}{a^3} - \frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)}{4a^3} + \frac{\left(b\left(b^2-3ac\right)\right)}{}$$
$$= -\frac{1}{4ax^4} + \frac{b}{2a^2x^2} + \frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}} + \frac{\left(b^2-ac\right)\log(x)}{a^3} - \frac{\left(b^2-ac\right)\log}{4}$$

Mathematica [A] time = 0.23, size = 188, normalized size = 1.65

$$\frac{-\frac{a^2}{x^4}+4\log(x)\left(b^2-ac\right)-\frac{\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}-3abc+b^3\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(-b^2\sqrt{b^2-4ac}+ac\sqrt{b^2-4ac}-3abc+b^3\right)\log\left(\sqrt{b}\right.}{\sqrt{b^2-4ac}}}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-(a^2/x^4) + (2*a*b)/x^2 + 4*(b^2 - a*c)*Log[x] - ((b^3 - 3*a*b*c + b^2*Sq
rt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^
2])/Sqrt[b^2 - 4*a*c] + ((b^3 - 3*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + a*c*Sqrt[
b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a
^3)
```

fricas [A] time = 0.78, size = 374, normalized size = 3.28

$$\left[-\frac{\left(b^3-3\,abc\right)\sqrt{b^2-4\,ac}\,x^4\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\left(b^4-5\,ab^2c+4\,a^2c^2\right)x^4\log\left(cx^4+bx^2+\right.}{4\left(a^3b^2-4\,a^4c\right)x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^4*log((2*c^2*x^4 + 2*b*c*x^2 + b
^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^4 -
 5*a*b^2*c + 4*a^2*c^2)*x^4*log(c*x^4 + b*x^2 + a) - 4*(b^4 - 5*a*b^2*c + 4
*a^2*c^2)*x^4*log(x) + a^2*b^2 - 4*a^3*c - 2*(a*b^3 - 4*a^2*b*c)*x^2)/((a^3
*b^2 - 4*a^4*c)*x^4), 1/4*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^4*arctan(
-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^4 - 5*a*b^2*c + 4*a^2
*c^2)*x^4*log(c*x^4 + b*x^2 + a) + 4*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^4*log(
x) - a^2*b^2 + 4*a^3*c + 2*(a*b^3 - 4*a^2*b*c)*x^2)/((a^3*b^2 - 4*a^4*c)*x^
4)]
```

giac [A] time = 0.55, size = 126, normalized size = 1.11

$$-\frac{\left(b^2-ac\right)\log\left(cx^4+bx^2+a\right)}{4\,a^3}+\frac{\left(b^2-ac\right)\log\left(x^2\right)}{2\,a^3}-\frac{\left(b^3-3\,abc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,a^3}-\frac{3\,b^2x^4-3\,acx^4-2\,abx^2+a^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(b^2 - a*c)*log(c*x^4 + b*x^2 + a)/a^3 + 1/2*(b^2 - a*c)*log(x^2)/a^3
- 1/2*(b^3 - 3*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 +
 4*a*c)*a^3) - 1/4*(3*b^2*x^4 - 3*a*c*x^4 - 2*a*b*x^2 + a^2)/(a^3*x^4)
```

maple [A] time = 0.01, size = 159, normalized size = 1.39

$$\frac{3bc\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\,a^2}-\frac{b^3\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\,a^3}-\frac{c\ln(x)}{a^2}+\frac{c\ln\left(c\,x^4+b\,x^2+a\right)}{4a^2}+\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln\left(c\,x^4+b\,x^2+a\right)}{4a^3}+\frac{b}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4/a^2*c*ln(c*x^4+b*x^2+a)-1/4/a^3*ln(c*x^4+b*x^2+a)*b^2+3/2/a^2/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*c-1/2/a^3/(4*a*c-b^2)^(1/2
)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3-1/4/a/x^4-1/a^2*ln(x)*c+1/a^3*l
n(x)*b^2+1/2*b/a^2/x^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 6.37, size = 2451, normalized size = 21.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(a + b*x^2 + c*x^4)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*
a^3*b^2)) - (1/(4*a) - (b*x^2)/(2*a^2))/x^4 - (log(x)*(a*c - b^2))/a^3 + (b
*atan((2*a^6*(4*a*c - b^2)*((((b*(3*a*c - b^2)*((4*a^4*b^4*c^2 - 8*a^5*b^2*
c^3)/a^6 - (2*a*b^2*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a^4*c - 4*a^3
*b^2)))/(4*a^3*(4*a*c - b^2)^(1/2)) - (b^3*c^2*(3*a*c - b^2)*(2*b^4 + 8*a^2
*c^2 - 10*a*b^2*c))/(2*a^2*(4*a*c - b^2)^(1/2)*(16*a^4*c - 4*a^3*b^2)))*(2*
b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*b^2)) + (b^5*c^2*(3*a*c
 - b^2)^3)/(16*a^8*(4*a*c - b^2)^(3/2)) + (b*(3*a*c - b^2)*((4*a^2*b^4*c^3
- 5*a^3*b^2*c^4)/a^6 + (((4*a^4*b^4*c^2 - 8*a^5*b^2*c^3)/a^6 - (2*a*b^2*c^2
*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a^4*c - 4*a^3*b^2))*(2*b^4 + 8*a^2*c
^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*b^2))))/(4*a^3*(4*a*c - b^2)^(1/2)))
*(3*b^6 - 10*a^3*c^3 + 27*a^2*b^2*c^2 - 18*a*b^4*c))/(c^2*(b^6*c^2 - 6*a*b^
4*c^3 + 9*a^2*b^2*c^4)*(6*b^6 - 25*a^3*c^3 + 54*a^2*b^2*c^2 - 36*a*b^4*c))
- (16*a^9*x^2*((3*b*(b^4 + 3*a^2*c^2 - 4*a*b^2*c)*((((5*a^3*b*c^5 - 6*a^2*b
^3*c^4)/a^6 - (((10*a^5*b*c^4 + 2*a^4*b^3*c^3)/a^6 + ((40*a^7*b*c^3 - 12*a^
6*b^3*c^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*a^6*(16*a^4*c - 4*a^3*b^2))
)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*b^2)))*(2*b^4 + 8*
a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*b^2)) - (b^3*c^5)/a^6 + (b*(3*a
*c - b^2)*((b*((10*a^5*b*c^4 + 2*a^4*b^3*c^3)/a^6 + ((40*a^7*b*c^3 - 12*a^6
*b^3*c^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*a^6*(16*a^4*c - 4*a^3*b^2)))
*(3*a*c - b^2))/(4*a^3*(4*a*c - b^2)^(1/2)) + (b*(40*a^7*b*c^3 - 12*a^6*b^3
*c^2)*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(8*a^9*(4*a*c - b^2)^
(1/2)*(16*a^4*c - 4*a^3*b^2))))/(4*a^3*(4*a*c - b^2)^(1/2)) + (b^2*(40*a^7*
b*c^3 - 12*a^6*b^3*c^2)*(3*a*c - b^2)^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(
32*a^12*(4*a*c - b^2)*(16*a^4*c - 4*a^3*b^2))))/(8*a^3*c^2*(6*b^6 - 25*a^3*
c^3 + 54*a^2*b^2*c^2 - 36*a*b^4*c)) + (((b^3*(40*a^7*b*c^3 - 12*a^6*b^3*c^2
)*(3*a*c - b^2)^3)/(64*a^15*(4*a*c - b^2)^(3/2)) - (((b*((10*a^5*b*c^4 + 2*
a^4*b^3*c^3)/a^6 + ((40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(2*b^4 + 8*a^2*c^2 - 10
*a*b^2*c))/(2*a^6*(16*a^4*c - 4*a^3*b^2)))*(3*a*c - b^2))/(4*a^3*(4*a*c - b
^2)^(1/2)) + (b*(40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(3*a*c - b^2)*(2*b^4 + 8*a^
2*c^2 - 10*a*b^2*c))/(8*a^9*(4*a*c - b^2)^(1/2)*(16*a^4*c - 4*a^3*b^2)))*(2
```

```
*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*b^2)) + (b*((5*a^3*b*c
^5 - 6*a^2*b^3*c^4)/a^6 - (((10*a^5*b*c^4 + 2*a^4*b^3*c^3)/a^6 + ((40*a^7*b
*c^3 - 12*a^6*b^3*c^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*a^6*(16*a^4*c -
 4*a^3*b^2)))*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*b^2)))
*(3*a*c - b^2))/(4*a^3*(4*a*c - b^2)^(1/2)))*(3*b^6 - 10*a^3*c^3 + 27*a^2*b
^2*c^2 - 18*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(6*b^6 - 25*a^3*c^3 +
54*a^2*b^2*c^2 - 36*a*b^4*c)))*(4*a*c - b^2)^(3/2))/(b^6*c^2 - 6*a*b^4*c^3
+ 9*a^2*b^2*c^4) + (6*a^6*b*(4*a*c - b^2)^(3/2)*(b^4 + 3*a^2*c^2 - 4*a*b^2*
c)*((b^4*c^4 - a*b^2*c^5)/a^6 + (((4*a^2*b^4*c^3 - 5*a^3*b^2*c^4)/a^6 + (((
4*a^4*b^4*c^2 - 8*a^5*b^2*c^3)/a^6 - (2*a*b^2*c^2*(2*b^4 + 8*a^2*c^2 - 10*a
*b^2*c))/(16*a^4*c - 4*a^3*b^2))*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a
^4*c - 4*a^3*b^2)))*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*
b^2)) - (b*((b*(3*a*c - b^2)*((4*a^4*b^4*c^2 - 8*a^5*b^2*c^3)/a^6 - (2*a*b^
2*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a^4*c - 4*a^3*b^2)))/(4*a^3*(4*
a*c - b^2)^(1/2)) - (b^3*c^2*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c)
)/(2*a^2*(4*a*c - b^2)^(1/2)*(16*a^4*c - 4*a^3*b^2)))*(3*a*c - b^2))/(4*a^3
*(4*a*c - b^2)^(1/2)) + (b^4*c^2*(3*a*c - b^2)^2*(2*b^4 + 8*a^2*c^2 - 10*a*
b^2*c))/(8*a^5*(4*a*c - b^2)*(16*a^4*c - 4*a^3*b^2))))/(c^2*(b^6*c^2 - 6*a*
b^4*c^3 + 9*a^2*b^2*c^4)*(6*b^6 - 25*a^3*c^3 + 54*a^2*b^2*c^2 - 36*a*b^4*c)
))*(3*a*c - b^2))/(2*a^3*(4*a*c - b^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.855 $\int \frac{x^6}{a+bx^2+cx^4} dx$

Optimal. Leaf size=203

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

```
[Out] -b*x/c^2+1/3*x^3/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2
))*(b^2-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b-(-4*a*c+b
^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))
*(b^2-a*c+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b+(-4*a*c+b^2
)^(1/2))^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1122, 1279, 1166, 205}

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2
)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*
c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
```

```
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^6}{a+bx^2+cx^4}\,dx = \frac{x^3}{3c}-\frac{\int \frac{x^2\left(3a+3bx^2\right)}{a+bx^2+cx^4}\,dx}{3c}$$

$$= -\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{\int \frac{3ab+3\left(b^2-ac\right)x^2}{a+bx^2+cx^4}\,dx}{3c^2}$$

$$= -\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{\left(b^2-ac-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2}+\frac{\left(b^2-ac+\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}}}{2c^2}$$

$$= -\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{\left(b^2-ac-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b^2-ac+\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{b+\sqrt{b}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.15, size = 250, normalized size = 1.23

$$\frac{\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}+3abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}-3abc+b^3\right)\tan}{\sqrt{2}\,c^{5/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((-b^3 + 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3 - 3
*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sq
rt[b + Sqrt[b^2 - 4*a*c]])
```

fricas [B] time = 1.18, size = 1564, normalized size = 7.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*c*x^3 - 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*
c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b
^2*c + a^4*c^2)*x + sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c
^3 - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4
*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^
3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*
c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))
) + 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c
^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*
c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*
c^2)*x - sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 - (b^4*c
^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2
*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 3*sqrt(
1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*
c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x + sq
rt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5 - 6*a*b^
```

```
2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b
^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) + 3*sqrt(1/2)*c^2*sq
rt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^
2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x - sqrt(1/2)*(b^
7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5 - 6*a*b^2*c^6 + 8*a
^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*
a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b
^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 6*b*x)/c^2
```

giac [B] time = 1.01, size = 2457, normalized size = 12.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5 -
 (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 -
 4*a*c)*a^2*c^4)*c^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2
 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3
*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 -
```

```
 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*abs(c))*arctan(2*sq
rt(1/2)*x/sqrt((b*c^3 + sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^4 - 8*a^2*
b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^7)*c
^2) + 1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*
c^5 - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(
b^2 - 4*a*c)*a^2*c^4)*c^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^
2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*
c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*abs(c))*arctan
(2*sqrt(1/2)*x/sqrt((b*c^3 - sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^4 - 8
*a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c
^7)*c^2) + 1/3*(c^2*x^3 - 3*b*c*x)/c^3
```

maple [B] time = 0.05, size = 467, normalized size = 2.30

$$-\frac{3\sqrt{2}\, ab \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}\; c}-\frac{3\sqrt{2}\, ab \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}\; c}+\frac{\sqrt{2}\, b^{3} \operatorname{arctanh}\left(\frac{\sqrt{2}}{\sqrt{\left(-b+\sqrt{-}\right.}}\right.}{2\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4a\right.}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3/c*x^3-b/c^2*x+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x
*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/2/c^2*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*b^2-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b+1/2/c^2/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a+1/2/c^2*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*b^2-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b+1/2/c^2/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{cx^3 - 3\,bx}{3\,c^2} - \frac{-\int \frac{(b^2-ac)x^2+ab}{cx^4+bx^2+a}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^3 - 3*b*x)/c^2 - integrate(-((b^2 - a*c)*x^2 + a*b)/(c*x^4 + b*x^2
 + a), x)/c^2
```

mupad [B] time = 5.01, size = 4127, normalized size = 20.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^3/(3*c) - atan(((((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16
*a*b*c^6)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3
*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7
 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(
-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c^3 + 9*
a^2*b^2*c^2 - 6*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a
^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c -
3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)
))^(1/2)*1i - (((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b
*c^6)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2
```

```
 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7 + b
^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*
b^2*c^2 - 6*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b
*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*
b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(
1/2)*1i)/((((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6
)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a
^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^
(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7 + b^4*(
-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c
- b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^
2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*
c^2 - 6*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3
 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)
 + (((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(b^
7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7 + b^4*(-(4*a*c
 - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^
3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 +
 b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*c^2 - 6
*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a
^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(
a^4*c - a^3*b^2))/c^3))*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^
3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2
*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2
)*2i - atan(((((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*
c^6)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 +
 a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3
)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*((b^4*(-(4*a
*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2
*c^2 - 6*a*b^4*c))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3
 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)
*1i - (((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*((
b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^
2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*((b^4*(-(4*a*c - b^
```

```
2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7
+ b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*c^2 -
6*a*b^4*c))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a
^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((
((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*((b^4*(-(
4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(
1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/
2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c
^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*c^2 - 6*a*b^4
*c))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*
c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((4*a*b^3*
c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*((b^4*(-(4*a*c - b^
2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7
+ b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7
 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b
^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b
^2*c^6)))^(1/2) + (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*c^2 - 6*a*b^4*c))/c^3)*
((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*
c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(a^4*c - a^3*b^2))
/c^3))*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2
 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i - (b*x)/c^2
```

sympy [A] time = 5.16, size = 194, normalized size = 0.96

$$-\frac{bx}{c^2}+\text{RootSum}\left(t^4\left(256a^2c^7-128ab^2c^6+16b^4c^5\right)+t^2\left(-80a^3bc^3+100a^2b^3c^2-36ab^5c+4b^7\right)+a^5,\left(t\mapsto t\log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2+a),x)
```

```
[Out] -b*x/c**2 + RootSum(_t**4*(256*a**2*c**7 - 128*a*b**2*c**6 + 16*b**4*c**5)
+ _t**2*(-80*a**3*b*c**3 + 100*a**2*b**3*c**2 - 36*a*b**5*c + 4*b**7) + a**
5, Lambda(_t, _t*log(x + (-64*_t**3*a**2*c**7 + 48*_t**3*a*b**2*c**6 - 8*_t
**3*b**4*c**5 + 14*_t*a**3*b*c**3 - 28*_t*a**2*b**3*c**2 + 14*_t*a*b**5*c -
 2*_t*b**7)/(a**4*c**2 - 3*a**3*b**2*c + a**2*b**4)))) + x**3/(3*c)
```

3.856 $\int \frac{x^4}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=179

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

```
[Out] x/c-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c-b^
2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arc
tan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*a*c
+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1122, 1166, 205}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2 + c*x^4),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
```

```
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{a+bx^2+cx^4}\,dx &= \frac{x}{c}-\frac{\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx}{c}\\ &= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}\\ &= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 202, normalized size = 1.13

$$-\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\,\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2 + c*x^4),x]
```

```
[Out] x/c - ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) - ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]])
```

fricas [B] time = 0.96, size = 1059, normalized size = 5.92

$$-\frac{\sqrt{\frac{1}{2}}\, c \sqrt{-\frac{b^{3}-3\,abc+\left(b^{2}c^{3}-4\,ac^{4}\right)\sqrt{\frac{b^{4}-2\,ab^{2}c+a^{2}c^{2}}{b^{2}c^{6}-4\,ac^{7}}}}{b^{2}c^{3}-4\,ac^{4}}}\, \log\left(-2\left(ab^{2}-a^{2}c\right)x+\sqrt{\frac{1}{2}}\left(b^{4}-5\,ab^{2}c+4\,a^{2}c^{2}-\left(b^{3}c^{3}-4\,abc^{4}\right)\sqrt{\frac{b^{4}}{}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2
 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4
)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b
*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^
7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 -
4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 -
4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2
 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^
7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-(b^3
- 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4
- 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) -
sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2
*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (
b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(
b^2*c^3 - 4*a*c^4))) - 2*x)/c
```

giac [B] time = 0.97, size = 2109, normalized size = 11.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] x/c + 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
```

(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt((b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt((b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*

```
c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2)
```

maple [B] time = 0.03, size = 343, normalized size = 1.92

$$\frac{\sqrt{2}\,a \operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,a \arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,b^2 \operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/c*x+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*a-1/2/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2-1/2/c*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*b+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a-1/2/(-4*a*c+b^2)^(1/2)/c
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{c}-\frac{\left(\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,b^4-8\,\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,ab^2c-2\,\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,b^3c-2\,b^4c+16\,\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,a^2c^2+8\,\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,abc^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^2 + a)/(c*x^4 + b*x^2 + a), x)/c
```

mupad [B] time = 0.65, size = 3026, normalized size = 16.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2 + c*x^4),x)
```

[Out] x/c - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*
(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4
*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(
-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x
*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4
*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a
*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^3 - 4
*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c
))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)
 + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 +
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2
*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 -
8*a*b^2*c^4)))^(1/2) + (2*a^2*b)/c))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c -
 (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a
^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c
^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 -
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2
*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4
*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^

```
3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)
))^(1/2)*1i)/((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4
)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)
*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(
4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2
*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*
b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b
^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*a^2*b)/c))*(-(b^5
- b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i
```

sympy [A] time = 5.51, size = 129, normalized size = 0.72

$$\operatorname{RootSum}\left(t^4\left(256a^2c^5-128ab^2c^4+16b^4c^3\right)+t^2\left(48a^2bc^2-28ab^3c+4b^5\right)+a^3,\left(t\mapsto t\log\left(x+\frac{32t^3abc^4-8t^3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(48*
a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + a**3, Lambda(_t, _t*log(x + (32*_t**3
*a*b*c**4 - 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b**4)
/(a**2*c - a*b**2)))) + x/c
```

3.857 $\int \frac{x^2}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b-(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/
2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1130, 205}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (Sqrt[b + Sqrt[b^2 - 4*
a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqr
t[c]*Sqrt[b^2 - 4*a*c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\int \frac{x^2}{a+bx^2+cx^4}\,dx = -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2\cdots}}$$

$$= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.08, size = 165, normalized size = 1.10

$$\frac{\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*
a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (
Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

fricas [B] time = 0.79, size = 559, normalized size = 3.73

$$\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}\left(b^2c-4\,ac^2\right)\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}\left(b^2\cdots\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sq
rt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) - 1/
2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c -
4*a*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqr
```

```
t(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) - 1/2
*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4
*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(
b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) + 1/2*s
qrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a
*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b
^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x)
```

giac [B] time = 1.05, size = 503, normalized size = 3.35

$$-\frac{\left(2\,b^2c^2-8\,ac^3-\sqrt{2}\,\sqrt{b^2-4\,ac}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,b^2+4\,\sqrt{2}\,\sqrt{b^2-4\,ac}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,ac+2\,\sqrt{2}\,\sqrt{b^2-4\,a}\right.}{2\left(b^4-8\,ab^2c-2\,b^3c+16\,a^2c^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a
*c)*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((b^4 - 8*a*
b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*abs(c)) + 1/2
*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*
c^2)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((b^4 - 8*a*b^2*
c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*abs(c))
```

maple [A] time = 0.02, size = 208, normalized size = 1.39

$$\frac{\sqrt{2}\,b\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\;\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)
```

```
^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b+1
/2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 4.46, size = 416, normalized size = 2.77

$$-2\,\mathrm{atanh}\left(\frac{\left(x\,\left(4\,a\,c^2-2\,b^2\,c\right)+\frac{x\,\left(8\,b^3\,c^2-32\,a\,b\,c^3\right)\,\left(b^3+\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c\right)}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}\right)\,\sqrt{-\frac{b^3+\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}}{a\,c}\right)\,\sqrt{-\frac{b^3+\sqrt{-(}}{8\,\left(16\,a^2\,c\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2 + c*x^4),x)
```

```
[Out] - 2*atanh(((x*(4*a*c^2 - 2*b^2*c) + (x*(8*b^3*c^2 - 32*a*b*c^3)*(b^3 + (-(4
*a*c - b^2)^3)^(1/2) - 4*a*b*c))/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-
(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2
*c^2)))^(1/2))/(a*c))*(-(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*
c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2) - 2*atanh(((x*(4*a*c^2 - 2*b^2*c) - (
x*(8*b^3*c^2 - 32*a*b*c^3)*((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c))/(8*(
b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*
b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(a*c))*(((-(4*a*c - b^2
)^3)^(1/2) - b^3 + 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2)
```

sympy [A] time = 2.62, size = 75, normalized size = 0.50

$$\mathrm{RootSum}\left(t^4\left(256a^2c^3-128ab^2c^2+16b^4c\right)+t^2\left(-16abc+4b^3\right)+a,\left(t\mapsto t\log\left(64t^3ac^2-16t^3b^2c-2tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3 - 128*a*b**2*c**2 + 16*b**4*c) + _t**2*(-16*a*
b*c + 4*b**3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 - 16*_t**3*b**2*c - 2*
_t*b + x)))
```

3.858 $\int \frac{1}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*
a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*2^(1/2)*c^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^
(1/2))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1093, 205}

$$\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a+bx^2+cx^4}\,dx = \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}}$$
$$= \frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.08, size = 129, normalized size = 0.86

$$\frac{\sqrt{2}\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/S
qrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

fricas [B] time = 0.66, size = 613, normalized size = 4.09

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2}{\sqrt{a^2}}}{ab^2-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqrt(
a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(
a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^
2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c - (a*b^
```

```
3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a
^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt(-(b - (a*b^2 -
4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*
(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b
^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*
sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*lo
g(2*c*x - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3
*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c
)))
```

giac [B] time = 0.58, size = 1024, normalized size = 6.83

$$\frac{\left(\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,b^4-8\,\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,ab^2c-2\,\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,b^3c-2\,b^4c+16\,\sqrt{2}\,\sqrt{bc+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*
b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^
2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs
(c)) + 1/4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 -
4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4
```

```
 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c
^3)*abs(c))
```

maple [A] time = 0.02, size = 116, normalized size = 0.77

$$-\frac{\sqrt{2}\, c \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a),x)
```

```
[Out] -c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 4.61, size = 763, normalized size = 5.09

$$-\operatorname{atan}\left(\frac{b^4\,x\,1\mathrm{i}+b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1\mathrm{i}+a^2\,c^2\,x\,16\mathrm{i}-}{4\,a\,b^4\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}-32\,a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2 + c*x^4),x)
```

```
[Out] - atan((b^4*x*1i + b*x*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/
2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i)/(4*a*b^4*(-(b^3 + (b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*
a^2*b^2*c))^(1/2) + 64*a^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2
```

```
- 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2
) - 32*a^2*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^
(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*(-(b^3 + (
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 +
 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i - atan((b^4*x*1i - b*x*(b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i
)/(4*a*b^4*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 +
 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) + 64*a^3*c^2*(((b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^
4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) - 32*a^2*b^2*c*(((b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2
 - 64*a^2*b^2*c))^(1/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i
```

sympy [A] time = 2.84, size = 87, normalized size = 0.58

$$\operatorname{RootSum}\left(t^4\left(256a^3c^2-128a^2b^2c+16ab^4\right)+t^2\left(-16abc+4b^3\right)+c,\left(t\mapsto t\log\left(x+\frac{32t^3a^2bc-8t^3ab^3+4tac-\right.\right.\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(-16*a*
b*c + 4*b**3) + c, Lambda(_t, _t*log(x + (32*_t**3*a**2*b*c - 8*_t**3*a*b**
3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

3.859 $\int \frac{1}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=174

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

```
[Out] -1/a/x-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(
1+b/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x
*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(1-b/(-4*a*c+b^2)^(1
/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1123, 1166, 205}

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(1/(a*x)) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sq
rt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2+cx^4\right)}\,dx = -\frac{1}{ax}+\frac{\int \frac{-b-cx^2}{a+bx^2+cx^4}\,dx}{a}$$

$$= -\frac{1}{ax}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,d}{2a}$$

$$= -\frac{1}{ax}-\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right.}{\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.38, size = 191, normalized size = 1.10

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2}{x}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*(2/x + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*
a*c]]))/a
```

fricas [B] time = 1.06, size = 1116, normalized size = 6.41

$$-\frac{\sqrt{\frac{1}{2}}\,ax\sqrt{-\frac{b^3-3\,abc+\left(a^3b^2-4\,a^4c\right)\sqrt{\frac{b^4-2\,ab^2c+a^2c^2}{a^6b^2-4\,a^7c}}}{a^3b^2-4\,a^4c}}\,\log\left(-2\left(b^2c^2-ac^3\right)x+\sqrt{\frac{1}{2}}\left(b^5-5\,ab^3c+4\,a^2bc^2-\left(a^3b^4-6\,a^4b^2c+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 -
2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2
*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a
^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c))
)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt(-(b^3 -
 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 -
4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqrt(1/2)*(b^5
 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^
2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^
2 - 4*a^4*c))) + sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*
log(-2*(b^2*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^
3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2
- 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*s
qrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqr
t(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*
c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c
)))/(a^3*b^2 - 4*a^4*c))) + 2)/(a*x)
```

giac [B] time = 0.97, size = 1839, normalized size = 10.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
```

```
b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + (2*b^4*c^2 - 16*a*b^2*c
^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*
(b^2 - 4*a*c)*a*c^3)*a^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5
 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 -
 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^
2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2)*abs(a))*arctan(2*sqrt(1/2)*
x/sqrt((a*b + sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*
a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(a)*abs(
c)) + 1/8*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + (2*b^4*c^2 - 16*a*
b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c -
16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2
 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 16*a^2*b^3*
c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^3*b*c^3 -
2*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^2*b*c^2)*abs(a))*arctan(2*sqrt(
1/2)*x/sqrt((a*b - sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3*b^4 - 8*a^4*b^2*c
 - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(a)
*abs(c)) - 1/(a*x)
```

maple [A] time = 0.02, size = 232, normalized size = 1.33

$$\frac{\sqrt{2}\, bc \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\, \sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\; a}+\frac{\sqrt{2}\, bc \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\, \sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\; a}+\frac{\sqrt{2}\, c \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\; a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x)+1/2*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*b-1/2*c/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+1/2*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*b-1/a/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 + b)/(c*x^4 + b*x^2 + a), x)/a - 1/(a*x)
```

mupad [B] time = 4.85, size = 2997, normalized size = 17.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2 + c*x^4)),x)
```

```
[Out] - atan(((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4
+ 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a
^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 -
8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a
^4*b^2*c)))^(1/2)*1i + (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 + b^2*(-(4*a
```

*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2
))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 - 4*a^4*b^
3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4
+ 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b
^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c
- b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b
*c^3 - 4*a^4*b^3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 + b^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2
))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - (x*(4*a^4*c^4 - 2*a^3*b^2
c^3) + (-(b^5 + b^2(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c -
a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/
2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5
+ b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 2*a^3*c^4))*(
-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - a
tan(((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*
b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a
^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*
b^2*c)))^(1/2)*1i + (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 - 4*a^4*b^3*c
^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a
^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5
- b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^
3 - 4*a^4*b^3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*(-(4*a*c - b
^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*
(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - (x*(4*a^4*c^4 - 2*a^3*b^2*c^

```
3) + (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*
(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*
(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3
)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 2*a^3*c^4))*(-(b
^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - 1/(a
*x)
```

sympy [A] time = 4.92, size = 148, normalized size = 0.85

$$\mathrm{RootSum}\left(t^4\left(256a^5c^2-128a^4b^2c+16a^3b^4\right)+t^2\left(48a^2bc^2-28ab^3c+4b^5\right)+c^3,\left(t\mapsto t\log\left(x+\frac{-64t^3a^5c^2+48\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2 - 128*a**4*b**2*c + 16*a**3*b**4) + _t**2*(48*
a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + c**3, Lambda(_t, _t*log(x + (-64*_t**
3*a**5*c**2 + 48*_t**3*a**4*b**2*c - 8*_t**3*a**3*b**4 - 10*_t*a**2*b*c**2
+ 10*_t*a*b**3*c - 2*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(a*x)
```

3.860 $\int \frac{1}{x^4(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=196

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

```
[Out] -1/3/a/x^3+b/a^2/x+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2
))*c^(1/2)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^2*2^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(
1/2)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^
(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1123, 1281, 1166, 205}

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(3*a*x^3) + b/(a^2*x) + (Sqrt[c]*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b
 - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
```

```
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^2+cx^4\right)}\,dx &= -\frac{1}{3ax^3}+\frac{\int \frac{-3b-3cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx}{3a}\\ &= -\frac{1}{3ax^3}+\frac{b}{a^2x}-\frac{\int \frac{-3\left(b^2-ac\right)-3bcx^2}{a+bx^2+cx^4}\,dx}{3a^2}\\ &= -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a^2}+\frac{\left(c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-\ldots}}}{2a^2}\\ &= -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{\sqrt{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\ldots\right)}{\sqrt{2}\,a^2\sqrt{b+\sqrt{b^2-4a\ldots}}}\end{aligned}$$

Mathematica [A] time = 0.14, size = 216, normalized size = 1.10

$$\frac{\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2a}{x^3}+\frac{6b}{x}}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a)/x^3 + (6*b)/x + (3*Sqrt[2]*Sqrt[c]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*
a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-b^2 + 2*a*c + b*Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*a^2)
```

fricas [B] time = 0.92, size = 1622, normalized size = 8.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(1/2)*a^2*x^3*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 -
 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)
/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(b^4*c^3 - 3*a*b^2*c^4
+ a^2*c^5)*x + sqrt(1/2)*(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3
 + 4*a^4*c^4 - (a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c
 + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))*sqrt(-
(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^
2 - 4*a^6*c))) - 3*sqrt(1/2)*a^2*x^3*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 +
 (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3
 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(b^4*c^3 - 3
*a*b^2*c^4 + a^2*c^5)*x - sqrt(1/2)*(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*
a^3*b^2*c^3 + 4*a^4*c^4 - (a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*sqrt((b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*
c)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^8 -
 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c
)))/(a^5*b^2 - 4*a^6*c))) + 3*sqrt(1/2)*a^2*x^3*sqrt(-(b^5 - 5*a*b^3*c + 5*
a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(
b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x + sqrt(1/2)*(b^8 - 8*a*b^6*c + 20*a^2*b^
4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4 + (a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)
```

```
*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^
2 - 4*a^11*c)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*
sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2
 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 3*sqrt(1/2)*a^2*x^3*sqrt(-(b^5 - 5*a
*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b
^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*
c))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x - sqrt(1/2)*(b^8 - 8*a*b^6*c
+ 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4 + (a^5*b^5 - 7*a^6*b^3*c + 12
*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^
4)/(a^10*b^2 - 4*a^11*c)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2
- 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4
)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 6*b*x^2 + 2*a)/(a^2*x^3)
```

giac [B] time = 1.16, size = 1640, normalized size = 8.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c
 - 2*b^6*c + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 10*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^4*c^2 + 18*a*b^4*c^2 + 2*b^5*c^2 - 16*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b*c^3 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 48*a^2*b^2*c
^3 - 14*a*b^3*c^3 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32*
a^3*c^4 + 24*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^5 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c
- 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 -
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 3*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*(b^2 - 4*a*c)
*b^4*c - 10*(b^2 - 4*a*c)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*
a*c)*a^2*c^3 + 6*(b^2 - 4*a*c)*a*b*c^3)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b +
sqrt(a^4*b^2 - 4*a^5*c))/(a^2*c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c +
16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c)) + 1/4*(sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 2*b^6*c +
24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 10*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^4*c^2 - 18*a*b^4*c^2 - 2*b^5*c^2 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 5*sq
```

```
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 48*a^2*b^2*c^3 + 14*a*b^3
*c^3 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 - 24*
a^2*b*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 -
 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 12*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 6*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^4*c + 10*(
b^2 - 4*a*c)*a*b^2*c^2 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3
- 6*(b^2 - 4*a*c)*a*b*c^3)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b - sqrt(a^4*b^2
- 4*a^5*c))/(a^2*c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 +
8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c)) + 1/3*(3*b*x^2 - a)/(a^2*x^3
)
```

maple [B] time = 0.02, size = 368, normalized size = 1.88

$$\frac{\sqrt{2}\, c^{2} \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right) c}}\right)}{\sqrt{-4ac+b^{2}}\, \sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right) c}\; a}+\frac{\sqrt{2}\, c^{2} \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right) c}}\right)}{\sqrt{-4ac+b^{2}}\, \sqrt{\left(b+\sqrt{-4ac+b^{2}}\right) c}\; a}-\frac{\sqrt{2}\, b^{2} c \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac}\right.}}\right.}{2\sqrt{-4ac+b^{2}}\, \sqrt{\left(-b+\sqrt{-4ac+}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/a^2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b+1/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)-1/2/a^2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2+1/2/a^2*c*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*c*x)*b+1/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-1/2/a^2*c
/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2-1/3/a/x^3+b/a^2/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((b*c*x^2 + b^2 - a*c)/(c*x^4 + b*x^2 + a), x)/a^2 + 1/3*(3*b*x^2
- a)/(a^2*x^3)
```

mupad [B] time = 0.79, size = 4160, normalized size = 21.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2 + c*x^4)),x)
```

```
[Out] - (1/(3*a) - (b*x^2)/a^2)/x^3 - atan(((((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7
 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b
^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6
*b^2*c)))^(1/2)*(16*a^10*c^4 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*((b^4*(-(
4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 4*a^8*b^4*c^2 - 20*a^9*b^2*c^3
) - x*(4*a^8*c^5 + 2*a^6*b^4*c^3 - 8*a^7*b^2*c^4))*((b^4*(-(4*a*c - b^2)^3)
^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(
1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7
*c^2 - 8*a^6*b^2*c)))^(1/2)*1i - (((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20
*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c
- 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*
c)))^(1/2)*(16*a^10*c^4 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*((b^4*(-(4*a*c
 - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c -
 b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^
4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 4*a^8*b^4*c^2 - 20*a^9*b^2*c^3) + x
*(4*a^8*c^5 + 2*a^6*b^4*c^3 - 8*a^7*b^2*c^4))*((b^4*(-(4*a*c - b^2)^3)^(1/2
) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2)
+ 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2
- 8*a^6*b^2*c)))^(1/2)*1i)/((((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*
b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a
*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^
(1/2)*(16*a^10*c^4 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*((b^4*(-(4*a*c - b^
2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 1
6*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 4*a^8*b^4*c^2 - 20*a^9*b^2*c^3) - x*(4*a
^8*c^5 + 2*a^6*b^4*c^3 - 8*a^7*b^2*c^4))*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b
^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a
*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a
^6*b^2*c)))^(1/2) + (((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 -
25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(
-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(1
6*a^10*c^4 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*((b^4*(-(4*a*c - b^2)^3)^(1
/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2
```

) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 4*a^8*b^4*c^2 - 20*a^9*b^2*c^3) + x*(4*a^8*c^5 + 2*a^6*b^4*c^3 - 8*a^7*b^2*c^4))*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) - 2*a^6*b*c^5))*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*2i - atan((((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(16*a^10*c^4 + 4*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)) - x*(4*a^8*c^5 + 2*a^6*b^4*c^3 - 8*a^7*b^2*c^4))*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*1i - ((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(16*a^10*c^4 + 4*a^8*b^4*c^2 - 20*a^9*b^2*c^3 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)) + x*(4*a^8*c^5 + 2*a^6*b^4*c^3 - 8*a^7*b^2*c^4))*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*1i)/(((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(16*a^10*c^4 + 4*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)) - x*(4*a^8*c^5 + 2*a^6*b^4*c^3 - 8*a^7*b^2*c^4))*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + ((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(16*a^10*c^4 + 4*a^8*b^4*c^2 - 20*a^9*b^2*c^3 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)) + x*(4*a^8*c^5 + 2*a^6*b^4*c^

```
3 - 8*a^7*b^2*c^4))*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 +
25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(
-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) -
2*a^6*b*c^5))*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2
*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*
c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*2i
```

sympy [A] time = 16.67, size = 211, normalized size = 1.08

$$\text{RootSum}\left(t^4\left(256a^7c^2 - 128a^6b^2c + 16a^5b^4\right) + t^2\left(-80a^3bc^3 + 100a^2b^3c^2 - 36ab^5c + 4b^7\right) + c^5, \left(t \mapsto t\log\left(x + \right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**7*c**2 - 128*a**6*b**2*c + 16*a**5*b**4) + _t**2*(-80
*a**3*b*c**3 + 100*a**2*b**3*c**2 - 36*a*b**5*c + 4*b**7) + c**5, Lambda(_t
, _t*log(x + (-96*_t**3*a**7*b*c**2 + 56*_t**3*a**6*b**3*c - 8*_t**3*a**5*b
**5 - 4*_t*a**4*c**4 + 32*_t*a**3*b**2*c**3 - 40*_t*a**2*b**4*c**2 + 16*_t*
a*b**6*c - 2*_t*b**8)/(a**2*c**5 - 3*a*b**2*c**4 + b**4*c**3)))) + (-a + 3*
b*x**2)/(3*a**2*x**3)
```

3.861 $\int \frac{x^7}{\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=132

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}}-\frac{bx^2}{2c\left(b^2-4ac\right)}+\frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\log\left(a+bx^2+cx^4\right)}{4c^2}$$

```
[Out] -1/2*b*x^2/c/(-4*a*c+b^2)+1/2*x^4*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+
1/2*b*(-6*a*c+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)
^(3/2)+1/4*ln(c*x^4+b*x^2+a)/c^2
```

Rubi [A] time = 0.17, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1114, 738, 773, 634, 618, 206, 628}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}}+\frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{bx^2}{2c\left(b^2-4ac\right)}+\frac{\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b*x^2)/(2*c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(2*c^2*(b^2 - 4*a*c)^(3/2)) + Log[a + b*x^2 + c*x^4]/(4*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^3}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\mathrm{Subst}\left(\int \frac{x(4a+bx)}{a+bx+cx^2}\,dx, x, x^2\right)}{2\left(b^2-4ac\right)}$$

$$= -\frac{bx^2}{2c\left(b^2-4ac\right)} + \frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\mathrm{Subst}\left(\int \frac{-ab+\left(-b^2+4ac\right)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c\left(b^2-4ac\right)}$$

$$= -\frac{bx^2}{2c\left(b^2-4ac\right)} + \frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\mathrm{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2} - \frac{\left(b\left(b^2\right.\right.}{}$$

$$= -\frac{bx^2}{2c\left(b^2-4ac\right)} + \frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log\left(a+bx^2+cx^4\right)}{4c^2} + \frac{\left(b\left(b^2-6ac\right)\right)}{}$$

$$= -\frac{bx^2}{2c\left(b^2-4ac\right)} + \frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}} + \frac{\log}{}$$

Mathematica [A] time = 0.17, size = 121, normalized size = 0.92

$$\frac{\frac{2\left(-2a^2c+ab\left(b-3cx^2\right)+b^3x^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{2b\left(b^2-6ac\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}} + \log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(-2*a^2*c + b^3*x^2 + a*b*(b - 3*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c
*x^4)) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2
 + 4*a*c)^(3/2) + Log[a + b*x^2 + c*x^4])/(4*c^2)
```

fricas [B] time = 1.20, size = 663, normalized size = 5.02

$$\left[\frac{2\,ab^4 - 12\,a^2b^2c + 16\,a^3c^2 + 2\left(b^5 - 7\,ab^3c + 12\,a^2bc^2\right)x^2 + \left(\left(b^3c - 6\,abc^2\right)x^4 + ab^3 - 6\,a^2bc + \left(b^4 - 6\,ab^2c\right)x^2\right)}{4\left(ab^4c^2 - 8\,a^2b^2c^3 + 16\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c
^2)*x^2 + ((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*
x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2
+ b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (a*b^4 - 8*a^2*b^2*c + 16*a^
3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*
b*c^2)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4
 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a
^2*b*c^4)*x^2), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 - 7*a*b^3
*c + 12*a^2*b*c^2)*x^2 + 2*((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (
b^4 - 6*a*b^2*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 +
4*a*c)/(b^2 - 4*a*c)) + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(c*x^4 +
 b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c
^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2)]
```

giac [A] time = 0.60, size = 152, normalized size = 1.15

$$-\frac{\left(b^{3}-6\,abc\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\left(b^{2}c^{2}-4\,ac^{3}\right)\sqrt{-b^{2}+4\,ac}}-\frac{b^{2}cx^{4}-4\,ac^{2}x^{4}-b^{3}x^{2}+2\,abcx^{2}-ab^{2}}{4\left(cx^{4}+bx^{2}+a\right)\left(b^{2}c^{2}-4\,ac^{3}\right)}+\frac{\log\left(cx^{4}+bx^{2}+a\right)}{4\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3 - 6*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4
*a*c^3)*sqrt(-b^2 + 4*a*c)) - 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 - b^3*x^2 + 2*a*
b*c*x^2 - a*b^2)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) + 1/4*log(c*x^4
+ b*x^2 + a)/c^2
```

maple [A] time = 0.02, size = 222, normalized size = 1.68

$$-\frac{3ab\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}c}+\frac{b^{3}\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{2\left(4ac-b^{2}\right)^{\frac{3}{2}}c^{2}}+\frac{a\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{\left(4ac-b^{2}\right)c}-\frac{b^{2}\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{4\left(4ac-b^{2}\right)c^{2}}+\frac{\frac{\left(3ac-b^{2}\right)b\,x^{2}}{\left(4ac-b^{2}\right)c^{2}}+\frac{\left(2ac-b^{2}\right)}{\left(4ac-b^{2}\right)}}{2c\,x^{4}+2b\,x^{2}+2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(b*(3*a*c-b^2)/c^2/(4*a*c-b^2)*x^2+a*(2*a*c-b^2)/(4*a*c-b^2)/c^2)/(c*x^
4+b*x^2+a)+1/c/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a-1/4/c^2/(4*a*c-b^2)*ln(c*x^4
```

```
+b*x^2+a)*b^2-3/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a
*b+1/2/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 5.10, size = 1336, normalized size = 10.12

$$\frac{\frac{a\left(2\,a\,c-b^2\right)}{2\,c^2\left(4\,a\,c-b^2\right)}+\frac{b\,x^2\left(3\,a\,c-b^2\right)}{2\,c^2\left(4\,a\,c-b^2\right)}}{c\,x^4+b\,x^2+a}-\frac{\ln\left(c\,x^4+b\,x^2+a\right)\left(-128\,a^3\,c^3+96\,a^2\,b^2\,c^2-24\,a\,b^4\,c+2\,b^6\right)}{2\left(256\,a^3\,c^5-192\,a^2\,b^2\,c^4+48\,a\,b^4\,c^3-4\,b^6\,c^2\right)}+\frac{b\,\mathrm{atan}\left(\frac{\left(8\,a\,c^3\left(4\,a\,c-b^2\right)\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((a*(2*a*c - b^2))/(2*c^2*(4*a*c - b^2)) + (b*x^2*(3*a*c - b^2))/(2*c^2*(4*
a*c - b^2)))/(a + b*x^2 + c*x^4) - (log(a + b*x^2 + c*x^4)*(2*b^6 - 128*a^3
*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4
*c^3 - 192*a^2*b^2*c^4)) + (b*atan(((8*a*c^3*(4*a*c - b^2)^3 - 2*b^2*c^2*(4
*a*c - b^2)^3)*(x^2*(((b*((6*b^3*c^2 - 28*a*b*c^3)/(4*a*c^3 - b^2*c^2) + ((
8*b^3*c^4 - 32*a*b*c^5)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)
)/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*
b^2*c^4)))*(6*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(3/2)) + (b*(8*b^3*c^4 - 32*
```

```
a*b*c^5)*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))
/(16*c^2*(4*a*c - b^2)^(3/2)*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 +
 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) - (b*((b^3 - 5*a*b*c)/
(4*a*c^3 - b^2*c^2) + (((6*b^3*c^2 - 28*a*b*c^3)/(4*a*c^3 - b^2*c^2) + ((8*
b^3*c^4 - 32*a*b*c^5)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/
(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^
2*c^4)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(256*a^3*c
^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) - (b^2*((b^3*c^4)/2 - 2*a
*b*c^5)*(6*a*c - b^2)^2)/(c^4*(4*a*c - b^2)^3*(4*a*c^3 - b^2*c^2))))/(2*a*(
4*a*c - b^2)^(3/2))) - ((b*(6*a*c - b^2)*(8*a + (8*a*c^2*(2*b^6 - 128*a^3*c
^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3
- 192*a^2*b^2*c^4)))/(8*c^2*(4*a*c - b^2)^(3/2)) + (a*b*(6*a*c - b^2)*(2*b^
6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/((4*a*c - b^2)^(3/2)*(256*a
^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) +
(b*(a/c^2 + ((8*a + (8*a*c^2*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b
^4*c))/(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))*(2*b^6 -
 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(256*a^3*c^5 - 4*b^6*c^2 +
48*a*b^4*c^3 - 192*a^2*b^2*c^4)) - (a*b^2*(6*a*c - b^2)^2)/(c^2*(4*a*c - b^
2)^3)))/(2*a*(4*a*c - b^2)^(3/2))))/(b^6 + 36*a^2*b^2*c^2 - 12*a*b^4*c))*(6
*a*c - b^2))/(2*c^2*(4*a*c - b^2)^(3/2))
```

sympy [B] time = 40.65, size = 745, normalized size = 5.64

$$\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{4c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{4c^2}\right)\log\left(x^2+\frac{-32a^2c^3\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{4c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{4c^2}\right)+8a^2c+}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2))*log(x**2 + (-32*a**2*c**3*(
-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)) + 8*a**2*c + 16*a*b**2*c**2*
(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)) - a*b**2 - 2*b**4*c*(-b*sqr
t(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c
**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)))/(6*a*b*c - b**3)) + (b*sqrt(-(4*a
*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6)) + 1/(4*c**2))*log(x**2 + (-32*a**2*c**3*(b*sqrt(-(4*a*c
 - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
```

```
a*b**4*c - b**6)) + 1/(4*c**2)) + 8*a**2*c + 16*a*b**2*c**2*(b*sqrt(-(4*a*c
 - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)) + 1/(4*c**2)) - a*b**2 - 2*b**4*c*(b*sqrt(-(4*a*c - b**2)
**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6)) + 1/(4*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x**2*(3*a
*b*c - b**3))/(8*a**2*c**3 - 2*a*b**2*c**2 + x**4*(8*a*c**4 - 2*b**2*c**3)
+ x**2*(8*a*b*c**3 - 2*b**3*c**2))
```

3.862 $\int \frac{x^5}{\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=78

$$\frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] 1/2*x^2*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2*a*arctanh((2*c*x^2+b)/(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1114, 722, 618, 206}

$$\frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^2*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*ArcTanh[(b
 + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
```

```
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$
$$= \frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{a\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{b^2-4ac}$$
$$= \frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}$$
$$= \frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.09, size = 93, normalized size = 1.19

$$\frac{2a\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}} + \frac{a\left(b-2cx^2\right)+b^2x^2}{2c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (b^2*x^2 + a*(b - 2*c*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a
*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

fricas [B] time = 0.79, size = 407, normalized size = 5.22

$$\left[-\frac{ab^3 - 4\,a^2bc + \left(b^4 - 6\,ab^2c + 8\,a^2c^2\right)x^2 + 2\left(ac^2x^4 + abcx^2 + a^2c\right)\sqrt{b^2 - 4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-\left(2\,cx^2+b\right)\sqrt{b^2}}{cx^4+bx^2+a}\right.}{2\left(ab^4c - 8\,a^2b^2c^2 + 16\,a^3c^3 + \left(b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4\right)x^4 + \left(b^5c - 8\,ab^3c^2 + 16\,a^2bc^3\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a*b^3 - 4*a^2*b*c + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x^2 + 2*(a*c^2*x^4
 + a*b*c*x^2 + a^2*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 -
2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4*c - 8
*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2), -1/2*(a*b^3 - 4*a^2*b*c + (b^4 - 6*a
*b^2*c + 8*a^2*c^2)*x^2 - 4*(a*c^2*x^4 + a*b*c*x^2 + a^2*c)*sqrt(-b^2 + 4*a
*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(a*b^4*c - 8*a
^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c
 - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)]
```

giac [A] time = 0.91, size = 96, normalized size = 1.23

$$-\frac{2\,a\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2 - 4\,ac\right)\sqrt{-b^2 + 4\,ac}} - \frac{b^2x^2 - 2\,acx^2 + ab}{2\left(cx^4 + bx^2 + a\right)\left(b^2c - 4\,ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -2*a*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*
a*c)) - 1/2*(b^2*x^2 - 2*a*c*x^2 + a*b)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c
^2))
```

maple [A] time = 0.01, size = 104, normalized size = 1.33

$$\frac{2a\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}} + \frac{\frac{ab}{\left(4ac-b^2\right)c} - \frac{\left(2ac-b^2\right)x^2}{\left(4ac-b^2\right)c}}{2c\,x^4 + 2b\,x^2 + 2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(-(2*a*c-b^2)/c/(4*a*c-b^2)*x^2+a*b/c/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2*a/
(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.18, size = 187, normalized size = 2.40

$$-\frac{\frac{x^2\,\left(2\,a\,c-b^2\right)}{2\,c\,\left(4\,a\,c-b^2\right)}-\frac{a\,b}{2\,c\,\left(4\,a\,c-b^2\right)}}{c\,x^4+b\,x^2+a}-\frac{2\,a\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{{\left(4\,a\,c-b^2\right)}^{3/2}}-\frac{x^2\,\left(\frac{4\,a\,c^2}{{\left(4\,a\,c-b^2\right)}^{7/2}}+\frac{4\,a\,\left(b^3\,c^2-4\,a\,b\,c^3\right)\,\left(b^3-4\,a\,b\,c\right)}{{\left(4\,a\,c-b^2\right)}^{13/2}}\right)\,{\left(4\,a\,c-b^2\right)}^4}{8\,a^2\,c^2}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((x^2*(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b)/(2*c*(4*a*c - b^2)))/(a
+ b*x^2 + c*x^4) - (2*a*atan((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*((4
*a*c^2)/(4*a*c - b^2)^(7/2) + (4*a*(b^3*c^2 - 4*a*b*c^3)*(b^3 - 4*a*b*c))/(
4*a*c - b^2)^(13/2))*(4*a*c - b^2)^4)/(8*a^2*c^2)))/(4*a*c - b^2)^(3/2)
```

sympy [B] time = 3.90, size = 282, normalized size = 3.62

$$-a\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2+\frac{-16a^3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8a^2b^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-ab^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+ab}{2ac}\right)+a\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**3*c**2*sqrt(-1/(4*a*c - b*
*2)**3) + 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - a*b**4*sqrt(-1/(4*a*c
```

```
- b**2)**3) + a*b)/(2*a*c)) + a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*a
**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)*
*3) + a*b**4*sqrt(-1/(4*a*c - b**2)**3) + a*b)/(2*a*c)) + (a*b + x**2*(-2*a
*c + b**2))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3 - 2*b**2*c**2) + x**
2*(8*a*b*c**2 - 2*b**3*c))
```

3.863 $\int \frac{x^3}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=75

$$\frac{2a+bx^2}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] 1/2*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*arctanh((2*c*x^2+b)/(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1114, 638, 618, 206}

$$\frac{2a+bx^2}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)\\ &= \frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2\left(b^2-4ac\right)}\\ &= \frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}\\ &= \frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 79, normalized size = 1.05

$$\frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTan[(b + 2*c*x^
2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

fricas [B] time = 0.94, size = 360, normalized size = 4.80

$$\left[\frac{2\,ab^2-8\,a^2c+\left(b^3-4\,abc\right)x^2-\left(bcx^4+b^2x^2+ab\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{2\left(ab^4-8\,a^2b^2c+16\,a^3c^2+\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x^4+\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x^2\right)},\ \frac{2\,ab^2-8\cdots}{2\left(ab\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*x^2 - (b*c*x^4 + b^2*x^2 + a*b)*s
qrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*s
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 +
 (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*
x^2), 1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*x^2 - 2*(b*c*x^4 + b^2*x^2 +
 a*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*
a*c)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^
3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]
```

giac [A] time = 0.62, size = 82, normalized size = 1.09

$$\frac{b \arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}+\frac{bx^2+2\,a}{2\,\left(cx^4+bx^2+a\right)\left(b^2-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c
)) + 1/2*(b*x^2 + 2*a)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))
```

maple [A] time = 0.01, size = 77, normalized size = 1.03

$$-\frac{b \arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{-b\,x^2-2a}{2\left(4ac-b^2\right)\left(c\,x^4+b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(-b*x^2-2*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)-b/(4*a*c-b^2)^(3/2)*arctan((2*
c*x^2+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.57, size = 178, normalized size = 2.37

$$\frac{b\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{\left(4\,a\,c-b^2\right)^{3/2}}-\frac{x^2\left(4\,a\,c-b^2\right)^4\left(\frac{b^2\,c^2}{a\left(4\,a\,c-b^2\right)^{7/2}}+\frac{b^2\left(2\,b^3\,c^2-8\,a\,b\,c^3\right)\left(b^3-4\,a\,b\,c\right)}{2\,a\left(4\,a\,c-b^2\right)^{13/2}}\right)}{2\,b^2\,c^2}\right)}{\left(4\,a\,c-b^2\right)^{3/2}}-\frac{\frac{a}{4\,a\,c-b^2}+\frac{b\,x^2}{2\left(4\,a\,c-b^2\right)}}{c\,x^4+b\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] (b*atan((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*(4*a*c - b^2)^4*((b^2*c^
2)/(a*(4*a*c - b^2)^(7/2)) + (b^2*(2*b^3*c^2 - 8*a*b*c^3)*(b^3 - 4*a*b*c))/
(2*a*(4*a*c - b^2)^(13/2))))/(2*b^2*c^2)))/(4*a*c - b^2)^(3/2) - (a/(4*a*c
- b^2) + (b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
```

sympy [B] time = 1.88, size = 269, normalized size = 3.59

$$\frac{b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2+\frac{-16a^2bc^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^3c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^2}{2bc}\right)}{2}-\frac{b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2+\frac{16a^2bc^2\sqrt{-\frac{1}{(4ac}}}{}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b
**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b*
*2)**3) + b**2)/(2*b*c))/2 - b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*a*
*2*b*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3
) + b**5*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c))/2 + (-2*a - b*x**2)/(8
*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3))
```

3.864 $\int \frac{x}{(a+bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=74

$$\frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] 1/2*(-2*c*x^2-b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2*c*arctanh((2*c*x^2+b)/(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1107, 614, 618, 206}

$$\frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b + 2*c*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*c*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{c\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{b^2-4ac}$$

$$= -\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{(2c)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}$$

$$= -\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.08, size = 79, normalized size = 1.07

$$-\frac{\frac{4c\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{b+2cx^2}{a+bx^2+cx^4}}{2\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*((b + 2*c*x^2)/(a + b*x^2 + c*x^4) + (4*c*ArcTan[(b + 2*c*x^2)/Sqrt[-b
^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c)
```

fricas [B] time = 0.87, size = 361, normalized size = 4.88

$$\left[-\frac{b^3-4\,abc+2\left(b^2c-4\,ac^2\right)x^2+2\left(c^2x^4+bcx^2+ac\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{2\left(ab^4-8\,a^2b^2c+16\,a^3c^2+\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x^4+\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x^2\right)}, -\frac{b^3-}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + 2*(c^2*x^4 + b*c*x^2 + a*c
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b
)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^
2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^
2)*x^2), -1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - 4*(c^2*x^4 + b*c*x
^2 + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2
- 4*a*c)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]
```

giac [A] time = 0.58, size = 82, normalized size = 1.11

$$-\frac{2\,c\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}-\frac{2\,cx^2+b}{2\left(cx^4+bx^2+a\right)\left(b^2-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -2*c*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*
a*c)) - 1/2*(2*c*x^2 + b)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))
```

maple [A] time = 0.01, size = 75, normalized size = 1.01

$$\frac{2c\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{2c\,x^2+b}{2\left(4ac-b^2\right)\left(c\,x^4+b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+2*c/(4*a*c-b^2)^(3/2)*arctan((2
*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.31, size = 172, normalized size = 2.32

$$\frac{\frac{b}{2\left(4\,a\,c-b^2\right)}+\frac{c\,x^2}{4\,a\,c-b^2}}{c\,x^4+b\,x^2+a}-\frac{2\,c\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{\left(4\,a\,c-b^2\right)^{3/2}}-\frac{x^2\,\left(4\,a\,c-b^2\right)^4\,\left(\frac{4\,c^4}{a\,\left(4\,a\,c-b^2\right)^{7/2}}+\frac{4\,c^2\,\left(b^3\,c^2-4\,a\,b\,c^3\right)\,\left(b^3-4\,a\,b\,c\right)}{a\,\left(4\,a\,c-b^2\right)^{13/2}}\right)}{8\,c^4}\right)}{\left(4\,a\,c-b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] (b/(2*(4*a*c - b^2)) + (c*x^2)/(4*a*c - b^2))/(a + b*x^2 + c*x^4) - (2*c*at
an((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*(4*a*c - b^2)^4*((4*c^4)/(a*(
4*a*c - b^2)^(7/2)) + (4*c^2*(b^3*c^2 - 4*a*b*c^3)*(b^3 - 4*a*b*c))/(a*(4*a
*c - b^2)^(13/2))))/(8*c^4)))/(4*a*c - b^2)^(3/2)
```

sympy [B] time = 2.78, size = 267, normalized size = 3.61

$$-c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2+\frac{-16a^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+bc}{2c^2}\right)+c\sqrt{-\frac{1}{\left(4ac-b^2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -c*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*c**3*sqrt(-1/(4*a*c - b*
*2)**3) + 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - b**4*c*sqrt(-1/(4*a*c
- b**2)**3) + b*c)/(2*c**2)) + c*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*
a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)
**3) + b**4*c*sqrt(-1/(4*a*c - b**2)**3) + b*c)/(2*c**2)) + (b + 2*c*x**2)/
(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3)
)
```

3.865 $\int \frac{1}{x(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=122

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] 1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*b*(-6*a*c+b^2)*a
rctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+ln(x)/a^2-1/4
*ln(c*x^4+b*x^2+a)/a^2
```

Rubi [A] time = 0.20, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1114, 740, 800, 634, 618, 206, 628}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (b*(b^2 -
 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2
)) + Log[x]/a^2 - Log[a + b*x^2 + c*x^4]/(4*a^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-b^2+4ac-bcx}{x\left(a+bx+cx^2\right)}\,dx,x,x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{-b^2+4ac}{ax}+\frac{b\left(b^2-5ac\right)+c\left(b^2-4ac\right)x}{a\left(a+bx+cx^2\right)}\right)\,dx,x,x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^2} - \frac{\text{Subst}\left(\int \frac{b\left(b^2-5ac\right)+c\left(b^2-4ac\right)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2a^2\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^2} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a^2} - \frac{\left(b\left(b^2-6ac\right.\right.}{}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^2} - \frac{\log\left(a+bx^2+cx^4\right)}{4a^2} + \frac{\left(b\left(b^2-6ac\right)\right)\text{Subs}}{2a}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}} + \frac{\log(x)}{a^2} - \frac{\log\left(a+\right.}{4}$$

Mathematica [A] time = 0.33, size = 207, normalized size = 1.70

$$\frac{\frac{2a\left(-2ac+b^2+bcx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(\sqrt{b^2-4ac}\right.}{\left(b^2-4ac\right)^{3/2}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*Log[
x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log
[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c -
b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^2)
```

fricas [B] time = 1.00, size = 813, normalized size = 6.66

$$\left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+2\left(ab^3c-4\,a^2bc^2\right)x^2+\left(\left(b^3c-6\,abc^2\right)x^4+ab^3-6\,a^2bc+\left(b^4-6\,ab^2c\right)x^2\right)\sqrt{b^2-4\,ac}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2 +
 ((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt
(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt
(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^
2)*log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c -
8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(x
))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^
4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), 1/4*(2*a*b^4 - 12
*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2 + 2*((b^3*c - 6*a*b
*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt(-b^2 + 4*a*c)*a
rctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c
 + 16*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^
2*b*c^2)*x^2)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*
a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)
]
```

giac [A] time = 0.56, size = 166, normalized size = 1.36

$$-\frac{\left(b^3-6\,abc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^2b^2-4\,a^3c\right)\sqrt{-b^2+4\,ac}}+\frac{b^2cx^4-4\,ac^2x^4+b^3x^2-2\,abcx^2+3\,ab^2-8\,a^2c}{4\left(cx^4+bx^2+a\right)\left(a^2b^2-4\,a^3c\right)}-\frac{\log\left(cx^4+bx^2+a\right)}{4\,a^2}+\frac{\log\left(x^2\cdots\right.}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3 - 6*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 - 4
*a^3*c)*sqrt(-b^2 + 4*a*c)) + 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 + b^3*x^2 - 2*a*
b*c*x^2 + 3*a*b^2 - 8*a^2*c)/((c*x^4 + b*x^2 + a)*(a^2*b^2 - 4*a^3*c)) - 1/
4*log(c*x^4 + b*x^2 + a)/a^2 + 1/2*log(x^2)/a^2
```

maple [B] time = 0.02, size = 253, normalized size = 2.07

$$-\frac{bc\,x^2}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a}-\frac{3bc\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}a}+\frac{b^3\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(4ac-b^2\right)^{\frac{3}{2}}a^2}-\frac{b^2}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/2/a/(c*x^4+b*x^2+a)*b*c/(4*a*c-b^2)*x^2+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*c-
1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^2-1/a/(4*a*c-b^2)*c*ln(c*x^4+b*x^2+a)+1
/4/a^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^2-3/a/(4*a*c-b^2)^(3/2)*arctan((2*c*
x^2+b)/(4*a*c-b^2)^(1/2))*b*c+1/2/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/
(4*a*c-b^2)^(1/2))*b^3+1/a^2*ln(x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 8.29, size = 5048, normalized size = 41.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] log(x)/a^2 + ((2*a*c - b^2)/(2*a*(4*a*c - b^2)) - (b*c*x^2)/(2*a*(4*a*c - b
^2)))/(a + b*x^2 + c*x^4) - (log(a + b*x^2 + c*x^4)*(2*b^6 - 128*a^3*c^3 +
96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c +
192*a^4*b^2*c^2)) + (b*atan((x^2*((((((b*((320*a^5*b*c^6 - 2*a^2*b^7*c^3 +
36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48
*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*
a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*
b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*
b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a^2
```

*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) + (b*((6*a*b^5*c^4 + 80*a^3*b*c^6 - 44*a^2*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (((320*a^5*b*c^6 - 2*a^2*b^7*c^3 + 36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(3/2)) + (b^3*(6*a*c - b^2)^3*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(64*a^6*(4*a*c - b^2)^(9/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(3*b^6 - 40*a^3*c^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*((((6*a*b^5*c^4 + 80*a^3*b*c^6 - 44*a^2*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (((320*a^5*b*c^6 - 2*a^2*b^7*c^3 + 36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) - (b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - (b*(6*a*c - b^2)*((b*((320*a^5*b*c^6 - 2*a^2*b^7*c^3 + 36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))))/(4*a^2*(4*a*c - b^2)^(3/2)) + (b^2*(6*a*c - b^2)^2*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(32*a^4*(4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))))/(8*a^3*c^2

```
*(4*a*c - b^2)^3*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)))*(16
*a^6*b^6*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b
^4*c*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*(4*a*c - b^2)^(9/2)))/(b^6*c^2 -
 12*a*b^4*c^3 + 36*a^2*b^2*c^4) + (((b*((4*a*b^4*c^3 - 17*a^2*b^2*c^4)/(a^3
*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) - (((4*a^2*b^6*c^2 - 36*a^3*b^4*c^3 + 80*a
^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5
*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4
*c))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*
a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a
*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6
*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(3/2)) - (((b*((4*a^2*b^6*c^2 - 36*a^3*b^
4*c^3 + 80*a^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*
c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^
2 - 24*a*b^4*c))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a
^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a^2*(4*a*c - b
^2)^(3/2)) + (b*(6*a*c - b^2)*(4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*
c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(8*a^2*(4*a*c - b
^2)^(3/2)*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 4
8*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24
*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) +
 (b^3*(6*a*c - b^2)^3*(4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4))/(6
4*a^6*(4*a*c - b^2)^(9/2)*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))*(16*a^6*b^
6*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4
*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*(4*a*c - b^2)^(9/2))*(3*b^6 - 40*a^3*c^
3 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(b^6*c^2 -
 12*a*b^4*c^3 + 36*a^2*b^2*c^4)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72
*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*(16*a^6*b^6*(4*a*c - b^2)^
(9/2) - 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2
) + 768*a^8*b^2*c^2*(4*a*c - b^2)^(9/2))*((((4*a*b^4*c^3 - 17*a^2*b^2*c^4)/
(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) - (((4*a^2*b^6*c^2 - 36*a^3*b^4*c^3 +
80*a^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32
*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a
*b^4*c))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 -
 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 -
24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))
)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*
a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) - (b^2*c^4)/(a^3*b^4 + 16*a^5*c^
2 - 8*a^4*b^2*c) + (b*(6*a*c - b^2)*((b*((4*a^2*b^6*c^2 - 36*a^3*b^4*c^3 +
80*a^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32
*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a
*b^4*c))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 -
 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(3/2
)) + (b*(6*a*c - b^2)*(4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*
b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(8*a^2*(4*a*c - b^2)^(3/2
)*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^
```

```
4*c + 192*a^4*b^2*c^2))))/(4*a^2*(4*a*c - b^2)^(3/2)) + (b^2*(6*a*c - b^2)^
2*(4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 +
96*a^2*b^2*c^2 - 24*a*b^4*c))/(32*a^4*(4*a*c - b^2)^3*(a^3*b^4 + 16*a^5*c^2
 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))
))/(8*a^3*c^2*(4*a*c - b^2)^3*(b^6*c^2 - 12*a*b^4*c^3 + 36*a^2*b^2*c^4)*(6*
b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)))*(6*a*c - b^2))/(2*a^2*(
4*a*c - b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.866 $\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=162

$$\frac{b\log\left(a+bx^2+cx^4\right)}{2a^3}-\frac{2b\log(x)}{a^3}-\frac{b^2-3ac}{a^2x^2\left(b^2-4ac\right)}-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}+\frac{-2ac+b^2+\ldots}{2ax^2\left(b^2-4ac\right)\left(a+\ldots\right.}$$

```
[Out] (3*a*c-b^2)/a^2/(-4*a*c+b^2)/x^2+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^2
/(c*x^4+b*x^2+a)-(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)
^(1/2))/a^3/(-4*a*c+b^2)^(3/2)-2*b*ln(x)/a^3+1/2*b*ln(c*x^4+b*x^2+a)/a^3
```

Rubi [A] time = 0.25, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1114, 740, 800, 634, 618, 206, 628}

$$-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}-\frac{b^2-3ac}{a^2x^2\left(b^2-4ac\right)}+\frac{b\log\left(a+bx^2+cx^4\right)}{2a^3}-\frac{2b\log(x)}{a^3}+\frac{-2ac+b^2\ldots}{2ax^2\left(b^2-4ac\right)\left(a\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*x^2)) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^
2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan
h[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x]
)/a^3 + (b*Log[a + b*x^2 + c*x^4])/(2*a^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-2\left(b^2-3ac\right)-2bcx}{x^2\left(a+bx+cx^2\right)}\,dx,x,x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{2\left(-b^2+3ac\right)}{ax^2} - \frac{2b\left(-b^2+4ac\right)}{a^2x} + \frac{2\left(-b^4+5ab^2c-3\right.}{a^2\left(a+\right.}\right.\right.}{2a\left(b^2-4ac\right)}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{2b\log(x)}{a^3} - \frac{\text{Subst}\left(\int \frac{-b^4+5a}{}\right.}{}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{2b\log(x)}{a^3} + \frac{b\,\text{Subst}\left(\int \frac{b+}{a+bx}\right.}{2a}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{2b\log(x)}{a^3} + \frac{b\log\left(a+bx^2\right.}{2a^3}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\left(b^4-6ab^2c+6a^2c^2\right)\tanh^{-}}{a^3\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.27, size = 248, normalized size = 1.53

$$\frac{\frac{\left(6a^2c^2-6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{\left(-6a^2c^2+6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}-b^4\right)\log\left(\sqrt{b^2-4ac}+b+2cx\right.}{\left(b^2-4ac\right)^{3/2}}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (-(a/x^2) - (a*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) - 4*b*Log[x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[
b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x
^2])/(b^2 - 4*a*c)^(3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 -
4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(
b^2 - 4*a*c)^(3/2))/(2*a^3)
```

fricas [B] time = 1.46, size = 1007, normalized size = 6.22

$$\left[-\frac{a^2b^4 - 8\,a^3b^2c + 16\,a^4c^2 + 2\left(ab^4c - 7\,a^2b^2c^2 + 12\,a^3c^3\right)x^4 + \left(2\,ab^5 - 15\,a^2b^3c + 28\,a^3bc^2\right)x^2 + \left(\left(b^4c - 6\,ab^2c^2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12
*a^3*c^3)*x^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x^2 + ((b^4*c - 6*a
*b^2*c^2 + 6*a^2*c^3)*x^6 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4 -
6*a^2*b^2*c + 6*a^3*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2
+ b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)
*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) + 4
*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*
c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(x))/((a^3*b^4*c -
8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^
4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2), -1/2*(a^2*b^4 - 8*a^3*b^2*c
+ 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^4 + (2*a*b^5 - 15
*a^2*b^3*c + 28*a^3*b*c^2)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^6 +
 (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x^
2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c
)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*
b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(c*x^4 + b*x^2
+ a) + 4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*
a^2*b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(x))/((a^3*
b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b
*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2)]
```

giac [A] time = 0.59, size = 182, normalized size = 1.12

$$\frac{\left(b^4 - 6\,ab^2c + 6\,a^2c^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^3b^2 - 4\,a^4c\right)\sqrt{-b^2 + 4\,ac}} - \frac{2\,b^2cx^4 - 6\,ac^2x^4 + 2\,b^3x^2 - 7\,abcx^2 + ab^2 - 4\,a^2c}{2\left(cx^6 + bx^4 + ax^2\right)\left(a^2b^2 - 4\,a^3c\right)} + \frac{b\log\left(cx^4 + bx^2 + \right.}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] (b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^
3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*b^2*c*x^4 - 6*a*c^2*x^4 + 2*b
```

```
^3*x^2 - 7*a*b*c*x^2 + a*b^2 - 4*a^2*c)/((c*x^6 + b*x^4 + a*x^2)*(a^2*b^2 -
 4*a^3*c)) + 1/2*b*log(c*x^4 + b*x^2 + a)/a^3 - b*log(x^2)/a^3
```

maple [B] time = 0.02, size = 352, normalized size = 2.17

$$-\frac{c^2x^2}{\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a}+\frac{b^2c\,x^2}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a^2}-\frac{6c^2\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}a}+\frac{6b^2c\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^2+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b
^2)*x^2*b^2-3/2/a/(c*x^4+b*x^2+a)*b/(4*a*c-b^2)*c+1/2/a^2/(c*x^4+b*x^2+a)*b
^3/(4*a*c-b^2)+2/a^2/(4*a*c-b^2)*c*ln(c*x^4+b*x^2+a)*b-1/2/a^3/(4*a*c-b^2)*
ln(c*x^4+b*x^2+a)*b^3-6/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^
(1/2))*c^2+6/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^
2*c-1/a^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4-1/2/a
^2/x^2-2/a^3*b*ln(x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 8.81, size = 5491, normalized size = 33.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] (log(a + b*x^2 + c*x^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))
/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)) - (1/(2*a) - (x
^2*(2*b^3 - 7*a*b*c))/(2*a^2*(4*a*c - b^2)) + (c*x^4*(3*a*c - b^2))/(a^2*(4
*a*c - b^2)))/(a*x^2 + b*x^4 + c*x^6) - (2*b*log(x))/a^3 + (atan(((2*a^9*b^
6*(4*a*c - b^2)^(9/2) - 128*a^12*c^3*(4*a*c - b^2)^(9/2) - 24*a^10*b^4*c*(4
```

*a*c - b^2)^(9/2) + 96*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))*(3*b^6 - 3*a^3*c^3 + 36*a^2*b^2*c^2 - 21*a*b^4*c)*((4*(2*b^5*c^4 - 12*a*b^3*c^5 + 18*a^2*b*c^6))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) + (((4*(9*a^5*c^6 - 4*a^2*b^6*c^3 + 29*a^3*b^4*c^4 - 54*a^4*b^2*c^5))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2 + 18*a^5*b^5*c^3 - 46*a^6*b^3*c^4))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (2*(a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/((a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)) + (((((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2 + 18*a^5*b^5*c^3 - 46*a^6*b^3*c^4))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (2*(a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/((a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) - ((a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(a^3*(4*a*c - b^2)^(3/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) - ((a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*a^6*(4*a*c - b^2)^3*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)*(36*a^4*c^6 + b^8*c^2 - 12*a*b^6*c^3 + 48*a^2*b^4*c^4 - 72*a^3*b^2*c^5)) - (x^2*((((4*(54*a^3*c^8 - 2*b^6*c^5 + 18*a*b^4*c^6 - 54*a^2*b^2*c^7))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((4*(276*a^5*b*c^7 - 6*a^2*b^7*c^4 + 65*a^3*b^5*c^5 - 233*a^4*b^3*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((4*(480*a^8*c^7 - a^4*b^8*c^3 + 6*a^5*b^6*c^4 + 30*a^6*b^4*c^5 - 272*a^7*b^2*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)) - (((((4*(480*a^8*c^7 - a^4*b^8*c^3 + 6*a^5*b^6*c^4 + 30*a^6*b^4*c^5 - 272*a^7*b^2*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6* b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/(a^3*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/(2*a^6*(4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(3*b^6 - 3*a^3*c^3 + 36*a^2*b^2*c^2 - 21*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)) - (b*((((((4*(480*a^8*c^7 - a^4*b^8*c^3 + 6*a^5*b^6*c^4 + 30*a^6*b^4*c^5 - 272*a^7*b^2*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/(a^3*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)) - (((4*(276*a^5*b*c^7 - 6*a^2*b^7*c^4 + 65*a^3*b^5*c^5 - 233*a^4*b^3*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((4*(480*a^8*c^7 - a^4*b^8*c^3 + 6*a^5*b^6*c^4 + 30*a^6*b^4*c^5 - 272*a^7*b^2*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^3*(640*a^10*b*c^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/(2*a^9*(4*a*c - b^2)^(9/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(3*b^6 - 49*a^3*c^3 + 72*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)))*(2*a^9*b^6*(4*a*c - b^2)^(9/2) - 128*a^12*c^3*(4*a*c - b^2)^(9/2) - 24*a^10*b^4*c*(4*a*c - b^2)^(9/2) + 96*a^11*b^2*c^2*(4*a*c - b^2)^(9/2)))/(36*a^4*c^6 + b^8*c^2 - 12*a*b^6*c^3 + 48*a^2*b^4*c^4 - 72*a^3*b^2*c^5) + (b*((((((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2 + 18*a^5*b^5*c^3 - 46*a^6*b^3*c^4))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (2*(a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/((a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) - ((a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))

```
/(a^3*(4*a*c - b^2)^(3/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 6
4*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b
^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*
c^2)) - (((4*(9*a^5*c^6 - 4*a^2*b^6*c^3 + 29*a^3*b^4*c^4 - 54*a^4*b^2*c^5))
/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2
+ 18*a^5*b^5*c^3 - 46*a^6*b^3*c^4))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) -
(2*(a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*
a^2*b^3*c^2 - 12*a*b^5*c))/((a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 -
 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2
*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^
2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) + ((a^7
*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^3)
/(2*a^9*(4*a*c - b^2)^(9/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))*(2*a^9*b
^6*(4*a*c - b^2)^(9/2) - 128*a^12*c^3*(4*a*c - b^2)^(9/2) - 24*a^10*b^4*c*(
4*a*c - b^2)^(9/2) + 96*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))*(3*b^6 - 49*a^3*c
^3 + 72*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(9*a^4*c^
4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)*(36*a^4*c^6 + b
^8*c^2 - 12*a*b^6*c^3 + 48*a^2*b^4*c^4 - 72*a^3*b^2*c^5)))*(b^4 + 6*a^2*c^2
 - 6*a*b^2*c))/(a^3*(4*a*c - b^2)^(3/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.867 $\int \frac{x^8}{(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=331

$$-\frac{\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}+$$

```
[Out] 1/2*(-10*a*c+3*b^2)*x/c^2/(-4*a*c+b^2)-1/2*b*x^3/c/(-4*a*c+b^2)+1/2*x^5*(b*
x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a
*c+b^2)^(1/2))^(1/2))*(3*b^3-13*a*b*c+(-20*a^2*c^2+19*a*b^2*c-3*b^4)/(-4*a*
c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4
*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^3-13*a*b*c+(20
*a^2*c^2-19*a*b^2*c+3*b^4)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)
/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.84, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1120, 1279, 1166, 205}

$$-\frac{\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((3*b^2 - 10*a*c)*x)/(2*c^2*(b^2 - 4*a*c)) - (b*x^3)/(2*c*(b^2 - 4*a*c)) +
(x^5*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3 - 13*a*
b*c - (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*S
qrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3 - 13*a*b*c + (3*b^4 - 19*a*b^2*c + 20
*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{x^4\left(10a+3bx^2\right)}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{bx^3}{2c\left(b^2-4ac\right)} + \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\int \frac{x^2\left(9ab+3\left(3b^2-10ac\right)x^2\right)}{a+bx^2+cx^4}\,dx}{6c\left(b^2-4ac\right)}$$

$$= \frac{\left(3b^2-10ac\right)x}{2c^2\left(b^2-4ac\right)} - \frac{bx^3}{2c\left(b^2-4ac\right)} + \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{3a\left(3b^2-10ac\right)+3b\left(3b^2-1\cdots\right.}{a+bx^2+cx^4}}{6c^2\left(b^2-4ac\right.\cdots}$$

$$= \frac{\left(3b^2-10ac\right)x}{2c^2\left(b^2-4ac\right)} - \frac{bx^3}{2c\left(b^2-4ac\right)} + \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(3b^3-13abc-\frac{3b^4-19\cdots}{\cdots}\right.}{4\cdots}$$

$$= \frac{\left(3b^2-10ac\right)x}{2c^2\left(b^2-4ac\right)} - \frac{bx^3}{2c\left(b^2-4ac\right)} + \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(3b^3-13abc-\frac{3b^4-19\cdots}{\cdots}\right.}{2\sqrt{2}\,c^{5/2}\left(b^2\cdots\right.}$$

Mathematica [A] time = 0.66, size = 327, normalized size = 0.99

$$\frac{-\frac{2\sqrt{c}\,x\left(2a^2c-ab\left(b-3cx^2\right)+b^3\left(-x^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\sqrt{2}\left(-20a^2c^2+19ab^2c-13abc\sqrt{b^2-4ac}+3b^3\sqrt{b^2-4ac}-3b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(20a^2c^2-19ab^2c-13\cdots\right.}{\cdots}}{4c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*Sqrt[c]*x - (2*Sqrt[c]*x*(2*a^2*c - b^3*x^2 - a*b*(b - 3*c*x^2)))/((b^2
- 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4 + 19*a*b^2*c - 20*a^2*c^2
+ 3*b^3*Sqrt[b^2 - 4*a*c] - 13*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) - (Sqrt[2]*(3*b^4 - 19*a*b^2*c + 20*a^2*c^2 + 3*b^3*Sqrt[b^2 -
 4*a*c] - 13*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c
^(5/2))
```

fricas [B] time = 1.10, size = 2856, normalized size = 8.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(b^2*c - 4*a*c^2)*x^5 + 2*(3*b^3 - 11*a*b*c)*x^3 + sqrt(1/2)*(a*b^2*
c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt
(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*
b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^
2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*
a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64
*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5
*c^3)*x + 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^
3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b^7*c^6 +
336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 918*a*b^
6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b
^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385
*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 6
4*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3
 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13))
)/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqrt(1/2)*(a*b
^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*s
qrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12
*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051
*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 +
48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 -
 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*
a^5*c^3)*x - 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818
*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b^7*c^6
 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 918*a
*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*
a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c +
385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7
- 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*
c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^1
3)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + sqrt(1/2)*(
a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2
)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 -
 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3
051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^
7 - 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 25
00*a^5*c^3)*x + 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8
818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 + (3*b^9*c^5 - 52*a*b^7*
c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 91
8*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 -
```

```
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c
 + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c
^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b
^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*
c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqrt(1/2
)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*
x^2)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^
5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c
+ 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c
^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2
*c^7 - 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 -
 2500*a^5*c^3)*x - 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2
- 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 + (3*b^9*c^5 - 52*a*b
^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 -
 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10
 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^
5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^
2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^
3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a
^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + 2*(3*
a*b^2 - 10*a^2*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^
3*c^2 - 4*a*b*c^3)*x^2)
```

giac [B] time = 1.17, size = 3339, normalized size = 10.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b*c*x^3 + a*b^2*x - 2*a^2*c*x)/((c*x^4 + b*x^2 + a)*(b^2
*c^2 - 4*a*c^3)) + x/c^2 + 1/16*(6*b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8
 - 928*a^3*b^3*c^9 + 640*a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^7*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^8*c^5 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^5*c^6 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^6*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^7*c^6 + 464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b^3*c^7 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b^4*c^7 + 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b^5*c^7 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^4*b*c^8 - 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b^2*c^8 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b^3*c^8 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
```

c)*c)*a^3*b*c^9 - 6*(b^2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*a*c)*a*b^5*c^7 - 19
2*(b^2 - 4*a*c)*a^2*b^3*c^8 + 160*(b^2 - 4*a*c)*a^3*b*c^9 - (6*b^5*c^2 - 50
*a*b^3*c^3 + 104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*b^4*c - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b
*c^2 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 +
13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 6*(b
^2 - 4*a*c)*b^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^3)*(b^2*c^2 - 4*a*c^3)^2 - 2*(
3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^5*c^4 + 6*a*b^6*c^4 + 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*b^2*c^5 + 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 3*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 68*a^2*b^4*c^5 - 160*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^3*b*c^6 - 22*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*
c^6 + 256*a^3*b^2*c^6 + 40*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^7
- 320*a^4*c^7 - 6*(b^2 - 4*a*c)*a*b^4*c^4 + 44*(b^2 - 4*a*c)*a^2*b^2*c^5 -
80*(b^2 - 4*a*c)*a^3*c^6)*abs(-b^2*c^2 + 4*a*c^3))*arctan(2*sqrt(1/2)*x/sqr
t((b^3*c^2 - 4*a*b*c^3 + sqrt((b^3*c^2 - 4*a*b*c^3)^2 - 4*(a*b^2*c^2 - 4*a^
2*c^3)*(b^2*c^3 - 4*a*c^4)))/(b^2*c^3 - 4*a*c^4)))/((a*b^6*c^5 - 12*a^2*b^4
*c^6 - 2*a*b^5*c^6 + 48*a^3*b^2*c^7 + 16*a^2*b^3*c^7 + a*b^4*c^7 - 64*a^4*c
^8 - 32*a^3*b*c^8 - 8*a^2*b^2*c^8 + 16*a^3*c^9)*abs(-b^2*c^2 + 4*a*c^3)*abs
(c)) - 1/16*(6*b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8 - 928*a^3*b^3*c^9 +
640*a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^7*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*
c^5 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5
*c^6 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c
^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^6 +
464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 +
192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7
+ 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 -
320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 1
60*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 -
96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 +
80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 6*
(b^2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*a*c)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*
b^3*c^8 + 160*(b^2 - 4*a*c)*a^3*b*c^9 - (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2
*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 +
25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2)*sqrt(

```
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b^3*c^2
+ 26*(b^2 - 4*a*c)*a*b*c^3)*(b^2*c^2 - 4*a*c^3)^2 + 2*(3*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^4*c^4 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 6*a*b
^6*c^4 + 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 44*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 3*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^4*c^5 + 68*a^2*b^4*c^5 - 160*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^
6 - 22*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 256*a^3*b^2*c^
6 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^7 + 320*a^4*c^7 + 6*(b
^2 - 4*a*c)*a*b^4*c^4 - 44*(b^2 - 4*a*c)*a^2*b^2*c^5 + 80*(b^2 - 4*a*c)*a^3
*c^6)*abs(-b^2*c^2 + 4*a*c^3))*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^2 - 4*a*b*c
^3 - sqrt((b^3*c^2 - 4*a*b*c^3)^2 - 4*(a*b^2*c^2 - 4*a^2*c^3)*(b^2*c^3 - 4*
a*c^4)))/(b^2*c^3 - 4*a*c^4)))/((a*b^6*c^5 - 12*a^2*b^4*c^6 - 2*a*b^5*c^6 +
 48*a^3*b^2*c^7 + 16*a^2*b^3*c^7 + a*b^4*c^7 - 64*a^4*c^8 - 32*a^3*b*c^8 -
8*a^2*b^2*c^8 + 16*a^3*c^9)*abs(-b^2*c^2 + 4*a*c^3)*abs(c))
```

maple [B] time = 0.04, size = 844, normalized size = 2.55

$$\frac{3ab\,x^3}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)c}-\frac{b^3x^3}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)c^2}+\frac{5\sqrt{2}\,a^2\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/c^2*x+3/2/c/(c*x^4+b*x^2+a)*b/(4*a*c-b^2)*x^3*a-1/2/c^2/(c*x^4+b*x^2+a)*b
^3/(4*a*c-b^2)*x^3+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*x-1/2/c^2/(c*x^4+b*x
^2+a)*a/(4*a*c-b^2)*x*b^2+13/4/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b-3/4/
c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3+5/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*a^2-19/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*a*b^2+3/4/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*
x)*b^4-13/4/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b+3/4/c^2/(4*a*c-b^2)*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)*b^3+5/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^
```

```
(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2-19
/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2+3/4/c^2/(4*a*c
-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(b^3-3\,abc\right)x^3+\left(ab^2-2\,a^2c\right)x}{2\left(ab^2c^2-4\,a^2c^3+\left(b^2c^3-4\,ac^4\right)x^4+\left(b^3c^2-4\,abc^3\right)x^2\right)}+\frac{-\int\frac{3\,ab^2-10\,a^2c+\left(3\,b^3-13\,abc\right)x^2}{cx^4+bx^2+a}\,dx}{2\left(b^2c^2-4\,ac^3\right)}+\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b^3 - 3*a*b*c)*x^3 + (a*b^2 - 2*a^2*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b
^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + 1/2*integrate(-(3*a*b^
2 - 10*a^2*c + (3*b^3 - 13*a*b*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c^2 - 4
*a*c^3) + x/c^2
```

mupad [B] time = 1.57, size = 7599, normalized size = 22.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((b*x^3*(3*a*c - b^2))/(2*(4*a*c - b^2)) + (a*x*(2*a*c - b^2))/(2*(4*a*c -
b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) - atan(((((10240*a^5*c^7 + 48*a*b^8*c^
3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6
- b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(-(9*b^13 + 9*b^4*(-(4*a*c
 - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 +
 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2
) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11
+ b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*
b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3
*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b
^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 -
 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3
*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*b^8 + 200*
a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5
+ b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 268
```

80*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 4
4800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*
a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10
*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2
*c^10)))^(1/2)*1i - (((10240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 422
4*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4
- 48*a^2*b^2*c^5)) + (x*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*
a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 4480
0*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b
^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^
6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^
10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7)
)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^
2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 3024
0*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 2
13*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^1
2*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c
^9 - 6144*a^5*b^2*c^10)))^(1/2) + (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2
- 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))
)*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b
^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2
c^2(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9
)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 -
1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i)/((((10
240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4
*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(
-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*
c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^
2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 128
0*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 -
192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b
*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5
*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 2
40*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))
^(1/2) - (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*
a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*
a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^
3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^
11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a
^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (((10240*a^5*c^7 + 48*a*b^8*c^3 -
736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b

^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (63*a^3*b^5 - 573*a^4*b^3*c + 1300*a^5*b*c^2)/(4*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5))))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*2i - atan(((((10240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i - (((10240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 422

4*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i)/((((10240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (((10240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4

```
*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*
c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13
 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10
656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a
*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(3
2*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^
6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (x*(9*b^8 + 200*a^4
*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*
a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 4480
0*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b
^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^
6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^
10)))^(1/2) + (63*a^3*b^5 - 573*a^4*b^3*c + 1300*a^5*b*c^2)/(4*(64*a^3*c^6
- b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5))))*(-(9*b^13 - 9*b^4*(-(4*a*c -
b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30
240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b
^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4
*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*2i + x/c^2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.868 $\int \frac{x^6}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=271

$$\frac{\left(-\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+\right.}$$

```
[Out] -1/2*b*x/c/(-4*a*c+b^2)+1/2*x^3*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/
4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-6*a*c-b*(-8*a
*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1
/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-
6*a*c+b*(-8*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)*2^(1/2)/(b+(-
4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1120, 1279, 1166, 205}

$$\frac{\left(-\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b*x)/(2*c*(b^2 - 4*a*c)) + (x^3*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^
2 + c*x^4)) + ((b^2 - 6*a*c - (b*(b^2 - 8*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 -
4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - 6*a*c + (b*(b^2 - 8*a*c))/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2
*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int \frac{x^2\left(6a+bx^2\right)}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{bx}{2c\left(b^2-4ac\right)}+\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\int \frac{ab+\left(b^2-6ac\right)x^2}{a+bx^2+cx^4}\,dx}{2c\left(b^2-4ac\right)}$$

$$= -\frac{bx}{2c\left(b^2-4ac\right)}+\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b^2-6ac-\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}$$

$$= -\frac{bx}{2c\left(b^2-4ac\right)}+\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b^2-6ac-\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4}}}\right.}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.50, size = 282, normalized size = 1.04

$$\frac{-\frac{2\sqrt{c}\,x\left(a\left(b-2cx^2\right)+b^2x^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}+8abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}-8abc+b^3\right)\tan^{-1}\left(\cdots\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4c^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a + b*x^2 + c*x^4)^2,x]

[Out] ((-2*Sqrt[c]*x*(b^2*x^2 + a*(b - 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x ^4)) + (Sqrt[2]*(-b^3 + 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3 - 8*a*b*c + b^2*Sqr t[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/ (4*c^(3/2))

fricas [B] time = 1.23, size = 2257, normalized size = 8.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] -1/4*(2*(b^2 - 2*a*c)*x^3 + 2*a*b*x - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60 *a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b ^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64 *a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5* a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x + 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 8 8*a^2*b^3*c^2 - 144*a^3*b*c^3 - (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c ^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqr t((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) + sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c ^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x - 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 - (b^8*c ^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((

```
b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 6
4*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c
^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6
*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^
4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a
*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*
a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^
4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*
a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a
*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x + 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88
*a^2*b^3*c^2 - 144*a^3*b*c^3 + (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 -
640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^
6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c +
 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt
((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 -
 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) + s
qrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^
2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 +
48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6
- 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 4
8*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x -
 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 + (b^8*c^
3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b
^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^
4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*
c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4
 + 48*a^2*b^2*c^5 - 64*a^3*c^6))))/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a
^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)
```

giac [B] time = 1.06, size = 2736, normalized size = 10.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*x^3 - 2*a*c*x^3 + a*b*x)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))
- 1/16*(2*b^8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*
```

c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6 - (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*abs(b^2*c - 4*a*c^2))*arctan(2*sqrt(1/2)*x/sqrt((b^3*c - 4*a*b*c^2 + sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3)))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*abs(b^2*c - 4*a*c^2)*abs(c)) + 1/16*(2*b^8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6 - (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*

```
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a
^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b
*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*abs(b^2*c - 4
*a*c^2))*arctan(2*sqrt(1/2)*x/sqrt((b^3*c - 4*a*b*c^2 - sqrt((b^3*c - 4*a*b
*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3)
))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3
*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*
abs(b^2*c - 4*a*c^2)*abs(c))
```

maple [B] time = 0.03, size = 602, normalized size = 2.22

$$\frac{2\sqrt{2}\, ab \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{2\sqrt{2}\, ab \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, b^3 \ldots}{4\left(4ac-b^2\right)\sqrt{-\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2*(2*a*c-b^2)/c/(4*a*c-b^2)*x^3+1/2/(4*a*c-b^2)*a*b/c*x)/(c*x^4+b*x^2+a
)-3/2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a+1/4/(4*a*c-b^2)/c*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*b^2+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b-1/4/(
4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3+3/2/(4*a*c-b^2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*c*x)*a-1/4/(4*a*c-b^2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2+2/(4*a*c-b^2)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b^2-2\,ac\right)x^3+abx}{2\left(\left(b^2c^2-4\,ac^3\right)x^4+ab^2c-4\,a^2c^2+\left(b^3c-4\,abc^2\right)x^2\right)}-\frac{-\int\frac{\left(b^2-6\,ac\right)x^2+ab}{cx^4+bx^2+a}\,dx}{2\left(b^2c-4\,ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((b^2 - 2*a*c)*x^3 + a*b*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2
*c^2 + (b^3*c - 4*a*b*c^2)*x^2) - 1/2*integrate(-((b^2 - 6*a*c)*x^2 + a*b)/
(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

mupad [B] time = 6.00, size = 6293, normalized size = 23.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((x^3*(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b*x)/(2*c*(4*a*c - b^2)))/(
a + b*x^2 + c*x^4) - atan(((((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*c
^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^
3)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*
b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^
8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16
*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 1
6*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^
5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*
c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^1
0*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^
2*c^8)))^(1/2) - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(b
^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2
7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 -
24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 614
4*a^5*b^2*c^8)))^(1/2)*1i - (((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*
c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c
^3)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2
*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*
c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b
^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(1
6*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c +
16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a
^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9
*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^
10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b
^2*c^8)))^(1/2) + (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(
b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2)
- 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 -
27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 -
```

24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 61
44*a^5*b^2*c^8)))^(1/2)*1i)/((((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5
*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*
c^3)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^
2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a
c - b^2)^9)^(1/2))/(32(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*
b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(
16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c +
16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*
a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^
9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b
^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*
b^2*c^8)))^(1/2) - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*
(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2)
- 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 -
27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3
- 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6
144*a^5*b^2*c^8)))^(1/2) + (((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*c
^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^
3)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*
b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c
- b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^
8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16
*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 1
6*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^
5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*
c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^1
0*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^
2*c^8)))^(1/2) + (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(b
^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2
7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 -
24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 614
4*a^5*b^2*c^8)))^(1/2) + (5*a^2*b^4 + 216*a^4*c^2 - 66*a^3*b^2*c)/(4*(b^6*c
- 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3))))*(-(b^11 + b^2*(-(4*a*c -
b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*
a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^
9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^
4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*2i - atan(((((16*a*b^7*c^2 - 1024*a^4
*b*c^5 - 192*a^2*b^5*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b
^4*c^2 + 48*a^2*b^2*c^3)) - (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 384
0*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*
b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a
*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^
5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b

^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*1i - (((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) + (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*1i)/((((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) + (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*

```
b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a
*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^
5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b
^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4*a*c - b
^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a
^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9
 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4
*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^
2 - 16*a*b^4*c))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4
*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3
+ 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096
*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 +
3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (5*a^2*b^4 + 216*a^4*c^2 - 6
6*a^3*b^2*c)/(4*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3))))*(-(
b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 15
04*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280
*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*2i
```

sympy [A] time = 51.73, size = 379, normalized size = 1.40

$$\frac{abx + x^3\left(-2ac + b^2\right)}{8a^2c^2 - 2ab^2c + x^4\left(8ac^3 - 2b^2c^2\right) + x^2\left(8abc^2 - 2b^3c\right)} + \text{RootSum}\left(t^4\left(1048576a^6c^9 - 1572864a^5b^2c^8 + 983040a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (a*b*x + x**3*(-2*a*c + b**2))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3 -
 2*b**2*c**2) + x**2*(8*a*b*c**2 - 2*b**3*c)) + RootSum(_t**4*(1048576*a**6
*c**9 - 1572864*a**5*b**2*c**8 + 983040*a**4*b**4*c**7 - 327680*a**3*b**6*c
**6 + 61440*a**2*b**8*c**5 - 6144*a*b**10*c**4 + 256*b**12*c**3) + _t**2*(-
61440*a**5*b*c**5 + 61440*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608*a**2
*b**7*c**2 - 432*a*b**9*c + 16*b**11) + 1296*a**5*c**2 - 360*a**4*b**2*c +
25*a**3*b**4, Lambda(_t, _t*log(x + (49152*_t**3*a**4*c**7 - 40960*_t**3*a*
*3*b**2*c**6 + 12288*_t**3*a**2*b**4*c**5 - 1536*_t**3*a*b**6*c**4 + 64*_t*
*3*b**8*c**3 - 1728*_t*a**3*b*c**3 + 656*_t*a**2*b**3*c**2 - 88*_t*a*b**5*c
 + 4*_t*b**7)/(324*a**3*c**2 - 81*a**2*b**2*c + 5*a*b**4))))
```

3.869 $\int \frac{x^4}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=237

$$\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] 1/2*x*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(1/2)
/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c-b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1
/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2+4*a*c+b*(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(3/2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1120, 1166, 205}

$$\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b - (b^2 + 4*a*
c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 +
 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
- 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*
c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
```

```
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{2a-bx^2}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= \frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4\left(b^2-4ac\right)^{3/2}} + \frac{\left(b^2+4\right.}{}$$

$$= \frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2+4\right.}{2\sqrt{2}}$$

Mathematica [A] time = 0.40, size = 235, normalized size = 0.99

$$\frac{1}{4}\left(\frac{2\left(2ax+bx^3\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-4ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+4ac+b^2\right.}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(2*a*x + b*x^3))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-b^2 -
 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
```

```
- 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sq
rt[2]*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*
a*c]]))/4
```

fricas [B] time = 1.13, size = 1668, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 -
4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^
3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))
/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*
x + sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*
a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3
 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/
(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) - sqrt(1/2)*((b^2*c
- 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b
*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12
*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b
^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x - sqrt(1/2)*(b^4 - 8*a*b^2*c +
16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*
c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^
2*c^3 - 64*a^3*c^4))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c
+ (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a
^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2
+ 4*a*c)*x + sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - 2*(b^7*c - 12*a*b^5*
c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b
^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^
2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) - sqrt(1/2)
*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3
 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 +
 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x - sqrt(1/2)*(b^4 - 8*a
*b^2*c + 16*a^2*c^2 - 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c
^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3
+ 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*
```

```
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 +
48*a^2*b^2*c^3 - 64*a^3*c^4))) + 4*a*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*
a^2*c + (b^3 - 4*a*b*c)*x^2)
```

giac [B] time = 1.06, size = 2132, normalized size = 9.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^3 + 2*a*x)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*b^7*c^2 -
 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^4 - (2*b^3
*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)
*b*c^2)*(b^2 - 4*a*c)^2 + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*
c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^
3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2
 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*abs(b^2 - 4*a*c))*arctan(2*s
qrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*
c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a
*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^
3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*b^
7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^4 -
```

```
 (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 -
 4*a*c)*b*c^2)*(b^2 - 4*a*c)^2 - 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b^4*c^2 + 16*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 16*a^2
*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32*a^3*c^4 -
 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*abs(b^2 - 4*a*c))*arc
tan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 -
 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^
2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4
- 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*abs(c))
```

maple [B] time = 0.03, size = 452, normalized size = 1.91

$$-\frac{\sqrt{2}\, a c \,\mathrm{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, a c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, b \cdots}{4\left(4ac-b^2\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2*b/(4*a*c-b^2)*x^3-a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4/(4*a*c-b^2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*b-1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*a-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2-1/4/(4*a
*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)*b-1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*a-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bx^3+2\,ax}{2\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)}+\frac{-\frac{\left(\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,b^4-8\,\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,ab^2c-2\,\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,b^3c\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^3 + 2*a*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b
*c)*x^2) + 1/2*integrate((b*x^2 - 2*a)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c
)
```

mupad [B] time = 5.91, size = 4973, normalized size = 20.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - atan(((((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4
)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(((-(4*a*c - b^
2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(
b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
+ 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3
- 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*((
(-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b
^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280
*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) - (x*(b^4*c + 8
*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b
^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*
(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*1i - (((2048*a^4*c^5 - 32*a
*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^
2*b^2*c^2 - 12*a*b^4*c)) + (x*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*
c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b
^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*
b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3
*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9
 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6
*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^
5 - 6144*a^5*b^2*c^6)))^(1/2) + (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b
```

```
^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b
*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*
b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5
*b^2*c^6)))^(1/2)*1i)/((((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1
536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*
(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3
*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 12
80*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 -
 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 -
8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*
c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2
*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)
- (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))
*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^
3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1
280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) - (4*a^2*b*c
^2 + 3*a*b^3*c)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((2
048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 -
64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(((-(4*a*c - b^2)^9)^(1/2)
- b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 409
6*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b
^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b
*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b
^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*
(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) + (x*(b^4*c + 8*a^2*c^3 + 2
*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2)
 - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 40
96*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*
b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 76
8*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7
- 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6
144*a^5*b^2*c^6)))^(1/2)*2i - atan(((((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^
2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*
b^4*c)) - (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5
*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^
2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)
*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 +
 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^
4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^1
0*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^
2*c^6)))^(1/2) - (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*
b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240
*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1
```

/2)*1i - (((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^
4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(b^9 + (-(4*
a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32
*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^
4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^
3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*
(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^
3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1
280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) + (x*(b^4*c
+ 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-
(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/
(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6
*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*1i)/((((2048*a^4*c^5 -
32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 4
8*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a
^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 2
4*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144
*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^
2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)
^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 40
96*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*
b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) - (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))
/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 76
8*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7
- 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6
144*a^5*b^2*c^6)))^(1/2) + (((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3
 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) +
 (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 51
2*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3
 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*
c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c
^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^
2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 2
40*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^
(1/2) + (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^
2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 +
 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*
c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) - (4*
a^2*b*c^2 + 3*a*b^3*c)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512
*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3
- 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*2i - ((a*
x)/(4*a*c - b^2) + (b*x^3)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)

sympy [A] time = 9.01, size = 296, normalized size = 1.25

$$\frac{-2ax-bx^3}{8a^2c-2ab^2+x^4\left(8ac^2-2b^2c\right)+x^2\left(8abc-2b^3\right)}+\text{RootSum}\left(t^4\left(1048576a^6c^7-1572864a^5b^2c^6+983040a^4b^4c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (-2*a*x - b*x**3)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*
(8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**6*c**7 - 1572864*a**5*b**2*
c**6 + 983040*a**4*b**4*c**5 - 327680*a**3*b**6*c**4 + 61440*a**2*b**8*c**3
 - 6144*a*b**10*c**2 + 256*b**12*c) + _t**2*(-12288*a**4*b*c**4 + 8192*a**3
*b**3*c**3 - 1536*a**2*b**5*c**2 + 16*b**9) + 16*a**3*c**2 + 24*a**2*b**2*c
 + 9*a*b**4, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4 - 12288*_t**3*a
**2*b**3*c**3 + 3072*_t**3*a*b**5*c**2 - 256*_t**3*b**7*c + 64*_t*a**2*c**2
 - 128*_t*a*b**2*c - 4*_t*b**4)/(4*a*c + 3*b**2))))
```

3.870 $\int \frac{x^2}{(a+bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=221

$$-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -1/2*x*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*arctan(x*2^(1/2)*c^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1119, 1166, 205}

$$-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(x*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(2*b -
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*b
+ Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
```

```
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\int \frac{b-2cx^2}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(c\left(1+\frac{2b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\left(b^2-4ac\right)}+\frac{\left(c\left(2b-\sqrt{b^2\right.\right.}{}$$

$$= -\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(2b\right.}{\sqrt{2}}$$

Mathematica [A] time = 0.43, size = 222, normalized size = 1.00

$$\frac{-bx-2cx^3}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}-2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}}}\right.}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (-(b*x) - 2*c*x^3)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(-2*b +
 Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*b
```

```
 + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

fricas [B] time = 0.98, size = 1680, normalized size = 7.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c
^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
)/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*
c^2)*x + 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6
*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3
*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) - sqrt(1/2)*
((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3
+ 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c +
48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2)*x - 1/2*sqrt(1/2)*(b^
5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256
*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(
-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqr
t(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^
4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a
*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a
^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^
4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^
3))*log((3*b^2*c + 4*a*c^2)*x + 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c
^2 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c - (a*b^6
- 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*
a^4*c^3))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2
- 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(
a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2
)*x - 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + (a*b^8 - 8*a^2*b^6*c
+ 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^
2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
```

```
^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) + 2*b*x)/((b^2*
c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

giac [B] time = 0.98, size = 1970, normalized size = 8.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] -1/2*(2*c*x^3 + b*x)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)) + 1/8*(4*b^6*c^2 -
 32*a*b^4*c^3 + 64*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^5*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*
c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
 - 4*(b^2 - 4*a*c)*b^4*c^2 + 16*(b^2 - 4*a*c)*a*b^2*c^3 - (2*b^2*c^2 - 8*a*
c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*(b^2 - 4*a*c)
^2 + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*
c - 2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b*c^3 - 32*a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c
^2)*abs(b^2 - 4*a*c))*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3
- 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/
((a*b^6 - 12*a^2*b^4*c - 2*a*b^5*c + 48*a^3*b^2*c^2 + 16*a^2*b^3*c^2 + a*b^
4*c^2 - 64*a^4*c^3 - 32*a^3*b*c^3 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*abs(b^2 - 4
*a*c)*abs(c)) - 1/8*(4*b^6*c^2 - 32*a*b^4*c^3 + 64*a^2*b^2*c^4 - 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 16*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*(b^2 - 4*a*c)*b^4*c^2 + 16*(b^2 - 4*a*
c)*a*b^2*c^3 - (2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*c^2 - 2*
```

```
(b^2 - 4*a*c)*c^2)*(b^2 - 4*a*c)^2 - (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*a*b^3*c^2 - 4*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 32*a^2*b*c^3 - 2*(b^2 - 4*a*
c)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2)*abs(b^2 - 4*a*c))*arctan(2*sqrt(1/2)*x/
sqrt((b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c -
 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6 - 12*a^2*b^4*c - 2*a*b^5*c + 48*a^3
*b^2*c^2 + 16*a^2*b^3*c^2 + a*b^4*c^2 - 64*a^4*c^3 - 32*a^3*b*c^3 - 8*a^2*b
^2*c^3 + 16*a^3*c^4)*abs(b^2 - 4*a*c)*abs(c))
```

maple [A] time = 0.08, size = 342, normalized size = 1.55

$$\frac{\sqrt{2}\, bc \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, bc \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, c \operatorname{arctanh}}{2\left(4ac-b^2\right)\sqrt{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2/(4*a*c-b^2)*x/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))+c/(4*a*c-b^2)/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b-1/2*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)+1/2/(4*a*c-b^2)*x/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)+c/(4*a*c-b^2)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b+1/2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^3 + b*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*x^2) - 1/2*integrate((2*c*x^2 - b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*
c)
```

mupad [B] time = 1.35, size = 4854, normalized size = 21.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] atan(((((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^
6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(((-(4*a*c - b^2)^9)^(1
/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 +
 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a
^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*
b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)
^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*
b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 +
3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) - (x*(4*a*c^4 - 5*b^2*c^3))/(b
^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*
c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2
*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*
b^2*c^5)))^(1/2)*1i - (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2
*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(((-(4*
a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^
3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*
b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^
5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(
((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*
b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 128
0*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) + (x*(4*a*c^4
- 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) -
b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*
a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4
*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*1i)/((4*a*c^4 + 3*b^2*c^3)/(2*(b^6 - 64*a^
3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a
^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^
4*c)) + (x*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^
2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b
^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8
*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c
^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*
b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240
*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1
/2) - (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c
- b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(
32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*
```

c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) + (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) + (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*2i + atan(((((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) - (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*1i - (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) + (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*1i)/((4*a*c^4 + 3*b^2*c^3)/(2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1

280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) - (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) + (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) + (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*2i + ((b*x)/(2*(4*a*c - b^2)) + (c*x^3)/(4*a*c - b^2))/(a + b*x^2 + c*x^4)

sympy [A] time = 20.75, size = 298, normalized size = 1.35

$$\frac{bx+2cx^3}{8a^2c-2ab^2+x^4\left(8ac^2-2b^2c\right)+x^2\left(8abc-2b^3\right)}+\mathrm{RootSum}\Bigg(t^4\left(1048576a^7c^6-1572864a^6b^2c^5+983040a^5b^4c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (b*x + 2*c*x**3)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(
8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**7*c**6 - 1572864*a**6*b**2*c
**5 + 983040*a**5*b**4*c**4 - 327680*a**4*b**6*c**3 + 61440*a**3*b**8*c**2
- 6144*a**2*b**10*c + 256*a*b**12) + _t**2*(-12288*a**4*b*c**4 + 8192*a**3*
b**3*c**3 - 1536*a**2*b**5*c**2 + 16*b**9) + 16*a**2*c**3 + 24*a*b**2*c**2
+ 9*b**4*c, Lambda(_t, _t*log(x + (16384*_t**3*a**5*c**4 - 8192*_t**3*a**4*
b**2*c**3 + 512*_t**3*a**2*b**6*c - 64*_t**3*a*b**8 - 128*_t*a**2*b*c**2 -
16*_t*a*b**3*c - 4*_t*b**5)/(4*a*c**2 + 3*b**2*c))))
```

3.871 $\int \frac{1}{\left(a+bx^2+cx^4\right)^2}\, dx$

Optimal. Leaf size=252

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\ldots\right.}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{\ldots}}$$

```
[Out] 1/2*x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/
2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c+b*(-4*a*c+b^2)
^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arcta
n(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c-b*(-4
*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1092, 1166, 205}

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\ldots\right.}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(-2),x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt
[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sq
rt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
```

```
x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{b^2-2ac-2\left(b^2-4ac\right)-bcx^2}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\left(b^2-4ac\right)^{3/2}} + \cdots$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \cdots$$

Mathematica [A] time = 0.42, size = 243, normalized size = 0.96

$$\frac{\frac{2x\left(-2ac+b^2+bcx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+12ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(-2),x]
```

```
[Out] ((2*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[
2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
```

```
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/(4*a)
```

fricas [B] time = 1.18, size = 2309, normalized size = 9.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c +
 (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6
 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*
a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6
- 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^
3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 67
2*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 4
48*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b
^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c
+ 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqr
t((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
- 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) -
sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b
*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c +
 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6
 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c +
48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*
x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 8
64*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 +
512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c
 + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 +
(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2
*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(
a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b
^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c +
 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)
*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3
*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sq
rt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*
```

```
b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x
^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c +
 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt
((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log(
(5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*
c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3*b^9 - 20*a^4*b^7
*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^
2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*
sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b
^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^
2*c^2 - 64*a^6*c^3))) + 2*(b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2
*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

giac [B] time = 0.86, size = 2682, normalized size = 10.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c*x^3 + b^2*x - 2*a*c*x)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1
/16*(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*
b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4 + (2*b^
3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c
)*b*c^2)*(a*b^2 - 4*a^2*c)^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b
^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sq
```

rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)*abs(a*b^2 - 4*a^2*c))*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2 - 4*a^2*c)*abs(c)) - 1/16*(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4 + (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 192*a^4*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c + 20*(b^2 - 4*a*c)*a^2*b^2*c^2 - 48*(b^2 - 4*a*c)*a^3*c^3)*abs(a*b^2 - 4*a^2*c))*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c - sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2 - 4*a^2*c)*abs(c))

maple [B] time = 0.06, size = 733, normalized size = 2.91

$$\frac{\sqrt{2}\, b^{2} c \arctanh\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{4 \sqrt{-4 a c+b^{2}}\, \left(4 a c-b^{2}\right) \sqrt{\left(-b+\sqrt{-4 a c+b^{2}}\right) c}\, a}+\frac{\sqrt{2}\, b^{2} c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{4 \sqrt{-4 a c+b^{2}}\, \left(4 a c-b^{2}\right) \sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}\, a}-\frac{3}{\sqrt{-4 a c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/4/(4*a*c-b^2)/a*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*b+c/(-4*a*c+b^2
)^(1/2)/(4*a*c-b^2)*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)-1/4/(-4*a*c+b^
2)^(1/2)/(4*a*c-b^2)/a*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*b^2+1/4*c/(
4*a*c-b^2)/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b-3*c^2/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)+1/4*c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*b^2-1/4/(4*a*c-b^2)/a*x/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)*b-c/(
-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*x/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)+1/4/
(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*x/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)*b
^2-1/4*c/(4*a*c-b^2)/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b-3*c^2/(-4*a*c+b^2)^(1/2)/(4*a*c
-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*c*x)+1/4*c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^3 + (b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^
3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((b*c*x^2 + b^2 - 6*a*c)/(c*x
^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

mupad [B] time = 6.00, size = 6404, normalized size = 25.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((x*(2*a*c - b^2))/(2*a*(4*a*c - b^2)) - (b*c*x^3)/(2*a*(4*a*c - b^2)))/(a
+ b*x^2 + c*x^4) + atan(((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 +
 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4
*c + 48*a^4*b^2*c^2)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^
5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*
c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b
^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^
2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4
*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c
 - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 38
40*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12
 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840
*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*
b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c
- b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 384
0*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12
+ 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*
a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*a^5*c^6 + 16*a*b^8*c^2
 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*
a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2
)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4
*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 409
6*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b
^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a
^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-
(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1
504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^
(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 128
0*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c
^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(
b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 15
04*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280
*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i)/((((6144*a^
5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^
5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11
+ b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^
3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*
b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16
*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2
```

- 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c *(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 2 40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^ (1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c* (-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 24 0*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^ 2)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2* b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^ 8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(10 24*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2 *b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2 7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 614 4*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2* b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27 *a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 2 4*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144 *a^8*b^2*c^5)))^(1/2) + (5*b^3*c^4 - 36*a*b*c^5)/(4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2 7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 614 4*a^8*b^2*c^5)))^(1/2)*2i + atan(((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2* b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 1 2*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 61 44*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2 *(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5 *c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32* (a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c ^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^4*c^ 3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2* (-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5* c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(

a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i)/((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*

```
c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))
/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)
^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3
*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^
9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c
^4 - 6144*a^8*b^2*c^5)))^(1/2) + (5*b^3*c^4 - 36*a*b*c^5)/(4*(a^2*b^6 - 64*
a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*(-(b^11 - b^2*(-(4*a*c - b^2)^9
)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^
3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a
^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*
c^4 - 6144*a^8*b^2*c^5)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.872 $\int \frac{1}{x^2(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=308

$$-\frac{3b^2-10ac}{2a^2x(b^2-4ac)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4\right.}{2\sqrt{2}\,a^2\left(b^2-\right.}$$

```
[Out] 1/2*(10*a*c-3*b^2)/a^2/(-4*a*c+b^2)/x+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2
)/x/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2))*c^(1/2)*(3*b^3-16*a*b*c+(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c
+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c-(-10*a*c+3*b^2)*(-
4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/
2)
```

Rubi [A] time = 1.44, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1121, 1281, 1166, 205}

$$-\frac{3b^2-10ac}{2a^2x(b^2-4ac)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4\right.}{2\sqrt{2}\,a^2\left(b^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b
^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 -
10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
(Sqrt[c]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-3b^2+10ac-3bcx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx}{2a\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{\int \frac{-b\left(3b^2-13ac\right)-c\left(3b^2-10ac\right)x^2}{a+bx^2+cx^4}\,dx}{2a^2\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(3b^2-10ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\frac{1}{\sqrt{b}}\right.\right.}{4a^2\left(b^2-\right.}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} - \frac{\sqrt{c}\left(3b^2-10ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\right.}{2\sqrt{2}\,a^2\left(b^2-4ac\right)}$$

Mathematica [A] time = 0.60, size = 302, normalized size = 0.98

$$\frac{-\frac{2x\left(-3abc-2ac^2x^2+b^3+b^2cx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4a\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (-4/x - (2*x*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4
*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2
]*Sqrt[c]*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a
*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

fricas [B] time = 1.38, size = 2912, normalized size = 9.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(3*b^2*c - 10*a*c^2)*x^4 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c
)*x^2 - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3
+ (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420
*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((8
1*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^
6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4
+ 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x + 1/2*sqrt(1/2)*(27*b^11 - 486*a*b^9*c
 + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^
5 - (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*
a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2
- 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c
^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*
b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8
 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b
^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4
*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^
5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a
*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7
*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550
*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 6
4*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-
(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x - 1/2*sq
rt(1/2)*(27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 144
08*a^4*b^3*c^4 - 5200*a^5*b*c^5 - (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*
c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 9
18*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*
c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*
c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*
b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^1
3*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) - sqrt(1/
2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*
a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a
^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b
^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^
11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^
7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*
c^5 - 2500*a^3*c^6)*x + 1/2*sqrt(1/2)*(27*b^11 - 486*a*b^9*c + 3330*a^2*b^7
*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5 + (3*a^5*b^10
 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1
280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*
c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^
3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^
6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c
```

```
+ 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^
4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2
*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 -
4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^
2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a
^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 +
625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 -
1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x - 1/2*sqrt(1/2)*(27*b^1
1 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4
- 5200*a^5*b*c^5 + (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*
b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 30
51*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c
+ 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3
*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^
3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a
^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b
^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))))/((a^2*b^2*c - 4*a^3*c^2
)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)
```

giac [B] time = 1.34, size = 3087, normalized size = 10.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^2*c*x^4 - 10*a*c^2*x^4 + 3*b^3*x^2 - 11*a*b*c*x^2 + 2*a*b^2 - 8*a
^2*c)/((c*x^5 + b*x^3 + a*x)*(a^2*b^2 - 4*a^3*c)) - 1/16*(6*a^4*b^8*c^2 - 8
0*a^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^4*b^6*c^2 + 5
6*(b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4 + (6*b^4*c^2 -
44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^
```

2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c)^2 + 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 - 37*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 6*a^2*b^7*c + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 50*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 74*a^3*b^5*c^2 - 208*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 104*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 25*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 304*a^4*b^3*c^3 + 52*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 + 416*a^5*b*c^4 + 6*(b^2 - 4*a*c)*a^2*b^5*c - 50*(b^2 - 4*a*c)*a^3*b^3*c^2 + 104*(b^2 - 4*a*c)*a^4*b*c^3)*abs(a^2*b^2 - 4*a^3*c))*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^3 - 4*a^3*b*c + sqrt((a^2*b^3 - 4*a^3*b*c)^2 - 4*(a^3*b^2 - 4*a^4*c)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*a^3*c^2)))/((a^5*b^6 - 12*a^6*b^4*c - 2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6*b^3*c^2 + a^5*b^4*c^2 - 64*a^8*c^3 - 32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7*c^4)*abs(a^2*b^2 - 4*a^3*c)*abs(c)) + 1/16*(6*a^4*b^8*c^2 - 80*a^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^4*b^6*c^2 + 56*(b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4 + (6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c)^2 - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7 - 37*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c + 6*a^2*b^7*c + 152*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 50*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 74*a^3*b^5*c^2 - 208*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 104*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 25*sqrt(2)*sqrt(b*c - sqrt

```
(b^2 - 4*a*c)*c)*a^3*b^3*c^3 + 304*a^4*b^3*c^3 + 52*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^4*b*c^4 - 416*a^5*b*c^4 - 6*(b^2 - 4*a*c)*a^2*b^5*c + 50
*(b^2 - 4*a*c)*a^3*b^3*c^2 - 104*(b^2 - 4*a*c)*a^4*b*c^3)*abs(a^2*b^2 - 4*a
^3*c))*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^3 - 4*a^3*b*c - sqrt((a^2*b^3 - 4*a
^3*b*c)^2 - 4*(a^3*b^2 - 4*a^4*c)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*
a^3*c^2)))/((a^5*b^6 - 12*a^6*b^4*c - 2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6
*b^3*c^2 + a^5*b^4*c^2 - 64*a^8*c^3 - 32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7
*c^4)*abs(a^2*b^2 - 4*a^3*c)*abs(c))
```

maple [B] time = 0.04, size = 712, normalized size = 2.31

$$-\frac{c^{2} x^{3}}{\left(c\,x^{4}+b\,x^{2}+a\right)\left(4ac-b^{2}\right)a}+\frac{b^{2} c\,x^{3}}{2\left(c\,x^{4}+b\,x^{2}+a\right)\left(4ac-b^{2}\right)a^{2}}+\frac{4\sqrt{2}\;b\,c^{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{\left(4ac-b^{2}\right)\sqrt{-4ac+b^{2}}\;\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b
^2)*x^3*b^2-3/2/a/(c*x^4+b*x^2+a)*b*c/(4*a*c-b^2)*x+1/2/a^2/(c*x^4+b*x^2+a)
*b^3/(4*a*c-b^2)*x+5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-3/4/a^2*c/(4
*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*b-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*c*x)*b^3-5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+3/4/a^2*c/(4*a*c-b
^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*b^2+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*c*x)*b-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3
-1/a^2/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(3\,b^{2}c-10\,ac^{2}\right)x^{4}+2\,ab^{2}-8\,a^{2}c+\left(3\,b^{3}-11\,abc\right)x^{2}}{2\left(\left(a^{2}b^{2}c-4\,a^{3}c^{2}\right)x^{5}+\left(a^{2}b^{3}-4\,a^{3}bc\right)x^{3}+\left(a^{3}b^{2}-4\,a^{4}c\right)x\right)}+\frac{-\int\frac{3\,b^{3}-13\,abc+\left(3\,b^{2}c-10\,ac^{2}\right)x^{2}}{cx^{4}+bx^{2}+a}\,dx}{2\left(a^{2}b^{2}-4\,a^{3}c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*b^2*c - 10*a*c^2)*x^4 + 2*a*b^2 - 8*a^2*c + (3*b^3 - 11*a*b*c)*x^2
)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a
^4*c)*x) + 1/2*integrate(-(3*b^3 - 13*a*b*c + (3*b^2*c - 10*a*c^2)*x^2)/(c*
x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)
```

mupad [B] time = 6.72, size = 7555, normalized size = 24.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] - atan((((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 207
7*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 -
 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c -
 b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8
*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(85
1968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4
 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 + x*(-(
9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^
2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*
(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/
2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*
a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b
*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680
*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) + x*(204800*a^
12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*
a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 - 9*b^4
*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*
b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2
)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b
^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 +
3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i - ((-(9*b^13 - 9*b^4*(-(4*
a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^
3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 +
4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a
^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c
^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*
a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 - x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9
)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^
4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a
*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^
```

6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) - x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i)/(((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 + x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) + x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) + ((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 - x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) - x*(204800*a^12*c^9 + 144*a^6*b^12*c^

3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^1
0*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2
) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*
c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*
c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24
*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*
a^10*b^2*c^5)))^(1/2) + 128000*a^10*c^9 + 504*a^6*b^8*c^5 - 8112*a^7*b^6*c^
6 + 48704*a^8*b^4*c^7 - 129280*a^9*b^2*c^8))*(-(9*b^13 - 9*b^4*(-(4*a*c - b
^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 302
40*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^
11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*
c^4 - 6144*a^10*b^2*c^5)))^(1/2)*2i - atan((((-(9*b^13 + 9*b^4*(-(4*a*c - b
^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 302
40*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^
11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*
c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 46
72*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^
5*c^6 - 1261568*a^13*b^3*c^7 + x*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2)
 + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c
^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c
 - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*
a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a
^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^
11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1
572864*a^15*b^3*c^7)) + x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^
10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458
752*a^11*b^2*c^8))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b
*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5
*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 2
40*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))
^(1/2)*1i - ((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 +
 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c
^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a
c - b^2)^9)^(1/2))/(32(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7
*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)
*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9
*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 - x
*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^
9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*
c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)
^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1
280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^

16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 32
7680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) - x*(20480
0*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143
360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 + 9
b^4(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*
a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c -
 b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a
^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^
3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i)/(((-(9*b^13 + 9*b^4*(
-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^
7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^
9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^1
2 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 38
40*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^
13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778
240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 + x*(-(9*b^13 + 9*b^4*(-(4*a*c - b^
2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 3024
0*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 2
13*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^1
1*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c
^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6
144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*
b^5*c^6 - 1572864*a^15*b^3*c^7)) + x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 -
3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^
4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) +
26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4
 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c -
51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6
*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10
*b^2*c^5)))^(1/2) + ((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6
*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a
^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*
c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c +
 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)
))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*
a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3
c^7 - x(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 207
7*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 +
 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c -
 b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8
*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(10
48576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*
c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) -
x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c
^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*

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b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2
- 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-
(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2)
)/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^
8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) + 128000*a^10*c^9
 + 504*a^6*b^8*c^5 - 8112*a^7*b^6*c^6 + 48704*a^8*b^4*c^7 - 129280*a^9*b^2*
c^8))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a
^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25
*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^
2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^
2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*2i - (
1/a + (b*x^2*(11*a*c - 3*b^2))/(2*a^2*(4*a*c - b^2)) + (c*x^4*(10*a*c - 3*b
^2))/(2*a^2*(4*a*c - b^2)))/(a*x + b*x^3 + c*x^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.873 $\int \frac{x^{11}}{\left(a+bx^2+cx^4\right)^3} \, dx$

Optimal. Leaf size=209

$$\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\left(b^2-4ac\right)^{5/2}}-\frac{bx^2\left(b^2-7ac\right)}{2c^2\left(b^2-4ac\right)^2}+\frac{x^4\left(bx^2\left(b^2-10ac\right)+a\left(b^2-16ac\right)\right)}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^8\left(2a\right.}{4\left(b^2-4ac\right)\left(\right.}$$

```
[Out] -1/2*b*(-7*a*c+b^2)*x^2/c^2/(-4*a*c+b^2)^2+1/4*x^8*(b*x^2+2*a)/(-4*a*c+b^2)
/(c*x^4+b*x^2+a)^2+1/4*x^4*(a*(-16*a*c+b^2)+b*(-10*a*c+b^2)*x^2)/c/(-4*a*c+
b^2)^2/(c*x^4+b*x^2+a)+1/2*b*(30*a^2*c^2-10*a*b^2*c+b^4)*arctanh((2*c*x^2+b
)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(5/2)+1/4*ln(c*x^4+b*x^2+a)/c^3
```

Rubi [A] time = 0.40, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1114, 738, 818, 773, 634, 618, 206, 628}

$$\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\left(b^2-4ac\right)^{5/2}}-\frac{bx^2\left(b^2-7ac\right)}{2c^2\left(b^2-4ac\right)^2}+\frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^4\left(bx^2\left(b^2-10ac\right)+\right.}{4c\left(b^2-4ac\right)^2\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(b*(b^2 - 7*a*c)*x^2)/(2*c^2*(b^2 - 4*a*c)^2) + (x^8*(2*a + b*x^2))/(4*(b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x^4*(a*(b^2 - 16*a*c) + b*(b^2 - 10*a*
c)*x^2))/(4*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b*(b^4 - 10*a*b^2*c +
 30*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)
^(5/2)) + Log[a + b*x^2 + c*x^4]/(4*c^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^5}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= \frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{x^3(8a+bx)}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4\left(b^2-4ac\right)}$$

$$= \frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^4\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x^2\right)}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\operatorname{Subst}\left(\int \frac{x\left(2a\left(b^2\right.\right.}{}\right.}{4c}$$

$$= -\frac{b\left(b^2-7ac\right)x^2}{2c^2\left(b^2-4ac\right)^2} + \frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^4\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x^2\right.}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= -\frac{b\left(b^2-7ac\right)x^2}{2c^2\left(b^2-4ac\right)^2} + \frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^4\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x^2\right.}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= -\frac{b\left(b^2-7ac\right)x^2}{2c^2\left(b^2-4ac\right)^2} + \frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^4\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x^2\right.}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= -\frac{b\left(b^2-7ac\right)x^2}{2c^2\left(b^2-4ac\right)^2} + \frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^4\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x^2\right.}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Mathematica [A] time = 0.32, size = 244, normalized size = 1.17

$$\frac{-\frac{2bc\left(30a^2c^2-10ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}} + \frac{2a^3c^2+a^2bc\left(5cx^2-4b\right)+ab^3\left(b-5cx^2\right)+b^5x^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{32a^3c^3-39a^2b^2c^2+50a^2bc^3x^2+11ab^4c-30ab^3c^2x^2-b^6+4}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-b^6 + 11*a*b^4*c - 39*a^2*b^2*c^2 + 32*a^3*c^3 + 4*b^5*c*x^2 - 30*a*b^3*
c^2*x^2 + 50*a^2*b*c^3*x^2)/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (2*a^3*
c^2 + b^5*x^2 + a*b^3*(b - 5*c*x^2) + a^2*b*c*(-4*b + 5*c*x^2))/((b^2 - 4*a
*c)*(a + b*x^2 + c*x^4)^2) - (2*b*c*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTan[
(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*Log[a + b*x^2 +
 c*x^4])/(4*c^4)
```

fricas [B] time = 1.13, size = 1631, normalized size = 7.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(2*b^7*c
- 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*x^6 + (3*b^8 - 31*a*b^6*c
+ 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(3*a*b^7 - 34*a^2*
b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*x^2 + ((b^5*c^2 - 10*a*b^3*c^3 + 3
0*a^2*b*c^4)*x^8 + a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*
b^4*c^2 + 30*a^2*b^2*c^3)*x^6 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*
b*c^3)*x^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x^2)*sqrt(b^2 - 4*a*
c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*
c))/(c*x^4 + b*x^2 + a)) + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^
5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a))/(a^2*b^6*
c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^
6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^8 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b
^3*c^6 - 64*a^3*b*c^7)*x^6 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*
a^3*b^2*c^6 - 128*a^4*c^7)*x^4 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3
*c^5 - 64*a^4*b*c^6)*x^2), 1/4*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2
- 96*a^5*c^3 + 2*(2*b^7*c - 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*
x^6 + (3*b^8 - 31*a*b^6*c + 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*
x^4 + 2*(3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*x^2 + 2*
((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^8 + a^2*b^5 - 10*a^3*b^3*c + 30*
a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^6 + (b^7 - 8*a*b^5*
c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b
^2*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b
^2 - 4*a*c)) + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8
+ a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^
```

```
5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4
*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3
*b^3*c^2 - 64*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a))/(a^2*b^6*c^3 - 12*a^3
*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b
^2*c^7 - 64*a^3*c^8)*x^8 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*
a^3*b*c^7)*x^6 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6
- 128*a^4*c^7)*x^4 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^
4*b*c^6)*x^2)]
```

giac [A] time = 1.84, size = 306, normalized size = 1.46

$$-\frac{\left(b^{5}-10\,ab^{3}c+30\,a^{2}bc^{2}\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\left(b^{4}c^{3}-8\,ab^{2}c^{4}+16\,a^{2}c^{5}\right)\sqrt{-b^{2}+4\,ac}}-\frac{3\,b^{4}c^{2}x^{8}-24\,ab^{2}c^{3}x^{8}+48\,a^{2}c^{4}x^{8}-2\,b^{5}cx^{6}+12\,ab^{3}c^{2}x^{6}-4\,a^{2}bc}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a
*c))/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(-b^2 + 4*a*c)) - 1/8*(3*b^4
*c^2*x^8 - 24*a*b^2*c^3*x^8 + 48*a^2*c^4*x^8 - 2*b^5*c*x^6 + 12*a*b^3*c^2*x
^6 - 4*a^2*b*c^3*x^6 - 3*b^6*x^4 + 20*a*b^4*c*x^4 - 22*a^2*b^2*c^2*x^4 + 32
*a^3*c^3*x^4 - 6*a*b^5*x^2 + 40*a^2*b^3*c*x^2 - 28*a^3*b*c^2*x^2 - 3*a^2*b^
4 + 18*a^3*b^2*c)/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*(c*x^4 + b*x^2 + a)
^2) + 1/4*log(c*x^4 + b*x^2 + a)/c^3
```

maple [B] time = 0.02, size = 547, normalized size = 2.62

$$-\frac{15a^{2}b\arctan\left(\frac{2cx^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}\;c}+\frac{5ab^{3}\arctan\left(\frac{2cx^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}\;c^{2}}-\frac{b^{5}\arctan\left(\frac{2cx^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{2\left(16a^{2}c^{2}-8ab^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}\;c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(1/c^2*b*(25*a^2*c^2-15*a*b^2*c+2*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1
/2*(32*a^3*c^3+11*a^2*b^2*c^2-19*a*b^4*c+3*b^6)/c^3/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^4+a*b*(31*a^2*c^2-22*a*b^2*c+3*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x^
2+3/2*a^2*(8*a^2*c^2-7*a*b^2*c+b^4)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+
b*x^2+a)^2+4/c/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*a^2-2/c^2/(16*a
^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*a*b^2+1/4/c^3/(16*a^2*c^2-8*a*b^2*c
+b^4)*ln(c*x^4+b*x^2+a)*b^4-15/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/
2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*b+5/c^2/(16*a^2*c^2-8*a*b^2*c+
```

```
b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^3-1/2/c^3/
(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)
^(1/2))*b^5
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 7.30, size = 2588, normalized size = 12.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^4*(3*b^6 + 32*a^3*c^3 + 11*a^2*b^2*c^2 - 19*a*b^4*c))/(4*c^3*(b^4 + 16*
a^2*c^2 - 8*a*b^2*c)) + (x^2*(3*a*b^5 - 22*a^2*b^3*c + 31*a^3*b*c^2))/(2*c^
3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a*(a*b^4 + 8*a^3*c^2 - 7*a^2*b^2*c))
/(4*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*x^6*(2*b^4 + 25*a^2*c^2 - 15*a
*b^2*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 +
 c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - (log((a/c^4 + ((c^3*(-(b^2*(b^4 + 30*a^
2*c^2 - 10*a*b^2*c)^2)/(c^6*(4*a*c - b^2)^5))^(1/2) - 1)*((8*a)/c + (2*(c^3
*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(c^6*(4*a*c - b^2)^5))^(1/2) - 1
)*(2*a + b*x^2))/c + (2*b*x^2*(3*b^4 + 62*a^2*c^2 - 26*a*b^2*c))/(c*(4*a*c
- b^2)^2)))/(4*c^3) + (x^2*(b^5 + 23*a^2*b*c^2 - 9*a*b^3*c))/(c^4*(4*a*c -
b^2)^2))*(a/c^4 - ((c^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(c^6*(4*a
*c - b^2)^5))^(1/2) + 1)*((8*a)/c - (2*(c^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a
*b^2*c)^2)/(c^6*(4*a*c - b^2)^5))^(1/2) + 1)*(2*a + b*x^2))/c + (2*b*x^2*(3
*b^4 + 62*a^2*c^2 - 26*a*b^2*c))/(c*(4*a*c - b^2)^2)))/(4*c^3) + (x^2*(b^5
+ 23*a^2*b*c^2 - 9*a*b^3*c))/(c^4*(4*a*c - b^2)^2)))*(2*b^10 - 2048*a^5*c^5
 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*
(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*
c^6 - 5120*a^4*b^2*c^7)) - (b*atan(((x^2*(((b*((6*b^5*c^3 - 52*a*b^3*c^4 +
124*a^2*b*c^5)/(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5) + ((8*b^5*c^6 - 64*a*b^
3*c^7 + 128*a^2*b*c^8)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*
b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(16*a^2*c^6 + b^4*c^4 - 8*a*b^
2*c^5)*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a
```

```
^3*b^4*c^6 - 5120*a^4*b^2*c^7)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(8*c^3*(4
*a*c - b^2)^(5/2)) + (b*(8*b^5*c^6 - 64*a*b^3*c^7 + 128*a^2*b*c^8)*(b^4 + 3
0*a^2*c^2 - 10*a*b^2*c)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3
*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(16*c^3*(4*a*c - b^2)^(5/2)*(16*
a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5)*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4
- 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)))/(a*(4*a*c - b^2)
^2) - (b*((b^5 + 23*a^2*b*c^2 - 9*a*b^3*c)/(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*
c^5) + (((6*b^5*c^3 - 52*a*b^3*c^4 + 124*a^2*b*c^5)/(16*a^2*c^6 + b^4*c^4 -
 8*a*b^2*c^5) + ((8*b^5*c^6 - 64*a*b^3*c^7 + 128*a^2*b*c^8)*(2*b^10 - 2048*
a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*
c))/(2*(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5)*(4096*a^5*c^8 - 4*b^10*c^3 + 80
*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)))*(2*b^
10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 -
 40*a*b^8*c))/(2*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^
5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)) - (b^2*((b^5*c^6)/2 - 4*a*b^3*c^7
 + 8*a^2*b*c^8)*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(c^6*(4*a*c - b^2)^5*(16
*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5))))/(2*a*(4*a*c - b^2)^(5/2))) + ((b*((8*a
)/c + (8*a*c^2*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3
+ 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4
 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7))*(b^4 + 30*a^2*c^
2 - 10*a*b^2*c))/(8*c^3*(4*a*c - b^2)^(5/2)) + (a*b*(b^4 + 30*a^2*c^2 - 10*
a*b^2*c)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560
*a^4*b^2*c^4 - 40*a*b^8*c))/(c*(4*a*c - b^2)^(5/2)*(4096*a^5*c^8 - 4*b^10*c
^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7))
)/(a*(4*a*c - b^2)^2) - (b*(a/c^4 + (((8*a)/c + (8*a*c^2*(2*b^10 - 2048*a^5
*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))
/(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4
*c^6 - 5120*a^4*b^2*c^7))*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a
^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(4096*a^5*c^8 - 4*b^10*c^3
+ 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)) -
(a*b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(c^4*(4*a*c - b^2)^5)))/(2*a*(4*a
*c - b^2)^(5/2)))*(32*a^2*c^6*(4*a*c - b^2)^5 + 2*b^4*c^4*(4*a*c - b^2)^5 -
 16*a*b^2*c^5*(4*a*c - b^2)^5))/(b^10 + 160*a^2*b^6*c^2 - 600*a^3*b^4*c^3 +
 900*a^4*b^2*c^4 - 20*a*b^8*c))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(2*c^3*(4*
a*c - b^2)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.874 $\int \frac{x^9}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=121

$$-\frac{6a^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}-\frac{3ax^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^6\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

```
[Out] 1/4*x^6*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-3/2*a*x^2*(b*x^2+2*a)/(-
4*a*c+b^2)^2/(c*x^4+b*x^2+a)-6*a^2*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/
(-4*a*c+b^2)^(5/2)
```

Rubi [A] time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1114, 722, 618, 206}

$$-\frac{6a^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{x^6\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3ax^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^6*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*a*x^2*(2*a
 + b*x^2))/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (6*a^2*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
```

```
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^9}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= \frac{x^6\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{(3a)\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{2\left(b^2-4ac\right)}$$

$$= \frac{x^6\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3ax^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\right.}{\left(b^2-4ac\right)^2}$$

$$= \frac{x^6\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3ax^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(6a^2\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x}\right.}{\left(b^2-4ac\right.}$$

$$= \frac{x^6\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3ax^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{6a^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.17, size = 194, normalized size = 1.60

$$\frac{1}{4}\left(\frac{24a^2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}} + \frac{a^2c\left(2cx^2-3b\right)+ab^2\left(b-4cx^2\right)+b^4x^2}{c^3\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)^2} + \frac{22a^2bc^2-20a^2c^3x^2-8ab^3c+16ab^2c^2x^2+}{c^3\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b^5 - 8*a*b^3*c + 22*a^2*b*c^2 - 2*b^4*c*x^2 + 16*a*b^2*c^2*x^2 - 20*a^2*
c^3*x^2)/(c^3*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b^4*x^2 + a*b^2*(b -
4*c*x^2) + a^2*c*(-3*b + 2*c*x^2))/(c^3*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^
2) + (24*a^2*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2)
)/4
```

fricas [B] time = 0.90, size = 973, normalized size = 8.04

$$\left[-\frac{a^2b^5 - 14\,a^3b^3c + 40\,a^4bc^2 + 2\left(b^6c - 12\,ab^4c^2 + 42\,a^2b^2c^3 - 40\,a^3c^4\right)x^6 + \left(b^7 - 12\,ab^5c + 30\,a^2b^3c^2 + 8\,a^3bc^3\right.}{4\left(a^2b^6c^2 - 12\,a^3b^4c^3 + 48\,a^4b^2c^4 - 64\,a^5c^5 + \left(b^6c^4 - 12\,ab^4c^5 + 48\,a^2b^2c^6 - 64\,a^3c^7\right)x^8 + 2\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*c - 12*a*b^4*c^2 + 42
*a^2*b^2*c^3 - 40*a^3*c^4)*x^6 + (b^7 - 12*a*b^5*c + 30*a^2*b^3*c^2 + 8*a^3
*b*c^3)*x^4 + 2*(a*b^6 - 14*a^2*b^4*c + 46*a^3*b^2*c^2 - 24*a^4*c^3)*x^2 -
12*(a^2*c^4*x^8 + 2*a^2*b*c^3*x^6 + 2*a^3*b*c^2*x^2 + a^4*c^2 + (a^2*b^2*c^
2 + 2*a^3*c^3)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*
a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a^2*b^6*c^2 -
 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 4
8*a^2*b^2*c^6 - 64*a^3*c^7)*x^8 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^
5 - 64*a^3*b*c^6)*x^6 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b
^2*c^5 - 128*a^4*c^6)*x^4 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4
- 64*a^4*b*c^5)*x^2), -1/4*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*
c - 12*a*b^4*c^2 + 42*a^2*b^2*c^3 - 40*a^3*c^4)*x^6 + (b^7 - 12*a*b^5*c + 3
0*a^2*b^3*c^2 + 8*a^3*b*c^3)*x^4 + 2*(a*b^6 - 14*a^2*b^4*c + 46*a^3*b^2*c^2
 - 24*a^4*c^3)*x^2 + 24*(a^2*c^4*x^8 + 2*a^2*b*c^3*x^6 + 2*a^3*b*c^2*x^2 +
a^4*c^2 + (a^2*b^2*c^2 + 2*a^3*c^3)*x^4)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^
2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 4
8*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*
a^3*c^7)*x^8 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x
^6 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 128*a^4*c^
6)*x^4 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b*c^5)*x^2
)]
```

giac [A] time = 1.87, size = 212, normalized size = 1.75

$$\frac{6\,a^2\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4 - 8\,ab^2c + 16\,a^2c^2\right)\sqrt{-b^2+4\,ac}} - \frac{2\,b^4cx^6 - 16\,ab^2c^2x^6 + 20\,a^2c^3x^6 + b^5x^4 - 8\,ab^3cx^4 - 2\,a^2bc^2x^4 + 2\,ab^4x^2 -}{4\left(b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4\right)\left(cx^4 + bx^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 6*a^2*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c
^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(2*b^4*c*x^6 - 16*a*b^2*c^2*x^6 + 20*a^2*c^3*
x^6 + b^5*x^4 - 8*a*b^3*c*x^4 - 2*a^2*b*c^2*x^4 + 2*a*b^4*x^2 - 20*a^2*b^2*
c*x^2 + 12*a^3*c^2*x^2 + a^2*b^3 - 10*a^3*b*c)/((b^4*c^2 - 8*a*b^2*c^3 + 16
*a^2*c^4)*(c*x^4 + b*x^2 + a)^2)
```

maple [B] time = 0.02, size = 267, normalized size = 2.21

$$\frac{6a^2\arctan\left(\frac{2c x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a b^2c+b^4\right)\sqrt{4ac-b^2}}+\frac{-\frac{\left(10a^2c^2-8a b^2c+b^4\right)x^6}{\left(16a^2c^2-8a b^2c+b^4\right)c}+\frac{\left(2a^2c^2+8a b^2c-b^4\right)b x^4}{2\left(16a^2c^2-8a b^2c+b^4\right)c^2}+\frac{\left(10ac-b^2\right)a^2b}{2\left(16a^2c^2-8a b^2c+b^4\right)c^2}-\frac{\left(6a^2c^2-10a b^2c+b^4\right.}{\left(16a^2c^2-8a b^2c+b\right.}}{2\left(c x^4+b x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(-1/c*(10*a^2*c^2-8*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2*b*(
2*a^2*c^2+8*a*b^2*c-b^4)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-a*(6*a^2*c^2-10
*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x^2+1/2*a^2*b*(10*a*c-b^2)/c^2
/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+6*a^2/(16*a^2*c^2-8*a*b^2*c+
b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.53, size = 444, normalized size = 3.67

$$\frac{6\,a^2\,\mathrm{atan}\left(\frac{\left(x^2\left(\frac{36\,a^3\,c^2}{{\left(4\,a\,c-b^2\right)}^{9/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{36\,a^3\,b\left(16\,a^2\,b\,c^4-8\,a\,b^3\,c^3+b^5\,c^2\right)}{{\left(4\,a\,c-b^2\right)}^{15/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)+\frac{72\,a^4\,b\,c^2}{{\left(4\,a\,c-b^2\right)}^{15/2}}\right)\left(b^4\,{\left(4\,a\,c-b^2\right)}^5+16\,a^2\,c^2\,{\left(4\,a\,c-b^2\right)}^5-8\,a\,b^2\,c\,\left(4\,a\,c-b\right.\right.}{72\,a^4\,c^2}\right.}{{\left(4\,a\,c-b^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (6*a^2*atan(((x^2*((36*a^3*c^2)/((4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c)) + (36*a^3*b*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4))/((4*a*c - b^2
)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (72*a^4*b*c^2)/(4*a*c - b^2)^(1
5/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c
- b^2)^5))/(72*a^4*c^2)))/(4*a*c - b^2)^(5/2) - ((x^6*(b^4 + 10*a^2*c^2 - 8
*a*b^2*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*(b^3 - 10*a*b*c))/(4
*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^4*(2*a^2*b*c^2 - b^5 + 8*a*b^3*c)
)/(4*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x^2*(b^4 + 6*a^2*c^2 - 10*a*b
^2*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c
^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

sympy [B] time = 4.64, size = 554, normalized size = 4.58

$$-3a^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x^2+\frac{-192a^5c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+144a^4b^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-36a^3b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3a^2b^6}{6a^2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -3*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**5*c**3*sqrt(-1/(4*a*
c - b**2)**5) + 144*a**4*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5) - 36*a**3*b**
4*c*sqrt(-1/(4*a*c - b**2)**5) + 3*a**2*b**6*sqrt(-1/(4*a*c - b**2)**5) + 3
*a**2*b)/(6*a**2*c)) + 3*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (192*a*
*5*c**3*sqrt(-1/(4*a*c - b**2)**5) - 144*a**4*b**2*c**2*sqrt(-1/(4*a*c - b*
*2)**5) + 36*a**3*b**4*c*sqrt(-1/(4*a*c - b**2)**5) - 3*a**2*b**6*sqrt(-1/(
4*a*c - b**2)**5) + 3*a**2*b)/(6*a**2*c)) + (10*a**3*b*c - a**2*b**3 + x**6
*(-20*a**2*c**3 + 16*a*b**2*c**2 - 2*b**4*c) + x**4*(2*a**2*b*c**2 + 8*a*b*
*3*c - b**5) + x**2*(-12*a**3*c**2 + 20*a**2*b**2*c - 2*a*b**4))/(64*a**4*c
**4 - 32*a**3*b**2*c**3 + 4*a**2*b**4*c**2 + x**8*(64*a**2*c**6 - 32*a*b**2
*c**5 + 4*b**4*c**4) + x**6*(128*a**2*b*c**5 - 64*a*b**3*c**4 + 8*b**5*c**3
) + x**4*(128*a**3*c**5 - 24*a*b**4*c**3 + 4*b**6*c**2) + x**2*(128*a**3*b*
c**4 - 64*a**2*b**3*c**3 + 8*a*b**5*c**2))
```

3.875 $\int \frac{x^7}{(a+bx^2+cx^4)^3} dx$

Optimal. Leaf size=119

$$\frac{3ab\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{3bx^2(2a+bx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{x^6(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

```
[Out] -1/4*x^6*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/4*b*x^2*(b*x^2+2*a)/(
-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3*a*b*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^(5/2)
```

Rubi [A] time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1114, 728, 722, 618, 206}

$$-\frac{x^6(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3bx^2(2a+bx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{3ab\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x^6*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*b*x^2*(2*
a + b*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*a*b*ArcTanh[(b + 2
*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
```

```
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 728

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] + Dist[(m*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(
d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{\left(a+bx+cx^2\right)^3}\,dx,x,x^2\right)$$

$$= -\frac{x^6\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{(3b)\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)}{4\left(b^2-4ac\right)}$$

$$= -\frac{x^6\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3bx^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{(3ab)\,\text{Subst}\left(\int \frac{1}{a+bx\cdots}\right.}{2\left(b^2-4a\cdots\right.}$$

$$= -\frac{x^6\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3bx^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{(3ab)\,\text{Subst}\left(\int \frac{1}{b^2-4\cdots}\right.}{\left(b^2-\cdots\right.}$$

$$= -\frac{x^6\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3bx^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3ab\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\cdots\right.}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.19, size = 137, normalized size = 1.15

$$-\frac{8a^3c+a^2\left(b^2+10bcx^2+16c^2x^4\right)+abx^2\left(2b^2+bcx^2+6c^2x^4\right)+b^4x^4}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)^2}-\frac{3ab\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(8*a^3*c + b^4*x^4 + a*b*x^2*(2*b^2 + b*c*x^2 + 6*c^2*x^4) + a^2*(b^2
+ 10*b*c*x^2 + 16*c^2*x^4))/(c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2) - (3*
a*b*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2)
```

fricas [B] time = 1.11, size = 892, normalized size = 7.50

$$\left[-\frac{6\left(ab^3c^2-4\,a^2bc^3\right)x^6+a^2b^4+4\,a^3b^2c-32\,a^4c^2+\left(b^6-3\,ab^4c+12\,a^2b^2c^2-64\,a^3c^3\right)x^4+2\left(ab^5+a^2b^3c-20\right.}{4\left(a^2b^6c-12\,a^3b^4c^2+48\,a^4b^2c^3-64\,a^5c^4+\left(b^6c^3-12\,ab^4c^4+48\,a^2b^2c^5-64\,a^3c^6\right)x^8+2\left(b^7c^2-12\,ab^5c^3+4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*(a*b^3*c^2 - 4*a^2*b*c^3)*x^6 + a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2
 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 - 64*a^3*c^3)*x^4 + 2*(a*b^5 + a^2*b^3
*c - 20*a^3*b*c^2)*x^2 - 6*(a*b*c^3*x^8 + 2*a*b^2*c^2*x^6 + 2*a^2*b^2*c*x^2
 + a^3*b*c + (a*b^3*c + 2*a^2*b*c^2)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4
+ 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
 + a)))/(a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^
3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^8 + 2*(b^7*c^2 - 12*a*b^5
*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^6 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*
b^4*c^3 + 32*a^3*b^2*c^4 - 128*a^4*c^5)*x^4 + 2*(a*b^7*c - 12*a^2*b^5*c^2 +
 48*a^3*b^3*c^3 - 64*a^4*b*c^4)*x^2), -1/4*(6*(a*b^3*c^2 - 4*a^2*b*c^3)*x^6
 + a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 -
 64*a^3*c^3)*x^4 + 2*(a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*x^2 - 12*(a*b*c^3*x
^8 + 2*a*b^2*c^2*x^6 + 2*a^2*b^2*c*x^2 + a^3*b*c + (a*b^3*c + 2*a^2*b*c^2)*
x^4)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a
*c)))/(a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3
- 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^8 + 2*(b^7*c^2 - 12*a*b^5*c
^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^6 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^
4*c^3 + 32*a^3*b^2*c^4 - 128*a^4*c^5)*x^4 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 4
8*a^3*b^3*c^3 - 64*a^4*b*c^4)*x^2)]
```

giac [A] time = 1.77, size = 171, normalized size = 1.44

$$-\frac{3\,ab\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)\sqrt{-b^{2}+4\,ac}}-\frac{6\,abc^{2}x^{6}+b^{4}x^{4}+ab^{2}cx^{4}+16\,a^{2}c^{2}x^{4}+2\,ab^{3}x^{2}+10\,a^{2}bcx^{2}+a^{2}b^{2}+8\,a^{3}}{4\left(b^{4}c-8\,ab^{2}c^{2}+16\,a^{2}c^{3}\right)\left(cx^{4}+bx^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3*a*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*
c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*a*b*c^2*x^6 + b^4*x^4 + a*b^2*c*x^4 + 16*
a^2*c^2*x^4 + 2*a*b^3*x^2 + 10*a^2*b*c*x^2 + a^2*b^2 + 8*a^3*c)/((b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*(c*x^4 + b*x^2 + a)^2)
```

maple [B] time = 0.02, size = 230, normalized size = 1.93

$$-\frac{3ab\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}}+\frac{-\frac{3abc\,x^{6}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{\left(5ac+b^{2}\right)ab\,x^{2}}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c}-\frac{\left(16a^{2}c^{2}+a\,b^{2}c+b^{4}\right)x^{4}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c}-\frac{\left(8ac+b^{2}\right)a^{2}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c}}{2\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(-3*a*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-1/2*(16*a^2*c^2+a*b^2*c+b^4)/c
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-(5*a*c+b^2)*a*b/c/(16*a^2*c^2-8*a*b^2*c+b^4
)*x^2-1/2*a^2*(8*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2-3
*a*b/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c
-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.44, size = 423, normalized size = 3.55

$$-\frac{\frac{x^2\left(5\,c\,a^2\,b+a\,b^3\right)}{2\,c\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x^4\left(16\,a^2\,c^2+a\,b^2\,c+b^4\right)}{4\,c\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\left(8\,c\,a^2+a\,b^2\right)}{4\,c\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{3\,a\,b\,c\,x^6}{2\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^4\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}-\frac{3\,a\,b\,\mathrm{atan}\left(\left(\left(x^2\left(\frac{9\,a}{\left(4\,a\,c-b^2\right)^{9/2}\left(16\right.}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] - ((x^2*(a*b^3 + 5*a^2*b*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^4*(b
^4 + 16*a^2*c^2 + a*b^2*c))/(4*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*(a*b^
2 + 8*a^2*c))/(4*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a*b*c*x^6)/(2*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2
+ 2*b*c*x^6) - (3*a*b*atan(((x^2*((9*a*b^2*c^2)/((4*a*c - b^2)^(9/2)*(b^4 +
 16*a^2*c^2 - 8*a*b^2*c)) + (9*a*b^3*(2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c
^4))/(2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (18*a^2*b^3
*c^2)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)
^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(18*a^2*b^2*c^2)))/(4*a*c - b^2)^(5/2)
```

sympy [B] time = 3.81, size = 524, normalized size = 4.40

$$\frac{3ab\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x^2+\frac{-192a^4bc^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+144a^3b^3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-36a^2b^5c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3ab^7\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3ab^2}{6abc}\right)}{2}-\frac{3ab\sqrt{-\frac{}{(}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**4*b*c**3*sqrt(-1/(4*a*
c - b**2)**5) + 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)**5) - 36*a**2*b**
5*c*sqrt(-1/(4*a*c - b**2)**5) + 3*a*b**7*sqrt(-1/(4*a*c - b**2)**5) + 3*a*
b**2)/(6*a*b*c))/2 - 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (192*a**4*
b*c**3*sqrt(-1/(4*a*c - b**2)**5) - 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**
2)**5) + 36*a**2*b**5*c*sqrt(-1/(4*a*c - b**2)**5) - 3*a*b**7*sqrt(-1/(4*a*
c - b**2)**5) + 3*a*b**2)/(6*a*b*c))/2 + (-8*a**3*c - a**2*b**2 - 6*a*b*c**
2*x**6 + x**4*(-16*a**2*c**2 - a*b**2*c - b**4) + x**2*(-10*a**2*b*c - 2*a*
b**3))/(64*a**4*c**3 - 32*a**3*b**2*c**2 + 4*a**2*b**4*c + x**8*(64*a**2*c*
*5 - 32*a*b**2*c**4 + 4*b**4*c**3) + x**6*(128*a**2*b*c**4 - 64*a*b**3*c**3
 + 8*b**5*c**2) + x**4*(128*a**3*c**4 - 24*a*b**4*c**2 + 4*b**6*c) + x**2*(
128*a**3*b*c**3 - 64*a**2*b**3*c**2 + 8*a*b**5*c))
```

3.876 $\int \frac{x^5}{(a+bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=130

$$-\frac{(2ac+b^2)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{x^2(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{x^2(2ac+b^2)+3ab}{2(b^2-4ac)^2(a+bx^2+cx^4)}$$

```
[Out] 1/4*x^2*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/2*(3*a*b+x^2*(2*a*c+b^
2))/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)-(2*a*c+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Rubi [A] time = 0.13, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1114, 738, 638, 618, 206}

$$\frac{x^2(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{x^2(2ac+b^2)+3ab}{2(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{(2ac+b^2)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^2*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*a*b + (b^2
 + 2*a*c)*x^2)/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((b^2 + 2*a*c)*Arc
Tanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
```

```
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= \frac{x^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{2a-2bx}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4\left(b^2-4ac\right)}$$

$$= \frac{x^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3ab+\left(b^2+2ac\right)x^2}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(b^2+2ac\right)\text{Subst}\left(\int \frac{}{a+\cdots}\right.}{2\left(b^2-4a\cdots\right.}$$

$$= \frac{x^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3ab+\left(b^2+2ac\right)x^2}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(b^2+2ac\right)\text{Subst}\left(\int \frac{}{b^2\cdots}\right.}{\left(b^2-\cdots\right.}$$

$$= \frac{x^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3ab+\left(b^2+2ac\right)x^2}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(b^2+2ac\right)\tanh^{-1}\left(\frac{b+\cdots}{\sqrt{b^2\cdots}}\right.}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.13, size = 145, normalized size = 1.12

$$\frac{1}{4}\left(\frac{4\left(2ac+b^2\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}+\frac{\left(2ac+b^2\right)\left(b+2cx^2\right)}{c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{a\left(b-2cx^2\right)+b^2x^2}{c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (((b^2 + 2*a*c)*(b + 2*c*x^2))/(c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b
^2*x^2 + a*(b - 2*c*x^2))/(c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (4*(b^
2 + 2*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/
4
```

fricas [B] time = 0.85, size = 907, normalized size = 6.98

$$\left[\frac{2\left(b^4c-2\,ab^2c^2-8\,a^2c^3\right)x^6+6\,a^2b^3-24\,a^3bc+3\left(b^5-2\,ab^3c-8\,a^2bc^2\right)x^4+2\left(5\,ab^4-22\,a^2b^2c+8\,a^3c^2\right)x^2+}{4\left(\left(b^6c^2-12\,ab^4c^3+48\,a^2b^2c^4-64\,a^3c^5\right)x^8+a^2b^6-12\,a^3b^4c+48\,a^4b^2c^2-64\,a^5c^3+2\left(b^7c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^6 + 6*a^2*b^3 - 24*a^3*b*c + 3*
(b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*x^4 + 2*(5*a*b^4 - 22*a^2*b^2*c + 8*a^3*c^2
)*x^2 + 2*((b^2*c^2 + 2*a*c^3)*x^8 + 2*(b^3*c + 2*a*b*c^2)*x^6 + (b^4 + 4*a
*b^2*c + 4*a^2*c^2)*x^4 + a^2*b^2 + 2*a^3*c + 2*(a*b^3 + 2*a^2*b*c)*x^2)*sq
rt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sq
rt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*
c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 -
 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7
 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2), 1/4*(2*(b^4*c - 2*a*
b^2*c^2 - 8*a^2*c^3)*x^6 + 6*a^2*b^3 - 24*a^3*b*c + 3*(b^5 - 2*a*b^3*c - 8*
a^2*b*c^2)*x^4 + 2*(5*a*b^4 - 22*a^2*b^2*c + 8*a^3*c^2)*x^2 - 4*((b^2*c^2 +
 2*a*c^3)*x^8 + 2*(b^3*c + 2*a*b*c^2)*x^6 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x
^4 + a^2*b^2 + 2*a^3*c + 2*(a*b^3 + 2*a^2*b*c)*x^2)*sqrt(-b^2 + 4*a*c)*arct
an(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c
^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2
*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4
)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*
x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)]
```

giac [A] time = 1.81, size = 161, normalized size = 1.24

$$\frac{\left(b^{2}+2\,ac\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)\sqrt{-b^{2}+4\,ac}}+\frac{2\,b^{2}cx^{6}+4\,ac^{2}x^{6}+3\,b^{3}x^{4}+6\,abcx^{4}+10\,ab^{2}x^{2}-4\,a^{2}cx^{2}+6\,a^{2}b}{4\left(cx^{4}+bx^{2}+a\right)^{2}\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] (b^2 + 2*a*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c +
16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(2*b^2*c*x^6 + 4*a*c^2*x^6 + 3*b^3*x^
4 + 6*a*b*c*x^4 + 10*a*b^2*x^2 - 4*a^2*c*x^2 + 6*a^2*b)/((c*x^4 + b*x^2 + a
)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

maple [B] time = 0.02, size = 270, normalized size = 2.08

$$\frac{2ac\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}}+\frac{b^{2}\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}}+\frac{\frac{\left(2ac+b^{2}\right)c\,x^{6}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{3\left(2ac+b^{2}\right)b\,x^{4}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{3}{16a^{2}c^{2}-}}{2\left(c\,x^{4}+b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*((2*a*c+b^2)*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+3/2*b*(2*a*c+b^2)/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^4-a*(2*a*c-5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+3*a^
2*b/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+2/(16*a^2*c^2-8*a*b^2*c+b
^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*c+1/(16*a^2*c
^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b
^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.46, size = 460, normalized size = 3.54

$$\frac{\frac{3\,a^2\,b}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x^2\,\left(5\,a\,b^2-2\,a^2\,c\right)}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{3\,b\,x^4\,\left(b^2+2\,a\,c\right)}{4\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{c\,x^6\,\left(b^2+2\,a\,c\right)}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^4\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}+\frac{\mathrm{atan}\left(\left(x^2\left(\frac{\left(b^2+2\,a\,c\right)\left(b^2\,c^2+2\,a\,c^3 \cdots}{a\left(4\,a\,c-b^2\right)^{9/2}\left(16\,a^2\,c^2-8\,a\,b \cdots}\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((3*a^2*b)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(5*a*b^2 - 2*a^2*c))/(
2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*x^4*(2*a*c + b^2))/(4*(b^4 + 16*a^
2*c^2 - 8*a*b^2*c)) + (c*x^6*(2*a*c + b^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + (atan(((
x^2*(((2*a*c + b^2)*(2*a*c^3 + b^2*c^2))/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a
^2*c^2 - 8*a*b^2*c)) + (b*(2*a*c + b^2)^2*(2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^
2*b*c^4))/(2*a*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (2*b
*c^2*(2*a*c + b^2)^2)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2*c
^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(8*a^2*c^4 + 2*b^4*c^2 + 8
*a*b^2*c^3))*(2*a*c + b^2))/(4*a*c - b^2)^(5/2)
```

sympy [B] time = 5.41, size = 580, normalized size = 4.46

$$-\frac{\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)\log\left(x^2+\frac{-64a^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)+48a^2b^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)-12ab^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)+2abc \cdots}{4ac^2+2b^2c}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x**2 + (-64*a**3*c**3*sqrt(-
1/(4*a*c - b**2)**5)*(2*a*c + b**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b*
*2)**5)*(2*a*c + b**2) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b*
*2) + 2*a*b*c + b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + b**3)/(4*a
*c**2 + 2*b**2*c))/2 + sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x**2 +
 (64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) - 48*a**2*b**2*c**
2*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + 12*a*b**4*c*sqrt(-1/(4*a*c -
b**2)**5)*(2*a*c + b**2) + 2*a*b*c - b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c
 + b**2) + b**3)/(4*a*c**2 + 2*b**2*c))/2 + (6*a**2*b + x**6*(4*a*c**2 + 2*
```

```
b**2*c) + x**4*(6*a*b*c + 3*b**3) + x**2*(-4*a**2*c + 10*a*b**2))/(64*a**4*
c**2 - 32*a**3*b**2*c + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*c**3 +
 4*b**4*c**2) + x**6*(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5*c) + x**4*(
128*a**3*c**3 - 24*a*b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2 - 64*a**2*b**
3*c + 8*a*b**5))
```

3.877 $\int \frac{x^3}{(a+bx^2+cx^4)^3} dx$

Optimal. Leaf size=113

$$\frac{3bc\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{2a+bx^2}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{3b(b+2cx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)}$$

```
[Out] 1/4*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-3/4*b*(2*c*x^2+b)/(-4*a*c+b^
2)^2/(c*x^4+b*x^2+a)+3*b*c*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(5/2)
```

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1114, 638, 614, 618, 206}

$$\frac{2a+bx^2}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{3b(b+2cx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{3bc\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a + b*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*b*(b + 2*c*x^2))
/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*b*c*ArcTanh[(b + 2*c*x^2)/Sqr
t[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= \frac{2a+bx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4\left(b^2-4ac\right)}$$

$$= \frac{2a+bx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3b\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{(3bc)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\right.}{2\left(b^2-4ac\right)^2}$$

$$= \frac{2a+bx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3b\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{(3bc)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\right.}{\left(b^2-4ac\right.}$$

$$= \frac{2a+bx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3b\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3bc\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.10, size = 114, normalized size = 1.01

$$\frac{-\frac{12bc\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{(b^2-4ac)(2a+bx^2)}{(a+bx^2+cx^4)^2}-\frac{3b(b+2cx^2)}{a+bx^2+cx^4}}{4(b^2-4ac)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (((b^2 - 4*a*c)*(2*a + b*x^2))/(a + b*x^2 + c*x^4)^2 - (3*b*(b + 2*c*x^2))/
(a + b*x^2 + c*x^4) - (12*b*c*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqr
t[-b^2 + 4*a*c])/(4*(b^2 - 4*a*c)^2)
```

fricas [B] time = 1.09, size = 808, normalized size = 7.15

$$\left[-\frac{6\left(b^3c^2-4\,abc^3\right)x^6+ab^4+4\,a^2b^2c-32\,a^3c^2+9\left(b^4c-4\,ab^2c^2\right)x^4+2\left(b^5+ab^3c-20\,a^2bc^2\right)x^2-}{4\left(\left(b^6c^2-12\,ab^4c^3+48\,a^2b^2c^4-64\,a^3c^5\right)x^8+a^2b^6-12\,a^3b^4c+48\,a^4b^2c^2-64\,a^5c^3+2\left(b^7c-12\,ab^5c^2+48\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*(b^3*c^2 - 4*a*b*c^3)*x^6 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(
b^4*c - 4*a*b^2*c^2)*x^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*x^2 - 6*(b*c^3*
x^8 + 2*b^2*c^2*x^6 + 2*a*b^2*c*x^2 + (b^3*c + 2*a*b*c^2)*x^4 + a^2*b*c)*sq
rt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sq
rt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*
c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 -
 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7
 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2), -1/4*(6*(b^3*c^2 - 4
*a*b*c^3)*x^6 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(b^4*c - 4*a*b^2*c^2)*
x^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*x^2 - 12*(b*c^3*x^8 + 2*b^2*c^2*x^6
+ 2*a*b^2*c*x^2 + (b^3*c + 2*a*b*c^2)*x^4 + a^2*b*c)*sqrt(-b^2 + 4*a*c)*arc
tan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^
4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)
*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)]
```

giac [A] time = 1.79, size = 143, normalized size = 1.27

$$-\frac{3\,bc\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}-\frac{6\,bc^2x^6+9\,b^2cx^4+2\,b^3x^2+10\,abcx^2+ab^2+8\,a^2c}{4\left(cx^4+bx^2+a\right)^2\left(b^4-8\,ab^2c+16\,a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3*b*c*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*
c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*b*c^2*x^6 + 9*b^2*c*x^4 + 2*b^3*x^2 + 10*
a*b*c*x^2 + a*b^2 + 8*a^2*c)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a
^2*c^2))
```

maple [A] time = 0.01, size = 142, normalized size = 1.26

$$-\frac{3bc\,x^2}{2\left(4ac-b^2\right)^2\left(c\,x^4+b\,x^2+a\right)}-\frac{3bc\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{5}{2}}}-\frac{3b^2}{4\left(4ac-b^2\right)^2\left(c\,x^4+b\,x^2+a\right)}+\frac{-b\,x^2-2a}{4\left(4ac-b^2\right)\left(c\,x^4+b\,x^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/4*(-b*x^2-2*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^2-3/2*b/(4*a*c-b^2)^2/(c*x^4+b
*x^2+a)*x^2*c-3/4*b^2/(4*a*c-b^2)^2/(c*x^4+b*x^2+a)-3*b/(4*a*c-b^2)^(5/2)*c
*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.39, size = 400, normalized size = 3.54

$$-\frac{\frac{8\,c\,a^2+a\,b^2}{4\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x^2\left(b^3+5\,a\,c\,b\right)}{2\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{9\,b^2\,c\,x^4}{4\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{3\,b\,c^2\,x^6}{2\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^4\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}-\frac{3\,b\,c\,\mathrm{atan}\left(\left(x^2\left(\frac{9\,b^2\,c}{a\left(4\,a\,c-b^2\right)^{9/2}\left(16\,a\right.}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] - ((a*b^2 + 8*a^2*c)/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(b^3 + 5*a*b
*c))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^2*c*x^4)/(4*(b^4 + 16*a^2*c^
2 - 8*a*b^2*c)) + (3*b*c^2*x^6)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2
*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - (3*b*c*atan(((x^2*((
9*b^2*c^4)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b^3*c^
2*(9*b^5*c^2 - 72*a*b^3*c^3 + 144*a^2*b*c^4))/(a*(4*a*c - b^2)^(15/2)*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c))) + (18*b^3*c^4)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*
c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(18*b
^2*c^4)))/(4*a*c - b^2)^(5/2)
```

sympy [B] time = 3.04, size = 491, normalized size = 4.35

$$\frac{3bc\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x^2+\frac{-192a^3bc^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+144a^2b^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-36ab^5c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3b^7c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3b^2c}{6bc^2}\right)}{2}-\frac{3bc\sqrt{-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**3*b*c**4*sqrt(-1/(4*a*
c - b**2)**5) + 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) - 36*a*b**5*c
**2*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*sqrt(-1/(4*a*c - b**2)**5) + 3*b*
*2*c)/(6*b*c**2))/2 - 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (192*a**3
*b*c**4*sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b*
*2)**5) + 36*a*b**5*c**2*sqrt(-1/(4*a*c - b**2)**5) - 3*b**7*c*sqrt(-1/(4*a
*c - b**2)**5) + 3*b**2*c)/(6*b*c**2))/2 + (-8*a**2*c - a*b**2 - 9*b**2*c*x
**4 - 6*b*c**2*x**6 + x**2*(-10*a*b*c - 2*b**3))/(64*a**4*c**2 - 32*a**3*b*
*2*c + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*c**3 + 4*b**4*c**2) + x
**6*(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5*c) + x**4*(128*a**3*c**3 - 2
4*a*b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2 - 64*a**2*b**3*c + 8*a*b**5))
```

3.878 $\int \frac{x}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=113

$$-\frac{6c^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{3c\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{b+2cx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

```
[Out] 1/4*(-2*c*x^2-b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/2*c*(2*c*x^2+b)/(-4*a*c+b
^2)^2/(c*x^4+b*x^2+a)-6*c^2*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c
+b^2)^(5/2)
```

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1107, 614, 618, 206}

$$-\frac{6c^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{3c\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{b+2cx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(b + 2*c*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*c*(b + 2*c*x^2)
)/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (6*c^2*ArcTanh[(b + 2*c*x^2)/Sq
rt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{b+2cx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{(3c)\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{2\left(b^2-4ac\right)}$$

$$= -\frac{b+2cx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3c\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3c^2\right)\text{Subst}\left(\int \frac{1}{a+bx\cdots}\right.}{\left(b^2-4ac\cdots\right.}$$

$$= -\frac{b+2cx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3c\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(6c^2\right)\text{Subst}\left(\int \frac{1}{b^2-4a\cdots}\right.}{\left(b^2-\cdots\right.}$$

$$= -\frac{b+2cx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3c\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{6c^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\cdots\right.}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.10, size = 106, normalized size = 0.94

$$\frac{\frac{24c^2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{\left(b+2cx^2\right)\left(-2c\left(5a+3cx^4\right)+b^2-6bcx^2\right)}{\left(a+bx^2+cx^4\right)^2}}{4\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (-(((b + 2*c*x^2)*(b^2 - 6*b*c*x^2 - 2*c*(5*a + 3*c*x^4)))/(a + b*x^2 + c*x
^4)^2) + (24*c^2*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*
c])/(4*(b^2 - 4*a*c)^2)
```

fricas [B] time = 0.82, size = 809, normalized size = 7.16

$$\left[\frac{12\left(b^2c^3-4\,ac^4\right)x^6-b^5+14\,ab^3c-40\,a^2bc^2+18\left(b^3c^2-4\,abc^3\right)x^4+4\left(b^4c+ab^2c^2-20\,a^2c^3\right)x^2+\cdots}{4\left(\left(b^6c^2-12\,ab^4c^3+48\,a^2b^2c^4-64\,a^3c^5\right)x^8+a^2b^6-12\,a^3b^4c+48\,a^4b^2c^2-64\,a^5c^3+2\left(b^7c-12\,ab^5c^2+48\,a^2b\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(12*(b^2*c^3 - 4*a*c^4)*x^6 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^
3*c^2 - 4*a*b*c^3)*x^4 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*x^2 + 12*(c^4*x
^8 + 2*b*c^3*x^6 + 2*a*b*c^2*x^2 + (b^2*c^2 + 2*a*c^3)*x^4 + a^2*c^2)*sqrt(
b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(
b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c
^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3
 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10
*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 -
12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2), 1/4*(12*(b^2*c^3 - 4*a*
c^4)*x^6 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*x^4 +
 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*x^2 - 24*(c^4*x^8 + 2*b*c^3*x^6 + 2*a*b
*c^2*x^2 + (b^2*c^2 + 2*a*c^3)*x^4 + a^2*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c^3 + 4
8*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6
+ (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 +
2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)]
```

giac [A] time = 1.77, size = 144, normalized size = 1.27

$$\frac{6\,c^2\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}+\frac{12\,c^3x^6+18\,bc^2x^4+4\,b^2cx^2+20\,ac^2x^2-b^3+10\,abc}{4\left(cx^4+bx^2+a\right)^2\left(b^4-8\,ab^2c+16\,a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 6*c^2*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c
^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(12*c^3*x^6 + 18*b*c^2*x^4 + 4*b^2*c*x^2 + 20
```

```
*a*c^2*x^2 - b^3 + 10*a*b*c)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a
^2*c^2))
```

maple [A] time = 0.01, size = 141, normalized size = 1.25

$$\frac{3c^2x^2}{\left(4ac-b^2\right)^2\left(c\,x^4+b\,x^2+a\right)}+\frac{6c^2\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{5}{2}}}+\frac{3bc}{2\left(4ac-b^2\right)^2\left(c\,x^4+b\,x^2+a\right)}+\frac{2c\,x^2+b}{4\left(4ac-b^2\right)\left(c\,x^4+b\,x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/4*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^2+3*c^2/(4*a*c-b^2)^2/(c*x^4+b*
x^2+a)*x^2+3/2*c/(4*a*c-b^2)^2/(c*x^4+b*x^2+a)*b+6*c^2/(4*a*c-b^2)^(5/2)*ar
ctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.34, size = 386, normalized size = 3.42

$$\frac{\frac{3\,c^3\,x^6}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}-\frac{b^3-10\,a\,b\,c}{4\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x^2\left(b^2\,c+5\,a\,c^2\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{9\,b\,c^2\,x^4}{2\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^4\left(b^2+2\,a\,c\right)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}+\frac{6\,c^2\,\mathrm{atan}\left(\left(x^2\left(\frac{36\,c^6}{a\left(4\,a\,c-b^2\right)^{9/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c\right.}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((3*c^3*x^6)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) - (b^3 - 10*a*b*c)/(4*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)) + (x^2*(5*a*c^2 + b^2*c))/(b^4 + 16*a^2*c^2 - 8*a*b^
2*c) + (9*b*c^2*x^4)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2)
 + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + (6*c^2*atan(((x^2*((36*c^6)/(a*
```

```
(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (36*b*c^4*(b^5*c^2 -
8*a*b^3*c^3 + 16*a^2*b*c^4))/(a*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c))) + (72*b*c^6)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2
*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(72*c^6)))/(4*a*c - b^2)
^(5/2)
```

sympy [B] time = 2.91, size = 481, normalized size = 4.26

$$-3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x^2+\frac{-192a^3c^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+144a^2b^2c^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-36ab^4c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3b^6c^2\sqrt{-}}{6c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -3*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**3*c**5*sqrt(-1/(4*a*
c - b**2)**5) + 144*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) - 36*a*b**4*c
**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**6*c**2*sqrt(-1/(4*a*c - b**2)**5) + 3
*b*c**2)/(6*c**3)) + 3*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (192*a**3
*c**5*sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2
)**5) + 36*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) - 3*b**6*c**2*sqrt(-1/(4*
a*c - b**2)**5) + 3*b*c**2)/(6*c**3)) + (10*a*b*c - b**3 + 18*b*c**2*x**4 +
 12*c**3*x**6 + x**2*(20*a*c**2 + 4*b**2*c))/(64*a**4*c**2 - 32*a**3*b**2*c
 + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*c**3 + 4*b**4*c**2) + x**6*
(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5*c) + x**4*(128*a**3*c**3 - 24*a*
b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2 - 64*a**2*b**3*c + 8*a*b**5))
```

3.879 $\int \frac{1}{x(a+bx^2+cx^4)^3} dx$

Optimal. Leaf size=200

$$-\frac{\log \left(a+bx^2+cx^4\right)}{4a^3}+\frac{\log(x)}{a^3}+\frac{16a^2c^2+2bcx^2\left(b^2-7ac\right)-15ab^2c+2b^4}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+}{\sqrt{b}}\right.}{2a^3\left(b^2-4ac\right)^{5/2}}$$

```
[Out] 1/4*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/4*(2*b^4-15*a*b^
2*c+16*a^2*c^2+2*b*c*(-7*a*c+b^2)*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1
/2*b*(30*a^2*c^2-10*a*b^2*c+b^4)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^
3/(-4*a*c+b^2)^(5/2)+ln(x)/a^3-1/4*ln(c*x^4+b*x^2+a)/a^3
```

Rubi [A] time = 0.30, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1114, 740, 822, 800, 634, 618, 206, 628}

$$\frac{16a^2c^2+2bcx^2\left(b^2-7ac\right)-15ab^2c+2b^4}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{5/2}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^3}+\text{lo}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*b^4
- 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*x^2)/(4*a^2*(b^2 - 4*a*c)^2
*(a + b*x^2 + c*x^4)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)) + Log[x]/a^3 - Log[a
+ b*x^2 + c*x^4]/(4*a^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^3}\,dx,x,x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{-2\left(b^2-4ac\right)-3bcx}{x\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)}{4a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subs}}{}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subs}}{}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\log(x}{a^3}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\log(x}{a^3}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\log(x}{a^3}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{b\left(b^4\right.}{}$$

Mathematica [A] time = 0.52, size = 342, normalized size = 1.71

$$\frac{\frac{a^2\left(-2ac+b^2+bcx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{a\left(16a^2c^2-15ab^2c-14abc^2x^2+2b^4+2b^3cx^2\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\left(16a^2c^2\sqrt{b^2-4ac}+30a^2bc^2-10ab^3c-8ab^2c\sqrt{b^2-4ac}+b^4\sqrt{b^2-4ac}+b^5\right)\log\left(-\right.}{\left(b^2-4ac\right)^{5/2}}}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((a^2*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (a*(
2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*x^2 - 14*a*b*c^2*x^2))/((b^2 - 4*
a*c)^2*(a + b*x^2 + c*x^4)) + 4*Log[x] - ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2
+ b^4*Sqrt[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2
 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2) + ((b^
5 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt[b^2 - 4*a*c] + 8*a*b^2*c*Sqrt[b^2
- 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2])/(b^2 - 4*a*c)^(5/2))/(4*a^3)
```

fricas [B] time = 2.39, size = 2017, normalized size = 10.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c^
2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^6 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*
a^3*b^2*c^3 - 64*a^4*c^4)*x^4 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 +
4*a^4*b*c^3)*x^2 + ((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^8 + a^2*b^5 -
 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^
6 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^4 + 2*(a*b^6 - 10*a
^2*b^4*c + 30*a^3*b^2*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^
2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a
^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b
^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^
2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^
4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a) + 4*((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*
b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8
 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b
^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)*log(x))/(a^5*b^6 -
12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^4*c^3
+ 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5
```

```
*b^3*c^3 - 64*a^6*b*c^4)*x^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 3
2*a^6*b^2*c^3 - 128*a^7*c^4)*x^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c
^2 - 64*a^7*b*c^3)*x^2), 1/4*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 -
96*a^5*c^3 + 2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^6 + (4*a*b^6*c
 - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*x^4 + 2*(a*b^7 - 10*a^2*b
^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*x^2 + 2*((b^5*c^2 - 10*a*b^3*c^3 + 30*
a^2*b*c^4)*x^8 + a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^
4*c^2 + 30*a^2*b^2*c^3)*x^6 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*
c^3)*x^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x^2)*sqrt(-b^2 + 4*a*c
)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*
a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^
3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^
4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)*
log(c*x^4 + b*x^2 + a) + 4*((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^
5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)*log(x))/(a^5*b^6 - 12*a^6*b^4*c +
 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c
^4 - 64*a^6*c^5)*x^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*
a^6*b*c^4)*x^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3
- 128*a^7*c^4)*x^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*
c^3)*x^2)]
```

giac [A] time = 1.88, size = 323, normalized size = 1.62

$$-\frac{\left(b^5 - 10\,ab^3c + 30\,a^2bc^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^3b^4 - 8\,a^4b^2c + 16\,a^5c^2\right)\sqrt{-b^2+4\,ac}} + \frac{3\,b^4c^2x^8 - 24\,ab^2c^3x^8 + 48\,a^2c^4x^8 + 6\,b^5cx^6 - 44\,ab^3c^2x^6 + 68\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a
*c))/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt(-b^2 + 4*a*c)) + 1/8*(3*b^4
*c^2*x^8 - 24*a*b^2*c^3*x^8 + 48*a^2*c^4*x^8 + 6*b^5*c*x^6 - 44*a*b^3*c^2*x
^6 + 68*a^2*b*c^3*x^6 + 3*b^6*x^4 - 10*a*b^4*c*x^4 - 58*a^2*b^2*c^2*x^4 + 1
28*a^3*c^3*x^4 + 10*a*b^5*x^2 - 72*a^2*b^3*c*x^2 + 92*a^3*b*c^2*x^2 + 9*a^2
*b^4 - 66*a^3*b^2*c + 96*a^4*c^2)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c*
x^4 + b*x^2 + a)^2) - 1/4*log(c*x^4 + b*x^2 + a)/a^3 + 1/2*log(x^2)/a^3
```

maple [B] time = 0.03, size = 822, normalized size = 4.11

$$-\frac{7b\,c^3x^6}{2\left(c\,x^4+b\,x^2+a\right)^2\left(16a^2c^2-8a\,b^2c+b^4\right)a}+\frac{b^3c^2x^6}{2\left(c\,x^4+b\,x^2+a\right)^2\left(16a^2c^2-8a\,b^2c+b^4\right)a^2}-\frac{29}{4\left(c\,x^4+b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] -7/2/a/(c*x^4+b*x^2+a)^2*b*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2/a^2/(c*x^
4+b*x^2+a)^2*b^3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+4/(c*x^4+b*x^2+a)^2*c^3
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-29/4/a/(c*x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^4*b^2+1/a^2/(c*x^4+b*x^2+a)^2*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x
^4*b^4-1/2/(c*x^4+b*x^2+a)^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*c^2-3/a/(c*x^
4+b*x^2+a)^2*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*c+1/2/a^2/(c*x^4+b*x^2+a)^2
*b^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+6*a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b
^2*c+b^4)*c^2-21/4/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b^2*c+3/4/a
/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b^4-4/a/(16*a^2*c^2-8*a*b^2*c
+b^4)*c^2*ln(c*x^4+b*x^2+a)+2/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c*ln(c*x^4+b*x
^2+a)*b^2-1/4/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*b^4-15/a/(16
*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1
/2))*b*c^2+5/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^2+b)/(4*a*c-b^2)^(1/2))*b^3*c-1/2/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^5+ln(x)/a^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 10.95, size = 9339, normalized size = 46.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2 + c*x^4)^3),x)
```

[Out] log(x)/a^3 + ((3*(b^4 + 8*a^2*c^2 - 7*a*b^2*c))/(4*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^4*(4*b^4*c + 16*a^2*c^3 - 29*a*b^2*c^2))/(4*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (b*x^2*(a^2*c^2 - b^4 + 6*a*b^2*c))/(2*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (b*c^2*x^6*(7*a*c - b^2))/(2*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - (log((((a^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*(4*a*c - b^2)^5))^(1/2) + 1)*((b^2*c^3*(4*b^6 - 497*a^3*c^3 + 302*a^2*b^2*c^2 - 61*a*b^4*c))/(a^4*(4*a*c - b^2)^4) - ((a^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*(4*a*c - b^2)^5))^(1/2) + 1)*((4*b^2*c^2*(b^4 + 23*a^2*c^2 - 9*a*b^2*c))/(a^2*(4*a*c - b^2)^2) + (b*c^2*(a^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*(4*a*c - b^2)^5))^(1/2) + 1)*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^3 + (2*b*c^3*x^2*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^2*(4*a*c - b^2)^2)))/(4*a^3) + (b*c^4*x^2*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*(4*a*c - b^2)^4)))/(4*a^3) - (b^2*c^4*(7*a*c - b^2)^2)/(a^6*(4*a*c - b^2)^4) + (b^3*c^5*x^2*(7*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^6))*(((a^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*(4*a*c - b^2)^5))^(1/2) - 1)*(((a^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*(4*a*c - b^2)^5))^(1/2) - 1)*((4*b^2*c^2*(b^4 + 23*a^2*c^2 - 9*a*b^2*c))/(a^2*(4*a*c - b^2)^2) - (b*c^2*(a^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*(4*a*c - b^2)^5))^(1/2) - 1)*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^3 + (2*b*c^3*x^2*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^2*(4*a*c - b^2)^2)))/(4*a^3) + (b^2*c^3*(4*b^6 - 497*a^3*c^3 + 302*a^2*b^2*c^2 - 61*a*b^4*c))/(a^4*(4*a*c - b^2)^4) + (b*c^4*x^2*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*(4*a*c - b^2)^4)))/(4*a^3) + (b^2*c^4*(7*a*c - b^2)^2)/(a^6*(4*a*c - b^2)^4) - (b^3*c^5*x^2*(7*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^6)))*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)) + (b*atan((((b*((4*a^2*b^8*c^3 - 61*a^3*b^6*c^4 + 302*a^4*b^4*c^5 - 497*a^5*b^2*c^6)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3) - (((4*a^4*b^10*c^2 - 68*a^5*b^8*c^3 + 444*a^6*b^6*c^4 - 1312*a^7*b^4*c^5 + 1472*a^8*b^2*c^6)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3) + ((4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*(4*a*c - b^2)^(5/2)) - (((b*((4*a^4*b^10*c^2 - 68*a^5*b^8*c^3 + 444*a^6*b^6*c^4 - 1312*a^7*b^4*c^5 + 1472*a^8*b^2*c^6)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3) + ((4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(a^6*b^8 + 256*a^1

0*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096
*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2
c^4)))(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*(4*a*c - b^2)^(5/2)) + (b*(
b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b
^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*b^10 - 2048*a^5*c^5 + 32
0*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(8*a^3*(
4*a*c - b^2)^(5/2)*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2
- 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*
c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(2*b^10 - 2048*a^5*c^5 + 320*a
^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(4*a^3*b
^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 51
20*a^7*b^2*c^4)) + (b^3*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^3*(4*a^7*b^10*c^2 -
64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6))
/(64*a^9*(4*a*c - b^2)^(15/2)*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a
^8*b^4*c^2 - 256*a^9*b^2*c^3)))*(3*b^8 + 160*a^4*c^4 + 180*a^2*b^4*c^2 - 32
5*a^3*b^2*c^3 - 39*a*b^6*c)*(16*a^9*b^12*(4*a*c - b^2)^(15/2) + 65536*a^15*
c^6*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^11
*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(15/2) + 6
1440*a^13*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c - b^2)^(
15/2)))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*
b^6*c^4 - 600*a^3*b^4*c^5 + 900*a^4*b^2*c^6)*(6*b^10 - 6400*a^5*c^5 + 960*a
^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*b^8*c)) - (x^2*(((
(((b*((5120*a^10*b*c^9 + 2*a^4*b^13*c^3 - 36*a^5*b^11*c^4 + 276*a^6*b^9*c^5
- 1216*a^7*b^7*c^6 + 3456*a^8*b^5*c^7 - 6144*a^9*b^3*c^8)/(a^6*b^12 + 4096
*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*
b^4*c^4 - 6144*a^11*b^2*c^5) - ((2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 -
1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c)*(163840*a^13*b*c^9 - 12*a
^6*b^15*c^2 + 328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97
280*a^10*b^7*c^6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(2*(4*a^3*b^
10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 512
0*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2
- 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^2
*c^2 - 10*a*b^2*c))/(4*a^3*(4*a*c - b^2)^(5/2)) - (b*(b^4 + 30*a^2*c^2 - 10
*a*b^2*c)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 256
0*a^4*b^2*c^4 - 40*a*b^8*c)*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*
b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 + 227
328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(8*a^3*(4*a*c - b^2)^(5/2)*(4*a^3*
b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5
120*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^
2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10 - 2
048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*
b^8*c))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 25
60*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)) + (b*((8960*a^7*b*c^9 - 6*a^2*b^11*c^4
+ 137*a^3*b^9*c^5 - 1217*a^4*b^7*c^6 + 5256*a^5*b^5*c^7 - 11024*a^6*b^3*c^8
)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^

6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((5120*a^10*b*c^9 + 2*a^4
*b^13*c^3 - 36*a^5*b^11*c^4 + 276*a^6*b^9*c^5 - 1216*a^7*b^7*c^6 + 3456*a^8
*b^5*c^7 - 6144*a^9*b^3*c^8)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 24
0*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) -
((2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^
2*c^4 - 40*a*b^8*c)*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*b^13*c^3
- 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 + 227328*a^11
*b^5*c^7 - 294912*a^12*b^3*c^8))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8
*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^12 + 409
6*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10
*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 -
1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(4*a^3*b^10 - 4096*a^
8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^
4)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*(4*a*c - b^2)^(5/2)) + (b^3*(b
^4 + 30*a^2*c^2 - 10*a*b^2*c)^3*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*
a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 +
227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(64*a^9*(4*a*c - b^2)^(15/2)*(
a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c
^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(3*b^8 + 160*a^4*c^4 + 180*a^
2*b^4*c^2 - 325*a^3*b^2*c^3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*
(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*
c^4 - 120*a*b^8*c)) + (3*b*((b^9*c^5 - 21*a*b^7*c^6 + 147*a^2*b^5*c^7 - 343
*a^3*b^3*c^8)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 -
1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((8960*a^7*b*
c^9 - 6*a^2*b^11*c^4 + 137*a^3*b^9*c^5 - 1217*a^4*b^7*c^6 + 5256*a^5*b^5*c^
7 - 11024*a^6*b^3*c^8)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*
b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((51
20*a^10*b*c^9 + 2*a^4*b^13*c^3 - 36*a^5*b^11*c^4 + 276*a^6*b^9*c^5 - 1216*a
^7*b^7*c^6 + 3456*a^8*b^5*c^7 - 6144*a^9*b^3*c^8)/(a^6*b^12 + 4096*a^12*c^6
- 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 -
6144*a^11*b^2*c^5) - ((2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*
b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c)*(163840*a^13*b*c^9 - 12*a^6*b^15*c
^2 + 328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*
b^7*c^6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(2*(4*a^3*b^10 - 4096
*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2
c^4)(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^
9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10 - 2048*a^5*c^5
+ 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*
(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*
c^3 + 5120*a^7*b^2*c^4)))*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a
^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(4*a^3*b^10 - 4096*a^8*c^5
- 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)) -
(b*((b*((5120*a^10*b*c^9 + 2*a^4*b^13*c^3 - 36*a^5*b^11*c^4 + 276*a^6*b^9*c
^5 - 1216*a^7*b^7*c^6 + 3456*a^8*b^5*c^7 - 6144*a^9*b^3*c^8)/(a^6*b^12 + 40
96*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^1

0*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2
- 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c)*(163840*a^13*b*c^9 - 12
*a^6*b^15*c^2 + 328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 -
97280*a^10*b^7*c^6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(2*(4*a^3*
b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5
120*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^
2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a
^2*c^2 - 10*a*b^2*c))/(4*a^3*(4*a*c - b^2)^(5/2)) - (b*(b^4 + 30*a^2*c^2 -
10*a*b^2*c)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2
560*a^4*b^2*c^4 - 40*a*b^8*c)*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*a^
7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 + 2
27328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(8*a^3*(4*a*c - b^2)^(5/2)*(4*a^
3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 +
5120*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*
c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30
*a^2*c^2 - 10*a*b^2*c))/(4*a^3*(4*a*c - b^2)^(5/2)) + (b^2*(b^4 + 30*a^2*c^
2 - 10*a*b^2*c)^2*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c
^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c)*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 +
328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c
^6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(32*a^6*(4*a*c - b^2)^5*(4
*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^
3 + 5120*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b
^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^6 -
45*a^3*c^3 + 40*a^2*b^2*c^2 - 11*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^6*(6*b
^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4
- 120*a*b^8*c)))*(16*a^9*b^12*(4*a*c - b^2)^(15/2) + 65536*a^15*c^6*(4*a*c
- b^2)^(15/2) - 384*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^11*b^8*c^2*(4
*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^13*b
^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c - b^2)^(15/2)))/(b^
10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 600*a^3*b^4*c^5 + 900*a^4*b^2*c^6
) + (3*b*(b^6 - 45*a^3*c^3 + 40*a^2*b^2*c^2 - 11*a*b^4*c)*((((4*a^2*b^8*c^3
- 61*a^3*b^6*c^4 + 302*a^4*b^4*c^5 - 497*a^5*b^2*c^6)/(a^6*b^8 + 256*a^10*
c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3) - (((4*a^4*b^10*c^2
- 68*a^5*b^8*c^3 + 444*a^6*b^6*c^4 - 1312*a^7*b^4*c^5 + 1472*a^8*b^2*c^6)/(
a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3) +
((4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 +
1024*a^11*b^2*c^6)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*
c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b
^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^
4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(2*b^10
- 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40
*a*b^8*c))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 -
2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^
6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(4*a^3*b^10 -
4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^

```
7*b^2*c^4)) - (b^6*c^4 - 14*a*b^4*c^5 + 49*a^2*b^2*c^6)/(a^6*b^8 + 256*a^10
*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3) + (b*((b*((4*a^4*b^
10*c^2 - 68*a^5*b^8*c^3 + 444*a^6*b^6*c^4 - 1312*a^7*b^4*c^5 + 1472*a^8*b^2
*c^6)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2
*c^3) + ((4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4
*c^5 + 1024*a^11*b^2*c^6)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a
^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(a^6*b^8 + 256*a^10*c^4 - 1
6*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5
- 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(
b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*(4*a*c - b^2)^(5/2)) + (b*(b^4 + 30*
a^2*c^2 - 10*a*b^2*c)*(4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 -
1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6
*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(8*a^3*(4*a*c - b
^2)^(5/2)*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9
*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 256
0*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3
*(4*a*c - b^2)^(5/2)) + (b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2*(4*a^7*b^10*
c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*
c^6)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4
*b^2*c^4 - 40*a*b^8*c))/(32*a^6*(4*a*c - b^2)^5*(a^6*b^8 + 256*a^10*c^4 - 1
6*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5
- 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(
16*a^9*b^12*(4*a*c - b^2)^(15/2) + 65536*a^15*c^6*(4*a*c - b^2)^(15/2) - 38
4*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^11*b^8*c^2*(4*a*c - b^2)^(15/2)
 - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^13*b^4*c^4*(4*a*c - b^
2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c - b^2)^(15/2)))/(8*a^3*c^2*(4*a*c - b
^2)^6*(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 600*a^3*b^4*c^5 + 900*a^
4*b^2*c^6)*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 77
75*a^4*b^2*c^4 - 120*a*b^8*c)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(2*a^3*(4*
a*c - b^2)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.880 $\int \frac{1}{x^3(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=255

$$\frac{3b\log\left(a+bx^2+cx^4\right)}{4a^4}-\frac{3b\log(x)}{a^4}-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3x^2\left(b^2-4ac\right)^2}+\frac{20a^2c^2+3bcx^2\left(b^2-6ac\right)-20ab^2c+3b^4}{4a^2x^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\left(-20a^3\right.}{}$$

```
[Out] -3/2*(-5*a*c+b^2)*(-2*a*c+b^2)/a^3/(-4*a*c+b^2)^2/x^2+1/4*(b*c*x^2-2*a*c+b^
2)/a/(-4*a*c+b^2)/x^2/(c*x^4+b*x^2+a)^2+1/4*(3*b^4-20*a*b^2*c+20*a^2*c^2+3*
b*c*(-6*a*c+b^2)*x^2)/a^2/(-4*a*c+b^2)^2/x^2/(c*x^4+b*x^2+a)-3/2*(-20*a^3*c
^3+30*a^2*b^2*c^2-10*a*b^4*c+b^6)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a
^4/(-4*a*c+b^2)^(5/2)-3*b*ln(x)/a^4+3/4*b*ln(c*x^4+b*x^2+a)/a^4
```

Rubi [A] time = 0.39, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1114, 740, 822, 800, 634, 618, 206, 628}

$$\frac{20a^2c^2+3bcx^2\left(b^2-6ac\right)-20ab^2c+3b^4}{4a^2x^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\left(b^2-4ac\right)^{5/2}}-\frac{3\left(b^2-5ac\right)\left(\right.}{2a^3x^2\left(b^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*x^2) + (b^2 - 2*a*c
 + b*c*x^2)/(4*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)^2) + (3*b^4 - 20*a*b
^2*c + 20*a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*x^2)/(4*a^2*(b^2 - 4*a*c)^2*x^2*(a
+ b*x^2 + c*x^4)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*Arc
Tanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2)) - (3*b*L
og[x])/a^4 + (3*b*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{-3b^2+10ac-4bcx}{x^2\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc\left(b^2-6ac\right)x^2}{4a^2\left(b^2-4ac\right)^2x^2\left(a+bx^2+cx^4\right)} + \frac{\text{Sul}}{}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc\left(b^2-6ac\right)x^2}{4a^2\left(b^2-4ac\right)^2x^2\left(a+bx^2+cx^4\right)} + \frac{\text{Sul}}{}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2x^2} + \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} + \frac{3b^4-20ab^2c+20a^2c}{4a^2\left(b^2-4ac\right)^2x}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2x^2} + \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} + \frac{3b^4-20ab^2c+20a^2c}{4a^2\left(b^2-4ac\right)^2x}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2x^2} + \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} + \frac{3b^4-20ab^2c+20a^2c}{4a^2\left(b^2-4ac\right)^2x}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2x^2} + \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} + \frac{3b^4-20ab^2c+20a^2c}{4a^2\left(b^2-4ac\right)^2x}$$

Mathematica [A] time = 0.62, size = 402, normalized size = 1.58

$$\frac{\frac{a^2\left(-3abc-2ac^2x^2+b^3+b^2cx^2\right)}{\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)^2}-\frac{a\left(46a^2bc^2+28a^2c^3x^2-29ab^3c-26ab^2c^2x^2+4b^5+4b^4cx^2\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\left(-20a^3c^3+30a^2b^2c^2+16a^2bc^2\sqrt{b^2-4ac}-10ab^4c+b^5\sqrt{b\cdots\right.}{\left(b^2-4a\cdots\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-2*a)/x^2 + (a^2*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((-b^2 + 4*a*
c)*(a + b*x^2 + c*x^4)^2) - (a*(4*b^5 - 29*a*b^3*c + 46*a^2*b*c^2 + 4*b^4*c
*x^2 - 26*a*b^2*c^2*x^2 + 28*a^2*c^3*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*
x^4)) - 12*b*Log[x] + (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3 +
b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2
 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2) + (3*(
-b^6 + 10*a*b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8
*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b
^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/(4*a^4)
```

fricas [B] time = 3.06, size = 2312, normalized size = 9.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c
^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*x^8 + 3*(4*a*b^7*c - 45*
a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*x^6 + 2*(3*a*b^8 - 30*a^2*b^
6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*x^4 + (9*a^2*b^7 - 104
*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*x^2 + 3*((b^6*c^2 - 10*a*b^4*
c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^10 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*
b^3*c^3 - 20*a^3*b*c^4)*x^8 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^
2*c^3 - 40*a^4*c^4)*x^6 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4
*b*c^3)*x^4 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*x^2)*s
qrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*s
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^
2*b^3*c^4 - 64*a^3*b*c^5)*x^10 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 -
 64*a^3*b^2*c^4)*x^8 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3
- 128*a^4*b*c^4)*x^6 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^
2*c^3)*x^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x^2)*
log(c*x^4 + b*x^2 + a) + 12*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*
a^3*b*c^5)*x^10 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4
)*x^8 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4
```

```
)*x^6 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^4 + (a
^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x^2)*log(x))/((a^4*b
^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*x^10 + 2*(a^4*b^7*c
- 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*x^8 + (a^4*b^8 - 10*a^5*b
^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*x^6 + 2*(a^5*b^7 - 12
*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*x^4 + (a^6*b^6 - 12*a^7*b^4*c +
 48*a^8*b^2*c^2 - 64*a^9*c^3)*x^2), -1/4*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5
*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 4
0*a^4*c^5)*x^8 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*
b*c^4)*x^6 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 -
200*a^5*c^4)*x^4 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b
*c^3)*x^2 + 6*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^10
+ 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^8 + (b^8 - 8*a
*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^6 + 2*(a*b^7 - 10*
a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x^4 + (a^2*b^6 - 10*a^3*b^4*c +
30*a^4*b^2*c^2 - 20*a^5*c^3)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*
sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3
*c^4 - 64*a^3*b*c^5)*x^10 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a
^3*b^2*c^4)*x^8 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128
*a^4*b*c^4)*x^6 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3
)*x^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x^2)*log(c
*x^4 + b*x^2 + a) + 12*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b
*c^5)*x^10 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^8
 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^6
 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^4 + (a^2*b^
7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x^2)*log(x))/((a^4*b^6*c^
2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*x^10 + 2*(a^4*b^7*c - 12*
a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*x^8 + (a^4*b^8 - 10*a^5*b^6*c
+ 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*x^6 + 2*(a^5*b^7 - 12*a^6*
b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*x^4 + (a^6*b^6 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2 - 64*a^9*c^3)*x^2)]
```

giac [A] time = 1.80, size = 382, normalized size = 1.50

$$\frac{3\left(b^{6}-10\,ab^{4}c+30\,a^{2}b^{2}c^{2}-20\,a^{3}c^{3}\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\left(a^{4}b^{4}-8\,a^{5}b^{2}c+16\,a^{6}c^{2}\right)\sqrt{-b^{2}+4\,ac}}-\frac{9\,b^{5}c^{2}x^{8}-72\,ab^{3}c^{3}x^{8}+144\,a^{2}bc^{4}x^{8}+18\,b^{6}cx^{6}-136\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/2*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x^2 + b)/s
qrt(-b^2 + 4*a*c))/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b^2 + 4*a*c)
) - 1/8*(9*b^5*c^2*x^8 - 72*a*b^3*c^3*x^8 + 144*a^2*b*c^4*x^8 + 18*b^6*c*x^
```

```
6 - 136*a*b^4*c^2*x^6 + 236*a^2*b^2*c^3*x^6 + 56*a^3*c^4*x^6 + 9*b^7*x^4 -
38*a*b^5*c*x^4 - 110*a^2*b^3*c^2*x^4 + 436*a^3*b*c^3*x^4 + 26*a*b^6*x^2 - 1
92*a^2*b^4*c*x^2 + 316*a^3*b^2*c^2*x^2 + 72*a^4*c^3*x^2 + 19*a^2*b^5 - 144*
a^3*b^3*c + 260*a^4*b*c^2)/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*(c*x^4 + b
*x^2 + a)^2) + 3/4*b*log(c*x^4 + b*x^2 + a)/a^4 - 3/2*b*log(x^2)/a^4 + 1/2*
(3*b*x^2 - a)/(a^4*x^2)
```

maple [B] time = 0.03, size = 1002, normalized size = 3.93

$$-\frac{7c^4x^6}{\left(c\,x^4+b\,x^2+a\right)^2\left(16a^2c^2-8a\,b^2c+b^4\right)a}+\frac{13b^2c^3x^6}{2\left(c\,x^4+b\,x^2+a\right)^2\left(16a^2c^2-8a\,b^2c+b^4\right)a^2}-\frac{b^4}{\left(c\,x^4+b\,x^2+a\right)^2\left(1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] -7/a/(c*x^4+b*x^2+a)^2*c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+13/2/a^2/(c*x^4+b
*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*b^2-1/a^3/(c*x^4+b*x^2+a)^2*c^
2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*b^4-37/2/a/(c*x^4+b*x^2+a)^2*b*c^3/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^4+55/4/a^2/(c*x^4+b*x^2+a)^2*b^3*c^2/(16*a^2*c^2-8*a*
b^2*c+b^4)*x^4-2/a^3/(c*x^4+b*x^2+a)^2*b^5*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4
-9/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*c^3-7/2/a/(c*x^4+b*x^2+
a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^2*c^2+6/a^2/(c*x^4+b*x^2+a)^2/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^2*b^4*c-1/a^3/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c
+b^4)*x^2*b^6-29/2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b*c^2+9/a/(
c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b^3*c-5/4/a^2/(c*x^4+b*x^2+a)^2
/(16*a^2*c^2-8*a*b^2*c+b^4)*b^5+12/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*ln(c*
x^4+b*x^2+a)*b-6/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c*ln(c*x^4+b*x^2+a)*b^3+3/4
/a^4/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*b^5-30/a/(16*a^2*c^2-8*a*
b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c^3+45/a
^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b
^2)^(1/2))*b^2*c^2-15/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arct
an((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4*c+3/2/a^4/(16*a^2*c^2-8*a*b^2*c+b^4)/
(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^6-1/2/a^3/x^2-3*b
*ln(x)/a^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 11.76, size = 10074, normalized size = 39.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2 + c*x^4)^3),x)
```

```
[Out] (log(((27*c^5*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b^2)^6) -
 ((3*b - 3*a^4*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*(4
*a*c - b^2)^5))^(1/2))*((9*c^3*(4*b^10 - 100*a^5*c^5 + 342*a^2*b^6*c^2 - 83
7*a^3*b^4*c^3 + 780*a^4*b^2*c^4 - 61*a*b^8*c))/(a^6*(4*a*c - b^2)^4) - ((3*
b - 3*a^4*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*(4*a*c
- b^2)^5))^(1/2))*((6*c^3*x^2*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4
*c))/(a^3*(4*a*c - b^2)^2) + (b*c^2*(3*b - 3*a^4*(-(b^6 - 20*a^3*c^3 + 30*a
^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 -
 10*a*c*x^2))/a^4 + (12*b*c^2*(b^6 - 10*a^3*c^3 + 23*a^2*b^2*c^2 - 9*a*b^4*
c))/(a^3*(4*a*c - b^2)^2)))/(4*a^4) + (9*b*c^4*x^2*(6*b^8 + 900*a^4*c^4 + 4
79*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*(4*a*c - b^2)^4)))/(4
*a^4) + (27*b*c^4*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^2)/(a^9*(4*a*c - b^2)^4))*
((27*c^5*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b^2)^6) - ((3*
b + 3*a^4*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*(4*a*c
- b^2)^5))^(1/2))*((9*c^3*(4*b^10 - 100*a^5*c^5 + 342*a^2*b^6*c^2 - 837*a^3
*b^4*c^3 + 780*a^4*b^2*c^4 - 61*a*b^8*c))/(a^6*(4*a*c - b^2)^4) - ((3*b + 3
*a^4*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*(4*a*c - b^2
)^5))^(1/2))*((6*c^3*x^2*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/
(a^3*(4*a*c - b^2)^2) + (b*c^2*(3*b + 3*a^4*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^
2*c^2 - 10*a*b^4*c)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a
*c*x^2))/a^4 + (12*b*c^2*(b^6 - 10*a^3*c^3 + 23*a^2*b^2*c^2 - 9*a*b^4*c))/(
a^3*(4*a*c - b^2)^2)))/(4*a^4) + (9*b*c^4*x^2*(6*b^8 + 900*a^4*c^4 + 479*a^
2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*(4*a*c - b^2)^4)))/(4*a^4)
 + (27*b*c^4*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^2)/(a^9*(4*a*c - b^2)^4)))*(6*b
^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^
4 - 120*a*b^9*c))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^
6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)) - (3*b*log(x))/a^4 - (1/(2*a)
 + (x^4*(3*b^6 + 50*a^3*c^3 + 7*a^2*b^2*c^2 - 18*a*b^4*c))/(2*a^3*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)) + (3*x^6*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3))/(4
*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(9*b^5 + 122*a^2*b*c^2 - 68*a*b
^3*c))/(4*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c^2*x^8*(b^4 + 10*a^2*c^
2 - 7*a*b^2*c))/(2*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^6*(2*a*c + b^2)
+ a^2*x^2 + c^2*x^10 + 2*a*b*x^4 + 2*b*c*x^8) - (3*atan((x^2*((((27000*a^6*
c^11 + 27*b^12*c^5 - 567*a*b^10*c^6 + 4779*a^2*b^8*c^7 - 20601*a^3*b^6*c^8
+ 47790*a^4*b^4*c^9 - 56700*a^5*b^2*c^10)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^
```

10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144
*a^14*b^2*c^5) - (((129600*a^9*b*c^10 + 54*a^3*b^13*c^4 - 1233*a^4*b^11*c^5
 + 11583*a^5*b^9*c^6 - 57204*a^6*b^7*c^7 + 156276*a^7*b^5*c^8 - 223200*a^8*
b^3*c^9)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 12
80*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((153600*a^13*c
^10 + 6*a^6*b^14*c^3 - 108*a^7*b^12*c^4 + 588*a^8*b^10*c^5 + 792*a^9*b^8*c^
6 - 22272*a^10*b^6*c^7 + 100608*a^11*b^4*c^8 - 199680*a^12*b^2*c^9)/(a^9*b^
12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3
 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - ((6*b^11 - 6144*a^5*b*c^5 + 960*
a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c)*(163840*a^
16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960
*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 294912*a^15*b^3*
c^8))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560
*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c
 + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2
c^5)))(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 768
0*a^4*b^3*c^4 - 120*a*b^9*c))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c
 + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*b^11 - 6144*a
^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^
9*c))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560
*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)) - (3*((3*((153600*a^13*c^10 + 6*a^6*b^14*
c^3 - 108*a^7*b^12*c^4 + 588*a^8*b^10*c^5 + 792*a^9*b^8*c^6 - 22272*a^10*b^
6*c^7 + 100608*a^11*b^4*c^8 - 199680*a^12*b^2*c^9)/(a^9*b^12 + 4096*a^15*c^
6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c
^4 - 6144*a^14*b^2*c^5) - ((6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 384
0*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c)*(163840*a^16*b*c^9 - 12*a^9
*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 9
7280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8))/(2*(4*a^4*b
^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 51
20*a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c
^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20
*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*(4*a*c - b^2)^(5/2)) - (3*(
b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11 - 6144*a^5*b*c^5 +
960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c)*(16384
0*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^11*c^4 + 2
4960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 294912*a^15*
b^3*c^8))/(8*a^4*(4*a*c - b^2)^(5/2)*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^
8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 40
96*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*
a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10
*a*b^4*c))/(4*a^4*(4*a*c - b^2)^(5/2)) + (9*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*
c^2 - 10*a*b^4*c)^2*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b
^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c)*(163840*a^16*b*c^9 - 12*a^9*b^15*c
^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^
13*b^7*c^6 + 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8))/(32*a^8*(4*a*c - b

^2)^5*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(3*b^8 + 10*a^4*c^4 + 120*a^2*b^4*c^2 - 145*a^3*b^2*c^3 - 33*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^6*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)) + (b*((((3*((153600*a^13*c^10 + 6*a^6*b^14*c^3 - 108*a^7*b^12*c^4 + 588*a^8*b^10*c^5 + 792*a^9*b^8*c^6 - 22272*a^10*b^6*c^7 + 100608*a^11*b^4*c^8 - 199680*a^12*b^2*c^9)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - ((6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c)*(163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*(4*a*c - b^2)^(5/2)) - (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c)*(163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8))/(8*a^4*(4*a*c - b^2)^(5/2)*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)) - (3*((129600*a^9*b*c^10 + 54*a^3*b^13*c^4 - 1233*a^4*b^11*c^5 + 11583*a^5*b^9*c^6 - 57204*a^6*b^7*c^7 + 156276*a^7*b^5*c^8 - 223200*a^8*b^3*c^9)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((153600*a^13*c^10 + 6*a^6*b^14*c^3 - 108*a^7*b^12*c^4 + 588*a^8*b^10*c^5 + 792*a^9*b^8*c^6 - 22272*a^10*b^6*c^7 + 100608*a^11*b^4*c^8 - 199680*a^12*b^2*c^9)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - ((6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c)*(163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*(

4*a*c - b^2)^(5/2)) + (27*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^3*(163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8))/(64*a^12*(4*a*c - b^2)^(15/2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(3*b^8 + 190*a^4*c^4 + 180*a^2*b^4*c^2 - 335*a^3*b^2*c^3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)))*(16*a^12*b^12*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*(4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*a*c - b^2)^(15/2)))/(10800*a^6*c^8 + 27*b^12*c^2 - 540*a*b^10*c^3 + 4320*a^2*b^8*c^4 - 17280*a^3*b^6*c^5 + 35100*a^4*b^4*c^6 - 32400*a^5*b^2*c^7) + (((27*b^9*c^4 - 378*a*b^7*c^5 + 2700*a^4*b*c^8 + 1863*a^2*b^5*c^6 - 3780*a^3*b^3*c^7)/(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) + (((900*a^8*c^8 - 36*a^3*b^10*c^3 + 549*a^4*b^8*c^4 - 3078*a^5*b^6*c^5 + 7533*a^6*b^4*c^6 - 7020*a^7*b^2*c^7)/(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) - (((1920*a^11*b*c^7 - 12*a^6*b^11*c^2 + 204*a^7*b^9*c^3 - 1332*a^8*b^7*c^4 + 4056*a^9*b^5*c^5 - 5376*a^10*b^3*c^6)/(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) - ((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^6)*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(2*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)) + (3*((3*((1920*a^11*b*c^7 - 12*a^6*b^11*c^2 + 204*a^7*b^9*c^3 - 1332*a^8*b^7*c^4 + 4056*a^9*b^5*c^5 - 5376*a^10*b^3*c^6)/(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) - ((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^6)*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(2*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*(4*a*c - b^2)^(5/2)) - (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^6)*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(8*a^4*(4*a*c - b^2)^(5/2)*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096

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*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2
*c^4)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*(4*a*c - b
^2)^(5/2)) - (9*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2*(4*a^10*
b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^
14*b^2*c^6)*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 +
 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(32*a^8*(4*a*c - b^2)^5*(a^9*b^8 + 256*a^
13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 -
4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8
*b^2*c^4)))*(3*b^8 + 10*a^4*c^4 + 120*a^2*b^4*c^2 - 145*a^3*b^2*c^3 - 33*a*
b^6*c)*(16*a^12*b^12*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*(4*a*c - b^2)^(1
5/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*(4*a*c - b^
2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^4*(4
*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*a*c - b^2)^(15/2)))/(8*a^3*c^2*(
4*a*c - b^2)^6*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 -
 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)*(10800*a^6*c^8 + 27*b^
12*c^2 - 540*a*b^10*c^3 + 4320*a^2*b^8*c^4 - 17280*a^3*b^6*c^5 + 35100*a^4*
b^4*c^6 - 32400*a^5*b^2*c^7)) - (b*((((3*((1920*a^11*b*c^7 - 12*a^6*b^11*c^
2 + 204*a^7*b^9*c^3 - 1332*a^8*b^7*c^4 + 4056*a^9*b^5*c^5 - 5376*a^10*b^3*c
^6)/(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^
2*c^3) - ((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13
*b^4*c^5 + 1024*a^14*b^2*c^6)*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 -
3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(2*(a^9*b^8 + 256*a^13*
c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 409
6*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^
2*c^4)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*(4*a*c -
b^2)^(5/2)) - (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(4*a^10*b
^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^1
4*b^2*c^6)*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 +
7680*a^4*b^3*c^4 - 120*a*b^9*c))/(8*a^4*(4*a*c - b^2)^(5/2)*(a^9*b^8 + 256*
a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10
- 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a
^8*b^2*c^4)))*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3
 + 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*
b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)) - (3*((900*
a^8*c^8 - 36*a^3*b^10*c^3 + 549*a^4*b^8*c^4 - 3078*a^5*b^6*c^5 + 7533*a^6*b
^4*c^6 - 7020*a^7*b^2*c^7)/(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^1
1*b^4*c^2 - 256*a^12*b^2*c^3) - (((1920*a^11*b*c^7 - 12*a^6*b^11*c^2 + 204*
a^7*b^9*c^3 - 1332*a^8*b^7*c^4 + 4056*a^9*b^5*c^5 - 5376*a^10*b^3*c^6)/(a^9
*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) -
 ((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5
 + 1024*a^14*b^2*c^6)*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3
*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(2*(a^9*b^8 + 256*a^13*c^4 - 16
*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^9*c^
5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))
*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b
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^3*c^4 - 120*a*b^9*c))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a
^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(b^6 - 20*a^3*c^3 + 30*
a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*(4*a*c - b^2)^(5/2)) + (27*(b^6 - 20*a^3*
c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^3*(4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 3
84*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^6))/(64*a^12*(4*a*c -
 b^2)^(15/2)*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 25
6*a^12*b^2*c^3)))*(3*b^8 + 190*a^4*c^4 + 180*a^2*b^4*c^2 - 335*a^3*b^2*c^3
- 39*a*b^6*c)*(16*a^12*b^12*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*(4*a*c -
b^2)^(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*(4*a
*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4
*c^4*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*a*c - b^2)^(15/2)))/(8*a^
3*c^2*(4*a*c - b^2)^(13/2)*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a
^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)*(10800*a^6
*c^8 + 27*b^12*c^2 - 540*a*b^10*c^3 + 4320*a^2*b^8*c^4 - 17280*a^3*b^6*c^5
+ 35100*a^4*b^4*c^6 - 32400*a^5*b^2*c^7)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c
^2 - 10*a*b^4*c))/(2*a^4*(4*a*c - b^2)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.881 $\int \frac{x^{10}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=400

$$\frac{3\left(-\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\tan^{-}}{8\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+}}$$

```
[Out] -3/8*b*(-8*a*c+b^2)*x/c^2/(-4*a*c+b^2)^2+1/8*(-28*a*c+b^2)*x^3/c/(-4*a*c+b^
2)^2+1/4*x^7*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/8*x^5*(12*a*b-(-2
8*a*c+b^2)*x^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3/16*arctan(x*2^(1/2)*c^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^4-9*a*b^2*c+28*a^2*c^2+(-44*a^2*b*c^2+11
*a*b^3*c-b^5)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)^2*2^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2)+3/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/
2))*(b^4-9*a*b^2*c+28*a^2*c^2+(44*a^2*b*c^2-11*a*b^3*c+b^5)/(-4*a*c+b^2)^(1
/2))/c^(5/2)/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 1.73, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1120, 1275, 1279, 1166, 205}

$$\frac{3\left(-\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\tan^{-}}{8\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (-3*b*(b^2 - 8*a*c)*x)/(8*c^2*(b^2 - 4*a*c)^2) + ((b^2 - 28*a*c)*x^3)/(8*c*
(b^2 - 4*a*c)^2) + (x^7*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
^2) + (x^5*(12*a*b - (b^2 - 28*a*c)*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c
*x^4)) + (3*(b^4 - 9*a*b^2*c + 28*a^2*c^2 - (b^5 - 11*a*b^3*c + 44*a^2*b*c^
2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(8*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*(b
^4 - 9*a*b^2*c + 28*a^2*c^2 + (b^5 - 11*a*b^3*c + 44*a^2*b*c^2)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]
*c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^{10}}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{x^6\left(14a+bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= \frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^5\left(12ab-\left(b^2-28ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\int \frac{x^4\left(60ab-3\left(b^2-28ac\right)x^2\right)}{a+bx^2+cx^4}\,dx}{8\left(b^2-4ac\right)^2}$$

$$= \frac{\left(b^2-28ac\right)x^3}{8c\left(b^2-4ac\right)^2} + \frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^5\left(12ab-\left(b^2-28ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\int \frac{x^2(\cdots}{\cdots}}{\cdots}$$

$$= -\frac{3b\left(b^2-8ac\right)x}{8c^2\left(b^2-4ac\right)^2} + \frac{\left(b^2-28ac\right)x^3}{8c\left(b^2-4ac\right)^2} + \frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^5\left(12ab-\left(b^2-\cdots\right.\right.}{8\left(b^2-4ac\right)^2\left(a+\cdots\right.}$$

$$= -\frac{3b\left(b^2-8ac\right)x}{8c^2\left(b^2-4ac\right)^2} + \frac{\left(b^2-28ac\right)x^3}{8c\left(b^2-4ac\right)^2} + \frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^5\left(12ab-\left(b^2-\cdots\right.\right.}{8\left(b^2-4ac\right)^2\left(a+\cdots\right.}$$

$$= -\frac{3b\left(b^2-8ac\right)x}{8c^2\left(b^2-4ac\right)^2} + \frac{\left(b^2-28ac\right)x^3}{8c\left(b^2-4ac\right)^2} + \frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^5\left(12ab-\left(b^2-\cdots\right.\right.}{8\left(b^2-4ac\right)^2\left(a+\cdots\right.}$$

Mathematica [A] time = 1.17, size = 455, normalized size = 1.14

$$\frac{-\frac{4\left(a^2cx\left(2cx^2-3b\right)+ab^2x\left(b-4cx^2\right)+b^4x^3\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3\sqrt{2}\sqrt{c}\left(28a^2c^2\sqrt{b^2-4ac}-44a^2bc^2+11ab^3c-9ab^2c\sqrt{b^2-4ac}+b^4\sqrt{b^2-4ac}-b^5\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*x*(2*b^5 - 17*a*b^3*c + 48*a^2*b*c^2 - 5*b^4*c*x^2 + 37*a*b^2*c^2*x^2 -
 44*a^2*c^3*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (4*(b^4*x^3 + a*b
^2*x*(b - 4*c*x^2) + a^2*c*x*(-3*b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 +
 c*x^4)^2) + (3*Sqrt[2]*Sqrt[c]*(-b^5 + 11*a*b^3*c - 44*a^2*b*c^2 + b^4*Sqr
t[b^2 - 4*a*c] - 9*a*b^2*c*Sqrt[b^2 - 4*a*c] + 28*a^2*c^2*Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(
```

```
5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(b^5 - 11*a*b^3*c +
44*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c] - 9*a*b^2*c*Sqrt[b^2 - 4*a*c] + 28*a^2
*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c^3)
```

fricas [B] time = 1.65, size = 4279, normalized size = 10.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*(5*b^4*c - 37*a*b^2*c^2 + 44*a^2*c^3)*x^7 + 2*(3*b^5 - 20*a*b^3*c
- 4*a^2*b*c^2)*x^5 + 2*(6*a*b^4 - 49*a^2*b^2*c + 28*a^3*c^2)*x^3 + 3*sqrt(1
/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3
 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6
*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)
*x^2)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^
4*b*c^4 + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 12
80*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 -
1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c
^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 2
0*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a
^5*c^10))*log(27*(21*a^2*b^8 - 447*a^3*b^6*c + 4189*a^4*b^4*c^2 - 19208*a^5
*b^2*c^3 + 38416*a^6*c^4)*x + 27/2*sqrt(1/2)*(b^13 - 31*a*b^11*c + 413*a^2*
b^9*c^2 - 3012*a^3*b^7*c^3 + 12496*a^4*b^5*c^4 - 27584*a^5*b^3*c^5 + 25088*
a^6*b*c^6 - (b^14*c^5 - 30*a*b^12*c^6 + 416*a^2*b^10*c^7 - 3360*a^3*b^8*c^8
 + 16640*a^4*b^6*c^9 - 49664*a^5*b^4*c^10 + 81920*a^6*b^2*c^11 - 57344*a^7*
c^12)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^
4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1
280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c
^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*
b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 2
2*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 -
 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 -
1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c
^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) - 3*sqrt(1/2)*((b^4*c^4 - 8*a*b^2*
c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c
^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)
*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(b^9 - 21*a*
b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (b^10*c^5 - 20
*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^
5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*
a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 +
 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^
```

6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(27*(21*a^2*b^8 - 447*a^3*b^6*c + 4189*a^4*b^4*c^2 - 19208*a^5*b^2*c^3 + 38416*a^6*c^4)*x - 27/2*sqrt(1/2)*(b^13 - 31*a*b^11*c + 413*a^2*b^9*c^2 - 3012*a^3*b^7*c^3 + 12496*a^4*b^5*c^4 - 27584*a^5*b^3*c^5 + 25088*a^6*b*c^6 - (b^14*c^5 - 30*a*b^12*c^6 + 416*a^2*b^10*c^7 - 3360*a^3*b^8*c^8 + 16640*a^4*b^6*c^9 - 49664*a^5*b^4*c^10 + 81920*a^6*b^2*c^11 - 57344*a^7*c^12)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) + 3*sqrt(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(27*(21*a^2*b^8 - 447*a^3*b^6*c + 4189*a^4*b^4*c^2 - 19208*a^5*b^2*c^3 + 38416*a^6*c^4)*x + 27/2*sqrt(1/2)*(b^13 - 31*a*b^11*c + 413*a^2*b^9*c^2 - 3012*a^3*b^7*c^3 + 12496*a^4*b^5*c^4 - 27584*a^5*b^3*c^5 + 25088*a^6*b*c^6 + (b^14*c^5 - 30*a*b^12*c^6 + 416*a^2*b^10*c^7 - 3360*a^3*b^8*c^8 + 16640*a^4*b^6*c^9 - 49664*a^5*b^4*c^10 + 81920*a^6*b^2*c^11 - 57344*a^7*c^12)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) - 3*sqrt(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5

```
- 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 102
4*a^5*c^10))*log(27*(21*a^2*b^8 - 447*a^3*b^6*c + 4189*a^4*b^4*c^2 - 19208*
a^5*b^2*c^3 + 38416*a^6*c^4)*x - 27/2*sqrt(1/2)*(b^13 - 31*a*b^11*c + 413*a
^2*b^9*c^2 - 3012*a^3*b^7*c^3 + 12496*a^4*b^5*c^4 - 27584*a^5*b^3*c^5 + 250
88*a^6*b*c^6 + (b^14*c^5 - 30*a*b^12*c^6 + 416*a^2*b^10*c^7 - 3360*a^3*b^8*
c^8 + 16640*a^4*b^6*c^9 - 49664*a^5*b^4*c^10 + 81920*a^6*b^2*c^11 - 57344*a
^7*c^12)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401
*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13
+ 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^
5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (b^10*c^5 - 20*a*b^8*c^6 + 160*a
^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8
- 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^1
0 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14
 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^
4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) + 6*(a^2*b^3 - 8*a^3*b*c)*x)/((
b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*
a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4
*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)
```

giac [B] time = 3.63, size = 2430, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/32*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 - 16*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6
*c - 2*b^7*c + 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 24*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^5*c^2 + 32*a*b^5*c^2 - 2*b^6*c^2 - 128*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^2*c^3 - 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 160
*a^2*b^3*c^3 + 28*a*b^4*c^3 + 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^4 + 256*a^3*b*c^4 - 192*a^2*b^2*c^4 + 448*a^3*c^5 + sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 14*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^4*c^2 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*c^3 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b*c^3 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^3 + 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*c^4 + 2*(b^2 - 4*a*c)*b^5*c - 24*(b^2 - 4*a*c)*a*b^3*c^2 + 2*(b^
```

```
2 - 4*a*c)*b^4*c^2 + 64*(b^2 - 4*a*c)*a^2*b*c^3 - 20*(b^2 - 4*a*c)*a*b^2*c^
3 + 112*(b^2 - 4*a*c)*a^2*c^4)*arctan(2*sqrt(1/2)*x/sqrt((b^5*c^2 - 8*a*b^3
*c^3 + 16*a^2*b*c^4 + sqrt((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)^2 - 4*(a*
b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))
)/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/((b^8*c^2 - 16*a*b^6*c^3 - 2*b^7*c
^3 + 96*a^2*b^4*c^4 + 24*a*b^5*c^4 + b^6*c^4 - 256*a^3*b^2*c^5 - 96*a^2*b^3
*c^5 - 12*a*b^4*c^5 + 256*a^4*c^6 + 128*a^3*b*c^6 + 48*a^2*b^2*c^6 - 64*a^3
*c^7)*abs(c)) + 3/32*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 - 16*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b^6*c + 2*b^7*c + 80*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^3*c^2 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 32*a*b^5*c^2 + 2*b^6*c^2 - 128*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 64*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^3*c^3 + 160*a^2*b^3*c^3 - 28*a*b^4*c^3 + 32*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^4 - 256*a^3*b*c^4 + 192*a^2*b^2*c^4 - 448*a^3*c^5 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 14*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 96*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^5*c + 24*(b^2 - 4*a*c)*a
*b^3*c^2 - 2*(b^2 - 4*a*c)*b^4*c^2 - 64*(b^2 - 4*a*c)*a^2*b*c^3 + 20*(b^2 -
 4*a*c)*a*b^2*c^3 - 112*(b^2 - 4*a*c)*a^2*c^4)*arctan(2*sqrt(1/2)*x/sqrt((b
^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4 - sqrt((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*
b*c^4)^2 - 4*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*(b^4*c^3 - 8*a*b^2*c^
4 + 16*a^2*c^5)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/((b^8*c^2 - 16*a*b
^6*c^3 - 2*b^7*c^3 + 96*a^2*b^4*c^4 + 24*a*b^5*c^4 + b^6*c^4 - 256*a^3*b^2*
c^5 - 96*a^2*b^3*c^5 - 12*a*b^4*c^5 + 256*a^4*c^6 + 128*a^3*b*c^6 + 48*a^2*
b^2*c^6 - 64*a^3*c^7)*abs(c)) - 1/8*(5*b^4*c*x^7 - 37*a*b^2*c^2*x^7 + 44*a^
2*c^3*x^7 + 3*b^5*x^5 - 20*a*b^3*c*x^5 - 4*a^2*b*c^2*x^5 + 6*a*b^4*x^3 - 49
*a^2*b^2*c*x^3 + 28*a^3*c^2*x^3 + 3*a^2*b^3*x - 24*a^3*b*c*x)/((b^4*c^2 - 8
*a*b^2*c^3 + 16*a^2*c^4)*(c*x^4 + b*x^2 + a)^2)
```

maple [B] time = 0.05, size = 1141, normalized size = 2.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (-1/8*(44*a^2*c^2-37*a*b^2*c+5*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c*x^7+1/8*b*
(4*a^2*c^2+20*a*b^2*c-3*b^4)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*a/c^2*(
28*a^2*c^2-49*a*b^2*c+6*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3/8*a^2*b*(8*a*
c-b^2)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2-21/4/(16*a^2*c^2
-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2+27/16/c/(16*a^2*c^2-8*a*b^2*c+b^4
)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*c*x)*a*b^2-3/16/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)*b^4+33/4/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*c*x)*a^2*b-33/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*a*b^3+3/16/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^5+21/4/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*c*x)*a^2-27/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2+3
/16/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4+33/4/(16*a^2*c^2-
8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*b-33/16/c/(16*a^2*
c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^3+3/16/c^2/(16
*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*b^4*c - 37*a*b^2*c^2 + 44*a^2*c^3)*x^7 + (3*b^5 - 20*a*b^3*c - 4*a
^2*b*c^2)*x^5 + (6*a*b^4 - 49*a^2*b^2*c + 28*a^3*c^2)*x^3 + 3*(a^2*b^3 - 8*
a^3*b*c)*x)/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3
*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6
*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^
3*b*c^4)*x^2) + 3/8*integrate((a*b^3 - 8*a^2*b*c + (b^4 - 9*a*b^2*c + 28*a^
2*c^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
```

mupad [B] time = 9.04, size = 10912, normalized size = 27.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] - ((x^3*(6*a*b^4 + 28*a^3*c^2 - 49*a^2*b^2*c))/(8*c^2*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)) + (x^7*(5*b^4 + 44*a^2*c^2 - 37*a*b^2*c))/(8*c*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c)) - (b*x^5*(4*a^2*c^2 - 3*b^4 + 20*a*b^2*c))/(8*c^2*(b^4 + 16*
a^2*c^2 - 8*a*b^2*c)) - (3*a^2*b*x*(8*a*c - b^2))/(8*c^2*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
- atan(((((3*(256*a*b^13*c^3 + 2097152*a^7*b*c^9 - 7168*a^2*b^11*c^4 + 8192
0*a^3*b^9*c^5 - 491520*a^4*b^7*c^6 + 1638400*a^5*b^5*c^7 - 2883584*a^6*b^3*
c^8))/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 128
0*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*((9*(b^4*(-(4*a*
c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3
*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 +
 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)
^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576
*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^
8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 19660
80*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256
*b^11*c^5 - 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840
*a^3*b^5*c^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c
^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2)
 - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*
a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c
^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17
*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*
c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12
*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 +
2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) - (x*(9*b^10 - 14112*a
^5*c^5 + 1881*a^2*b^6*c^2 - 9090*a^3*b^4*c^3 + 21312*a^4*b^2*c^4 - 198*a*b^
8*c))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*
b^2*c^6)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 -
769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*
c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*
a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2
)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*
b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 +
860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440
*a^9*b^2*c^14)))^(1/2)*1i - (((3*(256*a*b^13*c^3 + 2097152*a^7*b*c^9 - 7168
*a^2*b^11*c^4 + 81920*a^3*b^9*c^5 - 491520*a^4*b^7*c^6 + 1638400*a^5*b^5*c^
7 - 2883584*a^6*b^3*c^8))/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 2
40*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) +
 (x*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2
*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1
```

069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2
*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(
1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^
7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*
a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^
2*c^14)))^(1/2)*(256*b^11*c^5 - 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*
a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 +
b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*((9*(b^4*(-(4*
a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a
^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6
 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^1
5)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(10485
76*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*
c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 196
6080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) +
(x*(9*b^10 - 14112*a^5*c^5 + 1881*a^2*b^6*c^2 - 9090*a^3*b^4*c^3 + 21312*a^
4*b^2*c^4 - 198*a*b^8*c))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^
2*b^4*c^5 - 256*a^3*b^2*c^6)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 +
1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c
^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 30105
60*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*
b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a
*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258
048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^
8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*1i)/((((3*(256*a*b^13*c^3 + 2097
152*a^7*b*c^9 - 7168*a^2*b^11*c^4 + 81920*a^3*b^9*c^5 - 491520*a^4*b^7*c^6
+ 1638400*a^5*b^5*c^7 - 2883584*a^6*b^3*c^8))/(512*(4096*a^6*c^9 + b^12*c^3
 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 -
6144*a^5*b^2*c^8)) - (x*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320
*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 31
6864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*
b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(
-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c
^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5
*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c
^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5120*a*b^9*c^6 - 262144
*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + 327680*a^4*b^3*c^9))
/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c
^6)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a
^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 -
 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c
^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)
^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*
c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 86016
0*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*

b^2*c^14)))^(1/2) - (x*(9*b^10 - 14112*a^5*c^5 + 1881*a^2*b^6*c^2 - 9090*a^3*b^4*c^3 + 21312*a^4*b^2*c^4 - 198*a*b^8*c))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) - (3*(189*a^3*b^8 + 197568*a^7*c^4 - 3645*a^4*b^6*c + 29844*a^5*b^4*c^2 - 117936*a^6*b^2*c^3))/(256*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) + (((3*(256*a*b^13*c^3 + 2097152*a^7*b*c^9 - 7168*a^2*b^11*c^4 + 81920*a^3*b^9*c^5 - 491520*a^4*b^7*c^6 + 1638400*a^5*b^5*c^7 - 2883584*a^6*b^3*c^8))/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) + (x*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) + (x*(9*b^10 - 14112*a^5*c^5 + 1881*a^2*b^6*c^2 - 9090*a^3*b^4*c^3 + 21312*a^4*b^2*c^4 - 198*a*b^8*c))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 10698

24*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*2i - atan(((((3*(256*a*b^13*c^3 + 2097152*a^7*b*c^9 - 7168*a^2*b^11*c^4 + 81920*a^3*b^9*c^5 - 491520*a^4*b^7*c^6 + 1638400*a^5*b^5*c^7 - 2883584*a^6*b^3*c^8))/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) - (x*(9*b^10 - 14112*a^5*c^5 + 1881*a^2*b^6*c^2 - 9090*a^3*b^4*c^3 + 21312*a^4*b^2*c^4 - 198*a*b^8*c))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*1i - (((3*(256*a*b^13*c^3 + 2097152*a^7*b*c^9 - 7168*a^2*b^11*c^4 + 81920*a^3*b^9*c^5 - 491520*a^4*b^7*c^6 + 1638400*a^5*b^5*c^7 - 2883584*a^6*b^3*c^8))/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) + (x*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*

b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^
13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5120*a*b^9*c^6 - 262144*
a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + 327680*a^4*b^3*c^9))/
(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^
6)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a
^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 +
1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c
^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)
^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*
c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 86016
0*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*
b^2*c^14)))^(1/2) + (x*(9*b^10 - 14112*a^5*c^5 + 1881*a^2*b^6*c^2 - 9090*a^
3*b^4*c^3 + 21312*a^4*b^2*c^4 - 198*a*b^8*c))/(32*(256*a^4*c^7 + b^8*c^3 -
16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*(b^19 + b^4*(-(4*a*
c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c
^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 234393
6*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
- 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c
^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 537
60*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*
b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*1i)/((((3*(
256*a*b^13*c^3 + 2097152*a^7*b*c^9 - 7168*a^2*b^11*c^4 + 81920*a^3*b^9*c^5
- 491520*a^4*b^7*c^6 + 1638400*a^5*b^5*c^7 - 2883584*a^6*b^3*c^8))/(512*(40
96*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6
+ 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*(-(9*(b^19 + b^4*(-(4*a*c - b^
2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 6
3440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*
b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a
*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 +
b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4
*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^
12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5
120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8
+ 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b
^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 17
20320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4
- 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560
*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^
2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b
^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 25804
8*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*
b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) - (x*(9*b^10 - 14112*a^5*c^5 + 188
1*a^2*b^6*c^2 - 9090*a^3*b^4*c^3 + 21312*a^4*b^2*c^4 - 198*a*b^8*c))/(32*(2
56*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6))*(
-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^1

5*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 10698
24*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(
4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)
))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 -
7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*
b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^
14)))^(1/2) - (3*(189*a^3*b^8 + 197568*a^7*c^4 - 3645*a^4*b^6*c + 29844*a^5
*b^4*c^2 - 117936*a^6*b^2*c^3))/(256*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c
^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c
^8)) + (((3*(256*a*b^13*c^3 + 2097152*a^7*b*c^9 - 7168*a^2*b^11*c^4 + 81920
*a^3*b^9*c^5 - 491520*a^4*b^7*c^6 + 1638400*a^5*b^5*c^7 - 2883584*a^6*b^3*c
^8))/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280
*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) + (x*(-(9*(b^19 + b^4*
(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3
*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 -
2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)
^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576
*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^
8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 19660
80*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256
*b^11*c^5 - 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840
*a^3*b^5*c^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c
^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^1
5)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440
*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*
c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^1
7*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20
*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^1
2*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 +
2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) + (x*(9*b^10 - 14112*
a^5*c^5 + 1881*a^2*b^6*c^2 - 9090*a^3*b^4*c^3 + 21312*a^4*b^2*c^4 - 198*a*b
^8*c))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3
*b^2*c^6)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9
+ 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^
9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 4
9*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b
^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^
2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10
+ 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 26214
40*a^9*b^2*c^14)))^(1/2)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 172
0320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4
- 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*
a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2
c(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^
18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048

```
*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b
^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.882 $\int \frac{x^8}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=348

$$\frac{\left(-\frac{-40a^2c^2-18ab^2c+b^4}{\sqrt{b^2-4ac}}-16abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2c^2-18ab^2c+b^4}{\sqrt{b^2-4ac}}-16abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{}{4(}$$

```
[Out] -1/8*(20*a*c+b^2)*x/c/(-4*a*c+b^2)^2+1/4*x^5*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^
4+b*x^2+a)^2+1/8*x^3*(12*a*b+(20*a*c+b^2)*x^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+
a)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^3-16*a*b*
c+(40*a^2*c^2+18*a*b^2*c-b^4)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)^2*2^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c
+b^2)^(1/2))^(1/2))*(b^3-16*a*b*c+(-40*a^2*c^2-18*a*b^2*c+b^4)/(-4*a*c+b^2)
^(1/2))/c^(3/2)/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.89, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1120, 1275, 1279, 1166, 205}

$$\frac{\left(-\frac{-40a^2c^2-18ab^2c+b^4}{\sqrt{b^2-4ac}}-16abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2c^2-18ab^2c+b^4}{\sqrt{b^2-4ac}}-16abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{}{4(}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -((b^2 + 20*a*c)*x)/(8*c*(b^2 - 4*a*c)^2) + (x^5*(2*a + b*x^2))/(4*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)^2) + (x^3*(12*a*b + (b^2 + 20*a*c)*x^2))/(8*(b^2
- 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((b^3 - 16*a*b*c - (b^4 - 18*a*b^2*c - 40
*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
 4*a*c]]])/(8*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ ((b^3 - 16*a*b*c + (b^4 - 18*a*b^2*c - 40*a^2*c^2)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(3/2)*(b
^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{x^5\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{x^4\left(10a-bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= \frac{x^5\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^3\left(12ab+\left(b^2+20ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\int \frac{x^2\left(36ab+\left(b^2+20ac\right)x^2\right)}{a+bx^2+cx^4}\,d}{8\left(b^2-4ac\right)^2}$$

$$= -\frac{\left(b^2+20ac\right)x}{8c\left(b^2-4ac\right)^2} + \frac{x^5\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^3\left(12ab+\left(b^2+20ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \int$$

$$= -\frac{\left(b^2+20ac\right)x}{8c\left(b^2-4ac\right)^2} + \frac{x^5\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^3\left(12ab+\left(b^2+20ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(b\right.}{}$$

$$= -\frac{\left(b^2+20ac\right)x}{8c\left(b^2-4ac\right)^2} + \frac{x^5\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^3\left(12ab+\left(b^2+20ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(b\right.}{}$$

Mathematica [A] time = 0.96, size = 381, normalized size = 1.09

$$\frac{\frac{4\left(-2a^2cx+abx\left(b-3cx^2\right)+b^3x^3\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2x\left(-36a^2c^2+11ab^2c-16abc^2x^2-2b^4+b^3cx^2\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(40a^2c^2+18ab^2c-16abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}-b^4\right)\tan^{-1}\left(\right.}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}}{16c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*x*(-2*b^4 + 11*a*b^2*c - 36*a^2*c^2 + b^3*c*x^2 - 16*a*b*c^2*x^2))/((b^
2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (4*(-2*a^2*c*x + b^3*x^3 + a*b*x*(b - 3
*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[2]*Sqrt[c]*(-b^4 +
18*a*b^2*c + 40*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 16*a*b*c*Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^
(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(b^4 - 18*a*b^2*c - 4
0*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 16*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b +
 Sqrt[b^2 - 4*a*c]]))/(16*c^2)
```

fricas [B] time = 0.97, size = 3725, normalized size = 10.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(b^3*c - 16*a*b*c^2)*x^7 - 2*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*x^5 - 4
*(a*b^3 + 14*a^2*b*c)*x^3 + sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 +
16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8
*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2
 + 1680*a^3*b*c^3 + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^
4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c
^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4
*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 6
40*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log((35*a*b^6 - 1491*a^2
*b^4*c + 15000*a^3*b^2*c^2 + 10000*a^4*c^3)*x + 1/2*sqrt(1/2)*(b^10 - 17*a*
b^8*c - 392*a^2*b^6*c^2 + 5696*a^3*b^4*c^3 - 23680*a^4*b^2*c^4 + 32000*a^5*
c^5 - (b^13*c^3 - 72*a*b^11*c^4 + 1200*a^2*b^9*c^5 - 8960*a^3*b^7*c^6 + 345
60*a^4*b^5*c^7 - 67584*a^5*b^3*c^8 + 53248*a^6*b*c^9)*sqrt((b^4 - 50*a*b^2*
c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c
^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2
*b^3*c^2 + 1680*a^3*b*c^3 + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 64
0*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 6
25*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 +
1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6
*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))) - sqrt(1/2)*((b
^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4
*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2
+ 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(
b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (b^10*c^3 - 20*a*b^8*
c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*
sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^
6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 -
20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*
a^5*c^8))*log((35*a*b^6 - 1491*a^2*b^4*c + 15000*a^3*b^2*c^2 + 10000*a^4*c^
3)*x - 1/2*sqrt(1/2)*(b^10 - 17*a*b^8*c - 392*a^2*b^6*c^2 + 5696*a^3*b^4*c^
3 - 23680*a^4*b^2*c^4 + 32000*a^5*c^5 - (b^13*c^3 - 72*a*b^11*c^4 + 1200*a^
2*b^9*c^5 - 8960*a^3*b^7*c^6 + 34560*a^4*b^5*c^7 - 67584*a^5*b^3*c^8 + 5324
8*a^6*b*c^9)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7
 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))
*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (b^10*c^3 - 2
0*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a
```

```
^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 16
0*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^1
0*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7
 - 1024*a^5*c^8))) + sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 +
a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*
b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^
3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680
*a^3*b*c^3 - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 +
 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^
10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^
10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*
b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log((35*a*b^6 - 1491*a^2*b^4*c
+ 15000*a^3*b^2*c^2 + 10000*a^4*c^3)*x + 1/2*sqrt(1/2)*(b^10 - 17*a*b^8*c -
 392*a^2*b^6*c^2 + 5696*a^3*b^4*c^3 - 23680*a^4*b^2*c^4 + 32000*a^5*c^5 + (
b^13*c^3 - 72*a*b^11*c^4 + 1200*a^2*b^9*c^5 - 8960*a^3*b^7*c^6 + 34560*a^4*
b^5*c^7 - 67584*a^5*b^3*c^8 + 53248*a^6*b*c^9)*sqrt((b^4 - 50*a*b^2*c + 625
*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 12
80*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^
2 + 1680*a^3*b*c^3 - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b
^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*
c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^
4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 -
640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))) - sqrt(1/2)*((b^4*c^3
- 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 +
2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^
3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(b^7 - 3
5*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (b^10*c^3 - 20*a*b^8*c^4 + 1
60*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b
^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 -
 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^
8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8
))*log((35*a*b^6 - 1491*a^2*b^4*c + 15000*a^3*b^2*c^2 + 10000*a^4*c^3)*x -
1/2*sqrt(1/2)*(b^10 - 17*a*b^8*c - 392*a^2*b^6*c^2 + 5696*a^3*b^4*c^3 - 236
80*a^4*b^2*c^4 + 32000*a^5*c^5 + (b^13*c^3 - 72*a*b^11*c^4 + 1200*a^2*b^9*c
^5 - 8960*a^3*b^7*c^6 + 34560*a^4*b^5*c^7 - 67584*a^5*b^3*c^8 + 53248*a^6*b
*c^9)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*
a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-
(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (b^10*c^3 - 20*a*b^8
*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)
*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b
^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 -
 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024
*a^5*c^8))) - 2*(a^2*b^2 + 20*a^3*c)*x)/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^
5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3
+ 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c -
```

```
8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)
```

giac [B] time = 2.46, size = 4558, normalized size = 13.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/64*(2*b^13*c^4 - 68*a*b^11*c^5 + 688*a^2*b^9*c^6 - 2688*a^3*b^7*c^7 + 20
48*a^4*b^5*c^8 + 11264*a^5*b^3*c^9 - 20480*a^6*b*c^10 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^13*c^2 + 34*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^11*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^12*c^3 - 344*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^4 - 60*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^10*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^11*c^4 + 1344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^5 + 448*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 + 30*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^9*c^5 - 1024*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^6 - 896*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^6 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 - 5632*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^7 - 1536*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 + 448*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 + 10240*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^8 + 5120*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 + 768*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 - 2560*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^9 - 2*(b^2 - 4*a*c)*b^11*c
^4 + 60*(b^2 - 4*a*c)*a*b^9*c^5 - 448*(b^2 - 4*a*c)*a^2*b^7*c^6 + 896*(b^2
- 4*a*c)*a^3*b^5*c^7 + 1536*(b^2 - 4*a*c)*a^4*b^3*c^8 - 5120*(b^2 - 4*a*c)*
a^5*b*c^9 - (2*b^5*c^2 - 40*a*b^3*c^3 + 128*a^2*b*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^3*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 32*(b^2 - 4*a*c)*a*b*c^3)*(b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^8*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^3 - 2*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 - 2*a*b^8*c^3 - 192*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^4 - 24*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^5*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*
```

c^4 - 16*a^2*b^6*c^4 + 896*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*
c^5 + 288*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 12*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 + 384*a^3*b^4*c^5 - 1280*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^6 - 640*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^4*b*c^6 - 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^
2*c^6 - 1792*a^4*b^2*c^6 + 320*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*
c^7 + 2560*a^5*c^7 + 2*(b^2 - 4*a*c)*a*b^6*c^3 + 24*(b^2 - 4*a*c)*a^2*b^4*c
^4 - 288*(b^2 - 4*a*c)*a^3*b^2*c^5 + 640*(b^2 - 4*a*c)*a^4*c^6)*abs(b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3))*arctan(2*sqrt(1/2)*x/sqrt((b^5*c - 8*a*b^3*c^2
+ 16*a^2*b*c^3 + sqrt((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)^2 - 4*(a*b^4*c -
 8*a^2*b^2*c^2 + 16*a^3*c^3)*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)))/(b^4*c^
2 - 8*a*b^2*c^3 + 16*a^2*c^4)))/((a*b^10*c^3 - 20*a^2*b^8*c^4 - 2*a*b^9*c^4
 + 160*a^3*b^6*c^5 + 32*a^2*b^7*c^5 + a*b^8*c^5 - 640*a^4*b^4*c^6 - 192*a^3
*b^5*c^6 - 16*a^2*b^6*c^6 + 1280*a^5*b^2*c^7 + 512*a^4*b^3*c^7 + 96*a^3*b^4
*c^7 - 1024*a^6*c^8 - 512*a^5*b*c^8 - 256*a^4*b^2*c^8 + 256*a^5*c^9)*abs(b^
4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*abs(c)) + 1/64*(2*b^13*c^4 - 68*a*b^11*c^5
+ 688*a^2*b^9*c^6 - 2688*a^3*b^7*c^7 + 2048*a^4*b^5*c^8 + 11264*a^5*b^3*c^9
 - 20480*a^6*b*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^13*c^2 + 34*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b^11*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
b^12*c^3 - 344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^9*c^4 - 60*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^10*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^11*c
^4 + 1344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7
*c^5 + 448*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^
8*c^5 + 30*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^9*
c^5 - 1024*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^
5*c^6 - 896*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b
^6*c^6 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b^7*c^6 - 5632*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
5*b^3*c^7 - 1536*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^4*b^4*c^7 + 448*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^3*b^5*c^7 + 10240*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^6*b*c^8 + 5120*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^5*b^2*c^8 + 768*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^4*b^3*c^8 - 2560*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^5*b*c^9 - 2*(b^2 - 4*a*c)*b^11*c^4 + 60*(b^2 - 4*a*c)*a*b^9*c^5 -
448*(b^2 - 4*a*c)*a^2*b^7*c^6 + 896*(b^2 - 4*a*c)*a^3*b^5*c^7 + 1536*(b^2 -
 4*a*c)*a^4*b^3*c^8 - 5120*(b^2 - 4*a*c)*a^5*b*c^9 - (2*b^5*c^2 - 40*a*b^3*
c^3 + 128*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^5 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c -
64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 32
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*sqrt(2)*

```
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 32*(b^2 - 4*a*c)*a*b*c^3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)^2 +
 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^8*c^2 + 8*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^7*c^3 + 2*a*b^8*c^3 - 192*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^3*b^4*c^4 - 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^4 + sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 + 16*a^2*b^6*c^4 + 896*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^5 + 288*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^4*c^5 - 384*a^3*b^4*c^5 - 1280*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^5*c^6 - 640*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^6 - 144*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 + 1792*a^4*b^2*c^6 + 320*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^7 - 2560*a^5*c^7 - 2*(b^2 - 4*a*
c)*a*b^6*c^3 - 24*(b^2 - 4*a*c)*a^2*b^4*c^4 + 288*(b^2 - 4*a*c)*a^3*b^2*c^5
 - 640*(b^2 - 4*a*c)*a^4*c^6)*abs(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3))*arctan
(2*sqrt(1/2)*x/sqrt((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 - sqrt((b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)^2 - 4*(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*(b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)))/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)))/((
a*b^10*c^3 - 20*a^2*b^8*c^4 - 2*a*b^9*c^4 + 160*a^3*b^6*c^5 + 32*a^2*b^7*c^
5 + a*b^8*c^5 - 640*a^4*b^4*c^6 - 192*a^3*b^5*c^6 - 16*a^2*b^6*c^6 + 1280*a
^5*b^2*c^7 + 512*a^4*b^3*c^7 + 96*a^3*b^4*c^7 - 1024*a^6*c^8 - 512*a^5*b*c^
8 - 256*a^4*b^2*c^8 + 256*a^5*c^9)*abs(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*ab
s(c)) + 1/8*(b^3*c*x^7 - 16*a*b*c^2*x^7 - b^4*x^5 - 5*a*b^2*c*x^5 - 36*a^2*
c^2*x^5 - 2*a*b^3*x^3 - 28*a^2*b*c*x^3 - a^2*b^2*x - 20*a^3*c*x)/((b^4*c -
8*a*b^2*c^2 + 16*a^2*c^3)*(c*x^4 + b*x^2 + a)^2)
```

maple [B] time = 0.04, size = 953, normalized size = 2.74

$$-\frac{5\sqrt{2}\, a^2 c \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(16a^2c^2-8a\, b^2 c+b^4\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{5\sqrt{2}\, a^2 c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(16a^2c^2-8a\, b^2 c+b^4\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (-1/8*b*(16*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7-1/8*(36*a^2*c^2+5*a*b^2
*c+b^4)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/4*a/c*b*(14*a*c+b^2)/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^3-1/8*a^2*(20*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c
*x^4+b*x^2+a)^2+1/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b-1/16
/(16*a^2*c^2-8*a*b^2*c+b^4)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3-5/2/(16*a^2*c^2-8*a
```

```
*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2-9/8/(16*a^2*c^2-
8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2+1/16/(16*a^2*
c^2-8*a*b^2*c+b^4)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4-1/(16*a^2
*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b+1/16/(16*a^2*c^2-8*a*b^2*c+b^4)/
c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)*b^3-5/2/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)*a^2-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x)*a*b^2+1/16/(16*a^2*c^2-8*a*b^2*c+b^4)/c/(-4*a*c+b^2)^(1/
2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*b^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(b^3c - 16\,abc^2\right)x^7 - \left(b^4 + 5\,ab^2c + 36\,a^2c^2\right)x^5 - 2\left(ab^3 + 14\,a^2bc\right)x^3 - \left(a^2b^2\right.}{8\left(\left(b^4c^3 - 8\,ab^2c^4 + 16\,a^2c^5\right)x^8 + a^2b^4c - 8\,a^3b^2c^2 + 16\,a^4c^3 + 2\left(b^5c^2 - 8\,ab^3c^3 + 16\,a^2bc^4\right)x^6 + \left(b^6c - 6\,ab^4c^2 + \right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((b^3*c - 16*a*b*c^2)*x^7 - (b^4 + 5*a*b^2*c + 36*a^2*c^2)*x^5 - 2*(a*b
^3 + 14*a^2*b*c)*x^3 - (a^2*b^2 + 20*a^3*c)*x)/((b^4*c^3 - 8*a*b^2*c^4 + 16
*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b
^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*
b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2) - 1/8*integrate(-(a*b^2 + 20*a^2
*c + (b^3 - 16*a*b*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^4*c - 8*a*b^2*c^2 + 1
6*a^2*c^3)
```

mupad [B] time = 8.54, size = 9575, normalized size = 27.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] atan(((((5242880*a^7*c^8 - 256*a*b^12*c^2 + 61440*a^3*b^8*c^4 - 655360*a^4*
b^6*c^5 + 2949120*a^5*b^4*c^6 - 6291456*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^
6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c
```

^5 - 6144*a^5*b^2*c^6)) - (x*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720
320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4
+ 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*
c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 -
40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 -
 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*
a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4
 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b
^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3
*b^2*c^4)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1
140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c
^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a
c - b^2)^15)^(1/2))/(512(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 7
20*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*
c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 26
21440*a^9*b^2*c^12)))^(1/2) - (x*(b^8 + 800*a^4*c^4 + 314*a^2*b^4*c^2 + 208
*a^3*b^2*c^3 - 36*a*b^6*c))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^
2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 17
20320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^
4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^1
5*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3
- 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7
 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 294912
0*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*1i - (((5242880*a^7*c^8 - 25
6*a*b^12*c^2 + 61440*a^3*b^8*c^4 - 655360*a^4*b^6*c^5 + 2949120*a^5*b^4*c^6
 - 6291456*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a
^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + (x*
(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13
*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*
a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15
)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*
c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160
*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^
2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a
^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4
*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^17 + b^2*(-(
4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*
b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 186
3680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(10
48576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^
14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 19
66080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) +
 (x*(b^8 + 800*a^4*c^4 + 314*a^2*b^4*c^2 + 208*a^3*b^2*c^3 - 36*a*b^6*c))/(
32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4))
)*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^

13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 68096
0*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^
15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^1
6*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 8601
60*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*
b^2*c^12)))^(1/2)*1i)/((((5242880*a^7*c^8 - 256*a*b^12*c^2 + 61440*a^3*b^8*
c^4 - 655360*a^4*b^6*c^5 + 2949120*a^5*b^4*c^6 - 6291456*a^6*b^2*c^7)/(512*
(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - (x*(-(b^17 + b^2*(-(4*a*c - b^2)
^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 3
4880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3
*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c
^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 537
60*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^
6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3
 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c
^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*
b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720
320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4
+ 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*
c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 -
40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 -
 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*
a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) - (x*(b^8 + 800*a^4*c^4 + 314*
a^2*b^4*c^2 + 208*a^3*b^2*c^3 - 36*a*b^6*c))/(32*(b^8*c + 256*a^4*c^5 - 16*
a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^17 + b^2*(-(4*a*c - b^
2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 +
 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b
^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10
*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 5
3760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*
b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) + (((524288
0*a^7*c^8 - 256*a*b^12*c^2 + 61440*a^3*b^8*c^4 - 655360*a^4*b^6*c^5 + 29491
20*a^5*b^4*c^6 - 6291456*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^
10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b
^2*c^6)) + (x*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 +
 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7
*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4
*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 +
 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^1
0*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 -
2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4 - 262144*a^5*b
*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(32*(b
^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(
b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^

2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6
*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(
1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5
 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^
6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c
^12)))^(1/2) + (x*(b^8 + 800*a^4*c^4 + 314*a^2*b^4*c^2 + 208*a^3*b^2*c^3 -
36*a*b^6*c))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256
*a^3*b^2*c^4)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8
 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b
^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-
(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4
 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b
^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11
- 2621440*a^9*b^2*c^12)))^(1/2) - (35*a^2*b^7 - 1176*a^3*b^5*c + 6400*a^5*b
*c^3 + 9456*a^4*b^3*c^2)/(256*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*
a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6))))*(-
(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c
^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^
6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^
(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^
5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a
^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*
c^12)))^(1/2)*2i + atan(((((5242880*a^7*c^8 - 256*a*b^12*c^2 + 61440*a^3*b^
8*c^4 - 655360*a^4*b^6*c^5 + 2949120*a^5*b^4*c^6 - 6291456*a^6*b^2*c^7)/(51
2*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c
^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - (x*(-(b^17 - b^2*(-(4*a*c - b^
2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 +
 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b
^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10
*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 5
3760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*
b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c
^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5
*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^
2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 17
20320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^
4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^1
5*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3
- 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7
 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 294912
0*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) - (x*(b^8 + 800*a^4*c^4 + 31
4*a^2*b^4*c^2 + 208*a^3*b^2*c^3 - 36*a*b^6*c))/(32*(b^8*c + 256*a^4*c^5 - 1
6*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^17 - b^2*(-(4*a*c -
b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3
 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7

*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^
10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 +
 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^
7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*1i - (((5
242880*a^7*c^8 - 256*a*b^12*c^2 + 61440*a^3*b^8*c^4 - 655360*a^4*b^6*c^5 +
2949120*a^5*b^4*c^6 - 6291456*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24
*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*
a^5*b^2*c^6)) + (x*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*
c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^
5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c
*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*
c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^
5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^
11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4 - 262144*
a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(
32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4))
)*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^
13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 68096
0*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^
15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^1
6*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 8601
60*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*
b^2*c^12)))^(1/2) + (x*(b^8 + 800*a^4*c^4 + 314*a^2*b^4*c^2 + 208*a^3*b^2*c
^3 - 36*a*b^6*c))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3
 - 256*a^3*b^2*c^4)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*
b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*
a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a
c(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^1
8*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*
a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*
c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*1i)/((((5242880*a^7*c^8 - 256*a*b^12*c
^2 + 61440*a^3*b^8*c^4 - 655360*a^4*b^6*c^5 + 2949120*a^5*b^4*c^6 - 6291456
*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3
 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - (x*(-(b^17 -
b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 101
60*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^
6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(
512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680
*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c
^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^
(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5
 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*
a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^17 - b^2*(-(4*a*c - b^
2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 +
 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b

^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) - (x*(b^8 + 800*a^4*c^4 + 314*a^2*b^4*c^2 + 208*a^3*b^2*c^3 - 36*a*b^6*c))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) + (((5242880*a^7*c^8 - 256*a*b^12*c^2 + 61440*a^3*b^8*c^4 - 655360*a^4*b^6*c^5 + 2949120*a^5*b^4*c^6 - 6291456*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + (x*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) + (x*(b^8 + 800*a^4*c^4 + 314*a^2*b^4*c^2 + 208*a^3*b^2*c^3 - 36*a*b^6*c))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) - (35*a^2*b^7 - 1176*a^3*b^5*c + 6400*a^5*b*c^3 + 9456*a^4*b^3*c^2)/(256*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6))))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^

```
4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*
b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11
 - 2621440*a^9*b^2*c^12)))^(1/2)*2i - ((x^3*(a*b^3 + 14*a^2*b*c))/(4*c*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^7*(b^3 - 16*a*b*c))/(8*(b^4 + 16*a^2*c^2 -
 8*a*b^2*c)) + (x^5*(b^4 + 36*a^2*c^2 + 5*a*b^2*c))/(8*c*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)) + (a^2*x*(20*a*c + b^2))/(8*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
)/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.883 $\int \frac{x^6}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=298

$$\frac{3x\left(x^2\left(4ac+b^2\right)+4ab\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^3\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3\left(-\frac{b\left(12ac+b^2\right)}{\sqrt{b^2-4ac}}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\right.}{}$$

```
[Out] 1/4*x^3*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/8*x*(4*a*b+(4*a*c+b^2)
*x^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3/16*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a
*c+b^2)^(1/2))^(1/2))*(b^2+4*a*c-b*(12*a*c+b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c
+b^2)^2*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+3/16*arctan(x*2^(1/2)*
c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2+4*a*c+b*(12*a*c+b^2)/(-4*a*c+b^2
)^(1/2))/(-4*a*c+b^2)^2*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1120, 1275, 1166, 205}

$$\frac{x^3\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3x\left(x^2\left(4ac+b^2\right)+4ab\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\left(-\frac{b\left(12ac+b^2\right)}{\sqrt{b^2-4ac}}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^3*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*x*(4*a*b +
 (b^2 + 4*a*c)*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*(b^2 + 4*
a*c - (b*(b^2 + 12*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) + (3*(b^2 + 4*a*c + (b*(b^2 + 12*a*c))/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(
b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{x^3\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{x^2\left(6a-3bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= \frac{x^3\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x\left(4ab+\left(b^2+4ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\int \frac{12ab-3\left(b^2+4ac\right)x^2}{a+bx^2+cx^4}\,dx}{8\left(b^2-4ac\right)^2}$$

$$= \frac{x^3\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x\left(4ab+\left(b^2+4ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3\left(b^2+4ac-\frac{b\left(b^2+12ac\right.}{\sqrt{b^2-4ac}}\right.\right.}{16\left(b^2\right.}$$

$$= \frac{x^3\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x\left(4ab+\left(b^2+4ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3\left(b^2+4ac-\frac{b\left(b^2+12ac\right)}{\sqrt{b^2-4ac}}\right.}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2}$$

Mathematica [A] time = 0.85, size = 343, normalized size = 1.15

$$\frac{-\frac{4\left(ax\left(b-2cx^2\right)+b^2x^3\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{8abcx+24ac^2x^3+4b^3x+6b^2cx^3}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-12abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{3\sqrt{2}\sqrt{c}\left(b^2\sqrt{\right.\right.}{}}{16c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*b^3*x + 8*a*b*c*x + 6*b^2*c*x^3 + 24*a*c^2*x^3)/((b^2 - 4*a*c)^2*(a + b
*x^2 + c*x^4)) - (4*(b^2*x^3 + a*x*(b - 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^
2 + c*x^4)^2) + (3*Sqrt[2]*Sqrt[c]*(-b^3 - 12*a*b*c + b^2*Sqrt[b^2 - 4*a*c]
 + 4*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sq
rt[c]*(b^3 + 12*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)
*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c)
```

fricas [B] time = 1.06, size = 3128, normalized size = 10.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] 1/16*(6*(b^2*c + 4*a*c^2)*x^7 + 2*(5*b^3 + 16*a*b*c)*x^5 + 24*a^2*b*x + 2*(
19*a*b^2 - 4*a^2*c)*x^3 + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c +
 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c +
 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (b^10*c - 20*a
*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*
c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 128
0*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 6
40*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))*log(3*(5*b^4 + 40*a*b^2*
c + 16*a^2*c^2)*x + 3*sqrt(1/2)*(2*b^7 - 24*a*b^5*c + 96*a^2*b^3*c^2 - 128*
a^3*b*c^3 + (3*b^12*c - 56*a*b^10*c^2 + 400*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
+ 1280*a^4*b^4*c^5 + 2048*a^5*b^2*c^6 - 4096*a^6*c^7)/sqrt(b^10*c^2 - 20*a*
b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c
^7))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (b^10*c - 20*a*b^8*c^2 + 160*
a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*
c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 -
 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4
+ 1280*a^4*b^2*c^5 - 1024*a^5*c^6))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3
+ 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 -
8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 -
8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (
b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^
5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*
b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^
2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))*log(3*(5*b^
4 + 40*a*b^2*c + 16*a^2*c^2)*x - 3*sqrt(1/2)*(2*b^7 - 24*a*b^5*c + 96*a^2*b
^3*c^2 - 128*a^3*b*c^3 + (3*b^12*c - 56*a*b^10*c^2 + 400*a^2*b^8*c^3 - 1280
*a^3*b^6*c^4 + 1280*a^4*b^4*c^5 + 2048*a^5*b^2*c^6 - 4096*a^6*c^7)/sqrt(b^1
0*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6
 - 1024*a^5*c^7))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (b^10*c - 20*a*b
^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^
6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*
a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640
*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))) + 3*sqrt(1/2)*((b^4*c^2 -
 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6
 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4
+ 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*
a^2*b*c^2 - (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 12
80*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c
^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8
*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)
)*log(3*(5*b^4 + 40*a*b^2*c + 16*a^2*c^2)*x + 3*sqrt(1/2)*(2*b^7 - 24*a*b^5
*c + 96*a^2*b^3*c^2 - 128*a^3*b*c^3 - (3*b^12*c - 56*a*b^10*c^2 + 400*a^2*b
```

```
^8*c^3 - 1280*a^3*b^6*c^4 + 1280*a^4*b^4*c^5 + 2048*a^5*b^2*c^6 - 4096*a^6*
c^7)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 128
0*a^4*b^2*c^6 - 1024*a^5*c^7))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (b^
10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5
- 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^
4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*
b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))) - 3*sqrt(1/2
)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a
^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*
a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*
a*b^3*c + 80*a^2*b*c^2 - (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3
*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 +
160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10
*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 -
1024*a^5*c^6))*log(3*(5*b^4 + 40*a*b^2*c + 16*a^2*c^2)*x - 3*sqrt(1/2)*(2*b
^7 - 24*a*b^5*c + 96*a^2*b^3*c^2 - 128*a^3*b*c^3 - (3*b^12*c - 56*a*b^10*c^
2 + 400*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 1280*a^4*b^4*c^5 + 2048*a^5*b^2*c^
6 - 4096*a^6*c^7)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*
b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^
2*b*c^2 - (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280
*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4
 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c
^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)))
)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a
^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*
a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)
```

giac [B] time = 2.82, size = 1750, normalized size = 5.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3/16*(2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 16*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^4*c - 4*b^5*c + 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 16
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^3*c^2 + 32*a*b^3*c^2 + 6*b^4*c^2 - 8*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 64*a^2*b*c^3 - 16*a*b^2*c^3 - 32*a^2*c^4
- 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c + 16*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*
```

```
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 + 4*(b^2 - 4*a*c)*b^3*c - 16*(b^2 - 4*a*c)
*a*b*c^2 - 6*(b^2 - 4*a*c)*b^2*c^2 - 8*(b^2 - 4*a*c)*a*c^3)*arctan(2*sqrt(1
/2)*x/sqrt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + sqrt((b^5 - 8*a*b^3*c + 16*a^2
*b*c^2)^2 - 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*(b^4*c - 8*a*b^2*c^2 + 16*
a^2*c^3)))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/((b^8 - 16*a*b^6*c - 2*b^7*
c + 96*a^2*b^4*c^2 + 24*a*b^5*c^2 + b^6*c^2 - 256*a^3*b^2*c^3 - 96*a^2*b^3*
c^3 - 12*a*b^4*c^3 + 256*a^4*c^4 + 128*a^3*b*c^4 + 48*a^2*b^2*c^4 - 64*a^3*
c^5)*abs(c)) - 3/16*(2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 - 16*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 4*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^4*c + 4*b^5*c + 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 32*a*b^3*c^2 - 6*b^4*c^2 - 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 64*a^2*b*c^3 + 16*a*b^2*c^3
 + 32*a^2*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*b^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c
- 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 3*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 4*(b^2 - 4*a*c)*b^3*c + 16
*(b^2 - 4*a*c)*a*b*c^2 + 6*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*a
rctan(2*sqrt(1/2)*x/sqrt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - sqrt((b^5 - 8*a*
b^3*c + 16*a^2*b*c^2)^2 - 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*(b^4*c - 8*a
*b^2*c^2 + 16*a^2*c^3)))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/((b^8 - 16*a*
b^6*c - 2*b^7*c + 96*a^2*b^4*c^2 + 24*a*b^5*c^2 + b^6*c^2 - 256*a^3*b^2*c^3
 - 96*a^2*b^3*c^3 - 12*a*b^4*c^3 + 256*a^4*c^4 + 128*a^3*b*c^4 + 48*a^2*b^2
*c^4 - 64*a^3*c^5)*abs(c)) + 1/8*(3*b^2*c*x^7 + 12*a*c^2*x^7 + 5*b^3*x^5 +
16*a*b*c*x^5 + 19*a*b^2*x^3 - 4*a^2*c*x^3 + 12*a^2*b*x)/((c*x^4 + b*x^2 + a
)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

maple [B] time = 0.04, size = 753, normalized size = 2.53

$$\frac{9\sqrt{2}\, abc \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{9\sqrt{2}\, abc \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-\right.}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (3/8*c*(4*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8*b*(16*a*c+5*b^2)/(16*
a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*(4*a*c-19*b^2)*a/(16*a^2*c^2-8*a*b^2*c+b^4)*
```

```
x^3+3/2/(16*a^2*c^2-8*a*b^2*c+b^4)*a^2*b*x)/(c*x^4+b*x^2+a)^2-3/4/(16*a^2*c
^2-8*a*b^2*c+b^4)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a-3/16/(16*a^2*c^2-8*a*b^2*c+b^4)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*b^2+9/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*c*x)*a*b+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3+3/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*a+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2+9/4/(16*a
^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b+3/16/(16*
a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(b^2*c + 4*a*c^2)*x^7 + (5*b^3 + 16*a*b*c)*x^5 + 12*a^2*b*x + (19*a*
b^2 - 4*a^2*c)*x^3)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6
 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2
) + 3/8*integrate(((b^2 + 4*a*c)*x^2 - 4*a*b)/(c*x^4 + b*x^2 + a), x)/(b^4
- 8*a*b^2*c + 16*a^2*c^2)
```

mupad [B] time = 8.18, size = 8521, normalized size = 28.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] atan(((((3*(1024*a*b^11*c^2 - 1048576*a^6*b*c^7 - 20480*a^2*b^9*c^3 + 16384
0*a^3*b^7*c^4 - 655360*a^4*b^5*c^5 + 1310720*a^5*b^3*c^6))/(512*(b^12 + 409
6*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^
5*b^2*c^5 - 24*a*b^10*c)) - (x*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 819
20*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 10
24*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a
^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4
```

*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2) - (x*(9*b^6*c - 288*a^3*c^4 + 126*a*b^4*c^2 + 576*a^2*b^2*c^3))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)*1i - (((3*(1024*a*b^11*c^2 - 1048576*a^6*b*c^7 - 20480*a^2*b^9*c^3 + 163840*a^3*b^7*c^4 - 655360*a^4*b^5*c^5 + 1310720*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2) + (x*(9*b^6*c - 288*a^3*c^4 + 126*a*b^4*c^2 + 576*a^2*b^2*c^3))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)*1i)/((3*(576*a^4*c^4 + 540*a^2*b^4*c^2 + 1584*a^3*b^2*c^3 + 45*a*b^6*c))/(256*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (((3*(1024*a*b^11*c^2 - 1048576*a^6*b*c^7 - 20480*a^2*b^9*c^3 + 163840*a^3*b^7*c^4 - 655360*a^4*b^5*c^5 + 1310720*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 2

40*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 2
4*a*b^10*c)) - (x*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7
- 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^
5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40
*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 2
58048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8
*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 2
62144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c
^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c
)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11
*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*
b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 +
720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10
*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 262
1440*a^9*b^2*c^10)))^(1/2) - (x*(9*b^6*c - 288*a^3*c^4 + 126*a*b^4*c^2 + 57
6*a^2*b^2*c^3))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 -
16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 5
60*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 +
61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*
b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 2580
48*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^
4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2) + (((3*(1024*a*b^11*c^2 - 1048576*a^6
*b*c^7 - 20480*a^2*b^9*c^3 + 163840*a^3*b^7*c^4 - 655360*a^4*b^5*c^5 + 1310
720*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^
6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(-(9*(b^15
+ (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^
3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a
*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c
^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*
a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c
^10)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*
b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4
+ 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c -
b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11
520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512
*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*
b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 -
1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2) +
(x*(9*b^6*c - 288*a^3*c^4 + 126*a*b^4*c^2 + 576*a^2*b^2*c^3))/(32*(b^8 + 2
56*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 +
(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b
^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^
13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3
- 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6
*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10

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)))^(1/2)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*
a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61
440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^1
8*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*
a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c
^9 - 2621440*a^9*b^2*c^10)))^(1/2)*2i + ((x^3*(19*a*b^2 - 4*a^2*c))/(8*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c*x^7*(4*a*c + b^2))/(8*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)) + (3*a^2*b*x)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*x^5*(16
*a*c + 5*b^2))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2
 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + atan(((((3*(1024*a*b^11*c^2 - 1048576
*a^6*b*c^7 - 20480*a^2*b^9*c^3 + 163840*a^3*b^7*c^4 - 655360*a^4*b^5*c^5 +
1310720*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^
3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - (x*((9*((
-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160
*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 2
0*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^1
6*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 8601
60*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^
2*c^10)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a
^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c
^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^
15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 +
11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(5
12*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^
3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7
- 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)
 - (x*(9*b^6*c - 288*a^3*c^4 + 126*a*b^4*c^2 + 576*a^2*b^2*c^3))/(32*(b^8 +
 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a
*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*
b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b
^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3
 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^
6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^1
0)))^(1/2)*1i - (((3*(1024*a*b^11*c^2 - 1048576*a^6*b*c^7 - 20480*a^2*b^9*c
^3 + 163840*a^3*b^7*c^4 - 655360*a^4*b^5*c^5 + 1310720*a^5*b^3*c^6))/(512*(
b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4
 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*((9*((-(4*a*c - b^2)^15)^(1/2) - b
^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7
*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(b^20*c +
1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 +
53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7
*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)*(256*b^11*c^
2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*
c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^
3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7
```

*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5
*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^
11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*
c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949
120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2) + (x*(9*b^6*c - 288*a^3*c^4
 + 126*a*b^4*c^2 + 576*a^2*b^2*c^3))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^
2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 +
 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4
+ 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(b^20*c + 10485
76*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760
*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*
c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)*1i)/((((3*(1024*a
*b^11*c^2 - 1048576*a^6*b*c^7 - 20480*a^2*b^9*c^3 + 163840*a^3*b^7*c^4 - 65
5360*a^4*b^5*c^5 + 1310720*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^
2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b
^10*c)) - (x*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*
a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61
440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^1
8*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*
a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c
^9 - 2621440*a^9*b^2*c^10)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*
a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(
32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((
9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 -
4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6
 - 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2
*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 +
860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^
9*b^2*c^10)))^(1/2) - (x*(9*b^6*c - 288*a^3*c^4 + 126*a*b^4*c^2 + 576*a^2*b
^2*c^3))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b
^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b
^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a
^6*b^3*c^6 - 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2
 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b
^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 -
2621440*a^9*b^2*c^10)))^(1/2) + (((3*(1024*a*b^11*c^2 - 1048576*a^6*b*c^7 -
 20480*a^2*b^9*c^3 + 163840*a^3*b^7*c^4 - 655360*a^4*b^5*c^5 + 1310720*a^5*
b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 +
3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*((9*((-(4*a*c - b^
2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3
 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))
/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680
*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c
^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1
/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 -

```
163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*
b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) -
b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^
7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(b^20*c +
 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 +
 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^
7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2) + (x*(9*b^6
*c - 288*a^3*c^4 + 126*a*b^4*c^2 + 576*a^2*b^2*c^3))/(32*(b^8 + 256*a^4*c^4
 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15
)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11
520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512
*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*
b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 -
1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2) +
 (3*(576*a^4*c^4 + 540*a^2*b^4*c^2 + 1584*a^3*b^2*c^3 + 45*a*b^6*c))/(256*(
b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4
 - 6144*a^5*b^2*c^5 - 24*a*b^10*c))))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15
 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^
4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(b^20*c + 104
8576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 537
60*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^
6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)*2i
```

sympy [B] time = 23.39, size = 627, normalized size = 2.10

$$\frac{12a^2bx + x^7\left(12ac^2 + 3b^2c\right) + x^5\left(16abc + 5b^3\right) + x^3\left(-4a^2c + 1\right.}{128a^4c^2 - 64a^3b^2c + 8a^2b^4 + x^8\left(128a^2c^4 - 64ab^2c^3 + 8b^4c^2\right) + x^6\left(256a^2bc^3 - 128ab^3c^2 + 16b^5c\right) + x^4\left(256a^3c^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] (12*a**2*b*x + x**7*(12*a*c**2 + 3*b**2*c) + x**5*(16*a*b*c + 5*b**3) + x**
3*(-4*a**2*c + 19*a*b**2))/(128*a**4*c**2 - 64*a**3*b**2*c + 8*a**2*b**4 +
x**8*(128*a**2*c**4 - 64*a*b**2*c**3 + 8*b**4*c**2) + x**6*(256*a**2*b*c**3
 - 128*a*b**3*c**2 + 16*b**5*c) + x**4*(256*a**3*c**3 - 48*a*b**4*c + 8*b**
6) + x**2*(256*a**3*b*c**2 - 128*a**2*b**3*c + 16*a*b**5)) + RootSum(_t**4*
(68719476736*a**10*c**11 - 171798691840*a**9*b**2*c**10 + 193273528320*a**8
*b**4*c**9 - 128849018880*a**7*b**6*c**8 + 56371445760*a**6*b**8*c**7 - 169
11433728*a**5*b**10*c**6 + 3523215360*a**4*b**12*c**5 - 503316480*a**3*b**1
4*c**4 + 47185920*a**2*b**16*c**3 - 2621440*a*b**18*c**2 + 65536*b**20*c) +
 _t**2*(-188743680*a**7*b*c**7 + 141557760*a**6*b**3*c**6 - 2359296*a**5*b*
*5*c**5 - 26542080*a**4*b**7*c**4 + 9584640*a**3*b**9*c**3 - 1290240*a**2*b
**11*c**2 + 46080*a*b**13*c + 2304*b**15) + 20736*a**5*c**4 + 103680*a**4*b
**2*c**3 + 142560*a**3*b**4*c**2 + 32400*a**2*b**6*c + 2025*a*b**8, Lambda(
```

```
_t, _t*log(x + (33554432*_t**3*a**6*c**7 - 16777216*_t**3*a**5*b**2*c**6 -
10485760*_t**3*a**4*b**4*c**5 + 10485760*_t**3*a**3*b**6*c**4 - 3276800*_t*
*3*a**2*b**8*c**3 + 458752*_t**3*a*b**10*c**2 - 24576*_t**3*b**12*c - 64512
*_t*a**3*b*c**3 - 43776*_t*a**2*b**3*c**2 - 21312*_t*a*b**5*c - 144*_t*b**7
)/(432*a**2*c**2 + 1080*a*b**2*c + 135*b**4))))
```

3.884 $\int \frac{x^4}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=289

$$\frac{x\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(-4ac+7b^2+12bcx^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{b^2-}}}\right.}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] 1/4*x*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-1/8*x*(12*b*c*x^2-4*a*c+7*
b^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3/8*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^2+4*a*c-2*b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(5/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/8*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^2+4*a*c+2*b*(-4*a*c+b^2)^(1/2))/
(-4*a*c+b^2)^(5/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.71, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1120, 1178, 1166, 205}

$$\frac{x\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(-4ac+7b^2+12bcx^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{b^2-}}}\right.}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x*(7*b^2 - 4*a
*c + 12*b*c*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[c]*(3*b
^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]
]) - (3*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b
 + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{x\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{2a-5bx^2}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= \frac{x\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x\left(7b^2-4ac+12bcx^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\int \frac{3a\left(b^2+4ac\right)-12abcx^2}{a+bx^2+cx^4}\,dx}{8a\left(b^2-4ac\right)^2}$$

$$= \frac{x\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x\left(7b^2-4ac+12bcx^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3c\left(3b^2+4ac-2b\sqrt{\right.\right.}{8(}$$

$$= \frac{x\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x\left(7b^2-4ac+12bcx^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3\sqrt{c}\left(3b^2+4ac-2b\right.}{4\sqrt{2}\left(b^2-4a\right.}$$

Mathematica [A] time = 0.71, size = 285, normalized size = 0.99

$$\frac{1}{8}\left(\frac{2\left(2ax+bx^3\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{4acx-7b^2x-12bcx^3}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3\sqrt{2}\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{}{\sqrt{}}\right.}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*(2*a*x + b*x^3))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (-7*b^2*x + 4*
a*c*x - 12*b*c*x^3)/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt
[c]*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 - (3*Sqrt[2]*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/8
```

fricas [B] time = 1.09, size = 3128, normalized size = 10.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] -1/16*(24*b*c^2*x^7 + 2*(19*b^2*c - 4*a*c^2)*x^5 + 2*(5*b^3 + 16*a*b*c)*x^3 + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*x + 3/2*sqrt(1/2)*(b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4 - (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*x - 3/2*sqrt(1/2)*(b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4 - (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*x + 3/2*sqrt(1/2)*(b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4 + (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7

```
*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)/sqrt(a^2*b^1
0 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1
024*a^7*c^5))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^10 - 20*a^2*b^8
*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/s
qrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*
b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4
*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a
*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a
^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*
(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*
b*c^2 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a
^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 -
 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c
 + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))*lo
g(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*x - 3/2*sqrt(1/2)*(b^8 - 8*a*b^6*
c + 128*a^3*b^2*c^3 - 256*a^4*c^4 + (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2
 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^
6)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*
a^6*b^2*c^4 - 1024*a^7*c^5))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^
10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 -
1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*
c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^
6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))) + 6*(a*b^2 + 4
*a^2*c)*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c
^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^
4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)
```

giac [**B**] time = 2.64, size = 1861, normalized size = 6.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/32*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 4*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*
c - 2*b^6*c - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*a*b^4*c^2 + 2*b^5*c^2 + 64*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 32*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^2*b^2*c^3 + 16*a*b^3*c^3 - 16*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 128*a^3*c^4 - 96*a^2*b*c^4 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 48*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
```

```
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*(b^2 - 4*a*c)*b^4*c - 2*(b^2 - 4*a*c)*b^3*
c^2 - 32*(b^2 - 4*a*c)*a^2*c^3 - 24*(b^2 - 4*a*c)*a*b*c^3)*arctan(2*sqrt(1/
2)*x/sqrt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + sqrt((b^5 - 8*a*b^3*c + 16*a^2*
b*c^2)^2 - 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*(b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/((a*b^8 - 16*a^2*b^6*c - 2*a
*b^7*c + 96*a^3*b^4*c^2 + 24*a^2*b^5*c^2 + a*b^6*c^2 - 256*a^4*b^2*c^3 - 96
*a^3*b^3*c^3 - 12*a^2*b^4*c^3 + 256*a^5*c^4 + 128*a^4*b*c^4 + 48*a^3*b^2*c^
4 - 64*a^4*c^5)*abs(c)) + 3/32*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6
 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^5*c + 2*b^6*c - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 8*a
*b^4*c^2 - 2*b^5*c^2 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 +
 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a^2*b^2*c^3 - 16
*a*b^3*c^3 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 128*a^3*c
^4 + 96*a^2*b*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*
c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 48*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 24*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 12*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^
4*c + 2*(b^2 - 4*a*c)*b^3*c^2 + 32*(b^2 - 4*a*c)*a^2*c^3 + 24*(b^2 - 4*a*c)
*a*b*c^3)*arctan(2*sqrt(1/2)*x/sqrt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - sqrt(
(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)^2 - 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*(
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/((
a*b^8 - 16*a^2*b^6*c - 2*a*b^7*c + 96*a^3*b^4*c^2 + 24*a^2*b^5*c^2 + a*b^6*
c^2 - 256*a^4*b^2*c^3 - 96*a^3*b^3*c^3 - 12*a^2*b^4*c^3 + 256*a^5*c^4 + 128
*a^4*b*c^4 + 48*a^3*b^2*c^4 - 64*a^4*c^5)*abs(c)) - 1/8*(12*b*c^2*x^7 + 19*
b^2*c*x^5 - 4*a*c^2*x^5 + 5*b^3*x^3 + 16*a*b*c*x^3 + 3*a*b^2*x + 12*a^2*c*x
)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

maple [B] time = 0.04, size = 617, normalized size = 2.13

$$-\frac{3\sqrt{2}\, a\, c^{2} \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right) c}}\right)}{2\left(16a^{2}c^{2}-8a\, b^{2}c+b^{4}\right)\sqrt{-4ac+b^{2}}\, \sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right) c}}-\frac{3\sqrt{2}\, a\, c^{2} \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right) c}}\right)}{2\left(16a^{2}c^{2}-8a\, b^{2}c+b^{4}\right)\sqrt{-4ac+b^{2}}\, \sqrt{\left(b+\sqrt{-4\right.}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (-3/2*b*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8*c*(4*a*c-19*b^2)/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^5-1/8*b*(16*a*c+5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-3/8
*a*(4*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+3/4/(16*a^2*
c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b-3/2/(16*a^2*c^2-8*a*b^2*c+b^4)
*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a-9/8/(16*a^2*c^2-8*a*b^2*c+b
^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2-3/4/(16*a^2*c^2-8*a*b^2*
c+b^4)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*b-3/2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*a-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*c*x)*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(12*b*c^2*x^7 + (19*b^2*c - 4*a*c^2)*x^5 + (5*b^3 + 16*a*b*c)*x^3 + 3*
(a*b^2 + 4*a^2*c)*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c -
 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^
6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^
2) - 3/8*integrate((4*b*c*x^2 - b^2 - 4*a*c)/(c*x^4 + b*x^2 + a), x)/(b^4 -
 8*a*b^2*c + 16*a^2*c^2)
```

mupad [B] time = 7.59, size = 8397, normalized size = 29.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2 + c*x^4)^3,x)
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```
[Out] atan(((((3*(262144*a^6*c^8 - 64*b^12*c^2 + 1024*a*b^10*c^3 - 5120*a^2*b^8*c
^4 + 81920*a^4*b^4*c^6 - 262144*a^5*b^2*c^7))/(128*(b^12 + 4096*a^6*c^6 + 2
40*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 2
4*a*b^10*c)) - (x*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 +
 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5
 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*
a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 25
8048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*
```

b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)*(128*b^11*c^2 - 2560*a*b^9*c^3 - 13
1072*a^5*b*c^7 + 20480*a^2*b^7*c^4 - 81920*a^3*b^5*c^5 + 163840*a^4*b^3*c^6
))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c))
)*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^
2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3
*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720
*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^
5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 262144
0*a^10*b^2*c^9)))^(1/2) - (x*(144*a^2*c^5 + 117*b^4*c^3 + 72*a*b^2*c^4))/(1
6*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9
*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4
160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6
- 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*
b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 8
60160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^1
0*b^2*c^9)))^(1/2)*1i - (((3*(262144*a^6*c^8 - 64*b^12*c^2 + 1024*a*b^10*c^
3 - 5120*a^2*b^8*c^4 + 81920*a^4*b^4*c^6 - 262144*a^5*b^2*c^7))/(128*(b^12
+ 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 61
44*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 +
81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4
+ 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 10485
76*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760
*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*
c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)*(128*b^11*c^2 - 2
560*a*b^9*c^3 - 131072*a^5*b*c^7 + 20480*a^2*b^7*c^4 - 81920*a^3*b^5*c^5 +
163840*a^4*b^3*c^6))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*
c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7
+ 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c
^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 4
0*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 -
258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^
9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2) + (x*(144*a^2*c^5 + 117*b^4*c^3 +
72*a*b^2*c^4))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 -
16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 56
0*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 -
61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2
*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 25804
8*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4
*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)*1i)/((((3*(262144*a^6*c^8 - 64*b^12*c^
2 + 1024*a*b^10*c^3 - 5120*a^2*b^8*c^4 + 81920*a^4*b^4*c^6 - 262144*a^5*b^2
c^7))/(128(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 384
0*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - (x*((9*((-(4*a*c - b^2)^
15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 +
11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(5
12*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^

4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6
- 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)
*(128*b^11*c^2 - 2560*a*b^9*c^3 - 131072*a^5*b*c^7 + 20480*a^2*b^7*c^4 - 81
920*a^3*b^5*c^5 + 163840*a^4*b^3*c^6))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*
c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15
 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^
4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 104
8576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 537
60*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^
6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2) - (x*(144*a^2*c
^5 + 117*b^4*c^3 + 72*a*b^2*c^4))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 -
 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81
920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1
024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 1048576*
a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^
5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7
 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2) - (3*(45*b^5*c^3 + 3
60*a*b^3*c^4 + 144*a^2*b*c^5))/(64*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 -
 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (
((3*(262144*a^6*c^8 - 64*b^12*c^2 + 1024*a*b^10*c^3 - 5120*a^2*b^8*c^4 + 81
920*a^4*b^4*c^6 - 262144*a^5*b^2*c^7))/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*
b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^1
0*c)) + (x*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^
2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 6144
0*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^1
8*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^
6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8
 - 2621440*a^10*b^2*c^9)))^(1/2)*(128*b^11*c^2 - 2560*a*b^9*c^3 - 131072*a^
5*b*c^7 + 20480*a^2*b^7*c^4 - 81920*a^3*b^5*c^5 + 163840*a^4*b^3*c^6))/(16*
(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*(
(-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 416
0*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 -
20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^
16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860
160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*
b^2*c^9)))^(1/2) + (x*(144*a^2*c^5 + 117*b^4*c^3 + 72*a*b^2*c^4))/(16*(b^8
+ 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*
a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3
*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*
b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^
2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a
^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c
^9)))^(1/2)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560
*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 6
1440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*

b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)*2i - ((x^3*(5*b^3 + 16*a*b*c))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^5*(4*a*c^2 - 19*b^2*c))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*c^2*x^7)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a*x*(4*a*c + b^2))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + atan(((((3*(262144*a^6*c^8 - 64*b^12*c^2 + 1024*a*b^10*c^3 - 5120*a^2*b^8*c^4 + 81920*a^4*b^4*c^6 - 262144*a^5*b^2*c^7))/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - (x*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)*(128*b^11*c^2 - 2560*a*b^9*c^3 - 131072*a^5*b*c^7 + 20480*a^2*b^7*c^4 - 81920*a^3*b^5*c^5 + 163840*a^4*b^3*c^6))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2) - (x*(144*a^2*c^5 + 117*b^4*c^3 + 72*a*b^2*c^4))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)*1i - (((3*(262144*a^6*c^8 - 64*b^12*c^2 + 1024*a*b^10*c^3 - 5120*a^2*b^8*c^4 + 81920*a^4*b^4*c^6 - 262144*a^5*b^2*c^7))/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)*(128*b^11*c^2 - 2560*a*b^9*c^3 - 131072*a^5*b*c^7 + 20480*a^2*b^7*c^4 - 81920*a^3*b^5*c^5 + 163840*a^4*b^3*c^6))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^

7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2) + (x*(144*a^2*c^5 + 117*b^4*c^3 + 72*a*b^2*c^4))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)*1i)/((((3*(262144*a^6*c^8 - 64*b^12*c^2 + 1024*a*b^10*c^3 - 5120*a^2*b^8*c^4 + 81920*a^4*b^4*c^6 - 262144*a^5*b^2*c^7))/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - (x*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)*(128*b^11*c^2 - 2560*a*b^9*c^3 - 131072*a^5*b*c^7 + 20480*a^2*b^7*c^4 - 81920*a^3*b^5*c^5 + 163840*a^4*b^3*c^6))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2) - (x*(144*a^2*c^5 + 117*b^4*c^3 + 72*a*b^2*c^4))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2) - (3*(45*b^5*c^3 + 360*a*b^3*c^4 + 144*a^2*b*c^5))/(64*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (((3*(262144*a^6*c^8 - 64*b^12*c^2 + 1024*a*b^10*c^3 - 5120*a^2*b^8*c^4 + 81920*a^4*b^4*c^6 - 262144*a^5*b^2*c^7))/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)*(128*b^11*c^2 - 2560*a*b^9*c^3 - 131072*a^5*b*c^7 + 20480*a^2*b^7*c^4 - 81920*a^3*b^5*c^5 + 163840*a^4*b^3*c^6))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15

```
)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4
*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^2
0 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^
3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080
*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2) + (x*(14
4*a^2*c^5 + 117*b^4*c^3 + 72*a*b^2*c^4))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^
4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1
/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7
*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20 +
1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 +
53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8
*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)))*(-(9*(b^15
 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^
3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a
*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c
^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*
a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*
c^9)))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.885 $\int \frac{x^2}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=311

$$-\frac{x\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x\left(cx^2\left(20ac+b^2\right)+b\left(8ac+b^2\right)\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{b^2-}}}\right.}{8\sqrt{2}\,a\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] -1/4*x*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/8*x*(b*(8*a*c+b^2)+c*(2
0*a*c+b^2)*x^2)/a/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/16*arctan(x*2^(1/2)*c^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2+20*a*c+b*(-52*a*c+b^2)/(-4*a
*c+b^2)^(1/2))/a/(-4*a*c+b^2)^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/16*a
rctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2+20*a*c-b
*(-52*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2)
```

Rubi [A] time = 0.70, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1119, 1178, 1166, 205}

$$-\frac{x\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x\left(cx^2\left(20ac+b^2\right)+b\left(8ac+b^2\right)\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{b^2-}}}\right.}{8\sqrt{2}\,a\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*(b*(b^2 + 8
*a*c) + c*(b^2 + 20*a*c)*x^2))/(8*a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) +
(Sqrt[c]*(b^2 + 20*a*c + (b*(b^2 - 52*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*S
qrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b^2 + 20*a*c - (b*(b^2 - 52*a*c))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2+cx^4\right)^3}\,dx = -\frac{x\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\int \frac{b-10cx^2}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= -\frac{x\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x\left(b\left(b^2+8ac\right)+c\left(b^2+20ac\right)x^2\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\int \frac{-b\left(b^2-16ac\right)-c\left(\ldots\right.}{a+bx^2+\ldots}}{8a\left(b^2-\ldots\right.}$$

$$= -\frac{x\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x\left(b\left(b^2+8ac\right)+c\left(b^2+20ac\right)x^2\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\left(c\left(b^2+20ac\ldots\right.\right.}{\ldots}$$

$$= -\frac{x\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x\left(b\left(b^2+8ac\right)+c\left(b^2+20ac\right)x^2\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b^2+20ac\ldots\right.}{8\sqrt{2}\,a\left(\ldots\right.}$$

Mathematica [A] time = 0.85, size = 334, normalized size = 1.07

$$\frac{1}{16}\left(-\frac{4x\left(b+2cx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{2x\left(8abc+20ac^2x^2+b^3+b^2cx^2\right)}{a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}\ldots\right.}{a\left(b^2-4ac\right)^{5/2}\sqrt{\ldots}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*x*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*x*(b^3 + 8
*a*b*c + b^2*c*x^2 + 20*a*c^2*x^2))/(a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4))
 + (Sqrt[2]*Sqrt[c]*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b
^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^3 + 5
2*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/16
```

fricas [B] time = 1.30, size = 3777, normalized size = 12.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] 1/16*(2*(b^2*c^2 + 20*a*c^3)*x^7 + 4*(b^3*c + 14*a*b*c^2)*x^5 + 2*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*x^3 + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*x + 1/2*sqrt(1/2)*(b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5 - (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))) - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*x - 1/2*sqrt(1/2)*(b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5 - (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))) + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4

```
*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-
(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b
^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)
*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b
^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 -
 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024
*a^8*c^5))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3
*c^5)*x + 1/2*sqrt(1/2)*(b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5
*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5 + (a^3*b^14 - 38*a^4*b^12*c + 48
0*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 3
2768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a
^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*
c^4 - 1024*a^11*c^5)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3
*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 128
0*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^
10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 -
 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*
c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))) - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^
2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c -
 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4
 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(b^7 - 35*a*b^5*c +
280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c
^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^
2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4
*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*
a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((35*b
^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*x - 1/2*sqrt(1
/2)*(b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3
*c^4 - 25600*a^5*b*c^5 + (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 272
0*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 4
0960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8
*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)
))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 -
 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024
*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c +
160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a
^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c
^4 - 1024*a^8*c^5))) - 2*(a*b^3 - 16*a^2*b*c)*x)/((a*b^4*c^2 - 8*a^2*b^2*c^
3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a
^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2
*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)
```

giac [B] time = 2.45, size = 4270, normalized size = 13.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/64*(2*a^2*b^12*c^2 - 136*a^3*b^10*c^3 + 1856*a^4*b^8*c^4 - 10496*a^5*b^6*
c^5 + 27136*a^6*b^4*c^6 - 26624*a^7*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^12 + 68*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^10*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^11*c - 928*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c^2 - 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^9*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^10*c^2 + 5248*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^3 + 1344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^3 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^3 - 13568*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 - 5120*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^4 - 672*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^4 + 13312*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^5 + 6656*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 + 2560*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^5 - 3328*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^6 - 2*(b^2 - 4*a*c)*a^2
*b^10*c^2 + 128*(b^2 - 4*a*c)*a^3*b^8*c^3 - 1344*(b^2 - 4*a*c)*a^4*b^6*c^4
+ 5120*(b^2 - 4*a*c)*a^5*b^4*c^5 - 6656*(b^2 - 4*a*c)*a^6*b^2*c^6 + (2*b^4*
c^2 + 32*a*b^2*c^3 - 160*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^3*c + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*c^2 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 20
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 -
 4*a*c)*b^2*c^2 - 40*(b^2 - 4*a*c)*a*c^3)*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
)^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^9 - 28*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^8*c - 2*a*b^9*c + 240*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b
^5*c^2 + 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 + sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^2 + 56*a^2*b^7*c^2 - 832*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 288*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^3*b^4*c^3 - 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
5*c^3 - 480*a^3*b^5*c^3 + 1024*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*
b*c^4 + 512*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + 144*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + 1664*a^4*b^3*c^4 - 256*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 2048*a^5*b*c^5 + 2*(b^2 -
4*a*c)*a*b^7*c - 48*(b^2 - 4*a*c)*a^2*b^5*c^2 + 288*(b^2 - 4*a*c)*a^3*b^3*c
^3 - 512*(b^2 - 4*a*c)*a^4*b*c^4)*abs(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2))*ar
```

ctan(2*sqrt(1/2)*x/sqrt((a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + sqrt((a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)^2 - 4*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)))/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)))/((a^3*b^10 - 20*a^4*b^8*c - 2*a^3*b^9*c + 160*a^5*b^6*c^2 + 32*a^4*b^7*c^2 + a^3*b^8*c^2 - 640*a^6*b^4*c^3 - 192*a^5*b^5*c^3 - 16*a^4*b^6*c^3 + 1280*a^7*b^2*c^4 + 512*a^6*b^3*c^4 + 96*a^5*b^4*c^4 - 1024*a^8*c^5 - 512*a^7*b*c^5 - 256*a^6*b^2*c^5 + 256*a^7*c^6)*abs(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*abs(c)) - 1/64*(2*a^2*b^12*c^2 - 136*a^3*b^10*c^3 + 1856*a^4*b^8*c^4 - 10496*a^5*b^6*c^5 + 27136*a^6*b^4*c^6 - 26624*a^7*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^12 + 68*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^10*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^11*c - 928*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c^2 - 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^9*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^10*c^2 + 5248*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^3 + 1344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^3 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^3 - 13568*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 - 5120*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^4 - 672*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^4 + 13312*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^5 + 6656*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 + 2560*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^5 - 3328*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^6 - 2*(b^2 - 4*a*c)*a^2*b^10*c^2 + 128*(b^2 - 4*a*c)*a^3*b^8*c^3 - 1344*(b^2 - 4*a*c)*a^4*b^6*c^4 + 5120*(b^2 - 4*a*c)*a^5*b^4*c^5 - 6656*(b^2 - 4*a*c)*a^6*b^2*c^6 + (2*b^4*c^2 + 32*a*b^2*c^3 - 160*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 - 40*(b^2 - 4*a*c)*a*c^3)*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^9 - 28*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^8*c + 2*a*b^9*c + 240*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 + 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^2 - 56*a^2*b^7*c^2 - 832*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 288*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 + 480*a^3*b^5*c^3 + 1024*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 + 512*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + 144*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 1664*a^4*b^3*c

```
^4 - 256*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + 2048*a^5*b*c^5
 - 2*(b^2 - 4*a*c)*a*b^7*c + 48*(b^2 - 4*a*c)*a^2*b^5*c^2 - 288*(b^2 - 4*a*
c)*a^3*b^3*c^3 + 512*(b^2 - 4*a*c)*a^4*b*c^4)*abs(a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2))*arctan(2*sqrt(1/2)*x/sqrt((a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 - s
qrt((a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)^2 - 4*(a^2*b^4 - 8*a^3*b^2*c + 16*
a^4*c^2)*(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)))/(a*b^4*c - 8*a^2*b^2*c^2
+ 16*a^3*c^3)))/((a^3*b^10 - 20*a^4*b^8*c - 2*a^3*b^9*c + 160*a^5*b^6*c^2 +
 32*a^4*b^7*c^2 + a^3*b^8*c^2 - 640*a^6*b^4*c^3 - 192*a^5*b^5*c^3 - 16*a^4*
b^6*c^3 + 1280*a^7*b^2*c^4 + 512*a^6*b^3*c^4 + 96*a^5*b^4*c^4 - 1024*a^8*c^
5 - 512*a^7*b*c^5 - 256*a^6*b^2*c^5 + 256*a^7*c^6)*abs(a*b^4 - 8*a^2*b^2*c
+ 16*a^3*c^2)*abs(c)) + 1/8*(b^2*c^2*x^7 + 20*a*c^3*x^7 + 2*b^3*c*x^5 + 28*
a*b*c^2*x^5 + b^4*x^3 + 5*a*b^2*c*x^3 + 36*a^2*c^2*x^3 - a*b^3*x + 16*a^2*b
*c*x)/((a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*(c*x^4 + b*x^2 + a)^2)
```

maple [B] time = 0.16, size = 2958, normalized size = 9.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 52*c^4/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b+9*c^3/(
-4*a*c+b^2)^2/(4*a*c-b^2)^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2-9*c^3/(-4*a*c+b^2)^
2/(4*a*c-b^2)^2*a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2-27*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b
^2)^2*a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)*b^3-1/16*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/a*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x)*b^7+1/16*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/a*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*
x)*b^6-1/16*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^6+52*c
^4/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*a^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b-1/16*c/(-4*a*c
+b^2)^(5/2)/(4*a*c-b^2)^2/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^7-27*c^3/(-4*a*c+b^2)^(
5/2)/(4*a*c-b^2)^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3+1/16/(-4*a*c+b^2)^2/(4*a*c-b
^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2/a*x^3*b^6+1/16/(-4*a*c+b^2)^
2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2/a*x^3*b^6+20*c^3/(
-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*a^2*x^
3+3/4*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2
)/c)^2*x^3*b^5-56*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4
```

```
*a*c+b^2)^(1/2)/c)^2*x*a^3+3/4*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c+
1/2*(-4*a*c+b^2)^(1/2)/c)^2*x^3*b^4+3/4*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2
+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x^3*b^4+20*c^3/(-4*a*c+b^2)^2/(4*a*c-b
^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*a^2*x^3-3/4*c/(-4*a*c+b^2)^(
5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x^3*b^5+56*c^3/
(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x
*a^3-1/16/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1
/2)/c)^2/a*x^3*b^7-9*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*
a*c+b^2)^(1/2)/c)^2*a*x^3*b^2+12*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*
b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*a^2*b-6*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x
^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*a*b^3+12*c^2/(-4*a*c+b^2)^2/(4*a*c
-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*a^2*b-6*c/(-4*a*c+b^2)^2
/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*a*b^3+20*c^4/(-4*
a*c+b^2)^2/(4*a*c-b^2)^2*a^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+3/4*c^2/(-4*a*c+b^2)^2/(4*a
*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4+4*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^
2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*a^2*x^3*b-3*c^2/(-4*a*c+b^2)^(5/2)/(4
*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*a*x^3*b^3+42*c^2/(-4*a
*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*a^2*
b^2-21/2*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(
1/2)/c)^2*x*a*b^4+15/4*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*b^5-4*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2
)^(1/2)/c)^2*a^2*x^3*b+3*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-
1/2*(-4*a*c+b^2)^(1/2)/c)^2*a*x^3*b^3-42*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)
^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*a^2*b^2+21/2*c/(-4*a*c+b^2)^(
5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*a*b^4+15/4*c^
2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^5-20*c^4/(-4*a*c
+b^2)^2/(4*a*c-b^2)^2*a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-3/4*c^2/(-4*a*c+b^2)^2/(4*a
*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4-9*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2
+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*a*x^3*b^2+7/8/(-4*a*c+b^2)^(5/2)/(4*a*
c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*b^6+3/4/(-4*a*c+b^2)^2/
(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*b^5-7/8/(-4*a*c+b^
2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*b^6+1/16/
(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2/a
*x^3*b^7+3/4/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/
2)/c)^2*x*b^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((b^2*c^2 + 20*a*c^3)*x^7 + 2*(b^3*c + 14*a*b*c^2)*x^5 + (b^4 + 5*a*b^2
*c + 36*a^2*c^2)*x^3 - (a*b^3 - 16*a^2*b*c)*x)/((a*b^4*c^2 - 8*a^2*b^2*c^3
+ 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2
*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(
a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2) + 1/8*integrate((b^3 - 16*a*b*c
+ (b^2*c + 20*a*c^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)
```

mupad [B] time = 8.37, size = 9731, normalized size = 31.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((b*x*(16*a*c - b^2))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^3*(b^4 + 36*a
^2*c^2 + 5*a*b^2*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*x^5*(14*a*c^
2 + b^2*c))/(4*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^7*(20*a*c^2 + b^2*c
))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8
 + 2*a*b*x^2 + 2*b*c*x^6) + atan(((((256*a*b^13*c^2 + 4194304*a^7*b*c^8 - 9
216*a^2*b^11*c^3 + 122880*a^3*b^9*c^4 - 819200*a^4*b^7*c^5 + 2949120*a^5*b^
5*c^6 - 5505024*a^6*b^3*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c
+ 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)
) - (x*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a
^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 -
680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c -
b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^
5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 +
 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440
*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*
c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^
2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-
(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c
^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^
6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^
(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^
2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a
^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2
*c^9)))^(1/2) - (x*(800*a^3*c^6 - b^6*c^3 + 34*a*b^4*c^4 - 1472*a^2*b^2*c^5
))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2
*c^3)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*
```

a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 -
680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c -
b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a
^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5
+ 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 262144
0*a^12*b^2*c^9)))^(1/2)*1i - (((256*a*b^13*c^2 + 4194304*a^7*b*c^8 - 9216*a
^2*b^11*c^3 + 122880*a^3*b^9*c^4 - 819200*a^4*b^7*c^5 + 2949120*a^5*b^5*c^6
- 5505024*a^6*b^3*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240
*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (
x*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^
13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 68096
0*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^
15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^1
6*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 8601
60*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12
*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 -
40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8
+ 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17
+ b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 -
10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5
*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2)
)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7
680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^
8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)
))^(1/2) + (x*(800*a^3*c^6 - b^6*c^3 + 34*a*b^4*c^4 - 1472*a^2*b^2*c^5))/(3
2*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)
))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b
^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 6809
60*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)
^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^
16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860
160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^1
2*b^2*c^9)))^(1/2)*1i)/((((256*a*b^13*c^2 + 4194304*a^7*b*c^8 - 9216*a^2*b^
11*c^3 + 122880*a^3*b^9*c^4 - 819200*a^4*b^7*c^5 + 2949120*a^5*b^5*c^6 - 55
05024*a^6*b^3*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*
b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (x*(-(
b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^
2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6
*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(
1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2
- 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^
9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*
c^9)))^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 4096
0*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 25
6*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^

2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160
*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6
+ 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(51
2*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a
^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6
 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1
/2) - (x*(800*a^3*c^6 - b^6*c^3 + 34*a*b^4*c^4 - 1472*a^2*b^2*c^5))/(32*(a^
2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-
(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c
^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^
6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^
(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^
2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a
^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2
*c^9)))^(1/2) + (((256*a*b^13*c^2 + 4194304*a^7*b*c^8 - 9216*a^2*b^11*c^3 +
 122880*a^3*b^9*c^4 - 819200*a^4*b^7*c^5 + 2949120*a^5*b^5*c^6 - 5505024*a^
6*b^3*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2
- 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(-(b^17 + b
^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 1016
0*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6
 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(5
12*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*
a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^
6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(
1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^
7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^
4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a
*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^1
1*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 186368
0*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b
^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*
c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 19660
80*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (x
*(800*a^3*c^6 - b^6*c^3 + 34*a*b^4*c^4 - 1472*a^2*b^2*c^5))/(32*(a^2*b^8 +
256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 +
b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 101
60*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^
6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(
512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680
*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c
^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^
(1/2) - (8000*a^3*c^7 - 35*b^6*c^4 - 84*a*b^4*c^5 + 12720*a^2*b^2*c^6)/(256
*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*
c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5))))*(-(b^17 + b^2*(-(4*a*c - b^2)
^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 3

4880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3
*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 104
8576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 537
60*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b
^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*2i + atan((((
(256*a*b^13*c^2 + 4194304*a^7*b*c^8 - 9216*a^2*b^11*c^3 + 122880*a^3*b^9*c^
4 - 819200*a^4*b^7*c^5 + 2949120*a^5*b^5*c^6 - 5505024*a^6*b^3*c^7)/(512*(a
^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3
 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (x*(-(b^17 - b^2*(-(4*a*c - b^2)
^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 3
4880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3
*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 104
8576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 537
60*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b
^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b
*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5
*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c +
 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2
) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*
b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55
*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13
*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^
12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 +
2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) - (x*(800*a^3*c^6 - b^
6*c^3 + 34*a*b^4*c^4 - 1472*a^2*b^2*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a
^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2
)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 +
34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^
3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 10
48576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53
760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*
b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*1i - (((256*
a*b^13*c^2 + 4194304*a^7*b*c^8 - 9216*a^2*b^11*c^3 + 122880*a^3*b^9*c^4 - 8
19200*a^4*b^7*c^5 + 2949120*a^5*b^5*c^6 - 5505024*a^6*b^3*c^7)/(512*(a^2*b^
12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 38
40*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^
(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*
a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7
- 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*
a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^
7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^
7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7
- 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*
c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a
^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1

720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (x*(800*a^3*c^6 - b^6*c^3 + 34*a*b^4*c^4 - 1472*a^2*b^2*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*1i)/((((256*a*b^13*c^2 + 4194304*a^7*b*c^8 - 9216*a^2*b^11*c^3 + 122880*a^3*b^9*c^4 - 819200*a^4*b^7*c^5 + 2949120*a^5*b^5*c^6 - 5505024*a^6*b^3*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (x*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) - (x*(800*a^3*c^6 - b^6*c^3 + 34*a*b^4*c^4 - 1472*a^2*b^2*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (((256*a*b^13*c^2 + 4194304*a^7*b*c^8 - 9216*a^2*b^11*c^3 + 122880*a^3*b^9*c^4 - 819200*a^4*b^7*c^5 + 2949120*a^5*b^5*c^6 - 5505024*a^6*b^3*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 17203

```
20*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 +
 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c
 + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 4
0*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 -
258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a
^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^1
1*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*
a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 -
256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*
c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^
5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c
*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^1
8*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^
8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c
^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (x*(800*a^3*c^6 - b^6*c^3 + 34*a*b^4*c
^4 - 1472*a^2*b^2*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*
b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720
320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4
+ 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*
c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 -
40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 -
 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*
a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) - (8000*a^3*c^7 - 35*b^6*c^4 -
 84*a*b^4*c^5 + 12720*a^2*b^2*c^6)/(256*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b
^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^
2*c^5))))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 114
0*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5
 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c
 - b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720
*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^
5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621
440*a^12*b^2*c^9)))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.886 $\int \frac{1}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=355

$$\frac{x\left(3bcx^2\left(b^2-8ac\right)+\left(b^2-7ac\right)\left(3b^2-4ac\right)\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c+b\left(b^2-8ac\right)\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{\cdots}}{\sqrt{b-\cdots}}\right.}{8\sqrt{2}\,a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] 1/4*x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/8*x*((-7*a*c+b
^2)*(-4*a*c+3*b^2)+3*b*c*(-8*a*c+b^2)*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+
a)+3/16*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^4
-10*a*b^2*c+56*a^2*c^2+b*(-8*a*c+b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^
(5/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+3/16*arctan(x*2^(1/2)*c^(1/2)/(b
+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^3-8*a*b*c+(-56*a^2*c^2+10*a*b^2*c-b^
4)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1
/2)
```

Rubi [A] time = 1.84, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1092, 1178, 1166, 205}

$$\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c+b\left(b^2-8ac\right)\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{c}\left(-\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}-8abc+b^3\right)\cdots}{8\sqrt{2}\,a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-\cdots}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(-3),x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*
((b^2 - 7*a*c)*(3*b^2 - 4*a*c) + 3*b*c*(b^2 - 8*a*c)*x^2))/(8*a^2*(b^2 - 4*
a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 + b
*(b^2 - 8*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (3*Sqrt[c]*(b^3 - 8*a*b*c - (b^4 - 10*a*b^2*c + 56*a^2*c^2)/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt
[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{x\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{b^2-2ac-4\left(b^2-4ac\right)-5bcx^2}{\left(a+bx^2+cx^4\right)^2}\,dx}{4a\left(b^2-4ac\right)}$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(\left(b^2-7ac\right)\left(3b^2-4ac\right)+3bc\left(b^2-8ac\right)x^2\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \int \frac{3(b}{}$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(\left(b^2-7ac\right)\left(3b^2-4ac\right)+3bc\left(b^2-8ac\right)x^2\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(3c\right(}{}$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(\left(b^2-7ac\right)\left(3b^2-4ac\right)+3bc\left(b^2-8ac\right)x^2\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3\sqrt{c}}{}$$

Mathematica [A] time = 1.02, size = 372, normalized size = 1.05

$$\frac{\frac{2x\left(28a^2c^2-25ab^2c-24abc^2x^2+3b^4+3b^3cx^2\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3\sqrt{2}\sqrt{c}\left(56a^2c^2-10ab^2c-8abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{3\sqrt{2}\sqrt{c}\left(56a^2c^2-1\right.}{}}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(-3),x]
```

```
[Out] ((4*a*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2
*x*(3*b^4 - 25*a*b^2*c + 28*a^2*c^2 + 3*b^3*c*x^2 - 24*a*b*c^2*x^2))/((b^2
- 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c + 56
*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - S
qrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 - b^
3*Sqrt[b^2 - 4*a*c] + 8*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a
*c]]))/(16*a^2)
```

fricas [B] time = 1.60, size = 4323, normalized size = 12.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] 1/16*(6*(b^3*c^2 - 8*a*b*c^3)*x^7 + 2*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)
*x^5 + 2*(3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*x^3 - 3*sqrt(1/2)*((a^2*b^4*c^2
 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2
*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c +
32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(b^9
- 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^
10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 -
1024*a^10*c^5)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3
+ 2401*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^
4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160
*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log(27*
(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*
a^4*c^7)*x + 27/2*sqrt(1/2)*(b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a
^3*b^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 8
7808*a^7*c^7 - (a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*
c^3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a
^12*b*c^7)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 24
01*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^
3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*
b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160
*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((b^
8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b
^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c
^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*
b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))) + 3*sqrt(1/2)*((a^2*b^4*c^2 -
 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(
a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32
*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(b^9 -
21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10
 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 10
24*a^10*c^5)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 +
2401*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*
c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a
^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log(27*(2
1*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^
4*c^7)*x - 27/2*sqrt(1/2)*(b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a^3
*b^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 878
08*a^7*c^7 - (a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*c^
3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a^1
2*b*c^7)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401
*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3
+ 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^
5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160*a
```

^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((b^8
- 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^1
0 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4
 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^
4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))) - 3*sqrt(1/2)*((a^2*b^4*c^2 - 8
*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^
2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a
^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(b^9 - 21
*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a^5*b^10 -
 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024
*a^10*c^5)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 24
01*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^
3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7
*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log(27*(21*
b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*
c^7)*x + 27/2*sqrt(1/2)*(b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a^3*b
^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 87808
*a^7*c^7 + (a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*c^3
+ 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a^12*
b*c^7)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a
^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 +
1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*
c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a^5*b^10 - 20*a^6*b^8*c + 160*a^7
*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((b^8 -
22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^10
- 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 -
 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*
c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))) + 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a
^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*
b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5
*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(b^9 - 21*a
*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a^5*b^10 - 2
0*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a
^10*c^5)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401
*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3
+ 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b
^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log(27*(21*b^
8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^
7)*x - 27/2*sqrt(1/2)*(b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a^3*b^8
*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 87808*a
^7*c^7 + (a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*c^3 +
15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a^12*b*
c^7)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4
*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 12
80*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^

```
2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b
^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((b^8 - 22
*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^10 -
20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1
024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^
3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))) + 2*(5*a*b^4 - 37*a^2*b^2*c + 44*a^
3*c^2)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5
*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a
^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*
b*c^2)*x^2)
```

giac [B] time = 1.43, size = 2705, normalized size = 7.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/32*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8 - 17*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7
*c - 2*b^8*c + 116*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 26
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^6*c^2 + 34*a*b^6*c^2 + 2*b^7*c^2 - 368*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 -
 232*a^2*b^4*c^3 - 30*a*b^5*c^3 + 448*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 64*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 736*a^3*b^2*c^4 + 176*a^
2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 - 896*a^4*c
^5 - 352*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b^7 + 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c - 8
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 176*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 88*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 11*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 44*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 2*(b^2 - 4*a*c)
*b^6*c - 26*(b^2 - 4*a*c)*a*b^4*c^2 - 2*(b^2 - 4*a*c)*b^5*c^2 + 128*(b^2 -
4*a*c)*a^2*b^2*c^3 + 22*(b^2 - 4*a*c)*a*b^3*c^3 - 224*(b^2 - 4*a*c)*a^3*c^4
 - 88*(b^2 - 4*a*c)*a^2*b*c^4)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^5 - 8*a^3*b
^3*c + 16*a^4*b*c^2 + sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)^2 - 4*(a^
3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3))
)/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/((a^3*b^8 - 16*a^4*b^6*c - 2*a
```

```
^3*b^7*c + 96*a^5*b^4*c^2 + 24*a^4*b^5*c^2 + a^3*b^6*c^2 - 256*a^6*b^2*c^3
- 96*a^5*b^3*c^3 - 12*a^4*b^4*c^3 + 256*a^7*c^4 + 128*a^6*b*c^4 + 48*a^5*b^
2*c^4 - 64*a^6*c^5)*abs(c)) + 3/32*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*b^8 - 17*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c + 2*b^8*c + 116*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b^4*c^2 + 26*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
5*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 - 34*a*b^6*c^2 - 2*
b^7*c^2 - 368*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 232*a^2*b^4*c^3 + 30*a*b^5*c^3 + 448*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^3*b*c^4 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*
c^4 - 736*a^3*b^2*c^4 - 176*a^2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^3*c^5 + 896*a^4*c^5 + 352*a^3*b*c^5 + sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 - 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^6*c + 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^2*b^3*c^2 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^4*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^5*c^2 - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b*c^3 - 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^2*c^3 - 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^3*c^3 + 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^6*c + 26*(b^2 - 4*a*c)*a*b^4*c^2 + 2*(b^2
- 4*a*c)*b^5*c^2 - 128*(b^2 - 4*a*c)*a^2*b^2*c^3 - 22*(b^2 - 4*a*c)*a*b^3*c
^3 + 224*(b^2 - 4*a*c)*a^3*c^4 + 88*(b^2 - 4*a*c)*a^2*b*c^4)*arctan(2*sqrt(
1/2)*x/sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 - sqrt((a^2*b^5 - 8*a^3*b
^3*c + 16*a^4*b*c^2)^2 - 4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*c
- 8*a^3*b^2*c^2 + 16*a^4*c^3)))/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/
((a^3*b^8 - 16*a^4*b^6*c - 2*a^3*b^7*c + 96*a^5*b^4*c^2 + 24*a^4*b^5*c^2 +
a^3*b^6*c^2 - 256*a^6*b^2*c^3 - 96*a^5*b^3*c^3 - 12*a^4*b^4*c^3 + 256*a^7*c
^4 + 128*a^6*b*c^4 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*abs(c)) + 1/8*(3*b^3*c^2*
x^7 - 24*a*b*c^3*x^7 + 6*b^4*c*x^5 - 49*a*b^2*c^2*x^5 + 28*a^2*c^3*x^5 + 3*
b^5*x^3 - 20*a*b^3*c*x^3 - 4*a^2*b*c^2*x^3 + 5*a*b^4*x - 37*a^2*b^2*c*x + 4
4*a^3*c^2*x)/((a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*(c*x^4 + b*x^2 + a)^2)
```

maple [B] time = 0.13, size = 3360, normalized size = 9.46

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] -24*c^4/(-4*a*c+b^2)^2/(4*a*c-b^2)^2*a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b-3*c^2/(-4*a*c+b
```

```
^2)^2/(4*a*c-b^2)^2/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^5-168*c^5/(-4*a*c+b^2)^(5/2)/(4*
a*c-b^2)^2*a^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+27/8*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(
x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2/a*x^3*b^6-3*c/(-4*a*c+b^2)^2/(4*a*c
-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2/a*x^3*b^5+27*c^2/(-4*a*c+b
^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*a*x*b^2+27*c^2
/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*a*x*
b^2+15*c^3/(-4*a*c+b^2)^2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3-24*c^3/(-4*a
*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*a*x^3*b-57
/2*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4-15*c^3/(-4*a
*c+b^2)^2/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3-24*c^3/(-4*a*c+b^2)^2/(4*
a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*a*x^3*b+20*c^3/(-4*a*c+
b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*a^2*b*x+6
6*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/
c)^2*a*b^2*x^3-66*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4
*a*c+b^2)^(1/2)/c)^2*a*b^2*x^3-20*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2
+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*a^2*b*x-57/2*c^3/(-4*a*c+b^2)^(5/2)/(4
*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4-15*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)
^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*a*x*b^3-3*c/(-4*a*c+b^2)^2/(4*a
*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2/a*x^3*b^5-168*c^5/(-4*a*
c+b^2)^(5/2)/(4*a*c-b^2)^2*a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-27/8*c/(-4*a*c+b^2)^(5
/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2/a*x^3*b^6+15*c^2
/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*
a*x*b^3+27/8*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/a*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^6-
3/16*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/a^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^8+114*c^4/
(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2+27/8*c^2/(-4*a
*c+b^2)^(5/2)/(4*a*c-b^2)^2/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^6-3/16*c/(-4*a*c+b^2)
^(5/2)/(4*a*c-b^2)^2/a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^8+114*c^4/(-4*a*c+b^2)^(5/
2)/(4*a*c-b^2)^2*a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2+24*c^4/(-4*a*c+b^2)^2/(4*a*c-b^2)
^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)*b+3*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/a*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)*b^5-3/16*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/a^2*2^(1/2)/((-b+(-4*a
```

```
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*b^7+3/16*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/a^2*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^7-72*c
^4/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^
2*x^3*a^2+5/16/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^
2)^(1/2)/c)^2/a*x*b^7-3/16/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/
2*(-4*a*c+b^2)^(1/2)/c)^2/a^2*x^3*b^8+3/16/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^
2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2/a^2*x^3*b^7+3/16/(-4*a*c+b^2)^2/(4*a*
c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2/a^2*x^3*b^7+5/16/(-4*a*c+
b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2/a*x*b^6+5/16/
(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2/a*x*b
^6-5/16/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2
)/c)^2/a*x*b^7+3/16/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a
*c+b^2)^(1/2)/c)^2/a^2*x^3*b^8+72*c^4/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2
+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x^3*a^2+45/2*c^2/(-4*a*c+b^2)^(5/2)/(4
*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x^3*b^4-15/4*c/(-4*a*c
+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*b^5-44
*c^3/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*
a^2*x-44*c^3/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/
2)/c)^2*a^2*x+15*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+
b^2)^(1/2)/c)^2*x^3*b^3-21/4*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/
2*(-4*a*c+b^2)^(1/2)/c)^2*x*b^4-45/2*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(
x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*x^3*b^4+15*c^2/(-4*a*c+b^2)^2/(4*a*
c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x^3*b^3-21/4*c/(-4*a*c+b^
2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*b^4+15/4*c/(-
4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)^2*x*b
^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\left(b^{3}c^{2}-8\,abc^{3}\right)x^{7}+\left(6\,b^{4}c-49\,ab^{2}c^{2}+28\,a^{2}c^{3}\right)x^{5}+\left(3\,b^{5}-20\,ab^{3}c-4\,a^{2}bc^{2}\right)x^{3}+\left(5\,a\right.}{8\left(\left(a^{2}b^{4}c^{2}-8\,a^{3}b^{2}c^{3}+16\,a^{4}c^{4}\right)x^{8}+a^{4}b^{4}-8\,a^{5}b^{2}c+16\,a^{6}c^{2}+2\left(a^{2}b^{5}c-8\,a^{3}b^{3}c^{2}+16\,a^{4}bc^{3}\right)x^{6}+\left(a^{2}b^{6}-6\,a^{3}b^{4}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(b^3*c^2 - 8*a*b*c^3)*x^7 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*x^
5 + (3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*x^3 + (5*a*b^4 - 37*a^2*b^2*c + 44*a
^3*c^2)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^
5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (
a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5
*b*c^2)*x^2) - 3/8*integrate(-(b^4 - 9*a*b^2*c + 28*a^2*c^2 + (b^3*c - 8*a*
b*c^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)
```

mupad [B] time = 9.00, size = 10979, normalized size = 30.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x*(5*b^4 + 44*a^2*c^2 - 37*a*b^2*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
 + (x^5*(6*b^4*c + 28*a^2*c^3 - 49*a*b^2*c^2))/(8*a^2*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)) - (x^3*(4*a^2*b*c^2 - 3*b^5 + 20*a*b^3*c))/(8*a^2*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c)) + (3*c*x^7*(b^3*c - 8*a*b*c^2))/(8*a^2*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
- atan(((((3*(7340032*a^9*c^9 - 256*a^2*b^14*c^2 + 7424*a^3*b^12*c^3 - 9420
8*a^4*b^10*c^4 + 675840*a^5*b^8*c^5 - 2949120*a^6*b^6*c^6 + 7798784*a^7*b^4
*c^7 - 11534336*a^8*b^2*c^8))/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*
c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^
5)) - (x*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 7
69*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c
^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a
^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)
^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b
^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 8
60160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*
a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c
^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4
*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(
9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*
c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824
*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*
a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))
/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 76
80*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b
^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9
)))^(1/2) + (x*(14112*a^4*c^7 + 9*b^8*c^3 - 180*a*b^6*c^4 + 1530*a^2*b^4*c^
5 - 6192*a^3*b^2*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b
^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 17
20320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4
 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560
*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^
2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6
*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 25804
8*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13
*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*1i - (((3*(7340032*a^9*c^9 - 256*a
^2*b^14*c^2 + 7424*a^3*b^12*c^3 - 94208*a^4*b^10*c^4 + 675840*a^5*b^8*c^5 -
```

2949120*a^6*b^6*c^6 + 7798784*a^7*b^4*c^7 - 11534336*a^8*b^2*c^8))/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) - (x*(14112*a^4*c^7 + 9*b^8*c^3 - 180*a*b^6*c^4 + 1530*a^2*b^4*c^5 - 6192*a^3*b^2*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*1i)/((((3*(7340032*a^9*c^9 - 256*a^2*b^14*c^2 + 7424*a^3*b^12*c^3 - 94208*a^4*b^10*c^4 + 675840*a^5*b^8*c^5 - 2949120*a^6*b^6*c^6 + 7798784*a^7*b^4*c^7 - 11534336*a^8*b^2*c^8))/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (x*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 106982

4*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4
*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))
)/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7
680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*
b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^
9)))^(1/2) + (x*(14112*a^4*c^7 + 9*b^8*c^3 - 180*a*b^6*c^4 + 1530*a^2*b^4*c
^5 - 6192*a^3*b^2*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*
b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1
720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^
4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 301056
0*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b
^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^
6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 2580
48*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^1
3*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) + (((3*(7340032*a^9*c^9 - 256*a^2
*b^14*c^2 + 7424*a^3*b^12*c^3 - 94208*a^4*b^10*c^4 + 675840*a^5*b^8*c^5 - 2
949120*a^6*b^6*c^6 + 7798784*a^7*b^4*c^7 - 11534336*a^8*b^2*c^8))/(512*(a^4
*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3
+ 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(-(9*(b^19 + b^4*(-(4*a*c - b^
2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 6
3440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*
b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a
*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a
^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9
*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c
^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7
 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5
*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*
a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2)
 - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^1
1*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 30
10560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11
*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 4
0*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 -
258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120
*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) - (x*(14112*a^4*c^7 + 9*b^8*c
^3 - 180*a*b^6*c^4 + 1530*a^2*b^4*c^5 - 6192*a^3*b^2*c^6))/(32*(a^4*b^8 + 2
56*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19
+ b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 86
20*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7
*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^
2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a
^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b
^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 -
 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2

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) + (3*(189*b^7*c^5 - 3456*a*b^5*c^6 - 56448*a^3*b*c^8 + 22608*a^2*b^3*c^7)
)/(256*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a
^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5))))*(-(9*(b^19 + b^4*(-(4*
a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13
*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343
936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2
) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1
048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 5
3760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^
12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*2i - atan
(((((3*(7340032*a^9*c^9 - 256*a^2*b^14*c^2 + 7424*a^3*b^12*c^3 - 94208*a^4*
b^10*c^4 + 675840*a^5*b^8*c^5 - 2949120*a^6*b^6*c^6 + 7798784*a^7*b^4*c^7 -
 11534336*a^8*b^2*c^8))/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 24
0*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) -
(x*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*
b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 10
69824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*
(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1
/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2
 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a
^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^
2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40
960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 +
256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((9*(b^4*(
-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 86
20*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7
*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^
2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a
^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b
^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 -
 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2
) + (x*(14112*a^4*c^7 + 9*b^8*c^3 - 180*a*b^6*c^4 + 1530*a^2*b^4*c^5 - 6192
*a^3*b^2*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 -
 256*a^7*b^2*c^3)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9
*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864
*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*
c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*
a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c +
 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^
10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8
- 2621440*a^14*b^2*c^9)))^(1/2)*1i - (((3*(7340032*a^9*c^9 - 256*a^2*b^14*c
^2 + 7424*a^3*b^12*c^3 - 94208*a^4*b^10*c^4 + 675840*a^5*b^8*c^5 - 2949120*
a^6*b^6*c^6 + 7798784*a^7*b^4*c^7 - 11534336*a^8*b^2*c^8))/(512*(a^4*b^12 +
 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*
a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) -
```

b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4
*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7
- 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c
- 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10
 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^
4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 294
9120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a
^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 3
27680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*
c^2 - 256*a^7*b^2*c^3)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 172032
0*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 3
16864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8
*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*
(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^1
8*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^
10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4
*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) - (x*(14112*a^4*c^7 + 9*b^8*c^3 - 180*
a*b^6*c^4 + 1530*a^2*b^4*c^5 - 6192*a^3*b^2*c^6))/(32*(a^4*b^8 + 256*a^8*c^
4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((9*(b^4*(-(4*a*c -
b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^1
3*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 234
3936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/
2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 +
1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 +
53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a
^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*1i)/((3*
(189*b^7*c^5 - 3456*a*b^5*c^6 - 56448*a^3*b*c^8 + 22608*a^2*b^3*c^7))/(256*
(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*
c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (((3*(7340032*a^9*c^9 - 256*a
^2*b^14*c^2 + 7424*a^3*b^12*c^3 - 94208*a^4*b^10*c^4 + 675840*a^5*b^8*c^5 -
 2949120*a^6*b^6*c^6 + 7798784*a^7*b^4*c^7 - 11534336*a^8*b^2*c^8))/(512*(a
^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^
3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (x*((9*(b^4*(-(4*a*c - b^2)^15)
^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 -
63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7
*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*
a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*
a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^
9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*
c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^
7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^
5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96
*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19
 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^1
1*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 30

10560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) + (x*(14112*a^4*c^7 + 9*b^8*c^3 - 180*a*b^6*c^4 + 1530*a^2*b^4*c^5 - 6192*a^3*b^2*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) + (((3*(7340032*a^9*c^9 - 256*a^2*b^14*c^2 + 7424*a^3*b^12*c^3 - 94208*a^4*b^10*c^4 + 675840*a^5*b^8*c^5 - 2949120*a^6*b^6*c^6 + 7798784*a^7*b^4*c^7 - 11534336*a^8*b^2*c^8))/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) - (x*(14112*a^4*c^7 + 9*b^8*c^3 - 180*a*b^6*c^4 + 1530*a^2*b^4*c^5 - 6192*a^3*b^2*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^1

```
9 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^
11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3
010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 1
1*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 -
40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 -
 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 294912
0*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.887 $\int \frac{1}{x^2(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=425

$$-\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3x\left(b^2-4ac\right)^2}+\frac{36a^2c^2+bcx^2\left(5b^2-32ac\right)-35ab^2c+5b^4}{8a^2x\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right.\right.}{8\sqrt{2}\,a^3\left(b^2-4ac\right)^2\sqrt{}}$$

```
[Out] -3/8*(-12*a*c+5*b^2)*(-5*a*c+b^2)/a^3/(-4*a*c+b^2)^2/x+1/4*(b*c*x^2-2*a*c+b
^2)/a/(-4*a*c+b^2)/x/(c*x^4+b*x^2+a)^2+1/8*(5*b^4-35*a*b^2*c+36*a^2*c^2+b*c
*(-32*a*c+5*b^2)*x^2)/a^2/(-4*a*c+b^2)^2/x/(c*x^4+b*x^2+a)-3/16*arctan(x*2^
(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*((-12*a*c+5*b^2)*(-5*a*
c+b^2)+b*(124*a^2*c^2-47*a*b^2*c+5*b^4)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2
)^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/16*arctan(x*2^(1/2)*c^(1/2)/(b+(
-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*((-12*a*c+5*b^2)*(-5*a*c+b^2)+(-124*a^2*b
*c^2+47*a*b^3*c-5*b^5)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^2*2^(1/2)/(b+(-
4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.96, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1121, 1277, 1281, 1166, 205}

$$\frac{36a^2c^2+bcx^2\left(5b^2-32ac\right)-35ab^2c+5b^4}{8a^2x\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,a^3\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*x) + (b^2 - 2*a*
c + b*c*x^2)/(4*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)^2) + (5*b^4 - 35*a*b^
2*c + 36*a^2*c^2 + b*c*(5*b^2 - 32*a*c)*x^2)/(8*a^2*(b^2 - 4*a*c)^2*x*(a +
b*x^2 + c*x^4)) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4 -
47*a*b^2*c + 124*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) - (5*b^5 - 47*a*b
^3*c + 124*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4
*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^3}\,dx = \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{-5b^2+18ac-7bcx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx}{4a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)^2} + \frac{5b^4-35ab^2c+36a^2c^2+bc\left(5b^2-32ac\right)x^2}{8a^2\left(b^2-4ac\right)^2x\left(a+bx^2+cx^4\right)} + \frac{\int \frac{3}{}}{}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)^2} + \frac{5b^4-35ab^2c+36a}{8a^2\left(b^2-4ac\right)}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)^2} + \frac{5b^4-35ab^2c+36a}{8a^2\left(b^2-4ac\right)}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)^2} + \frac{5b^4-35ab^2c+36a}{8a^2\left(b^2-4ac\right)}$$

Mathematica [A] time = 1.76, size = 454, normalized size = 1.07

$$-\frac{\frac{3\sqrt{2}\sqrt{c}\left(60a^2c^2\sqrt{b^2-4ac}+124a^2bc^2-47ab^3c-37ab^2c\sqrt{b^2-4ac}+5b^4\sqrt{b^2-4ac}+5b^5\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{3\sqrt{2}\sqrt{c}\left(60a^2c^2\sqrt{b^2-4ac}-124a^2bc^2+4\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] -1/16*(16/x + (4*a*x*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a
*c)*(a + b*x^2 + c*x^4)^2) + (2*x*(7*b^5 - 52*a*b^3*c + 84*a^2*b*c^2 + 7*b^
4*c*x^2 - 47*a*b^2*c^2*x^2 + 52*a^2*c^3*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 +
 c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2 + 5*b^4*S
qrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c
)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^5 + 47*a*b^
3*c - 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c
] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a^3
```

fricas [B] time = 2.20, size = 4924, normalized size = 11.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*x^8 + 2*(30*b^5*c - 227*a*
b^3*c^2 + 392*a^2*b*c^3)*x^6 + 16*a^2*b^4 - 128*a^3*b^2*c + 256*a^4*c^2 + 2
*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*x^4 + 2*(25*a*b^5 - 1
94*a^2*b^3*c + 364*a^3*b*c^2)*x^2 + 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c
^3 + 16*a^5*c^4)*x^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^7 + (
a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6
*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x)*sqrt(-(25*b^11 - 495*
a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*
a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 +
 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725
*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5
 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*
b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 1
60*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))*log
(-27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^
4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*x + 27/2*sqrt(1/2)*(125*b^17
- 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^
9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1
324800*a^8*b*c^8 - (5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960
*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4
*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*sqrt((625*b^12 - 12250*a*b^10
*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a
^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 -
 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))*sqrt(-(25*b^11 - 4
95*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 184
80*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^
3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94
725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*
c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^
17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c
+ 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)))
 - 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^9 + 2*(a^3*b^5
*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^
3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^
2*c + 16*a^7*c^2)*x)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 1501
5*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^
8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^
```

5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))*log(-27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*x - 27/2*sqrt(1/2)*(125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8 - (5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))) + 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))*log(-27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*x + 27/2*sqrt(1/2)*(125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8 + (5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^

```
10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300
*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2
 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a
^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^
12*c^5))) - 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^9 + 2
*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^7 + (a^3*b^6 - 6*a^4*b^4*c +
32*a^6*c^3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 -
 8*a^6*b^2*c + 16*a^7*c^2)*x)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c
^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 -
20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 102
4*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^
3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*
b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*
c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^1
0*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))*log(-27*(4125*b^10*c^4 - 77
825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*
c^8 - 810000*a^5*c^9)*x - 27/2*sqrt(1/2)*(125*b^17 - 3775*a*b^15*c + 49360*
a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*
c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8 + (5*a^
7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a
^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c
^7 + 122880*a^15*c^8)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 -
 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c
^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280
*a^18*b^2*c^4 - 1024*a^19*c^5)))*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^
7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10
 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 -
1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310
*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^
14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b
^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*
a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))))/((a^3*b^4*c^2 - 8*a^4*
b^2*c^3 + 16*a^5*c^4)*x^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^
7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 1
6*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x)
```

giac [B] time = 2.62, size = 5273, normalized size = 12.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3/64*(10*a^6*b^14*c^2 - 254*a^7*b^12*c^3 + 2712*a^8*b^10*c^4 - 15552*a^9*b
^8*c^5 + 50432*a^10*b^6*c^6 - 87552*a^11*b^4*c^7 + 63488*a^12*b^2*c^8 - 5*s
```

qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^14 + 127*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^12*c + 10*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^13*c - 1356*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^10*c^2 - 214*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^11*c^2 - 5*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^12*c^2 + 777
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^8*c^3 + 1
856*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^9*c^3 +
107*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^10*c^3
- 25216*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b^6
*c^4 - 8128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b
^7*c^4 - 928*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*
b^8*c^4 + 43776*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^11*b^4*c^5 + 17920*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^10*b^5*c^5 + 4064*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^9*b^6*c^5 - 31744*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^12*b^2*c^6 - 15872*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^11*b^3*c^6 - 8960*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^10*b^4*c^6 + 7936*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^11*b^2*c^7 - 10*(b^2 - 4*a*c)*a^6*b^12*c^2 + 214*(b^2
- 4*a*c)*a^7*b^10*c^3 - 1856*(b^2 - 4*a*c)*a^8*b^8*c^4 + 8128*(b^2 - 4*a*c)
*a^9*b^6*c^5 - 17920*(b^2 - 4*a*c)*a^10*b^4*c^6 + 15872*(b^2 - 4*a*c)*a^11*
b^2*c^7 + (10*b^6*c^2 - 114*a*b^4*c^3 + 416*a^2*b^2*c^4 - 480*a^3*c^5 - 5*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 57*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 10*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 208*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 74*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 240*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 120*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 37*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 60*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 10*(b^2 - 4*a*c)*b^4*c^2 + 74*(b^2 - 4
*a*c)*a*b^2*c^3 - 120*(b^2 - 4*a*c)*a^2*c^4)*(a^3*b^4 - 8*a^4*b^2*c + 16*a^
5*c^2)^2 + 2*(5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^11 - 102*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^9*c - 10*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^10*c - 10*a^3*b^11*c + 836*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^5*b^7*c^2 + 164*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4
*b^8*c^2 + 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^9*c^2 + 204*a^4*
b^9*c^2 - 3440*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^3 - 1016*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^3 - 82*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^3 - 1672*a^5*b^7*c^3 + 7104*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^4 + 2816*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^6*b^4*c^4 + 508*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c
^4 + 6880*a^6*b^5*c^4 - 5888*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b*

c^5 - 2944*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^5 - 1408*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 - 14208*a^7*b^3*c^5 + 1472*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^6 + 11776*a^8*b*c^6 + 10*(b^
2 - 4*a*c)*a^3*b^9*c - 164*(b^2 - 4*a*c)*a^4*b^7*c^2 + 1016*(b^2 - 4*a*c)*a
^5*b^5*c^3 - 2816*(b^2 - 4*a*c)*a^6*b^3*c^4 + 2944*(b^2 - 4*a*c)*a^7*b*c^5)
*abs(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))*arctan(2*sqrt(1/2)*x/sqrt((a^3*b^
5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + sqrt((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2
)^2 - 4*(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*(a^3*b^4*c - 8*a^4*b^2*c^2 + 1
6*a^5*c^3)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/((a^7*b^10 - 20*a^8
*b^8*c - 2*a^7*b^9*c + 160*a^9*b^6*c^2 + 32*a^8*b^7*c^2 + a^7*b^8*c^2 - 640
*a^10*b^4*c^3 - 192*a^9*b^5*c^3 - 16*a^8*b^6*c^3 + 1280*a^11*b^2*c^4 + 512*
a^10*b^3*c^4 + 96*a^9*b^4*c^4 - 1024*a^12*c^5 - 512*a^11*b*c^5 - 256*a^10*b
^2*c^5 + 256*a^11*c^6)*abs(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*abs(c)) + 3/
64*(10*a^6*b^14*c^2 - 254*a^7*b^12*c^3 + 2712*a^8*b^10*c^4 - 15552*a^9*b^8*
c^5 + 50432*a^10*b^6*c^6 - 87552*a^11*b^4*c^7 + 63488*a^12*b^2*c^8 - 5*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^14 + 127*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^12*c + 10*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^13*c - 1356*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^10*c^2 - 214*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^11*c^2 - 5*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^12*c^2 + 7776*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^8*c^3 + 1856
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^9*c^3 + 10
7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^10*c^3 -
25216*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^6*c^
4 - 8128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^7*
c^4 - 928*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^8
*c^4 + 43776*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11
*b^4*c^5 + 17920*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^10*b^5*c^5 + 4064*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^9*b^6*c^5 - 31744*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^12*b^2*c^6 - 15872*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^11*b^3*c^6 - 8960*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^10*b^4*c^6 + 7936*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^11*b^2*c^7 - 10*(b^2 - 4*a*c)*a^6*b^12*c^2 + 214*(b^2 - 4
*a*c)*a^7*b^10*c^3 - 1856*(b^2 - 4*a*c)*a^8*b^8*c^4 + 8128*(b^2 - 4*a*c)*a^
9*b^6*c^5 - 17920*(b^2 - 4*a*c)*a^10*b^4*c^6 + 15872*(b^2 - 4*a*c)*a^11*b^2
*c^7 + (10*b^6*c^2 - 114*a*b^4*c^3 + 416*a^2*b^2*c^4 - 480*a^3*c^5 - 5*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 57*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 10*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 208*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 74*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 240*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 120*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt

```
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 37*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 60*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 10*(b^2 - 4*a*c)*b^4*c^2 + 74*(b^2 - 4*a*
c)*a*b^2*c^3 - 120*(b^2 - 4*a*c)*a^2*c^4)*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c
^2)^2 - 2*(5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^11 - 102*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^9*c - 10*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^3*b^10*c + 10*a^3*b^11*c + 836*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^5*b^7*c^2 + 164*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^
8*c^2 + 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^9*c^2 - 204*a^4*b^9
*c^2 - 3440*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^3 - 1016*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^3 - 82*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^4*b^7*c^3 + 1672*a^5*b^7*c^3 + 7104*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^4 + 2816*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^6*b^4*c^4 + 508*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^4
- 6880*a^6*b^5*c^4 - 5888*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b*c^5
 - 2944*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^5 - 1408*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 + 14208*a^7*b^3*c^5 + 1472*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b*c^6 - 11776*a^8*b*c^6 - 10*(b^2 -
 4*a*c)*a^3*b^9*c + 164*(b^2 - 4*a*c)*a^4*b^7*c^2 - 1016*(b^2 - 4*a*c)*a^5*
b^5*c^3 + 2816*(b^2 - 4*a*c)*a^6*b^3*c^4 - 2944*(b^2 - 4*a*c)*a^7*b*c^5)*ab
s(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))*arctan(2*sqrt(1/2)*x/sqrt((a^3*b^5 -
 8*a^4*b^3*c + 16*a^5*b*c^2 - sqrt((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)^2
 - 4*(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a
^5*c^3)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/((a^7*b^10 - 20*a^8*b^
8*c - 2*a^7*b^9*c + 160*a^9*b^6*c^2 + 32*a^8*b^7*c^2 + a^7*b^8*c^2 - 640*a^
10*b^4*c^3 - 192*a^9*b^5*c^3 - 16*a^8*b^6*c^3 + 1280*a^11*b^2*c^4 + 512*a^1
0*b^3*c^4 + 96*a^9*b^4*c^4 - 1024*a^12*c^5 - 512*a^11*b*c^5 - 256*a^10*b^2*
c^5 + 256*a^11*c^6)*abs(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*abs(c)) - 1/8*(
7*b^4*c^2*x^7 - 47*a*b^2*c^3*x^7 + 52*a^2*c^4*x^7 + 14*b^5*c*x^5 - 99*a*b^3
*c^2*x^5 + 136*a^2*b*c^3*x^5 + 7*b^6*x^3 - 43*a*b^4*c*x^3 + 25*a^2*b^2*c^2*
x^3 + 68*a^3*c^3*x^3 + 9*a*b^5*x - 66*a^2*b^3*c*x + 108*a^3*b*c^2*x)/((a^3*
b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c*x^4 + b*x^2 + a)^2) - 1/(a^3*x)
```

maple [B] time = 0.06, size = 1567, normalized size = 3.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] -17/2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*c^3+45/4/a/(16*a^2*c
^2-8*a*b^2*c+b^4)*c^3*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-45/4/a/(16*a^2*c^2-8*a*b^2*c+b^
4)*c^3*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*c*x)+43/8/a^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+
```

```
b^4)*x^3*b^4*c+33/4/a/(c*x^4+b*x^2+a)^2*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c-
25/8/a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^2*c^2+47/8/a^2/(c
*x^4+b*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7*b^2-17/a/(c*x^4+b*x^2+a)
^2*c^3*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-13/2/a/(c*x^4+b*x^2+a)^2*c^4/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^7-27/2/(c*x^4+b*x^2+a)^2*b/(16*a^2*c^2-8*a*b^2*c+b^4
)*x*c^2-141/16/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)*b^3+15/16/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*b^5+15/16/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*b^5-141/16/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(
-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3+93/4/a/(16*a^2*c^2-8*a*b^2*c+b^
4)*c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b+93/4/a/(16*a^2*c^2-8*a*b^
2*c+b^4)*c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b-111/16/a^2/(16*a^2*c^2
-8*a*b^2*c+b^4)*c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2+111/16/a^2/(16*a^2*c^2-8*a*b^
2*c+b^4)*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2+15/16/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*b^4-15/16/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*b^4-9/8/a^2/(c*x^4+b*x^2+a)^2*b^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x-7/8
/a^3/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^6-1/a^3/x+99/8/a^2/
(c*x^4+b*x^2+a)^2*c^2*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-7/8/a^3/(c*x^4+b*x
^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7*b^4-7/4/a^3/(c*x^4+b*x^2+a)^2*c*
b^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*x^8 + (30*b^5*c - 227*a*b^3
*c^2 + 392*a^2*b*c^3)*x^6 + 8*a^2*b^4 - 64*a^3*b^2*c + 128*a^4*c^2 + (15*b^
6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*x^4 + (25*a*b^5 - 194*a^2*b^
3*c + 364*a^3*b*c^2)*x^2)/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^9 +
 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^7 + (a^3*b^6 - 6*a^4*b^4*c
+ 32*a^6*c^3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4
 - 8*a^6*b^2*c + 16*a^7*c^2)*x) - 3/8*integrate((5*b^5 - 42*a*b^3*c + 92*a^
```

```
2*b*c^2 + (5*b^4*c - 37*a*b^2*c^2 + 60*a^2*c^3)*x^2)/(c*x^4 + b*x^2 + a), x
)/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)
```

mupad [B] time = 9.37, size = 12130, normalized size = 28.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2 + c*x^4)^3),x)
```

```
[Out] - atan(((x*(271790899200*a^20*c^14 - 230400*a^9*b^22*c^3 + 9861120*a^10*b^2
0*c^4 - 191038464*a^11*b^18*c^5 + 2207803392*a^12*b^16*c^6 - 16878108672*a^
13*b^14*c^7 + 89374851072*a^14*b^12*c^8 - 333226967040*a^15*b^10*c^9 + 8698
15812096*a^16*b^8*c^10 - 1543847804928*a^17*b^6*c^11 + 1747313491968*a^18*b
^4*c^12 - 1101055131648*a^19*b^2*c^13) + (-(9*(25*b^21 - 25*b^6*(-(4*a*c -
b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^1
5*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6
- 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*
a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c
 - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20
+ 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3
 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 19660
80*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(245
760*a^12*b^23*c^2 - 1185410973696*a^23*b*c^13 - 10911744*a^13*b^21*c^3 + 22
0397568*a^14*b^19*c^4 - 2673082368*a^15*b^17*c^5 + 21630025728*a^16*b^15*c^
6 - 122607894528*a^17*b^13*c^7 + 496773365760*a^18*b^11*c^8 - 1438679826432
*a^19*b^9*c^9 + 2918430277632*a^20*b^7*c^10 - 3949222428672*a^21*b^5*c^11 +
 3208340570112*a^22*b^3*c^12 + x*(-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)
^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 +
1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 439042
56*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*
(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^
15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 104857
6*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760
*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*
b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(10995116277
76*a^26*b*c^13 - 262144*a^15*b^23*c^2 + 11534336*a^16*b^21*c^3 - 230686720*
a^17*b^19*c^4 + 2768240640*a^18*b^17*c^5 - 22145925120*a^19*b^15*c^6 + 1240
17180672*a^20*b^13*c^7 - 496068722688*a^21*b^11*c^8 + 1417339207680*a^22*b^
9*c^9 - 2834678415360*a^23*b^7*c^10 + 3779571220480*a^24*b^5*c^11 - 3023656
976384*a^25*b^3*c^12)))*(-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) +
18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a
^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^
7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c
- b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
```

+ 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*1i + (x*(271790899200*a^20*c^14 - 230400*a^9*b^22*c^3 + 9861120*a^10*b^20*c^4 - 191038464*a^11*b^18*c^5 + 2207803392*a^12*b^16*c^6 - 16878108672*a^13*b^14*c^7 + 89374851072*a^14*b^12*c^8 - 333226967040*a^15*b^10*c^9 + 869815812096*a^16*b^8*c^10 - 1543847804928*a^17*b^6*c^11 + 1747313491968*a^18*b^4*c^12 - 1101055131648*a^19*b^2*c^13) + (-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(1185410973696*a^23*b*c^13 - 245760*a^12*b^23*c^2 + 10911744*a^13*b^21*c^3 - 220397568*a^14*b^19*c^4 + 2673082368*a^15*b^17*c^5 - 21630025728*a^16*b^15*c^6 + 122607894528*a^17*b^13*c^7 - 496773365760*a^18*b^11*c^8 + 1438679826432*a^19*b^9*c^9 - 2918430277632*a^20*b^7*c^10 + 3949222428672*a^21*b^5*c^11 - 3208340570112*a^22*b^3*c^12 + x*(-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(1099511627776*a^26*b*c^13 - 262144*a^15*b^23*c^2 + 11534336*a^16*b^21*c^3 - 230686720*a^17*b^19*c^4 + 2768240640*a^18*b^17*c^5 - 22145925120*a^19*b^15*c^6 + 124017180672*a^20*b^13*c^7 - 496068722688*a^21*b^11*c^8 + 1417339207680*a^22*b^9*c^9 - 2834678415360*a^23*b^7*c^10 + 3779571220480*a^24*b^5*c^11 - 3023656976384*a^25*b^3*c^12)))*(-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*1i)/((x*(271790899200*a^20*c^14 - 230400*a^9*b^22*c^3 + 9861120*a^10*b^20*c^4 - 191038464*a^11*b^18*c^5 + 2207803392*a^12*b^16*c^6 - 16878108672*a^13*b^14*c^7 + 89374851072*a^14*b^12*c^8 - 333226

967040*a^15*b^10*c^9 + 869815812096*a^16*b^8*c^10 - 1543847804928*a^17*b^6*
c^11 + 1747313491968*a^18*b^4*c^12 - 1101055131648*a^19*b^2*c^13) + (-(9*(2
5*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^
2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*
c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 -
52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c
 - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^
15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^
16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 +
860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440
*a^16*b^2*c^9)))^(1/2)*(1185410973696*a^23*b*c^13 - 245760*a^12*b^23*c^2 +
10911744*a^13*b^21*c^3 - 220397568*a^14*b^19*c^4 + 2673082368*a^15*b^17*c^5
 - 21630025728*a^16*b^15*c^6 + 122607894528*a^17*b^13*c^7 - 496773365760*a^
18*b^11*c^8 + 1438679826432*a^19*b^9*c^9 - 2918430277632*a^20*b^7*c^10 + 39
49222428672*a^21*b^5*c^11 - 3208340570112*a^22*b^3*c^12 + x*(-(9*(25*b^21 -
 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c
^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19
905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680
*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a
^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2
)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 -
 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a
^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^
2*c^9)))^(1/2)*(1099511627776*a^26*b*c^13 - 262144*a^15*b^23*c^2 + 11534336
*a^16*b^21*c^3 - 230686720*a^17*b^19*c^4 + 2768240640*a^18*b^17*c^5 - 22145
925120*a^19*b^15*c^6 + 124017180672*a^20*b^13*c^7 - 496068722688*a^21*b^11*
c^8 + 1417339207680*a^22*b^9*c^9 - 2834678415360*a^23*b^7*c^10 + 3779571220
480*a^24*b^5*c^11 - 3023656976384*a^25*b^3*c^12)))*(-(9*(25*b^21 - 25*b^6*(
-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 1880
95*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^
6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*
c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^
2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*
(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^1
0*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c
^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^
(1/2) - (x*(271790899200*a^20*c^14 - 230400*a^9*b^22*c^3 + 9861120*a^10*b^2
0*c^4 - 191038464*a^11*b^18*c^5 + 2207803392*a^12*b^16*c^6 - 16878108672*a^
13*b^14*c^7 + 89374851072*a^14*b^12*c^8 - 333226967040*a^15*b^10*c^9 + 8698
15812096*a^16*b^8*c^10 - 1543847804928*a^17*b^6*c^11 + 1747313491968*a^18*b
^4*c^12 - 1101055131648*a^19*b^2*c^13) + (-(9*(25*b^21 - 25*b^6*(-(4*a*c -
b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^1
5*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6
- 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*
a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c

- b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(245760*a^12*b^23*c^2 - 1185410973696*a^23*b*c^13 - 10911744*a^13*b^21*c^3 + 220397568*a^14*b^19*c^4 - 2673082368*a^15*b^17*c^5 + 21630025728*a^16*b^15*c^6 - 122607894528*a^17*b^13*c^7 + 496773365760*a^18*b^11*c^8 - 1438679826432*a^19*b^9*c^9 + 2918430277632*a^20*b^7*c^10 - 3949222428672*a^21*b^5*c^11 + 3208340570112*a^22*b^3*c^12 + x*(-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(1099511627776*a^26*b*c^13 - 262144*a^15*b^23*c^2 + 11534336*a^16*b^21*c^3 - 230686720*a^17*b^19*c^4 + 2768240640*a^18*b^17*c^5 - 22145925120*a^19*b^15*c^6 + 124017180672*a^20*b^13*c^7 - 496068722688*a^21*b^11*c^8 + 1417339207680*a^22*b^9*c^9 - 2834678415360*a^23*b^7*c^10 + 3779571220480*a^24*b^5*c^11 - 3023656976384*a^25*b^3*c^12)))*(-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2) + 191102976000*a^17*c^14 + 2851200*a^9*b^16*c^6 - 92568960*a^10*b^14*c^7 + 1312630272*a^11*b^12*c^8 - 10611136512*a^12*b^10*c^9 + 53445353472*a^13*b^8*c^10 - 171591892992*a^14*b^6*c^11 + 342580396032*a^15*b^4*c^12 - 388363714560*a^16*b^2*c^13))*(-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*2i - atan(((x*(271790899200*a^20*c^14 - 230400*a^9*b^22*c^3 + 9861120*a^10*b^20*c^4 - 191038464*a^11*b^18*c^5 + 2207803392*a^12*b^16*c^6 - 16878108672*a^13*b^14*c^7 + 89374851072*a^14*b^12*c^8 - 333226967040*a^15*b^10*c^9 + 869815812096*a^16*b^8*c^10 - 1543847804928*a^17 *b^6*c^11 + 1747313491968*a^18*b^4*c^12 - 1101055131648*a^19*b^2*c^13) + (-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(245760*a^12*b^23*c^2 - 1185410973696*a^23*b*c^13 - 10911744*a^13*b^21*c^3 + 220397568*a^14*b^19*c^4 - 2673082368*a^15*b^17*c^5 + 21630025728*a^16*b^15*c^6 - 122607894528*a^17*b^13*c^7 + 496773365760*a^18*b^11*c^8 - 1438679826432*a^19*b^9*c^9 + 2918430277632*a^20*b^7*c^10 - 3949222428672*a^21*b^5*c^11 + 3208340570112*a^22*b^3*c^12 + x*(-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(1099511627776*a^26*b*c^13 - 262144*a^15*b^23*c^2 + 11534336*a^16*b^21*c^3 - 230686720*a^17*b^19*c^4 + 2768240640*a^18*b^17*c^5 - 22145925120*a^19*b^15*c^6 + 124017180672*a^20*b^13*c^7 - 496068722688*a^21*b^11*c^8 + 1417339207680*a^22*b^9*c^9 - 2834678415360*a^23*b^7*c^10 + 3779571220480*a^24*b^5*c^11 - 3023656976384*a^25*b^3*c^12)))*(-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*1i + (x*(271790899200*a^20*c^14 - 230400*a^9*b^22*c^3 + 9861120*a^10*b^20*c^4 - 191038464*a^11*b^18*c^5 + 2207803392*a^12*b^16*c^6 - 16878108672*a^13*b^14*c^7 + 89374851072*a^14*b^12*c^8 - 333226967040*a^15*b^10*c^9 + 869815812096*a^16*b^8*c^10 - 1543847804928*a^17*b^6*c^11 + 1747313491968*a^18*b^4*c^12 - 1101055131648*a^19*b^2*c^13) + (-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a

^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(1185410973696*a^23*b*c^13 - 245760*a^12*b^23*c^2 + 10911744*a^13*b^21*c^3 - 220397568*a^14*b^19*c^4 + 2673082368*a^15*b^17*c^5 - 21630025728*a^16*b^15*c^6 + 122607894528*a^17*b^13*c^7 - 496773365760*a^18*b^11*c^8 + 1438679826432*a^19*b^9*c^9 - 2918430277632*a^20*b^7*c^10 + 3949222428672*a^21*b^5*c^11 - 3208340570112*a^22*b^3*c^12 + x*(-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(1099511627776*a^26*b*c^13 - 262144*a^15*b^23*c^2 + 11534336*a^16*b^21*c^3 - 230686720*a^17*b^19*c^4 + 2768240640*a^18*b^17*c^5 - 22145925120*a^19*b^15*c^6 + 124017180672*a^20*b^13*c^7 - 496068722688*a^21*b^11*c^8 + 1417339207680*a^22*b^9*c^9 - 2834678415360*a^23*b^7*c^10 + 3779571220480*a^24*b^5*c^11 - 3023656976384*a^25*b^3*c^12)))*(-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*1i)/((x*(271790899200*a^20*c^14 - 230400*a^9*b^22*c^3 + 9861120*a^10*b^20*c^4 - 191038464*a^11*b^18*c^5 + 2207803392*a^12*b^16*c^6 - 16878108672*a^13*b^14*c^7 + 89374851072*a^14*b^12*c^8 - 333226967040*a^15*b^10*c^9 + 869815812096*a^16*b^8*c^10 - 1543847804928*a^17*b^6*c^11 + 1747313491968*a^18*b^4*c^12 - 1101055131648*a^19*b^2*c^13) + (-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(1185410973696*a^23*b*c^13 - 245760*a^12*b^23*c^2 + 10911744*a^13*b^21*c^3 - 220397568*a^14*b^19*c^4 + 2673082368*a^15*b^17*c^5 - 21630025728*a^16*b^15*c^6 + 122607894528*a^17*b^13*c^7 - 496773365760*a^18*b^11*c^8 + 1438679826432*a^19*b^9*c^9 -

2918430277632*a^20*b^7*c^10 + 3949222428672*a^21*b^5*c^11 - 3208340570112*a^22*b^3*c^12 + x*(-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(1099511627776*a^26*b*c^13 - 262144*a^15*b^23*c^2 + 11534336*a^16*b^21*c^3 - 230686720*a^17*b^19*c^4 + 2768240640*a^18*b^17*c^5 - 22145925120*a^19*b^15*c^6 + 124017180672*a^20*b^13*c^7 - 496068722688*a^21*b^11*c^8 + 1417339207680*a^22*b^9*c^9 - 2834678415360*a^23*b^7*c^10 + 3779571220480*a^24*b^5*c^11 - 3023656976384*a^25*b^3*c^12)))*(-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2) - (x*(271790899200*a^20*c^14 - 230400*a^9*b^22*c^3 + 9861120*a^10*b^20*c^4 - 191038464*a^11*b^18*c^5 + 2207803392*a^12*b^16*c^6 - 16878108672*a^13*b^14*c^7 + 89374851072*a^14*b^12*c^8 - 333226967040*a^15*b^10*c^9 + 869815812096*a^16*b^8*c^10 - 1543847804928*a^17*b^6*c^11 + 1747313491968*a^18*b^4*c^12 - 1101055131648*a^19*b^2*c^13) + (-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(245760*a^12*b^23*c^2 - 1185410973696*a^23*b*c^13 - 10911744*a^13*b^21*c^3 + 220397568*a^14*b^19*c^4 - 2673082368*a^15*b^17*c^5 + 21630025728*a^16*b^15*c^6 - 122607894528*a^17*b^13*c^7 + 496773365760*a^18*b^11*c^8 - 1438679826432*a^19*b^9*c^9 + 2918430277632*a^20*b^7*c^10 - 3949222428672*a^21*b^5*c^11 + 3208340570112*a^22*b^3*c^12 + x*(-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)

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^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*
c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860
160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^
16*b^2*c^9)))^(1/2)*(1099511627776*a^26*b*c^13 - 262144*a^15*b^23*c^2 + 115
34336*a^16*b^21*c^3 - 230686720*a^17*b^19*c^4 + 2768240640*a^18*b^17*c^5 -
22145925120*a^19*b^15*c^6 + 124017180672*a^20*b^13*c^7 - 496068722688*a^21*
b^11*c^8 + 1417339207680*a^22*b^9*c^9 - 2834678415360*a^23*b^7*c^10 + 37795
71220480*a^24*b^5*c^11 - 3023656976384*a^25*b^3*c^12)))*(-(9*(25*b^21 + 25*
b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 -
 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 199056
00*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9
*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b
^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/
(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 768
0*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*
b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^
9)))^(1/2) + 191102976000*a^17*c^14 + 2851200*a^9*b^16*c^6 - 92568960*a^10*
b^14*c^7 + 1312630272*a^11*b^12*c^8 - 10611136512*a^12*b^10*c^9 + 534453534
72*a^13*b^8*c^10 - 171591892992*a^14*b^6*c^11 + 342580396032*a^15*b^4*c^12
- 388363714560*a^16*b^2*c^13))*(-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(
1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 12
99860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256
*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-
(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15
)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*
a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a
^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^
6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*2i - (1/a + (x
^4*(15*b^6 + 324*a^3*c^3 + 25*a^2*b^2*c^2 - 91*a*b^4*c))/(8*a^3*(b^4 + 16*a
^2*c^2 - 8*a*b^2*c)) + (b*x^6*(30*b^4*c + 392*a^2*c^3 - 227*a*b^2*c^2))/(8*
a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c*x^8*(5*b^4*c + 60*a^2*c^3 - 37*a
*b^2*c^2))/(8*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*x^2*(25*b^4 + 364*a^
2*c^2 - 194*a*b^2*c))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^5*(2*a*c +
 b^2) + a^2*x + c^2*x^9 + 2*a*b*x^3 + 2*b*c*x^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.888 $\int \frac{x^5}{a-bx^2+cx^4}\,dx$

Optimal. Leaf size=82

$$\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{b\log\left(a-bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

```
[Out] 1/2*x^2/c+1/4*b*ln(c*x^4-b*x^2+a)/c^2+1/2*(-2*a*c+b^2)*arctanh((-2*c*x^2+b)
/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1114, 703, 634, 618, 206, 628}

$$\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{b\log\left(a-bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a - b*x^2 + c*x^4),x]
```

```
[Out] x^2/(2*c) + ((b^2 - 2*a*c)*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2
*Sqrt[b^2 - 4*a*c]) + (b*Log[a - b*x^2 + c*x^4])/(4*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{a-bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{a-bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{x^2}{2c}+\frac{\text{Subst}\left(\int \frac{-a+bx}{a-bx+cx^2}\,dx,x,x^2\right)}{2c}\\
&= \frac{x^2}{2c}+\frac{b\,\text{Subst}\left(\int \frac{-b+2cx}{a-bx+cx^2}\,dx,x,x^2\right)}{4c^2}+\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{a-bx+cx^2}\,dx,x,x^2\right)}{4c^2}\\
&= \frac{x^2}{2c}+\frac{b\log\left(a-bx^2+cx^4\right)}{4c^2}-\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,-b+2cx^2\right)}{2c^2}\\
&= \frac{x^2}{2c}+\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{b\log\left(a-bx^2+cx^4\right)}{4c^2}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 80, normalized size = 0.98

$$\frac{\frac{2\left(b^2-2ac\right)\tan^{-1}\left(\frac{2cx^2-b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+b\log\left(a-bx^2+cx^4\right)+2cx^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a - b*x^2 + c*x^4),x]
```

```
[Out] (2*c*x^2 + (2*(b^2 - 2*a*c)*ArcTan[(-b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt
[-b^2 + 4*a*c] + b*Log[a - b*x^2 + c*x^4])/(4*c^2)
```

fricas [A] time = 0.80, size = 259, normalized size = 3.16

$$\left[\frac{2\left(b^{2}c-4\,ac^{2}\right)x^{2}-\left(b^{2}-2\,ac\right)\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{4}-2\,bcx^{2}+b^{2}-2\,ac+\left(2\,cx^{2}-b\right)\sqrt{b^{2}-4\,ac}}{cx^{4}-bx^{2}+a}\right)+\left(b^{3}-4\,abc\right)\log\left(cx^{4}-bx^{2}+\right.}{4\left(b^{2}c^{2}-4\,ac^{3}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c - 4*a*c^2)*x^2 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^4 - 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 - b)*sqrt(b^2 - 4*a*c))/(c*x^4 - b
*x^2 + a)) + (b^3 - 4*a*b*c)*log(c*x^4 - b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1
/4*(2*(b^2*c - 4*a*c^2)*x^2 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c)*log(c*x^4 -
 b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

giac [A] time = 0.53, size = 78, normalized size = 0.95

$$\frac{x^{2}}{2\,c}+\frac{b\log\left(cx^{4}-bx^{2}+a\right)}{4\,c^{2}}+\frac{\left(b^{2}-2\,ac\right)\arctan\left(\frac{2\,cx^{2}-b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\,\sqrt{-b^{2}+4\,ac}\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c + 1/4*b*log(c*x^4 - b*x^2 + a)/c^2 + 1/2*(b^2 - 2*a*c)*arctan((2*
c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

maple [A] time = 0.00, size = 116, normalized size = 1.41

$$-\frac{a\arctan\left(\frac{2c\,x^{2}-b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}\;c}+\frac{b^{2}\arctan\left(\frac{2c\,x^{2}-b}{\sqrt{4ac-b^{2}}}\right)}{2\sqrt{4ac-b^{2}}\;c^{2}}+\frac{x^{2}}{2c}+\frac{b\ln\left(c\,x^{4}-b\,x^{2}+a\right)}{4c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4-b*x^2+a),x)
```

[Out] 1/2/c*x^2+1/4*b*ln(c*x^4-b*x^2+a)/c^2-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
-b)/(4*a*c-b^2)^(1/2))*a+1/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2-b)/(4*a*
c-b^2)^(1/2))*b^2

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.74, size = 656, normalized size = 8.00

$$\frac{x^2}{2\,c}-\frac{\ln\left(c\,x^4-b\,x^2+a\right)\,\left(2\,b^3-8\,a\,b\,c\right)}{2\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}-\frac{\mathrm{atan}\left(\frac{2\,c^2\,\left(4\,a\,c-b^2\right)\,\left(\frac{\frac{\left(8\,a\,b+\frac{8\,a\,c^2\,\left(2\,b^3-8\,a\,b\,c\right)}{16\,a\,c^3-4\,b^2\,c^2}\right)\,\left(2\,a\,c-b^2\right)}{8\,c^2\,\sqrt{4\,a\,c-b^2}}+\frac{a\,\left(2\,b^3-8\,a\,b\,c\right)\,\left(2\,a\,c-b^2\right)}{\sqrt{4\,a\,c-b^2}\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}}{a}+x^2\,\left(\frac{\left(2\,a\,c-b \cdots\right.}{\cdots}\right.\right.}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a - b*x^2 + c*x^4),x)
```

```
[Out] x^2/(2*c) - (log(a - b*x^2 + c*x^4)*(2*b^3 - 8*a*b*c))/(2*(16*a*c^3 - 4*b^2
*c^2)) - (atan((2*c^2*(4*a*c - b^2)*((((8*a*b + (8*a*c^2*(2*b^3 - 8*a*b*c))
/(16*a*c^3 - 4*b^2*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (a*(2
*b^3 - 8*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2))
)/a + x^2*((((2*a*c - b^2)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b^3 - 8
*a*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(8*c^2*(4*a*c - b^2)^(1/2)) - (b*(2*b^3 -
 8*a*b*c)*(2*a*c - b^2))/(2*(4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2)))/a
+ (b*(((2*b^3 - 8*a*b*c)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b^3 - 8*a
```

```
*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(2*(16*a*c^3 - 4*b^2*c^2)) - (b^3 - a*b*c)/
c^2 + (b*(2*a*c - b^2)^2)/(2*c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))
) + (b*((a*b^2)/c^2 + ((2*b^3 - 8*a*b*c)*(8*a*b + (8*a*c^2*(2*b^3 - 8*a*b*c
))/(16*a*c^3 - 4*b^2*c^2)))/(2*(16*a*c^3 - 4*b^2*c^2)) - (a*(2*a*c - b^2)^2
)/(c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(b^4 + 4*a^2*c^2 - 4*a*
b^2*c))*(2*a*c - b^2))/(2*c^2*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 2.76, size = 311, normalized size = 3.79

$$\left(\frac{b}{4c^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4c^2\left(4ac-b^2\right)}\right)\log\left(x^2 + \frac{ab - 8ac^2\left(\frac{b}{4c^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4c^2\left(4ac-b^2\right)}\right) + 2b^2c\left(\frac{b}{4c^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4c^2\left(4ac-b^2\right)}\right)}{2ac-b^2}\right) + \left(\frac{b}{4c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4-b*x**2+a),x)
```

```
[Out] (b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*l
og(x**2 + (a*b - 8*a*c**2*(b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/
(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*
c - b**2)/(4*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (b/(4*c**2) + sqrt(-4
*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (a*b - 8*a*
c**2*(b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2
))) + 2*b**2*c*(b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*
a*c - b**2))))/(2*a*c - b**2)) + x**2/(2*c)
```

3.889 $\int \frac{x^3}{a-bx^2+cx^4}\,dx$

Optimal. Leaf size=64

$$\frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a-bx^2+cx^4\right)}{4c}$$

```
[Out] 1/4*ln(c*x^4-b*x^2+a)/c+1/2*b*arctanh((-2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c/(-
4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1114, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a-bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b*x^2 + c*x^4),x]
```

```
[Out] (b*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + Log[
a - b*x^2 + c*x^4]/(4*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a-bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{a-bx+cx^2}\,dx, x, x^2\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{-b+2cx}{a-bx+cx^2}\,dx, x, x^2\right)}{4c} + \frac{b\operatorname{Subst}\left(\int \frac{1}{a-bx+cx^2}\,dx, x, x^2\right)}{4c}\\
&= \frac{\log\left(a-bx^2+cx^4\right)}{4c} - \frac{b\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, -b+2cx^2\right)}{2c}\\
&= \frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{\log\left(a-bx^2+cx^4\right)}{4c}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 1.02

$$\frac{\frac{2b\tan^{-1}\left(\frac{2cx^2-b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \log\left(a-bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b*x^2 + c*x^4),x]
```

```
[Out] ((2*b*ArcTan[(-b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 - b*x^2 + c*x^4])/(4*c)
```

fricas [A] time = 0.81, size = 206, normalized size = 3.22

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^4-2\,bcx^2+b^2-2\,ac-\left(2\,cx^2-b\right)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right) + \left(b^2-4\,ac\right)\log\left(cx^4-bx^2+a\right)}{4\left(b^2c-4\,ac^2\right)}, -\frac{2\sqrt{-b^2+4\,ac}\,b\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 - 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x
^2 - b)*sqrt(b^2 - 4*a*c))/(c*x^4 - b*x^2 + a)) + (b^2 - 4*a*c)*log(c*x^4 -
 b*x^2 + a))/(b^2*c - 4*a*c^2), -1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x
^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log(c*x^4 - b*x^2
 + a))/(b^2*c - 4*a*c^2)]
```

giac [A] time = 0.57, size = 62, normalized size = 0.97

$$\frac{b \arctan\left(\frac{2 \, cx^2 - b}{\sqrt{-b^2 + 4 \, ac}}\right)}{2 \, \sqrt{-b^2 + 4 \, ac} \, c} + \frac{\log\left(cx^4 - bx^2 + a\right)}{4 \, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*b*arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/4
*log(c*x^4 - b*x^2 + a)/c
```

maple [A] time = 0.00, size = 63, normalized size = 0.98

$$\frac{b \arctan\left(\frac{2c\,x^2 - b}{\sqrt{4ac - b^2}}\right)}{2\sqrt{4ac - b^2}\; c} + \frac{\ln\left(c\,x^4 - b\,x^2 + a\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4-b*x^2+a),x)
```

```
[Out] 1/4*ln(c*x^4-b*x^2+a)/c+1/2*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2-b)/(4*a*c
-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.40, size = 120, normalized size = 1.88

$$\frac{4\,a\,c\,\ln\left(c\,x^4-b\,x^2+a\right)}{16\,a\,c^2-4\,b^2\,c}-\frac{b^2\,\ln\left(c\,x^4-b\,x^2+a\right)}{16\,a\,c^2-4\,b^2\,c}-\frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}-\frac{2\,c\,x^2}{\sqrt{4\,a\,c-b^2}}\right)}{2\,c\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a - b*x^2 + c*x^4),x)
```

```
[Out] (4*a*c*log(a - b*x^2 + c*x^4))/(16*a*c^2 - 4*b^2*c) - (b^2*log(a - b*x^2 +
c*x^4))/(16*a*c^2 - 4*b^2*c) - (b*atan(b/(4*a*c - b^2)^(1/2) - (2*c*x^2)/(4
*a*c - b^2)^(1/2)))/(2*c*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 1.45, size = 223, normalized size = 3.48

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)\log\left(x^2+\frac{8ac\left(-\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)-2a-2b^2\left(-\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4-b*x**2+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (8*a*c*(
-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) - 2*a - 2*b**2*(-b*s
qrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)))/b) + (b*sqrt(-4*a*c + b
**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (8*a*c*(b*sqrt(-4*a*c + b**
2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) - 2*a - 2*b**2*(b*sqrt(-4*a*c + b**2)/(4
*c*(4*a*c - b**2)) + 1/(4*c)))/b)
```

3.890 $\int \frac{x}{a-bx^2+cx^4}\,dx$

Optimal. Leaf size=35

$$\frac{\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

[Out] arctanh((-2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1107, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[x/(a - b*x^2 + c*x^4),x]

[Out] ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{a-bx^2+cx^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{a-bx+cx^2}\,dx, x, x^2\right)$$
$$= -\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, -b+2cx^2\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{-b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.17

$$\frac{\tan^{-1}\left(\frac{2cx^2-b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a - b*x^2 + c*x^4),x]
```

```
[Out] ArcTan[(-b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]/Sqrt[-b^2 + 4*a*c]
```

fricas [A] time = 0.80, size = 134, normalized size = 3.83

$$\left[\frac{\log\left(\frac{2\,c^2x^4-2\,bcx^2+b^2-2\,ac-\left(2\,cx^2-b\right)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)}{2\,\sqrt{b^2-4\,ac}}, -\frac{\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\left(2\,cx^2-b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*c^2*x^4 - 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 - b)*sqrt(b^2 - 4*
a*c))/(c*x^4 - b*x^2 + a))/sqrt(b^2 - 4*a*c), -sqrt(-b^2 + 4*a*c)*arctan(-(
2*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

giac [A] time = 0.57, size = 37, normalized size = 1.06

$$\frac{\arctan\left(\frac{2\,cx^2-b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

maple [A] time = 0.00, size = 38, normalized size = 1.09

$$\frac{\arctan\left(\frac{2c\,x^{2}-b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4-b*x^2+a),x)
```

```
[Out] 1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2-b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.30, size = 42, normalized size = 1.20

$$-\frac{\mathrm{atan}\left(\frac{a\,b-2\,a\,c\,x^{2}}{a\,\sqrt{4\,a\,c-b^{2}}}\right)}{\sqrt{4\,a\,c-b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a - b*x^2 + c*x^4),x)
```

```
[Out] -atan((a*b - 2*a*c*x^2)/(a*(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)^(1/2)
```

sympy [B] time = 0.72, size = 131, normalized size = 3.74

$$-\frac{\sqrt{-\frac{1}{4ac-b^{2}}}\log\left(x^{2}+\frac{-4ac\sqrt{-\frac{1}{4ac-b^{2}}}+b^{2}\sqrt{-\frac{1}{4ac-b^{2}}}-b}{2c}\right)}{2}+\frac{\sqrt{-\frac{1}{4ac-b^{2}}}\log\left(x^{2}+\frac{4ac\sqrt{-\frac{1}{4ac-b^{2}}}-b^{2}\sqrt{-\frac{1}{4ac-b^{2}}}-b}{2c}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4-b*x**2+a),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*
sqrt(-1/(4*a*c - b**2)) - b)/(2*c))/2 + sqrt(-1/(4*a*c - b**2))*log(x**2 +
(4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) - b)/(2*c))/2
```

3.891 $\int \frac{1}{x(a-bx^2+cx^4)}\,dx$

Optimal. Leaf size=70

$$\frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a-bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

```
[Out] ln(x)/a-1/4*ln(c*x^4-b*x^2+a)/a+1/2*b*arctanh((-2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/a/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1114, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a-bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b*x^2 + c*x^4)),x]
```

```
[Out] (b*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a - b*x^2 + c*x^4]/(4*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a-bx^2+cx^4\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\left(a-bx+cx^2\right)}\,dx,x,x^2\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2a}+\frac{\operatorname{Subst}\left(\int \frac{b-cx}{a-bx+cx^2}\,dx,x,x^2\right)}{2a}\\
&= \frac{\log(x)}{a}-\frac{\operatorname{Subst}\left(\int \frac{-b+2cx}{a-bx+cx^2}\,dx,x,x^2\right)}{4a}+\frac{b\operatorname{Subst}\left(\int \frac{1}{a-bx+cx^2}\,dx,x,x^2\right)}{4a}\\
&= \frac{\log(x)}{a}-\frac{\log\left(a-bx^2+cx^4\right)}{4a}-\frac{b\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,-b+2cx^2\right)}{2a}\\
&= \frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a-bx^2+cx^4\right)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 117, normalized size = 1.67

$$\frac{\left(b-\sqrt{b^2-4ac}\right)\log\left(-\sqrt{b^2-4ac}-b+2cx^2\right)-\left(\sqrt{b^2-4ac}+b\right)\log\left(\sqrt{b^2-4ac}-b+2cx^2\right)+4\log(x)\sqrt{b^2-}}{4a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b*x^2 + c*x^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[x] + (b - Sqrt[b^2 - 4*a*c])*Log[-b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2] - (b + Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2 - 4*a*c] + 2*c
*x^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

fricas [A] time = 0.98, size = 230, normalized size = 3.29

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^4-2\,bcx^2+b^2-2\,ac-\left(2\,cx^2-b\right)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)-\left(b^2-4\,ac\right)\log\left(cx^4-bx^2+a\right)+4\left(b^2-4\,ac\right)\log(x)}{4\left(ab^2-4\,a^2c\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 - 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x
^2 - b)*sqrt(b^2 - 4*a*c))/(c*x^4 - b*x^2 + a)) - (b^2 - 4*a*c)*log(c*x^4 -
 b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), -1/4*(2*sqrt(-b^2
+ 4*a*c)*b*arctan(-(2*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 -
 4*a*c)*log(c*x^4 - b*x^2 + a) - 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

giac [A] time = 0.57, size = 71, normalized size = 1.01

$$\frac{b\arctan\left(\frac{2\,cx^2-b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,a}-\frac{\log\left(cx^4-bx^2+a\right)}{4\,a}+\frac{\log\left(x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*b*arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/4
*log(c*x^4 - b*x^2 + a)/a + 1/2*log(x^2)/a
```

maple [A] time = 0.01, size = 69, normalized size = 0.99

$$\frac{b\arctan\left(\frac{2c\,x^2-b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;a}+\frac{\ln(x)}{a}-\frac{\ln\left(c\,x^4-b\,x^2+a\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4-b*x^2+a),x)
```

```
[Out] 1/a*ln(x)-1/4*ln(c*x^4-b*x^2+a)/a+1/2/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
-b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.89, size = 1015, normalized size = 14.50

$$\frac{\ln(x)}{a}+\frac{\ln\left(c\,x^4-b\,x^2+a\right)\left(8\,a\,c-2\,b^2\right)}{2\,\left(4\,a\,b^2-16\,a^2\,c\right)}-\frac{b\,\mathrm{atan}\left(\frac{16\,a^3\,x^2\left(\frac{\left(3\,b^3-8\,a\,b\,c\right)\left(\frac{\left(8\,a\,c-2\,b^2\right)^2\left(10\,b\,c^3-\frac{\left(12\,b^3\,c^2-40\,a\,b\,c^3\right)\left(8\,a\,c-2\,b^2\right)}{2\,\left(4\,a\,b^2-16\,a^2\,c\right)}\right)}{4\,\left(4\,a\,b^2-16\,a^2\,c\right)^2}-\frac{b^2\left(10\,b\,c^3-\frac{\left(12\,b \cdots\right.}{\cdots}\right.}{16 \cdots}\right.}{8\,a^3\,c^2\left(25\,a\,c-6\,b^2\right)}\cdots\right.}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - b*x^2 + c*x^4)),x)
```

```
[Out] log(x)/a + (log(a - b*x^2 + c*x^4)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c)
) - (b*atan((16*a^3*x^2*(((3*b^3 - 8*a*b*c)*(((8*a*c - 2*b^2)^2*(10*b*c^3 -
 ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(4*
(4*a*b^2 - 16*a^2*c)^2) - (b^2*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*
c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(16*a^2*(4*a*c - b^2)) + (b^2*(12*b^
```

```
3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(16*a^2*(4*a*b^2 - 16*a^2*c)*(4*a*c -
b^2))))/(8*a^3*c^2*(25*a*c - 6*b^2)) - ((3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*(
(b^3*(12*b^3*c^2 - 40*a*b*c^3))/(64*a^3*(4*a*c - b^2)^(3/2)) - (b*(12*b^3*c
^2 - 40*a*b*c^3)*(8*a*c - 2*b^2)^2)/(16*a*(4*a*b^2 - 16*a^2*c)^2*(4*a*c - b
^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a
*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c -
b^2)^(1/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2)))*(4*a*c - b^
2)^(3/2))/(b^2*c^2) - (2*(3*b^3 - 8*a*b*c)*(4*a*c - b^2)^(3/2)*(((8*a*c - 2
*b^2)^2*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))/(4*a*b^2 - 16*a^2*c)))/(
4*(4*a*b^2 - 16*a^2*c)^2) - (b^2*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))
/(4*a*b^2 - 16*a^2*c)))/(16*a^2*(4*a*c - b^2)) + (b^4*c^2*(8*a*c - 2*b^2))/
(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2))))/(b^2*c^4*(25*a*c - 6*b^2)) + (2*
(4*a*c - b^2)*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b^5*c^2)/(16*a^2*(4*a*c -
 b^2)^(3/2)) - (b^3*c^2*(8*a*c - 2*b^2)^2)/(4*(4*a*b^2 - 16*a^2*c)^2*(4*a*c
 - b^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^
2))/(4*a*b^2 - 16*a^2*c)))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2))))
/(b^2*c^4*(25*a*c - 6*b^2))))/(2*a*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 5.74, size = 253, normalized size = 3.61

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)\log\left(x^2+\frac{8a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)-2ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)+2ac-b^2}{bc}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4-b*x**2+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (8*a**2*
c*(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*b**2*(-b*sq
rt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*c - b**2)/(b*c)) +
(b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (8*a**2*c
*(b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*b**2*(b*sqrt(
-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*c - b**2)/(b*c)) + log
(x)/a
```

3.892 $\int \frac{1}{x^3(a-bx^2+cx^4)}\,dx$

Optimal. Leaf size=89

$$\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{b\log\left(a-bx^2+cx^4\right)}{4a^2}+\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

```
[Out] -1/2/a/x^2+b*ln(x)/a^2-1/4*b*ln(c*x^4-b*x^2+a)/a^2+1/2*(-2*a*c+b^2)*arctanh
((-2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1114, 709, 800, 634, 618, 206, 628}

$$\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{b\log\left(a-bx^2+cx^4\right)}{4a^2}+\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b*x^2 + c*x^4)),x]
```

```
[Out] -1/(2*a*x^2) + ((b^2 - 2*a*c)*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*
a^2*Sqrt[b^2 - 4*a*c]) + (b*Log[x])/a^2 - (b*Log[a - b*x^2 + c*x^4])/(4*a^2
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a-bx^2+cx^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a-bx+cx^2\right)}\,dx,x,x^2\right)$$
$$= -\frac{1}{2ax^2}+\frac{\text{Subst}\left(\int \frac{b-cx}{x\left(a-bx+cx^2\right)}\,dx,x,x^2\right)}{2a}$$
$$= -\frac{1}{2ax^2}+\frac{\text{Subst}\left(\int \left(\frac{b}{ax}-\frac{-b^2+ac+bcx}{a\left(a-bx+cx^2\right)}\right)dx,x,x^2\right)}{2a}$$
$$= -\frac{1}{2ax^2}+\frac{b\log(x)}{a^2}-\frac{\text{Subst}\left(\int \frac{-b^2+ac+bcx}{a-bx+cx^2}\,dx,x,x^2\right)}{2a^2}$$
$$= -\frac{1}{2ax^2}+\frac{b\log(x)}{a^2}-\frac{b\,\text{Subst}\left(\int \frac{-b+2cx}{a-bx+cx^2}\,dx,x,x^2\right)}{4a^2}+\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{a-bx+cx^2}\,dx,x\right.}{4a^2}$$
$$= -\frac{1}{2ax^2}+\frac{b\log(x)}{a^2}-\frac{b\log\left(a-bx^2+cx^4\right)}{4a^2}-\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,-b+\right.}{2a^2}$$
$$= -\frac{1}{2ax^2}+\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{b\log(x)}{a^2}-\frac{b\log\left(a-bx^2+cx^4\right)}{4a^2}$$

Mathematica [A] time = 0.14, size = 139, normalized size = 1.56

$$\frac{\frac{\left(-b\sqrt{b^2-4ac}-2ac+b^2\right)\log\left(-\sqrt{b^2-4ac}-b+2cx^2\right)}{\sqrt{b^2-4ac}}-\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\log\left(\sqrt{b^2-4ac}-b+2cx^2\right)}{\sqrt{b^2-4ac}}-\frac{2a}{x^2}+4b\log(x)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a)/x^2 + 4*b*Log[x] + ((b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c])*Log[-b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] - ((b^2 - 2*a*c + b*Sqrt[b^2
- 4*a*c])*Log[-b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

fricas [A] time = 0.92, size = 298, normalized size = 3.35

$$\left[-\frac{\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\,x^2\log\left(\frac{2\,c^2x^4-2\,bcx^2+b^2-2\,ac+\left(2\,cx^2-b\right)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)+\left(b^3-4\,abc\right)x^2\log\left(cx^4-bx^2+a\right)-4\left(b^3-\right.}{4\left(a^2b^2-4\,a^3c\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 - 2*b*c*x^2 + b^2
 - 2*a*c + (2*c*x^2 - b)*sqrt(b^2 - 4*a*c))/(c*x^4 - b*x^2 + a)) + (b^3 - 4
*a*b*c)*x^2*log(c*x^4 - b*x^2 + a) - 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2), -1/4*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*
a*c)*x^2*arctan(-(2*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^3 - 4
*a*b*c)*x^2*log(c*x^4 - b*x^2 + a) - 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2)]
```

giac [A] time = 0.59, size = 95, normalized size = 1.07

$$-\frac{b\log\left(cx^4-bx^2+a\right)}{4\,a^2}+\frac{b\log\left(x^2\right)}{2\,a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^2-b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,a^2}-\frac{bx^2+a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*b*log(c*x^4 - b*x^2 + a)/a^2 + 1/2*b*log(x^2)/a^2 + 1/2*(b^2 - 2*a*c)*
arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/2*(b*
x^2 + a)/(a^2*x^2)
```

maple [A] time = 0.01, size = 123, normalized size = 1.38

$$-\frac{c\arctan\left(\frac{2c\,x^2-b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;a}+\frac{b^2\arctan\left(\frac{2c\,x^2-b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;a^2}+\frac{b\ln(x)}{a^2}-\frac{b\ln\left(c\,x^4-b\,x^2+a\right)}{4a^2}-\frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4-b*x^2+a),x)
```

```
[Out] -1/2/a/x^2+1/a^2*b*ln(x)-1/4*b*ln(c*x^4-b*x^2+a)/a^2-1/a/(4*a*c-b^2)^(1/2)*
arctan((2*c*x^2-b)/(4*a*c-b^2)^(1/2))*c+1/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2
*c*x^2-b)/(4*a*c-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 5.84, size = 2032, normalized size = 22.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a - b*x^2 + c*x^4)),x)
```

```
[Out] (b*log(x))/a^2 - 1/(2*a*x^2) + (log(a - b*x^2 + c*x^4)*(2*b^3 - 8*a*b*c))/(
2*(16*a^3*c - 4*a^2*b^2)) + (atan((16*a^6*x^2*(((3*b^4 + a^2*c^2 - 9*a*b^2*
c)*(c^5/a^3 + ((2*b^3 - 8*a*b*c)*((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a
^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3
*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2))))/(2*(16
*a^3*c - 4*a^2*b^2)) - ((((2*a*c - b^2)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 +
 ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a
^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)) + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 -
 12*a^3*b^3*c^2)*(2*a*c - b^2))/(8*a^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^
2*b^2)))*(2*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(1/2)) - ((2*b^3 - 8*a*b*c)*(4
0*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2)^2)/(32*a^7*(4*a*c - b^2)*(16*a^
3*c - 4*a^2*b^2))))/(8*a^3*c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + ((((2*b^3
- 8*a*b*c)*(((2*a*c - b^2)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*
a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(4
*a^2*(4*a*c - b^2)^(1/2)) + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c
^2)*(2*a*c - b^2))/(8*a^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2))))/(2*
(16*a^3*c - 4*a^2*b^2)) - ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2)^3)
/(64*a^9*(4*a*c - b^2)^(3/2)) + (((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a
^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3
*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*a*c
- b^2))/(4*a^2*(4*a*c - b^2)^(1/2)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(
8*a^3*c^2*(4*a*c - b^2)^(1/2)*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)))*(4*a*c - b^2
)^(3/2))/(4*a^2*c^4 + b^4*c^2 - 4*a*b^2*c^3) + (2*a^3*(4*a*c - b^2)*(3*b^5
+ 13*a^2*b*c^2 - 15*a*b^3*c)*(((2*b^3 - 8*a*b*c)*((((4*a^3*b*c^3 - 4*a^2*b^
3*c^2)/a^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*a*c
 - b^2))/(4*a^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c -
b^2))/(2*a*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a
^2*b^2)) + ((2*a*c - b^2)*((a^2*c^4 - 4*a*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)
*((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a
^3*c - 4*a^2*b^2)))/(2*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)
) - (b^2*c^2*(2*a*c - b^2)^3)/(16*a^5*(4*a*c - b^2)^(3/2))))/(c^2*(a^2*c^2
- 6*b^4 + 24*a*b^2*c)*(4*a^2*c^4 + b^4*c^2 - 4*a*b^2*c^3)) - (2*a^3*(4*a*c
- b^2)^(3/2)*(3*b^4 + a^2*c^2 - 9*a*b^2*c)*((b*c^4)/a^3 - ((2*b^3 - 8*a*b*c
```

```
)*((a^2*c^4 - 4*a*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*((4*a^3*b*c^3 - 4*a^2*b
^3*c^2)/a^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2)))/(2*(
16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) + ((2*a*c - b^2)*((((4*
a^3*b*c^3 - 4*a^2*b^3*c^2)/a^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c
- 4*a^2*b^2))*(2*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*b^3
- 8*a*b*c)*(2*a*c - b^2))/(2*a*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))
)/(4*a^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2)^2)
/(8*a^3*(4*a*c - b^2)*(16*a^3*c - 4*a^2*b^2))))/(c^2*(a^2*c^2 - 6*b^4 + 24*
a*b^2*c)*(4*a^2*c^4 + b^4*c^2 - 4*a*b^2*c^3)))*(2*a*c - b^2))/(2*a^2*(4*a*c
 - b^2)^(1/2))
```

sympy [B] time = 142.97, size = 350, normalized size = 3.93

$$\left(-\frac{b}{4a^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2\left(4ac-b^2\right)}\right)\log\left(x^2 + \frac{-8a^3c\left(-\frac{b}{4a^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2\left(4ac-b^2\right)}\right) + 2a^2b^2\left(-\frac{b}{4a^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2\left(4ac-b^2\right)}\right) -}{2ac^2 - b^2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4-b*x**2+a),x)
```

```
[Out] (-b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)))*
log(x**2 + (-8*a**3*c*(-b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*
a**2*(4*a*c - b**2))) + 2*a**2*b**2*(-b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a
*c - b**2)/(4*a**2*(4*a*c - b**2))) - 3*a*b*c + b**3)/(2*a*c**2 - b**2*c))
+ (-b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2))
)*log(x**2 + (-8*a**3*c*(-b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(
4*a**2*(4*a*c - b**2))) + 2*a**2*b**2*(-b/(4*a**2) + sqrt(-4*a*c + b**2)*(2
*a*c - b**2)/(4*a**2*(4*a*c - b**2))) - 3*a*b*c + b**3)/(2*a*c**2 - b**2*c)
) - 1/(2*a*x**2) + b*log(x)/a**2
```

3.893 $\int \frac{x^4}{a-bx^2+cx^4}\,dx$

Optimal. Leaf size=179

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

```
[Out] x/c-1/2*arctanh(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c-b
^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*ar
ctanh(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*a
*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1122, 1166, 208}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a - b*x^2 + c*x^4),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
 ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
```

```
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{a-bx^2+cx^4}\,dx &= \frac{x}{c}-\frac{\int \frac{a-bx^2}{a-bx^2+cx^4}\,dx}{c}\\ &= \frac{x}{c}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{-\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{-\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}\\ &= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

Mathematica [A] time = 0.12, size = 208, normalized size = 1.16

$$\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}-b}}+\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}-b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a - b*x^2 + c*x^4),x]
```

```
[Out] x/c + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
-b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-b - Sqrt
[b^2 - 4*a*c]]) + ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[-b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*S
qrt[-b + Sqrt[b^2 - 4*a*c]])
```

fricas [B] time = 1.00, size = 1051, normalized size = 5.87

$$-\frac{\sqrt{\frac{1}{2}}\, c \sqrt{\frac{b^3-3\,abc+\left(b^2c^3-4\,ac^4\right)\sqrt{\frac{b^4-2\,ab^2c+a^2c^2}{b^2c^6-4\,ac^7}}}{b^2c^3-4\,ac^4}}\, \log\left(-2\left(ab^2-a^2c\right)x+\sqrt{\frac{1}{2}}\left(b^4-5\,ab^2c+4\,a^2c^2-\left(b^3c^3-4\,abc^4\right)\sqrt{\frac{b^4-2\,a}{b^2c^6}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt((b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a
*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2
- a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt((b^3 - 3*a*b*c
 + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)
))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt((b^3 - 3*a*b*c + (b^2*c^3 - 4*a
*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a
*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 -
(b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))
)*sqrt((b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2
)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt((b^3 - 3*a
*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*
c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a
*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)
/(b^2*c^6 - 4*a*c^7)))*sqrt((b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1
/2)*c*sqrt((b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x -
 sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt((b^3 - 3*a*b*c - (b^2*c^3
- 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3
- 4*a*c^4))) - 2*x)/c
```

giac [B] time = 0.98, size = 2153, normalized size = 12.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] x/c + 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
```

rt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c +
sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt
(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^
5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2
- 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*
a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)
*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 - 2*(s
qrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(-b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 2*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^3
*c^4 - 8*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(
-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(-b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3 -
 8*(b^2 - 4*a*c)*a^2*c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt(-(b*c + sqrt(b^
2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 + 2*a*b^3*c^4 + 16*a^3*
c^5 - 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*(2*b^5*c^4 - 12*a*b^3
*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*
c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c
)*a*b^3*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*
b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b
^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - (2*b^5*c^2 - 16*a*b^3*c^3 +
 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*
b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c
- 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b
^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 + 2*(sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3
 + 2*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a*b^4*c^3 + 16*
sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 8*sqrt(2)*sqrt(-b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^
5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4)*abs(c
))*arctan(2*sqrt(1/2)*x/sqrt(-(b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4

```
*c^3 - 8*a^2*b^2*c^4 + 2*a*b^3*c^4 + 16*a^3*c^5 - 8*a^2*b*c^5 + a*b^2*c^5 -
 4*a^2*c^6)*c^2)
```

maple [B] time = 0.03, size = 343, normalized size = 1.92

$$\frac{\sqrt{2}\, a \,\mathrm{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right) c}}\right)}{\sqrt{-4ac+b^{2}}\, \sqrt{\left(b+\sqrt{-4ac+b^{2}}\right) c}}+\frac{\sqrt{2}\, a \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right) c}}\right)}{\sqrt{-4ac+b^{2}}\, \sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right) c}}-\frac{\sqrt{2}\, b^{2} \,\mathrm{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right) c}}\right)}{2\sqrt{-4ac+b^{2}}\, \sqrt{\left(b+\sqrt{-4ac+b^{2}}\right) c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4-b*x^2+a),x)
```

```
[Out] 1/c*x-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*
a-1/2/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2+1/2/c*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*
x)*b+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a-1/2/(-4*a*c+b^2)^(1/2)/c*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{c}+\frac{-\frac{\left(\sqrt{2}\sqrt{-bc-\sqrt{b^2-4\,ac}\,c}\,b^4-8\,\sqrt{2}\sqrt{-bc-\sqrt{b^2-4\,ac}\,c}\,ab^2c+2\,\sqrt{2}\sqrt{-bc-\sqrt{b^2-4\,ac}\,c}\,b^3c+2\,b^4c+16\,\sqrt{2}\sqrt{-bc-\sqrt{b^2-4\,ac}\,c}\,a^2c^2-8\,\sqrt{2}\sqrt{-bc-\sqrt{b^2-4\,a}}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] x/c + integrate((b*x^2 - a)/(c*x^4 - b*x^2 + a), x)/c
```

mupad [B] time = 0.67, size = 3000, normalized size = 16.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a - b*x^2 + c*x^4),x)
```

```
[Out] x/c + atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*
((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*((
b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(
b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*
c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^
3*c^3 - 16*a*b*c^4)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7
*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2
*c^4)))^(1/2))/c)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a
*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c
^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*((b^5 + b^2*(-(4*a*c -
 b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^3 - 4*a*b^2
*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2
) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5
 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*((
b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*
a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*((b^5 + b^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2
))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*((b^5 + b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 -
4*a*b^2*c))/c)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^
3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)
))^(1/2) - (2*a^2*b)/c))*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*
a*b^2*c^4)))^(1/2)*2i + atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*
c^3 - 16*a*b*c^4)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a
*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c
^4)))^(1/2))/c)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b
^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4
)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*((b^5 - b^2*(-(4*a*c - b
^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*
(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3 - 4*a*b^2*c
^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*((b^
5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c
```

```
- b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16
*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*((b^5 - b^2*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*((b^5 - b^2*(-(4*a*
c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 -
 4*a*b^2*c))/c)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b
^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4
)))^(1/2) + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*
((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*((
b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(
b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*
c^3 - 8*a*b^2*c^4)))^(1/2) - (2*a^2*b)/c))*((b^5 - b^2*(-(4*a*c - b^2)^3)^(
1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*
c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i
```

sympy [A] time = 2.78, size = 129, normalized size = 0.72

$$\operatorname{RootSum}\left(t^4\left(256a^2c^5-128ab^2c^4+16b^4c^3\right)+t^2\left(-48a^2bc^2+28ab^3c-4b^5\right)+a^3,\left(t\mapsto t\log\left(x+\frac{-32t^3abc^4+8\cdots}{\cdots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4-b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(-48
*a**2*b*c**2 + 28*a*b**3*c - 4*b**5) + a**3, Lambda(_t, _t*log(x + (-32*_t*
*3*a*b*c**4 + 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b**
4)/(a**2*c - a*b**2)))) + x/c
```

3.894 $\int \frac{x^2}{a-bx^2+cx^4}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{b-\sqrt{b^2-4ac}}\ \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{\sqrt{b^2-4ac}+b}\ \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] 1/2*arctanh(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b-(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)-1/2*arctanh(x*2^(1/2)*c^(1
/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1
/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1130, 208}

$$\frac{\sqrt{b-\sqrt{b^2-4ac}}\ \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{\sqrt{b^2-4ac}+b}\ \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]) - (Sqrt[b + Sqrt[b^2 - 4*a*
c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt
[c]*Sqrt[b^2 - 4*a*c])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\int \frac{x^2}{a-bx^2+cx^4}\,dx = -\left(\frac{1}{2}\left(-1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{-\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx\right)+\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{-\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx$$

$$= \frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b+\sqrt{b^2-4ac}}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.11, size = 137, normalized size = 0.91

$$\frac{\sqrt{\sqrt{b^2-4ac}-b}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}-b}}\right)-\sqrt{-\sqrt{b^2-4ac}-b}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2 + c*x^4),x]
```

```
[Out] (-(Sqrt[-b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b - Sqrt[b
^2 - 4*a*c]]]) + Sqrt[-b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[-b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

fricas [B] time = 0.81, size = 551, normalized size = 3.67

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\,\log\left(\frac{\sqrt{\frac{1}{2}}\left(b^2c-4\,ac^2\right)\sqrt{\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\,\log\left(-\frac{\sqrt{\frac{1}{2}}\left(b^2c-4\right.}{\sqrt{b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt((b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt((b + (b^2*c - 4*a*c^2)/sqr
t(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) + 1/2
*sqrt(1/2)*sqrt((b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*
a*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt((b + (b^2*c - 4*a*c^2)/sqrt(b
^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) + 1/2*sq
```

```
rt(1/2)*sqrt((b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c
^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt((b - (b^2*c - 4*a*c^2)/sqrt(b^2*c
^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) - 1/2*sqrt(1
/2)*sqrt((b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))
*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt((b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2
- 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x)
```

giac [B] time = 1.06, size = 513, normalized size = 3.42

$$\frac{\left(2\,b^2c^2 - 8\,ac^3 - \sqrt{2}\,\sqrt{b^2-4\,ac}\,\sqrt{-bc-\sqrt{b^2-4\,ac}\,c}\,b^2 + 4\,\sqrt{2}\,\sqrt{b^2-4\,ac}\,\sqrt{-bc-\sqrt{b^2-4\,ac}\,c}\,ac - 2\,\sqrt{2}\,\sqrt{b^2-\right.}{2\left(b^4 - 8\,ab^2c + 2\,b^3c + 16\,a^2c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 -
 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*
c)*a*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b*c -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 -
4*a*c)*c^2)*arctan(2*sqrt(1/2)*x/sqrt(-(b + sqrt(b^2 - 4*a*c))/c))/((b^4 -
8*a*b^2*c + 2*b^3*c + 16*a^2*c^2 - 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*abs(c)) -
 1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2
- 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)
*c)*a*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b*c
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 -
 4*a*c)*c^2)*arctan(2*sqrt(1/2)*x/sqrt(-(b - sqrt(b^2 - 4*a*c))/c))/((b^4 -
 8*a*b^2*c + 2*b^3*c + 16*a^2*c^2 - 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*abs(c))
```

maple [A] time = 0.01, size = 208, normalized size = 1.39

$$-\frac{\sqrt{2}\,b\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4-b*x^2+a),x)
```

```
[Out] -1/2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*c*x)-1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
```

```
/2))*c)^(1/2)*arctanh(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b+1/2*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)-1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{cx^4 - bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^4 - b*x^2 + a), x)
```

mupad [B] time = 4.54, size = 416, normalized size = 2.77

$$-2\,\mathrm{atanh}\left(\frac{\left(x\,\left(4\,a\,c^2-2\,b^2\,c\right)+\frac{x\,\left(8\,b^3\,c^2-32\,a\,b\,c^3\right)\,\left(b^3+\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c\right)}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}\right)\,\sqrt{\frac{b^3+\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}}{a\,c}\right)\,\sqrt{\frac{b^3+\sqrt{-{\left(4\,a\,c\right.}}}{8\,\left(16\,a^2\,c^3-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^2 + c*x^4),x)
```

```
[Out] - 2*atanh(((x*(4*a*c^2 - 2*b^2*c) + (x*(8*b^3*c^2 - 32*a*b*c^3)*(b^3 + (-(4
*a*c - b^2)^3)^(1/2) - 4*a*b*c))/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((
b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*
c^2)))^(1/2))/(a*c))*((b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c
+ 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2) - 2*atanh(((x*(4*a*c^2 - 2*b^2*c) - (x*
(8*b^3*c^2 - 32*a*b*c^3)*((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c))/(8*(b^
4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b
*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(a*c))*(-((-(4*a*c - b^2
)^3)^(1/2) - b^3 + 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2)
```

sympy [A] time = 1.25, size = 75, normalized size = 0.50

$$\mathrm{RootSum}\left(t^4\left(256a^2c^3-128ab^2c^2+16b^4c\right)+t^2\left(16abc-4b^3\right)+a,\left(t\mapsto t\log\left(64t^3ac^2-16t^3b^2c+2tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4-b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3 - 128*a*b**2*c**2 + 16*b**4*c) + _t**2*(16*a*b
*c - 4*b**3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 - 16*_t**3*b**2*c + 2*_
t*b + x)))
```

3.895 $\int \frac{1}{a-bx^2+cx^4}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] arctanh(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4
*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctanh(x*2^(1/2)*c^(1/2)/(b+(
-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2
)^(1/2))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1093, 208}

$$\frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTanh[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b
 + Sqrt[b^2 - 4*a*c]])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a-bx^2+cx^4}\,dx = \frac{c\int \frac{1}{-\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{-\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\sqrt{2}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.08, size = 137, normalized size = 0.91

$$\frac{\sqrt{2}\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{-\sqrt{b^2-4ac}-b}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b - Sqrt[b^2 - 4*a*c]]]/
Sqrt[-b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^
2 - 4*a*c]]]/Sqrt[-b + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

fricas [B] time = 0.61, size = 605, normalized size = 4.03

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{ab^2-4}{\sqrt{a^2b^2-}}}{ab^2-4a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt((b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
- 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqrt(a
^2*b^2 - 4*a^3*c))*sqrt((b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*
b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt((b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 -
 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 -
```

```
 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt((b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b
^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt((b - (a*b^2 - 4*a^2
*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2
- 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt((b - (a*b^2 - 4
*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt((
b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x
 - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sq
rt((b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c)))
```

giac [B] time = 0.57, size = 1050, normalized size = 7.00

$$\frac{\left(\sqrt{2}\sqrt{-bc-\sqrt{b^2-4\,ac}\,c}\,b^4-8\,\sqrt{2}\sqrt{-bc-\sqrt{b^2-4\,ac}\,c}\,ab^2c+2\,\sqrt{2}\sqrt{-bc-\sqrt{b^2-4\,ac}\,c}\,b^3c+2\,b^4c+16\,\sqrt{2}\sqrt{-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(-b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^
3*c + 2*b^4*c + 16*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqr
t(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(-b*c - sqrt(b^
2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(-b*c - sq
rt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
-b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c -
 sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^
2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^
2 - 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x/sqrt(-(b + sqrt(b^2 - 4*a*c))/c))/((
a*b^4 - 8*a^2*b^2*c + 2*a*b^3*c + 16*a^3*c^2 - 8*a^2*b*c^2 + a*b^2*c^2 - 4*
a^2*c^3)*abs(c)) + 1/4*(sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sq
rt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(-b*c + sqrt
(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*c + 16*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*c^2 - 8*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*
sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqr
t(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4
*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x/sqrt(-(b - sqrt(b
^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c + 2*a*b^3*c + 16*a^3*c^2 - 8*a^2*b*c
^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c))
```

maple [A] time = 0.01, size = 116, normalized size = 0.77

$$-\frac{\sqrt{2}\, c \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4-b*x^2+a),x)
```

```
[Out] -c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{cx^4 - bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^4 - b*x^2 + a), x)
```

mupad [B] time = 0.49, size = 763, normalized size = 5.09

$$-\operatorname{atan}\left(\frac{b^4\, x\, 1\mathrm{i} + b\, x\, \sqrt{-64\, a^3\, c^3 + 48\, a^2\, b^2\, c^2 - 12\, a\, b^4\, c + b^6}\ 1\mathrm{i} + a^2\, c^2\, x\, 16\mathrm{i} - \cdots}{4\, a\, b^4 \sqrt{\frac{b^3+\sqrt{-64\, a^3\, c^3+48\, a^2\, b^2\, c^2-12\, a\, b^4\, c+b^6}\, -4\, a\, b\, c}{128\, a^3\, c^2-64\, a^2\, b^2\, c+8\, a\, b^4}} + 64\, a^3\, c^2 \sqrt{\frac{b^3+\sqrt{-64\, a^3\, c^3+48\, a^2\, b^2\, c^2-12\, a\, b^4\, c+b^6}\, -4\, a\, b\, c}{128\, a^3\, c^2-64\, a^2\, b^2\, c+8\, a\, b^4}} - 32\, a^2 \cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2 + c*x^4),x)
```

```
[Out] - atan((b^4*x*1i + b*x*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/
2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i)/(4*a*b^4*((b^3 + (b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a
^2*b^2*c))^(1/2) + 64*a^3*c^2*((b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)
- 32*a^2*b^2*c*((b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/
2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*((b^3 + (b^6
```

```
- 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128
*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i - atan((b^4*x*1i - b*x*(b^6 - 64*a^3*c^3
 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i)/(4
*a*b^4*(-((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*
a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) + 64*a^3*c^2*(-((b^6 -
 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4
+ 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) - 32*a^2*b^2*c*(-((b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2
- 64*a^2*b^2*c))^(1/2)))*(-((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i
```

sympy [A] time = 1.25, size = 87, normalized size = 0.58

$$\operatorname{RootSum}\left(t^4\left(256a^3c^2-128a^2b^2c+16ab^4\right)+t^2\left(16abc-4b^3\right)+c,\left(t\mapsto t\log\left(x+\frac{-32t^3a^2bc+8t^3ab^3+4tac-}{c}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4-b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(16*a*b
*c - 4*b**3) + c, Lambda(_t, _t*log(x + (-32*_t**3*a**2*b*c + 8*_t**3*a*b**
3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

3.896 $\int \frac{1}{x^2(a-bx^2+cx^4)}\,dx$

Optimal. Leaf size=172

$$\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

```
[Out] -1/a/x+1/2*arctanh(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*
(1+b/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctanh
(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(1-b/(-4*a*c+b^2)^
(1/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1123, 1166, 208}

$$\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2 + c*x^4)),x]
```

```
[Out] -(1/(a*x)) + (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^2\left(a-bx^2+cx^4\right)}\,dx = -\frac{1}{ax}+\frac{\int \frac{b-cx^2}{a-bx^2+cx^4}\,dx}{a}$$

$$= -\frac{1}{ax}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{-\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{-\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,d}{2a}$$

$$= -\frac{1}{ax}+\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4a}}}\right)}{\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.40, size = 199, normalized size = 1.16

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}-b}}+\frac{2}{x}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*(2/x + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[-b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - Sqrt[b^2
- 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[-b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b + Sqrt[b^2
- 4*a*c]]))/a
```

fricas [B] time = 0.81, size = 1108, normalized size = 6.44

$$\frac{\sqrt{\frac{1}{2}}\, ax\sqrt{\frac{b^3-3\,abc+\left(a^3b^2-4\,a^4c\right)\sqrt{\frac{b^4-2\,ab^2c+a^2c^2}{a^6b^2-4\,a^7c}}}{a^3b^2-4\,a^4c}}\,\log\left(-2\left(b^2c^2-ac^3\right)x+\sqrt{\frac{1}{2}}\left(b^5-5\,ab^3c+4\,a^2bc^2-\left(a^3b^4-6\,a^4b^2c+8\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*a*x*sqrt((b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c
^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4
*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*
sqrt((b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt((b^3 - 3*a
*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^
7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqrt(1/2)*(b^5 - 5
*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt((b^3 - 3*a*b*c + (a^3*b^2 - 4
*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4
*a^4*c))) + sqrt(1/2)*a*x*sqrt((b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b
^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2
*(b^2*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4
- 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^
7*c)))*sqrt((b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^
2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt((b^
3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2
 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqrt(1/2)*(
b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b
^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt((b^3 - 3*a*b*c - (a^3*
b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*
b^2 - 4*a^4*c))) - 2)/(a*x)
```

giac [B] time = 1.01, size = 1877, normalized size = 10.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt
```

(b^2 - 4*a*c)*c)*a^3*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 + 2*(sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 8*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^3*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^2*b*c^2)*abs(a))*arctan(2*sqrt(1/2)*x/sqrt(-(a*b + sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3*b^4 - 8*a^4*b^2*c + 2*a^3*b^3*c + 16*a^5*c^2 - 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(a)*abs(c)) - 1/8*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 - 2*(sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 8*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2)*abs(a))*arctan(2*sqrt(1/2)*x/sqrt(-(a*b - sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3*b^4 - 8*a^4*b^2*c + 2*a^3*b^3*c + 16*a^5*c^2 - 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(a)*abs(c)) - 1/(a*x)

maple [A] time = 0.02, size = 232, normalized size = 1.35

$$-\frac{\sqrt{2}\, bc \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\; a}-\frac{\sqrt{2}\, bc \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\; a}+\frac{\sqrt{2}\, c \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right.}}\right.}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4-b*x^2+a),x)
```

```
[Out] -1/a/x+1/2*c/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-1/2*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*b-1/2*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-1/2*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 - b)/(c*x^4 - b*x^2 + a), x)/a - 1/(a*x)
```

mupad [B] time = 4.93, size = 2979, normalized size = 17.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^2 + c*x^4)),x)
```

```
[Out] - atan(((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) - ((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/
2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 +
 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^
6*b*c^3 - 8*a^5*b^3*c^2)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*
a^4*b^2*c)))^(1/2)))*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*
b^2*c)))^(1/2)*1i + (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) - ((b^5 + b^2*(-(4*a*c -
```

b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(
8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 - 4*a^4*b^3*c^
2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*
a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*
c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) - ((b^5 + b^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 -
4*a^4*b^3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*((b^5 + b^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*((b^5 + b^2*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b
^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) - (
(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a
c - b^2)^3)^(1/2))/(8(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*
b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*((b^5 + b^2*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*((b^5 + b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 2*a^3*c^4))*((b^5 + b^2*(
-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)
^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - atan(((x*(4*a^
4*c^4 - 2*a^3*b^2*c^3) - ((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*
a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*b*c^3 - 8*a^5*
b^3*c^2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c +
a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/
2)))*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*
(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1
i + (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) - ((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*
a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 - 4*a^4*b^3*c^2 + x*(32*a^6*b*
c^3 - 8*a^5*b^3*c^2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*
b^2*c)))^(1/2)))*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*
b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*
c)))^(1/2)*1i)/((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) - ((b^5 - b^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 - 4*a^4*b^3*c^2 +
x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*
c^2 - 8*a^4*b^2*c)))^(1/2)))*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2
- 8*a^4*b^2*c)))^(1/2) - (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) - ((b^5 - b^2*(-(4*

```
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5
*b*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4
+ 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16
*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 2*a^3*c^4))*((b^5 - b^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*
b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - 1/(a*x)
```

sympy [A] time = 3.83, size = 148, normalized size = 0.86

$$\operatorname{RootSum}\left(t^4\left(256a^5c^2-128a^4b^2c+16a^3b^4\right)+t^2\left(-48a^2bc^2+28ab^3c-4b^5\right)+c^3,\left(t\mapsto t\log\left(x+\frac{-64t^3a^5c^2+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4-b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2 - 128*a**4*b**2*c + 16*a**3*b**4) + _t**2*(-48
*a**2*b*c**2 + 28*a*b**3*c - 4*b**5) + c**3, Lambda(_t, _t*log(x + (-64*_t*
*3*a**5*c**2 + 48*_t**3*a**4*b**2*c - 8*_t**3*a**3*b**4 + 10*_t*a**2*b*c**2
 - 10*_t*a*b**3*c + 2*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(a*x)
```

3.897 $\int \frac{x^5}{a-b+2ax^2+ax^4} \, dx$

Optimal. Leaf size=69

$$-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(ax^4+2ax^2+a-b\right)}{2a}+\frac{x^2}{2a}$$

```
[Out] 1/2*x^2/a-1/2*ln(a*x^4+2*a*x^2+a-b)/a-1/2*(a+b)*arctanh((x^2+1)*a^(1/2)/b^(
1/2))/a^(3/2)/b^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1114, 703, 634, 618, 206, 628}

$$-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(ax^4+2ax^2+a-b\right)}{2a}+\frac{x^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x^2/(2*a) - ((a + b)*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*a^(3/2)*Sqrt[
b]) - Log[a - b + 2*a*x^2 + a*x^4]/(2*a)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{a-b+2ax^2+ax^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{a-b+2ax+ax^2}\,dx,x,x^2\right)\\
&= \frac{x^2}{2a}+\frac{\operatorname{Subst}\left(\int \frac{-a+b-2ax}{a-b+2ax+ax^2}\,dx,x,x^2\right)}{2a}\\
&= \frac{x^2}{2a}-\frac{\operatorname{Subst}\left(\int \frac{2a+2ax}{a-b+2ax+ax^2}\,dx,x,x^2\right)}{2a}+\frac{(a+b)\operatorname{Subst}\left(\int \frac{1}{a-b+2ax+ax^2}\,dx,x,x^2\right)}{2a}\\
&= \frac{x^2}{2a}-\frac{\log\left(a-b+2ax^2+ax^4\right)}{2a}-\frac{(a+b)\operatorname{Subst}\left(\int \frac{1}{4ab-x^2}\,dx,x,2a\left(1+x^2\right)\right)}{a}\\
&= \frac{x^2}{2a}-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(a-b+2ax^2+ax^4\right)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 62, normalized size = 0.90

$$
\frac{x^2-\log\left(a\left(x^2+1\right)^2-b\right)}{2a}-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -1/2*((a + b)*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(a^(3/2)*Sqrt[b]) + (x^
2 - Log[-b + a*(1 + x^2)^2])/(2*a)
```

fricas [A] time = 0.80, size = 156, normalized size = 2.26

$$\left[\frac{2\,abx^2 - 2\,ab\log\left(ax^4 + 2\,ax^2 + a - b\right) + \sqrt{ab}\,(a+b)\log\left(\frac{ax^4+2\,ax^2-2\,\sqrt{ab}\,(x^2+1)+a+b}{ax^4+2\,ax^2+a-b}\right)}{4\,a^2b}, \frac{abx^2 - ab\log\left(ax^4 + 2\,ax^2 +\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*x^2 - 2*a*b*log(a*x^4 + 2*a*x^2 + a - b) + sqrt(a*b)*(a + b)*lo
g((a*x^4 + 2*a*x^2 - 2*sqrt(a*b)*(x^2 + 1) + a + b)/(a*x^4 + 2*a*x^2 + a -
b)))/(a^2*b), 1/2*(a*b*x^2 - a*b*log(a*x^4 + 2*a*x^2 + a - b) + sqrt(-a*b)*
(a + b)*arctan(sqrt(-a*b)/(a*x^2 + a)))/(a^2*b)]
```

giac [A] time = 0.26, size = 60, normalized size = 0.87

$$\frac{x^2}{2\,a} + \frac{(a+b)\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\,\sqrt{-ab}\,a} - \frac{\log\left(ax^4 + 2\,ax^2 + a - b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] 1/2*x^2/a + 1/2*(a + b)*arctan((a*x^2 + a)/sqrt(-a*b))/(sqrt(-a*b)*a) - 1/2
*log(a*x^4 + 2*a*x^2 + a - b)/a
```

maple [A] time = 0.00, size = 86, normalized size = 1.25

$$-\frac{b\,\mathrm{arctanh}\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}\;a} + \frac{x^2}{2a} - \frac{\mathrm{arctanh}\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}} - \frac{\ln\left(a\,x^4 + 2a\,x^2 + a - b\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] 1/2*x^2/a-1/2*ln(a*x^4+2*a*x^2+a-b)/a-1/2/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+
2*a)/(a*b)^(1/2))-1/2/a/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))*
b
```

maxima [A] time = 3.00, size = 74, normalized size = 1.07

$$\frac{x^2}{2\,a} + \frac{(a+b)\log\left(\frac{ax^2+a-\sqrt{ab}}{ax^2+a+\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a} - \frac{\log\left(ax^4+2\,ax^2+a-b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/a + 1/4*(a + b)*log((a*x^2 + a - sqrt(a*b))/(a*x^2 + a + sqrt(a*b))
)/(sqrt(a*b)*a) - 1/2*log(a*x^4 + 2*a*x^2 + a - b)/a
```

mupad [B] time = 0.39, size = 166, normalized size = 2.41

$$\frac{x^2}{2\,a} - \ln\left(a\,\sqrt{a^3\,b}-b\,\sqrt{a^3\,b}-a^2\,b\,x^2+a\,x^2\,\sqrt{a^3\,b}\right)\left(\frac{\frac{a^2}{2}+\frac{\sqrt{a^3\,b}}{4}}{a^3}+\frac{\sqrt{a^3\,b}}{4\,a^2\,b}\right) - \ln\left(a\,\sqrt{a^3\,b}-b\,\sqrt{a^3\,b}+a^2\,b\,x^2+a\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a - b + 2*a*x^2 + a*x^4),x)
```

```
[Out] x^2/(2*a) - log(a*(a^3*b)^(1/2) - b*(a^3*b)^(1/2) - a^2*b*x^2 + a*x^2*(a^3*
b)^(1/2))*((a^2/2 + (a^3*b)^(1/2)/4)/a^3 + (a^3*b)^(1/2)/(4*a^2*b)) - log(a
*(a^3*b)^(1/2) - b*(a^3*b)^(1/2) + a^2*b*x^2 + a*x^2*(a^3*b)^(1/2))*((a^2/2
 - (a^3*b)^(1/2)/4)/a^3 - (a^3*b)^(1/2)/(4*a^2*b))
```

sympy [B] time = 1.79, size = 138, normalized size = 2.00

$$\left(-\frac{1}{2a}-\frac{\sqrt{a^3b}\,(a+b)}{4a^3b}\right)\log\left(x^2+\frac{-4ab\left(-\frac{1}{2a}-\frac{\sqrt{a^3b}\,(a+b)}{4a^3b}\right)+a-b}{a+b}\right)+\left(-\frac{1}{2a}+\frac{\sqrt{a^3b}\,(a+b)}{4a^3b}\right)\log\left(x^2+\frac{-4ab\left(-\frac{1}{2a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] (-1/(2*a) - sqrt(a**3*b)*(a + b)/(4*a**3*b))*log(x**2 + (-4*a*b*(-1/(2*a) -
 sqrt(a**3*b)*(a + b)/(4*a**3*b)) + a - b)/(a + b)) + (-1/(2*a) + sqrt(a**3
*b)*(a + b)/(4*a**3*b))*log(x**2 + (-4*a*b*(-1/(2*a) + sqrt(a**3*b)*(a + b)
/(4*a**3*b)) + a - b)/(a + b)) + x**2/(2*a)
```

3.898 $\int \frac{x^3}{a-b+2ax^2+ax^4} dx$

Optimal. Leaf size=56

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{\log\left(ax^4+2ax^2+a-b\right)}{4a}$$

```
[Out] 1/4*ln(a*x^4+2*a*x^2+a-b)/a+1/2*arctanh((x^2+1)*a^(1/2)/b^(1/2))/a^(1/2)/b^
(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1114, 634, 618, 206, 628}

$$\frac{\log\left(ax^4+2ax^2+a-b\right)}{4a}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b]) + Log[a - b + 2*a*
x^2 + a*x^4]/(4*a)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a-b+2ax^2+ax^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{a-b+2ax+ax^2}\,dx, x, x^2\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{a-b+2ax+ax^2}\,dx, x, x^2\right)\right)+\frac{\operatorname{Subst}\left(\int \frac{2a+2ax}{a-b+2ax+ax^2}\,dx, x, x^2\right)}{4a}\\
&= \frac{\log\left(a-b+2ax^2+ax^4\right)}{4a}+\operatorname{Subst}\left(\int \frac{1}{4ab-x^2}\,dx, x, 2a\left(1+x^2\right)\right)\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{\log\left(a-b+2ax^2+ax^4\right)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.91

$$\frac{\log\left(a\left(x^2+1\right)^2-b\right)+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{\sqrt{b}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] ((2*Sqrt[a]*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/Sqrt[b] + Log[-b + a*(1 +
 x^2)^2])/(4*a)
```

fricas [A] time = 0.84, size = 134, normalized size = 2.39

$$\left[\frac{b\log\left(ax^4+2\,ax^2+a-b\right)+\sqrt{ab}\,\log\left(\frac{ax^4+2\,ax^2+2\,\sqrt{ab}\,\left(x^2+1\right)+a+b}{ax^4+2\,ax^2+a-b}\right)}{4\,ab},\frac{b\log\left(ax^4+2\,ax^2+a-b\right)-2\,\sqrt{-ab}\,\arctan\left(\right.}{4\,ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] [1/4*(b*log(a*x^4 + 2*a*x^2 + a - b) + sqrt(a*b)*log((a*x^4 + 2*a*x^2 + 2*s
qrt(a*b)*(x^2 + 1) + a + b)/(a*x^4 + 2*a*x^2 + a - b)))/(a*b), 1/4*(b*log(a
*x^4 + 2*a*x^2 + a - b) - 2*sqrt(-a*b)*arctan(sqrt(-a*b)/(a*x^2 + a)))/(a*b
)]
```

giac [A] time = 0.26, size = 46, normalized size = 0.82

$$-\frac{\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\,\sqrt{-ab}}+\frac{\log\left(ax^4+2\,ax^2+a-b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] -1/2*arctan((a*x^2 + a)/sqrt(-a*b))/sqrt(-a*b) + 1/4*log(a*x^4 + 2*a*x^2 +
a - b)/a
```

maple [A] time = 0.00, size = 49, normalized size = 0.88

$$\frac{\operatorname{arctanh}\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}}+\frac{\ln\left(a\,x^4+2a\,x^2+a-b\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] 1/4/a*ln(a*x^4+2*a*x^2+a-b)+1/2/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)
^(1/2))
```

maxima [A] time = 2.97, size = 60, normalized size = 1.07

$$-\frac{\log\left(\frac{ax^2+a-\sqrt{ab}}{ax^2+a+\sqrt{ab}}\right)}{4\,\sqrt{ab}}+\frac{\log\left(ax^4+2\,ax^2+a-b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] -1/4*log((a*x^2 + a - sqrt(a*b))/(a*x^2 + a + sqrt(a*b)))/sqrt(a*b) + 1/4*l
og(a*x^4 + 2*a*x^2 + a - b)/a
```

mupad [B] time = 0.17, size = 153, normalized size = 2.73

$$\frac{\ln\left(x^2\sqrt{a^3\,b}+a\,b-a^2-a^2\,x^2\right)}{4\,a}+\frac{\ln\left(x^2\sqrt{a^3\,b}-a\,b+a^2+a^2\,x^2\right)}{4\,a}-\frac{\ln\left(x^2\sqrt{a^3\,b}-a\,b+a^2+a^2\,x^2\right)\sqrt{a^3\,b}}{4\,a^2\,b}+\frac{\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a - b + 2*a*x^2 + a*x^4),x)
```

```
[Out] log(x^2*(a^3*b)^(1/2) + a*b - a^2 - a^2*x^2)/(4*a) + log(x^2*(a^3*b)^(1/2)
- a*b + a^2 + a^2*x^2)/(4*a) - (log(x^2*(a^3*b)^(1/2) - a*b + a^2 + a^2*x^2
)*(a^3*b)^(1/2))/(4*a^2*b) + (log(x^2*(a^3*b)^(1/2) + a*b - a^2 - a^2*x^2)*
(a^3*b)^(1/2))/(4*a^2*b)
```

sympy [B] time = 0.84, size = 110, normalized size = 1.96

$$\left(\frac{1}{4a}-\frac{\sqrt{a^3b}}{4a^2b}\right)\log\left(x^2+\frac{4ab\left(\frac{1}{4a}-\frac{\sqrt{a^3b}}{4a^2b}\right)+a-b}{a}\right)+\left(\frac{1}{4a}+\frac{\sqrt{a^3b}}{4a^2b}\right)\log\left(x^2+\frac{4ab\left(\frac{1}{4a}+\frac{\sqrt{a^3b}}{4a^2b}\right)+a-b}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] (1/(4*a) - sqrt(a**3*b)/(4*a**2*b))*log(x**2 + (4*a*b*(1/(4*a) - sqrt(a**3*
b)/(4*a**2*b)) + a - b)/a) + (1/(4*a) + sqrt(a**3*b)/(4*a**2*b))*log(x**2 +
 (4*a*b*(1/(4*a) + sqrt(a**3*b)/(4*a**2*b)) + a - b)/a)
```

3.899 $\int \frac{x}{a-b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

```
[Out] -1/2*arctanh((x^2+1)*a^(1/2)/b^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1107, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{a-b+2ax^2+ax^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{a-b+2ax+ax^2}\,dx, x, x^2\right)$$
$$= -\,\text{Subst}\left(\int \frac{1}{4ab-x^2}\,dx, x, 2a\left(1+x^2\right)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -1/2*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(Sqrt[a]*Sqrt[b])
```

fricas [A] time = 0.86, size = 91, normalized size = 2.94

$$\left[\frac{\sqrt{ab}\,\log\left(\frac{ax^4+2\,ax^2-2\,\sqrt{ab}\left(x^2+1\right)+a+b}{ax^4+2\,ax^2+a-b}\right)}{4\,ab}, \frac{\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}}{ax^2+a}\right)}{2\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a*b)*log((a*x^4 + 2*a*x^2 - 2*sqrt(a*b)*(x^2 + 1) + a + b)/(a*x^4
 + 2*a*x^2 + a - b))/(a*b), 1/2*sqrt(-a*b)*arctan(sqrt(-a*b)/(a*x^2 + a))/(
a*b)]
```

giac [A] time = 0.26, size = 23, normalized size = 0.74

$$\frac{\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\,\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] 1/2*arctan((a*x^2 + a)/sqrt(-a*b))/sqrt(-a*b)
```

maple [A] time = 0.00, size = 26, normalized size = 0.84

$$-\frac{\operatorname{arctanh}\left(\frac{2a\,x^{2}+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] -1/2/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))
```

maxima [A] time = 3.02, size = 37, normalized size = 1.19

$$\frac{\log\left(\frac{ax^{2}+a-\sqrt{ab}}{ax^{2}+a+\sqrt{ab}}\right)}{4\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] 1/4*log((a*x^2 + a - sqrt(a*b))/(a*x^2 + a + sqrt(a*b)))/sqrt(a*b)
```

mupad [B] time = 4.34, size = 31, normalized size = 1.00

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a}\ \sqrt{b}\ x^{2}}{a\,x^{2}+a-b}\right)}{2\,\sqrt{a}\ \sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a - b + 2*a*x^2 + a*x^4),x)
```

```
[Out] atanh((a^(1/2)*b^(1/2)*x^2)/(a - b + a*x^2))/(2*a^(1/2)*b^(1/2))
```

sympy [A] time = 0.34, size = 53, normalized size = 1.71

$$\frac{\sqrt{\frac{1}{ab}}\log\left(-b\sqrt{\frac{1}{ab}}+x^{2}+1\right)}{4}-\frac{\sqrt{\frac{1}{ab}}\log\left(b\sqrt{\frac{1}{ab}}+x^{2}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] sqrt(1/(a*b))*log(-b*sqrt(1/(a*b)) + x**2 + 1)/4 - sqrt(1/(a*b))*log(b*sqrt
(1/(a*b)) + x**2 + 1)/4
```

3.900 $\int \frac{1}{x\left(a-b+2ax^2+ax^4\right)}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a-b)}-\frac{\log\left(ax^4+2ax^2+a-b\right)}{4(a-b)}+\frac{\log(x)}{a-b}$$

```
[Out] ln(x)/(a-b)-1/4*ln(a*x^4+2*a*x^2+a-b)/(a-b)+1/2*arctanh((x^2+1)*a^(1/2)/b^(
1/2))*a^(1/2)/(a-b)/b^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1114, 705, 29, 634, 618, 206, 628}

$$-\frac{\log\left(ax^4+2ax^2+a-b\right)}{4(a-b)}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a-b)}+\frac{\log(x)}{a-b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (Sqrt[a]*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*(a - b)*Sqrt[b]) + Log[x]
/(a - b) - Log[a - b + 2*a*x^2 + a*x^4]/(4*(a - b))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a-b+2ax^2+ax^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(a-b+2ax+ax^2\right)}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2(a-b)}+\frac{\text{Subst}\left(\int \frac{-2a-ax}{a-b+2ax+ax^2}\,dx, x, x^2\right)}{2(a-b)}\\
&= \frac{\log(x)}{a-b}-\frac{\text{Subst}\left(\int \frac{2a+2ax}{a-b+2ax+ax^2}\,dx, x, x^2\right)}{4(a-b)}-\frac{a\,\text{Subst}\left(\int \frac{1}{a-b+2ax+ax^2}\,dx, x, x^2\right)}{2(a-b)}\\
&= \frac{\log(x)}{a-b}-\frac{\log\left(a-b+2ax^2+ax^4\right)}{4(a-b)}+\frac{a\,\text{Subst}\left(\int \frac{1}{4ab-x^2}\,dx, x, 2a\left(1+x^2\right)\right)}{a-b}\\
&= \frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2(a-b)\sqrt{b}}+\frac{\log(x)}{a-b}-\frac{\log\left(a-b+2ax^2+ax^4\right)}{4(a-b)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 90, normalized size = 1.17

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt{a}\left(x^2+1\right)-\sqrt{b}\right)+\left(\sqrt{b}-\sqrt{a}\right)\log\left(\sqrt{a}\left(x^2+1\right)+\sqrt{b}\right)-4\sqrt{b}\log(x)}{4\sqrt{b}(b-a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (-4*Sqrt[b]*Log[x] + (Sqrt[a] + Sqrt[b])*Log[-Sqrt[b] + Sqrt[a]*(1 + x^2)]
+ (-Sqrt[a] + Sqrt[b])*Log[Sqrt[b] + Sqrt[a]*(1 + x^2)])/(4*Sqrt[b]*(-a + b
))
```

fricas [A] time = 0.86, size = 151, normalized size = 1.96

$$\left[-\frac{\sqrt{\frac{a}{b}}\log\left(\frac{ax^4+2ax^2-2(bx^2+b)\sqrt{\frac{a}{b}}+a+b}{ax^4+2ax^2+a-b}\right)+\log\left(ax^4+2ax^2+a-b\right)-4\log(x)}{4(a-b)}, -\frac{2\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{-\frac{a}{b}}}{ax^2+a}\right)+\log\left(ax\right.}{4(a-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b)*sqrt(a/b) + a + b)/(a
*x^4 + 2*a*x^2 + a - b)) + log(a*x^4 + 2*a*x^2 + a - b) - 4*log(x))/(a - b)
, -1/4*(2*sqrt(-a/b)*arctan(b*sqrt(-a/b)/(a*x^2 + a)) + log(a*x^4 + 2*a*x^2
 + a - b) - 4*log(x))/(a - b)]
```

giac [A] time = 0.33, size = 71, normalized size = 0.92

$$-\frac{a\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\sqrt{-ab}(a-b)}-\frac{\log\left(ax^4+2ax^2+a-b\right)}{4(a-b)}+\frac{\log\left(x^2\right)}{2(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] -1/2*a*arctan((a*x^2 + a)/sqrt(-a*b))/(sqrt(-a*b)*(a - b)) - 1/4*log(a*x^4
+ 2*a*x^2 + a - b)/(a - b) + 1/2*log(x^2)/(a - b)
```

maple [A] time = 0.01, size = 71, normalized size = 0.92

$$\frac{a\,\mathrm{arctanh}\left(\frac{2ax^2+2a}{2\sqrt{ab}}\right)}{2(a-b)\sqrt{ab}}+\frac{\ln(x)}{a-b}-\frac{\ln\left(ax^4+2ax^2+a-b\right)}{4(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] ln(x)/(a-b)-1/4*ln(a*x^4+2*a*x^2+a-b)/(a-b)+1/2*a/(a-b)/(a*b)^(1/2)*arctanh
(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))
```

maxima [A] time = 3.13, size = 85, normalized size = 1.10

$$-\frac{a\log\left(\frac{ax^2+a-\sqrt{ab}}{ax^2+a+\sqrt{ab}}\right)}{4\sqrt{ab}\,(a-b)}-\frac{\log\left(ax^4+2\,ax^2+a-b\right)}{4\,(a-b)}+\frac{\log\left(x^2\right)}{2\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] -1/4*a*log((a*x^2 + a - sqrt(a*b))/(a*x^2 + a + sqrt(a*b)))/(sqrt(a*b)*(a -
 b)) - 1/4*log(a*x^4 + 2*a*x^2 + a - b)/(a - b) + 1/2*log(x^2)/(a - b)
```

mupad [B] time = 4.56, size = 183, normalized size = 2.38

$$\frac{\ln(x)}{a-b}-\frac{\ln\left(16\,a^4+20\,a^4\,x^2+\frac{\left(b-\sqrt{a\,b}\right)\left(x^2\left(16\,a^5+80\,b\,a^4\right)-16\,a^4\,b+16\,a^5\right)}{4\left(a\,b-b^2\right)}\right)\left(b-\sqrt{a\,b}\right)}{4\left(a\,b-b^2\right)}-\frac{\ln\left(16\,a^4+20\,a^4\,x^2+\frac{\left(b+\sqrt{a\,b}\right)\left(x^2\left(16\right.\right.}{}\right.}{4\left(a\,b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - b + 2*a*x^2 + a*x^4)),x)
```

```
[Out] log(x)/(a - b) - (log(16*a^4 + 20*a^4*x^2 + ((b - (a*b)^(1/2))*(x^2*(80*a^4
*b + 16*a^5) - 16*a^4*b + 16*a^5))/(4*(a*b - b^2)))*(b - (a*b)^(1/2)))/(4*(
a*b - b^2)) - (log(16*a^4 + 20*a^4*x^2 + ((b + (a*b)^(1/2))*(x^2*(80*a^4*b
+ 16*a^5) - 16*a^4*b + 16*a^5))/(4*(a*b - b^2)))*(b + (a*b)^(1/2)))/(4*(a*b
 - b^2))
```

sympy [B] time = 5.31, size = 184, normalized size = 2.39

$$\left(-\frac{1}{4\,(a-b)}-\frac{\sqrt{ab}}{4b\,(a-b)}\right)\log\left(x^2+\frac{4ab\left(-\frac{1}{4(a-b)}-\frac{\sqrt{ab}}{4b(a-b)}\right)+a-4b^2\left(-\frac{1}{4(a-b)}-\frac{\sqrt{ab}}{4b(a-b)}\right)+b}{a}\right)+\left(-\frac{1}{4\,(a-b)}+\frac{\sqrt{a}}{4b\,(a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] (-1/(4*(a - b)) - sqrt(a*b)/(4*b*(a - b)))*log(x**2 + (4*a*b*(-1/(4*(a - b)
) - sqrt(a*b)/(4*b*(a - b))) + a - 4*b**2*(-1/(4*(a - b)) - sqrt(a*b)/(4*b*
(a - b))) + b)/a) + (-1/(4*(a - b)) + sqrt(a*b)/(4*b*(a - b)))*log(x**2 + (
4*a*b*(-1/(4*(a - b)) + sqrt(a*b)/(4*b*(a - b))) + a - 4*b**2*(-1/(4*(a - b
)) + sqrt(a*b)/(4*b*(a - b))) + b)/a) + log(x)/(a - b)
```

3.901 $\int \frac{1}{x^3(a-b+2ax^2+ax^4)}\,dx$

Optimal. Leaf size=97

$$-\frac{1}{2x^2(a-b)}-\frac{\sqrt{a}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a-b)^2}+\frac{a\log\left(ax^4+2ax^2+a-b\right)}{2(a-b)^2}-\frac{2a\log(x)}{(a-b)^2}$$

```
[Out] -1/2/(a-b)/x^2-2*a*ln(x)/(a-b)^2+1/2*a*ln(a*x^4+2*a*x^2+a-b)/(a-b)^2-1/2*(a
+b)*arctanh((x^2+1)*a^(1/2)/b^(1/2))*a^(1/2)/(a-b)^2/b^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1114, 709, 800, 634, 618, 206, 628}

$$-\frac{1}{2x^2(a-b)}+\frac{a\log\left(ax^4+2ax^2+a-b\right)}{2(a-b)^2}-\frac{\sqrt{a}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a-b)^2}-\frac{2a\log(x)}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -1/(2*(a - b)*x^2) - (Sqrt[a]*(a + b)*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])
/(2*(a - b)^2*Sqrt[b]) - (2*a*Log[x])/(a - b)^2 + (a*Log[a - b + 2*a*x^2 +
a*x^4])/(2*(a - b)^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a-b+2ax^2+ax^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a-b+2ax+ax^2\right)}\,dx,x,x^2\right)$$

$$= -\frac{1}{2(a-b)x^2} + \frac{\text{Subst}\left(\int \frac{-2a-ax}{x\left(a-b+2ax+ax^2\right)}\,dx,x,x^2\right)}{2(a-b)}$$

$$= -\frac{1}{2(a-b)x^2} + \frac{\text{Subst}\left(\int \left(-\frac{2a}{(a-b)x} + \frac{a(3a+b+2ax)}{(a-b)\left(a-b+2ax+ax^2\right)}\right)\,dx,x,x^2\right)}{2(a-b)}$$

$$= -\frac{1}{2(a-b)x^2} - \frac{2a\log(x)}{(a-b)^2} + \frac{a\,\text{Subst}\left(\int \frac{3a+b+2ax}{a-b+2ax+ax^2}\,dx,x,x^2\right)}{2(a-b)^2}$$

$$= -\frac{1}{2(a-b)x^2} - \frac{2a\log(x)}{(a-b)^2} + \frac{a\,\text{Subst}\left(\int \frac{2a+2ax}{a-b+2ax+ax^2}\,dx,x,x^2\right)}{2(a-b)^2} + \frac{(a(a+b))\,\text{Subst}\left(\right.}{2(}$$

$$= -\frac{1}{2(a-b)x^2} - \frac{2a\log(x)}{(a-b)^2} + \frac{a\log\left(a-b+2ax^2+ax^4\right)}{2(a-b)^2} - \frac{(a(a+b))\,\text{Subst}\left(\int \frac{1}{4ab-}\right.}{(a-}$$

$$= -\frac{1}{2(a-b)x^2} - \frac{\sqrt{a}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2(a-b)^2\sqrt{b}} - \frac{2a\log(x)}{(a-b)^2} + \frac{a\log\left(a-b+2ax^2+\right.}{2(a-b)^2}$$

Mathematica [**A**] time = 0.09, size = 146, normalized size = 1.51

$$\frac{-8a\sqrt{b}\,x^2\log(x) + \sqrt{a}\,x^2\left(\sqrt{a}+\sqrt{b}\right)^2\log\left(\sqrt{a}\left(x^2+1\right)-\sqrt{b}\right) - \left(\sqrt{a}-\sqrt{b}\right)\left(\left(ax^2-\sqrt{a}\sqrt{b}\,x^2\right)\log\left(\sqrt{a}\left(x^2+1\right)\right.\right.}{4\sqrt{b}\,x^2(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (-8*a*Sqrt[b]*x^2*Log[x] + Sqrt[a]*(Sqrt[a] + Sqrt[b])^2*x^2*Log[-Sqrt[b] +
 Sqrt[a]*(1 + x^2)] - (Sqrt[a] - Sqrt[b])*(2*(Sqrt[a]*Sqrt[b] + b) + (a*x^2
 - Sqrt[a]*Sqrt[b]*x^2)*Log[Sqrt[b] + Sqrt[a]*(1 + x^2)]))/(4*(a - b)^2*Sqr
t[b]*x^2)
```

fricas [A] time = 0.89, size = 209, normalized size = 2.15

$$\left[\frac{(a+b)x^2\sqrt{\frac{a}{b}}\log\left(\frac{ax^4+2\,ax^2-2\left(bx^2+b\right)\sqrt{\frac{a}{b}}+a+b}{ax^4+2\,ax^2+a-b}\right)+2\,ax^2\log\left(ax^4+2\,ax^2+a-b\right)-8\,ax^2\log(x)-2\,a+2\,b}{4\left(a^2-2\,ab+b^2\right)x^2}, \frac{(a+b)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] [1/4*((a + b)*x^2*sqrt(a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b)*sqrt(a/b)
+ a + b)/(a*x^4 + 2*a*x^2 + a - b)) + 2*a*x^2*log(a*x^4 + 2*a*x^2 + a - b)
- 8*a*x^2*log(x) - 2*a + 2*b)/((a^2 - 2*a*b + b^2)*x^2), 1/2*((a + b)*x^2*s
qrt(-a/b)*arctan(b*sqrt(-a/b)/(a*x^2 + a)) + a*x^2*log(a*x^4 + 2*a*x^2 + a
- b) - 4*a*x^2*log(x) - a + b)/((a^2 - 2*a*b + b^2)*x^2)]
```

giac [A] time = 0.28, size = 126, normalized size = 1.30

$$\frac{a\log\left(ax^4+2\,ax^2+a-b\right)}{2\left(a^2-2\,ab+b^2\right)}-\frac{a\log\left(x^2\right)}{a^2-2\,ab+b^2}+\frac{\left(a^2+ab\right)\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\left(a^2-2\,ab+b^2\right)\sqrt{-ab}}+\frac{2\,ax^2-a+b}{2\left(a^2-2\,ab+b^2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] 1/2*a*log(a*x^4 + 2*a*x^2 + a - b)/(a^2 - 2*a*b + b^2) - a*log(x^2)/(a^2 -
2*a*b + b^2) + 1/2*(a^2 + a*b)*arctan((a*x^2 + a)/sqrt(-a*b))/((a^2 - 2*a*b
 + b^2)*sqrt(-a*b)) + 1/2*(2*a*x^2 - a + b)/((a^2 - 2*a*b + b^2)*x^2)
```

maple [A] time = 0.01, size = 122, normalized size = 1.26

$$-\frac{a^2\,\mathrm{arctanh}\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\left(a-b\right)^2\sqrt{ab}}-\frac{ab\,\mathrm{arctanh}\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\left(a-b\right)^2\sqrt{ab}}-\frac{2a\ln(x)}{\left(a-b\right)^2}+\frac{a\ln\left(a\,x^4+2a\,x^2+a-b\right)}{2\left(a-b\right)^2}-\frac{1}{2\left(a-b\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] -1/2/(a-b)/x^2-2*a*ln(x)/(a-b)^2+1/2*a*ln(a*x^4+2*a*x^2+a-b)/(a-b)^2-1/2/(a
-b)^2*a^2/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))-1/2/(a-b)^2*a/
(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))*b
```

maxima [A] time = 2.97, size = 123, normalized size = 1.27

$$\frac{a\log\left(ax^4+2\,ax^2+a-b\right)}{2\left(a^2-2\,ab+b^2\right)}-\frac{a\log\left(x^2\right)}{a^2-2\,ab+b^2}+\frac{\left(a^2+ab\right)\log\left(\frac{ax^2+a-\sqrt{ab}}{ax^2+a+\sqrt{ab}}\right)}{4\left(a^2-2\,ab+b^2\right)\sqrt{ab}}-\frac{1}{2\,(a-b)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] 1/2*a*log(a*x^4 + 2*a*x^2 + a - b)/(a^2 - 2*a*b + b^2) - a*log(x^2)/(a^2 -
2*a*b + b^2) + 1/4*(a^2 + a*b)*log((a*x^2 + a - sqrt(a*b))/(a*x^2 + a + sqr
t(a*b)))/((a^2 - 2*a*b + b^2)*sqrt(a*b)) - 1/2/((a - b)*x^2)
```

mupad [B] time = 4.87, size = 389, normalized size = 4.01

$$\frac{\ln\left(100\,a\,(a\,b)^{7/2}-198\,b\,(a\,b)^{7/2}-a^3\,(a\,b)^{5/2}+100\,b^3\,(a\,b)^{5/2}-b^5\,(a\,b)^{3/2}+a^2\,b^6-100\,a^3\,b^5+198\,a^4\,b^4-100\,a^5\,b}{a^2\,b-2\,a\,b^2+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a - b + 2*a*x^2 + a*x^4)),x)
```

```
[Out] (log(100*a*(a*b)^(7/2) - 198*b*(a*b)^(7/2) - a^3*(a*b)^(5/2) + 100*b^3*(a*b
)^(5/2) - b^5*(a*b)^(3/2) + a^2*b^6 - 100*a^3*b^5 + 198*a^4*b^4 - 100*a^5*b
^3 + a^6*b^2 + a^2*b^6*x^2 - 100*a^3*b^5*x^2 + 198*a^4*b^4*x^2 - 100*a^5*b^
3*x^2 + a^6*b^2*x^2)*((a*(a*b)^(1/2))/4 + b*(a/2 + (a*b)^(1/2)/4)))/(a^2*b
- 2*a*b^2 + b^3) - (2*a*log(x))/(a^2 - 2*a*b + b^2) - (log(198*b*(a*b)^(7/2
) - 100*a*(a*b)^(7/2) + a^3*(a*b)^(5/2) - 100*b^3*(a*b)^(5/2) + b^5*(a*b)^(
3/2) + a^2*b^6 - 100*a^3*b^5 + 198*a^4*b^4 - 100*a^5*b^3 + a^6*b^2 + a^2*b^
6*x^2 - 100*a^3*b^5*x^2 + 198*a^4*b^4*x^2 - 100*a^5*b^3*x^2 + a^6*b^2*x^2)*
((a*(a*b)^(1/2))/4 - b*(a/2 - (a*b)^(1/2)/4)))/(a^2*b - 2*a*b^2 + b^3) - 1/
(2*x^2*(a - b))
```

sympy [B] time = 32.93, size = 372, normalized size = 3.84

$$-\frac{2a\log(x)}{(a-b)^2}+\left(\frac{a}{2\,(a-b)^2}-\frac{\sqrt{ab}\,(a+b)}{4b\left(a^2-2ab+b^2\right)}\right)\log\left(x^2+\frac{-4a^2b\left(\frac{a}{2(a-b)^2}-\frac{\sqrt{ab}\,(a+b)}{4b\left(a^2-2ab+b^2\right)}\right)+a^2+8ab^2\left(\frac{a}{2(a-b)^2}-\frac{\sqrt{}}{4b(}\right.}{a^2+ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] -2*a*log(x)/(a - b)**2 + (a/(2*(a - b)**2) - sqrt(a*b)*(a + b)/(4*b*(a**2 -
 2*a*b + b**2)))*log(x**2 + (-4*a**2*b*(a/(2*(a - b)**2) - sqrt(a*b)*(a + b
)/(4*b*(a**2 - 2*a*b + b**2))) + a**2 + 8*a*b**2*(a/(2*(a - b)**2) - sqrt(a
*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**2))) + 3*a*b - 4*b**3*(a/(2*(a - b)**2)
 - sqrt(a*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**2))))/(a**2 + a*b)) + (a/(2*(a
 - b)**2) + sqrt(a*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**2)))*log(x**2 + (-4*a
**2*b*(a/(2*(a - b)**2) + sqrt(a*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**2))) +
a**2 + 8*a*b**2*(a/(2*(a - b)**2) + sqrt(a*b)*(a + b)/(4*b*(a**2 - 2*a*b +
b**2))) + 3*a*b - 4*b**3*(a/(2*(a - b)**2) + sqrt(a*b)*(a + b)/(4*b*(a**2 -
 2*a*b + b**2))))/(a**2 + a*b)) - 1/(x**2*(2*a - 2*b))
```

3.902 $\int \frac{x^4}{a-b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=114

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}+\frac{x}{a}$$

```
[Out] x/a+1/2*arctan(a^(1/4)*x/(a^(1/2)-b^(1/2))^(1/2))*(a^(1/2)-b^(1/2))^(3/2)/a
^(5/4)/b^(1/2)-1/2*arctan(a^(1/4)*x/(a^(1/2)+b^(1/2))^(1/2))*(a^(1/2)+b^(1/
2))^(3/2)/a^(5/4)/b^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1122, 1166, 205}

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x/a + ((Sqrt[a] - Sqrt[b])^(3/2)*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]
])/(2*a^(5/4)*Sqrt[b]) - ((Sqrt[a] + Sqrt[b])^(3/2)*ArcTan[(a^(1/4)*x)/Sqrt
[Sqrt[a] + Sqrt[b]]])/(2*a^(5/4)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{a-b+2ax^2+ax^4}\,dx &= \frac{x}{a}-\frac{\int \frac{a-b+2ax^2}{a-b+2ax^2+ax^4}\,dx}{a}\\ &= \frac{x}{a}-\frac{1}{2}\left(2-\frac{a+b}{\sqrt{a}\sqrt{b}}\right)\int \frac{1}{a-\sqrt{a}\sqrt{b}+ax^2}\,dx-\frac{1}{2}\left(2+\frac{a+b}{\sqrt{a}\sqrt{b}}\right)\int \frac{1}{a+\sqrt{a}\sqrt{b}+ax^2}\\ &= \frac{x}{a}+\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 144, normalized size = 1.26

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-\sqrt{a}\sqrt{b}}}\right)}{2a\sqrt{b}\sqrt{a-\sqrt{a}\sqrt{b}}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{2a\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x/a + ((Sqrt[a] - Sqrt[b])^2*ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]*Sqrt[b]]])
/(2*a*Sqrt[a - Sqrt[a]*Sqrt[b]]*Sqrt[b]) - ((Sqrt[a] + Sqrt[b])^2*ArcTan[(S
qrt[a]*x)/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(2*a*Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b
])
```

fricas [B] time = 0.84, size = 603, normalized size = 5.29

$$\frac{a\sqrt{-\frac{a^2b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}+a+3b}{a^2b}}\log\left(-\left(3a^2-2ab-b^2\right)x+\left(a^4b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}-3a^2b-ab^2\right)\sqrt{-\frac{a^2b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}+a+3b}{a^2b}}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] 1/4*(a*sqrt(-(a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)) + a + 3*b)/(a^2*b))
*log(-(3*a^2 - 2*a*b - b^2)*x + (a^4*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b))
- 3*a^2*b - a*b^2)*sqrt(-(a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)) + a + 3
*b)/(a^2*b))) - a*sqrt(-(a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)) + a + 3*
b)/(a^2*b))*log(-(3*a^2 - 2*a*b - b^2)*x - (a^4*b*sqrt((9*a^2 + 6*a*b + b^2
)/(a^5*b)) - 3*a^2*b - a*b^2)*sqrt(-(a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*
b)) + a + 3*b)/(a^2*b))) - a*sqrt((a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)
) - a - 3*b)/(a^2*b))*log(-(3*a^2 - 2*a*b - b^2)*x + (a^4*b*sqrt((9*a^2 + 6
*a*b + b^2)/(a^5*b)) + 3*a^2*b + a*b^2)*sqrt((a^2*b*sqrt((9*a^2 + 6*a*b + b
^2)/(a^5*b)) - a - 3*b)/(a^2*b))) + a*sqrt((a^2*b*sqrt((9*a^2 + 6*a*b + b^2
)/(a^5*b)) - a - 3*b)/(a^2*b))*log(-(3*a^2 - 2*a*b - b^2)*x - (a^4*b*sqrt((
9*a^2 + 6*a*b + b^2)/(a^5*b)) + 3*a^2*b + a*b^2)*sqrt((a^2*b*sqrt((9*a^2 +
6*a*b + b^2)/(a^5*b)) - a - 3*b)/(a^2*b))) + 4*x)/a
```

giac [B] time = 0.36, size = 511, normalized size = 4.48

$$-\frac{\left(3\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,a^4-\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,a^3b-4\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,a^2b^2-2\left(3\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,ab-4\sqrt{a^2}\right.\right.}{2\left(3a^6b-7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] -1/2*(3*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^4 - sqrt(a^2 + sqrt(a*b)*a)*sqr
t(a*b)*a^3*b - 4*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^2*b^2 - 2*(3*sqrt(a^2
+ sqrt(a*b)*a)*sqrt(a*b)*a*b - 4*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*b^2)*a^2
 + (3*sqrt(a^2 + sqrt(a*b)*a)*a^3*b - 7*sqrt(a^2 + sqrt(a*b)*a)*a^2*b^2 + 4
*sqrt(a^2 + sqrt(a*b)*a)*a*b^3)*abs(a))*arctan(x/sqrt((a^2 + sqrt(a^4 - (a^
2 - a*b)*a^2))/a^2))/(3*a^6*b - 7*a^5*b^2 + 4*a^4*b^3) + 1/2*(3*sqrt(a^2 -
sqrt(a*b)*a)*sqrt(a*b)*a^4 - sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a^3*b - 4*sq
rt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a^2*b^2 - 2*(3*sqrt(a^2 - sqrt(a*b)*a)*sqrt
(a*b)*a*b - 4*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*b^2)*a^2 - (3*sqrt(a^2 - sq
rt(a*b)*a)*a^3*b - 7*sqrt(a^2 - sqrt(a*b)*a)*a^2*b^2 + 4*sqrt(a^2 - sqrt(a*
b)*a)*a*b^3)*abs(a))*arctan(x/sqrt((a^2 - sqrt(a^4 - (a^2 - a*b)*a^2))/a^2)
)/(3*a^6*b - 7*a^5*b^2 + 4*a^4*b^3) + x/a
```

maple [B] time = 0.03, size = 210, normalized size = 1.84

$$-\frac{a\,\mathrm{arctanh}\left(\frac{ax}{\sqrt{\left(-a+\sqrt{ab}\right)a}}\right)}{2\sqrt{ab}\,\sqrt{\left(-a+\sqrt{ab}\right)a}}-\frac{a\arctan\left(\frac{ax}{\sqrt{\left(a+\sqrt{ab}\right)a}}\right)}{2\sqrt{ab}\,\sqrt{\left(a+\sqrt{ab}\right)a}}-\frac{b\,\mathrm{arctanh}\left(\frac{ax}{\sqrt{\left(-a+\sqrt{ab}\right)a}}\right)}{2\sqrt{ab}\,\sqrt{\left(-a+\sqrt{ab}\right)a}}-\frac{b\arctan\left(\frac{ax}{\sqrt{\left(a+\sqrt{ab}\right)a}}\right)}{2\sqrt{ab}\,\sqrt{\left(a+\sqrt{ab}\right)a}}+\frac{\mathrm{arctanh}\left(\frac{}{\sqrt{}}\right.}{\sqrt{\left(-a+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] x/a+1/(((a*b)^(1/2)-a)*a)^(1/2)*arctanh(a*x/(((a*b)^(1/2)-a)*a)^(1/2))-1/2/
(a*b)^(1/2)/(((a*b)^(1/2)-a)*a)^(1/2)*arctanh(a*x/(((a*b)^(1/2)-a)*a)^(1/2)
)*a-1/2/(a*b)^(1/2)/(((a*b)^(1/2)-a)*a)^(1/2)*arctanh(a*x/(((a*b)^(1/2)-a)*
a)^(1/2))*b-1/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(a*x/(((a*b)^(1/2)+a)*a)^(1/2
))-1/2/(a*b)^(1/2)/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(a*x/(((a*b)^(1/2)+a)*a)
^(1/2))*a-1/2/(a*b)^(1/2)/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(a*x/(((a*b)^(1/2
)+a)*a)^(1/2))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{a}-\frac{\frac{\left(3\sqrt{a^2+\sqrt{ab}\,a}\,a^2b-4\sqrt{a^2+\sqrt{ab}\,a}\,ab^2+3\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,a^2-4\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,ab\right)a\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a+\sqrt{-4(a-b)a+4a^2}}{a}}}\right)}{2\left(3a^4b-4a^3b^2\right)}+\frac{\left(3\sqrt{a^2-\sqrt{ab}\,a}\,a^2b-4\,\cdots\right.}{\cdots}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] x/a - integrate((2*a*x^2 + a - b)/(a*x^4 + 2*a*x^2 + a - b), x)/a
```

mupad [B] time = 4.79, size = 1097, normalized size = 9.62

$$\frac{x}{a}-2\,\mathrm{atanh}\left(\frac{24\,x\,\sqrt{a^5\,b^3}\,\sqrt{-\frac{3}{16\,a^2}-\frac{1}{16\,a\,b}-\frac{3\,\sqrt{a^5\,b^3}}{16\,a^4\,b^2}-\frac{\sqrt{a^5\,b^3}}{16\,a^5\,b}}}{4\,a\,b^2-\frac{6\,\sqrt{a^5\,b^3}}{a}-6\,a^2\,b+2\,b^3+\frac{2\,b^2\,\sqrt{a^5\,b^3}}{a^3}+\frac{4\,b\,\sqrt{a^5\,b^3}}{a^2}}+\frac{8\,x\,\sqrt{a^5\,b^3}\,\sqrt{-\frac{3}{16\,a^2}-\frac{1}{16\,a\,b}-\frac{3\,\sqrt{a^5\,b^3}}{16\,a^4\,b^2}-\cdots}}{\frac{4\,\sqrt{a^5\,b^3}}{a}-\frac{6\,\sqrt{a^5\,b^3}}{b}+2\,a\,b^2+4\,a^2\,b-6\,a^3+\cdots}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a - b + 2*a*x^2 + a*x^4),x)
```

```
[Out] x/a - 2*atanh((24*x*(a^5*b^3)^(1/2)*(- 3/(16*a^2) - 1/(16*a*b) - (3*(a^5*b^
3)^(1/2))/(16*a^4*b^2) - (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b^2 - (6*(
a^5*b^3)^(1/2))/a - 6*a^2*b + 2*b^3 + (2*b^2*(a^5*b^3)^(1/2))/a^3 + (4*b*(a
^5*b^3)^(1/2))/a^2) + (8*x*(a^5*b^3)^(1/2)*(- 3/(16*a^2) - 1/(16*a*b) - (3*
(a^5*b^3)^(1/2))/(16*a^4*b^2) - (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/((4*(a^5
*b^3)^(1/2))/a - (6*(a^5*b^3)^(1/2))/b + 2*a*b^2 + 4*a^2*b - 6*a^3 + (2*b*(
a^5*b^3)^(1/2))/a^2) - (8*a*b^2*x*(- 3/(16*a^2) - 1/(16*a*b) - (3*(a^5*b^3)
^(1/2))/(16*a^4*b^2) - (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b + (4*(a^5*
b^3)^(1/2))/a^2 - 6*a^2 + 2*b^2 - (6*(a^5*b^3)^(1/2))/(a*b) + (2*b*(a^5*b^3
```

```
)^(1/2))/a^3) - (24*a^2*b*x*(- 3/(16*a^2) - 1/(16*a*b) - (3*(a^5*b^3)^(1/2)
)/(16*a^4*b^2) - (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b + (4*(a^5*b^3)^(
1/2))/a^2 - 6*a^2 + 2*b^2 - (6*(a^5*b^3)^(1/2))/(a*b) + (2*b*(a^5*b^3)^(1/2
))/a^3))*(-(3*a*(a^5*b^3)^(1/2) + b*(a^5*b^3)^(1/2) + a^4*b + 3*a^3*b^2)/(1
6*a^5*b^2))^(1/2) + 2*atanh((24*x*(a^5*b^3)^(1/2)*((3*(a^5*b^3)^(1/2))/(16*
a^4*b^2) - 1/(16*a*b) - 3/(16*a^2) + (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/((6
*(a^5*b^3)^(1/2))/a + 4*a*b^2 - 6*a^2*b + 2*b^3 - (2*b^2*(a^5*b^3)^(1/2))/a
^3 - (4*b*(a^5*b^3)^(1/2))/a^2) - (8*x*(a^5*b^3)^(1/2)*((3*(a^5*b^3)^(1/2))
/(16*a^4*b^2) - 1/(16*a*b) - 3/(16*a^2) + (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2)
)/((4*(a^5*b^3)^(1/2))/a - (6*(a^5*b^3)^(1/2))/b - 2*a*b^2 - 4*a^2*b + 6*a^
3 + (2*b*(a^5*b^3)^(1/2))/a^2) + (8*a*b^2*x*((3*(a^5*b^3)^(1/2))/(16*a^4*b^
2) - 1/(16*a*b) - 3/(16*a^2) + (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b -
(4*(a^5*b^3)^(1/2))/a^2 - 6*a^2 + 2*b^2 + (6*(a^5*b^3)^(1/2))/(a*b) - (2*b*
(a^5*b^3)^(1/2))/a^3) + (24*a^2*b*x*((3*(a^5*b^3)^(1/2))/(16*a^4*b^2) - 1/(
16*a*b) - 3/(16*a^2) + (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b - (4*(a^5*
b^3)^(1/2))/a^2 - 6*a^2 + 2*b^2 + (6*(a^5*b^3)^(1/2))/(a*b) - (2*b*(a^5*b^3
)^(1/2))/a^3))*((3*a*(a^5*b^3)^(1/2) + b*(a^5*b^3)^(1/2) - a^4*b - 3*a^3*b^
2)/(16*a^5*b^2))^(1/2)
```

sympy [A] time = 2.02, size = 105, normalized size = 0.92

$$\operatorname{RootSum}\left(256t^4a^5b^2 + t^2\left(32a^4b + 96a^3b^2\right) + a^3 - 3a^2b + 3ab^2 - b^3, \left(t \mapsto t\log\left(x + \frac{64t^3a^4b + 4ta^3 + 24ta^2b +}{3a^2 - 2ab - b^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] RootSum(256*_t**4*a**5*b**2 + _t**2*(32*a**4*b + 96*a**3*b**2) + a**3 - 3*a
**2*b + 3*a*b**2 - b**3, Lambda(_t, _t*log(x + (64*_t**3*a**4*b + 4*_t*a**3
 + 24*_t*a**2*b + 4*_t*a*b**2)/(3*a**2 - 2*a*b - b**2)))) + x/a
```

3.903 $\int \frac{x^2}{a-b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=109

$$\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}-\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}$$

```
[Out] -1/2*arctan(a^(1/4)*x/(a^(1/2)-b^(1/2))^(1/2))*(a^(1/2)-b^(1/2))^(1/2)/a^(3
/4)/b^(1/2)+1/2*arctan(a^(1/4)*x/(a^(1/2)+b^(1/2))^(1/2))*(a^(1/2)+b^(1/2))
^(1/2)/a^(3/4)/b^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1130, 205}

$$\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}-\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -(Sqrt[Sqrt[a] - Sqrt[b]]*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a
^(3/4)*Sqrt[b]) + (Sqrt[Sqrt[a] + Sqrt[b]]*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a]
+ Sqrt[b]]])/(2*a^(3/4)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\int \frac{x^2}{a-b+2ax^2+ax^4}\,dx = -\left(\frac{1}{2}\left(-1+\frac{\sqrt{a}}{\sqrt{b}}\right)\int \frac{1}{a-\sqrt{a}\,\sqrt{b}+ax^2}\,dx\right)+\frac{1}{2}\left(1+\frac{\sqrt{a}}{\sqrt{b}}\right)\int \frac{1}{a+\sqrt{a}\,\sqrt{b}+ax^2}\,dx$$
$$= -\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}+\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}$$

Mathematica [**A**] time = 0.10, size = 128, normalized size = 1.17

$$\frac{\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}+a}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a-\sqrt{a}\,\sqrt{b}}}}{2\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] (-(((Sqrt[a] - Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]*Sqrt[b]]])/Sqrt
[a - Sqrt[a]*Sqrt[b]]) + ((Sqrt[a] + Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a + S
qrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]])/(2*Sqrt[a]*Sqrt[b])
```

fricas [**B**] time = 0.83, size = 267, normalized size = 2.45

$$\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\log\left(a^2b\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\sqrt{\frac{1}{a^3b}}+x\right)-\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\log\left(-a^2b\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\sqrt{\frac{1}{a^3b}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-(a*b*sqrt(1/(a^3*b)) + 1)/(a*b))*log(a^2*b*sqrt(-(a*b*sqrt(1/(a^3
*b)) + 1)/(a*b))*sqrt(1/(a^3*b)) + x) - 1/4*sqrt(-(a*b*sqrt(1/(a^3*b)) + 1)
/(a*b))*log(-a^2*b*sqrt(-(a*b*sqrt(1/(a^3*b)) + 1)/(a*b))*sqrt(1/(a^3*b)) +
 x) - 1/4*sqrt((a*b*sqrt(1/(a^3*b)) - 1)/(a*b))*log(a^2*b*sqrt((a*b*sqrt(1/
(a^3*b)) - 1)/(a*b))*sqrt(1/(a^3*b)) + x) + 1/4*sqrt((a*b*sqrt(1/(a^3*b)) -
 1)/(a*b))*log(-a^2*b*sqrt((a*b*sqrt(1/(a^3*b)) - 1)/(a*b))*sqrt(1/(a^3*b))
 + x)
```

giac [B] time = 0.36, size = 199, normalized size = 1.83

$$\frac{\left(3\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,a-4\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,b\right)|a|\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a+\sqrt{-4(a-b)a+4a^2}}{a}}}\right)}{2\left(3a^4b-4a^3b^2\right)}-\frac{\left(3\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,a-4\sqrt{a^2-\right.}{2\left(3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] 1/2*(3*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a - 4*sqrt(a^2 + sqrt(a*b)*a)*sqrt
(a*b)*b)*abs(a)*arctan(2*sqrt(1/2)*x/sqrt((2*a + sqrt(-4*(a - b)*a + 4*a^2)
)/a))/(3*a^4*b - 4*a^3*b^2) - 1/2*(3*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a -
4*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*b)*abs(a)*arctan(2*sqrt(1/2)*x/sqrt((2*
a - sqrt(-4*(a - b)*a + 4*a^2))/a))/(3*a^4*b - 4*a^3*b^2)
```

maple [A] time = 0.01, size = 134, normalized size = 1.23

$$\frac{a\,\mathrm{arctanh}\left(\frac{ax}{\sqrt{\left(-a+\sqrt{ab}\right)a}}\right)}{2\sqrt{ab}\,\sqrt{\left(-a+\sqrt{ab}\right)a}}+\frac{a\arctan\left(\frac{ax}{\sqrt{\left(a+\sqrt{ab}\right)a}}\right)}{2\sqrt{ab}\,\sqrt{\left(a+\sqrt{ab}\right)a}}-\frac{\mathrm{arctanh}\left(\frac{ax}{\sqrt{\left(-a+\sqrt{ab}\right)a}}\right)}{2\sqrt{\left(-a+\sqrt{ab}\right)a}}+\frac{\arctan\left(\frac{ax}{\sqrt{\left(a+\sqrt{ab}\right)a}}\right)}{2\sqrt{\left(a+\sqrt{ab}\right)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] -1/2/((-a+(a*b)^(1/2))*a)^(1/2)*arctanh(1/((-a+(a*b)^(1/2))*a)^(1/2)*a*x)+1
/2/(a*b)^(1/2)/((-a+(a*b)^(1/2))*a)^(1/2)*arctanh(1/((-a+(a*b)^(1/2))*a)^(1
/2)*a*x)*a+1/2/((a+(a*b)^(1/2))*a)^(1/2)*arctan(1/((a+(a*b)^(1/2))*a)^(1/2)
*a*x)+1/2/(a*b)^(1/2)/((a+(a*b)^(1/2))*a)^(1/2)*a*arctan(1/((a+(a*b)^(1/2))
*a)^(1/2)*a*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{ax^4+2\,ax^2+a-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a*x^4 + 2*a*x^2 + a - b), x)
```

mupad [B] time = 0.30, size = 216, normalized size = 1.98

$$-2\,\mathrm{atanh}\left(\frac{2\left(x\,\left(4\,a^3+4\,b\,a^2\right)-\frac{4\,a\,x\,\left(\sqrt{a^3\,b^3}+a^2\,b\right)}{b}\right)\sqrt{-\frac{\sqrt{a^3\,b^3}+a^2\,b}{16\,a^3\,b^2}}}{2\,a\,b-2\,a^2}\right)\sqrt{-\frac{\sqrt{a^3\,b^3}+a^2\,b}{16\,a^3\,b^2}}-2\,\mathrm{atanh}\left(\frac{2\left(x\,\left(4\,a^3+4\,b\,a\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b + 2*a*x^2 + a*x^4),x)
```

```
[Out] - 2*atanh((2*(x*(4*a^2*b + 4*a^3) - (4*a*x*((a^3*b^3)^(1/2) + a^2*b))/b)*(-
((a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2))/(2*a*b - 2*a^2))*(-((a^3*b^3
)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2) - 2*atanh((2*(x*(4*a^2*b + 4*a^3) + (4
*a*x*((a^3*b^3)^(1/2) - a^2*b))/b)*(((a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))
^(1/2))/(2*a*b - 2*a^2))*(((a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))^(1/2)
```

sympy [A] time = 0.60, size = 44, normalized size = 0.40

$$\mathrm{RootSum}\left(256t^4a^3b^2+32t^2a^2b+a-b,\left(t\mapsto t\log\left(-64t^3a^2b-4ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**2 + 32*_t**2*a**2*b + a - b, Lambda(_t, _t*log(-6
4*_t**3*a**2*b - 4*_t*a + x)))
```

3.904 $\int \frac{1}{a-b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=109

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{a}\,\sqrt{b}\,\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt[4]{a}\,\sqrt{b}\,\sqrt{\sqrt{a}+\sqrt{b}}}$$

```
[Out] 1/2*arctan(a^(1/4)*x/(a^(1/2)-b^(1/2))^(1/2))/a^(1/4)/b^(1/2)/(a^(1/2)-b^(1
/2))^(1/2)-1/2*arctan(a^(1/4)*x/(a^(1/2)+b^(1/2))^(1/2))/a^(1/4)/b^(1/2)/(a
^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1093, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{a}\,\sqrt{b}\,\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt[4]{a}\,\sqrt{b}\,\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b + 2*a*x^2 + a*x^4)^(-1),x]
```

```
[Out] ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*a^(1/4)*Sqrt[Sqrt[a] - Sqrt[
b]]*Sqrt[b]) - ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[b]]]/(2*a^(1/4)*Sqrt[
Sqrt[a] + Sqrt[b]]*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a-b+2ax^2+ax^4}\,dx = \frac{\sqrt{a}\int \frac{1}{a-\sqrt{a}\sqrt{b}+ax^2}\,dx}{2\sqrt{b}} - \frac{\sqrt{a}\int \frac{1}{a+\sqrt{a}\sqrt{b}+ax^2}\,dx}{2\sqrt{b}}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\sqrt{b}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}+\sqrt{b}}\sqrt{b}}$$

Mathematica [A] time = 0.06, size = 105, normalized size = 0.96

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{b}\sqrt{a-\sqrt{a}\sqrt{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{2\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b + 2*a*x^2 + a*x^4)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]*Sqrt[b]]]/(2*Sqrt[a - Sqrt[a]*Sqrt[b]]*
Sqrt[b]) - ArcTan[(Sqrt[a]*x)/Sqrt[a + Sqrt[a]*Sqrt[b]]]/(2*Sqrt[a + Sqrt[a
]*Sqrt[b]]*Sqrt[b])
```

fricas [B] time = 0.96, size = 553, normalized size = 5.07

$$-\frac{1}{4}\sqrt{-\frac{\frac{ab-b^2}{\sqrt{a^3b-2\,a^2b^2+ab^3}}+1}{ab-b^2}}\log\left(\left(b-\frac{a^2b-ab^2}{\sqrt{a^3b-2\,a^2b^2+ab^3}}\right)\sqrt{-\frac{\frac{ab-b^2}{\sqrt{a^3b-2\,a^2b^2+ab^3}}+1}{ab-b^2}}+x\right)+\frac{1}{4}\sqrt{-\frac{\frac{ab-b^2}{\sqrt{a^3b-2\,a^2b^2+ab^3}}+1}{ab-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) + 1)/(a*b - b^2))*l
og((b - (a^2*b - a*b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3))*sqrt(-((a*b - b^2)
/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) + 1)/(a*b - b^2)) + x) + 1/4*sqrt(-((a*b -
 b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) + 1)/(a*b - b^2))*log(-(b - (a^2*b -
a*b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3))*sqrt(-((a*b - b^2)/sqrt(a^3*b - 2*a
^2*b^2 + a*b^3) + 1)/(a*b - b^2)) + x) - 1/4*sqrt(((a*b - b^2)/sqrt(a^3*b -
 2*a^2*b^2 + a*b^3) - 1)/(a*b - b^2))*log((b + (a^2*b - a*b^2)/sqrt(a^3*b -
 2*a^2*b^2 + a*b^3))*sqrt(((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) - 1)
```

```
/(a*b - b^2)) + x) + 1/4*sqrt(((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3)
- 1)/(a*b - b^2))*log(-(b + (a^2*b - a*b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3)
)*sqrt(((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) - 1)/(a*b - b^2)) + x)
```

giac [B] time = 0.25, size = 299, normalized size = 2.74

$$\frac{\left(3\sqrt{a^2+\sqrt{ab}\,a}\,a^2b-4\sqrt{a^2+\sqrt{ab}\,a}\,ab^2-3\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,a^2+4\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,ab\right)|a|\arctan\left(\frac{2\sqrt{\ldots}}{\sqrt{\frac{2a+\sqrt{-4(\ldots}}{\ldots}}}\right)}{2\left(3a^5b-7a^4b^2+4a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] 1/2*(3*sqrt(a^2 + sqrt(a*b)*a)*a^2*b - 4*sqrt(a^2 + sqrt(a*b)*a)*a*b^2 - 3*
sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^2 + 4*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)
*a*b)*abs(a)*arctan(2*sqrt(1/2)*x/sqrt((2*a + sqrt(-4*(a - b)*a + 4*a^2))/a
))/(3*a^5*b - 7*a^4*b^2 + 4*a^3*b^3) + 1/2*(3*sqrt(a^2 - sqrt(a*b)*a)*a^2*b
 - 4*sqrt(a^2 - sqrt(a*b)*a)*a*b^2 + 3*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a^
2 - 4*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a*b)*abs(a)*arctan(2*sqrt(1/2)*x/sq
rt((2*a - sqrt(-4*(a - b)*a + 4*a^2))/a))/(3*a^5*b - 7*a^4*b^2 + 4*a^3*b^3)
```

maple [A] time = 0.01, size = 74, normalized size = 0.68

$$-\frac{a\,\mathrm{arctanh}\left(\frac{ax}{\sqrt{\left(-a+\sqrt{ab}\right)a}}\right)}{2\sqrt{ab}\,\sqrt{\left(-a+\sqrt{ab}\right)a}}-\frac{a\arctan\left(\frac{ax}{\sqrt{\left(a+\sqrt{ab}\right)a}}\right)}{2\sqrt{ab}\,\sqrt{\left(a+\sqrt{ab}\right)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] -1/2/(a*b)^(1/2)/((-a+(a*b)^(1/2))*a)^(1/2)*arctanh(1/((-a+(a*b)^(1/2))*a)^
(1/2)*a*x)*a-1/2/(a*b)^(1/2)/((a+(a*b)^(1/2))*a)^(1/2)*a*arctan(1/((a+(a*b)
^(1/2))*a)^(1/2)*a*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ax^4+2\,ax^2+a-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] integrate(1/(a*x^4 + 2*a*x^2 + a - b), x)
```

mupad [B] time = 5.78, size = 322, normalized size = 2.95

$$\frac{\ln\left(4\,a^3\,b\,\sqrt{-\frac{1}{a\,b+\sqrt{a\,b^3}}}-4\,a^3\,x+\frac{4\,a^4\,b\,x}{a\,b+\sqrt{a\,b^3}}\right)\sqrt{-\frac{1}{a\,b+\sqrt{a\,b^3}}}}{4}+\frac{\ln\left(4\,a^3\,x-4\,a^3\,b\,\sqrt{-\frac{1}{a\,b-\sqrt{a\,b^3}}}-\frac{4\,a^4\,b\,x}{a\,b-\sqrt{a\,b^3}}\right)\sqrt{-\frac{1}{a\,b-\sqrt{a\,b^3}}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b + 2*a*x^2 + a*x^4),x)
```

```
[Out] (log(4*a^3*b*(-1/(a*b + (a*b^3)^(1/2)))^(1/2) - 4*a^3*x + (4*a^4*b*x)/(a*b
+ (a*b^3)^(1/2)))*(-1/(a*b + (a*b^3)^(1/2)))^(1/2))/4 + (log(4*a^3*x - 4*a^
3*b*(-1/(a*b - (a*b^3)^(1/2)))^(1/2) - (4*a^4*b*x)/(a*b - (a*b^3)^(1/2)))*(
-1/(a*b - (a*b^3)^(1/2)))^(1/2))/4 - log(4*a^3*x + 4*a^3*b*(-1/(a*b + (a*b^
3)^(1/2)))^(1/2) - (4*a^4*b*x)/(a*b + (a*b^3)^(1/2)))*((a*b - (a*b^3)^(1/2)
)/(16*(a*b^3 - a^2*b^2)))^(1/2) - log(4*a^3*x + 16*a^3*b*(-1/(16*a*b - 16*(
a*b^3)^(1/2)))^(1/2) - (4*a^4*b*x)/(a*b - (a*b^3)^(1/2)))*((a*b + (a*b^3)^(
1/2))/(16*(a*b^3 - a^2*b^2)))^(1/2)
```

sympy [A] time = 0.95, size = 63, normalized size = 0.58

$$\operatorname{RootSum}\left(t^4\left(256a^2b^2-256ab^3\right)+32t^2ab+1,\left(t\mapsto t\log\left(-64t^3a^2b+64t^3ab^2-4ta-4tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] RootSum(_t**4*(256*a**2*b**2 - 256*a*b**3) + 32*_t**2*a*b + 1, Lambda(_t, _
t*log(-64*_t**3*a**2*b + 64*_t**3*a*b**2 - 4*_t*a - 4*_t*b + x)))
```

3.905 $\int \frac{1}{x^2(a-b+2ax^2+ax^4)} dx$

Optimal. Leaf size=121

$$-\frac{1}{x(a-b)}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{b}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{b}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}$$

```
[Out] -1/(a-b)/x-1/2*a^(1/4)*arctan(a^(1/4)*x/(a^(1/2)-b^(1/2))^(1/2))/(a^(1/2)-b
^(1/2))^(3/2)/b^(1/2)+1/2*a^(1/4)*arctan(a^(1/4)*x/(a^(1/2)+b^(1/2))^(1/2))
/b^(1/2)/(a^(1/2)+b^(1/2))^(3/2)
```

Rubi [A] time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1123, 1166, 205}

$$-\frac{1}{x(a-b)}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{b}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{b}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -(1/((a - b)*x)) - (a^(1/4)*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]])/(2
*(Sqrt[a] - Sqrt[b])^(3/2)*Sqrt[b]) + (a^(1/4)*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt
[a] + Sqrt[b]]])/(2*(Sqrt[a] + Sqrt[b])^(3/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a-b+2ax^2+ax^4\right)}\,dx &= -\frac{1}{(a-b)x}-\frac{\int \frac{-2a-ax^2}{a-b+2ax^2+ax^4}\,dx}{-a+b}\\ &= -\frac{1}{(a-b)x}-\frac{a\int \frac{1}{a-\sqrt{a}\sqrt{b}+ax^2}\,dx}{2\left(\sqrt{a}-\sqrt{b}\right)\sqrt{b}}+\frac{a\int \frac{1}{a+\sqrt{a}\sqrt{b}+ax^2}\,dx}{2\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}}\\ &= -\frac{1}{(a-b)x}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}}+\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.15, size = 143, normalized size = 1.18

$$\frac{\frac{\left(\sqrt{a}\sqrt{b}+a\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-\sqrt{a}\sqrt{b}}}\right)}{\sqrt{b}\sqrt{a-\sqrt{a}\sqrt{b}}}-\frac{\left(a-\sqrt{a}\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{2}{x}}{2(b-a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (2/x + ((a + Sqrt[a]*Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]*Sqrt[b]]]
)/(Sqrt[a - Sqrt[a]*Sqrt[b]]*Sqrt[b]) - ((a - Sqrt[a]*Sqrt[b])*ArcTan[(Sqrt
[a]*x)/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]))/(2*
(-a + b))
```

fricas [B] time = 0.79, size = 1612, normalized size = 13.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] 1/4*((a - b)*x*sqrt(-(a^2 + 3*a*b + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*sqr
t((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 +
15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4))*log((3*a
^2 + a*b)*x + (6*a^2*b + 2*a*b^2 - (a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*sqrt
((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 + 1
5*a^2*b^5 - 6*a*b^6 + b^7)))*sqrt(-(a^2 + 3*a*b + (a^3*b - 3*a^2*b^2 + 3*a*
b^3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 -
 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 -
b^4))) - (a - b)*x*sqrt(-(a^2 + 3*a*b + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)
*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^
4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4))*log(
(3*a^2 + a*b)*x - (6*a^2*b + 2*a*b^2 - (a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*
sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4
 + 15*a^2*b^5 - 6*a*b^6 + b^7)))*sqrt(-(a^2 + 3*a*b + (a^3*b - 3*a^2*b^2 +
3*a*b^3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b
^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^
3 - b^4))) + (a - b)*x*sqrt(-(a^2 + 3*a*b - (a^3*b - 3*a^2*b^2 + 3*a*b^3 -
b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^
3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4))*
log((3*a^2 + a*b)*x + (6*a^2*b + 2*a*b^2 + (a^4*b - 2*a^3*b^2 + 2*a*b^4 - b
^5)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3
*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))*sqrt(-(a^2 + 3*a*b - (a^3*b - 3*a^2*b^
2 + 3*a*b^3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a
^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*
a*b^3 - b^4))) - (a - b)*x*sqrt(-(a^2 + 3*a*b - (a^3*b - 3*a^2*b^2 + 3*a*b^
3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 2
0*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^
4))*log((3*a^2 + a*b)*x - (6*a^2*b + 2*a*b^2 + (a^4*b - 2*a^3*b^2 + 2*a*b^4
 - b^5)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20
*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))*sqrt(-(a^2 + 3*a*b - (a^3*b - 3*a^
2*b^2 + 3*a*b^3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 +
15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2
+ 3*a*b^3 - b^4))) - 4)/((a - b)*x)
```

giac [B] time = 0.38, size = 698, normalized size = 5.77

$$\frac{\left(\left(3\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,ab-4\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,b^2\right)(a-b)^2|a|-2\left(3\sqrt{a^2+\sqrt{ab}\,a}\,a^3b-7\sqrt{a^2+\sqrt{ab}\,a}\,a^2b^2+4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] 1/2*((3*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a*b - 4*sqrt(a^2 + sqrt(a*b)*a)*s
qrt(a*b)*b^2)*(a - b)^2*abs(a) - 2*(3*sqrt(a^2 + sqrt(a*b)*a)*a^3*b - 7*sqr
t(a^2 + sqrt(a*b)*a)*a^2*b^2 + 4*sqrt(a^2 + sqrt(a*b)*a)*a*b^3)*abs(a - b)*
abs(a) + (3*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^4 - 10*sqrt(a^2 + sqrt(a*b)
*a)*sqrt(a*b)*a^3*b + 11*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^2*b^2 - 4*sqrt
(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a*b^3)*abs(a))*arctan(x/sqrt((a^2 - a*b + sqr
t((a^2 - a*b)^2 - (a^2 - a*b)*(a^2 - 2*a*b + b^2)))/(a^2 - a*b)))/((3*a^6*b
 - 13*a^5*b^2 + 21*a^4*b^3 - 15*a^3*b^4 + 4*a^2*b^5)*abs(a - b)) - 1/2*((3*
sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a*b - 4*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)
*b^2)*(a - b)^2*abs(a) + 2*(3*sqrt(a^2 - sqrt(a*b)*a)*a^3*b - 7*sqrt(a^2 -
sqrt(a*b)*a)*a^2*b^2 + 4*sqrt(a^2 - sqrt(a*b)*a)*a*b^3)*abs(a - b)*abs(a) +
 (3*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a^4 - 10*sqrt(a^2 - sqrt(a*b)*a)*sqrt
(a*b)*a^3*b + 11*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a^2*b^2 - 4*sqrt(a^2 - s
qrt(a*b)*a)*sqrt(a*b)*a*b^3)*abs(a))*arctan(x/sqrt((a^2 - a*b - sqrt((a^2 -
 a*b)^2 - (a^2 - a*b)*(a^2 - 2*a*b + b^2)))/(a^2 - a*b)))/((3*a^6*b - 13*a^
5*b^2 + 21*a^4*b^3 - 15*a^3*b^4 + 4*a^2*b^5)*abs(a - b)) - 1/((a - b)*x)
```

maple [B] time = 0.01, size = 180, normalized size = 1.49

$$\frac{a^2 \operatorname{arctanh}\left(\frac{ax}{\sqrt{\left(-a+\sqrt{ab}\right)a}}\right)}{2\left(a-b\right)\sqrt{ab}\,\sqrt{\left(-a+\sqrt{ab}\right)a}}+\frac{a^2 \arctan\left(\frac{ax}{\sqrt{\left(a+\sqrt{ab}\right)a}}\right)}{2\left(a-b\right)\sqrt{ab}\,\sqrt{\left(a+\sqrt{ab}\right)a}}+\frac{a \operatorname{arctanh}\left(\frac{ax}{\sqrt{\left(-a+\sqrt{ab}\right)a}}\right)}{2\left(a-b\right)\sqrt{\left(-a+\sqrt{ab}\right)a}}-\frac{a \arctan\left(\frac{ax}{\sqrt{\left(a+\sqrt{ab}\right)a}}\right)}{2\left(a-b\right)\sqrt{\left(a+\sqrt{ab}\right)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] -1/(a-b)/x+1/2*a/(a-b)/((-a+(a*b)^(1/2))*a)^(1/2)*arctanh(1/((-a+(a*b)^(1/2
))*a)^(1/2)*a*x)+1/2*a^2/(a-b)/(a*b)^(1/2)/((-a+(a*b)^(1/2))*a)^(1/2)*arcta
nh(1/((-a+(a*b)^(1/2))*a)^(1/2)*a*x)-1/2*a/(a-b)/((a+(a*b)^(1/2))*a)^(1/2)*
arctan(1/((a+(a*b)^(1/2))*a)^(1/2)*a*x)+1/2*a^2/(a-b)/(a*b)^(1/2)/((a+(a*b)
^(1/2))*a)^(1/2)*arctan(1/((a+(a*b)^(1/2))*a)^(1/2)*a*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\frac{1}{2}\left(\frac{\left(6\sqrt{a^2+\sqrt{ab}\,a}\,a^2b-8\sqrt{a^2+\sqrt{ab}\,a}\,ab^2-3\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,a^2+\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,ab+4\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,b^2\right)a\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a+\sqrt{-4(a-b)a+4a^2}}{a}}}\right)}{3a^5b-7a^4b^2+4a^3b^3}+\frac{\left(6\right.}{}\right.}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] -a*integrate((x^2 + 2)/(a*x^4 + 2*a*x^2 + a - b), x)/(a - b) - 1/((a - b)*x
)
```

mupad [B] time = 5.12, size = 2774, normalized size = 22.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b + 2*a*x^2 + a*x^4)),x)
```

```
[Out] atan(((x*(8*a^7*b - 4*a^8 + 4*a^4*b^4 - 8*a^5*b^3) + (-(3*a*b^2 + a^2*b + 3
*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^
2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 - 128*a^5*b^4 + 192*a^6*b^3 - 128*a^7*b^2
 - x*(-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4
 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b - 64*a^4*b^6 + 320*a^5*b^5
- 640*a^6*b^4 + 640*a^7*b^3 - 320*a^8*b^2)))*(-(3*a*b^2 + a^2*b + 3*a*(a*b^
3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/
2)*1i + (x*(8*a^7*b - 4*a^8 + 4*a^4*b^4 - 8*a^5*b^3) - (-(3*a*b^2 + a^2*b +
 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*
b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 - 128*a^5*b^4 + 192*a^6*b^3 - 128*a^7*b
^2 + x*(-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b
^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b - 64*a^4*b^6 + 320*a^5*b^
5 - 640*a^6*b^4 + 640*a^7*b^3 - 320*a^8*b^2)))*(-(3*a*b^2 + a^2*b + 3*a*(a*
b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(
1/2)*1i)/(6*a^6*b - 2*a^7 + (x*(8*a^7*b - 4*a^8 + 4*a^4*b^4 - 8*a^5*b^3) +
(-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^
5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 - 128*a^5*b^4 + 192
*a^6*b^3 - 128*a^7*b^2 - x*(-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^
3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b - 64*
a^4*b^6 + 320*a^5*b^5 - 640*a^6*b^4 + 640*a^7*b^3 - 320*a^8*b^2)))*(-(3*a*b
^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^
2*b^3 + a^3*b^2)))^(1/2) - (x*(8*a^7*b - 4*a^8 + 4*a^4*b^4 - 8*a^5*b^3) - (
-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5
 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 - 128*a^5*b^4 + 192*
a^6*b^3 - 128*a^7*b^2 + x*(-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3
)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b - 64*a
^4*b^6 + 320*a^5*b^5 - 640*a^6*b^4 + 640*a^7*b^3 - 320*a^8*b^2)))*(-(3*a*b^
2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2
*b^3 + a^3*b^2)))^(1/2) + 2*a^4*b^3 - 6*a^5*b^2))*(-(3*a*b^2 + a^2*b + 3*a*
(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2))
)^(1/2)*2i - 1/(x*(a - b)) + atan(((x*(8*a^7*b - 4*a^8 + 4*a^4*b^4 - 8*a^5*
b^3) + (-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b
^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 - 128*a^5*b^
```

```
4 + 192*a^6*b^3 - 128*a^7*b^2 - x*(-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) -
b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*
b - 64*a^4*b^6 + 320*a^5*b^5 - 640*a^6*b^4 + 640*a^7*b^3 - 320*a^8*b^2)))*(
-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5
 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*1i + (x*(8*a^7*b - 4*a^8 + 4*a^4*b^4 - 8*a^
5*b^3) - (-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a
*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 - 128*a^5*
b^4 + 192*a^6*b^3 - 128*a^7*b^2 + x*(-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2)
- b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^
9*b - 64*a^4*b^6 + 320*a^5*b^5 - 640*a^6*b^4 + 640*a^7*b^3 - 320*a^8*b^2)))
*(-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b
^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*1i)/(6*a^6*b - 2*a^7 + (x*(8*a^7*b - 4*a^
8 + 4*a^4*b^4 - 8*a^5*b^3) + (-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*
b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 3
2*a^4*b^5 - 128*a^5*b^4 + 192*a^6*b^3 - 128*a^7*b^2 - x*(-(3*a*b^2 + a^2*b
- 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3
*b^2)))^(1/2)*(64*a^9*b - 64*a^4*b^6 + 320*a^5*b^5 - 640*a^6*b^4 + 640*a^7*
b^3 - 320*a^8*b^2)))*(-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/
2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2) - (x*(8*a^7*b - 4*a^8
 + 4*a^4*b^4 - 8*a^5*b^3) - (-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b
^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32
*a^4*b^5 - 128*a^5*b^4 + 192*a^6*b^3 - 128*a^7*b^2 + x*(-(3*a*b^2 + a^2*b -
 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*
b^2)))^(1/2)*(64*a^9*b - 64*a^4*b^6 + 320*a^5*b^5 - 640*a^6*b^4 + 640*a^7*b
^3 - 320*a^8*b^2)))*(-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2
))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2) + 2*a^4*b^3 - 6*a^5*b^
2))*(-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b^4
- b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*2i
```

sympy [A] time = 6.14, size = 134, normalized size = 1.11

$$\operatorname{RootSum}\left(t^4\left(256a^3b^2-768a^2b^3+768ab^4-256b^5\right)+t^2\left(32a^2b+96ab^2\right)+a,\left(t\mapsto t\log\left(x+\frac{64t^3a^4b-128t^3a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] RootSum(_t**4*(256*a**3*b**2 - 768*a**2*b**3 + 768*a*b**4 - 256*b**5) + _t*
*2*(32*a**2*b + 96*a*b**2) + a, Lambda(_t, _t*log(x + (64*_t**3*a**4*b - 12
8*_t**3*a**3*b**2 + 128*_t**3*a*b**4 - 64*_t**3*b**5 + 4*_t*a**3 + 40*_t*a*
*2*b + 20*_t*a*b**2)/(3*a**2 + a*b)))) - 1/(x*(a - b))
```

3.906 $\int \frac{x^5}{a+b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=69

$$\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(ax^4+2ax^2+a+b\right)}{2a}+\frac{x^2}{2a}$$

```
[Out] 1/2*x^2/a-1/2*ln(a*x^4+2*a*x^2+a+b)/a+1/2*(a-b)*arctan((x^2+1)*a^(1/2)/b^(1
/2))/a^(3/2)/b^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 703, 634, 618, 204, 628}

$$\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(ax^4+2ax^2+a+b\right)}{2a}+\frac{x^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x^2/(2*a) + ((a - b)*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*a^(3/2)*Sqrt[b
]) - Log[a + b + 2*a*x^2 + a*x^4]/(2*a)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{a+b+2ax^2+ax^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{a+b+2ax+ax^2}\,dx,x,x^2\right)\\
&= \frac{x^2}{2a}+\frac{\operatorname{Subst}\left(\int \frac{-a-b-2ax}{a+b+2ax+ax^2}\,dx,x,x^2\right)}{2a}\\
&= \frac{x^2}{2a}-\frac{\operatorname{Subst}\left(\int \frac{2a+2ax}{a+b+2ax+ax^2}\,dx,x,x^2\right)}{2a}+\frac{(a-b)\operatorname{Subst}\left(\int \frac{1}{a+b+2ax+ax^2}\,dx,x,x^2\right)}{2a}\\
&= \frac{x^2}{2a}-\frac{\log\left(a+b+2ax^2+ax^4\right)}{2a}-\frac{(a-b)\operatorname{Subst}\left(\int \frac{1}{-4ab-x^2}\,dx,x,2a\left(1+x^2\right)\right)}{a}\\
&= \frac{x^2}{2a}+\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(a+b+2ax^2+ax^4\right)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 62, normalized size = 0.90

$$\frac{\sqrt{a}\left(x^2-\log\left(a\left(x^2+1\right)^2+b\right)\right)+\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{\sqrt{b}}}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] (((a - b)*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/Sqrt[b] + Sqrt[a]*(x^2 - Log
[b + a*(1 + x^2)^2]))/(2*a^(3/2))
```

fricas [A] time = 0.91, size = 157, normalized size = 2.28

$$\left[\frac{2\,abx^2-2\,ab\log\left(ax^4+2\,ax^2+a+b\right)+\sqrt{-ab}\,(a-b)\log\left(\frac{ax^4+2\,ax^2+2\,\sqrt{-ab}\left(x^2+1\right)+a-b}{ax^4+2\,ax^2+a+b}\right)}{4\,a^2b},\ \frac{abx^2-ab\log\left(ax^4+2\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*x^2 - 2*a*b*log(a*x^4 + 2*a*x^2 + a + b) + sqrt(-a*b)*(a - b)*l
og((a*x^4 + 2*a*x^2 + 2*sqrt(-a*b)*(x^2 + 1) + a - b)/(a*x^4 + 2*a*x^2 + a
+ b)))/(a^2*b), 1/2*(a*b*x^2 - a*b*log(a*x^4 + 2*a*x^2 + a + b) - sqrt(a*b)
*(a - b)*arctan(sqrt(a*b)/(a*x^2 + a)))/(a^2*b)]
```

giac [A] time = 0.25, size = 58, normalized size = 0.84

$$\frac{x^2}{2\,a}+\frac{(a-b)\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a}-\frac{\log\left(ax^4+2\,ax^2+a+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] 1/2*x^2/a + 1/2*(a - b)*arctan((a*x^2 + a)/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*l
og(a*x^4 + 2*a*x^2 + a + b)/a
```

maple [A] time = 0.01, size = 84, normalized size = 1.22

$$-\frac{b\arctan\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}\,a}+\frac{x^2}{2a}+\frac{\arctan\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}}-\frac{\ln\left(a\,x^4+2a\,x^2+a+b\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] 1/2/a*x^2-1/2*ln(a*x^4+2*a*x^2+a+b)/a+1/2/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+2
*a)/(a*b)^(1/2))-1/2/a/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))*b
```

maxima [A] time = 3.03, size = 58, normalized size = 0.84

$$\frac{x^2}{2a}+\frac{(a-b)\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a}-\frac{\log\left(ax^4+2\,ax^2+a+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/a + 1/2*(a - b)*arctan((a*x^2 + a)/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*l
og(a*x^4 + 2*a*x^2 + a + b)/a
```

mupad [B] time = 0.18, size = 302, normalized size = 4.38

$$\frac{x^2}{2a}-\frac{\ln\left(a\,x^4+2\,a\,x^2+a+b\right)}{2\,a}-\frac{\operatorname{atan}\left(\frac{a\,b\left(x^2\left(\frac{\frac{\sqrt{a}\,(2\,a-2\,b)}{\sqrt{b}}+\frac{(a-b)\left(4\,a\,b-12\,a^2\right)}{4\,a^{3/2}\,\sqrt{b}}}{a+b}+\frac{\sqrt{a}\left(6\,a-2\,b-\frac{(a-b)^2}{b}+\frac{2\,a\,b-6\,a^2}{a}\right)}{\sqrt{b}\,(a+b)}\right)-\frac{\frac{(a-b)\left(16\,a\,b-\frac{8\,a^3+8\,b\,a^2}{a}+16\,a^2\right)}{4\,a^{3/2}\,\sqrt{b}}\cdots}{a+b}\cdots\right)}{a^2-2\,a\,b+b^2}\cdots\right)\cdots}{2\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b + 2*a*x^2 + a*x^4),x)
```

```
[Out] x^2/(2*a) - log(a + b + 2*a*x^2 + a*x^4)/(2*a) - (atan((a*b*(x^2*(((a^(1/2)
*(2*a - 2*b))/b^(1/2) + ((a - b)*(4*a*b - 12*a^2))/(4*a^(3/2)*b^(1/2)))/(a
+ b) + (a^(1/2)*(6*a - 2*b - (a - b)^2/b + (2*a*b - 6*a^2)/a))/(b^(1/2)*(a
+ b))) - (((a - b)*(16*a*b - (8*a^2*b + 8*a^3)/a + 16*a^2))/(4*a^(3/2)*b^(1
/2)) - ((16*a^2*b + 16*a^3)*(a - b))/(8*a^(5/2)*b^(1/2)))/(a + b) + (a^(1/2
)*(4*a + 4*b - (8*a*b - (8*a^2*b + 8*a^3)/(2*a) + 8*a^2)/a - ((a - b)^2*(a^
2*b + a^3))/(a^3*b)))/(b^(1/2)*(a + b))))/(a^2 - 2*a*b + b^2))*(a - b))/(2*
a^(3/2)*b^(1/2))
```

sympy [B] time = 1.60, size = 144, normalized size = 2.09

$$\left(-\frac{1}{2a}-\frac{\sqrt{-a^3b}\,(a-b)}{4a^3b}\right)\log\left(x^2+\frac{4ab\left(-\frac{1}{2a}-\frac{\sqrt{-a^3b}\,(a-b)}{4a^3b}\right)+a+b}{a-b}\right)+\left(-\frac{1}{2a}+\frac{\sqrt{-a^3b}\,(a-b)}{4a^3b}\right)\log\left(x^2+\frac{4ab\left(-\frac{1}{2a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] (-1/(2*a) - sqrt(-a**3*b)*(a - b)/(4*a**3*b))*log(x**2 + (4*a*b*(-1/(2*a) -
 sqrt(-a**3*b)*(a - b)/(4*a**3*b)) + a + b)/(a - b)) + (-1/(2*a) + sqrt(-a*
*3*b)*(a - b)/(4*a**3*b))*log(x**2 + (4*a*b*(-1/(2*a) + sqrt(-a**3*b)*(a -
b)/(4*a**3*b)) + a + b)/(a - b)) + x**2/(2*a)
```

3.907 $\int \frac{x^3}{a+b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=54

$$\frac{\log\left(ax^4+2ax^2+a+b\right)}{4a}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

```
[Out] 1/4*ln(a*x^4+2*a*x^2+a+b)/a-1/2*arctan((x^2+1)*a^(1/2)/b^(1/2))/a^(1/2)/b^(
1/2)
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 634, 618, 204, 628}

$$\frac{\log\left(ax^4+2ax^2+a+b\right)}{4a}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b]) + Log[a + b + 2*a*
x^2 + a*x^4]/(4*a)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a+b+2ax^2+ax^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{a+b+2ax+ax^2}\,dx, x, x^2\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{a+b+2ax+ax^2}\,dx, x, x^2\right)\right) + \frac{\operatorname{Subst}\left(\int \frac{2a+2ax}{a+b+2ax+ax^2}\,dx, x, x^2\right)}{4a}\\
&= \frac{\log\left(a+b+2ax^2+ax^4\right)}{4a} + \operatorname{Subst}\left(\int \frac{1}{-4ab-x^2}\,dx, x, 2a\left(1+x^2\right)\right)\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}} + \frac{\log\left(a+b+2ax^2+ax^4\right)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.91

$$\frac{\log\left(a\left(x^2+1\right)^2+b\right) - \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{\sqrt{b}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] ((-2*Sqrt[a]*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/Sqrt[b] + Log[b + a*(1 +
x^2)^2])/(4*a)
```

fricas [A] time = 0.87, size = 131, normalized size = 2.43

$$\left[\frac{b\log\left(ax^4+2\,ax^2+a+b\right)-\sqrt{-ab}\,\log\left(\frac{ax^4+2\,ax^2+2\,\sqrt{-ab}\,(x^2+1)+a-b}{ax^4+2\,ax^2+a+b}\right)}{4\,ab},\ \frac{b\log\left(ax^4+2\,ax^2+a+b\right)+2\,\sqrt{ab}\,\arctan\right.}{4\,ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] [1/4*(b*log(a*x^4 + 2*a*x^2 + a + b) - sqrt(-a*b)*log((a*x^4 + 2*a*x^2 + 2*
sqrt(-a*b)*(x^2 + 1) + a - b)/(a*x^4 + 2*a*x^2 + a + b)))/(a*b), 1/4*(b*log
(a*x^4 + 2*a*x^2 + a + b) + 2*sqrt(a*b)*arctan(sqrt(a*b)/(a*x^2 + a)))/(a*b
)]
```

giac [A] time = 0.23, size = 42, normalized size = 0.78

$$-\frac{\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\,\sqrt{ab}}+\frac{\log\left(ax^4+2\,ax^2+a+b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] -1/2*arctan((a*x^2 + a)/sqrt(a*b))/sqrt(a*b) + 1/4*log(a*x^4 + 2*a*x^2 + a
+ b)/a
```

maple [A] time = 0.00, size = 47, normalized size = 0.87

$$-\frac{\arctan\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}}+\frac{\ln\left(a\,x^4+2a\,x^2+a+b\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] 1/4/a*ln(a*x^4+2*a*x^2+a+b)-1/2/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+2*a)/(a*b)^
(1/2))
```

maxima [A] time = 2.87, size = 42, normalized size = 0.78

$$-\frac{\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\,\sqrt{ab}}+\frac{\log\left(ax^4+2\,ax^2+a+b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] -1/2*arctan((a*x^2 + a)/sqrt(a*b))/sqrt(a*b) + 1/4*log(a*x^4 + 2*a*x^2 + a
+ b)/a
```

mupad [B] time = 0.09, size = 85, normalized size = 1.57

$$\frac{\ln\left(a\,x^4+2\,a\,x^2+a+b\right)}{4\,a}-\frac{\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}}{a+b}+\frac{a^{3/2}}{\sqrt{b}\,\left(a+b\right)}+\frac{\sqrt{a}\,\sqrt{b}\,x^2}{a+b}+\frac{a^{3/2}\,x^2}{\sqrt{b}\,\left(a+b\right)}\right)}{2\,\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b + 2*a*x^2 + a*x^4),x)
```

```
[Out] log(a + b + 2*a*x^2 + a*x^4)/(4*a) - atan((a^(1/2)*b^(1/2))/(a + b) + a^(3/
2)/(b^(1/2)*(a + b)) + (a^(1/2)*b^(1/2)*x^2)/(a + b) + (a^(3/2)*x^2)/(b^(1/
2)*(a + b)))/(2*a^(1/2)*b^(1/2))
```

sympy [B] time = 0.60, size = 117, normalized size = 2.17

$$\left(\frac{1}{4a}-\frac{\sqrt{-a^3b}}{4a^2b}\right)\log\left(x^2+\frac{-4ab\left(\frac{1}{4a}-\frac{\sqrt{-a^3b}}{4a^2b}\right)+a+b}{a}\right)+\left(\frac{1}{4a}+\frac{\sqrt{-a^3b}}{4a^2b}\right)\log\left(x^2+\frac{-4ab\left(\frac{1}{4a}+\frac{\sqrt{-a^3b}}{4a^2b}\right)+a+b}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] (1/(4*a) - sqrt(-a**3*b)/(4*a**2*b))*log(x**2 + (-4*a*b*(1/(4*a) - sqrt(-a*
*3*b)/(4*a**2*b)) + a + b)/a) + (1/(4*a) + sqrt(-a**3*b)/(4*a**2*b))*log(x*
*2 + (-4*a*b*(1/(4*a) + sqrt(-a**3*b)/(4*a**2*b)) + a + b)/a)
```

3.908 $\int \frac{x}{a+b+2ax^2+ax^4} \, dx$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[Out] 1/2*arctan((x^2+1)*a^(1/2)/b^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1107, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{a+b+2ax^2+ax^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{a+b+2ax+ax^2}\,dx, x, x^2\right)$$
$$= -\,\text{Subst}\left(\int \frac{1}{-4ab-x^2}\,dx, x, 2a\left(1+x^2\right)\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b + 2*a*x^2 + a*x^4),x]
[Out] ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b])
```

fricas [A] time = 0.90, size = 91, normalized size = 2.94

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{ax^4+2\,ax^2-2\,\sqrt{-ab}\left(x^2+1\right)+a-b}{ax^4+2\,ax^2+a+b}\right)}{4\,ab}, -\frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{ax^2+a}\right)}{2\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
[Out] [-1/4*sqrt(-a*b)*log((a*x^4 + 2*a*x^2 - 2*sqrt(-a*b)*(x^2 + 1) + a - b)/(a*
x^4 + 2*a*x^2 + a + b))/(a*b), -1/2*sqrt(a*b)*arctan(sqrt(a*b)/(a*x^2 + a))
/(a*b)]
```

giac [A] time = 0.24, size = 21, normalized size = 0.68

$$\frac{\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\,\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] 1/2*arctan((a*x^2 + a)/sqrt(a*b))/sqrt(a*b)
```

maple [A] time = 0.00, size = 26, normalized size = 0.84

$$\frac{\arctan\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] 1/2/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))
```

maxima [A] time = 2.94, size = 21, normalized size = 0.68

$$\frac{\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] 1/2*arctan((a*x^2 + a)/sqrt(a*b))/sqrt(a*b)
```

mupad [B] time = 0.05, size = 24, normalized size = 0.77

$$\frac{\mathrm{atan}\left(\frac{\sqrt{a}+\sqrt{a}\,x^2}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b + 2*a*x^2 + a*x^4),x)
```

```
[Out] atan((a^(1/2) + a^(1/2)*x^2)/b^(1/2))/(2*a^(1/2)*b^(1/2))
```

sympy [B] time = 0.46, size = 60, normalized size = 1.94

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-b\sqrt{-\frac{1}{ab}}+x^2+1\right)}{4}+\frac{\sqrt{-\frac{1}{ab}}\log\left(b\sqrt{-\frac{1}{ab}}+x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-b*sqrt(-1/(a*b)) + x**2 + 1)/4 + sqrt(-1/(a*b))*log(b*
sqrt(-1/(a*b)) + x**2 + 1)/4
```

3.909 $\int \frac{1}{x(a+b+2ax^2+ax^4)}\,dx$

Optimal. Leaf size=69

$$-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}(a+b)}-\frac{\log\left(ax^4+2ax^2+a+b\right)}{4(a+b)}+\frac{\log(x)}{a+b}$$

```
[Out] ln(x)/(a+b)-1/4*ln(a*x^4+2*a*x^2+a+b)/(a+b)-1/2*arctan((x^2+1)*a^(1/2)/b^(1
/2))*a^(1/2)/(a+b)/b^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1114, 705, 29, 634, 618, 204, 628}

$$-\frac{\log\left(ax^4+2ax^2+a+b\right)}{4(a+b)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}(a+b)}+\frac{\log(x)}{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -(Sqrt[a]*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*Sqrt[b]*(a + b)) + Log[x]
/(a + b) - Log[a + b + 2*a*x^2 + a*x^4]/(4*(a + b))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+b+2ax^2+ax^4\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\left(a+b+2ax+ax^2\right)}\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2(a+b)}+\frac{\text{Subst}\left(\int \frac{-2a-ax}{a+b+2ax+ax^2}\,dx,x,x^2\right)}{2(a+b)}\\
&= \frac{\log(x)}{a+b}-\frac{\text{Subst}\left(\int \frac{2a+2ax}{a+b+2ax+ax^2}\,dx,x,x^2\right)}{4(a+b)}-\frac{a\,\text{Subst}\left(\int \frac{1}{a+b+2ax+ax^2}\,dx,x,x^2\right)}{2(a+b)}\\
&= \frac{\log(x)}{a+b}-\frac{\log\left(a+b+2ax^2+ax^4\right)}{4(a+b)}+\frac{a\,\text{Subst}\left(\int \frac{1}{-4ab-x^2}\,dx,x,2a\left(1+x^2\right)\right)}{a+b}\\
&= -\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a+b)}+\frac{\log(x)}{a+b}-\frac{\log\left(a+b+2ax^2+ax^4\right)}{4(a+b)}
\end{aligned}
$$

Mathematica [C] time = 0.06, size = 105, normalized size = 1.52

$$\frac{i\left(\sqrt{a}+i\sqrt{b}\right)\log\left(\sqrt{a}\,\left(x^2+1\right)-i\sqrt{b}\right)+\left(-\sqrt{b}-i\sqrt{a}\right)\log\left(\sqrt{a}\,\left(x^2+1\right)+i\sqrt{b}\right)+4\sqrt{b}\,\log(x)}{4\sqrt{b}\,(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (4*Sqrt[b]*Log[x] + I*(Sqrt[a] + I*Sqrt[b])*Log[(-I)*Sqrt[b] + Sqrt[a]*(1 +
 x^2)] + ((-I)*Sqrt[a] - Sqrt[b])*Log[I*Sqrt[b] + Sqrt[a]*(1 + x^2)])/(4*Sq
rt[b]*(a + b))
```

fricas [A] time = 0.95, size = 147, normalized size = 2.13

$$\left[\frac{\sqrt{-\frac{a}{b}}\,\log\left(\frac{ax^4+2\,ax^2-2\left(bx^2+b\right)\sqrt{-\frac{a}{b}}+a-b}{ax^4+2\,ax^2+a+b}\right)-\log\left(ax^4+2\,ax^2+a+b\right)+4\,\log(x)}{4\,(a+b)},\ \frac{2\,\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{\frac{a}{b}}}{ax^2+a}\right)-\log\left(ax^4\right.}{4\,(a+b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b)*sqrt(-a/b) + a - b)/(
a*x^4 + 2*a*x^2 + a + b)) - log(a*x^4 + 2*a*x^2 + a + b) + 4*log(x))/(a + b
), 1/4*(2*sqrt(a/b)*arctan(b*sqrt(a/b)/(a*x^2 + a)) - log(a*x^4 + 2*a*x^2 +
 a + b) + 4*log(x))/(a + b)]
```

giac [A] time = 0.23, size = 61, normalized size = 0.88

$$-\frac{a\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,(a+b)}-\frac{\log\left(ax^4+2\,ax^2+a+b\right)}{4\,(a+b)}+\frac{\log\left(x^2\right)}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] -1/2*a*arctan((a*x^2 + a)/sqrt(a*b))/(sqrt(a*b)*(a + b)) - 1/4*log(a*x^4 +
2*a*x^2 + a + b)/(a + b) + 1/2*log(x^2)/(a + b)
```

maple [A] time = 0.01, size = 63, normalized size = 0.91

$$-\frac{a\arctan\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\,(a+b)\,\sqrt{ab}}+\frac{\ln(x)}{a+b}-\frac{\ln\left(a\,x^4+2a\,x^2+a+b\right)}{4\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] -1/4*ln(a*x^4+2*a*x^2+a+b)/(a+b)-1/2*a/(a+b)/(a*b)^(1/2)*arctan(1/2*(2*a*x^
2+2*a)/(a*b)^(1/2))+ln(x)/(a+b)
```

maxima [A] time = 3.01, size = 61, normalized size = 0.88

$$-\frac{a\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\sqrt{ab}\,(a+b)}-\frac{\log\left(ax^4+2\,ax^2+a+b\right)}{4\,(a+b)}+\frac{\log\left(x^2\right)}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] -1/2*a*arctan((a*x^2 + a)/sqrt(a*b))/(sqrt(a*b)*(a + b)) - 1/4*log(a*x^4 +
2*a*x^2 + a + b)/(a + b) + 1/2*log(x^2)/(a + b)
```

mupad [B] time = 4.64, size = 71, normalized size = 1.03

$$\frac{\ln(x)}{a+b}-\frac{4\,b\,\ln\left(a\,x^4+2\,a\,x^2+a+b\right)}{16\,b^2+16\,a\,b}-\frac{\sqrt{a}\,\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{a}\,x^2}{\sqrt{b}}\right)}{2\,\sqrt{b}\,\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b + 2*a*x^2 + a*x^4)),x)
```

```
[Out] log(x)/(a + b) - (4*b*log(a + b + 2*a*x^2 + a*x^4))/(16*a*b + 16*b^2) - (a^
(1/2)*atan(a^(1/2)/b^(1/2) + (a^(1/2)*x^2)/b^(1/2)))/(2*b^(1/2)*(a + b))
```

sympy [B] time = 5.96, size = 194, normalized size = 2.81

$$\left(-\frac{1}{4\,(a+b)}-\frac{\sqrt{-ab}}{4b\,(a+b)}\right)\log\left(x^2+\frac{-4ab\left(-\frac{1}{4(a+b)}-\frac{\sqrt{-ab}}{4b(a+b)}\right)+a-4b^2\left(-\frac{1}{4(a+b)}-\frac{\sqrt{-ab}}{4b(a+b)}\right)-b}{a}\right)+\left(-\frac{1}{4\,(a+b)}+\frac{\sqrt{}}{4b\,(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] (-1/(4*(a + b)) - sqrt(-a*b)/(4*b*(a + b)))*log(x**2 + (-4*a*b*(-1/(4*(a +
b)) - sqrt(-a*b)/(4*b*(a + b))) + a - 4*b**2*(-1/(4*(a + b)) - sqrt(-a*b)/(
4*b*(a + b))) - b)/a) + (-1/(4*(a + b)) + sqrt(-a*b)/(4*b*(a + b)))*log(x**
2 + (-4*a*b*(-1/(4*(a + b)) + sqrt(-a*b)/(4*b*(a + b))) + a - 4*b**2*(-1/(4
*(a + b)) + sqrt(-a*b)/(4*b*(a + b))) - b)/a) + log(x)/(a + b)
```

3.910 $\int \frac{1}{x^3(a+b+2ax^2+ax^4)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2x^2(a+b)}+\frac{\sqrt{a}\,(a-b)\tan^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a+b)^2}+\frac{a\log\left(ax^4+2ax^2+a+b\right)}{2(a+b)^2}-\frac{2a\log(x)}{(a+b)^2}$$

```
[Out] -1/2/(a+b)/x^2-2*a*ln(x)/(a+b)^2+1/2*a*ln(a*x^4+2*a*x^2+a+b)/(a+b)^2+1/2*(a
-b)*arctan((x^2+1)*a^(1/2)/b^(1/2))*a^(1/2)/(a+b)^2/b^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1114, 709, 800, 634, 618, 204, 628}

$$-\frac{1}{2x^2(a+b)}+\frac{a\log\left(ax^4+2ax^2+a+b\right)}{2(a+b)^2}+\frac{\sqrt{a}\,(a-b)\tan^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a+b)^2}-\frac{2a\log(x)}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -1/(2*(a + b)*x^2) + (Sqrt[a]*(a - b)*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/
(2*Sqrt[b]*(a + b)^2) - (2*a*Log[x])/(a + b)^2 + (a*Log[a + b + 2*a*x^2 + a
*x^4])/(2*(a + b)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b+2ax^2+ax^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a+b+2ax+ax^2\right)}\,dx,x,x^2\right)$$

$$= -\frac{1}{2(a+b)x^2}+\frac{\text{Subst}\left(\int \frac{-2a-ax}{x\left(a+b+2ax+ax^2\right)}\,dx,x,x^2\right)}{2(a+b)}$$

$$= -\frac{1}{2(a+b)x^2}+\frac{\text{Subst}\left(\int \left(-\frac{2a}{(a+b)x}+\frac{a(3a-b+2ax)}{(a+b)\left(a+b+2ax+ax^2\right)}\right)dx,x,x^2\right)}{2(a+b)}$$

$$= -\frac{1}{2(a+b)x^2}-\frac{2a\log(x)}{(a+b)^2}+\frac{a\,\text{Subst}\left(\int \frac{3a-b+2ax}{a+b+2ax+ax^2}\,dx,x,x^2\right)}{2(a+b)^2}$$

$$= -\frac{1}{2(a+b)x^2}-\frac{2a\log(x)}{(a+b)^2}+\frac{a\,\text{Subst}\left(\int \frac{2a+2ax}{a+b+2ax+ax^2}\,dx,x,x^2\right)}{2(a+b)^2}+\frac{(a(a-b))\,\text{Subst}}{}$$

$$= -\frac{1}{2(a+b)x^2}-\frac{2a\log(x)}{(a+b)^2}+\frac{a\log\left(a+b+2ax^2+ax^4\right)}{2(a+b)^2}-\frac{(a(a-b))\,\text{Subst}\left(\int \right.}{(a}$$

$$= -\frac{1}{2(a+b)x^2}+\frac{\sqrt{a}\,(a-b)\tan^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a+b)^2}-\frac{2a\log(x)}{(a+b)^2}+\frac{a\log\left(a+b+2ax^2\right.}{2(a+b)^2}$$

Mathematica [C] time = 0.10, size = 163, normalized size = 1.83

$$\frac{\left(2a^{3/2}\sqrt{b}-ia^2+iab\right)\log\left(\sqrt{a}\,x^2+\sqrt{a}-i\sqrt{b}\right)}{4\sqrt{a}\,\sqrt{b}\,(a+b)^2}+\frac{\left(2a^{3/2}\sqrt{b}+ia^2-iab\right)\log\left(\sqrt{a}\,x^2+\sqrt{a}+i\sqrt{b}\right)}{4\sqrt{a}\,\sqrt{b}\,(a+b)^2}-\frac{1}{2x^2(a+b)}-$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -1/2*1/((a + b)*x^2) - (2*a*Log[x])/(a + b)^2 + (((-I)*a^2 + 2*a^(3/2)*Sqrt
[b] + I*a*b)*Log[Sqrt[a] - I*Sqrt[b] + Sqrt[a]*x^2])/(4*Sqrt[a]*Sqrt[b]*(a
+ b)^2) + ((I*a^2 + 2*a^(3/2)*Sqrt[b] - I*a*b)*Log[Sqrt[a] + I*Sqrt[b] + Sq
rt[a]*x^2])/(4*Sqrt[a]*Sqrt[b]*(a + b)^2)
```

fricas [A] time = 0.86, size = 208, normalized size = 2.34

$$\left[-\frac{(a-b)x^2\sqrt{-\frac{a}{b}}\,\log\left(\frac{ax^4+2\,ax^2-2\left(bx^2+b\right)\sqrt{-\frac{a}{b}}+a-b}{ax^4+2\,ax^2+a+b}\right)-2\,ax^2\log\left(ax^4+2\,ax^2+a+b\right)+8\,ax^2\log(x)+2\,a+2\,b}{4\left(a^2+2\,ab+b^2\right)x^2},-\frac{(a\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] [-1/4*((a - b)*x^2*sqrt(-a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b)*sqrt(-a/
b) + a - b)/(a*x^4 + 2*a*x^2 + a + b)) - 2*a*x^2*log(a*x^4 + 2*a*x^2 + a +
b) + 8*a*x^2*log(x) + 2*a + 2*b)/((a^2 + 2*a*b + b^2)*x^2), -1/2*((a - b)*x
^2*sqrt(a/b)*arctan(b*sqrt(a/b)/(a*x^2 + a)) - a*x^2*log(a*x^4 + 2*a*x^2 +
a + b) + 4*a*x^2*log(x) + a + b)/((a^2 + 2*a*b + b^2)*x^2)]
```

giac [A] time = 0.28, size = 125, normalized size = 1.40

$$\frac{a\log\left(ax^4+2\,ax^2+a+b\right)}{2\left(a^2+2\,ab+b^2\right)}-\frac{a\log\left(x^2\right)}{a^2+2\,ab+b^2}+\frac{\left(a^2-ab\right)\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\left(a^2+2\,ab+b^2\right)\sqrt{ab}}+\frac{2\,ax^2-a-b}{2\left(a^2+2\,ab+b^2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] 1/2*a*log(a*x^4 + 2*a*x^2 + a + b)/(a^2 + 2*a*b + b^2) - a*log(x^2)/(a^2 +
2*a*b + b^2) + 1/2*(a^2 - a*b)*arctan((a*x^2 + a)/sqrt(a*b))/((a^2 + 2*a*b
+ b^2)*sqrt(a*b)) + 1/2*(2*a*x^2 - a - b)/((a^2 + 2*a*b + b^2)*x^2)
```

maple [A] time = 0.01, size = 110, normalized size = 1.24

$$\frac{a^2\arctan\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\,(a+b)^2\,\sqrt{ab}}-\frac{ab\arctan\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\,(a+b)^2\,\sqrt{ab}}-\frac{2a\ln(x)}{(a+b)^2}+\frac{a\ln\left(a\,x^4+2a\,x^2+a+b\right)}{2\,(a+b)^2}-\frac{1}{2\,(a+b)\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] 1/2*a*ln(a*x^4+2*a*x^2+a+b)/(a+b)^2+1/2/(a+b)^2*a^2/(a*b)^(1/2)*arctan(1/2*
(2*a*x^2+2*a)/(a*b)^(1/2))-1/2/(a+b)^2*a/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+2*
a)/(a*b)^(1/2))*b-1/2/(a+b)/x^2-2*a*ln(x)/(a+b)^2
```

maxima [A] time = 2.90, size = 104, normalized size = 1.17

$$\frac{a\log\left(ax^4+2\,ax^2+a+b\right)}{2\left(a^2+2\,ab+b^2\right)}-\frac{a\log\left(x^2\right)}{a^2+2\,ab+b^2}+\frac{\left(a^2-ab\right)\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\left(a^2+2\,ab+b^2\right)\sqrt{ab}}-\frac{1}{2\,(a+b)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] 1/2*a*log(a*x^4 + 2*a*x^2 + a + b)/(a^2 + 2*a*b + b^2) - a*log(x^2)/(a^2 +
2*a*b + b^2) + 1/2*(a^2 - a*b)*arctan((a*x^2 + a)/sqrt(a*b))/((a^2 + 2*a*b
+ b^2)*sqrt(a*b)) - 1/2/((a + b)*x^2)
```

mupad [B] time = 7.39, size = 3313, normalized size = 37.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b + 2*a*x^2 + a*x^4)),x)
```

```
[Out] (8*a*b*log(((2*a^5)/(a + b)^3 - (a/(2*(a + b)^2) - (-(a*(a - b)^2)/(b*(a +
b)^4))^(1/2)/4)*((12*a^5*x^2)/(a + b)^2 - (a/(2*(a + b)^2) - (-(a*(a - b)^2
)/(b*(a + b)^4))^(1/2)/4)*((8*a^4*(3*a - b))/(a + b) + 16*a^4*(a/(2*(a + b)
^2) - (-(a*(a - b)^2)/(b*(a + b)^4))^(1/2)/4)*(a + b + a*x^2 - 5*b*x^2) + (
4*a^4*x^2*(7*a + 5*b))/(a + b)) + (a^4*(15*a - b))/(a + b)^2) + (a^5*x^2)/(
a + b)^3)*((2*a^5)/(a + b)^3 - (a/(2*(a + b)^2) + (-(a*(a - b)^2)/(b*(a + b
)^4))^(1/2)/4)*((12*a^5*x^2)/(a + b)^2 - (a/(2*(a + b)^2) + (-(a*(a - b)^2)
/(b*(a + b)^4))^(1/2)/4)*((8*a^4*(3*a - b))/(a + b) + 16*a^4*(a/(2*(a + b)^
2) + (-(a*(a - b)^2)/(b*(a + b)^4))^(1/2)/4)*(a + b + a*x^2 - 5*b*x^2) + (4
*a^4*x^2*(7*a + 5*b))/(a + b)) + (a^4*(15*a - b))/(a + b)^2) + (a^5*x^2)/(a
 + b)^3)))/(32*a*b^2 + 16*a^2*b + 16*b^3) - (2*a*log(x))/(2*a*b + a^2 + b^2
) - 1/(2*x^2*(a + b)) + (a^(1/2)*atan(((13*a^2 - 34*a*b + b^2)*((8*a*b*((14
*a^5*b + 15*a^6 - a^4*b^2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (8*a*b*((40*a^
6*b + 24*a^7 - 8*a^4*b^3 + 8*a^5*b^2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) + (8*
a*b*(64*a^7*b + 16*a^8 + 16*a^4*b^4 + 64*a^5*b^3 + 96*a^6*b^2))/((32*a*b^2
+ 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))))/(32*a*b^2 + 16*a^2*
b + 16*b^3)))/(32*a*b^2 + 16*a^2*b + 16*b^3) - (2*a^5)/(3*a*b^2 + 3*a^2*b +
 a^3 + b^3) + (a^(1/2)*((a^(1/2)*(a - b)*((40*a^6*b + 24*a^7 - 8*a^4*b^3 +
8*a^5*b^2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) + (8*a*b*(64*a^7*b + 16*a^8 + 16
*a^4*b^4 + 64*a^5*b^3 + 96*a^6*b^2))/((32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b
^2 + 3*a^2*b + a^3 + b^3))))/(4*b^(1/2)*(2*a*b + a^2 + b^2)) + (2*a^(3/2)*b
^(1/2)*(a - b)*(64*a^7*b + 16*a^8 + 16*a^4*b^4 + 64*a^5*b^3 + 96*a^6*b^2))/
((2*a*b + a^2 + b^2)*(32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^
3 + b^3)))*(a - b))/(4*b^(1/2)*(2*a*b + a^2 + b^2)) + (a^2*(a - b)^2*(64*a^
```

7*b + 16*a^8 + 16*a^4*b^4 + 64*a^5*b^3 + 96*a^6*b^2))/(2*(2*a*b + a^2 + b^2
)^2*(32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)))*(24*a*
b^(13/2) + 4*b^(15/2) + 4*a^6*b^(3/2) + 24*a^5*b^(5/2) + 60*a^4*b^(7/2) + 8
0*a^3*b^(9/2) + 60*a^2*b^(11/2)))/((a + b)^3*(98*a*b + a^2 + b^2)*(a^(13/2)
 - 2*a^(11/2)*b + a^(9/2)*b^2)) - (x^2*(((13*a^2 - 34*a*b + b^2)*(a^5/(3*a*
b^2 + 3*a^2*b + a^3 + b^3) - (8*a*b*((12*a^5*b + 12*a^6)/(3*a*b^2 + 3*a^2*b
 + a^3 + b^3) - (8*a*b*((76*a^6*b + 28*a^7 + 20*a^4*b^3 + 68*a^5*b^2)/(3*a*
b^2 + 3*a^2*b + a^3 + b^3) - (8*a*b*(32*a^7*b - 16*a^8 + 80*a^4*b^4 + 224*a
^5*b^3 + 192*a^6*b^2))/((32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b +
 a^3 + b^3))))/(32*a*b^2 + 16*a^2*b + 16*b^3)))/(32*a*b^2 + 16*a^2*b + 16*b
^3) - (a^(1/2)*((a^(1/2)*(a - b)*((76*a^6*b + 28*a^7 + 20*a^4*b^3 + 68*a^5*
b^2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (8*a*b*(32*a^7*b - 16*a^8 + 80*a^4*b
^4 + 224*a^5*b^3 + 192*a^6*b^2))/((32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 +
 3*a^2*b + a^3 + b^3))))/(4*b^(1/2)*(2*a*b + a^2 + b^2)) - (2*a^(3/2)*b^(1/
2)*(a - b)*(32*a^7*b - 16*a^8 + 80*a^4*b^4 + 224*a^5*b^3 + 192*a^6*b^2))/((
2*a*b + a^2 + b^2)*(32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3
 + b^3)))*(a - b))/(4*b^(1/2)*(2*a*b + a^2 + b^2)) + (a^2*(a - b)^2*(32*a^7*
b - 16*a^8 + 80*a^4*b^4 + 224*a^5*b^3 + 192*a^6*b^2))/(2*(2*a*b + a^2 + b^2
)^2*(32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))))/((a +
 b)^3*(98*a*b + a^2 + b^2)) + (a^(1/2)*(a^2 - 34*a*b + 13*b^2)*((8*a*b*((a^
(1/2)*(a - b)*((76*a^6*b + 28*a^7 + 20*a^4*b^3 + 68*a^5*b^2)/(3*a*b^2 + 3*a
^2*b + a^3 + b^3) - (8*a*b*(32*a^7*b - 16*a^8 + 80*a^4*b^4 + 224*a^5*b^3 +
192*a^6*b^2))/((32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^
3))))/(4*b^(1/2)*(2*a*b + a^2 + b^2)) - (2*a^(3/2)*b^(1/2)*(a - b)*(32*a^7*
b - 16*a^8 + 80*a^4*b^4 + 224*a^5*b^3 + 192*a^6*b^2))/((2*a*b + a^2 + b^2)*
(32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))))/(32*a*b^2
 + 16*a^2*b + 16*b^3) - (a^(1/2)*(a - b)*((12*a^5*b + 12*a^6)/(3*a*b^2 + 3*
a^2*b + a^3 + b^3) - (8*a*b*((76*a^6*b + 28*a^7 + 20*a^4*b^3 + 68*a^5*b^2)/
(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (8*a*b*(32*a^7*b - 16*a^8 + 80*a^4*b^4 +
224*a^5*b^3 + 192*a^6*b^2))/((32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^
2*b + a^3 + b^3))))/(32*a*b^2 + 16*a^2*b + 16*b^3)))/(4*b^(1/2)*(2*a*b + a^
2 + b^2)) + (a^(3/2)*(a - b)^3*(32*a^7*b - 16*a^8 + 80*a^4*b^4 + 224*a^5*b^
3 + 192*a^6*b^2))/(64*b^(3/2)*(2*a*b + a^2 + b^2)^3*(3*a*b^2 + 3*a^2*b + a^
3 + b^3))))/(b^(1/2)*(a + b)^3*(98*a*b + a^2 + b^2)))*(24*a*b^(13/2) + 4*b^
(15/2) + 4*a^6*b^(3/2) + 24*a^5*b^(5/2) + 60*a^4*b^(7/2) + 80*a^3*b^(9/2) +
 60*a^2*b^(11/2)))/(a^(13/2) - 2*a^(11/2)*b + a^(9/2)*b^2) + (a^(1/2)*((a^(
1/2)*(a - b)*((14*a^5*b + 15*a^6 - a^4*b^2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3)
 - (8*a*b*((40*a^6*b + 24*a^7 - 8*a^4*b^3 + 8*a^5*b^2)/(3*a*b^2 + 3*a^2*b +
 a^3 + b^3) + (8*a*b*(64*a^7*b + 16*a^8 + 16*a^4*b^4 + 64*a^5*b^3 + 96*a^6*
b^2))/((32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))))/(3
2*a*b^2 + 16*a^2*b + 16*b^3)))/(4*b^(1/2)*(2*a*b + a^2 + b^2)) - (8*a*b*((a
^(1/2)*(a - b)*((40*a^6*b + 24*a^7 - 8*a^4*b^3 + 8*a^5*b^2)/(3*a*b^2 + 3*a^
2*b + a^3 + b^3) + (8*a*b*(64*a^7*b + 16*a^8 + 16*a^4*b^4 + 64*a^5*b^3 + 96
*a^6*b^2))/((32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))
))/(4*b^(1/2)*(2*a*b + a^2 + b^2)) + (2*a^(3/2)*b^(1/2)*(a - b)*(64*a^7*b +

```
 16*a^8 + 16*a^4*b^4 + 64*a^5*b^3 + 96*a^6*b^2))/((2*a*b + a^2 + b^2)*(32*a
*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))))/(32*a*b^2 + 16
*a^2*b + 16*b^3) + (a^(3/2)*(a - b)^3*(64*a^7*b + 16*a^8 + 16*a^4*b^4 + 64*
a^5*b^3 + 96*a^6*b^2))/(64*b^(3/2)*(2*a*b + a^2 + b^2)^3*(3*a*b^2 + 3*a^2*b
 + a^3 + b^3)))*(a^2 - 34*a*b + 13*b^2)*(24*a*b^(13/2) + 4*b^(15/2) + 4*a^6
*b^(3/2) + 24*a^5*b^(5/2) + 60*a^4*b^(7/2) + 80*a^3*b^(9/2) + 60*a^2*b^(11/
2)))/(b^(1/2)*(a + b)^3*(98*a*b + a^2 + b^2)*(a^(13/2) - 2*a^(11/2)*b + a^(
9/2)*b^2)))*(a - b))/(2*b^(1/2)*(2*a*b + a^2 + b^2))
```

sympy [B] time = 41.75, size = 386, normalized size = 4.34

$$-\frac{2a\log(x)}{(a+b)^2}+\left(\frac{a}{2(a+b)^2}-\frac{\sqrt{-ab}\,(a-b)}{4b\left(a^2+2ab+b^2\right)}\right)\log\left(x^2+\frac{4a^2b\left(\frac{a}{2(a+b)^2}-\frac{\sqrt{-ab}\,(a-b)}{4b\left(a^2+2ab+b^2\right)}\right)+a^2+8ab^2\left(\frac{a}{2(a+b)^2}-\frac{\sqrt{}}{4b(}\right.}{a^2-ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] -2*a*log(x)/(a + b)**2 + (a/(2*(a + b)**2) - sqrt(-a*b)*(a - b)/(4*b*(a**2
+ 2*a*b + b**2)))*log(x**2 + (4*a**2*b*(a/(2*(a + b)**2) - sqrt(-a*b)*(a -
b)/(4*b*(a**2 + 2*a*b + b**2))) + a**2 + 8*a*b**2*(a/(2*(a + b)**2) - sqrt(
-a*b)*(a - b)/(4*b*(a**2 + 2*a*b + b**2))) - 3*a*b + 4*b**3*(a/(2*(a + b)**
2) - sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a*b + b**2))))/(a**2 - a*b)) + (a/(2
*(a + b)**2) + sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a*b + b**2)))*log(x**2 + (
4*a**2*b*(a/(2*(a + b)**2) + sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a*b + b**2))
) + a**2 + 8*a*b**2*(a/(2*(a + b)**2) + sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a
*b + b**2))) - 3*a*b + 4*b**3*(a/(2*(a + b)**2) + sqrt(-a*b)*(a - b)/(4*b*(
a**2 + 2*a*b + b**2))))/(a**2 - a*b)) - 1/(x**2*(2*a + 2*b))
```

3.911 $\int \frac{x^4}{a+b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=432

$$\frac{\left(-2\sqrt{a}\sqrt{a+b}+a+b\right)\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\left(-2\sqrt{a}\sqrt{a+b}+a+b\right)\log\left(\sqrt{2}\sqrt[4]{a}\right.}{4\sqrt{2}a^{5/4}\sqrt{a+b}}$$

```
[Out] x/a+1/8*ln(x^2*a^(1/2)+(a+b)^(1/2)-a^(1/4)*x*2^(1/2)*(-a^(1/2)+(a+b)^(1/2))
^(1/2))*(a+b-2*a^(1/2)*(a+b)^(1/2))/a^(5/4)*2^(1/2)/(a+b)^(1/2)/(-a^(1/2)+(
a+b)^(1/2))^(1/2)-1/8*ln(x^2*a^(1/2)+(a+b)^(1/2)+a^(1/4)*x*2^(1/2)*(-a^(1/2
)+(a+b)^(1/2))^(1/2))*(a+b-2*a^(1/2)*(a+b)^(1/2))/a^(5/4)*2^(1/2)/(a+b)^(1/
2)/(-a^(1/2)+(a+b)^(1/2))^(1/2)+1/4*arctan((-a^(1/4)*x*2^(1/2)+(-a^(1/2)+(a
+b)^(1/2))^(1/2))/(a^(1/2)+(a+b)^(1/2))^(1/2))*(a+b+2*a^(1/2)*(a+b)^(1/2))/
a^(5/4)*2^(1/2)/(a+b)^(1/2)/(a^(1/2)+(a+b)^(1/2))^(1/2)-1/4*arctan((a^(1/4)
*x*2^(1/2)+(-a^(1/2)+(a+b)^(1/2))^(1/2))/(a^(1/2)+(a+b)^(1/2))^(1/2))*(a+b+
2*a^(1/2)*(a+b)^(1/2))/a^(5/4)*2^(1/2)/(a+b)^(1/2)/(a^(1/2)+(a+b)^(1/2))^(1
/2)
```

Rubi [A] time = 0.89, antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1122, 1169, 634, 618, 204, 628}

$$\frac{\left(-2\sqrt{a}\sqrt{a+b}+a+b\right)\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\left(-2\sqrt{a}\sqrt{a+b}+a+b\right)\log\left(\sqrt{2}\sqrt[4]{a}\right.}{4\sqrt{2}a^{5/4}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x/a + ((a + b + 2*Sqrt[a]*Sqrt[a + b])*ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]]
 - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[a + b]]])/(2*Sqrt[2]*a^(5/4)*Sqrt
[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) - ((a + b + 2*Sqrt[a]*Sqrt[a + b])*Arc
Tan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[
a + b]]])/(2*Sqrt[2]*a^(5/4)*Sqrt[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) + ((a
 + b - 2*Sqrt[a]*Sqrt[a + b])*Log[Sqrt[a + b] - Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[
a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2]*a^(5/4)*Sqrt[a + b]*Sqrt[-Sq
rt[a] + Sqrt[a + b]]) - ((a + b - 2*Sqrt[a]*Sqrt[a + b])*Log[Sqrt[a + b] +
Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2]*a
^(5/4)*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{a+b+2ax^2+ax^4}\,dx = \frac{x}{a} - \frac{\int \frac{a+b+2ax^2}{a+b+2ax^2+ax^4}\,dx}{a}$$

$$= \frac{x}{a} - \frac{\int \frac{\frac{\sqrt{2}(a+b)\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}} - \left(a+b-2\sqrt{a}\sqrt{a+b}\right)x}{\frac{\sqrt{a+b}}{\sqrt{a}} - \frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}} + x^2}\,dx}{2\sqrt{2}\,a^{5/4}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}} - \frac{\int \frac{\frac{\sqrt{2}(a+b)\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}} + \left(a+b-2\sqrt{a}\sqrt{a+b}\right)x}{\frac{\sqrt{a+b}}{\sqrt{a}} + \frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}} + x^2}\,dx}{2\sqrt{2}\,a^{5/4}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}}$$

$$= \frac{x}{a} + \frac{\left(a+b-2\sqrt{a}\sqrt{a+b}\right)\int \frac{-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}} + 2x}{\frac{\sqrt{a+b}}{\sqrt{a}} - \frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}} + x^2}\,dx}{4\sqrt{2}\,a^{5/4}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}} - \frac{\left(a+b-2\sqrt{a}\sqrt{a+b}\right)\int \ldots}{4\sqrt{2}\,a^{5/4}\sqrt{a+b}\sqrt{\ldots}}$$

$$= \frac{x}{a} + \frac{\left(a+b-2\sqrt{a}\sqrt{a+b}\right)\log\left(\sqrt{a+b} - \sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x + \sqrt{a}\,x^2\right)}{4\sqrt{2}\,a^{5/4}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}} - \frac{(a - \ldots}{\ldots}$$

$$= \frac{x}{a} + \frac{\left(a+b+2\sqrt{a}\sqrt{a+b}\right)\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}} - \sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\,a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a}+\sqrt{a+b}}} - \frac{\left(a+b+2\sqrt{a}\sqrt{a+b}\right)\tan\ldots}{2\sqrt{2}\,a^{5/4}\sqrt{a+b}\ldots}$$

Mathematica [C] time = 0.10, size = 164, normalized size = 0.38

$$-\frac{i\left(\sqrt{a}-i\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right)}{2a\sqrt{b}\sqrt{a-i\sqrt{a}\sqrt{b}}} + \frac{i\left(\sqrt{a}+i\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{2a\sqrt{b}\sqrt{a+i\sqrt{a}\sqrt{b}}} + \frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x/a - ((I/2)*(Sqrt[a] - I*Sqrt[b])^2*ArcTan[(Sqrt[a]*x)/Sqrt[a - I*Sqrt[a]*
Sqrt[b]]])/(a*Sqrt[a - I*Sqrt[a]*Sqrt[b]]*Sqrt[b]) + ((I/2)*(Sqrt[a] + I*Sq
rt[b])^2*ArcTan[(Sqrt[a]*x)/Sqrt[a + I*Sqrt[a]*Sqrt[b]]])/(a*Sqrt[a + I*Sqr
t[a]*Sqrt[b]]*Sqrt[b])
```

fricas [A] time = 0.84, size = 615, normalized size = 1.42

$$\frac{a\sqrt{\frac{a^2b\sqrt{-\frac{9\,a^2-6\,ab+b^2}{a^5b}}+a-3\,b}{a^2b}}\,\log\left(-\left(3\,a^2+2\,ab-b^2\right)x+\left(a^4b\sqrt{-\frac{9\,a^2-6\,ab+b^2}{a^5b}}+3\,a^2b-ab^2\right)\sqrt{\frac{a^2b\sqrt{-\frac{9\,a^2-6\,ab+b^2}{a^5b}}+a-3\,b}{a^2b}}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] 1/4*(a*sqrt((a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) + a - 3*b)/(a^2*b))
*log(-(3*a^2 + 2*a*b - b^2)*x + (a^4*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b))
 + 3*a^2*b - a*b^2)*sqrt((a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) + a -
3*b)/(a^2*b))) - a*sqrt((a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) + a - 3
*b)/(a^2*b))*log(-(3*a^2 + 2*a*b - b^2)*x - (a^4*b*sqrt(-(9*a^2 - 6*a*b + b
^2)/(a^5*b)) + 3*a^2*b - a*b^2)*sqrt((a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^
5*b)) + a - 3*b)/(a^2*b))) - a*sqrt(-(a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^
5*b)) - a + 3*b)/(a^2*b))*log(-(3*a^2 + 2*a*b - b^2)*x + (a^4*b*sqrt(-(9*a^
2 - 6*a*b + b^2)/(a^5*b)) - 3*a^2*b + a*b^2)*sqrt(-(a^2*b*sqrt(-(9*a^2 - 6*
a*b + b^2)/(a^5*b)) - a + 3*b)/(a^2*b))) + a*sqrt(-(a^2*b*sqrt(-(9*a^2 - 6*
a*b + b^2)/(a^5*b)) - a + 3*b)/(a^2*b))*log(-(3*a^2 + 2*a*b - b^2)*x - (a^4
*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) - 3*a^2*b + a*b^2)*sqrt(-(a^2*b*sqr
t(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) - a + 3*b)/(a^2*b))) + 4*x)/a
```

giac [A] time = 0.35, size = 533, normalized size = 1.23

$$\frac{\left(3\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,a^4+\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,a^3b-4\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,a^2b^2+2\left(3\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] 1/2*(3*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a^4 + sqrt(a^2 + sqrt(-a*b)*a)*s
qrt(-a*b)*a^3*b - 4*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a^2*b^2 + 2*(3*sqrt
(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a*b + 4*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)
*b^2)*a^2 - (3*sqrt(a^2 + sqrt(-a*b)*a)*a^3*b + 7*sqrt(a^2 + sqrt(-a*b)*a)*
a^2*b^2 + 4*sqrt(a^2 + sqrt(-a*b)*a)*a*b^3)*abs(a))*arctan(x/sqrt((a^2 + sq
rt(a^4 - (a^2 + a*b)*a^2))/a^2))/(3*a^6*b + 7*a^5*b^2 + 4*a^4*b^3) - 1/2*(3
*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a^4 + sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a
*b)*a^3*b - 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a^2*b^2 + 2*(3*sqrt(a^2 -
```

```
 sqrt(-a*b)*a)*sqrt(-a*b)*a*b + 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*b^2)*
a^2 + (3*sqrt(a^2 - sqrt(-a*b)*a)*a^3*b + 7*sqrt(a^2 - sqrt(-a*b)*a)*a^2*b^
2 + 4*sqrt(a^2 - sqrt(-a*b)*a)*a*b^3)*abs(a))*arctan(x/sqrt((a^2 - sqrt(a^4
 - (a^2 + a*b)*a^2))/a^2))/(3*a^6*b + 7*a^5*b^2 + 4*a^4*b^3) + x/a
```

maple [B] time = 0.14, size = 1658, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] 1/a*x+1/8/a/b*ln(-x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-(a+b)^(1/2))*
(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/8/a^2/b*ln(-x^2*a^(1/2)+x*(2*(a
*(a+b))^(1/2)-2*a)^(1/2)-(a+b)^(1/2))*(a^2+a*b)^(1/2)*(a+b)^(1/2)*(2*(a^2+a
*b)^(1/2)-2*a)^(1/2)-1/4/a^(3/2)/b*ln(-x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a
)^(1/2)-(a+b)^(1/2))*(a^2+a*b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/4/a^(1
/2)/b*ln(-x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-(a+b)^(1/2))*(2*(a^2+
a*b)^(1/2)-2*a)^(1/2)+1/a/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/
2)*arctan((-2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/
2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(a+b)^(1/2)-1/4/a/b/(4*a^(1/2)*(a+b)^(1/2)
-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^
(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1
/2)-2*a)^(1/2)*(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/4/a^2/b/(4*a^(1/
2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*a^(1/2)*x+(2*(a*(a+b
))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(
2*(a*(a+b))^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)
-2*a)^(1/2)+1/2/a^(3/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/
2)*arctan((-2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/
2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1
/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/2/a^(1/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a
*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2)
)/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2
*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/8/a/b*ln(x^2*a^(1/2)+x*(2*(a*(a+b
))^(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-
1/8/a^2/b*ln(x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(a^2+
a*b)^(1/2)*(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/4/a^(3/2)/b*ln(x^2*a
^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(a^2+a*b)^(1/2)*(2*(a^2
+a*b)^(1/2)-2*a)^(1/2)+1/4/a^(1/2)/b*ln(x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*
a)^(1/2)+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/a/(4*a^(1/2)*(a+b)^(1
/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a
)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(a+b)^(1/2)+1
/4/a/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2
)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)
+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2
```

```
*a)^(1/2)+1/4/a^2/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arc
tan((2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a
*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*(a+
b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/2/a^(3/2)/b/(4*a^(1/2)*(a+b)^(1/2)
-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(
1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/
2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/2/a^(1/2)/b/(
4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*x+(2*(
a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1
/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{a}-\frac{\frac{\left(3\sqrt{a^2+\sqrt{-ab}\,a}\,a^2b+4\sqrt{a^2+\sqrt{-ab}\,a}\,ab^2-3\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,a^2-4\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,ab\right)a\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a+\sqrt{-4(a+b)a+4a^2}}{a}}}\right)}{2\left(3a^4b+4a^3b^2\right)}+\frac{\left(3\sqrt{a^2-\sqrt{-ab}\cdots\right.}{\cdots}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] x/a - integrate((2*a*x^2 + a + b)/(a*x^4 + 2*a*x^2 + a + b), x)/a
```

mupad [B] time = 4.65, size = 1147, normalized size = 2.66

$$\frac{x}{a}+2\,\mathrm{atanh}\left(\frac{24\,x\,\sqrt{-a^5\,b^3}\,\sqrt{\frac{1}{16\,a\,b}-\frac{3}{16\,a^2}+\frac{3\,\sqrt{-a^5\,b^3}}{16\,a^4\,b^2}-\frac{\sqrt{-a^5\,b^3}}{16\,a^5\,b}}}{\frac{6\,\sqrt{-a^5\,b^3}}{a}+4\,a\,b^2+6\,a^2\,b-2\,b^3-\frac{2\,b^2\,\sqrt{-a^5\,b^3}}{a^3}+\frac{4\,b\,\sqrt{-a^5\,b^3}}{a^2}}-\frac{8\,x\,\sqrt{-a^5\,b^3}\,\sqrt{\frac{1}{16\,a\,b}-\frac{3}{16\,a^2}+\frac{3\,\sqrt{-a^5\cdots}}{16\,a^4\,b^{\cdots}}}}{\frac{4\,\sqrt{-a^5\,b^3}}{a}+\frac{6\,\sqrt{-a^5\,b^3}}{b}-2\,a\,b^2+4\,a^2\,b+6\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b + 2*a*x^2 + a*x^4),x)
```

```
[Out] x/a + 2*atanh((24*x*(-a^5*b^3)^(1/2)*(1/(16*a*b) - 3/(16*a^2) + (3*(-a^5*b^
3)^(1/2))/(16*a^4*b^2) - (-a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/((6*(-a^5*b^3)
^(1/2))/a + 4*a*b^2 + 6*a^2*b - 2*b^3 - (2*b^2*(-a^5*b^3)^(1/2))/a^3 + (4*b
*(-a^5*b^3)^(1/2))/a^2) - (8*x*(-a^5*b^3)^(1/2)*(1/(16*a*b) - 3/(16*a^2) +
(3*(-a^5*b^3)^(1/2))/(16*a^4*b^2) - (-a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/((4
*(-a^5*b^3)^(1/2))/a + (6*(-a^5*b^3)^(1/2))/b - 2*a*b^2 + 4*a^2*b + 6*a^3 -
 (2*b*(-a^5*b^3)^(1/2))/a^2) - (8*a*b^2*x*(1/(16*a*b) - 3/(16*a^2) + (3*(-a
^5*b^3)^(1/2))/(16*a^4*b^2) - (-a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b +
(4*(-a^5*b^3)^(1/2))/a^2 + 6*a^2 - 2*b^2 + (6*(-a^5*b^3)^(1/2))/(a*b) - (2*
b*(-a^5*b^3)^(1/2))/a^3) + (24*a^2*b*x*(1/(16*a*b) - 3/(16*a^2) + (3*(-a^5*
```

```
b^3)^(1/2))/(16*a^4*b^2) - (-a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b + (4*
(-a^5*b^3)^(1/2))/a^2 + 6*a^2 - 2*b^2 + (6*(-a^5*b^3)^(1/2))/(a*b) - (2*b*(
-a^5*b^3)^(1/2))/a^3))*((3*a*(-a^5*b^3)^(1/2) - b*(-a^5*b^3)^(1/2) + a^4*b
- 3*a^3*b^2)/(16*a^5*b^2))^(1/2) + 2*atanh((24*x*(-a^5*b^3)^(1/2)*(1/(16*a*
b) - 3/(16*a^2) - (3*(-a^5*b^3)^(1/2))/(16*a^4*b^2) + (-a^5*b^3)^(1/2)/(16*
a^5*b))^(1/2))/((6*(-a^5*b^3)^(1/2))/a - 4*a*b^2 - 6*a^2*b + 2*b^3 - (2*b^2
*(-a^5*b^3)^(1/2))/a^3 + (4*b*(-a^5*b^3)^(1/2))/a^2) - (8*x*(-a^5*b^3)^(1/2
)*(1/(16*a*b) - 3/(16*a^2) - (3*(-a^5*b^3)^(1/2))/(16*a^4*b^2) + (-a^5*b^3)
^(1/2)/(16*a^5*b))^(1/2))/((4*(-a^5*b^3)^(1/2))/a + (6*(-a^5*b^3)^(1/2))/b
+ 2*a*b^2 - 4*a^2*b - 6*a^3 - (2*b*(-a^5*b^3)^(1/2))/a^2) - (8*a*b^2*x*(1/(
16*a*b) - 3/(16*a^2) - (3*(-a^5*b^3)^(1/2))/(16*a^4*b^2) + (-a^5*b^3)^(1/2)
/(16*a^5*b))^(1/2))/(4*a*b - (4*(-a^5*b^3)^(1/2))/a^2 + 6*a^2 - 2*b^2 - (6*
(-a^5*b^3)^(1/2))/(a*b) + (2*b*(-a^5*b^3)^(1/2))/a^3) + (24*a^2*b*x*(1/(16*
a*b) - 3/(16*a^2) - (3*(-a^5*b^3)^(1/2))/(16*a^4*b^2) + (-a^5*b^3)^(1/2)/(1
6*a^5*b))^(1/2))/(4*a*b - (4*(-a^5*b^3)^(1/2))/a^2 + 6*a^2 - 2*b^2 - (6*(-a
^5*b^3)^(1/2))/(a*b) + (2*b*(-a^5*b^3)^(1/2))/a^3))*(-(3*a*(-a^5*b^3)^(1/2)
 - b*(-a^5*b^3)^(1/2) - a^4*b + 3*a^3*b^2)/(16*a^5*b^2))^(1/2)
```

sympy [A] time = 2.20, size = 105, normalized size = 0.24

$$\operatorname{RootSum}\left(256t^4a^5b^2 + t^2\left(-32a^4b + 96a^3b^2\right) + a^3 + 3a^2b + 3ab^2 + b^3, \left(t \mapsto t\log\left(x + \frac{-64t^3a^4b + 4ta^3 - 24ta^2b}{3a^2 + 2ab - b^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] RootSum(256*_t**4*a**5*b**2 + _t**2*(-32*a**4*b + 96*a**3*b**2) + a**3 + 3*
a**2*b + 3*a*b**2 + b**3, Lambda(_t, _t*log(x + (-64*_t**3*a**4*b + 4*_t*a*
*3 - 24*_t*a**2*b + 4*_t*a*b**2)/(3*a**2 + 2*a*b - b**2)))) + x/a
```

3.912 $\int \frac{x^2}{a+b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=331

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\,x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}\,x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\tan^{-1}\left(\right.}{2\sqrt{2}\,a}$$

```
[Out] 1/8*ln(x^2*a^(1/2)+(a+b)^(1/2)-a^(1/4)*x*2^(1/2)*(-a^(1/2)+(a+b)^(1/2))^(1/
2))/a^(3/4)*2^(1/2)/(-a^(1/2)+(a+b)^(1/2))^(1/2)-1/8*ln(x^2*a^(1/2)+(a+b)^(
1/2)+a^(1/4)*x*2^(1/2)*(-a^(1/2)+(a+b)^(1/2))^(1/2))/a^(3/4)*2^(1/2)/(-a^(1
/2)+(a+b)^(1/2))^(1/2)-1/4*arctan((-a^(1/4)*x*2^(1/2)+(-a^(1/2)+(a+b)^(1/2)
)^(1/2))/(a^(1/2)+(a+b)^(1/2))^(1/2))/a^(3/4)*2^(1/2)/(a^(1/2)+(a+b)^(1/2))
^(1/2)+1/4*arctan((a^(1/4)*x*2^(1/2)+(-a^(1/2)+(a+b)^(1/2))^(1/2))/(a^(1/2)
+(a+b)^(1/2))^(1/2))/a^(3/4)*2^(1/2)/(a^(1/2)+(a+b)^(1/2))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1129, 634, 618, 204, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\,x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}\,x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\tan^{-1}\left(\right.}{2\sqrt{2}\,a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + S
qrt[a + b]]]/(2*Sqrt[2]*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[a + b]]) + ArcTan[(Sqrt
[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[a + b]]]/
(2*Sqrt[2]*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[a + b]]) + Log[Sqrt[a + b] - Sqrt[2]
*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2]/(4*Sqrt[2]*a^(3/4)*S
qrt[-Sqrt[a] + Sqrt[a + b]]) - Log[Sqrt[a + b] + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt
[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2]/(4*Sqrt[2]*a^(3/4)*Sqrt[-Sqrt[a] + Sqrt
[a + b]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1129

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m - 1)/(q
 - r*x + x^2), x], x] - Dist[1/(2*c*r), Int[x^(m - 1)/(q + r*x + x^2), x],
x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 1] && LtQ[m, 3
] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^2}{a+b+2ax^2+ax^4}\,dx = \frac{\int \frac{x}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}dx}{2\sqrt{2}\,a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}} - \frac{\int \frac{x}{\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}dx}{2\sqrt{2}\,a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}dx}{4a} + \frac{\int \frac{1}{\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}dx}{4a} + \frac{\int \frac{-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}+2}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}}}{4\sqrt{2}\,a^{3/4}\sqrt{-\sqrt{a}+\sqrt{}}}$$

$$= \frac{\log\left(\sqrt{a+b}-\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x+\sqrt{a}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}} - \frac{\log\left(\sqrt{a+b}+\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}}\right.}{4\sqrt{2}\,a^{3/4}\sqrt{-\sqrt{a}}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{\sqrt{a}+\sqrt{a+b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}+\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{\sqrt{a}+\sqrt{a+b}}} + \frac{\log\left(\sqrt{a+b}-\sqrt{2}\right.}{4\sqrt{2}\,a^{3}}$$

Mathematica [C] time = 0.12, size = 143, normalized size = 0.43

$$\frac{\frac{\left(\sqrt{b}+i\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a-i\sqrt{a}\sqrt{b}}}+\frac{\left(\sqrt{b}-i\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+i\sqrt{a}\sqrt{b}}}}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] (((I*Sqrt[a] + Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a - I*Sqrt[a]*Sqrt[b]]])/Sq
rt[a - I*Sqrt[a]*Sqrt[b]] + (((-I)*Sqrt[a] + Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sq
rt[a + I*Sqrt[a]*Sqrt[b]]])/Sqrt[a + I*Sqrt[a]*Sqrt[b]])/(2*Sqrt[a]*Sqrt[b]
)
```

fricas [A] time = 0.70, size = 279, normalized size = 0.84

$$\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\log\left(a^2b\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\sqrt{-\frac{1}{a^3b}}+x\right)-\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\log\left(-a^2b\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\sqrt{-\frac{1}{a^3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((a*b*sqrt(-1/(a^3*b)) + 1)/(a*b))*log(a^2*b*sqrt((a*b*sqrt(-1/(a^3
*b)) + 1)/(a*b))*sqrt(-1/(a^3*b)) + x) - 1/4*sqrt((a*b*sqrt(-1/(a^3*b)) + 1
)/(a*b))*log(-a^2*b*sqrt((a*b*sqrt(-1/(a^3*b)) + 1)/(a*b))*sqrt(-1/(a^3*b))
 + x) - 1/4*sqrt(-(a*b*sqrt(-1/(a^3*b)) - 1)/(a*b))*log(a^2*b*sqrt(-(a*b*sq
rt(-1/(a^3*b)) - 1)/(a*b))*sqrt(-1/(a^3*b)) + x) + 1/4*sqrt(-(a*b*sqrt(-1/(
a^3*b)) - 1)/(a*b))*log(-a^2*b*sqrt(-(a*b*sqrt(-1/(a^3*b)) - 1)/(a*b))*sqrt
(-1/(a^3*b)) + x)
```

giac [A] time = 0.34, size = 203, normalized size = 0.61

$$-\frac{\left(3\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,a+4\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,b\right)|a|\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a+\sqrt{-4(a+b)a+4a^2}}{a}}}\right)}{2\left(3a^4b+4a^3b^2\right)}+\frac{\left(3\sqrt{a^2-\sqrt{-ab}\,a}\,\sqrt{-ab}\,a+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] -1/2*(3*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a + 4*sqrt(a^2 + sqrt(-a*b)*a)*
sqrt(-a*b)*b)*abs(a)*arctan(2*sqrt(1/2)*x/sqrt((2*a + sqrt(-4*(a + b)*a + 4
*a^2))/a))/(3*a^4*b + 4*a^3*b^2) + 1/2*(3*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*
b)*a + 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*b)*abs(a)*arctan(2*sqrt(1/2)*x
/sqrt((2*a - sqrt(-4*(a + b)*a + 4*a^2))/a))/(3*a^4*b + 4*a^3*b^2)
```

maple [B] time = 0.06, size = 724, normalized size = 2.19

$$-\frac{\sqrt{-2a+2\sqrt{(a+b)\,a}}\;\sqrt{-2a+2\sqrt{a^2+ab}}\;\arctan\left(\frac{-2\sqrt{a}\,x+\sqrt{-2a+2\sqrt{(a+b)a}}}{\sqrt{2a+4\sqrt{a+b}\,\sqrt{a}-2\sqrt{(a+b)a}}}\right)}{4\sqrt{2a+4\sqrt{a+b}\,\sqrt{a}-2\sqrt{(a+b)\,a}}\;\sqrt{a}\,b}+\frac{\sqrt{-2a+2\sqrt{(a+b)\,a}}\;\sqrt{-2a+2\sqrt{}}}{4\sqrt{2a+4\sqrt{a+b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] 1/8*(a^2+a*b)^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)/a^(3/2)/b*ln(-a^(1/2)*x^
2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x-(a+b)^(1/2))-1/4/(2*a+4*(a+b)^(1/2)*a^(1
/2)-2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(a^2+a*b)^(1/2)
*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)/a^(3/2)/b*arctan((-2*a^(1/2)*x+(-2*a+2*((a+
b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))+1/
```

```
8*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)/a^(1/2)/b*ln(-a^(1/2)*x^2+(-2*a+2*((a+b)*a
)^(1/2))^(1/2)*x-(a+b)^(1/2))-1/4/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1
/2))^(1/2)*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)/a^
(1/2)/b*arctan((-2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(
1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))-1/8*(a^2+a*b)^(1/2)*(-2*a+2*(a^2+a*b
)^(1/2))^(1/2)/a^(3/2)/b*ln(a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x+(a
+b)^(1/2))+1/4/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*
((a+b)*a)^(1/2))^(1/2)*(a^2+a*b)^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)/a^(3/
2)/b*arctan((2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)
*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))-1/8*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)/a^(1/
2)/b*ln(a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x+(a+b)^(1/2))+1/4/(2*a+
4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*((a+b)*a)^(1/2))^(1/
2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)/a^(1/2)/b*arctan((2*a^(1/2)*x+(-2*a+2*((a
+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{ax^4 + 2\,ax^2 + a + b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a*x^4 + 2*a*x^2 + a + b), x)
```

mupad [B] time = 0.28, size = 222, normalized size = 0.67

$$-2\,\mathrm{atanh}\left(\frac{2\left(x\,\left(4\,a^2\,b-4\,a^3\right)+\frac{4\,a\,x\,\left(\sqrt{-a^3\,b^3}+a^2\,b\right)}{b}\right)\sqrt{\frac{\sqrt{-a^3\,b^3}+a^2\,b}{16\,a^3\,b^2}}}{2\,a^2+2\,b\,a}\right)\sqrt{\frac{\sqrt{-a^3\,b^3}\,+a^2\,b}{16\,a^3\,b^2}}-2\,\mathrm{atanh}\left(\frac{2\left(x\,\left(4\,a^2\,b-\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b + 2*a*x^2 + a*x^4),x)
```

```
[Out] - 2*atanh((2*(x*(4*a^2*b - 4*a^3) + (4*a*x*((-a^3*b^3)^(1/2) + a^2*b))/b)*(
((-a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2))/(2*a*b + 2*a^2))*(((-a^3*b^
3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2) - 2*atanh((2*(x*(4*a^2*b - 4*a^3) - (
4*a*x*((-a^3*b^3)^(1/2) - a^2*b))/b)*(-((-a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b
^2))^(1/2))/(2*a*b + 2*a^2))*(-((-a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))^(1/
2)
```

sympy [A] time = 0.83, size = 44, normalized size = 0.13

$$\operatorname{RootSum}\left(256t^4a^3b^2 - 32t^2a^2b + a + b, \left(t \mapsto t\log\left(64t^3a^2b - 4ta + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**2 - 32*_t**2*a**2*b + a + b, Lambda(_t, _t*log(64
*_t**3*a**2*b - 4*_t*a + x)))
```

3.913 $\int \frac{1}{a+b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=359

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\tan}{2\sqrt{2}}$$

```
[Out] -1/8*ln(x^2*a^(1/2)+(a+b)^(1/2)-a^(1/4)*x*2^(1/2)*(-a^(1/2)+(a+b)^(1/2))^(1
/2))/a^(1/4)*2^(1/2)/(a+b)^(1/2)/(-a^(1/2)+(a+b)^(1/2))^(1/2)+1/8*ln(x^2*a^
(1/2)+(a+b)^(1/2)+a^(1/4)*x*2^(1/2)*(-a^(1/2)+(a+b)^(1/2))^(1/2))/a^(1/4)*2
^(1/2)/(a+b)^(1/2)/(-a^(1/2)+(a+b)^(1/2))^(1/2)-1/4*arctan((-a^(1/4)*x*2^(1
/2)+(-a^(1/2)+(a+b)^(1/2))^(1/2))/(a^(1/2)+(a+b)^(1/2))^(1/2))/a^(1/4)*2^(1
/2)/(a+b)^(1/2)/(a^(1/2)+(a+b)^(1/2))^(1/2)+1/4*arctan((a^(1/4)*x*2^(1/2)+(
-a^(1/2)+(a+b)^(1/2))^(1/2))/(a^(1/2)+(a+b)^(1/2))^(1/2))/a^(1/4)*2^(1/2)/(
a+b)^(1/2)/(a^(1/2)+(a+b)^(1/2))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\tan}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b + 2*a*x^2 + a*x^4)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + S
qrt[a + b]]]/(2*Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) +
ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sq
rt[a + b]]]/(2*Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) - L
og[Sqrt[a + b] - Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x
^2]/(4*Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]]) + Log[Sqrt
[a + b] + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2]/(4*
Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a+b+2ax^2+ax^4}\,dx = \frac{\int \frac{\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}-x}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}\,dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}} + \frac{\int \frac{\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}+x}{\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}\,dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}\,dx}{4\sqrt{a}\sqrt{a+b}} + \frac{\int \frac{1}{\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}\,dx}{4\sqrt{a}\sqrt{a+b}} - \frac{\int \frac{-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{}}}{\sqrt[4]{a}}}}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{-\sqrt{}}}$$

$$= -\frac{\log\left(\sqrt{a+b}-\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x+\sqrt{a}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}} + \frac{\log\left(\sqrt{a+b}+\sqrt{2}\sqrt[4]{a}\sqrt{-}\right.}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{-}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a}+\sqrt{a+b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}+\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a}+\sqrt{a+b}}} - \frac{\log\left(\sqrt{a+}\right.}{4}$$

Mathematica [C] time = 0.07, size = 119, normalized size = 0.33

$$\frac{i\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{b}\sqrt{a+i\sqrt{a}\sqrt{b}}} - \frac{i\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{b}\sqrt{a-i\sqrt{a}\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b + 2*a*x^2 + a*x^4)^(-1),x]
```

```
[Out] ((-1/2*I)*ArcTan[(Sqrt[a]*x)/Sqrt[a - I*Sqrt[a]*Sqrt[b]]])/(Sqrt[a - I*Sqrt
[a]*Sqrt[b]]*Sqrt[b]) + ((I/2)*ArcTan[(Sqrt[a]*x)/Sqrt[a + I*Sqrt[a]*Sqrt[b
]]])/(Sqrt[a + I*Sqrt[a]*Sqrt[b]]*Sqrt[b])
```

fricas [B] time = 0.83, size = 567, normalized size = 1.58

$$\frac{1}{4}\sqrt{\frac{\left(ab+b^2\right)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}+1}{ab+b^2}}\log\left(\left(\left(a^2b+ab^2\right)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}+b\right)\sqrt{\frac{\left(ab+b^2\right)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}}{ab+b^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(((a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + 1)/(a*b + b^2)
)*log(((a^2*b + a*b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + b)*sqrt(((a*b
 + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + 1)/(a*b + b^2)) + x) - 1/4*s
qrt(((a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + 1)/(a*b + b^2))*log
(-((a^2*b + a*b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + b)*sqrt(((a*b + b
^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + 1)/(a*b + b^2)) + x) - 1/4*sqrt(
-((a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - 1)/(a*b + b^2))*log(((
a^2*b + a*b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - b)*sqrt(-((a*b + b^2)
*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - 1)/(a*b + b^2)) + x) + 1/4*sqrt(-((
a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - 1)/(a*b + b^2))*log(-((a^
2*b + a*b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - b)*sqrt(-((a*b + b^2)*s
qrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - 1)/(a*b + b^2)) + x)
```

giac [A] time = 0.25, size = 307, normalized size = 0.86

$$\frac{\left(3\sqrt{a^2+\sqrt{-ab}\,a}\,a^2b+4\sqrt{a^2+\sqrt{-ab}\,a}\,ab^2+3\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,a^2+4\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,ab\right)|a|\arctan\left(-\right.}{2\left(3a^5b+7a^4b^2+4a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] 1/2*(3*sqrt(a^2 + sqrt(-a*b)*a)*a^2*b + 4*sqrt(a^2 + sqrt(-a*b)*a)*a*b^2 +
3*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a^2 + 4*sqrt(a^2 + sqrt(-a*b)*a)*sqrt
(-a*b)*a*b)*abs(a)*arctan(2*sqrt(1/2)*x/sqrt((2*a + sqrt(-4*(a + b)*a + 4*a
^2))/a))/(3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3) + 1/2*(3*sqrt(a^2 - sqrt(-a*b)*a
)*a^2*b + 4*sqrt(a^2 - sqrt(-a*b)*a)*a*b^2 - 3*sqrt(a^2 - sqrt(-a*b)*a)*sqr
t(-a*b)*a^2 - 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a*b)*abs(a)*arctan(2*sq
rt(1/2)*x/sqrt((2*a - sqrt(-4*(a + b)*a + 4*a^2))/a))/(3*a^5*b + 7*a^4*b^2
+ 4*a^3*b^3)
```

maple [B] time = 0.08, size = 913, normalized size = 2.54

$$-\frac{\arctan\left(\frac{-2\sqrt{a}\,x+\sqrt{-2a+2\sqrt{(a+b)a}}}{\sqrt{2a+4\sqrt{a+b}\,\sqrt{a}-2\sqrt{(a+b)a}}}\right)}{\sqrt{a+b}\,\sqrt{2a+4\sqrt{a+b}\,\sqrt{a}-2\sqrt{(a+b)\,a}}}+\frac{\arctan\left(\frac{2\sqrt{a}\,x+\sqrt{-2a+2\sqrt{(a+b)a}}}{\sqrt{2a+4\sqrt{a+b}\,\sqrt{a}-2\sqrt{(a+b)a}}}\right)}{\sqrt{a+b}\,\sqrt{2a+4\sqrt{a+b}\,\sqrt{a}-2\sqrt{(a+b)\,a}}}+\frac{\sqrt{-2a+2\sqrt{(a+b)\,a}}}{4\sqrt{a+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] -1/8/(a+b)^(1/2)/a/b*ln(-a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x-(a+b)
^(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)-1/8/(a+b)^(1/2)/b*ln
(-a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x-(a+b)^(1/2))*(-2*a+2*(a^2+a*
b)^(1/2))^(1/2)-1/(a+b)^(1/2)/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))
^(1/2)*arctan((-2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1
/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))+1/4/(a+b)^(1/2)/a/b/(2*a+4*(a+b)^(1/2
)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((-2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(
1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2*(
(a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)+1/4/(a
+b)^(1/2)/b/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((-2*
a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+
b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))
^(1/2)+1/8/(a+b)^(1/2)/a/b*ln(a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x+
(a+b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)+1/8/(a+b)^(1/2)
/b*ln(a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x+(a+b)^(1/2))*(-2*a+2*(a^
2+a*b)^(1/2))^(1/2)+1/(a+b)^(1/2)/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1
/2))^(1/2)*arctan((2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)
^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))-1/4/(a+b)^(1/2)/a/b/(2*a+4*(a+b)^(
1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((2*a^(1/2)*x+(-2*a+2*((a+b)*a)
^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2
*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)-1/4/
(a+b)^(1/2)/b/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((2
*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a
+b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2)
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ax^4 + 2\,ax^2 + a + b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] integrate(1/(a*x^4 + 2*a*x^2 + a + b), x)
```

mupad [B] time = 5.16, size = 986, normalized size = 2.75

$$2\,\mathrm{atanh}\left(\frac{8\,a^3\,x\,\sqrt{\frac{a\,b}{16\,\left(a^2\,b^2+a\,b^3\right)}-\frac{\sqrt{-a\,b^3}}{16\,\left(a^2\,b^2+a\,b^3\right)}}}{\frac{2\,a^4\,b^2}{a^2\,b^2+a\,b^3}-\frac{2\,a^3\,b\,\sqrt{-a\,b^3}}{a^2\,b^2+a\,b^3}}-\frac{8\,a^5\,b^2\,x\,\sqrt{\frac{a\,b}{16\,\left(a^2\,b^2+a\,b^3\right)}-\frac{\sqrt{-a\,b^3}}{16\,\left(a^2\,b^2+a\,b^3\right)}}}{\frac{2\,a^5\,b^5}{a^2\,b^2+a\,b^3}+\frac{2\,a^6\,b^4}{a^2\,b^2+a\,b^3}-\frac{2\,a^4\,b^4\,\sqrt{-a\,b^3}}{a^2\,b^2+a\,b^3}-\frac{2\,a^5\,b^3\,\sqrt{-a\,b^3}}{a^2\,b^2+a\,b^3}}+\frac{8\,a^4\,b\,x\,\sqrt{\cdots}}{\frac{2\,a^5\,b^5}{a^2\,b^2+a\,b^3}+\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b + 2*a*x^2 + a*x^4),x)
```

```
[Out] 2*atanh((8*a^3*x*((a*b)/(16*(a*b^3 + a^2*b^2)) - (-a*b^3)^(1/2)/(16*(a*b^3
+ a^2*b^2)))^(1/2))/((2*a^4*b^2)/(a*b^3 + a^2*b^2) - (2*a^3*b*(-a*b^3)^(1/2
))/(a*b^3 + a^2*b^2)) - (8*a^5*b^2*x*((a*b)/(16*(a*b^3 + a^2*b^2)) - (-a*b^
3)^(1/2)/(16*(a*b^3 + a^2*b^2)))^(1/2))/((2*a^5*b^5)/(a*b^3 + a^2*b^2) + (2
*a^6*b^4)/(a*b^3 + a^2*b^2) - (2*a^4*b^4*(-a*b^3)^(1/2))/(a*b^3 + a^2*b^2)
- (2*a^5*b^3*(-a*b^3)^(1/2))/(a*b^3 + a^2*b^2)) + (8*a^4*b*x*((a*b)/(16*(a*
b^3 + a^2*b^2)) - (-a*b^3)^(1/2)/(16*(a*b^3 + a^2*b^2)))^(1/2)*(-a*b^3)^(1/
2))/((2*a^5*b^5)/(a*b^3 + a^2*b^2) + (2*a^6*b^4)/(a*b^3 + a^2*b^2) - (2*a^4
*b^4*(-a*b^3)^(1/2))/(a*b^3 + a^2*b^2) - (2*a^5*b^3*(-a*b^3)^(1/2))/(a*b^3
+ a^2*b^2)))*((a*b - (-a*b^3)^(1/2))/(16*(a*b^3 + a^2*b^2)))^(1/2) - 2*atan
h((8*a^5*b^2*x*((-a*b^3)^(1/2)/(16*(a*b^3 + a^2*b^2)) + (a*b)/(16*(a*b^3 +
a^2*b^2)))^(1/2))/((2*a^5*b^5)/(a*b^3 + a^2*b^2) + (2*a^6*b^4)/(a*b^3 + a^2
*b^2) + (2*a^4*b^4*(-a*b^3)^(1/2))/(a*b^3 + a^2*b^2) + (2*a^5*b^3*(-a*b^3)^
(1/2))/(a*b^3 + a^2*b^2)) - (8*a^3*x*((-a*b^3)^(1/2)/(16*(a*b^3 + a^2*b^2))
 + (a*b)/(16*(a*b^3 + a^2*b^2)))^(1/2))/((2*a^4*b^2)/(a*b^3 + a^2*b^2) + (2
*a^3*b*(-a*b^3)^(1/2))/(a*b^3 + a^2*b^2)) + (8*a^4*b*x*((-a*b^3)^(1/2)/(16*
(a*b^3 + a^2*b^2)) + (a*b)/(16*(a*b^3 + a^2*b^2)))^(1/2)*(-a*b^3)^(1/2))/((
2*a^5*b^5)/(a*b^3 + a^2*b^2) + (2*a^6*b^4)/(a*b^3 + a^2*b^2) + (2*a^4*b^4*(
-a*b^3)^(1/2))/(a*b^3 + a^2*b^2) + (2*a^5*b^3*(-a*b^3)^(1/2))/(a*b^3 + a^2*
b^2)))*((a*b + (-a*b^3)^(1/2))/(16*(a*b^3 + a^2*b^2)))^(1/2)
```

sympy [A] time = 1.24, size = 63, normalized size = 0.18

$$\operatorname{RootSum}\left(t^4\left(256a^2b^2+256ab^3\right)-32t^2ab+1,\left(t\mapsto t\log\left(64t^3a^2b+64t^3ab^2-4ta+4tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] RootSum(_t**4*(256*a**2*b**2 + 256*a*b**3) - 32*_t**2*a*b + 1, Lambda(_t, _
t*log(64*_t**3*a**2*b + 64*_t**3*a*b**2 - 4*_t*a + 4*_t*b + x)))
```

3.914 $\int \frac{1}{x^2\left(a+b+2ax^2+ax^4\right)}\,dx$

Optimal. Leaf size=433

$$\frac{\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}\,x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}\,x^2\right)}{4\sqrt{2}\,(a+b)^{3/2}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(\sqrt{2}\sqrt[4]{a}\,x\sqrt{\cdots}\right.}{4\sqrt{2}\,(a+b)^{3/2}\sqrt{\cdots}}$$

```
[Out] -1/(a+b)/x+1/8*a^(1/4)*ln(x^2*a^(1/2)+(a+b)^(1/2)-a^(1/4)*x*2^(1/2)*(-a^(1/
2)+(a+b)^(1/2))^(1/2))*(2*a^(1/2)-(a+b)^(1/2))/(a+b)^(3/2)*2^(1/2)/(-a^(1/2
)+(a+b)^(1/2))^(1/2)-1/8*a^(1/4)*ln(x^2*a^(1/2)+(a+b)^(1/2)+a^(1/4)*x*2^(1/
2)*(-a^(1/2)+(a+b)^(1/2))^(1/2))*(2*a^(1/2)-(a+b)^(1/2))/(a+b)^(3/2)*2^(1/2
)/(-a^(1/2)+(a+b)^(1/2))^(1/2)+1/4*a^(1/4)*arctan((-a^(1/4)*x*2^(1/2)+(-a^(
1/2)+(a+b)^(1/2))^(1/2))/(a^(1/2)+(a+b)^(1/2))^(1/2))*(2*a^(1/2)+(a+b)^(1/2
))/(a+b)^(3/2)*2^(1/2)/(a^(1/2)+(a+b)^(1/2))^(1/2)-1/4*a^(1/4)*arctan((a^(1
/4)*x*2^(1/2)+(-a^(1/2)+(a+b)^(1/2))^(1/2))/(a^(1/2)+(a+b)^(1/2))^(1/2))*(2
*a^(1/2)+(a+b)^(1/2))/(a+b)^(3/2)*2^(1/2)/(a^(1/2)+(a+b)^(1/2))^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1123, 1169, 634, 618, 204, 628}

$$\frac{\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}\,x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}\,x^2\right)}{4\sqrt{2}\,(a+b)^{3/2}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(\sqrt{2}\sqrt[4]{a}\,x\sqrt{\cdots}\right.}{4\sqrt{2}\,(a+b)^{3/2}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -(1/((a + b)*x)) + (a^(1/4)*(2*Sqrt[a] + Sqrt[a + b])*ArcTan[(Sqrt[-Sqrt[a]
 + Sqrt[a + b]] - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[a + b]]])/(2*Sqrt[
2]*(a + b)^(3/2)*Sqrt[Sqrt[a] + Sqrt[a + b]]) - (a^(1/4)*(2*Sqrt[a] + Sqrt[
a + b])*ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt
[a] + Sqrt[a + b]]])/(2*Sqrt[2]*(a + b)^(3/2)*Sqrt[Sqrt[a] + Sqrt[a + b]])
+ (a^(1/4)*(2*Sqrt[a] - Sqrt[a + b])*Log[Sqrt[a + b] - Sqrt[2]*a^(1/4)*Sqrt
[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2]*(a + b)^(3/2)*Sqrt[-S
qrt[a] + Sqrt[a + b]]) - (a^(1/4)*(2*Sqrt[a] - Sqrt[a + b])*Log[Sqrt[a + b]
 + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2
]*(a + b)^(3/2)*Sqrt[-Sqrt[a] + Sqrt[a + b]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b+2ax^2+ax^4\right)}\,dx = -\frac{1}{(a+b)x} + \frac{\int \frac{-2a-ax^2}{a+b+2ax^2+ax^4}\,dx}{a+b}$$

$$= -\frac{1}{(a+b)x} + \frac{\int \frac{-2\sqrt{2}\,a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}-\left(-2a+\sqrt{a}\,\sqrt{a+b}\right)x}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\,\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}\,dx}{2\sqrt{2}\,\sqrt[4]{a}\,(a+b)^{3/2}\sqrt{-\sqrt{a}+\sqrt{a+b}}} + \frac{\int \frac{-2\sqrt{2}\,a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}\cdots}{\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}\,\sqrt{-\sqrt{a}+\cdots}}{\sqrt[4]{a}}\cdots}\cdots}{2\sqrt{2}\,\sqrt[4]{a}\,(a+b)^{3/2}\sqrt{\cdots}}$$

$$= -\frac{1}{(a+b)x} + \frac{\left(\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\right)\int \frac{-\frac{\sqrt{2}\,\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}+2x}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\,\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}\,dx}{4\sqrt{2}\,(a+b)^{3/2}\sqrt{-\sqrt{a}+\sqrt{a+b}}} - \frac{\left(\sqrt[4]{a}\left(2\sqrt{a}\cdots\right.\right.}{4\cdots}$$

$$= -\frac{1}{(a+b)x} + \frac{\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(\sqrt{a+b}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x+\cdots\right.}{4\sqrt{2}\,(a+b)^{3/2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}\cdots$$

$$= -\frac{1}{(a+b)x} + \frac{\sqrt[4]{a}\left(2\sqrt{a}+\sqrt{a+b}\right)\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\,(a+b)^{3/2}\sqrt{\sqrt{a}+\sqrt{a+b}}} - \frac{\sqrt[4]{a}\left(2\sqrt{a}+\cdots\right.}{2\sqrt{\cdots}}$$

Mathematica [C] time = 0.15, size = 174, normalized size = 0.40

$$\frac{1}{x(-a-b)} + \frac{\left(-\sqrt{a}\,\sqrt{b}+ia\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-i\sqrt{a}\,\sqrt{b}}}\right)}{2\sqrt{b}\,\sqrt{a-i\sqrt{a}\,\sqrt{b}}\,(a+b)} + \frac{\left(-\sqrt{a}\,\sqrt{b}-ia\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a+i\sqrt{a}\,\sqrt{b}}}\right)}{2\sqrt{b}\,\sqrt{a+i\sqrt{a}\,\sqrt{b}}\,(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] 1/((-a - b)*x) + ((I*a - Sqrt[a]*Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a - I*Sqr
t[a]*Sqrt[b]]])/(2*Sqrt[a - I*Sqrt[a]*Sqrt[b]]*Sqrt[b]*(a + b)) + (((-I)*a
- Sqrt[a]*Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a + I*Sqrt[a]*Sqrt[b]]])/(2*Sqrt
[a + I*Sqrt[a]*Sqrt[b]]*Sqrt[b]*(a + b))
```

fricas [B] time = 0.78, size = 1582, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] 1/4*((a + b)*x*sqrt((a^2 - 3*a*b + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*sqrt
(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 +
15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4))*log(-(3*
a^2 - a*b)*x + (6*a^2*b - 2*a*b^2 + (a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5)*sqr
t(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 +
 15*a^2*b^5 + 6*a*b^6 + b^7)))*sqrt((a^2 - 3*a*b + (a^3*b + 3*a^2*b^2 + 3*a
*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3
 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^3
+ b^4))) - (a + b)*x*sqrt((a^2 - 3*a*b + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4
)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*
b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4))*lo
g(-(3*a^2 - a*b)*x - (6*a^2*b - 2*a*b^2 + (a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^
5)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3
*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))*sqrt((a^2 - 3*a*b + (a^3*b + 3*a^2*b^2
 + 3*a*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a
^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*
a*b^3 + b^4))) + (a + b)*x*sqrt((a^2 - 3*a*b - (a^3*b + 3*a^2*b^2 + 3*a*b^3
 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 2
0*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^
4))*log(-(3*a^2 - a*b)*x + (6*a^2*b - 2*a*b^2 - (a^4*b + 2*a^3*b^2 - 2*a*b^
4 - b^5)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 +
20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))*sqrt((a^2 - 3*a*b - (a^3*b + 3*a
^2*b^2 + 3*a*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2
+ 15*a^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^
2 + 3*a*b^3 + b^4))) - (a + b)*x*sqrt((a^2 - 3*a*b - (a^3*b + 3*a^2*b^2 + 3
*a*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b
^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^
3 + b^4))*log(-(3*a^2 - a*b)*x - (6*a^2*b - 2*a*b^2 - (a^4*b + 2*a^3*b^2 -
2*a*b^4 - b^5)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*
b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))*sqrt((a^2 - 3*a*b - (a^3*b
 + 3*a^2*b^2 + 3*a*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^
5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*
a^2*b^2 + 3*a*b^3 + b^4))) - 4)/((a + b)*x)
```

giac [B] time = 0.38, size = 742, normalized size = 1.71

$$\left(\left(3\sqrt{a^2-\sqrt{-ab}\,a}\,\sqrt{-ab}\,ab+4\sqrt{a^2-\sqrt{-ab}\,a}\,\sqrt{-ab}\,b^2\right)(a+b)^2|a|-2\left(3\sqrt{a^2-\sqrt{-ab}\,a}\,a^3b+7\sqrt{a^2-\sqrt{-ab}\,a}\,a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] 1/2*((3*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a*b + 4*sqrt(a^2 - sqrt(-a*b)*a
)*sqrt(-a*b)*b^2)*(a + b)^2*abs(a) - 2*(3*sqrt(a^2 - sqrt(-a*b)*a)*a^3*b +
7*sqrt(a^2 - sqrt(-a*b)*a)*a^2*b^2 + 4*sqrt(a^2 - sqrt(-a*b)*a)*a*b^3)*abs(
a)*abs(-a - b) - (3*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a^4 + 10*sqrt(a^2 -
 sqrt(-a*b)*a)*sqrt(-a*b)*a^3*b + 11*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a^
2*b^2 + 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a*b^3)*abs(a))*arctan(2*sqrt(
1/2)*x/sqrt((2*a^2 + 2*a*b + sqrt(-4*(a^2 + 2*a*b + b^2)*(a^2 + a*b) + 4*(a
^2 + a*b)^2))/(a^2 + a*b)))/((3*a^6*b + 13*a^5*b^2 + 21*a^4*b^3 + 15*a^3*b^
4 + 4*a^2*b^5)*abs(-a - b)) - 1/2*((3*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a
*b + 4*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*b^2)*(a + b)^2*abs(a) + 2*(3*sqr
t(a^2 + sqrt(-a*b)*a)*a^3*b + 7*sqrt(a^2 + sqrt(-a*b)*a)*a^2*b^2 + 4*sqrt(a
^2 + sqrt(-a*b)*a)*a*b^3)*abs(a)*abs(-a - b) - (3*sqrt(a^2 + sqrt(-a*b)*a)*
sqrt(-a*b)*a^4 + 10*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a^3*b + 11*sqrt(a^2
 + sqrt(-a*b)*a)*sqrt(-a*b)*a^2*b^2 + 4*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)
*a*b^3)*abs(a))*arctan(2*sqrt(1/2)*x/sqrt((2*a^2 + 2*a*b - sqrt(-4*(a^2 + 2
*a*b + b^2)*(a^2 + a*b) + 4*(a^2 + a*b)^2))/(a^2 + a*b)))/((3*a^6*b + 13*a^
5*b^2 + 21*a^4*b^3 + 15*a^3*b^4 + 4*a^2*b^5)*abs(-a - b)) - 1/((a + b)*x)
```

maple [B] time = 0.07, size = 3318, normalized size = 7.66

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] 1/4*a^(3/2)/(a+b)^2/b/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*a
rctan((-2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1
/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a
*b)^(1/2))^(1/2)-1/2*a^2/(a+b)^(5/2)/b/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*
a)^(1/2))^(1/2)*arctan((-2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4
*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/
2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)-1/4/a^(1/2)/(a+b)^2/(2*a+4*(a+b)^(1/2)*a^
```

(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)+1/2*a^2/(a+b)^(5/2)/b/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)-1/4*a^(3/2)/(a+b)^2/b/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)+1/4/a^(1/2)/(a+b)^2/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((-2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)-1/4/(a+b)^(5/2)*ln(a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x+(a+b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)-1/8*a^(1/2)/(a+b)^2*ln(-a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x-(a+b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)+1/8*a^(1/2)/(a+b)^2*ln(a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x+(a+b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)-1/4*a/(a+b)^(5/2)*ln(a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x+(a+b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)-2*a^2/(a+b)^(5/2)/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))+1/4*a/(a+b)^(5/2)*ln(-a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x-(a+b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)+2*a^2/(a+b)^(5/2)/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((-2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))+1/4/(a+b)^(5/2)*ln(-a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x-(a+b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)-1/(a+b)/x-1/2*a/(a+b)^(5/2)/b/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((-2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)+1/2*a/(a+b)^(5/2)/b/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)-1/4*a^(1/2)/(a+b)^2/b/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)+1/4*a^(1/2)/(a+b)^2/b/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((-2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)-2*a/(a+b)^(5/2)*b/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))+1/8/a^(1/2)/(a+b)^2*ln(a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x+(a+b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)-1/8/a^(1/2)/(a+b)^2*ln(-a^(1/2)*x^2+(-

```
2*a+2*((a+b)*a)^(1/2))^(1/2)*x-(a+b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*
(a^2+a*b)^(1/2)-1/8*a^(3/2)/(a+b)^2/b*ln(-a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/
2))^(1/2)*x-(a+b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)+1/4*a^2/(a+b)^(5/2)
/b*ln(-a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x-(a+b)^(1/2))*(-2*a+2*(a
^2+a*b)^(1/2))^(1/2)+2*a/(a+b)^(5/2)*b/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*
a)^(1/2))^(1/2)*arctan((-2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4
*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))+1/2*a/(a+b)^(5/2)/(2*a+4*(a+
b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((2*a^(1/2)*x+(-2*a+2*((a+b
)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2
*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)-1/4*a^(1/2)/(a+b
)^2/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((2*a^(1/2)*x
+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/
2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)-1/
8*a^(1/2)/(a+b)^2/b*ln(-a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x-(a+b)^
(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)-1/2/(a+b)^(5/2)/(2*a+
4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((-2*a^(1/2)*x+(-2*a+2
*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2
))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^
(1/2)+1/2/(a+b)^(5/2)/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*a
rctan((2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/
2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*
b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)-1/2*a/(a+b)^(5/2)/(2*a+4*(a+b)^(1/2)*a^(1/2
)-2*((a+b)*a)^(1/2))^(1/2)*arctan((-2*a^(1/2)*x+(-2*a+2*((a+b)*a)^(1/2))^(1
/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2))*(-2*a+2*((a+b)*a)
^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)+1/4*a/(a+b)^(5/2)/b*ln(-a^(1/2
)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x-(a+b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/2)
)^(1/2)*(a^2+a*b)^(1/2)-1/4*a/(a+b)^(5/2)/b*ln(a^(1/2)*x^2+(-2*a+2*((a+b)*a
)^(1/2))^(1/2)*x+(a+b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2
)+1/8*a^(1/2)/(a+b)^2/b*ln(a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x+(a+
b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)*(a^2+a*b)^(1/2)+1/4*a^(1/2)/(a+b)^
2/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2))^(1/2)*arctan((-2*a^(1/2)*x+
(-2*a+2*((a+b)*a)^(1/2))^(1/2))/(2*a+4*(a+b)^(1/2)*a^(1/2)-2*((a+b)*a)^(1/2
))^(1/2))*(-2*a+2*((a+b)*a)^(1/2))^(1/2)*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)+1/8
*a^(3/2)/(a+b)^2/b*ln(a^(1/2)*x^2+(-2*a+2*((a+b)*a)^(1/2))^(1/2)*x+(a+b)^(1
/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)-1/4*a^2/(a+b)^(5/2)/b*ln(a^(1/2)*x^2+(-
2*a+2*((a+b)*a)^(1/2))^(1/2)*x+(a+b)^(1/2))*(-2*a+2*(a^2+a*b)^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\frac{1}{2}\left(\frac{\left(6\sqrt{a^2+\sqrt{-ab}\,a}\,a^2b+8\sqrt{a^2+\sqrt{-ab}\,a}\,ab^2+3\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,a^2+\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,ab-4\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,b^2\right)a\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a+\sqrt{-4(a+b)a+4}}{a}}}\right.}{3a^5b+7a^4b^2+4a^3b^3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] -a*integrate((x^2 + 2)/(a*x^4 + 2*a*x^2 + a + b), x)/(a + b) - 1/((a + b)*x
)
```

mupad [B] time = 5.27, size = 2848, normalized size = 6.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b + 2*a*x^2 + a*x^4)),x)
```

```
[Out] - 1/(x*(a + b)) - atan((((-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^
3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*
a^4*b^5 + 128*a^5*b^4 + 192*a^6*b^3 + 128*a^7*b^2 + x*(-(3*a*b^2 - a^2*b -
3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3
*b^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6 + 320*a^5*b^5 + 640*a^6*b^4 + 640*a^7*
b^3 + 320*a^8*b^2)) - x*(8*a^7*b + 4*a^8 - 4*a^4*b^4 - 8*a^5*b^3))*(-(3*a*b
^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*
a^2*b^3 + a^3*b^2)))^(1/2)*1i - ((-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) +
b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8
*b + 32*a^4*b^5 + 128*a^5*b^4 + 192*a^6*b^3 + 128*a^7*b^2 - x*(-(3*a*b^2 -
a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b
^3 + a^3*b^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6 + 320*a^5*b^5 + 640*a^6*b^4 +
640*a^7*b^3 + 320*a^8*b^2)) + x*(8*a^7*b + 4*a^8 - 4*a^4*b^4 - 8*a^5*b^3))*
(-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 +
b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*1i)/(6*a^6*b + 2*a^7 + ((-(3*a*b^2 - a^2
*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3
+ a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 + 128*a^5*b^4 + 192*a^6*b^3 + 128
*a^7*b^2 + x*(-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(1
6*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6 + 32
0*a^5*b^5 + 640*a^6*b^4 + 640*a^7*b^3 + 320*a^8*b^2)) - x*(8*a^7*b + 4*a^8
```

- 4*a^4*b^4 - 8*a^5*b^3))*(-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b ^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2) + ((-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2 *b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 + 128*a^5*b^4 + 192*a^6*b^3 + 128*a^7*b^2 - x*(-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6 + 320*a^5*b^5 + 640*a^6*b^4 + 640*a^7*b^3 + 320*a^8*b^2)) + x*(8*a^7*b + 4 *a^8 - 4*a^4*b^4 - 8*a^5*b^3))*(-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b* (-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2) + 2*a^4*b ^3 + 6*a^5*b^2))*(-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*2i - atan((((-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2 *b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 + 128*a^5*b^4 + 192*a^6*b^3 + 128*a^7*b^2 + x*(-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6 + 320*a^5*b^5 + 640*a^6*b^4 + 640*a^7*b^3 + 320*a^8*b^2)) - x*(8*a^7*b + 4 *a^8 - 4*a^4*b^4 - 8*a^5*b^3))*(-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b* (-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*1i - ((-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 + 128*a^5*b^4 + 192* a^6*b^3 + 128*a^7*b^2 - x*(-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b ^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b + 64 *a^4*b^6 + 320*a^5*b^5 + 640*a^6*b^4 + 640*a^7*b^3 + 320*a^8*b^2)) + x*(8*a ^7*b + 4*a^8 - 4*a^4*b^4 - 8*a^5*b^3))*(-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1 /2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*1 i)/(6*a^6*b + 2*a^7 + ((-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3) ^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^ 4*b^5 + 128*a^5*b^4 + 192*a^6*b^3 + 128*a^7*b^2 + x*(-(3*a*b^2 - a^2*b + 3* a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b ^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6 + 320*a^5*b^5 + 640*a^6*b^4 + 640*a^7*b^ 3 + 320*a^8*b^2)) - x*(8*a^7*b + 4*a^8 - 4*a^4*b^4 - 8*a^5*b^3))*(-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^ 2*b^3 + a^3*b^2)))^(1/2) + ((-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a *b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 + 128*a^5*b^4 + 192*a^6*b^3 + 128*a^7*b^2 - x*(-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6 + 320*a^5*b^5 + 640*a^6*b^4 + 640*a ^7*b^3 + 320*a^8*b^2)) + x*(8*a^7*b + 4*a^8 - 4*a^4*b^4 - 8*a^5*b^3))*(-(3* a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2) + 2*a^4*b^3 + 6*a^5*b^2))*(-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a ^3*b^2)))^(1/2)*2i

sympy [A] time = 4.54, size = 134, normalized size = 0.31

$$\mathrm{RootSum}\left(t^4\left(256a^3b^2+768a^2b^3+768ab^4+256b^5\right)+t^2\left(-32a^2b+96ab^2\right)+a,\left(t\mapsto t\log\left(x+\frac{-64t^3a^4b-128}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] RootSum(_t**4*(256*a**3*b**2 + 768*a**2*b**3 + 768*a*b**4 + 256*b**5) + _t*
*2*(-32*a**2*b + 96*a*b**2) + a, Lambda(_t, _t*log(x + (-64*_t**3*a**4*b -
128*_t**3*a**3*b**2 + 128*_t**3*a*b**4 + 64*_t**3*b**5 + 4*_t*a**3 - 40*_t*
a**2*b + 20*_t*a*b**2)/(3*a**2 - a*b)))) - 1/(x*(a + b))
```

3.915 $\int \frac{x}{1+x^2+x^4}\,dx$

Optimal. Leaf size=20

$$\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/3*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1107, 618, 204}

$$\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^2 + x^4),x]
```

```
[Out] ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{1+x^2+x^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right)$$
$$= -\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^2\right)$$
$$= \frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^2 + x^4),x]
[Out] ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3]
```

fricas [A] time = 0.91, size = 18, normalized size = 0.90

$$\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

giac [A] time = 0.15, size = 18, normalized size = 0.90

$$\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

maple [A] time = 0.00, size = 19, normalized size = 0.95

$$\frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^{2}+1\right)\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+x^2+1),x)
[Out] 1/3*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

maxima [A] time = 2.84, size = 18, normalized size = 0.90

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{2}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

mupad [B] time = 0.06, size = 20, normalized size = 1.00

$$\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}x^{2}}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2 + x^4 + 1),x)
[Out] (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/3))/3
```

sympy [A] time = 0.17, size = 26, normalized size = 1.30

$$\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}x^{2}}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+x**2+1),x)
[Out] sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/3
```

3.916 $\int \frac{x}{10+2x^2+x^4}\,dx$

Optimal. Leaf size=14

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(x^2+1\right)\right)$$

```
[Out] 1/6*arctan(1/3*x^2+1/3)
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1107, 618, 204}

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(10 + 2*x^2 + x^4),x]
```

```
[Out] ArcTan[(1 + x^2)/3]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{10+2x^2+x^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{10+2x+x^2}\,dx, x, x^2\right)$$
$$= -\,\text{Subst}\left(\int \frac{1}{-36-x^2}\,dx, x, 2\left(1+x^2\right)\right)$$
$$= \frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(1+x^2\right)\right)$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(10 + 2*x^2 + x^4),x]
[Out] ArcTan[(1 + x^2)/3]/6
```

fricas [A] time = 0.77, size = 10, normalized size = 0.71

$$\frac{1}{6}\,\arctan\left(\frac{1}{3}\,x^2+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+10),x, algorithm="fricas")
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

giac [A] time = 0.59, size = 10, normalized size = 0.71

$$\frac{1}{6}\,\arctan\left(\frac{1}{3}\,x^2+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+10),x, algorithm="giac")
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$\frac{\arctan\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+2*x^2+10),x)
```

```
[Out] 1/6*arctan(1/3*x^2+1/3)
```

maxima [A] time = 2.92, size = 10, normalized size = 0.71

$$\frac{1}{6}\arctan\left(\frac{1}{3}x^2+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+10),x, algorithm="maxima")
```

```
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

mupad [B] time = 0.06, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*x^2 + x^4 + 10),x)
```

```
[Out] atan(x^2/3 + 1/3)/6
```

sympy [A] time = 0.12, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+2*x**2+10),x)
```

```
[Out] atan(x**2/3 + 1/3)/6
```

3.917 $\int \frac{x^2}{20+9x^2+x^4} \, dx$

Optimal. Leaf size=23

$$\sqrt{5} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right) - 2\tan^{-1}\left(\frac{x}{2}\right)$$

```
[Out] -2*arctan(1/2*x)+arctan(1/5*x*5^(1/2))*5^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1130, 203}

$$\sqrt{5} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right) - 2\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(20 + 9*x^2 + x^4),x]
```

```
[Out] -2*ArcTan[x/2] + Sqrt[5]*ArcTan[x/Sqrt[5]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{20+9x^2+x^4} \, dx &= -\left(4\int \frac{1}{4+x^2} \, dx\right) + 5\int \frac{1}{5+x^2} \, dx\\ &= -2\tan^{-1}\left(\frac{x}{2}\right) + \sqrt{5} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)-2\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(20 + 9*x^2 + x^4),x]
[Out] -2*ArcTan[x/2] + Sqrt[5]*ArcTan[x/Sqrt[5]]
```

fricas [A] time = 0.79, size = 18, normalized size = 0.78

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-2\arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+9*x^2+20),x, algorithm="fricas")
[Out] sqrt(5)*arctan(1/5*sqrt(5)*x) - 2*arctan(1/2*x)
```

giac [A] time = 0.19, size = 18, normalized size = 0.78

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-2\arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+9*x^2+20),x, algorithm="giac")
[Out] sqrt(5)*arctan(1/5*sqrt(5)*x) - 2*arctan(1/2*x)
```

maple [A] time = 0.01, size = 19, normalized size = 0.83

$$-2\arctan\left(\frac{x}{2}\right)+\sqrt{5}\,\arctan\left(\frac{\sqrt{5}\,x}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4+9*x^2+20),x)
[Out] -2*arctan(1/2*x)+arctan(1/5*x*5^(1/2))*5^(1/2)
```

maxima [A] time = 2.98, size = 18, normalized size = 0.78

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-2\arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+9*x^2+20),x, algorithm="maxima")
```

```
[Out] sqrt(5)*arctan(1/5*sqrt(5)*x) - 2*arctan(1/2*x)
```

mupad [B] time = 4.37, size = 18, normalized size = 0.78

$$\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,x}{5}\right)-2\,\mathrm{atan}\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(9*x^2 + x^4 + 20),x)
```

```
[Out] 5^(1/2)*atan((5^(1/2)*x)/5) - 2*atan(x/2)
```

sympy [A] time = 0.21, size = 20, normalized size = 0.87

$$-2\,\mathrm{atan}\left(\frac{x}{2}\right)+\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,x}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4+9*x**2+20),x)
```

```
[Out] -2*atan(x/2) + sqrt(5)*atan(sqrt(5)*x/5)
```

3.918 $\int \frac{x^2}{1-x^2+x^4} \, dx$

Optimal. Leaf size=74

$$\frac{\log \left(x^2-\sqrt{3} x+1\right)}{4 \sqrt{3}}-\frac{\log \left(x^2+\sqrt{3} x+1\right)}{4 \sqrt{3}}-\frac{1}{2} \tan^{-1}\left(\sqrt{3}-2 x\right)+\frac{1}{2} \tan^{-1}\left(2 x+\sqrt{3}\right)$$

```
[Out] 1/2*arctan(2*x-3^(1/2))+1/2*arctan(2*x+3^(1/2))+1/12*ln(1+x^2-x*3^(1/2))*3^
(1/2)-1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1127, 1161, 618, 204, 1164, 628}

$$\frac{\log \left(x^2-\sqrt{3} x+1\right)}{4 \sqrt{3}}-\frac{\log \left(x^2+\sqrt{3} x+1\right)}{4 \sqrt{3}}-\frac{1}{2} \tan^{-1}\left(\sqrt{3}-2 x\right)+\frac{1}{2} \tan^{-1}\left(2 x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 - x^2 + x^4),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x]/2 + ArcTan[Sqrt[3] + 2*x]/2 + Log[1 - Sqrt[3]*x + x^
2]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{1-x^2+x^4}\,dx &= -\left(\frac{1}{2}\int \frac{1-x^2}{1-x^2+x^4}\,dx\right)+\frac{1}{2}\int \frac{1+x^2}{1-x^2+x^4}\,dx \\
&= \frac{1}{4}\int \frac{1}{1-\sqrt{3}\,x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+\sqrt{3}\,x+x^2}\,dx+\frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}\,x-x^2}\,dx}{4\sqrt{3}}+\frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}\,x-x^2}\,dx}{4\sqrt{3}} \\
&= \frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,-\sqrt{3}+2x\right)-\frac{1}{2}\,\text{Su} \\
&= -\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{2}\tan^{-1}\left(\sqrt{3}+2x\right)+\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.14, size = 94, normalized size = 1.27

$$\frac{\sqrt{-1-i\sqrt{3}}\,\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)+\sqrt{-1+i\sqrt{3}}\,\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{2\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(1 - x^2 + x^4),x]
```

```
[Out] (Sqrt[-1 - I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x)/2] + Sqrt[-1
 + I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x)/2])/(2*Sqrt[6])
```

fricas [B] time = 0.57, size = 159, normalized size = 2.15

$$-\frac{1}{6}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)-\frac{1}{6}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-x^2+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sq
rt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) - 1/6*sqrt(6)*
sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(3)
*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) - 1/24*sqrt(6)*sqrt(2)*log
(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + 1/24*sqrt(6)*sqrt(2)*log(-sqrt(6)*sqrt(2)
*x + 2*x^2 + 2)
```

giac [A] time = 0.17, size = 56, normalized size = 0.76

$$-\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{1}{2}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{2}\arctan\left(2x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-x^2+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2 - sqrt(3)*x +
 1) + 1/2*arctan(2*x + sqrt(3)) + 1/2*arctan(2*x - sqrt(3))
```

maple [A] time = 0.02, size = 57, normalized size = 0.77

$$\frac{\arctan\left(2x-\sqrt{3}\right)}{2}+\frac{\arctan\left(2x+\sqrt{3}\right)}{2}+\frac{\sqrt{3}\,\ln\left(x^2-\sqrt{3}\,x+1\right)}{12}-\frac{\sqrt{3}\,\ln\left(x^2+\sqrt{3}\,x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4-x^2+1),x)
```

```
[Out] 1/2*arctan(2*x-3^(1/2))+1/2*arctan(2*x+3^(1/2))+1/12*3^(1/2)*ln(x^2-3^(1/2)
*x+1)-1/12*3^(1/2)*ln(x^2+3^(1/2)*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^4 - x^2 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^4 - x^2 + 1), x)
```

mupad [B] time = 0.08, size = 44, normalized size = 0.59

$$-\operatorname{atan}\left(\frac{x}{2} - \frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \operatorname{atan}\left(\frac{x}{2} + \frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4 - x^2 + 1),x)
```

```
[Out] atan(x/2 + (3^(1/2)*x*1i)/2)*((3^(1/2)*1i)/6 + 1/2) - atan(x/2 - (3^(1/2)*x
*1i)/2)*((3^(1/2)*1i)/6 - 1/2)
```

sympy [A] time = 0.31, size = 63, normalized size = 0.85

$$\frac{\sqrt{3}\log\left(x^2 - \sqrt{3}x + 1\right)}{12} - \frac{\sqrt{3}\log\left(x^2 + \sqrt{3}x + 1\right)}{12} + \frac{\operatorname{atan}\left(2x - \sqrt{3}\right)}{2} + \frac{\operatorname{atan}\left(2x + \sqrt{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4-x**2+1),x)
```

```
[Out] sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/12
 + atan(2*x - sqrt(3))/2 + atan(2*x + sqrt(3))/2
```

3.919 $\int \frac{x^2}{2-2x^2+x^4}\,dx$

Optimal. Leaf size=188

$$\frac{\log\left(x^2-\sqrt{2\left(1+\sqrt{2}\right)}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2\left(1+\sqrt{2}\right)}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)$$

```
[Out] -1/4*arctan((-2*x+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^
(1/2)+1/4*arctan((2*x+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/
2))^(1/2)+1/4*ln(x^2+2^(1/2)-x*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)-1/4
*ln(x^2+2^(1/2)+x*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1127, 1161, 618, 204, 1164, 628}

$$\frac{\log\left(x^2-\sqrt{2\left(1+\sqrt{2}\right)}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2\left(1+\sqrt{2}\right)}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 - 2*x^2 + x^4),x]
```

```
[Out] -(Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*x)/Sqrt[2*(-1 + S
qrt[2])]])/2 + (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*x)/
Sqrt[2*(-1 + Sqrt[2])]])/2 + Log[Sqrt[2] - Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(
4*Sqrt[2*(1 + Sqrt[2])]) - Log[Sqrt[2] + Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(4*
Sqrt[2*(1 + Sqrt[2])])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{2-2x^2+x^4}\,dx = -\left(\frac{1}{2}\int \frac{\sqrt{2}-x^2}{2-2x^2+x^4}\,dx\right)+\frac{1}{2}\int \frac{\sqrt{2}+x^2}{2-2x^2+x^4}\,dx$$

$$= \frac{1}{4}\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx+\frac{1}{4}\int \frac{1}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx+\frac{\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}+}{-\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}}}{4\sqrt{2\left(1+\sqrt{}\right.}}$$

$$= \frac{\log\left(\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{1}{2}\,\text{Subst}\left(\int \frac{}{2\left(1-\sqrt{}\right.}\right.$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}}+\frac{\log\left(\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(\right.}{}$$

Mathematica [C] time = 0.03, size = 39, normalized size = 0.21

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-1-i}}\right)}{(-1-i)^{3/2}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-1+i}}\right)}{(-1+i)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 - 2*x^2 + x^4),x]
```

```
[Out] -(ArcTan[x/Sqrt[-1 - I]]/(-1 - I)^(3/2)) - ArcTan[x/Sqrt[-1 + I]]/(-1 + I)^
(3/2)
```

fricas [A] time = 0.86, size = 247, normalized size = 1.31

$$\frac{1}{16}\cdot 2^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+2x^2+2\sqrt{2}\right)-\frac{1}{16}\cdot 2^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(-2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-2*x^2+2),x, algorithm="fricas")
```

```
[Out] 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(2^(3/4)*x*sqrt(2*sqrt(2)
 + 4) + 2*x^2 + 2*sqrt(2)) - 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)
*log(-2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2)) - 1/4*2^(3/4)*sqrt
```

```
(2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 1/2*2^(1/4)*sqr
t(2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2))*sqrt(2*sqrt(2) + 4) -
sqrt(2) - 1) - 1/4*2^(3/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*x*sqrt(2
*sqrt(2) + 4) + 1/2*2^(1/4)*sqrt(-2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2
*sqrt(2))*sqrt(2*sqrt(2) + 4) + sqrt(2) + 1)
```

giac [A] time = 0.85, size = 147, normalized size = 0.78

$$\frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x+2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\right)}{2\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\right)}{2\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{2\sqrt{2}-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-2*x^2+2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2*x + 2^(1/4)*sqrt(sqrt(2) + 2)
)/sqrt(-sqrt(2) + 2)) + 1/4*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2*x - 2
^(1/4)*sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(2*sqrt(2) - 2)*log
(x^2 + 2^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(2)) + 1/8*sqrt(2*sqrt(2) - 2)*log
(x^2 - 2^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(2))
```

maple [B] time = 0.10, size = 308, normalized size = 1.64

$$\frac{\sqrt{2}\,\left(2+2\sqrt{2}\right)\arctan\left(\frac{2x-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}}-\frac{\left(2+2\sqrt{2}\right)\arctan\left(\frac{2x-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}}+\frac{\sqrt{2}\,\left(2+2\sqrt{2}\right)\arctan\left(\frac{2x+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4-2*x^2+2),x)
```

```
[Out] -1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(x^2+2^(1/2)+x*(2+2*2^(1/2))^(1/2))+1/4*
2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*x+(2+2*2^(1/2))^(1/2))
/(-2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*ln(x^2+2^(1/2)+x*(2+2*2^(1/2
))^(1/2))-1/4*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*x+(2+2*2^(1/2))^
(1/2))/(-2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(x^2+2^(1/2)
-x*(2+2*2^(1/2))^(1/2))+1/4*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arct
an((2*x-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/8*(2+2*2^(1/2))^(1/2)*
ln(x^2+2^(1/2)-x*(2+2*2^(1/2))^(1/2))-1/4*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2
)*arctan((2*x-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^4-2\,x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-2*x^2+2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^4 - 2*x^2 + 2), x)
```

mupad [B] time = 4.37, size = 101, normalized size = 0.54

$$\operatorname{atanh}\left(32\,x\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+2\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)+\operatorname{atanh}\left(32\,x\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4 - 2*x^2 + 2),x)
```

```
[Out] atanh(32*x*((- 2^(1/2)/32 - 1/32)^(1/2) + (2^(1/2)/32 - 1/32)^(1/2))^3)*(2*
(- 2^(1/2)/32 - 1/32)^(1/2) + 2*(2^(1/2)/32 - 1/32)^(1/2)) + atanh(32*x*((-
 2^(1/2)/32 - 1/32)^(1/2) - (2^(1/2)/32 - 1/32)^(1/2))^3)*(2*(- 2^(1/2)/32
- 1/32)^(1/2) - 2*(2^(1/2)/32 - 1/32)^(1/2))
```

sympy [A] time = 0.83, size = 24, normalized size = 0.13

$$\operatorname{RootSum}\left(128t^4+16t^2+1,\left(t\mapsto t\log\left(64t^3+4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4-2*x**2+2),x)
```

```
[Out] RootSum(128*_t**4 + 16*_t**2 + 1, Lambda(_t, _t*log(64*_t**3 + 4*_t + x)))
```

3.920 $\int x^7\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=171

$$\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{9/2}}-\frac{b\left(7b^2-12ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^4}+\frac{\left(-32ac+35b^2-\right.}{}$$

```
[Out] 1/10*x^4*(c*x^4+b*x^2+a)^(3/2)/c+1/480*(-42*b*c*x^2-32*a*c+35*b^2)*(c*x^4+b
*x^2+a)^(3/2)/c^3+1/512*b*(-12*a*c+7*b^2)*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^2
+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(9/2)-1/256*b*(-12*a*c+7*b^2)*(2*c*x^2
+b)*(c*x^4+b*x^2+a)^(1/2)/c^4
```

Rubi [A] time = 0.16, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 742, 779, 612, 621, 206}

$$\frac{\left(-32ac+35b^2-42bcx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{480c^3}-\frac{b\left(7b^2-12ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^4}+\frac{b\left(7b^2-12ac\right)\left(b^2-4a\right.}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] -(b*(7*b^2 - 12*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*c^4) + (x^
4*(a + b*x^2 + c*x^4)^(3/2))/(10*c) + ((35*b^2 - 32*a*c - 42*b*c*x^2)*(a +
b*x^2 + c*x^4)^(3/2))/(480*c^3) + (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh
[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(512*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int x^7\sqrt{a+bx^2+cx^4}\,dx=\frac{1}{2}\text{Subst}\left(\int x^3\sqrt{a+bx+cx^2}\,dx,x,x^2\right)$$
$$=\frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{10c}+\frac{\text{Subst}\left(\int x\left(-2a-\frac{7bx}{2}\right)\sqrt{a+bx+cx^2}\,dx,x,x^2\right)}{10c}$$
$$=\frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{10c}+\frac{\left(35b^2-32ac-42bcx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{480c^3}-\frac{\left(b\left(7b^2-12ac\right)\right.}{}$$
$$=-\frac{b\left(7b^2-12ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^4}+\frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{10c}+\frac{\left(35b^2-32ac-\right.}{}$$
$$=-\frac{b\left(7b^2-12ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^4}+\frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{10c}+\frac{\left(35b^2-32ac-\right.}{}$$
$$=-\frac{b\left(7b^2-12ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^4}+\frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{10c}+\frac{\left(35b^2-32ac-\right.}{}$$

Mathematica [A] time = 0.15, size = 164, normalized size = 0.96

$$\frac{-\frac{\left(32ac-35b^2+42bcx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{5\left(12abc-7b^3\right)\left(2\sqrt{c}\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\right)}{256c^{7/2}}+x^4\left(a+bx^2+cx\right.}{10c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x^4*(a + b*x^2 + c*x^4)^(3/2) - ((-35*b^2 + 32*a*c + 42*b*c*x^2)*(a + b*x^
2 + c*x^4)^(3/2))/(48*c^2) + (5*(-7*b^3 + 12*a*b*c)*(2*Sqrt[c]*(b + 2*c*x^2
)*Sqrt[a + b*x^2 + c*x^4] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*
Sqrt[a + b*x^2 + c*x^4])]))/(256*c^(7/2)))/(10*c)
```

fricas [A] time = 0.94, size = 367, normalized size = 2.15

$$\left[\frac{15\left(7\,b^5-40\,ab^3c+48\,a^2bc^2\right)\sqrt{c}\,\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)+4\left(384\right.}{15}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/15360*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^4 - 8
*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) +
 4*(384*c^5*x^8 + 48*b*c^4*x^6 - 105*b^4*c + 460*a*b^2*c^2 - 256*a^2*c^3 -
8*(7*b^2*c^3 - 16*a*c^4)*x^4 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x^2)*sqrt(c*x^4
 + b*x^2 + a))/c^5, -1/7680*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(-c
)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*
x^2 + a*c)) - 2*(384*c^5*x^8 + 48*b*c^4*x^6 - 105*b^4*c + 460*a*b^2*c^2 - 2
56*a^2*c^3 - 8*(7*b^2*c^3 - 16*a*c^4)*x^4 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x^
2)*sqrt(c*x^4 + b*x^2 + a))/c^5]
```

giac [A] time = 0.25, size = 172, normalized size = 1.01

$$\frac{1}{3840}\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6\left(8x^2+\frac{b}{c}\right)x^2-\frac{7b^2c^2-16ac^3}{c^4}\right)x^2+\frac{35b^3c-116abc^2}{c^4}\right)x^2-\frac{105b^4-460ab^2c+25}{c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*(8*x^2 + b/c)*x^2 - (7*b^2*c^2 - 16
*a*c^3)/c^4)*x^2 + (35*b^3*c - 116*a*b*c^2)/c^4)*x^2 - (105*b^4 - 460*a*b^2
*c + 256*a^2*c^2)/c^4) - 1/512*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*log(abs(
-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(9/2)
```

maple [A] time = 0.02, size = 296, normalized size = 1.73

$$\frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}x^4}{10c}+\frac{3\sqrt{cx^4+bx^2+a}\,abx^2}{32c^2}-\frac{7\sqrt{cx^4+bx^2+a}\,b^3x^2}{128c^3}+\frac{3a^2b\ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{32c^{\frac{5}{2}}}-\frac{5a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/10*x^4*(c*x^4+b*x^2+a)^(3/2)/c-7/80*b/c^2*x^2*(c*x^4+b*x^2+a)^(3/2)+7/96*
b^2/c^3*(c*x^4+b*x^2+a)^(3/2)-7/128*b^3/c^3*(c*x^4+b*x^2+a)^(1/2)*x^2-7/256
*b^4/c^4*(c*x^4+b*x^2+a)^(1/2)-5/64*b^3/c^(7/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c
*x^4+b*x^2+a)^(1/2))*a+7/512*b^5/c^(9/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*
x^2+a)^(1/2))+3/32*b/c^2*a*(c*x^4+b*x^2+a)^(1/2)*x^2+3/64*b^2/c^3*a*(c*x^4+
b*x^2+a)^(1/2)+3/32*b/c^(5/2)*a^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^
(1/2))-1/15*a/c^2*(c*x^4+b*x^2+a)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 5.31, size = 315, normalized size = 1.84

$$\frac{x^4 \left(c\,x^4 + b\,x^2 + a\right)^{3/2}}{10\,c} + \frac{7\,b \left(\frac{a \left(\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\sqrt{c\,x^4+b\,x^2+a} + \frac{\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c} - \frac{x^2 \left(c\,x^4+b\,x^2+a\right)^{3/2}}{4\,c} + \frac{5\,b \left(\frac{\left(8\,c\left(c\,x^4+a\right)-3\,b^2+\cdots\right.}{\cdots}\right.}{\cdots} \cdots \right)}{20\,c} \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (x^4*(a + b*x^2 + c*x^4)^(3/2))/(10*c) + (7*b*((a*((b/(4*c) + x^2/2)*(a + b
*x^2 + c*x^4)^(1/2) + (log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2
))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (x^2*(a + b*x^2 + c*x^4)^(3/2))/(4*
c) + (5*b*(((8*c*(a + c*x^4) - 3*b^2 + 2*b*c*x^2)*(a + b*x^2 + c*x^4)^(1/2)
)/(24*c^2) + (log(2*(a + b*x^2 + c*x^4)^(1/2) + (b + 2*c*x^2)/c^(1/2))*(b^3
 - 4*a*b*c))/(16*c^(5/2))))/(8*c)))/(20*c) - (a*(((8*c*(a + c*x^4) - 3*b^2
+ 2*b*c*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(24*c^2) + (log(2*(a + b*x^2 + c*x^
4)^(1/2) + (b + 2*c*x^2)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(5*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \sqrt{a + bx^2 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**7*sqrt(a + b*x**2 + c*x**4), x)
```

3.921 $\int x^5\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=153

$$-\frac{(b^2-4ac)(5b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{7/2}}+\frac{(5b^2-4ac)(b+2cx^2)\sqrt{a+bx^2+cx^4}}{128c^3}-\frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48c^2}+$$

```
[Out] -5/48*b*(c*x^4+b*x^2+a)^(3/2)/c^2+1/8*x^2*(c*x^4+b*x^2+a)^(3/2)/c-1/256*(-4
*a*c+b^2)*(-4*a*c+5*b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1
/2))/c^(7/2)+1/128*(-4*a*c+5*b^2)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c^3
```

Rubi [A] time = 0.13, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 742, 640, 612, 621, 206}

$$\frac{(5b^2-4ac)(b+2cx^2)\sqrt{a+bx^2+cx^4}}{128c^3}-\frac{(b^2-4ac)(5b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{7/2}}-\frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48c^2}+$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((5*b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*c^3) - (5*b*(a
 + b*x^2 + c*x^4)^(3/2))/(48*c^2) + (x^2*(a + b*x^2 + c*x^4)^(3/2))/(8*c) -
 ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b
*x^2 + c*x^4])])/(256*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int x^5\sqrt{a+bx^2+cx^4}\,dx=\frac{1}{2}\text{Subst}\left(\int x^2\sqrt{a+bx+cx^2}\,dx,x,x^2\right)$$
$$=\frac{x^2\left(a+bx^2+cx^4\right)^{3/2}}{8c}+\frac{\text{Subst}\left(\int\left(-a-\frac{5bx}{2}\right)\sqrt{a+bx+cx^2}\,dx,x,x^2\right)}{8c}$$
$$=-\frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{x^2\left(a+bx^2+cx^4\right)^{3/2}}{8c}+\frac{\left(5b^2-4ac\right)\text{Subst}\left(\int\sqrt{a+bx+cx^2}\right.}{32c^2}$$
$$=\frac{\left(5b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^3}-\frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{x^2\left(a+bx^2+cx^4\right)^{3/2}}{8c}$$
$$=\frac{\left(5b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^3}-\frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{x^2\left(a+bx^2+cx^4\right)^{3/2}}{8c}$$
$$=\frac{\left(5b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^3}-\frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{x^2\left(a+bx^2+cx^4\right)^{3/2}}{8c}$$

Mathematica [A] time = 0.07, size = 136, normalized size = 0.89

$$\frac{2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(b\left(8c^2x^4-52ac\right)+24c^2x^2\left(a+2cx^4\right)+15b^3-10b^2cx^2\right)-3\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}}{768c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(15*b^3 - 10*b^2*c*x^2 + 24*c^2*x^2*(a +
 2*c*x^4) + b*(-52*a*c + 8*c^2*x^4)) - 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*
ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(768*c^(7/2))
```

fricas [A] time = 0.97, size = 303, normalized size = 1.98

$$\left[\frac{3\left(5b^4-24ab^2c+16a^2c^2\right)\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2+4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c}-4ac\right)+4\left(48c^4x^6\right.}{1536c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/1536*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c
*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(
48*c^4*x^6 + 8*b*c^3*x^4 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)
*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^4, 1/768*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c
^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2
*x^4 + b*c*x^2 + a*c)) + 2*(48*c^4*x^6 + 8*b*c^3*x^4 + 15*b^3*c - 52*a*b*c^
2 - 2*(5*b^2*c^2 - 12*a*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^4]
```

giac [A] time = 0.23, size = 134, normalized size = 0.88

$$\frac{1}{384}\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6x^2+\frac{b}{c}\right)x^2-\frac{5b^2c-12ac^2}{c^3}\right)x^2+\frac{15b^3-52abc}{c^3}\right)+\frac{\left(5b^4-24ab^2c+16a^2c^2\right)\log\left(\left|-\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*x^2 + b/c)*x^2 - (5*b^2*c - 12*a*c^2
)/c^3)*x^2 + (15*b^3 - 52*a*b*c)/c^3) + 1/256*(5*b^4 - 24*a*b^2*c + 16*a^2*
c^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(7/
2)
```

maple [A] time = 0.02, size = 247, normalized size = 1.61

$$-\frac{\sqrt{c\,x^4+b\,x^2+a}\;a\,x^2}{16c}+\frac{5\sqrt{c\,x^4+b\,x^2+a}\;b^2x^2}{64c^2}-\frac{a^2\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{16c^{\frac{3}{2}}}+\frac{3a\,b^2\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\right.}{32c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/8*x^2*(c*x^4+b*x^2+a)^(3/2)/c-5/48*b*(c*x^4+b*x^2+a)^(3/2)/c^2+5/64*b^2/c
^2*(c*x^4+b*x^2+a)^(1/2)*x^2+5/128*b^3/c^3*(c*x^4+b*x^2+a)^(1/2)+3/32*b^2/c
^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a-5/256*b^4/c^(7/2)*
ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/16*a/c*(c*x^4+b*x^2+a)^(1
/2)*x^2-1/32*a/c^2*(c*x^4+b*x^2+a)^(1/2)*b-1/16*a^2/c^(3/2)*ln((c*x^2+1/2*b
)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 4.64, size = 193, normalized size = 1.26

$$\frac{x^2\left(c\,x^4+b\,x^2+a\right)^{3/2}}{8\,c}-\frac{a\left(\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\sqrt{c\,x^4+b\,x^2+a}+\frac{\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{8\,c}-\frac{5\,b\left(\frac{\left(8\,c\left(c\,x^4+a\right)-3\,b^2+2\,b\,c\,x^2\right)}{24\,c^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (x^2*(a + b*x^2 + c*x^4)^(3/2))/(8*c) - (a*((b/(4*c) + x^2/2)*(a + b*x^2 +
c*x^4)^(1/2) + (log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2))*(a*c
 - b^2/4))/(2*c^(3/2))))/(8*c) - (5*b*(((8*c*(a + c*x^4) - 3*b^2 + 2*b*c*x^
2)*(a + b*x^2 + c*x^4)^(1/2))/(24*c^2) + (log(2*(a + b*x^2 + c*x^4)^(1/2) +
 (b + 2*c*x^2)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(16*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\sqrt{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(a + b*x**2 + c*x**4), x)
```

3.922 $\int x^3\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=108

$$\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}}-\frac{b\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{16c^2}+\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6c}$$

```
[Out] 1/6*(c*x^4+b*x^2+a)^(3/2)/c+1/32*b*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^2+b)/c^(
1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(5/2)-1/16*b*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2
)/c^2
```

Rubi [A] time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 640, 612, 621, 206}

$$\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}}-\frac{b\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{16c^2}+\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] -(b*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c^2) + (a + b*x^2 + c*x^4)^(
3/2)/(6*c) + (b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x
^2 + c*x^4])])/(32*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int x^3\sqrt{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int x\sqrt{a+bx+cx^2}\,dx, x, x^2\right)$$
$$= \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6c} - \frac{b\,\text{Subst}\left(\int \sqrt{a+bx+cx^2}\,dx, x, x^2\right)}{4c}$$
$$= -\frac{b\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{16c^2} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6c} + \frac{\left(b\left(b^2-4ac\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+c}}\right.}{32c^2}$$
$$= -\frac{b\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{16c^2} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6c} + \frac{\left(b\left(b^2-4ac\right)\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx\right.}{16c^2}$$
$$= -\frac{b\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{16c^2} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6c} + \frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+}}\right.}{32c^{5/2}}$$

Mathematica [A] time = 0.05, size = 101, normalized size = 0.94

$$\frac{2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(8c\left(a+cx^4\right)-3b^2+2bcx^2\right)+3b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{96c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2 + 2*b*c*x^2 + 8*c*(a + c*x^4)) +
 3*b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]
)])/(96*c^(5/2))
```

fricas [A] time = 0.78, size = 237, normalized size = 2.19

$$\left[-\frac{3\left(b^{3}-4\,abc\right)\sqrt{c}\,\log\left(-8\,c^{2}x^{4}-8\,bcx^{2}-b^{2}+4\,\sqrt{cx^{4}+bx^{2}+a}\left(2\,cx^{2}+b\right)\sqrt{c}\,-4\,ac\right)-4\left(8\,c^{3}x^{4}+2\,bc^{2}x^{2}-\right.}{192\,c^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(b^3 - 4*a*b*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqr
t(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^4 + 2*b*c^
2*x^2 - 3*b^2*c + 8*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^3, -1/96*(3*(b^3 - 4*
a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(
c^2*x^4 + b*c*x^2 + a*c)) - 2*(8*c^3*x^4 + 2*b*c^2*x^2 - 3*b^2*c + 8*a*c^2)
*sqrt(c*x^4 + b*x^2 + a))/c^3]
```

giac [A] time = 0.22, size = 98, normalized size = 0.91

$$\frac{1}{48}\,\sqrt{cx^{4}+bx^{2}+a}\left(2\left(4\,x^{2}+\frac{b}{c}\right)x^{2}-\frac{3\,b^{2}-8\,ac}{c^{2}}\right)-\frac{\left(b^{3}-4\,abc\right)\log\left(\left|-2\left(\sqrt{c}\,x^{2}-\sqrt{cx^{4}+bx^{2}+a}\right)\sqrt{c}\,-b\right|\right)}{32\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)*x^2 - (3*b^2 - 8*a*c)/c^2) -
1/32*(b^3 - 4*a*b*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqr
t(c) - b))/c^(5/2)
```

maple [A] time = 0.01, size = 139, normalized size = 1.29

$$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\;b\,x^{2}}{8c}-\frac{ab\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{8c^{\frac{3}{2}}}+\frac{b^{3}\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{32c^{\frac{5}{2}}}-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\;b}{16c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2+a)^(1/2),x)
```

[Out] 1/6*(c*x^4+b*x^2+a)^(3/2)/c-1/8*b/c*x^2*(c*x^4+b*x^2+a)^(1/2)-1/16*b^2/c^2*(c*x^4+b*x^2+a)^(1/2)-1/8*b/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a+1/32*b^3/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [B] time = 4.52, size = 87, normalized size = 0.81

$$\frac{\left(8\,c\,\left(c\,x^4+a\right)-3\,b^2+2\,b\,c\,x^2\right)\,\sqrt{c\,x^4+b\,x^2+a}}{48\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^4+b\,x^2+a}+\frac{2\,c\,x^2+b}{\sqrt{c}}\right)\,\left(b^3-4\,a\,b\,c\right)}{32\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a + b*x^2 + c*x^4)^(1/2),x)

[Out] ((8*c*(a + c*x^4) - 3*b^2 + 2*b*c*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(48*c^2) + (log(2*(a + b*x^2 + c*x^4)^(1/2) + (b + 2*c*x^2)/c^(1/2))*(b^3 - 4*a*b*c))/(32*c^(5/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3} \sqrt{a + b x^{2} + c x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral(x**3*sqrt(a + b*x**2 + c*x**4), x)

3.923 $\int x\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=83

$$\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c}-\frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}}$$

```
[Out] -1/16*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c
^(3/2)+1/8*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1107, 612, 621, 206}

$$\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c}-\frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c) - ((b^2 - 4*a*c)*ArcTanh[(b +
 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \sqrt{a+bx+cx^2}\,dx, x, x^2\right)\\ &= \frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c} - \frac{(b^2-4ac)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{16c}\\ &= \frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c} - \frac{(b^2-4ac)\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{8c}\\ &= \frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c} - \frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 1.00

$$\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c} - \frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c) - ((b^2 - 4*a*c)*ArcTanh[(b +
 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(3/2))
```

fricas [A] time = 0.88, size = 197, normalized size = 2.37

$$\left[-\frac{(b^2-4\,ac)\sqrt{c}\,\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{c}\,-4\,ac\right)-4\,\sqrt{cx^4+bx^2+a}\,(2\,c^2x^2\right.}{32\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x
^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^4 + b*x^2 + a)*
(2*c^2*x^2 + b*c))/c^2, 1/16*((b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4
+ b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*sqrt(c*x
^4 + b*x^2 + a)*(2*c^2*x^2 + b*c))/c^2]
```

giac [A] time = 0.20, size = 76, normalized size = 0.92

$$\frac{1}{8}\sqrt{cx^4+bx^2+a}\left(2x^2+\frac{b}{c}\right)+\frac{(b^2-4ac)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{16c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + 1/16*(b^2 - 4*a*c)*log(abs(-2*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3/2)
```

maple [A] time = 0.01, size = 101, normalized size = 1.22

$$\frac{a\ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{4\sqrt{c}}-\frac{b^2\ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{16c^{\frac{3}{2}}}+\frac{(2cx^2+b)\sqrt{cx^4+bx^2+a}}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/8*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c+1/4/c^(1/2)*ln((c*x^2+1/2*b)/c^(1/2
)+(c*x^4+b*x^2+a)^(1/2))*a-1/16/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x
^2+a)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 4.62, size = 72, normalized size = 0.87

$$\frac{\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\,\sqrt{c\,x^4+b\,x^2+a}}{2}+\frac{\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{4\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((b/(4*c) + x^2/2)*(a + b*x^2 + c*x^4)^(1/2))/2 + (log((a + b*x^2 + c*x^4)^
(1/2) + (b/2 + c*x^2)/c^(1/2))*(a*c - b^2/4))/(4*c^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*x**2 + c*x**4), x)
```

3.924 $\int \frac{\sqrt{a+bx^2+cx^4}}{x}\,dx$

Optimal. Leaf size=109

$$\frac{1}{2}\sqrt{a+bx^2+cx^4}-\frac{1}{2}\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))*a^(1/2)+1/4*b*a
rctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+1/2*(c*x^4+b*
x^2+a)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 734, 843, 621, 206, 724}

$$\frac{1}{2}\sqrt{a+bx^2+cx^4}-\frac{1}{2}\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x,x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/2 - (Sqrt[a]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[
a + b*x^2 + c*x^4])])/2 + (b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^
2 + c*x^4])])/(4*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\sqrt{a+bx^2+cx^4} - \frac{1}{4}\operatorname{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\sqrt{a+bx^2+cx^4} + \frac{1}{2}a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right) + \frac{1}{4}b\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx}\cdots}\right.$$
$$= \frac{1}{2}\sqrt{a+bx^2+cx^4} - a\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right) + \frac{1}{2}b\operatorname{Subst}\left(\int \frac{1}{4c-x\cdots}\right.$$
$$= \frac{1}{2}\sqrt{a+bx^2+cx^4} - \frac{1}{2}\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right) + \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}}$$

Mathematica [A] time = 0.04, size = 106, normalized size = 0.97

$$\frac{1}{4}\left(2\sqrt{a+bx^2+cx^4} - 2\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right) + \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x,x]
```

```
[Out] (2*Sqrt[a + b*x^2 + c*x^4] - 2*Sqrt[a]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqr
t[a + b*x^2 + c*x^4])] + (b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2
 + c*x^4])])/Sqrt[c])/4
```

fricas [A] time = 0.93, size = 566, normalized size = 5.19

$$\left[\frac{b\sqrt{c}\log\left(-8c^2x^4 - 8bcx^2 - b^2 - 4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c} - 4ac\right) + 2\sqrt{a}c\log\left(-\frac{\left(b^2+4ac\right)x^4+8abx^2-4\sqrt{cx^4\cdots}}{x^4\cdots}\right.}{8c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/8*(b*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a
)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 2*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^4 + 8*
```

```
a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4
*sqrt(c*x^4 + b*x^2 + a)*c)/c, -1/4*(b*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x
^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - sqrt(a)*c*log(-
((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sq
rt(a) + 8*a^2)/x^4) - 2*sqrt(c*x^4 + b*x^2 + a)*c)/c, 1/8*(4*sqrt(-a)*c*arc
tan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 +
 a^2)) + b*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2
+ a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^4 + b*x^2 + a)*c)/c, 1/4*(
2*sqrt(-a)*c*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c
*x^4 + a*b*x^2 + a^2)) - b*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c
*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*sqrt(c*x^4 + b*x^2 + a)*c
)/c]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Value
```

maple [A] time = 0.01, size = 91, normalized size = 0.83

$$-\frac{\sqrt{a}\,\ln\left(\frac{b\,x^{2}+2 a+2 \sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{a}}{x^{2}}\right)}{2}+\frac{b \ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{4 \sqrt{c}}+\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x,x)
```

```
[Out] 1/2*(c*x^4+b*x^2+a)^(1/2)+1/4*b*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1
/2))/c^(1/2)-1/2*a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2
)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 4.42, size = 88, normalized size = 0.81

$$\frac{\sqrt{c\,x^4+b\,x^2+a}}{2}-\frac{\sqrt{a}\,\ln\left(\frac{b}{2}+\frac{a}{x^2}+\frac{\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2}+\frac{b\,\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{4\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/x,x)
```

```
[Out] (a + b*x^2 + c*x^4)^(1/2)/2 - (a^(1/2)*log(b/2 + a/x^2 + (a^(1/2)*(a + b*x^
2 + c*x^4)^(1/2))/x^2))/2 + (b*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2
)/c^(1/2)))/(4*c^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x, x)
```

3.925 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^3} dx$

Optimal. Leaf size=112

$$-\frac{\sqrt{a+bx^2+cx^4}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}}+\frac{1}{2}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)$$

```
[Out] -1/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(1/2)+1/2*a
rctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*c^(1/2)-1/2*(c*x^4+b*
x^2+a)^(1/2)/x^2
```

Rubi [A] time = 0.10, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 732, 843, 621, 206, 724}

$$-\frac{\sqrt{a+bx^2+cx^4}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}}+\frac{1}{2}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^3,x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*x^2) - (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt
[a + b*x^2 + c*x^4])])/(4*Sqrt[a]) + (Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt
[c]*Sqrt[a + b*x^2 + c*x^4])])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{a+bx^2+cx^4}}{2x^2} + \frac{1}{4}\text{Subst}\left(\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{a+bx^2+cx^4}}{2x^2} + \frac{1}{4}b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right) + \frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+}}\right.$$
$$= -\frac{\sqrt{a+bx^2+cx^4}}{2x^2} - \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right) + c\,\text{Subst}\left(\int \frac{1}{4c-x^2}\right.$$
$$= -\frac{\sqrt{a+bx^2+cx^4}}{2x^2} - \frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}} + \frac{1}{2}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)$$

Mathematica [A] time = 0.05, size = 112, normalized size = 1.00

$$-\frac{\sqrt{a+bx^2+cx^4}}{2x^2} - \frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}} + \frac{1}{2}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^3,x]
```

```
[Out] -1/2*Sqrt[a + b*x^2 + c*x^4]/x^2 - (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt
[a + b*x^2 + c*x^4])])/(4*Sqrt[a]) + (Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt
[c]*Sqrt[a + b*x^2 + c*x^4])])/2
```

fricas [A] time = 0.82, size = 601, normalized size = 5.37

$$\left[\frac{2\,a\sqrt{c}\,x^2\log\left(-8\,c^2x^4 - 8\,bcx^2 - b^2 - 4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c} - 4\,ac\right) + \sqrt{a}\,bx^2\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2-4\,\sqrt{}}{}\right.}{8\,ax^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*sqrt(c)*x^2*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x
^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + sqrt(a)*b*x^2*log(-((b^2 + 4*a*c)*
```

```
x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/
x^4) - 4*sqrt(c*x^4 + b*x^2 + a)*a)/(a*x^2), -1/8*(4*a*sqrt(-c)*x^2*arctan(
1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c
)) - sqrt(a)*b*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x
^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*sqrt(c*x^4 + b*x^2 + a)*a)/
(a*x^2), 1/4*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*
a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + a*sqrt(c)*x^2*log(-8*c^2*x^4 - 8*b
*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 2
*sqrt(c*x^4 + b*x^2 + a)*a)/(a*x^2), 1/4*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*
x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*a*sq
rt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x
^4 + b*c*x^2 + a*c)) - 2*sqrt(c*x^4 + b*x^2 + a)*a)/(a*x^2)]
```

giac [A] time = 0.29, size = 148, normalized size = 1.32

$$\frac{b \arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}}-\frac{1}{2}\sqrt{c}\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)+\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)b+}{2\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a) -
1/2*sqrt(c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b)
) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a*sqrt(c))/((sqrt(c)
*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)
```

maple [A] time = 0.01, size = 140, normalized size = 1.25

$$\frac{\sqrt{c\,x^4+b\,x^2+a}\;c\,x^2}{2a}-\frac{b\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\;\sqrt{a}}{x^2}\right)}{4\sqrt{a}}+\frac{\sqrt{c}\;\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2}+\frac{\sqrt{c\,x^4+b\,x^2+a}\;b}{2a}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^3,x)
```

```
[Out] -1/2/a/x^2*(c*x^4+b*x^2+a)^(3/2)+1/2*b/a*(c*x^4+b*x^2+a)^(1/2)-1/4*b/a^(1/2
)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)+1/2*c/a*(c*x^4+b*x^2+
a)^(1/2)*x^2+1/2*c^(1/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 4.55, size = 91, normalized size = 0.81

$$\frac{\sqrt{c}\,\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{2}-\frac{\sqrt{c\,x^4+b\,x^2+a}}{2\,x^2}-\frac{b\,\ln\left(\frac{b}{2}+\frac{a}{x^2}+\frac{\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{4\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/x^3,x)
```

```
[Out] (c^(1/2)*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2)))/2 - (a + b
*x^2 + c*x^4)^(1/2)/(2*x^2) - (b*log(b/2 + a/x^2 + (a^(1/2)*(a + b*x^2 + c*
x^4)^(1/2))/x^2))/(4*a^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**3, x)
```

3.926 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^5}\,dx$

Optimal. Leaf size=88

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{3/2}}-\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{8ax^4}$$

```
[Out] 1/16*(-4*a*c+b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^
(3/2)-1/8*(b*x^2+2*a)*(c*x^4+b*x^2+a)^(1/2)/a/x^4
```

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1114, 720, 724, 206}

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{3/2}}-\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{8ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^5,x]
```

```
[Out] -((2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*a*x^4) + ((b^2 - 4*a*c)*ArcTanh
[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^2+cx^4}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx,x,x^2\right)\\ &= -\frac{\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{8ax^4}-\frac{\left(b^2-4ac\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{16a}\\ &= -\frac{\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{8ax^4}+\frac{\left(b^2-4ac\right)\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{8a}\\ &= -\frac{\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{8ax^4}+\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 1.00

$$\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{3/2}}-\frac{\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{8ax^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^5,x]
```

```
[Out] -1/8*((2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(a*x^4) + ((b^2 - 4*a*c)*ArcTa
nh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(3/2))
```

fricas [A] time = 0.92, size = 215, normalized size = 2.44

$$\left[-\frac{\left(b^2-4\,ac\right)\sqrt{a}\,x^4\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)+4\,\sqrt{cx^4+bx^2+a}\left(abx^2+2\,a^2\right)}{32\,a^2x^4},-\frac{\left(b^2-4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2 - 4*a*c)*sqrt(a)*x^4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*s
qrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*sqrt(c*x^4 +
 b*x^2 + a)*(a*b*x^2 + 2*a^2))/(a^2*x^4), -1/16*((b^2 - 4*a*c)*sqrt(-a)*x^4
*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x
^2 + a^2)) + 2*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2))/(a^2*x^4)]
```

giac [B] time = 0.22, size = 241, normalized size = 2.74

$$-\frac{\left(b^2-4\,ac\right)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}\,a}+\frac{\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3b^2+4\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3ac+8\left(\cdots\right.}{8\left(\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/8*(b^2 - 4*a*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a)
)/(sqrt(-a)*a) + 1/8*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*b^2 + 4*(sq
rt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a*c + 8*(sqrt(c)*x^2 - sqrt(c*x^4 +
b*x^2 + a))^2*a*b*sqrt(c) + (sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a*b^2 +
 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^2*c)/(((sqrt(c)*x^2 - sqrt(c*x
^4 + b*x^2 + a))^2 - a)^2*a)
```

maple [B] time = 0.01, size = 193, normalized size = 2.19

$$-\frac{\sqrt{c\,x^4+b\,x^2+a}\;bc\,x^2}{8a^2}-\frac{c\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\;\sqrt{a}}{x^2}\right)}{4\sqrt{a}}+\frac{b^2\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\;\sqrt{a}}{x^2}\right)}{16a^{\frac{3}{2}}}+\frac{\sqrt{c\,x^4+b\,x^2+a}\;c}{4a}-\frac{\sqrt{c\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^5,x)
```

```
[Out] -1/4/a/x^4*(c*x^4+b*x^2+a)^(3/2)+1/8*b/a^2/x^2*(c*x^4+b*x^2+a)^(3/2)-1/8*b^
2/a^2*(c*x^4+b*x^2+a)^(1/2)+1/16*b^2/a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a
)^(1/2)*a^(1/2))/x^2)-1/8*b/a^2*c*(c*x^4+b*x^2+a)^(1/2)*x^2+1/4*c/a*(c*x^4+
b*x^2+a)^(1/2)-1/4*c/a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))
/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/x^5,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(1/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**5, x)
```

3.927 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^7} \, dx$

Optimal. Leaf size=116

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{5/2}}+\frac{b\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{16a^2x^4}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}$$

```
[Out] -1/6*(c*x^4+b*x^2+a)^(3/2)/a/x^6-1/32*b*(-4*a*c+b^2)*arctanh(1/2*(b*x^2+2*a
)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(5/2)+1/16*b*(b*x^2+2*a)*(c*x^4+b*x^2+a)
^(1/2)/a^2/x^4
```

Rubi [A] time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 730, 720, 724, 206}

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{5/2}}+\frac{b\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{16a^2x^4}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^7,x]
```

```
[Out] (b*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*a^2*x^4) - (a + b*x^2 + c*x^4
)^(3/2)/(6*a*x^6) - (b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[
a + b*x^2 + c*x^4])])/(32*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^2+cx^4}}{x^7}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^4}\,dx,x,x^2\right)\\ &= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}-\frac{b\operatorname{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx,x,x^2\right)}{4a}\\ &= \frac{b\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{16a^2x^4}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}+\frac{\left(b\left(b^2-4ac\right)\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\right.}{32a^2}\\ &= \frac{b\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{16a^2x^4}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}-\frac{\left(b\left(b^2-4ac\right)\right)\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\right.}{16a^2}\\ &= \frac{b\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{16a^2x^4}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right.}{32a^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 108, normalized size = 0.93

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{5/2}}-\frac{\sqrt{a+bx^2+cx^4}\left(8a^2+2ax^2\left(b+4cx^2\right)-3b^2x^4\right)}{48a^2x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^7,x]
```

```
[Out] -1/48*(Sqrt[a + b*x^2 + c*x^4]*(8*a^2 - 3*b^2*x^4 + 2*a*x^2*(b + 4*c*x^2)))
/(a^2*x^6) - (b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x
^2 + c*x^4])])/(32*a^(5/2))
```

fricas [A] time = 0.95, size = 261, normalized size = 2.25

$$\left[-\frac{3\left(b^3-4\,abc\right)\sqrt{a}\,x^6\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)+4\left(2\,a^2bx^2-\left(3\,ab^2-8\,a^2c\right)x^4+8\,a^3\right)}{192\,a^3x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(b^3 - 4*a*b*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2
+ 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*(2*a^2*
b*x^2 - (3*a*b^2 - 8*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^3*x^6)
, 1/96*(3*(b^3 - 4*a*b*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(
b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*(2*a^2*b*x^2 - (3*a*b^
2 - 8*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^3*x^6)]
```

giac [B] time = 0.30, size = 359, normalized size = 3.09

$$\frac{\left(b^3-4\,abc\right)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{16\,\sqrt{-a}\,a^2}-\frac{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^5b^3-12\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^5abc-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/16*(b^3 - 4*a*b*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-
a))/(sqrt(-a)*a^2) - 1/48*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*b^3
- 12*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a*b*c - 48*(sqrt(c)*x^2 - sq
rt(c*x^4 + b*x^2 + a))^4*a^2*c^(3/2) - 8*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))^3*a*b^3 - 48*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^2*b*c - 48*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^2*b^2*sqrt(c) - 3*(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2 + a))*a^2*b^3 - 36*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 +
a))*a^3*b*c - 16*a^4*c^(3/2))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 -
 a)^3*a^2)
```

maple [B] time = 0.01, size = 222, normalized size = 1.91

$$\frac{\sqrt{c\,x^4+b\,x^2+a}\;b^2c\,x^2}{16a^3}+\frac{bc\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\;\sqrt{a}}{x^2}\right)}{8a^{\frac{3}{2}}}-\frac{b^3\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\;\sqrt{a}}{x^2}\right)}{32a^{\frac{5}{2}}}-\frac{\sqrt{c\,x^4+b\,x^2+a}\;bc}{8a^2}+\frac{\sqrt{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^7,x)
```

```
[Out] -1/6*(c*x^4+b*x^2+a)^(3/2)/a/x^6+1/8*b/a^2/x^4*(c*x^4+b*x^2+a)^(3/2)-1/16*b
^2/a^3/x^2*(c*x^4+b*x^2+a)^(3/2)+1/16*b^3/a^3*(c*x^4+b*x^2+a)^(1/2)-1/32*b^
3/a^(5/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)+1/16*b^2/a^3*
c*(c*x^4+b*x^2+a)^(1/2)*x^2-1/8*b/a^2*c*(c*x^4+b*x^2+a)^(1/2)+1/8*b/a^(3/2)
*c*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/x^7,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(1/2)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**7, x)
```

3.928 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^9}\,dx$

Optimal. Leaf size=161

$$\frac{(b^2-4ac)(5b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{256a^{7/2}}-\frac{(5b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{128a^3x^4}+\frac{5b(a+bx^2+cx^4)^{3/2}}{48a^2x^6}-$$

```
[Out] -1/8*(c*x^4+b*x^2+a)^(3/2)/a/x^8+5/48*b*(c*x^4+b*x^2+a)^(3/2)/a^2/x^6+1/256
*(-4*a*c+b^2)*(-4*a*c+5*b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a
)^(1/2))/a^(7/2)-1/128*(-4*a*c+5*b^2)*(b*x^2+2*a)*(c*x^4+b*x^2+a)^(1/2)/a^3
/x^4
```

Rubi [A] time = 0.15, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 744, 806, 720, 724, 206}

$$-\frac{(5b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{128a^3x^4}+\frac{(b^2-4ac)(5b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{256a^{7/2}}+\frac{5b(a+bx^2+cx^4)^{3/2}}{48a^2x^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^9,x]
```

```
[Out] -((5*b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*a^3*x^4) - (a
 + b*x^2 + c*x^4)^(3/2)/(8*a*x^8) + (5*b*(a + b*x^2 + c*x^4)^(3/2))/(48*a^2
*x^6) + ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqr
t[a + b*x^2 + c*x^4])])/(256*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
```

```
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^9}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^5}\,dx,x,x^2\right)$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{8ax^8} - \frac{\text{Subst}\left(\int \frac{\left(\frac{5b}{2}+cx\right)\sqrt{a+bx+cx^2}}{x^4}\,dx,x,x^2\right)}{8a}$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{8ax^8} + \frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48a^2x^6} + \frac{\left(5b^2-4ac\right)\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx,x,x^2\right.}{32a^2}$$

$$= -\frac{\left(5b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^3x^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{8ax^8} + \frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48a^2x^6}$$

$$= -\frac{\left(5b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^3x^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{8ax^8} + \frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48a^2x^6}$$

$$= -\frac{\left(5b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^3x^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{8ax^8} + \frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48a^2x^6}$$

Mathematica [A] time = 0.10, size = 141, normalized size = 0.88

$$\frac{3\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right) - \frac{2\sqrt{a}\sqrt{a+bx^2+cx^4}\left(48a^3+8a^2x^2\left(b+3cx^2\right)-2abx^4\left(5b+26cx^2\right)+15b^3x^6\right)}{x^8}}{768a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^9,x]
```

```
[Out] ((-2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]*(48*a^3 + 15*b^3*x^6 + 8*a^2*x^2*(b +
3*c*x^2) - 2*a*b*x^4*(5*b + 26*c*x^2)))/x^8 + 3*(5*b^4 - 24*a*b^2*c + 16*a^
2*c^2)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(768*a^(
7/2))
```

fricas [A] time = 0.76, size = 325, normalized size = 2.02

$$\left[\frac{3\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)\sqrt{a}\,x^8\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)-4\left(\left(15\,ab^3-52\,a^2bc\right)x^6+\right.}{1536\,a^4x^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/1536*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^8*log(-((b^2 + 4*a*c
)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2
)/x^4) - 4*((15*a*b^3 - 52*a^2*b*c)*x^6 + 8*a^3*b*x^2 - 2*(5*a^2*b^2 - 12*a
^3*c)*x^4 + 48*a^4)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^8), -1/768*(3*(5*b^4 -
24*a*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^8*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b
*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((15*a*b^3 - 52*a^2*b*c
)*x^6 + 8*a^3*b*x^2 - 2*(5*a^2*b^2 - 12*a^3*c)*x^4 + 48*a^4)*sqrt(c*x^4 + b
*x^2 + a))/(a^4*x^8)]
```

giac [B] time = 0.28, size = 617, normalized size = 3.83

$$-\frac{\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{128\,\sqrt{-a}\,a^3}+\frac{15\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^7b^4-72\left(\sqrt{c}\,x^2-\sqrt{cx^4+b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/128*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4
+ b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^3) + 1/384*(15*(sqrt(c)*x^2 - sqrt(c*x^
4 + b*x^2 + a))^7*b^4 - 72*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a*b^2*
c + 48*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a^2*c^2 - 55*(sqrt(c)*x^2
- sqrt(c*x^4 + b*x^2 + a))^5*a*b^4 + 264*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))^5*a^2*b^2*c + 336*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^3*c^2 +
 1152*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^3*b*c^(3/2) + 73*(sqrt(c)
*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^2*b^4 + 648*(sqrt(c)*x^2 - sqrt(c*x^4 +
 b*x^2 + a))^3*a^3*b^2*c + 336*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^
4*c^2 + 384*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^3*b^3*sqrt(c) + 256
*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^4*b*c^(3/2) + 15*(sqrt(c)*x^2
- sqrt(c*x^4 + b*x^2 + a))*a^3*b^4 + 312*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))*a^4*b^2*c + 48*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^5*c^2 + 128*
a^5*b*c^(3/2))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^4*a^3)
```

maple [B] time = 0.02, size = 387, normalized size = 2.40

$$\frac{\sqrt{c\,x^4+b\,x^2+a}\;b\,c^2x^2}{32a^3}-\frac{5\sqrt{c\,x^4+b\,x^2+a}\;b^3c\,x^2}{128a^4}+\frac{c^2\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\;\sqrt{a}}{x^2}\right)}{16a^{\frac{3}{2}}}-\frac{3b^2c\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\;\sqrt{a}}{x^2}\right)}{32a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^9,x)
```

```
[Out] -1/8*(c*x^4+b*x^2+a)^(3/2)/a/x^8+5/48*b*(c*x^4+b*x^2+a)^(3/2)/a^2/x^6-5/64*
b^2/a^3/x^4*(c*x^4+b*x^2+a)^(3/2)+5/128*b^3/a^4/x^2*(c*x^4+b*x^2+a)^(3/2)-5
/128*b^4/a^4*(c*x^4+b*x^2+a)^(1/2)+5/256*b^4/a^(7/2)*ln((b*x^2+2*a+2*(c*x^4
+b*x^2+a)^(1/2)*a^(1/2))/x^2)-5/128*b^3/a^4*c*(c*x^4+b*x^2+a)^(1/2)*x^2+7/6
4*b^2/a^3*c*(c*x^4+b*x^2+a)^(1/2)-3/32*b^2/a^(5/2)*c*ln((b*x^2+2*a+2*(c*x^4
+b*x^2+a)^(1/2)*a^(1/2))/x^2)+1/16*c/a^2/x^4*(c*x^4+b*x^2+a)^(3/2)-1/32*c/a
^3*b/x^2*(c*x^4+b*x^2+a)^(3/2)+1/32*c^2/a^3*b*(c*x^4+b*x^2+a)^(1/2)*x^2-1/1
6*c^2/a^2*(c*x^4+b*x^2+a)^(1/2)+1/16*c^2/a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x
^2+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/x^9,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(1/2)/x^9, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**9,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**9, x)
```

3.929 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^{11}} \, dx$

Optimal. Leaf size=199

$$-\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{512a^{9/2}}+\frac{b\left(7b^2-12ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^4x^4}-\frac{\left(35b^2-32ac\right)\left(a\right.}{480a}$$

```
[Out] -1/10*(c*x^4+b*x^2+a)^(3/2)/a/x^10+7/80*b*(c*x^4+b*x^2+a)^(3/2)/a^2/x^8-1/4
80*(-32*a*c+35*b^2)*(c*x^4+b*x^2+a)^(3/2)/a^3/x^6-1/512*b*(-12*a*c+7*b^2)*(
-4*a*c+b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(9/2)+
1/256*b*(-12*a*c+7*b^2)*(b*x^2+2*a)*(c*x^4+b*x^2+a)^(1/2)/a^4/x^4
```

Rubi [A] time = 0.23, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1114, 744, 834, 806, 720, 724, 206}

$$-\frac{\left(35b^2-32ac\right)\left(a+bx^2+cx^4\right)^{3/2}}{480a^3x^6}+\frac{b\left(7b^2-12ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^4x^4}-\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh^{-1}\left(\right.}{512a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^11,x]
```

```
[Out] (b*(7*b^2 - 12*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*a^4*x^4) -
(a + b*x^2 + c*x^4)^(3/2)/(10*a*x^10) + (7*b*(a + b*x^2 + c*x^4)^(3/2))/(80
*a^2*x^8) - ((35*b^2 - 32*a*c)*(a + b*x^2 + c*x^4)^(3/2))/(480*a^3*x^6) - (
b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a +
b*x^2 + c*x^4])])/(512*a^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
```

```
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
```

```
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^{11}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}} - \frac{\operatorname{Subst}\left(\int \frac{\left(\frac{7b}{2}+2cx\right)\sqrt{a+bx+cx^2}}{x^5}\,dx, x, x^2\right)}{10a}$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}} + \frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{80a^2x^8} + \frac{\operatorname{Subst}\left(\int \frac{\left(\frac{1}{4}\left(35b^2-32ac\right)+\frac{7bcx}{2}\right)\sqrt{a+bx+cx^2}}{x^4}\,dx, x\right.}{40a^2}$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}} + \frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{80a^2x^8} - \frac{\left(35b^2-32ac\right)\left(a+bx^2+cx^4\right)^{3/2}}{480a^3x^6} - \frac{\left(b\left(7b\right.\right.}{}$$

$$= \frac{b\left(7b^2-12ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^4x^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}} + \frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{80a^2x^8}$$

$$= \frac{b\left(7b^2-12ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^4x^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}} + \frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{80a^2x^8}$$

$$= \frac{b\left(7b^2-12ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^4x^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}} + \frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{80a^2x^8}$$

Mathematica [A] time = 0.12, size = 173, normalized size = 0.87

$$-\frac{b\left(48a^2c^2-40ab^2c+7b^4\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{512a^{9/2}} - \frac{\sqrt{a+bx^2+cx^4}\left(384a^4+16a^3\left(3bx^2+8cx^4\right)-8a^2\left(7b^2x^4\right.\right.}{3840a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^11,x]
```

```
[Out] -1/3840*(Sqrt[a + b*x^2 + c*x^4]*(384*a^4 - 105*b^4*x^8 + 10*a*b^2*x^6*(7*b
 + 46*c*x^2) + 16*a^3*(3*b*x^2 + 8*c*x^4) - 8*a^2*(7*b^2*x^4 + 29*b*c*x^6 +
 32*c^2*x^8)))/(a^4*x^10) - (b*(7*b^4 - 40*a*b^2*c + 48*a^2*c^2)*ArcTanh[(2
*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(512*a^(9/2))
```

fricas [A] time = 1.10, size = 389, normalized size = 1.95

$$\left[\frac{15\left(7\,b^5-40\,ab^3c+48\,a^2bc^2\right)\sqrt{a}\,x^{10}\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)+4\left(\left(105\,ab^4-460\,a^2b\right.\right.}{15360\,a^5x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] [1/15360*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(a)*x^10*log(-((b^2 +
4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) +
8*a^2)/x^4) + 4*((105*a*b^4 - 460*a^2*b^2*c + 256*a^3*c^2)*x^8 - 48*a^4*b*x
^2 - 2*(35*a^2*b^3 - 116*a^3*b*c)*x^6 - 384*a^5 + 8*(7*a^3*b^2 - 16*a^4*c)*
x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^5*x^10), 1/7680*(15*(7*b^5 - 40*a*b^3*c +
48*a^2*b*c^2)*sqrt(-a)*x^10*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a
)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((105*a*b^4 - 460*a^2*b^2*c + 256
*a^3*c^2)*x^8 - 48*a^4*b*x^2 - 2*(35*a^2*b^3 - 116*a^3*b*c)*x^6 - 384*a^5 +
 8*(7*a^3*b^2 - 16*a^4*c)*x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^5*x^10)]
```

giac [B] time = 0.36, size = 842, normalized size = 4.23

$$\frac{\left(7\,b^5-40\,ab^3c+48\,a^2bc^2\right)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{256\,\sqrt{-a}\,a^4}-\frac{105\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^9b^5-600\left(\sqrt{c}\,x^2-\sqrt{cx^4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/256*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4
 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^4) - 1/3840*(105*(sqrt(c)*x^2 - sqrt(c
*x^4 + b*x^2 + a))^9*b^5 - 600*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^9*a*
b^3*c + 720*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^9*a^2*b*c^2 - 490*(sqrt
(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a*b^5 + 2800*(sqrt(c)*x^2 - sqrt(c*x^4
 + b*x^2 + a))^7*a^2*b^3*c - 3360*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7
*a^3*b*c^2 - 7680*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^6*a^4*c^(5/2) + 8
96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^2*b^5 - 5120*(sqrt(c)*x^2 -
sqrt(c*x^4 + b*x^2 + a))^5*a^3*b^3*c - 15360*(sqrt(c)*x^2 - sqrt(c*x^4 + b*
x^2 + a))^5*a^4*b*c^2 - 24320*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^4
*b^2*c^(3/2) - 2560*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^5*c^(5/2) -
 790*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^3*b^5 - 9200*(sqrt(c)*x^2
- sqrt(c*x^4 + b*x^2 + a))^3*a^4*b^3*c - 12000*(sqrt(c)*x^2 - sqrt(c*x^4 +
```

```
b*x^2 + a))^3*a^5*b*c^2 - 3840*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^
4*b^4*sqrt(c) - 5120*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^5*b^2*c^(3
/2) - 2560*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^6*c^(5/2) - 105*(sqr
t(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^4*b^5 - 3240*(sqrt(c)*x^2 - sqrt(c*x^
4 + b*x^2 + a))*a^5*b^3*c - 720*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^6
*b*c^2 - 1280*a^6*b^2*c^(3/2) + 512*a^7*c^(5/2))/(((sqrt(c)*x^2 - sqrt(c*x^
4 + b*x^2 + a))^2 - a)^5*a^4)
```

maple [B] time = 0.02, size = 442, normalized size = 2.22

$$-\frac{3\sqrt{c\,x^{4}+b\,x^{2}+a}\;b^{2}c^{2}x^{2}}{64a^{4}}+\frac{7\sqrt{c\,x^{4}+b\,x^{2}+a}\;b^{4}c\,x^{2}}{256a^{5}}-\frac{3b\,c^{2}\ln\left(\frac{b\,x^{2}+2a+2\sqrt{c\,x^{4}+b\,x^{2}+a}\;\sqrt{a}}{x^{2}}\right)}{32a^{\frac{5}{2}}}+\frac{5b^{3}c\ln\left(\frac{b\,x^{2}+2a+2\sqrt{c\,x^{4}+b}}{x^{2}}\right.}{64a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^11,x)
```

```
[Out] -1/10*(c*x^4+b*x^2+a)^(3/2)/a/x^10+7/80*b*(c*x^4+b*x^2+a)^(3/2)/a^2/x^8-7/9
6*b^2/a^3/x^6*(c*x^4+b*x^2+a)^(3/2)+7/128*b^3/a^4/x^4*(c*x^4+b*x^2+a)^(3/2)
-7/256*b^4/a^5/x^2*(c*x^4+b*x^2+a)^(3/2)+7/256*b^5/a^5*(c*x^4+b*x^2+a)^(1/2
)-7/512*b^5/a^(9/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)+7/2
56*b^4/a^5*c*(c*x^4+b*x^2+a)^(1/2)*x^2-13/128*b^3/a^4*c*(c*x^4+b*x^2+a)^(1/
2)+5/64*b^3/a^(7/2)*c*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-3
/32*b/a^3*c/x^4*(c*x^4+b*x^2+a)^(3/2)+3/64*b^2/a^4*c/x^2*(c*x^4+b*x^2+a)^(3
/2)-3/64*b^2/a^4*c^2*(c*x^4+b*x^2+a)^(1/2)*x^2+3/32*b/a^3*c^2*(c*x^4+b*x^2+
a)^(1/2)-3/32*b/a^(5/2)*c^2*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/
x^2)+1/15*c/a^2/x^6*(c*x^4+b*x^2+a)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/x^11,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(1/2)/x^11, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**11,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**11, x)
```

3.930 $\int x^4\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=395

$$\frac{bx\left(8b^2-29ac\right)\sqrt{a+bx^2+cx^4}}{105c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{a}\,b\left(8b^2-29ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{105c^{11/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] -2/105*(-5*a*c+2*b^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^2+1/35*x^3*(5*c*x^2+b)*(c*x
^4+b*x^2+a)^(1/2)/c+1/105*b*(-29*a*c+8*b^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^(5/2)
/(a^(1/2)+x^2*c^(1/2))-1/105*a^(1/4)*b*(-29*a*c+8*b^2)*(cos(2*arctan(c^(1/4
)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arc
tan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/
2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2+a
)^(1/2)+1/210*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/
a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(8*b^3-29*a*b*c+2*(-5*a*c+2*b
^2)*a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(11/
4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1116, 1279, 1197, 1103, 1195}

$$-\frac{2x\left(2b^2-5ac\right)\sqrt{a+bx^2+cx^4}}{105c^2}+\frac{bx\left(8b^2-29ac\right)\sqrt{a+bx^2+cx^4}}{105c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}\left(2b^2-5ac\right)-29abc+8b^3\right)\left(\sqrt{a}\ldots\right.}{210c^{1\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (-2*(2*b^2 - 5*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(105*c^2) + (b*(8*b^2 - 29*a
*c)*x*Sqrt[a + b*x^2 + c*x^4])/(105*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) + (x^3
*(b + 5*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(35*c) - (a^(1/4)*b*(8*b^2 - 29*a*c
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
105*c^(11/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(8*b^3 - 29*a*b*c + 2*Sqrt
[a]*Sqrt[c]*(2*b^2 - 5*a*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
 b/(Sqrt[a]*Sqrt[c]))/4])/(210*c^(11/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1116

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(a + b*x^2 + c*x^4)^p*(2*b*p + c*(m + 4*p - 1)*x^
2))/(c*(m + 4*p + 1)*(m + 4*p - 1)), x] - Dist[(2*p*d^2)/(c*(m + 4*p + 1)*(
m + 4*p - 1)), Int[(d*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p - 1)*Simp[a*b*(m -
1) - (2*a*c*(m + 4*p - 1) - b^2*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && GtQ[m, 1] && IntegerQ[
2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int x^4\sqrt{a+bx^2+cx^4}\,dx = \frac{x^3\left(b+5cx^2\right)\sqrt{a+bx^2+cx^4}}{35c} - \frac{\int \frac{x^2\left(3ab+2\left(2b^2-5ac\right)x^2\right)}{\sqrt{a+bx^2+cx^4}}dx}{35c}$$

$$= -\frac{2\left(2b^2-5ac\right)x\sqrt{a+bx^2+cx^4}}{105c^2} + \frac{x^3\left(b+5cx^2\right)\sqrt{a+bx^2+cx^4}}{35c} + \frac{\int \frac{2a\left(2b^2-5ac\right)+b\left(8b^2-\right.}{\sqrt{a+bx^2+cx^4}}}{105c^2}$$

$$= -\frac{2\left(2b^2-5ac\right)x\sqrt{a+bx^2+cx^4}}{105c^2} + \frac{x^3\left(b+5cx^2\right)\sqrt{a+bx^2+cx^4}}{35c} - \frac{\left(\sqrt{a}\,b\left(8b^2-29ac\right.\right.}{105}$$

$$= -\frac{2\left(2b^2-5ac\right)x\sqrt{a+bx^2+cx^4}}{105c^2} + \frac{b\left(8b^2-29ac\right)x\sqrt{a+bx^2+cx^4}}{105c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{x^3\left(b+5cx^2\right)\sqrt{}}{35}$$

Mathematica [C] time = 1.55, size = 538, normalized size = 1.36

$$\frac{4cx\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(10a^2c+a\left(-4b^2+13bcx^2+25c^2x^4\right)-4b^3x^2-b^2cx^4+18bc^2x^6+15c^3x^8\right)-i\left(-20a^2c^2+37ab^2c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(10*a^2*c - 4*b^3*x^2 - b^2*c*x^4 +
18*b*c^2*x^6 + 15*c^3*x^8 + a*(-4*b^2 + 13*b*c*x^2 + 25*c^2*x^4)) + I*b*(8*
b^2 - 29*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b -
 Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a
*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-8*b^4 + 37
*a*b^2*c - 20*a^2*c^2 + 8*b^3*Sqrt[b^2 - 4*a*c] - 29*a*b*c*Sqrt[b^2 - 4*a*c
])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*
b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*Arc
Sinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b
 - Sqrt[b^2 - 4*a*c])])/(420*c^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b
*x^2 + c*x^4])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^4+bx^2+a}\,x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4 + bx^2 + a}\, x^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*x^4, x)
```

maple [A] time = 0.05, size = 476, normalized size = 1.21

$$\frac{\sqrt{c\,x^4+b\,x^2+a}\;x^5}{7}+\frac{\sqrt{c\,x^4+b\,x^2+a}\;b\,x^3}{35c}-\frac{\left(\frac{2a}{7}-\frac{4b^2}{35c}\right)\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\;a\,\mathrm{Elli}}{12\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\;\sqrt{c\,x^4+b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/7*x^5*(c*x^4+b*x^2+a)^(1/2)+1/35*b/c*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(2/7*a
-4/35*b^2/c)/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12*(2/7*a-4/35*b^2/c)/c*a*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1
/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^
(1/2))/a/c)^(1/2))-1/2*(-3/35*b/c*a-2/3*(2/7*a-4/35*b^2/c)/c*b)*a*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*
(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b
^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(
-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4 + bx^2 + a}\, x^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \sqrt{c\,x^4+b\,x^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^4*(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{4}\sqrt{a+bx^{2}+cx^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(a + b*x**2 + c*x**4), x)
```

3.931 $\int x^2\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=342

$$-\frac{2x\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}\,b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\right.\right.}{30c^{7/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] 1/15*x*(3*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c-2/15*(-3*a*c+b^2)*x*(c*x^4+b*x^2
+a)^(1/2)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))+2/15*a^(1/4)*(-3*a*c+b^2)*(cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elliptic
E(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2
)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x
^4+b*x^2+a)^(1/2)-1/30*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/c
os(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),
1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*b^2-6*a*c+b*a^(1/
2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+
b*x^2+a)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1116, 1197, 1103, 1195}

$$-\frac{2x\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}\,b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\right.\right.}{30c^{7/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (-2*(b^2 - 3*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^(3/2)*(Sqrt[a] + Sqrt[c]
*x^2)) + (x*(b + 3*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(15*c) + (2*a^(1/4)*(b^2
 - 3*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]
))/4])/(15*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]) - (a^(1/4)*(2*b^2 + Sqrt[a]*b*S
qrt[c] - 6*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*S
qrt[c]))/4])/(30*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
```

```
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1116

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(a + b*x^2 + c*x^4)^p*(2*b*p + c*(m + 4*p - 1)*x^
2))/(c*(m + 4*p + 1)*(m + 4*p - 1)), x] - Dist[(2*p*d^2)/(c*(m + 4*p + 1)*(
m + 4*p - 1)), Int[(d*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p - 1)*Simp[a*b*(m -
1) - (2*a*c*(m + 4*p - 1) - b^2*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && GtQ[m, 1] && IntegerQ[
2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int x^2\sqrt{a+bx^2+cx^4}\,dx = \frac{x\left(b+3cx^2\right)\sqrt{a+bx^2+cx^4}}{15c} - \frac{\int \frac{ab+2\left(b^2-3ac\right)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{15c}$$

$$= \frac{x\left(b+3cx^2\right)\sqrt{a+bx^2+cx^4}}{15c} + \frac{\left(2\sqrt{a}\left(b^2-3ac\right)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{15c^{3/2}} - \frac{\left(\sqrt{a}\left(\sqrt{a}\,b+\frac{2(}{}\right.\right.}{}$$

$$= -\frac{2\left(b^2-3ac\right)x\sqrt{a+bx^2+cx^4}}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{x\left(b+3cx^2\right)\sqrt{a+bx^2+cx^4}}{15c} + \frac{2\sqrt[4]{a}\left(b^2-3ac\right)\left(\sqrt{\right.}{}$$

Mathematica [C] time = 1.25, size = 479, normalized size = 1.40

$$\frac{-i\left(b^2-3ac\right)\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b + 3*c*x^2)*(a + b*x^2 + c*x^4) -
I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4
*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*
a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*
c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])
])/(30*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^4+bx^2+a}\,x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4 + bx^2 + a}\, x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*x^2, x)
```

maple [A] time = 0.01, size = 417, normalized size = 1.22

$$\frac{\sqrt{c\,x^4+b\,x^2+a}\;x^3}{5}-\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\;ab\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)}{ac}\ldots}}{2}\ldots\right)}{60\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\;\sqrt{c\,x^4+b\,x^2+a}\;c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/5*x^3*(c*x^4+b*x^2+a)^(1/2)+1/15*b/c*x*(c*x^4+b*x^2+a)^(1/2)-1/60*a*b/c*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+
4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Ell
ipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^
2)^(1/2))/a*b/c-4)^(1/2))-1/2*(2/5*a-2/15*b^2/c)*a*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+
b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(El
lipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b
^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4 + bx^2 + a}\, x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{c\,x^4+b\,x^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^2*(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*x**2 + c*x**4), x)
```

3.932 $\int \sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=309

$$\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\,b\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}}{3c^{3/4}\sqrt{a+}}$$

```
[Out] 1/3*x*(c*x^4+b*x^2+a)^(1/2)+1/3*b*x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(1/2)+
x^2*c^(1/2))-1/3*a^(1/4)*b*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2
*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*
(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2
)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)+1/6*a^(1/4)*(cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elliptic
F(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2
)+x^2*c^(1/2))*(b+2*a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))
^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1091, 1197, 1103, 1195}

$$\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\,b\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}}{3c^{3/4}\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x^2 + c*x^4])/3 + (b*x*Sqrt[a + b*x^2 + c*x^4])/(3*Sqrt[c]*(S
qrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*b*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(
1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
 b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
```

```
0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \sqrt{a+bx^2+cx^4}\,dx = \frac{1}{3}x\sqrt{a+bx^2+cx^4} + \frac{1}{3}\int \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{1}{3}x\sqrt{a+bx^2+cx^4} + \frac{1}{3}\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx - \frac{\left(\sqrt{a}\,b\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}}{3\sqrt{c}}$$

$$= \frac{1}{3}x\sqrt{a+bx^2+cx^4} + \frac{bx\sqrt{a+bx^2+cx^4}}{3\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\sqrt[4]{a}\,b\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\right.}{3c^{3/4}\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] time = 0.82, size = 445, normalized size = 1.44

$$\frac{-i\left(b\sqrt{b^2-4ac}+4ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+ib(}{12c\sqrt{-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4) + I*b*(-b + Sqrt
[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Ell
ipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2
- 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b
- 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSi
nh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b -
 Sqrt[b^2 - 4*a*c])])/(12*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2
+ c*x^4])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^4+bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a), x)
```

maple [A] time = 0.01, size = 379, normalized size = 1.23

$$-\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)+\mathrm{Ellipt}\right.}{6\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,\left(b+\sqrt{-4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*x*(c*x^4+b*x^2+a)^(1/2)+1/6*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2
+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-1/6*b*a*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1
/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a
*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1
/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2^(1/2)*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\sqrt{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4), x)
```

3.933 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^2}\,dx$

Optimal. Leaf size=303

$$\frac{2\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}\,x^2}-\frac{\sqrt{a+bx^2+cx^4}}{x}+\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\cdots}\right.\right.}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

```
[Out] -(c*x^4+b*x^2+a)^(1/2)/x+2*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/(a^(1/2)+x^2*c^(
1/2))-2*a^(1/4)*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-
b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x
^2*c^(1/2))^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)+1/2*(cos(2*arctan(c^(1/4)*x/a^(1
/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1
/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(b+2
*a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c
^(1/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1117, 1197, 1103, 1195}

$$\frac{2\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}\,x^2}-\frac{\sqrt{a+bx^2+cx^4}}{x}+\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\cdots}\right.\right.}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^2,x]
```

```
[Out] -(Sqrt[a + b*x^2 + c*x^4]/x) + (2*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[
a] + Sqrt[c]*x^2) - (2*a^(1/4)*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*
x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1
/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/Sqrt[a + b*x^2 + c*x^4] + ((b + 2*Sqrt[
a]*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqr
t[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[
c]))/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
```

```
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1117

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^p)/(d*(m + 1)), x] - Dist[(2*p)
/(d^2*(m + 1)), Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^2}\,dx = -\frac{\sqrt{a+bx^2+cx^4}}{x} + \int \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{x} + \left(b+2\sqrt{a}\sqrt{c}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx - \left(2\sqrt{a}\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx}}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{x} + \frac{2\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}\,x^2} - \frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\,\mathrm{t}\right.}{\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] time = 0.81, size = 435, normalized size = 1.44

$$\frac{-i\sqrt{2}\,x\sqrt{b^2-4ac}\,\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+ix\left(\sqrt{b^2-4ac}\right.}{2x\sqrt{\frac{c}{\sqrt{b^2-4ac}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^2,x]
```

```
[Out] (-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4) + I*(-b + Sqrt[b^2
- 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Ellipt
icE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4
*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*Sqrt[2]*Sqrt[b^2 - 4*a*c]*x*Sqrt[(b - S
qrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[
c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*
c])])/(2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^2, x)
```

maple [A] time = 0.01, size = 381, normalized size = 1.26

$$-\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)+\mathrm{Ellipti}\right.}{\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,\left(b+\sqrt{-4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^2,x)
```

```
[Out] -(c*x^4+b*x^2+a)^(1/2)/x+1/4*b*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-
2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)
^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-c*a*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*
(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)
^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(
b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/x^2,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(1/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**2, x)
```

3.934 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^4}\,dx$

Optimal. Leaf size=341

$$\frac{\sqrt[4]{c}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}}{3a^{3/4}\sqrt{a+}}$$

```
[Out] -1/3*(c*x^4+b*x^2+a)^(1/2)/x^3-1/3*b*(c*x^4+b*x^2+a)^(1/2)/a/x+1/3*b*x*c^(1
/2)*(c*x^4+b*x^2+a)^(1/2)/a/(a^(1/2)+x^2*c^(1/2))-1/3*b*c^(1/4)*(cos(2*arct
an(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(
sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+
x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^4
+b*x^2+a)^(1/2)+1/6*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(
2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2
*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(b+2*a^(1/2)*c^(1/2))*(
(c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^4+b*x^2+a)^(1/2
)
```

Rubi [A] time = 0.16, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1117, 1281, 1197, 1103, 1195}

$$\frac{\sqrt[4]{c}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}}{3a^{3/4}\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^4,x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(3*x^3) - (b*Sqrt[a + b*x^2 + c*x^4])/(3*a*x) + (b
*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(3*a*(Sqrt[a] + Sqrt[c]*x^2)) - (b*c^(1
/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])
/(3*a^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + ((b + 2*Sqrt[a]*Sqrt[c])*c^(1/4)*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*El
lipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*a^(
3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
```

```
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1117

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^p)/(d*(m + 1)), x] - Dist[(2*p)
/(d^2*(m + 1)), Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^4}\,dx = -\frac{\sqrt{a+bx^2+cx^4}}{3x^3} + \frac{1}{3}\int \frac{b+2cx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{3x^3} - \frac{b\sqrt{a+bx^2+cx^4}}{3ax} - \frac{\int \frac{-2ac-bcx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3a}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{3x^3} - \frac{b\sqrt{a+bx^2+cx^4}}{3ax} - \frac{\left(b\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{3\sqrt{a}} + \frac{1}{3}\left(\left(\frac{b}{\sqrt{a}}+2\sqrt{c}\right)\sqrt{c}\right)$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{3x^3} - \frac{b\sqrt{a+bx^2+cx^4}}{3ax} + \frac{b\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{3a\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{}}{}$$

Mathematica [C] time = 0.90, size = 459, normalized size = 1.35

$$\frac{-4\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(a+bx^2\right)\left(a+bx^2+cx^4\right) - ix^3\left(b\sqrt{b^2-4ac}+4ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}F\left(i\right.}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^4,x]
```

```
[Out] (-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2)*(a + b*x^2 + c*x^4) + I*b*(
-b + Sqrt[b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 -
 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (
b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[
b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b
^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(12*a*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])
]*x^3*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^4, x)
```

maple [A] time = 0.02, size = 404, normalized size = 1.18

$$-\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\left(-\operatorname{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)+\mathrm{Ellipt}\right.}{6\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,\left(b+\sqrt{-4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^4,x)
```

```
[Out] -1/3*(c*x^4+b*x^2+a)^(1/2)/x^3-1/3*b*(c*x^4+b*x^2+a)^(1/2)/a/x+1/6*c*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1
/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Elliptic
F(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1
/2))/a*b/c-4)^(1/2))-1/6*b*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*
(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(
1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1
/2))-EllipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-
4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/x^4,x)
[Out] int((a + b*x^2 + c*x^4)^(1/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**4,x)
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**4, x)
```

3.935 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^6}\,dx$

Optimal. Leaf size=397

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}\,b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt[4]{c}\left(b^2-3ac\right)\left(\sqrt{a}\right.}{}$$

```
[Out] -1/5*(c*x^4+b*x^2+a)^(1/2)/x^5-1/15*b*(c*x^4+b*x^2+a)^(1/2)/a/x^3+2/15*(-3*
a*c+b^2)*(c*x^4+b*x^2+a)^(1/2)/a^2/x-2/15*(-3*a*c+b^2)*x*c^(1/2)*(c*x^4+b*x
^2+a)^(1/2)/a^2/(a^(1/2)+x^2*c^(1/2))+2/15*c^(1/4)*(-3*a*c+b^2)*(cos(2*arct
an(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(
sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+
x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(c*x^4
+b*x^2+a)^(1/2)-1/30*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos
(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/
2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*b^2-6*a*c+b*a^(1/2)
*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(c*x^4+b*
x^2+a)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1117, 1281, 1197, 1103, 1195}

$$\frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{15a^2x}-\frac{2\sqrt{c}\,x\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{15a^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{a}\,b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+}{(\sqrt{a}}}}{30a^{7/4}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^6,x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(5*x^5) - (b*Sqrt[a + b*x^2 + c*x^4])/(15*a*x^3) +
 (2*(b^2 - 3*a*c)*Sqrt[a + b*x^2 + c*x^4])/(15*a^2*x) - (2*Sqrt[c]*(b^2 - 3
*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(15*a^2*(Sqrt[a] + Sqrt[c]*x^2)) + (2*c^(1
/4)*(b^2 - 3*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
 + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]
*Sqrt[c]))/4])/(15*a^(7/4)*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(2*b^2 + Sqr
t[a]*b*Sqrt[c] - 6*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(S
qrt[a]*Sqrt[c]))/4])/(30*a^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1117

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^p)/(d*(m + 1)), x] - Dist[(2*p)
/(d^2*(m + 1)), Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^6}\,dx = -\frac{\sqrt{a+bx^2+cx^4}}{5x^5} + \frac{1}{5}\int \frac{b+2cx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{5x^5} - \frac{b\sqrt{a+bx^2+cx^4}}{15ax^3} - \frac{\int \frac{2(b^2-3ac)+bcx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx}{15a}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{5x^5} - \frac{b\sqrt{a+bx^2+cx^4}}{15ax^3} + \frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{15a^2x} + \frac{\int \frac{-abc-2c(b^2-3a}{\sqrt{a+bx^2+cx^4}}}{15a^2}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{5x^5} - \frac{b\sqrt{a+bx^2+cx^4}}{15ax^3} + \frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{15a^2x} + \frac{\left(2\sqrt{c}\left(b^2-3a\right.\right.}{1}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{5x^5} - \frac{b\sqrt{a+bx^2+cx^4}}{15ax^3} + \frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{15a^2x} - \frac{2\sqrt{c}\left(b^2-3ac\right.}{15a^2\left(\sqrt{\right.}}$$

Mathematica [C] time = 1.34, size = 530, normalized size = 1.34

$$\frac{-2\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(3a^3+a^2\left(4bx^2+9cx^4\right)+a\left(-b^2x^4+7bcx^6+6c^2x^8\right)-2b^2x^6\left(b+cx^2\right)\right)-ix^5\left(b^2-3ac\right)\left(\sqrt{b^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^6,x]
```

```
[Out] (-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(3*a^3 - 2*b^2*x^6*(b + c*x^2) + a^2*(4
*b*x^2 + 9*c*x^4) + a*(-(b^2*x^4) + 7*b*c*x^6 + 6*c^2*x^8)) - I*(b^2 - 3*a*
c)*(-b + Sqrt[b^2 - 4*a*c])*x^5*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x
], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*a*b*c + b
^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2 - 4*a*c])*x^5*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] +
4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(3
0*a^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x^5*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^2+a}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^6, x)
```

maple [A] time = 0.02, size = 452, normalized size = 1.14

$$-\frac{\sqrt{2}\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\;bc\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2}, \frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)}{60\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\;a} - \left(3ac-b^2 \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^6,x)
```

```
[Out] -1/5*(c*x^4+b*x^2+a)^(1/2)/x^5-1/15*b*(c*x^4+b*x^2+a)^(1/2)/a/x^3-2/15*(3*a
*c-b^2)/a^2*(c*x^4+b*x^2+a)^(1/2)/x-1/60*b*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)
^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-1
/15*c*(3*a*c-b^2)/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a
*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x
^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-Elli
pticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2
)^(1/2))/a*b/c-4)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4 + b\,x^2 + a}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/x^6,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(1/2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**6, x)
```

3.936 $\int x^7 \left(a+bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=223

$$-\frac{3b\left(b^2-4ac\right)^2\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4096c^{11/2}}+\frac{3b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{2048c^5}-\frac{b\left(3b\right.}{}$$

```
[Out] -1/256*b*(-4*a*c+3*b^2)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(3/2)/c^4+1/14*x^4*(c*x
^4+b*x^2+a)^(5/2)/c+1/560*(-30*b*c*x^2-16*a*c+21*b^2)*(c*x^4+b*x^2+a)^(5/2)
/c^3-3/4096*b*(-4*a*c+b^2)^2*(-4*a*c+3*b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)
/(c*x^4+b*x^2+a)^(1/2))/c^(11/2)+3/2048*b*(-4*a*c+b^2)*(-4*a*c+3*b^2)*(2*c*
x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c^5
```

Rubi [A] time = 0.21, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 742, 779, 612, 621, 206}

$$\frac{\left(-16ac+21b^2-30bcx^2\right)\left(a+bx^2+cx^4\right)^{5/2}}{560c^3}-\frac{b\left(3b^2-4ac\right)\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{256c^4}+\frac{3b\left(b^2-4ac\right)\left(3b^2-4a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (3*b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(
2048*c^5) - (b*(3*b^2 - 4*a*c)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(25
6*c^4) + (x^4*(a + b*x^2 + c*x^4)^(5/2))/(14*c) + ((21*b^2 - 16*a*c - 30*b*
c*x^2)*(a + b*x^2 + c*x^4)^(5/2))/(560*c^3) - (3*b*(b^2 - 4*a*c)^2*(3*b^2 -
 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4096*c
^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int x^7 \left(a+bx^2+cx^4\right)^{3/2} dx = \frac{1}{2}\text{Subst}\left(\int x^3 \left(a+bx+cx^2\right)^{3/2} dx, x, x^2\right)$$

$$= \frac{x^4 \left(a+bx^2+cx^4\right)^{5/2}}{14c} + \frac{\text{Subst}\left(\int x\left(-2a-\frac{9bx}{2}\right)\left(a+bx+cx^2\right)^{3/2} dx, x, x^2\right)}{14c}$$

$$= \frac{x^4 \left(a+bx^2+cx^4\right)^{5/2}}{14c} + \frac{\left(21b^2-16ac-30bcx^2\right)\left(a+bx^2+cx^4\right)^{5/2}}{560c^3} - \frac{\left(b\left(3b^2-4ac\right)\right.}{}$$

$$= -\frac{b\left(3b^2-4ac\right)\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{256c^4} + \frac{x^4\left(a+bx^2+cx^4\right)^{5/2}}{14c} + \frac{\left(21b^2-16a\right.}{}$$

$$= \frac{3b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{2048c^5} - \frac{b\left(3b^2-4ac\right)\left(b+2cx^2\right)\left(a\right.}{256c^4}$$

$$= \frac{3b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{2048c^5} - \frac{b\left(3b^2-4ac\right)\left(b+2cx^2\right)\left(a\right.}{256c^4}$$

$$= \frac{3b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{2048c^5} - \frac{b\left(3b^2-4ac\right)\left(b+2cx^2\right)\left(a\right.}{256c^4}$$

Mathematica [A] time = 0.25, size = 192, normalized size = 0.86

$$\frac{-\frac{\left(16ac-21b^2+30bcx^2\right)\left(a+bx^2+cx^4\right)^{5/2}}{40c^2} + \frac{7\left(4abc-3b^3\right)\left(2\sqrt{c}\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}\left(4c\left(5a+2cx^4\right)-3b^2+8bcx^2\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\right.}{2048c^{9/2}}}{14c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^4*(a + b*x^2 + c*x^4)^(5/2) - ((-21*b^2 + 16*a*c + 30*b*c*x^2)*(a + b*x^
2 + c*x^4)^(5/2))/(40*c^2) + (7*(-3*b^3 + 4*a*b*c)*(2*Sqrt[c]*(b + 2*c*x^2)
*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2 + 8*b*c*x^2 + 4*c*(5*a + 2*c*x^4)) + 3*(b^
2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]))/(
2048*c^(9/2)))/(14*c)
```

fricas [A] time = 0.93, size = 535, normalized size = 2.40

$$\left[-\frac{105\left(3b^7-28ab^5c+80a^2b^3c^2-64a^3bc^3\right)\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2-4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c}-4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/286720*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(c
)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b
)*sqrt(c) - 4*a*c) - 4*(5120*c^7*x^12 + 6400*b*c^6*x^10 + 128*(b^2*c^5 + 64
*a*c^6)*x^8 + 315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4
- 16*(9*b^3*c^4 - 44*a*b*c^5)*x^6 + 8*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^2
*c^5)*x^4 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*x^2)*sqrt(c*x^
4 + b*x^2 + a))/c^6, 1/143360*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 6
4*a^3*b*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt
(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(5120*c^7*x^12 + 6400*b*c^6*x^10 + 128*
(b^2*c^5 + 64*a*c^6)*x^8 + 315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 -
2048*a^3*c^4 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^6 + 8*(21*b^4*c^3 - 124*a*b^2*
c^4 + 128*a^2*c^5)*x^4 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*x
^2)*sqrt(c*x^4 + b*x^2 + a))/c^6]
```

giac [B] time = 0.40, size = 669, normalized size = 3.00

$$\frac{1}{7680}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6\left(8x^2+\frac{b}{c}\right)x^2-\frac{7b^2c^2-16ac^3}{c^4}\right)x^2+\frac{35b^3c-116abc^2}{c^4}\right)x^2-\frac{105b^4-460ab^2c+}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/7680*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*(8*x^2 + b/c)*x^2 - (7*b^2*c^2 -
 16*a*c^3)/c^4)*x^2 + (35*b^3*c - 116*a*b*c^2)/c^4)*x^2 - (105*b^4 - 460*a*
b^2*c + 256*a^2*c^2)/c^4) - 15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*log(abs(
-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(9/2))*a + 1/307
20*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(2*(8*(10*x^2 + b/c)*x^2 - (9*b^2*c^3 -
 20*a*c^4)/c^5)*x^2 + (21*b^3*c^2 - 68*a*b*c^3)/c^5)*x^2 - (105*b^4*c - 448
*a*b^2*c^2 + 240*a^2*c^3)/c^5)*x^2 + (315*b^5 - 1680*a*b^3*c + 1808*a^2*b*c
^2)/c^5) + 15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*log(abs
(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(11/2))*b + 1/4
30080*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(2*(8*(10*(12*x^2 + b/c)*x^2 - (11*b
^2*c^4 - 24*a*c^5)/c^6)*x^2 + (99*b^3*c^3 - 316*a*b*c^4)/c^6)*x^2 - (231*b^
4*c^2 - 972*a*b^2*c^3 + 512*a^2*c^4)/c^6)*x^2 + (1155*b^5*c - 6048*a*b^3*c^
2 + 6352*a^2*b*c^3)/c^6)*x^2 - (3465*b^6 - 21840*a*b^4*c + 34608*a^2*b^2*c^
2 - 8192*a^3*c^3)/c^6) - 105*(33*b^7 - 252*a*b^5*c + 560*a^2*b^3*c^2 - 320*
a^3*b*c^3)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))
/c^(13/2))*c
```

maple [B] time = 0.04, size = 534, normalized size = 2.39

$$\frac{\sqrt{c\,x^4+b\,x^2+a}\;c\,x^{12}}{14}+\frac{5\sqrt{c\,x^4+b\,x^2+a}\;b\,x^{10}}{56}+\frac{4\sqrt{c\,x^4+b\,x^2+a}\;a\,x^8}{35}+\frac{\sqrt{c\,x^4+b\,x^2+a}\;b^2x^8}{560c}+\frac{11\sqrt{c\,x^4+b\,x^2+}}{1120c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/560*b^2*x^8/c*(c*x^4+b*x^2+a)^(1/2)-9/4480*b^3/c^2*x^6*(c*x^4+b*x^2+a)^(1
/2)+3/1280*b^4/c^3*x^4*(c*x^4+b*x^2+a)^(1/2)-3/1024*b^5/c^4*x^2*(c*x^4+b*x^
2+a)^(1/2)+1/70*a^2*x^4/c*(c*x^4+b*x^2+a)^(1/2)+49/640*a^2*b^2/c^3*(c*x^4+b
*x^2+a)^(1/2)+3/64*a^3*b/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(
1/2))-9/256*a*b^4/c^4*(c*x^4+b*x^2+a)^(1/2)+21/1024*a*b^5/c^(9/2)*ln((c*x^2
+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-15/256*a^2*b^3/c^(7/2)*ln((c*x^2+1/2
*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+4/35*a*x^8*(c*x^4+b*x^2+a)^(1/2)+9/2048*
b^6/c^5*(c*x^4+b*x^2+a)^(1/2)+1/14*c*x^12*(c*x^4+b*x^2+a)^(1/2)+11/1120*a*b
*x^6/c*(c*x^4+b*x^2+a)^(1/2)-31/2240*a*b^2/c^2*x^4*(c*x^4+b*x^2+a)^(1/2)+13
/640*a*b^3/c^3*x^2*(c*x^4+b*x^2+a)^(1/2)-73/2240*a^2*b/c^2*x^2*(c*x^4+b*x^2
+a)^(1/2)+5/56*b*x^10*(c*x^4+b*x^2+a)^(1/2)-9/4096*b^7/c^(11/2)*ln((c*x^2+1
/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/35*a^3/c^2*(c*x^4+b*x^2+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^7\left(c\,x^4+b\,x^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^7*(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**7*(a + b*x**2 + c*x**4)**(3/2), x)
```

3.937 $\int x^5 \left(a+bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=204

$$\frac{\left(b^2-4ac\right)^2\left(7b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2048c^{9/2}}-\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{1024c^4}+\frac{\left(7b^2-4ac\right)}{}$$

```
[Out] 1/384*(-4*a*c+7*b^2)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(3/2)/c^3-7/120*b*(c*x^4+b
*x^2+a)^(5/2)/c^2+1/12*x^2*(c*x^4+b*x^2+a)^(5/2)/c+1/2048*(-4*a*c+b^2)^2*(-
4*a*c+7*b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(9/2)
-1/1024*(-4*a*c+b^2)*(-4*a*c+7*b^2)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c^4
```

Rubi [A] time = 0.18, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 742, 640, 612, 621, 206}

$$\frac{\left(7b^2-4ac\right)\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{384c^3}-\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{1024c^4}+\frac{\left(b^2-4ac\right)^2\left(7b^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(102
4*c^4) + ((7*b^2 - 4*a*c)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(384*c^3
) - (7*b*(a + b*x^2 + c*x^4)^(5/2))/(120*c^2) + (x^2*(a + b*x^2 + c*x^4)^(5
/2))/(12*c) + ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqr
t[c]*Sqrt[a + b*x^2 + c*x^4])])/(2048*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int x^5 \left(a+bx^2+cx^4\right)^{3/2} dx = \frac{1}{2}\text{Subst}\left(\int x^2 \left(a+bx+cx^2\right)^{3/2} dx, x, x^2\right)$$
$$= \frac{x^2 \left(a+bx^2+cx^4\right)^{5/2}}{12c} + \frac{\text{Subst}\left(\int \left(-a-\frac{7bx}{2}\right)\left(a+bx+cx^2\right)^{3/2} dx, x, x^2\right)}{12c}$$
$$= -\frac{7b\left(a+bx^2+cx^4\right)^{5/2}}{120c^2} + \frac{x^2\left(a+bx^2+cx^4\right)^{5/2}}{12c} + \frac{\left(7b^2-4ac\right)\text{Subst}\left(\int \left(a+bx+cx\right.\right.}{48c^2}$$
$$= \frac{\left(7b^2-4ac\right)\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{384c^3} - \frac{7b\left(a+bx^2+cx^4\right)^{5/2}}{120c^2} + \frac{x^2\left(a+bx^2+c\right.}{12c}$$
$$= -\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{1024c^4} + \frac{\left(7b^2-4ac\right)\left(b+2cx^2\right)\left(a+b\right.}{384c^3}$$
$$= -\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{1024c^4} + \frac{\left(7b^2-4ac\right)\left(b+2cx^2\right)\left(a+b\right.}{384c^3}$$
$$= -\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{1024c^4} + \frac{\left(7b^2-4ac\right)\left(b+2cx^2\right)\left(a+b\right.}{384c^3}$$

Mathematica [A] time = 0.16, size = 175, normalized size = 0.86

$$\frac{\frac{\left(7b^2-4ac\right)\left(2\sqrt{c}\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}\left(4c\left(5a+2cx^4\right)-3b^2+8bcx^2\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\right)}{512c^{7/2}} + x^2\left(a+bx^2+cx^4\right)^{5/2} - \frac{7b\left(a+bx^2\right.}{10}}{12c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((-7*b*(a + b*x^2 + c*x^4)^(5/2))/(10*c) + x^2*(a + b*x^2 + c*x^4)^(5/2) +
((7*b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2 +
 8*b*c*x^2 + 4*c*(5*a + 2*c*x^4)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^2)
/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]))/(512*c^(7/2)))/(12*c)
```

fricas [A] time = 0.94, size = 451, normalized size = 2.21

$$\left[-\frac{15\left(7b^6-60ab^4c+144a^2b^2c^2-64a^3c^3\right)\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2+4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c}-4a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/61440*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)*l
og(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*s
qrt(c) - 4*a*c) - 4*(1280*c^6*x^10 + 1664*b*c^5*x^8 + 16*(3*b^2*c^4 + 140*a
*c^5)*x^6 - 105*b^5*c + 760*a*b^3*c^2 - 1296*a^2*b*c^3 - 8*(7*b^3*c^3 - 36*
a*b*c^4)*x^4 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x^2)*sqrt(c*x^4
 + b*x^2 + a))/c^5, -1/30720*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64
*a^3*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c
)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*(1280*c^6*x^10 + 1664*b*c^5*x^8 + 16*(3*b^
2*c^4 + 140*a*c^5)*x^6 - 105*b^5*c + 760*a*b^3*c^2 - 1296*a^2*b*c^3 - 8*(7*
b^3*c^3 - 36*a*b*c^4)*x^4 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x^
2)*sqrt(c*x^4 + b*x^2 + a))/c^5]
```

giac [B] time = 0.40, size = 535, normalized size = 2.62

$$\frac{1}{768}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6x^2+\frac{b}{c}\right)x^2-\frac{5b^2c-12ac^2}{c^3}\right)x^2+\frac{15b^3-52abc}{c^3}\right)+\frac{3\left(5b^4-24ab^2c+16a^2c^2\right)\mathrm{lo}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/768*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*x^2 + b/c)*x^2 - (5*b^2*c - 12*a*
c^2)/c^3)*x^2 + (15*b^3 - 52*a*b*c)/c^3) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c
^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(7/2
))*a + 1/7680*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*(8*x^2 + b/c)*x^2 - (7*b^
2*c^2 - 16*a*c^3)/c^4)*x^2 + (35*b^3*c - 116*a*b*c^2)/c^4)*x^2 - (105*b^4 -
 460*a*b^2*c + 256*a^2*c^2)/c^4) - 15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*l
og(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(9/2))*b
+ 1/30720*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(2*(8*(10*x^2 + b/c)*x^2 - (9*b^
2*c^3 - 20*a*c^4)/c^5)*x^2 + (21*b^3*c^2 - 68*a*b*c^3)/c^5)*x^2 - (105*b^4*
c - 448*a*b^2*c^2 + 240*a^2*c^3)/c^5)*x^2 + (315*b^5 - 1680*a*b^3*c + 1808*
a^2*b*c^2)/c^5) + 15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*
log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(11/2))*
c
```

maple [B] time = 0.02, size = 432, normalized size = 2.12

$$\frac{\sqrt{c\,x^4+b\,x^2+a}\;c\,x^{10}}{12}+\frac{13\sqrt{c\,x^4+b\,x^2+a}\;b\,x^8}{120}+\frac{7\sqrt{c\,x^4+b\,x^2+a}\;a\,x^6}{48}+\frac{\sqrt{c\,x^4+b\,x^2+a}\;b^2x^6}{320c}+\frac{3\sqrt{c\,x^4+b\,x^2}}{160c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 7/1536*b^4/c^3*x^2*(c*x^4+b*x^2+a)^(1/2)+1/320*b^2*x^6/c*(c*x^4+b*x^2+a)^(1
/2)-7/1920*b^3/c^2*x^4*(c*x^4+b*x^2+a)^(1/2)+1/32*a^2*x^2/c*(c*x^4+b*x^2+a)
^(1/2)-15/512*a*b^4/c^(7/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
+9/128*a^2*b^2/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-27/3
20*a^2*b/c^2*(c*x^4+b*x^2+a)^(1/2)+19/384*a*b^3/c^3*(c*x^4+b*x^2+a)^(1/2)+1
3/120*b*x^8*(c*x^4+b*x^2+a)^(1/2)+3/160*a*b*x^4/c*(c*x^4+b*x^2+a)^(1/2)-9/3
20*a*b^2/c^2*x^2*(c*x^4+b*x^2+a)^(1/2)-1/32*a^3/c^(3/2)*ln((c*x^2+1/2*b)/c^
(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/12*c*x^10*(c*x^4+b*x^2+a)^(1/2)+7/48*a*x^6*(
c*x^4+b*x^2+a)^(1/2)-7/1024*b^5/c^4*(c*x^4+b*x^2+a)^(1/2)+7/2048*b^6/c^(9/2
)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5 \left(c\,x^4 + b\,x^2 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^5*(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**5*(a + b*x**2 + c*x**4)**(3/2), x)
```

3.938 $\int x^3 \left(a+bx^2+cx^4\right)^{3/2} \, dx$

Optimal. Leaf size=150

$$-\frac{3b\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}}+\frac{3b\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right.}{32c^2}$$

```
[Out] -1/32*b*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(3/2)/c^2+1/10*(c*x^4+b*x^2+a)^(5/2)/c-
3/512*b*(-4*a*c+b^2)^2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2
))/c^(7/2)+3/256*b*(-4*a*c+b^2)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c^3
```

Rubi [A] time = 0.12, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 640, 612, 621, 206}

$$\frac{3b\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{3b\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}}-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^3}{32c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (3*b*(b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*c^3) - (b*(b
 + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(32*c^2) + (a + b*x^2 + c*x^4)^(5/2)
/(10*c) - (3*b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*
x^2 + c*x^4])])/(512*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int x^3\left(a+bx^2+cx^4\right)^{3/2}dx=\frac{1}{2}\text{Subst}\left(\int x\left(a+bx+cx^2\right)^{3/2}dx,x,x^2\right)$$

$$=\frac{\left(a+bx^2+cx^4\right)^{5/2}}{10c}-\frac{b\,\text{Subst}\left(\int\left(a+bx+cx^2\right)^{3/2}dx,x,x^2\right)}{4c}$$

$$=-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(a+bx^2+cx^4\right)^{5/2}}{10c}+\frac{\left(3b\left(b^2-4ac\right)\right)\text{Subst}\left(\int\sqrt{\ldots}\right.}{64c^{2}\ldots}$$

$$=\frac{3b\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{(a+b\ldots}{\ldots}$$

$$=\frac{3b\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{(a+b\ldots}{\ldots}$$

$$=\frac{3b\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{(a+b\ldots}{\ldots}$$

Mathematica [A] time = 0.14, size = 149, normalized size = 0.99

$$-\frac{3b\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)-2\sqrt{c}\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}\right)}{512c^{7/2}}-\frac{b\left(b+2cx^2\right)\left(a+bx^2+c\ldots\right.}{32c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -1/32*(b*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/c^2 + (a + b*x^2 + c*x^4)
^(5/2)/(10*c) - (3*b*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x^2)*Sqrt[a + b*x^2
 + c*x^4] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 +
 c*x^4])]))/(512*c^(7/2))
```

fricas [A] time = 0.79, size = 361, normalized size = 2.41

$$\left[\frac{15\left(b^5-8\,ab^3c+16\,a^2bc^2\right)\sqrt{c}\,\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}\,-4\,ac\right)+4\left(128\,c^5x\right.}{5120}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/5120*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c
*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(
128*c^5*x^8 + 176*b*c^4*x^6 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 + 8*(b
^2*c^3 + 32*a*c^4)*x^4 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2
 + a))/c^4, 1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(-c)*arctan(1/2
*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c))
+ 2*(128*c^5*x^8 + 176*b*c^4*x^6 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 +
 8*(b^2*c^3 + 32*a*c^4)*x^4 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^2)*sqrt(c*x^4 +
b*x^2 + a))/c^4]
```

giac [B] time = 0.39, size = 414, normalized size = 2.76

$$\frac{1}{96}\left(2\,\sqrt{cx^4+bx^2+a}\left(2\left(4\,x^2+\frac{b}{c}\right)x^2-\frac{3\,b^2-8\,ac}{c^2}\right)-\frac{3\left(b^3-4\,abc\right)\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}\,-b\right|\right.}{c^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)*x^2 - (3*b^2 - 8*a*c)/c^2)
 - 3*(b^3 - 4*a*b*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqr
t(c) - b))/c^(5/2))*a + 1/768*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*x^2 + b/c
)*x^2 - (5*b^2*c - 12*a*c^2)/c^3)*x^2 + (15*b^3 - 52*a*b*c)/c^3) + 3*(5*b^4
 - 24*a*b^2*c + 16*a^2*c^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 +
a))*sqrt(c) - b))/c^(7/2))*b + 1/7680*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*(
8*x^2 + b/c)*x^2 - (7*b^2*c^2 - 16*a*c^3)/c^4)*x^2 + (35*b^3*c - 116*a*b*c^
2)/c^4)*x^2 - (105*b^4 - 460*a*b^2*c + 256*a^2*c^2)/c^4) - 15*(7*b^5 - 40*a
```

```
*b^3*c + 48*a^2*b*c^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*s
qrt(c) - b))/c^(9/2))*c
```

maple [B] time = 0.02, size = 316, normalized size = 2.11

$$\frac{\sqrt{c\,x^4+b\,x^2+a}\;c\,x^8}{10}+\frac{11\sqrt{c\,x^4+b\,x^2+a}\;b\,x^6}{80}+\frac{\sqrt{c\,x^4+b\,x^2+a}\;a\,x^4}{5}+\frac{\sqrt{c\,x^4+b\,x^2+a}\;b^2x^4}{160c}+\frac{7\sqrt{c\,x^4+b\,x^2+a}}{160c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/160*b^2*x^4/c*(c*x^4+b*x^2+a)^(1/2)-1/128*b^3/c^2*x^2*(c*x^4+b*x^2+a)^(1/
2)+3/64*a*b^3/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-3/32*
a^2*b/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-5/64*a*b^2/c^
2*(c*x^4+b*x^2+a)^(1/2)+7/160*a*b*x^2/c*(c*x^4+b*x^2+a)^(1/2)+1/10*c*x^8*(c
*x^4+b*x^2+a)^(1/2)+1/5*a*x^4*(c*x^4+b*x^2+a)^(1/2)+3/256*b^4/c^3*(c*x^4+b*
x^2+a)^(1/2)-3/512*b^5/c^(7/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/
2))+1/10*a^2/c*(c*x^4+b*x^2+a)^(1/2)+11/80*b*x^6*(c*x^4+b*x^2+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 4.88, size = 223, normalized size = 1.49

$$\frac{\left(c\,x^4+b\,x^2+a\right)^{5/2}}{10\,c}-\frac{b\left(\frac{3\,a\left(\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)\left(\frac{a}{2\,\sqrt{c}}-\frac{b^2}{8\,c^{3/2}}\right)+\frac{\left(2\,c\,x^2+b\right)\,\sqrt{c\,x^4+b\,x^2+a}}{4\,c}\right)}{4}+\frac{x^2\left(c\,x^4+b\,x^2+a\right)^{3/2}}{4}-\frac{3\,b^2\left(\ln\left(\sqrt{c\,x^4+b\,x^2\right.\right.}{}\right.}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] (a + b*x^2 + c*x^4)^(5/2)/(10*c) - (b*((3*a*(log((a + b*x^2 + c*x^4)^(1/2)
+ (b/2 + c*x^2)/c^(1/2))*(a/(2*c^(1/2)) - b^2/(8*c^(3/2))) + ((b + 2*c*x^2)
*(a + b*x^2 + c*x^4)^(1/2))/(4*c)))/4 + (x^2*(a + b*x^2 + c*x^4)^(3/2))/4 -
 (3*b^2*(log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2))*(a/(2*c^(1/
2)) - b^2/(8*c^(3/2))) + ((b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(4*c)))/
(16*c) + (b*(a + b*x^2 + c*x^4)^(3/2))/(8*c)))/(4*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a + bx^2 + cx^4\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*x**2 + c*x**4)**(3/2), x)
```

3.939 $\int x\left(a+bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=124

$$\frac{3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}}-\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c}$$

```
[Out] 1/16*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(3/2)/c+3/256*(-4*a*c+b^2)^2*arctanh(1/2*(
2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(5/2)-3/128*(-4*a*c+b^2)*(2*c*x
^2+b)*(c*x^4+b*x^2+a)^(1/2)/c^2
```

Rubi [A] time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1107, 612, 621, 206}

$$-\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^2}+\frac{3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}}+\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*c^2) + ((b +
2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(16*c) + (3*(b^2 - 4*a*c)^2*ArcTanh[(b
+ 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(256*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int x\left(a+bx^2+cx^4\right)^{3/2}\,dx=\frac{1}{2}\,\text{Subst}\left(\int\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^2\right)$$

$$=\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c}-\frac{\left(3\left(b^2-4ac\right)\right)\text{Subst}\left(\int\sqrt{a+bx+cx^2}\,dx,x,x^2\right)}{32c}$$

$$=-\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c}+\frac{\left(3\left(b^2\right.\right.}{}$$

$$=-\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c}+\frac{\left(3\left(b^2\right.\right.}{}$$

$$=-\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c}+\frac{3\left(b^2-\right.}{}$$

Mathematica [A] time = 0.09, size = 126, normalized size = 1.02

$$\frac{\frac{3\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)-2\sqrt{c}\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}\right)}{8c^{3/2}}+2\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2) + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(
b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/
(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]))/(8*c^(3/2)))/(32*c)
```

fricas [A] time = 0.78, size = 297, normalized size = 2.40

$$\left[\frac{3\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{c}\,\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}\,-4\,ac\right)+4\left(16\,c^4x^6+\right.}{512\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/512*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2
 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(16*c
^4*x^6 + 24*b*c^3*x^4 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x^2)*
sqrt(c*x^4 + b*x^2 + a))/c^3, -1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt
(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b
*c*x^2 + a*c)) - 2*(16*c^4*x^6 + 24*b*c^3*x^4 - 3*b^3*c + 20*a*b*c^2 + 2*(b
^2*c^2 + 20*a*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^3]
```

giac [B] time = 0.39, size = 317, normalized size = 2.56

$$\frac{1}{16}\left(2\sqrt{cx^4+bx^2+a}\left(2x^2+\frac{b}{c}\right)+\frac{\left(b^2-4ac\right)\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}}\right)a+\frac{1}{96}\left(2\sqrt{cx^4+bx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + (b^2 - 4*a*c)*log(abs(-2*(s
qrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3/2))*a + 1/96*(2*sq
rt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)*x^2 - (3*b^2 - 8*a*c)/c^2) - 3*(b^3
- 4*a*b*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))
/c^(5/2))*b + 1/768*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*x^2 + b/c)*x^2 - (5
*b^2*c - 12*a*c^2)/c^3)*x^2 + (15*b^3 - 52*a*b*c)/c^3) + 3*(5*b^4 - 24*a*b^
2*c + 16*a^2*c^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c
) - b))/c^(7/2))*c
```

maple [B] time = 0.02, size = 242, normalized size = 1.95

$$\frac{\sqrt{c\,x^4+b\,x^2+a}\;c\,x^6}{8}+\frac{3\sqrt{c\,x^4+b\,x^2+a}\;b\,x^4}{16}+\frac{5\sqrt{c\,x^4+b\,x^2+a}\;a\,x^2}{16}+\frac{\sqrt{c\,x^4+b\,x^2+a}\;b^2x^2}{64c}+\frac{3a^2\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{\right.}{16\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 5/16*a*x^2*(c*x^4+b*x^2+a)^(1/2)-3/128*b^3/c^2*(c*x^4+b*x^2+a)^(1/2)+3/256*
b^4/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/64*b^2*x^2/c*
(c*x^4+b*x^2+a)^(1/2)+5/32*a*b/c*(c*x^4+b*x^2+a)^(1/2)-3/32*a*b^2/c^(3/2)*l
n((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+3/16*a^2*ln((c*x^2+1/2*b)/c^
```

```
(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+3/16*b*x^4*(c*x^4+b*x^2+a)^(1/2)+1/8*c
*x^6*(c*x^4+b*x^2+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 4.96, size = 115, normalized size = 0.93

$$\frac{\left(c\,x^2+\frac{b}{2}\right)\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{8\,c}+\frac{\left(3\,a\,c-\frac{3\,b^2}{4}\right)\,\left(\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\,\sqrt{c\,x^4+b\,x^2+a}+\frac{\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b/2 + c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(8*c) + ((3*a*c - (3*b^2)/4)*((b/
(4*c) + x^2/2)*(a + b*x^2 + c*x^4)^(1/2) + (log((a + b*x^2 + c*x^4)^(1/2) +
 (b/2 + c*x^2)/c^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(8*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^2+cx^4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x*(a + b*x**2 + c*x**4)**(3/2), x)
```

3.940 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x} \, dx$

Optimal. Leaf size=155

$$-\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}}+\frac{\left(8ac+b^2+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16c}$$

```
[Out] 1/6*(c*x^4+b*x^2+a)^(3/2)-1/2*a^(3/2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^
4+b*x^2+a)^(1/2))-1/32*b*(-12*a*c+b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x
^4+b*x^2+a)^(1/2))/c^(3/2)+1/16*(2*b*c*x^2+8*a*c+b^2)*(c*x^4+b*x^2+a)^(1/2)
/c
```

Rubi [A] time = 0.18, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1114, 734, 814, 843, 621, 206, 724}

$$-\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}}+\frac{\left(8ac+b^2+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x,x]
```

```
[Out] ((b^2 + 8*a*c + 2*b*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c) + (a + b*x^2 + c
*x^4)^(3/2)/6 - (a^(3/2)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 +
c*x^4])])/2 - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b
*x^2 + c*x^4])])/(32*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x}\,dx, x, x^2\right)$$

$$= \frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2} - \frac{1}{4}\,\text{Subst}\left(\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x}\,dx, x, x^2\right)$$

$$= \frac{\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16c} + \frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2} + \frac{\text{Subst}\left(\int \frac{8a^2c-\frac{1}{2}b\left(b^2-12ac\right)}{x\sqrt{a+bx+cx^2}}\right.}{16c}$$

$$= \frac{\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16c} + \frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2} + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\right.$$

$$= \frac{\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16c} + \frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2} - a^2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,\right.$$

$$= \frac{\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16c} + \frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2} - \frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{2a+}{2\sqrt{a}\sqrt{a+}}\right.$$

Mathematica [A] time = 0.15, size = 143, normalized size = 0.92

$$\frac{1}{96}\left(-48a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right) - \frac{3b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{c^{3/2}} + \frac{2\sqrt{a+bx^2+cx^4}\left(8c\left(4a\right.\right.}{c}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x,x]
```

```
[Out] ((2*Sqrt[a + b*x^2 + c*x^4]*(3*b^2 + 14*b*c*x^2 + 8*c*(4*a + c*x^4)))/c - 4
8*a^(3/2)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])] - (3*b
*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])
/c^(3/2))/96
```

fricas [A] time = 1.06, size = 727, normalized size = 4.69

$$\left[\frac{48\,a^{\frac{3}{2}}c^2\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right) - 3\left(b^3-12\,abc\right)\sqrt{c}\,\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{c}\right.}{192\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/192*(48*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 +
 b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 3*(b^3 - 12*a*b*c)*sqrt(c
)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b
)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^4 + 14*b*c^2*x^2 + 3*b^2*c + 32*a*c^2)*sqrt
(c*x^4 + b*x^2 + a))/c^2, 1/96*(24*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^4 + 8*
a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 3
*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)
*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(8*c^3*x^4 + 14*b*c^2*x^2 + 3*b^2*
c + 32*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^2, 1/192*(96*sqrt(-a)*a*c^2*arctan
(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^
2)) - 3*(b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(
c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^4 + 14*b*c^2
*x^2 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^2, 1/96*(48*sqrt(-a)*
a*c^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 +
a*b*x^2 + a^2)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2
 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(8*c^3*x^4 + 14
*b*c^2*x^2 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.index.cc index_m operator + Erro
r: Bad Argument Value
```

maple [A] time = 0.02, size = 192, normalized size = 1.24

$$\frac{\sqrt{c x^{4}+b x^{2}+a}\, c x^{4}}{6}+\frac{7 \sqrt{c x^{4}+b x^{2}+a}\, b x^{2}}{24}-\frac{a^{\frac{3}{2}} \ln \left(\frac{b x^{2}+2 a+2 \sqrt{c x^{4}+b x^{2}+a}\, \sqrt{a}}{x^{2}}\right)}{2}+\frac{3 a b \ln \left(\frac{c x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^{4}+b x^{2}+a}\right)}{8 \sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x,x)
```

```
[Out] 1/6*c*x^4*(c*x^4+b*x^2+a)^(1/2)+7/24*b*x^2*(c*x^4+b*x^2+a)^(1/2)+1/16/c*b^2
*(c*x^4+b*x^2+a)^(1/2)-1/32/c^(3/2)*b^3*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x
^2+a)^(1/2))+3/8*a*b*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2
)+2/3*a*(c*x^4+b*x^2+a)^(1/2)-1/2*a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(
1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{2} + cx^{4}\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x, x)
```

3.941 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=150

$$\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{2x^2}+\frac{3}{8}\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}-\frac{3}{4}\sqrt{a}\,b\tanh^{-1}\left(\frac{}{2\sqrt{a}}\right.$$

```
[Out] -1/2*(c*x^4+b*x^2+a)^(3/2)/x^2-3/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4
+b*x^2+a)^(1/2))*a^(1/2)+3/16*(4*a*c+b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(
c*x^4+b*x^2+a)^(1/2))/c^(1/2)+3/8*(2*c*x^2+3*b)*(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1114, 732, 814, 843, 621, 206, 724}

$$\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{2x^2}+\frac{3}{8}\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}-\frac{3}{4}\sqrt{a}\,b\tanh^{-1}\left(\frac{}{2\sqrt{a}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (3*(3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/8 - (a + b*x^2 + c*x^4)^(3/2)/(
2*x^2) - (3*Sqrt[a]*b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x
^4])])/4 + (3*(b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2
 + c*x^4])])/(16*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^3}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{2x^2} + \frac{3}{4}\operatorname{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x}\,dx, x, x^2\right)$$

$$= \frac{3}{8}\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{2x^2} - \frac{3\operatorname{Subst}\left(\int \frac{-4abc-c\left(b^2+4ac\right)x}{x\sqrt{a+bx+cx^2}}\,dx,\right.}{16c}$$

$$= \frac{3}{8}\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{2x^2} + \frac{1}{4}(3ab)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+}}\right.$$

$$= \frac{3}{8}\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{2x^2} - \frac{1}{2}(3ab)\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx,\right.$$

$$= \frac{3}{8}\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{2x^2} - \frac{3}{4}\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2}}\right.$$

Mathematica [A] time = 0.12, size = 134, normalized size = 0.89

$$\frac{1}{16}\left(\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}} + \frac{2\sqrt{a+bx^2+cx^4}\left(-4a+5bx^2+2cx^4\right)}{x^2} - 12\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+}{2\sqrt{a}\sqrt{a+}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] ((2*Sqrt[a + b*x^2 + c*x^4]*(-4*a + 5*b*x^2 + 2*c*x^4))/x^2 - 12*Sqrt[a]*b*
ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])] + (3*(b^2 + 4*a*
c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[c])/16
```

fricas [A] time = 0.84, size = 713, normalized size = 4.75

$$\left[\frac{12\sqrt{a}\,bcx^2\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right) + 3\left(b^2+4\,ac\right)\sqrt{c}\,x^2\log\left(-8\,c^2x^4-8\,bcx^2-b^2-\right.}{32\,cx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/32*(12*sqrt(a)*b*c*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^
4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 3*(b^2 + 4*a*c)*sqrt(c
)*x^2*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2
 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^4 + 5*b*c*x^2 - 4*a*c)*sqrt(c*x^4 + b*x
^2 + a))/(c*x^2), 1/16*(6*sqrt(a)*b*c*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x
^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 3*(b^2
 + 4*a*c)*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqr
t(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(2*c^2*x^4 + 5*b*c*x^2 - 4*a*c)*sqrt(c
*x^4 + b*x^2 + a))/(c*x^2), 1/32*(24*sqrt(-a)*b*c*x^2*arctan(1/2*sqrt(c*x^4
 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 3*(b^2 +
4*a*c)*sqrt(c)*x^2*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2
+ a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^4 + 5*b*c*x^2 - 4*a*c)*sqr
t(c*x^4 + b*x^2 + a))/(c*x^2), 1/16*(12*sqrt(-a)*b*c*x^2*arctan(1/2*sqrt(c*
x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 3*(b^2
 + 4*a*c)*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqr
t(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(2*c^2*x^4 + 5*b*c*x^2 - 4*a*c)*sqrt(c
*x^4 + b*x^2 + a))/(c*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[1,0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{
-1,[1]%%%}]%%},[2,1,0]%%%}+%%%{%%{[1,0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2,0]%%%}
 / %%%{%%%{1,[1]%%%},[4,0,0]%%%}+%%%{%%%{-2,[1]%%%},[2,1,0]%%%}+%%%{%%%{1,[
1]%%%},[0,2,0]%%%} Error: Bad Argument Value
```

maple [A] time = 0.02, size = 170, normalized size = 1.13

$$\frac{\sqrt{c\,x^4+b\,x^2+a}\;c\,x^2}{4}+\frac{3a\sqrt{c}\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{4}-\frac{3\sqrt{a}\,b\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{4}+\frac{3b^2\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\cdots\right.}{1\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^3,x)
```

```
[Out] 1/4*c*x^2*(c*x^4+b*x^2+a)^(1/2)+5/8*b*(c*x^4+b*x^2+a)^(1/2)+3/16*b^2*ln((c*
x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+3/4*a*c^(1/2)*ln((c*x^2+1
/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/2*a/x^2*(c*x^4+b*x^2+a)^(1/2)-3/4*a^
(1/2)*b*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2+a\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^3,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{2}+cx^{4}\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**3,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**3, x)
```

3.942 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=151

$$-\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{a}}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{4x^4}-\frac{3\left(b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{8x^2}+\frac{3}{4}b\sqrt{c}\,\tanh^{-1}\left(\frac{b}{2\sqrt{c}\sqrt{a}}\right.$$

```
[Out] -1/4*(c*x^4+b*x^2+a)^(3/2)/x^4-3/16*(4*a*c+b^2)*arctanh(1/2*(b*x^2+2*a)/a^(
1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(1/2)+3/4*b*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(
c*x^4+b*x^2+a)^(1/2))*c^(1/2)-3/8*(-2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/x^2
```

Rubi [A] time = 0.16, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1114, 732, 812, 843, 621, 206, 724}

$$-\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{a}}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{4x^4}-\frac{3\left(b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{8x^2}+\frac{3}{4}b\sqrt{c}\,\tanh^{-1}\left(\frac{b}{2\sqrt{c}\sqrt{a}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (-3*(b - 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*x^2) - (a + b*x^2 + c*x^4)^(3
/2)/(4*x^4) - (3*(b^2 + 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*
x^2 + c*x^4])])/(16*Sqrt[a]) + (3*b*Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c
]*Sqrt[a + b*x^2 + c*x^4])])/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^5}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{4x^4} + \frac{3}{8}\operatorname{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{3\left(b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{8x^2} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{4x^4} - \frac{3}{16}\operatorname{Subst}\left(\int \frac{-b^2-4ac-4bcx}{x\sqrt{a+bx+cx^2}}\right.$$

$$= -\frac{3\left(b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{8x^2} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{4x^4} + \frac{1}{4}(3bc)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\right.$$

$$= -\frac{3\left(b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{8x^2} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{4x^4} + \frac{1}{2}(3bc)\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x,\right.$$

$$= -\frac{3\left(b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{8x^2} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{4x^4} - \frac{3\left(b^2+4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+}}\right.}{16\sqrt{a}}$$

Mathematica [A] time = 0.17, size = 134, normalized size = 0.89

$$\frac{1}{16}\left(-\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{a}} - \frac{2\sqrt{a+bx^2+cx^4}\left(2a+5bx^2-4cx^4\right)}{x^4} + 12b\sqrt{c}\tanh^{-1}\left(\frac{b+2c}{2\sqrt{c}\sqrt{a+b}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] ((-2*(2*a + 5*b*x^2 - 4*c*x^4)*Sqrt[a + b*x^2 + c*x^4])/x^4 - (3*(b^2 + 4*a
*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[a] + 1
2*b*Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/16
```

fricas [A] time = 1.06, size = 713, normalized size = 4.72

$$\left[\frac{12\,ab\sqrt{c}\,x^4\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)+3\left(b^2+4\,ac\right)\sqrt{a}\,x^4\log\left(-\frac{\left(b^2+4\right.}{}\right.}{32\,ax^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/32*(12*a*b*sqrt(c)*x^4*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 +
 b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 3*(b^2 + 4*a*c)*sqrt(a)*x^4*lo
g(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)
*sqrt(a) + 8*a^2)/x^4) + 4*(4*a*c*x^4 - 5*a*b*x^2 - 2*a^2)*sqrt(c*x^4 + b*x
^2 + a))/(a*x^4), -1/32*(24*a*b*sqrt(-c)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2
+ a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 3*(b^2 + 4*a*c)*sq
rt(a)*x^4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(
b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(4*a*c*x^4 - 5*a*b*x^2 - 2*a^2)*sqrt
(c*x^4 + b*x^2 + a))/(a*x^4), 1/16*(6*a*b*sqrt(c)*x^4*log(-8*c^2*x^4 - 8*b*
c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 3*
(b^2 + 4*a*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)
*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*(4*a*c*x^4 - 5*a*b*x^2 - 2*a^2)*sq
rt(c*x^4 + b*x^2 + a))/(a*x^4), -1/16*(12*a*b*sqrt(-c)*x^4*arctan(1/2*sqrt(
c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 3*(b
^2 + 4*a*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*s
qrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*(4*a*c*x^4 - 5*a*b*x^2 - 2*a^2)*sqrt
(c*x^4 + b*x^2 + a))/(a*x^4)]
```

giac [B] time = 0.45, size = 302, normalized size = 2.00

$$-\frac{3}{4}b\sqrt{c}\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)+\frac{1}{2}\sqrt{cx^4+bx^2+a}\,c+\frac{3\left(b^2+4ac\right)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right.}{8\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] -3/4*b*sqrt(c)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) +
b)) + 1/2*sqrt(c*x^4 + b*x^2 + a)*c + 3/8*(b^2 + 4*a*c)*arctan(-(sqrt(c)*x^
2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a) + 1/8*(5*(sqrt(c)*x^2 - sqr
t(c*x^4 + b*x^2 + a))^3*b^2 + 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a
*c + 16*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a*b*sqrt(c) - 3*(sqrt(c)*
x^2 - sqrt(c*x^4 + b*x^2 + a))*a*b^2 + 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))*a^2*c - 8*a^2*b*sqrt(c))/((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 -
 a)^2
```

maple [A] time = 0.02, size = 174, normalized size = 1.15

$$-\frac{3\sqrt{a}\,c\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{4}-\frac{3b^2\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{16\sqrt{a}}+\frac{3b\sqrt{c}\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{4}+\frac{\sqrt{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^5,x)
```

```
[Out] 1/2*c*(c*x^4+b*x^2+a)^(1/2)+3/4*b*c^(1/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b
*x^2+a)^(1/2))-1/4*a/x^4*(c*x^4+b*x^2+a)^(1/2)-5/8*b/x^2*(c*x^4+b*x^2+a)^(1
/2)-3/16/a^(1/2)*b^2*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-3/
4*a^(1/2)*c*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2+a\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^5,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**5,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**5, x)
```

3.943 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^7} \, dx$

Optimal. Leaf size=163

$$\frac{b(b^2-12ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{3/2}}-\frac{(x^2(8ac+b^2)+2ab)\sqrt{a+bx^2+cx^4}}{16ax^4}+\frac{1}{2}c^{3/2}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+c}}\right.$$

```
[Out] -1/6*(c*x^4+b*x^2+a)^(3/2)/x^6+1/32*b*(-12*a*c+b^2)*arctanh(1/2*(b*x^2+2*a)
/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(3/2)+1/2*c^(3/2)*arctanh(1/2*(2*c*x^2+b)
/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))-1/16*(2*a*b+(8*a*c+b^2)*x^2)*(c*x^4+b*x^2+a
)^(1/2)/a/x^4
```

Rubi [A] time = 0.18, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1114, 732, 810, 843, 621, 206, 724}

$$\frac{b(b^2-12ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{3/2}}-\frac{(x^2(8ac+b^2)+2ab)\sqrt{a+bx^2+cx^4}}{16ax^4}+\frac{1}{2}c^{3/2}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+c}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] -((2*a*b + (b^2 + 8*a*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*a*x^4) - (a + b*
x^2 + c*x^4)^(3/2)/(6*x^6) + (b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqr
t[a]*Sqrt[a + b*x^2 + c*x^4])])/(32*a^(3/2)) + (c^(3/2)*ArcTanh[(b + 2*c*x^
2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^7}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6x^6} + \frac{1}{4}\,\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^2\right)\sqrt{a+bx^2+cx^4}}{16ax^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6x^6} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}b\left(b^2-12ac\right)-}{x\sqrt{a+bx+c}}\right.}{16a}$$

$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^2\right)\sqrt{a+bx^2+cx^4}}{16ax^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6x^6} + \frac{1}{2}c^2\,\text{Subst}\left(\int \frac{}{\sqrt{a+}}\right.$$

$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^2\right)\sqrt{a+bx^2+cx^4}}{16ax^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6x^6} + c^2\,\text{Subst}\left(\int \frac{1}{4c-x^2}\right.$$

$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^2\right)\sqrt{a+bx^2+cx^4}}{16ax^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6x^6} + \frac{b\left(b^2-12ac\right)\tanh^{-1}}{32a^{3/}}$$

Mathematica [A] time = 0.22, size = 149, normalized size = 0.91

$$\frac{1}{96}\left(\frac{3b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{a^{3/2}} - \frac{2\sqrt{a+bx^2+cx^4}\left(8a^2+14abx^2+32acx^4+3b^2x^4\right)}{ax^6} + 48c^{3/2}\tanh\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] ((-2*Sqrt[a + b*x^2 + c*x^4]*(8*a^2 + 14*a*b*x^2 + 3*b^2*x^4 + 32*a*c*x^4))
/(a*x^6) + (3*b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*
x^2 + c*x^4])])/a^(3/2) + 48*c^(3/2)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[
a + b*x^2 + c*x^4])])/96
```

fricas [A] time = 1.16, size = 771, normalized size = 4.73

$$\left[\frac{48\,a^2c^{\frac{3}{2}}x^6\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)-3\left(b^3-12\,abc\right)\sqrt{a}\,x^6\log\left(-\frac{(b^2}{}\right.}{192\,a^2x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/192*(48*a^2*c^(3/2)*x^6*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4
+ b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b*c)*sqrt(a)*x^
6*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 +
2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(14*a^2*b*x^2 + (3*a*b^2 + 32*a^2*c)*x^4 + 8
*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^2*x^6), -1/192*(96*a^2*sqrt(-c)*c*x^6*arc
tan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 +
 a*c)) + 3*(b^3 - 12*a*b*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2
 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*(14*a^
2*b*x^2 + (3*a*b^2 + 32*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^2*x
^6), 1/96*(24*a^2*c^(3/2)*x^6*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x
^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b*c)*sqrt(-a
)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 +
a*b*x^2 + a^2)) - 2*(14*a^2*b*x^2 + (3*a*b^2 + 32*a^2*c)*x^4 + 8*a^3)*sqrt(
c*x^4 + b*x^2 + a))/(a^2*x^6), -1/96*(48*a^2*sqrt(-c)*c*x^6*arctan(1/2*sqrt
(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 3*(
b^3 - 12*a*b*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*
a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*(14*a^2*b*x^2 + (3*a*b^2 + 32*a^
2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^2*x^6)]
```

giac [B] time = 0.68, size = 412, normalized size = 2.53

$$-\frac{1}{2}\,c^{\frac{3}{2}}\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)-\frac{\left(b^3-12\,abc\right)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{16\,\sqrt{-a}\,a}+\frac{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/2*c^(3/2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b
)) - 1/16*(b^3 - 12*a*b*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/
sqrt(-a))/(sqrt(-a)*a) + 1/48*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*
b^3*sqrt(c) + 60*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a*b*c^(3/2) + 48
*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a*b^2*c + 96*(sqrt(c)*x^2 - sqrt
(c*x^4 + b*x^2 + a))^4*a^2*c^2 + 8*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^
3*a*b^3*sqrt(c) - 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^3*c^2 - 3*
(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^2*b^3*sqrt(c) + 36*(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2 + a))*a^3*b*c^(3/2) + 64*a^4*c^2)/(((sqrt(c)*x^2 - sqrt
(c*x^4 + b*x^2 + a))^2 - a)^3*a*sqrt(c))
```

maple [A] time = 0.02, size = 202, normalized size = 1.24

$$-\frac{3bc\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{8\sqrt{a}}+\frac{b^3\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{32a^{\frac{3}{2}}}+\frac{c^{\frac{3}{2}}\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2}-\frac{\sqrt{c\,x^4+b}}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^7,x)
```

```
[Out] 1/2*c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-7/24*b/x^4*(c*x
^4+b*x^2+a)^(1/2)-1/16/a*b^2/x^2*(c*x^4+b*x^2+a)^(1/2)+1/32/a^(3/2)*b^3*ln(
(b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-3/8*b*c/a^(1/2)*ln((b*x^2+
2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-2/3*c/x^2*(c*x^4+b*x^2+a)^(1/2)-1
/6*a/x^6*(c*x^4+b*x^2+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2+a\right)^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^7,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**7,x)
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**7, x)
```

3.944 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=133

$$-\frac{3(b^2-4ac)^2\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{256a^{5/2}}+\frac{3(b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{128a^2x^4}-\frac{(2a+bx^2)(a+bx^2+cx^4)^{3/2}}{16ax^8}$$

```
[Out] -1/16*(b*x^2+2*a)*(c*x^4+b*x^2+a)^(3/2)/a/x^8-3/256*(-4*a*c+b^2)^2*arctanh(
1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(5/2)+3/128*(-4*a*c+b^2)*(
b*x^2+2*a)*(c*x^4+b*x^2+a)^(1/2)/a^2/x^4
```

Rubi [A] time = 0.12, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1114, 720, 724, 206}

$$\frac{3(b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{128a^2x^4}-\frac{3(b^2-4ac)^2\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{256a^{5/2}}-\frac{(2a+bx^2)(a+bx^2+cx^4)^{3/2}}{16ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] (3*(b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*a^2*x^4) - ((2
*a + b*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(16*a*x^8) - (3*(b^2 - 4*a*c)^2*ArcT
anh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(256*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^9}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx,x,x^2\right)$$

$$= -\frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16ax^8} - \frac{\left(3\left(b^2-4ac\right)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx,x,x^2\right)}{32a}$$

$$= \frac{3\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^2x^4} - \frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16ax^8} + \frac{\left(3\left(b^2-4a\right.\right.}{}$$

$$= \frac{3\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^2x^4} - \frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16ax^8} - \frac{\left(3\left(b^2-4a\right.\right.}{}$$

$$= \frac{3\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^2x^4} - \frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16ax^8} - \frac{3\left(b^2-4ac\right.}{}$$

Mathematica [A] time = 0.17, size = 138, normalized size = 1.04

$$-\frac{\frac{3\left(b^2-4ac\right)\left(x^4\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)-2\sqrt{a}\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}\right)}{8a^{3/2}x^4}+\frac{2\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{x^8}}{32a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] -1/32*((2*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(3/2))/x^8 + (3*(b^2 - 4*a*c)*(
-2*Sqrt[a]*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4] + (b^2 - 4*a*c)*x^4*ArcTan
h[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]))/(8*a^(3/2)*x^4))/a
```

fricas [A] time = 1.05, size = 319, normalized size = 2.40

$$\left[\frac{3\left(b^4-8ab^2c+16a^2c^2\right)\sqrt{a}\,x^8\log\left(-\frac{\left(b^2+4ac\right)x^4+8abx^2-4\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{a}+8a^2}{x^4}\right)+4\left(\left(3ab^3-20a^2bc\right)x^6-24\right.}{512a^3x^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/512*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^8*log(-((b^2 + 4*a*c)*x^
4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^
4) + 4*((3*a*b^3 - 20*a^2*b*c)*x^6 - 24*a^3*b*x^2 - 2*(a^2*b^2 + 20*a^3*c)*
x^4 - 16*a^4)*sqrt(c*x^4 + b*x^2 + a))/(a^3*x^8), 1/256*(3*(b^4 - 8*a*b^2*c
 + 16*a^2*c^2)*sqrt(-a)*x^8*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a
)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((3*a*b^3 - 20*a^2*b*c)*x^6 - 24*
a^3*b*x^2 - 2*(a^2*b^2 + 20*a^3*c)*x^4 - 16*a^4)*sqrt(c*x^4 + b*x^2 + a))/(
a^3*x^8)]
```

giac [B] time = 0.49, size = 606, normalized size = 4.56

$$\frac{3\left(b^4-8ab^2c+16a^2c^2\right)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{128\sqrt{-a}\,a^2}-\frac{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^7b^4-24\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 3/128*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*
x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) - 1/128*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b
*x^2 + a))^7*b^4 - 24*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a*b^2*c - 8
0*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a^2*c^2 - 256*(sqrt(c)*x^2 - sq
rt(c*x^4 + b*x^2 + a))^6*a^2*b*c^(3/2) - 11*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x
^2 + a))^5*a*b^4 - 168*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^2*b^2*c
- 48*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^3*c^2 - 128*(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2 + a))^4*a^2*b^3*sqrt(c) - 11*(sqrt(c)*x^2 - sqrt(c*x^4
+ b*x^2 + a))^3*a^2*b^4 - 168*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^3
*b^2*c - 48*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^4*c^2 - 256*(sqrt(c
```

```
)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^4*b*c^(3/2) + 3*(sqrt(c)*x^2 - sqrt(c*
x^4 + b*x^2 + a))*a^3*b^4 - 24*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^4*
b^2*c - 80*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^5*c^2)/(((sqrt(c)*x^2
- sqrt(c*x^4 + b*x^2 + a))^2 - a)^4*a^2)
```

maple [B] time = 0.02, size = 260, normalized size = 1.95

$$-\frac{3c^2\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{16\sqrt{a}}+\frac{3b^2c\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{32a^{\frac{3}{2}}}-\frac{3b^4\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{256a^{\frac{5}{2}}}-\frac{5\sqrt{c\,x^4+b\,\cdots}}{32a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^9,x)
```

```
[Out] -1/64*b^2/a/x^4*(c*x^4+b*x^2+a)^(1/2)+3/128*b^3/a^2/x^2*(c*x^4+b*x^2+a)^(1/
2)-3/16*c^2/a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-5/1
6*c/x^4*(c*x^4+b*x^2+a)^(1/2)-3/256*b^4/a^(5/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^
2+a)^(1/2)*a^(1/2))/x^2)-1/8*a/x^8*(c*x^4+b*x^2+a)^(1/2)-5/32/a*c*b/x^2*(c*
x^4+b*x^2+a)^(1/2)+3/32/a^(3/2)*c*b^2*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)
*a^(1/2))/x^2)-3/16*b/x^6*(c*x^4+b*x^2+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2+a\right)^{3/2}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^9,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^9, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**9,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**9, x)
```

3.945 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^{11}}\,dx$

Optimal. Leaf size=162

$$\frac{3b\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{512a^{7/2}}-\frac{3b\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^3x^4}+\frac{b\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32a^2x^8}$$

```
[Out] 1/32*b*(b*x^2+2*a)*(c*x^4+b*x^2+a)^(3/2)/a^2/x^8-1/10*(c*x^4+b*x^2+a)^(5/2)
/a/x^10+3/512*b*(-4*a*c+b^2)^2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2
+a)^(1/2))/a^(7/2)-3/256*b*(-4*a*c+b^2)*(b*x^2+2*a)*(c*x^4+b*x^2+a)^(1/2)/a
^3/x^4
```

Rubi [A] time = 0.14, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 730, 720, 724, 206}

$$-\frac{3b\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^3x^4}+\frac{3b\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{512a^{7/2}}+\frac{b\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)}{32a^2x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^11,x]
```

```
[Out] (-3*b*(b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*a^3*x^4) +
(b*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(32*a^2*x^8) - (a + b*x^2 + c*x
^4)^(5/2)/(10*a*x^10) + (3*b*(b^2 - 4*a*c)^2*ArcTanh[(2*a + b*x^2)/(2*Sqrt[
a]*Sqrt[a + b*x^2 + c*x^4])])/(512*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
```

```
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^{11}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{5/2}}{10ax^{10}} - \frac{b\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx, x, x^2\right)}{4a}$$

$$= \frac{b\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32a^2x^8} - \frac{\left(a+bx^2+cx^4\right)^{5/2}}{10ax^{10}} + \frac{\left(3b\left(b^2-4ac\right)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\right.}{64a^2}$$

$$= -\frac{3b\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^3x^4} + \frac{b\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32a^2x^8} - \frac{\left(a+bx\right.}{1}$$

$$= -\frac{3b\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^3x^4} + \frac{b\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32a^2x^8} - \frac{\left(a+bx\right.}{1}$$

$$= -\frac{3b\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^3x^4} + \frac{b\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32a^2x^8} - \frac{\left(a+bx\right.}{1}$$

Mathematica [A] time = 0.14, size = 167, normalized size = 1.03

$$\frac{b\left(16a^{3/2}\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2} - 3x^4\left(b^2-4ac\right)\left(2\sqrt{a}\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4} - x^4\left(b^2-4ac\right)\tanh^{-1}\left(\right.\right.\right.}{512a^{7/2}x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^11,x]
```

```
[Out] -1/10*(a + b*x^2 + c*x^4)^(5/2)/(a*x^10) + (b*(16*a^(3/2)*(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(3/2) - 3*(b^2 - 4*a*c)*x^4*(2*Sqrt[a]*(2*a + b*x^2)*Sqrt[
a + b*x^2 + c*x^4] - (b^2 - 4*a*c)*x^4*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqr
t[a + b*x^2 + c*x^4])])))/(512*a^(7/2)*x^8)
```

fricas [A] time = 1.06, size = 383, normalized size = 2.36

$$\left[\frac{15\left(b^5-8\,ab^3c+16\,a^2bc^2\right)\sqrt{a}\,x^{10}\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right) - 4\left(\left(15\,ab^4-100\,a^2b^2c+1\right.\right.}{5120\,a^4x^{10}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] [1/5120*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(a)*x^10*log(-((b^2 + 4*a*
c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^
2)/x^4) - 4*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x^8 + 176*a^4*b*x^2 -
 2*(5*a^2*b^3 - 28*a^3*b*c)*x^6 + 128*a^5 + 8*(a^3*b^2 + 32*a^4*c)*x^4)*sqr
t(c*x^4 + b*x^2 + a))/(a^4*x^10), -1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c
^2)*sqrt(-a)*x^10*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)
/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x
^8 + 176*a^4*b*x^2 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^6 + 128*a^5 + 8*(a^3*b^2
+ 32*a^4*c)*x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^10)]
```

giac [B] time = 0.53, size = 832, normalized size = 5.14

$$-\frac{3\left(b^5-8\,ab^3c+16\,a^2bc^2\right)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{256\,\sqrt{-a}\,a^3}+\frac{15\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^9b^5-120\left(\sqrt{c}\,x^2-\sqrt{cx^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] -3/256*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 +
 b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^3) + 1/1280*(15*(sqrt(c)*x^2 - sqrt(c*x^
4 + b*x^2 + a))^9*b^5 - 120*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^9*a*b^3
*c + 240*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^9*a^2*b*c^2 + 1280*(sqrt(c
)*x^2 - sqrt(c*x^4 + b*x^2 + a))^8*a^3*c^(5/2) - 70*(sqrt(c)*x^2 - sqrt(c*x
^4 + b*x^2 + a))^7*a*b^5 + 560*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a^
2*b^3*c + 2720*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a^3*b*c^2 + 5120*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^6*a^3*b^2*c^(3/2) + 128*(sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 + a))^5*a^2*b^5 + 2560*(sqrt(c)*x^2 - sqrt(c*x^4 + b*
x^2 + a))^5*a^3*b^3*c + 3840*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^4*
b*c^2 + 1280*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^3*b^4*sqrt(c) + 25
60*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^4*b^2*c^(3/2) + 2560*(sqrt(c
)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^5*c^(5/2) + 70*(sqrt(c)*x^2 - sqrt(c*x
^4 + b*x^2 + a))^3*a^3*b^5 + 2000*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3
*a^4*b^3*c + 2400*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^5*b*c^2 + 256
0*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^5*b^2*c^(3/2) - 15*(sqrt(c)*x
^2 - sqrt(c*x^4 + b*x^2 + a))*a^4*b^5 + 120*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x
^2 + a))*a^5*b^3*c + 1040*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^6*b*c^2
 + 256*a^7*c^(5/2))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^5*a^3)
```

maple [B] time = 0.02, size = 337, normalized size = 2.08

$$\frac{3b\,c^2\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{32a^{\frac{3}{2}}}-\frac{3b^3c\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{64a^{\frac{5}{2}}}+\frac{3b^5\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{512a^{\frac{7}{2}}}-\frac{\sqrt{c\,x^4+b\,x}}{10a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^11,x)
```

```
[Out] -1/160/a*b^2/x^6*(c*x^4+b*x^2+a)^(1/2)+1/128/a^2*b^3/x^4*(c*x^4+b*x^2+a)^(1
/2)-3/256/a^3*b^4/x^2*(c*x^4+b*x^2+a)^(1/2)+3/512/a^(7/2)*b^5*ln((b*x^2+2*a
+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-1/10/a*c^2/x^2*(c*x^4+b*x^2+a)^(1/2)
+3/32*b*c^2/a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-3/6
4*b^3*c/a^(5/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-7/160*b
*c/a/x^4*(c*x^4+b*x^2+a)^(1/2)+5/64*b^2*c/a^2/x^2*(c*x^4+b*x^2+a)^(1/2)-1/1
0*a/x^10*(c*x^4+b*x^2+a)^(1/2)-11/80*b/x^8*(c*x^4+b*x^2+a)^(1/2)-1/5*c/x^6*
(c*x^4+b*x^2+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{x^{11}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^11,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^11, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{2} + cx^{4}\right)^{\frac{3}{2}}}{x^{11}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**11,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**11, x)
```

3.946 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=216

$$-\frac{(b^2-4ac)^2(7b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2048a^{9/2}}+\frac{(b^2-4ac)(7b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{1024a^4x^4}-\frac{(7b^2-4ac}{}$$

```
[Out] -1/384*(-4*a*c+7*b^2)*(b*x^2+2*a)*(c*x^4+b*x^2+a)^(3/2)/a^3/x^8-1/12*(c*x^4
+b*x^2+a)^(5/2)/a/x^12+7/120*b*(c*x^4+b*x^2+a)^(5/2)/a^2/x^10-1/2048*(-4*a*
c+b^2)^2*(-4*a*c+7*b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/
2))/a^(9/2)+1/1024*(-4*a*c+b^2)*(-4*a*c+7*b^2)*(b*x^2+2*a)*(c*x^4+b*x^2+a)^
(1/2)/a^4/x^4
```

Rubi [A] time = 0.22, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 744, 806, 720, 724, 206}

$$-\frac{(7b^2-4ac)(2a+bx^2)(a+bx^2+cx^4)^{3/2}}{384a^3x^8}+\frac{(b^2-4ac)(7b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{1024a^4x^4}-\frac{(b^2-4ac)^2(7b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^13,x]
```

```
[Out] ((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(1024
*a^4*x^4) - ((7*b^2 - 4*a*c)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(384*
a^3*x^8) - (a + b*x^2 + c*x^4)^(5/2)/(12*a*x^12) + (7*b*(a + b*x^2 + c*x^4)
^(5/2))/(120*a^2*x^10) - ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(2*a + b*
x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2048*a^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^{13}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^7}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{5/2}}{12ax^{12}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{7b}{2}+cx\right)\left(a+bx+cx^2\right)^{3/2}}{x^6}\,dx, x, x^2\right)}{12a}$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{5/2}}{12ax^{12}} + \frac{7b\left(a+bx^2+cx^4\right)^{5/2}}{120a^2x^{10}} + \frac{\left(7b^2-4ac\right)\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx, x,\right.}{48a^2}$$

$$= -\frac{\left(7b^2-4ac\right)\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{384a^3x^8} - \frac{\left(a+bx^2+cx^4\right)^{5/2}}{12ax^{12}} + \frac{7b\left(a+bx^2+cx^4\right)^{5}}{120a^2x^{10}}$$

$$= \frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{1024a^4x^4} - \frac{\left(7b^2-4ac\right)\left(2a+bx^2\right)\left(a+bx^2+\right.}{384a^3x^8}$$

$$= \frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{1024a^4x^4} - \frac{\left(7b^2-4ac\right)\left(2a+bx^2\right)\left(a+bx^2+\right.}{384a^3x^8}$$

$$= \frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{1024a^4x^4} - \frac{\left(7b^2-4ac\right)\left(2a+bx^2\right)\left(a+bx^2+\right.}{384a^3x^8}$$

Mathematica [A] time = 0.21, size = 206, normalized size = 0.95

$$-\frac{\frac{\left(\frac{7b^2}{2}-2ac\right)\left(16a^{3/2}\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}-3x^4\left(b^2-4ac\right)\left(2\sqrt{a}\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}-x^4\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)\right)\right)}{256a^{7/2}x^8} + \frac{\left(a+bx^2+cx^4\right)^{5/2}}{x^{12}}}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^13,x]
```

```
[Out] -1/12*((a + b*x^2 + c*x^4)^(5/2)/x^12 - (7*b*(a + b*x^2 + c*x^4)^(5/2))/(10
*a*x^10) + (((7*b^2)/2 - 2*a*c)*(16*a^(3/2)*(2*a + b*x^2)*(a + b*x^2 + c*x^
4)^(3/2) - 3*(b^2 - 4*a*c)*x^4*(2*Sqrt[a]*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*
x^4] - (b^2 - 4*a*c)*x^4*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 +
c*x^4])])))/(256*a^(7/2)*x^8))/a
```

fricas [A] time = 1.54, size = 473, normalized size = 2.19

$$\left[-\frac{15\left(7\,b^{6}-60\,ab^{4}c+144\,a^{2}b^{2}c^{2}-64\,a^{3}c^{3}\right)\sqrt{a}\,x^{12}\log\left(-\frac{\left(b^{2}+4\,ac\right)x^{4}+8\,abx^{2}+4\,\sqrt{cx^{4}+bx^{2}+a}\left(bx^{2}+2\,a\right)\sqrt{a}+8\,a^{2}}{x^{4}}\right)-4\left(\left(105\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] [-1/61440*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(a)*x
^12*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
+ 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*((105*a*b^5 - 760*a^2*b^3*c + 1296*a^3*b*c
^2)*x^10 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a^4*c^2)*x^8 - 1664*a^5*b*x^
2 + 8*(7*a^3*b^3 - 36*a^4*b*c)*x^6 - 1280*a^6 - 16*(3*a^4*b^2 + 140*a^5*c)*
x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^5*x^12), 1/30720*(15*(7*b^6 - 60*a*b^4*c +
 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^4 + b*x^2
+ a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((105*a*b^5 - 76
0*a^2*b^3*c + 1296*a^3*b*c^2)*x^10 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a^
4*c^2)*x^8 - 1664*a^5*b*x^2 + 8*(7*a^3*b^3 - 36*a^4*b*c)*x^6 - 1280*a^6 - 1
6*(3*a^4*b^2 + 140*a^5*c)*x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^5*x^12)]
```

giac [B] time = 0.70, size = 1235, normalized size = 5.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] 1/1024*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*arctan(-(sqrt(c)
*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^4) - 1/15360*(105*(sq
rt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^11*b^6 - 900*(sqrt(c)*x^2 - sqrt(c*x^4
 + b*x^2 + a))^11*a*b^4*c + 2160*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^11
*a^2*b^2*c^2 - 960*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^11*a^3*c^3 - 595
*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^9*a*b^6 + 5100*(sqrt(c)*x^2 - sqrt
(c*x^4 + b*x^2 + a))^9*a^2*b^4*c - 12240*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))^9*a^3*b^2*c^2 - 15040*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^9*a^4*c
^3 - 76800*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^8*a^4*b*c^(5/2) + 1386*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a^2*b^6 - 11880*(sqrt(c)*x^2 - sqr
t(c*x^4 + b*x^2 + a))^7*a^3*b^4*c - 97440*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
 + a))^7*a^4*b^2*c^2 - 24960*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a^5*
c^3 - 112640*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^6*a^4*b^3*c^(3/2) - 61
440*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^6*a^5*b*c^(5/2) - 1686*(sqrt(c)
*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^3*b^6 - 42600*(sqrt(c)*x^2 - sqrt(c*x^4
```

```
 + b*x^2 + a))^5*a^4*b^4*c - 128160*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))
^5*a^5*b^2*c^2 - 24960*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^6*c^3 -
15360*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^4*b^5*sqrt(c) - 61440*(sq
rt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^5*b^3*c^(3/2) - 92160*(sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 + a))^4*a^6*b*c^(5/2) - 595*(sqrt(c)*x^2 - sqrt(c*x^4
 + b*x^2 + a))^3*a^4*b^6 - 25620*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*
a^5*b^4*c - 58320*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^6*b^2*c^2 - 1
5040*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^7*c^3 - 30720*(sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 + a))^2*a^6*b^3*c^(3/2) - 12288*(sqrt(c)*x^2 - sqrt(c
*x^4 + b*x^2 + a))^2*a^7*b*c^(5/2) + 105*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))*a^5*b^6 - 900*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^6*b^4*c - 132
00*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^7*b^2*c^2 - 960*(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2 + a))*a^8*c^3 - 3072*a^8*b*c^(5/2))/(((sqrt(c)*x^2 - sq
rt(c*x^4 + b*x^2 + a))^2 - a)^6*a^4)
```

maple [B] time = 0.03, size = 457, normalized size = 2.12

$$\frac{c^3 \ln\left(\frac{b x^2+2a+2\sqrt{c x^4+b x^2+a}\,\sqrt{a}}{x^2}\right)}{32 a^{\frac{3}{2}}}-\frac{9 b^2 c^2 \ln\left(\frac{b x^2+2a+2\sqrt{c x^4+b x^2+a}\,\sqrt{a}}{x^2}\right)}{128 a^{\frac{5}{2}}}+\frac{15 b^4 c \ln\left(\frac{b x^2+2a+2\sqrt{c x^4+b x^2+a}\,\sqrt{a}}{x^2}\right)}{512 a^{\frac{7}{2}}}-\frac{7 b^6 \ln\left(\frac{b x^2 \cdots}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^13,x)
```

```
[Out] -1/320/a*b^2/x^8*(c*x^4+b*x^2+a)^(1/2)+7/1920/a^2*b^3/x^6*(c*x^4+b*x^2+a)^(
1/2)-7/1536/a^3*b^4/x^4*(c*x^4+b*x^2+a)^(1/2)+7/1024/a^4*b^5/x^2*(c*x^4+b*x
^2+a)^(1/2)-7/2048/a^(9/2)*b^6*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2
))/x^2)-9/128*c^2*b^2/a^(5/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2)
)/x^2)-1/32*c^2/a/x^4*(c*x^4+b*x^2+a)^(1/2)-1/12*a/x^12*(c*x^4+b*x^2+a)^(1/
2)-7/48*c/x^8*(c*x^4+b*x^2+a)^(1/2)-13/120*b/x^10*(c*x^4+b*x^2+a)^(1/2)+1/3
2*c^3/a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)+27/320*c^
2*b/a^2/x^2*(c*x^4+b*x^2+a)^(1/2)+15/512/a^(7/2)*b^4*c*ln((b*x^2+2*a+2*(c*x
^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)-19/384/a^3*b^3*c/x^2*(c*x^4+b*x^2+a)^(1/2)+
9/320/a^2*b^2*c/x^4*(c*x^4+b*x^2+a)^(1/2)-3/160/a*b*c/x^6*(c*x^4+b*x^2+a)^(
1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4+b\,x^2+a\right)^{3/2}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^13,x)
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^13, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**13,x)
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**13, x)
```

3.947 $\int x^4 \left(a+bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=495

$$-\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{c}\left(60a^2c^2-51ab^2c+8b^4\right)+8b\left(2b^2-9ac\right)\left(b^2-3ac\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{2310c^{15/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] 1/33*x^3*(3*c*x^2+b)*(c*x^4+b*x^2+a)^(3/2)/c+1/1155*(60*a^2*c^2-51*a*b^2*c+
8*b^4)*x*(c*x^4+b*x^2+a)^(1/2)/c^3-1/385*x^3*(b*(a*c+2*b^2)+10*c*(-3*a*c+b^
2)*x^2)*(c*x^4+b*x^2+a)^(1/2)/c^2-8/1155*b*(-9*a*c+2*b^2)*(-3*a*c+b^2)*x*(c
*x^4+b*x^2+a)^(1/2)/c^(7/2)/(a^(1/2)+x^2*c^(1/2))+8/1155*a^(1/4)*b*(-9*a*c+
2*b^2)*(-3*a*c+b^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan
(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^
(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c
^(1/2))^2)^(1/2)/c^(15/4)/(c*x^4+b*x^2+a)^(1/2)-1/2310*a^(1/4)*(cos(2*arcta
n(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(s
in(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x
^2*c^(1/2))*(8*b*(-9*a*c+2*b^2)*(-3*a*c+b^2)+(60*a^2*c^2-51*a*b^2*c+8*b^4)*
a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(15/4)/(
c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1116, 1273, 1279, 1197, 1103, 1195}

$$\frac{x\left(60a^2c^2-51ab^2c+8b^4\right)\sqrt{a+bx^2+cx^4}}{1155c^3}-\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{c}\left(60a^2c^2-51ab^2c+8b^4\right)+8b\left(2b^2-9ac\right)\left(b^2-3ac\right)\right)\left(\sqrt{\right.}{2310c^{15/4}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((8*b^4 - 51*a*b^2*c + 60*a^2*c^2)*x*Sqrt[a + b*x^2 + c*x^4])/(1155*c^3) -
(8*b*(2*b^2 - 9*a*c)*(b^2 - 3*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(1155*c^(7/2)
*(Sqrt[a] + Sqrt[c]*x^2)) - (x^3*(b*(2*b^2 + a*c) + 10*c*(b^2 - 3*a*c)*x^2)
*Sqrt[a + b*x^2 + c*x^4])/(385*c^2) + (x^3*(b + 3*c*x^2)*(a + b*x^2 + c*x^4
)^(3/2))/(33*c) + (8*a^(1/4)*b*(2*b^2 - 9*a*c)*(b^2 - 3*a*c)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(1155*c^(15/4)*Sq
rt[a + b*x^2 + c*x^4]) - (a^(1/4)*(8*b*(2*b^2 - 9*a*c)*(b^2 - 3*a*c) + Sqrt
```

```
[a]*Sqrt[c]*(8*b^4 - 51*a*b^2*c + 60*a^2*c^2))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt
[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)
*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2310*c^(15/4)*Sqrt[a + b*x^2 +
 c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1116

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(a + b*x^2 + c*x^4)^p*(2*b*p + c*(m + 4*p - 1)*x^
2))/(c*(m + 4*p + 1)*(m + 4*p - 1)), x] - Dist[(2*p*d^2)/(c*(m + 4*p + 1)*(
m + 4*p - 1)), Int[(d*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p - 1)*Simp[a*b*(m -
1) - (2*a*c*(m + 4*p - 1) - b^2*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && GtQ[m, 1] && IntegerQ[
2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1273

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*
p + 3)), x] + Dist[(2*p)/(c*(4*p + m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a +
b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c*
```

```
e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &&
 NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p
] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int x^4 \left(a+bx^2+cx^4\right)^{3/2} dx = \frac{x^3 \left(b+3cx^2\right) \left(a+bx^2+cx^4\right)^{3/2}}{33c} - \frac{\int x^2 \left(3ab+6\left(b^2-3ac\right)x^2\right)\sqrt{a+bx^2+cx^4}\, dx}{33c}$$

$$= -\frac{x^3 \left(b\left(2b^2+ac\right)+10c\left(b^2-3ac\right)x^2\right)\sqrt{a+bx^2+cx^4}}{385c^2} + \frac{x^3\left(b+3cx^2\right)\left(a+bx^2+c\right.}{33c}$$

$$= \frac{\left(8b^4-51ab^2c+60a^2c^2\right)x\sqrt{a+bx^2+cx^4}}{1155c^3} - \frac{x^3\left(b\left(2b^2+ac\right)+10c\left(b^2-3ac\right)x^2\right)\surd}{385c^2}$$

$$= \frac{\left(8b^4-51ab^2c+60a^2c^2\right)x\sqrt{a+bx^2+cx^4}}{1155c^3} - \frac{x^3\left(b\left(2b^2+ac\right)+10c\left(b^2-3ac\right)x^2\right)\surd}{385c^2}$$

$$= \frac{\left(8b^4-51ab^2c+60a^2c^2\right)x\sqrt{a+bx^2+cx^4}}{1155c^3} - \frac{8b\left(2b^2-9ac\right)\left(b^2-3ac\right)x\sqrt{a+bx^2+}}{1155c^{7/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}$$

Mathematica [C] time = 2.21, size = 657, normalized size = 1.33

$$\frac{-4ib\left(27a^2c^2-15ab^2c+2b^4\right)\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(60*a^3*c^2 + a^2*c*(-51*b^2 + 92*b*
c*x^2 + 255*c^2*x^4) + a*(8*b^4 - 57*b^3*c*x^2 - 14*b^2*c^2*x^4 + 367*b*c^3
*x^6 + 300*c^4*x^8) + x^2*(8*b^5 + 2*b^4*c*x^2 - b^3*c^2*x^4 + 145*b^2*c^3*
x^6 + 245*b*c^4*x^8 + 105*c^5*x^10)) - (4*I)*b*(2*b^4 - 15*a*b^2*c + 27*a^2
*c^2)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^
2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x]
, (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-8*b^6 + 68*a*b^4*c
 - 159*a^2*b^2*c^2 + 60*a^3*c^3 + 8*b^5*Sqrt[b^2 - 4*a*c] - 60*a*b^3*c*Sqrt
[b^2 - 4*a*c] + 108*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c
*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt
[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(2310
*c^4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^8 + bx^6 + ax^4\right)\sqrt{cx^4 + bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^8 + b*x^6 + a*x^4)*sqrt(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*x^4, x)
```

maple [A] time = 0.01, size = 674, normalized size = 1.36

$$\frac{\sqrt{c\,x^4+b\,x^2+a}\;c\,x^9}{11}+\frac{4\sqrt{c\,x^4+b\,x^2+a}\;b\,x^7}{33}+\frac{\left(\frac{13ac}{11}+\frac{b^2}{33}\right)\sqrt{c\,x^4+b\,x^2+a}\;x^5}{7c}+\frac{\left(\frac{38ab}{33}-\frac{6\left(\frac{13ac}{11}+\frac{b^2}{33}\right)b}{7c}\right)\sqrt{c\,x^4+b\,x^2}}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/11*c*x^9*(c*x^4+b*x^2+a)^(1/2)+4/33*b*x^7*(c*x^4+b*x^2+a)^(1/2)+1/7*(13/1
1*a*c+1/33*b^2)/c*x^5*(c*x^4+b*x^2+a)^(1/2)+1/5*(38/33*a*b-6/7*(13/11*a*c+1
/33*b^2)/c*b)/c*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(a^2-5/7*(13/11*a*c+1/33*b^2)
/c*a-4/5*(38/33*a*b-6/7*(13/11*a*c+1/33*b^2)/c*b)/c*b)/c*x*(c*x^4+b*x^2+a)^
(1/2)-1/12*(a^2-5/7*(13/11*a*c+1/33*b^2)/c*a-4/5*(38/33*a*b-6/7*(13/11*a*c+
1/33*b^2)/c*b)/c*b)/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(
-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/
(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-1/2*(-3/5*(38/33*a*b-6/7
*(13/11*a*c+1/33*b^2)/c*b)/c*a-2/3*(a^2-5/7*(13/11*a*c+1/33*b^2)/c*a-4/5*(3
8/33*a*b-6/7*(13/11*a*c+1/33*b^2)/c*b)/c*b)/c*b)*a*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+
b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(El
lipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b
^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^4+bx^2+a\right)^{\frac{3}{2}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \left(c\,x^4+b\,x^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int(x^4*(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{4} \left(a + bx^{2} + cx^{4}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral(x**4*(a + b*x**2 + c*x**4)**(3/2), x)
```

3.948 $\int x^2 \left(a+bx^2+cx^4\right)^{3/2} \, dx$

Optimal. Leaf size=443

$$\frac{x\left(84a^2c^2-57ab^2c+8b^4\right)\sqrt{a+bx^2+cx^4}}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{\sqrt[4]{a}\left(84a^2c^2-57ab^2c+4\sqrt{a}\,b\sqrt{c}\left(b^2-6ac\right)+8b^4\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{}}{630c^{11/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] 1/63*x*(7*c*x^2+3*b)*(c*x^4+b*x^2+a)^(3/2)/c-1/315*x*(b*(-9*a*c+4*b^2)+6*c*
(-7*a*c+2*b^2)*x^2)*(c*x^4+b*x^2+a)^(1/2)/c^2+1/315*(84*a^2*c^2-57*a*b^2*c+
8*b^4)*x*(c*x^4+b*x^2+a)^(1/2)/c^(5/2)/(a^(1/2)+x^2*c^(1/2))-1/315*a^(1/4)*
(84*a^2*c^2-57*a*b^2*c+8*b^4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/co
s(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1
/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(
1/2)+x^2*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)+1/630*a^(1/4)*(co
s(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*El
lipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(
a^(1/2)+x^2*c^(1/2))*(8*b^4-57*a*b^2*c+84*a^2*c^2+4*b*(-6*a*c+b^2)*a^(1/2)*
c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*
x^2+a)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1116, 1176, 1197, 1103, 1195}

$$\frac{x\left(84a^2c^2-57ab^2c+8b^4\right)\sqrt{a+bx^2+cx^4}}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{\sqrt[4]{a}\left(84a^2c^2-57ab^2c+4\sqrt{a}\,b\sqrt{c}\left(b^2-6ac\right)+8b^4\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{}}{630c^{11/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*x*Sqrt[a + b*x^2 + c*x^4])/(315*c^(5/2)*
(Sqrt[a] + Sqrt[c]*x^2)) - (x*(b*(4*b^2 - 9*a*c) + 6*c*(2*b^2 - 7*a*c)*x^2)
*Sqrt[a + b*x^2 + c*x^4])/(315*c^2) + (x*(3*b + 7*c*x^2)*(a + b*x^2 + c*x^4
)^(3/2))/(63*c) - (a^(1/4)*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(315*c^(11/4)*Sqr
t[a + b*x^2 + c*x^4]) + (a^(1/4)*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2 + 4*Sqrt[
a]*b*Sqrt[c]*(b^2 - 6*a*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
b/(Sqrt[a]*Sqrt[c]))/4])/(630*c^(11/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1116

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(a + b*x^2 + c*x^4)^p*(2*b*p + c*(m + 4*p - 1)*x^
2))/(c*(m + 4*p + 1)*(m + 4*p - 1)), x] - Dist[(2*p*d^2)/(c*(m + 4*p + 1)*(
m + 4*p - 1)), Int[(d*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p - 1)*Simp[a*b*(m -
1) - (2*a*c*(m + 4*p - 1) - b^2*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && GtQ[m, 1] && IntegerQ[
2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int x^2\left(a+bx^2+cx^4\right)^{3/2}\,dx=\frac{x\left(3b+7cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{63c}-\frac{\int\left(ab+2\left(2b^2-7ac\right)x^2\right)\sqrt{a+bx^2+cx^4}\,dx}{21c}$$

$$=-\frac{x\left(b\left(4b^2-9ac\right)+6c\left(2b^2-7ac\right)x^2\right)\sqrt{a+bx^2+cx^4}}{315c^2}+\frac{x\left(3b+7cx^2\right)\left(a+bx^2+c\right.}{63c}$$

$$=-\frac{x\left(b\left(4b^2-9ac\right)+6c\left(2b^2-7ac\right)x^2\right)\sqrt{a+bx^2+cx^4}}{315c^2}+\frac{x\left(3b+7cx^2\right)\left(a+bx^2+c\right.}{63c}$$

$$=\frac{\left(8b^4-57ab^2c+84a^2c^2\right)x\sqrt{a+bx^2+cx^4}}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{x\left(b\left(4b^2-9ac\right)+6c\left(2b^2-7ac\right)x^2\right)\sqrt{}}{315c^2}$$

Mathematica [C] time = 1.89, size = 602, normalized size = 1.36

$$\frac{i\left(84a^2c^2-57ab^2c+8b^4\right)\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b}{b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(-4*b^4*x^2 - b^3*c*x^4 + 53*b^2*c^2
*x^6 + 85*b*c^3*x^8 + 35*c^4*x^10 + a^2*c*(24*b + 77*c*x^2) + a*(-4*b^3 + 2
7*b^2*c*x^2 + 151*b*c^2*x^4 + 112*c^3*x^6)) + I*(8*b^4 - 57*a*b^2*c + 84*a^
2*c^2)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x
], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-8*b^5 + 65*a*b^3*
c - 132*a^2*b*c^2 + 8*b^4*Sqrt[b^2 - 4*a*c] - 57*a*b^2*c*Sqrt[b^2 - 4*a*c]
+ 84*a^2*c^2*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x
], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(1260*c^3*Sqrt[c/(b +
Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(cx^6+bx^4+ax^2\right)\sqrt{cx^4+bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^4 + a*x^2)*sqrt(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*x^2, x)
```

maple [A] time = 0.01, size = 545, normalized size = 1.23

$$\frac{\sqrt{c\,x^4 + b\,x^2 + a}\;c\,x^7}{9} + \frac{10\sqrt{c\,x^4 + b\,x^2 + a}\;b\,x^5}{63} + \frac{\left(\frac{11ac}{9} + \frac{b^2}{21}\right)\sqrt{c\,x^4 + b\,x^2 + a}\;x^3}{5c} - \frac{\left(\frac{76ab}{63} - \frac{4\left(\frac{11ac}{9} + \frac{b^2}{21}\right)b}{5c}\right)\sqrt{2}\,\sqrt{-\frac{2(-}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/9*c*x^7*(c*x^4+b*x^2+a)^(1/2)+10/63*b*x^5*(c*x^4+b*x^2+a)^(1/2)+1/5*(11/9
*a*c+1/21*b^2)/c*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(76/63*a*b-4/5*(11/9*a*c+1/2
1*b^2)/c*b)/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12*(76/63*a*b-4/5*(11/9*a*c+1/21*b^
2)/c*b)/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^
(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2
+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*
(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-1/2*(a^2-3/5*(11/9*a*c+1/21*b^2)/c*a
-2/3*(76/63*a*b-4/5*(11/9*a*c+1/21*b^2)/c*b)/c*b)*a*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c
+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(E
llipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+
b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(c x^4 + b x^2 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^2*(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + bx^2 + cx^4\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x**2 + c*x**4)**(3/2), x)
```

3.949 $\int \left(a+bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=381

$$-\frac{2bx\left(b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{c}\left(b^2-20ac\right)+2b\left(b^2-8ac\right)\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan\right.}{70c^{7/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] 1/7*x*(c*x^4+b*x^2+a)^(3/2)+1/35*x*(3*b*c*x^2+10*a*c+b^2)*(c*x^4+b*x^2+a)^(
1/2)/c-2/35*b*(-8*a*c+b^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^(3/2)/(a^(1/2)+x^2*c^(
1/2))+2/35*a^(1/4)*b*(-8*a*c+b^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2
)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4)
)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/
(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)-1/70*a^(1/4)*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*
EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))
*(a^(1/2)+x^2*c^(1/2))*(2*b*(-8*a*c+b^2)+(-20*a*c+b^2)*a^(1/2)*c^(1/2))*((c
*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1091, 1176, 1197, 1103, 1195}

$$-\frac{2bx\left(b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{c}\left(b^2-20ac\right)+2b\left(b^2-8ac\right)\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan\right.}{70c^{7/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*b*(b^2 - 8*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(35*c^(3/2)*(Sqrt[a] + Sqrt[
c]*x^2)) + (x*(b^2 + 10*a*c + 3*b*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(35*c) +
(x*(a + b*x^2 + c*x^4)^(3/2))/7 + (2*a^(1/4)*b*(b^2 - 8*a*c)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(35*c^(7/4)*Sqrt[
a + b*x^2 + c*x^4]) - (a^(1/4)*(Sqrt[a]*Sqrt[c]*(b^2 - 20*a*c) + 2*b*(b^2 -
 8*a*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c])
)/4])/(70*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \left(a+bx^2+cx^4\right)^{3/2} dx = \frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}+\frac{3}{7}\int \left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}\, dx$$

$$= \frac{x\left(b^2+10ac+3bcx^2\right)\sqrt{a+bx^2+cx^4}}{35c}+\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}+\frac{\int \frac{-a\left(b^2-20ac\right)-2b\left(b^2-8\right.}{\sqrt{a+bx^2+cx^4}}}{35c}$$

$$= \frac{x\left(b^2+10ac+3bcx^2\right)\sqrt{a+bx^2+cx^4}}{35c}+\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}+\frac{\left(2\sqrt{a}\,b\left(b^2-8ac\right)\right)}{35c^{3}}$$

$$= -\frac{2b\left(b^2-8ac\right)x\sqrt{a+bx^2+cx^4}}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{x\left(b^2+10ac+3bcx^2\right)\sqrt{a+bx^2+cx^4}}{35c}+\frac{1}{7}x\left(a+\right.$$

Mathematica [C] time = 1.52, size = 533, normalized size = 1.40

$$\frac{2cx\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(15a^2c+a\left(b^2+23bcx^2+20c^2x^4\right)+x^2\left(b^3+9b^2cx^2+13bc^2x^4+5c^3x^6\right)\right)+i\left(-20a^2c^2+9ab^2c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(15*a^2*c + a*(b^2 + 23*b*c*x^2 + 20
*c^2*x^4) + x^2*(b^3 + 9*b^2*c*x^2 + 13*b*c^2*x^4 + 5*c^3*x^6)) - I*b*(b^2
- 8*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt
[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]
*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^4 + 9*a*b^2*c
 - 20*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[
b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2
]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
 - 4*a*c])])/(70*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4
])
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^4+bx^2+a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2), x)
```

maple [A] time = 0.01, size = 471, normalized size = 1.24

$$\frac{\sqrt{c\,x^4+b\,x^2+a}\;c\,x^5}{7}+\frac{8\sqrt{c\,x^4+b\,x^2+a}\;b\,x^3}{35}-\frac{\left(\frac{46ab}{35}-\frac{2\left(\frac{9ac}{7}+\frac{3b^2}{35}\right)b}{3c}\right)\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}\cdots}}{2\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/7*c*x^5*(c*x^4+b*x^2+a)^(1/2)+8/35*b*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(9/7*a
*c+3/35*b^2)/c*x*(c*x^4+b*x^2+a)^(1/2)+1/4*(a^2-1/3*(9/7*a*c+3/35*b^2)/c*a)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^
2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*E
llipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+
b^2)^(1/2))/a*b/c-4)^(1/2))-1/2*(46/35*a*b-2/3*(9/7*a*c+3/35*b^2)/c*b)*a*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4
)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(
-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2^(1/2)*((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2
)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c\,x^4+b\,x^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2),x)
[Out] int((a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2+cx^4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral((a + b*x**2 + c*x**4)**(3/2), x)
```

3.950 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=361

$$\frac{\sqrt[4]{a}\left(8\sqrt{a}\,b\sqrt{c}+12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{10c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\left(12ac+b^2\right)\left(\sqrt{a}+\right.}{}$$

```
[Out] -(c*x^4+b*x^2+a)^(3/2)/x+1/5*x*(6*c*x^2+7*b)*(c*x^4+b*x^2+a)^(1/2)+1/5*(12*
a*c+b^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-1/5*a^(1/4)*
(12*a*c+b^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4
)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c
^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))
^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)+1/10*a^(1/4)*(cos(2*arctan(c^(1/4)*
x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arcta
n(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2)
)*(b^2+12*a*c+8*b*a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2
)^(1/2)/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1117, 1176, 1197, 1103, 1195}

$$\frac{\sqrt[4]{a}\left(8\sqrt{a}\,b\sqrt{c}+12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{10c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\left(12ac+b^2\right)\left(\sqrt{a}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^2,x]
```

```
[Out] ((b^2 + 12*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(5*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^
2)) + (x*(7*b + 6*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/5 - (a + b*x^2 + c*x^4)^(
3/2)/x - (a^(1/4)*(b^2 + 12*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 +
c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
(2 - b/(Sqrt[a]*Sqrt[c]))/4])/(5*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4
)*(b^2 + 8*Sqrt[a]*b*Sqrt[c] + 12*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*
x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1
/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(10*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1117

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^p)/(d*(m + 1)), x] - Dist[(2*p)
/(d^2*(m + 1)), Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^2}\,dx = -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{x} + 3\int \left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}\,dx$$

$$= \frac{1}{5}x\left(7b+6cx^2\right)\sqrt{a+bx^2+cx^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x} + \frac{\int \frac{8abc+c\left(b^2+12ac\right)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{5c}$$

$$= \frac{1}{5}x\left(7b+6cx^2\right)\sqrt{a+bx^2+cx^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x} - \frac{\left(\sqrt{a}\left(b^2+12ac\right)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}$$

$$= \frac{\left(b^2+12ac\right)x\sqrt{a+bx^2+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{1}{5}x\left(7b+6cx^2\right)\sqrt{a+bx^2+cx^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x}$$

Mathematica [C] time = 1.27, size = 505, normalized size = 1.40

$$\frac{4c\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(-5a^2-3abx^2-4acx^4+2b^2x^4+3bcx^6+c^2x^8\right)+ix\left(12ac+b^2\right)\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^2,x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-5*a^2 - 3*a*b*x^2 + 2*b^2*x^4 - 4*a*
c*x^4 + 3*b*c*x^6 + c^2*x^8) + I*(b^2 + 12*a*c)*(-b + Sqrt[b^2 - 4*a*c])*x*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b -
 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSin
h[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b -
Sqrt[b^2 - 4*a*c])] - I*(-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 12*a*c*Sq
rt[b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]
*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[
b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(20*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]
)]*x*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^2, x)
```

maple [A] time = 0.02, size = 430, normalized size = 1.19

$$\frac{\sqrt{c\,x^4+b\,x^2+a}\;c\,x^3}{5}+\frac{2\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\;\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\;+4\,ab\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{2\left(b\right.}}{}\right.}{5\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\;\sqrt{c\,x^4+b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^2,x)
```

```
[Out] -a*(c*x^4+b*x^2+a)^(1/2)/x+1/5*c*x^3*(c*x^4+b*x^2+a)^(1/2)+2/5*b*x*(c*x^4+b
*x^2+a)^(1/2)+2/5*a*b*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4
*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c
*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-1/2*(12/5*a*c+1/5*b^2)*a*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+
4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+
(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2^(1/2)*((-
b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/
2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4+b\,x^2+a\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^2,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**2,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**2, x)
```

3.951 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=353

$$\frac{\left(8\sqrt{a}\,b\sqrt{c}+4ac+3b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{\left(3b-2cx^2\right)\sqrt{a+bx^2+c}}{3x}$$

```
[Out] -1/3*(c*x^4+b*x^2+a)^(3/2)/x^3-1/3*(-2*c*x^2+3*b)*(c*x^4+b*x^2+a)^(1/2)/x+8
/3*b*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/(a^(1/2)+x^2*c^(1/2))-8/3*a^(1/4)*b*c^
(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(
1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))
^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/
2)/(c*x^4+b*x^2+a)^(1/2)+1/6*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos
(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/
2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(3*b^2+4*a*c+8*b*a^(1/
2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)
/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1117, 1271, 1197, 1103, 1195}

$$\frac{\left(8\sqrt{a}\,b\sqrt{c}+4ac+3b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3x^3}-\frac{(3b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] (8*b*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(3*(Sqrt[a] + Sqrt[c]*x^2)) - ((3*b
 - 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(3*x) - (a + b*x^2 + c*x^4)^(3/2)/(3*x
^3) - (8*a^(1/4)*b*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)
/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b
/(Sqrt[a]*Sqrt[c]))/4])/(3*Sqrt[a + b*x^2 + c*x^4]) + ((3*b^2 + 8*Sqrt[a]*b
*Sqrt[c] + 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
 + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]
*Sqrt[c]))/4])/(6*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1117

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^p)/(d*(m + 1)), x] - Dist[(2*p)
/(d^2*(m + 1)), Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1271

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(d*(m
+ 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(2*p)/(f^
2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^4}\,dx = -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3x^3} + \int \frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{x^2}\,dx$$

$$= -\frac{\left(3b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{3x} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{3x^3} - \frac{1}{3}\int \frac{-3b^2-4ac-8bcx^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= -\frac{\left(3b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{3x} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{3x^3} - \frac{1}{3}\left(8\sqrt{a}\,b\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx}}$$

$$= \frac{8b\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{3\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\left(3b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{3x} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{3x^3} - \frac{8\sqrt[4]{a}\,b}{}$$

Mathematica [C] time = 0.95, size = 473, normalized size = 1.34

$$\frac{2\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(-a^2-5abx^2-4b^2x^4-3bcx^6+c^2x^8\right)-ix^3\left(4b\sqrt{b^2-4ac}+4ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4}}{b-}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] (2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-a^2 - 5*a*b*x^2 - 4*b^2*x^4 - 3*b*c*x^
6 + c^2*x^8) + (4*I)*b*(-b + Sqrt[b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] +
4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I
*(-b^2 + 4*a*c + 4*b*Sqrt[b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)
/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
- 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(6*Sqrt[c/
(b + Sqrt[b^2 - 4*a*c])]*x^3*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^4,x, algorithm="fricas")

[Out] integral((c*x^4 + b*x^2 + a)^(3/2)/x^4, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^4,x, algorithm="giac")

[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^4, x)

maple [A] time = 0.02, size = 428, normalized size = 1.21

$$-\frac{4\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2}, \frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)+\mathrm{Ellipt}}{3\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,\left(b+\sqrt{-4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)^(3/2)/x^4,x)

[Out] -1/3*a*(c*x^4+b*x^2+a)^(1/2)/x^3-4/3*b*(c*x^4+b*x^2+a)^(1/2)/x+1/3*c*x*(c*x^4+b*x^2+a)^(1/2)+1/4*(4/3*a*c+b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-4/3*b*c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4+b\,x^2+a\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^4,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**4,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**4, x)
```

3.952 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=400

$$\frac{\sqrt[4]{c}\left(8\sqrt{a}\,b\sqrt{c}+12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{10a^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{c}\left(12ac+b^2\right)\left(\sqrt{a}+\right.}{}$$

[Out] -1/5*(c*x^4+b*x^2+a)^(3/2)/x^5-1/5*(12*a*c+b^2)*(c*x^4+b*x^2+a)^(1/2)/a/x-1/5*(-6*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/x^3+1/5*(12*a*c+b^2)*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/(a^(1/2)+x^2*c^(1/2))-1/5*c^(1/4)*(12*a*c+b^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^4+b*x^2+a)^(1/2)+1/10*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(b^2+12*a*c+8*b*a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^4+b*x^2+a)^(1/2)

Rubi [A] time = 0.25, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1117, 1271, 1281, 1197, 1103, 1195}

$$\frac{\sqrt[4]{c}\left(8\sqrt{a}\,b\sqrt{c}+12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{10a^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{c}\left(12ac+b^2\right)\left(\sqrt{a}+\right.}{}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^6,x]

[Out] -((b^2 + 12*a*c)*Sqrt[a + b*x^2 + c*x^4])/(5*a*x) + (Sqrt[c]*(b^2 + 12*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(5*a*(Sqrt[a] + Sqrt[c]*x^2)) - ((b - 6*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(5*x^3) - (a + b*x^2 + c*x^4)^(3/2)/(5*x^5) - (c^(1/4)*(b^2 + 12*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(5*a^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (c^(1/4)*(b^2 + 8*Sqrt[a]*b*Sqrt[c] + 12*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(10*a^(3/4)*Sqrt[a + b*x^2 + c*x^4])

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1117

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^p)/(d*(m + 1)), x] - Dist[(2*p)
/(d^2*(m + 1)), Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1271

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(d*(m
+ 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(2*p)/(f^
2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^6}\,dx = -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{5x^5}+\frac{3}{5}\int \frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{x^4}\,dx$$

$$= -\frac{\left(b-6cx^2\right)\sqrt{a+bx^2+cx^4}}{5x^3}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{5x^5}-\frac{1}{5}\int \frac{-b^2-12ac-8bcx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx$$

$$= -\frac{\left(b^2+12ac\right)\sqrt{a+bx^2+cx^4}}{5ax}-\frac{\left(b-6cx^2\right)\sqrt{a+bx^2+cx^4}}{5x^3}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{5x^5}+\int$$

$$= -\frac{\left(b^2+12ac\right)\sqrt{a+bx^2+cx^4}}{5ax}-\frac{\left(b-6cx^2\right)\sqrt{a+bx^2+cx^4}}{5x^3}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{5x^5}-\frac{(\sqrt{}}{}$$

$$= -\frac{\left(b^2+12ac\right)\sqrt{a+bx^2+cx^4}}{5ax}+\frac{\sqrt{c}\left(b^2+12ac\right)x\sqrt{a+bx^2+cx^4}}{5a\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\left(b-6cx^2\right)\sqrt{a+b}}{5x^3}$$

Mathematica [C] time = 1.36, size = 527, normalized size = 1.32

$$-4\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(a^3+a^2\left(3bx^2+8cx^4\right)+a\left(3b^2x^4+9bcx^6+7c^2x^8\right)+b^2x^6\left(b+cx^2\right)\right)+ix^5\left(12ac+b^2\right)\left(\sqrt{b^2-4a}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] (-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a^3 + b^2*x^6*(b + c*x^2) + a^2*(3*b*x
^2 + 8*c*x^4) + a*(3*b^2*x^4 + 9*b*c*x^6 + 7*c^2*x^8)) + I*(b^2 + 12*a*c)*(
-b + Sqrt[b^2 - 4*a*c])*x^5*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 -
```

```
 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (
b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^3 + 4*a*b*c + b^2*S
qrt[b^2 - 4*a*c] + 12*a*c*Sqrt[b^2 - 4*a*c])*x^5*Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c
*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt
[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(20*a
*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x^5*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^6, x)
```

maple [A] time = 0.02, size = 450, normalized size = 1.12

$$\frac{2\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\;bc\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\;x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)}{5\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\;\sqrt{c\,x^4+b\,x^2+a}} - \left(c^2+\frac{(7}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^6,x)
```

```
[Out] -1/5*a*(c*x^4+b*x^2+a)^(1/2)/x^5-2/5*b*(c*x^4+b*x^2+a)^(1/2)/x^3-1/5*(7*a*c
+b^2)/a*(c*x^4+b*x^2+a)^(1/2)/x+2/5*b*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)
```

```
^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))
/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-1/2*(c^2
+1/5*c*(7*a*c+b^2)/a)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(
-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/
(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(-
4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-
EllipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c
+b^2)^(1/2))/a*b/c-4)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^6,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4 + b\,x^2 + a\right)^{3/2}}{x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^6,x)
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**6,x)
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**6, x)
```

3.953 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=447

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}\left(b^2-20ac\right)+2b\left(b^2-8ac\right)\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{70a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2b\sqrt[4]{c}}{}$$

```
[Out] -1/7*(c*x^4+b*x^2+a)^(3/2)/x^7-1/35*(-20*a*c+b^2)*(c*x^4+b*x^2+a)^(1/2)/a/x
^3+2/35*b*(-8*a*c+b^2)*(c*x^4+b*x^2+a)^(1/2)/a^2/x-3/35*(10*c*x^2+b)*(c*x^4
+b*x^2+a)^(1/2)/x^5-2/35*b*(-8*a*c+b^2)*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^2
/(a^(1/2)+x^2*c^(1/2))+2/35*b*c^(1/4)*(-8*a*c+b^2)*(cos(2*arctan(c^(1/4)*x/
a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(
c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*
((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(c*x^4+b*x^2+a)^(1/
2)-1/70*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(
1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2
)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*b*(-8*a*c+b^2)+(-20*a*c+b^2)*a^(
1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(c*x^
4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1117, 1271, 1281, 1197, 1103, 1195}

$$\frac{2b\left(b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{35a^2x}-\frac{2b\sqrt{c}\,x\left(b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{35a^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}\left(b^2-20ac\right)+2b\left(b^2-8ac\right)\right)\left(\sqrt{a}\right.}{70a^{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] -((b^2 - 20*a*c)*Sqrt[a + b*x^2 + c*x^4])/(35*a*x^3) + (2*b*(b^2 - 8*a*c)*S
qrt[a + b*x^2 + c*x^4])/(35*a^2*x) - (2*b*Sqrt[c]*(b^2 - 8*a*c)*x*Sqrt[a +
b*x^2 + c*x^4])/(35*a^2*(Sqrt[a] + Sqrt[c]*x^2)) - (3*(b + 10*c*x^2)*Sqrt[a
 + b*x^2 + c*x^4])/(35*x^5) - (a + b*x^2 + c*x^4)^(3/2)/(7*x^7) + (2*b*c^(1
/4)*(b^2 - 8*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
 + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]
*Sqrt[c]))/4])/(35*a^(7/4)*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(Sqrt[a]*Sqr
t[c]*(b^2 - 20*a*c) + 2*b*(b^2 - 8*a*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
```

```
b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(70*a^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1117

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^p)/(d*(m + 1)), x] - Dist[(2*p)
/(d^2*(m + 1)), Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1271

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(d*(m
+ 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(2*p)/(f^
2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^8}\,dx = -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{7x^7} + \frac{3}{7}\int \frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{x^6}\,dx$$

$$= -\frac{3\left(b+10cx^2\right)\sqrt{a+bx^2+cx^4}}{35x^5} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{7x^7} + \frac{3}{35}\int \frac{b^2-20ac-8bcx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx$$

$$= -\frac{\left(b^2-20ac\right)\sqrt{a+bx^2+cx^4}}{35ax^3} - \frac{3\left(b+10cx^2\right)\sqrt{a+bx^2+cx^4}}{35x^5} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{7x^7}$$

$$= -\frac{\left(b^2-20ac\right)\sqrt{a+bx^2+cx^4}}{35ax^3} + \frac{2b\left(b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{35a^2x} - \frac{3\left(b+10cx^2\right)\sqrt{a+}}{35x^5}$$

$$= -\frac{\left(b^2-20ac\right)\sqrt{a+bx^2+cx^4}}{35ax^3} + \frac{2b\left(b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{35a^2x} - \frac{3\left(b+10cx^2\right)\sqrt{a+}}{35x^5}$$

$$= -\frac{\left(b^2-20ac\right)\sqrt{a+bx^2+cx^4}}{35ax^3} + \frac{2b\left(b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{35a^2x} - \frac{2b\sqrt{c}\left(b^2-8ac\right)x}{35a^2\left(\sqrt{a}+\right.}$$

Mathematica [C] time = 1.56, size = 572, normalized size = 1.28

$$\frac{ix^7\left(-20a^2c^2+9ab^2c-8abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}-b^4\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}F\left(i\sinh^{-1}\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] (-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(5*a^4 - 2*b^3*x^8*(b + c*x^2) + a^3*(1
3*b*x^2 + 20*c*x^4) + a*b*x^6*(-b^2 + 17*b*c*x^2 + 16*c^2*x^4) + 3*a^2*(3*b
^2*x^4 + 13*b*c*x^6 + 5*c^2*x^8)) - I*b*(b^2 - 8*a*c)*(-b + Sqrt[b^2 - 4*a*
c])*x^7*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqr
t[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[
I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c
])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^4 + 9*a*b^2*c - 20*a^2*c^2 + b^3*Sqrt[b
^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*x^7*Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2
)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(70*a^2*S
qrt[c/(b + Sqrt[b^2 - 4*a*c])]*x^7*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)/x^8, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^8, x)
```

maple [A] time = 0.02, size = 495, normalized size = 1.11

$$-\frac{\left(8ac-b^2\right)\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\left(-\text{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right.\right.}{35\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,\left(b+\sqrt{-4ac+b^2}\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^8,x)
```

```
[Out] -1/7*a*(c*x^4+b*x^2+a)^(1/2)/x^7-8/35*b*(c*x^4+b*x^2+a)^(1/2)/x^5-1/35*(15*
a*c+b^2)/a*(c*x^4+b*x^2+a)^(1/2)/x^3-2/35*b*(8*a*c-b^2)/a^2*(c*x^4+b*x^2+a)
^(1/2)/x+1/4*(c^2-1/35*c*(15*a*c+b^2)/a)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2)
)/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-1/35*b*
c*(8*a*c-b^2)/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b
^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b
*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-Elliptic
E(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1
/2))/a*b/c-4)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4 + b\,x^2 + a\right)^{3/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^8,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**8,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**8, x)
```

3.954 $\int \sqrt{3-2x^2-x^4}\,dx$

Optimal. Leaf size=48

$$\frac{1}{3}\sqrt{-x^4-2x^2+3}\,x+\frac{4F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}-\frac{2E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] -2/3*EllipticE(x,1/3*I*3^(1/2))*3^(1/2)+4/3*EllipticF(x,1/3*I*3^(1/2))*3^(1
/2)+1/3*x*(-x^4-2*x^2+3)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1091, 1180, 524, 424, 419}

$$\frac{1}{3}\sqrt{-x^4-2x^2+3}\,x+\frac{4F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}-\frac{2E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 2*x^2 - x^4],x]
```

```
[Out] (x*Sqrt[3 - 2*x^2 - x^4])/3 - (2*EllipticE[ArcSin[x], -1/3])/Sqrt[3] + (4*E
llipticF[ArcSin[x], -1/3])/Sqrt[3]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
```

```
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{3-2x^2-x^4}\,dx &= \frac{1}{3}x\sqrt{3-2x^2-x^4}+\frac{1}{3}\int \frac{6-2x^2}{\sqrt{3-2x^2-x^4}}\,dx \\
&= \frac{1}{3}x\sqrt{3-2x^2-x^4}+\frac{2}{3}\int \frac{6-2x^2}{\sqrt{2-2x^2}\,\sqrt{6+2x^2}}\,dx \\
&= \frac{1}{3}x\sqrt{3-2x^2-x^4}-\frac{2}{3}\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx+8\int \frac{1}{\sqrt{2-2x^2}\,\sqrt{6+2x^2}}\,dx \\
&= \frac{1}{3}x\sqrt{3-2x^2-x^4}-\frac{2E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}+\frac{4F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 59, normalized size = 1.23

$$\frac{1}{3}\left(\sqrt{-x^4-2x^2+3}\,x-4iF\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)-2iE\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 2*x^2 - x^4],x]
```

```
[Out] (x*Sqrt[3 - 2*x^2 - x^4] - (2*I)*EllipticE[I*ArcSinh[x/Sqrt[3]], -3] - (4*I
)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3])/3
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-x^4-2\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 - 2*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4-2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4-2*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 - 2*x^2 + 3), x)
```

maple [B] time = 0.02, size = 114, normalized size = 2.38

$$\frac{\sqrt{-x^4-2x^2+3}\;x}{3}+\frac{2\sqrt{-x^2+1}\;\sqrt{3x^2+9}\;\text{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}+\frac{2\sqrt{-x^2+1}\;\sqrt{3x^2+9}\;\left(-\text{EllipticE}\left(x,\frac{i\sqrt{3}}{3}\right)+\text{Ellip}\right.}{3\sqrt{-x^4-2x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4-2*x^2+3)^(1/2),x)
```

```
[Out] 1/3*x*(-x^4-2*x^2+3)^(1/2)+2/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3
)^(1/2)*EllipticF(x,1/3*I*3^(1/2))+2/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4
-2*x^2+3)^(1/2)*(EllipticF(x,1/3*I*3^(1/2))-EllipticE(x,1/3*I*3^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4-2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4-2*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 - 2*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{-x^4-2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3 - x^4 - 2*x^2)^(1/2),x)
```

```
[Out] int((3 - x^4 - 2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4 - 2x^2 + 3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4-2*x**2+3)**(1/2),x)
```

```
[Out] Integral(sqrt(-x**4 - 2*x**2 + 3), x)
```

3.955 $\int \frac{x^7}{\sqrt{a+bx^2+cx^4}} \, dx$

Optimal. Leaf size=121

$$-\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{7/2}}+\frac{\left(-16ac+15b^2-10bcx^2\right)\sqrt{a+bx^2+cx^4}}{48c^3}+\frac{x^4\sqrt{a+bx^2+cx^4}}{6c}$$

```
[Out] -1/32*b*(-12*a*c+5*b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/
2))/c^(7/2)+1/6*x^4*(c*x^4+b*x^2+a)^(1/2)/c+1/48*(-10*b*c*x^2-16*a*c+15*b^2
)*(c*x^4+b*x^2+a)^(1/2)/c^3
```

Rubi [A] time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 742, 779, 621, 206}

$$\frac{\left(-16ac+15b^2-10bcx^2\right)\sqrt{a+bx^2+cx^4}}{48c^3}-\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{7/2}}+\frac{x^4\sqrt{a+bx^2+cx^4}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x^4*Sqrt[a + b*x^2 + c*x^4])/(6*c) + ((15*b^2 - 16*a*c - 10*b*c*x^2)*Sqrt[
a + b*x^2 + c*x^4])/(48*c^3) - (b*(5*b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
```

```
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= \frac{x^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{\text{Subst}\left(\int \frac{x\left(-2a-\frac{5bx}{2}\right)}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{6c}$$

$$= \frac{x^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{\left(15b^2-16ac-10bcx^2\right)\sqrt{a+bx^2+cx^4}}{48c^3} - \frac{\left(b\left(5b^2-12ac\right)\right)\text{Subst}}{32}$$

$$= \frac{x^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{\left(15b^2-16ac-10bcx^2\right)\sqrt{a+bx^2+cx^4}}{48c^3} - \frac{\left(b\left(5b^2-12ac\right)\right)\text{Subst}}{}$$

$$= \frac{x^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{\left(15b^2-16ac-10bcx^2\right)\sqrt{a+bx^2+cx^4}}{48c^3} - \frac{b\left(5b^2-12ac\right)\tanh^{-1}}{32c^{7/2}}$$

Mathematica [A] time = 0.06, size = 104, normalized size = 0.86

$$\frac{\left(36abc-15b^3\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)+2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(8c\left(cx^4-2a\right)+15b^2-10bcx^2\right)}{96c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(15*b^2 - 10*b*c*x^2 + 8*c*(-2*a + c*x^4
)) + (-15*b^3 + 36*a*b*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 +
 c*x^4])])/(96*c^(7/2))
```

fricas [A] time = 0.79, size = 241, normalized size = 1.99

$$\left[-\frac{3\left(5b^3-12abc\right)\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2-4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c}-4ac\right)-4\left(8c^3x^4-10bc^2x^2\right.}{192c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*
sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^4 - 10*
b*c^2*x^2 + 15*b^2*c - 16*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^4, 1/96*(3*(5*b
^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sq
rt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(8*c^3*x^4 - 10*b*c^2*x^2 + 15*b^2*c
- 16*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^4]
```

giac [A] time = 0.21, size = 103, normalized size = 0.85

$$\frac{1}{48}\sqrt{cx^4+bx^2+a}\left(2x^2\left(\frac{4x^2}{c}-\frac{5b}{c^2}\right)+\frac{15b^2-16ac}{c^3}\right)+\frac{\left(5b^3-12abc\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right.\right.}{32c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c*x^4 + b*x^2 + a)*(2*x^2*(4*x^2/c - 5*b/c^2) + (15*b^2 - 16*a*c)
/c^3) + 1/32*(5*b^3 - 12*a*b*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^
2 + a))*sqrt(c) - b))/c^(7/2)
```

maple [A] time = 0.02, size = 162, normalized size = 1.34

$$\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\;x^{4}}{6c}-\frac{5\sqrt{c\,x^{4}+b\,x^{2}+a}\;b\,x^{2}}{24c^{2}}+\frac{3ab\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{8c^{\frac{5}{2}}}-\frac{5b^{3}\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+}\right.}{32c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*x^4*(c*x^4+b*x^2+a)^(1/2)/c-5/24*b/c^2*x^2*(c*x^4+b*x^2+a)^(1/2)+5/16*b
^2/c^3*(c*x^4+b*x^2+a)^(1/2)-5/32*b^3/c^(7/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x
^4+b*x^2+a)^(1/2))+3/8*b/c^(5/2)*a*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)
^(1/2))-1/3*a/c^2*(c*x^4+b*x^2+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{7}}{\sqrt{c\,x^{4}+b\,x^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^7/(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{\sqrt{a+bx^{2}+cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**7/sqrt(a + b*x**2 + c*x**4), x)
```

3.956 $\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=104

$$\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{a+bx^2+cx^4}}{4c}$$

```
[Out] 1/16*(-4*a*c+3*b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/
c^(5/2)-3/8*b*(c*x^4+b*x^2+a)^(1/2)/c^2+1/4*x^2*(c*x^4+b*x^2+a)^(1/2)/c
```

Rubi [A] time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 742, 640, 621, 206}

$$\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{a+bx^2+cx^4}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (-3*b*Sqrt[a + b*x^2 + c*x^4])/(8*c^2) + (x^2*Sqrt[a + b*x^2 + c*x^4])/(4*c
) + ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(16*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{x^2\sqrt{a+bx^2+cx^4}}{4c} + \frac{\text{Subst}\left(\int \frac{-a-\frac{3bx}{2}}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4c}\\
&= -\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{a+bx^2+cx^4}}{4c} + \frac{\left(3b^2-4ac\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{16c^2}\\
&= -\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{a+bx^2+cx^4}}{4c} + \frac{\left(3b^2-4ac\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{8c^2}\\
&= -\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{a+bx^2+cx^4}}{4c} + \frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 88, normalized size = 0.85

$$\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)+2\sqrt{c}\,\left(2cx^2-3b\right)\sqrt{a+bx^2+cx^4}}{16c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*(-3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4] + (3*b^2 - 4*a*c)*ArcTa
nh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(5/2))
```

fricas [A] time = 0.69, size = 203, normalized size = 1.95

$$\left[-\frac{\left(3\,b^2-4\,ac\right)\sqrt{c}\,\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)-4\,\sqrt{cx^4+bx^2+a}\left(2\,c^2x^2\right.}{32\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c
*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^4 + b*x^2 + a
)*(2*c^2*x^2 - 3*b*c))/c^3, -1/16*((3*b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt
(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*s
qrt(c*x^4 + b*x^2 + a)*(2*c^2*x^2 - 3*b*c))/c^3]
```

giac [A] time = 0.24, size = 82, normalized size = 0.79

$$\frac{1}{8}\,\sqrt{cx^4+bx^2+a}\left(\frac{2\,x^2}{c}-\frac{3\,b}{c^2}\right)-\frac{\left(3\,b^2-4\,ac\right)\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{16\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*x^2/c - 3*b/c^2) - 1/16*(3*b^2 - 4*a*c)*log(
abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(5/2)
```

maple [A] time = 0.01, size = 116, normalized size = 1.12

$$\frac{\sqrt{c\,x^4+b\,x^2+a}\;x^2}{4c}-\frac{a\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{4c^{\frac{3}{2}}}+\frac{3b^2\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{16c^{\frac{5}{2}}}-\frac{3\sqrt{c\,x^4+b\,x^2+a}\;b}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*x^2*(c*x^4+b*x^2+a)^(1/2)/c-3/8*b*(c*x^4+b*x^2+a)^(1/2)/c^2+3/16*b^2/c^
(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/4*a/c^(3/2)*ln((c*x
^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^5/(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(a + b*x**2 + c*x**4), x)
```

3.957 $\int \frac{x^3}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{a+bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

```
[Out] -1/4*b*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(3/2)+1/2*(
c*x^4+b*x^2+a)^(1/2)/c
```

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1114, 640, 621, 206}

$$\frac{\sqrt{a+bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
+ b*x^2 + c*x^4])])/(4*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{a+bx^2+cx^4}}{2c} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4c}$$
$$= \frac{\sqrt{a+bx^2+cx^4}}{2c} - \frac{b\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2c}$$
$$= \frac{\sqrt{a+bx^2+cx^4}}{2c} - \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

Mathematica [A] time = 0.01, size = 68, normalized size = 1.00

$$\frac{\sqrt{a+bx^2+cx^4}}{2c} - \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
+ b*x^2 + c*x^4])])/(4*c^(3/2))
```

fricas [A] time = 0.70, size = 161, normalized size = 2.37

$$\left[\frac{b\sqrt{c}\,\log\left(-8\,c^2x^4 - 8\,bcx^2 - b^2 + 4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c} - 4\,ac\right) + 4\,\sqrt{cx^4+bx^2+a}\,c}{8\,c^2}, \frac{b\sqrt{-c}\,\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(b*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a
)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^4 + b*x^2 + a)*c)/c^2, 1/4*(b
*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^
4 + b*c*x^2 + a*c)) + 2*sqrt(c*x^4 + b*x^2 + a)*c)/c^2]
```

giac [A] time = 0.21, size = 61, normalized size = 0.90

$$\frac{b\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{4c^{\frac{3}{2}}}+\frac{\sqrt{cx^4+bx^2+a}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3
/2) + 1/2*sqrt(c*x^4 + b*x^2 + a)/c
```

maple [A] time = 0.01, size = 56, normalized size = 0.82

$$-\frac{b\ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{4c^{\frac{3}{2}}}+\frac{\sqrt{cx^4+bx^2+a}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*(c*x^4+b*x^2+a)^(1/2)/c-1/4*b/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b
*x^2+a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 4.43, size = 55, normalized size = 0.81

$$\frac{\sqrt{cx^4+bx^2+a}}{2c}-\frac{b\ln\left(\sqrt{cx^4+bx^2+a}+\frac{cx^2+\frac{b}{2}}{\sqrt{c}}\right)}{4c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (a + b*x^2 + c*x^4)^(1/2)/(2*c) - (b*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 +
 c*x^2)/c^(1/2)))/(4*c^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a + bx^2 + cx^4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a + b*x**2 + c*x**4), x)
```

3.958 $\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

```
[Out] 1/2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1107, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$
$$= \text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x^2 + c*x^4],x]
[Out] ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[c])
```

fricas [A] time = 0.82, size = 118, normalized size = 2.74

$$\left[\frac{\log\left(-8\,c^2x^4 - 8\,bcx^2 - b^2 - 4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c} - 4\,ac\right)}{4\,\sqrt{c}}, -\frac{\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right)}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] [1/4*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2
+ b)*sqrt(c) - 4*a*c)/sqrt(c), -1/2*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2
+ a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c))/c]
```

giac [A] time = 0.21, size = 40, normalized size = 0.93

$$-\frac{\log\left(\left|-2\left(\sqrt{c}\,x^2 - \sqrt{cx^4+bx^2+a}\right)\sqrt{c} - b\right|\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/sqrt(
c)
```

maple [A] time = 0.01, size = 35, normalized size = 0.81

$$\frac{\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 4.69, size = 34, normalized size = 0.79

$$\frac{\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2))/(2*c^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(x/sqrt(a + b*x**2 + c*x**4), x)
```

3.959 $\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1114, 724, 206}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[a])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$
$$= -\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/Sqrt[a]
```

fricas [A] time = 0.86, size = 124, normalized size = 2.82

$$\left[\frac{\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)}{4\,\sqrt{a}}, \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
 + 2*a)*sqrt(a) + 8*a^2)/x^4)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c*x^4 +
 b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2))/a]
```

giac [A] time = 0.25, size = 38, normalized size = 0.86

$$\frac{\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.01, size = 39, normalized size = 0.89

$$-\frac{\ln\left(\frac{b\,x^{2}+2a+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\,\sqrt{a}}{x^{2}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2/a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 4.44, size = 44, normalized size = 1.00

$$-\frac{\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a}}-\frac{\ln\left(2\,a+2\,\sqrt{a}\,\,\sqrt{c\,x^4+b\,x^2+a}\,\,+b\,x^2\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] - log(1/x^2)/(2*a^(1/2)) - log(2*a + 2*a^(1/2)*(a + b*x^2 + c*x^4)^(1/2) +
b*x^2)/(2*a^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+bx^{2}+cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(1/(x*sqrt(a + b*x**2 + c*x**4)), x)
```

3.960 $\int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=72

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}-\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}$$

```
[Out] 1/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(3/2)-1/2*(c
*x^4+b*x^2+a)^(1/2)/a/x^2
```

Rubi [A] time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1114, 730, 724, 206}

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}-\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*a*x^2) + (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sq
rt[a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
```

```
m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}-\frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4a}\\ &= -\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}+\frac{b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2a}\\ &= -\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}+\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 72, normalized size = 1.00

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}-\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*Sqrt[a + b*x^2 + c*x^4]/(a*x^2) + (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*
Sqrt[a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

fricas [A] time = 0.92, size = 179, normalized size = 2.49

$$\left[\frac{\sqrt{a}\,bx^2\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)-4\,\sqrt{cx^4+bx^2+a}\,a}{8\,a^2x^2}, -\frac{\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(b}{2\,(acx^4+a}\right.}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*
x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*sqrt(c*x^4 + b*x^2 + a)*a)
/(a^2*x^2), -1/4*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
+ 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*sqrt(c*x^4 + b*x^2 + a)*a)/(
a^2*x^2)]
```

giac [A] time = 0.43, size = 114, normalized size = 1.58

$$-\frac{b \arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}\,a}+\frac{\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)b+2\,a\sqrt{c}}{2\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*
a) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a*sqrt(c))/(((sqrt(
c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*a)
```

maple [A] time = 0.01, size = 63, normalized size = 0.88

$$\frac{b\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{4a^{\frac{3}{2}}}-\frac{\sqrt{c\,x^4+b\,x^2+a}}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*(c*x^4+b*x^2+a)^(1/2)/a/x^2+1/4*b/a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2
+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 4.48, size = 56, normalized size = 0.78

$$\frac{b\,\mathrm{atanh}\left(\frac{\frac{b\,x^2}{2}+a}{\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}\right)}{4\,a^{3/2}}-\frac{\sqrt{c\,x^4+b\,x^2+a}}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] (b*atanh((a + (b*x^2)/2)/(a^(1/2)*(a + b*x^2 + c*x^4)^(1/2))))/(4*a^(3/2))
- (a + b*x^2 + c*x^4)^(1/2)/(2*a*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + b*x**2 + c*x**4)), x)
```

3.961 $\int \frac{1}{x^5 \sqrt{a+bx^2+cx^4}} \, dx$

Optimal. Leaf size=108

$$-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{\sqrt{a+bx^2+cx^4}}{4ax^4}$$

```
[Out] -1/16*(-4*a*c+3*b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))
/a^(5/2)-1/4*(c*x^4+b*x^2+a)^(1/2)/a/x^4+3/8*b*(c*x^4+b*x^2+a)^(1/2)/a^2/x^
2
```

Rubi [A] time = 0.11, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 744, 806, 724, 206}

$$-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{\sqrt{a+bx^2+cx^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(4*a*x^4) + (3*b*Sqrt[a + b*x^2 + c*x^4])/(8*a^2*x
^2) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*
x^4])])/(16*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
```

```
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{4ax^4}-\frac{\text{Subst}\left(\int \frac{\frac{3b}{2}+cx}{x^2\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4a}\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{4ax^4}+\frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2}+\frac{\left(3b^2-4ac\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{16a^2}\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{4ax^4}+\frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{\left(3b^2-4ac\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{8a^2}\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{4ax^4}+\frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 91, normalized size = 0.84

$$\frac{(4ac-3b^2)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{(3bx^2-2a)\sqrt{a+bx^2+cx^4}}{8a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-2*a + 3*b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*a^2*x^4) + ((-3*b^2 + 4*a*c)*
ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(5/2))
```

fricas [A] time = 0.85, size = 221, normalized size = 2.05

$$\left[-\frac{(3b^2-4ac)\sqrt{a}x^4\log\left(-\frac{(b^2+4ac)x^4+8abx^2+4\sqrt{cx^4+bx^2+a}(bx^2+2a)\sqrt{a}+8a^2}{x^4}\right)-4\sqrt{cx^4+bx^2+a}(3abx^2-2a^2)}{32a^3x^4}, \frac{(3b^2 \cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2 - 4*a*c)*sqrt(a)*x^4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4
*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*sqrt(c*x^4
 + b*x^2 + a)*(3*a*b*x^2 - 2*a^2))/(a^3*x^4), 1/16*((3*b^2 - 4*a*c)*sqrt(-a
)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 +
a*b*x^2 + a^2)) + 2*sqrt(c*x^4 + b*x^2 + a)*(3*a*b*x^2 - 2*a^2))/(a^3*x^4)]
```

giac [B] time = 0.30, size = 221, normalized size = 2.05

$$\frac{(3b^2-4ac)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{8\sqrt{-a}a^2}-\frac{3\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3b^2-4\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3ac-5\left(\cdots\right.}{8\left(\left(\sqrt{c}x^2-\sqrt{cx^4 \cdots}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2 - 4*a*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a
))/(sqrt(-a)*a^2) - 1/8*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*b^2 -
4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a*c - 5*(sqrt(c)*x^2 - sqrt(c*x
^4 + b*x^2 + a))*a*b^2 - 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^2*c -
8*a^2*b*sqrt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^2*a^2)
```

maple [A] time = 0.01, size = 127, normalized size = 1.18

$$\frac{c\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{4a^{\frac{3}{2}}}-\frac{3b^2\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{16a^{\frac{5}{2}}}+\frac{3\sqrt{c\,x^4+b\,x^2+a}\,b}{8a^2x^2}-\frac{\sqrt{c\,x^4+b\,x^2+a}}{4a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/4*(c*x^4+b*x^2+a)^(1/2)/a/x^4+3/8*b*(c*x^4+b*x^2+a)^(1/2)/a^2/x^2-3/16*b
^2/a^(5/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)+1/4*c/a^(3/2
)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^5*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(a + b*x**2 + c*x**4)), x)
```

3.962 $\int \frac{1}{x^7\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=145

$$\frac{b(5b^2-12ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}-\frac{(15b^2-16ac)\sqrt{a+bx^2+cx^4}}{48a^3x^2}+\frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4}-\frac{\sqrt{a+bx^2+cx^4}}{6ax^6}$$

```
[Out] 1/32*b*(-12*a*c+5*b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2
))/a^(7/2)-1/6*(c*x^4+b*x^2+a)^(1/2)/a/x^6+5/24*b*(c*x^4+b*x^2+a)^(1/2)/a^2
/x^4-1/48*(-16*a*c+15*b^2)*(c*x^4+b*x^2+a)^(1/2)/a^3/x^2
```

Rubi [A] time = 0.17, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 744, 834, 806, 724, 206}

$$-\frac{(15b^2-16ac)\sqrt{a+bx^2+cx^4}}{48a^3x^2}+\frac{b(5b^2-12ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}+\frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4}-\frac{\sqrt{a+bx^2+cx^4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(6*a*x^6) + (5*b*Sqrt[a + b*x^2 + c*x^4])/(24*a^2*
x^4) - ((15*b^2 - 16*a*c)*Sqrt[a + b*x^2 + c*x^4])/(48*a^3*x^2) + (b*(5*b^2
 - 12*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(32*
a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
```

```
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^7\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{6ax^6} - \frac{\text{Subst}\left(\int \frac{\frac{5b}{2}+2cx}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{6a}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4} + \frac{\text{Subst}\left(\int \frac{\frac{1}{4}\left(15b^2-16ac\right)+\frac{5bcx}{2}}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{12a^2}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{\left(15b^2-16ac\right)\sqrt{a+bx^2+cx^4}}{48a^3x^2} - \frac{\left(b\left(5b^2-1\right.\right.}{}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{\left(15b^2-16ac\right)\sqrt{a+bx^2+cx^4}}{48a^3x^2} + \frac{\left(b\left(5b^2-1\right.\right.}{}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{\left(15b^2-16ac\right)\sqrt{a+bx^2+cx^4}}{48a^3x^2} + \frac{b\left(5b^2-12\right.}{}$$

Mathematica [A] time = 0.08, size = 112, normalized size = 0.77

$$\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}} + \frac{\sqrt{a+bx^2+cx^4}\left(-8a^2+2a\left(5bx^2+8cx^4\right)-15b^2x^4\right)}{48a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-8*a^2 - 15*b^2*x^4 + 2*a*(5*b*x^2 + 8*c*x^4)))/(
48*a^3*x^6) + (b*(5*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a +
 b*x^2 + c*x^4])])/(32*a^(7/2))
```

fricas [A] time = 0.92, size = 265, normalized size = 1.83

$$\left[-\frac{3\left(5\,b^3-12\,abc\right)\sqrt{a}\,x^6\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)-4\left(10\,a^2bx^2-\left(15\,ab^2-16\,a^2c\right)x^4-\right.}{192\,a^4x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3 - 12*a*b*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x
^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(10*
a^2*b*x^2 - (15*a*b^2 - 16*a^2*c)*x^4 - 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^
4*x^6), -1/96*(3*(5*b^3 - 12*a*b*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*
x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*(10*a^2*b*x^
2 - (15*a*b^2 - 16*a^2*c)*x^4 - 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^6)]
```

giac [B] time = 0.26, size = 335, normalized size = 2.31

$$-\frac{\left(5\,b^{3}-12\,abc\right)\arctan\left(-\frac{\sqrt{c}\,x^{2}-\sqrt{cx^{4}+bx^{2}+a}}{\sqrt{-a}}\right)}{16\,\sqrt{-a}\,a^{3}}+\frac{15\left(\sqrt{c}\,x^{2}-\sqrt{cx^{4}+bx^{2}+a}\right)^{5}b^{3}-36\left(\sqrt{c}\,x^{2}-\sqrt{cx^{4}+bx^{2}+a}\right)^{5}a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*(5*b^3 - 12*a*b*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sq
rt(-a))/(sqrt(-a)*a^3) + 1/48*(15*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5
*b^3 - 36*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a*b*c - 40*(sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 + a))^3*a*b^3 + 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))^3*a^2*b*c + 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^3*c^(3/2)
+ 33*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^2*b^3 + 36*(sqrt(c)*x^2 - sq
rt(c*x^4 + b*x^2 + a))*a^3*b*c + 48*a^3*b^2*sqrt(c) - 32*a^4*c^(3/2))/(((sq
rt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^3*a^3)
```

maple [A] time = 0.02, size = 176, normalized size = 1.21

$$-\frac{3bc\ln\left(\frac{b\,x^{2}+2a+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{a}}{x^{2}}\right)}{8a^{\frac{5}{2}}}+\frac{5b^{3}\ln\left(\frac{b\,x^{2}+2a+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{a}}{x^{2}}\right)}{32a^{\frac{7}{2}}}+\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\;c}{3a^{2}x^{2}}-\frac{5\sqrt{c\,x^{4}+b\,x^{2}+a}\;b^{2}}{16a^{3}x^{2}}+\frac{5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/6*(c*x^4+b*x^2+a)^(1/2)/a/x^6+5/24*b*(c*x^4+b*x^2+a)^(1/2)/a^2/x^4-5/16*
b^2/a^3/x^2*(c*x^4+b*x^2+a)^(1/2)+5/32*b^3/a^(7/2)*ln((b*x^2+2*a+2*(c*x^4+b
*x^2+a)^(1/2)*a^(1/2))/x^2)-3/8*b/a^(5/2)*c*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)
^(1/2)*a^(1/2))/x^2)+1/3*c/a^2/x^2*(c*x^4+b*x^2+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^7\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^7*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**7*sqrt(a + b*x**2 + c*x**4)), x)
```

3.963 $\int \frac{x^4}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=313

$$-\frac{2bx\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{c}+2b\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt[4]{a}\,b\,(}{}$$

```
[Out] 1/3*x*(c*x^4+b*x^2+a)^(1/2)/c-2/3*b*x*(c*x^4+b*x^2+a)^(1/2)/c^(3/2)/(a^(1/2
)+x^2*c^(1/2))+2/3*a^(1/4)*b*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos
(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/
2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1
/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)-1/6*a^(1/4)*(cos(2*
arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellipt
icF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1
/2)+x^2*c^(1/2))*(2*b+a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2
))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1122, 1197, 1103, 1195}

$$-\frac{2bx\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{c}+2b\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt[4]{a}\,b\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x^2 + c*x^4])/(3*c) - (2*b*x*Sqrt[a + b*x^2 + c*x^4])/(3*c^(3
/2)*(Sqrt[a] + Sqrt[c]*x^2)) + (2*a^(1/4)*b*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]
) - (a^(1/4)*(2*b + Sqrt[a]*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^
2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4
)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
```

```
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{x\sqrt{a+bx^2+cx^4}}{3c} - \frac{\int \frac{a+2bx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3c}$$

$$= \frac{x\sqrt{a+bx^2+cx^4}}{3c} + \frac{\left(2\sqrt{a}\,b\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{3c^{3/2}} - \frac{\left(\sqrt{a}\,\left(2b+\sqrt{a}\,\sqrt{c}\right)\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{3c^{3/2}}$$

$$= \frac{x\sqrt{a+bx^2+cx^4}}{3c} - \frac{2bx\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{2\sqrt[4]{a}\,b\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\right.}{3c^{7/4}\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] time = 0.86, size = 444, normalized size = 1.42

$$\frac{i\left(b\sqrt{b^2-4ac}+ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-ib\left(\cdots\right.}{6c^2\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4) - I*b*(-b + Sqrt
[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Ell
ipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2
- 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^2 + a*c + b*Sqrt[b^2 - 4*a*c])*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b -
2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh
[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - S
qrt[b^2 - 4*a*c])])/(6*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 +
 c*x^4])
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

maple [A] time = 0.01, size = 388, normalized size = 1.24

$$\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)+\mathrm{EllipticF}\right.}{3\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,\left(b+\sqrt{-4ac+b^2}\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*x*(c*x^4+b*x^2+a)^(1/2)/c-1/12/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/
a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))+1/3*b/c*a
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^
2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(
b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(
1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^4/(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(a + b*x**2 + c*x**4), x)
```

3.964 $\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=267

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}}{\sqrt[4]{a}}\right.\right.}{c^{3/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*(cos(2*arctan
(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(si
n(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^
2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b
*x^2+a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*
arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(
2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)
+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1139, 1103, 1195}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}}{\sqrt[4]{a}}\right.\right.}{c^{3/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*E
llipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(3
/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b
*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(
1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1139

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{a}\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}} - \frac{\sqrt{a}\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}}$$

$$= \frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] time = 0.13, size = 278, normalized size = 1.04

$$\frac{i\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-F\left(i\sinh^{-1}\left(\right.\right.\right.}{2\sqrt{2}c\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((I/2)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticE
[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*
c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt
```

```
[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqr
t[2]*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^2}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

maple [A] time = 0.01, size = 216, normalized size = 0.81

$$-\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)+\mathrm{Elliptic}\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,\left(b+\sqrt{-4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2)
)/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(
1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2^(
1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/
c-4)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^2/(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*x**2 + c*x**4), x)
```

3.965 $\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=114

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+bx^2+cx^4}}$$

```
[Out] 1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/
4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(
1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)
/a^(1/4)/c^(1/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2
*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx=\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] time = 0.10, size = 186, normalized size = 1.63

$$-\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])
])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*x^4 + b*x^2 + a), x)
```

maple [A] time = 0.01, size = 144, normalized size = 1.26

$$\frac{\sqrt{2}\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\;\mathrm{EllipticF}\left(\frac{\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a
*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2
)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a
*c+b^2)^(1/2))/a*b/c-4)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x^4 + b\,x^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(1/(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*x**2 + c*x**4), x)
```

3.966 $\int \frac{1}{x^2\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=294

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\cdots\right.\right.}{a^{3/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] -(c*x^4+b*x^2+a)^(1/2)/a/x+x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/(a^(1/2)+x^2*c
^(1/2))-c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(
1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2
)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/
2))^2)^(1/2)/a^(3/4)/(c*x^4+b*x^2+a)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4
)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arc
tan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/
2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^4+b*x^2+a)
^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1123, 12, 1139, 1103, 1195}

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\cdots\right.\right.}{a^{3/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[a + b*x^2 + c*x^4]/(a*x)) + (Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(a*(
Sqrt[a] + Sqrt[c]*x^2)) - (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)]
, (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (c^(1/4
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
2*a^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1139

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\sqrt{a+bx^2+cx^4}}{ax} + \frac{\int \frac{cx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{ax} + \frac{c\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{ax} + \frac{\sqrt{c}\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}} - \frac{\sqrt{c}\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{ax} + \frac{\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{a\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan\right.}{a^{3/4}\sqrt{a+bx^2+c}}$$

Mathematica [C] time = 0.48, size = 298, normalized size = 1.01

$$\frac{-\frac{4\left(a+bx^2+cx^4\right)}{x} + \frac{i\sqrt{2}\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4}}}\right.\right.\right.}{\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}}}{4a\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-4*(a + b*x^2 + c*x^4))/x + (I*Sqrt[2]*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(
b - Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 -
 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[
I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c
])/(b - Sqrt[b^2 - 4*a*c])]))/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[a
+ b*x^2 + c*x^4])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}}{cx^6+bx^4+ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^4 + b*x^2 + a)/(c*x^6 + b*x^4 + a*x^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)

maple [A] time = 0.01, size = 239, normalized size = 0.81

$$-\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)+\mathrm{Ellipti}\cdots\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,\left(b+\sqrt{-4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(c*x^4+b*x^2+a)^(1/2),x)

[Out] -(c*x^4+b*x^2+a)^(1/2)/a/x-1/2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*
(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+
4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2
)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4
)^(1/2))-EllipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(
b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int(1/(x^2*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(a + b*x**2 + c*x**4)), x)
```

3.967 $\int \frac{1}{x^4\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=345

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}+2b\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+c}{\left(\sqrt{a}+\sqrt{c}\right.}}}{3a^{7/4}\sqrt{a}}$$

```
[Out] -1/3*(c*x^4+b*x^2+a)^(1/2)/a/x^3+2/3*b*(c*x^4+b*x^2+a)^(1/2)/a^2/x-2/3*b*x*
c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^2/(a^(1/2)+x^2*c^(1/2))+2/3*b*c^(1/4)*(cos(
2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elli
pticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^
(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/
(c*x^4+b*x^2+a)^(1/2)-1/6*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2
)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4)
)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*b+a^(1/2)*c^(1
/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(c*x^4+b*x^2+a
)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1123, 1281, 1197, 1103, 1195}

$$\frac{2b\sqrt{a+bx^2+cx^4}}{3a^2x}-\frac{2b\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{3a^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}+2b\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2\right.\right.}{6a^{7/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(3*a*x^3) + (2*b*Sqrt[a + b*x^2 + c*x^4])/(3*a^2*x
) - (2*b*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(3*a^2*(Sqrt[a] + Sqrt[c]*x^2))
 + (2*b*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*S
qrt[c]))/4])/(3*a^(7/4)*Sqrt[a + b*x^2 + c*x^4]) - ((2*b + Sqrt[a]*Sqrt[c])
*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c])
)/4])/(6*a^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
```

```
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{\int \frac{-2b-cx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx}{3a}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{2b\sqrt{a+bx^2+cx^4}}{3a^2x} - \frac{\int \frac{ac+2bcx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3a^2}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{2b\sqrt{a+bx^2+cx^4}}{3a^2x} + \frac{\left(2b\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{3a^{3/2}} - \frac{\left(\left(2b+\sqrt{a}\sqrt{c}\right)\right.}{3}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{2b\sqrt{a+bx^2+cx^4}}{3a^2x} - \frac{2b\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{3a^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{2b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x\right.}{}$$

Mathematica [C] time = 0.89, size = 459, normalized size = 1.33

$$\frac{-2\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(a-2bx^2\right)\left(a+bx^2+cx^4\right)+ix^3\left(b\sqrt{b^2-4ac}+ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}F\left(i\right.}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a - 2*b*x^2)*(a + b*x^2 + c*x^4) - I*b
*(-b + Sqrt[b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2
 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x],
 (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^2 + a*c + b*Sqrt[
b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b
^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(6*a^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]
)]*x^3*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}}{cx^8+bx^6+ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)/(c*x^8 + b*x^6 + a*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}\, x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

maple [A] time = 0.02, size = 413, normalized size = 1.20

$$\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)+\mathrm{Ellipti}\cdots\right.}{3\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,\left(b+\sqrt{-4ac+b^2}\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/3*(c*x^4+b*x^2+a)^(1/2)/a/x^3+2/3*b*(c*x^4+b*x^2+a)^(1/2)/a^2/x-1/12*c/a
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^
2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*E
llipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+
b^2)^(1/2))/a*b/c-4)^(1/2))+1/3*b*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a
*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*
b/c-4)^(1/2))-EllipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2
*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}\, x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4 \sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int(1/(x^4*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(1/(x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

3.968 $\int \frac{x^7}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=124

$$-\frac{b\left(12ac+5b^2\right)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{32c^{7/2}}-\frac{\left(16ac+15b^2+10bcx^2\right)\sqrt{a+bx^2-cx^4}}{48c^3}-\frac{x^4\sqrt{a+bx^2-cx^4}}{6c}$$

```
[Out] -1/32*b*(12*a*c+5*b^2)*arctan(1/2*(-2*c*x^2+b)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/
2))/c^(7/2)-1/6*x^4*(-c*x^4+b*x^2+a)^(1/2)/c-1/48*(10*b*c*x^2+16*a*c+15*b^2
)*(-c*x^4+b*x^2+a)^(1/2)/c^3
```

Rubi [A] time = 0.11, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1114, 742, 779, 621, 204}

$$-\frac{\left(16ac+15b^2+10bcx^2\right)\sqrt{a+bx^2-cx^4}}{48c^3}-\frac{b\left(12ac+5b^2\right)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{32c^{7/2}}-\frac{x^4\sqrt{a+bx^2-cx^4}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -(x^4*Sqrt[a + b*x^2 - c*x^4])/(6*c) - ((15*b^2 + 16*a*c + 10*b*c*x^2)*Sqrt
[a + b*x^2 - c*x^4])/(48*c^3) - (b*(5*b^2 + 12*a*c)*ArcTan[(b - 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])])/(32*c^(7/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
```

```
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx-cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{x^4\sqrt{a+bx^2-cx^4}}{6c} - \frac{\text{Subst}\left(\int \frac{x\left(-2a-\frac{5bx}{2}\right)}{\sqrt{a+bx-cx^2}}\,dx, x, x^2\right)}{6c}$$

$$= -\frac{x^4\sqrt{a+bx^2-cx^4}}{6c} - \frac{\left(15b^2+16ac+10bcx^2\right)\sqrt{a+bx^2-cx^4}}{48c^3} + \frac{\left(b\left(5b^2+12ac\right)\right)\text{Subst}}{32}$$

$$= -\frac{x^4\sqrt{a+bx^2-cx^4}}{6c} - \frac{\left(15b^2+16ac+10bcx^2\right)\sqrt{a+bx^2-cx^4}}{48c^3} + \frac{\left(b\left(5b^2+12ac\right)\right)\text{Subst}}{}$$

$$= -\frac{x^4\sqrt{a+bx^2-cx^4}}{6c} - \frac{\left(15b^2+16ac+10bcx^2\right)\sqrt{a+bx^2-cx^4}}{48c^3} - \frac{b\left(5b^2+12ac\right)\tan^{-1}\left(\right.}{32c^{7/2}}$$

Mathematica [A] time = 0.08, size = 107, normalized size = 0.86

$$\frac{-2\sqrt{c}\sqrt{a+bx^2-cx^4}\left(8c\left(2a+cx^4\right)+15b^2+10bcx^2\right)-3b\left(12ac+5b^2\right)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{96c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (-2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4]*(15*b^2 + 10*b*c*x^2 + 8*c*(2*a + c*x^4
)) - 3*b*(5*b^2 + 12*a*c)*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 -
c*x^4])])/(96*c^(7/2))
```

fricas [A] time = 0.92, size = 249, normalized size = 2.01

$$\left[-\frac{3\left(5\,b^3+12\,abc\right)\sqrt{-c}\,\log\left(8\,c^2x^4-8\,bcx^2+b^2-4\,\sqrt{-cx^4+bx^2+a}\left(2\,cx^2-b\right)\sqrt{-c}\,-4\,ac\right)+4\left(8\,c^3x^4+10\,b\right.}{192\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3 + 12*a*b*c)*sqrt(-c)*log(8*c^2*x^4 - 8*b*c*x^2 + b^2 - 4*
sqrt(-c*x^4 + b*x^2 + a)*(2*c*x^2 - b)*sqrt(-c) - 4*a*c) + 4*(8*c^3*x^4 + 1
0*b*c^2*x^2 + 15*b^2*c + 16*a*c^2)*sqrt(-c*x^4 + b*x^2 + a))/c^4, -1/96*(3*
(5*b^3 + 12*a*b*c)*sqrt(c)*arctan(1/2*sqrt(-c*x^4 + b*x^2 + a)*(2*c*x^2 - b
)*sqrt(c)/(c^2*x^4 - b*c*x^2 - a*c)) + 2*(8*c^3*x^4 + 10*b*c^2*x^2 + 15*b^2
*c + 16*a*c^2)*sqrt(-c*x^4 + b*x^2 + a))/c^4]
```

giac [A] time = 0.26, size = 112, normalized size = 0.90

$$-\frac{1}{48}\sqrt{-cx^4+bx^2+a}\left(2\,x^2\left(\frac{4\,x^2}{c}+\frac{5\,b}{c^2}\right)+\frac{15\,b^2+16\,ac}{c^3}\right)-\frac{\left(5\,b^3+12\,abc\right)\log\left(\left|2\left(\sqrt{-c}\,x^2-\sqrt{-cx^4+bx^2+a}\right)\sqrt{\right.\right.}{32\,\sqrt{-c}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*sqrt(-c*x^4 + b*x^2 + a)*(2*x^2*(4*x^2/c + 5*b/c^2) + (15*b^2 + 16*a*
c)/c^3) - 1/32*(5*b^3 + 12*a*b*c)*log(abs(2*(sqrt(-c)*x^2 - sqrt(-c*x^4 + b
*x^2 + a))*sqrt(-c) + b))/(sqrt(-c)*c^3)
```

maple [A] time = 0.02, size = 168, normalized size = 1.35

$$-\frac{\sqrt{-c\,x^{4}+b\,x^{2}+a}\;x^{4}}{6c}-\frac{5\sqrt{-c\,x^{4}+b\,x^{2}+a}\;b\,x^{2}}{24c^{2}}+\frac{3ab\arctan\left(\frac{\left(x^{2}-\frac{b}{2c}\right)\sqrt{c}}{\sqrt{-c\,x^{4}+b\,x^{2}+a}}\right)}{8c^{\frac{5}{2}}}+\frac{5b^{3}\arctan\left(\frac{\left(x^{2}-\frac{b}{2c}\right)\sqrt{c}}{\sqrt{-c\,x^{4}+b\,x^{2}+a}}\right)}{32c^{\frac{7}{2}}}-\frac{\sqrt{-c\,x^{4}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/6*x^4*(-c*x^4+b*x^2+a)^(1/2)/c-5/24*b/c^2*x^2*(-c*x^4+b*x^2+a)^(1/2)-5/1
6*b^2/c^3*(-c*x^4+b*x^2+a)^(1/2)+5/32*b^3/c^(7/2)*arctan(c^(1/2)*(x^2-1/2*b
/c)/(-c*x^4+b*x^2+a)^(1/2))+3/8*b/c^(5/2)*a*arctan(c^(1/2)*(x^2-1/2*b/c)/(-
c*x^4+b*x^2+a)^(1/2))-1/3/c^2*a*(-c*x^4+b*x^2+a)^(1/2)
```

maxima [A] time = 2.43, size = 153, normalized size = 1.23

$$-\frac{\sqrt{-cx^{4}+bx^{2}+a}\,x^{4}}{6\,c}-\frac{5\,\sqrt{-cx^{4}+bx^{2}+a}\,bx^{2}}{24\,c^{2}}-\frac{5\,b^{3}\arcsin\left(-\frac{2\,cx^{2}-b}{\sqrt{b^{2}+4\,ac}}\right)}{32\,c^{\frac{7}{2}}}-\frac{3\,ab\arcsin\left(-\frac{2\,cx^{2}-b}{\sqrt{b^{2}+4\,ac}}\right)}{8\,c^{\frac{5}{2}}}-\frac{5\,\sqrt{-cx^{4}+bx^{2}}}{16\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(-c*x^4 + b*x^2 + a)*x^4/c - 5/24*sqrt(-c*x^4 + b*x^2 + a)*b*x^2/c
^2 - 5/32*b^3*arcsin(-(2*c*x^2 - b)/sqrt(b^2 + 4*a*c))/c^(7/2) - 3/8*a*b*ar
csin(-(2*c*x^2 - b)/sqrt(b^2 + 4*a*c))/c^(5/2) - 5/16*sqrt(-c*x^4 + b*x^2 +
 a)*b^2/c^3 - 1/3*sqrt(-c*x^4 + b*x^2 + a)*a/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{7}}{\sqrt{-c\,x^{4}+b\,x^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2 - c*x^4)^(1/2),x)
```

```
[Out] int(x^7/(a + b*x^2 - c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{\sqrt{a+bx^{2}-cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**7/sqrt(a + b*x**2 - c*x**4), x)
```

3.969 $\int \frac{x^5}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=107

$$-\frac{(4ac+3b^2)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{16c^{5/2}}-\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2}-\frac{x^2\sqrt{a+bx^2-cx^4}}{4c}$$

```
[Out] -1/16*(4*a*c+3*b^2)*arctan(1/2*(-2*c*x^2+b)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2))
/c^(5/2)-3/8*b*(-c*x^4+b*x^2+a)^(1/2)/c^2-1/4*x^2*(-c*x^4+b*x^2+a)^(1/2)/c
```

Rubi [A] time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1114, 742, 640, 621, 204}

$$-\frac{(4ac+3b^2)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{16c^{5/2}}-\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2}-\frac{x^2\sqrt{a+bx^2-cx^4}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (-3*b*Sqrt[a + b*x^2 - c*x^4])/(8*c^2) - (x^2*Sqrt[a + b*x^2 - c*x^4])/(4*c
) - ((3*b^2 + 4*a*c)*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4
])])/(16*c^(5/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{a+bx^2-cx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx-cx^2}}\,dx,x,x^2\right)\\
&= -\frac{x^2\sqrt{a+bx^2-cx^4}}{4c}-\frac{\text{Subst}\left(\int \frac{-a-\frac{3bx}{2}}{\sqrt{a+bx-cx^2}}\,dx,x,x^2\right)}{4c}\\
&= -\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2}-\frac{x^2\sqrt{a+bx^2-cx^4}}{4c}+\frac{\left(3b^2+4ac\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx-cx^2}}\,dx,x,x^2\right)}{16c^2}\\
&= -\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2}-\frac{x^2\sqrt{a+bx^2-cx^4}}{4c}+\frac{\left(3b^2+4ac\right)\text{Subst}\left(\int \frac{1}{-4c-x^2}\,dx,x,\frac{b-2cx^2}{\sqrt{a+bx^2-cx^4}}\right)}{8c^2}\\
&= -\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2}-\frac{x^2\sqrt{a+bx^2-cx^4}}{4c}-\frac{\left(3b^2+4ac\right)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{16c^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 89, normalized size = 0.83

$$-\frac{\left(4ac+3b^2\right)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{16c^{5/2}}-\frac{\left(3b+2cx^2\right)\sqrt{a+bx^2-cx^4}}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -1/8*((3*b + 2*c*x^2)*Sqrt[a + b*x^2 - c*x^4])/c^2 - ((3*b^2 + 4*a*c)*ArcTa
n[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])])/(16*c^(5/2))
```

fricas [A] time = 0.81, size = 211, normalized size = 1.97

$$\left[-\frac{\left(3\,b^2+4\,ac\right)\sqrt{-c}\,\log\left(8\,c^2x^4-8\,bcx^2+b^2-4\,\sqrt{-cx^4+bx^2+a}\left(2\,cx^2-b\right)\sqrt{-c}\,-4\,ac\right)+4\,\sqrt{-cx^4+bx^2+a}\left(2\right.}{32\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2 + 4*a*c)*sqrt(-c)*log(8*c^2*x^4 - 8*b*c*x^2 + b^2 - 4*sqrt(-
c*x^4 + b*x^2 + a)*(2*c*x^2 - b)*sqrt(-c) - 4*a*c) + 4*sqrt(-c*x^4 + b*x^2
+ a)*(2*c^2*x^2 + 3*b*c))/c^3, -1/16*((3*b^2 + 4*a*c)*sqrt(c)*arctan(1/2*sq
rt(-c*x^4 + b*x^2 + a)*(2*c*x^2 - b)*sqrt(c)/(c^2*x^4 - b*c*x^2 - a*c)) + 2
*sqrt(-c*x^4 + b*x^2 + a)*(2*c^2*x^2 + 3*b*c))/c^3]
```

giac [A] time = 0.21, size = 91, normalized size = 0.85

$$-\frac{1}{8}\,\sqrt{-cx^4+bx^2+a}\left(\frac{2\,x^2}{c}+\frac{3\,b}{c^2}\right)-\frac{\left(3\,b^2+4\,ac\right)\log\left(\left|2\left(\sqrt{-c}\,x^2-\sqrt{-cx^4+bx^2+a}\right)\sqrt{-c}\,+b\right|\right)}{16\,\sqrt{-c}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-c*x^4 + b*x^2 + a)*(2*x^2/c + 3*b/c^2) - 1/16*(3*b^2 + 4*a*c)*lo
g(abs(2*(sqrt(-c)*x^2 - sqrt(-c*x^4 + b*x^2 + a))*sqrt(-c) + b))/(sqrt(-c)*
c^2)
```

maple [A] time = 0.02, size = 120, normalized size = 1.12

$$-\frac{\sqrt{-c\,x^4+b\,x^2+a}\;x^2}{4c}+\frac{a\arctan\left(\frac{\left(x^2-\frac{b}{2c}\right)\sqrt{c}}{\sqrt{-c\,x^4+b\,x^2+a}}\right)}{4c^{\frac{3}{2}}}+\frac{3b^2\arctan\left(\frac{\left(x^2-\frac{b}{2c}\right)\sqrt{c}}{\sqrt{-c\,x^4+b\,x^2+a}}\right)}{16c^{\frac{5}{2}}}-\frac{3\sqrt{-c\,x^4+b\,x^2+a}\;b}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/4*x^2*(-c*x^4+b*x^2+a)^(1/2)/c-3/8*b*(-c*x^4+b*x^2+a)^(1/2)/c^2+3/16*b^2
/c^(5/2)*arctan((x^2-1/2*b/c)/(-c*x^4+b*x^2+a)^(1/2)*c^(1/2))+1/4*a/c^(3/2)
*arctan((x^2-1/2*b/c)/(-c*x^4+b*x^2+a)^(1/2)*c^(1/2))
```

maxima [A] time = 2.42, size = 105, normalized size = 0.98

$$-\frac{\sqrt{-cx^4+bx^2+a}\,x^2}{4\,c}-\frac{3\,b^2\arcsin\left(-\frac{2\,cx^2-b}{\sqrt{b^2+4\,ac}}\right)}{16\,c^{\frac{5}{2}}}-\frac{a\arcsin\left(-\frac{2\,cx^2-b}{\sqrt{b^2+4\,ac}}\right)}{4\,c^{\frac{3}{2}}}-\frac{3\,\sqrt{-cx^4+bx^2+a}\,b}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-c*x^4 + b*x^2 + a)*x^2/c - 3/16*b^2*arcsin(-(2*c*x^2 - b)/sqrt(b
^2 + 4*a*c))/c^(5/2) - 1/4*a*arcsin(-(2*c*x^2 - b)/sqrt(b^2 + 4*a*c))/c^(3/
2) - 3/8*sqrt(-c*x^4 + b*x^2 + a)*b/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2 - c*x^4)^(1/2),x)
```

```
[Out] int(x^5/(a + b*x^2 - c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(a + b*x**2 - c*x**4), x)
```

3.970 $\int \frac{x^3}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=70

$$-\frac{b\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{4c^{3/2}}-\frac{\sqrt{a+bx^2-cx^4}}{2c}$$

```
[Out] -1/4*b*arctan(1/2*(-2*c*x^2+b)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2))/c^(3/2)-1/2*
(-c*x^4+b*x^2+a)^(1/2)/c
```

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1114, 640, 621, 204}

$$-\frac{b\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{4c^{3/2}}-\frac{\sqrt{a+bx^2-cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -Sqrt[a + b*x^2 - c*x^4]/(2*c) - (b*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
+ b*x^2 - c*x^4])])/(4*c^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+bx-cx^2}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{a+bx^2-cx^4}}{2c} + \frac{b\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx-cx^2}}\,dx, x, x^2\right)}{4c}$$
$$= -\frac{\sqrt{a+bx^2-cx^4}}{2c} + \frac{b\operatorname{Subst}\left(\int \frac{1}{-4c-x^2}\,dx, x, \frac{b-2cx^2}{\sqrt{a+bx^2-cx^4}}\right)}{2c}$$
$$= -\frac{\sqrt{a+bx^2-cx^4}}{2c} - \frac{b\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{4c^{3/2}}$$

Mathematica [A] time = 0.02, size = 70, normalized size = 1.00

$$-\frac{b\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{4c^{3/2}} - \frac{\sqrt{a+bx^2-cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -1/2*Sqrt[a + b*x^2 - c*x^4]/c - (b*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
+ b*x^2 - c*x^4])])/(4*c^(3/2))
```

fricas [A] time = 0.83, size = 169, normalized size = 2.41

$$\left[-\frac{b\sqrt{-c}\,\log\left(8\,c^2x^4 - 8\,bcx^2 + b^2 - 4\,\sqrt{-cx^4+bx^2+a}\left(2\,cx^2-b\right)\sqrt{-c} - 4\,ac\right) + 4\,\sqrt{-cx^4+bx^2+a}\,c}{8\,c^2}, -\frac{b\sqrt{c}\ \text{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(b*sqrt(-c)*log(8*c^2*x^4 - 8*b*c*x^2 + b^2 - 4*sqrt(-c*x^4 + b*x^2 +
 a)*(2*c*x^2 - b)*sqrt(-c) - 4*a*c) + 4*sqrt(-c*x^4 + b*x^2 + a)*c)/c^2, -1
/4*(b*sqrt(c)*arctan(1/2*sqrt(-c*x^4 + b*x^2 + a)*(2*c*x^2 - b)*sqrt(c)/(c^
2*x^4 - b*c*x^2 - a*c)) + 2*sqrt(-c*x^4 + b*x^2 + a)*c)/c^2]
```

giac [A] time = 0.27, size = 70, normalized size = 1.00

$$-\frac{b\log\left(\left|2\left(\sqrt{-c}\,x^2-\sqrt{-cx^4+bx^2+a}\right)\sqrt{-c}\,+b\right|\right)}{4\,\sqrt{-c}\,c}-\frac{\sqrt{-cx^4+bx^2+a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*b*log(abs(2*(sqrt(-c)*x^2 - sqrt(-c*x^4 + b*x^2 + a))*sqrt(-c) + b))/(
sqrt(-c)*c) - 1/2*sqrt(-c*x^4 + b*x^2 + a)/c
```

maple [A] time = 0.01, size = 58, normalized size = 0.83

$$\frac{b\arctan\left(\frac{\left(x^2-\frac{b}{2c}\right)\sqrt{c}}{\sqrt{-c\,x^4+b\,x^2+a}}\right)}{4c^{\frac{3}{2}}}-\frac{\sqrt{-c\,x^4+b\,x^2+a}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*(-c*x^4+b*x^2+a)^(1/2)/c+1/4*b/c^(3/2)*arctan((x^2-1/2*b/c)/(-c*x^4+b*
x^2+a)^(1/2)*c^(1/2))
```

maxima [A] time = 2.47, size = 50, normalized size = 0.71

$$-\frac{b\arcsin\left(-\frac{2\,cx^2-b}{\sqrt{b^2+4\,ac}}\right)}{4\,c^{\frac{3}{2}}}-\frac{\sqrt{-cx^4+bx^2+a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*b*arcsin(-(2*c*x^2 - b)/sqrt(b^2 + 4*a*c))/c^(3/2) - 1/2*sqrt(-c*x^4 +
 b*x^2 + a)/c
```

mupad [B] time = 4.59, size = 62, normalized size = 0.89

$$-\frac{\sqrt{-c\,x^4+b\,x^2+a}}{2\,c}-\frac{b\,\ln\left(\frac{\frac{b}{2}-c\,x^2}{\sqrt{-c}}+\sqrt{-c\,x^4+b\,x^2+a}\right)}{4\,{\left(-c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2 - c*x^4)^(1/2),x)
```

```
[Out] - (a + b*x^2 - c*x^4)^(1/2)/(2*c) - (b*log((b/2 - c*x^2)/(-c)^(1/2) + (a +
b*x^2 - c*x^4)^(1/2)))/(4*(-c)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a + bx^2 - cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a + b*x**2 - c*x**4), x)
```

3.971 $\int \frac{x}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=44

$$-\frac{\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{c}}$$

[Out] -1/2*arctan(1/2*(-2*c*x^2+b)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2))/c^(1/2)

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1107, 621, 204}

$$-\frac{\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[a + b*x^2 - c*x^4],x]

[Out] -ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])]/(2*Sqrt[c])

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx-cx^2}}\,dx, x, x^2\right)$$
$$= \text{Subst}\left(\int \frac{1}{-4c-x^2}\,dx, x, \frac{b-2cx^2}{\sqrt{a+bx^2-cx^4}}\right)$$
$$= -\frac{\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{c}}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -1/2*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])]/Sqrt[c]
```

fricas [A] time = 0.94, size = 124, normalized size = 2.82

$$\left[-\frac{\sqrt{-c}\log\left(8\,c^2x^4-8\,bcx^2+b^2-4\,\sqrt{-cx^4+bx^2+a}\left(2\,cx^2-b\right)\sqrt{-c}-4\,ac\right)}{4\,c}, -\frac{\arctan\left(\frac{\sqrt{-cx^4+bx^2+a}\left(2\,cx^2-b\right)\sqrt{c}}{2\left(c^2x^4-bcx^2-ac\right)}\right)}{2\,\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-c)*log(8*c^2*x^4 - 8*b*c*x^2 + b^2 - 4*sqrt(-c*x^4 + b*x^2 + a)
*(2*c*x^2 - b)*sqrt(-c) - 4*a*c)/c, -1/2*arctan(1/2*sqrt(-c*x^4 + b*x^2 + a
)*(2*c*x^2 - b)*sqrt(c)/(c^2*x^4 - b*c*x^2 - a*c))/sqrt(c)]
```

giac [A] time = 0.21, size = 45, normalized size = 1.02

$$-\frac{\log\left(\left|2\left(\sqrt{-c}\,x^2-\sqrt{-cx^4+bx^2+a}\right)\sqrt{-c}+b\right|\right)}{2\,\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(2*(sqrt(-c)*x^2 - sqrt(-c*x^4 + b*x^2 + a))*sqrt(-c) + b))/sqr
t(-c)
```

maple [A] time = 0.01, size = 36, normalized size = 0.82

$$\frac{\arctan\left(\frac{\left(x^{2}-\frac{b}{2c}\right)\sqrt{c}}{\sqrt{-c\,x^{4}+b\,x^{2}+a}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2/c^(1/2)*arctan((x^2-1/2*b/c)/(-c*x^4+b*x^2+a)^(1/2)*c^(1/2))
```

maxima [A] time = 2.39, size = 28, normalized size = 0.64

$$-\frac{\arcsin\left(-\frac{2\,cx^{2}-b}{\sqrt{b^{2}+4\,ac}}\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*arcsin(-(2*c*x^2 - b)/sqrt(b^2 + 4*a*c))/sqrt(c)
```

mupad [B] time = 4.79, size = 40, normalized size = 0.91

$$\frac{\ln\left(\frac{\frac{b}{2}-c\,x^{2}}{\sqrt{-c}}+\sqrt{-c\,x^{4}+b\,x^{2}+a}\right)}{2\,\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2 - c*x^4)^(1/2),x)
```

```
[Out] log((b/2 - c*x^2)/(-c)^(1/2) + (a + b*x^2 - c*x^4)^(1/2))/(2*(-c)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx^{2}-cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(x/sqrt(a + b*x**2 - c*x**4), x)
```

3.972 $\int \frac{1}{x\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=47

$$-\frac{\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\,\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctan(1/2*(-b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2-a)^(1/2))/a^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1114, 724, 204}

$$-\frac{\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\,\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] -ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c*x^4])]/(2*Sqrt[a])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x\sqrt{-a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx+cx^2}}\,dx, x, x^2\right)$$
$$= -\,\text{Subst}\left(\int \frac{1}{-4a-x^2}\,dx, x, \frac{-2a+bx^2}{\sqrt{-a+bx^2+cx^4}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{-2a+bx^2}{2\sqrt{a}\,\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{bx^2-2a}{2\sqrt{a}\,\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-a + b*x^2 + c*x^4]),x]
[Out] ArcTan[(-2*a + b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c*x^4])]/(2*Sqrt[a])
```

fricas [A] time = 0.82, size = 129, normalized size = 2.74

$$\left[-\frac{\sqrt{-a}\,\log\left(\frac{\left(b^2-4\,ac\right)x^4-8\,abx^2-4\,\sqrt{cx^4+bx^2-a}\,\left(bx^2-2\,a\right)\sqrt{-a}+8\,a^2}{x^4}\right)}{4\,a}, \frac{\arctan\left(\frac{\sqrt{cx^4+bx^2-a}\,\left(bx^2-2\,a\right)\sqrt{a}}{2\left(acx^4+abx^2-a^2\right)}\right)}{2\,\sqrt{a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
[Out] [-1/4*sqrt(-a)*log(((b^2 - 4*a*c)*x^4 - 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 -
a)*(b*x^2 - 2*a)*sqrt(-a) + 8*a^2)/x^4)/a, 1/2*arctan(1/2*sqrt(c*x^4 + b*x^
2 - a)*(b*x^2 - 2*a)*sqrt(a)/(a*c*x^4 + a*b*x^2 - a^2))/sqrt(a)]
```

giac [A] time = 0.21, size = 36, normalized size = 0.77

$$\frac{\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))/sqrt(a))/sqrt(a)
```

maple [A] time = 0.02, size = 45, normalized size = 0.96

$$-\frac{\ln\left(\frac{b\,x^{2}-2a+2\sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}{x^{2}}\right)}{2\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] -1/2/(-a)^(1/2)*ln((-2*a+b*x^2+2*(-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)
```

maxima [A] time = 2.33, size = 36, normalized size = 0.77

$$-\frac{\arcsin\left(-\frac{b}{\sqrt{b^{2}+4\,a c}}+\frac{2\,a}{\sqrt{b^{2}+4\,a c}\,x^{2}}\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/sqrt(a)
```

mupad [B] time = 4.52, size = 52, normalized size = 1.11

$$-\frac{\ln\left(\frac{1}{x^{2}}\right)}{2\,\sqrt{-a}}-\frac{\ln\left(2\,\sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}\,-2\,a+b\,x^{2}\right)}{2\,\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 - a + c*x^4)^(1/2)),x)
```

```
[Out] - log(1/x^2)/(2*(-a)^(1/2)) - log(2*(-a)^(1/2)*(b*x^2 - a + c*x^4)^(1/2) -
2*a + b*x^2)/(2*(-a)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-a+bx^{2}+cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-a + b*x**2 + c*x**4)), x)
```

3.973 $\int \frac{1}{x^3\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}-\frac{b\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{4a^{3/2}}$$

```
[Out] -1/4*b*arctan(1/2*(-b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2-a)^(1/2))/a^(3/2)+1/2*(
c*x^4+b*x^2-a)^(1/2)/a/x^2
```

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1114, 730, 724, 204}

$$\frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}-\frac{b\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{4a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(2*a*x^2) - (b*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqr
t[-a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
```

```
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{-a+bx^2+cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{-a+bx+cx^2}}\,dx,x,x^2\right)\\
&= \frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}+\frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx+cx^2}}\,dx,x,x^2\right)}{4a}\\
&= \frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}-\frac{b\,\text{Subst}\left(\int \frac{1}{-4a-x^2}\,dx,x,\frac{-2a+bx^2}{\sqrt{-a+bx^2+cx^4}}\right)}{2a}\\
&= \frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}-\frac{b\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{4a^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 76, normalized size = 0.99

$$
\frac{b\tan^{-1}\left(\frac{bx^2-2a}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{4a^{3/2}}+\frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(2*a*x^2) + (b*ArcTan[(-2*a + b*x^2)/(2*Sqrt[a]*Sq
rt[-a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

fricas [A] time = 0.84, size = 188, normalized size = 2.44

$$
\left[-\frac{\sqrt{-a}\,bx^2\log\left(\frac{\left(b^2-4\,ac\right)x^4-8\,abx^2-4\,\sqrt{cx^4+bx^2-a}\left(bx^2-2\,a\right)\sqrt{-a}+8\,a^2}{x^4}\right)-4\,\sqrt{cx^4+bx^2-a}\,a}{8\,a^2x^2},\frac{\sqrt{a}\,bx^2\arctan\left(\frac{\sqrt{cx^4+bx^2-a}\left(bx^2-\right.}{2\left(acx^4+abx^2-\right.}\right.}{4\,a^2}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a)*b*x^2*log(((b^2 - 4*a*c)*x^4 - 8*a*b*x^2 - 4*sqrt(c*x^4 + b
*x^2 - a)*(b*x^2 - 2*a)*sqrt(-a) + 8*a^2)/x^4) - 4*sqrt(c*x^4 + b*x^2 - a)*
a)/(a^2*x^2), 1/4*(sqrt(a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 - a)*(b*x^2
- 2*a)*sqrt(a)/(a*c*x^4 + a*b*x^2 - a^2)) + 2*sqrt(c*x^4 + b*x^2 - a)*a)/(a
^2*x^2)]
```

giac [A] time = 0.22, size = 111, normalized size = 1.44

$$\frac{b \arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}}{\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}}-\frac{\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}\right)b-2\,a\sqrt{c}}{2\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}\right)^2+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*b*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))/sqrt(a))/a^(3/2) - 1/
2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))*b - 2*a*sqrt(c))/(((sqrt(c)*x^2
- sqrt(c*x^4 + b*x^2 - a))^2 + a)*a)
```

maple [A] time = 0.01, size = 74, normalized size = 0.96

$$-\frac{b \ln\left(\frac{b\,x^2-2a+2\sqrt{-a}\,\sqrt{c\,x^4+b\,x^2-a}}{x^2}\right)}{4\sqrt{-a}\,a}+\frac{\sqrt{c\,x^4+b\,x^2-a}}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] 1/2*(c*x^4+b*x^2-a)^(1/2)/a/x^2-1/4*b/a/(-a)^(1/2)*ln((b*x^2-2*a+2*(-a)^(1/
2)*(c*x^4+b*x^2-a)^(1/2))/x^2)
```

maxima [A] time = 2.41, size = 62, normalized size = 0.81

$$-\frac{b \arcsin\left(-\frac{b}{\sqrt{b^2+4\,ac}}+\frac{2\,a}{\sqrt{b^2+4\,ac}\,x^2}\right)}{4\,a^{\frac{3}{2}}}+\frac{\sqrt{cx^4+bx^2-a}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*b*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/a^(3/2) +
 1/2*sqrt(c*x^4 + b*x^2 - a)/(a*x^2)
```

mupad [B] time = 4.55, size = 64, normalized size = 0.83

$$\frac{\sqrt{c\,x^4+b\,x^2-a}}{2\,a\,x^2}-\frac{b\,\mathrm{atanh}\left(\frac{a-\frac{b\,x^2}{2}}{\sqrt{-a}\,\sqrt{c\,x^4+b\,x^2-a}}\right)}{4\,{\left(-a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^2 - a + c*x^4)^(1/2)),x)
```

```
[Out] (b*x^2 - a + c*x^4)^(1/2)/(2*a*x^2) - (b*atanh((a - (b*x^2)/2)/((-a)^(1/2)*
(b*x^2 - a + c*x^4)^(1/2))))/(4*(-a)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\sqrt{-a+bx^{2}+cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(-a + b*x**2 + c*x**4)), x)
```

3.974 $\int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=115

$$-\frac{(4ac+3b^2)\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2}+\frac{\sqrt{-a+bx^2+cx^4}}{4ax^4}$$

```
[Out] -1/16*(4*a*c+3*b^2)*arctan(1/2*(-b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2-a)^(1/2))/
a^(5/2)+1/4*(c*x^4+b*x^2-a)^(1/2)/a/x^4+3/8*b*(c*x^4+b*x^2-a)^(1/2)/a^2/x^2
```

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1114, 744, 806, 724, 204}

$$-\frac{(4ac+3b^2)\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2}+\frac{\sqrt{-a+bx^2+cx^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(4*a*x^4) + (3*b*Sqrt[-a + b*x^2 + c*x^4])/(8*a^2*
x^2) - ((3*b^2 + 4*a*c)*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c
*x^4])])/(16*a^(5/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
```

```
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{-a+bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{\sqrt{-a+bx^2+cx^4}}{4ax^4} + \frac{\text{Subst}\left(\int \frac{\frac{3b}{2}+cx}{x^2\sqrt{-a+bx+cx^2}}\,dx, x, x^2\right)}{4a}\\
&= \frac{\sqrt{-a+bx^2+cx^4}}{4ax^4} + \frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2} + \frac{\left(3b^2+4ac\right)\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx+cx^2}}\,dx, x, \right.}{16a^2}\\
&= \frac{\sqrt{-a+bx^2+cx^4}}{4ax^4} + \frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2} - \frac{\left(3b^2+4ac\right)\text{Subst}\left(\int \frac{1}{-4a-x^2}\,dx, x, \frac{-2a+}{\sqrt{-a+b}}\right.}{8a^2}\\
&= \frac{\sqrt{-a+bx^2+cx^4}}{4ax^4} + \frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2} - \frac{\left(3b^2+4ac\right)\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{16a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 95, normalized size = 0.83

$$\frac{\left(4ac+3b^2\right)\tan^{-1}\left(\frac{bx^2-2a}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{\left(2a+3bx^2\right)\sqrt{-a+bx^2+cx^4}}{8a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] ((2*a + 3*b*x^2)*Sqrt[-a + b*x^2 + c*x^4])/(8*a^2*x^4) + ((3*b^2 + 4*a*c)*A
rcTan[(-2*a + b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c*x^4])])/(16*a^(5/2))
```

fricas [A] time = 0.75, size = 230, normalized size = 2.00

$$\left[-\frac{\left(3\,b^2+4\,ac\right)\sqrt{-a}\,x^4\log\left(\frac{\left(b^2-4\,ac\right)x^4-8\,abx^2-4\,\sqrt{cx^4+bx^2-a}\left(bx^2-2\,a\right)\sqrt{-a}+8\,a^2}{x^4}\right)-4\,\sqrt{cx^4+bx^2-a}\left(3\,abx^2+2\,a^2\right)}{32\,a^3x^4}, \ (3\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2 + 4*a*c)*sqrt(-a)*x^4*log(((b^2 - 4*a*c)*x^4 - 8*a*b*x^2 - 4
*sqrt(c*x^4 + b*x^2 - a)*(b*x^2 - 2*a)*sqrt(-a) + 8*a^2)/x^4) - 4*sqrt(c*x^
4 + b*x^2 - a)*(3*a*b*x^2 + 2*a^2))/(a^3*x^4), 1/16*((3*b^2 + 4*a*c)*sqrt(a
)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2 - a)*(b*x^2 - 2*a)*sqrt(a)/(a*c*x^4 + a
*b*x^2 - a^2)) + 2*sqrt(c*x^4 + b*x^2 - a)*(3*a*b*x^2 + 2*a^2))/(a^3*x^4)]
```

giac [B] time = 0.23, size = 224, normalized size = 1.95

$$\frac{\left(3\,b^2+4\,ac\right)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}}{\sqrt{a}}\right)}{8\,a^{\frac{5}{2}}}-\frac{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}\right)^3b^2+4\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}\right)^3ac+5\,(\ldots}{8\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4\ldots}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2 + 4*a*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))/sqrt(a)
)/a^(5/2) - 1/8*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^3*b^2 + 4*(sqrt(
c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^3*a*c + 5*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x
^2 - a))*a*b^2 - 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))*a^2*c - 8*a^2*b*
sqrt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^2 + a)^2*a^2)
```

maple [A] time = 0.01, size = 149, normalized size = 1.30

$$-\frac{c\ln\left(\frac{b\,x^2-2a+2\sqrt{-a}\,\sqrt{c\,x^4+b\,x^2-a}}{x^2}\right)}{4\sqrt{-a}\,a}-\frac{3b^2\ln\left(\frac{b\,x^2-2a+2\sqrt{-a}\,\sqrt{c\,x^4+b\,x^2-a}}{x^2}\right)}{16\sqrt{-a}\,a^2}+\frac{3\sqrt{c\,x^4+b\,x^2-a}\,b}{8a^2x^2}+\frac{\sqrt{c\,x^4+b\,x^2-a}}{4a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] 1/4*(c*x^4+b*x^2-a)^(1/2)/a/x^4+3/8*b*(c*x^4+b*x^2-a)^(1/2)/a^2/x^2-3/16*b^
2/a^2/(-a)^(1/2)*ln((b*x^2-2*a+2*(-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)-1/4
*c/a/(-a)^(1/2)*ln((b*x^2-2*a+2*(-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)
```

maxima [A] time = 2.43, size = 126, normalized size = 1.10

$$-\frac{3\,b^2\arcsin\left(-\frac{b}{\sqrt{b^2+4\,ac}}+\frac{2\,a}{\sqrt{b^2+4\,ac}\,x^2}\right)}{16\,a^{\frac{5}{2}}}-\frac{c\arcsin\left(-\frac{b}{\sqrt{b^2+4\,ac}}+\frac{2\,a}{\sqrt{b^2+4\,ac}\,x^2}\right)}{4\,a^{\frac{3}{2}}}+\frac{3\sqrt{cx^4+bx^2-a}\,b}{8\,a^2x^2}+\frac{\sqrt{cx^4+bx^2-a}}{4\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/16*b^2*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/a^(5/2
) - 1/4*c*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/a^(3/2
) + 3/8*sqrt(c*x^4 + b*x^2 - a)*b/(a^2*x^2) + 1/4*sqrt(c*x^4 + b*x^2 - a)/(
a*x^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5\,\sqrt{c\,x^4+b\,x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(b*x^2 - a + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^5*(b*x^2 - a + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(-a + b*x**2 + c*x**4)), x)
```

3.975 $\int \frac{1}{x^7\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=154

$$-\frac{b\left(12ac+5b^2\right)\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{32a^{7/2}}+\frac{\left(16ac+15b^2\right)\sqrt{-a+bx^2+cx^4}}{48a^3x^2}+\frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4}+\frac{\sqrt{-a+bx^2+cx^4}}{6ax^6}$$

```
[Out] -1/32*b*(12*a*c+5*b^2)*arctan(1/2*(-b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2-a)^(1/2
))/a^(7/2)+1/6*(c*x^4+b*x^2-a)^(1/2)/a/x^6+5/24*b*(c*x^4+b*x^2-a)^(1/2)/a^2
/x^4+1/48*(16*a*c+15*b^2)*(c*x^4+b*x^2-a)^(1/2)/a^3/x^2
```

Rubi [A] time = 0.17, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1114, 744, 834, 806, 724, 204}

$$\frac{\left(16ac+15b^2\right)\sqrt{-a+bx^2+cx^4}}{48a^3x^2}-\frac{b\left(12ac+5b^2\right)\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{32a^{7/2}}+\frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4}+\frac{\sqrt{-a+bx^2+cx^4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(6*a*x^6) + (5*b*Sqrt[-a + b*x^2 + c*x^4])/(24*a^2
*x^4) + ((15*b^2 + 16*a*c)*Sqrt[-a + b*x^2 + c*x^4])/(48*a^3*x^2) - (b*(5*b
^2 + 12*a*c)*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c*x^4])])/(3
2*a^(7/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
```

```
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^7\sqrt{-a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{-a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{-a+bx^2+cx^4}}{6ax^6} + \frac{\text{Subst}\left(\int \frac{\frac{5b}{2}+2cx}{x^3\sqrt{-a+bx+cx^2}}\,dx, x, x^2\right)}{6a}$$

$$= \frac{\sqrt{-a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4} + \frac{\text{Subst}\left(\int \frac{\frac{1}{4}\left(15b^2+16ac\right)+\frac{5bcx}{2}}{x^2\sqrt{-a+bx+cx^2}}\,dx, x, x^2\right)}{12a^2}$$

$$= \frac{\sqrt{-a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4} + \frac{\left(15b^2+16ac\right)\sqrt{-a+bx^2+cx^4}}{48a^3x^2} + \frac{\left(b\left(5\right.\right.}{}$$

$$= \frac{\sqrt{-a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4} + \frac{\left(15b^2+16ac\right)\sqrt{-a+bx^2+cx^4}}{48a^3x^2} - \frac{\left(b\left(5\right.\right.}{}$$

$$= \frac{\sqrt{-a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4} + \frac{\left(15b^2+16ac\right)\sqrt{-a+bx^2+cx^4}}{48a^3x^2} - \frac{b\left(5\right.}{}$$

Mathematica [A] time = 0.08, size = 116, normalized size = 0.75

$$\frac{b\left(12ac+5b^2\right)\tan^{-1}\left(\frac{bx^2-2a}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{32a^{7/2}} + \frac{\sqrt{-a+bx^2+cx^4}\left(8a^2+2a\left(5bx^2+8cx^4\right)+15b^2x^4\right)}{48a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[-a + b*x^2 + c*x^4]*(8*a^2 + 15*b^2*x^4 + 2*a*(5*b*x^2 + 8*c*x^4)))/(
48*a^3*x^6) + (b*(5*b^2 + 12*a*c)*ArcTan[(-2*a + b*x^2)/(2*Sqrt[a]*Sqrt[-a
+ b*x^2 + c*x^4])])/(32*a^(7/2))
```

fricas [A] time = 0.66, size = 272, normalized size = 1.77

$$\left[-\frac{3\left(5\,b^3+12\,abc\right)\sqrt{-a}\,x^6\log\left(\frac{\left(b^2-4\,ac\right)x^4-8\,abx^2-4\,\sqrt{cx^4+bx^2-a}\left(bx^2-2\,a\right)\sqrt{-a}+8\,a^2}{x^4}\right)-4\left(10\,a^2bx^2+\left(15\,ab^2+16\,a^2c\right)x^4\right.}{192\,a^4x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3 + 12*a*b*c)*sqrt(-a)*x^6*log(((b^2 - 4*a*c)*x^4 - 8*a*b*x
^2 - 4*sqrt(c*x^4 + b*x^2 - a)*(b*x^2 - 2*a)*sqrt(-a) + 8*a^2)/x^4) - 4*(10
*a^2*b*x^2 + (15*a*b^2 + 16*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 - a))/(a
^4*x^6), 1/96*(3*(5*b^3 + 12*a*b*c)*sqrt(a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x
^2 - a)*(b*x^2 - 2*a)*sqrt(a)/(a*c*x^4 + a*b*x^2 - a^2)) + 2*(10*a^2*b*x^2
+ (15*a*b^2 + 16*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 - a))/(a^4*x^6)]
```

giac [B] time = 0.27, size = 344, normalized size = 2.23

$$\frac{\left(5\,b^3+12\,abc\right)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}}{\sqrt{a}}\right)}{16\,a^{\frac{7}{2}}}-\frac{15\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}\right)^5 b^3+36\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}\right)^5 abc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(5*b^3 + 12*a*b*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))/sqr
t(a))/a^(7/2) - 1/48*(15*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^5*b^3 + 36
*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^5*a*b*c + 40*(sqrt(c)*x^2 - sqrt(c
*x^4 + b*x^2 - a))^3*a*b^3 + 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^3*a
^2*b*c - 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^2*a^3*c^(3/2) + 33*(sqr
t(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))*a^2*b^3 - 36*(sqrt(c)*x^2 - sqrt(c*x^4
+ b*x^2 - a))*a^3*b*c - 48*a^3*b^2*sqrt(c) - 32*a^4*c^(3/2))/(((sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 - a))^2 + a)^3*a^3)
```

maple [A] time = 0.02, size = 202, normalized size = 1.31

$$-\frac{3bc\ln\left(\frac{b\,x^2-2a+2\sqrt{-a}\,\sqrt{c\,x^4+b\,x^2-a}}{x^2}\right)}{8\sqrt{-a}\,a^2}-\frac{5b^3\ln\left(\frac{b\,x^2-2a+2\sqrt{-a}\,\sqrt{c\,x^4+b\,x^2-a}}{x^2}\right)}{32\sqrt{-a}\,a^3}+\frac{\sqrt{c\,x^4+b\,x^2-a}\,c}{3a^2x^2}+\frac{5\sqrt{c\,x^4+b\,x^2-a}\,b^2}{16a^3x^2}+\frac{5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] 1/6*(c*x^4+b*x^2-a)^(1/2)/a/x^6+5/24*b*(c*x^4+b*x^2-a)^(1/2)/a^2/x^4+5/16*b
^2/a^3/x^2*(c*x^4+b*x^2-a)^(1/2)-5/32*b^3/a^3/(-a)^(1/2)*ln((b*x^2-2*a+2*(-
a)^(1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)-3/8*b/a^2*c/(-a)^(1/2)*ln((b*x^2-2*a+2
*(-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)+1/3*c/a^2/x^2*(c*x^4+b*x^2-a)^(1/2)
```

maxima [A] time = 2.26, size = 179, normalized size = 1.16

$$-\frac{5\,b^3\arcsin\left(-\frac{b}{\sqrt{b^2+4\,ac}}+\frac{2\,a}{\sqrt{b^2+4\,ac}\,x^2}\right)}{32\,a^{\frac{7}{2}}}-\frac{3\,bc\arcsin\left(-\frac{b}{\sqrt{b^2+4\,ac}}+\frac{2\,a}{\sqrt{b^2+4\,ac}\,x^2}\right)}{8\,a^{\frac{5}{2}}}+\frac{5\sqrt{cx^4+bx^2-a}\,b^2}{16\,a^3x^2}+\frac{\sqrt{cx^4+bx^2-}}{3\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/32*b^3*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/a^(7/2
) - 3/8*b*c*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/a^(5
/2) + 5/16*sqrt(c*x^4 + b*x^2 - a)*b^2/(a^3*x^2) + 1/3*sqrt(c*x^4 + b*x^2 -
 a)*c/(a^2*x^2) + 5/24*sqrt(c*x^4 + b*x^2 - a)*b/(a^2*x^4) + 1/6*sqrt(c*x^4
 + b*x^2 - a)/(a*x^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^7\,\sqrt{c\,x^4+b\,x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(b*x^2 - a + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^7*(b*x^2 - a + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\sqrt{-a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/(x**7*sqrt(-a + b*x**2 + c*x**4)), x)
```

3.976 $\int \frac{x^4}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=409

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(-b\sqrt{4ac+b^2}+ac+b^2\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}}-$$

```
[Out] -1/3*x*(-c*x^4+b*x^2+a)^(1/2)/c-1/6*b*EllipticE(x*2^(1/2)*c^(1/2)/(b+(4*a*c
+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(b-
(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(
1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/c^(5/2)*2^(1/2)/(-c*x^4
+b*x^2+a)^(1/2)+1/6*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2)
,((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(b^2+a*c-b*(4*a*c+b^2
)^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2
)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/c^(5/2)*2^(1/2)/(-c*x^4+b*x^2+a)^
(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1122, 1202, 524, 424, 419}

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(-b\sqrt{4ac+b^2}+ac+b^2\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -(x*Sqrt[a + b*x^2 - c*x^4])/(3*c) - (b*(b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sq
rt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*
x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(3*
Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(b^
2 + a*c - b*Sqrt[b^2 + 4*a*c])*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*
Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 +
 4*a*c])])/(3*Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x^4])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
```

```
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{x\sqrt{a+bx^2-cx^4}}{3c} + \frac{\int \frac{a+2bx^2}{\sqrt{a+bx^2-cx^4}}\,dx}{3c}$$

$$= -\frac{x\sqrt{a+bx^2-cx^4}}{3c} + \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{a+2bx^2}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{3c\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{x\sqrt{a+bx^2-cx^4}}{3c} - \frac{\left(b\left(b-\sqrt{b^2+4ac}\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}}{3c^2\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{x\sqrt{a+bx^2-cx^4}}{3c} - \frac{b\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}{3\sqrt{2}\,c^{5/2}\sqrt{a+bx^2-cx^4}}$$

Mathematica [C] time = 0.77, size = 459, normalized size = 1.12

$$\frac{i\sqrt{2}\left(b\sqrt{4ac+b^2}-ac-b^2\right)\sqrt{\frac{\sqrt{4ac+b^2}+b-2cx^2}{\sqrt{4ac+b^2}+b}}\sqrt{\frac{\sqrt{4ac+b^2}-b+2cx^2}{\sqrt{4ac+b^2}-b}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)-i\sqrt{2}}{6c^2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (2*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x*(-a - b*x^2 + c*x^4) - I*Sqrt[2]*
b*(-b + Sqrt[b^2 + 4*a*c])*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b + Sqrt
[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 + 4*
a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (
b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + I*Sqrt[2]*(-b^2 - a*c + b
*Sqrt[b^2 + 4*a*c])*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b + Sqrt[b^2 +
4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*
EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqr
t[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(6*c^2*Sqrt[-(c/(b + Sqrt[b^2 + 4
*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-cx^4+bx^2+a}\,x^4}{cx^4-bx^2-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 + b*x^2 + a)*x^4/(c*x^4 - b*x^2 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(-c*x^4 + b*x^2 + a), x)
```

maple [A] time = 0.05, size = 391, normalized size = 0.96

$$-\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}+4}\,\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{-\frac{2\left(b+\sqrt{4ac+b^2}\right)b}{ac}-4}}{2}\right)+\mathrm{Elliptic}\cdots\right.}{3\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4+b\,x^2+a}\,\left(b+\sqrt{4ac+b^2}\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/3*x*(-c*x^4+b*x^2+a)^(1/2)/c+1/12/c*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)
^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/3*b/c*a*2
^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^
(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*
a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1
/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(
4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(-c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2 - c*x^4)^(1/2),x)
```

```
[Out] int(x^4/(a + b*x^2 - c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(a + b*x**2 - c*x**4), x)
```

3.977 $\int \frac{x^2}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=377

$$\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}-\frac{\left(b-\sqrt{4\ldots}\right.}{\ldots}$$

```
[Out] -1/4*EllipticE(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2
)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-
(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+
b^2)^(1/2)))^(1/2)/c^(3/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)+1/4*EllipticF(x*2
^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c
+b^2)^(1/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2))
)^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)
/c^(3/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1140, 493, 424, 419}

$$\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}-\frac{\left(b-\sqrt{4\ldots}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b
 - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Elliptic
E[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 +
4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2)*Sqrt[a + b*x^2 - c*x^4
]) + ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2
)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Elli
pticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^
2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2)*Sqrt[a + b*x^2 - c
*x^4])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
```

```
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 1140

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/
(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[x^2/(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqr
t[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && NegQ[c/a]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{x^2}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{\sqrt{a+bx^2-cx^4}}$$

$$= \frac{\left(\left(b-\sqrt{b^2+4ac}\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{2c\sqrt{a+bx^2-cx^4}} - \frac{\Big(\Big(}{}$$

$$= -\frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4}}}\right.\right.}{2\sqrt{2}\,c^{3/2}\sqrt{a+bx^2-cx^4}}$$

Mathematica [C] time = 0.12, size = 271, normalized size = 0.72

$$-\frac{i\left(\sqrt{4ac+b^2}-b\right)\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}-b}+1}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)\middle|-\frac{b+\sqrt{b^2+4ac}}{\sqrt{b^2+4ac}-b}\right)-F\left(i\sinh\ldots\right.\right.}{2\sqrt{2}c\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] ((-1/2*I)*(-b + Sqrt[b^2 + 4*a*c])*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*
c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[
2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], -((b + Sqrt[b^2 + 4*a*c])/(-b + S
qrt[b^2 + 4*a*c]))] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 +
4*a*c]))]*x], -((b + Sqrt[b^2 + 4*a*c])/(-b + Sqrt[b^2 + 4*a*c]))]))/(Sqrt[
2]*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-cx^4+bx^2+a}\,x^2}{cx^4-bx^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 + b*x^2 + a)*x^2/(c*x^4 - b*x^2 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

maple [A] time = 0.01, size = 217, normalized size = 0.58

$$-\frac{\sqrt{2}\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}+4}\sqrt{\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}+4}\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}x}{2},\frac{\sqrt{-\frac{2\left(b+\sqrt{4ac+b^2}\right)b}{ac}-4}}{2}\right)+\mathrm{Elliptic}\ldots\right.}{2\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\sqrt{-c\,x^4+b\,x^2+a}\left(b+\sqrt{4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(4*a*c+b^2)^(1/2))/
a*x^2+4)^(1/2)*(2*(b+(4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(-c*x^4+b*x^2+a)^(1/
2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^
(1/2)*x,1/2*(-2*(b+(4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2^(1/2)
*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(-2*(b+(4*a*c+b^2)^(1/2))/a*b/c-4)^
(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{-c\,x^4 + b\,x^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2 - c*x^4)^(1/2),x)
```

```
[Out] int(x^2/(a + b*x^2 - c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a + bx^2 - cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*x**2 - c*x**4), x)
```

3.978 $\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=169

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{a+bx^2-cx^4}}$$

```
[Out] 1/2*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)
^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/
2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)*2^(1
/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1104, 419}

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sq
rt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4
*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1104

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b +
q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (
2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] &
```

```
& NegQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{\sqrt{a+bx^2-cx^4}}$$
$$= \frac{\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{a+bx^2-cx^4}}$$

Mathematica [C] time = 0.08, size = 177, normalized size = 1.05

$$-\frac{i\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}-b}+1}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\,x\right)|-\frac{b+\sqrt{b^2+4ac}}{\sqrt{b^2+4ac}-b}\right)}{\sqrt{2}\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
Sqrt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*
a*c]))]*x], -((b + Sqrt[b^2 + 4*a*c])/(-b + Sqrt[b^2 + 4*a*c]))])/(Sqrt[2]*
Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-cx^4+bx^2+a}}{cx^4-bx^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 + b*x^2 + a)/(c*x^4 - b*x^2 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-c*x^4 + b*x^2 + a), x)
```

maple [A] time = 0.01, size = 145, normalized size = 0.86

$$\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}+4}\,\operatorname{EllipticF}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{-\frac{2\left(b+\sqrt{4ac+b^2}\right)b}{ac}-4}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4+b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c*x^4+b*x^2+a)^(1/2),x)
[Out] 1/4*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(4*a*c+b^2)^(1/2))/a*x
^2+4)^(1/2)*(2*(b+(4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*
EllipticF(1/2*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(-2*(b+(4*a*c+
b^2)^(1/2))/a*b/c-4)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2 - c*x^4)^(1/2),x)
[Out] int(1/(a + b*x^2 - c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx^2 - cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*x**2 - c*x**4), x)
```

3.979 $\int \frac{1}{x^2\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=408

$$-\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}\,a\sqrt{c}\sqrt{a+bx^2-cx^4}}+\frac{\left(b-\right.}{}$$

```
[Out] -(-c*x^4+b*x^2+a)^(1/2)/a/x+1/4*EllipticE(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^
(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(b-(4*a*c
+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^
(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/a*2^(1/2)/c^(1/2)/(-c*x^4+b*x
^2+a)^(1/2)-1/4*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b
+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-
2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2
/(b+(4*a*c+b^2)^(1/2)))^(1/2)/a*2^(1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1123, 12, 1140, 493, 424, 419}

$$-\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}\,a\sqrt{c}\sqrt{a+bx^2-cx^4}}+\frac{\left(b-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] -(Sqrt[a + b*x^2 - c*x^4]/(a*x)) + ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b
^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)
/(b + Sqrt[b^2 + 4*a*c])]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt
[2]*a*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4]) - ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b +
Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*
c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(
2*Sqrt[2]*a*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1140

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/
(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[x^2/(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqr
t[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && NegQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx^2-cx^4}}\,dx = -\frac{\sqrt{a+bx^2-cx^4}}{ax} - \frac{\int \frac{cx^2}{\sqrt{a+bx^2-cx^4}}\,dx}{a}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{ax} - \frac{c\int \frac{x^2}{\sqrt{a+bx^2-cx^4}}\,dx}{a}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{ax} - \frac{\left(c\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{x^2}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}}{a\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{ax} - \frac{\left(\left(b-\sqrt{b^2+4ac}\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2}}}}}{2a\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{ax} + \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+}}}}{2\sqrt{2}\,a\sqrt{c}\sqrt{a+bx^2-cx^4}}$$

Mathematica [C] time = 0.43, size = 283, normalized size = 0.69

$$\frac{\frac{i\left(\sqrt{4ac+b^2}-b\right)\sqrt{\frac{4cx^2}{\sqrt{4ac+b^2}-b}+2}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)\middle|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)-F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)\middle|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)\right)}{\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}} -}{4a\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] ((-4*a)/x - 4*b*x + 4*c*x^3 + (I*(-b + Sqrt[b^2 + 4*a*c])*Sqrt[2 + (4*c*x^2
)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*(El
lipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[
b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-
(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*
a*c])]))/Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))])/(4*a*Sqrt[a + b*x^2 - c*x^4])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-cx^4+bx^2+a}}{cx^6-bx^4-ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 + b*x^2 + a)/(c*x^6 - b*x^4 - a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-cx^4 + bx^2 + a}\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^2), x)
```

maple [A] time = 0.01, size = 241, normalized size = 0.59

$$\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}+4}\,\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,x}{2}, \frac{\sqrt{-\frac{2\left(b+\sqrt{4ac+b^2}\right)b}{ac}-4}}{2}\right)+\mathrm{EllipticF}\left(\cdots\right)\right)}{2\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4+b\,x^2+a}\,\left(b+\sqrt{4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -(-c*x^4+b*x^2+a)^(1/2)/a/x+1/2*c*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*
(-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(4*a*c+b^2)^(1/2))/a*x^2+4)
^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*
((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(-2*(b+(4*a*c+b^2)^(1/2))/a*b/c-4)^(
1/2))-EllipticE(1/2*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(-2*(b+(
4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-cx^4 + bx^2 + a}\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2 - c*x^4)^(1/2)),x)
[Out] int(1/(x^2*(a + b*x^2 - c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(a + b*x**2 - c*x**4)), x)
```

3.980 $\int \frac{1}{x^4\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=445

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(-b\sqrt{4ac+b^2}+ac+b^2\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}a^2\sqrt{c}\sqrt{a+bx^2-cx^4}}-$$

```
[Out] -1/3*(-c*x^4+b*x^2+a)^(1/2)/a/x^3+2/3*b*(-c*x^4+b*x^2+a)^(1/2)/a^2/x-1/6*b*
EllipticE(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/
2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*
c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^
(1/2)))^(1/2)/a^2*2^(1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)+1/6*EllipticF(x*2^
(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+
b^2)^(1/2)))^(1/2))*(b^2+a*c-b*(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)
^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2))
)^(1/2)/a^2*2^(1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 445, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1123, 1281, 1202, 524, 424, 419}

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(-b\sqrt{4ac+b^2}+ac+b^2\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}a^2\sqrt{c}\sqrt{a+bx^2-cx^4}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2 - c*x^4]/(3*a*x^3) + (2*b*Sqrt[a + b*x^2 - c*x^4])/(3*a^2*x
) - (b*(b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^
2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Ell
ipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b
^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])]])/(3*Sqrt[2]*a^2*Sqrt[c]*Sqrt[a + b*x^
2 - c*x^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(b^2 + a*c - b*Sqrt[b^2 + 4*a*c]
)*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[
b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a
*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(3*Sqrt[2]*a^2*Sqr
t[c]*Sqrt[a + b*x^2 - c*x^4])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+bx^2-cx^4}}\,dx = -\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}+\frac{\int \frac{-2b+cx^2}{x^2\sqrt{a+bx^2-cx^4}}\,dx}{3a}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}+\frac{2b\sqrt{a+bx^2-cx^4}}{3a^2x}-\frac{\int \frac{-ac-2bcx^2}{\sqrt{a+bx^2-cx^4}}\,dx}{3a^2}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}+\frac{2b\sqrt{a+bx^2-cx^4}}{3a^2x}-\frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{\ }{\sqrt{1-\frac{\ }{b-}}}}{3a^2\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}+\frac{2b\sqrt{a+bx^2-cx^4}}{3a^2x}-\frac{\left(b\left(b-\sqrt{b^2+4ac}\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{\ }{b-}}\right.}{3a^2\sqrt{a+bx^2-cx}}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}+\frac{2b\sqrt{a+bx^2-cx^4}}{3a^2x}-\frac{b\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{\ }{b-}}}{3\sqrt{2}\,a}$$

Mathematica [C] time = 0.76, size = 472, normalized size = 1.06

$$\frac{-2\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\left(a-2bx^2\right)\left(a+bx^2-cx^4\right)+i\sqrt{2}\,x^3\left(b\sqrt{4ac+b^2}-ac-b^2\right)\sqrt{\frac{\sqrt{4ac+b^2}+b-2cx^2}{\sqrt{4ac+b^2}+b}}\sqrt{\frac{\sqrt{4ac+b^2}-b+2cx^2}{\sqrt{4ac+b^2}-b}}\,F}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] (-2*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*(a - 2*b*x^2)*(a + b*x^2 - c*x^4) -
I*Sqrt[2]*b*(-b + Sqrt[b^2 + 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*
x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b +
 Sqrt[b^2 + 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4
*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + I*Sqrt[2]*(
-b^2 - a*c + b*Sqrt[b^2 + 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2
)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sq
rt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*
c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(6*a^2*Sqrt[-(c
/(b + Sqrt[b^2 + 4*a*c]))]*x^3*Sqrt[a + b*x^2 - c*x^4])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-cx^4+bx^2+a}}{cx^8-bx^6-ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 + b*x^2 + a)/(c*x^8 - b*x^6 - a*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-cx^4+bx^2+a}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^4), x)
```

maple [A] time = 0.02, size = 417, normalized size = 0.94

$$-\frac{\sqrt{2}\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}+4}\,\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{-\frac{2\left(b+\sqrt{4ac+b^2}\right)b}{ac}-4}}{2}\right)+\mathrm{Elliptic}\cdots\right.}{3\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4+b\,x^2+a}\,\left(b+\sqrt{4ac+b^2}\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/3*(-c*x^4+b*x^2+a)^(1/2)/a/x^3+2/3*b*(-c*x^4+b*x^2+a)^(1/2)/a^2/x+1/12*c
/a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(4*a*c+b^2)^(1/2))/a*x^
2+4)^(1/2)*(2*(b+(4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*E
llipticF(1/2*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(-2*(b+(4*a*c+b
^2)^(1/2))/a*b/c-4)^(1/2))-1/3*b*c/a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/
2)*(-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(4*a*c+b^2)^(1/2))/a*x^2
+4)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/
2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(-2*(b+(4*a*c+b^2)^(1/2))/a*b/c-4
)^(1/2))-EllipticE(1/2*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(-2*(
b+(4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-cx^4 + bx^2 + a}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\sqrt{-c\,x^4 + b\,x^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2 - c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^2 - c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{a + bx^2 - cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a + b*x**2 - c*x**4)), x)
```

3.981 $\int \frac{x^9}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=190

$$\frac{3(5b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{7/2}}-\frac{\left(b(15b^2-52ac)-2cx^2(5b^2-12ac)\right)\sqrt{a+bx^2+cx^4}}{8c^3(b^2-4ac)}-\frac{bx^4\sqrt{a+bx^2+c}}{c(b^2-4ac)}$$

```
[Out] 3/16*(-4*a*c+5*b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/
c^(7/2)+x^6*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-b*x^4*(c*x^4+b*x
^2+a)^(1/2)/c/(-4*a*c+b^2)-1/8*(b*(-52*a*c+15*b^2)-2*c*(-12*a*c+5*b^2)*x^2)
*(c*x^4+b*x^2+a)^(1/2)/c^3/(-4*a*c+b^2)
```

Rubi [A] time = 0.24, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 738, 832, 779, 621, 206}

$$-\frac{\left(b(15b^2-52ac)-2cx^2(5b^2-12ac)\right)\sqrt{a+bx^2+cx^4}}{8c^3(b^2-4ac)}+\frac{3(5b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{7/2}}+\frac{x^6(2a+}{(b^2-4ac)\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^6*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*x^4*Sqrt[a
 + b*x^2 + c*x^4])/(c*(b^2 - 4*a*c)) - ((b*(15*b^2 - 52*a*c) - 2*c*(5*b^2 -
 12*a*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c^3*(b^2 - 4*a*c)) + (3*(5*b^2 -
4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(7
/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^9}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{x^2(6a+3bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{b^2-4ac}$$

$$= \frac{x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{bx^4\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} - \frac{\text{Subst}\left(\int \frac{x\left(-6ab-\frac{3}{2}\left(5b^2-12ac\right)x\right)}{\sqrt{a+bx+cx^2}}\,dx,\right.}{3c\left(b^2-4ac\right)}$$

$$= \frac{x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{bx^4\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} - \frac{\left(b\left(15b^2-52ac\right)-2c\left(5b^2-12a\right.\right.}{8c^3\left(b^2-4a\right.}$$

$$= \frac{x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{bx^4\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} - \frac{\left(b\left(15b^2-52ac\right)-2c\left(5b^2-12a\right.\right.}{8c^3\left(b^2-4a\right.}$$

$$= \frac{x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{bx^4\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} - \frac{\left(b\left(15b^2-52ac\right)-2c\left(5b^2-12a\right.\right.}{8c^3\left(b^2-4a\right.}$$

Mathematica [A] time = 0.19, size = 181, normalized size = 0.95

$$\frac{\frac{2\sqrt{c}\left(4a^2c\left(6cx^2-13b\right)+a\left(15b^3-62b^2cx^2-20bc^2x^4+8c^3x^6\right)+b^2x^2\left(15b^2+5bcx^2-2c^2x^4\right)\right)}{\sqrt{a+bx^2+cx^4}} - 3\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2}}\right.}{16c^{7/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(4*a^2*c*(-13*b + 6*c*x^2) + b^2*x^2*(15*b^2 + 5*b*c*x^2 - 2*c^
2*x^4) + a*(15*b^3 - 62*b^2*c*x^2 - 20*b*c^2*x^4 + 8*c^3*x^6)))/Sqrt[a + b*
x^2 + c*x^4] - 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(7/2)*(-b^2 + 4*a*c))
```

fricas [A] time = 1.04, size = 591, normalized size = 3.11

$$\left[-\frac{3\left(5ab^4-24a^2b^2c+16a^3c^2+\left(5b^4c-24ab^2c^2+16a^2c^3\right)x^4+\left(5b^5-24ab^3c+16a^2bc^2\right)x^2\right)\sqrt{c}\log\left(-8c^2x\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2 + (5*b^4*c - 24*a*b^2*c^2 +
16*a^2*c^3)*x^4 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^2)*sqrt(c)*log(-8*c
^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c)
- 4*a*c) - 4*(2*(b^2*c^3 - 4*a*c^4)*x^6 - 15*a*b^3*c + 52*a^2*b*c^2 - 5*(b^
3*c^2 - 4*a*b*c^3)*x^4 - (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x^2)*sqrt(c
*x^4 + b*x^2 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^4 + (b^3*
c^4 - 4*a*b*c^5)*x^2), -1/16*(3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2 + (5*b
^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)
*x^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c
^2*x^4 + b*c*x^2 + a*c)) - 2*(2*(b^2*c^3 - 4*a*c^4)*x^6 - 15*a*b^3*c + 52*a
^2*b*c^2 - 5*(b^3*c^2 - 4*a*b*c^3)*x^4 - (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*
c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*
c^6)*x^4 + (b^3*c^4 - 4*a*b*c^5)*x^2)]
```

giac [A] time = 0.27, size = 215, normalized size = 1.13

$$\frac{\left(\left(\frac{2\left(b^2c^2-4\,ac^3\right)x^2}{b^2c^3-4\,ac^4}-\frac{5\left(b^3c-4\,abc^2\right)}{b^2c^3-4\,ac^4}\right)x^2-\frac{15\,b^4-62\,ab^2c+24\,a^2c^2}{b^2c^3-4\,ac^4}\right)x^2-\frac{15\,ab^3-52\,a^2bc}{b^2c^3-4\,ac^4}}{8\sqrt{cx^4+bx^2+a}}-\frac{3\left(5\,b^2-4\,ac\right)\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+\right.\right.\right.}{16\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(((2*(b^2*c^2 - 4*a*c^3)*x^2/(b^2*c^3 - 4*a*c^4) - 5*(b^3*c - 4*a*b*c^2
)/(b^2*c^3 - 4*a*c^4))*x^2 - (15*b^4 - 62*a*b^2*c + 24*a^2*c^2)/(b^2*c^3 -
4*a*c^4))*x^2 - (15*a*b^3 - 52*a^2*b*c)/(b^2*c^3 - 4*a*c^4))/sqrt(c*x^4 + b
*x^2 + a) - 3/16*(5*b^2 - 4*a*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x
^2 + a))*sqrt(c) - b))/c^(7/2)
```

maple [B] time = 0.02, size = 354, normalized size = 1.86

$$\frac{x^6}{4\sqrt{c\,x^4+b\,x^2+a}\;c}-\frac{13a\,b^2x^2}{4\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}\;c^2}+\frac{15b^4x^2}{16\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}\;c^3}-\frac{5b\,x^4}{8\sqrt{c\,x^4+b\,x^2+a}\;c^2}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/4*x^6/c/(c*x^4+b*x^2+a)^(1/2)-5/8*b/c^2*x^4/(c*x^4+b*x^2+a)^(1/2)-15/16*b
^2/c^3*x^2/(c*x^4+b*x^2+a)^(1/2)+15/32*b^3/c^4/(c*x^4+b*x^2+a)^(1/2)+15/16*
```

```
b^4/c^3/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2+15/32*b^5/c^4/(4*a*c-b^2)/(c*
x^4+b*x^2+a)^(1/2)+15/16*b^2/c^(7/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+
a)^(1/2))-13/8*b/c^3*a/(c*x^4+b*x^2+a)^(1/2)-13/4*b^2/c^2*a/(4*a*c-b^2)/(c*
x^4+b*x^2+a)^(1/2)*x^2-13/8*b^3/c^3*a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)+3/4
/c^2*a*x^2/(c*x^4+b*x^2+a)^(1/2)-3/4/c^(5/2)*a*ln((c*x^2+1/2*b)/c^(1/2)+(c*
x^4+b*x^2+a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^9/(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**9/(a + b*x**2 + c*x**4)**(3/2), x)
```

3.982 $\int \frac{x^7}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{\left(-8ac+3b^2-2bcx^2\right)\sqrt{a+bx^2+cx^4}}{2c^2\left(b^2-4ac\right)}+\frac{x^4\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{3b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{5/2}}$$

```
[Out] -3/4*b*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(5/2)+x^4*(
b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)+1/2*(-2*b*c*x^2-8*a*c+3*b^2)*
(c*x^4+b*x^2+a)^(1/2)/c^2/(-4*a*c+b^2)
```

Rubi [A] time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 738, 779, 621, 206}

$$\frac{\left(-8ac+3b^2-2bcx^2\right)\sqrt{a+bx^2+cx^4}}{2c^2\left(b^2-4ac\right)}+\frac{x^4\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{3b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^4*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + ((3*b^2 - 8*a
*c - 2*b*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(2*c^2*(b^2 - 4*a*c)) - (3*b*ArcTa
nh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
```

```
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{x^4\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\operatorname{Subst}\left(\int \frac{x(4a+2bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{b^2-4ac}$$

$$= \frac{x^4\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} + \frac{\left(3b^2-8ac-2bcx^2\right)\sqrt{a+bx^2+cx^4}}{2c^2\left(b^2-4ac\right)} - \frac{(3b)\operatorname{Subst}\left(\int \right.}{}$$

$$= \frac{x^4\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} + \frac{\left(3b^2-8ac-2bcx^2\right)\sqrt{a+bx^2+cx^4}}{2c^2\left(b^2-4ac\right)} - \frac{(3b)\operatorname{Subst}\left(\int \right.}{}$$

$$= \frac{x^4\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} + \frac{\left(3b^2-8ac-2bcx^2\right)\sqrt{a+bx^2+cx^4}}{2c^2\left(b^2-4ac\right)} - \frac{3b\tanh^{-1}\left(\frac{}{2\sqrt{c}\ldots}\right.}{4c^{5/2}}$$

Mathematica [A] time = 0.12, size = 137, normalized size = 1.02

$$\frac{\frac{2\sqrt{c}\left(8a^2c+a\left(-3b^2+10bcx^2+4c^2x^4\right)-b^2x^2\left(3b+cx^2\right)\right)}{\sqrt{a+bx^2+cx^4}}+3b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{5/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(8*a^2*c - b^2*x^2*(3*b + c*x^2) + a*(-3*b^2 + 10*b*c*x^2 + 4*c
^2*x^4)))/Sqrt[a + b*x^2 + c*x^4] + 3*b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)
/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(5/2)*(-b^2 + 4*a*c))
```

fricas [A] time = 0.74, size = 459, normalized size = 3.43

$$\left[\frac{3\left(\left(b^3c-4\,abc^2\right)x^4+ab^3-4\,a^2bc+\left(b^4-4\,ab^2c\right)x^2\right)\sqrt{c}\,\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right.\right.}{8\left(ab^2c^3-4\,a^2c^4+\left(b^2c^4-4\,ac^5\right)x^4+\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*((b^3*c - 4*a*b*c^2)*x^4 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x^
2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*
c*x^2 + b)*sqrt(c) - 4*a*c) + 4*((b^2*c^2 - 4*a*c^3)*x^4 + 3*a*b^2*c - 8*a^
2*c^2 + (3*b^3*c - 10*a*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a*b^2*c^3 - 4
*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2), 1/4*(3*((b
^3*c - 4*a*b*c^2)*x^4 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x^2)*sqrt(-c)
*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x
^2 + a*c)) + 2*((b^2*c^2 - 4*a*c^3)*x^4 + 3*a*b^2*c - 8*a^2*c^2 + (3*b^3*c
- 10*a*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c
^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2)]
```

giac [A] time = 0.27, size = 154, normalized size = 1.15

$$\frac{\left(\frac{\left(b^2c-4\,ac^2\right)x^2}{b^2c^2-4\,ac^3}+\frac{3\,b^3-10\,abc}{b^2c^2-4\,ac^3}\right)x^2+\frac{3\,ab^2-8\,a^2c}{b^2c^2-4\,ac^3}}{2\,\sqrt{cx^4+bx^2+a}}+\frac{3\,b\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{4\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(((b^2*c - 4*a*c^2)*x^2/(b^2*c^2 - 4*a*c^3) + (3*b^3 - 10*a*b*c)/(b^2*c
^2 - 4*a*c^3))*x^2 + (3*a*b^2 - 8*a^2*c)/(b^2*c^2 - 4*a*c^3))/sqrt(c*x^4 +
b*x^2 + a) + 3/4*b*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(
c) - b))/c^(5/2)
```

maple [B] time = 0.02, size = 264, normalized size = 1.97

$$\frac{2ab\,x^2}{\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}\;c}-\frac{3b^3x^2}{4\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}\;c^2}+\frac{x^4}{2\sqrt{c\,x^4+b\,x^2+a}\;c}+\frac{a\,b^2}{\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/2*x^4/c/(c*x^4+b*x^2+a)^(1/2)+3/4*b/c^2*x^2/(c*x^4+b*x^2+a)^(1/2)-3/8*b^2
/c^3/(c*x^4+b*x^2+a)^(1/2)-3/4*b^3/c^2/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^
2-3/8*b^4/c^3/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)-3/4*b/c^(5/2)*ln((c*x^2+1/2
*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/c^2*a/(c*x^4+b*x^2+a)^(1/2)+2/c*a*b/(4
*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2+1/c^2*a*b^2/(4*a*c-b^2)/(c*x^4+b*x^2+a)
^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^7/(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**7/(a + b*x**2 + c*x**4)**(3/2), x)
```

3.983 $\int \frac{x^5}{(a+bx^2+cx^4)^{3/2}} \, dx$

Optimal. Leaf size=115

$$\frac{x^2(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{b\sqrt{a+bx^2+cx^4}}{c(b^2-4ac)} + \frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}}$$

```
[Out] 1/2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(3/2)+x^2*(b*x
^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-b*(c*x^4+b*x^2+a)^(1/2)/c/(-4*a*
c+b^2)
```

Rubi [A] time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 738, 640, 621, 206}

$$\frac{x^2(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{b\sqrt{a+bx^2+cx^4}}{c(b^2-4ac)} + \frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^2*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*Sqrt[a + b
*x^2 + c*x^4])/(c*(b^2 - 4*a*c)) + ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
+ b*x^2 + c*x^4])]/(2*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)\\
&= \frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\text{Subst}\left(\int \frac{2a+bx}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{b^2-4ac}\\
&= \frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{b\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2c}\\
&= \frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{b\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)}+\frac{\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{c}\\
&= \frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{b\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 107, normalized size = 0.93

$$\frac{\frac{2\sqrt{c}\left(a\left(b-2cx^2\right)+b^2x^2\right)}{\sqrt{a+bx^2+cx^4}}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(b^2*x^2 + a*(b - 2*c*x^2)))/Sqrt[a + b*x^2 + c*x^4] - (b^2 - 4
*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*c^(3/2
)*(-b^2 + 4*a*c))
```

fricas [A] time = 0.99, size = 387, normalized size = 3.37

$$\left[\frac{\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)\sqrt{c}\,\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}\right.}{4\left(ab^2c^2-4\,a^2c^3+\left(b^2c^3-4\,ac^4\right)x^4+\left(b^3c^2-4\,abc^3\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(
c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 +
b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^4 + b*x^2 + a)*(a*b*c + (b^2*c - 2*a*c^2)*
x^2))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c
^3)*x^2), -1/2*(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*
x^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^
2*x^4 + b*c*x^2 + a*c)) + 2*sqrt(c*x^4 + b*x^2 + a)*(a*b*c + (b^2*c - 2*a*c
^2)*x^2))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a
*b*c^3)*x^2)]
```

giac [A] time = 0.26, size = 101, normalized size = 0.88

$$-\frac{\frac{\left(b^2-2\,ac\right)x^2}{b^2c-4\,ac^2}+\frac{ab}{b^2c-4\,ac^2}}{\sqrt{cx^4+bx^2+a}}-\frac{\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{2\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -((b^2 - 2*a*c)*x^2/(b^2*c - 4*a*c^2) + a*b/(b^2*c - 4*a*c^2))/sqrt(c*x^4 +
 b*x^2 + a) - 1/2*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c
) - b))/c^(3/2)
```

maple [A] time = 0.02, size = 149, normalized size = 1.30

$$\frac{b^2x^2}{2\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}\;c}+\frac{b^3}{4\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}\;c^2}-\frac{x^2}{2\sqrt{c\,x^4+b\,x^2+a}\;c}+\frac{\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+}\right.}{2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/2*x^2/c/(c*x^4+b*x^2+a)^(1/2)+1/4*b/c^2/(c*x^4+b*x^2+a)^(1/2)+1/2*b^2/c/
(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2+1/4*b^3/c^2/(4*a*c-b^2)/(c*x^4+b*x^2+
a)^(1/2)+1/2/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 4.76, size = 84, normalized size = 0.73

$$\frac{\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{2\,c^{3/2}}+\frac{\frac{a\,b}{2}-x^2\,\left(a\,c-\frac{b^2}{2}\right)}{2\,c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^4+b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2))/(2*c^(3/2)) + ((a*b)
/2 - x^2*(a*c - b^2/2))/(2*c*(a*c - b^2/4)*(a + b*x^2 + c*x^4)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**5/(a + b*x**2 + c*x**4)**(3/2), x)
```

3.984 $\int \frac{x^3}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2a+bx^2}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1114, 636}

$$\frac{2a+bx^2}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*a + b*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)\\ &= \frac{2a+bx^2}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 36, normalized size = 1.00

$$\frac{2a+bx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*a + b*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

fricas [A] time = 0.95, size = 67, normalized size = 1.86

$$\frac{\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)}{(b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^
2*c + (b^3 - 4*a*b*c)*x^2)
```

giac [A] time = 0.28, size = 44, normalized size = 1.22

$$\frac{\frac{bx^2}{b^2-4\,ac}+\frac{2\,a}{b^2-4\,ac}}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (b*x^2/(b^2 - 4*a*c) + 2*a/(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a)
```

maple [A] time = 0.01, size = 38, normalized size = 1.06

$$-\frac{b\,x^2+2a}{\sqrt{c\,x^4+b\,x^2+a}\,\left(4ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -(b*x^2+2*a)/(c*x^4+b*x^2+a)^(1/2)/(4*a*c-b^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 4.47, size = 37, normalized size = 1.03

$$-\frac{b\,x^2+2\,a}{\left(4\,a\,c-b^2\right)\,\sqrt{c\,x^4+b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] -(2*a + b*x^2)/((4*a*c - b^2)*(a + b*x^2 + c*x^4)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**3/(a + b*x**2 + c*x**4)**(3/2), x)
```

3.985 $\int \frac{x}{\left(a+bx^2+cx^4\right)^{3/2}} \, dx$

Optimal. Leaf size=36

$$-\frac{b+2cx^2}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (-2*c*x^2-b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1107, 613}

$$-\frac{b+2cx^2}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((b + 2*c*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]))
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(a+bx^2+cx^4\right)^{3/2}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}} \, dx, x, x^2\right) \\
&= -\frac{b+2cx^2}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 1.03

$$\frac{b+2cx^2}{\left(4ac-b^2\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (b + 2*c*x^2)/((-b^2 + 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

fricas [A] time = 0.90, size = 67, normalized size = 1.86

$$-\frac{\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)}{\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a
^2*c + (b^3 - 4*a*b*c)*x^2)
```

giac [A] time = 0.20, size = 45, normalized size = 1.25

$$-\frac{\frac{2\,cx^2}{b^2-4\,ac}+\frac{b}{b^2-4\,ac}}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(2*c*x^2/(b^2 - 4*a*c) + b/(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a)
```

maple [A] time = 0.00, size = 36, normalized size = 1.00

$$\frac{2c\,x^2+b}{\sqrt{c\,x^4+b\,x^2+a}\,\left(4ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] (2*c*x^2+b)/(c*x^4+b*x^2+a)^(1/2)/(4*a*c-b^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 4.36, size = 35, normalized size = 0.97

$$\frac{2\,c\,x^2+b}{\left(4\,a\,c-b^2\right)\,\sqrt{c\,x^4+b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] (b + 2*c*x^2)/((4*a*c - b^2)*(a + b*x^2 + c*x^4)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x/(a + b*x**2 + c*x**4)**(3/2), x)
```

3.986 $\int \frac{1}{x\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\,\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(3/2)+(b*c*x^
2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 740, 12, 724, 206}

$$\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\,\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - ArcTanh
[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*a^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)\\
&= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\operatorname{Subst}\left(\int \frac{-\frac{b^2}{2}+2ac}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{a\left(b^2-4ac\right)}\\
&= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} + \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2a}\\
&= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{a}\\
&= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 89, normalized size = 1.00

$$\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - ArcTanh
[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*a^(3/2))
```

fricas [B] time = 1.03, size = 389, normalized size = 4.37

$$\left[\frac{\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)\sqrt{a}\,\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)+4\left(abcx^2\right.}{4\left(a^3b^2-4\,a^4c+\left(a^2b^2c-4\,a^3c^2\right)x^4+\left(a^2b^3-4\,a^3bc\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(
a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 +
 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*(a*b*c*x^2 + a*b^2 - 2*a^2*c)*sqrt(c*x^4 +
b*x^2 + a))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 - 4
*a^3*b*c)*x^2), 1/2*(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a
)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*(a*b*c*x^2 + a*b^2 - 2*a^2*c)*sqrt(c*x^4 +
 b*x^2 + a))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 -
4*a^3*b*c)*x^2)]
```

giac [A] time = 0.20, size = 110, normalized size = 1.24

$$\frac{\frac{abcx^2}{a^2b^2-4\,a^3c}+\frac{ab^2-2\,a^2c}{a^2b^2-4\,a^3c}}{\sqrt{cx^4+bx^2+a}}+\frac{\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (a*b*c*x^2/(a^2*b^2 - 4*a^3*c) + (a*b^2 - 2*a^2*c)/(a^2*b^2 - 4*a^3*c))/sqr
t(c*x^4 + b*x^2 + a) + arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt
(-a))/(sqrt(-a)*a)
```

maple [A] time = 0.01, size = 99, normalized size = 1.11

$$-\frac{\left(2c\,x^{2}+b\right)b}{2\left(4ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}\;a}-\frac{\ln\left(\frac{b\,x^{2}+2a+2\sqrt{c\,x^{4}+b\,x^{2}+a}\;\sqrt{a}}{x^{2}}\right)}{2a^{\frac{3}{2}}}+\frac{1}{2\sqrt{c\,x^{4}+b\,x^{2}+a}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/2/a/(c*x^4+b*x^2+a)^(1/2)-1/2*b/a*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)
^(1/2)-1/2/a^(3/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(c\,x^{4}+b\,x^{2}+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(1/(x*(a + b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^{2}+cx^{4}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.987 $\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=139

$$\frac{3b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}}-\frac{\left(3b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{2a^2x^2\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{ax^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] 3/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(5/2)+(b*c*x
^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^2/(c*x^4+b*x^2+a)^(1/2)-1/2*(-8*a*c+3*b^2)*(
c*x^4+b*x^2+a)^(1/2)/a^2/(-4*a*c+b^2)/x^2
```

Rubi [A] time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1114, 740, 806, 724, 206}

$$-\frac{\left(3b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{2a^2x^2\left(b^2-4ac\right)}+\frac{3b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}}+\frac{-2ac+b^2+bcx^2}{ax^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^2*Sqrt[a + b*x^2 + c*x^4]) - ((3
*b^2 - 8*a*c)*Sqrt[a + b*x^2 + c*x^4])/(2*a^2*(b^2 - 4*a*c)*x^2) + (3*b*Arc
Tanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^2\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}\left(-3b^2+8ac\right)-bcx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^2\sqrt{a+bx^2+cx^4}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{2a^2\left(b^2-4ac\right)x^2} - \frac{(3b)\,\text{Subst}\left(\int \frac{}{x\sqrt{a+}}\right.}{4a^2}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^2\sqrt{a+bx^2+cx^4}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{2a^2\left(b^2-4ac\right)x^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{4a-x}\right.}{2}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^2\sqrt{a+bx^2+cx^4}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{2a^2\left(b^2-4ac\right)x^2} + \frac{3b\tanh^{-1}\left(\frac{2a+}{2\sqrt{a}\sqrt{a+}}\right.}{4a^{5/2}}$$

Mathematica [A] time = 0.08, size = 137, normalized size = 0.99

$$\frac{\frac{2\sqrt{a}\left(-4a^2c+a\left(b^2-10bcx^2-8c^2x^4\right)+3b^2x^2\left(b+cx^2\right)\right)}{x^2\sqrt{a+bx^2+cx^4}} - 3b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] ((2*Sqrt[a]*(-4*a^2*c + 3*b^2*x^2*(b + c*x^2) + a*(b^2 - 10*b*c*x^2 - 8*c^2
*x^4)))/(x^2*Sqrt[a + b*x^2 + c*x^4]) - 3*b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*
x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(5/2)*(-b^2 + 4*a*c))
```

fricas [A] time = 0.99, size = 485, normalized size = 3.49

$$\left[\frac{3\left(\left(b^3c-4\,abc^2\right)x^6+\left(b^4-4\,ab^2c\right)x^4+\left(ab^3-4\,a^2bc\right)x^2\right)\sqrt{a}\,\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)-}{8\left(\left(a^3b^2c-4\,a^4c^2\right)x^6+\left(a^3b^3-4\,a^4bc\right)x^4+\left(a^4b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*((b^3*c - 4*a*b*c^2)*x^6 + (b^4 - 4*a*b^2*c)*x^4 + (a*b^3 - 4*a^2*b
*c)*x^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2
 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*((3*a*b^2*c - 8*a^2*c^2)*x^4
+ a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^2)*sqrt(c*x^4 + b*x^2 + a))/
((a^3*b^2*c - 4*a^4*c^2)*x^6 + (a^3*b^3 - 4*a^4*b*c)*x^4 + (a^4*b^2 - 4*a^5
*c)*x^2), -1/4*(3*((b^3*c - 4*a*b*c^2)*x^6 + (b^4 - 4*a*b^2*c)*x^4 + (a*b^3
 - 4*a^2*b*c)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a
)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((3*a*b^2*c - 8*a^2*c^2)*x^4 + a^
2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^
3*b^2*c - 4*a^4*c^2)*x^6 + (a^3*b^3 - 4*a^4*b*c)*x^4 + (a^4*b^2 - 4*a^5*c)*
x^2)]
```

giac [A] time = 0.28, size = 200, normalized size = 1.44

$$-\frac{\frac{\left(a^2b^2c-2\,a^3c^2\right)x^2}{a^4b^2-4\,a^5c}+\frac{a^2b^3-3\,a^3bc}{a^4b^2-4\,a^5c}}{\sqrt{cx^4+bx^2+a}}-\frac{3\,b\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,a^2}+\frac{\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)b+2\,a\sqrt{c}}{2\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -((a^2*b^2*c - 2*a^3*c^2)*x^2/(a^4*b^2 - 4*a^5*c) + (a^2*b^3 - 3*a^3*b*c)/(
a^4*b^2 - 4*a^5*c))/sqrt(c*x^4 + b*x^2 + a) - 3/2*b*arctan(-(sqrt(c)*x^2 -
sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + 1/2*((sqrt(c)*x^2 - sqr
t(c*x^4 + b*x^2 + a))*b + 2*a*sqrt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))^2 - a)*a^2)
```

maple [A] time = 0.02, size = 195, normalized size = 1.40

$$\frac{3b^2c\,x^2}{2\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}\;a^2}+\frac{3b^3}{4\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}\;a^2}-\frac{2\left(2c\,x^2+b\right)c}{\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}\;a}+\frac{3b\ln\left(\frac{b\,x^2+2a}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/2/a/x^2/(c*x^4+b*x^2+a)^(1/2)-3/4*b/a^2/(c*x^4+b*x^2+a)^(1/2)+3/2*b^2/a^
2/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2*c+3/4*b^3/a^2/(4*a*c-b^2)/(c*x^4+b*
x^2+a)^(1/2)+3/4*b/a^(5/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x
^2)-2*c/a*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(1/(x^3*(a + b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.988 $\int \frac{1}{x^5\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=195

$$-\frac{3\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{7/2}}+\frac{b\left(15b^2-52ac\right)\sqrt{a+bx^2+cx^4}}{8a^3x^2\left(b^2-4ac\right)}-\frac{\left(5b^2-12ac\right)\sqrt{a+bx^2+cx^4}}{4a^2x^4\left(b^2-4ac\right)}+\frac{}{ax^4\left(b\right.}$$

```
[Out] -3/16*(-4*a*c+5*b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))
/a^(7/2)+(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^4/(c*x^4+b*x^2+a)^(1/2)-1/4*(
-12*a*c+5*b^2)*(c*x^4+b*x^2+a)^(1/2)/a^2/(-4*a*c+b^2)/x^4+1/8*b*(-52*a*c+15
*b^2)*(c*x^4+b*x^2+a)^(1/2)/a^3/(-4*a*c+b^2)/x^2
```

Rubi [A] time = 0.21, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1114, 740, 834, 806, 724, 206}

$$\frac{b\left(15b^2-52ac\right)\sqrt{a+bx^2+cx^4}}{8a^3x^2\left(b^2-4ac\right)}-\frac{\left(5b^2-12ac\right)\sqrt{a+bx^2+cx^4}}{4a^2x^4\left(b^2-4ac\right)}-\frac{3\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{7/2}}+\frac{-}{ax^4\left(b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^4*Sqrt[a + b*x^2 + c*x^4]) - ((5
*b^2 - 12*a*c)*Sqrt[a + b*x^2 + c*x^4])/(4*a^2*(b^2 - 4*a*c)*x^4) + (b*(15*
b^2 - 52*a*c)*Sqrt[a + b*x^2 + c*x^4])/(8*a^3*(b^2 - 4*a*c)*x^2) - (3*(5*b^
2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*
a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{x^3\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^4\sqrt{a+bx^2+cx^4}} - \frac{\mathrm{Subst}\left(\int \frac{\frac{1}{2}\left(-5b^2+12ac\right)-2bcx}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^4\sqrt{a+bx^2+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^2+cx^4}}{4a^2\left(b^2-4ac\right)x^4} + \frac{\mathrm{Subst}\left(\int \frac{-\frac{1}{4}b(15}{}\right.}{2}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^4\sqrt{a+bx^2+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^2+cx^4}}{4a^2\left(b^2-4ac\right)x^4} + \frac{b\left(15b^2-52ac\right)}{8a^3\left(b^2-\right.}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^4\sqrt{a+bx^2+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^2+cx^4}}{4a^2\left(b^2-4ac\right)x^4} + \frac{b\left(15b^2-52ac\right)}{8a^3\left(b^2-\right.}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^4\sqrt{a+bx^2+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^2+cx^4}}{4a^2\left(b^2-4ac\right)x^4} + \frac{b\left(15b^2-52ac\right)}{8a^3\left(b^2-\right.}$$

Mathematica [A] time = 0.13, size = 179, normalized size = 0.92

$$\frac{3\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)+\frac{2\sqrt{a}\left(-8a^3c+2a^2\left(b^2+10bcx^2-12c^2x^4\right)+abx^2\left(-5b^2+62bcx^2+52c^2x^4\right)-15b^3x^4\left(\right.\right.}{x^4\sqrt{a+bx^2+cx^4}}}{16a^{7/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] ((2*Sqrt[a]*(-8*a^3*c - 15*b^3*x^4*(b + c*x^2) + 2*a^2*(b^2 + 10*b*c*x^2 -
12*c^2*x^4) + a*b*x^2*(-5*b^2 + 62*b*c*x^2 + 52*c^2*x^4)))/(x^4*Sqrt[a + b*
x^2 + c*x^4]) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*ArcTanh[(2*a + b*x^2)/(
2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(7/2)*(-b^2 + 4*a*c))
```

fricas [A] time = 1.40, size = 615, normalized size = 3.15

$$\left[-\frac{3\left(\left(5\,b^4c-24\,ab^2c^2+16\,a^2c^3\right)x^8+\left(5\,b^5-24\,ab^3c+16\,a^2bc^2\right)x^6+\left(5\,ab^4-24\,a^2b^2c+16\,a^3c^2\right)x^4\right)\sqrt{a}\,\log\left(-\right.}{32\left(\left(a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^8 + (5*b^5 - 24*a*b^3*c
+ 16*a^2*b*c^2)*x^6 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^4)*sqrt(a)*lo
g(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)
*sqrt(a) + 8*a^2)/x^4) - 4*((15*a*b^3*c - 52*a^2*b*c^2)*x^6 - 2*a^3*b^2 + 8
*a^4*c + (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*x^4 + 5*(a^2*b^3 - 4*a^3*b*
c)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^4*b^2*c - 4*a^5*c^2)*x^8 + (a^4*b^3 -
4*a^5*b*c)*x^6 + (a^5*b^2 - 4*a^6*c)*x^4), 1/16*(3*((5*b^4*c - 24*a*b^2*c^2
 + 16*a^2*c^3)*x^8 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^6 + (5*a*b^4 - 2
4*a^2*b^2*c + 16*a^3*c^2)*x^4)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*
(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((15*a*b^3*c - 52*a^2
*b*c^2)*x^6 - 2*a^3*b^2 + 8*a^4*c + (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*
x^4 + 5*(a^2*b^3 - 4*a^3*b*c)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^4*b^2*c - 4
*a^5*c^2)*x^8 + (a^4*b^3 - 4*a^5*b*c)*x^6 + (a^5*b^2 - 4*a^6*c)*x^4)]
```

giac [A] time = 0.38, size = 350, normalized size = 1.79

$$\frac{\frac{\left(a^3b^3c-3\,a^4bc^2\right)x^2}{a^6b^2-4\,a^7c}+\frac{a^3b^4-4\,a^4b^2c+2\,a^5c^2}{a^6b^2-4\,a^7c}}{\sqrt{cx^4+bx^2+a}}+\frac{3\left(5\,b^2-4\,ac\right)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}\,a^3}-\frac{7\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3b^2-4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((a^3*b^3*c - 3*a^4*b*c^2)*x^2/(a^6*b^2 - 4*a^7*c) + (a^3*b^4 - 4*a^4*b^2*c
 + 2*a^5*c^2)/(a^6*b^2 - 4*a^7*c))/sqrt(c*x^4 + b*x^2 + a) + 3/8*(5*b^2 - 4
*a*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a
^3) - 1/8*(7*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*b^2 - 4*(sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 + a))^3*a*c + 8*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a
))^2*a*b*sqrt(c) - 9*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a*b^2 - 4*(sqr
t(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^2*c - 16*a^2*b*sqrt(c))/(((sqrt(c)*x^
2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^2*a^3)
```

maple [A] time = 0.02, size = 314, normalized size = 1.61

$$\frac{13b\,c^2x^2}{2\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}\;a^2}-\frac{15b^3c\,x^2}{8\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}\;a^3}+\frac{13b^2c}{4\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}\;a^2}-\frac{}{16\left(4ac-b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/4/a/x^4/(c*x^4+b*x^2+a)^(1/2)+5/8*b/a^2/x^2/(c*x^4+b*x^2+a)^(1/2)+15/16*
b^2/a^3/(c*x^4+b*x^2+a)^(1/2)-15/8*b^3/a^3/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2
)*x^2*c-15/16*b^4/a^3/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)-15/16*b^2/a^(7/2)*l
n((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)+13/2*b/a^2*c^2/(4*a*c-b^
2)/(c*x^4+b*x^2+a)^(1/2)*x^2+13/4*b^2/a^2*c/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/
2)-3/4*c/a^2/(c*x^4+b*x^2+a)^(1/2)+3/4*c/a^(5/2)*ln((b*x^2+2*a+2*(c*x^4+b*x
^2+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(1/(x^5*(a + b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{5} \left(a + bx^{2} + cx^{4}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.989 $\int \frac{x^6}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=408

$$\frac{2x\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{c^{3/2}\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{\sqrt[4]{a}\left(\sqrt{a}\,b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\right.\right.}{2c^{7/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] x^3*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-b*x*(c*x^4+b*x^2+a)^(1/2
)/c/(-4*a*c+b^2)+2*(-3*a*c+b^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^(3/2)/(-4*a*c+b^2
)/(a^(1/2)+x^2*c^(1/2))-2*a^(1/4)*(-3*a*c+b^2)*(cos(2*arctan(c^(1/4)*x/a^(1
/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1
/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*
x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(-4*a*c+b^2)/(c*x^4+b*x
^2+a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-
b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*b^2-6*a*c+b*a^(1/2)*c^(1
/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(-4*a*c+b^2)/(
c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1120, 1279, 1197, 1103, 1195}

$$\frac{2x\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{c^{3/2}\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{\sqrt[4]{a}\left(\sqrt{a}\,b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\right.\right.}{2c^{7/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^3*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*x*Sqrt[a +
 b*x^2 + c*x^4])/(c*(b^2 - 4*a*c)) + (2*(b^2 - 3*a*c)*x*Sqrt[a + b*x^2 + c*
x^4])/(c^(3/2)*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)) - (2*a^(1/4)*(b^2 - 3
*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4
])/(c^(7/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(2*b^2 + Sqrt
[a]*b*Sqrt[c] - 6*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sq
rt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4])/(2*c^(7/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{x^3\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} + \frac{\int \frac{x^2\left(6a+3bx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx}{-b^2+4ac}$$

$$= \frac{x^3\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{bx\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} + \frac{\int \frac{3ab+6\left(b^2-3ac\right)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3c\left(b^2-4ac\right)}$$

$$= \frac{x^3\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{bx\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} + \frac{\left(\sqrt{a}\left(2b-3\sqrt{a}\sqrt{c}\right)\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}}{\left(b-2\sqrt{a}\sqrt{c}\right)c^{3/2}}$$

$$= \frac{x^3\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{bx\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} + \frac{2\left(b^2-3ac\right)x\sqrt{a+bx^2+cx^4}}{c^{3/2}\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{2}{}$$

Mathematica [C] time = 1.23, size = 489, normalized size = 1.20

$$\frac{2cx\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(a\left(b-2cx^2\right)+b^2x^2\right) - i\left(b^2-3ac\right)\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b^2*x^2 + a*(b - 2*c*x^2)) - I*(b^2
 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqr
t[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])
]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*a*b*c
+ b^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4
*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sq
rt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(2*
c^2*(-b^2 + 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\,x^6}{c^2x^8+2\,bcx^6+\left(b^2+2\,ac\right)x^4+2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*x^6/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x
^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/(c*x^4 + b*x^2 + a)^(3/2), x)
```

maple [A] time = 0.02, size = 482, normalized size = 1.18

$$-\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\;ab\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)}{4\left(4ac-b^2\right)\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\;c}-\frac{\left(\frac{1}{c}+\frac{2\cdots}{(4\cdots}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -2*c*(1/2/c^2*(2*a*c-b^2)/(4*a*c-b^2)*x^3-1/2*a*b/c^2/(4*a*c-b^2)*x)/((x^4+
b/c*x^2+1/c*a)*c)^(1/2)-1/4/(4*a*c-b^2)*a*b/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^
(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-1/
2*(1/c+(2*a*c-b^2)/c/(4*a*c-b^2))*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x
^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(
1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/
c-4)^(1/2))-EllipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(
2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x^6/(c*x^4 + b*x^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int(x^6/(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral(x**6/(a + b*x**2 + c*x**4)**(3/2), x)
```

3.990 $\int \frac{x^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=342

$$\frac{\sqrt[4]{a}\,b\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{x\left(2a\right.}{\left(b^2-4ac\right)\sqrt{a}}$$

```
[Out] x*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-b*x*(c*x^4+b*x^2+a)^(1/2)/
(-4*a*c+b^2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))+a^(1/4)*b*(cos(2*arctan(c^(1/4)*
x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arcta
n(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2)
)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(-4*a*c+b^2)/(c*x
^4+b*x^2+a)^(1/2)-1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/co
s(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1
/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(
1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(-2*a^(1/2)*c^(1/2)+b)/(c*x^4+b*x^2+a)^(
1/2)
```

Rubi [A] time = 0.13, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1120, 1197, 1103, 1195}

$$\frac{\sqrt[4]{a}\,b\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{x\left(2a\right.}{\left(b^2-4ac\right)\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*x*Sqrt[a + b
*x^2 + c*x^4])/(Sqrt[c]*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*b
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c
^(3/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (a^(1/4)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*(b - 2*Sqrt[a]*Sqr
t[c])*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{x\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} + \frac{\int \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{-b^2+4ac}$$

$$= \frac{x\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\sqrt{a}\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{c}} + \frac{\left(\sqrt{a}\,b\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}\left(b^2-4ac\right)}$$

$$= \frac{x\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{\sqrt[4]{a}\,b\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\cdots}}{c^{3/4}\left(\cdots\right.}$$

Mathematica [C] time = 0.84, size = 452, normalized size = 1.32

$$\frac{4cx\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(2a+bx^2\right)+i\left(b\sqrt{b^2-4ac}+4ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\cdots}\right.\right.}{4c\left(b^2-4ac\right)\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(2*a + b*x^2) - I*b*(-b + Sqrt[b^2 -
 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sq
rt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE
[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*
c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sq
rt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqr
t[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[
b^2 - 4*a*c])])/(4*c*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a +
 b*x^2 + c*x^4])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\,x^4}{c^2x^8+2\,bcx^6+\left(b^2+2\,ac\right)x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*x^4/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x
^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(c*x^4 + b*x^2 + a)^(3/2), x)
```

maple [A] time = 0.02, size = 450, normalized size = 1.32

$$-\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)+\mathrm{Ellipti}\right.}{2\left(4ac-b^2\right)\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,\left(b+\sqrt{-4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -2*c*(1/2*b/(4*a*c-b^2)/c*x^3+a/c/(4*a*c-b^2)*x)/((x^4+b/c*x^2+a/c)*c)^(1/2
)+1/2*a/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a
*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x
^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x
,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-1/2*b/(4*a*c-b^2)*a*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2
)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c
+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2
*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2^(1/2)*((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(c*x^4 + b*x^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^4/(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**4/(a + b*x**2 + c*x**4)**(3/2), x)
```

3.991 $\int \frac{x^2}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=341

$$\frac{2\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{(b^2-4ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{x\left(b+2cx^2\right)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{2\sqrt[4]{a}\,\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2\right.\right.}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

```
[Out] -x*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)+2*x*c^(1/2)*(c*x^4+b*x^2+
a)^(1/2)/(-4*a*c+b^2)/(a^(1/2)+x^2*c^(1/2))-2*a^(1/4)*c^(1/4)*(cos(2*arctan
(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(si
n(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^
2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/(-4*a*c+b^2)/(c*
x^4+b*x^2+a)^(1/2)+1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arc
tan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b
/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^
2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/(-2*a^(1/2)*c^(1/2)+b)/(c*x^4+b*x^2+a)^
(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1119, 1197, 1103, 1195}

$$\frac{2\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{(b^2-4ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{x\left(b+2cx^2\right)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{2\sqrt[4]{a}\,\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2\right.\right.}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((x*(b + 2*c*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])) + (2*Sqrt[c]*x
*Sqrt[a + b*x^2 + c*x^4])/((b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)) - (2*a^(1
/4)*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqr
t[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[
c]))/4])/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + ((Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*(b - 2*Sqrt[a]*
Sqrt[c])*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = -\frac{x\left(b+2cx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\int \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{-b^2+4ac}$$

$$= -\frac{x\left(b+2cx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\left(2\sqrt{a}\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{b^2-4ac} - \frac{\left(b+2\sqrt{a}\sqrt{c}\right)\int \frac{1}{\sqrt{a+bx^2+c}}}{-b^2+4ac}$$

$$= -\frac{x\left(b+2cx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} + \frac{2\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{}{(}}}{\left(b^2-\right.}$$

Mathematica [C] time = 0.78, size = 437, normalized size = 1.28

$$\frac{-2x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(b+2cx^2\right) - i\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b}{b}\right.}{2\left(b^2-4ac\right)\sqrt{\frac{c}{\sqrt{b^2-4ac}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b + 2*c*x^2) + I*(-b + Sqrt[b^2 - 4*
a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*
ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])
/(b - Sqrt[b^2 - 4*a*c])] - I*Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(b - Sqrt[b^2
- 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(2
*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\,x^2}{c^2x^8+2\,bcx^6+\left(b^2+2\,ac\right)x^4+2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*x^2/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x
^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(c*x^4 + b*x^2 + a)^(3/2), x)
```

maple [A] time = 0.01, size = 446, normalized size = 1.31

$$\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2}, \frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)+\mathrm{Ellipti}\right.}{\left(4ac-b^2\right)\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,\left(b+\sqrt{-4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -2*c*(-1/(4*a*c-b^2)*x^3-1/2*b/(4*a*c-b^2)/c*x)/((x^4+b/c*x^2+a/c)*c)^(1/2)
-1/4*b/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*
c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^
4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,
1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))+c/(4*a*c-b^2)*a*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*
(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)
^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(
b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-EllipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^4 + b*x^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^2/(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(a + bx^{2} + cx^{4}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**2/(a + b*x**2 + c*x**4)**(3/2), x)
```

3.992 $\int \frac{1}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=353

$$\frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{a^{3/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\cdots\right.\right.}{2a^{3/4}\left(b-2\sqrt{a}\,\sqrt{c}\right)\sqrt{a+bx^2\cdots}}$$

```
[Out] x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-b*x*c^(1/2)*(c*x
^4+b*x^2+a)^(1/2)/a/(-4*a*c+b^2)/(a^(1/2)+x^2*c^(1/2))+b*c^(1/4)*(cos(2*arc
tan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE
(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)
+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(-4*a
*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))
^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x
/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b
*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(-2*a^(1/2)*c^(1/2)+b)/(c*x^
4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1092, 1197, 1103, 1195}

$$\frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{a^{3/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\cdots\right.\right.}{2a^{3/4}\left(b-2\sqrt{a}\,\sqrt{c}\right)\sqrt{a+bx^2\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(-3/2),x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*
Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(a*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)
) + (b*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sq
rt[c]))/4])/(a^(3/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3
/4)*(b - 2*Sqrt[a]*Sqrt[c])*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{x\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\int \frac{2ac+bcx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\sqrt{c}\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}\left(b-2\sqrt{a}\sqrt{c}\right)} + \frac{\left(b\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}\left(b^2-4ac\right)}$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{b\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{a\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{\cdots}{(\cdots}}}{a^{3/4}\left(\cdots\right.}$$

Mathematica [C] time = 0.84, size = 456, normalized size = 1.29

$$-\frac{-4x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(-2ac+b^2+bcx^2\right)-i\left(b\sqrt{b^2-4ac}+4ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}F\left(i\sinh\cdots\right.}{4a\left(b^2-\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(-3/2),x]
```

```
[Out] -1/4*(-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b^2 - 2*a*c + b*c*x^2) + I*b*(-
b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*
c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + S
qrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[b^2 -
 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sq
rt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF
[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*
c])/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c
])]*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}}{c^2x^8+2\,bcx^6+\left(b^2+2\,ac\right)x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x^4 +
 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(-3/2), x)
```

maple [A] time = 0.01, size = 481, normalized size = 1.36

$$-\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2}, \frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)+\mathrm{Elliptic}\right.}{2\left(4ac-b^2\right)\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,\left(b+\sqrt{-4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -2*c*(1/2*b/a/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c*x)/((x^4+b/c*
x^2+a/c)*c)^(1/2)+1/4*(1/a-(2*a*c-b^2)/a/(4*a*c-b^2))*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a
*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(
1/2))-1/2*b/(4*a*c-b^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b
+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2
)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2)
)-EllipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a
*c+b^2)^(1/2))/a*b/c-4)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(1/(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^{2}+cx^{4}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(-3/2), x)
```

3.993 $\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=428

$$\frac{\sqrt[4]{c}\left(\sqrt{a}\,b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{7/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{2\sqrt[4]{c}\left(b^2-3ac\right)\left(\sqrt{a}+\cdots\right.}{a\cdots}$$

```
[Out] (b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x/(c*x^4+b*x^2+a)^(1/2)-2*(-3*a*c+b^2)*(
c*x^4+b*x^2+a)^(1/2)/a^2/(-4*a*c+b^2)/x+2*(-3*a*c+b^2)*x*c^(1/2)*(c*x^4+b*x
^2+a)^(1/2)/a^2/(-4*a*c+b^2)/(a^(1/2)+x^2*c^(1/2))-2*c^(1/4)*(-3*a*c+b^2)*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*
EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))
*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7
/4)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/
a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(
c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*
(2*b^2-6*a*c+b*a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(
1/2)/a^(7/4)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 428, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1121, 1281, 1197, 1103, 1195}

$$-\frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{a^2x\left(b^2-4ac\right)}+\frac{2\sqrt{c}\,x\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{a^2\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{a}\,b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+\cdots}{\left(\sqrt{a}\cdots\right.}}}{2a^{7/4}\left(b^2-4ac\right)\sqrt{a\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x*Sqrt[a + b*x^2 + c*x^4]) - (2*(b
^2 - 3*a*c)*Sqrt[a + b*x^2 + c*x^4])/(a^2*(b^2 - 4*a*c)*x) + (2*Sqrt[c]*(b^
2 - 3*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(a^2*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]
*x^2)) - (2*c^(1/4)*(b^2 - 3*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 +
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
 (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(7/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^
4]) + (c^(1/4)*(2*b^2 + Sqrt[a]*b*Sqrt[c] - 6*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(
1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(7/4)*(b^2 - 4*a*c)*Sq
rt[a + b*x^2 + c*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x\sqrt{a+bx^2+cx^4}} - \frac{\int \frac{-2\left(b^2-3ac\right)-bcx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x\sqrt{a+bx^2+cx^4}} - \frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{a^2\left(b^2-4ac\right)x} + \frac{\int \frac{abc+2c\left(b^2-3ac\right)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a^2\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x\sqrt{a+bx^2+cx^4}} - \frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{a^2\left(b^2-4ac\right)x} - \frac{\left(2\sqrt{c}\left(b^2-3ac\right)\right)\int}{a^{3/2}\left(b^2-\right.}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x\sqrt{a+bx^2+cx^4}} - \frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{a^2\left(b^2-4ac\right)x} + \frac{2\sqrt{c}\left(b^2-3ac\right)x\sqrt{}}{a^2\left(b^2-4ac\right)\left(\sqrt{}\right.}$$

Mathematica [C] time = 1.33, size = 515, normalized size = 1.20

$$-\frac{2\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(-4a^2c+a\left(b^2-7bcx^2-6c^2x^4\right)+2b^2x^2\left(b+cx^2\right)\right)-ix\left(b^2-3ac\right)\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2}{\sqrt{b^2-4ac}+b}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -1/2*(2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-4*a^2*c + 2*b^2*x^2*(b + c*x^2) +
 a*(b^2 - 7*b*c*x^2 - 6*c^2*x^4)) - I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c]
)*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2
*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*Ar
cSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(
b - Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 3*a*c
*Sqrt[b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c
])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sq
rt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(a^2*(b^2 - 4*a*c)*Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*x*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}}{c^2x^{10}+2\,bcx^8+\left(b^2+2\,ac\right)x^6+2\,abx^4+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)/(c^2*x^10 + 2*b*c*x^8 + (b^2 + 2*a*c)*x^6
+ 2*a*b*x^4 + a^2*x^2), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x^2), x)
```

maple [A]   time = 0.02, size = 536, normalized size = 1.25

$$-\frac{\left(\frac{\left(2ac-b^2\right)c}{\left(4ac-b^2\right)a^2} + \frac{c}{a^2}\right)\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\left(-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2}, \frac{\sqrt{\frac{2\left(b+\sqrt{-4ac\cdots}\right.}{ac}}}{2}\cdots\right.\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,\left(b+\sqrt{-4ac+b^2}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -2*c*(1/2*(2*a*c-b^2)/(4*a*c-b^2)/a^2*x^3+1/2*b*(3*a*c-b^2)/a^2/(4*a*c-b^2)
/c*x)/((x^4+b/c*x^2+a/c)*c)^(1/2)-1/a^2*(c*x^4+b*x^2+a)^(1/2)/x+1/4*(-1/a^2
*b+b*(3*a*c-b^2)/a^2/(4*a*c-b^2))*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
*(-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2
+4)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))-1/2*(c*(2*a*c-
b^2)/(4*a*c-b^2)/a^2+c/a^2)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2
*(-b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^
(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*2^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(-4*a*c+b^2)^(1/2))/a*b/c-4)^(
1/2))-EllipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+(
-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 \left(c\, x^4 + b\, x^2 + a\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.994 $\int \frac{x^4}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=50

$$\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}$$

```
[Out] -2/3*b*(c*x^4+b*x^2)^(1/2)/c^2/x+1/3*x*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A] time = 0.07, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3, 2016, 1588}

$$\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] (-2*b*Sqrt[b*x^2 + c*x^4])/(3*c^2*x) + (x*Sqrt[b*x^2 + c*x^4])/(3*c)
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{x\sqrt{bx^2+cx^4}}{3c} - \frac{(2b)\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{3c}$$
$$= -\frac{2b\sqrt{bx^2+cx^4}}{3c^2x} + \frac{x\sqrt{bx^2+cx^4}}{3c}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.68

$$\frac{(cx^2-2b)\sqrt{x^2(b+cx^2)}}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
[Out] ((-2*b + c*x^2)*Sqrt[x^2*(b + c*x^2)])/(3*c^2*x)
```

fricas [A] time = 0.74, size = 30, normalized size = 0.60

$$\frac{\sqrt{cx^4+bx^2}\,(cx^2-2\,b)}{3\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 - 2*b)/(c^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/sqrt(c*x^4 + b*x^2), x)
```

maple [A] time = 0.00, size = 37, normalized size = 0.74

$$-\frac{\left(c\,x^{2}+b\right)\left(-c\,x^{2}+2b\right)x}{3\sqrt{c\,x^{4}+b\,x^{2}}\;c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2)^(1/2),x)
[Out] -1/3*(c*x^2+b)*(-c*x^2+2*b)*x/c^2/(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.26, size = 34, normalized size = 0.68

$$\frac{c^{2}x^{4}-bcx^{2}-2\,b^{2}}{3\,\sqrt{cx^{2}+b}\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*(c^2*x^4 - b*c*x^2 - 2*b^2)/(sqrt(c*x^2 + b)*c^2)
```

mupad [B] time = 4.60, size = 33, normalized size = 0.66

$$-\frac{\sqrt{c\,x^{4}+b\,x^{2}}\;\left(\frac{2\,b}{3\,c^{2}}-\frac{x^{2}}{3\,c}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2 + c*x^4)^(1/2),x)
[Out] -((b*x^2 + c*x^4)^(1/2)*((2*b)/(3*c^2) - x^2/(3*c)))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{4}}{\sqrt{x^{2}\left(b+cx^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x**4/sqrt(x**2*(b + c*x**2)), x)
```

3.995 $\int \frac{x^3}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

```
[Out] -1/2*b*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(3/2)+1/2*(c*x^4+b*x^2)^(
1/2)/c
```

Rubi [A] time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3, 2018, 640, 620, 206}

$$\frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/
(2*c^(3/2))
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx &= \int \frac{x^3}{\sqrt{bx^2+cx^4}}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right) \\
&= \frac{\sqrt{bx^2+cx^4}}{2c} - \frac{b\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{4c} \\
&= \frac{\sqrt{bx^2+cx^4}}{2c} - \frac{b\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{2c} \\
&= \frac{\sqrt{bx^2+cx^4}}{2c} - \frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 73, normalized size = 1.26

$$\frac{x\left(\sqrt{c}\,x\left(b+cx^2\right)-b\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b+cx^2}}\right)\right)}{2c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2) - b*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b +
c*x^2]]))/(2*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

fricas [A] time = 0.72, size = 114, normalized size = 1.97

$$\left[\frac{b\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}\,c}{4\,c^2},\ \frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}\,c}{2\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*sqrt(
c*x^4 + b*x^2)*c)/c^2, 1/2*(b*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/
(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*c)/c^2]
```

giac [A] time = 0.20, size = 59, normalized size = 1.02

$$\frac{b\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}-b\right|\right)}{4\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^4+bx^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/c^(3/2)
+ 1/2*sqrt(c*x^4 + b*x^2)/c
```

maple [A] time = 0.01, size = 64, normalized size = 1.10

$$\frac{\sqrt{c\,x^2+b}\,\left(-bc\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)+\sqrt{c\,x^2+b}\,c^{\frac{3}{2}}x\right)x}{2\sqrt{c\,x^4+b\,x^2}\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(c*x^2+b)^(1/2)*(x*(c*x^2+b)^(1/2)*c^(3/2)-b*ln(c^(1/2)*x+(c*x^2+b)^(
1/2))*c)/(c*x^4+b*x^2)^(1/2)/c^(5/2)
```

maxima [A] time = 1.16, size = 52, normalized size = 0.90

$$-\frac{b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{4\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^4+bx^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(3/2) + 1/2*sqrt(
c*x^4 + b*x^2)/c
```

mupad [B] time = 4.61, size = 53, normalized size = 0.91

$$\frac{\sqrt{c\,x^4+b\,x^2}}{2\,c}-\frac{b\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{4\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (b*x^2 + c*x^4)^(1/2)/(2*c) - (b*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4
)^(1/2)))/(4*c^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(b + c*x**2)), x)
```

3.996 $\int \frac{x^2}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}} \, dx$

Optimal. Leaf size=22

$$\frac{\sqrt{bx^2+cx^4}}{cx}$$

```
[Out] (c*x^4+b*x^2)^(1/2)/c/x
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3, 1588}

$$\frac{\sqrt{bx^2+cx^4}}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/(c*x)
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}} \, dx &= \int \frac{x^2}{\sqrt{bx^2+cx^4}} \, dx\\ &= \frac{\sqrt{bx^2+cx^4}}{cx}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{\sqrt{x^2\left(b+cx^2\right)}}{cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
[Out] Sqrt[x^2*(b + c*x^2)]/(c*x)
```

fricas [A] time = 0.80, size = 20, normalized size = 0.91

$$\frac{\sqrt{cx^4+bx^2}}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x^4 + b*x^2)/(c*x)
```

giac [A] time = 0.18, size = 31, normalized size = 1.41

$$-\frac{2\sqrt{b}}{\left(\sqrt{c+\frac{b}{x^2}}-\frac{\sqrt{b}}{x}\right)^2-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] -2*sqrt(b)/((sqrt(c + b/x^2) - sqrt(b)/x)^2 - c)
```

maple [A] time = 0.00, size = 26, normalized size = 1.18

$$\frac{\left(c\,x^2+b\right)x}{\sqrt{c\,x^4+b\,x^2}\;c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2)^(1/2),x)
[Out] (c*x^2+b)/(c*x^4+b*x^2)^(1/2)/c*x
```

maxima [A] time = 1.22, size = 13, normalized size = 0.59

$$\frac{\sqrt{cx^2 + b}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2 + b)/c
```

mupad [B] time = 4.37, size = 20, normalized size = 0.91

$$\frac{\sqrt{c\,x^4 + b\,x^2}}{c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (b*x^2 + c*x^4)^(1/2)/(c*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x**2*(b + c*x**2)), x)
```

3.997 $\int \frac{x}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

```
[Out] arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3, 2013, 620, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]/Sqrt[c]
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
```

```
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx &= \int \frac{x}{\sqrt{bx^2+cx^4}}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\ &= \operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 52, normalized size = 1.68

$$\frac{x\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b+cx^2}}\right)}{\sqrt{c}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b + c*x^2]])/(Sqrt[c]*Sqrt[x^2*
(b + c*x^2)])
```

fricas [A] time = 0.78, size = 74, normalized size = 2.39

$$\left[\frac{\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{2\,\sqrt{c}}, -\frac{\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c), -sqrt(-c)*a
rctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b))/c]
```

giac [A] time = 0.19, size = 39, normalized size = 1.26

$$-\frac{\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}-b\right|\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/sqrt(c)
```

maple [A] time = 0.00, size = 44, normalized size = 1.42

$$\frac{\sqrt{c\,x^2+b}\;x\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+b}\right)}{\sqrt{c\,x^4+b\,x^2}\;\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x*(c*x^2+b)^(1/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))/c^(1/
2)
```

maxima [A] time = 1.06, size = 32, normalized size = 1.03

$$\frac{\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c)
```

mupad [B] time = 4.56, size = 33, normalized size = 1.06

$$\frac{\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2))/(2*c^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2 \left(b + cx^2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2*(b + c*x**2)), x)
```

3.998 $\int \frac{1}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=30

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

```
[Out] -arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3, 2008, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/Sqrt[b])
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{bx^2+cx^4}}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.01, size = 52, normalized size = 1.73

$$-\frac{x\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] -((x*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[x^2*(b
 + c*x^2)]))
```

fricas [A] time = 0.80, size = 80, normalized size = 2.67

$$\left[\frac{\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)}{2\,\sqrt{b}}, \frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3)/sqrt(b), sqr
t(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x))/b]
```

giac [A] time = 0.19, size = 46, normalized size = 1.53

$$-\frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} + \frac{\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(b)/sqrt(-b))*sgn(x)/sqrt(-b) + arctan(sqrt(c*x^2 + b)/sqrt(-b)
)/(sqrt(-b)*sgn(x))
```

maple [B] time = 0.00, size = 50, normalized size = 1.67

$$-\frac{\sqrt{c\,x^{2}+b}\;x\ln\left(\frac{2b+2\sqrt{c\,x^{2}+b}\;\sqrt{b}}{x}\right)}{\sqrt{c\,x^{4}+b\,x^{2}}\;\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/(c*x^4+b*x^2)^(1/2)*x*(c*x^2+b)^(1/2)/b^(1/2)*ln(2*(b+(c*x^2+b)^(1/2)*b^
(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^{4}+bx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^4 + b*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{c\,x^{4}+b\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(1/(b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^{2}+cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(1/sqrt(b*x**2 + c*x**4), x)
```

3.999 $\int \frac{1}{x\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=23

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

```
[Out] -(c*x^4+b*x^2)^(1/2)/b/x^2
```

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3, 2014}

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[b*x^2 + c*x^4]/(b*x^2))
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx &= \int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx\\ &= -\frac{\sqrt{bx^2+cx^4}}{bx^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
[Out] -(Sqrt[x^2*(b + c*x^2)]/(b*x^2))
```

fricas [A] time = 0.83, size = 21, normalized size = 0.91

$$-\frac{\sqrt{cx^4+bx^2}}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] -sqrt(c*x^4 + b*x^2)/(b*x^2)
```

giac [A] time = 0.18, size = 25, normalized size = 1.09

$$\frac{1}{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] 1/(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))
```

maple [A] time = 0.00, size = 26, normalized size = 1.13

$$-\frac{c\,x^2+b}{\sqrt{c\,x^4+b\,x^2}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2)^(1/2),x)
[Out] -(c*x^2+b)/b/(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.02, size = 21, normalized size = 0.91

$$-\frac{\sqrt{cx^4+bx^2}}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^4 + b*x^2)/(b*x^2)
```

mupad [B] time = 4.31, size = 21, normalized size = 0.91

$$-\frac{\sqrt{c\,x^4+b\,x^2}}{b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] -(b*x^2 + c*x^4)^(1/2)/(b*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**2*(b + c*x**2))), x)
```

3.1000 $\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=59

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

```
[Out] 1/2*c*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(3/2)-1/2*(c*x^4+b*x^2)^(1/2
)/b/x^3
```

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3, 2025, 2008, 206}

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(2*b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]
])/(2*b^(3/2))
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx &= \int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx\\ &= -\frac{\sqrt{bx^2+cx^4}}{2bx^3}-\frac{c\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{2b}\\ &= -\frac{\sqrt{bx^2+cx^4}}{2bx^3}+\frac{c\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{2b}\\ &= -\frac{\sqrt{bx^2+cx^4}}{2bx^3}+\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 68, normalized size = 1.15

$$\frac{c\sqrt{x^2\left(b+cx^2\right)}\left(\frac{\tanh^{-1}\left(\sqrt{\frac{cx^2}{b}+1}\right)}{2\sqrt{\frac{cx^2}{b}+1}}-\frac{b}{2cx^2}\right)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] (c*Sqrt[x^2*(b + c*x^2)]*(-1/2*b/(c*x^2) + ArcTanh[Sqrt[1 + (c*x^2)/b]]/(2*
Sqrt[1 + (c*x^2)/b])))/(b^2*x)
```

fricas [A] time = 0.90, size = 133, normalized size = 2.25

$$\left[\frac{\sqrt{b}\,cx^3\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)-2\,\sqrt{cx^4+bx^2}\,b}{4\,b^2x^3},-\frac{\sqrt{-b}\,cx^3\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}\,b}{2\,b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*c*x^3*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^
3) - 2*sqrt(c*x^4 + b*x^2)*b)/(b^2*x^3), -1/2*(sqrt(-b)*c*x^3*arctan(sqrt(c
*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*b)/(b^2*x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,
2,4]%%%}] at parameters values [64.3995612673,65,-85]Warning, choosing root
 of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,2,4]%%%}] at para
meters values [66.1769613782,93,91]-1/2/b/x*sqrt(b*(1/x)^2+c)-2*c/4/b/sqrt(
b)*ln(abs(sqrt(b*(1/x)^2+c)-sqrt(b)/x))
```

maple [A] time = 0.01, size = 73, normalized size = 1.24

$$-\frac{\sqrt{c\,x^{2}+b}\,\left(-b c\,x^{2} \ln \left(\frac{2 b+2 \sqrt{c\,x^{2}+b}\,\sqrt{b}}{x}\right)+\sqrt{c\,x^{2}+b}\,b^{\frac{3}{2}}\right)}{2 \sqrt{c\,x^{4}+b\,x^{2}}\,b^{\frac{5}{2}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/2/x*(c*x^2+b)^(1/2)*(-c*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^2*b+(c*x^2
+b)^(1/2)*b^(3/2))/(c*x^4+b*x^2)^(1/2)/b^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^{4}+bx^{2}}\,x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^2), x)
```

mupad [B] time = 4.64, size = 76, normalized size = 1.29

$$-\frac{\left(\frac{\sqrt{c}\,x^2\,\sqrt{c+\frac{b}{x^2}}}{2\,b}+\frac{c^{3/2}\,x^3\,\mathrm{asin}\left(\frac{\sqrt{b}\,1{}\mathrm{i}}{\sqrt{c}\,x}\right)\,1{}\mathrm{i}}{2\,b^{3/2}}\right)\,\sqrt{\frac{b}{c\,x^2}+1}}{x\,\sqrt{c\,x^4+b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] -(((c^(1/2)*x^2*(c + b/x^2)^(1/2))/(2*b) + (c^(3/2)*x^3*asin((b^(1/2)*1i)/(
c^(1/2)*x))*1i)/(2*b^(3/2)))*(b/(c*x^2) + 1)^(1/2))/(x*(b*x^2 + c*x^4)^(1/2
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(b + c*x**2))), x)
```

3.1001 $\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=52

$$\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}-\frac{\sqrt{bx^2+cx^4}}{3bx^4}$$

```
[Out] -1/3*(c*x^4+b*x^2)^(1/2)/b/x^4+2/3*c*(c*x^4+b*x^2)^(1/2)/b^2/x^2
```

Rubi [A] time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3, 2016, 2014}

$$\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}-\frac{\sqrt{bx^2+cx^4}}{3bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(3*b*x^4) + (2*c*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^2)
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx$$
$$= -\frac{\sqrt{bx^2+cx^4}}{3bx^4} - \frac{(2c)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{3b}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{3bx^4} + \frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.67

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(2cx^2-b\right)}{3b^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
[Out] (Sqrt[x^2*(b + c*x^2)]*(-b + 2*c*x^2))/(3*b^2*x^4)
```

fricas [A] time = 0.90, size = 31, normalized size = 0.60

$$\frac{\sqrt{cx^4+bx^2}\left(2\,cx^2-b\right)}{3\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(2*c*x^2 - b)/(b^2*x^4)
```

giac [A] time = 0.19, size = 57, normalized size = 1.10

$$\frac{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}\,+b}{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] 1/3*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) + b)/(sqrt(c)*x^2 - sqrt
(c*x^4 + b*x^2))^3
```

maple [A] time = 0.00, size = 37, normalized size = 0.71

$$-\frac{\left(c\,x^{2}+b\right)\left(-2c\,x^{2}+b\right)}{3\sqrt{c\,x^{4}+b\,x^{2}}\;b^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2)^(1/2),x)
[Out] -1/3*(c*x^2+b)*(-2*c*x^2+b)/x^2/b^2/(c*x^4+b*x^2)^(1/2)
```

maxima [A] time = 1.09, size = 44, normalized size = 0.85

$$\frac{2\,\sqrt{cx^{4}+bx^{2}}\,c}{3\,b^{2}x^{2}}-\frac{\sqrt{cx^{4}+bx^{2}}}{3\,bx^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/3*sqrt(c*x^4 + b*x^2)*c/(b^2*x^2) - 1/3*sqrt(c*x^4 + b*x^2)/(b*x^4)
```

mupad [B] time = 4.47, size = 29, normalized size = 0.56

$$-\frac{\left(b-2\,c\,x^{2}\right)\,\sqrt{c\,x^{4}+b\,x^{2}}}{3\,b^{2}\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^2 + c*x^4)^(1/2)),x)
[Out] -((b - 2*c*x^2)*(b*x^2 + c*x^4)^(1/2))/(3*b^2*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\sqrt{x^{2}\left(b+cx^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(x**2*(b + c*x**2))), x)
```

3.1002 $\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=87

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

```
[Out] -3/8*c^2*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(5/2)-1/4*(c*x^4+b*x^2)^(
1/2)/b/x^5+3/8*c*(c*x^4+b*x^2)^(1/2)/b^2/x^3
```

Rubi [A] time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3, 2025, 2008, 206}

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(4*b*x^5) + (3*c*Sqrt[b*x^2 + c*x^4])/(8*b^2*x^3) - (3
*c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(5/2))
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx &= \int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx\\ &= -\frac{\sqrt{bx^2+cx^4}}{4bx^5}-\frac{(3c)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{4b}\\ &= -\frac{\sqrt{bx^2+cx^4}}{4bx^5}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}+\frac{\left(3c^2\right)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b^2}\\ &= -\frac{\sqrt{bx^2+cx^4}}{4bx^5}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{\left(3c^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{8b^2}\\ &= -\frac{\sqrt{bx^2+cx^4}}{4bx^5}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 44, normalized size = 0.51

$$-\frac{c^2\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{cx^2}{b}+1\right)}{b^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -((c^2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (c*x^2)/b])
/(b^3*x))
```

fricas [A] time = 0.83, size = 163, normalized size = 1.87

$$\left[\frac{3\sqrt{b}\,c^2x^5\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\left(3\,bcx^2-2\,b^2\right)}{16\,b^3x^5},\frac{3\,\sqrt{-b}\,c^2x^5\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4}}{8\,b^3x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*c^2*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b
))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 - 2*b^2))/(b^3*x^5), 1/8*(3*sqrt
(-b)*c^2*x^5*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^
4 + b*x^2)*(3*b*c*x^2 - 2*b^2))/(b^3*x^5)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,
2,4]%%%}] at parameters values [64.3995612673,65,-85]Warning, choosing root
 of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%},0,%%%{1,[0,2,4]%%%}] at para
meters values [66.1769613782,93,91]2*(-2*b^2/16/b^3/x/x+3*b*c/16/b^3)/x*sqr
t(b*(1/x)^2+c)+6*c^2/16/b^2/sqrt(b)*ln(abs(sqrt(b*(1/x)^2+c)-sqrt(b)/x))
```

maple [A] time = 0.01, size = 94, normalized size = 1.08

$$-\frac{\sqrt{c\,x^{2}+b}\,\left(3 b\,c^{2} x^{4} \ln\left(\frac{2 b+2 \sqrt{c\,x^{2}+b}\,\sqrt{b}}{x}\right)-3 \sqrt{c\,x^{2}+b}\,b^{\frac{3}{2}} c\,x^{2}+2 \sqrt{c\,x^{2}+b}\,b^{\frac{5}{2}}\right)}{8 \sqrt{c\,x^{4}+b\,x^{2}}\,b^{\frac{7}{2}} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/8*(c*x^2+b)^(1/2)*(3*ln(2*(b+(c*x^2+b)^(1/2)*b^(1/2))/x)*x^4*b*c^2-3*(c*
x^2+b)^(1/2)*b^(3/2)*x^2*c+2*(c*x^2+b)^(1/2)*b^(5/2))/x^3/(c*x^4+b*x^2)^(1/
2)/b^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^2}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(b*x^2 + c*x^4)^(1/2)),x)
[Out] int(1/(x^4*(b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(1/(x**4*sqrt(x**2*(b + c*x**2))), x)
```

3.1003 $\int \frac{x^4}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=108

$$\frac{x\sqrt{a+cx^4}}{3c}-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}}$$

```
[Out] 1/3*x*(c*x^4+a)^(1/2)/c-1/6*a^(3/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1
/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/
4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)
^(1/2)/c^(5/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4, 321, 220}

$$\frac{x\sqrt{a+cx^4}}{3c}-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[a + c*x^4])/(3*c) - (a^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2]
)/(6*c^(5/4)*Sqrt[a + c*x^4])
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx &= \int \frac{x^4}{\sqrt{a+cx^4}}\,dx\\ &= \frac{x\sqrt{a+cx^4}}{3c}-\frac{a\int \frac{1}{\sqrt{a+cx^4}}\,dx}{3c}\\ &= \frac{x\sqrt{a+cx^4}}{3c}-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 62, normalized size = 0.57

$$\frac{x\left(-a\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+a+cx^4\right)}{3c\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] (x*(a + c*x^4 - a*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c
*x^4)/a)]))/(3*c*Sqrt[a + c*x^4])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\sqrt{cx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^4/sqrt(c*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/sqrt(c*x^4 + a), x)
```

maple [C] time = 0.05, size = 91, normalized size = 0.84

$$-\frac{\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,a\,\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,c}+\frac{\sqrt{c\,x^4+a}\,x}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+a)^(1/2),x)
[Out] 1/3*x*(c*x^4+a)^(1/2)/c-1/3*a/c/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*
(I/a^(1/2)*c^(1/2))^(1/2),I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/sqrt(c*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{c\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + c*x^4)^(1/2),x)
```

```
[Out] int(x^4/(a + c*x^4)^(1/2), x)
```

sympy [C] time = 1.90, size = 37, normalized size = 0.34

$$\frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+a)**(1/2),x)
```

```
[Out] x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt
(a)*gamma(9/4))
```

3.1004 $\int \frac{x^3}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}} dx$

Optimal. Leaf size=18

$$\frac{\sqrt{a+cx^4}}{2c}$$

[Out] 1/2*(c*x^4+a)^(1/2)/c

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {4, 261}

$$\frac{\sqrt{a+cx^4}}{2c}$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]

[Out] Sqrt[a + c*x^4]/(2*c)

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}} dx &= \int \frac{x^3}{\sqrt{a+cx^4}} dx\\ &= \frac{\sqrt{a+cx^4}}{2c}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\sqrt{a+cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] Sqrt[a + c*x^4]/(2*c)
```

fricas [A] time = 0.62, size = 14, normalized size = 0.78

$$\frac{\sqrt{cx^4 + a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(c*x^4 + a)/c
```

giac [A] time = 0.19, size = 14, normalized size = 0.78

$$\frac{\sqrt{cx^4 + a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^4 + a)/c
```

maple [A] time = 0.01, size = 15, normalized size = 0.83

$$\frac{\sqrt{c\,x^4 + a}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/2*(c*x^4+a)^(1/2)/c
```

maxima [A] time = 0.97, size = 14, normalized size = 0.78

$$\frac{\sqrt{cx^4 + a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^4 + a)/c
```

mupad [**B**] time = 4.66, size = 14, normalized size = 0.78

$$\frac{\sqrt{c\,x^4+a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + c*x^4)^(1/2),x)
[Out] (a + c*x^4)^(1/2)/(2*c)
```

sympy [**A**] time = 0.89, size = 22, normalized size = 1.22

$$\begin{cases} \frac{\sqrt{a+cx^4}}{2c} & \text{for } c \neq 0 \\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+a)**(1/2),x)
[Out] Piecewise((sqrt(a + c*x**4)/(2*c), Ne(c, 0)), (x**4/(4*sqrt(a)), True))
```

3.1005 $\int \frac{x^2}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=210

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{x}{\sqrt{c}}$$

```
[Out] x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*(cos(2*arctan(c^(1/
4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*ar
ctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(
1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)+1/2*a^(1/4)*(cos(2*arcta
n(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(s
in(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+
a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {4, 305, 220, 1196}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{x}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(Sqrt[a] +
 Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (a^(1/4)*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4])
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \int \frac{x^2}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{\sqrt{a}\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}} - \frac{\sqrt{a}\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}}$$

$$= \frac{x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{c^{3/4}\sqrt{a+cx^4}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.24

$$\frac{x^3\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)}{3\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

[Out] (x^3*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/(3*Sqrt[a + c*x^4])

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\sqrt{cx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^4+a)^(1/2),x, algorithm="fricas")

[Out] integral(x^2/sqrt(c*x^4 + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{cx^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate(x^2/sqrt(c*x^4 + a), x)

maple [C] time = 0.01, size = 97, normalized size = 0.46

$$\frac{i\sqrt{-\frac{i\sqrt{c}\, x^2}{\sqrt{a}}+1}\, \sqrt{\frac{i\sqrt{c}\, x^2}{\sqrt{a}}+1}\, \left(-\mathrm{EllipticE}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, x, i\right)+\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, x, i\right)\right)\sqrt{a}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, \sqrt{c\, x^4+a}\, \sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(c*x^4+a)^(1/2),x)

[Out] I*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)-EllipticE((I/a^(1/2)*c^(1/2))^(1/2)*x,I))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{cx^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(c*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{c\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + c*x^4)^(1/2),x)
```

```
[Out] int(x^2/(a + c*x^4)^(1/2), x)
```

sympy [C] time = 0.91, size = 37, normalized size = 0.18

$$\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+a)**(1/2),x)
```

```
[Out] x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt
(a)*gamma(7/4))
```

3.1006 $\int \frac{x}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=30

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

```
[Out] 1/2*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {4, 275, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]]/(2*Sqrt[c])
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx &= \int \frac{x}{\sqrt{a+cx^4}}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{a+cx^4}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]]/(2*Sqrt[c])
```

fricas [A] time = 0.83, size = 63, normalized size = 2.10

$$\left[\frac{\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\,\sqrt{c}\,x^2-a\right)}{4\,\sqrt{c}},-\frac{\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x^2}{\sqrt{cx^4+a}}\right)}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a)/sqrt(c), -1/2*sqrt(-
c)*arctan(sqrt(-c)*x^2/sqrt(c*x^4 + a))/c]
```

giac [A] time = 0.16, size = 25, normalized size = 0.83

$$-\frac{\log\left(\left|-\sqrt{c}\,x^2+\sqrt{cx^4+a}\right|\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-sqrt(c)*x^2 + sqrt(c*x^4 + a)))/sqrt(c)
```

maple [A] time = 0.01, size = 24, normalized size = 0.80

$$\frac{\ln \left(\sqrt{c}\, x^{2}+\sqrt{c\,x^{4}+a}\right)}{2 \sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/2*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2))/c^(1/2)
```

maxima [B] time = 2.43, size = 45, normalized size = 1.50

$$-\frac{\log\left(-\frac{\sqrt{c}-\frac{\sqrt{cx^4+a}}{x^2}}{\sqrt{c}+\frac{\sqrt{cx^4+a}}{x^2}}\right)}{4\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*log(-(sqrt(c) - sqrt(c*x^4 + a)/x^2)/(sqrt(c) + sqrt(c*x^4 + a)/x^2))/
sqrt(c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\sqrt{c\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + c*x^4)^(1/2),x)
```

```
[Out] int(x/(a + c*x^4)^(1/2), x)
```

sympy [A] time = 1.10, size = 20, normalized size = 0.67

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+a)**(1/2),x)
```

```
[Out] asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c))
```

3.1007 $\int \frac{1}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}}$$

```
[Out] 1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/
4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c
^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/(c*x^4+a)
^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {4, 220}

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]
)
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \int \frac{1}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}}$$

Mathematica [C] time = 0.03, size = 74, normalized size = 0.84

$$-\frac{i\sqrt{\frac{cx^4}{a}+1}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
[Out] ((-I)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x],
 -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*Sqrt[a + c*x^4])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{cx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(c*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(c*x^4 + a), x)
```

maple [C] time = 0.00, size = 70, normalized size = 0.80

$$\frac{\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\ \mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x, i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(
1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^4 + a), x)
```

mupad [B] time = 4.35, size = 37, normalized size = 0.42

$$\frac{x\,\sqrt{\frac{c\,x^4}{a}+1}\ {}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{\sqrt{c\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^4)^(1/2),x)
```

```
[Out] (x*((c*x^4)/a + 1)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(c*x^4)/a))/(a + c*x^4
)^(1/2)
```

sympy [C] time = 0.86, size = 36, normalized size = 0.41

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**(1/2),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)
*gamma(5/4))
```

3.1008 $\int \frac{1}{x\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=27

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctanh((c*x^4+a)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {4, 266, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]]/(2*Sqrt[a])
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx &= \int \frac{1}{x\sqrt{a+cx^4}}\,dx \\
&= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^4\right) \\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^4}\right)}{2c} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$
-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -1/2*ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]]/Sqrt[a]
```

fricas [A] time = 0.68, size = 63, normalized size = 2.33

$$
\left[\frac{\log\left(\frac{cx^4-2\,\sqrt{cx^4+a}\,\sqrt{a}+2\,a}{x^4}\right)}{4\,\sqrt{a}}, \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^4+a}\,\sqrt{-a}}{a}\right)}{2\,a}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log((c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(a) + 2*a)/x^4)/sqrt(a), 1/2*sqrt(-
a)*arctan(sqrt(c*x^4 + a)*sqrt(-a)/a)/a]
```

giac [A] time = 0.15, size = 23, normalized size = 0.85

$$\frac{\arctan\left(\frac{\sqrt{cx^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] 1/2*arctan(sqrt(c*x^4 + a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.01, size = 29, normalized size = 1.07

$$-\frac{\ln\left(\frac{2a+2\sqrt{c\,x^4+a}\,\sqrt{a}}{x^2}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+a)^(1/2),x)
[Out] -1/2/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^4+a)^(1/2))/x^2)
```

maxima [A] time = 2.29, size = 37, normalized size = 1.37

$$\frac{\log\left(\frac{\sqrt{cx^4+a}-\sqrt{a}}{\sqrt{cx^4+a}+\sqrt{a}}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] 1/4*log((sqrt(c*x^4 + a) - sqrt(a))/(sqrt(c*x^4 + a) + sqrt(a)))/sqrt(a)
```

mupad [B] time = 4.55, size = 19, normalized size = 0.70

$$-\frac{\mathrm{atanh}\left(\frac{\sqrt{c\,x^4+a}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^4)^(1/2)),x)
```

```
[Out] -atanh((a + c*x^4)^(1/2)/a^(1/2))/(2*a^(1/2))
```

sympy **[A]** time = 1.25, size = 22, normalized size = 0.81

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{c}\,x^2}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+a)**(1/2),x)
```

```
[Out] -asinh(sqrt(a)/(sqrt(c)*x**2))/(2*sqrt(a))
```

3.1009 $\int \frac{1}{x^2\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=232

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt{a\ldots}}{a\ldots}$$

```
[Out] -(c*x^4+a)^(1/2)/a/x+x*c^(1/2)*(c*x^4+a)^(1/2)/a/(a^(1/2)+x^2*c^(1/2))-c^(1
/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/
4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c
^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^4+a)^(1/2)+1
/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*
x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2
)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^4+a)^
(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4, 325, 305, 220, 1196}

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+cx^4}}+\frac{\sqrt{c}\ldots}{a\left(\sqrt{\ldots}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[a + c*x^4]/(a*x)) + (Sqrt[c]*x*Sqrt[a + c*x^4])/(a*(Sqrt[a] + Sqrt[c
]*x^2)) - (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a
+ c*x^4]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*Sqr
t[a + c*x^4])
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx &= \int \frac{1}{x^2\sqrt{a+cx^4}}\,dx \\
&= -\frac{\sqrt{a+cx^4}}{ax}+\frac{c\int \frac{x^2}{\sqrt{a+cx^4}}\,dx}{a} \\
&= -\frac{\sqrt{a+cx^4}}{ax}+\frac{\sqrt{c}\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\sqrt{a}}-\frac{\sqrt{c}\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{a}} \\
&= -\frac{\sqrt{a+cx^4}}{ax}+\frac{\sqrt{c}\,x\sqrt{a+cx^4}}{a\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E}{a^{3/4}\sqrt{a+cx^4}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.21

$$-\frac{\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{cx^4}{a}\right)}{x\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]

[Out] -((Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((c*x^4)/a)])/(x*Sqrt[a + c*x^4]))

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+a}}{cx^6+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^4+a)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^4 + a)/(c*x^6 + a*x^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(c*x^4 + a)*x^2), x)

maple [C] time = 0.01, size = 115, normalized size = 0.50

$$\frac{i\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\left(-\text{EllipticE}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)+\text{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)\right)\sqrt{c}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{a}}-\frac{\sqrt{c\,x^4+a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(c*x^4+a)^(1/2),x)

```
[Out] -(c*x^4+a)^(1/2)/a/x+I*c^(1/2)/a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2
)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*(Ell
ipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)-EllipticE((I/a^(1/2)*c^(1/2))^(1/2)*x
,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + a}\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*x^2), x)
```

mupad [B] time = 4.56, size = 40, normalized size = 0.17

$$-\frac{\sqrt{\frac{a}{c\,x^4}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{7}{4};\ -\frac{a}{c\,x^4}\right)}{3\,x\,\sqrt{c\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^4)^(1/2)),x)
```

```
[Out] -((a/(c*x^4) + 1)^(1/2)*hypergeom([1/2, 3/4], 7/4, -a/(c*x^4)))/(3*x*(a + c
*x^4)^(1/2))
```

sympy [C] time = 1.11, size = 39, normalized size = 0.17

$$\frac{\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{4} \end{matrix}\right| \frac{cx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\, x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+a)**(1/2),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)
*x*gamma(3/4))
```

3.1010 $\int \frac{1}{x^3\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=21

$$-\frac{\sqrt{a+cx^4}}{2ax^2}$$

```
[Out] -1/2*(c*x^4+a)^(1/2)/a/x^2
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {4, 264}

$$-\frac{\sqrt{a+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + c*x^4]/(2*a*x^2)
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx &= \int \frac{1}{x^3\sqrt{a+cx^4}}\,dx\\ &= -\frac{\sqrt{a+cx^4}}{2ax^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$-\frac{\sqrt{a+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
[Out] -1/2*Sqrt[a + c*x^4]/(a*x^2)
```

fricas [A] time = 0.79, size = 17, normalized size = 0.81

$$-\frac{\sqrt{cx^4 + a}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(c*x^4 + a)/(a*x^2)
```

giac [A] time = 0.17, size = 31, normalized size = 1.48

$$\frac{\sqrt{c}}{\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + a}\right)^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] sqrt(c)/((sqrt(c)*x^2 - sqrt(c*x^4 + a))^2 - a)
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\sqrt{c\,x^4 + a}}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+a)^(1/2),x)
[Out] -1/2*(c*x^4+a)^(1/2)/a/x^2
```

maxima [A] time = 1.08, size = 17, normalized size = 0.81

$$-\frac{\sqrt{cx^4 + a}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(c*x^4 + a)/(a*x^2)
```

mupad [B] time = 4.51, size = 17, normalized size = 0.81

$$-\frac{\sqrt{c\,x^4+a}}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + c*x^4)^(1/2)),x)
```

```
[Out] -(a + c*x^4)^(1/2)/(2*a*x^2)
```

sympy [A] time = 0.84, size = 20, normalized size = 0.95

$$-\frac{\sqrt{c}\sqrt{\frac{a}{cx^4}+1}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+a)**(1/2),x)
```

```
[Out] -sqrt(c)*sqrt(a/(c*x**4) + 1)/(2*a)
```

3.1011 $\int \frac{1}{x^4\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=110

$$-\frac{c^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+cx^4}}-\frac{\sqrt{a+cx^4}}{3ax^3}$$

```
[Out] -1/3*(c*x^4+a)^(1/2)/a/x^3-1/6*c^(3/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)
^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^
(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))
^2)^(1/2)/a^(5/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4, 325, 220}

$$-\frac{c^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+cx^4}}-\frac{\sqrt{a+cx^4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + c*x^4]/(3*a*x^3) - (c^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2
])/(6*a^(5/4)*Sqrt[a + c*x^4])
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \int \frac{1}{x^4\sqrt{a+cx^4}}\,dx$$
$$= -\frac{\sqrt{a+cx^4}}{3ax^3} - \frac{c\int \frac{1}{\sqrt{a+cx^4}}\,dx}{3a}$$
$$= -\frac{\sqrt{a+cx^4}}{3ax^3} - \frac{c^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+cx^4}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.46

$$-\frac{\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\frac{cx^4}{a}\right)}{3x^3\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -1/3*(Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((c*x^4)/a)])/
(x^3*Sqrt[a + c*x^4])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+a}}{cx^8+ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)/(c*x^8 + a*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + a}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*x^4), x)
```

maple [C] time = 0.01, size = 93, normalized size = 0.85

$$-\frac{\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,c\,\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x, i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,a}-\frac{\sqrt{c\,x^4+a}}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+a)^(1/2),x)
```

```
[Out] -1/3*(c*x^4+a)^(1/2)/a/x^3-1/3*c/a/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^
(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*Elliptic
F((I/a^(1/2)*c^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + a}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\sqrt{c\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^4*(a + c*x^4)^(1/2)), x)
```

sympy [C] time = 1.07, size = 41, normalized size = 0.37

$$\frac{\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+a)**(1/2),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)
*x**3*gamma(1/4))
```

3.1012 $\int \frac{x^4}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=73

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{3ax\sqrt{a+bx^2}}{8b^2}+\frac{x^3\sqrt{a+bx^2}}{4b}$$

```
[Out] 3/8*a^2*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)-3/8*a*x*(b*x^2+a)^(1/2)/
b^2+1/4*x^3*(b*x^2+a)^(1/2)/b
```

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {5, 321, 217, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{3ax\sqrt{a+bx^2}}{8b^2}+\frac{x^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] (-3*a*x*Sqrt[a + b*x^2])/(8*b^2) + (x^3*Sqrt[a + b*x^2])/(4*b) + (3*a^2*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \int \frac{x^4}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{x^3\sqrt{a+bx^2}}{4b} - \frac{(3a)\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{4b}$$
$$= -\frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b} + \frac{\left(3a^2\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8b^2}$$
$$= -\frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b} + \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{8b^2}$$
$$= -\frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b} + \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.85

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right) + \sqrt{b}\,x\sqrt{a+bx^2}\,\left(2bx^2-3a\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(-3*a + 2*b*x^2) + 3*a^2*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/(8*b^(5/2))
```

fricas [A] time = 0.75, size = 124, normalized size = 1.70

$$\left[\frac{3\,a^2\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(2\,b^2x^3-3\,abx\right)\sqrt{bx^2+a}}{16\,b^3}, -\frac{3\,a^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(2\,b^2x^3\right.}{8\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*b^2*x^3 - 3*a*b*x)*sqrt(b*x^2 + a))/b^3, -1/8*(3*a^2*sqrt(-b)*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) - (2*b^2*x^3 - 3*a*b*x)*sqrt(b*x^2 + a))/b^3]
```

giac [A] time = 0.29, size = 54, normalized size = 0.74

$$\frac{1}{8}\sqrt{bx^2+a}\,x\left(\frac{2\,x^2}{b}-\frac{3\,a}{b^2}\right)-\frac{3\,a^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*x*(2*x^2/b - 3*a/b^2) - 3/8*a^2*log(abs(-sqrt(b)*x + sq
rt(b*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 59, normalized size = 0.81

$$\frac{\sqrt{b\,x^2+a}\;x^3}{4b}+\frac{3a^2\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{5}{2}}}-\frac{3\sqrt{b\,x^2+a}\;ax}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*x^3*(b*x^2+a)^(1/2)/b-3/8*a*x*(b*x^2+a)^(1/2)/b^2+3/8*a^2/b^(5/2)*ln(b^
(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 0.97, size = 51, normalized size = 0.70

$$\frac{\sqrt{bx^2+a}\,x^3}{4\,b}-\frac{3\,\sqrt{bx^2+a}\,ax}{8\,b^2}+\frac{3\,a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(b*x^2 + a)*x^3/b - 3/8*sqrt(b*x^2 + a)*a*x/b^2 + 3/8*a^2*arcsinh(b
*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^4/(a + b*x^2)^(1/2), x)
```

sympy [A] time = 4.61, size = 95, normalized size = 1.30

$$-\frac{3a^{\frac{3}{2}}x}{8b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}\,x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{x^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*x/(8*b**2*sqrt(1 + b*x**2/a)) - sqrt(a)*x**3/(8*b*sqrt(1 + b*x*
*2/a)) + 3*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) + x**5/(4*sqrt(a)*sqr
t(1 + b*x**2/a))
```

3.1013 $\int \frac{x^3}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}} \, dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^2\right)^{3/2}}{3b^2}-\frac{a\sqrt{a+bx^2}}{b^2}$$

```
[Out] 1/3*(b*x^2+a)^(3/2)/b^2-a*(b*x^2+a)^(1/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {5, 266, 43}

$$\frac{\left(a+bx^2\right)^{3/2}}{3b^2}-\frac{a\sqrt{a+bx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] -((a*Sqrt[a + b*x^2])/b^2) + (a + b*x^2)^(3/2)/(3*b^2)
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \int \frac{x^3}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{a}{b\sqrt{a+bx}} + \frac{\sqrt{a+bx}}{b}\right)dx, x, x^2\right)$$
$$= -\frac{a\sqrt{a+bx^2}}{b^2} + \frac{\left(a+bx^2\right)^{3/2}}{3b^2}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{\left(bx^2-2a\right)\sqrt{a+bx^2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] ((-2*a + b*x^2)*Sqrt[a + b*x^2])/(3*b^2)
```

fricas [A] time = 0.76, size = 23, normalized size = 0.64

$$\frac{\sqrt{bx^2+a}\left(bx^2-2\,a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(b*x^2 + a)*(b*x^2 - 2*a)/b^2
```

giac [A] time = 0.15, size = 30, normalized size = 0.83

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{3\,b^2} - \frac{\sqrt{bx^2+a}\,a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] 1/3*(b*x^2 + a)^(3/2)/b^2 - sqrt(b*x^2 + a)*a/b^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.69

$$-\frac{\sqrt{b\,x^{2}+a}\,\left(-b\,x^{2}+2a\right)}{3b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(1/2),x)
[Out] -1/3*(b*x^2+a)^(1/2)*(-b*x^2+2*a)/b^2
```

maxima [A] time = 1.02, size = 33, normalized size = 0.92

$$\frac{\sqrt{bx^{2}+a}\,x^{2}}{3\,b}-\frac{2\,\sqrt{bx^{2}+a}\,a}{3\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(b*x^2 + a)*x^2/b - 2/3*sqrt(b*x^2 + a)*a/b^2
```

mupad [B] time = 4.60, size = 24, normalized size = 0.67

$$-\frac{\sqrt{b\,x^{2}+a}\,\left(2\,a-b\,x^{2}\right)}{3\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(1/2),x)
[Out] -((a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2)
```

sympy [A] time = 0.55, size = 44, normalized size = 1.22

$$\begin{cases} -\frac{2a\sqrt{a+bx^{2}}}{3b^{2}}+\frac{x^{2}\sqrt{a+bx^{2}}}{3b} & \text{for}\ b\neq 0\\ \frac{x^{4}}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(1/2),x)
[Out] Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)/(3*b), Ne
(b, 0)), (x**4/(4*sqrt(a)), True))
```

3.1014 $\int \frac{x^2}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=49

$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

```
[Out] -1/2*a*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/2*x*(b*x^2+a)^(1/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {5, 321, 217, 206}

$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(
3/2))
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{x^2}{\sqrt{a+bx^2}}\,dx\\ &= \frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}\\ &= \frac{x\sqrt{a+bx^2}}{2b}-\frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b}\\ &= \frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.00

$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(
3/2))
```

fricas [A] time = 0.91, size = 93, normalized size = 1.90

$$\left[\frac{2\sqrt{bx^2+a}\,bx+a\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{4\,b^2},\ \frac{\sqrt{bx^2+a}\,bx+a\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*x + a*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a))/b^2, 1/2*(sqrt(b*x^2 + a)*b*x + a*sqrt(-b)*arctan(sqrt(-b)*x/
sqrt(b*x^2 + a)))/b^2]
```

giac [A] time = 0.18, size = 40, normalized size = 0.82

$$\frac{\sqrt{bx^2+a}\,x}{2\,b}+\frac{a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x/b + 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3
/2)
```

maple [A] time = 0.01, size = 39, normalized size = 0.80

$$-\frac{a\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}+\frac{\sqrt{b\,x^2+a}\;x}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.02, size = 31, normalized size = 0.63

$$\frac{\sqrt{bx^2+a}\,x}{2\,b}-\frac{a\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x/b - 1/2*a*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

mupad [B] time = 4.64, size = 56, normalized size = 1.14

$$\begin{cases} \frac{x^3}{3\,\sqrt{a}} & \text{if } b=0 \\ \frac{x\,\sqrt{b\,x^2+a}}{2\,b}-\frac{a\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}} & \text{if } b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(1/2),x)
```

```
[Out] piecewise(b == 0, x^3/(3*a^(1/2)), b ~= 0, (x*(a + b*x^2)^(1/2))/(2*b) - (a
*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)))
```

sympy **[A]** time = 2.91, size = 42, normalized size = 0.86

$$\frac{\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2)
)
```

3.1015 $\int \frac{x}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=15

$$\frac{\sqrt{a+bx^2}}{b}$$

[Out] (b*x^2+a)^(1/2)/b

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {5, 261}

$$\frac{\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]

[Out] Sqrt[a + b*x^2]/b

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{x}{\sqrt{a+bx^2}}\,dx\\ &= \frac{\sqrt{a+bx^2}}{b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] Sqrt[a + b*x^2]/b
```

fricas [A] time = 0.80, size = 13, normalized size = 0.87

$$\frac{\sqrt{bx^2 + a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2 + a)/b
```

giac [A] time = 0.15, size = 13, normalized size = 0.87

$$\frac{\sqrt{bx^2 + a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(b*x^2 + a)/b
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\sqrt{b\,x^2 + a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(1/2),x)
```

```
[Out] (b*x^2+a)^(1/2)/b
```

maxima [A] time = 1.05, size = 13, normalized size = 0.87

$$\frac{\sqrt{bx^2 + a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b*x^2 + a)/b
```

mupad [B] time = 4.33, size = 13, normalized size = 0.87

$$\frac{\sqrt{b\,x^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)/b
```

sympy [A] time = 0.40, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sqrt{a+bx^2}}{b} & \text{for } b \neq 0 \\ \frac{x^2}{2\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + b*x**2)/b, Ne(b, 0)), (x**2/(2*sqrt(a)), True))
```

3.1016 $\int \frac{1}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}} \, dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \int \frac{1}{\sqrt{a+bx^2}}\,dx$$
$$= \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Mathematica [**A**] time = 0.00, size = 25, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

fricas [**A**] time = 0.80, size = 59, normalized size = 2.36

$$\left[\frac{\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a))/b]
```

giac [**A**] time = 0.18, size = 23, normalized size = 0.92

$$-\frac{\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b)
```

maple [A] time = 0.00, size = 21, normalized size = 0.84

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2),x)
[Out] ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)
```

maxima [A] time = 1.02, size = 13, normalized size = 0.52

$$\frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

mupad [B] time = 0.12, size = 20, normalized size = 0.80

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(1/2),x)
[Out] log(b^(1/2)*x + (a + b*x^2)^(1/2))/b^(1/2)
```

sympy [A] time = 1.20, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2),x)
[Out] asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

3.1017 $\int \frac{1}{x\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=25

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {5, 266, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{x\sqrt{a+bx^2}}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

fricas [A] time = 0.59, size = 60, normalized size = 2.40

$$\left[\frac{\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)}{2\,\sqrt{a}}, \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)/sqrt(a), sqrt(-a)*
arctan(sqrt(-a)/sqrt(b*x^2 + a))/a]
```

giac [A] time = 0.16, size = 22, normalized size = 0.88

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.00, size = 29, normalized size = 1.16

$$-\frac{\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(1/2),x)
[Out] -1/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

maxima [A] time = 1.08, size = 17, normalized size = 0.68

$$-\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] -arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a)
```

mupad [B] time = 4.57, size = 19, normalized size = 0.76

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(1/2)),x)
```

```
[Out] -atanh((a + b*x^2)^(1/2)/a^(1/2))/a^(1/2)
```

sympy [A] time = 1.20, size = 19, normalized size = 0.76

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(1/2),x)
```

```
[Out] -asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a)
```

3.1018 $\int \frac{1}{x^2\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=19

$$-\frac{\sqrt{a+bx^2}}{ax}$$

[Out] -(b*x^2+a)^(1/2)/a/x

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {5, 264}

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]

[Out] -(Sqrt[a + b*x^2]/(a*x))

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{x^2\sqrt{a+bx^2}}\,dx\\ &= -\frac{\sqrt{a+bx^2}}{ax}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] -(Sqrt[a + b*x^2]/(a*x))
```

fricas [A] time = 0.74, size = 17, normalized size = 0.89

$$-\frac{\sqrt{bx^2+a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] -sqrt(b*x^2 + a)/(a*x)
```

giac [A] time = 0.18, size = 30, normalized size = 1.58

$$\frac{2\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$-\frac{\sqrt{b\,x^2+a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(1/2),x)
[Out] -(b*x^2+a)^(1/2)/a/x
```

maxima [A] time = 1.01, size = 17, normalized size = 0.89

$$-\frac{\sqrt{bx^2+a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b*x^2 + a)/(a*x)
```

mupad [B] time = 0.04, size = 17, normalized size = 0.89

$$-\frac{\sqrt{b\,x^2+a}}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(1/2)),x)
[Out] -(a + b*x^2)^(1/2)/(a*x)
```

sympy [A] time = 0.75, size = 19, normalized size = 1.00

$$-\frac{\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(1/2),x)
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/a
```

3.1019 $\int \frac{1}{x^3\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=50

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{\sqrt{a+bx^2}}{2ax^2}$$

```
[Out] 1/2*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)-1/2*(b*x^2+a)^(1/2)/a/x^2
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {5, 266, 51, 63, 208}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2]/(2*a*x^2) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2
))
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{x^3\sqrt{a+bx^2}}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{a+bx^2}}{2ax^2} - \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{4a} \\
&= -\frac{\sqrt{a+bx^2}}{2ax^2} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{2a} \\
&= -\frac{\sqrt{a+bx^2}}{2ax^2} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 61, normalized size = 1.22

$$
\frac{b\sqrt{a+bx^2}\left(\frac{\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{2\sqrt{\frac{bx^2}{a}+1}}-\frac{a}{2bx^2}\right)}{a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] (b*Sqrt[a + b*x^2]*(-1/2*a/(b*x^2) + ArcTanh[Sqrt[1 + (b*x^2)/a]]/(2*Sqrt[1
 + (b*x^2)/a])))/a^2
```

fricas [A] time = 0.70, size = 105, normalized size = 2.10

$$\left[\frac{\sqrt{a} b x^2 \log\left(-\frac{b x^2 + 2 \sqrt{b x^2 + a} \sqrt{a} + 2 a}{x^2}\right) - 2 \sqrt{b x^2 + a}\, a}{4 a^2 x^2}, -\frac{\sqrt{-a} b x^2 \arctan\left(\frac{\sqrt{-a}}{\sqrt{b x^2 + a}}\right) + \sqrt{b x^2 + a}\, a}{2 a^2 x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*b*x^2*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2
*sqrt(b*x^2 + a)*a)/(a^2*x^2), -1/2*(sqrt(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b*
x^2 + a)) + sqrt(b*x^2 + a)*a)/(a^2*x^2)]
```

giac [A] time = 0.17, size = 51, normalized size = 1.02

$$-\frac{\frac{b^2 \arctan\left(\frac{\sqrt{b x^2 + a}}{\sqrt{-a}}\right)}{\sqrt{-a}\, a} + \frac{\sqrt{b x^2 + a}\, b}{a x^2}}{2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x^2 + a)*b
/(a*x^2))/b
```

maple [A] time = 0.01, size = 48, normalized size = 0.96

$$\frac{b \ln\left(\frac{2a + 2\sqrt{b\,x^2 + a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}} - \frac{\sqrt{b\,x^2 + a}}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*b/a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))
/x)
```

maxima [A] time = 1.03, size = 36, normalized size = 0.72

$$\frac{b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{\sqrt{b x^2 + a}}{2\,a x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/2*sqrt(b*x^2 + a)/(a*x^2)
```

mupad [B] time = 4.54, size = 38, normalized size = 0.76

$$\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{\sqrt{b\,x^2+a}}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(1/2)),x)
```

```
[Out] (b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2)) - (a + b*x^2)^(1/2)/(2*a*x
^2)
```

sympy [A] time = 3.47, size = 42, normalized size = 0.84

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**
(3/2))
```

3.1020 $\int \frac{1}{x^4\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=44

$$\frac{2b\sqrt{a+bx^2}}{3a^2x}-\frac{\sqrt{a+bx^2}}{3ax^3}$$

```
[Out] -1/3*(b*x^2+a)^(1/2)/a/x^3+2/3*b*(b*x^2+a)^(1/2)/a^2/x
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {5, 271, 264}

$$\frac{2b\sqrt{a+bx^2}}{3a^2x}-\frac{\sqrt{a+bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2]/(3*a*x^3) + (2*b*Sqrt[a + b*x^2])/(3*a^2*x)
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \int \frac{1}{x^4\sqrt{a+bx^2}}\,dx$$
$$= -\frac{\sqrt{a+bx^2}}{3ax^3} - \frac{(2b)\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx}{3a}$$
$$= -\frac{\sqrt{a+bx^2}}{3ax^3} + \frac{2b\sqrt{a+bx^2}}{3a^2x}$$

Mathematica [**A**] time = 0.01, size = 29, normalized size = 0.66

$$-\frac{(a-2bx^2)\sqrt{a+bx^2}}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] -1/3*((a - 2*b*x^2)*Sqrt[a + b*x^2])/(a^2*x^3)
```

fricas [**A**] time = 0.80, size = 27, normalized size = 0.61

$$\frac{(2\,bx^2-a)\sqrt{bx^2+a}}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] 1/3*(2*b*x^2 - a)*sqrt(b*x^2 + a)/(a^2*x^3)
```

giac [**A**] time = 0.28, size = 55, normalized size = 1.25

$$\frac{4\left(3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)b^{\frac{3}{2}}}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] 4/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*b^(3/2)/((sqrt(b)*x - sqrt(b*x^
2 + a))^2 - a)^3
```

maple [A] time = 0.00, size = 26, normalized size = 0.59

$$-\frac{\sqrt{b\,x^{2}+a}\,\left(-2b\,x^{2}+a\right)}{3a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3*(b*x^2+a)^(1/2)*(-2*b*x^2+a)/a^2/x^3
```

maxima [A] time = 1.00, size = 36, normalized size = 0.82

$$\frac{2\,\sqrt{bx^{2}+a}\,b}{3\,a^{2}x}-\frac{\sqrt{bx^{2}+a}}{3\,ax^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(b*x^2 + a)*b/(a^2*x) - 1/3*sqrt(b*x^2 + a)/(a*x^3)
```

mupad [B] time = 4.55, size = 25, normalized size = 0.57

$$-\frac{\sqrt{b\,x^{2}+a}\,\left(a-2\,b\,x^{2}\right)}{3\,a^{2}\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(1/2)),x)
```

```
[Out] -((a + b*x^2)^(1/2)*(a - 2*b*x^2))/(3*a^2*x^3)
```

sympy [A] time = 1.05, size = 46, normalized size = 1.05

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{3ax^{2}}+\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(3*a**2)
```

3.1021 $\int \frac{x^4}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=16

$$\frac{x^5}{3\sqrt{cx^4}}$$

[Out] 1/3*x^5/(c*x^4)^(1/2)

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1, 15, 30}

$$\frac{x^5}{3\sqrt{cx^4}}$$

Antiderivative was successfully verified.

[In] Int[x^4/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]

[Out] x^5/(3*Sqrt[c*x^4])

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{x^4}{\sqrt{cx^4}}\,dx$$
$$= \frac{x^2\int x^2\,dx}{\sqrt{cx^4}}$$
$$= \frac{x^5}{3\sqrt{cx^4}}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x^5}{3\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
[Out] x^5/(3*Sqrt[c*x^4])
```

fricas [A] time = 0.83, size = 13, normalized size = 0.81

$$\frac{\sqrt{cx^4}\,x}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(c*x^4)*x/c
```

giac [A] time = 0.15, size = 8, normalized size = 0.50

$$\frac{x^3}{3\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4)^(1/2),x, algorithm="giac")
[Out] 1/3*x^3/sqrt(c)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{x^5}{3\sqrt{c\,x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(c*x^4)^(1/2),x)

[Out] 1/3*x^5/(c*x^4)^(1/2)

maxima [A] time = 0.96, size = 12, normalized size = 0.75

$$\frac{x^5}{3\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4)^(1/2),x, algorithm="maxima")

[Out] 1/3*x^5/sqrt(c*x^4)

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{x^4}{\sqrt{c\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(c*x^4)^(1/2),x)

[Out] int(x^4/(c*x^4)^(1/2), x)

sympy [A] time = 0.63, size = 15, normalized size = 0.94

$$\frac{x^5}{3\sqrt{c}\,\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(c*x**4)**(1/2),x)

[Out] x**5/(3*sqrt(c)*sqrt(x**4))

3.1022 $\int \frac{x^3}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=16

$$\frac{x^4}{2\sqrt{cx^4}}$$

[Out] 1/2*x^4/(c*x^4)^(1/2)

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1, 15, 30}

$$\frac{x^4}{2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]

[Out] x^4/(2*Sqrt[c*x^4])

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{x^3}{\sqrt{cx^4}}\,dx$$
$$= \frac{x^2 \int x\,dx}{\sqrt{cx^4}}$$
$$= \frac{x^4}{2\sqrt{cx^4}}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x^4}{2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
[Out] x^4/(2*Sqrt[c*x^4])
```

fricas [A] time = 0.80, size = 12, normalized size = 0.75

$$\frac{\sqrt{cx^4}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(c*x^4)/c
```

giac [A] time = 0.15, size = 8, normalized size = 0.50

$$\frac{x^2}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4)^(1/2),x, algorithm="giac")
[Out] 1/2*x^2/sqrt(c)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{x^4}{2\sqrt{c\,x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4)^(1/2),x)
```

```
[Out] 1/2*x^4/(c*x^4)^(1/2)
```

maxima [A] time = 1.04, size = 12, normalized size = 0.75

$$\frac{\sqrt{cx^4}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^4)/c
```

mupad [B] time = 4.50, size = 10, normalized size = 0.62

$$\frac{\sqrt{x^4}}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4)^(1/2),x)
```

```
[Out] (x^4)^(1/2)/(2*c^(1/2))
```

sympy [A] time = 0.53, size = 15, normalized size = 0.94

$$\frac{x^4}{2\sqrt{c}\,\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4)**(1/2),x)
```

```
[Out] x**4/(2*sqrt(c)*sqrt(x**4))
```

3.1023 $\int \frac{x^2}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=13

$$\frac{x^3}{\sqrt{cx^4}}$$

```
[Out] x^3/(c*x^4)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1, 15, 8}

$$\frac{x^3}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] x^3/Sqrt[c*x^4]
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{x^2}{\sqrt{cx^4}}\,dx$$
$$= \frac{x^2 \int 1\,dx}{\sqrt{cx^4}}$$
$$= \frac{x^3}{\sqrt{cx^4}}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{x^3}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
[Out] x^3/Sqrt[c*x^4]
```

fricas [A] time = 0.59, size = 14, normalized size = 1.08

$$\frac{\sqrt{cx^4}}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x^4)/(c*x)
```

giac [A] time = 0.17, size = 5, normalized size = 0.38

$$\frac{x}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4)^(1/2),x, algorithm="giac")
[Out] x/sqrt(c)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\frac{x^3}{\sqrt{c\,x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4)^(1/2),x)
[Out] x^3/(c*x^4)^(1/2)
```

maxima [A] time = 0.98, size = 11, normalized size = 0.85

$$\frac{x^3}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4)^(1/2),x, algorithm="maxima")
[Out] x^3/sqrt(c*x^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{x^2}{\sqrt{c\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4)^(1/2),x)
[Out] int(x^2/(c*x^4)^(1/2), x)
```

sympy [A] time = 0.48, size = 14, normalized size = 1.08

$$\frac{x^3}{\sqrt{c}\,\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4)**(1/2),x)
[Out] x**3/(sqrt(c)*sqrt(x**4))
```

3.1024 $\int \frac{x}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=15

$$\frac{x^2\log(x)}{\sqrt{cx^4}}$$

```
[Out] x^2*ln(x)/(c*x^4)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.143, Rules used = {1, 15, 29}

$$\frac{x^2\log(x)}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] (x^2*Log[x])/Sqrt[c*x^4]
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{x}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{x}{\sqrt{cx^4}}\,dx$$
$$= \frac{x^2 \int \frac{1}{x}\,dx}{\sqrt{cx^4}}$$
$$= \frac{x^2 \log(x)}{\sqrt{cx^4}}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{x^2 \log(x)}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] (x^2*Log[x])/Sqrt[c*x^4]
```

fricas [A] time = 0.69, size = 16, normalized size = 1.07

$$\frac{\sqrt{cx^4}\,\log(x)}{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4)*log(x)/(c*x^2)
```

giac [A] time = 0.15, size = 7, normalized size = 0.47

$$\frac{\log(|x|)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(x))/sqrt(c)
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{x^2 \ln(x)}{\sqrt{c\,x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4)^(1/2),x)
```

```
[Out] x^2*ln(x)/(c*x^4)^(1/2)
```

maxima [A] time = 0.98, size = 13, normalized size = 0.87

$$\frac{x^2 \log(x)}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] x^2*log(x)/sqrt(c*x^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x}{\sqrt{c\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4)^(1/2),x)
```

```
[Out] int(x/(c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4)**(1/2),x)
```

```
[Out] Integral(x/sqrt(c*x**4), x)
```

3.1025 $\int \frac{1}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=12

$$-\frac{x}{\sqrt{cx^4}}$$

```
[Out] -x/(c*x^4)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1, 15, 30}

$$-\frac{x}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] -(x/Sqrt[c*x^4])
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4}}\,dx$$
$$= \frac{x^2 \int \frac{1}{x^2}\,dx}{\sqrt{cx^4}}$$
$$= -\frac{x}{\sqrt{cx^4}}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{x}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
[Out] -(x/Sqrt[c*x^4])
```

fricas [A] time = 0.81, size = 15, normalized size = 1.25

$$-\frac{\sqrt{cx^4}}{cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4)^(1/2),x, algorithm="fricas")
[Out] -sqrt(c*x^4)/(c*x^3)
```

giac [A] time = 0.15, size = 8, normalized size = 0.67

$$-\frac{1}{\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4)^(1/2),x, algorithm="giac")
[Out] -1/(sqrt(c)*x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$-\frac{x}{\sqrt{c\,x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4)^(1/2),x)
```

```
[Out] -x/(c*x^4)^(1/2)
```

maxima [A] time = 0.98, size = 10, normalized size = 0.83

$$-\frac{x}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -x/sqrt(c*x^4)
```

mupad [B] time = 4.30, size = 13, normalized size = 1.08

$$-\frac{\sqrt{x^4}}{\sqrt{c}\, x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4)^(1/2),x)
```

```
[Out] -(x^4)^(1/2)/(c^(1/2)*x^3)
```

sympy [A] time = 0.47, size = 14, normalized size = 1.17

$$-\frac{x}{\sqrt{c}\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4)**(1/2),x)
```

```
[Out] -x/(sqrt(c)*sqrt(x**4))
```

3.1026 $\int \frac{1}{x\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2\sqrt{cx^4}}$$

```
[Out] -1/2/(c*x^4)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1, 15, 30}

$$-\frac{1}{2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -1/(2*Sqrt[c*x^4])
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{1}{x\sqrt{cx^4}}\,dx$$
$$= \frac{x^2\int \frac{1}{x^3}\,dx}{\sqrt{cx^4}}$$
$$= -\frac{1}{2\sqrt{cx^4}}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$-\frac{1}{2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
[Out] -1/2*1/Sqrt[c*x^4]
```

fricas [A] time = 0.81, size = 15, normalized size = 1.15

$$-\frac{\sqrt{cx^4}}{2\,cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(c*x^4)/(c*x^4)
```

giac [A] time = 0.16, size = 8, normalized size = 0.62

$$-\frac{1}{2\,\sqrt{c}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4)^(1/2),x, algorithm="giac")
[Out] -1/2/(sqrt(c)*x^2)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$-\frac{1}{2\sqrt{c\,x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4)^(1/2),x)
```

```
[Out] -1/2/(c*x^4)^(1/2)
```

maxima [A] time = 1.03, size = 9, normalized size = 0.69

$$-\frac{1}{2\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2/sqrt(c*x^4)
```

mupad [B] time = 4.34, size = 10, normalized size = 0.77

$$-\frac{1}{2\sqrt{c}\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c*x^4)^(1/2)),x)
```

```
[Out] -1/(2*c^(1/2)*(x^4)^(1/2))
```

sympy [A] time = 0.51, size = 15, normalized size = 1.15

$$-\frac{1}{2\sqrt{c}\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4)**(1/2),x)
```

```
[Out] -1/(2*sqrt(c)*sqrt(x**4))
```

3.1027 $\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{3x\sqrt{cx^4}}$$

```
[Out] -1/3/x/(c*x^4)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1, 15, 30}

$$-\frac{1}{3x\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -1/(3*x*Sqrt[c*x^4])
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{1}{x^2\sqrt{cx^4}}\,dx$$
$$= \frac{x^2 \int \frac{1}{x^4}\,dx}{\sqrt{cx^4}}$$
$$= -\frac{1}{3x\sqrt{cx^4}}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{3x\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
[Out] -1/3*1/(x*Sqrt[c*x^4])
```

fricas [A] time = 0.54, size = 15, normalized size = 0.94

$$-\frac{\sqrt{cx^4}}{3\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(c*x^4)/(c*x^5)
```

giac [A] time = 0.16, size = 8, normalized size = 0.50

$$-\frac{1}{3\,\sqrt{c}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4)^(1/2),x, algorithm="giac")
[Out] -1/3/(sqrt(c)*x^3)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{1}{3\sqrt{c\,x^4}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4)^(1/2),x)
```

```
[Out] -1/3/x/(c*x^4)^(1/2)
```

maxima [A] time = 1.08, size = 12, normalized size = 0.75

$$-\frac{1}{3\sqrt{cx^4}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3/(sqrt(c*x^4)*x)
```

mupad [B] time = 4.31, size = 13, normalized size = 0.81

$$-\frac{1}{3\sqrt{c}\,x\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c*x^4)^(1/2)),x)
```

```
[Out] -1/(3*c^(1/2)*x*(x^4)^(1/2))
```

sympy [A] time = 0.56, size = 17, normalized size = 1.06

$$-\frac{1}{3\sqrt{c}x\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4)**(1/2),x)
```

```
[Out] -1/(3*sqrt(c)*x*sqrt(x**4))
```

3.1028 $\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{4x^2\sqrt{cx^4}}$$

```
[Out] -1/4/x^2/(c*x^4)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1, 15, 30}

$$-\frac{1}{4x^2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -1/(4*x^2*Sqrt[c*x^4])
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{1}{x^3\sqrt{cx^4}}\,dx$$
$$= \frac{x^2 \int \frac{1}{x^5}\,dx}{\sqrt{cx^4}}$$
$$= -\frac{1}{4x^2\sqrt{cx^4}}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.06

$$-\frac{cx^2}{4\left(cx^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
[Out] -1/4*(c*x^2)/(c*x^4)^(3/2)
```

fricas [A] time = 0.48, size = 15, normalized size = 0.94

$$-\frac{\sqrt{cx^4}}{4\,cx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4)^(1/2),x, algorithm="fricas")
[Out] -1/4*sqrt(c*x^4)/(c*x^6)
```

giac [A] time = 0.16, size = 8, normalized size = 0.50

$$-\frac{1}{4\sqrt{c}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4)^(1/2),x, algorithm="giac")
[Out] -1/4/(sqrt(c)*x^4)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{1}{4\sqrt{c\,x^4}\;x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4)^(1/2),x)
```

```
[Out] -1/4/x^2/(c*x^4)^(1/2)
```

maxima [A] time = 1.05, size = 12, normalized size = 0.75

$$-\frac{1}{4\sqrt{cx^4}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4/(sqrt(c*x^4)*x^2)
```

mupad [B] time = 4.27, size = 13, normalized size = 0.81

$$-\frac{1}{4\sqrt{c}\,x^2\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c*x^4)^(1/2)),x)
```

```
[Out] -1/(4*c^(1/2)*x^2*(x^4)^(1/2))
```

sympy [A] time = 0.66, size = 19, normalized size = 1.19

$$-\frac{1}{4\sqrt{c}\,x^2\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4)**(1/2),x)
```

```
[Out] -1/(4*sqrt(c)*x**2*sqrt(x**4))
```

3.1029 $\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{5x^3\sqrt{cx^4}}$$

```
[Out] -1/5/x^3/(c*x^4)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1, 15, 30}

$$-\frac{1}{5x^3\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -1/(5*x^3*Sqrt[c*x^4])
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{1}{x^4\sqrt{cx^4}}\,dx$$
$$= \frac{x^2 \int \frac{1}{x^6}\,dx}{\sqrt{cx^4}}$$
$$= -\frac{1}{5x^3\sqrt{cx^4}}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{cx}{5\left(cx^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
[Out] -1/5*(c*x)/(c*x^4)^(3/2)
```

fricas [A] time = 0.83, size = 15, normalized size = 0.94

$$-\frac{\sqrt{cx^4}}{5\,cx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4)^(1/2),x, algorithm="fricas")
[Out] -1/5*sqrt(c*x^4)/(c*x^7)
```

giac [A] time = 0.15, size = 8, normalized size = 0.50

$$-\frac{1}{5\sqrt{c}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4)^(1/2),x, algorithm="giac")
[Out] -1/5/(sqrt(c)*x^5)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{1}{5\sqrt{c\,x^4}\;x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4)^(1/2),x)
```

```
[Out] -1/5/x^3/(c*x^4)^(1/2)
```

maxima [A] time = 1.06, size = 12, normalized size = 0.75

$$-\frac{1}{5\sqrt{cx^4}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5/(sqrt(c*x^4)*x^3)
```

mupad [B] time = 4.33, size = 13, normalized size = 0.81

$$-\frac{1}{5\sqrt{c}\,x^3\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c*x^4)^(1/2)),x)
```

```
[Out] -1/(5*c^(1/2)*x^3*(x^4)^(1/2))
```

sympy [A] time = 0.71, size = 19, normalized size = 1.19

$$-\frac{1}{5\sqrt{c}\,x^3\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4)**(1/2),x)
```

```
[Out] -1/(5*sqrt(c)*x**3*sqrt(x**4))
```

3.1030 $\int \frac{x^4}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{x^5}{5\sqrt{a}}$$

```
[Out] 1/5*x^5/a^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2, 12, 30}

$$\frac{x^5}{5\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x^5/(5*Sqrt[a])
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \int \frac{x^4}{\sqrt{a}}\,dx$$
$$= \frac{\int x^4\,dx}{\sqrt{a}}$$
$$= \frac{x^5}{5\sqrt{a}}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{x^5}{5\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] x^5/(5*Sqrt[a])
```

fricas [A] time = 0.83, size = 8, normalized size = 0.67

$$\frac{x^5}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/a^(1/2),x, algorithm="fricas")
[Out] 1/5*x^5/sqrt(a)
```

giac [A] time = 0.15, size = 8, normalized size = 0.67

$$\frac{x^5}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/a^(1/2),x, algorithm="giac")
[Out] 1/5*x^5/sqrt(a)
```

maple [A] time = 0.00, size = 9, normalized size = 0.75

$$\frac{x^5}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/a^(1/2),x)
```

```
[Out] 1/5*x^5/a^(1/2)
```

maxima [A] time = 1.01, size = 8, normalized size = 0.67

$$\frac{x^5}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/a^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*x^5/sqrt(a)
```

mupad [B] time = 0.02, size = 8, normalized size = 0.67

$$\frac{x^5}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/a^(1/2),x)
```

```
[Out] x^5/(5*a^(1/2))
```

sympy [A] time = 0.07, size = 8, normalized size = 0.67

$$\frac{x^5}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/a**(1/2),x)
```

```
[Out] x**5/(5*sqrt(a))
```

3.1031 $\int \frac{x^3}{\sqrt{a+(2+2c-2(1+c))x^4}} \, dx$

Optimal. Leaf size=12

$$\frac{x^4}{4\sqrt{a}}$$

```
[Out] 1/4*x^4/a^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2, 12, 30}

$$\frac{x^4}{4\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x^4/(4*Sqrt[a])
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \int \frac{x^3}{\sqrt{a}}\,dx = \frac{\int x^3\,dx}{\sqrt{a}} = \frac{x^4}{4\sqrt{a}}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{x^4}{4\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] x^4/(4*Sqrt[a])
```

fricas [A] time = 0.65, size = 8, normalized size = 0.67

$$\frac{x^4}{4\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/a^(1/2),x, algorithm="fricas")
[Out] 1/4*x^4/sqrt(a)
```

giac [A] time = 0.18, size = 8, normalized size = 0.67

$$\frac{x^4}{4\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/a^(1/2),x, algorithm="giac")
[Out] 1/4*x^4/sqrt(a)
```

maple [A] time = 0.00, size = 9, normalized size = 0.75

$$\frac{x^4}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/a^(1/2),x)
```

```
[Out] 1/4*x^4/a^(1/2)
```

maxima [A] time = 1.04, size = 8, normalized size = 0.67

$$\frac{x^4}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/a^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4/sqrt(a)
```

mupad [B] time = 0.03, size = 8, normalized size = 0.67

$$\frac{x^4}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/a^(1/2),x)
```

```
[Out] x^4/(4*a^(1/2))
```

sympy [A] time = 0.06, size = 8, normalized size = 0.67

$$\frac{x^4}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/a**(1/2),x)
```

```
[Out] x**4/(4*sqrt(a))
```

3.1032 $\int \frac{x^2}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{x^3}{3\sqrt{a}}$$

```
[Out] 1/3*x^3/a^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2, 12, 30}

$$\frac{x^3}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x^3/(3*Sqrt[a])
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \int \frac{x^2}{\sqrt{a}}\,dx$$
$$= \frac{\int x^2\,dx}{\sqrt{a}}$$
$$= \frac{x^3}{3\sqrt{a}}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{x^3}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] x^3/(3*Sqrt[a])
```

fricas [A] time = 0.84, size = 8, normalized size = 0.67

$$\frac{x^3}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/a^(1/2),x, algorithm="fricas")
[Out] 1/3*x^3/sqrt(a)
```

giac [A] time = 0.15, size = 8, normalized size = 0.67

$$\frac{x^3}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/a^(1/2),x, algorithm="giac")
[Out] 1/3*x^3/sqrt(a)
```

maple [A] time = 0.00, size = 9, normalized size = 0.75

$$\frac{x^3}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/a^(1/2),x)
```

```
[Out] 1/3*x^3/a^(1/2)
```

maxima [A] time = 0.95, size = 8, normalized size = 0.67

$$\frac{x^3}{3\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/a^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^3/sqrt(a)
```

mupad [B] time = 0.01, size = 8, normalized size = 0.67

$$\frac{x^3}{3\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/a^(1/2),x)
```

```
[Out] x^3/(3*a^(1/2))
```

sympy [A] time = 0.07, size = 8, normalized size = 0.67

$$\frac{x^3}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/a**(1/2),x)
```

```
[Out] x**3/(3*sqrt(a))
```

3.1033 $\int \frac{x}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{x^2}{2\sqrt{a}}$$

[Out] 1/2*x^2/a^(1/2)

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2, 12, 30}

$$\frac{x^2}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]

[Out] x^2/(2*Sqrt[a])

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{a + (2 + 2c - 2(1 + c))x^4}}\,dx = \int \frac{x}{\sqrt{a}}\,dx$$
$$= \frac{\int x\,dx}{\sqrt{a}}$$
$$= \frac{x^2}{2\sqrt{a}}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{x^2}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] x^2/(2*Sqrt[a])
```

fricas [A] time = 0.82, size = 8, normalized size = 0.67

$$\frac{x^2}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/a^(1/2),x, algorithm="fricas")
[Out] 1/2*x^2/sqrt(a)
```

giac [A] time = 0.15, size = 8, normalized size = 0.67

$$\frac{x^2}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/a^(1/2),x, algorithm="giac")
[Out] 1/2*x^2/sqrt(a)
```

maple [A] time = 0.00, size = 9, normalized size = 0.75

$$\frac{x^2}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/a^(1/2),x)
```

```
[Out] 1/2*x^2/a^(1/2)
```

maxima [A] time = 1.07, size = 8, normalized size = 0.67

$$\frac{x^2}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/a^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/sqrt(a)
```

mupad [B] time = 0.02, size = 8, normalized size = 0.67

$$\frac{x^2}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/a^(1/2),x)
```

```
[Out] x^2/(2*a^(1/2))
```

sympy [A] time = 0.08, size = 8, normalized size = 0.67

$$\frac{x^2}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/a**(1/2),x)
```

```
[Out] x**2/(2*sqrt(a))
```

3.1034 $\int \frac{1}{\sqrt{a+(2+2c-2(1+c))x^4}} dx$

Optimal. Leaf size=7

$$\frac{x}{\sqrt{a}}$$

```
[Out] x/a^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2, 8}

$$\frac{x}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x/Sqrt[a]
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a+(2+2c-2(1+c))x^4}} dx &= \int \frac{1}{\sqrt{a}} dx \\ &= \frac{x}{\sqrt{a}} \end{aligned}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$\frac{x}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x/Sqrt[a]
```

fricas [A] time = 0.81, size = 5, normalized size = 0.71

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a^(1/2),x, algorithm="fricas")
```

```
[Out] x/sqrt(a)
```

giac [A] time = 0.15, size = 5, normalized size = 0.71

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a^(1/2),x, algorithm="giac")
```

```
[Out] x/sqrt(a)
```

maple [A] time = 0.00, size = 6, normalized size = 0.86

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/a^(1/2),x)
```

```
[Out] x/a^(1/2)
```

maxima [A] time = 0.98, size = 5, normalized size = 0.71

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a^(1/2),x, algorithm="maxima")
```

```
[Out] x/sqrt(a)
```

mupad [B] time = 0.00, size = 5, normalized size = 0.71

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/a^(1/2),x)
```

```
[Out] x/a^(1/2)
```

sympy [A] time = 0.13, size = 5, normalized size = 0.71

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a**(1/2),x)
```

```
[Out] x/sqrt(a)
```

3.1035 $\int \frac{1}{x\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=8

$$\frac{\log(x)}{\sqrt{a}}$$

```
[Out] ln(x)/a^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2, 12, 29}

$$\frac{\log(x)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] Log[x]/Sqrt[a]
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \int \frac{1}{\sqrt{a}\,x}\,dx$$
$$= \frac{\int \frac{1}{x}\,dx}{\sqrt{a}}$$
$$= \frac{\log(x)}{\sqrt{a}}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{\log(x)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] Log[x]/Sqrt[a]
```

fricas [A] time = 0.62, size = 6, normalized size = 0.75

$$\frac{\log(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/a^(1/2),x, algorithm="fricas")
[Out] log(x)/sqrt(a)
```

giac [A] time = 0.15, size = 7, normalized size = 0.88

$$\frac{\log(|x|)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/a^(1/2),x, algorithm="giac")
[Out] log(abs(x))/sqrt(a)
```

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$\frac{\ln(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/a^(1/2),x)
```

```
[Out] ln(x)/a^(1/2)
```

maxima [A] time = 1.03, size = 6, normalized size = 0.75

$$\frac{\log(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/a^(1/2),x, algorithm="maxima")
```

```
[Out] log(x)/sqrt(a)
```

mupad [B] time = 4.24, size = 6, normalized size = 0.75

$$\frac{\ln(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^(1/2)*x),x)
```

```
[Out] log(x)/a^(1/2)
```

sympy [A] time = 0.08, size = 7, normalized size = 0.88

$$\frac{\log{\left(x \right)}}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/a**(1/2),x)
```

```
[Out] log(x)/sqrt(a)
```

3.1036 $\int \frac{1}{x^2\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=10

$$-\frac{1}{\sqrt{a}\,x}$$

```
[Out] -1/x/a^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2, 12, 30}

$$-\frac{1}{\sqrt{a}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -(1/(Sqrt[a]*x))
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\int \frac{1}{\sqrt{a}\,x^2}\,dx$$
$$=\frac{\int \frac{1}{x^2}\,dx}{\sqrt{a}}$$
$$=-\frac{1}{\sqrt{a}\,x}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$-\frac{1}{\sqrt{a}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] -(1/(Sqrt[a]*x))
```

fricas [A] time = 0.79, size = 8, normalized size = 0.80

$$-\frac{1}{\sqrt{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/a^(1/2),x, algorithm="fricas")
[Out] -1/(sqrt(a)*x)
```

giac [A] time = 0.18, size = 8, normalized size = 0.80

$$-\frac{1}{\sqrt{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/a^(1/2),x, algorithm="giac")
[Out] -1/(sqrt(a)*x)
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$-\frac{1}{\sqrt{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/a^(1/2),x)
```

```
[Out] -1/x/a^(1/2)
```

maxima [A] time = 0.88, size = 8, normalized size = 0.80

$$-\frac{1}{\sqrt{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/a^(1/2),x, algorithm="maxima")
```

```
[Out] -1/(sqrt(a)*x)
```

mupad [B] time = 0.03, size = 8, normalized size = 0.80

$$-\frac{1}{\sqrt{a}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^(1/2)*x^2),x)
```

```
[Out] -1/(a^(1/2)*x)
```

sympy [A] time = 0.08, size = 8, normalized size = 0.80

$$-\frac{1}{\sqrt{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/a**(1/2),x)
```

```
[Out] -1/(sqrt(a)*x)
```

3.1037 $\int \frac{1}{x^3\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{2\sqrt{a}\,x^2}$$

```
[Out] -1/2/x^2/a^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2, 12, 30}

$$-\frac{1}{2\sqrt{a}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -1/(2*Sqrt[a]*x^2)
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \int \frac{1}{\sqrt{a}\,x^3}\,dx$$
$$= \frac{\int \frac{1}{x^3}\,dx}{\sqrt{a}}$$
$$= -\frac{1}{2\sqrt{a}\,x^2}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{2\sqrt{a}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] -1/2*1/(Sqrt[a]*x^2)
```

fricas [A] time = 0.79, size = 8, normalized size = 0.67

$$-\frac{1}{2\,\sqrt{a}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/a^(1/2),x, algorithm="fricas")
[Out] -1/2/(sqrt(a)*x^2)
```

giac [A] time = 0.15, size = 8, normalized size = 0.67

$$-\frac{1}{2\,\sqrt{a}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/a^(1/2),x, algorithm="giac")
[Out] -1/2/(sqrt(a)*x^2)
```

maple [A] time = 0.00, size = 9, normalized size = 0.75

$$-\frac{1}{2\sqrt{a}\;x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/a^(1/2),x)
```

```
[Out] -1/2/x^2/a^(1/2)
```

maxima [A] time = 1.04, size = 8, normalized size = 0.67

$$-\frac{1}{2\sqrt{a}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/a^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2/(sqrt(a)*x^2)
```

mupad [B] time = 4.40, size = 8, normalized size = 0.67

$$-\frac{1}{2\sqrt{a}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^(1/2)*x^3),x)
```

```
[Out] -1/(2*a^(1/2)*x^2)
```

sympy [A] time = 0.08, size = 12, normalized size = 1.00

$$-\frac{1}{2\sqrt{a}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/a**(1/2),x)
```

```
[Out] -1/(2*sqrt(a)*x**2)
```

3.1038 $\int \frac{1}{x^4\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{3\sqrt{a}\,x^3}$$

```
[Out] -1/3/x^3/a^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2, 12, 30}

$$-\frac{1}{3\sqrt{a}\,x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -1/(3*Sqrt[a]*x^3)
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \int \frac{1}{\sqrt{a}\,x^4}\,dx$$
$$= \frac{\int \frac{1}{x^4}\,dx}{\sqrt{a}}$$
$$= -\frac{1}{3\sqrt{a}\,x^3}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{3\sqrt{a}\,x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] -1/3*1/(Sqrt[a]*x^3)
```

fricas [A] time = 0.83, size = 8, normalized size = 0.67

$$-\frac{1}{3\,\sqrt{a}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/a^(1/2),x, algorithm="fricas")
[Out] -1/3/(sqrt(a)*x^3)
```

giac [A] time = 0.18, size = 8, normalized size = 0.67

$$-\frac{1}{3\,\sqrt{a}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/a^(1/2),x, algorithm="giac")
[Out] -1/3/(sqrt(a)*x^3)
```

maple [A] time = 0.00, size = 9, normalized size = 0.75

$$-\frac{1}{3\sqrt{a}\;x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/a^(1/2),x)
[Out] -1/3/x^3/a^(1/2)
```

maxima [A] time = 1.00, size = 8, normalized size = 0.67

$$-\frac{1}{3\sqrt{a}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/a^(1/2),x, algorithm="maxima")
[Out] -1/3/(sqrt(a)*x^3)
```

mupad [B] time = 4.33, size = 8, normalized size = 0.67

$$-\frac{1}{3\sqrt{a}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^(1/2)*x^4),x)
[Out] -1/(3*a^(1/2)*x^3)
```

sympy [A] time = 0.07, size = 12, normalized size = 1.00

$$-\frac{1}{3\sqrt{a}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/a**(1/2),x)
[Out] -1/(3*sqrt(a)*x**3)
```

3.1039 $\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticF(x,1/3*I*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}(x)|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 2*x^2 - x^4],x]
```

```
[Out] EllipticF[ArcSin[x], -1/3]/Sqrt[3]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx &= 2\int \frac{1}{\sqrt{2-2x^2}\,\sqrt{6+2x^2}}\,dx\\ &= \frac{F\left(\sin^{-1}(x)|-\frac{1}{3}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 18, normalized size = 1.50

$$-iF\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 2*x^2 - x^4],x]
[Out] (-I)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3]
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4-2\,x^2+3}}{x^4+2\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^4 - 2*x^2 + 3)/(x^4 + 2*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4-2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4-2*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-x^4 - 2*x^2 + 3), x)
```

maple [B] time = 0.01, size = 43, normalized size = 3.58

$$\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\operatorname{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4-2*x^2+3)^(1/2),x)
[Out] 1/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*EllipticF(x,1/3*I*3
^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4 - 2\,x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4-2*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-x^4 - 2*x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{-x^4 - 2\,x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3 - x^4 - 2*x^2)^(1/2),x)
[Out] int(1/(3 - x^4 - 2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4 - 2x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4-2*x**2+3)**(1/2),x)
[Out] Integral(1/sqrt(-x**4 - 2*x**2 + 3), x)
```

3.1040 $\int \frac{1}{\sqrt{-1+5x^2-x^4}}\,dx$

Optimal. Leaf size=39

$$-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}\,x\right)|\frac{1}{42}\left(21+5\sqrt{21}\right)\right)}{\sqrt[4]{21}}$$

```
[Out] -1/21*(x^2/(5+21^(1/2)))^(1/2)/x*(5+21^(1/2))^(1/2)*EllipticF((1-2*x^2/(5+2
1^(1/2)))^(1/2),1/42*(882+210*21^(1/2))^(1/2))*21^(3/4)
```

Rubi [A] time = 0.07, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 420}

$$-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}\,x\right)|\frac{1}{42}\left(21+5\sqrt{21}\right)\right)}{\sqrt[4]{21}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 + 5*x^2 - x^4],x]
```

```
[Out] -(EllipticF[ArcCos[Sqrt[2/(5 + Sqrt[21])]*x], (21 + 5*Sqrt[21])/42]/21^(1/4
))
```

Rule 420

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> -
Simp[EllipticF[ArcCos[Rt[-(d/c), 2]*x], (b*c)/(b*c - a*d)]/(Sqrt[c]*Rt[-(d/
c), 2]*Sqrt[a - (b*c)/d]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ
[c, 0] && GtQ[a - (b*c)/d, 0]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+5x^2-x^4}}\,dx = 2\int \frac{1}{\sqrt{5+\sqrt{21}-2x^2}\,\sqrt{-5+\sqrt{21}+2x^2}}\,dx$$
$$= -\frac{F\left(\cos^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}\,x\right)\middle|\frac{1}{42}\left(21+5\sqrt{21}\right)\right)}{\sqrt[4]{21}}$$

Mathematica [B] time = 0.11, size = 87, normalized size = 2.23

$$\frac{\sqrt{-2x^2-\sqrt{21}+5}\,\sqrt{\left(\sqrt{21}-5\right)x^2+2}\,F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}\,x\right)\middle|\frac{23}{2}-\frac{5\sqrt{21}}{2}\right)}{2\sqrt{-x^4+5x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + 5*x^2 - x^4],x]
```

```
[Out] (Sqrt[5 - Sqrt[21] - 2*x^2]*Sqrt[2 + (-5 + Sqrt[21])*x^2]*EllipticF[ArcSin[
Sqrt[(5 + Sqrt[21])/2]*x], 23/2 - (5*Sqrt[21])/2])/(2*Sqrt[-1 + 5*x^2 - x^4
])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+5\,x^2-1}}{x^4-5\,x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+5*x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 5*x^2 - 1)/(x^4 - 5*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+5\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+5*x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^4 + 5*x^2 - 1), x)
```

maple [A] time = 0.08, size = 82, normalized size = 2.10

$$\frac{\sqrt{-\left(\frac{5}{2}-\frac{\sqrt{21}}{2}\right)x^{2}+1}\,\sqrt{-\left(\frac{5}{2}+\frac{\sqrt{21}}{2}\right)x^{2}+1}\,\operatorname{EllipticF}\left(\left(\frac{\sqrt{7}}{2}-\frac{\sqrt{3}}{2}\right)x,\frac{5}{2}+\frac{\sqrt{21}}{2}\right)}{\left(\frac{\sqrt{7}}{2}-\frac{\sqrt{3}}{2}\right)\sqrt{-x^{4}+5x^{2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+5*x^2-1)^(1/2),x)
```

```
[Out] 1/(1/2*7^(1/2)-1/2*3^(1/2))*(1-(5/2-1/2*21^(1/2))*x^2)^(1/2)*(1-(5/2+1/2*21
^(1/2))*x^2)^(1/2)/(-x^4+5*x^2-1)^(1/2)*EllipticF(x*(1/2*7^(1/2)-1/2*3^(1/2
)),5/2+1/2*21^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^{4}+5\,x^{2}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+5*x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-x^4 + 5*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{-x^{4}+5\,x^{2}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - x^4 - 1)^(1/2),x)
```

```
[Out] int(1/(5*x^2 - x^4 - 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^{4}+5x^{2}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+5*x**2-1)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**4 + 5*x**2 - 1), x)
```

3.1041 $\int x^{5/2}\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=31

$$\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}+\frac{2}{15}cx^{15/2}$$

```
[Out] 2/7*a*x^(7/2)+2/11*b*x^(11/2)+2/15*c*x^(15/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}+\frac{2}{15}cx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*a*x^(7/2))/7 + (2*b*x^(11/2))/11 + (2*c*x^(15/2))/15
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2+cx^4\right)\,dx &= \int\left(ax^{5/2}+bx^{9/2}+cx^{13/2}\right)\,dx\\ &= \frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}+\frac{2}{15}cx^{15/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.81

$$\frac{2x^{7/2}\left(165a+105bx^2+77cx^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(7/2)*(165*a + 105*b*x^2 + 77*c*x^4))/1155
```

fricas [**A**] time = 0.75, size = 24, normalized size = 0.77

$$\frac{2}{1155}\left(77\,cx^{7}+105\,bx^{5}+165\,ax^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] 2/1155*(77*c*x^7 + 105*b*x^5 + 165*a*x^3)*sqrt(x)
```

giac [**A**] time = 0.16, size = 19, normalized size = 0.61

$$\frac{2}{15}\,cx^{\frac{15}{2}}+\frac{2}{11}\,bx^{\frac{11}{2}}+\frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 2/15*c*x^(15/2) + 2/11*b*x^(11/2) + 2/7*a*x^(7/2)
```

maple [**A**] time = 0.00, size = 22, normalized size = 0.71

$$\frac{2\left(77c\,x^{4}+105b\,x^{2}+165a\right)x^{\frac{7}{2}}}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2+a),x)
[Out] 2/1155*x^(7/2)*(77*c*x^4+105*b*x^2+165*a)
```

maxima [**A**] time = 0.98, size = 19, normalized size = 0.61

$$\frac{2}{15}\,cx^{\frac{15}{2}}+\frac{2}{11}\,bx^{\frac{11}{2}}+\frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] 2/15*c*x^(15/2) + 2/11*b*x^(11/2) + 2/7*a*x^(7/2)
```

mupad [**B**] time = 4.29, size = 21, normalized size = 0.68

$$\frac{2\,x^{7/2}\,\left(77\,c\,x^4+105\,b\,x^2+165\,a\right)}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(7/2)*(165*a + 105*b*x^2 + 77*c*x^4))/1155
```

sympy [A] time = 6.72, size = 29, normalized size = 0.94

$$\frac{2ax^{\frac{7}{2}}}{7} + \frac{2bx^{\frac{11}{2}}}{11} + \frac{2cx^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] 2*a*x**(7/2)/7 + 2*b*x**(11/2)/11 + 2*c*x**(15/2)/15
```

3.1042 $\int x^{3/2}\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=31

$$\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}+\frac{2}{13}cx^{13/2}$$

```
[Out] 2/5*a*x^(5/2)+2/9*b*x^(9/2)+2/13*c*x^(13/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}+\frac{2}{13}cx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*a*x^(5/2))/5 + (2*b*x^(9/2))/9 + (2*c*x^(13/2))/13
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2+cx^4\right)\,dx&=\int\left(ax^{3/2}+bx^{7/2}+cx^{11/2}\right)\,dx\\&=\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}+\frac{2}{13}cx^{13/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.81

$$\frac{2}{585}x^{5/2}\left(117a+65bx^2+45cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(5/2)*(117*a + 65*b*x^2 + 45*c*x^4))/585
```

fricas [A] time = 0.80, size = 24, normalized size = 0.77

$$\frac{2}{585}\left(45\,cx^{6}+65\,bx^{4}+117\,ax^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] 2/585*(45*c*x^6 + 65*b*x^4 + 117*a*x^2)*sqrt(x)
```

giac [A] time = 0.21, size = 19, normalized size = 0.61

$$\frac{2}{13}\,cx^{\frac{13}{2}}+\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 2/13*c*x^(13/2) + 2/9*b*x^(9/2) + 2/5*a*x^(5/2)
```

maple [A] time = 0.00, size = 22, normalized size = 0.71

$$\frac{2\left(45c\,x^{4}+65b\,x^{2}+117a\right)x^{\frac{5}{2}}}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2+a),x)
[Out] 2/585*x^(5/2)*(45*c*x^4+65*b*x^2+117*a)
```

maxima [A] time = 1.04, size = 19, normalized size = 0.61

$$\frac{2}{13}\,cx^{\frac{13}{2}}+\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] 2/13*c*x^(13/2) + 2/9*b*x^(9/2) + 2/5*a*x^(5/2)
```

mupad [B] time = 0.04, size = 21, normalized size = 0.68

$$\frac{2\,x^{5/2}\,\left(45\,c\,x^4+65\,b\,x^2+117\,a\right)}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(5/2)*(117*a + 65*b*x^2 + 45*c*x^4))/585
```

sympy [A] time = 2.67, size = 29, normalized size = 0.94

$$\frac{2ax^{\frac{5}{2}}}{5} + \frac{2bx^{\frac{9}{2}}}{9} + \frac{2cx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] 2*a*x**(5/2)/5 + 2*b*x**(9/2)/9 + 2*c*x**(13/2)/13
```

3.1043 $\int \sqrt{x} \left(a+bx^2+cx^4\right) dx$

Optimal. Leaf size=31

$$\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}$$

```
[Out] 2/3*a*x^(3/2)+2/7*b*x^(7/2)+2/11*c*x^(11/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*a*x^(3/2))/3 + (2*b*x^(7/2))/7 + (2*c*x^(11/2))/11
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2+cx^4\right) dx &= \int \left(a\sqrt{x}+bx^{5/2}+cx^{9/2}\right) dx\\ &= \frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.81

$$\frac{2}{231}x^{3/2}\left(77a+33bx^2+21cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(3/2)*(77*a + 33*b*x^2 + 21*c*x^4))/231
```

fricas [A] time = 0.76, size = 22, normalized size = 0.71

$$\frac{2}{231} \left(21 \, c x^{5} + 33 \, b x^{3} + 77 \, a x\right) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] 2/231*(21*c*x^5 + 33*b*x^3 + 77*a*x)*sqrt(x)
```

giac [A] time = 0.17, size = 19, normalized size = 0.61

$$\frac{2}{11} \, c x^{\frac{11}{2}} + \frac{2}{7} \, b x^{\frac{7}{2}} + \frac{2}{3} \, a x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 2/11*c*x^(11/2) + 2/7*b*x^(7/2) + 2/3*a*x^(3/2)
```

maple [A] time = 0.00, size = 22, normalized size = 0.71

$$\frac{2 \left(21 c \,x^{4}+33 b \,x^{2}+77 a \right) x^{\frac{3}{2}}}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2+a),x)
[Out] 2/231*x^(3/2)*(21*c*x^4+33*b*x^2+77*a)
```

maxima [A] time = 1.01, size = 19, normalized size = 0.61

$$\frac{2}{11} \, c x^{\frac{11}{2}} + \frac{2}{7} \, b x^{\frac{7}{2}} + \frac{2}{3} \, a x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] 2/11*c*x^(11/2) + 2/7*b*x^(7/2) + 2/3*a*x^(3/2)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.68

$$\frac{2\,x^{3/2}\,\left(21\,c\,x^4+33\,b\,x^2+77\,a\right)}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(3/2)*(77*a + 33*b*x^2 + 21*c*x^4))/231
```

sympy [A] time = 2.10, size = 29, normalized size = 0.94

$$\frac{2ax^{\frac{3}{2}}}{3}+\frac{2bx^{\frac{7}{2}}}{7}+\frac{2cx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] 2*a*x**(3/2)/3 + 2*b*x**(7/2)/7 + 2*c*x**(11/2)/11
```

3.1044 $\int \frac{a+bx^2+cx^4}{\sqrt{x}}\,dx$

Optimal. Leaf size=29

$$2a\sqrt{x}+\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

```
[Out] 2/5*b*x^(5/2)+2/9*c*x^(9/2)+2*a*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$2a\sqrt{x}+\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/Sqrt[x],x]
```

```
[Out] 2*a*Sqrt[x] + (2*b*x^(5/2))/5 + (2*c*x^(9/2))/9
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{\sqrt{x}}\,dx &= \int \left(\frac{a}{\sqrt{x}}+bx^{3/2}+cx^{7/2}\right)dx\\ &= 2a\sqrt{x}+\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.86

$$\frac{2}{45}\sqrt{x}\left(45a+9bx^2+5cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(45*a + 9*b*x^2 + 5*c*x^4))/45
```

fricas [A] time = 0.50, size = 21, normalized size = 0.72

$$\frac{2}{45}\left(5\,cx^4+9\,bx^2+45\,a\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(1/2),x, algorithm="fricas")
[Out] 2/45*(5*c*x^4 + 9*b*x^2 + 45*a)*sqrt(x)
```

giac [A] time = 0.15, size = 19, normalized size = 0.66

$$\frac{2}{9}\,cx^{\frac{9}{2}}+\frac{2}{5}\,bx^{\frac{5}{2}}+2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(1/2),x, algorithm="giac")
[Out] 2/9*c*x^(9/2) + 2/5*b*x^(5/2) + 2*a*sqrt(x)
```

maple [A] time = 0.00, size = 22, normalized size = 0.76

$$\frac{2\left(5c\,x^4+9b\,x^2+45a\right)\sqrt{x}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^(1/2),x)
[Out] 2/45*x^(1/2)*(5*c*x^4+9*b*x^2+45*a)
```

maxima [A] time = 1.03, size = 19, normalized size = 0.66

$$\frac{2}{9}\,cx^{\frac{9}{2}}+\frac{2}{5}\,bx^{\frac{5}{2}}+2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(1/2),x, algorithm="maxima")
[Out] 2/9*c*x^(9/2) + 2/5*b*x^(5/2) + 2*a*sqrt(x)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.72

$$\frac{2\,\sqrt{x}\,\left(5\,c\,x^4+9\,b\,x^2+45\,a\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^(1/2),x)
```

```
[Out] (2*x^(1/2)*(45*a + 9*b*x^2 + 5*c*x^4))/45
```

sympy [A] time = 0.82, size = 27, normalized size = 0.93

$$2a\sqrt{x} + \frac{2bx^{\frac{5}{2}}}{5} + \frac{2cx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**(1/2),x)
```

```
[Out] 2*a*sqrt(x) + 2*b*x**(5/2)/5 + 2*c*x**(9/2)/9
```

3.1045 $\int \frac{a+bx^2+cx^4}{x^{3/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{2a}{\sqrt{x}}+\frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}$$

```
[Out] 2/3*b*x^(3/2)+2/7*c*x^(7/2)-2*a/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$-\frac{2a}{\sqrt{x}}+\frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^(3/2),x]
```

```
[Out] (-2*a)/Sqrt[x] + (2*b*x^(3/2))/3 + (2*c*x^(7/2))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^{3/2}}\,dx &= \int \left(\frac{a}{x^{3/2}}+b\sqrt{x}+cx^{5/2}\right)dx\\ &= -\frac{2a}{\sqrt{x}}+\frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.86

$$\frac{2\left(-21a+7bx^2+3cx^4\right)}{21\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^(3/2),x]
```

```
[Out] (2*(-21*a + 7*b*x^2 + 3*c*x^4))/(21*Sqrt[x])
```

fricas [A] time = 0.75, size = 21, normalized size = 0.72

$$\frac{2\left(3\,cx^4 + 7\,bx^2 - 21\,a\right)}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(3/2),x, algorithm="fricas")
[Out] 2/21*(3*c*x^4 + 7*b*x^2 - 21*a)/sqrt(x)
```

giac [A] time = 0.15, size = 19, normalized size = 0.66

$$\frac{2}{7}\,cx^{\frac{7}{2}} + \frac{2}{3}\,bx^{\frac{3}{2}} - \frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(3/2),x, algorithm="giac")
[Out] 2/7*c*x^(7/2) + 2/3*b*x^(3/2) - 2*a/sqrt(x)
```

maple [A] time = 0.00, size = 22, normalized size = 0.76

$$-\frac{2\left(-3c\,x^4 - 7b\,x^2 + 21a\right)}{21\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^(3/2),x)
[Out] -2/21*(-3*c*x^4-7*b*x^2+21*a)/x^(1/2)
```

maxima [A] time = 1.09, size = 19, normalized size = 0.66

$$\frac{2}{7}\,cx^{\frac{7}{2}} + \frac{2}{3}\,bx^{\frac{3}{2}} - \frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(3/2),x, algorithm="maxima")
[Out] 2/7*c*x^(7/2) + 2/3*b*x^(3/2) - 2*a/sqrt(x)
```

mupad [B] time = 0.04, size = 21, normalized size = 0.72

$$\frac{6\,c\,x^4+14\,b\,x^2-42\,a}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^(3/2),x)
```

```
[Out] (14*b*x^2 - 42*a + 6*c*x^4)/(21*x^(1/2))
```

sympy [A] time = 1.04, size = 27, normalized size = 0.93

$$-\frac{2a}{\sqrt{x}}+\frac{2bx^{\frac{3}{2}}}{3}+\frac{2cx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**(3/2),x)
```

```
[Out] -2*a/sqrt(x) + 2*b*x**(3/2)/3 + 2*c*x**(7/2)/7
```

3.1046 $\int \frac{a+bx^2+cx^4}{x^{5/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{2a}{3x^{3/2}}+2b\sqrt{x}+\frac{2}{5}cx^{5/2}$$

```
[Out] -2/3*a/x^(3/2)+2/5*c*x^(5/2)+2*b*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$-\frac{2a}{3x^{3/2}}+2b\sqrt{x}+\frac{2}{5}cx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^(5/2),x]
```

```
[Out] (-2*a)/(3*x^(3/2)) + 2*b*Sqrt[x] + (2*c*x^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^{5/2}}\,dx &= \int \left(\frac{a}{x^{5/2}}+\frac{b}{\sqrt{x}}+cx^{3/2}\right)dx\\ &= -\frac{2a}{3x^{3/2}}+2b\sqrt{x}+\frac{2}{5}cx^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.86

$$\frac{2\left(-5a+15bx^2+3cx^4\right)}{15x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^(5/2),x]
```

```
[Out] (2*(-5*a + 15*b*x^2 + 3*c*x^4))/(15*x^(3/2))
```

fricas [A] time = 0.59, size = 21, normalized size = 0.72

$$\frac{2\left(3\,cx^{4}+15\,bx^{2}-5\,a\right)}{15\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(5/2),x, algorithm="fricas")
[Out] 2/15*(3*c*x^4 + 15*b*x^2 - 5*a)/x^(3/2)
```

giac [A] time = 0.33, size = 19, normalized size = 0.66

$$\frac{2}{5}\,cx^{\frac{5}{2}}+2\,b\sqrt{x}-\frac{2\,a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(5/2),x, algorithm="giac")
[Out] 2/5*c*x^(5/2) + 2*b*sqrt(x) - 2/3*a/x^(3/2)
```

maple [A] time = 0.00, size = 22, normalized size = 0.76

$$-\frac{2\left(-3c\,x^{4}-15b\,x^{2}+5a\right)}{15x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^(5/2),x)
[Out] -2/15*(-3*c*x^4-15*b*x^2+5*a)/x^(3/2)
```

maxima [A] time = 1.02, size = 19, normalized size = 0.66

$$\frac{2}{5}\,cx^{\frac{5}{2}}+2\,b\sqrt{x}-\frac{2\,a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(5/2),x, algorithm="maxima")
[Out] 2/5*c*x^(5/2) + 2*b*sqrt(x) - 2/3*a/x^(3/2)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.72

$$\frac{6\,c\,x^{4}+30\,b\,x^{2}-10\,a}{15\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^(5/2),x)
```

```
[Out] (30*b*x^2 - 10*a + 6*c*x^4)/(15*x^(3/2))
```

sympy [**A**] time = 1.28, size = 27, normalized size = 0.93

$$-\frac{2a}{3x^{\frac{3}{2}}}+2b\sqrt{x}+\frac{2cx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**(5/2),x)
```

```
[Out] -2*a/(3*x**(3/2)) + 2*b*sqrt(x) + 2*c*x**(5/2)/5
```

3.1047 $\int \frac{a+bx^2+cx^4}{x^{7/2}} \, dx$

Optimal. Leaf size=29

$$-\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}+\frac{2}{3}cx^{3/2}$$

```
[Out] -2/5*a/x^(5/2)+2/3*c*x^(3/2)-2*b/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$-\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}+\frac{2}{3}cx^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^(7/2),x]
```

```
[Out] (-2*a)/(5*x^(5/2)) - (2*b)/Sqrt[x] + (2*c*x^(3/2))/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^{7/2}} \, dx &= \int \left(\frac{a}{x^{7/2}}+\frac{b}{x^{3/2}}+c\sqrt{x}\right) dx\\ &= -\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}+\frac{2}{3}cx^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.86

$$\frac{2\left(-3a-15bx^2+5cx^4\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^(7/2),x]
```

```
[Out] (2*(-3*a - 15*b*x^2 + 5*c*x^4))/(15*x^(5/2))
```

fricas [A] time = 0.80, size = 21, normalized size = 0.72

$$\frac{2\left(5\,cx^{4}-15\,bx^{2}-3\,a\right)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(7/2),x, algorithm="fricas")
[Out] 2/15*(5*c*x^4 - 15*b*x^2 - 3*a)/x^(5/2)
```

giac [A] time = 0.21, size = 20, normalized size = 0.69

$$\frac{2}{3}\,cx^{\frac{3}{2}}-\frac{2\left(5\,bx^{2}+a\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(7/2),x, algorithm="giac")
[Out] 2/3*c*x^(3/2) - 2/5*(5*b*x^2 + a)/x^(5/2)
```

maple [A] time = 0.00, size = 22, normalized size = 0.76

$$-\frac{2\left(-5c\,x^{4}+15b\,x^{2}+3a\right)}{15x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^(7/2),x)
[Out] -2/15*(-5*c*x^4+15*b*x^2+3*a)/x^(5/2)
```

maxima [A] time = 1.08, size = 20, normalized size = 0.69

$$\frac{2}{3}\,cx^{\frac{3}{2}}-\frac{2\left(5\,bx^{2}+a\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(7/2),x, algorithm="maxima")
[Out] 2/3*c*x^(3/2) - 2/5*(5*b*x^2 + a)/x^(5/2)
```

mupad [B] time = 4.33, size = 21, normalized size = 0.72

$$-\frac{-10\,c\,x^4+30\,b\,x^2+6\,a}{15\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^(7/2),x)
```

```
[Out] -(6*a + 30*b*x^2 - 10*c*x^4)/(15*x^(5/2))
```

sympy [A] time = 1.87, size = 27, normalized size = 0.93

$$-\frac{2a}{5x^{\frac{5}{2}}}-\frac{2b}{\sqrt{x}}+\frac{2cx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**(7/2),x)
```

```
[Out] -2*a/(5*x**(5/2)) - 2*b/sqrt(x) + 2*c*x**(3/2)/3
```

3.1048 $\int x^{5/2} \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=64

$$\frac{2}{7}a^2x^{7/2}+\frac{2}{15}x^{15/2}\left(2ac+b^2\right)+\frac{4}{11}abx^{11/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

```
[Out] 2/7*a^2*x^(7/2)+4/11*a*b*x^(11/2)+2/15*(2*a*c+b^2)*x^(15/2)+4/19*b*c*x^(19/
2)+2/23*c^2*x^(23/2)
```

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$\frac{2}{7}a^2x^{7/2}+\frac{2}{15}x^{15/2}\left(2ac+b^2\right)+\frac{4}{11}abx^{11/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a^2*x^(7/2))/7 + (4*a*b*x^(11/2))/11 + (2*(b^2 + 2*a*c)*x^(15/2))/15 + (
4*b*c*x^(19/2))/19 + (2*c^2*x^(23/2))/23
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2+cx^4\right)^2\,dx &= \int \left(a^2x^{5/2}+2abx^{9/2}+\left(b^2+2ac\right)x^{13/2}+2bcx^{17/2}+c^2x^{21/2}\right)dx\\ &= \frac{2}{7}a^2x^{7/2}+\frac{4}{11}abx^{11/2}+\frac{2}{15}\left(b^2+2ac\right)x^{15/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}\end{aligned}$$

Mathematica [A] time = 3.70, size = 64, normalized size = 1.00

$$\frac{2}{7}a^2x^{7/2}+\frac{2}{15}x^{15/2}\left(2ac+b^2\right)+\frac{4}{11}abx^{11/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a^2*x^(7/2))/7 + (4*a*b*x^(11/2))/11 + (2*(b^2 + 2*a*c)*x^(15/2))/15 + (
4*b*c*x^(19/2))/19 + (2*c^2*x^(23/2))/23
```

fricas [A] time = 0.86, size = 49, normalized size = 0.77

$$\frac{2}{504735}\left(21945\,c^2x^{11}+53130\,bcx^9+33649\left(b^2+2\,ac\right)x^7+91770\,abx^5+72105\,a^2x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 2/504735*(21945*c^2*x^11 + 53130*b*c*x^9 + 33649*(b^2 + 2*a*c)*x^7 + 91770*
a*b*x^5 + 72105*a^2*x^3)*sqrt(x)
```

giac [A] time = 0.21, size = 46, normalized size = 0.72

$$\frac{2}{23}\,c^2x^{\frac{23}{2}}+\frac{4}{19}\,bcx^{\frac{19}{2}}+\frac{2}{15}\,b^2x^{\frac{15}{2}}+\frac{4}{15}\,acx^{\frac{15}{2}}+\frac{4}{11}\,abx^{\frac{11}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*b^2*x^(15/2) + 4/15*a*c*x^(15/
2) + 4/11*a*b*x^(11/2) + 2/7*a^2*x^(7/2)
```

maple [A] time = 0.01, size = 49, normalized size = 0.77

$$\frac{2\left(21945c^2x^8+53130bc\,x^6+67298ac\,x^4+33649b^2x^4+91770ab\,x^2+72105a^2\right)x^{\frac{7}{2}}}{504735}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2/504735*x^(7/2)*(21945*c^2*x^8+53130*b*c*x^6+67298*a*c*x^4+33649*b^2*x^4+9
1770*a*b*x^2+72105*a^2)
```

maxima [A] time = 1.03, size = 44, normalized size = 0.69

$$\frac{2}{23}\,c^2x^{\frac{23}{2}}+\frac{4}{19}\,bcx^{\frac{19}{2}}+\frac{2}{15}\left(b^2+2\,ac\right)x^{\frac{15}{2}}+\frac{4}{11}\,abx^{\frac{11}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*(b^2 + 2*a*c)*x^(15/2) + 4/11*
a*b*x^(11/2) + 2/7*a^2*x^(7/2)
```

mupad [B] time = 4.41, size = 45, normalized size = 0.70

$$x^{15/2}\left(\frac{2\,b^2}{15}+\frac{4\,a\,c}{15}\right)+\frac{2\,a^2\,x^{7/2}}{7}+\frac{2\,c^2\,x^{23/2}}{23}+\frac{4\,a\,b\,x^{11/2}}{11}+\frac{4\,b\,c\,x^{19/2}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(15/2)*((4*a*c)/15 + (2*b^2)/15) + (2*a^2*x^(7/2))/7 + (2*c^2*x^(23/2))/2
3 + (4*a*b*x^(11/2))/11 + (4*b*c*x^(19/2))/19
```

sympy [A] time = 22.35, size = 70, normalized size = 1.09

$$\frac{2a^2x^{\frac{7}{2}}}{7}+\frac{4abx^{\frac{11}{2}}}{11}+\frac{4acx^{\frac{15}{2}}}{15}+\frac{2b^2x^{\frac{15}{2}}}{15}+\frac{4bcx^{\frac{19}{2}}}{19}+\frac{2c^2x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(7/2)/7 + 4*a*b*x**(11/2)/11 + 4*a*c*x**(15/2)/15 + 2*b**2*x**(15
/2)/15 + 4*b*c*x**(19/2)/19 + 2*c**2*x**(23/2)/23
```

3.1049 $\int x^{3/2} \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=64

$$\frac{2}{5}a^2x^{5/2}+\frac{2}{13}x^{13/2}\left(2ac+b^2\right)+\frac{4}{9}abx^{9/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}$$

```
[Out] 2/5*a^2*x^(5/2)+4/9*a*b*x^(9/2)+2/13*(2*a*c+b^2)*x^(13/2)+4/17*b*c*x^(17/2)
+2/21*c^2*x^(21/2)
```

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$\frac{2}{5}a^2x^{5/2}+\frac{2}{13}x^{13/2}\left(2ac+b^2\right)+\frac{4}{9}abx^{9/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a^2*x^(5/2))/5 + (4*a*b*x^(9/2))/9 + (2*(b^2 + 2*a*c)*x^(13/2))/13 + (4*
b*c*x^(17/2))/17 + (2*c^2*x^(21/2))/21
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] && !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2+cx^4\right)^2 \, dx &= \int \left(a^2x^{3/2}+2abx^{7/2}+\left(b^2+2ac\right)x^{11/2}+2bcx^{15/2}+c^2x^{19/2}\right) dx\\ &= \frac{2}{5}a^2x^{5/2}+\frac{4}{9}abx^{9/2}+\frac{2}{13}\left(b^2+2ac\right)x^{13/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}\end{aligned}$$

Mathematica [A] time = 0.06, size = 66, normalized size = 1.03

$$2\left(\frac{1}{5}a^2x^{5/2}+\frac{1}{13}x^{13/2}\left(2ac+b^2\right)+\frac{2}{9}abx^{9/2}+\frac{2}{17}bcx^{17/2}+\frac{1}{21}c^2x^{21/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] 2*((a^2*x^(5/2))/5 + (2*a*b*x^(9/2))/9 + ((b^2 + 2*a*c)*x^(13/2))/13 + (2*b
*c*x^(17/2))/17 + (c^2*x^(21/2))/21)
```

fricas [A] time = 0.80, size = 49, normalized size = 0.77

$$\frac{2}{69615}\left(3315\,c^2x^{10} + 8190\,bcx^8 + 5355\left(b^2 + 2\,ac\right)x^6 + 15470\,abx^4 + 13923\,a^2x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*c^2*x^10 + 8190*b*c*x^8 + 5355*(b^2 + 2*a*c)*x^6 + 15470*a*b*
x^4 + 13923*a^2*x^2)*sqrt(x)
```

giac [A] time = 0.17, size = 46, normalized size = 0.72

$$\frac{2}{21}\,c^2x^{\frac{21}{2}} + \frac{4}{17}\,bcx^{\frac{17}{2}} + \frac{2}{13}\,b^2x^{\frac{13}{2}} + \frac{4}{13}\,acx^{\frac{13}{2}} + \frac{4}{9}\,abx^{\frac{9}{2}} + \frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*b^2*x^(13/2) + 4/13*a*c*x^(13/
2) + 4/9*a*b*x^(9/2) + 2/5*a^2*x^(5/2)
```

maple [A] time = 0.01, size = 49, normalized size = 0.77

$$\frac{2\left(3315c^2x^8 + 8190bc\,x^6 + 10710ac\,x^4 + 5355b^2x^4 + 15470ab\,x^2 + 13923a^2\right)x^{\frac{5}{2}}}{69615}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2/69615*x^(5/2)*(3315*c^2*x^8+8190*b*c*x^6+10710*a*c*x^4+5355*b^2*x^4+15470
*a*b*x^2+13923*a^2)
```

maxima [A] time = 1.16, size = 44, normalized size = 0.69

$$\frac{2}{21}\,c^2x^{\frac{21}{2}} + \frac{4}{17}\,bcx^{\frac{17}{2}} + \frac{2}{13}\left(b^2 + 2\,ac\right)x^{\frac{13}{2}} + \frac{4}{9}\,abx^{\frac{9}{2}} + \frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*(b^2 + 2*a*c)*x^(13/2) + 4/9*a
*b*x^(9/2) + 2/5*a^2*x^(5/2)
```

mupad [B] time = 0.03, size = 45, normalized size = 0.70

$$x^{13/2}\left(\frac{2\,b^2}{13}+\frac{4\,a\,c}{13}\right)+\frac{2\,a^2\,x^{5/2}}{5}+\frac{2\,c^2\,x^{21/2}}{21}+\frac{4\,a\,b\,x^{9/2}}{9}+\frac{4\,b\,c\,x^{17/2}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(13/2)*((4*a*c)/13 + (2*b^2)/13) + (2*a^2*x^(5/2))/5 + (2*c^2*x^(21/2))/2
1 + (4*a*b*x^(9/2))/9 + (4*b*c*x^(17/2))/17
```

sympy [A] time = 12.36, size = 70, normalized size = 1.09

$$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4abx^{\frac{9}{2}}}{9}+\frac{4acx^{\frac{13}{2}}}{13}+\frac{2b^2x^{\frac{13}{2}}}{13}+\frac{4bcx^{\frac{17}{2}}}{17}+\frac{2c^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 4*a*c*x**(13/2)/13 + 2*b**2*x**(13/2
)/13 + 4*b*c*x**(17/2)/17 + 2*c**2*x**(21/2)/21
```

3.1050 $\int \sqrt{x} \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=64

$$\frac{2}{3}a^2x^{3/2}+\frac{2}{11}x^{11/2}\left(2ac+b^2\right)+\frac{4}{7}abx^{7/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}$$

```
[Out] 2/3*a^2*x^(3/2)+4/7*a*b*x^(7/2)+2/11*(2*a*c+b^2)*x^(11/2)+4/15*b*c*x^(15/2)
+2/19*c^2*x^(19/2)
```

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$\frac{2}{3}a^2x^{3/2}+\frac{2}{11}x^{11/2}\left(2ac+b^2\right)+\frac{4}{7}abx^{7/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a^2*x^(3/2))/3 + (4*a*b*x^(7/2))/7 + (2*(b^2 + 2*a*c)*x^(11/2))/11 + (4*
b*c*x^(15/2))/15 + (2*c^2*x^(19/2))/19
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2+cx^4\right)^2\,dx &= \int \left(a^2\sqrt{x}+2abx^{5/2}+\left(b^2+2ac\right)x^{9/2}+2bcx^{13/2}+c^2x^{17/2}\right)dx\\ &= \frac{2}{3}a^2x^{3/2}+\frac{4}{7}abx^{7/2}+\frac{2}{11}\left(b^2+2ac\right)x^{11/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}\end{aligned}$$

Mathematica [A] time = 3.38, size = 50, normalized size = 0.78

$$\frac{2x^{3/2}\left(7315a^2+1995x^4\left(2ac+b^2\right)+6270abx^2+2926bcx^6+1155c^2x^8\right)}{21945}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(3/2)*(7315*a^2 + 6270*a*b*x^2 + 1995*(b^2 + 2*a*c)*x^4 + 2926*b*c*x^6
 + 1155*c^2*x^8))/21945
```

fricas [A] time = 0.87, size = 47, normalized size = 0.73

$$\frac{2}{21945}\left(1155\,c^2x^9+2926\,bcx^7+1995\left(b^2+2\,ac\right)x^5+6270\,abx^3+7315\,a^2x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*c^2*x^9 + 2926*b*c*x^7 + 1995*(b^2 + 2*a*c)*x^5 + 6270*a*b*x^
3 + 7315*a^2*x)*sqrt(x)
```

giac [A] time = 0.15, size = 46, normalized size = 0.72

$$\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{15}\,bcx^{\frac{15}{2}}+\frac{2}{11}\,b^2x^{\frac{11}{2}}+\frac{4}{11}\,acx^{\frac{11}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*b^2*x^(11/2) + 4/11*a*c*x^(11/
2) + 4/7*a*b*x^(7/2) + 2/3*a^2*x^(3/2)
```

maple [A] time = 0.01, size = 49, normalized size = 0.77

$$\frac{2\left(1155c^2x^8+2926bc\,x^6+3990ac\,x^4+1995b^2x^4+6270ab\,x^2+7315a^2\right)x^{\frac{3}{2}}}{21945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2/21945*x^(3/2)*(1155*c^2*x^8+2926*b*c*x^6+3990*a*c*x^4+1995*b^2*x^4+6270*a
*b*x^2+7315*a^2)
```

maxima [A] time = 1.11, size = 44, normalized size = 0.69

$$\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{15}\,bcx^{\frac{15}{2}}+\frac{2}{11}\left(b^2+2\,ac\right)x^{\frac{11}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*(b^2 + 2*a*c)*x^(11/2) + 4/7*a
*b*x^(7/2) + 2/3*a^2*x^(3/2)
```

mupad [B] time = 0.03, size = 45, normalized size = 0.70

$$x^{11/2}\left(\frac{2\,b^2}{11}+\frac{4\,a\,c}{11}\right)+\frac{2\,a^2\,x^{3/2}}{3}+\frac{2\,c^2\,x^{19/2}}{19}+\frac{4\,a\,b\,x^{7/2}}{7}+\frac{4\,b\,c\,x^{15/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(11/2)*((4*a*c)/11 + (2*b^2)/11) + (2*a^2*x^(3/2))/3 + (2*c^2*x^(19/2))/1
9 + (4*a*b*x^(7/2))/7 + (4*b*c*x^(15/2))/15
```

sympy [A] time = 3.45, size = 63, normalized size = 0.98

$$\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4abx^{\frac{7}{2}}}{7}+\frac{4bcx^{\frac{15}{2}}}{15}+\frac{2c^2x^{\frac{19}{2}}}{19}+\frac{2x^{\frac{11}{2}}\left(2ac+b^2\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(3/2)/3 + 4*a*b*x**(7/2)/7 + 4*b*c*x**(15/2)/15 + 2*c**2*x**(19/2
)/19 + 2*x**(11/2)*(2*a*c + b**2)/11
```

3.1051 $\int \frac{(a+bx^2+cx^4)^2}{\sqrt{x}}\,dx$

Optimal. Leaf size=62

$$2a^2\sqrt{x}+\frac{2}{9}x^{9/2}\left(2ac+b^2\right)+\frac{4}{5}abx^{5/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}$$

```
[Out] 4/5*a*b*x^(5/2)+2/9*(2*a*c+b^2)*x^(9/2)+4/13*b*c*x^(13/2)+2/17*c^2*x^(17/2)
+2*a^2*x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$2a^2\sqrt{x}+\frac{2}{9}x^{9/2}\left(2ac+b^2\right)+\frac{4}{5}abx^{5/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/Sqrt[x],x]
```

```
[Out] 2*a^2*Sqrt[x] + (4*a*b*x^(5/2))/5 + (2*(b^2 + 2*a*c)*x^(9/2))/9 + (4*b*c*x^
(13/2))/13 + (2*c^2*x^(17/2))/17
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{\sqrt{x}}\,dx &= \int \left(\frac{a^2}{\sqrt{x}}+2abx^{3/2}+\left(b^2+2ac\right)x^{7/2}+2bcx^{11/2}+c^2x^{15/2}\right)dx\\ &= 2a^2\sqrt{x}+\frac{4}{5}abx^{5/2}+\frac{2}{9}\left(b^2+2ac\right)x^{9/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 1.02

$$2\left(a^2\sqrt{x}+\frac{1}{9}x^{9/2}\left(2ac+b^2\right)+\frac{2}{5}abx^{5/2}+\frac{2}{13}bcx^{13/2}+\frac{1}{17}c^2x^{17/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/Sqrt[x],x]
```

```
[Out] 2*(a^2*Sqrt[x] + (2*a*b*x^(5/2))/5 + ((b^2 + 2*a*c)*x^(9/2))/9 + (2*b*c*x^(
13/2))/13 + (c^2*x^(17/2))/17)
```

fricas [A] time = 0.84, size = 46, normalized size = 0.74

$$\frac{2}{9945}\left(585\,c^{2}x^{8}+1530\,bcx^{6}+1105\left(b^{2}+2\,ac\right)x^{4}+3978\,abx^{2}+9945\,a^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*c^2*x^8 + 1530*b*c*x^6 + 1105*(b^2 + 2*a*c)*x^4 + 3978*a*b*x^2
+ 9945*a^2)*sqrt(x)
```

giac [A] time = 0.15, size = 46, normalized size = 0.74

$$\frac{2}{17}\,c^{2}x^{\frac{17}{2}}+\frac{4}{13}\,bcx^{\frac{13}{2}}+\frac{2}{9}\,b^{2}x^{\frac{9}{2}}+\frac{4}{9}\,acx^{\frac{9}{2}}+\frac{4}{5}\,abx^{\frac{5}{2}}+2\,a^{2}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2) + 4/9*a*c*x^(9/2) +
 4/5*a*b*x^(5/2) + 2*a^2*sqrt(x)
```

maple [A] time = 0.01, size = 49, normalized size = 0.79

$$\frac{2\left(585c^{2}x^{8}+1530bc\,x^{6}+2210ac\,x^{4}+1105b^{2}x^{4}+3978ab\,x^{2}+9945a^{2}\right)\sqrt{x}}{9945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^(1/2),x)
```

```
[Out] 2/9945*x^(1/2)*(585*c^2*x^8+1530*b*c*x^6+2210*a*c*x^4+1105*b^2*x^4+3978*a*b
*x^2+9945*a^2)
```

maxima [A] time = 1.14, size = 48, normalized size = 0.77

$$\frac{2}{17}\,c^{2}x^{\frac{17}{2}}+\frac{4}{13}\,bcx^{\frac{13}{2}}+\frac{2}{9}\,b^{2}x^{\frac{9}{2}}+2\,a^{2}\sqrt{x}+\frac{4}{45}\left(5\,cx^{\frac{9}{2}}+9\,bx^{\frac{5}{2}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2) + 2*a^2*sqrt(x) + 4
/45*(5*c*x^(9/2) + 9*b*x^(5/2))*a
```

mupad [B] time = 0.03, size = 45, normalized size = 0.73

$$x^{9/2}\left(\frac{2\,b^2}{9}+\frac{4\,a\,c}{9}\right)+2\,a^2\,\sqrt{x}+\frac{2\,c^2\,x^{17/2}}{17}+\frac{4\,a\,b\,x^{5/2}}{5}+\frac{4\,b\,c\,x^{13/2}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^(1/2),x)
```

```
[Out] x^(9/2)*((4*a*c)/9 + (2*b^2)/9) + 2*a^2*x^(1/2) + (2*c^2*x^(17/2))/17 + (4*
a*b*x^(5/2))/5 + (4*b*c*x^(13/2))/13
```

sympy [A] time = 5.00, size = 68, normalized size = 1.10

$$2a^2\sqrt{x}+\frac{4abx^{\frac{5}{2}}}{5}+\frac{4acx^{\frac{9}{2}}}{9}+\frac{2b^2x^{\frac{9}{2}}}{9}+\frac{4bcx^{\frac{13}{2}}}{13}+\frac{2c^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**(1/2),x)
```

```
[Out] 2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 4*a*c*x**(9/2)/9 + 2*b**2*x**(9/2)/9 +
4*b*c*x**(13/2)/13 + 2*c**2*x**(17/2)/17
```

3.1052 $\int \frac{(a+bx^2+cx^4)^2}{x^{3/2}} dx$

Optimal. Leaf size=62

$$-\frac{2a^2}{\sqrt{x}}+\frac{2}{7}x^{7/2}\left(2ac+b^2\right)+\frac{4}{3}abx^{3/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}$$

```
[Out] 4/3*a*b*x^(3/2)+2/7*(2*a*c+b^2)*x^(7/2)+4/11*b*c*x^(11/2)+2/15*c^2*x^(15/2)
-2*a^2/x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$-\frac{2a^2}{\sqrt{x}}+\frac{2}{7}x^{7/2}\left(2ac+b^2\right)+\frac{4}{3}abx^{3/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^(3/2),x]
```

```
[Out] (-2*a^2)/Sqrt[x] + (4*a*b*x^(3/2))/3 + (2*(b^2 + 2*a*c)*x^(7/2))/7 + (4*b*c
*x^(11/2))/11 + (2*c^2*x^(15/2))/15
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{3/2}}\,dx &= \int \left(\frac{a^2}{x^{3/2}}+2ab\sqrt{x}+\left(b^2+2ac\right)x^{5/2}+2bcx^{9/2}+c^2x^{13/2}\right)dx\\ &= -\frac{2a^2}{\sqrt{x}}+\frac{4}{3}abx^{3/2}+\frac{2}{7}\left(b^2+2ac\right)x^{7/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.87

$$\frac{2\left(-1155a^2+110a\left(7bx^2+3cx^4\right)+165b^2x^4+210bcx^6+77c^2x^8\right)}{1155\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^(3/2),x]
```

```
[Out] (2*(-1155*a^2 + 165*b^2*x^4 + 210*b*c*x^6 + 77*c^2*x^8 + 110*a*(7*b*x^2 + 3
*c*x^4)))/(1155*Sqrt[x])
```

fricas [A] time = 0.81, size = 46, normalized size = 0.74

$$\frac{2\left(77\,c^{2}x^{8}+210\,bcx^{6}+165\left(b^{2}+2\,ac\right)x^{4}+770\,abx^{2}-1155\,a^{2}\right)}{1155\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*c^2*x^8 + 210*b*c*x^6 + 165*(b^2 + 2*a*c)*x^4 + 770*a*b*x^2 - 11
55*a^2)/sqrt(x)
```

giac [A] time = 0.15, size = 46, normalized size = 0.74

$$\frac{2}{15}\,c^{2}x^{\frac{15}{2}}+\frac{4}{11}\,bcx^{\frac{11}{2}}+\frac{2}{7}\,b^{2}x^{\frac{7}{2}}+\frac{4}{7}\,acx^{\frac{7}{2}}+\frac{4}{3}\,abx^{\frac{3}{2}}-\frac{2\,a^{2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*b^2*x^(7/2) + 4/7*a*c*x^(7/2) +
 4/3*a*b*x^(3/2) - 2*a^2/sqrt(x)
```

maple [A] time = 0.01, size = 49, normalized size = 0.79

$$-\frac{2\left(-77c^{2}x^{8}-210bc\,x^{6}-330ac\,x^{4}-165b^{2}x^{4}-770ab\,x^{2}+1155a^{2}\right)}{1155\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^(3/2),x)
```

```
[Out] -2/1155*(-77*c^2*x^8-210*b*c*x^6-330*a*c*x^4-165*b^2*x^4-770*a*b*x^2+1155*a
^2)/x^(1/2)
```

maxima [A] time = 1.11, size = 44, normalized size = 0.71

$$\frac{2}{15}\,c^{2}x^{\frac{15}{2}}+\frac{4}{11}\,bcx^{\frac{11}{2}}+\frac{2}{7}\left(b^{2}+2\,ac\right)x^{\frac{7}{2}}+\frac{4}{3}\,abx^{\frac{3}{2}}-\frac{2\,a^{2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*(b^2 + 2*a*c)*x^(7/2) + 4/3*a*b
*x^(3/2) - 2*a^2/sqrt(x)
```

mupad [B] time = 0.03, size = 45, normalized size = 0.73

$$x^{7/2}\left(\frac{2\,b^2}{7}+\frac{4\,a\,c}{7}\right)-\frac{2\,a^2}{\sqrt{x}}+\frac{2\,c^2\,x^{15/2}}{15}+\frac{4\,a\,b\,x^{3/2}}{3}+\frac{4\,b\,c\,x^{11/2}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^(3/2),x)
```

```
[Out] x^(7/2)*((4*a*c)/7 + (2*b^2)/7) - (2*a^2)/x^(1/2) + (2*c^2*x^(15/2))/15 + (
4*a*b*x^(3/2))/3 + (4*b*c*x^(11/2))/11
```

sympy [A] time = 5.65, size = 68, normalized size = 1.10

$$-\frac{2a^2}{\sqrt{x}}+\frac{4abx^{\frac{3}{2}}}{3}+\frac{4acx^{\frac{7}{2}}}{7}+\frac{2b^2x^{\frac{7}{2}}}{7}+\frac{4bcx^{\frac{11}{2}}}{11}+\frac{2c^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**(3/2),x)
```

```
[Out] -2*a**2/sqrt(x) + 4*a*b*x**(3/2)/3 + 4*a*c*x**(7/2)/7 + 2*b**2*x**(7/2)/7 +
 4*b*c*x**(11/2)/11 + 2*c**2*x**(15/2)/15
```

3.1053 $\int \frac{(a+bx^2+cx^4)^2}{x^{5/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2a^2}{3x^{3/2}}+\frac{2}{5}x^{5/2}\left(2ac+b^2\right)+4ab\sqrt{x}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}$$

```
[Out] -2/3*a^2/x^(3/2)+2/5*(2*a*c+b^2)*x^(5/2)+4/9*b*c*x^(9/2)+2/13*c^2*x^(13/2)+
4*a*b*x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$-\frac{2a^2}{3x^{3/2}}+\frac{2}{5}x^{5/2}\left(2ac+b^2\right)+4ab\sqrt{x}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^(5/2),x]
```

```
[Out] (-2*a^2)/(3*x^(3/2)) + 4*a*b*Sqrt[x] + (2*(b^2 + 2*a*c)*x^(5/2))/5 + (4*b*c
*x^(9/2))/9 + (2*c^2*x^(13/2))/13
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{5/2}}\,dx &= \int \left(\frac{a^2}{x^{5/2}}+\frac{2ab}{\sqrt{x}}+\left(b^2+2ac\right)x^{3/2}+2bcx^{7/2}+c^2x^{11/2}\right)dx\\ &= -\frac{2a^2}{3x^{3/2}}+4ab\sqrt{x}+\frac{2}{5}\left(b^2+2ac\right)x^{5/2}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}\end{aligned}$$

Mathematica [A] time = 0.07, size = 53, normalized size = 0.85

$$\frac{-390a^2+468a\left(5bx^2+cx^4\right)+234b^2x^4+260bcx^6+90c^2x^8}{585x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^(5/2),x]
```

```
[Out] (-390*a^2 + 234*b^2*x^4 + 260*b*c*x^6 + 90*c^2*x^8 + 468*a*(5*b*x^2 + c*x^4
))/(585*x^(3/2))
```

fricas [A] time = 0.72, size = 46, normalized size = 0.74

$$\frac{2\left(45\,c^2x^8+130\,bcx^6+117\left(b^2+2\,ac\right)x^4+1170\,abx^2-195\,a^2\right)}{585\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*c^2*x^8 + 130*b*c*x^6 + 117*(b^2 + 2*a*c)*x^4 + 1170*a*b*x^2 - 19
5*a^2)/x^(3/2)
```

giac [A] time = 0.15, size = 46, normalized size = 0.74

$$\frac{2}{13}\,c^2x^{\frac{13}{2}}+\frac{4}{9}\,bcx^{\frac{9}{2}}+\frac{2}{5}\,b^2x^{\frac{5}{2}}+\frac{4}{5}\,acx^{\frac{5}{2}}+4\,ab\sqrt{x}-\frac{2\,a^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*b^2*x^(5/2) + 4/5*a*c*x^(5/2) + 4
*a*b*sqrt(x) - 2/3*a^2/x^(3/2)
```

maple [A] time = 0.01, size = 49, normalized size = 0.79

$$-\frac{2\left(-45c^2x^8-130bc\,x^6-234ac\,x^4-117b^2x^4-1170ab\,x^2+195a^2\right)}{585x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^(5/2),x)
```

```
[Out] -2/585*(-45*c^2*x^8-130*b*c*x^6-234*a*c*x^4-117*b^2*x^4-1170*a*b*x^2+195*a^
2)/x^(3/2)
```

maxima [A] time = 1.13, size = 44, normalized size = 0.71

$$\frac{2}{13}\,c^2x^{\frac{13}{2}}+\frac{4}{9}\,bcx^{\frac{9}{2}}+\frac{2}{5}\left(b^2+2\,ac\right)x^{\frac{5}{2}}+4\,ab\sqrt{x}-\frac{2\,a^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*(b^2 + 2*a*c)*x^(5/2) + 4*a*b*sqr
t(x) - 2/3*a^2/x^(3/2)
```

mupad [B] time = 0.03, size = 45, normalized size = 0.73

$$x^{5/2}\left(\frac{2\,b^2}{5}+\frac{4\,a\,c}{5}\right)-\frac{2\,a^2}{3\,x^{3/2}}+\frac{2\,c^2\,x^{13/2}}{13}+4\,a\,b\,\sqrt{x}+\frac{4\,b\,c\,x^{9/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^(5/2),x)
```

```
[Out] x^(5/2)*((4*a*c)/5 + (2*b^2)/5) - (2*a^2)/(3*x^(3/2)) + (2*c^2*x^(13/2))/13
 + 4*a*b*x^(1/2) + (4*b*c*x^(9/2))/9
```

sympy [A] time = 6.89, size = 68, normalized size = 1.10

$$-\frac{2a^2}{3x^{\frac{3}{2}}}+4ab\sqrt{x}+\frac{4acx^{\frac{5}{2}}}{5}+\frac{2b^2x^{\frac{5}{2}}}{5}+\frac{4bcx^{\frac{9}{2}}}{9}+\frac{2c^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**(5/2),x)
```

```
[Out] -2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 4*a*c*x**(5/2)/5 + 2*b**2*x**(5/2)/5
 + 4*b*c*x**(9/2)/9 + 2*c**2*x**(13/2)/13
```

3.1054 $\int \frac{(a+bx^2+cx^4)^2}{x^{7/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2a^2}{5x^{5/2}}+\frac{2}{3}x^{3/2}\left(2ac+b^2\right)-\frac{4ab}{\sqrt{x}}+\frac{4}{7}bcx^{7/2}+\frac{2}{11}c^2x^{11/2}$$

```
[Out] -2/5*a^2/x^(5/2)+2/3*(2*a*c+b^2)*x^(3/2)+4/7*b*c*x^(7/2)+2/11*c^2*x^(11/2)-
4*a*b/x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$-\frac{2a^2}{5x^{5/2}}+\frac{2}{3}x^{3/2}\left(2ac+b^2\right)-\frac{4ab}{\sqrt{x}}+\frac{4}{7}bcx^{7/2}+\frac{2}{11}c^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^(7/2),x]
```

```
[Out] (-2*a^2)/(5*x^(5/2)) - (4*a*b)/Sqrt[x] + (2*(b^2 + 2*a*c)*x^(3/2))/3 + (4*b
*c*x^(7/2))/7 + (2*c^2*x^(11/2))/11
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{7/2}}\,dx &= \int \left(\frac{a^2}{x^{7/2}}+\frac{2ab}{x^{3/2}}+\left(b^2+2ac\right)\sqrt{x}+2bcx^{5/2}+c^2x^{9/2}\right)dx\\ &= -\frac{2a^2}{5x^{5/2}}-\frac{4ab}{\sqrt{x}}+\frac{2}{3}\left(b^2+2ac\right)x^{3/2}+\frac{4}{7}bcx^{7/2}+\frac{2}{11}c^2x^{11/2}\end{aligned}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 0.81

$$\frac{2\left(-231a^2+385x^4\left(2ac+b^2\right)-2310abx^2+330bcx^6+105c^2x^8\right)}{1155x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^(7/2),x]
```

```
[Out] (2*(-231*a^2 - 2310*a*b*x^2 + 385*(b^2 + 2*a*c)*x^4 + 330*b*c*x^6 + 105*c^2
*x^8))/(1155*x^(5/2))
```

fricas [A] time = 0.78, size = 46, normalized size = 0.74

$$\frac{2\left(105\,c^2x^8 + 330\,bcx^6 + 385\left(b^2 + 2\,ac\right)x^4 - 2310\,abx^2 - 231\,a^2\right)}{1155\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*c^2*x^8 + 330*b*c*x^6 + 385*(b^2 + 2*a*c)*x^4 - 2310*a*b*x^2 -
231*a^2)/x^(5/2)
```

giac [A] time = 0.16, size = 47, normalized size = 0.76

$$\frac{2}{11}\,c^2x^{\frac{11}{2}} + \frac{4}{7}\,bcx^{\frac{7}{2}} + \frac{2}{3}\,b^2x^{\frac{3}{2}} + \frac{4}{3}\,acx^{\frac{3}{2}} - \frac{2\left(10\,abx^2 + a^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*b^2*x^(3/2) + 4/3*a*c*x^(3/2) - 2
/5*(10*a*b*x^2 + a^2)/x^(5/2)
```

maple [A] time = 0.01, size = 49, normalized size = 0.79

$$-\frac{2\left(-105c^2x^8 - 330bc\,x^6 - 770ac\,x^4 - 385b^2x^4 + 2310ab\,x^2 + 231a^2\right)}{1155x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^(7/2),x)
```

```
[Out] -2/1155*(-105*c^2*x^8-330*b*c*x^6-770*a*c*x^4-385*b^2*x^4+2310*a*b*x^2+231*
a^2)/x^(5/2)
```

maxima [A] time = 1.12, size = 45, normalized size = 0.73

$$\frac{2}{11}\,c^2x^{\frac{11}{2}} + \frac{4}{7}\,bcx^{\frac{7}{2}} + \frac{2}{3}\left(b^2 + 2\,ac\right)x^{\frac{3}{2}} - \frac{2\left(10\,abx^2 + a^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*(b^2 + 2*a*c)*x^(3/2) - 2/5*(10*a
*b*x^2 + a^2)/x^(5/2)
```

mupad [B] time = 0.05, size = 48, normalized size = 0.77

$$x^{3/2}\left(\frac{2\,b^2}{3}+\frac{4\,a\,c}{3}\right)-\frac{\frac{2\,a^2}{5}+4\,b\,a\,x^2}{x^{5/2}}+\frac{2\,c^2\,x^{11/2}}{11}+\frac{4\,b\,c\,x^{7/2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^(7/2),x)
```

```
[Out] x^(3/2)*((4*a*c)/3 + (2*b^2)/3) - ((2*a^2)/5 + 4*a*b*x^2)/x^(5/2) + (2*c^2*
x^(11/2))/11 + (4*b*c*x^(7/2))/7
```

sympy [A] time = 9.11, size = 68, normalized size = 1.10

$$-\frac{2a^2}{5x^{\frac{5}{2}}}-\frac{4ab}{\sqrt{x}}+\frac{4acx^{\frac{3}{2}}}{3}+\frac{2b^2x^{\frac{3}{2}}}{3}+\frac{4bcx^{\frac{7}{2}}}{7}+\frac{2c^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**(7/2),x)
```

```
[Out] -2*a**2/(5*x**(5/2)) - 4*a*b/sqrt(x) + 4*a*c*x**(3/2)/3 + 2*b**2*x**(3/2)/3
 + 4*b*c*x**(7/2)/7 + 2*c**2*x**(11/2)/11
```

3.1055 $\int x^{5/2} \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=103

$$\frac{2}{7}a^3x^{7/2}+\frac{6}{11}a^2bx^{11/2}+\frac{6}{23}cx^{23/2}\left(ac+b^2\right)+\frac{2}{19}bx^{19/2}\left(6ac+b^2\right)+\frac{2}{5}ax^{15/2}\left(ac+b^2\right)+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

```
[Out] 2/7*a^3*x^(7/2)+6/11*a^2*b*x^(11/2)+2/5*a*(a*c+b^2)*x^(15/2)+2/19*b*(6*a*c+
b^2)*x^(19/2)+6/23*c*(a*c+b^2)*x^(23/2)+2/9*b*c^2*x^(27/2)+2/31*c^3*x^(31/2
)
```

Rubi [A] time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$\frac{6}{11}a^2bx^{11/2}+\frac{2}{7}a^3x^{7/2}+\frac{6}{23}cx^{23/2}\left(ac+b^2\right)+\frac{2}{19}bx^{19/2}\left(6ac+b^2\right)+\frac{2}{5}ax^{15/2}\left(ac+b^2\right)+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(7/2))/7 + (6*a^2*b*x^(11/2))/11 + (2*a*(b^2 + a*c)*x^(15/2))/5 +
(2*b*(b^2 + 6*a*c)*x^(19/2))/19 + (6*c*(b^2 + a*c)*x^(23/2))/23 + (2*b*c^2*
x^(27/2))/9 + (2*c^3*x^(31/2))/31
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int x^{5/2}\left(a+bx^2+cx^4\right)^3\,dx=\int\left(a^3x^{5/2}+3a^2bx^{9/2}+3a\left(b^2+ac\right)x^{13/2}+b\left(b^2+6ac\right)x^{17/2}+3c\left(b^2+ac\right)x^{21/2}+\right.$$
$$=\frac{2}{7}a^3x^{7/2}+\frac{6}{11}a^2bx^{11/2}+\frac{2}{5}a\left(b^2+ac\right)x^{15/2}+\frac{2}{19}b\left(b^2+6ac\right)x^{19/2}+\frac{6}{23}c\left(b^2+ac\right)x^{2}$$

Mathematica [A] time = 3.72, size = 103, normalized size = 1.00

$$\frac{2}{7}a^3x^{7/2}+\frac{6}{11}a^2bx^{11/2}+\frac{6}{23}cx^{23/2}\left(ac+b^2\right)+\frac{2}{19}bx^{19/2}\left(6ac+b^2\right)+\frac{2}{5}ax^{15/2}\left(ac+b^2\right)+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(7/2))/7 + (6*a^2*b*x^(11/2))/11 + (2*a*(b^2 + a*c)*x^(15/2))/5 +
(2*b*(b^2 + 6*a*c)*x^(19/2))/19 + (6*c*(b^2 + a*c)*x^(23/2))/23 + (2*b*c^2*
x^(27/2))/9 + (2*c^3*x^(31/2))/31
```

fricas [A] time = 0.57, size = 86, normalized size = 0.83

$$\frac{2}{46940355}\left(1514205\,c^{3}x^{15} + 5215595\,bc^{2}x^{13} + 6122655\left(b^{2}c + ac^{2}\right)x^{11} + 2470545\left(b^{3} + 6\,abc\right)x^{9} + 12801915\,a^{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 2/46940355*(1514205*c^3*x^15 + 5215595*b*c^2*x^13 + 6122655*(b^2*c + a*c^2)
*x^11 + 2470545*(b^3 + 6*a*b*c)*x^9 + 12801915*a^2*b*x^5 + 9388071*(a*b^2 +
 a^2*c)*x^7 + 6705765*a^3*x^3)*sqrt(x)
```

giac [A] time = 0.16, size = 87, normalized size = 0.84

$$\frac{2}{31}\,c^{3}x^{\frac{31}{2}} + \frac{2}{9}\,bc^{2}x^{\frac{27}{2}} + \frac{6}{23}\,b^{2}cx^{\frac{23}{2}} + \frac{6}{23}\,ac^{2}x^{\frac{23}{2}} + \frac{2}{19}\,b^{3}x^{\frac{19}{2}} + \frac{12}{19}\,abcx^{\frac{19}{2}} + \frac{2}{5}\,ab^{2}x^{\frac{15}{2}} + \frac{2}{5}\,a^{2}cx^{\frac{15}{2}} + \frac{6}{11}\,a^{2}bx^{\frac{11}{2}} + \frac{2}{7}\,a^{3}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*b^2*c*x^(23/2) + 6/23*a*c^2*x
^(23/2) + 2/19*b^3*x^(19/2) + 12/19*a*b*c*x^(19/2) + 2/5*a*b^2*x^(15/2) + 2
/5*a^2*c*x^(15/2) + 6/11*a^2*b*x^(11/2) + 2/7*a^3*x^(7/2)
```

maple [A] time = 0.01, size = 90, normalized size = 0.87

$$\frac{2\left(1514205c^{3}x^{12} + 5215595b\,c^{2}x^{10} + 6122655a\,c^{2}x^{8} + 6122655b^{2}c\,x^{8} + 14823270abc\,x^{6} + 2470545b^{3}x^{6} + 9388\right.}{46940355}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2/46940355*x^(7/2)*(1514205*c^3*x^12+5215595*b*c^2*x^10+6122655*a*c^2*x^8+6
122655*b^2*c*x^8+14823270*a*b*c*x^6+2470545*b^3*x^6+9388071*a^2*c*x^4+93880
71*a*b^2*x^4+12801915*a^2*b*x^2+6705765*a^3)
```

maxima [A] time = 1.00, size = 81, normalized size = 0.79

$$\frac{2}{31}\,c^{3}x^{\frac{31}{2}}+\frac{2}{9}\,bc^{2}x^{\frac{27}{2}}+\frac{6}{23}\left(b^{2}c+ac^{2}\right)x^{\frac{23}{2}}+\frac{2}{19}\left(b^{3}+6\,abc\right)x^{\frac{19}{2}}+\frac{6}{11}\,a^{2}bx^{\frac{11}{2}}+\frac{2}{5}\left(ab^{2}+a^{2}c\right)x^{\frac{15}{2}}+\frac{2}{7}\,a^{3}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*(b^2*c + a*c^2)*x^(23/2) + 2/
19*(b^3 + 6*a*b*c)*x^(19/2) + 6/11*a^2*b*x^(11/2) + 2/5*(a*b^2 + a^2*c)*x^(
15/2) + 2/7*a^3*x^(7/2)
```

mupad [B] time = 0.04, size = 76, normalized size = 0.74

$$x^{19/2}\left(\frac{2\,b^{3}}{19}+\frac{12\,a\,c\,b}{19}\right)+\frac{2\,a^{3}\,x^{7/2}}{7}+\frac{2\,c^{3}\,x^{31/2}}{31}+\frac{6\,a^{2}\,b\,x^{11/2}}{11}+\frac{2\,b\,c^{2}\,x^{27/2}}{9}+\frac{2\,a\,x^{15/2}\,\left(b^{2}+a\,c\right)}{5}+\frac{6\,c\,x^{23/2}\,\left(b^{2}+a\,c\right)}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^(19/2)*((2*b^3)/19 + (12*a*b*c)/19) + (2*a^3*x^(7/2))/7 + (2*c^3*x^(31/2)
)/31 + (6*a^2*b*x^(11/2))/11 + (2*b*c^2*x^(27/2))/9 + (2*a*x^(15/2)*(a*c +
b^2))/5 + (6*c*x^(23/2)*(a*c + b^2))/23
```

sympy [A] time = 60.63, size = 129, normalized size = 1.25

$$\frac{2a^{3}x^{\frac{7}{2}}}{7}+\frac{6a^{2}bx^{\frac{11}{2}}}{11}+\frac{2a^{2}cx^{\frac{15}{2}}}{5}+\frac{2ab^{2}x^{\frac{15}{2}}}{5}+\frac{12abcx^{\frac{19}{2}}}{19}+\frac{6ac^{2}x^{\frac{23}{2}}}{23}+\frac{2b^{3}x^{\frac{19}{2}}}{19}+\frac{6b^{2}cx^{\frac{23}{2}}}{23}+\frac{2bc^{2}x^{\frac{27}{2}}}{9}+\frac{2c^{3}x^{\frac{31}{2}}}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(7/2)/7 + 6*a**2*b*x**(11/2)/11 + 2*a**2*c*x**(15/2)/5 + 2*a*b**2
*x**(15/2)/5 + 12*a*b*c*x**(19/2)/19 + 6*a*c**2*x**(23/2)/23 + 2*b**3*x**(1
9/2)/19 + 6*b**2*c*x**(23/2)/23 + 2*b*c**2*x**(27/2)/9 + 2*c**3*x**(31/2)/3
1
```

3.1056 $\int x^{3/2}\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=103

$$\frac{2}{5}a^3x^{5/2}+\frac{2}{3}a^2bx^{9/2}+\frac{2}{7}cx^{21/2}\left(ac+b^2\right)+\frac{2}{17}bx^{17/2}\left(6ac+b^2\right)+\frac{6}{13}ax^{13/2}\left(ac+b^2\right)+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

```
[Out] 2/5*a^3*x^(5/2)+2/3*a^2*b*x^(9/2)+6/13*a*(a*c+b^2)*x^(13/2)+2/17*b*(6*a*c+b
^2)*x^(17/2)+2/7*c*(a*c+b^2)*x^(21/2)+6/25*b*c^2*x^(25/2)+2/29*c^3*x^(29/2)
```

Rubi [A] time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$\frac{2}{3}a^2bx^{9/2}+\frac{2}{5}a^3x^{5/2}+\frac{2}{7}cx^{21/2}\left(ac+b^2\right)+\frac{2}{17}bx^{17/2}\left(6ac+b^2\right)+\frac{6}{13}ax^{13/2}\left(ac+b^2\right)+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(5/2))/5 + (2*a^2*b*x^(9/2))/3 + (6*a*(b^2 + a*c)*x^(13/2))/13 + (
2*b*(b^2 + 6*a*c)*x^(17/2))/17 + (2*c*(b^2 + a*c)*x^(21/2))/7 + (6*b*c^2*x^
(25/2))/25 + (2*c^3*x^(29/2))/29
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2+cx^4\right)^3\,dx &= \int \left(a^3x^{3/2}+3a^2bx^{7/2}+3a\left(b^2+ac\right)x^{11/2}+b\left(b^2+6ac\right)x^{15/2}+3c\left(b^2+ac\right)x^{19/2}+\right.\\ &= \frac{2}{5}a^3x^{5/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{13}a\left(b^2+ac\right)x^{13/2}+\frac{2}{17}b\left(b^2+6ac\right)x^{17/2}+\frac{2}{7}c\left(b^2+ac\right)x^{21}\end{aligned}$$

Mathematica [A] time = 0.10, size = 105, normalized size = 1.02

$$2\left(\frac{1}{5}a^3x^{5/2}+\frac{1}{3}a^2bx^{9/2}+\frac{1}{7}cx^{21/2}\left(ac+b^2\right)+\frac{1}{17}bx^{17/2}\left(6ac+b^2\right)+\frac{3}{13}ax^{13/2}\left(ac+b^2\right)+\frac{3}{25}bc^2x^{25/2}+\frac{1}{29}c^3x^{29/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] 2*((a^3*x^(5/2))/5 + (a^2*b*x^(9/2))/3 + (3*a*(b^2 + a*c)*x^(13/2))/13 + (b
*(b^2 + 6*a*c)*x^(17/2))/17 + (c*(b^2 + a*c)*x^(21/2))/7 + (3*b*c^2*x^(25/2
))/25 + (c^3*x^(29/2))/29)
```

fricas [A] time = 0.72, size = 86, normalized size = 0.83

$$\frac{2}{3364725}\left(116025\,c^3x^{14}+403767\,bc^2x^{12}+480675\left(b^2c+ac^2\right)x^{10}+197925\left(b^3+6\,abc\right)x^8+1121575\,a^2bx^4+77\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 2/3364725*(116025*c^3*x^14 + 403767*b*c^2*x^12 + 480675*(b^2*c + a*c^2)*x^1
0 + 197925*(b^3 + 6*a*b*c)*x^8 + 1121575*a^2*b*x^4 + 776475*(a*b^2 + a^2*c)
*x^6 + 672945*a^3*x^2)*sqrt(x)
```

giac [A] time = 0.17, size = 87, normalized size = 0.84

$$\frac{2}{29}\,c^3x^{\frac{29}{2}}+\frac{6}{25}\,bc^2x^{\frac{25}{2}}+\frac{2}{7}\,b^2cx^{\frac{21}{2}}+\frac{2}{7}\,ac^2x^{\frac{21}{2}}+\frac{2}{17}\,b^3x^{\frac{17}{2}}+\frac{12}{17}\,abcx^{\frac{17}{2}}+\frac{6}{13}\,ab^2x^{\frac{13}{2}}+\frac{6}{13}\,a^2cx^{\frac{13}{2}}+\frac{2}{3}\,a^2bx^{\frac{9}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*b^2*c*x^(21/2) + 2/7*a*c^2*x^
(21/2) + 2/17*b^3*x^(17/2) + 12/17*a*b*c*x^(17/2) + 6/13*a*b^2*x^(13/2) + 6
/13*a^2*c*x^(13/2) + 2/3*a^2*b*x^(9/2) + 2/5*a^3*x^(5/2)
```

maple [A] time = 0.01, size = 90, normalized size = 0.87

$$\frac{2\left(116025c^3x^{12}+403767b\,c^2x^{10}+480675a\,c^2x^8+480675b^2c\,x^8+1187550abc\,x^6+197925b^3x^6+776475a^2c\,x^4\right.}{3364725}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2/3364725*x^(5/2)*(116025*c^3*x^12+403767*b*c^2*x^10+480675*a*c^2*x^8+48067
5*b^2*c*x^8+1187550*a*b*c*x^6+197925*b^3*x^6+776475*a^2*c*x^4+776475*a*b^2*
x^4+1121575*a^2*b*x^2+672945*a^3)
```

maxima [A] time = 1.06, size = 81, normalized size = 0.79

$$\frac{2}{29}\,c^3x^{\frac{29}{2}}+\frac{6}{25}\,bc^2x^{\frac{25}{2}}+\frac{2}{7}\left(b^2c+ac^2\right)x^{\frac{21}{2}}+\frac{2}{17}\left(b^3+6\,abc\right)x^{\frac{17}{2}}+\frac{2}{3}\,a^2bx^{\frac{9}{2}}+\frac{6}{13}\left(ab^2+a^2c\right)x^{\frac{13}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*(b^2*c + a*c^2)*x^(21/2) + 2/
17*(b^3 + 6*a*b*c)*x^(17/2) + 2/3*a^2*b*x^(9/2) + 6/13*(a*b^2 + a^2*c)*x^(1
3/2) + 2/5*a^3*x^(5/2)
```

mupad [B] time = 0.04, size = 76, normalized size = 0.74

$$x^{17/2}\left(\frac{2\,b^3}{17}+\frac{12\,a\,c\,b}{17}\right)+\frac{2\,a^3\,x^{5/2}}{5}+\frac{2\,c^3\,x^{29/2}}{29}+\frac{2\,a^2\,b\,x^{9/2}}{3}+\frac{6\,b\,c^2\,x^{25/2}}{25}+\frac{6\,a\,x^{13/2}\,\left(b^2+a\,c\right)}{13}+\frac{2\,c\,x^{21/2}\,\left(b^2+a\,c\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^(17/2)*((2*b^3)/17 + (12*a*b*c)/17) + (2*a^3*x^(5/2))/5 + (2*c^3*x^(29/2)
)/29 + (2*a^2*b*x^(9/2))/3 + (6*b*c^2*x^(25/2))/25 + (6*a*x^(13/2)*(a*c + b
^2))/13 + (2*c*x^(21/2)*(a*c + b^2))/7
```

sympy [A] time = 39.32, size = 129, normalized size = 1.25

$$\frac{2a^3x^{\frac{5}{2}}}{5}+\frac{2a^2bx^{\frac{9}{2}}}{3}+\frac{6a^2cx^{\frac{13}{2}}}{13}+\frac{6ab^2x^{\frac{13}{2}}}{13}+\frac{12abcx^{\frac{17}{2}}}{17}+\frac{2ac^2x^{\frac{21}{2}}}{7}+\frac{2b^3x^{\frac{17}{2}}}{17}+\frac{2b^2cx^{\frac{21}{2}}}{7}+\frac{6bc^2x^{\frac{25}{2}}}{25}+\frac{2c^3x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(5/2)/5 + 2*a**2*b*x**(9/2)/3 + 6*a**2*c*x**(13/2)/13 + 6*a*b**2*
x**(13/2)/13 + 12*a*b*c*x**(17/2)/17 + 2*a*c**2*x**(21/2)/7 + 2*b**3*x**(17
/2)/17 + 2*b**2*c*x**(21/2)/7 + 6*b*c**2*x**(25/2)/25 + 2*c**3*x**(29/2)/29
```

3.1057 $\int \sqrt{x} \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=103

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{19}cx^{19/2}\left(ac+b^2\right)+\frac{2}{15}bx^{15/2}\left(6ac+b^2\right)+\frac{6}{11}ax^{11/2}\left(ac+b^2\right)+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

```
[Out] 2/3*a^3*x^(3/2)+6/7*a^2*b*x^(7/2)+6/11*a*(a*c+b^2)*x^(11/2)+2/15*b*(6*a*c+b
^2)*x^(15/2)+6/19*c*(a*c+b^2)*x^(19/2)+6/23*b*c^2*x^(23/2)+2/27*c^3*x^(27/2
)
```

Rubi [A] time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$\frac{6}{7}a^2bx^{7/2}+\frac{2}{3}a^3x^{3/2}+\frac{6}{19}cx^{19/2}\left(ac+b^2\right)+\frac{2}{15}bx^{15/2}\left(6ac+b^2\right)+\frac{6}{11}ax^{11/2}\left(ac+b^2\right)+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*b*x^(7/2))/7 + (6*a*(b^2 + a*c)*x^(11/2))/11 + (
2*b*(b^2 + 6*a*c)*x^(15/2))/15 + (6*c*(b^2 + a*c)*x^(19/2))/19 + (6*b*c^2*x
^(23/2))/23 + (2*c^3*x^(27/2))/27
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] && !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2+cx^4\right)^3 \, dx &= \int \left(a^3\sqrt{x}+3a^2bx^{5/2}+3a\left(b^2+ac\right)x^{9/2}+b\left(b^2+6ac\right)x^{13/2}+3c\left(b^2+ac\right)x^{17/2}+3\right.\\ &= \frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{11}a\left(b^2+ac\right)x^{11/2}+\frac{2}{15}b\left(b^2+6ac\right)x^{15/2}+\frac{6}{19}c\left(b^2+ac\right)x^{19/}\end{aligned}$$

Mathematica [A] time = 3.38, size = 103, normalized size = 1.00

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{19}cx^{19/2}\left(ac+b^2\right)+\frac{2}{15}bx^{15/2}\left(6ac+b^2\right)+\frac{6}{11}ax^{11/2}\left(ac+b^2\right)+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*b*x^(7/2))/7 + (6*a*(b^2 + a*c)*x^(11/2))/11 + (
2*b*(b^2 + 6*a*c)*x^(15/2))/15 + (6*c*(b^2 + a*c)*x^(19/2))/19 + (6*b*c^2*x
^(23/2))/23 + (2*c^3*x^(27/2))/27
```

fricas [A] time = 0.61, size = 84, normalized size = 0.82

$$\frac{2}{4542615}\left(168245\,c^3x^{13} + 592515\,bc^2x^{11} + 717255\left(b^2c + ac^2\right)x^9 + 302841\left(b^3 + 6\,abc\right)x^7 + 1946835\,a^2bx^3 + 12\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 2/4542615*(168245*c^3*x^13 + 592515*b*c^2*x^11 + 717255*(b^2*c + a*c^2)*x^9
 + 302841*(b^3 + 6*a*b*c)*x^7 + 1946835*a^2*b*x^3 + 1238895*(a*b^2 + a^2*c)
*x^5 + 1514205*a^3*x)*sqrt(x)
```

giac [A] time = 0.16, size = 87, normalized size = 0.84

$$\frac{2}{27}\,c^3x^{\frac{27}{2}} + \frac{6}{23}\,bc^2x^{\frac{23}{2}} + \frac{6}{19}\,b^2cx^{\frac{19}{2}} + \frac{6}{19}\,ac^2x^{\frac{19}{2}} + \frac{2}{15}\,b^3x^{\frac{15}{2}} + \frac{4}{5}\,abcx^{\frac{15}{2}} + \frac{6}{11}\,ab^2x^{\frac{11}{2}} + \frac{6}{11}\,a^2cx^{\frac{11}{2}} + \frac{6}{7}\,a^2bx^{\frac{7}{2}} + \frac{2}{3}\,a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*b^2*c*x^(19/2) + 6/19*a*c^2*
x^(19/2) + 2/15*b^3*x^(15/2) + 4/5*a*b*c*x^(15/2) + 6/11*a*b^2*x^(11/2) + 6
/11*a^2*c*x^(11/2) + 6/7*a^2*b*x^(7/2) + 2/3*a^3*x^(3/2)
```

maple [A] time = 0.01, size = 90, normalized size = 0.87

$$\frac{2\left(168245c^3x^{12} + 592515b\,c^2x^{10} + 717255a\,c^2x^8 + 717255b^2c\,x^8 + 1817046abc\,x^6 + 302841b^3x^6 + 1238895a^2c\right.}{4542615}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2/4542615*x^(3/2)*(168245*c^3*x^12+592515*b*c^2*x^10+717255*a*c^2*x^8+71725
5*b^2*c*x^8+1817046*a*b*c*x^6+302841*b^3*x^6+1238895*a^2*c*x^4+1238895*a*b^
2*x^4+1946835*a^2*b*x^2+1514205*a^3)
```

maximA [A] time = 1.04, size = 81, normalized size = 0.79

$$\frac{2}{27}\,c^{3}x^{\frac{27}{2}}+\frac{6}{23}\,bc^{2}x^{\frac{23}{2}}+\frac{6}{19}\left(b^{2}c+ac^{2}\right)x^{\frac{19}{2}}+\frac{2}{15}\left(b^{3}+6\,abc\right)x^{\frac{15}{2}}+\frac{6}{7}\,a^{2}bx^{\frac{7}{2}}+\frac{6}{11}\left(ab^{2}+a^{2}c\right)x^{\frac{11}{2}}+\frac{2}{3}\,a^{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*(b^2*c + a*c^2)*x^(19/2) + 2
/15*(b^3 + 6*a*b*c)*x^(15/2) + 6/7*a^2*b*x^(7/2) + 6/11*(a*b^2 + a^2*c)*x^(
11/2) + 2/3*a^3*x^(3/2)
```

mupad [B] time = 0.04, size = 76, normalized size = 0.74

$$x^{15/2}\left(\frac{2\,b^{3}}{15}+\frac{4\,a\,c\,b}{5}\right)+\frac{2\,a^{3}\,x^{3/2}}{3}+\frac{2\,c^{3}\,x^{27/2}}{27}+\frac{6\,a^{2}\,b\,x^{7/2}}{7}+\frac{6\,b\,c^{2}\,x^{23/2}}{23}+\frac{6\,a\,x^{11/2}\,\left(b^{2}+a\,c\right)}{11}+\frac{6\,c\,x^{19/2}\,\left(b^{2}+a\,c\right)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^(15/2)*((2*b^3)/15 + (4*a*b*c)/5) + (2*a^3*x^(3/2))/3 + (2*c^3*x^(27/2))/
27 + (6*a^2*b*x^(7/2))/7 + (6*b*c^2*x^(23/2))/23 + (6*a*x^(11/2)*(a*c + b^2
))/11 + (6*c*x^(19/2)*(a*c + b^2))/19
```

sympy [A] time = 6.05, size = 112, normalized size = 1.09

$$\frac{2a^{3}x^{\frac{3}{2}}}{3}+\frac{6a^{2}bx^{\frac{7}{2}}}{7}+\frac{6bc^{2}x^{\frac{23}{2}}}{23}+\frac{2c^{3}x^{\frac{27}{2}}}{27}+\frac{2x^{\frac{19}{2}}\left(3ac^{2}+3b^{2}c\right)}{19}+\frac{2x^{\frac{15}{2}}\left(6abc+b^{3}\right)}{15}+\frac{2x^{\frac{11}{2}}\left(3a^{2}c+3ab^{2}\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(3/2)/3 + 6*a**2*b*x**(7/2)/7 + 6*b*c**2*x**(23/2)/23 + 2*c**3*x*
*(27/2)/27 + 2*x**(19/2)*(3*a*c**2 + 3*b**2*c)/19 + 2*x**(15/2)*(6*a*b*c +
b**3)/15 + 2*x**(11/2)*(3*a**2*c + 3*a*b**2)/11
```

3.1058 $\int \frac{(a+bx^2+cx^4)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=101

$$2a^3\sqrt{x}+\frac{6}{5}a^2bx^{5/2}+\frac{6}{17}cx^{17/2}\left(ac+b^2\right)+\frac{2}{13}bx^{13/2}\left(6ac+b^2\right)+\frac{2}{3}ax^{9/2}\left(ac+b^2\right)+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

```
[Out] 6/5*a^2*b*x^(5/2)+2/3*a*(a*c+b^2)*x^(9/2)+2/13*b*(6*a*c+b^2)*x^(13/2)+6/17*
c*(a*c+b^2)*x^(17/2)+2/7*b*c^2*x^(21/2)+2/25*c^3*x^(25/2)+2*a^3*x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$\frac{6}{5}a^2bx^{5/2}+2a^3\sqrt{x}+\frac{6}{17}cx^{17/2}\left(ac+b^2\right)+\frac{2}{13}bx^{13/2}\left(6ac+b^2\right)+\frac{2}{3}ax^{9/2}\left(ac+b^2\right)+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/Sqrt[x],x]
```

```
[Out] 2*a^3*Sqrt[x] + (6*a^2*b*x^(5/2))/5 + (2*a*(b^2 + a*c)*x^(9/2))/3 + (2*b*(b
^2 + 6*a*c)*x^(13/2))/13 + (6*c*(b^2 + a*c)*x^(17/2))/17 + (2*b*c^2*x^(21/2
))/7 + (2*c^3*x^(25/2))/25
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^3}{\sqrt{x}}\,dx &= \int \left(\frac{a^3}{\sqrt{x}}+3a^2bx^{3/2}+3a\left(b^2+ac\right)x^{7/2}+b\left(b^2+6ac\right)x^{11/2}+3c\left(b^2+ac\right)x^{15/2}+3bc^2x\right.\\ &= 2a^3\sqrt{x}+\frac{6}{5}a^2bx^{5/2}+\frac{2}{3}a\left(b^2+ac\right)x^{9/2}+\frac{2}{13}b\left(b^2+6ac\right)x^{13/2}+\frac{6}{17}c\left(b^2+ac\right)x^{17/2}+\frac{2}{7}\end{aligned}$$

Mathematica [A] time = 0.07, size = 102, normalized size = 1.01

$$2\left(a^3\sqrt{x}+\frac{3}{5}a^2bx^{5/2}+\frac{3}{17}cx^{17/2}\left(ac+b^2\right)+\frac{1}{13}bx^{13/2}\left(6ac+b^2\right)+\frac{1}{3}ax^{9/2}\left(ac+b^2\right)+\frac{1}{7}bc^2x^{21/2}+\frac{1}{25}c^3x^{25/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/Sqrt[x],x]
```

```
[Out] 2*(a^3*Sqrt[x] + (3*a^2*b*x^(5/2))/5 + (a*(b^2 + a*c)*x^(9/2))/3 + (b*(b^2
+ 6*a*c)*x^(13/2))/13 + (3*c*(b^2 + a*c)*x^(17/2))/17 + (b*c^2*x^(21/2))/7
+ (c^3*x^(25/2))/25)
```

fricas [A] time = 0.83, size = 83, normalized size = 0.82

$$\frac{2}{116025}\left(4641\,c^3x^{12}+16575\,bc^2x^{10}+20475\,\left(b^2c+ac^2\right)x^8+8925\,\left(b^3+6\,abc\right)x^6+69615\,a^2bx^2+38675\,\left(ab^2+a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/116025*(4641*c^3*x^12 + 16575*b*c^2*x^10 + 20475*(b^2*c + a*c^2)*x^8 + 89
25*(b^3 + 6*a*b*c)*x^6 + 69615*a^2*b*x^2 + 38675*(a*b^2 + a^2*c)*x^4 + 1160
25*a^3)*sqrt(x)
```

giac [A] time = 0.17, size = 87, normalized size = 0.86

$$\frac{2}{25}\,c^3x^{\frac{25}{2}}+\frac{2}{7}\,bc^2x^{\frac{21}{2}}+\frac{6}{17}\,b^2cx^{\frac{17}{2}}+\frac{6}{17}\,ac^2x^{\frac{17}{2}}+\frac{2}{13}\,b^3x^{\frac{13}{2}}+\frac{12}{13}\,abcx^{\frac{13}{2}}+\frac{2}{3}\,ab^2x^{\frac{9}{2}}+\frac{2}{3}\,a^2cx^{\frac{9}{2}}+\frac{6}{5}\,a^2bx^{\frac{5}{2}}+2\,a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 6/17*a*c^2*x
^(17/2) + 2/13*b^3*x^(13/2) + 12/13*a*b*c*x^(13/2) + 2/3*a*b^2*x^(9/2) + 2/
3*a^2*c*x^(9/2) + 6/5*a^2*b*x^(5/2) + 2*a^3*sqrt(x)
```

maple [A] time = 0.01, size = 90, normalized size = 0.89

$$\frac{2\left(4641c^3x^{12}+16575b\,c^2x^{10}+20475a\,c^2x^8+20475b^2c\,x^8+53550abc\,x^6+8925b^3x^6+38675a^2c\,x^4+38675a\,b^2\right.}{116025}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^(1/2),x)
```

```
[Out] 2/116025*x^(1/2)*(4641*c^3*x^12+16575*b*c^2*x^10+20475*a*c^2*x^8+20475*b^2*
c*x^8+53550*a*b*c*x^6+8925*b^3*x^6+38675*a^2*c*x^4+38675*a*b^2*x^4+69615*a^
2*b*x^2+116025*a^3)
```

maxima [A] time = 1.04, size = 88, normalized size = 0.87

$$\frac{2}{25}\,c^3x^{\frac{25}{2}}+\frac{2}{7}\,bc^2x^{\frac{21}{2}}+\frac{6}{17}\,b^2cx^{\frac{17}{2}}+\frac{2}{13}\,b^3x^{\frac{13}{2}}+2\,a^3\sqrt{x}+\frac{2}{15}\left(5\,cx^{\frac{9}{2}}+9\,bx^{\frac{5}{2}}\right)a^2+\frac{2}{663}\left(117\,c^2x^{\frac{17}{2}}+306\,bcx^{\frac{13}{2}}+221\,b^2x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 2/13*b^3*x^(
13/2) + 2*a^3*sqrt(x) + 2/15*(5*c*x^(9/2) + 9*b*x^(5/2))*a^2 + 2/663*(117*c
^2*x^(17/2) + 306*b*c*x^(13/2) + 221*b^2*x^(9/2))*a
```

mupad [B] time = 0.03, size = 76, normalized size = 0.75

$$x^{13/2}\left(\frac{2\,b^3}{13}+\frac{12\,a\,c\,b}{13}\right)+2\,a^3\,\sqrt{x}+\frac{2\,c^3\,x^{25/2}}{25}+\frac{6\,a^2\,b\,x^{5/2}}{5}+\frac{2\,b\,c^2\,x^{21/2}}{7}+\frac{2\,a\,x^{9/2}\,\left(b^2+a\,c\right)}{3}+\frac{6\,c\,x^{17/2}\,\left(b^2+a\,c\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x^(1/2),x)
```

```
[Out] x^(13/2)*((2*b^3)/13 + (12*a*b*c)/13) + 2*a^3*x^(1/2) + (2*c^3*x^(25/2))/25
 + (6*a^2*b*x^(5/2))/5 + (2*b*c^2*x^(21/2))/7 + (2*a*x^(9/2)*(a*c + b^2))/3
 + (6*c*x^(17/2)*(a*c + b^2))/17
```

sympy [A] time = 23.50, size = 128, normalized size = 1.27

$$2a^3\sqrt{x}+\frac{6a^2bx^{\frac{5}{2}}}{5}+\frac{2a^2cx^{\frac{9}{2}}}{3}+\frac{2ab^2x^{\frac{9}{2}}}{3}+\frac{12abcx^{\frac{13}{2}}}{13}+\frac{6ac^2x^{\frac{17}{2}}}{17}+\frac{2b^3x^{\frac{13}{2}}}{13}+\frac{6b^2cx^{\frac{17}{2}}}{17}+\frac{2bc^2x^{\frac{21}{2}}}{7}+\frac{2c^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**(1/2),x)
```

```
[Out] 2*a**3*sqrt(x) + 6*a**2*b*x**(5/2)/5 + 2*a**2*c*x**(9/2)/3 + 2*a*b**2*x**(9
/2)/3 + 12*a*b*c*x**(13/2)/13 + 6*a*c**2*x**(17/2)/17 + 2*b**3*x**(13/2)/13
 + 6*b**2*c*x**(17/2)/17 + 2*b*c**2*x**(21/2)/7 + 2*c**3*x**(25/2)/25
```

3.1059 $\int \frac{(a+bx^2+cx^4)^3}{x^{3/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{2a^3}{\sqrt{x}}+2a^2bx^{3/2}+\frac{2}{5}cx^{15/2}\left(ac+b^2\right)+\frac{2}{11}bx^{11/2}\left(6ac+b^2\right)+\frac{6}{7}ax^{7/2}\left(ac+b^2\right)+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}$$

```
[Out] 2*a^2*b*x^(3/2)+6/7*a*(a*c+b^2)*x^(7/2)+2/11*b*(6*a*c+b^2)*x^(11/2)+2/5*c*(
a*c+b^2)*x^(15/2)+6/19*b*c^2*x^(19/2)+2/23*c^3*x^(23/2)-2*a^3/x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$2a^2bx^{3/2}-\frac{2a^3}{\sqrt{x}}+\frac{2}{5}cx^{15/2}\left(ac+b^2\right)+\frac{2}{11}bx^{11/2}\left(6ac+b^2\right)+\frac{6}{7}ax^{7/2}\left(ac+b^2\right)+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^(3/2),x]
```

```
[Out] (-2*a^3)/Sqrt[x] + 2*a^2*b*x^(3/2) + (6*a*(b^2 + a*c)*x^(7/2))/7 + (2*b*(b^
2 + 6*a*c)*x^(11/2))/11 + (2*c*(b^2 + a*c)*x^(15/2))/5 + (6*b*c^2*x^(19/2))
/19 + (2*c^3*x^(23/2))/23
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^3}{x^{3/2}}\,dx &= \int \left(\frac{a^3}{x^{3/2}}+3a^2b\sqrt{x}+3a\left(b^2+ac\right)x^{5/2}+b\left(b^2+6ac\right)x^{9/2}+3c\left(b^2+ac\right)x^{13/2}+3bc^2x^{17/}\right.\\ &= -\frac{2a^3}{\sqrt{x}}+2a^2bx^{3/2}+\frac{6}{7}a\left(b^2+ac\right)x^{7/2}+\frac{2}{11}b\left(b^2+6ac\right)x^{11/2}+\frac{2}{5}c\left(b^2+ac\right)x^{15/2}+\frac{6}{19}bc^2\end{aligned}$$

Mathematica [A] time = 0.08, size = 100, normalized size = 1.01

$$2\left(-\frac{a^3}{\sqrt{x}}+a^2bx^{3/2}+\frac{1}{5}cx^{15/2}\left(ac+b^2\right)+\frac{1}{11}bx^{11/2}\left(6ac+b^2\right)+\frac{3}{7}ax^{7/2}\left(ac+b^2\right)+\frac{3}{19}bc^2x^{19/2}+\frac{1}{23}c^3x^{23/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x^(3/2),x]
```

```
[Out] 2*(-(a^3/Sqrt[x]) + a^2*b*x^(3/2) + (3*a*(b^2 + a*c)*x^(7/2))/7 + (b*(b^2 +
 6*a*c)*x^(11/2))/11 + (c*(b^2 + a*c)*x^(15/2))/5 + (3*b*c^2*x^(19/2))/19 +
 (c^3*x^(23/2))/23)
```

fricas [A] time = 0.68, size = 83, normalized size = 0.84

$$\frac{2\left(7315\,c^3x^{12} + 26565\,bc^2x^{10} + 33649\left(b^2c + ac^2\right)x^8 + 15295\left(b^3 + 6\,abc\right)x^6 + 168245\,a^2bx^2 + 72105\left(ab^2 + a^2c\right.\right.}{168245\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/168245*(7315*c^3*x^12 + 26565*b*c^2*x^10 + 33649*(b^2*c + a*c^2)*x^8 + 15
295*(b^3 + 6*a*b*c)*x^6 + 168245*a^2*b*x^2 + 72105*(a*b^2 + a^2*c)*x^4 - 16
8245*a^3)/sqrt(x)
```

giac [A] time = 0.16, size = 87, normalized size = 0.88

$$\frac{2}{23}\,c^3x^{\frac{23}{2}} + \frac{6}{19}\,bc^2x^{\frac{19}{2}} + \frac{2}{5}\,b^2cx^{\frac{15}{2}} + \frac{2}{5}\,ac^2x^{\frac{15}{2}} + \frac{2}{11}\,b^3x^{\frac{11}{2}} + \frac{12}{11}\,abcx^{\frac{11}{2}} + \frac{6}{7}\,ab^2x^{\frac{7}{2}} + \frac{6}{7}\,a^2cx^{\frac{7}{2}} + 2\,a^2bx^{\frac{3}{2}} - \frac{2\,a^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*b^2*c*x^(15/2) + 2/5*a*c^2*x^
(15/2) + 2/11*b^3*x^(11/2) + 12/11*a*b*c*x^(11/2) + 6/7*a*b^2*x^(7/2) + 6/7
*a^2*c*x^(7/2) + 2*a^2*b*x^(3/2) - 2*a^3/sqrt(x)
```

maple [A] time = 0.01, size = 90, normalized size = 0.91

$$-\frac{2\left(-7315c^3x^{12} - 26565b\,c^2x^{10} - 33649a\,c^2x^8 - 33649b^2c\,x^8 - 91770abc\,x^6 - 15295b^3x^6 - 72105a^2c\,x^4 - 7210\right.}{168245\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^(3/2),x)
```

```
[Out] -2/168245*(-7315*c^3*x^12-26565*b*c^2*x^10-33649*a*c^2*x^8-33649*b^2*c*x^8-
91770*a*b*c*x^6-15295*b^3*x^6-72105*a^2*c*x^4-72105*a*b^2*x^4-168245*a^2*b*
x^2+168245*a^3)/x^(1/2)
```

maxima [A] time = 0.98, size = 81, normalized size = 0.82

$$\frac{2}{23}\,c^{3}x^{\frac{23}{2}}+\frac{6}{19}\,bc^{2}x^{\frac{19}{2}}+\frac{2}{5}\,\left(b^{2}c+ac^{2}\right)x^{\frac{15}{2}}+\frac{2}{11}\,\left(b^{3}+6\,abc\right)x^{\frac{11}{2}}+2\,a^{2}bx^{\frac{3}{2}}+\frac{6}{7}\,\left(ab^{2}+a^{2}c\right)x^{\frac{7}{2}}-\frac{2\,a^{3}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*(b^2*c + a*c^2)*x^(15/2) + 2/
11*(b^3 + 6*a*b*c)*x^(11/2) + 2*a^2*b*x^(3/2) + 6/7*(a*b^2 + a^2*c)*x^(7/2)
 - 2*a^3/sqrt(x)
```

mupad [B] time = 0.04, size = 76, normalized size = 0.77

$$x^{11/2}\,\left(\frac{2\,b^3}{11}+\frac{12\,a\,c\,b}{11}\right)-\frac{2\,a^3}{\sqrt{x}}+\frac{2\,c^3\,x^{23/2}}{23}+2\,a^2\,b\,x^{3/2}+\frac{6\,b\,c^2\,x^{19/2}}{19}+\frac{6\,a\,x^{7/2}\,\left(b^2+a\,c\right)}{7}+\frac{2\,c\,x^{15/2}\,\left(b^2+a\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x^(3/2),x)
```

```
[Out] x^(11/2)*((2*b^3)/11 + (12*a*b*c)/11) - (2*a^3)/x^(1/2) + (2*c^3*x^(23/2))/
23 + 2*a^2*b*x^(3/2) + (6*b*c^2*x^(19/2))/19 + (6*a*x^(7/2)*(a*c + b^2))/7
+ (2*c*x^(15/2)*(a*c + b^2))/5
```

sympy [A] time = 19.83, size = 126, normalized size = 1.27

$$-\frac{2a^{3}}{\sqrt{x}}+2a^{2}bx^{\frac{3}{2}}+\frac{6a^{2}cx^{\frac{7}{2}}}{7}+\frac{6ab^{2}x^{\frac{7}{2}}}{7}+\frac{12abcx^{\frac{11}{2}}}{11}+\frac{2ac^{2}x^{\frac{15}{2}}}{5}+\frac{2b^{3}x^{\frac{11}{2}}}{11}+\frac{2b^{2}cx^{\frac{15}{2}}}{5}+\frac{6bc^{2}x^{\frac{19}{2}}}{19}+\frac{2c^{3}x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**(3/2),x)
```

```
[Out] -2*a**3/sqrt(x) + 2*a**2*b*x**(3/2) + 6*a**2*c*x**(7/2)/7 + 6*a*b**2*x**(7/
2)/7 + 12*a*b*c*x**(11/2)/11 + 2*a*c**2*x**(15/2)/5 + 2*b**3*x**(11/2)/11 +
 2*b**2*c*x**(15/2)/5 + 6*b*c**2*x**(19/2)/19 + 2*c**3*x**(23/2)/23
```

3.1060 $\int \frac{(a+bx^2+cx^4)^3}{x^{5/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{2a^3}{3x^{3/2}}+6a^2b\sqrt{x}+\frac{6}{13}cx^{13/2}\left(ac+b^2\right)+\frac{2}{9}bx^{9/2}\left(6ac+b^2\right)+\frac{6}{5}ax^{5/2}\left(ac+b^2\right)+\frac{6}{17}bc^2x^{17/2}+\frac{2}{21}c^3x^{21/2}$$

```
[Out] -2/3*a^3/x^(3/2)+6/5*a*(a*c+b^2)*x^(5/2)+2/9*b*(6*a*c+b^2)*x^(9/2)+6/13*c*(
a*c+b^2)*x^(13/2)+6/17*b*c^2*x^(17/2)+2/21*c^3*x^(21/2)+6*a^2*b*x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$6a^2b\sqrt{x}-\frac{2a^3}{3x^{3/2}}+\frac{6}{13}cx^{13/2}\left(ac+b^2\right)+\frac{2}{9}bx^{9/2}\left(6ac+b^2\right)+\frac{6}{5}ax^{5/2}\left(ac+b^2\right)+\frac{6}{17}bc^2x^{17/2}+\frac{2}{21}c^3x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^(5/2),x]
```

```
[Out] (-2*a^3)/(3*x^(3/2)) + 6*a^2*b*Sqrt[x] + (6*a*(b^2 + a*c)*x^(5/2))/5 + (2*b
*(b^2 + 6*a*c)*x^(9/2))/9 + (6*c*(b^2 + a*c)*x^(13/2))/13 + (6*b*c^2*x^(17/
2))/17 + (2*c^3*x^(21/2))/21
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^3}{x^{5/2}}\,dx=\int \left(\frac{a^3}{x^{5/2}}+\frac{3a^2b}{\sqrt{x}}+3a\left(b^2+ac\right)x^{3/2}+b\left(b^2+6ac\right)x^{7/2}+3c\left(b^2+ac\right)x^{11/2}+3bc^2x^{15/2}$$

$$=-\frac{2a^3}{3x^{3/2}}+6a^2b\sqrt{x}+\frac{6}{5}a\left(b^2+ac\right)x^{5/2}+\frac{2}{9}b\left(b^2+6ac\right)x^{9/2}+\frac{6}{13}c\left(b^2+ac\right)x^{13/2}+\frac{6}{17}b$$

Mathematica [A] time = 0.07, size = 103, normalized size = 1.02

$$2\left(-\frac{a^3}{3x^{3/2}}+3a^2b\sqrt{x}+\frac{3}{13}cx^{13/2}\left(ac+b^2\right)+\frac{1}{9}bx^{9/2}\left(6ac+b^2\right)+\frac{3}{5}ax^{5/2}\left(ac+b^2\right)+\frac{3}{17}bc^2x^{17/2}+\frac{1}{21}c^3x^{21/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x^(5/2),x]
```

```
[Out] 2*(-1/3*a^3/x^(3/2) + 3*a^2*b*Sqrt[x] + (3*a*(b^2 + a*c)*x^(5/2))/5 + (b*(b
^2 + 6*a*c)*x^(9/2))/9 + (3*c*(b^2 + a*c)*x^(13/2))/13 + (3*b*c^2*x^(17/2))
/17 + (c^3*x^(21/2))/21)
```

fricas [A] time = 0.75, size = 83, normalized size = 0.82

$$\frac{2\left(3315\,c^3x^{12}+12285\,bc^2x^{10}+16065\left(b^2c+ac^2\right)x^8+7735\left(b^3+6\,abc\right)x^6+208845\,a^2bx^2+41769\left(ab^2+a^2c\right)x^4\right.}{69615\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*c^3*x^12 + 12285*b*c^2*x^10 + 16065*(b^2*c + a*c^2)*x^8 + 773
5*(b^3 + 6*a*b*c)*x^6 + 208845*a^2*b*x^2 + 41769*(a*b^2 + a^2*c)*x^4 - 2320
5*a^3)/x^(3/2)
```

giac [A] time = 0.20, size = 87, normalized size = 0.86

$$\frac{2}{21}\,c^3x^{\frac{21}{2}}+\frac{6}{17}\,bc^2x^{\frac{17}{2}}+\frac{6}{13}\,b^2cx^{\frac{13}{2}}+\frac{6}{13}\,ac^2x^{\frac{13}{2}}+\frac{2}{9}\,b^3x^{\frac{9}{2}}+\frac{4}{3}\,abcx^{\frac{9}{2}}+\frac{6}{5}\,ab^2x^{\frac{5}{2}}+\frac{6}{5}\,a^2cx^{\frac{5}{2}}+6\,a^2b\sqrt{x}-\frac{2\,a^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*b^2*c*x^(13/2) + 6/13*a*c^2*
x^(13/2) + 2/9*b^3*x^(9/2) + 4/3*a*b*c*x^(9/2) + 6/5*a*b^2*x^(5/2) + 6/5*a^
2*c*x^(5/2) + 6*a^2*b*sqrt(x) - 2/3*a^3/x^(3/2)
```

maple [A] time = 0.01, size = 90, normalized size = 0.89

$$-\frac{2\left(-3315c^3x^{12}-12285b\,c^2x^{10}-16065a\,c^2x^8-16065b^2c\,x^8-46410abc\,x^6-7735b^3x^6-41769a^2c\,x^4-41769a\,b\right.}{69615x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^(5/2),x)
```

```
[Out] -2/69615*(-3315*c^3*x^12-12285*b*c^2*x^10-16065*a*c^2*x^8-16065*b^2*c*x^8-4
6410*a*b*c*x^6-7735*b^3*x^6-41769*a^2*c*x^4-41769*a*b^2*x^4-208845*a^2*b*x^
2+23205*a^3)/x^(3/2)
```

maxima [A] time = 1.06, size = 81, normalized size = 0.80

$$\frac{2}{21}\,c^3x^{\frac{21}{2}}+\frac{6}{17}\,bc^2x^{\frac{17}{2}}+\frac{6}{13}\left(b^2c+ac^2\right)x^{\frac{13}{2}}+\frac{2}{9}\left(b^3+6\,abc\right)x^{\frac{9}{2}}+6\,a^2b\sqrt{x}+\frac{6}{5}\left(ab^2+a^2c\right)x^{\frac{5}{2}}-\frac{2\,a^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*(b^2*c + a*c^2)*x^(13/2) + 2
/9*(b^3 + 6*a*b*c)*x^(9/2) + 6*a^2*b*sqrt(x) + 6/5*(a*b^2 + a^2*c)*x^(5/2)
- 2/3*a^3/x^(3/2)
```

mupad [B] time = 0.04, size = 76, normalized size = 0.75

$$x^{9/2}\left(\frac{2\,b^3}{9}+\frac{4\,a\,c\,b}{3}\right)-\frac{2\,a^3}{3\,x^{3/2}}+\frac{2\,c^3\,x^{21/2}}{21}+6\,a^2\,b\,\sqrt{x}+\frac{6\,b\,c^2\,x^{17/2}}{17}+\frac{6\,a\,x^{5/2}\,\left(b^2+a\,c\right)}{5}+\frac{6\,c\,x^{13/2}\,\left(b^2+a\,c\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x^(5/2),x)
```

```
[Out] x^(9/2)*((2*b^3)/9 + (4*a*b*c)/3) - (2*a^3)/(3*x^(3/2)) + (2*c^3*x^(21/2))/
21 + 6*a^2*b*x^(1/2) + (6*b*c^2*x^(17/2))/17 + (6*a*x^(5/2)*(a*c + b^2))/5
+ (6*c*x^(13/2)*(a*c + b^2))/13
```

sympy [A] time = 25.44, size = 128, normalized size = 1.27

$$-\frac{2a^3}{3x^{\frac{3}{2}}}+6a^2b\sqrt{x}+\frac{6a^2cx^{\frac{5}{2}}}{5}+\frac{6ab^2x^{\frac{5}{2}}}{5}+\frac{4abcx^{\frac{9}{2}}}{3}+\frac{6ac^2x^{\frac{13}{2}}}{13}+\frac{2b^3x^{\frac{9}{2}}}{9}+\frac{6b^2cx^{\frac{13}{2}}}{13}+\frac{6bc^2x^{\frac{17}{2}}}{17}+\frac{2c^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**(5/2),x)
```

```
[Out] -2*a**3/(3*x**(3/2)) + 6*a**2*b*sqrt(x) + 6*a**2*c*x**(5/2)/5 + 6*a*b**2*x*
*(5/2)/5 + 4*a*b*c*x**(9/2)/3 + 6*a*c**2*x**(13/2)/13 + 2*b**3*x**(9/2)/9 +
 6*b**2*c*x**(13/2)/13 + 6*b*c**2*x**(17/2)/17 + 2*c**3*x**(21/2)/21
```

3.1061 $\int \frac{(a+bx^2+cx^4)^3}{x^{7/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{2a^3}{5x^{5/2}}-\frac{6a^2b}{\sqrt{x}}+\frac{6}{11}cx^{11/2}\left(ac+b^2\right)+\frac{2}{7}bx^{7/2}\left(6ac+b^2\right)+2ax^{3/2}\left(ac+b^2\right)+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

```
[Out] -2/5*a^3/x^(5/2)+2*a*(a*c+b^2)*x^(3/2)+2/7*b*(6*a*c+b^2)*x^(7/2)+6/11*c*(a*
c+b^2)*x^(11/2)+2/5*b*c^2*x^(15/2)+2/19*c^3*x^(19/2)-6*a^2*b/x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$-\frac{6a^2b}{\sqrt{x}}-\frac{2a^3}{5x^{5/2}}+\frac{6}{11}cx^{11/2}\left(ac+b^2\right)+\frac{2}{7}bx^{7/2}\left(6ac+b^2\right)+2ax^{3/2}\left(ac+b^2\right)+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^(7/2),x]
```

```
[Out] (-2*a^3)/(5*x^(5/2)) - (6*a^2*b)/Sqrt[x] + 2*a*(b^2 + a*c)*x^(3/2) + (2*b*(
b^2 + 6*a*c)*x^(7/2))/7 + (6*c*(b^2 + a*c)*x^(11/2))/11 + (2*b*c^2*x^(15/2)
)/5 + (2*c^3*x^(19/2))/19
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] && !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^3}{x^{7/2}}\,dx &= \int \left(\frac{a^3}{x^{7/2}}+\frac{3a^2b}{x^{3/2}}+3a\left(b^2+ac\right)\sqrt{x}+b\left(b^2+6ac\right)x^{5/2}+3c\left(b^2+ac\right)x^{9/2}+3bc^2x^{13/2}+\right.\\ &= -\frac{2a^3}{5x^{5/2}}-\frac{6a^2b}{\sqrt{x}}+2a\left(b^2+ac\right)x^{3/2}+\frac{2}{7}b\left(b^2+6ac\right)x^{7/2}+\frac{6}{11}c\left(b^2+ac\right)x^{11/2}+\frac{2}{5}bc^2x^{15/2}\end{aligned}$$

Mathematica [A] time = 0.08, size = 100, normalized size = 1.01

$$2\left(-\frac{a^3}{5x^{5/2}}-\frac{3a^2b}{\sqrt{x}}+\frac{3}{11}cx^{11/2}\left(ac+b^2\right)+\frac{1}{7}bx^{7/2}\left(6ac+b^2\right)+ax^{3/2}\left(ac+b^2\right)+\frac{1}{5}bc^2x^{15/2}+\frac{1}{19}c^3x^{19/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x^(7/2),x]
```

```
[Out] 2*(-1/5*a^3/x^(5/2) - (3*a^2*b)/Sqrt[x] + a*(b^2 + a*c)*x^(3/2) + (b*(b^2 +
 6*a*c)*x^(7/2))/7 + (3*c*(b^2 + a*c)*x^(11/2))/11 + (b*c^2*x^(15/2))/5 + (
c^3*x^(19/2))/19)
```

fricas [A] time = 0.75, size = 83, normalized size = 0.84

$$\frac{2\left(385\,c^3x^{12}+1463\,bc^2x^{10}+1995\left(b^2c+ac^2\right)x^8+1045\left(b^3+6\,abc\right)x^6-21945\,a^2bx^2+7315\left(ab^2+a^2c\right)x^4-14\cdots}{7315\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/7315*(385*c^3*x^12 + 1463*b*c^2*x^10 + 1995*(b^2*c + a*c^2)*x^8 + 1045*(b
^3 + 6*a*b*c)*x^6 - 21945*a^2*b*x^2 + 7315*(a*b^2 + a^2*c)*x^4 - 1463*a^3)/
x^(5/2)
```

giac [A] time = 0.17, size = 88, normalized size = 0.89

$$\frac{2}{19}\,c^3x^{\frac{19}{2}}+\frac{2}{5}\,bc^2x^{\frac{15}{2}}+\frac{6}{11}\,b^2cx^{\frac{11}{2}}+\frac{6}{11}\,ac^2x^{\frac{11}{2}}+\frac{2}{7}\,b^3x^{\frac{7}{2}}+\frac{12}{7}\,abcx^{\frac{7}{2}}+2\,ab^2x^{\frac{3}{2}}+2\,a^2cx^{\frac{3}{2}}-\frac{2\left(15\,a^2bx^2+a^3\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*b^2*c*x^(11/2) + 6/11*a*c^2*x
^(11/2) + 2/7*b^3*x^(7/2) + 12/7*a*b*c*x^(7/2) + 2*a*b^2*x^(3/2) + 2*a^2*c*
x^(3/2) - 2/5*(15*a^2*b*x^2 + a^3)/x^(5/2)
```

maple [A] time = 0.01, size = 90, normalized size = 0.91

$$-\frac{2\left(-385c^3x^{12}-1463b\,c^2x^{10}-1995a\,c^2x^8-1995b^2c\,x^8-6270abc\,x^6-1045b^3x^6-7315a^2c\,x^4-7315a\,b^2x^4+2\cdots}{7315x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^(7/2),x)
```

```
[Out] -2/7315*(-385*c^3*x^12-1463*b*c^2*x^10-1995*a*c^2*x^8-1995*b^2*c*x^8-6270*a
*b*c*x^6-1045*b^3*x^6-7315*a^2*c*x^4-7315*a*b^2*x^4+21945*a^2*b*x^2+1463*a^
3)/x^(5/2)
```

maxima [A] time = 1.05, size = 82, normalized size = 0.83

$$\frac{2}{19}\,c^3x^{\frac{19}{2}}+\frac{2}{5}\,bc^2x^{\frac{15}{2}}+\frac{6}{11}\,\left(b^2c+ac^2\right)x^{\frac{11}{2}}+\frac{2}{7}\,\left(b^3+6\,abc\right)x^{\frac{7}{2}}+2\,\left(ab^2+a^2c\right)x^{\frac{3}{2}}-\frac{2\left(15\,a^2bx^2+a^3\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*(b^2*c + a*c^2)*x^(11/2) + 2/
7*(b^3 + 6*a*b*c)*x^(7/2) + 2*(a*b^2 + a^2*c)*x^(3/2) - 2/5*(15*a^2*b*x^2 +
 a^3)/x^(5/2)
```

mupad [B] time = 0.04, size = 79, normalized size = 0.80

$$x^{7/2}\left(\frac{2\,b^3}{7}+\frac{12\,a\,c\,b}{7}\right)-\frac{\frac{2\,a^3}{5}+6\,b\,a^2\,x^2}{x^{5/2}}+\frac{2\,c^3\,x^{19/2}}{19}+\frac{2\,b\,c^2\,x^{15/2}}{5}+2\,a\,x^{3/2}\,\left(b^2+a\,c\right)+\frac{6\,c\,x^{11/2}\,\left(b^2+a\,c\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x^(7/2),x)
```

```
[Out] x^(7/2)*((2*b^3)/7 + (12*a*b*c)/7) - ((2*a^3)/5 + 6*a^2*b*x^2)/x^(5/2) + (2
*c^3*x^(19/2))/19 + (2*b*c^2*x^(15/2))/5 + 2*a*x^(3/2)*(a*c + b^2) + (6*c*x
^(11/2)*(a*c + b^2))/11
```

sympy [A] time = 31.86, size = 124, normalized size = 1.25

$$-\frac{2a^3}{5x^{\frac{5}{2}}}-\frac{6a^2b}{\sqrt{x}}+2a^2cx^{\frac{3}{2}}+2ab^2x^{\frac{3}{2}}+\frac{12abcx^{\frac{7}{2}}}{7}+\frac{6ac^2x^{\frac{11}{2}}}{11}+\frac{2b^3x^{\frac{7}{2}}}{7}+\frac{6b^2cx^{\frac{11}{2}}}{11}+\frac{2bc^2x^{\frac{15}{2}}}{5}+\frac{2c^3x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**(7/2),x)
```

```
[Out] -2*a**3/(5*x**(5/2)) - 6*a**2*b/sqrt(x) + 2*a**2*c*x**(3/2) + 2*a*b**2*x**(
3/2) + 12*a*b*c*x**(7/2)/7 + 6*a*c**2*x**(11/2)/11 + 2*b**3*x**(7/2)/7 + 6*
b**2*c*x**(11/2)/11 + 2*b*c**2*x**(15/2)/5 + 2*c**3*x**(19/2)/19
```

3.1062 $\int \frac{x^{9/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=389

$$-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

```
[Out] 2/3*x^(3/2)/c-1/2*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1
/4))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-b-(-4*a*c+b^2)^(
1/2))^(1/4)+1/2*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/
4))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-b-(-4*a*c+b^2)^(1
/2))^(1/4)-1/2*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-b+(-4*a*c+b^2)^(1/2)
)^(1/4)+1/2*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*
(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-b+(-4*a*c+b^2)^(1/2))^
(1/4)
```

Rubi [A] time = 0.86, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1115, 1367, 1510, 298, 205, 208}

$$-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(3/2))/(3*c) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*
c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(7/4)*(-b - Sq
rt[b^2 - 4*a*c])^(1/4)) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^
(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(7/4)*(-
b + Sqrt[b^2 - 4*a*c])^(1/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcT
anh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(
7/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c
])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3
/4)*c^(7/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^{9/2}}{a+bx^2+cx^4}\,dx = 2\,\text{Subst}\left(\int \frac{x^{10}}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)$$
$$= \frac{2x^{3/2}}{3c} - \frac{2\,\text{Subst}\left(\int \frac{x^2(3a+3bx^4)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{3c}$$
$$= \frac{2x^{3/2}}{3c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{\frac{b}{2}+\frac{1}{2}}\right.}{c}$$
$$= \frac{2x^{3/2}}{3c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{2}\,c^{3/2}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\right.}{}$$
$$= \frac{2x^{3/2}}{3c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{(b+}{}$$

Mathematica [C] time = 0.05, size = 80, normalized size = 0.21

$$\frac{4x^{3/2} - 3\text{RootSum}\left[\#1^8c + \#1^4b + a\&, \frac{\#1^4b\log(\sqrt{x}-\#1)+a\log(\sqrt{x}-\#1)}{2\#1^5c+\#1b}\&\right]}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (4*x^(3/2) - 3*RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[Sqrt[x] - #1] + b*Log
[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(6*c)
```

fricas [B] time = 8.23, size = 6649, normalized size = 17.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(12*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*
c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^
2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^
```

6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*arctan(1/2*((b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 - (b^6*c^7 - 10*a*b^4*c^8 + 32*a^2*b^2*c^9 - 32*a^3*c^10)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt((a^10*b^12 - 10*a^11*b^10*c + 37*a^12*b^8*c^2 - 62*a^13*b^6*c^3 + 46*a^14*b^4*c^4 - 12*a^15*b^2*c^5 + a^16*c^6)*x - 1/2*sqrt(1/2)*(a^7*b^17 - 17*a^8*b^15*c + 119*a^9*b^13*c^2 - 441*a^10*b^11*c^3 + 924*a^11*b^9*c^4 - 1078*a^12*b^7*c^5 + 637*a^13*b^5*c^6 - 151*a^14*b^3*c^7 + 12*a^15*b*c^8 - (a^7*b^14*c^7 - 18*a^8*b^12*c^8 + 131*a^9*b^10*c^9 - 491*a^10*b^8*c^10 + 997*a^11*b^6*c^11 - 1052*a^12*b^4*c^12 + 496*a^13*b^2*c^13 - 64*a^14*c^14)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9))) + (a^5*b^15 - 14*a^6*b^13*c + 77*a^7*b^11*c^2 - 210*a^8*b^9*c^3 + 294*a^9*b^7*c^4 - 196*a^10*b^5*c^5 + 49*a^11*b^3*c^6 - 4*a^12*b*c^7 - (a^5*b^12*c^7 - 15*a^6*b^10*c^8 + 88*a^7*b^8*c^9 - 253*a^8*b^6*c^10 + 362*a^9*b^4*c^11 - 224*a^10*b^2*c^12 + 32*a^11*c^13)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(x))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))/(a^7*b^12 - 10*a^8*b^10*c + 37*a^9*b^8*c^2 - 62*a^10*b^6*c^3 + 46*a^11*b^4*c^4 - 12*a^12*b^2*c^5 + a^13*c^6)) - 12*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*arctan(-1/2*((b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 + (b^6*c^7 - 10*a*b^4*c^8 + 32*a^2*b^2*c^9 - 32*a^3*c^10)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt((a^10*b^12 - 10*a^11*b^10*c + 37*a^12*b^8*c^2 - 62*a^13*b^6*c^3 + 46*a^14*b^4*c^4 - 12*a^15*b^2*c^5 + a^16*c^6)*x - 1/2*sqrt(1/2)*(a^7*b^17 - 17*a^8*b^15*c + 119*a^9*b^13*c^2 - 441*a^10*b^11*c^3 + 924*a^11*b^9*c^4 - 1078*a^12*b^7*c^5 + 637*a^13*b^5*c^6 - 151*a^14*b^3*c^7 + 12*a^15*b*c^8 + (a^7*b^14*c^7 - 18*a^8*b^12*c^8 + 131*a^9*b^10*c^9 - 491*a^10*b^8*c^10 + 997*a^11*b^6*c^11 - 1052*a^12*b^4*c^12 + 496*a^13*b^2*c^13 - 64*a^14*c^14)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^

3*c^17)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9))) + (a^5*b^15 - 14*a^6*b^13*c + 77*a^7*b^11*c^2 - 210*a^8*b^9*c^3 + 294*a^9*b^7*c^4 - 196*a^10*b^5*c^5 + 49*a^11*b^3*c^6 - 4*a^12*b*c^7 + (a^5*b^12*c^7 - 15*a^6*b^10*c^8 + 88*a^7*b^8*c^9 - 253*a^8*b^6*c^10 + 362*a^9*b^4*c^11 - 224*a^10*b^2*c^12 + 32*a^11*c^13)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9))))/(a^7*b^12 - 10*a^8*b^10*c + 37*a^9*b^8*c^2 - 62*a^10*b^6*c^3 + 46*a^11*b^4*c^4 - 12*a^12*b^2*c^5 + a^13*c^6)) - 3*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*log(1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 - (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*sqrt(x)) + 3*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*log(-1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 - (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9

- 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*sqrt(x)) - 3*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*log(1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 + (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*sqrt(x)) + 3*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*log(-1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 + (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c

```
^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2
*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6
*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2
*c^8 + 16*a^2*c^9)) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*sqr
t(x)) + 4*x^(3/2))/c
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 19.14Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.06, size = 65, normalized size = 0.17

$$\frac{2x^{\frac{3}{2}}}{3c}-\frac{\left(\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{6} b+\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{2} a\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)+\sqrt{x}\right.}{2c\left(2\,\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{7} c+\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{3} b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 2/3/c*x^(3/2)-1/2/c*sum((_R^6*b+_R^2*a)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R
=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,x^{\frac{3}{2}}}{3\,c}-\int\frac{bx^{\frac{5}{2}}+a\sqrt{x}}{c^{2}x^{4}+bcx^{2}+ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)/c - integrate((b*x^(5/2) + a*sqrt(x))/(c^2*x^4 + b*c*x^2 + a*c)
, x)
```

mupad [B] time = 5.82, size = 12789, normalized size = 32.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] atan(((((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^
3*c^5))/c^3 - (256*x^(1/2)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5
*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*
a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6
*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6
*c^5 + 160*a^4*b^4*c^6 - 512*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^
2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^
3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*
a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11
 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) + (2
56*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a
*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280
*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^
2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a
^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/
4)*1i - (((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*
b^3*c^5))/c^3 + (256*x^(1/2)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a
^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(
4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
 - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b
^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b
^6*c^5 + 160*a^4*b^4*c^6 - 512*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c -
b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*
b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^
11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) -
(256*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4
*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 2
80*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*
c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256
*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(
1/4)*1i)/((((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^
5*b^3*c^5))/c^3 - (256*x^(1/2)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112
*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(
-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/
2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a
*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3
*b^6*c^5 + 160*a^4*b^4*c^6 - 512*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c
- b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^
4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(
```

-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) + (256*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) + (((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^5))/c^3 + (256*x^(1/2)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 + 160*a^4*b^4*c^6 - 512*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) - (256*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) - (256*(a^8*c - a^7*b^2))/c^3))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*2i + atan(((((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^5))/c^3 - (256*x^(1/2)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 + 160*a^4*b^4*c^6 - 512*a^5*b^2*c^7))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) + (256*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i - (((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^5))

/c^3 + (256*x^(1/2)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 + 160*a^4*b^4*c^6 - 512*a^5*b^2*c^7))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) - (256*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i)/((((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^5))/c^3 - (256*x^(1/2)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 + 160*a^4*b^4*c^6 - 512*a^5*b^2*c^7))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) + (256*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) + (((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^5))/c^3 + (256*x^(1/2)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 + 160*a^4*b^4*c^6 - 512*a^5*b^2*c^7))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) - (256*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4

*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^1
1 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) - (
256*(a^8*c - a^7*b^2))/c^3))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a
^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(
4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
 + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b
^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*2i + 2*atan(((((128*(51
2*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^5))/c^3 - (x
^(1/2)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*
c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2)
- 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c
- b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^
9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 + 160*a^4*b^4*c
^6 - 512*a^5*b^2*c^7)*256i)/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 1
12*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3
*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(
1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16
*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i - (256*x^(1/2)*(
a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)
^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4
 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^
8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) - (((128*
(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^5))/c^3 +
 (x^(1/2)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b
^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/
2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a
c - b^2)^5)^(1/2))/(32(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4
*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 + 160*a^4*b^
4*c^6 - 512*a^5*b^2*c^7)*256i)/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2)
- 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*
c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5
)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 -
 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i + (256*x^(1/2
)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)
^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*
c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*
c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 +
 b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4))/((((1
28*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^5))/c^
3 - (x^(1/2)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^
2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(
4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*
b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 + 160*a^4

*b^4*c^6 - 512*a^5*b^2*c^7)*256i)/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i - (256*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i + (((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^5))/c^3 + (x^(1/2)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 + 160*a^4*b^4*c^6 - 512*a^5*b^2*c^7)*256i)/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i + (256*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i + (256*(a^8*c - a^7*b^2))/c^3))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) + 2*atan(((((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^5))/c^3 - (x^(1/2)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 + 160*a^4*b^4*c^6 - 512*a^5*b^2*c^7)*256i)/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i - (256*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4

*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^1
1 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) - (
((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^5))
/c^3 + (x^(1/2)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86
*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^
5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*
(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a
^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 + 160*
a^4*b^4*c^6 - 512*a^5*b^2*c^7)*256i)/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^
(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4
+ a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8
*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i + (256*
x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c
- b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^
4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*
c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4))
/((((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^
5))/c^3 - (x^(1/2)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 +
86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^
2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4
c(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 9
6*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 + 1
60*a^4*b^4*c^6 - 512*a^5*b^2*c^7)*256i)/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^
5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c
^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c
- b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 +
b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i - (2
56*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a
*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280
*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^
2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a
^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/
4)*1i + (((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*
b^3*c^5))/c^3 + (x^(1/2)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b
*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*
c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5
*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c
^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c
^5 + 160*a^4*b^4*c^6 - 512*a^5*b^2*c^7)*256i)/c^3)*(-(b^11 - b^6*(-(4*a*c -
b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4
*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-
(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c
^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1

```
i + (256*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*
(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3
 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*
b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*
(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)
))^(1/4)*1i + (256*(a^8*c - a^7*b^2))/c^3))*(-(b^11 - b^6*(-(4*a*c - b^2)^5
)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^
4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c
- b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b
^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) + (2*x^(
3/2))/(3*c)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.1063 $\int \frac{x^{7/2}}{a+bx^2+cx^4} dx$

Optimal. Leaf size=385

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+$$

```
[Out] 1/2*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*
c+b^2)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/
2*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c
+b^2)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/2
*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^
2)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/2*ar
ctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)
/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+2*x^(1/2
)/c
```

Rubi [A] time = 0.80, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1115, 1367, 1422, 212, 208, 205}

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[x])/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1
/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(5/4)*(-b - Sqrt[b
^2 - 4*a*c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4
)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(5/4)*(-b +
Sqrt[b^2 - 4*a*c])^(3/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[
(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(5/4)
*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*A
rcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*
c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^{7/2}}{a+bx^2+cx^4}\,dx = 2\,\text{Subst}\left(\int \frac{x^8}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)$$

$$= \frac{2\sqrt{x}}{c} - \frac{2\,\text{Subst}\left(\int \frac{a+bx^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{c}$$

$$= \frac{2\sqrt{x}}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{\frac{b}{2}+\frac{1}{2}}\right.}{c}$$

$$= \frac{2\sqrt{x}}{c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, \sqrt{x}\right)}{c\sqrt{-b+\sqrt{b^2-4ac}}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\right.}{c\sqrt{-}}$$

$$= \frac{2\sqrt{x}}{c} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\,c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\,c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b+\right.}{}$$

Mathematica [C] time = 0.04, size = 80, normalized size = 0.21

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4b\log\left(\sqrt{x}-\#1\right)+a\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]-4\sqrt{x}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -1/2*(-4*Sqrt[x] + RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[Sqrt[x] - #1] + b
*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/c
```

fricas [B] time = 3.18, size = 5319, normalized size = 13.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 -
8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
```

))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(-1/4*(sqrt(1/2)*(b^11 - 13
*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^
5 + (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b
^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*
sqrt(4*(a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*x
 + 2*sqrt(1/2)*(b^12 - 12*a*b^10*c + 55*a^2*b^8*c^2 - 120*a^3*b^6*c^3 + 125
*a^4*b^4*c^4 - 54*a^5*b^2*c^5 + 8*a^6*c^6 + (b^11*c^5 - 15*a*b^9*c^6 + 85*a
^2*b^7*c^7 - 220*a^3*b^5*c^8 + 240*a^4*b^3*c^9 - 64*a^5*b*c^10)*sqrt((b^8 -
 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4
*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c
^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^
4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^1
2 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*a*
b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*
b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)) -
 2*sqrt(1/2)*(a*b^15 - 16*a^2*b^13*c + 103*a^3*b^11*c^2 - 340*a^4*b^9*c^3 +
 605*a^5*b^7*c^4 - 554*a^6*b^5*c^5 + 224*a^7*b^3*c^6 - 32*a^8*b*c^7 + (a*b^
14*c^5 - 19*a^2*b^12*c^6 + 147*a^3*b^10*c^7 - 590*a^4*b^8*c^8 + 1290*a^5*b^
6*c^9 - 1464*a^6*b^4*c^10 + 736*a^7*b^2*c^11 - 128*a^8*c^12)*sqrt((b^8 - 6*
a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^
11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(x)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^
2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^
2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(sqrt(1/
2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c
^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*
c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*
c^6 + 16*a^2*c^7)))/(a^5*b^8 - 6*a^6*b^6*c + 11*a^7*b^4*c^2 - 6*a^8*b^2*c^3
 + a^9*c^4)) - 4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b
^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*
a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(1/4*(sqrt(1/2)*(b
^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*
a^5*b*c^5 - (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 3
52*a^4*b^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a
^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*
c^13)))*sqrt(4*(a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^
6*c^4)*x + 2*sqrt(1/2)*(b^12 - 12*a*b^10*c + 55*a^2*b^8*c^2 - 120*a^3*b^6*c
^3 + 125*a^4*b^4*c^4 - 54*a^5*b^2*c^5 + 8*a^6*c^6 - (b^11*c^5 - 15*a*b^9*c^
6 + 85*a^2*b^7*c^7 - 220*a^3*b^5*c^8 + 240*a^4*b^3*c^9 - 64*a^5*b*c^10)*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*b^3*c + 5
*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 1

1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)) - 2*sqrt(1/2)*(a*b^15 - 16*a^2*b^13*c + 103*a^3*b^11*c^2 - 340*a^4*b^9*c^3 + 605*a^5*b^7*c^4 - 554*a^6*b^5*c^5 + 224*a^7*b^3*c^6 - 32*a^8*b*c^7 - (a*b^14*c^5 - 19*a^2*b^12*c^6 + 147*a^3*b^10*c^7 - 590*a^4*b^8*c^8 + 1290*a^5*b^6*c^9 - 1464*a^6*b^4*c^10 + 736*a^7*b^2*c^11 - 128*a^8*c^12)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/(a^5*b^8 - 6*a^6*b^6*c + 11*a^7*b^4*c^2 - 6*a^8*b^2*c^3 + a^9*c^4)) - c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log(2*(a*b^4 - 3*a^2*b^2*c + a^3*c^2)*sqrt(x) + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) + c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log(2*(a*b^4 - 3*a^2*b^2*c + a^3*c^2)*sqrt(x) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^1

```
1 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))
*log(2*(a*b^4 - 3*a^2*b^2*c + a^3*c^2)*sqrt(x) + (b^6 - 7*a*b^4*c + 13*a^2*
b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*
a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^
11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c
 + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c
 + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48
*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) + c*
sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^
6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^
4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^
5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log(2*(a*b^4 - 3*a^2*b^2*c + a^3*c^2)*sqrt(
x) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*c^6
 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(s
qrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16
*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)
/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*
a*b^2*c^6 + 16*a^2*c^7)))) - 4*sqrt(x))/c
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 14.51Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.01, size = 64, normalized size = 0.17

$$\frac{\left(-\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^4 b-a\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{2c\left(2\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7 c+\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3 b\right)}+\frac{2\sqrt{x}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 2/c*x^(1/2)+1/2/c*sum((-_R^4*b-a)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=Root
Of(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}}}{cx^4 + bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 6.86, size = 10449, normalized size = 27.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] atan(((((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (256
*x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*
c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a
*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
+ 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4
 - 128*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^
4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 1
3*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5
- 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (256*x^(1/2)*(
a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2
) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^
2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4
*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1
i - (((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (256*x
^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^
2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b
^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 +
96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 -
 128*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
+ 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*
a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 -
16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (256*x^(1/2)*(a^
4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2)
+ 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)
^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c
^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)
/((((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (256*x^(
```

1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
- 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2
c(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96
*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 1
28*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 +
61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*
b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16
*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (256*x^(1/2)*(a^4*
b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) +
80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5
)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9
+ b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (((
512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (256*x^(1/2)*
(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120
*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-
(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*
b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^
4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^
2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c
- 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^
6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (256*x^(1/2)*(a^4*b^4 +
2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^
4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/
2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^
8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)))*(-(b^9 +
b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3
*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c -
b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(1/4)*2i + atan(((((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b
^4*c + 13*a^5*b^2*c^2))/c - (256*x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1
/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a
^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)
*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*a
*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*
c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1
/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b
^2*c^8)))^(1/4) - (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b
^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3
*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a
c - b^2)^5)^(1/2))/(32(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*
c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4
*c + 13*a^5*b^2*c^2))/c + (256*x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2
) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^
2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4

```
*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(
256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*a*c
 - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^
2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2
))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
*c^8)))^(1/4) + (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9
 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b
^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c
 - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^
7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/((((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c
 + 13*a^5*b^2*c^2))/c - (256*x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2)
+ 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)
^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c
^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(25
6*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*a*c -
 b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*
(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))
/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c
^8)))^(1/4) - (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 -
 b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3
*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c -
 b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(1/4) + (((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13
*a^5*b^2*c^2))/c + (256*x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*
a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(
1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 +
b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5
*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)
^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*
a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*
(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))
^(1/4) + (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
- a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)
^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256
*a^3*b^2*c^8)))^(1/4)))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^
4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 1
3*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5
- 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i - 2*atan(((((
512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x^(1/2)*(-(b
^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3
*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a
*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*
c^7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^
3*c^5)*256i)/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a
```

^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*
c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b
^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (256*x^(1/2)*(a^4*b
^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 8
0*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)
^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9
+ b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (((5
12*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x^(1/2)*(-(b^
9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*
b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*
c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c
^7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3
*c^5)*256i)/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^
2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c
 - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^
6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (256*x^(1/2)*(a^4*b^
4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80
*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 +
 b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4))/((((51
2*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x^(1/2)*(-(b^9
 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b
^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c
 - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^
7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*
c^5)*256i)/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2
*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c
- 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6
*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (256*x^(1/2)*(a^4*b^4
 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*
a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(
1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 +
b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (((
512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x^(1/2)*(-(b
^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3
*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a
c - b^2)^5)^(1/2))/(32(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*
c^7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^
3*c^5)*256i)/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a
^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*
c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b
^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (256*x^(1/2)*(a^4*b
^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 8
0*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)
^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9

```
+ b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i))*(
-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*
a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(
4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b
^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - 2*atan(((((512*(a^3*b^6 - 4*a^6*c^3 - 7
*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^
(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c
- b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256
*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/
4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5)*256i)/c)*(-(b^9 - b^4
*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2
)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 25
6*a^3*b^2*c^8)))^(1/4)*1i + (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c
))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^
2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 9
6*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (((512*(a^3*b^6 - 4*a^6*c^3 - 7*
a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(
1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*
a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4
)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5)*256i)/c)*(-(b^9 - b^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
- a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)
^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256
*a^3*b^2*c^8)))^(1/4)*1i - (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c)
)/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
- 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2
*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96
*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4))/((((512*(a^3*b^6 - 4*a^6*c^3 - 7*a
^4*b^4*c + 13*a^5*b^2*c^2))/c - (x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1
/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a
^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)
*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5)*256i)/c)*(-(b^9 - b^4*(
-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 -
 a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^
5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*
a^3*b^2*c^8)))^(1/4)*1i + (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))
/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*
c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*
a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (((512*(a^3*b^6 - 4*a^6*c^3 - 7
*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^
```

```
(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c
- b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256
*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/
4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5)*256i)/c)*(-(b^9 - b^4
*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2
)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 25
6*a^3*b^2*c^8)))^(1/4)*1i - (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c
))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^
2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 9
6*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i))*(-(b^9 - b^4*(-(4*a*c - b^2)^
5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a
*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(
256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^
(1/4) + (2*x^(1/2))/c
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.1064 $\int \frac{x^{5/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=331

$$-\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tan}{2^{3/4}c^{3/4}\sqrt{b^2}}$$

```
[Out] -1/2*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*
a*c+b^2)^(1/2))^(3/4)*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(1/2)+1/2*arctanh(2^(1/4
)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*a*c+b^2)^(1/2))^(3
/4)*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(1/2)+1/2*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(
-b+(-4*a*c+b^2)^(1/2))^(1/4))*(-b+(-4*a*c+b^2)^(1/2))^(3/4)*2^(1/4)/c^(3/4)
/(-4*a*c+b^2)^(1/2)-1/2*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1
/2))^(1/4))*(-b+(-4*a*c+b^2)^(1/2))^(3/4)*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(1/2
)
```

Rubi [A] time = 0.44, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1115, 1374, 298, 205, 208}

$$-\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tan}{2^{3/4}c^{3/4}\sqrt{b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(((-b - Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sq
rt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c])) + ((-b + Sqrt
[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*
c])^(1/4)])/(2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])
^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(
2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan
h[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(3/
4)*Sqrt[b^2 - 4*a*c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^{5/2}}{a+bx^2+cx^4}\,dx = 2\,\text{Subst}\left(\int \frac{x^6}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)$$

$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)+\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\Bigg(\int$$

$$= -\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\,\sqrt{c}\,x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{2}\,\sqrt{c}}+\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\Bigg(\int \frac{}{\sqrt{-b+\sqrt{}}}}{\sqrt{2}\,\sqrt{}}$$

$$= -\frac{\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}$$

Mathematica [C] time = 0.03, size = 48, normalized size = 0.15

$$\frac{1}{2}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^3\log\left(\sqrt{x}-\#1\right)}{2\#1^4c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[Sqrt[x] - #1]*#1^3)/(b + 2*c*#1^4) & ]
/2
```

fricas [B] time = 1.66, size = 4058, normalized size = 12.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^
5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^
8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*arctan(1/2*((b^4 -
 5*a*b^2*c + 4*a^2*c^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9
)))*sqrt((a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*x - 1/2*sqrt(1/2)*(a^3*b^7 - 6*a
^4*b^5*c + 9*a^5*b^3*c^2 - 4*a^6*b*c^3 + (a^3*b^8*c^3 - 13*a^4*b^6*c^4 + 60
*a^5*b^4*c^5 - 112*a^6*b^2*c^6 + 64*a^7*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^3 - 3*
a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*
b^2*c^4 + 16*a^2*c^5))) + (a^2*b^6 - 6*a^3*b^4*c + 9*a^4*b^2*c^2 - 4*a^5*c^
3 + (a^2*b^7*c^3 - 9*a^3*b^5*c^4 + 24*a^4*b^3*c^5 - 16*a^5*b*c^6)*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c
^9)))*sqrt(x))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4
 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 4
8*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/(a^3*b
^4 - 2*a^4*b^2*c + a^5*c^2)) + 2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4
*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6
- 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16
*a^2*c^5)))*arctan(-1/2*((b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^5*c^3 - 8*a*b^3*
c^4 + 16*a^2*b*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^
7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt((a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*x
 - 1/2*sqrt(1/2)*(a^3*b^7 - 6*a^4*b^5*c + 9*a^5*b^3*c^2 - 4*a^6*b*c^3 - (a^
3*b^8*c^3 - 13*a^4*b^6*c^4 + 60*a^5*b^4*c^5 - 112*a^6*b^2*c^6 + 64*a^7*c^7)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
```

- 64*a^3*c^9)))*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(sqrt(1/2)*sqrt(
-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*
c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))) + (a^2*b^6 - 6*a^3*b^4*c + 9*a^4*b^2*c^2
- 4*a^5*c^3 - (a^2*b^7*c^3 - 9*a^3*b^5*c^4 + 24*a^4*b^3*c^5 - 16*a^5*b*c^6)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*
a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^
4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
)))/(a^3*b^4 - 2*a^4*b^2*c + a^5*c^2)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*
a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*
b^2*c^4 + 16*a^2*c^5)))*log(1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2
 - 16*a^3*b*c^3 - (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^
6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 +
 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*
c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 -
 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*
a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*
a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*sqrt(x
)) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*
a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*
b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(-1/2*sqr
t(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 - (b^8*c^3 - 14*a*b
^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^
2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqr
t(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c + (
b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c
^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 +
 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*sqrt(x)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3
- 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^
2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 -
8*a*b^2*c^4 + 16*a^2*c^5)))*log(1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3
*c^2 - 16*a^3*b*c^3 + (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^
2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c
^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (
b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c
^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 +
 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*
sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 -

```
 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*sq
rt(x)) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 +
 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*
a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(-1/2
*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 + (b^8*c^3 - 14
*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))
*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c
 - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b
^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c
^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*sqrt(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{cx^4 + bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)/(c*x^4 + b*x^2 + a), x)
```

maple [C] time = 0.01, size = 45, normalized size = 0.14

$$\frac{\mathrm{RootOf}\left(c\_Z^8 + b\_Z^4 + a\right)^6 \ln\left(-\mathrm{RootOf}\left(c\_Z^8 + b\_Z^4 + a\right) + \sqrt{x}\right)}{4\,\mathrm{RootOf}\left(c\_Z^8 + b\_Z^4 + a\right)^7 c + 2\,\mathrm{RootOf}\left(c\_Z^8 + b\_Z^4 + a\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*sum(_R^6/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{cx^4 + bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 6.51, size = 8093, normalized size = 24.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] - atan(((x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2) + (-(b^7 + b^2*(-(4*a*c -
b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c -
 b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5
- 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 + x^(1/2)*(-(b^7 + b^2*(-(4*a*c -
 b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c
- b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5
 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*
b^2*c^5) + 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4))*(-(b^7 + b^2*(-(4*a*c - b
^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c -
b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 -
 256*a^3*b^2*c^6)))^(1/4)*1i + (x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2) - (
-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a
*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^
6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 - x^(1/2)*
(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*
a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b
^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 8192*a
^3*b^4*c^4 - 65536*a^4*b^2*c^5) + 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4))*(-
(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*
b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6
*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i)/((x^(1/2)*(256*a^3*b^3
*c - 768*a^4*b*c^2) - (-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3
+ 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*
c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(3
2768*a^5*c^5 - x^(1/2)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3
 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4
*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(
131072*a^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5) + 2048*a^3*b^4*c^3 -
 16384*a^4*b^2*c^4))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 +
 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c
^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4) - (
x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2) + (-(b^7 + b^2*(-(4*a*c - b^2)^5)^(
1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^
(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3
*b^2*c^6)))^(3/4)*(32768*a^5*c^5 + x^(1/2)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^
(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)
^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^
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3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5)
+ 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1
/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(
1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*
b^2*c^6)))^(1/4) + 256*a^4*b*c))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48
*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(3
2*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)
))^(1/4)*2i - atan(((x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2) + (-(b^7 - b^2
*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*
c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*
a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 + x^(1/2)*(-(b^7 - b^
2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a
c(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96
*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4
- 65536*a^4*b^2*c^5) + 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4))*(-(b^7 - b^2*
(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c
*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a
^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i + (x^(1/2)*(256*a^3*b^3*c - 768*a^
4*b*c^2) - (-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^
3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c
^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^
5 - x^(1/2)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b
^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*
c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*
c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5) + 2048*a^3*b^4*c^3 - 16384*a^4*
b^2*c^4))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3
*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^
3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i)/((x^(1/2)*
(256*a^3*b^3*c - 768*a^4*b*c^2) - (-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 4
8*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(
32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6
)))^(3/4)*(32768*a^5*c^5 - x^(1/2)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) -
48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/
(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^
6)))^(1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5) + 2048*a
^3*b^4*c^3 - 16384*a^4*b^2*c^4))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48
*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(3
2*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)
))^(1/4) - (x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2) + (-(b^7 - b^2*(-(4*a*c
 - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*
c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c
^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 + x^(1/2)*(-(b^7 - b^2*(-(4*a*
c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a
c - b^2)^5)^(1/2))/(32(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*
c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a

^4*b^2*c^5) + 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4))*(-(b^7 - b^2*(-(4*a*c
- b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c
 - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^
5 - 256*a^3*b^2*c^6)))^(1/4) + 256*a^4*b*c))*(-(b^7 - b^2*(-(4*a*c - b^2)^5
)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^
5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*
a^3*b^2*c^6)))^(1/4)*2i - 2*atan(((x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2)
+ (-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 1
1*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a
*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 - x^(1/
2)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 -
11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*
a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 819
2*a^3*b^4*c^4 - 65536*a^4*b^2*c^5)*1i + 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^
4)*1i)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^
2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 -
 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4) + (x^(1/2)*(256*a
^3*b^3*c - 768*a^4*b*c^2) - (-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*
b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(25
6*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3
/4)*(32768*a^5*c^5 + x^(1/2)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3
*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(2
56*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(
1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5)*1i + 2048*a^3*
b^4*c^3 - 16384*a^4*b^2*c^4)*1i)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48
*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(3
2*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)
))^(1/4))/((x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2) + (-(b^7 + b^2*(-(4*a*c
 - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*
c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c
^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 - x^(1/2)*(-(b^7 + b^2*(-(4*a*
c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a
c - b^2)^5)^(1/2))/(32(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*
c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a
^4*b^2*c^5)*1i + 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4)*1i)*(-(b^7 + b^2*(-(
4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-
(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*
b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i - (x^(1/2)*(256*a^3*b^3*c - 768*a^4*b
c^2) - (-(b^7 + b^2(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c
^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3
- 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 +
 x^(1/2)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*
c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3
 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6
 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5)*1i + 2048*a^3*b^4*c^3 - 16384*a^4*

b^2*c^4)*1i)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*
b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8
*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i + 256*a^
4*b*c))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c
^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3
- 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4) - 2*atan(((x^(1/
2)*(256*a^3*b^3*c - 768*a^4*b*c^2) + (-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2)
- 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2)
)/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*
c^6)))^(3/4)*(32768*a^5*c^5 - x^(1/2)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2)
 - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2
))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2
c^6)))^(1/4)(131072*a^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5)*1i +
2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4)*1i)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(
1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^
(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3
*b^2*c^6)))^(1/4) + (x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2) - (-(b^7 - b^2
*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*
c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*
a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 + x^(1/2)*(-(b^7 - b^
2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a
c(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96
*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4
- 65536*a^4*b^2*c^5)*1i + 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4)*1i)*(-(b^7
- b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c
 + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4
+ 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4))/((x^(1/2)*(256*a^3*b^3*c - 768
*a^4*b*c^2) + (-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2
*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^
8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5
c^5 - x^(1/2)(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^
2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b
^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a
^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5)*1i + 2048*a^3*b^4*c^3 - 1638
4*a^4*b^2*c^4)*1i)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 4
0*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7
 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i -
(x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2) - (-(b^7 - b^2*(-(4*a*c - b^2)^5)^
(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)
^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^
3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 + x^(1/2)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)
^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5
)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a
^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5)
*1i + 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4)*1i)*(-(b^7 - b^2*(-(4*a*c - b^2

```
)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^
2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 2
56*a^3*b^2*c^6)))^(1/4)*1i + 256*a^4*b*c))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^
(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)
^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^
3*b^2*c^6)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.1065 $\int \frac{x^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=331

$$\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{b^2-4ac}}+\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\,\tanh^{-1}\left(\right.}{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{b^2-4ac}}$$

```
[Out] 1/2*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*a
*c+b^2)^(1/2))^(1/4)*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(1/2)+1/2*arctanh(2^(1/4)
*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*a*c+b^2)^(1/2))^(1/
4)*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(1/2)-1/2*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-
b+(-4*a*c+b^2)^(1/2))^(1/4))*(-b+(-4*a*c+b^2)^(1/2))^(1/4)*2^(3/4)/c^(1/4)/
(-4*a*c+b^2)^(1/2)-1/2*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/
2))^(1/4))*(-b+(-4*a*c+b^2)^(1/2))^(1/4)*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1115, 1374, 212, 208, 205}

$$\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{b^2-4ac}}+\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\,\tanh^{-1}\left(\right.}{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt
[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^
2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])
^(1/4)])/(2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])^(1
/4)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(
1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(1/4)*ArcTanh[(
2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(1/4)*
Sqrt[b^2 - 4*a*c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^{3/2}}{a+bx^2+cx^4}\,dx = 2\,\text{Subst}\left(\int \frac{x^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)$$

$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right) + \left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\Bigg(\int$$

$$= \frac{\sqrt{-b-\sqrt{b^2-4ac}}\ \text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}} + \frac{\sqrt{-b-\sqrt{b^2-4ac}}\ \text{Subst}\Bigg(}{\sqrt{}}$$

$$= \frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\ \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[4]{-b+\sqrt{b^2-4ac}}\ \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[4]{-}}{}$$

Mathematica [C] time = 0.03, size = 46, normalized size = 0.14

$$\frac{1}{2}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1\log\left(\sqrt{x}-\#1\right)}{2\#1^4c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[Sqrt[x] - #1]*#1)/(b + 2*c*#1^4) & ]/2
```

fricas [B] time = 1.29, size = 2482, normalized size = 7.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -2*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^
2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*
a^2*c^3)))*arctan(1/2*(sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (b^7*c - 1
2*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(sqrt(1/2)*(b^2 - 4*a*c)*sqrt(-(b + (b^4*
c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
- 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) + x)*sqrt(-(b + (b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 6
4*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) - sqrt(1/2)*(b^4 - 8*a*b^2*
c + 16*a^2*c^2 - (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqr
t(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(x)*sqrt(-(b +
 (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(sqrt(1/2)*sq
rt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 4
8*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/a) + 2*sq
rt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 1
2*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3)))*arctan(-1/2*(sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + (b^7*c - 12*a*
b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5))*sqrt(sqrt(1/2)*(b^2 - 4*a*c)*sqrt(-(b - (b^4*c -
8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64
*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) + x)*sqrt(-(b - (b^4*c - 8*a
*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^
3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) - sqrt(1/2)*(b^4 - 8*a*b^2*c +
16*a^2*c^2 + (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^
6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(x)*sqrt(-(b - (b^
4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^
```

```
4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(sqrt(1/2)*sqrt(-
(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^
2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/a) + 1/2*sqrt
(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3
)))*log((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5) + sqrt(x)) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b
 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*
b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(-(b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 1
6*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c
^4 - 64*a^3*c^5) + sqrt(x)) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c
^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log((b^4*c - 8*a*b^2*c^2 + 16*a^2
*c^3)*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5) +
sqrt(x)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)
/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b
^2*c^2 + 16*a^2*c^3)))*log(-(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/
2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5) + sqrt(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{cx^4 + bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/(c*x^4 + b*x^2 + a), x)
```

maple [C] time = 0.01, size = 45, normalized size = 0.14

$$\frac{\mathrm{RootOf}\left(c\_Z^8 + b\_Z^4 + a\right)^4 \ln\left(-\mathrm{RootOf}\left(c\_Z^8 + b\_Z^4 + a\right) + \sqrt{x}\right)}{4\,\mathrm{RootOf}\left(c\_Z^8 + b\_Z^4 + a\right)^7 c + 2\,\mathrm{RootOf}\left(c\_Z^8 + b\_Z^4 + a\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*sum(_R^4/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{cx^4 + bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 6.02, size = 8229, normalized size = 24.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] atan(((x^(1/2)*(512*a^3*c^4 - 256*a^2*b^2*c^3) + (-(b^5 + (-(4*a*c - b^2)^5
)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2
 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^
(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 +
 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c
^4 + 98304*a^3*b^4*c^5 - 393216*a^4*b^2*c^6) - x^(1/2)*(65536*a^4*b*c^6 + 4
096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 1
6*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b
^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 2048*a^3*b*c^4 - 512*a^2*b^3*c^3))*(-(b
^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*
a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i + (x^
(1/2)*(512*a^3*c^4 - 256*a^2*b^2*c^3) - (-(b^5 + (-(4*a*c - b^2)^5)^(1/2) +
 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2
*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 1
6*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b
^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4 + 9830
4*a^3*b^4*c^5 - 393216*a^4*b^2*c^6) + x^(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b
^5*c^4 - 32768*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c
^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 -
256*a^3*b^2*c^4)))^(3/4) + 2048*a^3*b*c^4 - 512*a^2*b^3*c^3))*(-(b^5 + (-(4
*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 -
 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i)/((x^(1/2)*(51
2*a^3*c^4 - 256*a^2*b^2*c^3) + (-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b
*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3
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- 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c ^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4 + 98304*a^3*b^4 *c^5 - 393216*a^4*b^2*c^6) - x^(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a* b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b ^2*c^4)))^(3/4) + 2048*a^3*b*c^4 - 512*a^2*b^3*c^3))*(-(b^5 + (-(4*a*c - b^ 2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6 *c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) - (x^(1/2)*(512*a^3*c^4 - 256*a^2*b^2*c^3) - (-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b ^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^ 2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3 *c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2* c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4 + 98304*a^3*b^4*c^5 - 39321 6*a^4*b^2*c^6) + x^(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^ 3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3 /4) + 2048*a^3*b*c^4 - 512*a^2*b^3*c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^ 2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2* b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*2i - 2*atan(((x^(1/2)*(512*a^3*c^4 - 256 *a^2*b^2*c^3) + (-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3* c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c ^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c) /(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4 + 98304*a^3*b^4*c^5 - 393216*a ^4*b^2*c^6)*1i + x^(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^ 3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3 /4)*1i - 2048*a^3*b*c^4 + 512*a^2*b^3*c^3)*1i)*(-(b^5 + (-(4*a*c - b^2)^5)^ (1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) + (x^(1/2)*(512*a^3*c^4 - 256*a^ 2*b^2*c^3) - (-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/ (32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(3 2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4))) ^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4 + 98304*a^3*b^4*c^5 - 393216*a^4* b^2*c^6)*1i - x^(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c ^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8 *c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) *1i - 2048*a^3*b*c^4 + 512*a^2*b^3*c^3)*1i)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/ 2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96 *a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4))/((x^(1/2)*(512*a^3*c^4 - 256*a^2*b ^2*c^3) + (-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32

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*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^
(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(
b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1
/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4 + 98304*a^3*b^4*c^5 - 393216*a^4*b^2
*c^6)*1i + x^(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5)
)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c
+ 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i
 - 2048*a^3*b*c^4 + 512*a^2*b^3*c^3)*1i)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2)
+ 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^
2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i - (x^(1/2)*(512*a^3*c^4 - 256*a^2*b
^2*c^3) - (-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32
*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^
(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(
b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1
/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4 + 98304*a^3*b^4*c^5 - 393216*a^4*b^2
*c^6)*1i - x^(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5)
)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c
+ 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i
 - 2048*a^3*b*c^4 + 512*a^2*b^3*c^3)*1i)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2)
+ 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^
2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2)
 + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a
^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) - atan(((x^(1/2)*(512*a^3*c^4 - 256*a
^2*b^2*c^3) - ((x^(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3
*c^5) + (-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(
b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1
/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4 + 98304*a^3*b^4*c^5 - 393216*a^4*b^2
*c^6))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b
^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/
4) + 2048*a^3*b*c^4 - 512*a^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) +
16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*
b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*
a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4
*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i + (x^(1/2)*(512*a^3*c^4 - 256*a^2*b^2*c^
3) - ((x^(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5) - (
-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 2
56*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(5242
88*a^5*c^7 - 8192*a^2*b^6*c^4 + 98304*a^3*b^4*c^5 - 393216*a^4*b^2*c^6))*(-
(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 25
6*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) - 2048
*a^3*b*c^4 + 512*a^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*
c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 -
 256*a^3*b^2*c^4)))^(1/4))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 25
6*a^3*b^2*c^4)))^(1/4)*1i)/((x^(1/2)*(512*a^3*c^4 - 256*a^2*b^2*c^3) - ((x^
```

(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5) + (-(b^5 - (
-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^
5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^
7 - 8192*a^2*b^6*c^4 + 98304*a^3*b^4*c^5 - 393216*a^4*b^2*c^6))*(-(b^5 - (-
(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5
 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 2048*a^3*b*c^
4 - 512*a^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a
*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*
b^2*c^4)))^(1/4))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^
3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2
c^4)))^(1/4) - (x^(1/2)(512*a^3*c^4 - 256*a^2*b^2*c^3) - ((x^(1/2)*(65536
*a^4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5) - (-(b^5 - (-(4*a*c - b^
2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6
*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2
*b^6*c^4 + 98304*a^3*b^4*c^5 - 393216*a^4*b^2*c^6))*(-(b^5 - (-(4*a*c - b^2
)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*
c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) - 2048*a^3*b*c^4 + 512*a^2*
b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*
(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(
1/4))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^
8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4
)))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*
c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*
2i + 2*atan(((x^(1/2)*(512*a^3*c^4 - 256*a^2*b^2*c^3) + ((x^(1/2)*(65536*a^
4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5) - (-(b^5 - (-(4*a*c - b^2)^
5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^
2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^
6*c^4 + 98304*a^3*b^4*c^5 - 393216*a^4*b^2*c^6)*1i)*(-(b^5 - (-(4*a*c - b^2
)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*
c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i + 2048*a^3*b*c^4 - 512*a
^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(
32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4))
)^(1/4)*1i)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(
32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4))
)^(1/4) + (x^(1/2)*(512*a^3*c^4 - 256*a^2*b^2*c^3) + ((x^(1/2)*(65536*a^4*b
*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5) + (-(b^5 - (-(4*a*c - b^2)^5)^
(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 +
 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c
^4 + 98304*a^3*b^4*c^5 - 393216*a^4*b^2*c^6)*1i)*(-(b^5 - (-(4*a*c - b^2)^5
)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2
 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i - 2048*a^3*b*c^4 + 512*a^2*
b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*
(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(
1/4)*1i)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*
(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(

```
1/4))/((x^(1/2)*(512*a^3*c^4 - 256*a^2*b^2*c^3) + ((x^(1/2)*(65536*a^4*b*c^
6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5) - (-(b^5 - (-(4*a*c - b^2)^5)^(1/
2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96
*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4
+ 98304*a^3*b^4*c^5 - 393216*a^4*b^2*c^6)*1i)*(-(b^5 - (-(4*a*c - b^2)^5)^(
1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 +
96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i + 2048*a^3*b*c^4 - 512*a^2*b^3
*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^
8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4
)*1i)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^
8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4
)*1i - (x^(1/2)*(512*a^3*c^4 - 256*a^2*b^2*c^3) + ((x^(1/2)*(65536*a^4*b*c^
6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5) + (-(b^5 - (-(4*a*c - b^2)^5)^(1/
2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96
*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4
+ 98304*a^3*b^4*c^5 - 393216*a^4*b^2*c^6)*1i)*(-(b^5 - (-(4*a*c - b^2)^5)^(
1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 +
96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i - 2048*a^3*b*c^4 + 512*a^2*b^3
*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^
8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4
)*1i)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^
8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4
)*1i))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b
^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/
4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.1066 $\int \frac{\sqrt{x}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=331

$$-\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\,\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\,\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\,\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\,\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

```
[Out] -2^(1/4)*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/
4))/(-b-(-4*a*c+b^2)^(1/2))^(1/4)/(-4*a*c+b^2)^(1/2)+2^(1/4)*c^(1/4)*arctan
h(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))/(-b-(-4*a*c+b^2)^(
1/2))^(1/4)/(-4*a*c+b^2)^(1/2)+2^(1/4)*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/
2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2)
)^(1/4)-2^(1/4)*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1
/2))^(1/4))/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A] time = 0.37, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1115, 1375, 298, 205, 208}

$$-\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\,\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\,\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\,\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\,\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((2^(1/4)*c^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c]
)^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) + (2^(1/4)*c^
(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sq
rt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (2^(1/4)*c^(1/4)*ArcTanh[
(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*
c]*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (2^(1/4)*c^(1/4)*ArcTanh[(2^(1/4)*c^(1
/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt
[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{a+bx^2+cx^4}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(2c)\,\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}} - \frac{(2c)\,\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(\sqrt{2}\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}} - \frac{\left(\sqrt{2}\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\cdots}\right.}{\sqrt{b^2-4ac}}$$

$$= -\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\cdots}{\sqrt[4]{-b-\cdots}}\right.}{\sqrt{b^2-4ac}\,\sqrt[4]{-b-\sqrt{b^2\cdots}}}$$

Mathematica [C] time = 0.03, size = 47, normalized size = 0.14

$$\frac{1}{2}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2 + c*x^4),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[Sqrt[x] - #1]/(b*#1 + 2*c*#1^5) & ]/2
```

fricas [B] time = 1.17, size = 2769, normalized size = 8.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -2*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^
6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2)))*arctan(((a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*sqrt(c^2*x - 1/2*sqrt
(1/2)*(b^3*c - 4*a*b*c^2 - (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*
a^4*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-
(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^
4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))/sqrt(a^2*b^6
- 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3) - (a*b^4*c - 8*a^2*b^2*c^2 +
16*a^3*c^3)*sqrt(x)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 1
6*a^3*c^2)))/c) + 2*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3
*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 -
8*a^2*b^2*c + 16*a^3*c^2)))*arctan(((a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*sqrt
(c^2*x - 1/2*sqrt(1/2)*(b^3*c - 4*a*b*c^2 + (a*b^6*c - 12*a^2*b^4*c^2 + 48*
a^3*b^2*c^3 - 64*a^4*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64
*a^5*c^3))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*
a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
)))*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b
^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)))/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3) - (a
*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b - (a*b
^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))/sqrt(a^2*b^6 - 12*a^3*
b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/c) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b + (a
*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(1/2*sqrt(1/2)*(b
```

```
^4 - 8*a*b^2*c + 16*a^2*c^2 - (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a
^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(
sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a
^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)
))*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*
c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*s
qrt(x)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/
sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*
b^2*c + 16*a^3*c^2)))*log(-1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (a
*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*
b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*
a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a
^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b + (a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*sqrt(x)) - 1/2*sqrt(sqrt(1/2)*sqr
t(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48
*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(1/2*sq
rt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*
c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3))*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 1
6*a^3*c^2)))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c
^2)) + c*sqrt(x)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^
4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(-1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^
2*c^2 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b - (
a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*
c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b - (a*b^4 -
 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*sqrt(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/(c*x^4 + b*x^2 + a), x)
```

maple [C] time = 0.01, size = 45, normalized size = 0.14

$$\frac{\operatorname{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{2}\ln\left(-\operatorname{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)+\sqrt{x}\right)}{4\operatorname{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{7}c+2\operatorname{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2+a),x)
[Out] 1/2*sum(_R^2/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(x)/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 5.31, size = 6133, normalized size = 18.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x^2 + c*x^4),x)
[Out] 2*atan((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*
(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(
3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3*c^5 - x^(1/2)*(-(b^5
 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^
5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(131072*a^
4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^6)*1i)*1i - 2
56*a*b*c^5*x^(1/2))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*
b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b
^2*c^3)))^(1/4) - ((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b
^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^
2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3*c^5 + x^(1
/2)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^
8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*
(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^6)*
1i)*1i + 256*a*b*c^5*x^(1/2))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*
c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 -
```

256*a^4*b^2*c^3)))^(1/4))/(((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c
^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 -
256*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3*
c^5 - x^(1/2)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)
/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3
)))^(1/4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3
*b^2*c^6)*1i)*1i - 256*a*b*c^5*x^(1/2))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) +
16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3
*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i - 256*a*c^5 + ((-(b^5 - (-(4*a*c - b
^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*
b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a
^3*b*c^6 - 16384*a^2*b^3*c^5 + x^(1/2)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) +
16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*
b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960
*a^2*b^4*c^5 - 131072*a^3*b^2*c^6)*1i)*1i + 256*a*b*c^5*x^(1/2))*(-(b^5 - (
-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^
4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i))*(-(b^5 -
(-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c
^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4) - atan((((-(b
^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*
a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(2048*a*
b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3*c^5 + x^(1/2)*(-(b^5 - (-(4*a*c -
b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^
2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(131072*a^4*c^7 - 4096*
a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^6)) - 256*a*b*c^5*x^(1/2))
*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 +
256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i
- ((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8
+ 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(
2048*a*b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3*c^5 - x^(1/2)*(-(b^5 - (-(
4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4
- 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(131072*a^4*c^7
- 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^6)) + 256*a*b*c^5*x
^(1/2))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(
a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1
/4)*1i)/(256*a*c^5 + ((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*
a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4
*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3*c^5 + x
^(1/2)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a
*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/
4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^
6)) - 256*a*b*c^5*x^(1/2))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
- 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 25
6*a^4*b^2*c^3)))^(1/4) + ((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
- 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256

*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3*c^5
 - x^(1/2)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(3
2*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))
^(1/4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^
2*c^6)) + 256*a*b*c^5*x^(1/2))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b
*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2
- 256*a^4*b^2*c^3)))^(1/4)))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c
^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 -
256*a^4*b^2*c^3)))^(1/4)*2i - atan((((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16
*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^
4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 + x^(1/2
)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8
+ 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(1
31072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^6) -
16384*a^2*b^3*c^5) - 256*a*b*c^5*x^(1/2))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2)
 + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a
^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i - ((-(b^5 + (-(4*a*c - b^2)^5)^(1/
2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96
*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 -
 x^(1/2)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*
(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(
1/4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*
c^6) - 16384*a^2*b^3*c^5) + 256*a*b*c^5*x^(1/2))*(-(b^5 + (-(4*a*c - b^2)^5
)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c
 + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i)/(256*a*c^5 + ((-(b^5 + (-(
4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4
- 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4
+ 32768*a^3*b*c^6 + x^(1/2)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^
2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 2
56*a^4*b^2*c^3)))^(1/4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^
5 - 131072*a^3*b^2*c^6) - 16384*a^2*b^3*c^5) - 256*a*b*c^5*x^(1/2))*(-(b^5
+ (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5
*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4) + ((-(b^5 +
 (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*
c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*
c^4 + 32768*a^3*b*c^6 - x^(1/2)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*
b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2
 - 256*a^4*b^2*c^3)))^(1/4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^
4*c^5 - 131072*a^3*b^2*c^6) - 16384*a^2*b^3*c^5) + 256*a*b*c^5*x^(1/2))*(-(
b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256
*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)))*(-(b^
5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a
^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*2i + 2*at
an((((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b
^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)

```
*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - x^(1/2)*(-(b^5 + (-(4*a*c - b^2)^5)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 9
6*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 +
 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^6)*1i - 16384*a^2*b^3*c^5)*1i - 256*a
*b*c^5*x^(1/2))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*
c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c
^3)))^(1/4) - ((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c
)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^
3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 + x^(1/2)*(-(b^5 + (-(4*a*c -
b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2
*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(131072*a^4*c^7 - 4096*a
*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^6)*1i - 16384*a^2*b^3*c^5)*
1i + 256*a*b*c^5*x^(1/2))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
- 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256
*a^4*b^2*c^3)))^(1/4))/(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 -
 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*
a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - x^(1/2)*(-(b^5 + (
-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^
4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(131072*a^4*c^
7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^6)*1i - 16384*a^2
*b^3*c^5)*1i - 256*a*b*c^5*x^(1/2))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*
a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4
*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i - 256*a*c^5 + ((-(b^5 + (-(4*a*c - b^2)^
5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*
c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b
*c^6 + x^(1/2)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c
)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^
3)))^(1/4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^
3*b^2*c^6)*1i - 16384*a^2*b^3*c^5)*1i + 256*a*b*c^5*x^(1/2))*(-(b^5 + (-(4*
a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 -
16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i))*(-(b^5 + (-(4
*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 -
 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.1067 $\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=331

$$\frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}c^{3/4}\tanh^{-1}\ldots}{\sqrt{b^2-4ac}\left(\sqrt{\ldots}\right.}$$

```
[Out] 2^(3/4)*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2))^(1/2))^(1/4
))/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(-4*a*c+b^2)^(1/2)+2^(3/4)*c^(3/4)*arctanh
(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))/(-b-(-4*a*c+b^2)^(1
/2))^(3/4)/(-4*a*c+b^2)^(1/2)-2^(3/4)*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2
)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))
^(3/4)-2^(3/4)*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/
2))^(1/4))/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A] time = 0.42, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1115, 1347, 212, 208, 205}

$$\frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}c^{3/4}\tanh^{-1}\ldots}{\sqrt{b^2-4ac}\left(\sqrt{\ldots}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2^(3/4)*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^
(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (2^(3/4)*c^(3/
4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[
b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (2^(3/4)*c^(3/4)*ArcTanh[(2^
(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*
(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (2^(3/4)*c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^
2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(2c)\,\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}} - \frac{(2c)\,\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}}$$

$$= \frac{(2c)\,\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}} + \frac{(2c)\,\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}\,x^2}\right.}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4a}}}$$

$$= \frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{2^{3/4}c^{3/4}\tanh^{-}}{\sqrt{b^2-4ac}\left(-b\right.}$$

Mathematica [C] time = 0.03, size = 49, normalized size = 0.15

$$\frac{1}{2}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[Sqrt[x] - #1]/(b*#1^3 + 2*c*#1^7) & ]/2
```

fricas [B] time = 1.41, size = 4045, normalized size = 12.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^
2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*arctan(-1/4*(sqrt(
1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 - (a^3*b^8 - 14*a^4*b
^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3 + 128*a^7*c^4)*sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(
4*(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x + 2*sqrt(1/2)*(b^8 - 8*a*b^6*c + 21*a
```

```
^2*b^4*c^2 - 22*a^3*b^2*c^3 + 8*a^4*c^4 - (a^3*b^9 - 13*a^4*b^7*c + 60*a^5*
b^5*c^2 - 112*a^6*b^3*c^3 + 64*a^7*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^3 - 3*a*b
*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b
^2*c + 16*a^5*c^2)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5
*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2
*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)) + 2*sqrt(1/2)*(b
^9*c - 10*a*b^7*c^2 + 33*a^2*b^5*c^3 - 40*a^3*b^3*c^4 + 16*a^4*b*c^5 - (a^3
*b^10*c - 15*a^4*b^8*c^2 + 86*a^5*b^6*c^3 - 232*a^6*b^4*c^4 + 288*a^7*b^2*c
^5 - 128*a^8*c^6)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c
+ 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(x)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 -
8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^
7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*
c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*
c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))/(b^4*c^3 - 2*a*b
^2*c^4 + a^2*c^5)) + 2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a
^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b
^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))
)*arctan(1/4*(sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 +
(a^3*b^8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3 + 128*a^7*c^4)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
64*a^9*c^3)))*sqrt(4*(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x + 2*sqrt(1/2)*(b^8
 - 8*a*b^6*c + 21*a^2*b^4*c^2 - 22*a^3*b^2*c^3 + 8*a^4*c^4 + (a^3*b^9 - 13*
a^4*b^7*c + 60*a^5*b^5*c^2 - 112*a^6*b^3*c^3 + 64*a^7*b*c^4)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))
*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))
/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c
 - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a
^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2
*c + 16*a^5*c^2)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c
^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c
^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)) + 2*sqrt(1/2)*(b^9
*c - 10*a*b^7*c^2 + 33*a^2*b^5*c^3 - 40*a^3*b^3*c^4 + 16*a^4*b*c^5 + (a^3*b
^10*c - 15*a^4*b^8*c^2 + 86*a^5*b^6*c^3 - 232*a^6*b^4*c^4 + 288*a^7*b^2*c^5
 - 128*a^8*c^6)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c +
48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c
- (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^
6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*
c + 16*a^5*c^2)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^
2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))/(b^4*c^3 - 2*a*b^2
*c^4 + a^2*c^5)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a
```

```
^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b
^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))
)*log(-2*(b^2*c - a*c^2)*sqrt(x) + (b^4 - 5*a*b^2*c + 4*a^2*c^2 - (a^3*b^5
- 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 1
2*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*
a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^
4*b^2*c + 16*a^5*c^2)))) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b
^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 -
12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a
^5*c^2)))*log(-2*(b^2*c - a*c^2)*sqrt(x) - (b^4 - 5*a*b^2*c + 4*a^2*c^2 - (
a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6
*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(
b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^
4 - 8*a^4*b^2*c + 16*a^5*c^2)))) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c
- (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^
6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*
c + 16*a^5*c^2)))*log(-2*(b^2*c - a*c^2)*sqrt(x) + (b^4 - 5*a*b^2*c + 4*a^2
*c^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c
^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)
*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))
/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 -
3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*
c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*
a^4*b^2*c + 16*a^5*c^2)))*log(-2*(b^2*c - a*c^2)*sqrt(x) - (b^4 - 5*a*b^2*c
 + 4*a^2*c^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c
 + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(s
qrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((
b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^
9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4 + bx^2 + a\right)\sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(x)), x)
```

maple [C] time = 0.01, size = 42, normalized size = 0.13

$$\frac{\ln\left(-\operatorname{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{4\operatorname{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7c+2\operatorname{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*sum(1/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\sqrt{x}}{a}-\int\frac{cx^{\frac{7}{2}}+bx^{\frac{3}{2}}}{acx^4+abx^2+a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)/a - integrate((c*x^(7/2) + b*x^(3/2))/(a*c*x^4 + a*b*x^2 + a^2),
x)
```

mupad [B] time = 6.26, size = 10401, normalized size = 31.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] - atan((((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*
c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4
 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(2048*a*c^7 - 5
12*b^2*c^6 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2
*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^
7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^
7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6) + x^(1/2
)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c^6))
*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11
*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^
4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)) + 512*c^7*x^(1/2))*(-(
b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b
^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^
6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i - ((-(b^7 + b^2*(-(4*a*c
```

- b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(2048*a*c^7 - 512*b^2*c^6 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6) - x^(1/2)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)) - 512*c^7*x^(1/2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i)/(((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(2048*a*c^7 - 512*b^2*c^6 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6) + x^(1/2)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)) + 512*c^7*x^(1/2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4) + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(2048*a*c^7 - 512*b^2*c^6 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6) - x^(1/2)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)) - 512*c^7*x^(1/2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*2i - atan((((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 +

256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(204
8*a*c^7 - 512*b^2*c^6 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c
^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b
^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)
*(8192*a*b^7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^
6) + x^(1/2)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^
2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b
^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*
c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)) + 512*c^7*x
^(1/2))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c
^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4
- 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i - ((-(b^7 - b
^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c +
a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 9
6*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(2048*a*c^7 - 512*b^2*c^6 + ((-(b^
7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5
*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*
c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^7*c^4 - 524288*a^4*
b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6) - x^(1/2)*(4096*b^7*c^4 - 4
5056*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*
a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4
*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^
4*c^2 - 256*a^6*b^2*c^3)))^(3/4)) - 512*c^7*x^(1/2))*(-(b^7 - b^2*(-(4*a*c
- b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c
- b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^
2 - 256*a^6*b^2*c^3)))^(1/4)*1i)/(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) -
48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/
(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^
3)))^(1/4)*(2048*a*c^7 - 512*b^2*c^6 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/
2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1
/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b
^2*c^3)))^(1/4)*(8192*a*b^7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 39
3216*a^3*b^3*c^6) + x^(1/2)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*
c^7 + 163840*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b
*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3
*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/
4)) + 512*c^7*x^(1/2))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3
+ 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8
+ 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4) +
((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 1
1*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a
^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(2048*a*c^7 - 512*b^2*
c^6 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^
2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 -
16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^7*c^4 -

524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6) - x^(1/2)*(4096
*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c^6))*(-(b^7
 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*
c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c
 + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)) - 512*c^7*x^(1/2))*(-(b^7 - b
^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c +
a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 9
6*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^
(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)
^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^
6*b^2*c^3)))^(1/4)*2i - 2*atan((((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48
*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(3
2*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)
))^(1/4)*(512*b^2*c^6 - 2048*a*c^7 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2)
 - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2
))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2
c^3)))^(1/4)(8192*a*b^7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 3932
16*a^3*b^3*c^6)*1i + x^(1/2)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b
*c^7 + 163840*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*
b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^
3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3
/4)*1i)*1i - 512*c^7*x^(1/2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^
3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(
a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^
(1/4) - ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*
c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4
 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(512*b^2*c^6 -
2048*a*c^7 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2
*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^
7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^
7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6)*1i - x^(
1/2)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c^
6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 -
 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16
*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i)*1i + 512*c^7*x^(
1/2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2
 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 -
16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4))/(((-(b^7 + b^2*(-
(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(
-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5
*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(512*b^2*c^6 - 2048*a*c^7 + ((-(b^7 + b
^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c -
a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 9
6*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^7*c^4 - 524288*a^4*b*c^7
 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6)*1i + x^(1/2)*(4096*b^7*c^4 - 450

56*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*
c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a
c - b^2)^5)^(1/2))/(32(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*
c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i)*1i - 512*c^7*x^(1/2))*(-(b^7 + b^2*(-(4*
a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4
*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^
4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2
) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/
2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^
2*c^3)))^(1/4)*(512*b^2*c^6 - 2048*a*c^7 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)
^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5
)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a
^6*b^2*c^3)))^(1/4)*(8192*a*b^7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5
+ 393216*a^3*b^3*c^6)*1i - x^(1/2)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608
*a^3*b*c^7 + 163840*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 4
8*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(
32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3
)))^(3/4)*1i)*1i + 512*c^7*x^(1/2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) -
48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))
/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c
^3)))^(1/4)*1i))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*
a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256
*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4) - 2*ata
n((((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 -
11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16
*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(512*b^2*c^6 - 2048*
a*c^7 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*
c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4
- 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^7*c^4
- 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6)*1i + x^(1/2)*
(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c^6))*(
-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a
*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*
b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i)*1i - 512*c^7*x^(1/2))
(-(b^7 - b^2(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11
*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^
4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4) - ((-(b^7 - b^2*(-(4*a*
c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a
c - b^2)^5)^(1/2))/(32(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*
c^2 - 256*a^6*b^2*c^3)))^(1/4)*(512*b^2*c^6 - 2048*a*c^7 + ((-(b^7 - b^2*(-
(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(
-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5
*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^7*c^4 - 524288*a^4*b*c^7 - 98
304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6)*1i - x^(1/2)*(4096*b^7*c^4 - 45056*a*
b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b

```
^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c -
b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 -
 256*a^6*b^2*c^3)))^(3/4)*1i)*1i + 512*c^7*x^(1/2))*(-(b^7 - b^2*(-(4*a*c -
 b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c
- b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2
 - 256*a^6*b^2*c^3)))^(1/4))/(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a
^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*
(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))
^(1/4)*(512*b^2*c^6 - 2048*a*c^7 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) -
 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))
/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c
^3)))^(1/4)*(8192*a*b^7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 393216
*a^3*b^3*c^6)*1i + x^(1/2)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*c
^7 + 163840*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*
c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*
b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4
)*1i)*1i - 512*c^7*x^(1/2))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*
b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^
3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1
/4)*1i + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3
*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^
4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(512*b^2*c^6 -
 2048*a*c^7 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^
2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a
^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b
^7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6)*1i - x^
(1/2)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c
^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2
- 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 1
6*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i)*1i + 512*c^7*x^
(1/2))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^
2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 -
 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i))*(-(b^7 - b^2
*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*
c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*
a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.1068 $\int \frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=371

$$-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\cdots\right.}{2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-\cdots}}$$

```
[Out] -1/2*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*
(1-b/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/2*c^(1/4
)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(1-b/(-4*a
*c+b^2)^(1/2))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/2*c^(1/4)*arctan(2
^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-4*a*c+b^2)^(1/
2))*2^(1/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/2*c^(1/4)*arctanh(2^(1/4)*c^(
1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-4*a*c+b^2)^(1/2))*2^(1/4
)/a/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-2/a/x^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1115, 1368, 1510, 298, 205, 208}

$$-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\cdots\right.}{2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -2/(a*Sqrt[x]) - (c^(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*
c])^(1/4)) - (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqr
t[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^
(1/4)) + (c^(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x
])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/
4)) + (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/
(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4+cx^8\right)}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{2}{a\sqrt{x}} + \frac{2\,\text{Subst}\left(\int \frac{x^2\left(-b-cx^4\right)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{a}$$
$$= -\frac{2}{a\sqrt{x}} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{a} - \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\cdots}{\cdots}$$
$$= -\frac{2}{a\sqrt{x}} + \frac{\left(\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{2}\,a} - \frac{\left(\sqrt{c}\left(1-\cdots\right.\right.}{\cdots}$$
$$= -\frac{2}{a\sqrt{x}} - \frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\sqrt[4]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\cdots}{\sqrt[4]{-b+\cdots}}\right.}{2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

Mathematica [C] time = 0.05, size = 78, normalized size = 0.21

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{\#1^4c\log\left(\sqrt{x}-\#1\right)+b\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]+\frac{4}{\sqrt{x}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*(4/Sqrt[x] + RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[Sqrt[x] - #1] + c*
Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/a
```

fricas [B] time = 3.52, size = 5384, normalized size = 14.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(4*a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4
- 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*
```

b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*arctan(1/2*((b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt((b^8*c^8 - 6*a*b^6*c^9 + 11*a^2*b^4*c^10 - 6*a^3*b^2*c^11 + a^4*c^12)*x - 1/2*sqrt(1/2)*(b^13*c^5 - 13*a*b^11*c^6 + 65*a^2*b^9*c^7 - 155*a^3*b^7*c^8 + 175*a^4*b^5*c^9 - 79*a^5*b^3*c^10 + 12*a^6*b*c^11 + (a^5*b^12*c^5 - 16*a^6*b^10*c^6 + 100*a^7*b^8*c^7 - 305*a^8*b^6*c^8 + 460*a^9*b^4*c^9 - 304*a^10*b^2*c^10 + 64*a^11*c^11)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))) - (b^10*c^4 - 10*a*b^8*c^5 + 35*a^2*b^6*c^6 - 50*a^3*b^4*c^7 + 25*a^4*b^2*c^8 - 4*a^5*c^9 + (a^5*b^9*c^4 - 11*a^6*b^7*c^5 + 41*a^7*b^5*c^6 - 56*a^8*b^3*c^7 + 16*a^9*b*c^8)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(x))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))/(b^8*c^5 - 6*a*b^6*c^6 + 11*a^2*b^4*c^7 - 6*a^3*b^2*c^8 + a^4*c^9)) - 4*a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*arctan(-1/2*((b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt((b^8*c^8 - 6*a*b^6*c^9 + 11*a^2*b^4*c^10 - 6*a^3*b^2*c^11 + a^4*c^12)*x - 1/2*sqrt(1/2)*(b^13*c^5 - 13*a*b^11*c^6 + 65*a^2*b^9*c^7 - 155*a^3*b^7*c^8 + 175*a^4*b^5*c^9 - 79*a^5*b^3*c^10 + 12*a^6*b*c^11 - (a^5*b^12*c^5 - 16*a^6*b^10*c^6 + 100*a^7*b^8*c^7 - 305*a^8*b^6*c^8 + 460*a^9*b^4*c^9 - 304*a^10*b^2*c^10 + 64*a^11*c^11)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))) - (b^10*c^4 - 10*a*b^8*c^5 + 35*a^2*b^6*c^6 - 50*a^3*b^4*c^7 + 25*a^4*b^2*c^8 - 4*a^5*c^9 - (a^5*b^9*c^4 - 11*a^6*b^7*c^5 + 41*a^7*b^5*c^6 - 56*a^8*b^3*c^7 + 16*a^9*b*c^8)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 -

12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))))/(b^8*c^5 - 6*a*b^6*c^6 + 11*a^2*b^4*c^7 - 6*a^3*b^2*c^8 + a^4*c^9)) - a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*log(1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 - (a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)) + (b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*sqrt(x)) + a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*log(-1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 - (a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)) + (b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*sqrt(x)) - a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*log(1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 + (a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -

```
6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a
^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(-(b^5 - 5*a*b^3*c +
5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c +
11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^
12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)) + (b^4*c^
4 - 3*a*b^2*c^5 + a^2*c^6)*sqrt(x)) + a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b
^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c
+ 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*l
og(-1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 1
28*a^4*b^3*c^4 - 32*a^5*b*c^5 + (a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 -
 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*a*b^6*c +
11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^
12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b
*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*
b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*
c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(-(b^5 - 5*
a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*
a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4
*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))
 + (b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*sqrt(x)) + 4*sqrt(x))/(a*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 7.01Unable to convert to re
al 1/4 Error: Bad Argument Value
```

maple [C] time = 0.01, size = 65, normalized size = 0.18

$$-\frac{\left(\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^6 c+\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^2 b\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{2a\left(2\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7 c+\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3 b\right)}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/a*sum((_R^6*c+_R^2*b)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8
*c+_Z^4*b+a))-2/a/x^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2}{a\sqrt{x}} - \int \frac{cx^{\frac{5}{2}} + b\sqrt{x}}{acx^4 + abx^2 + a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -2/(a*sqrt(x)) - integrate((c*x^(5/2) + b*sqrt(x))/(a*c*x^4 + a*b*x^2 + a^2
), x)
```

mupad [B] time = 5.74, size = 10573, normalized size = 28.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] 2*atan((((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*
c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a
*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c
+ 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 - x^(1/2)*(-(b^
9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*
b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*
c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c
^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*
a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7)*1i + 2048*a^11*b^
8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*b^2*c^7)*1i +
 256*a^11*b*c^8*x^(1/2))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4
 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - ((-(b^9 + b^4
*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2
)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 25
6*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 + x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^
2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(3
2*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)
))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800
*a^14*b^4*c^6 - 327680*a^15*b^2*c^7)*1i + 2048*a^11*b^8*c^4 - 22528*a^12*b^
6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*b^2*c^7)*1i - 256*a^11*b*c^8*x^(1/
2))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*
```

c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*
a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4))/(((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(
1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*
b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4
)*(32768*a^15*c^8 - x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*
b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
 - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*
c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16
*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 32768
0*a^15*b^2*c^7)*1i + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^
4*c^6 - 114688*a^14*b^2*c^7)*1i + 256*a^11*b*c^8*x^(1/2))*(-(b^9 + b^4*(-(4
*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^
2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^
(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8
*b^2*c^3)))^(1/4)*1i + ((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^
4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 1
3*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4
- 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8
+ x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5
*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*
a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c
 + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b
^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7)*1i
 + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^1
4*b^2*c^7)*1i - 256*a^11*b*c^8*x^(1/2))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/
2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b
^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^
8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*
1i))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
- 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2
c(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96
*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - atan((((-(b^9 - b^4*(-(4*a*c - b^
2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(3
2*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)
))^(3/4)*(32768*a^15*c^8 + x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) +
80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5
)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 2
56*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(1310
72*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6
- 327680*a^15*b^2*c^7) + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^1
3*b^4*c^6 - 114688*a^14*b^2*c^7) + 256*a^11*b*c^8*x^(1/2))*(-(b^9 - b^4*(-(
4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a
^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)

^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^
8*b^2*c^3)))^(1/4)*1i - ((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4
 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8
 - x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^
5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3
*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*
c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*
b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7) +
 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*
b^2*c^7) - 256*a^11*b*c^8*x^(1/2))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) +
80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5
)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 2
56*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i)/(
((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 12
0*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(
-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7
*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 + x^(1/2)*(-(b^9 - b^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)
^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256
*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6
*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7) + 2048*a^11*b^8*c^4 - 225
28*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*b^2*c^7) + 256*a^11*b*c^
8*x^(1/2))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^
5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3
*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*
c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) + ((-(b^9 - b^4*(-(4*a*c - b^
2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(3
2*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)
))^(3/4)*(32768*a^15*c^8 - x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) +
80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5
)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 2
56*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(1310
72*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6
 - 327680*a^15*b^2*c^7) + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^1
3*b^4*c^6 - 114688*a^14*b^2*c^7) - 256*a^11*b*c^8*x^(1/2))*(-(b^9 - b^4*(-(
4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a
^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)
^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^
8*b^2*c^3)))^(1/4)))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 +
 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a
*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 1

6*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*2i - atan((((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 + x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7) + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*b^2*c^7) + 256*a^11*b*c^8*x^(1/2))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i - ((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 - x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7) + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*b^2*c^7) - 256*a^11*b*c^8*x^(1/2))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i)/(((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 + x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7) + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*b^2*c^7) + 256*a^11*b*c^8*x^(1/2))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) + ((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2

- 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 - x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7) + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*b^2*c^7) - 256*a^11*b*c^8*x^(1/2))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*2i + 2*atan((((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 - x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7)*1i + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*b^2*c^7)*1i + 256*a^11*b*c^8*x^(1/2))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - ((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 + x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7)*1i + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*b^2*c^7)*1i - 256*a^11*b*c^8*x^(1/2))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4))/(((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))

```
^(3/4)*(32768*a^15*c^8 - x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80
*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256
*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072
*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 -
327680*a^15*b^2*c^7)*1i + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^
13*b^4*c^6 - 114688*a^14*b^2*c^7)*1i + 256*a^11*b*c^8*x^(1/2))*(-(b^9 - b^4
*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2
)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 25
6*a^8*b^2*c^3)))^(1/4)*1i + ((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4
*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2
) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9
*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15
*c^8 + x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^
2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c
 + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*
b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a
^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^
7)*1i + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 11468
8*a^14*b^2*c^7)*1i - 256*a^11*b*c^8*x^(1/2))*(-(b^9 - b^4*(-(4*a*c - b^2)^5
)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*
c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a
^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(
1/4)*1i))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5
*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*
a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c
 + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - 2/(a*x^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.1069 $\int \frac{1}{x^{5/2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=371

$$\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\cdots\right.}{\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)\cdots}$$

```
[Out] -2/3/a/x^(3/2)+1/2*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^
(1/2))^(1/4))*(1-b/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(3
/4)+1/2*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/
4))*(1-b/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/2*c^
(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-
4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/2*c^(3/4)*arcta
nh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-4*a*c+b^2)
^(1/2))*2^(3/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A] time = 0.51, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1115, 1368, 1422, 212, 208, 205}

$$\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\cdots\right.}{\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -2/(3*a*x^(3/2)) + (c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/
4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b - Sqrt[b^2 - 4*
a*c])^(3/4)) + (c^(3/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*S
qrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c]
)^(3/4)) + (c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt
[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) + (c^(3/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x]
)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4
))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2+cx^4\right)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4+cx^8\right)}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{2}{3ax^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{-3b-3cx^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{3a}$$

$$= -\frac{2}{3ax^{3/2}} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{a} - \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right.\right.}{}$$

$$= -\frac{2}{3ax^{3/2}} + \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, \sqrt{x}\right)}{a\sqrt{-b-\sqrt{b^2-4ac}}} + \frac{\left(c\left(1-\frac{}{\sqrt{}}\right.\right.}{}$$

$$= -\frac{2}{3ax^{3/2}} + \frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\,a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}}{\sqrt[4]{-b}}\right.}{\sqrt[4]{2}\,a\left(-b+\sqrt{b^2-4ac}\right.}$$

Mathematica [C] time = 0.05, size = 82, normalized size = 0.22

$$-\frac{3\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{\#1^4c\log\left(\sqrt{x}-\#1\right)+b\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]+\frac{4}{x^{3/2}}}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/6*(4/x^(3/2) + 3*RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[Sqrt[x] - #1] +
c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/a
```

fricas [B] time = 6.25, size = 6671, normalized size = 17.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(12*a*x^2*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^
3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 3
```

7*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)
/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*
a^8*b^2*c + 16*a^9*c^2)))*arctan(-1/4*(sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*
a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^
6*b^2*c^6 - 16*a^7*c^7 + (a^7*b^11 - 17*a^8*b^9*c + 113*a^9*b^7*c^2 - 364*a
^10*b^5*c^3 + 560*a^11*b^3*c^4 - 320*a^12*b*c^5)*sqrt((b^12 - 10*a*b^10*c +
37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^
6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(4*(b^1
2*c^4 - 10*a*b^10*c^5 + 37*a^2*b^8*c^6 - 62*a^3*b^6*c^7 + 46*a^4*b^4*c^8 -
12*a^5*b^2*c^9 + a^6*c^10)*x + 2*sqrt(1/2)*(b^18 - 18*a*b^16*c + 135*a^2*b^
14*c^2 - 546*a^3*b^12*c^3 + 1288*a^4*b^10*c^4 - 1792*a^5*b^8*c^5 + 1421*a^6
*b^6*c^6 - 592*a^7*b^4*c^7 + 114*a^8*b^2*c^8 - 8*a^9*c^9 + (a^7*b^15 - 19*a
^8*b^13*c + 148*a^9*b^11*c^2 - 605*a^10*b^9*c^3 + 1374*a^11*b^7*c^4 - 1672*
a^12*b^5*c^5 + 928*a^13*b^3*c^6 - 128*a^14*b*c^7)*sqrt((b^12 - 10*a*b^10*c
+ 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c
^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(b^7
- 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a
^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4
*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^
2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*sqrt(-(b^7 -
7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*
c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^
4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c
^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)) + 2*sqrt(1/2)*(b^
20*c^2 - 21*a*b^18*c^3 + 188*a^2*b^16*c^4 - 935*a^3*b^14*c^5 + 2821*a^4*b^1
2*c^6 - 5292*a^5*b^10*c^7 + 6083*a^6*b^8*c^8 - 4071*a^7*b^6*c^9 + 1449*a^8*
b^4*c^10 - 248*a^9*b^2*c^11 + 16*a^10*c^12 + (a^7*b^17*c^2 - 22*a^8*b^15*c^
3 + 204*a^9*b^13*c^4 - 1032*a^10*b^11*c^5 + 3075*a^11*b^9*c^6 - 5417*a^12*b
^7*c^7 + 5324*a^13*b^5*c^8 - 2480*a^14*b^3*c^9 + 320*a^15*b*c^10)*sqrt((b^1
2 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5
*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c
^3)))*sqrt(x)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*
b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 -
62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12
*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*
a^9*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b
*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a
^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a
^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8
*b^2*c + 16*a^9*c^2)))/(b^12*c^7 - 10*a*b^10*c^8 + 37*a^2*b^8*c^9 - 62*a^3*
b^6*c^10 + 46*a^4*b^4*c^11 - 12*a^5*b^2*c^12 + a^6*c^13)) - 12*a*x^2*sqrt(s
qrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 -
8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a
^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15
*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c

^2)))*arctan(1/4*(sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 - (a^7*b^11 - 17*a^8*b^9*c + 113*a^9*b^7*c^2 - 364*a^10*b^5*c^3 + 560*a^11*b^3*c^4 - 320*a^12*b*c^5)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(4*(b^12*c^4 - 10*a*b^10*c^5 + 37*a^2*b^8*c^6 - 62*a^3*b^6*c^7 + 46*a^4*b^4*c^8 - 12*a^5*b^2*c^9 + a^6*c^10)*x + 2*sqrt(1/2)*(b^18 - 18*a*b^16*c + 135*a^2*b^14*c^2 - 546*a^3*b^12*c^3 + 1288*a^4*b^10*c^4 - 1792*a^5*b^8*c^5 + 1421*a^6*b^6*c^6 - 592*a^7*b^4*c^7 + 114*a^8*b^2*c^8 - 8*a^9*c^9 - (a^7*b^15 - 19*a^8*b^13*c + 148*a^9*b^11*c^2 - 605*a^10*b^9*c^3 + 1374*a^11*b^7*c^4 - 1672*a^12*b^5*c^5 + 928*a^13*b^3*c^6 - 128*a^14*b*c^7)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)) + 2*sqrt(1/2)*(b^20*c^2 - 21*a*b^18*c^3 + 188*a^2*b^16*c^4 - 935*a^3*b^14*c^5 + 2821*a^4*b^12*c^6 - 5292*a^5*b^10*c^7 + 6083*a^6*b^8*c^8 - 4071*a^7*b^6*c^9 + 1449*a^8*b^4*c^10 - 248*a^9*b^2*c^11 + 16*a^10*c^12 - (a^7*b^17*c^2 - 22*a^8*b^15*c^3 + 204*a^9*b^13*c^4 - 1032*a^10*b^11*c^5 + 3075*a^11*b^9*c^6 - 5417*a^12*b^7*c^7 + 5324*a^13*b^5*c^8 - 2480*a^14*b^3*c^9 + 320*a^15*b*c^10)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))/(b^12*c^7 - 10*a*b^10*c^8 + 37*a^2*b^8*c^9 - 62*a^3*b^6*c^10 + 46*a^4*b^4*c^11 - 12*a^5*b^2*c^12 + a^6*c^13)) - 3*a*x^2*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3

+ 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 4
8*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*log(
-2*(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*sqrt(x) + (b^9 - 9*a*b
^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 - (a^7*b^6 - 10*a^8*b^
4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c
^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6
- 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^
7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*
a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^
4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b
^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))) + 3*a*x^2*s
qrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*
b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 -
62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12
*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*
a^9*c^2)))*log(-2*(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*sqrt(x)
- (b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 - (a^7*
b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c
+ 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*
c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt
(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*
a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*
b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^
4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)
))) - 3*a*x^2*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^
3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 3
7*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)
/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*
a^8*b^2*c + 16*a^9*c^2)))*log(-2*(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a
^3*c^5)*sqrt(x) + (b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^
4*b*c^4 + (a^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^1
2 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5
*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c
^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3
- (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^
8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b
^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*
c + 16*a^9*c^2)))) + 3*a*x^2*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2
*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 -
10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2
*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3))
)/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*log(-2*(b^6*c^2 - 5*a*b^4*c^3 + 6*
a^2*b^2*c^4 - a^3*c^5)*sqrt(x) - (b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3
*b^3*c^3 + 4*a^4*b*c^4 + (a^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10
*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b

```
^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*
c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2
 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^1
0*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a
^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b
^4 - 8*a^8*b^2*c + 16*a^9*c^2)))) - 4*sqrt(x))/(a*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 13.91Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.01, size = 64, normalized size = 0.17

$$\frac{\left(-\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{4} c-b\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)+\sqrt{x}\right)}{2a\left(2\,\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{7} c+\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{3} b\right)}-\frac{2}{3a\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/a*sum((-_R^4*c-b)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z
^4*b+a))-2/3/a/x^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(3\,b\sqrt{x}+\frac{a}{x^{\frac{3}{2}}}\right)}{3\,a^{2}}+\int\frac{bcx^{\frac{7}{2}}+\left(b^{2}-ac\right)x^{\frac{3}{2}}}{a^{2}cx^{4}+a^{2}bx^{2}+a^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -2/3*(3*b*sqrt(x) + a/x^(3/2))/a^2 + integrate((b*c*x^(7/2) + (b^2 - a*c)*x
^(3/2))/(a^2*c*x^4 + a^2*b*x^2 + a^3), x)
```

mupad [B] time = 8.64, size = 16557, normalized size = 44.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan(((x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) - (-(b^11 + b^6*(-(4*a*c -
 b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4
*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-
(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 +
 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(
(x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 2498
56*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(
1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 -
 a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b
^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*
c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^1
7*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917
504*a^16*b^2*c^7))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 +
 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^
2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4
*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 9
6*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 4096*a^11*b*c^9 - 512*a^9*b^5*c
^7 + 3072*a^10*b^3*c^8))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b
*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*
c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5
*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6
*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i + (x^(1/2)*(512*a^10*c^1
0 - 256*a^9*b^2*c^9) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c
^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c
- b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a
*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c
 + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8
+ 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^1
4*b^3*c^7) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^
2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(
4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*
b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 -
106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7))*(-(b^11 +
 b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^
5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6
*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))
/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2
*c^3)))^(3/4) + 4096*a^11*b*c^9 + 512*a^9*b^5*c^7 - 3072*a^10*b^3*c^8))*(-(
b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*
a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9
```

*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i)/((x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 4096*a^11*b*c^9 - 512*a^9*b^5*c^7 + 3072*a^10*b^3*c^8))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) - (x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) + 4096*a^11*b*c^9 + 512*a^9*b^5*c^7 - 3072*a^10*b^3*c^8))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*

b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2
) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^
3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)
^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4
 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*2i + atan(((x^
(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^
(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4
+ a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11
*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*
(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b
^5*c^6 - 491520*a^14*b^3*c^7) + (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 11
2*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*
(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1
/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*
a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8
192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*
b^2*c^7))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b
^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/
2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a
c - b^2)^5)^(1/2))/(32(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4
*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 4096*a^11*b*c^9 - 512*a^9*b^5*c^7 + 3072
*a^10*b^3*c^8))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86
*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^
5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*
(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a
^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i + (x^(1/2)*(512*a^10*c^10 - 256*a
^9*b^2*c^9) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a
^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)
^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-
(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9
*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^
11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7
) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2
 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 1
5*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b
^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 -
 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^
14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7))*(-(b^11 - b^6*(-(4
*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 2
80*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*
c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7
*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(
3/4) + 4096*a^11*b*c^9 + 512*a^9*b^5*c^7 - 3072*a^10*b^3*c^8))*(-(b^11 - b^
6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c

^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i)/((x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) + (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 4096*a^11*b*c^9 - 512*a^9*b^5*c^7 + 3072*a^10*b^3*c^8))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) - (x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) + 4096*a^11*b*c^9 + 512*a^9*b^5*c^7 - 3072*a^10*b^3*c^8))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b

^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*2i - 2*atan(((x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7)*1i)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i - 4096*a^11*b*c^9 - 512*a^9*b^5*c^7 + 3072*a^10*b^3*c^8)*1i)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) + (x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7)*1i)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 4096*a^11*b*c^9 + 512*a^9*b^5*c^7 - 3072*a^10*b^3*c^8)*1i)*(-(b

^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a ^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9* c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^1 0*b^2*c^3)))^(1/4))/((x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^ 5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6 *a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2)) /(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2 *c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12 *b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) - (-(b^11 + b^6*(-(4* a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 28 0*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c ^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7* b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1 /4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^1 5*b^4*c^6 - 917504*a^16*b^2*c^7)*1i)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3 *c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^ 5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i - 4096*a^11* b*c^9 - 512*a^9*b^5*c^7 + 3072*a^10*b^3*c^8)*1i)*(-(b^11 + b^6*(-(4*a*c - b ^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b ^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4 *a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 2 56*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i - (x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3 *c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a *c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256 *a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a ^13*b^5*c^6 - 491520*a^14*b^3*c^7) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3 *c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^ 5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^ 8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504* a^16*b^2*c^7)*1i)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4* c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96 *a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 4096*a^11*b*c^9 + 512*a^9*b^5 *c^7 - 3072*a^10*b^3*c^8)*1i)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112* a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-

(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) - 2*atan(((x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) + (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7)*1i)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i - 4096*a^11*b*c^9 - 512*a^9*b^5*c^7 + 3072*a^10*b^3*c^8)*1i)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) + (x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) + (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) + (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7)*1i)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4 *c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 4096*a^11*b*c^9 + 512*a^9*b^5*c^7 - 3072*a^10*b^3*c^8)*1i)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4))/((x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) + (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7)*1i)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i - 4096*a^11*b*c^9 - 512*a^9*b^5*c^7 + 3072*a^10*b^3*c^8)*1i)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i - (x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) + (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) + (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7)*1i)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 4096*a^11*b*c^9 + 512*a^9*b^5*c^7 - 3072*a^10*b^3*c^8)*1i)*(-(b^11 - b^6*(-(4*

```
a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 28
0*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c
^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*
b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1
/4)*1i))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^
7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2
) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*
c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*
c^2 - 256*a^10*b^2*c^3)))^(1/4) - 2/(3*a*x^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.1070 $\int \frac{1}{x^{7/2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=412

$$\frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}}{\sqrt[4]{-\sqrt{}}}\right.}{2^{3/4}a^2\sqrt[4]{-\sqrt{b^2-4ac}-}}$$

```
[Out] -2/5/a/x^(5/2)+1/2*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^
(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a^2/(-b-(-4*a*c+b^
2)^(1/2))^(1/4)-1/2*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2
)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a^2/(-b-(-4*a*c+
b^2)^(1/2))^(1/4)+1/2*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^
2)^(1/2))^(1/4))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a^2/(-b+(-4*a*
c+b^2)^(1/2))^(1/4)-1/2*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c
+b^2)^(1/2))^(1/4))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a^2/(-b+(-4
*a*c+b^2)^(1/2))^(1/4)+2*b/a^2/x^(1/2)
```

Rubi [A] time = 0.98, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1115, 1368, 1504, 1510, 298, 205, 208}

$$\frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}}{\sqrt[4]{-\sqrt{}}}\right.}{2^{3/4}a^2\sqrt[4]{-\sqrt{b^2-4ac}-}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -2/(5*a*x^(5/2)) + (2*b)/(a^2*Sqrt[x]) + (c^(1/4)*(b - (b^2 - 2*a*c)/Sqrt[b
^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4
)])/(2^(3/4)*a^2*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(b + (b^2 - 2*a
*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*
a*c])^(1/4)])/(2^(3/4)*a^2*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(b -
(b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sq
rt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a^2*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (c^
(1/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x
])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a^2*(-b + Sqrt[b^2 - 4*a*c])^(
1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
```

```
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2+cx^4\right)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^6\left(a+bx^4+cx^8\right)}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{2}{5ax^{5/2}} + \frac{2\,\text{Subst}\left(\int \frac{-5b-5cx^4}{x^2\left(a+bx^4+cx^8\right)}\,dx, x, \sqrt{x}\right)}{5a}$$

$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} - \frac{2\,\text{Subst}\left(\int \frac{x^2\left(-5\left(b^2-ac\right)-5bcx^4\right)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{5a^2}$$

$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} + \frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{a^2} + \frac{\left(c\left(b\right.\right.}{}$$

$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} - \frac{\left(\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{2}\,a^2} -$$

$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} + \frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}a^2\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan}{2^{3/4}a^2\sqrt[4]{-b+}}$$

Mathematica [C] time = 0.07, size = 107, normalized size = 0.26

$$-\frac{-5\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{\#1^4bc\log\left(\sqrt{x}-\#1\right)-ac\log\left(\sqrt{x}-\#1\right)+b^2\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]+\frac{4a}{x^{5/2}}-\frac{20b}{\sqrt{x}}}{10a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/10*((4*a)/x^(5/2) - (20*b)/Sqrt[x] - 5*RootSum[a + b*#1^4 + c*#1^8 & , (
b^2*Log[Sqrt[x] - #1] - a*c*Log[Sqrt[x] - #1] + b*c*Log[Sqrt[x] - #1]*#1^4)
/(b*#1 + 2*c*#1^5) & ])/a^2
```

fricas [B] time = 20.36, size = 7995, normalized size = 19.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/10*(20*a^2*x^3*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 3
0*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((
b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 -
 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 -
12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c +
16*a^11*c^2)))*arctan(1/2*((b^11 - 11*a*b^9*c + 43*a^2*b^7*c^2 - 70*a^3*b^5
*c^3 + 41*a^4*b^3*c^4 - 4*a^5*b*c^5 - (a^9*b^6 - 10*a^10*b^4*c + 32*a^11*b^
2*c^2 - 32*a^12*c^3)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b
^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*
c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))
*sqrt((b^16*c^14 - 14*a*b^14*c^15 + 79*a^2*b^12*c^16 - 230*a^3*b^10*c^17 +
367*a^4*b^8*c^18 - 314*a^5*b^6*c^19 + 130*a^6*b^4*c^20 - 20*a^7*b^2*c^21 +
a^8*c^22)*x - 1/2*sqrt(1/2)*(b^23*c^9 - 23*a*b^21*c^10 + 230*a^2*b^19*c^11
- 1311*a^3*b^17*c^12 + 4692*a^4*b^15*c^13 - 10947*a^5*b^13*c^14 + 16731*a^6
*b^11*c^15 - 16380*a^7*b^9*c^16 + 9711*a^8*b^7*c^17 - 3109*a^9*b^5*c^18 + 4
25*a^10*b^3*c^19 - 20*a^11*b*c^20 - (a^9*b^18*c^9 - 22*a^10*b^16*c^10 + 205
*a^11*b^14*c^11 - 1050*a^12*b^12*c^12 + 3206*a^13*b^10*c^13 - 5909*a^14*b^8
*c^14 + 6333*a^15*b^6*c^15 - 3580*a^16*b^4*c^16 + 880*a^17*b^2*c^17 - 64*a^
18*c^18)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 36
7*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^
8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(-(b^9
- 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*
a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*
a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7
*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c
^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2))) - (b^19*c^7 - 18*a*b^17*c^8
+ 135*a^2*b^15*c^9 - 546*a^3*b^13*c^10 + 1287*a^4*b^11*c^11 - 1782*a^5*b^9*
c^12 + 1386*a^6*b^7*c^13 - 540*a^7*b^5*c^14 + 81*a^8*b^3*c^15 - 4*a^9*b*c^1
6 - (a^9*b^14*c^7 - 17*a^10*b^12*c^8 + 117*a^11*b^10*c^9 - 416*a^12*b^8*c^1
0 + 805*a^13*b^6*c^11 - 810*a^14*b^4*c^12 + 352*a^15*b^2*c^13 - 32*a^16*c^1
4)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*
b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^
18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(x))*sqrt(sqr
t(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c
^4 + (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a
^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^
6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b
```

^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))/(b^16*c^9 - 14*a*b^14*c^10 + 79*a^2*b^12*c^11 - 230*a^3*b^10*c^12 + 367*a^4*b^8*c^13 - 314*a^5*b^6*c^14 + 130*a^6*b^4*c^15 - 20*a^7*b^2*c^16 + a^8*c^17)) - 20*a^2*x^3*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*arctan(-1/2*((b^11 - 11*a*b^9*c + 43*a^2*b^7*c^2 - 70*a^3*b^5*c^3 + 41*a^4*b^3*c^4 - 4*a^5*b*c^5 + (a^9*b^6 - 10*a^10*b^4*c + 32*a^11*b^2*c^2 - 32*a^12*c^3)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt((b^16*c^14 - 14*a*b^14*c^15 + 79*a^2*b^12*c^16 - 230*a^3*b^10*c^17 + 367*a^4*b^8*c^18 - 314*a^5*b^6*c^19 + 130*a^6*b^4*c^20 - 20*a^7*b^2*c^21 + a^8*c^22)*x - 1/2*sqrt(1/2)*(b^23*c^9 - 23*a*b^21*c^10 + 230*a^2*b^19*c^11 - 1311*a^3*b^17*c^12 + 4692*a^4*b^15*c^13 - 10947*a^5*b^13*c^14 + 16731*a^6*b^11*c^15 - 16380*a^7*b^9*c^16 + 9711*a^8*b^7*c^17 - 3109*a^9*b^5*c^18 + 425*a^10*b^3*c^19 - 20*a^11*b*c^20 + (a^9*b^18*c^9 - 22*a^10*b^16*c^10 + 205*a^11*b^14*c^11 - 1050*a^12*b^12*c^12 + 3206*a^13*b^10*c^13 - 5909*a^14*b^8*c^14 + 6333*a^15*b^6*c^15 - 3580*a^16*b^4*c^16 + 880*a^17*b^2*c^17 - 64*a^18*c^18)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2))) - (b^19*c^7 - 18*a*b^17*c^8 + 135*a^2*b^15*c^9 - 546*a^3*b^13*c^10 + 1287*a^4*b^11*c^11 - 1782*a^5*b^9*c^12 + 1386*a^6*b^7*c^13 - 540*a^7*b^5*c^14 + 81*a^8*b^3*c^15 - 4*a^9*b*c^16 + (a^9*b^14*c^7 - 17*a^10*b^12*c^8 + 117*a^11*b^10*c^9 - 416*a^12*b^8*c^10 + 805*a^13*b^6*c^11 - 810*a^14*b^4*c^12 + 352*a^15*b^2*c^13 - 32*a^16*c^14)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*

a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2))))/(b^16*c^9 - 14*a*b^14
*c^10 + 79*a^2*b^12*c^11 - 230*a^3*b^10*c^12 + 367*a^4*b^8*c^13 - 314*a^5*b
^6*c^14 + 130*a^6*b^4*c^15 - 20*a^7*b^2*c^16 + a^8*c^17)) - 5*a^2*x^3*sqrt(
sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*
b*c^4 + (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 7
9*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130
*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^2
0*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*log(1/2
sqrt(1/2)(b^18 - 20*a*b^16*c + 168*a^2*b^14*c^2 - 768*a^3*b^12*c^3 + 2068
*a^4*b^10*c^4 - 3312*a^5*b^8*c^5 + 3024*a^6*b^6*c^6 - 1409*a^7*b^4*c^7 + 26
4*a^8*b^2*c^8 - 16*a^9*c^9 - (a^9*b^13 - 19*a^10*b^11*c + 146*a^11*b^9*c^2
- 575*a^12*b^7*c^3 + 1204*a^13*b^5*c^4 - 1232*a^14*b^3*c^5 + 448*a^15*b*c^6
)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b
^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^1
8*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(sqrt(1/2)*sqr
t(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*
b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^
2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6
- 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 6
4*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*sqrt(-(b^9 - 9*a*b^7
*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*a^10*b^2*
c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*
c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7
+ a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^
9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)) + (b^8*c^7 - 7*a*b^6*c^8 + 15*a^2*b^4*
c^9 - 10*a^3*b^2*c^10 + a^4*c^11)*sqrt(x)) + 5*a^2*x^3*sqrt(sqrt(1/2)*sqrt(
-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^
4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2
- 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 -
20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*
a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*log(-1/2*sqrt(1/2)*(b^
18 - 20*a*b^16*c + 168*a^2*b^14*c^2 - 768*a^3*b^12*c^3 + 2068*a^4*b^10*c^4
- 3312*a^5*b^8*c^5 + 3024*a^6*b^6*c^6 - 1409*a^7*b^4*c^7 + 264*a^8*b^2*c^8
- 16*a^9*c^9 - (a^9*b^13 - 19*a^10*b^11*c + 146*a^11*b^9*c^2 - 575*a^12*b^7
*c^3 + 1204*a^13*b^5*c^4 - 1232*a^14*b^3*c^5 + 448*a^15*b*c^6)*sqrt((b^16 -
 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a
^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^1
9*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*
b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*a^10*b
^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^
10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c
^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/
(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^
5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^
2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*

b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)) + (b^8*c^7 - 7*a*b^6*c^8 + 15*a^2*b^4*c^9 - 10*a^3*b^2*c^10 + a^4*c^11)*sqrt(x)) - 5*a^2*x^3*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*log(1/2*sqrt(1/2)*(b^18 - 20*a*b^16*c + 168*a^2*b^14*c^2 - 768*a^3*b^12*c^3 + 2068*a^4*b^10*c^4 - 3312*a^5*b^8*c^5 + 3024*a^6*b^6*c^6 - 1409*a^7*b^4*c^7 + 264*a^8*b^2*c^8 - 16*a^9*c^9 + (a^9*b^13 - 19*a^10*b^11*c + 146*a^11*b^9*c^2 - 575*a^12*b^7*c^3 + 1204*a^13*b^5*c^4 - 1232*a^14*b^3*c^5 + 448*a^15*b*c^6)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)) + (b^8*c^7 - 7*a*b^6*c^8 + 15*a^2*b^4*c^9 - 10*a^3*b^2*c^10 + a^4*c^11)*sqrt(x)) + 5*a^2*x^3*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*log(-1/2*sqrt(1/2)*(b^18 - 20*a*b^16*c + 168*a^2*b^14*c^2 - 768*a^3*b^12*c^3 + 2068*a^4*b^10*c^4 - 3312*a^5*b^8*c^5 + 3024*a^6*b^6*c^6 - 1409*a^7*b^4*c^7 + 264*a^8*b^2*c^8 - 16*a^9*c^9 + (a^9*b^13 - 19*a^10*b^11*c + 146*a^11*b^9*c^2 - 575*a^12*b^7*c^3 + 1204*a^13*b^5*c^4 - 1232*a^14*b^3*c^5 + 448*a^15*b*c^6)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c +

```
 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 1
30*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a
^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)) + (b^8
*c^7 - 7*a*b^6*c^8 + 15*a^2*b^4*c^9 - 10*a^3*b^2*c^10 + a^4*c^11)*sqrt(x))
+ 4*(5*b*x^2 - a)*sqrt(x))/(a^2*x^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 14.71Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.01, size = 82, normalized size = 0.20

$$\frac{\left(\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^6 bc+\left(-ac+b^2\right)\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^2\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\right.}{2a^2\left(2\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7 c+\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3 b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/a^2*sum((b*c*_R^6+(-a*c+b^2)*_R^2)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R
=RootOf(_Z^8*c+_Z^4*b+a))-2/5/a/x^(5/2)+2*b/a^2/x^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(\frac{5\,b}{\sqrt{x}}-\frac{a}{x^{\frac{5}{2}}}\right)}{5\,a^2}+\int\frac{bcx^{\frac{5}{2}}+\left(b^2-ac\right)\sqrt{x}}{a^2cx^4+a^2bx^2+a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/5*(5*b/sqrt(x) - a/x^(5/2))/a^2 + integrate((b*c*x^(5/2) + (b^2 - a*c)*sq
rt(x))/(a^2*c*x^4 + a^2*b*x^2 + a^3), x)
```

mupad [B] time = 6.48, size = 15149, normalized size = 36.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9
*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a
*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2)
- 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1
/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^
12*b^2*c^3)))^(3/4)*(x^(1/2)*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a
^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^
3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c
*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96
*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10
*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 655
360*a^27*b^2*c^8) - 131072*a^26*b*c^9 + 2048*a^21*b^11*c^4 - 28672*a^22*b^9
*c^5 + 151552*a^23*b^7*c^6 - 368640*a^24*b^5*c^7 + 393216*a^25*b^3*c^8) + x
^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 + b^8*(-(4*a*c - b^2)
^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5
*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 1
5*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 1
6*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*1i + ((-(b^13 +
b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^
7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2
) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*
(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8
 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/
4)*(131072*a^26*b*c^9 + x^(1/2)*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 14
4*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5
*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*
(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^
6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c +
 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b
^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 -
655360*a^27*b^2*c^8) - 2048*a^21*b^11*c^4 + 28672*a^22*b^9*c^5 - 151552*a^2
3*b^7*c^6 + 368640*a^24*b^5*c^7 - 393216*a^25*b^3*c^8) + x^(1/2)*(768*a^21*
b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a
^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^
3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c
*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96
*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*1i)/(256*a^20*c^12 - ((-(b^13 + b
^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7
```

*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(x^(1/2)*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8) - 131072*a^26*b*c^9 + 2048*a^21*b^11*c^4 - 28672*a^22*b^9*c^5 + 151552*a^23*b^7*c^6 - 368640*a^24*b^5*c^7 + 393216*a^25*b^3*c^8) + x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4) + ((-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(131072*a^26*b*c^9 + x^(1/2)*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8) - 2048*a^21*b^11*c^4 + 28672*a^22*b^9*c^5 - 151552*a^23*b^7*c^6 + 368640*a^24*b^5*c^7 - 393216*a^25*b^3*c^8) + x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)))*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*2i - (2/(5*a) - (2*b*x^2)/a^2)/x^(5/2) + atan((((-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b

^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(x^(1/2)*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8) - 131072*a^26*b*c^9 + 2048*a^21*b^11*c^4 - 28672*a^22*b^9*c^5 + 151552*a^23*b^7*c^6 - 368640*a^24*b^5*c^7 + 393216*a^25*b^3*c^8) + x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*1i + ((-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(131072*a^26*b*c^9 + x^(1/2)*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8) - 2048*a^21*b^11*c^4 + 28672*a^22*b^9*c^5 - 151552*a^23*b^7*c^6 + 368640*a^24*b^5*c^7 - 393216*a^25*b^3*c^8) + x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*1i)/(256*a^20*c^12 - ((-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(x^(1/2)*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a

^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8) - 131072*a^26*b*c^9 + 2048*a^21*b^11*c^4 - 28672*a^22*b^9*c^5 + 151552*a^23*b^7*c^6 - 368640*a^24*b^5*c^7 + 393216*a^25*b^3*c^8) + x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4) + ((-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(131072*a^26*b*c^9 + x^(1/2)*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8) - 2048*a^21*b^11*c^4 + 28672*a^22*b^9*c^5 - 151552*a^23*b^7*c^6 + 368640*a^24*b^5*c^7 - 393216*a^25*b^3*c^8) + x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)))*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*2i - 2*atan((((-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(x^(1/2)*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 3

90*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8)*1i - 131072*a^26*b*c^9 + 2048*a^21*b^11*c^4 - 28672*a^22*b^9*c^5 + 151552*a^23*b^7*c^6 - 368640*a^24*b^5*c^7 + 393216*a^25*b^3*c^8)*1i - x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4) + ((-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(131072*a^26*b*c^9 + x^(1/2)*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8)*1i - 2048*a^21*b^11*c^4 + 28672*a^22*b^9*c^5 - 151552*a^23*b^7*c^6 + 368640*a^24*b^5*c^7 - 393216*a^25*b^3*c^8)*1i - x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4))/(256*a^20*c^12 + ((-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(x^(1/2)*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^

6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8)*1i - 131072*a^26*b*c^9 + 2048*a^21*b^11*c^4 - 28672*a^22*b^9*c^5 + 151552*a^23*b^7*c^6 - 368640*a^24*b^5*c^7 + 393216*a^25*b^3*c^8)*1i - x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*1i - ((-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(131072*a^26*b*c^9 + x^(1/2)*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8)*1i - 2048*a^21*b^11*c^4 + 28672*a^22*b^9*c^5 - 151552*a^23*b^7*c^6 + 368640*a^24*b^5*c^7 - 393216*a^25*b^3*c^8)*1i - x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*1i))*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4) - 2*atan((((-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(x^(1/2)*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b

^6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8)*1i - 131072*a^26*b*c^9 + 2048*a^21*b^11*c^4 - 28672*a^22*b^9*c^5 + 151552*a^23*b^7*c^6 - 368640*a^24*b^5*c^7 + 393216*a^25*b^3*c^8)*1i - x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4) + ((-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(131072*a^26*b*c^9 + x^(1/2)*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8)*1i - 2048*a^21*b^11*c^4 + 28672*a^22*b^9*c^5 - 151552*a^23*b^7*c^6 + 368640*a^24*b^5*c^7 - 393216*a^25*b^3*c^8)*1i - x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4))/(256*a^20*c^12 + ((-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(x^(1/2)*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8)*1i - 131072*a^26*b*c^9 + 2048*a^21*b^11*c^4 - 28672*a^22*b^9*c^5 + 151552*a^23*b^7*c^6 - 368640*a^24*b^5*c^7 + 393216*a^25*b^3*c^8)*1i - x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2

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)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3
*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32
*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c
^3)))^(1/4)*1i - ((-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 +
115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4
*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2
)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c -
 b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c
^2 - 256*a^12*b^2*c^3)))^(3/4)*(131072*a^26*b*c^9 + x^(1/2)*(-(b^13 - b^8*(
-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3
 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 1
7*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-(4*
a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 25
6*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(1
31072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*
c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8)*1i - 2048*a^21*b^11*c^4 +
28672*a^22*b^9*c^5 - 151552*a^23*b^7*c^6 + 368640*a^24*b^5*c^7 - 393216*a^2
5*b^3*c^8)*1i - x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 - b^
8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*
c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*c - b^2)^5)^(1/2)
- 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a^3*b^2*c^3*(-
(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 +
 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)
*1i))*(-(b^13 - b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*
c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 - a^4*c^4*(-(4*a*
c - b^2)^5)^(1/2) - 17*a*b^11*c - 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) +
 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) + 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/
2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^1
2*b^2*c^3)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.1071 $\int \frac{x^{13/2}}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=544

$$-\frac{bx^{3/2}}{2c\left(b^2-4ac\right)}+\frac{x^{7/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(\left(3b^2-14ac\right)\sqrt{b^2-4ac}-20abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{7/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\cdots$$

```
[Out] -1/2*b*x^(3/2)/c/(-4*a*c+b^2)+1/2*x^(7/2)*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b
*x^2+a)-1/8*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(
3*b^3-20*a*b*c-(-14*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-4*a*c+
b^2)^(3/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/8*arctanh(2^(1/4)*c^(1/4)*x^(1/2
)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(3*b^3-20*a*b*c-(-14*a*c+3*b^2)*(-4*a*c+b^
2)^(1/2))*2^(1/4)/c^(7/4)/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+
1/8*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(3*b^3-20
*a*b*c+(-14*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-4*a*c+b^2)^(3/
2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/8*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-
4*a*c+b^2)^(1/2))^(1/4))*(3*b^3-20*a*b*c+(-14*a*c+3*b^2)*(-4*a*c+b^2)^(1/2)
)*2^(1/4)/c^(7/4)/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A] time = 2.58, antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1115, 1365, 1502, 1510, 298, 205, 208}

$$\frac{\left(\left(3b^2-14ac\right)\sqrt{b^2-4ac}-20abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{7/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(-\left(3b^2-14ac\right)\sqrt{b^2-4ac}-20abc+3b^3\right)\tan^{-1}\cdots}{4\,2^{3/4}c^{7/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b*x^(3/2))/(2*c*(b^2 - 4*a*c)) + (x^(7/2)*(2*a + b*x^2))/(2*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)) + ((3*b^3 - 20*a*b*c + (3*b^2 - 14*a*c)*Sqrt[b^2 - 4*
a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*
2^(3/4)*c^(7/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ((3*b
^3 - 20*a*b*c - (3*b^2 - 14*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(7/4)*(b^2 - 4*a*c)
^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ((3*b^3 - 20*a*b*c + (3*b^2 - 14*a
*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4
```

```
*a*c])^(1/4)])/(4*2^(3/4)*c^(7/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*
c])^(1/4)) + ((3*b^3 - 20*a*b*c - (3*b^2 - 14*a*c)*Sqrt[b^2 - 4*a*c])*ArcTa
nh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^
(7/4)*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
```

```
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^{13/2}}{\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^{14}}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{7/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{x^6\left(14a+3bx^4\right)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2\left(b^2-4ac\right)}$$

$$= -\frac{bx^{3/2}}{2c\left(b^2-4ac\right)} + \frac{x^{7/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(9ab+3\left(3b^2-14ac\right)x^4\right)}{a+bx^4+cx^8}\,dx, x, \sqrt{\right.}{6c\left(b^2-4ac\right)}$$

$$= -\frac{bx^{3/2}}{2c\left(b^2-4ac\right)} + \frac{x^{7/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(3b^2-14ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\frac{20abc}{\sqrt{b^2-4ac}}\right)\text{Su}}{4c\left(b^2-\right.}$$

$$= -\frac{bx^{3/2}}{2c\left(b^2-4ac\right)} + \frac{x^{7/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(3b^2-14ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\frac{20abc}{\sqrt{b^2-4ac}}\right)\text{Su}}{4\sqrt{2}\,c^{3/2}}$$

$$= -\frac{bx^{3/2}}{2c\left(b^2-4ac\right)} + \frac{x^{7/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(3b^3-20abc+\left(3b^2-14ac\right)\sqrt{b^2-4}\right.}{4\,2^{3/4}c^{7/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-}}$$

Mathematica [C] time = 0.28, size = 144, normalized size = 0.26

$$\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{-14\#1^4ac\log\left(\sqrt{x}-\#1\right)+3\#1^4b^2\log\left(\sqrt{x}-\#1\right)+3ab\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]-\frac{4x^{3/2}\left(a\left(b-2cx^2\right)+b^2x^2\right)}{a+bx^2+cx^4}}{8c\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-4*x^(3/2)*(b^2*x^2 + a*(b - 2*c*x^2)))/(a + b*x^2 + c*x^4) + RootSum[a +
 b*#1^4 + c*#1^8 & , (3*a*b*Log[Sqrt[x] - #1] + 3*b^2*Log[Sqrt[x] - #1]*#1^
4 - 14*a*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(8*c*(b^2 - 4*a*c
))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 46.99Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.02, size = 149, normalized size = 0.27

$$\frac{\left(\left(14ac-3b^2\right)\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^6-3\,\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^2ab\right)\ln\left(-\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right.\right.}{8\left(4ac-b^2\right)c\left(2\,\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7c+\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*(2*a*c-b^2)/(4*a*c-b^2)/c*x^(7/2)+1/4/(4*a*c-b^2)*a*b/c*x^(3/2))/(c
*x^4+b*x^2+a)+1/8/c/(4*a*c-b^2)*sum(((14*a*c-3*b^2)*_R^6-3*_R^2*a*b)/(2*_R^
7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b^2-2\,ac\right)x^{\frac{7}{2}}+abx^{\frac{3}{2}}}{2\left(\left(b^2c^2-4\,ac^3\right)x^4+ab^2c-4\,a^2c^2+\left(b^3c-4\,abc^2\right)x^2\right)}+\int\frac{\left(3\,b^2-14\,ac\right)x^{\frac{5}{2}}+3\,ab\sqrt{x}}{4\left(\left(b^2c^2-4\,ac^3\right)x^4+ab^2c-4\,a^2c^2+\left(b^3c-4\,abc^2\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((b^2 - 2*a*c)*x^(7/2) + a*b*x^(3/2))/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2
*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2) + integrate(1/4*((3*b^2 - 14*a*c)
*x^(5/2) + 3*a*b*sqrt(x))/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 +
(b^3*c - 4*a*b*c^2)*x^2), x)
```

mupad [B] time = 7.01, size = 28774, normalized size = 52.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((x^(7/2)*(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b*x^(3/2))/(2*c*(4*a*c
- b^2)))/(a + b*x^2 + c*x^4) - atan(((((46036680704*a^12*c^12 - 110592*a^3*
b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 788037632*a^6*b^1
2*c^6 - 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 - 59401830400*a^9
*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 104991817728*a^11*b^2*c^11)/(128*(1
6384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*
c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9)) - (x^(1/2)
*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 901
26*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a
^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a
^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c
^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c -
 b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*
c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^
22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 8
11008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 3244
0320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 503316
48*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*c^13 + 36864*a^3*b^16*c^5 - 1302
528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 185991168*a^6*b^10*c^8 + 1061683
200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 + 8883535872*a^9*b^4*c^11 - 11576
```

279040*a^10*b^2*c^12))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/4) - (x^(1/2)*(9801*a^5*b^11 - 256905*a^6*b^9*c - 29042496*a^10*b*c^5 + 2642841*a^7*b^7*c^2 - 13243020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*1i - (((46036680704*a^12*c^12 - 110592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 788037632*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 - 59401830400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 104991817728*a^11*b^2*c^11)/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9)) + (x^(1/2)*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*c^13 + 36864*a^3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 185991168*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 + 8883535872*a^9*b^4*c^11 - 11576279040*a^10*b^2*c^12))/(16

*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*
c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^15)^(
1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b
^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^1
1*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5
*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) +
4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2
c^3(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8
192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14
080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a
^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^
9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/4) + (x^
(1/2)*(9801*a^5*b^11 - 256905*a^6*b^9*c - 29042496*a^10*b*c^5 + 2642841*a^7
*b^7*c^2 - 13243020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4))/(16*(4096*a^6*c^9
+ b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*
b^4*c^7 - 6144*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23
+ 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588
384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 8531747
84*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 20386938
88*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c +
10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c -
b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a
^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^
10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 -
12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69
206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*1i)/((((46036680704*
a^12*c^12 - 110592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*
c^5 + 788037632*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^
8*c^8 - 59401830400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 104991817728
*a^11*b^2*c^11)/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b
^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a
^6*b^2*c^9)) - (x^(1/2)*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 7418
01984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^
4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7
*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^1
0*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746
*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^
15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^
19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 1
26720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 129761
28*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016
*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*c^13 + 36
864*a^3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 185991168
*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 + 88835358
72*a^9*b^4*c^11 - 11576279040*a^10*b^2*c^12))/(16*(4096*a^6*c^9 + b^12*c^3

- 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6
144*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984
*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^1
5*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*
c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3
*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*
b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(
1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 +
b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720
*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^
7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10
*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/4) - (x^(1/2)*(9801*a^5*b^11 - 256
905*a^6*b^9*c - 29042496*a^10*b*c^5 + 2642841*a^7*b^7*c^2 - 13243020*a^8*b^
5*c^3 + 31945648*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4
 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8
)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 -
90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 6470457
6*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 179962675
2*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^
4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*
c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b
^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a
*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11
- 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 3
2440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 503
31648*a^11*b^2*c^18)))^(1/4) + (((46036680704*a^12*c^12 - 110592*a^3*b^18*c
^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 788037632*a^6*b^12*c^6
- 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 - 59401830400*a^9*b^6*c
^9 + 104312340480*a^10*b^4*c^10 - 104991817728*a^11*b^2*c^11)/(128*(16384*a
^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 -
8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9)) + (x^(1/2)*((81*
b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2
*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^1
3*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7
*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(
4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^
15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4
*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8
 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*
a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a
^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^1
1*b^2*c^18)))^(1/4)*(6576668672*a^11*c^13 + 36864*a^3*b^16*c^5 - 1302528*a^
4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 185991168*a^6*b^10*c^8 + 1061683200*a^
7*b^8*c^9 - 3886022656*a^8*b^6*c^10 + 8883535872*a^9*b^4*c^11 - 11576279040
*a^10*b^2*c^12))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8

*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*((81*b^8*(
-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19
*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5
 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8
+ 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c
 - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(
1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c
- b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 10
56*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b
^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^
8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2
c^18)))^(3/4) + (x^(1/2)(9801*a^5*b^11 - 256905*a^6*b^9*c - 29042496*a^10
*b*c^5 + 2642841*a^7*b^7*c^2 - 13243020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4)
)/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3
*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^
15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*
a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^
6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^
9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/
2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^
3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2
))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9
 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784
704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 576716
80*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)
- (107811*a^7*b^9 - 2531925*a^8*b^7*c + 128002112*a^11*b*c^4 + 22295196*a^9
*b^5*c^2 - 87242736*a^10*b^3*c^3)/(64*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^1
2*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*
b^4*c^8 - 28672*a^6*b^2*c^9))))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^2
3 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 105
88384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 85317
4784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 203869
3888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c
 + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c
 - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216
*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*
c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13
- 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 +
69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*2i - atan(((((4603
6680704*a^12*c^12 - 110592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a
^5*b^14*c^5 + 788037632*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 2140196044
8*a^8*b^8*c^8 - 59401830400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 1049
91817728*a^11*b^2*c^11)/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 3
36*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 -
 28672*a^6*b^2*c^9)) - (x^(1/2)*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/

2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10
588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 8531
74784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 20386
93888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*
c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*
c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(1677721
6*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18
*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13
 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 +
 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*
c^13 + 36864*a^3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 -
185991168*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 +
 8883535872*a^9*b^4*c^11 - 11576279040*a^10*b^2*c^12))/(16*(4096*a^6*c^9 +
b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^
4*c^7 - 6144*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) -
 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 105883
84*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 85317478
4*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 203869388
8*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c +
10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c -
b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^
12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^1
0 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 1
2976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 692
06016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/4) - (x^(1/2)*(9801*a^5*
b^11 - 256905*a^6*b^9*c - 29042496*a^10*b*c^5 + 2642841*a^7*b^7*c^2 - 13243
020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*
a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a
^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^1
1*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^
4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7
+ 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^1
0 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*
c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2)
 - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24
*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4
*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^
10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4
*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*1i - (((46036680704*a^12*c^12 - 110
592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 788037632
*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 - 5940183
0400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 104991817728*a^11*b^2*c^11)
/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*
a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9)) +
(x^(1/2)*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c

^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 6
4704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 179
9626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9
604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(
-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15
93*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7
- 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16
*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^
14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17
 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*c^13 + 36864*a^3*b^16*c
^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 185991168*a^6*b^10*c^8
+ 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 + 8883535872*a^9*b^4*c^1
1 - 11576279040*a^10*b^2*c^12))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^
4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^
8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11
+ 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704
576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626
752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*
a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*
a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a
*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48
*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^1
1 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 +
 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 5
0331648*a^11*b^2*c^18)))^(3/4) + (x^(1/2)*(9801*a^5*b^11 - 256905*a^6*b^9*c
 - 29042496*a^10*b*c^5 + 2642841*a^7*b^7*c^2 - 13243020*a^8*b^5*c^3 + 31945
648*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^
8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*(-(81*b^2
3 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^
19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c
^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^
8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a
*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)
^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*
c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 +
1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5
*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*
b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b
^2*c^18)))^(1/4)*1i)/((((46036680704*a^12*c^12 - 110592*a^3*b^18*c^3 + 4423
680*a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 788037632*a^6*b^12*c^6 - 5065015
296*a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 - 59401830400*a^9*b^6*c^9 + 1043
12340480*a^10*b^4*c^10 - 104991817728*a^11*b^2*c^11)/(128*(16384*a^7*c^10 -
 b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*
b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9)) - (x^(1/2)*(-(81*b^23 + 8
1*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^

2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*c^13 + 36864*a^3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 185991168*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 + 8883535872*a^9*b^4*c^11 - 11576279040*a^10*b^2*c^12))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/4) - (x^(1/2)*(9801*a^5*b^11 - 256905*a^6*b^9*c - 29042496*a^10*b*c^5 + 2642841*a^7*b^7*c^2 - 13243020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4) + (((46036680704*a^12*c^12 - 110592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 788037632*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 - 59401830400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 104991817728*a^11*b^2*c^11)/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9)) + (x^(1/2)*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 +

10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 8
53174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 20
38693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^
21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4
*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(1677
7216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b
^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c
^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^1
6 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*a^
11*c^13 + 36864*a^3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7
 - 185991168*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^1
0 + 8883535872*a^9*b^4*c^11 - 11576279040*a^10*b^2*c^12))/(16*(4096*a^6*c^9
 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4
*b^4*c^7 - 6144*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2
) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 105
88384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 85317
4784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 203869
3888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c
 + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c
 - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216
*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*
c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13
 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 +
69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/4) + (x^(1/2)*(9801*a
^5*b^11 - 256905*a^6*b^9*c - 29042496*a^10*b*c^5 + 2642841*a^7*b^7*c^2 - 13
243020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^12*c^3 -
24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 614
4*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*
a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15
*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c
^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*
c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b
^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1
/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b
^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*
a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7
*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*
b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4) - (107811*a^7*b^9 - 2531925*a^8*
b^7*c + 128002112*a^11*b*c^4 + 22295196*a^9*b^5*c^2 - 87242736*a^10*b^3*c^3
)/(64*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*
a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9))))*
(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 901
26*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a
^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a
^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c

^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*2i - 2*atan(((((46036680704*a^12*c^12 - 110592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 788037632*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 - 59401830400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 104991817728*a^11*b^2*c^11)/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9)) - (x^(1/2)*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*c^13 + 36864*a^3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 185991168*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 + 8883535872*a^9*b^4*c^11 - 11576279040*a^10*b^2*c^12)*1i)/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))) *((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/4)*1i + (x^(1/2)*(9801*a^5*b^11 - 256905*a^6*b^9*c - 29042496*a^10*b*c^5 + 2642841*a^7*b^7*c^2 - 13243020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c

- b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4) - (((46036680704*a^12*c^12 - 110592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 788037632*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 - 59401830400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 104991817728*a^11*b^2*c^11)/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9)) + (x^(1/2)*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*c^13 + 36864*a^3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 185991168*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 + 8883535872*a^9*b^4*c^11 - 11576279040*a^10*b^2*c^12)*1i)/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/4)*1i - (x^(1/2)*(9801*a^5*b^11 - 256905*a^6*b^9*c - 29042496*a^10*b*c^5 + 2642841*a^7*b^7*c^2 - 13243020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023

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*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3
*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*
(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*
a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b
^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^
6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4))/((((4603
6680704*a^12*c^12 - 110592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a
^5*b^14*c^5 + 788037632*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 2140196044
8*a^8*b^8*c^8 - 59401830400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 1049
91817728*a^11*b^2*c^11)/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 3
36*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 -
 28672*a^6*b^2*c^9)) - (x^(1/2)*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^2
3 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 105
88384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 85317
4784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 203869
3888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c
 + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c
 - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216
*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*
c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13
- 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 +
69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*c
^13 + 36864*a^3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 1
85991168*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 +
8883535872*a^9*b^4*c^11 - 11576279040*a^10*b^2*c^12)*1i)/(16*(4096*a^6*c^9
+ b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*
b^4*c^7 - 6144*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23
+ 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588
384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 8531747
84*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 20386938
88*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c +
 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c -
 b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a
^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^
10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 -
12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69
206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/4)*1i + (x^(1/2)*(9801*
a^5*b^11 - 256905*a^6*b^9*c - 29042496*a^10*b*c^5 + 2642841*a^7*b^7*c^2 - 1
3243020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^12*c^3 -
 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 61
44*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*
a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15
*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c
^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*
c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b
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^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*1i + (((46036680704*a^12*c^12 - 110592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 788037632*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 - 59401830400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 104991817728*a^11*b^2*c^11)/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9)) + (x^(1/2)*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*c^13 + 36864*a^3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 185991168*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 + 8883535872*a^9*b^4*c^11 - 11576279040*a^10*b^2*c^12)*1i)/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/4)*1i - (x^(1/2)*(9801*a^5*b^11 - 256905*a^6*b^9*c - 29042496*a^10*b*c^5 + 2642841*a^7*b^7*c^2 - 13243020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^

2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*1i + (107811*a^7*b^9 - 2531925*a^8*b^7*c + 128002112*a^11*b*c^4 + 22295196*a^9*b^5*c^2 - 87242736*a^10*b^3*c^3)/(64*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9))))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4) - 2*atan(((((46036680704*a^12*c^12 - 110592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 788037632*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 - 59401830400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 104991817728*a^11*b^2*c^11)/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9)) - (x^(1/2)*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*c^13 + 36864*a^3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 185991168*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 + 8883535872*a^9*b^4*c^11 - 11576279040*a^10*b^2*c^12)*1i)/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 2

6313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^1
5)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b
^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12
 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 -
 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))
^(3/4)*1i + (x^(1/2)*(9801*a^5*b^11 - 256905*a^6*b^9*c - 29042496*a^10*b*c^
5 + 2642841*a^7*b^7*c^2 - 13243020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4))/(16
*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*
c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c -
 b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*
b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^
11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^
5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) -
 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^
2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(
8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 1
4080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*
a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a
^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4) - ((
(46036680704*a^12*c^12 - 110592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783
040*a^5*b^14*c^5 + 788037632*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 21401
960448*a^8*b^8*c^8 - 59401830400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 -
 104991817728*a^11*b^2*c^11)/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^
4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*
c^8 - 28672*a^6*b^2*c^9)) + (x^(1/2)*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15
)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3
 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 -
 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 +
2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*
b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-
(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16
777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3
*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12
*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c
^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*
a^11*c^13 + 36864*a^3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c
^7 - 185991168*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c
^10 + 8883535872*a^9*b^4*c^11 - 11576279040*a^10*b^2*c^12)*1i)/(16*(4096*a^
6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 384
0*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)
^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3
+ 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 -
853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2
038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b
^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(

4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(167
77216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*
b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*
c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^
16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/4)*1i - (x^(1/2)
*(9801*a^5*b^11 - 256905*a^6*b^9*c - 29042496*a^10*b*c^5 + 2642841*a^7*b^7*
c^2 - 13243020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^1
2*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c
^7 - 6144*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 74
1801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*
a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a
^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a
^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 107
46*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2
)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*
c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 +
 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 1297
6128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 692060
16*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4))/((((46036680704*a^12*c^
12 - 110592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 7
88037632*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 -
 59401830400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 104991817728*a^11*b
^2*c^11)/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5
 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*
c^9)) - (x^(1/2)*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*
a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15
*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c
^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*
c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b
^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1
/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b
^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*
a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7
*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*
b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*c^13 + 36864*a^
3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 185991168*a^6*b
^10*c^8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 + 8883535872*a^9
*b^4*c^11 - 11576279040*a^10*b^2*c^12)*1i)/(16*(4096*a^6*c^9 + b^12*c^3 - 2
4*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144
*a^5*b^2*c^8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a
^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*
c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^
7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c
^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^
4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/

2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^
24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a
^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*
b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b
^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/4)*1i + (x^(1/2)*(9801*a^5*b^11 - 25
6905*a^6*b^9*c - 29042496*a^10*b*c^5 + 2642841*a^7*b^7*c^2 - 13243020*a^8*b
^5*c^3 + 31945648*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^
4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^
8)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11
+ 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704
576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626
752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*
a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*
a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a
*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48
*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^1
1 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 +
32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 5
0331648*a^11*b^2*c^18)))^(1/4)*1i + (((46036680704*a^12*c^12 - 110592*a^3*b
^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 788037632*a^6*b^12
*c^6 - 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 - 59401830400*a^9*
b^6*c^9 + 104312340480*a^10*b^4*c^10 - 104991817728*a^11*b^2*c^11)/(128*(16
384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c
^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9)) + (x^(1/2)*
(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 901
26*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a
^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a
^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c
^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c -
b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*
c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^
22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 8
11008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 3244
0320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 503316
48*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*c^13 + 36864*a^3*b^16*c^5 - 1302
528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 185991168*a^6*b^10*c^8 + 1061683
200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 + 8883535872*a^9*b^4*c^11 - 11576
279040*a^10*b^2*c^12)*1i)/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 24
0*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*(
-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 9012
6*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^
5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^
8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^
4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c -
b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c

```
*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^2
2*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 81
1008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440
320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 5033164
8*a^11*b^2*c^18)))^(3/4)*1i - (x^(1/2)*(9801*a^5*b^11 - 256905*a^6*b^9*c -
29042496*a^10*b*c^5 + 2642841*a^7*b^7*c^2 - 13243020*a^8*b^5*c^3 + 31945648
*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c
^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*(-(81*b^23 +
 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*
c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5
+ 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 -
 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c
- b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1
/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c -
 b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 105
6*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^
14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8
*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*
c^18)))^(1/4)*1i + (107811*a^7*b^9 - 2531925*a^8*b^7*c + 128002112*a^11*b*c
^4 + 22295196*a^9*b^5*c^2 - 87242736*a^10*b^3*c^3)/(64*(16384*a^7*c^10 - b^
14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6
*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9))))*(-(81*b^23 + 81*b^8*(-(4*a
*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*
a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^
6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^
9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/
2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^
3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2
))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9
 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784
704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 576716
80*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.1072 $\int \frac{x^{11/2}}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=520

$$-\frac{\left(\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}-10a\cdots\right.}{4\sqrt[4]{2}\,c^{5/4}\left(b^2\cdots\right.}$$

```
[Out] 1/2*x^(5/2)*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/8*arctan(2^(1/4)*c^(
1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b^2-10*a*c+b*(-12*a*c+b^2)/(-4
*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-4*a*c+b^2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)
-1/8*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b^2-10
*a*c+b*(-12*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-4*a*c+b^2)/(-b-(
-4*a*c+b^2)^(1/2))^(3/4)-1/8*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2
)^(1/2))^(1/4))*(b^2-10*a*c-b*(-12*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(
5/4)/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-1/8*arctanh(2^(1/4)*c^(1/4)
*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b^2-10*a*c-b*(-12*a*c+b^2)/(-4*a*c
+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-1/2
*b*x^(1/2)/c/(-4*a*c+b^2)
```

Rubi [A] time = 1.37, antiderivative size = 520, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1115, 1365, 1502, 1422, 212, 208, 205}

$$-\frac{\left(\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}-10a\cdots\right.}{4\sqrt[4]{2}\,c^{5/4}\left(b^2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b*Sqrt[x])/(2*c*(b^2 - 4*a*c)) + (x^(5/2)*(2*a + b*x^2))/(2*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)) - ((b^2 - 10*a*c + (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a*
c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^
(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - ((b^2 - 10*a*
c - (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/
(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b + Sqr
t[b^2 - 4*a*c])^(3/4)) - ((b^2 - 10*a*c + (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a
*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*
```

```
2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - ((b^2 - 10*
a*c - (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x
])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b +
Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
```

```
*c] ||  !IGtQ[n/2, 0])
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{x^{11/2}}{\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^{12}}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{5/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{x^4\left(10a+bx^4\right)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2\left(b^2-4ac\right)}$$

$$= -\frac{b\sqrt{x}}{2c\left(b^2-4ac\right)} + \frac{x^{5/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{ab+\left(b^2-10ac\right)x^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2c\left(b^2-4ac\right)}$$

$$= -\frac{b\sqrt{x}}{2c\left(b^2-4ac\right)} + \frac{x^{5/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^2-10ac-\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{\frac{b}{2}-\frac{1}{2}\cdots}\right.}{4c\left(b^2-4ac\right)}$$

$$= -\frac{b\sqrt{x}}{2c\left(b^2-4ac\right)} + \frac{x^{5/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2-10ac+\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{\sqrt{-\cdots}}\right.}{4c\left(b^2-4ac\right)\sqrt{-b-\cdots}}$$

$$= -\frac{b\sqrt{x}}{2c\left(b^2-4ac\right)} + \frac{x^{5/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2-10ac+\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\cdots}{\sqrt[4]{-b-\cdots}}\right.}{4\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)\left(-b-\sqrt{b^2-4a\cdots}\right.}$$

Mathematica [C] time = 0.27, size = 144, normalized size = 0.28

$$\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{-10\#1^4ac\log\left(\sqrt{x}-\#1\right)+\#1^4b^2\log\left(\sqrt{x}-\#1\right)+ab\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]-\frac{4\sqrt{x}\left(a\left(b-2cx^2\right)+b^2x^2\right)}{a+bx^2+cx^4}}{8c\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-4*Sqrt[x]*(b^2*x^2 + a*(b - 2*c*x^2)))/(a + b*x^2 + c*x^4) + RootSum[a +
 b*#1^4 + c*#1^8 & , (a*b*Log[Sqrt[x] - #1] + b^2*Log[Sqrt[x] - #1]*#1^4 -
10*a*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(8*c*(b^2 - 4*a*c))
```

fricas [B] time = 82.62, size = 11906, normalized size = 22.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2
)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*
b^3*c^3 + 18000*a^4*b*c^4 + (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1
280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqr
t((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b
^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11
 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^
5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^1
8 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a
^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)))*arctan
(-1/2*(sqrt(1/2)*(b^22 - 91*a*b^20*c + 3683*a^2*b^18*c^2 - 87230*a^3*b^16*c
^3 + 1338850*a^4*b^14*c^4 - 13940024*a^5*b^12*c^5 + 100253344*a^6*b^10*c^6
- 497651072*a^7*b^8*c^7 + 1672046080*a^8*b^6*c^8 - 3627264000*a^9*b^4*c^9 +
 4582400000*a^10*b^2*c^10 - 2560000000*a^11*c^11 - (b^25*c^5 - 70*a*b^23*c^
6 + 2192*a^2*b^21*c^7 - 40672*a^3*b^19*c^8 + 498432*a^4*b^17*c^9 - 4254720*
a^5*b^15*c^10 + 25976832*a^6*b^13*c^11 - 114475008*a^7*b^11*c^12 + 36195532
8*a^8*b^9*c^13 - 802029568*a^9*b^7*c^14 + 1183842304*a^10*b^5*c^15 - 104647
8848*a^11*b^3*c^16 + 419430400*a^12*b*c^17)*sqrt((b^12 - 78*a*b^10*c + 2571
*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5
 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*
a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*
c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))*sqrt(
(81*a^2*b^16 - 8118*a^3*b^14*c + 358651*a^4*b^12*c^2 - 9129750*a^5*b^10*c^3
 + 146540625*a^6*b^8*c^4 - 1519250000*a^7*b^6*c^5 + 9937500000*a^8*b^4*c^6
```

- 37500000000*a^9*b^2*c^7 + 62500000000*a^10*c^8)*x + 1/2*sqrt(1/2)*(b^22 - 112*a*b^20*c + 5735*a^2*b^18*c^2 - 176820*a^3*b^16*c^3 + 3634845*a^4*b^14*c^4 - 52073994*a^5*b^12*c^5 + 527503968*a^6*b^10*c^6 - 3751826400*a^7*b^8*c^7 + 18208800000*a^8*b^6*c^8 - 56920000000*a^9*b^4*c^9 + 102400000000*a^10*b^2*c^10 - 80000000000*a^11*c^11 - (b^25*c^5 - 91*a*b^23*c^6 + 3641*a^2*b^21*c^7 - 84776*a^3*b^19*c^8 + 1280016*a^4*b^17*c^9 - 13215744*a^5*b^15*c^10 + 95875584*a^6*b^13*c^11 - 493891584*a^7*b^11*c^12 + 1798938624*a^8*b^9*c^13 - 4533059584*a^9*b^7*c^14 + 7523860480*a^10*b^5*c^15 - 7405568000*a^11*b^3*c^16 + 3276800000*a^12*b*c^17)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)) - sqrt(1/2)*(9*a*b^30 - 1270*a^2*b^28*c + 82813*a^3*b^26*c^2 - 3305978*a^4*b^24*c^3 + 90231255*a^5*b^22*c^4 - 1780615316*a^6*b^20*c^5 + 26199812170*a^7*b^18*c^6 - 292147074792*a^8*b^16*c^7 + 2484388440192*a^9*b^14*c^8 - 16082985454080*a^10*b^12*c^9 + 78485701504000*a^11*b^10*c^10 - 283191078400000*a^12*b^8*c^11 + 730734080000000*a^13*b^6*c^12 - 1272576000000000*a^14*b^4*c^13 + 1337600000000000*a^15*b^2*c^14 - 640000000000000*a^16*c^15 - (9*a*b^33*c^5 - 1081*a^2*b^31*c^6 + 59923*a^3*b^29*c^7 - 2033390*a^4*b^27*c^8 + 47234960*a^5*b^25*c^9 - 795781312*a^6*b^23*c^10 + 10050046208*a^7*b^21*c^11 - 96993186304*a^8*b^19*c^12 + 722648002560*a^9*b^17*c^13 - 4169749463040*a^10*b^15*c^14 + 18574068219904*a^11*b^13*c^15 - 63226237812736*a^12*b^11*c^16 + 161327426306048*a^13*b^9*c^17 - 298510607974400*a^14*b^7*c^18 + 378064076800000*a^15*b^5*c^19 - 293076992000000*a^16*b^3*c^20 + 104857600000000*a^17*b*c^21)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^1

4 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 58982
4*a^8*b^2*c^18 - 262144*a^9*c^19)))*sqrt(x)*sqrt(-(b^9 - 45*a*b^7*c + 765*a
^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (b^12*c^5 - 24*a*b^10*c^6
 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^1
0 + 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*
b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18
*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4
*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^1
7 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 24
0*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4
096*a^6*c^11)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 -
5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^
8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*
c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470
625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*
b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 -
129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^
8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7
 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)
))/(6561*a^5*b^20 - 803358*a^6*b^18*c + 44473131*a^7*b^16*c^2 - 1466261550*
a^8*b^14*c^3 + 31889850625*a^9*b^12*c^4 - 478129875000*a^10*b^10*c^5 + 5004
993750000*a^11*b^8*c^6 - 36117500000000*a^12*b^6*c^7 + 171937500000000*a^13
*b^4*c^8 - 487500000000000*a^14*b^2*c^9 + 625000000000000*a^15*c^10)) - 4*(
(b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sq
rt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 +
 18000*a^4*b*c^4 - (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b
^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 -
 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 -
2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^
2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^1
5 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 26214
4*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^
8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)))*arctan(1/2*(sqr
t(1/2)*(b^22 - 91*a*b^20*c + 3683*a^2*b^18*c^2 - 87230*a^3*b^16*c^3 + 13388
50*a^4*b^14*c^4 - 13940024*a^5*b^12*c^5 + 100253344*a^6*b^10*c^6 - 49765107
2*a^7*b^8*c^7 + 1672046080*a^8*b^6*c^8 - 3627264000*a^9*b^4*c^9 + 458240000
0*a^10*b^2*c^10 - 2560000000*a^11*c^11 + (b^25*c^5 - 70*a*b^23*c^6 + 2192*a
^2*b^21*c^7 - 40672*a^3*b^19*c^8 + 498432*a^4*b^17*c^9 - 4254720*a^5*b^15*c
^10 + 25976832*a^6*b^13*c^11 - 114475008*a^7*b^11*c^12 + 361955328*a^8*b^9*
c^13 - 802029568*a^9*b^7*c^14 + 1183842304*a^10*b^5*c^15 - 1046478848*a^11*
b^3*c^16 + 419430400*a^12*b*c^17)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c
^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000
*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c
^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589
824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))*sqrt((81*a^2*b^

16 - 8118*a^3*b^14*c + 358651*a^4*b^12*c^2 - 9129750*a^5*b^10*c^3 + 1465406
25*a^6*b^8*c^4 - 1519250000*a^7*b^6*c^5 + 9937500000*a^8*b^4*c^6 - 37500000
000*a^9*b^2*c^7 + 62500000000*a^10*c^8)*x + 1/2*sqrt(1/2)*(b^22 - 112*a*b^2
0*c + 5735*a^2*b^18*c^2 - 176820*a^3*b^16*c^3 + 3634845*a^4*b^14*c^4 - 5207
3994*a^5*b^12*c^5 + 527503968*a^6*b^10*c^6 - 3751826400*a^7*b^8*c^7 + 18208
800000*a^8*b^6*c^8 - 56920000000*a^9*b^4*c^9 + 102400000000*a^10*b^2*c^10 -
 80000000000*a^11*c^11 + (b^25*c^5 - 91*a*b^23*c^6 + 3641*a^2*b^21*c^7 - 84
776*a^3*b^19*c^8 + 1280016*a^4*b^17*c^9 - 13215744*a^5*b^15*c^10 + 95875584
*a^6*b^13*c^11 - 493891584*a^7*b^11*c^12 + 1798938624*a^8*b^9*c^13 - 453305
9584*a^9*b^7*c^14 + 7523860480*a^10*b^5*c^15 - 7405568000*a^11*b^3*c^16 + 3
276800000*a^12*b*c^17)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*
a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(
b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256
*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4
*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))*sqrt(-(b^9 - 45*a*b^7*c +
765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (b^12*c^5 - 24*a*b^1
0*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^
2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950
*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/
(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 3225
6*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^
4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6
 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^1
0 + 4096*a^6*c^11)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c
^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (b^12*c^5 - 24*a*b^10*c^6 + 240*a
^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096
*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3
+ 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 -
36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^
14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 5898
24*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^
8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*
c^11)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 1800
0*a^4*b*c^4 - (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^
8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a
*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 26250
00*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^1
4*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 3
44064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9
*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3
840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)) - sqrt(1/2)*(9*a*b^30
 - 1270*a^2*b^28*c + 82813*a^3*b^26*c^2 - 3305978*a^4*b^24*c^3 + 90231255*a
^5*b^22*c^4 - 1780615316*a^6*b^20*c^5 + 26199812170*a^7*b^18*c^6 - 29214707
4792*a^8*b^16*c^7 + 2484388440192*a^9*b^14*c^8 - 16082985454080*a^10*b^12*c
^9 + 78485701504000*a^11*b^10*c^10 - 283191078400000*a^12*b^8*c^11 + 730734

080000000*a^13*b^6*c^12 - 1272576000000000*a^14*b^4*c^13 + 1337600000000000
*a^15*b^2*c^14 - 640000000000000*a^16*c^15 + (9*a*b^33*c^5 - 1081*a^2*b^31*
c^6 + 59923*a^3*b^29*c^7 - 2033390*a^4*b^27*c^8 + 47234960*a^5*b^25*c^9 - 7
95781312*a^6*b^23*c^10 + 10050046208*a^7*b^21*c^11 - 96993186304*a^8*b^19*c
^12 + 722648002560*a^9*b^17*c^13 - 4169749463040*a^10*b^15*c^14 + 185740682
19904*a^11*b^13*c^15 - 63226237812736*a^12*b^11*c^16 + 161327426306048*a^13
*b^9*c^17 - 298510607974400*a^14*b^7*c^18 + 378064076800000*a^15*b^5*c^19 -
293076992000000*a^16*b^3*c^20 + 104857600000000*a^17*b*c^21)*sqrt((b^12 -
78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2
625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2
*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15
+ 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144
*a^9*c^19)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c
^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (b^12*c^5 - 24*a*b^10*c^6 + 240*a
^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096
*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3
+ 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 -
36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^
14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 5898
24*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^
8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*
c^11)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 1800
0*a^4*b*c^4 - (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^
8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a
*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 26250
00*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^1
4*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 3
44064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9
*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3
840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)))/(6561*a^5*b^20 - 803
358*a^6*b^18*c + 44473131*a^7*b^16*c^2 - 1466261550*a^8*b^14*c^3 + 31889850
625*a^9*b^12*c^4 - 478129875000*a^10*b^10*c^5 + 5004993750000*a^11*b^8*c^6
- 36117500000000*a^12*b^6*c^7 + 171937500000000*a^13*b^4*c^8 - 487500000000
000*a^14*b^2*c^9 + 625000000000000*a^15*c^10)) - ((b^2*c^2 - 4*a*c^3)*x^4 +
a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 -
45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (b^12*
c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9
- 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b
^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 625
0000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^
12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 -
589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 -
24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 61
44*a^5*b^2*c^10 + 4096*a^6*c^11)))*log((9*a*b^8 - 451*a^2*b^6*c + 8625*a^3*
b^4*c^2 - 75000*a^4*b^2*c^3 + 250000*a^5*c^4)*sqrt(x) + 1/2*(b^11 - 47*a*b^

9*c + 853*a^2*b^7*c^2 - 7324*a^3*b^5*c^3 + 28400*a^4*b^3*c^4 - 40000*a^5*b*
c^5 - (b^14*c^5 - 44*a*b^12*c^6 + 720*a^2*b^10*c^7 - 6080*a^3*b^8*c^8 + 294
40*a^4*b^6*c^9 - 82944*a^5*b^4*c^10 + 126976*a^6*b^2*c^11 - 81920*a^7*c^12)
*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a
^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*
c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 12902
4*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2
*c^18 - 262144*a^9*c^19)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2
*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (b^12*c^5 - 24*a*b^10*c^6 +
240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10
+ 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^
6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c
^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b
^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17
+ 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*
a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 409
6*a^6*c^11)))) + ((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c -
4*a*b*c^2)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 -
5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^
8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*
c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470
625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*
b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 -
129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^
8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7
- 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)
))*log((9*a*b^8 - 451*a^2*b^6*c + 8625*a^3*b^4*c^2 - 75000*a^4*b^2*c^3 + 25
0000*a^5*c^4)*sqrt(x) - 1/2*(b^11 - 47*a*b^9*c + 853*a^2*b^7*c^2 - 7324*a^3
*b^5*c^3 + 28400*a^4*b^3*c^4 - 40000*a^5*b*c^5 - (b^14*c^5 - 44*a*b^12*c^6
+ 720*a^2*b^10*c^7 - 6080*a^3*b^8*c^8 + 29440*a^4*b^6*c^9 - 82944*a^5*b^4*c
^10 + 126976*a^6*b^2*c^11 - 81920*a^7*c^12)*sqrt((b^12 - 78*a*b^10*c + 2571
*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5
+ 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*
a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*
c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))*sqrt(
sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18
000*a^4*b*c^4 + (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*
c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78
*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 262
5000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b
^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 +
344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a
^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 +
3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)))) - ((b^2*c^2 - 4*a
*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(sqrt(1/2)*s

qrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c
^4 - (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*
a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c +
 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^
2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 -
5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6
*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))/
(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b
^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)))*log((9*a*b^8 - 451*a^2*b^6*c
+ 8625*a^3*b^4*c^2 - 75000*a^4*b^2*c^3 + 250000*a^5*c^4)*sqrt(x) + 1/2*(b^1
1 - 47*a*b^9*c + 853*a^2*b^7*c^2 - 7324*a^3*b^5*c^3 + 28400*a^4*b^3*c^4 - 4
0000*a^5*b*c^5 + (b^14*c^5 - 44*a*b^12*c^6 + 720*a^2*b^10*c^7 - 6080*a^3*b^
8*c^8 + 29440*a^4*b^6*c^9 - 82944*a^5*b^4*c^10 + 126976*a^6*b^2*c^11 - 8192
0*a^7*c^12)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3
 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 -
 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c
^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589
824*a^8*b^2*c^18 - 262144*a^9*c^19)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*
c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (b^12*c^5 - 24*a
*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^
5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 4
5950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c
^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 +
32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^
7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10
*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2
*c^10 + 4096*a^6*c^11)))) + ((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2
+ (b^3*c - 4*a*b*c^2)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2
*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (b^12*c^5 - 24*a*b^10*c^6 +
 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10
+ 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^
6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c
^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b
^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17
+ 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*
a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 409
6*a^6*c^11)))*log((9*a*b^8 - 451*a^2*b^6*c + 8625*a^3*b^4*c^2 - 75000*a^4*b
^2*c^3 + 250000*a^5*c^4)*sqrt(x) - 1/2*(b^11 - 47*a*b^9*c + 853*a^2*b^7*c^2
 - 7324*a^3*b^5*c^3 + 28400*a^4*b^3*c^4 - 40000*a^5*b*c^5 + (b^14*c^5 - 44*
a*b^12*c^6 + 720*a^2*b^10*c^7 - 6080*a^3*b^8*c^8 + 29440*a^4*b^6*c^9 - 8294
4*a^5*b^4*c^10 + 126976*a^6*b^2*c^11 - 81920*a^7*c^12)*sqrt((b^12 - 78*a*b^
10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*
a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c
^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 3440
64*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^

```
19)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b
^3*c^3 + 18000*a^4*b*c^4 - (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 12
80*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt
((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^
4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11
+ 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5
*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18
 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^
3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)))) + 4*((
b^2 - 2*a*c)*x^2 + a*b)*sqrt(x))/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2
*c^2 + (b^3*c - 4*a*b*c^2)*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 47.04Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.02, size = 146, normalized size = 0.28

$$\frac{\left(\left(10ac-b^2\right)\operatorname{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^4-ab\right)\ln\left(-\operatorname{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{8\left(4ac-b^2\right)c\left(2\operatorname{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7c+\operatorname{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3b\right)}+\frac{\frac{ab\sqrt{x}}{2\left(4ac-b^2\right)c}-\frac{\left(2ac-b^2\right)x^{\frac{5}{2}}}{2\left(4ac-b^2\right)c}}{c\,x^4+b\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*(2*a*c-b^2)/(4*a*c-b^2)/c*x^(5/2)+1/4/(4*a*c-b^2)*a*b/c*x^(1/2))/(c
*x^4+b*x^2+a)+1/8/c/(4*a*c-b^2)*sum(((10*a*c-b^2)*_R^4-a*b)/(2*_R^7*c+_R^3*
b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bx^{\frac{9}{2}}+2\,ax^{\frac{5}{2}}}{2\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)}+\int-\frac{bx^{\frac{7}{2}}+10\,ax^{\frac{3}{2}}}{4\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^(9/2) + 2*a*x^(5/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b
^3 - 4*a*b*c)*x^2) + integrate(-1/4*(b*x^(7/2) + 10*a*x^(3/2))/((b^2*c - 4*
a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2), x)
```

mupad [B] time = 11.85, size = 31964, normalized size = 61.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] 2*atan(((((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a^7*b*c^4 + 6549*a^5*b^5*c^2
- 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^
3 - 256*a^3*b^2*c^4)) + ((x^(1/2)*(1006632960*a^10*b*c^11 + 4096*a^3*b^15*c
^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6 + 53739520*a^6*b^9*c^7 - 29
8844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1493172224*a^9*b^3*c^10))/(16
*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^
4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - ((-(b^21 + b^6*(-(4*a*c - b^2)^
15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 +
 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 505036
80*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c
^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2
)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^
17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 12
6720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128
*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a
^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*(167772160*a^9*c^11 + 40960*
a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^5*b^8*c^7 - 52428800*a^6*b^6
*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^8*b^2*c^10)*1i)/(2*(b^8*c + 256*
a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^21 + b^6*
(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 3632
0*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*
b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*
c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^
2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(
16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a
^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12
*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c
^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(3/4)*1i)*(-(b^21
+ b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2
- 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 1506457
6*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^
9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*
```

b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(
8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 1
4080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^
6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9
*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*1i - (
x^(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5 - 3744*a^5*b^8*c + 66322*a^6*b^6*c^2
 - 547800*a^7*b^4*c^3 + 1980000*a^8*b^2*c^4))/(16*(b^12*c + 4096*a^6*c^7 -
24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 614
4*a^5*b^2*c^6)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*
c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 30013
44*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a
^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2)
 - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(
4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^
6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a
^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^
8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11
*b^2*c^16)))^(1/4) - (((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a^7*b*c^4 + 6549
*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 +
96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) - ((x^(1/2)*(1006632960*a^10*b*c^11 + 40
96*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6 + 53739520*a^6
*b^9*c^7 - 298844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1493172224*a^9*b
^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 12
80*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + ((-(b^21 + b^6*(-(
4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a
^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9
*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9
 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(
-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(167
77216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*
b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^
11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14
 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*(167772160*a^9*
c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^5*b^8*c^7 - 524
28800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^8*b^2*c^10)*1i)/(2*
(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(
-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^
17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 +
15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 13467
6480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 5
25*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(
1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*
c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 378
4704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671
680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(3/4)

1i)(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*1i + (x^(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5 - 3744*a^5*b^8*c + 66322*a^6*b^6*c^2 - 547800*a^7*b^4*c^3 + 1980000*a^8*b^2*c^4))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4))/((((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a^7*b*c^4 + 6549*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) + ((x^(1/2)*(1006632960*a^10*b*c^11 + 4096*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6 + 53739520*a^6*b^9*c^7 - 298844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1493172224*a^9*b^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - ((-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*(167772160*a^9*c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^5*b^8*c^7 - 52428800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^8*b^2*c^10)*1i)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c

- b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(3/4)*1i)*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*1i - (x^(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5 - 3744*a^5*b^8*c + 66322*a^6*b^6*c^2 - 547800*a^7*b^4*c^3 + 1980000*a^8*b^2*c^4))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*1i + (((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a^7*b*c^4 + 6549*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) - ((x^(1/2)*(1006632960*a^10*b*c^11 + 4096*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6 + 53739520*a^6*b^9*c^7 - 298844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1493172224*a^9*b^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + ((-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*(167772160*a^9*c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^5*b^8*c^7 - 52428800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^8*b^2*c^10)*1i)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728

000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*
c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 +
108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2
)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*
a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 4
8*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9
 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 +
32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50
331648*a^11*b^2*c^16)))^(3/4)*1i)*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) +
 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4
*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*
c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c
 - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
 - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c
^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^
16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c
^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^1
5 - 50331648*a^11*b^2*c^16)))^(1/4)*1i + (x^(1/2)*(81*a^4*b^10 - 2000000*a^
9*c^5 - 3744*a^5*b^8*c + 66322*a^6*b^6*c^2 - 547800*a^7*b^4*c^3 + 1980000*a
^8*b^2*c^4))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 -
 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))*(-(b^21 + b^6*(-
(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*
a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^
9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^
9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*
(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16
777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3
*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c
^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^1
4 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*1i))*(-(b^21 +
 b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 -
 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576
*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9
*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b
^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8
192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14
080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6
*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*
b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4) - atan(
((((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a^7*b*c^4 + 6549*a^5*b^5*c^2 - 47800
*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256
*a^3*b^2*c^4)) + ((x^(1/2)*(1006632960*a^10*b*c^11 + 4096*a^3*b^15*c^4 + 14
7456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6 + 53739520*a^6*b^9*c^7 - 298844160
*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1493172224*a^9*b^3*c^10))/(16*(b^12*
c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 384

0*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + ((-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/
2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160
*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*
b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4
*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(
1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^
24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^
4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^
10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4
*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*(167772160*a^9*c^11 + 40960*a^3*b^1
2*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^5*b^8*c^7 - 52428800*a^6*b^6*c^8 +
157286400*a^7*b^4*c^9 - 251658240*a^8*b^2*c^10))/(2*(b^8*c + 256*a^4*c^5 -
16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^21 + b^6*(-(4*a*c -
 b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15
*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 -
50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500
*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c
 - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a
^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^
8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12
976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 6920
6016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(3/4))*(-(b^21 + b^6*(-(4*a*
c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b
^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6
 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2
500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*
a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(1677721
6*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18
*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 -
 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 6
9206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4) + (x^(1/2)*(81*a^4*
b^10 - 2000000*a^9*c^5 - 3744*a^5*b^8*c + 66322*a^6*b^6*c^2 - 547800*a^7*b^
4*c^3 + 1980000*a^8*b^2*c^4))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 +
240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))
(-(b^21 + b^6(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*
b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5
+ 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134
676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c +
 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)
^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^2
0*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3
784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 576
71680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/
4)*1i - (((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a^7*b*c^4 + 6549*a^5*b^5*c^2
- 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^

3 - 256*a^3*b^2*c^4)) - ((x^(1/2)*(1006632960*a^10*b*c^11 + 4096*a^3*b^15*c
^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6 + 53739520*a^6*b^9*c^7 - 29
8844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1493172224*a^9*b^3*c^10))/(16
*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^
4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - ((-(b^21 + b^6*(-(4*a*c - b^2)^
15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 +
 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 505036
80*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c
^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2
)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^
17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 12
6720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128
*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a
^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*(167772160*a^9*c^11 + 40960*
a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^5*b^8*c^7 - 52428800*a^6*b^6
*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^8*b^2*c^10))/(2*(b^8*c + 256*a^4
*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^21 + b^6*(-(
4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a
^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9
*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9
 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(
-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(167
77216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*
b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^
11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14
 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(3/4))*(-(b^21 + b^6*
(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 3632
0*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*
b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*
c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^
2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(
16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a
^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12
*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c
^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4) - (x^(1/2)*(
81*a^4*b^10 - 2000000*a^9*c^5 - 3744*a^5*b^8*c + 66322*a^6*b^6*c^2 - 547800
*a^7*b^4*c^3 + 1980000*a^8*b^2*c^4))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10
*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2
c^6)))(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 20
85*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^
11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^
8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b
^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b
^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*
a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c

^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^1
3 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16
)))^(1/4)*1i)/((((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a^7*b*c^4 + 6549*a^5*b
^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2
*b^4*c^3 - 256*a^3*b^2*c^4)) + ((x^(1/2)*(1006632960*a^10*b*c^11 + 4096*a^3
*b^15*c^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6 + 53739520*a^6*b^9*c
^7 - 298844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1493172224*a^9*b^3*c^1
0))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3
*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + ((-(b^21 + b^6*(-(4*a*c
- b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^1
5*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 -
50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 250
0*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*
c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*
a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c
^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 1
2976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 692
06016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*(167772160*a^9*c^11 +
40960*a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^5*b^8*c^7 - 52428800*
a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^8*b^2*c^10))/(2*(b^8*c +
256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^21 +
b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 -
36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*
a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*
b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^
2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(81
92*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 140
80*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*
b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b
^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(3/4))*(-(b^21
+ b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2
- 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 150645
76*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a
^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2
*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/
(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 -
14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a
^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^
9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4) + (x^
(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5 - 3744*a^5*b^8*c + 66322*a^6*b^6*c^2 -
547800*a^7*b^4*c^3 + 1980000*a^8*b^2*c^4))/(16*(b^12*c + 4096*a^6*c^7 - 24
*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*
a^5*b^2*c^6)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^
10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344
*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8

*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4) + (((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a^7*b*c^4 + 6549*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) - ((x^(1/2)*(1006632960*a^10*b*c^11 + 4096*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6 + 53739520*a^6*b^9*c^7 - 298844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1493172224*a^9*b^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - ((-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*(167772160*a^9*c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^5*b^8*c^7 - 52428800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^8*b^2*c^10))/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(3/4))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4) - (x^(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5 - 3744*a^5*b^8*c + 66322*a^6*b^6*c^2 - 547800*a^7*b^4*c^3 + 1980000*a^8*b^2*c^4))/(16*(b^12*c + 4096*a^6*c^7

- 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6
144*a^5*b^2*c^6)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*
b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 300
1344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160
*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/
2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(
-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*
c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008
*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*
a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^
11*b^2*c^16)))^(1/4)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a
^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 -
 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 10838
0160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)
^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4
c(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b
^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 81
1008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440
320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 5033164
8*a^11*b^2*c^16)))^(1/4)*2i - atan(((((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a
^7*b*c^4 + 6549*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 -
16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) + ((x^(1/2)*(1006632960*a
^10*b*c^11 + 4096*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6
 + 53739520*a^6*b^9*c^7 - 298844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1
493172224*a^9*b^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a
^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + ((-
(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^1
7*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 1
5064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676
480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 52
5*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1
/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c
^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784
704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 576716
80*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*
(167772160*a^9*c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^
5*b^8*c^7 - 52428800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^8*b^
2*c^10))/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*
b^2*c^4)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 +
 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5
*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5
*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*
a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c
- b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 10
56*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^1

4*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(3/4))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4) + (x^(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5 - 3744*a^5*b^8*c + 66322*a^6*b^6*c^2 - 547800*a^7*b^4*c^3 + 1980000*a^8*b^2*c^4))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*1i - (((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a^7*b*c^4 + 6549*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) - ((x^(1/2)*(1006632960*a^10*b*c^11 + 4096*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6 + 53739520*a^6*b^9*c^7 - 298844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1493172224*a^9*b^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - ((-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*(167772160*a^9*c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^5*b^8*c^7 - 52428800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^8*b^2*c^10))/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*

a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2
) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-
(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c
^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*
a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a
^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^1
1*b^2*c^16)))^(3/4))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^1
0*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3
001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 1083801
60*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(
1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c
*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^2
2*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 8110
08*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 3244032
0*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*
a^11*b^2*c^16)))^(1/4) - (x^(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5 - 3744*a^5
*b^8*c + 66322*a^6*b^6*c^2 - 547800*a^7*b^4*c^3 + 1980000*a^8*b^2*c^4))/(16
*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^
4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15
)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 4
04160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680
*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3
*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^
15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17
 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 1267
20*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a
^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^1
0*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*1i)/((((9*a^3*b^9 - 397*a^4*b^
7*c + 130000*a^7*b*c^4 + 6549*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c +
256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) + ((x^(1/2)
*(1006632960*a^10*b*c^11 + 4096*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 491520
0*a^5*b^11*c^6 + 53739520*a^6*b^9*c^7 - 298844160*a^7*b^7*c^8 + 918552576*a
^8*b^5*c^9 - 1493172224*a^9*b^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^
10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b
^2*c^6)) + ((-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10
+ 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^
5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^
5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69
*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c
 - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1
056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^
14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8
*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*
c^16)))^(1/4)*(167772160*a^9*c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^10*c^
6 + 9830400*a^5*b^8*c^7 - 52428800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 - 25

1658240*a^8*b^2*c^10))/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*
c^3 - 256*a^3*b^2*c^4)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000
*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4
 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108
380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^1
5)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b
^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a
*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 -
811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 324
40320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331
648*a^11*b^2*c^16)))^(3/4))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728
000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*
c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 +
108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2
)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*
a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 4
8*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9
 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 +
32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50
331648*a^11*b^2*c^16)))^(1/4) + (x^(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5 - 3
744*a^5*b^8*c + 66322*a^6*b^6*c^2 - 547800*a^7*b^4*c^3 + 1980000*a^8*b^2*c^
4))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3
*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))*(-(b^21 - b^6*(-(4*a*c -
b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*
c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 5
0503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*
a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c
 - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^
12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8
 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 129
76128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206
016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4) + (((9*a^3*b^9 - 397*a^
4*b^7*c + 130000*a^7*b*c^4 + 6549*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*
c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) - ((x^(
1/2)*(1006632960*a^10*b*c^11 + 4096*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 49
15200*a^5*b^11*c^6 + 53739520*a^6*b^9*c^7 - 298844160*a^7*b^7*c^8 + 9185525
76*a^8*b^5*c^9 - 1493172224*a^9*b^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*
a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a
^5*b^2*c^6)) - ((-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c
^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 300134
4*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^
8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2)
 - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4
*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6
 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^

5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8
*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*
b^2*c^16)))^(1/4)*(167772160*a^9*c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^1
0*c^6 + 9830400*a^5*b^8*c^7 - 52428800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9
- 251658240*a^8*b^2*c^10))/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*
b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 7372
8000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13
*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 +
108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^
2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39
*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 -
48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^
9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 +
32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 5
0331648*a^11*b^2*c^16)))^(3/4))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 7
3728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b
^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^
7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c -
b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) +
39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5
- 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16
*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^1
2 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15
- 50331648*a^11*b^2*c^16)))^(1/4) - (x^(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5
- 3744*a^5*b^8*c + 66322*a^6*b^6*c^2 - 547800*a^7*b^4*c^3 + 1980000*a^8*b^
2*c^4))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280
*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))*(-(b^21 - b^6*(-(4*a*
c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b
^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6
- 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2
500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*
a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(1677721
6*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18
*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 -
12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 6
9206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)))*(-(b^21 - b^6*(-(
4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a
^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9
*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9
+ 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(
-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(167
77216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*
b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^
11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14
+ 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*2i - ((x^(5/2)*

(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b*x^(1/2))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + 2*atan(((((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a^7*b*c^4 + 6549*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) + ((x^(1/2)*(1006632960*a^10*b*c^11 + 4096*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6 + 53739520*a^6*b^9*c^7 - 298844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1493172224*a^9*b^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - ((-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*(167772160*a^9*c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^5*b^8*c^7 - 52428800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^8*b^2*c^10)*1i)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(3/4)*1i)*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*1i - (x^(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5 - 3744*a^5*b^8*c + 66322*a^6*b^6*c^2 - 547800*a^7*b^4*c^3 + 1980000*a^8*b^2*c^4))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a

*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/
2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24
*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*
b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10
*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c
^15 - 50331648*a^11*b^2*c^16)))^(1/4) - (((9*a^3*b^9 - 397*a^4*b^7*c + 1300
00*a^7*b*c^4 + 6549*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^
5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) - ((x^(1/2)*(10066329
60*a^10*b*c^11 + 4096*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11
*c^6 + 53739520*a^6*b^9*c^7 - 298844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9
 - 1493172224*a^9*b^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 2
40*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) +
 ((-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2
*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5
 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 13
4676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c
- 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15
)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^
20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 +
3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57
671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1
/4)*(167772160*a^9*c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 983040
0*a^5*b^8*c^7 - 52428800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^
8*b^2*c^10)*1i)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 2
56*a^3*b^2*c^4)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b
*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001
344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*
a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2
) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-
(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c
^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*
a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a
^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^1
1*b^2*c^16)))^(3/4)*1i)*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*
a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4
- 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 1083
80160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15
)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^
4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*
b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 8
11008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 3244
0320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 503316
48*a^11*b^2*c^16)))^(1/4)*1i + (x^(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5 - 37
44*a^5*b^8*c + 66322*a^6*b^6*c^2 - 547800*a^7*b^4*c^3 + 1980000*a^8*b^2*c^4
))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*

b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))*(-(b^21 - b^6*(-(4*a*c - b
^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c
^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50
503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a
^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c -
 b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^1
2*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8
+ 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 1297
6128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 692060
16*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4))/((((9*a^3*b^9 - 397*a^4
*b^7*c + 130000*a^7*b*c^4 + 6549*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c
 + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) + ((x^(1
/2)*(1006632960*a^10*b*c^11 + 4096*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 491
5200*a^5*b^11*c^6 + 53739520*a^6*b^9*c^7 - 298844160*a^7*b^7*c^8 + 91855257
6*a^8*b^5*c^9 - 1493172224*a^9*b^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*a
*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^
5*b^2*c^6)) - ((-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^
10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344
*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8
*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) -
 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*
a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6
+ 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5
*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*
b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b
^2*c^16)))^(1/4)*(167772160*a^9*c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^10
*c^6 + 9830400*a^5*b^8*c^7 - 52428800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 -
 251658240*a^8*b^2*c^10)*1i)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^
2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73
728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^
13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7
 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c -
b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) +
39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5
- 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*
c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12
 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 -
 50331648*a^11*b^2*c^16)))^(3/4)*1i)*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2
) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*
a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b
^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*
a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1
/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^2
4*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4
*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^1

0*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*1i - (x^(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5 - 3744*a^5*b^8*c + 66322*a^6*b^6*c^2 - 547800*a^7*b^4*c^3 + 1980000*a^8*b^2*c^4))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*1i + (((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a^7*b*c^4 + 6549*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) - ((x^(1/2)*(1006632960*a^10*b*c^11 + 4096*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6 + 53739520*a^6*b^9*c^7 - 298844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1493172224*a^9*b^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + ((-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*(167772160*a^9*c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^5*b^8*c^7 - 52428800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^8*b^2*c^10)*1i)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(3/4)*1i)*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 25

```
00*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a
*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216
*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*
c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 -
12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69
206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*1i + (x^(1/2)*(81*a^
4*b^10 - 2000000*a^9*c^5 - 3744*a^5*b^8*c + 66322*a^6*b^6*c^2 - 547800*a^7*
b^4*c^3 + 1980000*a^8*b^2*c^4))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2
+ 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)
))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^
2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^
5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 1
34676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c
 - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^1
5)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b
^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 +
 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 5
7671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(
1/4)*1i))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 +
2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*
b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*
c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a
*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c -
 b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 105
6*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14
*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c
^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^
16)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.1073 $\int \frac{x^{9/2}}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=471

$$\frac{\left(b\sqrt{b^2-4ac}+12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(b-\frac{12ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{(b\sqrt{b^2-4ac}+12ac+}{4\,2^{3/4}c^{3/4}(b^2-4ac}$$

```
[Out] 1/2*x^(3/2)*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/8*arctan(2^(1/4)*c^(
1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-12*a*c-b^2)/(-4*a*c+b^2)^(
1/2))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-1/8*arctan
h(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-12*a*c-b^2)/(
-4*a*c+b^2)^(1/2))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(1/
4)+1/8*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b^2+1
2*a*c+b*(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+
b^2)^(1/2))^(1/4)-1/8*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2
))^(1/4))*(b^2+12*a*c+b*(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(3
/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A] time = 0.92, antiderivative size = 471, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1115, 1365, 1510, 298, 205, 208}

$$\frac{\left(b\sqrt{b^2-4ac}+12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(b-\frac{12ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{(b\sqrt{b^2-4ac}+12ac+}{4\,2^{3/4}c^{3/4}(b^2-4ac}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^(3/2)*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^2 + 12*
a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2
- 4*a*c])^(1/4)])/(4*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4
*a*c])^(1/4)) + ((b - (b^2 + 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(
1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(3/4)*(b^2 - 4*
a*c)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ((b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c]
)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(
3/4)*c^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ((b - (b
^2 + 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqr
```

```
t[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4
*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^{9/2}}{\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^{10}}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{3/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(6a-bx^4\right)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2\left(b^2-4ac\right)}$$

$$= \frac{x^{3/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2+12ac-b\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx,\right.}{4\left(b^2-4ac\right)^{3/2}}$$

$$= \frac{x^{3/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^2+12ac-b\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\right.}{4\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}}$$

$$= \frac{x^{3/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^2+12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\left(b^2\right.}{4}$$

Mathematica [C] time = 0.21, size = 124, normalized size = 0.26

$$\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4b\log\left(\sqrt{x}-\#1\right)-6a\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]}{8\left(b^2-4ac\right)} - \frac{-2ax^{3/2}-bx^{7/2}}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(-2*a*x^(3/2) - b*x^(7/2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + RootS
um[a + b*#1^4 + c*#1^8 & , (-6*a*Log[Sqrt[x] - #1] + b*Log[Sqrt[x] - #1]*#1
^4)/(b*#1 + 2*c*#1^5) & ]/(8*(b^2 - 4*a*c))
```

fricas [B] time = 72.30, size = 11032, normalized size = 23.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] -1/8*(4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqr
t(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b
^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4
*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b
^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 57
6*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c
^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262
144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*
c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*arctan(-1/2*((b
^9 + 19*a*b^7*c + 124*a^2*b^5*c^2 - 2160*a^3*b^3*c^3 + 5184*a^4*b*c^4 - (b^
14*c^3 - 12*a*b^12*c^4 - 48*a^2*b^10*c^5 + 1600*a^3*b^8*c^6 - 11520*a^4*b^6
*c^7 + 39936*a^5*b^4*c^8 - 69632*a^6*b^2*c^9 + 49152*a^7*c^10)*sqrt((b^8 +
54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c
^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*
c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 58
9824*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt((117649*a^4*b^20 + 9983358*a^5*
b^18*c + 404714961*a^6*b^16*c^2 + 9897860448*a^7*b^14*c^3 + 158656107456*a^
8*b^12*c^4 + 1707655509504*a^9*b^10*c^5 + 12338818573824*a^10*b^8*c^6 + 588
12305154048*a^11*b^6*c^7 + 177024646692864*a^12*b^4*c^8 + 304679870005248*a
^13*b^2*c^9 + 228509902503936*a^14*c^10)*x - 1/2*sqrt(1/2)*(2401*a^3*b^25 +
 294294*a^4*b^23*c + 13335105*a^5*b^21*c^2 + 323354360*a^6*b^19*c^3 + 42692
53584*a^7*b^17*c^4 + 24537890304*a^8*b^15*c^5 - 79436754432*a^9*b^13*c^6 -
1621756588032*a^10*b^11*c^7 - 3506876964864*a^11*b^9*c^8 + 27305557622784*a
^12*b^7*c^9 + 100201644490752*a^13*b^5*c^10 - 142936235311104*a^14*b^3*c^11
 - 677066377789440*a^15*b*c^12 - (2401*a^3*b^30*c^3 - 49049*a^4*b^28*c^4 -
1432760*a^5*b^26*c^5 - 6473264*a^6*b^24*c^6 + 373184512*a^7*b^22*c^7 - 3191
85152*a^8*b^20*c^8 - 27408852992*a^9*b^18*c^9 + 93871525888*a^10*b^16*c^10
+ 774145638400*a^11*b^14*c^11 - 4486009651200*a^12*b^12*c^12 - 559078126387
2*a^13*b^10*c^13 + 81717925773312*a^14*b^8*c^14 - 108093958520832*a^15*b^6*
c^15 - 454721122861056*a^16*b^4*c^16 + 1497904875307008*a^17*b^2*c^17 - 128
3918464548864*a^18*c^18)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*
a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8
- 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^
6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))
*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 2
4*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144
*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17
496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*
c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 34406
4*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^1
5)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*
a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9))) - (343*a^2*b^19 + 21070*a^
3*b^17*c + 600271*a^4*b^15*c^2 + 8903196*a^5*b^13*c^3 + 62719920*a^6*b^11*c
```

^4 - 15909696*a^7*b^9*c^5 - 2396812032*a^8*b^7*c^6 - 6953610240*a^9*b^5*c^7 + 19591041024*a^10*b^3*c^8 + 78364164096*a^11*b*c^9 - (343*a^2*b^24*c^3 + 10437*a^3*b^22*c^4 + 90132*a^4*b^20*c^5 - 1028432*a^5*b^18*c^6 - 14041152*a^6*b^16*c^7 + 70390272*a^7*b^14*c^8 + 646137856*a^8*b^12*c^9 - 3121520640*a^9*b^10*c^10 - 11091935232*a^10*b^8*c^11 + 68335239168*a^11*b^6*c^12 + 24652283904*a^12*b^4*c^13 - 557256278016*a^13*b^2*c^14 + 743008370688*a^14*c^15)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt(x))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))/(2401*a^3*b^16 + 179046*a^4*b^14*c + 6354369*a^5*b^12*c^2 + 131902344*a^6*b^10*c^3 + 1713103344*a^7*b^8*c^4 + 13740938496*a^8*b^6*c^5 + 65167421184*a^9*b^4*c^6 + 166523848704*a^10*b^2*c^7 + 176319369216*a^11*c^8)) - 4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*arctan(1/2*((b^9 + 19*a*b^7*c + 124*a^2*b^5*c^2 - 2160*a^3*b^3*c^3 + 5184*a^4*b*c^4 + (b^14*c^3 - 12*a*b^12*c^4 - 48*a^2*b^10*c^5 + 1600*a^3*b^8*c^6 - 11520*a^4*b^6*c^7 + 39936*a^5*b^4*c^8 - 69632*a^6*b^2*c^9 + 49152*a^7*c^10)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt((117649*a^4*b^20 + 9983358*a^5*b^18*c + 404714961*a^6*b^16*c^2 + 9897860448*a^7*b^14*c^3 + 158656107456*a^8*b^12*c^4 + 1707655509504*a^9*b^10*c^5 + 12338818573824*a^10*b^8*c^6 + 58812305154048*a^11*b^6*c^7 + 177024646692864*a^12*b^4*c^8 + 304679870005248*a^13*b^2*c^9 + 228509902503936*a^14*c^10)*x - 1/2*sqrt(1/2)*(2401*a^3*b^25 + 294294*a^4*b^23*c + 13335105*a^5*b^21*c^2 + 323354360*a^6*b^19*c^3 + 4269253584*a^7*b^17*c^4 + 24537890304*a^8*b^15*c^5 - 79436754432*a^9*b^13*c^6 - 1621756588032*a^10*b^11*c^7 - 3506876964864*a^11*b^9*c^8 + 27305557622784*a^12*b^7*c^9 + 100201644490752*a^13*b^5*c^10 - 142936235311104*a^14*b^3*c^11 - 677066377789440*a^15*b*c^12 + (2401*a^

3*b^30*c^3 - 49049*a^4*b^28*c^4 - 1432760*a^5*b^26*c^5 - 6473264*a^6*b^24*c^6 + 373184512*a^7*b^22*c^7 - 319185152*a^8*b^20*c^8 - 27408852992*a^9*b^18*c^9 + 93871525888*a^10*b^16*c^10 + 774145638400*a^11*b^14*c^11 - 4486009651200*a^12*b^12*c^12 - 5590781263872*a^13*b^10*c^13 + 81717925773312*a^14*b^8*c^14 - 108093958520832*a^15*b^6*c^15 - 454721122861056*a^16*b^4*c^16 + 1497904875307008*a^17*b^2*c^17 - 1283918464548864*a^18*c^18)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9))) - (343*a^2*b^19 + 21070*a^3*b^17*c + 600271*a^4*b^15*c^2 + 8903196*a^5*b^13*c^3 + 62719920*a^6*b^11*c^4 - 15909696*a^7*b^9*c^5 - 2396812032*a^8*b^7*c^6 - 6953610240*a^9*b^5*c^7 + 19591041024*a^10*b^3*c^8 + 78364164096*a^11*b*c^9 + (343*a^2*b^24*c^3 + 10437*a^3*b^22*c^4 + 90132*a^4*b^20*c^5 - 1028432*a^5*b^18*c^6 - 14041152*a^6*b^16*c^7 + 70390272*a^7*b^14*c^8 + 646137856*a^8*b^12*c^9 - 3121520640*a^9*b^10*c^10 - 11091935232*a^10*b^8*c^11 + 68335239168*a^11*b^6*c^12 + 24652283904*a^12*b^4*c^13 - 557256278016*a^13*b^2*c^14 + 743008370688*a^14*c^15)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9))))/(2401*a^3*b^16 + 179046*a^4*b^14*c + 6354369*a^5*b^12*c^2 + 131902344*a^6*b^10

*c^3 + 1713103344*a^7*b^8*c^4 + 13740938496*a^8*b^6*c^5 + 65167421184*a^9*b
^4*c^6 + 166523848704*a^10*b^2*c^7 + 176319369216*a^11*c^8)) + ((b^2*c - 4*
a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b
^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*
c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*
c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b
^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 537
6*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6
*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^1
2*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c
^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*log(1/2*sqrt(1/2)*(b^18 + 25*a*b^16
*c - 146*a^2*b^14*c^2 - 5320*a^3*b^12*c^3 - 2464*a^4*b^10*c^4 + 1076096*a^5
*b^8*c^5 - 10483200*a^6*b^6*c^6 + 44181504*a^7*b^4*c^7 - 89579520*a^8*b^2*c
^8 + 71663616*a^9*c^9 - (b^23*c^3 - 20*a*b^21*c^4 + 432*a^2*b^19*c^5 - 1171
2*a^3*b^17*c^6 + 195072*a^4*b^15*c^7 - 1935360*a^5*b^13*c^8 + 12214272*a^6*
b^11*c^9 - 50823168*a^7*b^9*c^10 + 139788288*a^8*b^7*c^11 - 245628928*a^9*b
^5*c^12 + 250609664*a^10*b^3*c^13 - 113246208*a^11*b*c^14)*sqrt((b^8 + 54*a
*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 -
36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10
- 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824
*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c +
168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8
*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^
9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a
^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 +
32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^
7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10
*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2
*c^8 + 4096*a^6*c^9)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3
*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3
840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c +
1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^
16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 12902
4*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2
*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 12
80*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)) + (34
3*a^2*b^10 + 14553*a^3*b^8*c + 281232*a^4*b^6*c^2 + 2496096*a^5*b^4*c^3 + 1
0077696*a^6*b^2*c^4 + 15116544*a^7*c^5)*sqrt(x)) - ((b^2*c - 4*a*c^2)*x^4 +
a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^
5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a^
2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a
^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104
976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c
^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 5898
24*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a

*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*log(-1/2*sqrt(1/2)*(b^18 + 25*a*b^16*c - 146*a^2*b^14*c^2 - 5320*a^3*b^12*c^3 - 2464*a^4*b^10*c^4 + 1076096*a^5*b^8*c^5 - 10483200*a^6*b^6*c^6 + 44181504*a^7*b^4*c^7 - 89579520*a^8*b^2*c^8 + 71663616*a^9*c^9 - (b^23*c^3 - 20*a*b^21*c^4 + 432*a^2*b^19*c^5 - 11712*a^3*b^17*c^6 + 195072*a^4*b^15*c^7 - 1935360*a^5*b^13*c^8 + 12214272*a^6*b^11*c^9 - 50823168*a^7*b^9*c^10 + 139788288*a^8*b^7*c^11 - 245628928*a^9*b^5*c^12 + 250609664*a^10*b^3*c^13 - 113246208*a^11*b*c^14)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)) + (343*a^2*b^10 + 14553*a^3*b^8*c + 281232*a^4*b^6*c^2 + 2496096*a^5*b^4*c^3 + 10077696*a^6*b^2*c^4 + 15116544*a^7*c^5)*sqrt(x)) + ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*log(1/2*sqrt(1/2)*(b^18 + 25*a*b^16*c - 146*a^2*b^14*c^2 - 5320*a^3*b^12*c^3 - 2464*a^4*b^10*c^4 + 1076096*a^5*b^8*c^5 - 10483200*a^6*b^6*c^6 + 44181504*a^7*b^4*c^7 - 89579520*a^8*b^2*c^8 + 71663616*a^9*c^9 + (b^23*c^3 - 20*a*b^21*c^4 + 432*a^2*b^19*c^5 - 11712*a^3*b^17*c^6 + 195072*a^4*b^15*c^7 - 1935360*a^5*b^13*c^8 + 12214272*a^6*b^11*c^9 - 50823168*a^7*b^9*c^10 + 139788288*a^8*b^7*c^11 - 245628928*a^9*b^5*c^12 + 250609664*a^10*b^3*c^13 - 113246208*a^11*b*c^14)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^

2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11
+ 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*
a^9*c^15)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024
*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6
 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6
*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*
a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 1
29024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8
*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5
- 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*
sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24
*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*
a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 174
96*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c
^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064
*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15
)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a
^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)) + (343*a^2*b^10 + 14553*a^3*
b^8*c + 281232*a^4*b^6*c^2 + 2496096*a^5*b^4*c^3 + 10077696*a^6*b^2*c^4 + 1
5116544*a^7*c^5)*sqrt(x)) - ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3
 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 +
 3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^
6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*
a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6
- 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^1
0 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 58982
4*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8
*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^
9)))*log(-1/2*sqrt(1/2)*(b^18 + 25*a*b^16*c - 146*a^2*b^14*c^2 - 5320*a^3*b
^12*c^3 - 2464*a^4*b^10*c^4 + 1076096*a^5*b^8*c^5 - 10483200*a^6*b^6*c^6 +
44181504*a^7*b^4*c^7 - 89579520*a^8*b^2*c^8 + 71663616*a^9*c^9 + (b^23*c^3
- 20*a*b^21*c^4 + 432*a^2*b^19*c^5 - 11712*a^3*b^17*c^6 + 195072*a^4*b^15*c
^7 - 1935360*a^5*b^13*c^8 + 12214272*a^6*b^11*c^9 - 50823168*a^7*b^9*c^10 +
 139788288*a^8*b^7*c^11 - 245628928*a^9*b^5*c^12 + 250609664*a^10*b^3*c^13
- 113246208*a^11*b*c^14)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*
a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8
- 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^
6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))
*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3
- (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4
*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a
^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7
+ 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b
^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 -
 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*

```
b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*sqrt(-(b^7
+ 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4
 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8
 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*
c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a
^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^
12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c
^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7
- 6144*a^5*b^2*c^8 + 4096*a^6*c^9)) + (343*a^2*b^10 + 14553*a^3*b^8*c + 281
232*a^4*b^6*c^2 + 2496096*a^5*b^4*c^3 + 10077696*a^6*b^2*c^4 + 15116544*a^7
*c^5)*sqrt(x)) - 4*(b*x^3 + 2*a*x)*sqrt(x))/((b^2*c - 4*a*c^2)*x^4 + a*b^2
- 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 47.21Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.02, size = 120, normalized size = 0.25

$$-\frac{\left(\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^6 b-6\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^2 a\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{8\left(4ac-b^2\right)\left(2\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7 c+\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3 b\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*b/(4*a*c-b^2)*x^(7/2)-1/2*a/(4*a*c-b^2)*x^(3/2))/(c*x^4+b*x^2+a)-1/
8/(4*a*c-b^2)*sum((_R^6*b-6*_R^2*a)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=Ro
otOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bx^{\frac{7}{2}}+2\,ax^{\frac{3}{2}}}{2\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)}-\int-\frac{bx^{\frac{5}{2}}-6\,a\sqrt{x}}{4\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^(7/2) + 2*a*x^(3/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b
^3 - 4*a*b*c)*x^2) - integrate(-1/4*(b*x^(5/2) - 6*a*sqrt(x))/((b^2*c - 4*a
*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2), x)
```

mupad [B] time = 6.44, size = 23808, normalized size = 50.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((a*x^(3/2))/(4*a*c - b^2) + (b*x^(7/2))/(2*(4*a*c - b^2)))/(a + b*x^2 +
c*x^4) - atan(((((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*
b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*b
^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 1638
4*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*
a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*((b^4*(-(4*a*c -
 b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^
13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10
665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)
^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216
*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*
c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12
976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 6920
6016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11
- 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 3329
22880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 26508
00128*a^9*b^2*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*
b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a
*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^
3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6
- 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)
^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(1677
7216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b
^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9
- 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 +
69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/4) + (x^(1/2)*(49*a^3
*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b
^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 38
40*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15
)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 +
 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a
^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) +
```

3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*1i - (((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*b^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/4) - (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*1i)/((279936*a^8*c^5 + 343*a^4*b^8*c + 7350*a^5*b^6*c^2 + 58968*a^6*b^4*c^3 + 209952*a^7*b^2*c^4)/(64*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a

^7*b^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/4) + (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4) + (((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*b^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8

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*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*
c^14)))^(1/4)*(1207959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^1
2*c^5 - 56229888*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*
c^8 + 2390753280*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10))/(16*(b^12 + 4096*a
^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b
^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a
^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 58521
6*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b
^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-
(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c
^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*
a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8
*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*
b^2*c^14)))^(3/4) - (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*
c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240
*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*
a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 9
6*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^
5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324
*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2
)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^
2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8
 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 -
57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^
(1/4)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^
2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 +
3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2
*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15
)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^
20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3
784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 5767
1680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4
)*2i - 2*atan(((((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*
b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*b
^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 1638
4*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*
a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*((b^4*(-(4*a*c -
 b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^
13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10
665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)
^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216
*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*
c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12
976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 6920
6016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11
```

- 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 3329
22880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 26508
00128*a^9*b^2*c^10)*1i)/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a
^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(
4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752
*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c
^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b
^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(1
6777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^
3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c
^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12
 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/4)*1i - (x^(1/2)*(
49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712
*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^
3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b
^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13
*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 1066
5984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(
1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a
^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^
6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 1297
6128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 692060
16*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4) - (((5435817984*a^10*b*c
^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 - 32833536*a^5*b^11*c^5 + 32
3747840*a^6*b^9*c^6 - 1714421760*a^7*b^7*c^7 + 5121245184*a^8*b^5*c^8 - 817
0504192*a^9*b^3*c^9)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a
^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*
a*b^12*c)) + (x^(1/2)*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9
*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*
a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3
*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4
*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4
 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^
5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b
^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^
2*c^14)))^(1/4)*(1207959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b
^12*c^5 - 56229888*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^
6*c^8 + 2390753280*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10)*1i)/(16*(b^12 + 4
096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*
a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386
304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 -
585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*
a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2
c(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b
^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 81

1008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 3244032
0*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*
a^11*b^2*c^14)))^(3/4)*1i + (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*
a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c
^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c
^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b
*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^
5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c
^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a
c - b^2)^15)^(1/2))/(8192(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 +
 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*
b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8
*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*
c^14)))^(1/4))/((((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4
*b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*
b^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 163
84*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504
*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*((b^4*(-(4*a*c
 - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b
^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 1
0665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15
)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(1677721
6*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18
*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 1
2976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 692
06016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11
 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 332
922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 2650
800128*a^9*b^2*c^10)*1i)/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*
a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-
(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 275
2*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*
c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c -
b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(
16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a
^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*
c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^1
2 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/4)*1i - (x^(1/2)*
(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 1771
2*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c
^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c -
b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^1
3*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 106
65984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^
(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*

a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c
^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 129
76128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206
016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*1i - (279936*a^8*c^5 +
343*a^4*b^8*c + 7350*a^5*b^6*c^2 + 58968*a^6*b^4*c^3 + 209952*a^7*b^2*c^4)/
(64*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*
b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (((543581
7984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 - 32833536*a^5*
b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*b^7*c^7 + 5121245184*a^8*
b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10
*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*
b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 -
 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*
c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 178
91328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27
*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 -
48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^
7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 3
2440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 503
31648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11 - 204800*a^3*b^14*c^4 +
5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163
919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10)*1i)/
(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^
4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) -
 b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^
4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^
7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17
*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24
*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*
b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c
^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^1
3 - 50331648*a^11*b^2*c^14)))^(3/4)*1i + (x^(1/2)*(49*a^3*b^9*c + 15552*a^7
*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12
 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6
144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 1
2386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^
4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891
328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a
*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48
*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7
 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 324
40320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331
648*a^11*b^2*c^14)))^(1/4)*1i))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12
386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4
 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 178913

28*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*
b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*
a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 -
811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 3244
0320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 503316
48*a^11*b^2*c^14)))^(1/4) - atan(((((5435817984*a^10*b*c^10 - 4096*a^3*b^15
*c^3 + 1425408*a^4*b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6
- 1714421760*a^7*b^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9
)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a
^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/
2)*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^
15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350
528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2
*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1
/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c
^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 37847
04*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680
*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1
207959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888
*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280
*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2
*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^
10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a
^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 -
3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^
2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^1
5)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b
^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 +
3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 576
71680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/
4) + (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*
b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 -
1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(
b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2
+ 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^
6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*
a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(
8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 1
4080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6
*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b
^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*1i - (((
5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 - 3283353
6*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*b^7*c^7 + 512124518
4*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 16384*a^7*c^7 + 336*a^
2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 2867

2*a^6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/4) - (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*1i)/((279936*a^8*c^5 + 343*a^4*b^8*c + 7350*a^5*b^6*c^2 + 58968*a^6*b^4*c^3 + 209952*a^7*b^2*c^4)/(64*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*b^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c

^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/4) + (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4) + (((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*b^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2

) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12
*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 +
 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 1297612
8*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*
a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/4) - (x^(1/2)*(49*a^3*b^9*c +
15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/
(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^
4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)
^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a
^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c
^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^1
7*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^2
4*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4
*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*
c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^
13 - 50331648*a^11*b^2*c^14)))^(1/4))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1
/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*
b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7
- 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c
 + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c
^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^
16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^1
0 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13
- 50331648*a^11*b^2*c^14)))^(1/4)*2i - 2*atan(((((5435817984*a^10*b*c^10 -
4096*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 - 32833536*a^5*b^11*c^5 + 32374784
0*a^6*b^9*c^6 - 1714421760*a^7*b^7*c^7 + 5121245184*a^8*b^5*c^8 - 817050419
2*a^9*b^3*c^9)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8
*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12
c)) - (x^(1/2)(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^
9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b
^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8
+ 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c
- b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 10
56*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^1
4*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^
11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^1
4)))^(1/4)*(1207959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c
^5 - 56229888*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8
 + 2390753280*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10)*1i)/(16*(b^12 + 4096*a
^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b
^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*
a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 5852
16*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*
b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(
-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*

c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008
*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^
8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11
*b^2*c^14)))^(3/4)*1i - (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*
b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 +
 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 -
 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^
9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b
^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8
+ 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c
- b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 10
56*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^1
4*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^
11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^1
4)))^(1/4) - (((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^
13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*b^7
*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 16384*
a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^
5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*(-(b^19 + b^4*(-(4
*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^1
3*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 106
65984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^
(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*
a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c
^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 129
76128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206
016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11 -
 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 33292
2880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 265080
0128*a^9*b^2*c^10)*1i)/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^
3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 +
b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752
*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c
^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b
^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(1
6777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^
3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c
^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12
 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/4)*1i + (x^(1/2)*(
49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712
*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^
3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4
*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^1
3*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 106
65984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^

(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4))/((((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*b^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10)*1i)/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/4)*1i - (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*1i - (279936*a^8*c^5 + 343*a^4*b^8*c + 7350*a^5*b^6*c^2 + 58968*a^6*b^4*c^3 + 209952*a^7*b^2*c^4)/(64*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 -

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 1714421760*a^7*b^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/
(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4
*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)
*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15
*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 335052
8*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(
-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2
))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5
 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704
*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a
^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(120
7959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a
^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a
^8*b^4*c^9 - 2650800128*a^9*b^2*c^10)*1i)/(16*(b^12 + 4096*a^6*c^6 + 240*a^
2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b
^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*
a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5
- 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a
^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^
15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*
b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 +
 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57
671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3
/4)*1i + (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*
a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^
2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))
*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15
*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 335052
8*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(
-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2
))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5
 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704
*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a
^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*1i))
*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15
*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 335052
8*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(
-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2
))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5
 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704
*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a
^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.1074 $\int \frac{x^{7/2}}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=483

$$\frac{\sqrt{x}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(3b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(-3b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2}\right.}$$

```
[Out] 1/8*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(3*b^2+4*
a*c-3*b*(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c+
b^2)^(1/2))^(3/4)+1/8*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2
))^(1/4))*(3*b^2+4*a*c-3*b*(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)
^(3/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-1/8*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b
-(-4*a*c+b^2)^(1/2))^(1/4))*(3*b^2+4*a*c+3*b*(-4*a*c+b^2)^(1/2))*2^(3/4)/c^
(1/4)/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-1/8*arctanh(2^(1/4)*
c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(3*b^2+4*a*c+3*b*(-4*a*c+b^2
)^(1/2))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1
/2*(b*x^2+2*a)*x^(1/2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)
```

Rubi [A] time = 1.03, antiderivative size = 483, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1115, 1365, 1422, 212, 208, 205}

$$\frac{\sqrt{x}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(3b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(-3b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (Sqrt[x]*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^2 + 4
*a*c + 3*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b
^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2
- 4*a*c])^(3/4)) + ((3*b^2 + 4*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)
*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(1/4)*(b^2
- 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - ((3*b^2 + 4*a*c + 3*b*Sqrt
[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(
1/4)])/(4*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4
)) + ((3*b^2 + 4*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt
```

```
[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(3/2
)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^8}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{2a-3bx^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2\left(b^2-4ac\right)}$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(3b^2+4ac-3b\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx,\right.}{4\left(b^2-4ac\right)^{3/2}}$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(3b^2+4ac-3b\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}}\right.}{4\left(b^2-4ac\right)^{3/2}\sqrt{-b+\sqrt{b^2-4ac}}}$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(3b^2+4ac+3b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{(3b}{4}$$

Mathematica [C] time = 0.21, size = 127, normalized size = 0.26

$$\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{3\#1^4b\log\left(\sqrt{x}-\#1\right)-2a\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]}{8\left(b^2-4ac\right)} - \frac{-2a\sqrt{x}-bx^{5/2}}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(-2*a*Sqrt[x] - b*x^(5/2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + RootS
um[a + b*#1^4 + c*#1^8 & , (-2*a*Log[Sqrt[x] - #1] + 3*b*Log[Sqrt[x] - #1]*
#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(8*(b^2 - 4*a*c))
```

fricas [B] time = 8.62, size = 9245, normalized size = 19.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqr
t(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^
10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b
^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2
 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^
6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a
^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3
- 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*
arctan(-1/2*(sqrt(1/2)*(2187*b^15 - 47412*a*b^13*c + 423536*a^2*b^11*c^2 -
1990720*a^3*b^9*c^3 + 5177600*a^4*b^7*c^4 - 7052288*a^5*b^5*c^5 + 3985408*a
^6*b^3*c^6 - 180224*a^7*b*c^7 - (27*b^22*c - 820*a*b^20*c^2 + 10064*a^2*b^1
8*c^3 - 57024*a^3*b^16*c^4 + 44544*a^4*b^14*c^5 + 1505280*a^5*b^12*c^6 - 10
838016*a^6*b^10*c^7 + 38436864*a^7*b^8*c^8 - 79233024*a^8*b^6*c^9 + 9201254
4*a^9*b^4*c^10 - 49283072*a^10*b^2*c^11 + 4194304*a^11*c^12)*sqrt((6561*b^4
 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 -
 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b
^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt
((1476225*b^8 + 641520*a*b^6*c + 30816*a^2*b^4*c^2 - 8448*a^3*b^2*c^3 + 256
*a^4*c^4)*x + sqrt(1/2)*(111537*b^12 - 1375704*a*b^10*c + 5803760*a^2*b^8*c
^2 - 8961280*a^3*b^6*c^3 + 2522880*a^4*b^4*c^4 - 186368*a^5*b^2*c^5 + 4096*
a^6*c^6 + 8*(81*b^19*c - 2596*a*b^17*c^2 + 36416*a^2*b^15*c^3 - 292096*a^3*
b^13*c^4 + 1465856*a^4*b^11*c^5 - 4716544*a^5*b^9*c^6 + 9519104*a^6*b^7*c^7
 - 11075584*a^7*b^5*c^8 + 5832704*a^8*b^3*c^9 - 262144*a^9*b*c^10)*sqrt((65
61*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14
*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064
*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11))
)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 2
40*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4
096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^
16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024
*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^1
0 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3
*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*sqrt(-(81*
b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c
^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)
*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576
*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7
 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a
^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3
840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)) + sqrt(1/2)*(2657205*b^
19 - 57028212*a*b^17*c + 502044480*a^2*b^15*c^2 - 2306152704*a^3*b^13*c^3 +
 5758457344*a^4*b^11*c^4 - 7169792000*a^5*b^9*c^5 + 2897625088*a^6*b^7*c^6
+ 946012160*a^7*b^5*c^7 - 111345664*a^8*b^3*c^8 + 2883584*a^9*b*c^9 - (3280
```

5*b^26*c - 989172*a*b^24*c^2 + 12010848*a^2*b^22*c^3 - 66614144*a^3*b^20*c^4 + 38905600*a^4*b^18*c^5 + 1841587200*a^5*b^16*c^6 - 12771508224*a^6*b^14*c^7 + 43815469056*a^7*b^12*c^8 - 85947383808*a^8*b^10*c^9 + 90262732800*a^9*b^8*c^10 - 34319892480*a^10*b^6*c^11 - 9386852352*a^11*b^4*c^12 + 1895825408*a^12*b^2*c^13 - 67108864*a^13*c^14)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(x)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))/(332150625*a*b^12 + 321489000*a^2*b^10*c + 107535600*a^3*b^8*c^2 + 12061440*a^4*b^6*c^3 - 463104*a^5*b^4*c^4 - 104448*a^6*b^2*c^5 + 4096*a^7*c^6)) - 4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*arctan(1/2*(sqrt(1/2)*(2187*b^15 - 47412*a*b^13*c + 423536*a^2*b^11*c^2 - 1990720*a^3*b^9*c^3 + 5177600*a^4*b^7*c^4 - 7052288*a^5*b^5*c^5 + 3985408*a^6*b^3*c^6 - 180224*a^7*b*c^7 + (27*b^22*c - 820*a*b^20*c^2 + 10064*a^2*b^18*c^3 - 57024*a^3*b^16*c^4 + 44544*a^4*b^14*c^5 + 1505280*a^5*b^12*c^6 - 10838016*a^6*b^10*c^7 + 38436864*a^7*b^8*c^8 - 79233024*a^8*b^6*c^9 + 92012544*a^9*b^4*c^10 - 49283072*a^10*b^2*c^11 + 4194304*a^11*c^12)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt((1476225*b^8 + 641520*a*b^6*c + 30816*a^2*b^4*c^2 - 8448*a^3*b^2*c^3 + 256*a^4*c^4)*x + sqrt(1/2)*(111537*b^12 - 1375704*a*b^10*c + 5803760*a^2*b^8*c^2 - 8961280*a^3*b^6*c^3 + 2522880*a^4*b^4*c^4 - 186368*a^5*b^2*c^5 + 4096*a^6*c^6 - 8*(81*b^19*c - 2596*a*b^17*c^2 + 36416*a^2*b^15*c^3 - 292096*a^3*b^13*c^4 + 1465856*a^4*b^11*c^5 - 4

716544*a^5*b^9*c^6 + 9519104*a^6*b^7*c^7 - 11075584*a^7*b^5*c^8 + 5832704*a
^8*b^3*c^9 - 262144*a^9*b*c^10)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/
(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^
4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 +
 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(-(81*b^5 + 760*a*b^3*c - 240
*a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 +
 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a
*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^
3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 -
 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24
*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*
a^5*b^2*c^6 + 4096*a^6*c^7)))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c -
240*a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^
4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 64
8*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376
*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^
8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c -
 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 61
44*a^5*b^2*c^6 + 4096*a^6*c^7)))*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^
2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4
*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c +
16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^
5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a
^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c
^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c
^6 + 4096*a^6*c^7)) + sqrt(1/2)*(2657205*b^19 - 57028212*a*b^17*c + 5020444
80*a^2*b^15*c^2 - 2306152704*a^3*b^13*c^3 + 5758457344*a^4*b^11*c^4 - 71697
92000*a^5*b^9*c^5 + 2897625088*a^6*b^7*c^6 + 946012160*a^7*b^5*c^7 - 111345
664*a^8*b^3*c^8 + 2883584*a^9*b*c^9 + (32805*b^26*c - 989172*a*b^24*c^2 + 1
2010848*a^2*b^22*c^3 - 66614144*a^3*b^20*c^4 + 38905600*a^4*b^18*c^5 + 1841
587200*a^5*b^16*c^6 - 12771508224*a^6*b^14*c^7 + 43815469056*a^7*b^12*c^8 -
 85947383808*a^8*b^10*c^9 + 90262732800*a^9*b^8*c^10 - 34319892480*a^10*b^6
*c^11 - 9386852352*a^11*b^4*c^12 + 1895825408*a^12*b^2*c^13 - 67108864*a^13
*c^14)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3
 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b
^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 26
2144*a^9*c^11)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a
^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3
840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b
^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*
b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 5
89824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a
*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^
5*b^2*c^6 + 4096*a^6*c^7)))*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (
b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*

c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))/(332150625*a*b^12 + 321489000*a^2*b^10*c + 107535600*a^3*b^8*c^2 + 12061440*a^4*b^6*c^3 - 463104*a^5*b^4*c^4 - 104448*a^6*b^2*c^5 + 4096*a^7*c^6)) + ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*log(-(1215*b^4 + 264*a*b^2*c - 16*a^2*c^2)*sqrt(x) + (81*b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 + 4*(b^13*c - 24*a*b^11*c^2 + 240*a^2*b^9*c^3 - 1280*a^3*b^7*c^4 + 3840*a^4*b^5*c^5 - 6144*a^5*b^3*c^6 + 4096*a^6*b*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))) - ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*log(-(1215*b^4 + 264*a*b^2*c - 16*a^2*c^2)*sqrt(x) - (81*b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 + 4*(b^13*c - 24*a*b^11*c^2 + 240*a^2*b^9*c^3 - 1280*a^3*b^7*c^4 + 3840*a^4*b^5*c^5 - 6144*a^5*b^3*c^6 + 4096*a^6*b*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 6

48*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 537
6*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c
^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c
- 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6
144*a^5*b^2*c^6 + 4096*a^6*c^7)))) + ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2
*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*
a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 +
3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*
b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3
*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 -
589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*
a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a
^5*b^2*c^6 + 4096*a^6*c^7)))*log(-(1215*b^4 + 264*a*b^2*c - 16*a^2*c^2)*sqr
t(x) + (81*b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 - 4*(b^13*c -
24*a*b^11*c^2 + 240*a^2*b^9*c^3 - 1280*a^3*b^7*c^4 + 3840*a^4*b^5*c^5 - 614
4*a^5*b^3*c^6 + 4096*a^6*b*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/
(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^
4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 +
589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 76
0*a*b^3*c - 240*a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 128
0*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6
561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^1
4*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 34406
4*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)
))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*
b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))) - ((b^2*c - 4*a*c^2)*x^4 + a*
b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b
^3*c - 240*a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3
*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b
^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4
- 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6
*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b
^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c
^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*log(-(1215*b^4 + 264*a*b^2*c - 16*a
^2*c^2)*sqrt(x) - (81*b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 - 4
*(b^13*c - 24*a*b^11*c^2 + 240*a^2*b^9*c^3 - 1280*a^3*b^7*c^4 + 3840*a^4*b^
5*c^5 - 6144*a^5*b^3*c^6 + 4096*a^6*b*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 1
6*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5
+ 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^
7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(sqrt(1/2)*sqrt(-(
81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^
8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c
^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 +
576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*
c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 26214

```
4*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
+ 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))) - 4*(b*x^2 + 2*a)*
sqrt(x))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 47.2Unable to convert to re
al 1/4 Error: Bad Argument Value
```

maple [C] time = 0.02, size = 118, normalized size = 0.24

$$\frac{\left(-3\,\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{4}b+2a\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)+\sqrt{x}\right)}{8\left(4ac-b^{2}\right)\left(2\,\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{7}c+\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{3}b\right)}+\frac{-\frac{b\,x^{\frac{5}{2}}}{2\left(4ac-b^{2}\right)}-\frac{a\sqrt{x}}{4ac-b^{2}}}{c\,x^{4}+b\,x^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*b/(4*a*c-b^2)*x^(5/2)-1/2*a/(4*a*c-b^2)*x^(1/2))/(c*x^4+b*x^2+a)+1/
8/(4*a*c-b^2)*sum((-3*_R^4*b+2*a)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=Root
Of(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,cx^{\frac{9}{2}}+bx^{\frac{5}{2}}}{2\left(\left(b^{2}c-4\,ac^{2}\right)x^{4}+ab^{2}-4\,a^{2}c+\left(b^{3}-4\,abc\right)x^{2}\right)}-\int-\frac{2\,cx^{\frac{7}{2}}+5\,bx^{\frac{3}{2}}}{4\left(\left(b^{2}c-4\,ac^{2}\right)x^{4}+ab^{2}-4\,a^{2}c+\left(b^{3}-4\,abc\right)x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^(9/2) + b*x^(5/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (
b^3 - 4*a*b*c)*x^2) - integrate(-1/4*(2*c*x^(7/2) + 5*b*x^(3/2))/((b^2*c -
4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2), x)
```

mupad [B] time = 10.88, size = 26432, normalized size = 54.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] atan((((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*a^2*b^15*c^4 + 2605056*a^3
*b^13*c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5*b^9*c^7 - 581959680*a^6*b
^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^10))/(16*(b^12 + 409
6*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^
5*b^2*c^5 - 24*a*b^10*c)) - ((-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2)
- 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9
*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 11
84*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^1
2*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^
4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10
*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^1
1 - 50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^10 + 20480*a^2*b^13*c
^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 26214400*a^5*b^7*c^7 + 786
43200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*
b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^
2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 7
19360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7
*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c +
 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^
4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 1297
6128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016
*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/4) - (405*a^2*b^6*c^3 - 32*a^
5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^
4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)
^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719
360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b
^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 1
6777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4
+ 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 129761
28*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a
^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4) - (x^(1/2)*(128*a^6*c^7 + 20
25*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*a^4*b^4*c^5 + 864*a^5*b^2*c^6))/(16
*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c
^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^
15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 7193
60*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^
3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16
777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 +
 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 1297612
8*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^
10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*1i + ((((x^(1/2)*(603979776*a
```

^9*b*c^11 - 102400*a^2*b^15*c^4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^11*
c^6 + 166461440*a^5*b^9*c^7 - 581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*c^
9 - 1325400064*a^8*b^3*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 -
1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((
-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b
^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5
259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c -
b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*
a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c
^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 -
57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^
(1/4)*(83886080*a^8*b*c^10 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 491
5200*a^4*b^9*c^6 - 26214400*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*
a^7*b^3*c^9))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16
*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^
8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^
5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a
c(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^2
2*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 8110
08*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a
^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11
*b^2*c^12)))^(3/4) + (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 + 96*a
^4*b^2*c^5)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a
*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8
+ 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*
b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c
*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*
c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008
*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8
*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b
^2*c^12)))^(1/4) - (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*a^3*b^6*c
^4 + 1224*a^4*b^4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6 + 240*a^
2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b
^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 +
960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b
^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*
(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c
^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*
a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*
b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^
2*c^12)))^(1/4)*1i)/(((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*a^2*b^15*c^
4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5*b^9*c^7 -
581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^10))/
(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^
4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((-(81*b^17 - 81*b^2*(-(4*a*c -

b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^10 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 26214400*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/4) - (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4) - (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*a^4*b^4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4) - ((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*a^2*b^15*c^4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5*b^9*c^7 - 581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 81100

8*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^10 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 26214400*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/4) + (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4) - (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*a^4*b^4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*2i - ((a*x^(1/2))/(4*a*c - b^2) + (b*x^(5/2))/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + atan((((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*a^2*b^15*c^4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5*b^9*c^7 - 581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c))
- ((-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a

^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^10 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 26214400*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/4) - (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4) - (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*a^4*b^4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*1i + ((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*a^2*b^15*c^4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5*b^9*c^7 - 581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7

- 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69
206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^10
 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 2621440
0*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9))/(2*(b^8 + 25
6*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 + 8
1*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 844
80*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^
5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/
2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3
- 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*
a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*
b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/4) + (405*
a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8 + 256*
a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 + 81*
b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480
*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*
c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2)
)/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 -
14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^
6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^
6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4) - (x^(1/2
)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*a^4*b^4*c^5 + 86
4*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c
^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b
^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*
a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c
^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))
/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 1
4080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6
*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6
*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*1i)/(((((x
^(1/2)*(603979776*a^9*b*c^11 - 102400*a^2*b^15*c^4 + 2605056*a^3*b^13*c^5 -
 28114944*a^4*b^11*c^6 + 166461440*a^5*b^9*c^7 - 581959680*a^6*b^7*c^8 + 11
95376640*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^10))/(16*(b^12 + 4096*a^6*c^6 +
 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 -
 24*a*b^10*c)) - ((-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^
8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727
936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c
 - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48
*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5
- 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 3244
0320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 5033164
8*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^10 + 20480*a^2*b^13*c^4 - 491520
*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 26214400*a^5*b^7*c^7 + 78643200*a^6*b
^5*c^8 - 125829120*a^7*b^3*c^9))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 2

56*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2
) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b
^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 -
1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a
^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*
a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^
10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c
^11 - 50331648*a^11*b^2*c^12)))^(3/4) - (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918
*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256
*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2)
- 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9
*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 11
84*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^1
2*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^
4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10
*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^1
1 - 50331648*a^11*b^2*c^12)))^(1/4) - (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*
c^3 - 270*a^3*b^6*c^4 + 1224*a^4*b^4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 40
96*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a
^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) -
983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*
c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 118
4*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12
*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4
*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*
c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11
- 50331648*a^11*b^2*c^12)))^(1/4) - ((((x^(1/2)*(603979776*a^9*b*c^11 - 10
2400*a^2*b^15*c^4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^11*c^6 + 16646144
0*a^5*b^9*c^7 - 581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064
*a^8*b^3*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c
^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((-(81*b^17 + 81
b^2(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 8448
0*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5
*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2
))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 -
14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a
^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b
^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*(8388608
0*a^8*b*c^10 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c
^6 - 26214400*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9))/
(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-
(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^
13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 52
59264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c -
b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a

^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^
6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 5
7671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(
3/4) + (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2
*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(8
1*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13
*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259
264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^
2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2
*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6
+ 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 576
71680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/
4) - (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*a^4*
b^4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 12
80*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81
*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*
c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 52592
64*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2
)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*
b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 +
 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 5767
1680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4
)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*
a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^
5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*
a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 +
1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b
^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c
^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^1
2)))^(1/4)*2i + 2*atan((((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*a^2*b^15
*c^4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5*b^9*c^7
 - 581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^10
))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4
*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((-(81*b^17 - 81*b^2*(-(4*a*c
 - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^
3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 458752
0*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^2
4*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^
18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 -
 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 692
06016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^10
+ 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 26214400
*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9)*1i)/(2*(b^8 +
256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 -
 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 8

4480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*
b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(
1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^
3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 378470
4*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^
9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/4)*1i +
(405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8 +
256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17
- 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 +
84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6
*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^
(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c
^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 37847
04*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a
^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*1i +
(x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*a^4*b^4*
c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a
^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^1
7 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2
+ 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a
^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15
)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20
*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 378
4704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680
*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4) +
((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*a^2*b^15*c^4 + 2605056*a^3*b^13*
c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5*b^9*c^7 - 581959680*a^6*b^7*c^8
+ 1195376640*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^10))/(16*(b^12 + 4096*a^6*
c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*
c^5 - 24*a*b^10*c)) + ((-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9830
40*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 +
2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b
^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13
- 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16
*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 +
32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50
331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^10 + 20480*a^2*b^13*c^4 - 4
91520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 26214400*a^5*b^7*c^7 + 78643200*
a^6*b^5*c^8 - 125829120*a^7*b^3*c^9)*1i)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4
*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^
15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 7193
60*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^
3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16
777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 +
126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 1297612

8*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/4)*1i - (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*1i + (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*a^4*b^4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4))/(((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*a^2*b^15*c^4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5*b^9*c^7 - 581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^10 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 26214400*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9)*1i)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/4)*1i + (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*

b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 272793
6*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c +
 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a
*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 -
811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 324403
20*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*
a^11*b^2*c^12)))^(1/4)*1i + (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*
a^3*b^6*c^4 + 1224*a^4*b^4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6
 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5
 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^
8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727
936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c
 + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48
*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5
 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 3244
0320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 5033164
8*a^11*b^2*c^12)))^(1/4)*1i - ((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*a^
2*b^15*c^4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5*b
^9*c^7 - 581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*b^
3*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 38
40*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((-(81*b^17 - 81*b^2*(-
(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b
^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 +
4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(819
2*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*
a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12
*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10
 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b
*c^10 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 26
214400*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9)*1i)/(2*(
b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*
b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c
^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 525926
4*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)
^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b
^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 +
3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671
680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/4)
*1i - (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*
(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81
*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*
c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 52592
64*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2
)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*
b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 +

3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 5767
1680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4
)*1i + (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*a^
4*b^4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 -
1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(
81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^1
3*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 525
9264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b
^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^
2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6
 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57
671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1
/4)*1i))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 +
 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b
^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*
(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c
^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*
a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*
b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^
2*c^12)))^(1/4) + 2*atan((((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*a^2*b^
15*c^4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5*b^9*c
^7 - 581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^
10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a
^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((-(81*b^17 + 81*b^2*(-(4*a
*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*
c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587
520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b
^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*
b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7
 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 6
9206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^1
0 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 262144
00*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9)*1i)/(2*(b^8
+ 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17
 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 +
 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^
6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)
^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*
c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784
704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*
a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/4)*1i
+ (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8
 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^1
7 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2
+ 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a

^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15
)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20
*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 378
4704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680
*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*1i
 + (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*a^4*b^
4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280
*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b
^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^
2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264
*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^
15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^
20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3
784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 576716
80*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)
 + ((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*a^2*b^15*c^4 + 2605056*a^3*b^1
3*c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5*b^9*c^7 - 581959680*a^6*b^7*c
^8 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^10))/(16*(b^12 + 4096*a^
6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^
2*c^5 - 24*a*b^10*c)) + ((-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 98
3040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4
 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a
*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^
13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^
16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8
 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 -
50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^10 + 20480*a^2*b^13*c^4 -
 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 26214400*a^5*b^7*c^7 + 7864320
0*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9)*1i)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b
^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2
)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 71
9360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*
b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c +
16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4
 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976
128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*
a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/4)*1i - (405*a^2*b^6*c^3 - 32*
a^5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*
b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^
2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 7
19360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7
*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c +
 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^
4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 1297
6128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016

*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*1i + (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*a^4*b^4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4))/(((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*a^2*b^15*c^4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5*b^9*c^7 - 581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^10 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 26214400*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9)*1i)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/4)*1i + (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*1i + (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*a^4*b^4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*

a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 27
27936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15
*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 -
48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^
5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32
440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331
648*a^11*b^2*c^12)))^(1/4)*1i - ((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*
a^2*b^15*c^4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5
*b^9*c^7 - 581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*
b^3*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 +
3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((-(81*b^17 + 81*b^2*
(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3
*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6
+ 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8
192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 1408
0*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^
12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^
10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8
*b*c^10 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 -
26214400*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9)*1i)/(2
*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(8
1*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13
*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259
264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^
2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2
*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6
+ 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 576
71680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/
4)*1i - (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(
2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(
81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^1
3*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 525
9264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b
^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^
2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6
 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57
671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1
/4)*1i + (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*
a^4*b^4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2
- 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(
-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b
^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5
259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c -
 b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*
a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c

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^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 -
57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^
(1/4)*1i))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8
 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5
*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*
c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22
*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 81100
8*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^
8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*
b^2*c^12)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.1075 $\int \frac{x^{5/2}}{\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=450

$$-\frac{x^{3/2}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt[4]{c}\left(\sqrt{b^2-4ac}+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(4b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{\cdots}}{\sqrt[4]{\sqrt{b^2\cdots}}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}\cdots}}$$

```
[Out] -1/2*x^(3/2)*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*c^(1/4)*arctan(2^
(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(4*b-(-4*a*c+b^2)^(1/2
))*2^(1/4)/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-1/4*c^(1/4)*arc
tanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(4*b-(-4*a*c+b^
2)^(1/2))*2^(1/4)/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-1/4*c^(1
/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(4*b+(-4*
a*c+b^2)^(1/2))*2^(1/4)/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/
4*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(4
*b+(-4*a*c+b^2)^(1/2))*2^(1/4)/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(
1/4)
```

Rubi [A] time = 0.71, antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1115, 1364, 1510, 298, 205, 208}

$$-\frac{x^{3/2}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt[4]{c}\left(\sqrt{b^2-4ac}+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(4b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{\cdots}}{\sqrt[4]{\sqrt{b^2\cdots}}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(x^(3/2)*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (c^(1/4)*(
4*b + Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 -
4*a*c])^(1/4)])/(2*2^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/
4)) + (c^(1/4)*(4*b - Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b
^2 - 4*a*c])^(1/4)) + (c^(1/4)*(4*b + Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c
^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*(b^2 - 4*a*c)^(
3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(4*b - Sqrt[b^2 - 4*a*c])*A
rcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4
)*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1364

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(n - 1)*(d*x)^(m - n + 1)*(b + 2*c*x^n)*(a + b*x^n + c*
x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] - Dist[d^n/(n*(p + 1)*(b^2
- 4*a*c)), Int[(d*x)^(m - n)*(b*(m - n + 1) + 2*c*(m + 2*n*(p + 1) + 1)*x^n
)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] && GtQ[m, n -
1] && LeQ[m, 2*n - 1]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(3b-2cx^4\right)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2\left(b^2-4ac\right)}$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(4b-\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x,\right.}{2\left(b^2-4ac\right)^{3/2}}$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(\sqrt{c}\left(4b-\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}}\right.}{2\sqrt{2}\left(b^2-4ac\right)^{3/2}}$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\sqrt[4]{c}\left(4b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(4b\right.}{2\,2^{3}}$$

Mathematica [C] time = 0.21, size = 109, normalized size = 0.24

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{2\#1^4c\log\left(\sqrt{x}-\#1\right)-3b\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]+\frac{4x^{3/2}\left(b+2cx^2\right)}{a+bx^2+cx^4}}{8\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/8*((4*x^(3/2)*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + RootSum[a + b*#1^4 +
c*#1^8 & , (-3*b*Log[Sqrt[x] - #1] + 2*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*
c*#1^5) & ])/(b^2 - 4*a*c)
```

fricas [B] time = 22.26, size = 9757, normalized size = 21.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

[Out] -1/8*(4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqr
t(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*
b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b
^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18
 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^
4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a
^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2
- 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*
arctan(-((81*b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 - 4*(a*b^13
- 24*a^2*b^11*c + 240*a^3*b^9*c^2 - 1280*a^4*b^7*c^3 + 3840*a^5*b^5*c^4 - 6
144*a^6*b^3*c^5 + 4096*a^7*b*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2
)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*
a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7
 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt((74733890625*b^16*c^2 + 11
2193100000*a*b^14*c^3 + 68088600000*a^2*b^12*c^4 + 20761920000*a^3*b^10*c^5
 + 3063744000*a^4*b^8*c^6 + 113909760*a^5*b^6*c^7 - 19021824*a^6*b^4*c^8 -
1179648*a^7*b^2*c^9 + 65536*a^8*c^10)*x - 1/2*sqrt(1/2)*(2989355625*b^21*c
- 23678649000*a*b^19*c^2 + 7135160400*a^2*b^17*c^3 + 277460328960*a^3*b^15*
c^4 - 338956033536*a^4*b^13*c^5 - 492326940672*a^5*b^11*c^6 - 183476674560*
a^6*b^9*c^7 - 21980119040*a^7*b^7*c^8 + 750059520*a^8*b^5*c^9 + 190316544*a
^9*b^3*c^10 - 7340032*a^10*b*c^11 + (36905625*a*b^28*c - 1159839000*a^2*b^2
6*c^2 + 15854324400*a^3*b^24*c^3 - 122710429440*a^4*b^22*c^4 + 584418357504
*a^5*b^20*c^5 - 1728949905408*a^6*b^18*c^6 + 2983008514048*a^7*b^16*c^7 - 2
317983285248*a^8*b^14*c^8 - 462348419072*a^9*b^12*c^9 + 1339972648960*a^10*
b^10*c^10 + 254402363392*a^11*b^8*c^11 - 161849802752*a^12*b^6*c^12 - 51220
840448*a^13*b^4*c^13 - 2550136832*a^14*b^2*c^14 + 268435456*a^15*c^15)*sqrt
((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*
b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 34
4064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c
^9)))*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c
 + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5
 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*
a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 12
9024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^
2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280
*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6))) + (221
43375*b^14*c - 161619300*a*b^12*c^2 + 233100720*a^2*b^10*c^3 + 224213184*a^
3*b^8*c^4 + 48450816*a^4*b^6*c^5 + 185344*a^5*b^4*c^6 - 487424*a^6*b^2*c^7
+ 16384*a^7*c^8 - 4*(273375*a*b^21*c - 6355800*a^2*b^19*c^2 + 60732720*a^3*
b^17*c^3 - 301810176*a^4*b^15*c^4 + 798453248*a^5*b^13*c^5 - 951914496*a^6*
b^11*c^6 + 38461440*a^7*b^9*c^7 + 557711360*a^8*b^7*c^8 + 179503104*a^9*b^5
*c^9 + 11010048*a^10*b^3*c^10 - 1048576*a^11*b*c^11)*sqrt((6561*b^4 - 648*a
*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^
5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 -
 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(x))*sqr

t(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))/(332150625*b^12*c + 321489000*a*b^10*c^2 + 107535600*a^2*b^8*c^3 + 12061440*a^3*b^6*c^4 - 463104*a^4*b^4*c^5 - 104448*a^5*b^2*c^6 + 4096*a^6*c^7)) - 4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*arctan(((81*b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 + 4*(a*b^13 - 24*a^2*b^11*c + 240*a^3*b^9*c^2 - 1280*a^4*b^7*c^3 + 3840*a^5*b^5*c^4 - 6144*a^6*b^3*c^5 + 4096*a^7*b*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt((74733890625*b^16*c^2 + 112193100000*a*b^14*c^3 + 68088600000*a^2*b^12*c^4 + 20761920000*a^3*b^10*c^5 + 3063744000*a^4*b^8*c^6 + 113909760*a^5*b^6*c^7 - 19021824*a^6*b^4*c^8 - 1179648*a^7*b^2*c^9 + 65536*a^8*c^10)*x - 1/2*sqrt(1/2)*(2989355625*b^21*c - 23678649000*a*b^19*c^2 + 7135160400*a^2*b^17*c^3 + 277460328960*a^3*b^15*c^4 - 338956033536*a^4*b^13*c^5 - 492326940672*a^5*b^11*c^6 - 183476674560*a^6*b^9*c^7 - 21980119040*a^7*b^7*c^8 + 750059520*a^8*b^5*c^9 + 190316544*a^9*b^3*c^10 - 7340032*a^10*b*c^11 - (36905625*a*b^28*c - 1159839000*a^2*b^26*c^2 + 15854324400*a^3*b^24*c^3 - 122710429440*a^4*b^22*c^4 + 584418357504*a^5*b^20*c^5 - 1728949905408*a^6*b^18*c^6 + 2983008514048*a^7*b^16*c^7 - 2317983285248*a^8*b^14*c^8 - 462348419072*a^9*b^12*c^9 + 1339972648960*a^10*b^10*c^10 + 254402363392*a^11*b^8*c^11 - 161849802752*a^12*b^6*c^12 - 51220840448*a^13*b^4*c^13 - 2550136832*a^14*b^2*c^14 + 268435456*a^15*c^15)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*sqrt(sqrt(1/2)

*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 24
0*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 40
96*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b
^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*
a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8
 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*
b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6))) + (22143375
*b^14*c - 161619300*a*b^12*c^2 + 233100720*a^2*b^10*c^3 + 224213184*a^3*b^8
*c^4 + 48450816*a^4*b^6*c^5 + 185344*a^5*b^4*c^6 - 487424*a^6*b^2*c^7 + 163
84*a^7*c^8 + 4*(273375*a*b^21*c - 6355800*a^2*b^19*c^2 + 60732720*a^3*b^17*
c^3 - 301810176*a^4*b^15*c^4 + 798453248*a^5*b^13*c^5 - 951914496*a^6*b^11*
c^6 + 38461440*a^7*b^9*c^7 + 557711360*a^8*b^7*c^8 + 179503104*a^9*b^5*c^9
+ 11010048*a^10*b^3*c^10 - 1048576*a^11*b*c^11)*sqrt((6561*b^4 - 648*a*b^2*
c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^1
2*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 5898
24*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(x)*sqrt(sqrt
(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c
 + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5
 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*
a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 12
9024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^
2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280
*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6))))/(3321
50625*b^12*c + 321489000*a*b^10*c^2 + 107535600*a^2*b^8*c^3 + 12061440*a^3*
b^6*c^4 - 463104*a^4*b^4*c^5 - 104448*a^5*b^2*c^6 + 4096*a^6*c^7)) - ((b^2*
c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sq
rt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a
^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*
a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16
*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7
*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 -
262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6
*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*log(1/2*sqrt(1
/2)*(2187*b^15 - 47412*a*b^13*c + 423536*a^2*b^11*c^2 - 1990720*a^3*b^9*c^3
 + 5177600*a^4*b^7*c^4 - 7052288*a^5*b^5*c^5 + 3985408*a^6*b^3*c^6 - 180224
*a^7*b*c^7 - (27*a*b^22 - 820*a^2*b^20*c + 10064*a^3*b^18*c^2 - 57024*a^4*b
^16*c^3 + 44544*a^5*b^14*c^4 + 1505280*a^6*b^12*c^5 - 10838016*a^7*b^10*c^6
 + 38436864*a^8*b^8*c^7 - 79233024*a^9*b^6*c^8 + 92012544*a^10*b^4*c^9 - 49
283072*a^11*b^2*c^10 + 4194304*a^12*c^11)*sqrt((6561*b^4 - 648*a*b^2*c + 16
*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3
+ 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9
*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(sqrt(1/2)*sqrt(-(8
1*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8
*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^
6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 5

76*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c
^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144
*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 +
3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*sqrt(-(81*b^5 + 760*
a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*
a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((656
1*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*
c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*
a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))
/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^
4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)) - (273375*b^8*c + 205200*a*b^6*c^
2 + 47520*a^2*b^4*c^3 + 2304*a^3*b^2*c^4 - 256*a^4*c^5)*sqrt(x)) + ((b^2*c
- 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt
(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a^3
*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^
7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c
+ 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b
^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 26
2144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c
^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*log(-1/2*sqrt(1/
2)*(2187*b^15 - 47412*a*b^13*c + 423536*a^2*b^11*c^2 - 1990720*a^3*b^9*c^3
+ 5177600*a^4*b^7*c^4 - 7052288*a^5*b^5*c^5 + 3985408*a^6*b^3*c^6 - 180224*
a^7*b*c^7 - (27*a*b^22 - 820*a^2*b^20*c + 10064*a^3*b^18*c^2 - 57024*a^4*b^
16*c^3 + 44544*a^5*b^14*c^4 + 1505280*a^6*b^12*c^5 - 10838016*a^7*b^10*c^6
+ 38436864*a^8*b^8*c^7 - 79233024*a^9*b^6*c^8 + 92012544*a^10*b^4*c^9 - 492
83072*a^11*b^2*c^10 + 4194304*a^12*c^11)*sqrt((6561*b^4 - 648*a*b^2*c + 16*
a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 +
32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*
b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(sqrt(1/2)*sqrt(-(81
*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*
c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6
)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 57
6*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^
5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*
a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 +
3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*sqrt(-(81*b^5 + 760*a
*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a
^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561
*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c
^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a
^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/
(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4
*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)) - (273375*b^8*c + 205200*a*b^6*c^2
+ 47520*a^2*b^4*c^3 + 2304*a^3*b^2*c^4 - 256*a^4*c^5)*sqrt(x)) - ((b^2*c -
4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(

-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*
b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7
*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c
+ 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^
8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262
144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^
3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*log(1/2*sqrt(1/2)
*(2187*b^15 - 47412*a*b^13*c + 423536*a^2*b^11*c^2 - 1990720*a^3*b^9*c^3 +
5177600*a^4*b^7*c^4 - 7052288*a^5*b^5*c^5 + 3985408*a^6*b^3*c^6 - 180224*a^
7*b*c^7 + (27*a*b^22 - 820*a^2*b^20*c + 10064*a^3*b^18*c^2 - 57024*a^4*b^16
*c^3 + 44544*a^5*b^14*c^4 + 1505280*a^6*b^12*c^5 - 10838016*a^7*b^10*c^6 +
38436864*a^8*b^8*c^7 - 79233024*a^9*b^6*c^8 + 92012544*a^10*b^4*c^9 - 49283
072*a^11*b^2*c^10 + 4194304*a^12*c^11)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^
2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 3
2256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^
4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(sqrt(1/2)*sqrt(-(81*b
^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^
2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*
sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*
a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5
+ 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^
11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 38
40*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*sqrt(-(81*b^5 + 760*a*b
^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4
*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b
^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2
 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8
*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a
*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c
^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)) - (273375*b^8*c + 205200*a*b^6*c^2 +
 47520*a^2*b^4*c^3 + 2304*a^3*b^2*c^4 - 256*a^4*c^5)*sqrt(x)) + ((b^2*c - 4
*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(
81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^
8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c
^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c +
576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*
c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 26214
4*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3
+ 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*log(-1/2*sqrt(1/2)*
(2187*b^15 - 47412*a*b^13*c + 423536*a^2*b^11*c^2 - 1990720*a^3*b^9*c^3 + 5
177600*a^4*b^7*c^4 - 7052288*a^5*b^5*c^5 + 3985408*a^6*b^3*c^6 - 180224*a^7
*b*c^7 + (27*a*b^22 - 820*a^2*b^20*c + 10064*a^3*b^18*c^2 - 57024*a^4*b^16*
c^3 + 44544*a^5*b^14*c^4 + 1505280*a^6*b^12*c^5 - 10838016*a^7*b^10*c^6 + 3
8436864*a^8*b^8*c^7 - 79233024*a^9*b^6*c^8 + 92012544*a^10*b^4*c^9 - 492830
72*a^11*b^2*c^10 + 4194304*a^12*c^11)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2

```
*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32
256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4
*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(sqrt(1/2)*sqrt(-(81*b^
5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2
 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*s
qrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a
^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 +
 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^1
1*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 384
0*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*sqrt(-(81*b^5 + 760*a*b^
3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*
b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^
4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2
- 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*
b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a*
b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^
4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)) - (273375*b^8*c + 205200*a*b^6*c^2 +
47520*a^2*b^4*c^3 + 2304*a^3*b^2*c^4 - 256*a^4*c^5)*sqrt(x)) + 4*(2*c*x^3 +
 b*x)*sqrt(x))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x
^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 47.37Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.02, size = 121, normalized size = 0.27

$$\frac{\left(2\,\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{6}c-3\,\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{2}b\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)+\sqrt{x}\right)}{8\left(4ac-b^{2}\right)\left(2\,\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{7}c+\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{3}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(1/2*c/(4*a*c-b^2)*x^(7/2)+1/4*b/(4*a*c-b^2)*x^(3/2))/(c*x^4+b*x^2+a)+1/8
/(4*a*c-b^2)*sum((2*_R^6*c-3*_R^2*b)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=R
ootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,cx^{\frac{7}{2}}+bx^{\frac{3}{2}}}{2\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)}+\int-\frac{2\,cx^{\frac{5}{2}}-3\,b\sqrt{x}}{4\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^(7/2) + b*x^(3/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (
b^3 - 4*a*b*c)*x^2) + integrate(-1/4*(2*c*x^(5/2) - 3*b*sqrt(x))/((b^2*c -
4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2), x)
```

mupad [B] time = 6.06, size = 21913, normalized size = 48.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((b*x^(3/2))/(2*(4*a*c - b^2)) + (c*x^(7/2))/(4*a*c - b^2))/(a + b*x^2 + c*
x^4) - atan(((((110592*a*b^16*c^4 - 134217728*a^9*c^12 - 2433024*a^2*b^14*c
^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b^10*c^7 + 133693440*a^5*b^8*c^8
+ 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^4*c^10 + 1107296256*a^8*b^2*c^11
)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a
^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/
2)*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a
^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5
 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a
*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1
056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^
14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^
8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11
)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 - 909312*a^2*b^14*c^5 + 94
69952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879360*a^5*b^8*c^8 - 362807
296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 301989888*a^8*b^2*c^11))/(16*(b^
12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 -
 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^
(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a
^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^
7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 167772
16*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126
720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^
8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b
```

^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4) + (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*1i - (((110592*a*b^16*c^4 - 134217728*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b^10*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 - 909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 301989888*a^8*b^2*c^11))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4) - (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*1i)/((16875*a*b^7*c^5 + 320*a^4*b*c^8 + 13500*a^2*b^5*c^6 + 3600*a^3*b^3*c^7)/(64*(b^14 - 16384*a^7*c^7 + 336*

a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28
672*a^6*b^2*c^6 - 28*a*b^12*c)) + (((110592*a*b^16*c^4 - 134217728*a^9*c^12
 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b^10*c^7 + 1
33693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^4*c^10 + 11
07296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240
*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 2
8*a*b^12*c)) - (x^(1/2)*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983
040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4
 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*
b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^1
2 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^1
6*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7
 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 5
0331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 - 909
312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879360
*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 301989888*a
^8*b^2*c^11))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3
 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2
*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^
3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6
 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(
8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 140
80*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b
^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*
c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4) + (x^(1/2)*(
576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3*c^7))/(16*
(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^
4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^1
5)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 71936
0*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3
*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 167
77216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 +
126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128
*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^1
1*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4) + (((110592*a*b^16*c^4 - 13421
7728*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4
*b^10*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 1031798784*a^7*
b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^
10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^
6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15
)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360
*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*
c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 1677
7216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 1
26720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*

a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11
*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b
^16*c^4 - 909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^
7 + 180879360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10
- 301989888*a^8*b^2*c^11))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 128
0*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*
b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c
^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 525926
4*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)
^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b
^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 +
3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671
680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4)
- (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b
^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 38
40*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4
*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^1
1*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 45
87520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*
(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^
4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c
^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 +
69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)))*(-(81*b^17 + 81
b^2(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 8448
0*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5
*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2
))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 -
14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a
^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*
b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*2i - 2*a
tan(((((110592*a*b^16*c^4 - 134217728*a^9*c^12 - 2433024*a^2*b^14*c^5 + 212
00896*a^3*b^12*c^6 - 87687168*a^4*b^10*c^7 + 133693440*a^5*b^8*c^8 + 211812
352*a^6*b^6*c^9 - 1031798784*a^7*b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(
b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c
^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*(-(81
*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*
c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 52592
64*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2
)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*
b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 +
3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 5767
1680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4
)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 - 909312*a^2*b^14*c^5 + 9469952*a^
3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879360*a^5*b^8*c^8 - 362807296*a^6*
b^6*c^9 + 427819008*a^7*b^4*c^10 - 301989888*a^8*b^2*c^11)*1i)/(16*(b^12 +

4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4)*1i - (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4) - (((110592*a*b^16*c^4 - 134217728*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b^10*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 - 909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 301989888*a^8*b^2*c^11)*1i)/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4)*1i + (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(

1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^
4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7
 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 1677721
6*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 1267
20*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8
*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^
4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4))/((((110592*a*b^16*c^4 - 134217728
*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b^1
0*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^4*
c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c
^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^
2*c^6 - 28*a*b^12*c)) - (x^(1/2)*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1
/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4
*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7
 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216
*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 12672
0*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*
b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4
*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*
c^4 - 909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 +
180879360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 30
1989888*a^8*b^2*c^11)*1i)/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280
*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b
^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^
2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264
*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^
15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^
20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3
784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 576716
80*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4)*
1i - (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3
*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 +
3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-
(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b
^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 +
4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(819
2*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*
a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12
*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9
 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*1i - (16875*a*b
^7*c^5 + 320*a^4*b*c^8 + 13500*a^2*b^5*c^6 + 3600*a^3*b^3*c^7)/(64*(b^14 -
16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21
504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (((110592*a*b^16*c^4
 - 134217728*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687
168*a^4*b^10*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 10317987

84*a^7*b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 - 909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 301989888*a^8*b^2*c^11)*1i)/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4)*1i + (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*1i))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4) - 2*atan(((((110592*a*b^16*c^4 - 134217728*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b^10*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^

7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(
-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*
c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a
^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b
^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2
c^11)))^(1/4)(134217728*a^9*c^12 + 36864*a*b^16*c^4 - 909312*a^2*b^14*c^5
 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879360*a^5*b^8*c^8 - 3
62807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 301989888*a^8*b^2*c^11)*1i)
/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b
^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b
^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 -
719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^
7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24
+ 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c
^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 129
76128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 6920601
6*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4)*1i - (x^(1/2)*(576*a^4*b*
c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3*c^7))/(16*(b^12 + 40
96*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a
^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) -
 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*
c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 118
4*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13
*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5
*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*
c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10
 - 50331648*a^12*b^2*c^11)))^(1/4) - (((110592*a*b^16*c^4 - 134217728*a^9*c
^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b^10*c^7
+ 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^4*c^10 +
 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2
240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6
- 28*a*b^12*c)) + (x^(1/2)*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) -
983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c
^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184
*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*
c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*
b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c
^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10
- 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 -
909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879
360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 30198988
8*a^8*b^2*c^11)*1i)/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b
^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 +
81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84
480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b

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^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1
/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2
 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704
*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^1
0*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4)*1i + (
x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3*c
^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a
^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c
 - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^
3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 458752
0*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b
^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^
18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 -
 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 692
06016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4))/((((110592*a*b^16*c^
4 - 134217728*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 876
87168*a^4*b^10*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 103179
8784*a^7*b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 3
36*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 +
 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*(-(81*b^17 + 81*b^2*(-(4*a*c
- b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3
 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520
*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^
24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^1
8*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 -
12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 6920
6016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 +
36864*a*b^16*c^4 - 909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^
4*b^10*c^7 + 180879360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*
b^4*c^10 - 301989888*a^8*b^2*c^11)*1i)/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b
^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10
*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 96
0*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*
c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(
4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c
+ 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6
*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8
*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c
^11)))^(3/4)*1i - (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c
^6 + 3920*a^3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a
^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^1
7 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2
+ 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a
^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15
)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20
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*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 378
4704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680
*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*1i
 - (16875*a*b^7*c^5 + 320*a^4*b*c^8 + 13500*a^2*b^5*c^6 + 3600*a^3*b^3*c^7)
/(64*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4
*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (((11059
2*a*b^16*c^4 - 134217728*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^1
2*c^6 - 87687168*a^4*b^10*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c
^9 - 1031798784*a^7*b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*
a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^
5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*(-(81*b^17 + 81*b^
2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a
^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^
6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/
(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14
080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*
b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6
*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*
a^9*c^12 + 36864*a*b^16*c^4 - 909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 -
53870592*a^4*b^10*c^7 + 180879360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427
819008*a^7*b^4*c^10 - 301989888*a^8*b^2*c^11)*1i)/(16*(b^12 + 4096*a^6*c^6
+ 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5
- 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8
*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 27279
36*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c
- 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*
a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 -
 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440
320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648
*a^12*b^2*c^11)))^(3/4)*1i + (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 510
0*a^2*b^5*c^6 + 3920*a^3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c
^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c))
)*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^
2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5
- 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*
c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 10
56*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^1
4*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8
 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)
))^(1/4)*1i))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*
c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*
a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4
*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2
*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 81
1008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320

*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4) - atan(((((110592*a*b^16*c^4 - 134217728*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b^10*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 - 909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 301989888*a^8*b^2*c^11))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4) + (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*1i - (((110592*a*b^16*c^4 - 134217728*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b^10*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^

8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 - 909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 301989888*a^8*b^2*c^11))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4) - (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*1i)/((16875*a*b^7*c^5 + 320*a^4*b*c^8 + 13500*a^2*b^5*c^6 + 3600*a^3*b^3*c^7)/(64*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (((110592*a*b^16*c^4 - 134217728*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b^10*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 - 909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 301989888*a^8*b^2*c^11))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b

^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^
3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5
 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57
671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3
/4) + (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^
3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 +
 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(
-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*
b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 +
 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(81
92*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080
*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^1
2*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^
9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4) + (((110592*a*
b^16*c^4 - 134217728*a^9*c^12 - 2433024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^
6 - 87687168*a^4*b^10*c^7 + 133693440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 -
 1031798784*a^7*b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*
c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^
4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) + (x^(1/2)*(-(81*b^17 - 81*b^2*(-
(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b
^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 +
4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(819
2*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*
a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12
*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9
 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*
c^12 + 36864*a*b^16*c^4 - 909312*a^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 5387
0592*a^4*b^10*c^7 + 180879360*a^5*b^8*c^8 - 362807296*a^6*b^6*c^9 + 4278190
08*a^7*b^4*c^10 - 301989888*a^8*b^2*c^11))/(16*(b^12 + 4096*a^6*c^6 + 240*a
^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*
b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8
 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*
b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c
*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^2
2*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008
*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9
*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b
^2*c^11)))^(3/4) - (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*
c^6 + 3920*a^3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*
a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^
17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2
 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*
a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^1
5)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^2
0*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 37

```
84704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 5767168
0*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)))
*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2
*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 -
 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c
 - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 105
6*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14
*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8
- 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11))
)^(1/4)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.1076 $\int \frac{x^{3/2}}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=442

$$\frac{c^{3/4}\left(\frac{4b}{\sqrt{b^2-4ac}}+3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{4b}{\sqrt{b^2-4ac}}+3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\ldots}{\sqrt[4]{-\sqrt{\ldots}}}\right.}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}\ldots\right.}$$

```
[Out] 1/4*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(
3+4*b/(-4*a*c+b^2)^(1/2))*2^(3/4)/(-4*a*c+b^2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4
)+1/4*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)
)*(3+4*b/(-4*a*c+b^2)^(1/2))*2^(3/4)/(-4*a*c+b^2)/(-b-(-4*a*c+b^2)^(1/2))^(
3/4)+1/4*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/
4))*(3-4*b/(-4*a*c+b^2)^(1/2))*2^(3/4)/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))
^(3/4)+1/4*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^
(1/4))*(3-4*b/(-4*a*c+b^2)^(1/2))*2^(3/4)/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/
2))^(3/4)-1/2*(2*c*x^2+b)*x^(1/2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)
```

Rubi [A] time = 0.70, antiderivative size = 442, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1115, 1364, 1422, 212, 208, 205}

$$\frac{c^{3/4}\left(\frac{4b}{\sqrt{b^2-4ac}}+3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{4b}{\sqrt{b^2-4ac}}+3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\ldots}{\sqrt[4]{-\sqrt{\ldots}}}\right.}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(Sqrt[x]*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (c^(3/4)*(
3 + (4*b)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^
2 - 4*a*c])^(1/4)])/(2*2^(1/4)*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)
) + (c^(3/4)*(3 - (4*b)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])
/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*(b^2 - 4*a*c)*(-b + Sqrt[b^2 -
 4*a*c])^(3/4)) + (c^(3/4)*(3 + (4*b)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c
^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*(b^2 - 4*a*c)*(
-b - Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(3 - (4*b)/Sqrt[b^2 - 4*a*c])*Arc
Tanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*
(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1364

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(n - 1)*(d*x)^(m - n + 1)*(b + 2*c*x^n)*(a + b*x^n + c*
x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] - Dist[d^n/(n*(p + 1)*(b^2
- 4*a*c)), Int[(d*x)^(m - n)*(b*(m - n + 1) + 2*c*(m + 2*n*(p + 1) + 1)*x^n
)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] && GtQ[m, n -
1] && LeQ[m, 2*n - 1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\sqrt{x}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{b-6cx^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2\left(b^2-4ac\right)}$$

$$= -\frac{\sqrt{x}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{2\left(b^2-4ac\right)} -$$

$$= -\frac{\sqrt{x}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(3+\frac{4b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x,\right.}{2\left(b^2-4ac\right)\sqrt{-b-\sqrt{b^2-4ac}}}$$

$$= -\frac{\sqrt{x}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{c^{3/4}\left(3+\frac{4b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\left(3-\frac{4}{\sqrt{b^2}}\right.}{2\sqrt[4]{2}\left(b^2-\right.}$$

Mathematica [C] time = 0.23, size = 111, normalized size = 0.25

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{6\#1^4c\log\left(\sqrt{x}-\#1\right)-b\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]+\frac{4\sqrt{x}\left(b+2cx^2\right)}{a+bx^2+cx^4}}{8\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/8*((4*Sqrt[x]*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + RootSum[a + b*#1^4 +
c*#1^8 & , (-(b*Log[Sqrt[x] - #1]) + 6*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 +
2*c*#1^7) & ])/(b^2 - 4*a*c)
```

fricas [B] time = 19.51, size = 10570, normalized size = 23.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

[Out] -1/8*(4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqr
t(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a
^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4
*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b
^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 57
6*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*
c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262
144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*
c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*arctan(1/2*(sqr
t(1/2)*(b^18 + 25*a*b^16*c - 146*a^2*b^14*c^2 - 5320*a^3*b^12*c^3 - 2464*a^
4*b^10*c^4 + 1076096*a^5*b^8*c^5 - 10483200*a^6*b^6*c^6 + 44181504*a^7*b^4*
c^7 - 89579520*a^8*b^2*c^8 + 71663616*a^9*c^9 - (a^3*b^23 - 20*a^4*b^21*c +
 432*a^5*b^19*c^2 - 11712*a^6*b^17*c^3 + 195072*a^7*b^15*c^4 - 1935360*a^8*
b^13*c^5 + 12214272*a^9*b^11*c^6 - 50823168*a^10*b^9*c^7 + 139788288*a^11*b
^7*c^8 - 245628928*a^12*b^5*c^9 + 250609664*a^13*b^3*c^10 - 113246208*a^14*
b*c^11)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104
976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c
^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 5898
24*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt((49*b^12*c^
2 + 3150*a*b^10*c^3 + 95985*a^2*b^8*c^4 + 1621296*a^3*b^6*c^5 + 15746400*a^
4*b^4*c^6 + 75582720*a^5*b^2*c^7 + 136048896*a^6*c^8)*x + 1/2*sqrt(1/2)*(b^
18 + 52*a*b^16*c + 1269*a^2*b^14*c^2 + 14294*a^3*b^12*c^3 + 48608*a^4*b^10*
c^4 - 679392*a^5*b^8*c^5 - 4209408*a^6*b^6*c^6 - 4105728*a^7*b^4*c^7 + 2149
90848*a^8*b^2*c^8 - 483729408*a^9*c^9 - (a^3*b^23 + 7*a^4*b^21*c - 152*a^5*
b^19*c^2 - 2960*a^6*b^17*c^3 + 44032*a^7*b^15*c^4 + 60928*a^8*b^13*c^5 - 44
44160*a^9*b^11*c^6 + 36855808*a^10*b^9*c^7 - 153681920*a^11*b^7*c^8 + 36352
8192*a^12*b^5*c^9 - 467140608*a^13*b^3*c^10 + 254803968*a^14*b*c^11)*sqrt((
b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(
a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^1
0*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^
7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(-(b^7 + 21*a*b^5*c + 168*
a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2
 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sq
rt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^
4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256
*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^
4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c
 + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5
 + 4096*a^9*c^6)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^
3 + (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a
^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377
*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*
c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^1
1*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8
 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^

6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)) - sqrt(1/2
)*(7*b^24*c + 400*a*b^22*c^2 + 7843*a^2*b^20*c^3 + 22574*a^3*b^18*c^4 - 139
5688*a^4*b^16*c^5 - 11961472*a^5*b^14*c^6 + 98703360*a^6*b^12*c^7 + 1408361
472*a^7*b^10*c^8 - 12100202496*a^8*b^8*c^9 + 1218281472*a^9*b^6*c^10 + 2412
19731456*a^10*b^4*c^11 - 812665405440*a^11*b^2*c^12 + 835884417024*a^12*c^1
3 - (7*a^3*b^29*c + 85*a^4*b^27*c^2 + 1764*a^5*b^25*c^3 - 37920*a^6*b^23*c^
4 - 103296*a^7*b^21*c^5 - 2564352*a^8*b^19*c^6 + 145468416*a^9*b^17*c^7 - 1
602797568*a^10*b^15*c^8 + 6543507456*a^11*b^13*c^9 + 7533166592*a^12*b^11*c
^10 - 193399619584*a^13*b^9*c^11 + 890247315456*a^14*b^7*c^12 - 20785209016
32*a^15*b^5*c^13 + 2556193406976*a^16*b^3*c^14 - 1320903770112*a^17*b*c^15)
*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4
*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32
256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13
*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(x)*sqrt(-(b^7 + 21
*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 2
40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4
096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3
+ 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b
^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 -
589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 -
24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 61
44*a^8*b^2*c^5 + 4096*a^9*c^6)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 1
68*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c
^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)
*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4
*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32
256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13
*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10
*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c
^5 + 4096*a^9*c^6)))/(2401*b^16*c^3 + 179046*a*b^14*c^4 + 6354369*a^2*b^12*
c^5 + 131902344*a^3*b^10*c^6 + 1713103344*a^4*b^8*c^7 + 13740938496*a^5*b^6
*c^8 + 65167421184*a^6*b^4*c^9 + 166523848704*a^7*b^2*c^10 + 176319369216*a
^8*c^11)) - 4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^
2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^
3 - (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a
^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377
*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*
c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^1
1*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8
- 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^
6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*arctan(-1
/2*(sqrt(1/2)*(b^18 + 25*a*b^16*c - 146*a^2*b^14*c^2 - 5320*a^3*b^12*c^3 -
2464*a^4*b^10*c^4 + 1076096*a^5*b^8*c^5 - 10483200*a^6*b^6*c^6 + 44181504*a
^7*b^4*c^7 - 89579520*a^8*b^2*c^8 + 71663616*a^9*c^9 + (a^3*b^23 - 20*a^4*b
^21*c + 432*a^5*b^19*c^2 - 11712*a^6*b^17*c^3 + 195072*a^7*b^15*c^4 - 19353

60*a^8*b^13*c^5 + 12214272*a^9*b^11*c^6 - 50823168*a^10*b^9*c^7 + 139788288
*a^11*b^7*c^8 - 245628928*a^12*b^5*c^9 + 250609664*a^13*b^3*c^10 - 11324620
8*a^14*b*c^11)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^
3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9
*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6
 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt((49*
b^12*c^2 + 3150*a*b^10*c^3 + 95985*a^2*b^8*c^4 + 1621296*a^3*b^6*c^5 + 1574
6400*a^4*b^4*c^6 + 75582720*a^5*b^2*c^7 + 136048896*a^6*c^8)*x + 1/2*sqrt(1
/2)*(b^18 + 52*a*b^16*c + 1269*a^2*b^14*c^2 + 14294*a^3*b^12*c^3 + 48608*a^
4*b^10*c^4 - 679392*a^5*b^8*c^5 - 4209408*a^6*b^6*c^6 - 4105728*a^7*b^4*c^7
 + 214990848*a^8*b^2*c^8 - 483729408*a^9*c^9 + (a^3*b^23 + 7*a^4*b^21*c - 1
52*a^5*b^19*c^2 - 2960*a^6*b^17*c^3 + 44032*a^7*b^15*c^4 + 60928*a^8*b^13*c
^5 - 4444160*a^9*b^11*c^6 + 36855808*a^10*b^9*c^7 - 153681920*a^11*b^7*c^8
+ 363528192*a^12*b^5*c^9 - 467140608*a^13*b^3*c^10 + 254803968*a^14*b*c^11)
*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4
*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32
256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13
*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(-(b^7 + 21*a*b^5*c
 + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b
^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*
c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976
*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3
+ 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*
a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*
b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b
^2*c^5 + 4096*a^9*c^6)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b
^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 128
0*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b
^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a
^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10
*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7
 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240
*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 409
6*a^9*c^6)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (
a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^
4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*
b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 5
76*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8
*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 26
2144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6
*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)) - sqrt(1/2)*(7*
b^24*c + 400*a*b^22*c^2 + 7843*a^2*b^20*c^3 + 22574*a^3*b^18*c^4 - 1395688*
a^4*b^16*c^5 - 11961472*a^5*b^14*c^6 + 98703360*a^6*b^12*c^7 + 1408361472*a
^7*b^10*c^8 - 12100202496*a^8*b^8*c^9 + 1218281472*a^9*b^6*c^10 + 241219731
456*a^10*b^4*c^11 - 812665405440*a^11*b^2*c^12 + 835884417024*a^12*c^13 + (

7*a^3*b^29*c + 85*a^4*b^27*c^2 + 1764*a^5*b^25*c^3 - 37920*a^6*b^23*c^4 - 1
03296*a^7*b^21*c^5 - 2564352*a^8*b^19*c^6 + 145468416*a^9*b^17*c^7 - 160279
7568*a^10*b^15*c^8 + 6543507456*a^11*b^13*c^9 + 7533166592*a^12*b^11*c^10 -
193399619584*a^13*b^9*c^11 + 890247315456*a^14*b^7*c^12 - 2078520901632*a^
15*b^5*c^13 + 2556193406976*a^16*b^3*c^14 - 1320903770112*a^17*b*c^15)*sqrt
((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)
/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a
^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*
c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(
-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*
b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b
^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^
3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 -
5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12
*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(
a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^
4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^
2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 -
1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt
((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)
/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a
^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*
c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c +
240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 +
4096*a^9*c^6)))/(2401*b^16*c^3 + 179046*a*b^14*c^4 + 6354369*a^2*b^12*c^5 +
131902344*a^3*b^10*c^6 + 1713103344*a^4*b^8*c^7 + 13740938496*a^5*b^6*c^8
+ 65167421184*a^6*b^4*c^9 + 166523848704*a^7*b^2*c^10 + 176319369216*a^8*c^
11)) + ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt
(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^
3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*
c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^
4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576
*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c
^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 2621
44*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c
^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*log((7*b^6*c + 2
25*a*b^4*c^2 + 3240*a^2*b^2*c^3 + 11664*a^3*c^4)*sqrt(x) + 1/2*(b^9 + 19*a*
b^7*c + 124*a^2*b^5*c^2 - 2160*a^3*b^3*c^3 + 5184*a^4*b*c^4 - (a^3*b^14 - 1
2*a^4*b^12*c - 48*a^5*b^10*c^2 + 1600*a^6*b^8*c^3 - 11520*a^7*b^6*c^4 + 399
36*a^8*b^4*c^5 - 69632*a^9*b^2*c^6 + 49152*a^10*c^7)*sqrt((b^8 + 54*a*b^6*c
+ 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^
7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129
024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*
b^2*c^8 - 262144*a^15*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a
^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 -

1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))) - ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*log((7*b^6*c + 225*a*b^4*c^2 + 3240*a^2*b^2*c^3 + 11664*a^3*c^4)*sqrt(x) - 1/2*(b^9 + 19*a*b^7*c + 124*a^2*b^5*c^2 - 2160*a^3*b^3*c^3 + 5184*a^4*b*c^4 - (a^3*b^14 - 12*a^4*b^12*c - 48*a^5*b^10*c^2 + 1600*a^6*b^8*c^3 - 11520*a^7*b^6*c^4 + 39936*a^8*b^4*c^5 - 69632*a^9*b^2*c^6 + 49152*a^10*c^7)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))) + ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*log((7*b^6*c + 225*a*b^4*c^2 + 3240*a^2*b^2*c^3 + 11664*a^3*c^4)*sqrt(x) + 1/2*(b^9 + 19*a*b^7*c + 124*a^2*b^5*c^2 - 2160*a^3*b^3*c^3 + 5184*a^4*b*c^4 + (a^3*b^14 - 12*a^4*b^12*c - 48*a^5*b^10*c^2 + 1600*a^6*b^8*c^3 - 11520*a^7*b^6*c^4 + 39936*a^8*b^4*c^5 - 69632*a^9*b^2*c^6 + 49152*a^10*c^7)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 +

```
 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a
^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(sqrt(1/2)*sqrt(
-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*
b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b
^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^
3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 -
5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12
*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(
a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^
4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))) - ((b^2*c - 4*a*c^2)*x^4 + a*b^
2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c +
 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8
*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^
6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a
^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 +
32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^
13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^
10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2
*c^5 + 4096*a^9*c^6)))*log((7*b^6*c + 225*a*b^4*c^2 + 3240*a^2*b^2*c^3 + 11
664*a^3*c^4)*sqrt(x) - 1/2*(b^9 + 19*a*b^7*c + 124*a^2*b^5*c^2 - 2160*a^3*b
^3*c^3 + 5184*a^4*b*c^4 + (a^3*b^14 - 12*a^4*b^12*c - 48*a^5*b^10*c^2 + 160
0*a^6*b^8*c^3 - 11520*a^7*b^6*c^4 + 39936*a^8*b^4*c^5 - 69632*a^9*b^2*c^6 +
 49152*a^10*c^7)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*
c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a
^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c
^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(sq
rt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b
^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4
 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c
^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^
8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5
+ 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*
a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3
+ 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))) + 4*(2*c*x^2 + b)*
sqrt(x))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 47.03Unable to convert to r
```

```
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.02, size = 118, normalized size = 0.27

$$\frac{\left(6\operatorname{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^4 c-b\right)\ln\left(-\operatorname{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{8\left(4ac-b^2\right)\left(2\operatorname{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7 c+\operatorname{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3 b\right)}+\frac{\frac{2c\,x^{\frac{5}{2}}}{8ac-2b^2}+\frac{2b\sqrt{x}}{16ac-4b^2}}{c\,x^4+b\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(1/2*c/(4*a*c-b^2)*x^(5/2)+1/4*b/(4*a*c-b^2)*x^(1/2))/(c*x^4+b*x^2+a)+1/8
/(4*a*c-b^2)*sum((6*_R^4*c-b)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_
Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bcx^{\frac{9}{2}}+\left(b^2-2\,ac\right)x^{\frac{5}{2}}}{2\left(\left(ab^2c-4\,a^2c^2\right)x^4+a^2b^2-4\,a^3c+\left(ab^3-4\,a^2bc\right)x^2\right)}+\int-\frac{bcx^{\frac{7}{2}}+\left(b^2+6\,ac\right)x^{\frac{3}{2}}}{4\left(\left(ab^2c-4\,a^2c^2\right)x^4+a^2b^2-4\,a^3c+\left(ab^3-4\,a^2bc\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^(9/2) + (b^2 - 2*a*c)*x^(5/2))/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*
b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + integrate(-1/4*(b*c*x^(7/2) + (b
^2 + 6*a*c)*x^(3/2))/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^
3 - 4*a^2*b*c)*x^2), x)
```

mupad [B] time = 10.63, size = 28713, normalized size = 64.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] atan(((((((((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*
a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5
+ 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a
^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^
15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*
b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 +
 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57
```

671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) - (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(3/4) + (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4) - (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i - ((((((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11 +

4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) + (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(3/4) + (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4) + (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i)/((((((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6* c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) - (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(3/4) + (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4) - (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4) + ((((((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*

b^2*c^3 - 16*a*b^6*c)) + (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 10066
3296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b
^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^
11))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^
4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/
2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 5529
6*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^
5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*
b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a
^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*
a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b
^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4
*c^10 - 50331648*a^14*b^2*c^11)))^(3/4) + (2232*a*b^3*c^7 - 7*b^5*c^6 + 116
64*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16
*a*b^6*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 9
6*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^
5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324
*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2
)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^
5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5
 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 -
57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^
(1/4) + (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a
^2*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 +
 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)
^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^
3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 1066598
4*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2
) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16
777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 +
 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 1297612
8*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*
a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)))*((b^4*(-(4*a*c - b^2)^15)
^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 +
55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^
7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) +
3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 167772
16*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126
720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^
10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13
*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*2i + 2*atan(((((((((b^4*(-(4*a*c
 - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*
b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 -
10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^1
5)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^

24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^1 8*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69 206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b ^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^ 10)*1i)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6 *c)) - (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688* a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 40 96*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a ^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 123863 04*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 5 85216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a ^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2* c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4* b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811 008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320 *a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a ^14*b^2*c^11)))^(3/4)*1i - (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((b ^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6 *b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a *c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8 192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14 080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9* b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b ^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i + (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2*b^2*c^9)) /(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^ 4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^ 4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^ 7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17 *c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15* c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7* b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10* c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^1 0 - 50331648*a^14*b^2*c^11)))^(1/4) - ((((((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^ 4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^ 7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17 *c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15* c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*

b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*
c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^1
0 - 50331648*a^14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11 + 4096*a*b^14*c^4 -
73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a
^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10)*1i)/(2*(b^8 + 2
56*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) + (x^(1/2)*(20
48*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a^2*b^13*c^6
+ 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485
760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*c^6 + 240*a^
2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b
^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a
^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 +
3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^
2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^1
5)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b
^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 +
3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 576
71680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(3/
4)*1i - (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^
4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((b^4*(-(4*a*c - b^2)^
15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3
+ 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984
*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
+ 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 167
77216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 +
126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128
*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a
^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i - (x^(1/2)*(1225*b^6*c^7
- 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2*b^2*c^9))/(8*(b^12 + 4096*a^
6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^
2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^
9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216
*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^
3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(
4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*
c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a
^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11
*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b
^2*c^11)))^(1/4))/(((((((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^
9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216
*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^
3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(
4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*
c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a
^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11

*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b
^2*c^11)))^(1/4)*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5
 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 692060
16*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10)*1i)/(2*(b^8 + 256*a^4*c^4 + 96*a^2
*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) - (x^(1/2)*(2048*b^17*c^4 - 30720
*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*
c^7 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 -
75497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^
3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4
*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*
a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^
6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^
2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^
3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6
*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^
6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9
+ 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(3/4)*1i - (2232*a*b^3
*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2
- 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 -
12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c
^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 1789
1328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*
a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 4
8*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4
 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32
440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 5033
1648*a^14*b^2*c^11)))^(1/4)*1i + (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^10 +
10836*a*b^4*c^8 + 14256*a^2*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8
*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c
)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^
15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350
528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2
*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1
/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c
^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 37847
04*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680
*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i
 + ((((((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2
*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3
350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*
c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)
^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^2
0*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 37
84704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671
680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)

*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10)*1i)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) + (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(3/4)*1i - (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i - (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4) + ((b*x^(1/2))/(2*(4*a*c - b^2)) + (c*x^(5/2))/(4*a*c - b^2))/(a + b*x^2 + c*x^4) + atan(((((

(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*
c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528
*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-
(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)
)/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2
- 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*
a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^
12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*(1006
63296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6
 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 13421
7728*a^7*b^2*c^10))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^
3 - 16*a*b^6*c)) - (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a
^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8
 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(
8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*
c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(
1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4
*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7
 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*
c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c
^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b
^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c
^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10
 - 50331648*a^14*b^2*c^11)))^(3/4) + (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^
2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^
6*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^
2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 -
3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2
c^2(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15
)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^
20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3
784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 5767
1680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4
) - (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2*b
^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 384
0*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c -
b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 -
 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a
^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) -
 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777
216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 12
6720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a
^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^1
3*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i - (((((-(b^19 + b^4*(-(4*a*
c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c

^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 106659
84*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/
2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 1
6777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3
+ 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 129761
28*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016
*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11 + 4096
*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7
 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10))/(
2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) + (x
^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a^2
*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7*c
^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*c^
6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^
5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b
*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^
5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c
^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a
c - b^2)^15)^(1/2))/(8192(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c +
 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*
b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^
8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*
c^11)))^(3/4) + (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(2*(b^8 + 25
6*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(b^19 + b^4*
(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3
*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 +
 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^
15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b
^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^
18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 -
 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 6
9206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4) + (x^(1/2)*(1225*b^
6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2*b^2*c^9))/(8*(b^12 + 4
096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*
a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 1238
6304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 +
 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328
*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^
2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^
4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 8
11008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 324403
20*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648
*a^14*b^2*c^11)))^(1/4)*1i)/((((((-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) +
12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c
^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 1789

1328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*
a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 4
8*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4
 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32
440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 5033
1648*a^14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a
^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c
^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10))/(2*(b^8 + 256*a^4*c^4
 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) - (x^(1/2)*(2048*b^17*c^4
 - 30720*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a
^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5
*c^10 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 -
 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-
(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^
2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a
^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4
*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/
(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 -
14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^
9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12
*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(3/4) + (2232
*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^
4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(
1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4
*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7
 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*
c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c
^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b
^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c
^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10
 - 50331648*a^14*b^2*c^11)))^(1/4) - (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^1
0 + 10836*a*b^4*c^8 + 14256*a^2*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2
*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^
10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a
^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 -
 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^
2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^1
5)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b
^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 +
3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 576
71680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/
4) + (((((-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*
a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5
 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a
^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^

15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*
b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 +
3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57
671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1
/4)*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3
*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^
9 - 134217728*a^7*b^2*c^10))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a
^3*b^2*c^3 - 16*a*b^6*c)) + (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 10
0663296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^
4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3
c^11))/(8(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840
*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b
^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 -
55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^
7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) -
3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 167772
16*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126
720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^
10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13
*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(3/4) + (2232*a*b^3*c^7 - 7*b^5*c^6 +
11664*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3
- 16*a*b^6*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^
9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b
^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8
+ 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c
- b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 10
56*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^1
4*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c
^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^1
1)))^(1/4) + (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14
256*a^2*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*
c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-
(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b
^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 1
0665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15
)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^2
4 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18
*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 1
2976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 692
06016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)))*(-(b^19 + b^4*(-(4*
a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13
*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 1066
5984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(
1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 +
16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^

3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 1297
6128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 692060
16*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*2i + 2*atan((((((-(b^19
 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2
752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^
7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c
- b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192
*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080
*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^1
2*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*
c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*(100663296*a
^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 6553
60*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^
7*b^2*c^10)*1i)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 -
16*a*b^6*c)) - (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a^8*b
*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 2
6738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(8*(b
^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4
- 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2)
 + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^1
1*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 1
7891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c +
27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12
- 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*
c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 +
 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 5
0331648*a^14*b^2*c^11)))^(3/4)*1i - (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2
*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6
c)))(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2
*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3
350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*
c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)
^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^2
0*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 37
84704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671
680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)
1i + (x^(1/2)(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2
*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3
840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c
- b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3
 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984
*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
 - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 167
77216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 +
126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128

*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a
^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4) - (((((-(b^19 + b^4*(-(4*a*c
 - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^
3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 1066598
4*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2
) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16
777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 +
 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 1297612
8*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*
a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11 + 4096*
a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7
 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10)*1i)
/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) +
(x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a
^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7
*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*
c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*
c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9
*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*
a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3
*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4
*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c
 + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^
8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*
b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^
2*c^11)))^(3/4)*1i - (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(2*(b^8
 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(b^19 +
 b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 275
2*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*
c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c -
b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(
a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a
^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*
c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^
9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i - (x^(1/2)*
(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2*b^2*c^9))/(8*(
b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4
 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2
) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^
11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 -
17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c +
 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12
 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16
*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7
+ 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 -

50331648*a^14*b^2*c^11)))^(1/4))/((((((-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10)*1i)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) - (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(3/4)*1i - (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i + (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i + (((((-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 1238630

4*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 58
5216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^
8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c
*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b
^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 8110
08*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*
a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^
14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12
*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69
206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10)*1i)/(2*(b^8 + 256*a^4*c^4 + 96
*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) + (x^(1/2)*(2048*b^17*c^4 - 3
0720*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b
^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^1
0 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 128
0*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(b^1
9 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 +
2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b
^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c
 - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(819
2*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 1408
0*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^
12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6
*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(3/4)*1i - (2232*
a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4
*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1
/2) + 12386304*a^9*b*c^9 - 96*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*
b^11*c^4 + 585216*a^5*b^9*c^5 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7
- 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c
 + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^
12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^
16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^
7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10
- 50331648*a^14*b^2*c^11)))^(1/4)*1i - (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c
^10 + 10836*a*b^4*c^8 + 14256*a^2*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a
^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*
b^10*c)))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96
*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5
 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*
a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)
^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5
*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5
+ 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 5
7671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(
1/4)*1i))*(-(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) + 12386304*a^9*b*c^9 - 96
*a^2*b^15*c^2 + 2752*a^3*b^13*c^3 - 55296*a^4*b^11*c^4 + 585216*a^5*b^9*c^5

```
 - 3350528*a^6*b^7*c^6 + 10665984*a^7*b^5*c^7 - 17891328*a^8*b^3*c^8 + 324*
a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)
^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5
*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5
+ 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 5
7671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(
1/4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

3.1077 $\int \frac{\sqrt{x}}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=489

$$\frac{x^{3/2}\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt[4]{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b^2-20ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}-$$

```
[Out] 1/2*x^(3/2)*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/8*c^(1/4)*
arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(20*a*c-b^
2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-4*a*c+b^2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)
-1/8*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))
*(b+(20*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-4*a*c+b^2)/(-b-(-4*a*c+b^2
)^(1/2))^(1/4)+1/8*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^
(1/2))^(1/4))*(b+(-20*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-4*a*c+b^2)/(
-b+(-4*a*c+b^2)^(1/2))^(1/4)-1/8*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-
b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-20*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a
/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A] time = 1.00, antiderivative size = 489, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1115, 1366, 1510, 298, 205, 208}

$$\frac{x^{3/2}\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt[4]{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b^2-20ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
 (c^(1/4)*(b - (b^2 - 20*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sq
rt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a*(b^2 - 4*a*c)*(-b - Sq
rt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(b + (b^2 - 20*a*c)/Sqrt[b^2 - 4*a*c])*A
rcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)
*a*(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(b - (b^2 - 20*
a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 -
4*a*c])^(1/4)])/(4*2^(3/4)*a*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))
- (c^(1/4)*(b + (b^2 - 20*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*
```

Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a*(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1366

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^n)*(a + b*x^n + c*x^(
2*n))^(p + 1))/(a*d*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2
 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(m + n*(p
+ 1) + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(m + n*(2*p + 3) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0
] && IGtQ[n, 0] && ILtQ[p, -1]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(-b^2+10ac-bcx^4\right)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2a\left(b^2-4ac\right)}$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{4a\left(b^2-4ac\right)}$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(\sqrt{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx\right.}{4\sqrt{2}\,a\left(b^2-4ac\right)}$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt[4]{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(b+\frac{b^2}{\sqrt{b}}\right.}{4\,2^{3/4}a\left(b^2\right.}$$

Mathematica [C] time = 0.24, size = 149, normalized size = 0.30

$$-\frac{\left(a+bx^2+cx^4\right)\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{\#1^4bc\log\left(\sqrt{x}-\#1\right)-10ac\log\left(\sqrt{x}-\#1\right)+b^2\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]+4x^{3/2}\left(-2ac+b^2\right.}{8a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/8*(4*x^(3/2)*(b^2 - 2*a*c + b*c*x^2) + (a + b*x^2 + c*x^4)*RootSum[a + b
*#1^4 + c*#1^8 & , (b^2*Log[Sqrt[x] - #1] - 10*a*c*Log[Sqrt[x] - #1] + b*c*
Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(a*(-b^2 + 4*a*c)*(a + b*x^2
 + c*x^4))
```

fricas [B] time = 114.21, size = 12411, normalized size = 25.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] -1/8*(4*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3* b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1 280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqr t((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b ^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^ 15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^ 8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a ^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))*arctan (1/2*((b^11 - 47*a*b^9*c + 853*a^2*b^7*c^2 - 7324*a^3*b^5*c^3 + 28400*a^4*b ^3*c^4 - 40000*a^5*b*c^5 - (a^5*b^14 - 44*a^6*b^12*c + 720*a^7*b^10*c^2 - 6 080*a^8*b^8*c^3 + 29440*a^9*b^6*c^4 - 82944*a^10*b^4*c^5 + 126976*a^11*b^2* c^6 - 81920*a^12*c^7)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a ^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a ^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256* a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4 *c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt((531441*b^24*c^8 - 768 81798*a*b^22*c^9 + 5113978011*a^2*b^20*c^10 - 206852401350*a^3*b^18*c^11 + 5667080000625*a^4*b^16*c^12 - 110792866500000*a^5*b^14*c^13 + 1584936775000 000*a^6*b^12*c^14 - 16715805000000000*a^7*b^10*c^15 + 128988375000000000*a^ 8*b^8*c^16 - 710150000000000000*a^9*b^6*c^17 + 2647500000000000000*a^10*b^4 *c^18 - 6000000000000000000*a^11*b^2*c^19 + 6250000000000000000*a^12*c^20)* x - 1/2*sqrt(1/2)*(6561*b^31*c^5 - 1032993*a*b^29*c^6 + 75634965*a^2*b^27*c ^7 - 3414264975*a^3*b^25*c^8 + 106186248955*a^4*b^23*c^9 - 2407919378459*a^ 5*b^21*c^10 + 41083864936232*a^6*b^19*c^11 - 536376931701360*a^7*b^17*c^12 + 5394460343808000*a^8*b^15*c^13 - 41720627697600000*a^9*b^13*c^14 + 245614 092480000000*a^10*b^11*c^15 - 1078472304000000000*a^11*b^9*c^16 + 341052480 0000000000*a^12*b^7*c^17 - 7314160000000000000*a^13*b^5*c^18 + 948800000000 0000000*a^14*b^3*c^19 - 5600000000000000000*a^15*b*c^20 - (6561*a^5*b^34*c^ 5 - 895212*a^6*b^32*c^6 + 56697732*a^7*b^30*c^7 - 2212069617*a^8*b^28*c^8 + 59497163992*a^9*b^26*c^9 - 1169816993840*a^10*b^24*c^10 + 17397456159488*a ^11*b^22*c^11 - 199763116583168*a^12*b^20*c^12 + 1791922585643008*a^13*b^18 *c^13 - 12624164431147008*a^14*b^16*c^14 + 69835076189159424*a^15*b^14*c^15 - 301610411758387200*a^16*b^12*c^16 + 1004700278784000000*a^17*b^10*c^17 - 2527971917824000000*a^18*b^8*c^18 + 4641908326400000000*a^19*b^6*c^19 - 58 64652800000000000*a^20*b^4*c^20 + 4554752000000000000*a^21*b^2*c^21 - 16384 00000000000000*a^22*c^22)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 459 50*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32 256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17 *b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^

6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^1
0*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45
950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^
6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 3
2256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^1
7*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^1
0*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2
*c^5 + 4096*a^11*c^6))) - (729*b^23*c^4 - 86994*a*b^21*c^5 + 4700619*a^2*b^
19*c^6 - 151648714*a^3*b^17*c^7 + 3240737969*a^4*b^15*c^8 - 48070563100*a^5
*b^13*c^9 + 503690450000*a^6*b^11*c^10 - 3715387000000*a^7*b^9*c^11 + 18824
300000000*a^8*b^7*c^12 - 62050000000000*a^9*b^5*c^13 + 119000000000000*a^10
*b^3*c^14 - 100000000000000*a^11*b*c^15 - (729*a^5*b^26*c^4 - 84807*a^6*b^2
4*c^5 + 4445469*a^7*b^22*c^6 - 138927340*a^8*b^20*c^7 + 2884712240*a^9*b^18
*c^8 - 41968650816*a^10*b^16*c^9 + 439511597568*a^11*b^14*c^10 - 3350499342
336*a^12*b^12*c^11 + 18578963128320*a^13*b^10*c^12 - 74005426176000*a^14*b^
8*c^13 + 205936435200000*a^15*b^6*c^14 - 379514880000000*a^16*b^4*c^15 + 41
5744000000000*a^17*b^2*c^16 - 204800000000000*a^18*c^17)*sqrt((b^12 - 78*a*
b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 262500
0*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^1
4*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 34
4064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19
*c^9)))*sqrt(x))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 -
 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b
^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^1
1*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 47
0625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a
^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 -
 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a
^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^
2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6
)))/(6561*b^20*c^5 - 803358*a*b^18*c^6 + 44473131*a^2*b^16*c^7 - 1466261550
*a^3*b^14*c^8 + 31889850625*a^4*b^12*c^9 - 478129875000*a^5*b^10*c^10 + 500
4993750000*a^6*b^8*c^11 - 36117500000000*a^7*b^6*c^12 + 171937500000000*a^8
*b^4*c^13 - 487500000000000*a^9*b^2*c^14 + 625000000000000*a^10*c^15)) - 4*
((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*s
qrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3
+ 18000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*
b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12
- 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 -
 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a
^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c
^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 2621
44*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c
^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))*arctan(-1/2*((
b^11 - 47*a*b^9*c + 853*a^2*b^7*c^2 - 7324*a^3*b^5*c^3 + 28400*a^4*b^3*c^4

- 40000*a^5*b*c^5 + (a^5*b^14 - 44*a^6*b^12*c + 720*a^7*b^10*c^2 - 6080*a^8*b^8*c^3 + 29440*a^9*b^6*c^4 - 82944*a^10*b^4*c^5 + 126976*a^11*b^2*c^6 - 81920*a^12*c^7)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt((531441*b^24*c^8 - 76881798*a*b^22*c^9 + 5113978011*a^2*b^20*c^10 - 206852401350*a^3*b^18*c^11 + 5667080000625*a^4*b^16*c^12 - 110792866500000*a^5*b^14*c^13 + 1584936775000000*a^6*b^12*c^14 - 16715805000000000*a^7*b^10*c^15 + 128988375000000000*a^8*b^8*c^16 - 710150000000000000*a^9*b^6*c^17 + 2647500000000000000*a^10*b^4*c^18 - 6000000000000000000*a^11*b^2*c^19 + 6250000000000000000*a^12*c^20)*x - 1/2*sqrt(1/2)*(6561*b^31*c^5 - 1032993*a*b^29*c^6 + 75634965*a^2*b^27*c^7 - 3414264975*a^3*b^25*c^8 + 106186248955*a^4*b^23*c^9 - 2407919378459*a^5*b^21*c^10 + 41083864936232*a^6*b^19*c^11 - 536376931701360*a^7*b^17*c^12 + 5394460343808000*a^8*b^15*c^13 - 41720627697600000*a^9*b^13*c^14 + 245614092480000000*a^10*b^11*c^15 - 1078472304000000000*a^11*b^9*c^16 + 3410524800000000000*a^12*b^7*c^17 - 7314160000000000000*a^13*b^5*c^18 + 9488000000000000000*a^14*b^3*c^19 - 5600000000000000000*a^15*b*c^20 + (6561*a^5*b^34*c^5 - 895212*a^6*b^32*c^6 + 56697732*a^7*b^30*c^7 - 2212069617*a^8*b^28*c^8 + 59497163992*a^9*b^26*c^9 - 1169816993840*a^10*b^24*c^10 + 17397456159488*a^11*b^22*c^11 - 199763116583168*a^12*b^20*c^12 + 1791922585643008*a^13*b^18*c^13 - 12624164431147008*a^14*b^16*c^14 + 69835076189159424*a^15*b^14*c^15 - 301610411758387200*a^16*b^12*c^16 + 1004700278784000000*a^17*b^10*c^17 - 2527971917824000000*a^18*b^8*c^18 + 4641908326400000000*a^19*b^6*c^19 - 5864652800000000000*a^20*b^4*c^20 + 4554752000000000000*a^21*b^2*c^21 - 1638400000000000000*a^22*c^22)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 -

129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a
^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^
2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6
))) - (729*b^23*c^4 - 86994*a*b^21*c^5 + 4700619*a^2*b^19*c^6 - 151648714*a
^3*b^17*c^7 + 3240737969*a^4*b^15*c^8 - 48070563100*a^5*b^13*c^9 + 50369045
0000*a^6*b^11*c^10 - 3715387000000*a^7*b^9*c^11 + 18824300000000*a^8*b^7*c^
12 - 62050000000000*a^9*b^5*c^13 + 119000000000000*a^10*b^3*c^14 - 10000000
0000000*a^11*b*c^15 + (729*a^5*b^26*c^4 - 84807*a^6*b^24*c^5 + 4445469*a^7*
b^22*c^6 - 138927340*a^8*b^20*c^7 + 2884712240*a^9*b^18*c^8 - 41968650816*a
^10*b^16*c^9 + 439511597568*a^11*b^14*c^10 - 3350499342336*a^12*b^12*c^11 +
18578963128320*a^13*b^10*c^12 - 74005426176000*a^14*b^8*c^13 + 20593643520
0000*a^15*b^6*c^14 - 379514880000000*a^16*b^4*c^15 + 415744000000000*a^17*b
^2*c^16 - 204800000000000*a^18*c^17)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^
8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250
000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^
12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 -
589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt(x)*sqrt
(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 1
8000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6
*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 7
8*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 26
25000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12
*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5
+ 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*
a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3
+ 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6))))/(6561*b^20*c^5 -
803358*a*b^18*c^6 + 44473131*a^2*b^16*c^7 - 1466261550*a^3*b^14*c^8 + 3188
9850625*a^4*b^12*c^9 - 478129875000*a^5*b^10*c^10 + 5004993750000*a^6*b^8*c
^11 - 36117500000000*a^7*b^6*c^12 + 171937500000000*a^8*b^4*c^13 - 48750000
0000000*a^9*b^2*c^14 + 625000000000000*a^10*c^15)) - ((a*b^2*c - 4*a^2*c^2)
*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b
^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a
^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4
*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a
^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 +
6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^
13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c
^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^
12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4
- 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))*log(1/2*sqrt(1/2)*(b^22 - 91*a*b^20*
c + 3683*a^2*b^18*c^2 - 87230*a^3*b^16*c^3 + 1338850*a^4*b^14*c^4 - 1394002
4*a^5*b^12*c^5 + 100253344*a^6*b^10*c^6 - 497651072*a^7*b^8*c^7 + 167204608
0*a^8*b^6*c^8 - 3627264000*a^9*b^4*c^9 + 4582400000*a^10*b^2*c^10 - 2560000
000*a^11*c^11 - (a^5*b^25 - 70*a^6*b^23*c + 2192*a^7*b^21*c^2 - 40672*a^8*b
^19*c^3 + 498432*a^9*b^17*c^4 - 4254720*a^10*b^15*c^5 + 25976832*a^11*b^13*

c^6 - 114475008*a^12*b^11*c^7 + 361955328*a^13*b^9*c^8 - 802029568*a^14*b^7
*c^9 + 1183842304*a^15*b^5*c^10 - 1046478848*a^16*b^3*c^11 + 419430400*a^17
*b*c^12)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 +
470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36
*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4
 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824
*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c +
 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^6*
b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*
b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 4595
0*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)
/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 322
56*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*
b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*
c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c
^5 + 4096*a^11*c^6)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*
b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1
280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqr
t((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b
^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c
 + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^
15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^
8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a
^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)) + (729*
b^12*c^4 - 52731*a*b^10*c^5 + 1600425*a^2*b^8*c^6 - 26110000*a^3*b^6*c^7 +
241500000*a^4*b^4*c^8 - 1200000000*a^5*b^2*c^9 + 2500000000*a^6*c^10)*sqrt(
x)) + ((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*
x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^
3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 128
0*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt(
(b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4
*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c +
 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15
*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8
 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8
*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))*log(-1/2
sqrt(1/2)(b^22 - 91*a*b^20*c + 3683*a^2*b^18*c^2 - 87230*a^3*b^16*c^3 + 1
338850*a^4*b^14*c^4 - 13940024*a^5*b^12*c^5 + 100253344*a^6*b^10*c^6 - 4976
51072*a^7*b^8*c^7 + 1672046080*a^8*b^6*c^8 - 3627264000*a^9*b^4*c^9 + 45824
00000*a^10*b^2*c^10 - 2560000000*a^11*c^11 - (a^5*b^25 - 70*a^6*b^23*c + 21
92*a^7*b^21*c^2 - 40672*a^8*b^19*c^3 + 498432*a^9*b^17*c^4 - 4254720*a^10*b
^15*c^5 + 25976832*a^11*b^13*c^6 - 114475008*a^12*b^11*c^7 + 361955328*a^13
*b^9*c^8 - 802029568*a^14*b^7*c^9 + 1183842304*a^15*b^5*c^10 - 1046478848*a
^16*b^3*c^11 + 419430400*a^17*b*c^12)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b
^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 625

0000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)) + (729*b^12*c^4 - 52731*a*b^10*c^5 + 1600425*a^2*b^8*c^6 - 26110000*a^3*b^6*c^7 + 241500000*a^4*b^4*c^8 - 1200000000*a^5*b^2*c^9 + 2500000000*a^6*c^10)*sqrt(x)) - ((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))*log(1/2*sqrt(1/2)*(b^22 - 91*a*b^20*c + 3683*a^2*b^18*c^2 - 87230*a^3*b^16*c^3 + 1338850*a^4*b^14*c^4 - 13940024*a^5*b^12*c^5 + 100253344*a^6*b^10*c^6 - 497651072*a^7*b^8*c^7 + 1672046080*a^8*b^6*c^8 - 3627264000*a^9*b^4*c^9 + 4582400000*a^10*b^2*c^10 - 2560000000*a^11*c^11 + (a^5*b^25 - 70*a^6*b^23*c + 2192*a^7*b^21*c^2 - 40672*a^8*b^19*c^3 + 498432*a^9*b^17*c^4 - 4254720*a^10*b^15*c^5 + 25976832*a^11*b^13*c^6 - 114475008*a^12*b^11*c^7 + 361955328*a^13*b^9*c^8 - 802029568*a^14*b^7*c^9 + 1183842304*a^15*b^5*c^10 - 1046478848*a^16*b^3*c^11 + 419430400*a^17*b*c^12)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 -

5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)) + (729*b^12*c^4 - 52731*a*b^10*c^5 + 1600425*a^2*b^8*c^6 - 26110000*a^3*b^6*c^7 + 241500000*a^4*b^4*c^8 - 1200000000*a^5*b^2*c^9 + 2500000000*a^6*c^10)*sqrt(x)) + ((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))*log(-1/2*sqrt(1/2)*(b^22 - 91*a*b^20*c + 3683*a^2*b^18*c^2 - 87230*a^3*b^16*c^3 + 1338850*a^4*b^14*c^4 - 13940024*a^5*b^12*c^5 + 100253344*a^6*b^10*c^6 - 497651072*a^7*b^8*c^7 + 1672046080*a^8*b^6*c^8 - 3627264000*a^9*b^4*c^9 + 4582400000*a^10*b^2*c^10 - 2560000000*a^11*c^11 + (a^5*b^25 - 70*a^6*b^23*c + 2192*a^7*b^21*c^2 - 40672*a^8*b^19*c^3 + 498432*a^9*b^17*c^4 - 4254720*a^10*b^15*c^5 + 25976832*a^11*b^13*c^6 - 114475008*a^12*b^11*c^7 + 361955328*a^13*b^9*c^8 - 802029568*a^14*b^7*c^9 + 1183842304*a^15*b^5*c^10 - 1046478848*a^16*b^3*c^11 + 419430400*a^17*b*c^12)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 62500

```
00*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^1
2*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 5
89824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 2
4*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144
*a^10*b^2*c^5 + 4096*a^11*c^6)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2
- 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*
b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^
11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 4
70625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*
a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4
- 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*
a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c
^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^
6)) + (729*b^12*c^4 - 52731*a*b^10*c^5 + 1600425*a^2*b^8*c^6 - 26110000*a^3
*b^6*c^7 + 241500000*a^4*b^4*c^8 - 1200000000*a^5*b^2*c^9 + 2500000000*a^6*
c^10)*sqrt(x)) - 4*(b*c*x^3 + (b^2 - 2*a*c)*x)*sqrt(x))/((a*b^2*c - 4*a^2*c
^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 47.23Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.02, size = 146, normalized size = 0.30

$$-\frac{\left(\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{6} bc+\left(-10ac+b^{2}\right)\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{2}\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+\right.\right.}{8\left(4ac-b^{2}\right)a\left(2\,\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{7} c+\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{3} b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*b/a/(4*a*c-b^2)*c*x^(7/2)+1/4*(2*a*c-b^2)/(4*a*c-b^2)/a*x^(3/2))/(c
*x^4+b*x^2+a)-1/8/a/(4*a*c-b^2)*sum((_R^6*b*c+(-10*a*c+b^2)*_R^2)/(2*_R^7*c
+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bcx^{\frac{7}{2}} + \left(b^2 - 2\,ac\right)x^{\frac{3}{2}}}{2\left(\left(ab^2c - 4\,a^2c^2\right)x^4 + a^2b^2 - 4\,a^3c + \left(ab^3 - 4\,a^2bc\right)x^2\right)} - \int -\frac{bcx^{\frac{5}{2}} + \left(b^2 - 10\,ac\right)\sqrt{x}}{4\left(\left(ab^2c - 4\,a^2c^2\right)x^4 + a^2b^2 - 4\,a^3c + \left(ab^3 - 4\,a^2bc\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^(7/2) + (b^2 - 2*a*c)*x^(3/2))/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*
b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) - integrate(-1/4*(b*c*x^(5/2) + (b
^2 - 10*a*c)*sqrt(x))/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b
^3 - 4*a^2*b*c)*x^2), x)
```

mupad [B] time = 6.56, size = 26373, normalized size = 53.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] atan(((((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 28528
64*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 18574213
12*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 19386
073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^
4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 2867
2*a^8*b^2*c^6)) - (x^(1/2)*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 737280
00*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c
^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 1
08380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)
^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a
*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48
*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4
- 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 3
2440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 503
31648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 1966
08*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 320471040*
a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 762524467
2*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12))/(16*(a^2*b^12 + 4096*a^8*c^6 - 2
4*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144
*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c
^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 300134
4*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^
8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2)
```

- 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(3/4) + (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^3*b*c^11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*1i - (((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 2852864*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 1857421312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 19386073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^8*b^2*c^6)) + (x^(1/2)*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 196608*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 320471040*a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 7625244672*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11 b^2*c^11)))^(3/4) - (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^3*b*c^11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*1i)/((((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 2852864*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 1857421312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 19386073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^8*b^2*c^6)) - (x^(1/2)*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 196608*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 320471040*a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 7625244672*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(3/4) + (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^3*b*c^11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 +

15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 13467
6480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 5
25*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(
1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*
c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 378
4704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671
680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)
 - (5000000*a^3*c^12 - 3645*b^6*c^9 + 121500*a*b^4*c^10 - 1350000*a^2*b^2*c
^11)/(32*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^10*c^2 - 224
0*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^8*b^2*c^6))
+ (((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 2852864*a
^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 1857421312*a
^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 193860730
88*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^
10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^
8*b^2*c^6)) + (x^(1/2)*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a
^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 -
 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 10838
0160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)
^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4
c(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6
*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 81
1008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440
320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 5033164
8*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 196608*a
^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 320471040*a^5*
b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 7625244672*a^
8*b^4*c^11 - 7751073792*a^9*b^2*c^12))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^
3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7
*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10
+ 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^
5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^
5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69
*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c
 - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1
056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b
^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^
8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*
c^11)))^(3/4) - (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^3*b*c^11 -
 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^
4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^
21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c
^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 1506
4576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480
*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a

^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)
)/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2
- 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704
*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*
a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)))*(
-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^
17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 +
15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 13467
6480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 5
25*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(
1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*
c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 378
4704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671
680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)
*2i + atan(((((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 +
2852864*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 18
57421312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 +
19386073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c +
336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5
+ 28672*a^8*b^2*c^6)) - (x^(1/2)*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) +
73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*
b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c
^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c
- b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
+ 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^1
2 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^1
6*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c
^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10
- 50331648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4
+ 196608*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 3204
71040*a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 762
5244672*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12))/(16*(a^2*b^12 + 4096*a^8*c
^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4
- 6144*a^7*b^2*c^5)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^
10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 -
3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380
160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^
(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*
c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*
b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811
008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 324403
20*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648
*a^16*b^2*c^11)))^(3/4) + (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^
3*b*c^11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*
c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^

5)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*
a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*
c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 -
134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19
*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)
^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7
*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5
+ 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 -
57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))
^(1/4)*1i - (((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 +
2852864*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 18
57421312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 +
19386073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c +
336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5
+ 28672*a^8*b^2*c^6)) + (x^(1/2)*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) +
73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*
b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c
^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c
- b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
+ 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^1
2 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^1
6*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c
^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10
- 50331648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4
+ 196608*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 3204
71040*a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 762
5244672*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12))/(16*(a^2*b^12 + 4096*a^8*c
^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4
- 6144*a^7*b^2*c^5)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^
10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 -
3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380
160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^
(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*
c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*
b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811
008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 324403
20*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648
*a^16*b^2*c^11)))^(3/4) - (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^
3*b*c^11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*
c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^
5)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*
a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*
c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 -
134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19
*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)

^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7
*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5
+ 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 -
57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))
^(1/4)*1i)/((((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 +
2852864*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 18
57421312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 +
19386073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c +
336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5
+ 28672*a^8*b^2*c^6)) - (x^(1/2)*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) +
73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*
b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c
^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c
- b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
+ 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^1
2 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^1
6*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c
^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10
- 50331648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4
+ 196608*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 3204
71040*a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 762
5244672*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12))/(16*(a^2*b^12 + 4096*a^8*c
^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4
- 6144*a^7*b^2*c^5)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^
10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 -
3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380
160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^
(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*
c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*
b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811
008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 324403
20*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648
*a^16*b^2*c^11)))^(3/4) + (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^
3*b*c^11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*
c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^
5)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*
a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*
c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 -
134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19
*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)
^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7
*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5
+ 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 -
57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))
^(1/4) - (5000000*a^3*c^12 - 3645*b^6*c^9 + 121500*a*b^4*c^10 - 1350000*a^2

*b^2*c^11)/(32*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^8*b^2*c^6)) + (((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 2852864*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 1857421312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 19386073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^8*b^2*c^6)) + (x^(1/2)*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 196608*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 320471040*a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 7625244672*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(3/4) - (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^3*b*c^11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 -

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134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19
*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)
^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7
*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5
 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 -
 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))
^(1/4)*2i + 2*atan(((((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*
b*c^13 + 2852864*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*
c^8 - 1857421312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^
5*c^11 + 19386073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b
^12*c + 336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*
b^4*c^5 + 28672*a^8*b^2*c^6)) - (x^(1/2)*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^
(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404
160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a
^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(
-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15
)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*
a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720
*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^1
2*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*
b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b
^18*c^4 + 196608*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^
7 - 320471040*a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^
10 + 7625244672*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12)*1i)/(16*(a^2*b^12 +
 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a
^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) +
73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*
b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c
^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c
- b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
- 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^1
2 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^1
6*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c
^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10
 - 50331648*a^16*b^2*c^11)))^(3/4)*1i - (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c
^9 + 600000*a^3*b*c^11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6
- 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6
144*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*
b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 300
1344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160
*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/
2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(
-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^2
2*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008
*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*
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a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4) - (((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 2852864*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 1857421312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 19386073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^8*b^2*c^6)) + (x^(1/2)*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 196608*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 320471040*a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 7625244672*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12)*1i)/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(3/4)*1i + (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^3*b*c^11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4))/((5000000*a^3*c^12 - 3645*b^6*c^9 + 121500*a*b^4*c^10 - 1350000*a^2*b^2*c^11)/(32*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^8*b^2*c^6)) + (((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400 *a^9*b*c^13 + 2852864*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 1857421312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 19386073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^8*b^2*c^6)) - (x^(1/2)*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 196608*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 320471040*a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 7625244672*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12)*1i)/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(3/4)*1i - (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^3*b*c^11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*1i + (((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 2852864*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 1857421312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 19386073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^8*b^2*c^6)) + (x^(1/2)*(-(b^21 + b^6*(-(4*a*c

- b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 196608*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 320471040*a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 7625244672*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12)*1i)/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(3/4)*1i + (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^3*b*c^11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*1i))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4) + 2*atan(((((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 2852864*a^2*b^15*c^6 - 39247872*a^3*b

^13*c^7 + 335708160*a^4*b^11*c^8 - 1857421312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 19386073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^8*b^2*c^6)) - (x^(1/2)*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 196608*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 320471040*a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 7625244672*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12)*1i)/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(3/4)*1i - (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^3*b*c^11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4) - (((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 2852864*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 1857421312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 19386073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^8*b^2*c^6)) + (x^(1/2)*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^1

7*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 1
5064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676
480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 52
5*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1
/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c
^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784
704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 576716
80*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*
(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 196608*a^2*b^16*c^5 - 4005888*a^3
*b^14*c^6 + 45580288*a^4*b^12*c^7 - 320471040*a^5*b^10*c^8 + 1448607744*a^6
*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 7625244672*a^8*b^4*c^11 - 7751073792*a
^9*b^2*c^12)*1i)/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8
*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 -
b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 -
36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*
a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*
b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^
2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(81
92*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 140
80*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11
*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*
b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(3/4)*1i + (x^
(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^3*b*c^11 - 98000*a^2*b^3*c^10
))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^
5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 - b^6*(-(4*a*c -
 b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15
*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 -
50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 2500
*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a*c
 - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 +
 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^
3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12
976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 6920
6016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4))/((5000000*a^3*c^12 -
3645*b^6*c^9 + 121500*a*b^4*c^10 - 1350000*a^2*b^2*c^11)/(32*(a^2*b^14 - 16
384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^
6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^8*b^2*c^6)) + (((2048*b^19*c^4 - 11
6736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 2852864*a^2*b^15*c^6 - 39247872*
a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 1857421312*a^5*b^9*c^9 + 6670516224
*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 19386073088*a^8*b^3*c^12)/(64*(a
^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^10*c^2 - 2240*a^5*b^8*c
^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^8*b^2*c^6)) - (x^(1/2)*
(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b
^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 +
 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 1346

76480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c -
525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^
(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20
*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 37
84704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 5767
1680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4
)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 196608*a^2*b^16*c^5 - 4005888*a
^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 320471040*a^5*b^10*c^8 + 1448607744*a
^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 7625244672*a^8*b^4*c^11 - 7751073792
*a^9*b^2*c^12)*1i)/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b
^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21
- b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2
- 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 1506457
6*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^
9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*
b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(
8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 1
4080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^
11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^1
4*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(3/4)*1i - (
x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^3*b*c^11 - 98000*a^2*b^3*c^
10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*
a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 - b^6*(-(4*a*c
 - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^
15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6
- 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 + 25
00*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*(-(4*a
*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24
 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*
c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 -
12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69
206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*1i + (((2048*b^19*c^
4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 2852864*a^2*b^15*c^6 - 392
47872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 1857421312*a^5*b^9*c^9 + 6670
516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 19386073088*a^8*b^3*c^12)/
(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^4*b^10*c^2 - 2240*a^5
*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 28672*a^8*b^2*c^6)) + (x^
(1/2)*(-(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085
*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11
*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8
- 134676480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^1
9*c - 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2
)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^
7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^
5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8

```
- 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11))
)^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 196608*a^2*b^16*c^5 - 400
5888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 320471040*a^5*b^10*c^8 + 144860
7744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 7625244672*a^8*b^4*c^11 - 7751
073792*a^9*b^2*c^12)*1i)/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240
*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-
(b^21 - b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^1
7*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 1
5064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676
480*a^9*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 52
5*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1
/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c
^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784
704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 576716
80*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(3/4)*
1i + (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^3*b*c^11 - 98000*a^2*
b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 -
 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 - b^6*(-
(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*
a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^
9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^
9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b^2*c^2*
(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^
5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8
*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*
c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^
9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*1i))*(-(b^21 -
 b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 -
 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576
*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9
*b^3*c^9 + 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c - 525*a^2*b
^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8
192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14
080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^1
1*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14
*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4) + ((x^(
3/2)*(2*a*c - b^2))/(2*a*(4*a*c - b^2)) - (b*c*x^(7/2))/(2*a*(4*a*c - b^2))
)/(a + b*x^2 + c*x^4)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.1078 $\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=503

$$\frac{c^{3/4}\left(-3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}a\left(b^2-4ac\right)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}a\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+$$

```
[Out] 1/8*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(
3*b^2-28*a*c-3*b*(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-4*a*c+b^2)^(3/2)/(-b-(-4*a
*c+b^2)^(1/2))^(3/4)+1/8*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*
c+b^2)^(1/2))^(1/4))*(3*b^2-28*a*c-3*b*(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-4*a*
c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-1/8*c^(3/4)*arctan(2^(1/4)*c^(1/
4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(3*b^2-28*a*c+3*b*(-4*a*c+b^2)^(1
/2))*2^(3/4)/a/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-1/8*c^(3/4)
*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(3*b^2-28*a
*c+3*b*(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c+b^2)^(1
/2))^(3/4)+1/2*(b*c*x^2-2*a*c+b^2)*x^(1/2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)
```

Rubi [A] time = 1.28, antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1115, 1345, 1422, 212, 208, 205}

$$\frac{c^{3/4}\left(-3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}a\left(b^2-4ac\right)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}a\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
 (c^(3/4)*(3*b^2 - 28*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*
Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*a*(b^2 - 4*a*c)^(3/2)*
(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(3*b^2 - 28*a*c + 3*b*Sqrt[b^2 -
 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/
(4*2^(1/4)*a*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)
*(3*b^2 - 28*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])
/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*a*(b^2 - 4*a*c)^(3/2)*(-b - Sq
rt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(3*b^2 - 28*a*c + 3*b*Sqrt[b^2 - 4*a*c])
```

```
*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1
/4)*a*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1345

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(
x*(b^2 - 2*a*c + b*c*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^
2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + n*
(p + 1)*(b^2 - 4*a*c) + b*c*(n*(2*p + 3) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^
(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*
c, 0] && ILtQ[p, -1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{b^2-2ac-4\left(b^2-4ac\right)-3bcx^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2a\left(b^2-4ac\right)}$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(3b^2-28ac-3b\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2}}\right.}{4a\left(b^2-4ac\right)^{3/2}}$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(3b^2-28ac-3b\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{}{\sqrt{-b-\sqrt{b}}}\right.}{4a\left(b^2-4ac\right)^{3/2}\sqrt{-b-\sqrt{b^2-}}}$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{c^{3/4}\left(3b^2-28ac-3b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{}}{\sqrt[4]{-b-\sqrt{b^2-}}}\right.}{4\sqrt[4]{2}\,a\left(b^2-4ac\right)^{3/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}$$

Mathematica [C] time = 0.25, size = 153, normalized size = 0.30

$$-\frac{\left(a+bx^2+cx^4\right)\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{3\#1^4bc\log\left(\sqrt{x}-\#1\right)-14ac\log\left(\sqrt{x}-\#1\right)+3b^2\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]+4\sqrt{x}\left(-2ac+\right.}{8a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/8*(4*Sqrt[x]*(b^2 - 2*a*c + b*c*x^2) + (a + b*x^2 + c*x^4)*RootSum[a + b
*#1^4 + c*#1^8 & , (3*b^2*Log[Sqrt[x] - #1] - 14*a*c*Log[Sqrt[x] - #1] + 3*
b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(a*(-b^2 + 4*a*c)*(a +
 b*x^2 + c*x^4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 47.36Unable to convert to r
eal 1/4 Error: Bad Argument Value
```

maple [C] time = 0.02, size = 144, normalized size = 0.29

$$\frac{\left(-3\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^4 bc+14ac-3b^2\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{8\left(4ac-b^2\right)a\left(2\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7 c+\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3 b\right)}+\frac{-\frac{bc\,x^{\frac{5}{2}}}{2\left(4ac-b^2\right)a}+\frac{\left(2ac-b^{2}\right.}{2\left(4ac-\right.}}{c\,x^4+b\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*b/a/(4*a*c-b^2)*c*x^(5/2)+1/4*(2*a*c-b^2)/(4*a*c-b^2)/a*x^(1/2))/(c
*x^4+b*x^2+a)+1/8/a/(4*a*c-b^2)*sum((-3*_R^4*b*c+14*a*c-3*b^2)/(2*_R^7*c+_R
^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(3\,b^2c-14\,ac^2\right)x^{\frac{9}{2}}+\left(3\,b^3-13\,abc\right)x^{\frac{5}{2}}+4\left(ab^2-4\,a^2c\right)\sqrt{x}}{2\left(a^3b^2-4\,a^4c+\left(a^2b^2c-4\,a^3c^2\right)x^4+\left(a^2b^3-4\,a^3bc\right)x^2\right)}-\int\frac{\left(3\,b^2c-14\,ac^2\right)x^{\frac{7}{2}}+\left(3\,b^3-17\,abc\right)x^{\frac{3}{2}}}{4\left(a^3b^2-4\,a^4c+\left(a^2b^2c-4\,a^3c^2\right)x^4+\left(a^2b^3-4\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((3*b^2*c - 14*a*c^2)*x^(9/2) + (3*b^3 - 13*a*b*c)*x^(5/2) + 4*(a*b^2 -
 4*a^2*c)*sqrt(x))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*
b^3 - 4*a^3*b*c)*x^2) - integrate(1/4*((3*b^2*c - 14*a*c^2)*x^(7/2) + (3*b^
3 - 17*a*b*c)*x^(3/2))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (
a^2*b^3 - 4*a^3*b*c)*x^2), x)
```

mupad [B] time = 7.29, size = 35171, normalized size = 69.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] ((x^(1/2)*(2*a*c - b^2))/(2*a*(4*a*c - b^2)) - (b*c*x^(5/2))/(2*a*(4*a*c -
b^2)))/(a + b*x^2 + c*x^4) + atan(((((((81*b^8*(-(4*a*c - b^2)^15)^(1/2) -
 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^
3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6
+ 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 -
 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a
*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(
-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a
^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^
10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^
12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6
*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*(285212672*
a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 4620288*a^6*b^11*c
^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9 -
 478150656*a^10*b^3*c^10))/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^
6*b^4*c^2 - 256*a^7*b^2*c^3)) - (x^(1/2)*(12683575296*a^11*b*c^13 - 36864*a
^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6 + 233816064*a^5*
b^13*c^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9 - 16436428800*a
^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12))/(16*(a
^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^
3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/
2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^1
7*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*
c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c
^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 40
23*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c
^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(819
2*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 1408
0*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^1
3*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16
*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4) - (5378
24*a^4*c^11 + 891*b^8*c^7 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 510384*a
^3*b^2*c^10)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 25
6*a^7*b^2*c^3)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a
^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*
c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^
7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c
```

^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4) + (x^(1/2)*(15059072*a^4*c^13 + 9801*b^8*c^9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*a^3*b^2*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*1i - ((((((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*(285212672*a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 4620288*a^6*b^11*c^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9 - 478150656*a^10*b^3*c^10))/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) + (x^(1/2)*(12683575296*a^11*b*c^13 - 36864*a^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6 + 233816064*a^5*b^13*c^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9 - 16436428800*a^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/

2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^1
9*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a
^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14
*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b
^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4) - (537824*a^4*c^11 + 891*b^8*c^7
 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 510384*a^3*b^2*c^10)/(2*(a^4*b^8 +
 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((81*b^8*
(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^1
9*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^
5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8
 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*
c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^
(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c
 - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1
056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12
*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*
b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^
2*c^11)))^(1/4) - (x^(1/2)*(15059072*a^4*c^13 + 9801*b^8*c^9 - 227502*a*b^6
*c^10 + 2092104*a^2*b^4*c^11 - 8989344*a^3*b^2*c^12))/(16*(a^4*b^12 + 4096*
a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^
4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 +
741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 1058838
4*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784
*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888
*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 1
0746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b
^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 1
6777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3
 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12
976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 6920
6016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*1i)/(((((((81*b^8*(-(4
*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^
2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 -
279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2
494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c -
b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2
) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b
^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*
a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^1
4*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*
c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^
11)))^(1/4)*(285212672*a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c
^5 - 4620288*a^6*b^11*c^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 +
342884352*a^9*b^5*c^9 - 478150656*a^10*b^3*c^10))/(2*(a^4*b^8 + 256*a^8*c^4
 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) - (x^(1/2)*(1268357529

6*a^11*b*c^13 - 36864*a^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6 + 233816064*a^5*b^13*c^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9 - 16436428800*a^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4) - (537824*a^4*c^11 + 891*b^8*c^7 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 510384*a^3*b^2*c^10)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4) + (x^(1/2)*(15059072*a^4*c^13 + 9801*b^8*c^9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*a^3*b^2*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4) + ((((((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 20386

93888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*
c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*
c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^2
4 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^1
8*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6
 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 +
 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*(285212672*a^11*b
*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 4620288*a^6*b^11*c^6 + 3
2440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9 - 47815
0656*a^10*b^3*c^10))/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*
c^2 - 256*a^7*b^2*c^3)) + (x^(1/2)*(12683575296*a^11*b*c^13 - 36864*a^2*b^1
9*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6 + 233816064*a^5*b^13*c
^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9 - 16436428800*a^8*b^7
*c^10 + 28575793152*a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12))/(16*(a^4*b^1
2 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 38
40*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 8
1*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3
- 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 +
853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2
038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b
^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(
4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7
*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10
*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12
*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c
^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4) - (537824*a^4
*c^11 + 891*b^8*c^7 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 510384*a^3*b^2
c^10)/(2(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*
b^2*c^3)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*
c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 +
64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 17
99626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 +
9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*
(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1
593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12
 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^
16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*
c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^1
0 - 50331648*a^18*b^2*c^11)))^(1/4) - (x^(1/2)*(15059072*a^4*c^13 + 9801*b^
8*c^9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*a^3*b^2*c^12))/(
16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b
^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15
)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^
3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*
b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*

b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2)
 + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*
b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))
/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 -
 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 378470
4*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680
*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)))*
((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 9012
6*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^
5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^
8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^
4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c -
b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c
*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b
^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 81
1008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440
320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 5033164
8*a^18*b^2*c^11)))^(1/4)*2i + atan(((((((-(81*b^23 + 81*b^8*(-(4*a*c - b^2)
^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*
c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^
6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9
 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023
*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3
*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*
(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*
a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*
b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b
^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*(28521267
2*a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 4620288*a^6*b^11
*c^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9
 - 478150656*a^10*b^3*c^10))/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*
a^6*b^4*c^2 - 256*a^7*b^2*c^3)) - (x^(1/2)*(12683575296*a^11*b*c^13 - 36864
*a^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6 + 233816064*a^
5*b^13*c^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9 - 16436428800
*a^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12))/(16*
(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*
c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*c -
 b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*
b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^
11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^
5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) -
 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^
2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(
8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 1
4080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*

a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a
^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4) - (5
37824*a^4*c^11 + 891*b^8*c^7 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 51038
4*a^3*b^2*c^10)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 -
256*a^7*b^2*c^3)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 7418019
84*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b
^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^
9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b
^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^
2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)
^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216
*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 1267
20*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*
a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^
17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4) + (x^(1/2)*(15059072*a^4*c^13
+ 9801*b^8*c^9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*a^3*b^
2*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 -
1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(81*b^23 + 81*b
^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 -
1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279
571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494
119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2
)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) -
26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)
^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9
*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c
^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8
- 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)
))^(1/4)*1i - (((((-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984
*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^1
5*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*
c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3
*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*
b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(
1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a
^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720
*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^
14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17
*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*(285212672*a^11*b*c^11 - 12288*
a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 4620288*a^6*b^11*c^6 + 32440320*a^7*b^
9*c^7 - 136314880*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9 - 478150656*a^10*b^3*
c^10))/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*
b^2*c^3)) + (x^(1/2)*(12683575296*a^11*b*c^13 - 36864*a^2*b^19*c^4 + 141312
0*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6 + 233816064*a^5*b^13*c^7 - 145942118
4*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9 - 16436428800*a^8*b^7*c^10 + 285757

93152*a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12))/(16*(a^4*b^12 + 4096*a^10*
c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4
 - 6144*a^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 7418
01984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^
4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7
*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^1
0*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746
*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^
15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777
216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 1
26720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 129761
28*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016
*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4) - (537824*a^4*c^11 + 891*b
^8*c^7 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 510384*a^3*b^2*c^10)/(2*(a^
4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-
(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126
*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5
*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8
*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4
*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b
^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*
(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^
22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811
008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 324403
20*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648
*a^18*b^2*c^11)))^(1/4) - (x^(1/2)*(15059072*a^4*c^13 + 9801*b^8*c^9 - 2275
02*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*a^3*b^2*c^12))/(16*(a^4*b^12
 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 384
0*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)
^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3
+ 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 -
853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2
038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b
^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(
4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7
*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10
*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12
*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c
^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*1i)/((((((-(8
1*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a
^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b
^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b
^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(
-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2
)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-

(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22
*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 81100
8*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320
*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a
^18*b^2*c^11)))^(1/4)*(285212672*a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*
a^5*b^13*c^5 - 4620288*a^6*b^11*c^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*
b^7*c^8 + 342884352*a^9*b^5*c^9 - 478150656*a^10*b^3*c^10))/(2*(a^4*b^8 + 2
56*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) - (x^(1/2)*(
12683575296*a^11*b*c^13 - 36864*a^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891
968*a^4*b^15*c^6 + 233816064*a^5*b^13*c^7 - 1459421184*a^6*b^11*c^8 + 60238
06976*a^7*b^9*c^9 - 16436428800*a^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 2
8705816576*a^10*b^3*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c +
240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))
*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90
126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*
a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*
a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*
c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c
- b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6
c(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8
*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 -
811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 324
40320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331
648*a^18*b^2*c^11)))^(3/4) - (537824*a^4*c^11 + 891*b^8*c^7 - 19548*a*b^6*c
^8 + 155358*a^2*b^4*c^9 - 510384*a^3*b^2*c^10)/(2*(a^4*b^8 + 256*a^8*c^4 -
16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(81*b^23 + 81*b^8*(-(4
*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 120162
3*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*
a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*
a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(
1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*
a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1
/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c
^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 37
84704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 5767
1680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4
) + (x^(1/2)*(15059072*a^4*c^13 + 9801*b^8*c^9 - 227502*a*b^6*c^10 + 209210
4*a^2*b^4*c^11 - 8989344*a^3*b^2*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*
a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a
^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^1
1*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^
4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7
+ 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^1
0 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*
c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2)

- 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4) + (((((-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*(285212672*a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 4620288*a^6*b^11*c^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9 - 478150656*a^10*b^3*c^10))/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) + (x^(1/2)*(12683575296*a^11*b*c^13 - 36864*a^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6 + 233816064*a^5*b^13*c^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9 - 16436428800*a^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4) - (537824*a^4*c^11 + 891*b^8*c^7 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 510384*a^3*b^2*c^10)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 129

76128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206
016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4) - (x^(1/2)*(15059072*a^
4*c^13 + 9801*b^8*c^9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*
a^3*b^2*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*
c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(81*b^23
+ 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19
*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5
 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8
- 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c
 - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(
1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c
- b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 10
56*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*
b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b
^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2
c^11)))^(1/4)))(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*
a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15
*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c
^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*
c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b
^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1
/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^
19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*
a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^1
4*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*
b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*2i + 2*atan(((((((((81*b^8*(-(4*a
*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2
+ 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 27
9571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 249
4119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^
2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2)
- 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2
)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^
9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*
c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^
8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11
)))^(1/4)*(285212672*a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c^5
 - 4620288*a^6*b^11*c^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 + 34
2884352*a^9*b^5*c^9 - 478150656*a^10*b^3*c^10)*1i)/(2*(a^4*b^8 + 256*a^8*c^
4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) - (x^(1/2)*(126835752
96*a^11*b*c^13 - 36864*a^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b
^15*c^6 + 233816064*a^5*b^13*c^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7
*b^9*c^9 - 16436428800*a^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 2870581657
6*a^10*b^3*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b
^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8

*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4)*1i + (537824*a^4*c^11 + 891*b^8*c^7 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 510384*a^3*b^2*c^10)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*1i - (x^(1/2)*(15059072*a^4*c^13 + 9801*b^8*c^9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*a^3*b^2*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4) - ((((((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b

^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*(285212672*a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 4620288*a^6*b^11*c^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9 - 478150656*a^10*b^3*c^10)*1i)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) + (x^(1/2)*(12683575296*a^11*b*c^13 - 36864*a^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6 + 233816064*a^5*b^13*c^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9 - 16436428800*a^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4)*1i + (537824*a^4*c^11 + 891*b^8*c^7 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 510384*a^3*b^2*c^10)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*1i + (x^(1/2)*(15059072*a^4*c^13 + 9801*b^8*c^9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*a^3*b^2*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b

^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4))/(((((((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*(285212672*a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 4620288*a^6*b^11*c^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9 - 478150656*a^10*b^3*c^10)*1i)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) - (x^(1/2)*(12683575296*a^11*b*c^13 - 36864*a^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6 + 233816064*a^5*b^13*c^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9 - 16436428800*a^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4)*1i + (537824*a^4*c^11 + 891*b^8*c^7 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 510384*a^3*b^2*c^10)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*1i - (x^(1/2)*(15059072*a^4*c^13 + 9801*b^8*c^

9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*a^3*b^2*c^12))/(16*(
a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c
^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1
/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^
17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11
*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*
c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4
023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*
c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(81
92*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 140
80*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^
13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^1
6*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*1i + (
(((((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 9
0126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576
*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752
*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4
c^4(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c
 - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^
6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^
8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 -
 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32
440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 5033
1648*a^18*b^2*c^11)))^(1/4)*(285212672*a^11*b*c^11 - 12288*a^4*b^15*c^4 + 3
64544*a^5*b^13*c^5 - 4620288*a^6*b^11*c^6 + 32440320*a^7*b^9*c^7 - 13631488
0*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9 - 478150656*a^10*b^3*c^10)*1i)/(2*(a^
4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) + (
x^(1/2)*(12683575296*a^11*b*c^13 - 36864*a^2*b^19*c^4 + 1413120*a^3*b^17*c^
5 - 23891968*a^4*b^15*c^6 + 233816064*a^5*b^13*c^7 - 1459421184*a^6*b^11*c^
8 + 6023806976*a^7*b^9*c^9 - 16436428800*a^8*b^7*c^10 + 28575793152*a^9*b^5
*c^11 - 28705816576*a^10*b^3*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*
b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b
^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c
^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 6
4704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 179
9626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9
604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(
-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15
93*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12
 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^1
6*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c
^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10
 - 50331648*a^18*b^2*c^11)))^(3/4)*1i + (537824*a^4*c^11 + 891*b^8*c^7 - 19
548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 510384*a^3*b^2*c^10)/(2*(a^4*b^8 + 256
*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((81*b^8*(-(4

*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*1i + (x^(1/2)*(15059072*a^4*c^13 + 9801*b^8*c^9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*a^3*b^2*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*1i))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4) + 2*atan(((((((-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*(285212672*a^11*b*c^11 - 122

88*a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 4620288*a^6*b^11*c^6 + 32440320*a^7
*b^9*c^7 - 136314880*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9 - 478150656*a^10*b
^3*c^10)*1i)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 25
6*a^7*b^2*c^3)) - (x^(1/2)*(12683575296*a^11*b*c^13 - 36864*a^2*b^19*c^4 +
1413120*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6 + 233816064*a^5*b^13*c^7 - 145
9421184*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9 - 16436428800*a^8*b^7*c^10 +
28575793152*a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12))/(16*(a^4*b^12 + 4096
*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b
^4*c^4 - 6144*a^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2)
- 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588
384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 8531747
84*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 20386938
88*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c +
10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c -
b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 +
16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c
^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 -
12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69
206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4)*1i + (537824*a^4*c^1
1 + 891*b^8*c^7 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 510384*a^3*b^2*c^1
0)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*
c^3)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^1
1 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 647
04576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 17996
26752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 960
4*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(
4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593
*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 -
48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*
c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7
+ 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 -
50331648*a^18*b^2*c^11)))^(1/4)*1i - (x^(1/2)*(15059072*a^4*c^13 + 9801*b^
8*c^9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*a^3*b^2*c^12))/(
16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b
^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*
c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a
^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6
*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9
*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2
) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3
*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2)
)/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2
- 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 37847
04*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 5767168
0*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4) -

(((((-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*(285212672*a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 4620288*a^6*b^11*c^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9 - 478150656*a^10*b^3*c^10)*1i)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) + (x^(1/2)*(12683575296*a^11*b*c^13 - 36864*a^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6 + 233816064*a^5*b^13*c^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9 - 16436428800*a^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4)*1i + (537824*a^4*c^11 + 891*b^8*c^7 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 510384*a^3*b^2*c^10)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*1i + (x^(1/2)*(15059072*a^4*c^13 + 9801*b^8*c^9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*a^3*b^2*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^

8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4))/((((((-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*(285212672*a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 4620288*a^6*b^11*c^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9 - 478150656*a^10*b^3*c^10)*1i)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) - (x^(1/2)*(12683575296*a^11*b*c^13 - 36864*a^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6 + 233816064*a^5*b^13*c^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9 - 16436428800*a^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4)*1i + (537824*a^4*c^11 + 891*b^8*c^7 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 510384*a^3*b^2*c^10)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a

^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a
^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1
/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a
^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/
2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^
2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 378
4704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671
680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)
1i - (x^(1/2)(15059072*a^4*c^13 + 9801*b^8*c^9 - 227502*a*b^6*c^10 + 2092
104*a^2*b^4*c^11 - 8989344*a^3*b^2*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 2
4*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144
*a^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a
^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*
c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^
7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c
^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^
4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/
2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^1
9*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a
^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14
*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b
^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*1i + (((((-(81*b^23 + 81*b^8*(-(4
*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 120162
3*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*
a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*
a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(
1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*
a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1
/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c
^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 37
84704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 5767
1680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4
)*(285212672*a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 46202
88*a^6*b^11*c^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 + 342884352*
a^9*b^5*c^9 - 478150656*a^10*b^3*c^10)*1i)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a
^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) + (x^(1/2)*(12683575296*a^11*
b*c^13 - 36864*a^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6
+ 233816064*a^5*b^13*c^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9
 - 16436428800*a^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 28705816576*a^10*b
^3*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 -
 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(81*b^23 + 81*
b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2
- 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 27
9571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 249
4119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^

2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^ 9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14* c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^ 8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4)*1i + (537824*a^4*c^11 + 891*b^8*c^7 - 19548*a*b^6*c^8 + 155358*a^ 2*b^4*c^9 - 510384*a^3*b^2*c^10)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a* b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(- (4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^ 7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^1 0*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^1 2*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6* c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*1i + (x^(1/2)*(15059072*a^4*c^13 + 9801*b^8*c^9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c ^11 - 8989344*a^3*b^2*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 6470457 6*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 853174784*a^7*b^9*c^7 + 179962675 2*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 2038693888*a^10*b^3*c^10 + 9604*a^ 4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a* c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b ^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a ^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 3 2440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 503 31648*a^18*b^2*c^11)))^(1/4)*1i))*(-(81*b^23 + 81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 741801984*a^11*b*c^11 + 90126*a^2*b^19*c^2 - 1201623*a^3*b^17*c^3 + 10588384*a^4*b^15*c^4 - 64704576*a^5*b^13*c^5 + 279571968*a^6*b^11*c^6 - 85 3174784*a^7*b^9*c^7 + 1799626752*a^8*b^7*c^8 - 2494119936*a^9*b^5*c^9 + 203 8693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) - 4023*a*b^2 1*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4* a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b ^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b ^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c ^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.1079 $\int \frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=573

$$-\frac{5b^2-18ac}{2a^2\sqrt{x}\left(b^2-4ac\right)}+\frac{\sqrt[4]{c}\left(-\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a^2\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\left(5b^2-18ac\right)\sqrt{b\right.}}{4\,2^{3/4}a^2}$$

```
[Out] 1/8*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(
5*b^3-28*a*b*c-(-18*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))*2^(1/4)/a^2/(-4*a*c+b^2)
^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/8*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^
(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(5*b^3-28*a*b*c-(-18*a*c+5*b^2)*(-4*a*
c+b^2)^(1/2))*2^(1/4)/a^2/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-
1/8*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(
5*b^3-28*a*b*c+(-18*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))*2^(1/4)/a^2/(-4*a*c+b^2)
^(3/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/8*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^
(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(5*b^3-28*a*b*c+(-18*a*c+5*b^2)*(-4*a*
c+b^2)^(1/2))*2^(1/4)/a^2/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+
1/2*(18*a*c-5*b^2)/a^2/(-4*a*c+b^2)/x^(1/2)+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a
*c+b^2)/(c*x^4+b*x^2+a)/x^(1/2)
```

Rubi [A] time = 2.45, antiderivative size = 573, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1115, 1366, 1504, 1510, 298, 205, 208}

$$-\frac{5b^2-18ac}{2a^2\sqrt{x}\left(b^2-4ac\right)}+\frac{\sqrt[4]{c}\left(-\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a^2\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\left(5b^2-18ac\right)\sqrt{b\right.}}{4\,2^{3/4}a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(5*b^2 - 18*a*c)/(2*a^2*(b^2 - 4*a*c)*Sqrt[x]) + (b^2 - 2*a*c + b*c*x^2)/(
2*a*(b^2 - 4*a*c)*Sqrt[x]*(a + b*x^2 + c*x^4)) + (c^(1/4)*(5*b^3 - 28*a*b*c
 - (5*b^2 - 18*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a^2*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt
[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(5*b^3 - 28*a*b*c + (5*b^2 - 18*a*c)*Sqrt[
b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/
4)])/(4*2^(3/4)*a^2*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (
c^(1/4)*(5*b^3 - 28*a*b*c - (5*b^2 - 18*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^
```

```
(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a^2*(b^2
 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(5*b^3 - 28*a*b*
c + (5*b^2 - 18*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a^2*(b^2 - 4*a*c)^(3/2)*(-b + Sq
rt[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1366

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^n)*(a + b*x^n + c*x^(
2*n))^(p + 1))/(a*d*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2
 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(m + n*(p
+ 1) + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(m + n*(2*p + 3) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0
] && IGtQ[n, 0] && ILtQ[p, -1]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
```

```
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-5b^2+18ac-5bcx^4}{x^2\left(a+bx^4+cx^8\right)}\,dx, x, \sqrt{x}\right)}{2a\left(b^2-4ac\right)}$$

$$= -\frac{5b^2-18ac}{2a^2\left(b^2-4ac\right)\sqrt{x}} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(-b\left(5b^2-23ac \cdots\right.\right.}{a+bx \cdots} \cdots\right)}{2a^2\left(b \cdots\right.}$$

$$= -\frac{5b^2-18ac}{2a^2\left(b^2-4ac\right)\sqrt{x}} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(5b^2-18ac+\frac{5b^3 \cdots}{\sqrt{b^2-4 \cdots}}\right.\right.}{\cdots}$$

$$= -\frac{5b^2-18ac}{2a^2\left(b^2-4ac\right)\sqrt{x}} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx^2+cx^4\right)} + \frac{\left(\sqrt{c}\left(5b^2-18ac+\frac{5 \cdots}{\sqrt{b^2 \cdots}}\right.\right.}{\cdots}$$

$$= -\frac{5b^2-18ac}{2a^2\left(b^2-4ac\right)\sqrt{x}} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx^2+cx^4\right)} - \frac{\sqrt[4]{c}\left(5b^2-18ac-\frac{5b \cdots}{\sqrt{b^2- \cdots}}\right.}{4\,2^{3/4}a^2\left(b^2 \cdots\right.}$$

Mathematica [C] time = 0.34, size = 190, normalized size = 0.33

$$-\frac{\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{-18\#1^4ac^2\log\left(\sqrt{x}-\#1\right)+5\#1^4b^2c\log\left(\sqrt{x}-\#1\right)-23abc\log\left(\sqrt{x}-\#1\right)+5b^3\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]}{b^2-4ac}+\frac{4x^{3/2}\left(-3abc-2ac^2x^2+b^3+b^2cx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{16}{\sqrt{x}}}{8a^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(3/2)*(a + b*x^2 + c*x^4)^2),x]

[Out] -1/8*(16/Sqrt[x] + (4*x^(3/2)*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + RootSum[a + b*#1^4 + c*#1^8 & , (5*b^3*Log[Sqrt[x] - #1] - 23*a*b*c*Log[Sqrt[x] - #1] + 5*b^2*c*Log[Sqrt[x] - #1]*#1^4 - 18*a*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) &]/(b^2 - 4*a*c))/a^2

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 50.43Unable to convert to real 1/4 Error: Bad Argument Value

maple [C] time = 0.03, size = 245, normalized size = 0.43

$$-\frac{c^2x^{\frac{7}{2}}}{\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a}+\frac{b^2c\,x^{\frac{7}{2}}}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a^2}-\frac{3bc\,x^{\frac{3}{2}}}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a}+\frac{b^3}{2\left(c\,x^4+b\,x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^(7/2)+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a
*c-b^2)*x^(7/2)*b^2-3/2/a/(c*x^4+b*x^2+a)*b/(4*a*c-b^2)*x^(3/2)*c+1/2/a^2/(
c*x^4+b*x^2+a)*b^3/(4*a*c-b^2)*x^(3/2)-1/8/a^2/(4*a*c-b^2)*sum((c*(18*a*c-5
*b^2)*_R^6+b*(23*a*c-5*b^2)*_R^2)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=Root
Of(_Z^8*c+_Z^4*b+a))-2/a^2/x^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(5\,b^2c-18\,ac^2\right)x^{\frac{7}{2}}+\left(5\,b^3-19\,abc\right)x^{\frac{3}{2}}+\frac{4\left(ab^2-4\,a^2c\right)}{\sqrt{x}}}{2\left(a^3b^2-4\,a^4c+\left(a^2b^2c-4\,a^3c^2\right)x^4+\left(a^2b^3-4\,a^3bc\right)x^2\right)}-\int\frac{\left(5\,b^2c-18\,ac^2\right)x^{\frac{5}{2}}+\left(5\,b^3-23\,abc\right)\sqrt{x}}{4\left(a^3b^2-4\,a^4c+\left(a^2b^2c-4\,a^3c^2\right)x^4+\left(a^2b^3-4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((5*b^2*c - 18*a*c^2)*x^(7/2) + (5*b^3 - 19*a*b*c)*x^(3/2) + 4*(a*b^2
- 4*a^2*c)/sqrt(x))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2
*b^3 - 4*a^3*b*c)*x^2) - integrate(1/4*((5*b^2*c - 18*a*c^2)*x^(5/2) + (5*b
^3 - 23*a*b*c)*sqrt(x))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 +
(a^2*b^3 - 4*a^3*b*c)*x^2), x)
```

mupad [B] time = 11.42, size = 31145, normalized size = 54.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] atan(((x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 260
4992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b^1
7*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175
670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^17 - 1197821248143
360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 1520311317037056*a^30*
b^3*c^20) + (-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a
^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17
*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^
11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^
10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)
^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181
990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b
^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 +
16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*
c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 -
```

12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 6
9206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(32768000*a^21*b^34
*c^4 - 25649407252758528*a^38*c^21 - 2123366400*a^22*b^32*c^5 + 64398295040
*a^23*b^30*c^6 - 1213399564288*a^24*b^28*c^7 + 15898363035648*a^25*b^26*c^8
 - 153599583715328*a^26*b^24*c^9 + 1132021560639488*a^27*b^22*c^10 - 649291
7279490048*a^28*b^20*c^11 + 29298398985191424*a^29*b^18*c^12 - 104398826088
955904*a^30*b^16*c^13 + 293000581579014144*a^31*b^14*c^14 - 641705669216436
224*a^32*b^12*c^15 + 1077743462209552384*a^33*b^10*c^16 - 13483557107143802
88*a^34*b^8*c^17 + 1198053158392168448*a^35*b^6*c^18 - 695801744382230528*a
^36*b^4*c^19 + 223957324438437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 - 625
b^10(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*
c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5
 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*
c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^1
1*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 684
75*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^
2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c
*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*
b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 -
811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 324
40320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331
648*a^20*b^2*c^11)))^(1/4)*(91197892454252544*a^40*c^21 - 52428800*a^23*b^3
4*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*c^6 + 19860742471
68*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 257340683059200*a^28*b^24
*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648*a^30*b^20*c^11 -
 51694329453871104*a^31*b^18*c^12 + 188531248770056192*a^32*b^16*c^13 - 543
721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*b^12*c^15 - 21466
20531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b^8*c^17 - 2657721
914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*c^19 - 6124895493
22387456*a^39*b^2*c^20)))*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2)
+ 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 714
83001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 599
6689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 +
21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*
a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^1
5)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^
4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(819
2*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14
080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a
^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^
18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*1i +
(x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 260499200
0*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b^17*c^13
 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175670703
423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^17 - 1197821248143360*a^

28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 1520311317037056*a^30*b^3*c^20) + (-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(25649407252758528*a^38*c^21 - 32768000*a^21*b^34*c^4 + 2123366400*a^22*b^32*c^5 - 64398295040*a^23*b^30*c^6 + 1213399564288*a^24*b^28*c^7 - 15898363035648*a^25*b^26*c^8 + 153599583715328*a^26*b^24*c^9 - 1132021560639488*a^27*b^22*c^10 + 6492917279490048*a^28*b^20*c^11 - 29298398985191424*a^29*b^18*c^12 + 104398826088955904*a^30*b^16*c^13 - 293000581579014144*a^31*b^14*c^14 + 641705669216436224*a^32*b^12*c^15 - 1077743462209552384*a^33*b^10*c^16 + 1348355710714380288*a^34*b^8*c^17 - 1198053158392168448*a^35*b^6*c^18 + 695801744382230528*a^36*b^4*c^19 - 223957324438437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(9119789245425254 4*a^40*c^21 - 52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*c^6 + 1986074247168*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 257340683059200*a^28*b^24*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648*a^30*b^20*c^11 - 51694329453871104*a^31*b^18*c^12 + 188531248770056192*a^32*b^16*c^13 - 543721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*b^12*c^15 - 2146620531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b^8*c^17 - 2657721914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*c^19 - 612489549322387456*a^39*b^2*c^20)))*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c -

b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*1i)/((x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 2604992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b^17*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^17 - 1197821248143360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 1520311317037056*a^30*b^3*c^20) + (-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(32768000*a^21*b^34*c^4 - 25649407252758528*a^38*c^21 - 2123366400*a^22*b^32*c^5 + 64398295040*a^23*b^30*c^6 - 1213399564288*a^24*b^28*c^7 + 15898363035648*a^25*b^26*c^8 - 153599583715328*a^26*b^24*c^9 + 1132021560639488*a^27*b^22*c^10 - 6492917279490048*a^28*b^20*c^11 + 29298398985191424*a^29*b^18*c^12 - 104398826088955904*a^30*b^16*c^13 + 293000581579014144*a^31*b^14*c^14 - 641705669216436224*a^32*b^12*c^15 + 1077743462209552384*a^33*b^10*c^16 - 1348355710714380288*a^34*b^8*c^17 + 1198053158392168448*a^35*b^6*c^18 - 695801744382230528*a^36*b^4*c^19 + 223957324438437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(9119789245425254 4*a^40*c^21 - 52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*c^6 + 1986074247168*a^26*b^28

*c^7 - 26302715265024*a^27*b^26*c^8 + 257340683059200*a^28*b^24*c^9 - 19246
94567550976*a^29*b^22*c^10 + 11230133666971648*a^30*b^20*c^11 - 51694329453
871104*a^31*b^18*c^12 + 188531248770056192*a^32*b^16*c^13 - 543721556635811
840*a^33*b^14*c^14 + 1229750704231415808*a^34*b^12*c^15 - 2146620531372195 8
40*a^35*b^10*c^16 + 2815880065059913728*a^36*b^8*c^17 - 2657721914474102784
*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*c^19 - 612489549322387456*a^3
9*b^2*c^20)))*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360
*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^
17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*
b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*
a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^1
5)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 1
81990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c -
b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24
+ 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^1
8*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6
- 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 +
69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4) - (x^(1/2)*(60233
2119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 2604992000*a^21*b^21*c^1
1 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b^17*c^13 - 655774764236
8*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175670703423488*a^26*b^1
1*c^16 + 548447002296320*a^27*b^9*c^17 - 1197821248143360*a^28*b^7*c^18 + 1
742819580444672*a^29*b^5*c^19 - 1520311317037056*a^30*b^3*c^20) + (-(625*b^
25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a
^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*
b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*
a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 1257504
7680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23
*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*
a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875
*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 -
48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^1
6*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c
^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10
- 50331648*a^20*b^2*c^11)))^(3/4)*(2564940725275852 8*a^38*c^21 - 32768000*
a^21*b^34*c^4 + 2123366400*a^22*b^32*c^5 - 64398295040*a^23*b^30*c^6 + 1213
399564288*a^24*b^28*c^7 - 15898363035648*a^25*b^26*c^8 + 153599583715328*a^
26*b^24*c^9 - 1132021560639488*a^27*b^22*c^10 + 6492917279490048*a^28*b^20*
c^11 - 29298398985191424*a^29*b^18*c^12 + 104398826088955904*a^30*b^16*c^13
- 293000581579014144*a^31*b^14*c^14 + 641705669216436224*a^32*b^12*c^15 -
1077743462209552384*a^33*b^10*c^16 + 1348355710714380288*a^34*b^8*c^17 - 11
98053158392168448*a^35*b^6*c^18 + 695801744382230528*a^36*b^4*c^19 - 223957
324438437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)
^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^1
9*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^1

3*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(91197892454252544*a^40*c^21 - 52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*c^6 + 1986074247168*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 257340683059200*a^28*b^24*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648*a^30*b^20*c^11 - 51694329453871104*a^31*b^18*c^12 + 188531248770056192*a^32*b^16*c^13 - 543721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*b^12*c^15 - 2146620531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b^8*c^17 - 2657721914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*c^19 - 612489549322387456*a^39*b^2*c^20)))*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4) - 89161004482560*a^29*b*c^21 + 175760000*a^20*b^19*c^12 - 6846528000*a^21*b^17*c^13 + 118362316800*a^22*b^15*c^14 - 1191953858560*a^23*b^13*c^15 + 7705795952640*a^24*b^11*c^16 - 33166059110400*a^25*b^9*c^17 + 95038786764800*a^26*b^7*c^18 - 174846482841600*a^27*b^5*c^19 + 187403222384640*a^28*b^3*c^20))*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*2i - (2/a - (x^2*(5*b^3 - 19*a*b*c))/(2*a^2*(4*a*c -

b^2)) + (c*x^4*(18*a*c - 5*b^2))/(2*a^2*(4*a*c - b^2)))/(a*x^(1/2) + b*x^(5/2) + c*x^(9/2)) + atan(((x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 2604992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b^17*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^17 - 1197821248143360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 1520311317037056*a^30*b^3*c^20) + (-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(32768000*a^21*b^34*c^4 - 2564940725275852 8*a^38*c^21 - 2123366400*a^22*b^32*c^5 + 64398295040*a^23*b^30*c^6 - 1213399564288*a^24*b^28*c^7 + 15898363035648*a^25*b^26*c^8 - 153599583715328*a^26*b^24*c^9 + 1132021560639488*a^27*b^22*c^10 - 6492917279490048*a^28*b^20*c^11 + 29298398985191424*a^29*b^18*c^12 - 104398826088955904*a^30*b^16*c^13 + 293000581579014144*a^31*b^14*c^14 - 641705669216436224*a^32*b^12*c^15 + 1077743462209552384*a^33*b^10*c^16 - 1348355710714380288*a^34*b^8*c^17 + 1198053158392168448*a^35*b^6*c^18 - 695801744382230528*a^36*b^4*c^19 + 223957324438437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(91197892454252544*a^40*c^21 - 52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*c^6 + 1986074247168*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 257340683059200*a^28*b^24*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648*a^30*b^20*c^11 - 51694329453871104*a^31*b^18*c^12 + 188531248770056192*a^32*b^16*c^13 - 543721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*b^12*c^15 - 2146620531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b^8*c^17 - 2657721914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^

4*c^19 - 612489549322387456*a^39*b^2*c^20)))*(-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*1i + (x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 2604992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b^17*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^17 - 1197821248143360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 1520311317037056*a^30*b^3*c^20) + (-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(25649407252758528*a^38*c^21 - 32768000*a^21*b^34*c^4 + 2123366400*a^22*b^32*c^5 - 64398295040*a^23*b^30*c^6 + 1213399564288*a^24*b^28*c^7 - 15898363035648*a^25*b^26*c^8 + 153599583715328*a^26*b^24*c^9 - 1132021560639488*a^27*b^22*c^10 + 6492917279490048*a^28*b^20*c^11 - 29298398985191424*a^29*b^18*c^12 + 104398826088955904*a^30*b^16*c^13 - 293000581579014144*a^31*b^14*c^14 + 641705669216436224*a^32*b^12*c^15 - 1077743462209552384*a^33*b^10*c^16 + 1348355710714380288*a^34*b^8*c^17 - 1198053158392168448*a^35*b^6*c^18 + 695801744382230528*a^36*b^4*c^19 - 223957324438437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^

12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^1
3*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b
^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4
*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(9119789245425254*a^40*c^21 - 5242
8800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*c^6
+ 1986074247168*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 257340683059
200*a^28*b^24*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648*a^3
0*b^20*c^11 - 51694329453871104*a^31*b^18*c^12 + 188531248770056192*a^32*b^
16*c^13 - 543721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*b^12
*c^15 - 2146620531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b^8*c
^17 - 2657721914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*c^19
 - 612489549322387456*a^39*b^2*c^20)))*(-(625*b^25 + 625*b^10*(-(4*a*c - b^
2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b
^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b
^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a
^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*
a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4
*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 1718
01*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15
)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*
b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^
5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8
 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11))
)^(1/4)*1i)/((x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^1
0 + 2604992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a
^23*b^17*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^1
5 - 175670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^17 - 119782
1248143360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 152031131703705
6*a^30*b^3*c^20) + (-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 31054
23360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a
^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920
*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 2148301
2096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b
^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2
) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*
a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*
b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^1
2*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^1
2*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*
c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(32768000*a^
21*b^34*c^4 - 25649407252758528*a^38*c^21 - 2123366400*a^22*b^32*c^5 + 6439
8295040*a^23*b^30*c^6 - 1213399564288*a^24*b^28*c^7 + 15898363035648*a^25*b
^26*c^8 - 153599583715328*a^26*b^24*c^9 + 1132021560639488*a^27*b^22*c^10 -
 6492917279490048*a^28*b^20*c^11 + 29298398985191424*a^29*b^18*c^12 - 10439
8826088955904*a^30*b^16*c^13 + 293000581579014144*a^31*b^14*c^14 - 64170566

9216436224*a^32*b^12*c^15 + 1077743462209552384*a^33*b^10*c^16 - 1348355710
714380288*a^34*b^8*c^17 + 1198053158392168448*a^35*b^6*c^18 - 6958017443822
30528*a^36*b^4*c^19 + 223957324438437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^2
5 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^
2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b
^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a
^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047
680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*
c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a
*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*
a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 4
8*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16
*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^
7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10
- 50331648*a^20*b^2*c^11)))^(1/4)*(91197892454252544*a^40*c^21 - 52428800*a
^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*c^6 + 1986
074247168*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 257340683059200*a^
28*b^24*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648*a^30*b^20
*c^11 - 51694329453871104*a^31*b^18*c^12 + 188531248770056192*a^32*b^16*c^1
3 - 543721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*b^12*c^15
- 2146620531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b^8*c^17 -
2657721914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*c^19 - 612
489549322387456*a^39*b^2*c^20)))*(-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)
^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^
3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^
6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7
*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^
5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c -
b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4
*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2
))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c
^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 37
84704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 5767
1680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4
) - (x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 26049
92000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b^17*
c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 17567
0703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^17 - 119782124814336
0*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 1520311317037056*a^30*b^
3*c^20) + (-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^1
2*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c
^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11
*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10
*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(
1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 18199

0*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2
)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16
777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^
3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 1
2976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 692
06016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(2564940725275852 8*a^
38*c^21 - 32768000*a^21*b^34*c^4 + 2123366400*a^22*b^32*c^5 - 64398295040*a
^23*b^30*c^6 + 1213399564288*a^24*b^28*c^7 - 15898363035648*a^25*b^26*c^8 +
153599583715328*a^26*b^24*c^9 - 1132021560639488*a^27*b^22*c^10 + 64929172
79490048*a^28*b^20*c^11 - 29298398985191424*a^29*b^18*c^12 + 10439882608895
5904*a^30*b^16*c^13 - 293000581579014144*a^31*b^14*c^14 + 64170566921643622
4*a^32*b^12*c^15 - 1077743462209552384*a^33*b^10*c^16 + 1348355710714380288
*a^34*b^8*c^17 - 1198053158392168448*a^35*b^6*c^18 + 695801744382230528*a^3
6*b^4*c^19 - 223957324438437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 + 625*b
^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^
2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 +
1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^
8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*
b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475
*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)
^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(
-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^
22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 81
1008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440
320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 5033164
8*a^20*b^2*c^11)))^(1/4)*(9119789245425254 4*a^40*c^21 - 52428800*a^23*b^34*
c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*c^6 + 1986074247168
*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 257340683059200*a^28*b^24*c
^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648*a^30*b^20*c^11 - 5
1694329453871104*a^31*b^18*c^12 + 188531248770056192*a^32*b^16*c^13 - 54372
1556635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*b^12*c^15 - 2146620
531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b^8*c^17 - 265772191
4474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*c^19 - 612489549322
387456*a^39*b^2*c^20)))*(-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) +
3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483
001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 59966
89920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21
483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*
c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)
^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*
(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*
(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 1408
0*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^1
5*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18
*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4) - 89161

004482560*a^29*b*c^21 + 175760000*a^20*b^19*c^12 - 6846528000*a^21*b^17*c^13 + 118362316800*a^22*b^15*c^14 - 1191953858560*a^23*b^13*c^15 + 7705795952640*a^24*b^11*c^16 - 33166059110400*a^25*b^9*c^17 + 95038786764800*a^26*b^7*c^18 - 174846482841600*a^27*b^5*c^19 + 187403222384640*a^28*b^3*c^20))*(-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*2i + 2*atan(((x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 2604992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b^17*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^17 - 1197821248143360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 1520311317037056*a^30*b^3*c^20) - (-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(32768000*a^21*b^34*c^4 - 25649407252758528*a^38*c^21 - 2123366400*a^22*b^32*c^5 + 64398295040*a^23*b^30*c^6 - 1213399564288*a^24*b^28*c^7 + 15898363035648*a^25*b^26*c^8 - 153599583715328*a^26*b^24*c^9 + 1132021560639488*a^27*b^22*c^10 - 6492917279490048*a^28*b^20*c^11 + 29298398985191424*a^29*b^18*c^12 - 104398826088955904*a^30*b^16*c^13 + 293000581579014144*a^31*b^14*c^14 - 641705669216436224*a^32*b^12*c^15 + 1077743462209552384*a^33*b^10*c^16 - 1348355710714380288*a^34*b^8*c^17 + 1198053158392168448*a^35*b^6*c^18 - 695801744382230528*a^36*b^4*c^19 + 223957324438437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*

(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(9119789245425254 4*a^40*c^21 - 52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*c^6 + 1986074247168*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 257340683059200*a^28*b^24*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648*a^30*b^20*c^11 - 51694329453871104*a^31*b^18*c^12 + 188531248770056192*a^32*b^16*c^13 - 543721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*b^12*c^15 - 2146620531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b^8*c^17 - 2657721914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*c^19 - 612489549322387456*a^39*b^2*c^20)*1i)*1i)*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4) + (x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 2604992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b^17*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^17 - 1197821248143360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 1520311317037056*a^30*b^3*c^20) - (-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(25649407252758528*a^38*c^21 - 32768000*a^21*b^34*c^4 + 2123366400*a^22*b^32*c^5 - 64398295040*a^23*b^30*c^6 + 1213399564288 *a^24*b^28*c^7 - 15898363035648*a^25*b^26*c^8 + 153599583715328*a^26*b^24*c^9 - 1132021560639488*a^27*b^22*c^10 + 6492917279490048*a^28*b^20*c^11 - 29298398985191424*a^29*b^18*c^12 + 104398826088955904*a^30*b^16*c^13 - 293000581579014144*a^31*b^14*c^14 + 641705669216436224*a^32*b^12*c^15 - 1077743462209552384*a^33*b^10*c^16 + 1348355710714380288*a^34*b^8*c^17 - 1198053158392168448*a^35*b^6*c^18 + 695801744382230528*a^36*b^4*c^19 - 223957324438437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(9119789245425254 4*a^40*c^21 - 52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*c^6 + 1986074247168*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 257340683059200*a^28*b^24*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648*a^30*b^20*c^11 - 51694329453871104*a^31*b^18*c^12 + 188531248770056192*a^32*b^16*c^13 - 543721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*b^12*c^15 - 2146620531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b^8*c^17 - 2657721914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*c^19 - 612489549322387456*a^39*b^2*c^20)*1i)*1i)*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4))/((x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 2604992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b^17*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^17 - 1197821248143360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 1520311317037056*a^30*b^3*c^20) - (-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1

898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8
- 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^
3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a
^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^1
5)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(
4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22
*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 8110
08*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 3244032
0*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*
a^20*b^2*c^11)))^(3/4)*(32768000*a^21*b^34*c^4 - 25649407252758528*a^38*c^2
1 - 2123366400*a^22*b^32*c^5 + 64398295040*a^23*b^30*c^6 - 1213399564288*a^
24*b^28*c^7 + 15898363035648*a^25*b^26*c^8 - 153599583715328*a^26*b^24*c^9
+ 1132021560639488*a^27*b^22*c^10 - 6492917279490048*a^28*b^20*c^11 + 29298
398985191424*a^29*b^18*c^12 - 104398826088955904*a^30*b^16*c^13 + 293000581
579014144*a^31*b^14*c^14 - 641705669216436224*a^32*b^12*c^15 + 107774346220
9552384*a^33*b^10*c^16 - 1348355710714380288*a^34*b^8*c^17 + 11980531583921
68448*a^35*b^6*c^18 - 695801744382230528*a^36*b^4*c^19 + 223957324438437888
*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) +
3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 7148
3001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996
689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 2
1483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a
*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15
)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4
*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192
*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 140
80*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^
15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^1
8*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(91197
892454252544*a^40*c^21 - 52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5
- 104457043968*a^25*b^30*c^6 + 1986074247168*a^26*b^28*c^7 - 26302715265024
*a^27*b^26*c^8 + 257340683059200*a^28*b^24*c^9 - 1924694567550976*a^29*b^22
*c^10 + 11230133666971648*a^30*b^20*c^11 - 51694329453871104*a^31*b^18*c^12
+ 188531248770056192*a^32*b^16*c^13 - 543721556635811840*a^33*b^14*c^14 +
1229750704231415808*a^34*b^12*c^15 - 2146620531372195840*a^35*b^10*c^16 + 2
815880065059913728*a^36*b^8*c^17 - 2657721914474102784*a^37*b^6*c^18 + 1675
831642591068160*a^38*b^4*c^19 - 612489549322387456*a^39*b^2*c^20)*1i)*1i)*(
-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 +
638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478
624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 1352
4825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 -
12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 2962
5*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c
^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2)
+ 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21

*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*1i - (x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 2604992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b^17*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^17 - 1197821248143360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 1520311317037056*a^30*b^3*c^20) - (-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(25649407252758528*a^38*c^21 - 32768000*a^21*b^34*c^4 + 2123366400*a^22*b^32*c^5 - 64398295040*a^23*b^30*c^6 + 1213399564288*a^24*b^28*c^7 - 15898363035648*a^25*b^26*c^8 + 153599583715328*a^26*b^24*c^9 - 1132021560639488*a^27*b^22*c^10 + 6492917279490048*a^28*b^20*c^11 - 29298398985191424*a^29*b^18*c^12 + 104398826088955904*a^30*b^16*c^13 - 293000581579014144*a^31*b^14*c^14 + 641705669216436224*a^32*b^12*c^15 - 1077743462209552384*a^33*b^10*c^16 + 1348355710714380288*a^34*b^8*c^17 - 1198053158392168448*a^35*b^6*c^18 + 695801744382230528*a^36*b^4*c^19 - 223957324438437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(91197892454252544*a^40*c^21 - 52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*c^6 + 1986074247168*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 257340683059200*a^28*b^24*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648*a^30*b^20*c^11 - 51694329453871104*a^31*b^18*c^12 + 188531248770056192*a^32*b^16*c^13 - 543721556635811840*a^33*b^14*c^14 +

1229750704231415808*a^34*b^12*c^15 - 2146620531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b^8*c^17 - 2657721914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*c^19 - 612489549322387456*a^39*b^2*c^20)*1i)*1i)*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*1i - 89161004482560*a^29*b*c^21 + 175760000*a^20*b^19*c^12 - 6846528000*a^21*b^17*c^13 + 118362316800*a^22*b^15*c^14 - 1191953858560*a^23*b^13*c^15 + 7705795952640*a^24*b^11*c^16 - 33166059110400*a^25*b^9*c^17 + 95038786764800*a^26*b^7*c^18 - 174846482841600*a^27*b^5*c^19 + 187403222384640*a^28*b^3*c^20))*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4) + 2*atan(((x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 2604992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b^17*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^17 - 1197821248143360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 1520311317037056*a^30*b^3*c^20) - (-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 +

3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(32768000*a^21*b^34*c^4 - 25649407252758528*a^38*c^21 - 2123366400*a^22*b^32*c^5 + 64398295040*a^23*b^30*c^6 - 1213399564288*a^24*b^28*c^7 + 15898363035648*a^25*b^26*c^8 - 153599583715328*a^26*b^24*c^9 + 1132021560639488*a^27*b^22*c^10 - 6492917279490048*a^28*b^20*c^11 + 29298398985191424*a^29*b^18*c^12 - 104398826088955904*a^30*b^16*c^13 + 293000581579014144*a^31*b^14*c^14 - 641705669216436224*a^32*b^12*c^15 + 1077743462209552384*a^33*b^10*c^16 - 1348355710714380288*a^34*b^8*c^17 + 1198053158392168448*a^35*b^6*c^18 - 695801744382230528*a^36*b^4*c^19 + 223957324438437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(91197892454252544*a^40*c^21 - 52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*c^6 + 1986074247168*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 257340683059200*a^28*b^24*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648*a^30*b^20*c^11 - 51694329453871104*a^31*b^18*c^12 + 188531248770056192*a^32*b^16*c^13 - 543721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*b^12*c^15 - 2146620531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b^8*c^17 - 2657721914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*c^19 - 612489549322387456*a^39*b^2*c^20)*1i)*1i)*(-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4) + (x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 2604992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b^17*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*

c^17 - 1197821248143360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 15
20311317037056*a^30*b^3*c^20) - (-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^
(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3
 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6
 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*
c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5
*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c -
b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*
b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2)
)/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^
2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 378
4704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671
680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)
*(25649407252758528*a^38*c^21 - 32768000*a^21*b^34*c^4 + 2123366400*a^22*b^
32*c^5 - 64398295040*a^23*b^30*c^6 + 1213399564288*a^24*b^28*c^7 - 15898363
035648*a^25*b^26*c^8 + 153599583715328*a^26*b^24*c^9 - 1132021560639488*a^2
7*b^22*c^10 + 6492917279490048*a^28*b^20*c^11 - 29298398985191424*a^29*b^18
*c^12 + 104398826088955904*a^30*b^16*c^13 - 293000581579014144*a^31*b^14*c^
14 + 641705669216436224*a^32*b^12*c^15 - 1077743462209552384*a^33*b^10*c^16
 + 1348355710714380288*a^34*b^8*c^17 - 1198053158392168448*a^35*b^6*c^18 +
695801744382230528*a^36*b^4*c^19 - 223957324438437888*a^37*b^2*c^20 + x^(1/
2)*(-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^1
2 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 43
4478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 +
13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^
10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) -
29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b
^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(
1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*
a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126
720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128
*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a
^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(91197892454252544*a^40*c^21
 - 52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^
30*c^6 + 1986074247168*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 25734
0683059200*a^28*b^24*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971
648*a^30*b^20*c^11 - 51694329453871104*a^31*b^18*c^12 + 188531248770056192*
a^32*b^16*c^13 - 543721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^
34*b^12*c^15 - 2146620531372195840*a^35*b^10*c^16 + 2815880065059913728*a^3
6*b^8*c^17 - 2657721914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b
^4*c^19 - 612489549322387456*a^39*b^2*c^20)*1i)*1i)*(-(625*b^25 + 625*b^10*
(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 -
8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 189
8983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 -
21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*

c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2
*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)
^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*
a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c
 + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008
*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*
a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^
20*b^2*c^11)))^(1/4))/((x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^2
0*b^23*c^10 + 2604992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 7491
18545920*a^23*b^17*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^2
5*b^13*c^15 - 175670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^1
7 - 1197821248143360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 15203
11317037056*a^30*b^3*c^20) - (-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/
2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 +
71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 -
5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9
 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-
(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2
)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2
c^4(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(
8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 -
 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 378470
4*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680
*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(3
2768000*a^21*b^34*c^4 - 25649407252758528*a^38*c^21 - 2123366400*a^22*b^32*
c^5 + 64398295040*a^23*b^30*c^6 - 1213399564288*a^24*b^28*c^7 + 15898363035
648*a^25*b^26*c^8 - 153599583715328*a^26*b^24*c^9 + 1132021560639488*a^27*b
^22*c^10 - 6492917279490048*a^28*b^20*c^11 + 29298398985191424*a^29*b^18*c^
12 - 104398826088955904*a^30*b^16*c^13 + 293000581579014144*a^31*b^14*c^14
- 641705669216436224*a^32*b^12*c^15 + 1077743462209552384*a^33*b^10*c^16 -
1348355710714380288*a^34*b^8*c^17 + 1198053158392168448*a^35*b^6*c^18 - 695
801744382230528*a^36*b^4*c^19 + 223957324438437888*a^37*b^2*c^20 + x^(1/2)*
(-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 +
 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 43447
8624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 135
24825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10
- 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 296
25*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*
c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2
) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^2
1*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720
*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^
16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19
*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(91197892454252544*a^40*c^21 -
52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*

c^6 + 1986074247168*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 25734068
3059200*a^28*b^24*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648
*a^30*b^20*c^11 - 51694329453871104*a^31*b^18*c^12 + 188531248770056192*a^3
2*b^16*c^13 - 543721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*
b^12*c^15 - 2146620531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b
^8*c^17 - 2657721914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*
c^19 - 612489549322387456*a^39*b^2*c^20)*1i)*1i)*(-(625*b^25 + 625*b^10*(-(
4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 826
4990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 189898
3360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 211
22310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^1
1 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^
6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1
/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c
 - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c +
1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^
14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^1
7*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*
b^2*c^11)))^(1/4)*1i - (x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^2
0*b^23*c^10 + 2604992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 7491
18545920*a^23*b^17*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^2
5*b^13*c^15 - 175670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^1
7 - 1197821248143360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 15203
11317037056*a^30*b^3*c^20) - (-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/
2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 +
71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 -
5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9
 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-
(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2
)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2
c^4(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(
8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 -
 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 378470
4*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680
*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(2
5649407252758528*a^38*c^21 - 32768000*a^21*b^34*c^4 + 2123366400*a^22*b^32*
c^5 - 64398295040*a^23*b^30*c^6 + 1213399564288*a^24*b^28*c^7 - 15898363035
648*a^25*b^26*c^8 + 153599583715328*a^26*b^24*c^9 - 1132021560639488*a^27*b
^22*c^10 + 6492917279490048*a^28*b^20*c^11 - 29298398985191424*a^29*b^18*c^
12 + 104398826088955904*a^30*b^16*c^13 - 293000581579014144*a^31*b^14*c^14
+ 641705669216436224*a^32*b^12*c^15 - 1077743462209552384*a^33*b^10*c^16 +
1348355710714380288*a^34*b^8*c^17 - 1198053158392168448*a^35*b^6*c^18 + 695
801744382230528*a^36*b^4*c^19 - 223957324438437888*a^37*b^2*c^20 + x^(1/2)*
(-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 +
 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 43447

8624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 135
24825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10
- 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 296
25*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*
c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2
) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^2
1*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720
*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^
16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19
*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(91197892454252544*a^40*c^21 -
52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*
c^6 + 1986074247168*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 25734068
3059200*a^28*b^24*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648
*a^30*b^20*c^11 - 51694329453871104*a^31*b^18*c^12 + 188531248770056192*a^3
2*b^16*c^13 - 543721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*
b^12*c^15 - 2146620531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b
^8*c^17 - 2657721914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*
c^19 - 612489549322387456*a^39*b^2*c^20)*1i)*1i)*(-(625*b^25 + 625*b^10*(-(
4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 826
4990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 189898
3360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 211
22310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^1
1 - 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^
6*c^2*(-(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1
/2) + 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c
 - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c +
1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^
14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^1
7*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*
b^2*c^11)))^(1/4)*1i - 89161004482560*a^29*b*c^21 + 175760000*a^20*b^19*c^1
2 - 6846528000*a^21*b^17*c^13 + 118362316800*a^22*b^15*c^14 - 1191953858560
*a^23*b^13*c^15 + 7705795952640*a^24*b^11*c^16 - 33166059110400*a^25*b^9*c^
17 + 95038786764800*a^26*b^7*c^18 - 174846482841600*a^27*b^5*c^19 + 1874032
22384640*a^28*b^3*c^20))*(-(625*b^25 + 625*b^10*(-(4*a*c - b^2)^15)^(1/2) +
 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 7148
3001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996
689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 2
1483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 - 26244*a^5*c^5*(-(4*a
*c - b^2)^15)^(1/2) - 29625*a*b^23*c + 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15
)^(1/2) - 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) + 171801*a^4*b^2*c^4
*(-(4*a*c - b^2)^15)^(1/2) - 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192
*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 140
80*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^
15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^1
8*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.1080 $\int \frac{x^{15/2}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=621

$$-\frac{3\left(\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(-\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)^2\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[Out] 1/4*x^(9/2)*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/16*x^(5/2)*(8*a*b+
(12*a*c+b^2)*x^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)-3/64*arctan(2^(1/4)*c^(1/4
)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b^3-28*a*b*c+(-24*a^2*c^2-30*a*b^
2*c+b^4)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-4*a*c+b^2)^2/(-b-(-4*a*c+b^2
)^(1/2))^(3/4)-3/64*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))
^(1/4))*(b^3-28*a*b*c+(-24*a^2*c^2-30*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2))*2^(3
/4)/c^(5/4)/(-4*a*c+b^2)^2/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-3/64*arctan(2^(1/4
)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b^3-28*a*b*c+(24*a^2*c^2+
30*a*b^2*c-b^4)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-4*a*c+b^2)^2/(-b+(-4*
a*c+b^2)^(1/2))^(3/4)-3/64*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)
^(1/2))^(1/4))*(b^3-28*a*b*c+(24*a^2*c^2+30*a*b^2*c-b^4)/(-4*a*c+b^2)^(1/2)
)*2^(3/4)/c^(5/4)/(-4*a*c+b^2)^2/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-3/16*(12*a*c
+b^2)*x^(1/2)/c/(-4*a*c+b^2)^2
```

Rubi [A] time = 1.77, antiderivative size = 621, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1115, 1365, 1498, 1502, 1422, 212, 208, 205}

$$-\frac{3\left(\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(-\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)^2\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(15/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (-3*(b^2 + 12*a*c)*Sqrt[x])/(16*c*(b^2 - 4*a*c)^2) + (x^(9/2)*(2*a + b*x^2)
)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*x^(5/2)*(8*a*b + (b^2 + 12*a
*c)*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (3*(b^3 - 28*a*b*c + (
b^4 - 30*a*b^2*c - 24*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*S
qrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)^
2*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*(b^3 - 28*a*b*c - (b^4 - 30*a*b^2*c
```

```
- 24*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqr
t[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2
- 4*a*c])^(3/4)) - (3*(b^3 - 28*a*b*c + (b^4 - 30*a*b^2*c - 24*a^2*c^2)/Sqr
t[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^
(1/4)])/(32*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(3/4))
 - (3*(b^3 - 28*a*b*c - (b^4 - 30*a*b^2*c - 24*a^2*c^2)/Sqrt[b^2 - 4*a*c])*
ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1
/4)*c^(5/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1498

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Simp[(f^(n - 1)*(f*x)^(m - n + 1)*(a +
 b*x^n + c*x^(2*n))^(p + 1)*(b*d - 2*a*e - (b*e - 2*c*d)*x^n))/(n*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[f^n/(n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - n)*(
a + b*x^n + c*x^(2*n))^(p + 1)*Simp[(n - m - 1)*(b*d - 2*a*e) + (2*n*p + 2*
n + m + 1)*(b*e - 2*c*d)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m,
n - 1] && IntegerQ[p]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{x^{15/2}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^{16}}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{9/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{x^8\left(18a-3bx^4\right)}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}$$

$$= \frac{x^{9/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{5/2}\left(8ab+\left(b^2+12ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{x^4\left(-120ab-3\right.}{a+bx^4}\right.}{16\left(b^2\right.}$$

$$= -\frac{3\left(b^2+12ac\right)\sqrt{x}}{16c\left(b^2-4ac\right)^2} + \frac{x^{9/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{5/2}\left(8ab+\left(b^2+12ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} -$$

$$= -\frac{3\left(b^2+12ac\right)\sqrt{x}}{16c\left(b^2-4ac\right)^2} + \frac{x^{9/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{5/2}\left(8ab+\left(b^2+12ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} -$$

$$= -\frac{3\left(b^2+12ac\right)\sqrt{x}}{16c\left(b^2-4ac\right)^2} + \frac{x^{9/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{5/2}\left(8ab+\left(b^2+12ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} -$$

$$= -\frac{3\left(b^2+12ac\right)\sqrt{x}}{16c\left(b^2-4ac\right)^2} + \frac{x^{9/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{5/2}\left(8ab+\left(b^2+12ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} -$$

Mathematica [C] time = 0.44, size = 254, normalized size = 0.41

$$\frac{3c\left(a+bx^2+cx^4\right)^2\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{-28\#1^4abc\log\left(\sqrt{x}-\#1\right)+\#1^4b^3\log\left(\sqrt{x}-\#1\right)+12a^2c\log\left(\sqrt{x}-\#1\right)+ab^2\log\left(\sqrt{x}-\#\right.}{2\#1^7c+\#1^3b}\right.}{64c^2\left(b^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(15/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (4*Sqrt[x]*(-4*b^4 + 21*a*b^2*c - 68*a^2*c^2 + b^3*c*x^2 - 28*a*b*c^2*x^2)*
(a + b*x^2 + c*x^4) + 16*(b^2 - 4*a*c)*Sqrt[x]*(-2*a^2*c + b^3*x^2 + a*b*(b
```

```
 - 3*c*x^2)) + 3*c*(a + b*x^2 + c*x^4)^2*RootSum[a + b*#1^4 + c*#1^8 & , (a
*b^2*Log[Sqrt[x] - #1] + 12*a^2*c*Log[Sqrt[x] - #1] + b^3*Log[Sqrt[x] - #1]
*#1^4 - 28*a*b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(64*c^2*(
b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 192.56Unable to convert to
real 1/4 Error: Bad Argument Value
```

maple [C] time = 0.04, size = 275, normalized size = 0.44

$$\frac{3\left(\left(-28ac+b^2\right)\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^4 b+12a^2c+a\,b^2\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{64\left(16a^2c^2-8a\,b^2c+b^4\right)c\left(2\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7 c+\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3 b\right)}+\frac{-\frac{(28}{16(16a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(-3/32*a^2*(12*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2)-3/16*a/c*b*(
8*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)-1/32*(68*a^2*c^2+7*a*b^2*c+3*
b^4)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(9/2)-1/32*b*(28*a*c-b^2)/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^(13/2))/(c*x^4+b*x^2+a)^2+3/64/c/(16*a^2*c^2-8*a*b^2*c+b^4)
*sum((b*(-28*a*c+b^2)*_R^4+12*a^2*c+a*b^2)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)
),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\left(b^2c+12\,ac^2\right)x^{\frac{17}{2}}+\left(7\,b^3+44\,abc\right)x^{\frac{13}{2}}+24\,a^2bx^{\frac{5}{2}}+\left(35\,ab^2+4\,a^2c\right)x^{\frac{9}{2}}}{16\left(\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)x^8+2\left(b^5c-8\,ab^3c^2+16\,a^2bc^3\right)x^6+a^2b^4-8\,a^3b^2c+16\,a^4c^2+\left(b^6-6\,ab^4c+32\,a^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(b^2*c + 12*a*c^2)*x^(17/2) + (7*b^3 + 44*a*b*c)*x^(13/2) + 24*a^2*
b*x^(5/2) + (35*a*b^2 + 4*a^2*c)*x^(9/2))/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2
*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3
*c + 16*a^3*b*c^2)*x^2) - integrate(3/32*((b^2 + 12*a*c)*x^(7/2) + 40*a*b*x
^(3/2))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

mupad [B] time = 9.35, size = 50970, normalized size = 82.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] atan(((((3*(3159*a^3*b^14 - 20155392*a^10*c^7 - 367497*a^4*b^12*c + 1590021
9*a^5*b^10*c^2 - 299549340*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 + 284032396
8*a^8*b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^9*c^10 -
36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 -
 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*
b^2*c^9)) + ((3*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*
a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c
^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19
*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a
^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 +
 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*
a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 400
9*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)
^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(10995116277
76*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^3
4*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*
c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^
9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2
113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560
*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c
^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558
138880*a^19*b^2*c^24)))^(1/4)*(703687441776640*a^13*b*c^15 + 671088640*a^3*
b^21*c^5 - 26843545600*a^4*b^19*c^6 + 483183820800*a^5*b^17*c^7 - 515396075
5200*a^6*b^15*c^8 + 36077725286400*a^7*b^13*c^9 - 173173081374720*a^8*b^11*
c^10 + 577243604582400*a^9*b^9*c^11 - 1319413953331200*a^10*b^7*c^12 + 1979
120929996800*a^11*b^5*c^13 - 1759218604441600*a^12*b^3*c^14))/(65536*(b^18*
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c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4
+ 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7
*b^4*c^8 + 589824*a^8*b^2*c^9)) - (9*x^(1/2)*(16777216*a^3*b^25*c^4 - 31243
722414882816*a^15*b*c^16 + 23890755584*a^4*b^23*c^5 - 1000190509056*a^5*b^2
1*c^6 + 18747532247040*a^6*b^19*c^7 - 209186382151680*a^7*b^17*c^8 + 154495
1275978752*a^8*b^15*c^9 - 7925554690916352*a^9*b^13*c^10 + 2878301539192012
8*a^10*b^11*c^11 - 73870688712130560*a^11*b^9*c^12 + 130973825100677120*a^1
2*b^7*c^13 - 152242778028376064*a^13*b^5*c^14 + 103864266406232064*a^14*b^3
c^15))/(4194304(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^
20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3
784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 576716
80*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*
(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^
2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^2
3*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8
*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354
024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^
7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a
^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c
 - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6
c(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^
5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^
32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*
b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 19373070
7456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16
*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 1664
7293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a
^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24))
)^(3/4))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*
c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 14
0233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 +
43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^1
3*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 756253
1438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^
3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b
^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1
/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20
*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 +
 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 -
1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*
c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 21134258
99520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b
^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 1
9585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*
a^19*b^2*c^24)))^(1/4) - (9*x^(1/2)*(123201*a^4*b^16 + 483729408*a^12*c^8 -

14619852*a^5*b^14*c + 653342274*a^6*b^12*c^2 - 13105503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5 + 9199443456*a^10*b^4*c^6 + 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*1i - (((3*(3159*a^3*b^14 - 20155392*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + ((3*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*(703687441776640*a^13*b*c^15 + 671088640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6 + 48318382

0800*a^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8 + 36077725286400*a^7*b^13*c^9 - 173173081374720*a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11 - 1319413953331200*a^10*b^7*c^12 + 1979120929996800*a^11*b^5*c^13 - 1759218604441600*a^12*b^3*c^14))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + (9*x^(1/2)*(16777216*a^3*b^25*c^4 - 31243722414882816*a^15*b*c^16 + 23890755584*a^4*b^23*c^5 - 1000190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7 - 209186382151680*a^7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 7925554690916352*a^9*b^13*c^10 + 28783015391920128*a^10*b^11*c^11 - 73870688712130560*a^11*b^9*c^12 + 130973825100677120*a^12*b^7*c^13 - 152242778028376064*a^13*b^5*c^14 + 103864266406232064*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(3/4))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14

*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 208
09116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a
^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4) + (9*x^(1/2)*(123201*a^
4*b^16 + 483729408*a^12*c^8 - 14619852*a^5*b^14*c + 653342274*a^6*b^12*c^2
- 13105503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6
*c^5 + 9199443456*a^10*b^4*c^6 + 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c
 + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*
c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12
976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 692060
16*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b
^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*
b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b
^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528
*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 -
3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a
^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^2
5)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 5464
8*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(
1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040
*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^
30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a
^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 70
4475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^
13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^2
0 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 130567005
79840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*1i)/((((3*(3159*
a^3*b^14 - 20155392*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a^5*b^10*c^2 -
299549340*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 1
64042496*a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 5
76*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c
^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + ((3*(
-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 105
09*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^
5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 4337679974
4*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 9
86354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^
13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1
296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(
4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*
a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^
40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a
^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680
*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193
730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12
*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 -

16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869
760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c
^24)))^(1/4)*(703687441776640*a^13*b*c^15 + 671088640*a^3*b^21*c^5 - 268435
45600*a^4*b^19*c^6 + 483183820800*a^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8
 + 36077725286400*a^7*b^13*c^9 - 173173081374720*a^8*b^11*c^10 + 5772436045
82400*a^9*b^9*c^11 - 1319413953331200*a^10*b^7*c^12 + 1979120929996800*a^11
*b^5*c^13 - 1759218604441600*a^12*b^3*c^14))/(65536*(b^18*c - 262144*a^9*c^
10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*
c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824
*a^8*b^2*c^9)) - (9*x^(1/2)*(16777216*a^3*b^25*c^4 - 31243722414882816*a^15
*b*c^16 + 23890755584*a^4*b^23*c^5 - 1000190509056*a^5*b^21*c^6 + 187475322
47040*a^6*b^19*c^7 - 209186382151680*a^7*b^17*c^8 + 1544951275978752*a^8*b^
15*c^9 - 7925554690916352*a^9*b^13*c^10 + 28783015391920128*a^10*b^11*c^11
- 73870688712130560*a^11*b^9*c^12 + 130973825100677120*a^12*b^7*c^13 - 1522
42778028376064*a^13*b^5*c^14 + 103864266406232064*a^14*b^3*c^15))/(4194304*
(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^
3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c
^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 +
 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*
a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 3942
48*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 142436889
6*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 10849
3078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*
c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 82122626
82624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c -
 b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2)
 - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2
)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6
 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096
*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 82555
69920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^
15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 520227913
7280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b
^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13
056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(3/4))*(-(81*(b
^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*
b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*
c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b
^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 98635402
4448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*
c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4
c^4(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c -
 b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c
*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5
- 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32

*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4) - (9*x^(1/2)*(123201*a^4*b^16 + 483729408*a^12*c^8 - 14619852*a^5*b^14*c + 653342274*a^6*b^12*c^2 - 13105503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5 + 9199443456*a^10*b^4*c^6 + 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4) + (((3*(3159*a^3*b^14 - 20155392*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + ((3*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*

a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029
706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18
*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404
558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a
^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23
- 5497558138880*a^19*b^2*c^24)))^(1/4)*(703687441776640*a^13*b*c^15 + 67108
8640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6 + 483183820800*a^5*b^17*c^7 -
5153960755200*a^6*b^15*c^8 + 36077725286400*a^7*b^13*c^9 - 173173081374720*
a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11 - 1319413953331200*a^10*b^7*c^
12 + 1979120929996800*a^11*b^5*c^13 - 1759218604441600*a^12*b^3*c^14))/(655
36*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*
b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 5
89824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + (9*x^(1/2)*(16777216*a^3*b^25*c^
4 - 31243722414882816*a^15*b*c^16 + 23890755584*a^4*b^23*c^5 - 100019050905
6*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7 - 209186382151680*a^7*b^17*c^8
 + 1544951275978752*a^8*b^15*c^9 - 7925554690916352*a^9*b^13*c^10 + 2878301
5391920128*a^10*b^11*c^11 - 73870688712130560*a^11*b^9*c^12 + 1309738251006
77120*a^12*b^7*c^13 - 152242778028376064*a^13*b^5*c^14 + 103864266406232064
*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 10
56*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^1
4*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9
 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)
))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 +
 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 14023372
8*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 433767
99744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10
 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 756253143859
2*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15
 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2
*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) -
107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25
+ b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 12403
20*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 127008
7680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 +
 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*
a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^
19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 1958505
0869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b
^2*c^24)))^(3/4))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 47110422528
0*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25
*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^
19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656
*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12
 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 421376557056
0*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4

009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^
2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(109951162
7776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b
^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^2
8*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*
a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 +
 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 104045582745
60*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8
*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 54975
58138880*a^19*b^2*c^24)))^(1/4) + (9*x^(1/2)*(123201*a^4*b^16 + 483729408*a
^12*c^8 - 14619852*a^5*b^14*c + 653342274*a^6*b^12*c^2 - 13105503216*a^7*b^
10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5 + 9199443456*a^
10*b^4*c^6 + 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c + 16777216*a^12*c^1
3 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^1
6*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8
 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 5
0331648*a^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 4711
04225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a
^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992
*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 1315
1174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b
^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 42137
65570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^3
1*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a
*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(10
99511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 7296
0*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*
a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029
706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18
*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404
558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a
^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23
 - 5497558138880*a^19*b^2*c^24)))^(1/4)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^
25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27
*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*
c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9
*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840
358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*
b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(
1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^
3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)
))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2
*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c
^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b
^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475

299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b
^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 +
20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 1305670057984
0*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*2i - ((3*x^(1/2)*(12
*a^3*c + a^2*b^2))/(16*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*x^(5/2)*(a*b^
3 + 8*a^2*b*c))/(8*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*x^(13/2)*(28*a*c
- b^2))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^(9/2)*(3*b^4 + 68*a^2*c^2
+ 7*a*b^2*c))/(16*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a
^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + atan(((((3*(3159*a^3*b^14 - 2015539
2*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340*a^6*b^8*
c^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*a^9*b^2*c
^6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 -
5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^
6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + ((3*(-(81*(b^33 - b^8*(
-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 -
394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 14243
68896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 1
08493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b
^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212
262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a
*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(
1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c -
b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38
*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 1587
6096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8
255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^2
0*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 52022
79137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^
15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22
+ 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*(7036
87441776640*a^13*b*c^15 + 671088640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6
+ 483183820800*a^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8 + 36077725286400*
a^7*b^13*c^9 - 173173081374720*a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11
- 1319413953331200*a^10*b^7*c^12 + 1979120929996800*a^11*b^5*c^13 - 175921
8604441600*a^12*b^3*c^14))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2
+ 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b
^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (
9*x^(1/2)*(16777216*a^3*b^25*c^4 - 31243722414882816*a^15*b*c^16 + 23890755
584*a^4*b^23*c^5 - 1000190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7
- 209186382151680*a^7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 792555469
0916352*a^9*b^13*c^10 + 28783015391920128*a^10*b^11*c^11 - 7387068871213056
0*a^11*b^9*c^12 + 130973825100677120*a^12*b^7*c^13 - 152242778028376064*a^1
3*b^5*c^14 + 103864266406232064*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216
*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 12672
0*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*

b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4
*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/
2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 +
9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9
732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c
^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 38403582197
76*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^1
4 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) -
157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c
^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(335
54432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c
^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 1
58760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^1
3 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*
a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^1
8 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 2080911
6549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*
b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(3/4))*(-(81*(b^33 - b^8*(-(4*a*c
 - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*
a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a
^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 10849307
8528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^1
1 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 82122626826
24*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^
2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) +
54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^2
5)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 +
3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^
5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 82555699
20*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15
- 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 520227913728
0*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10
*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056
700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4) - (9*x^(1/2)
*(123201*a^4*b^16 + 483729408*a^12*c^8 - 14619852*a^5*b^14*c + 653342274*a^
6*b^12*c^2 - 13105503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210
048*a^9*b^6*c^5 + 9199443456*a^10*b^4*c^6 + 6261608448*a^11*b^2*c^7))/(4194
304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 1408
0*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^
12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^
10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(
-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 -
394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 14243
68896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 1
08493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b

^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*1i - (((3*(3159*a^3*b^14 - 20155392*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + ((3*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*(703687441776640*a^13*b*c^15 + 671088640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6 + 483183820800*a^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8 + 36077725286400*a^7*b^13*c^9 - 173173081374720*a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11 - 1319413953331200*a^10*b^7*c^12 + 1979120929996800*a^11*b^5*c^13 - 1759218604441600*a^12*b^3*c^14))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + (9*x^(1/2)*(16777216*a^3*b^25*c^4 - 31243722414882816*a^15*b*c^16 + 23890755584*a^4*b^23*c^5 - 1000190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7 - 209186382151680*a^7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 7925554690916352*a^9*b^13*c^10 + 28783015391920128*a^10*b^11*c^11 - 73870688712130560*a^11*b^9*c^12 + 130973825100677120*a^12*b^7* c^13 - 152242778028376064*a^13*b^5*c^14 + 103864266406232064*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(3/4))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4) + (9*x^(1/2)*(123201*a^4*b^16 + 483729408*a^12*c^8 - 14619852*a^5*b^14*c + 653342274*a^6*b^12*c^2 - 13105503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5 + 9199443456*a^10*b^4*c^6 + 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b

*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 1
40233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 +
43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^
13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 75625
31438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b
^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*
b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(
1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(109951162776*a^2
0*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8
+ 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 -
1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22
*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425
899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*
b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 -
19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880
*a^19*b^2*c^24)))^(1/4)*1i)/((((3*(3159*a^3*b^14 - 20155392*a^10*c^7 - 3674
97*a^4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340*a^6*b^8*c^3 + 1945179360*
a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536*(b^18
*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4
+ 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^
7*b^4*c^8 + 589824*a^8*b^2*c^9)) + ((3*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25
)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c
^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^
6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b
^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 384035
8219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^
5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/
2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*
b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))
/(33554432*(109951162776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b
^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^1
0 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^2
4*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 70447529
9840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^1
4*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20
809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*
a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*(703687441776640*a^13*
b*c^15 + 671088640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6 + 483183820800*a
^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8 + 36077725286400*a^7*b^13*c^9 - 17
3173081374720*a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11 - 13194139533312
00*a^10*b^7*c^12 + 1979120929996800*a^11*b^5*c^13 - 1759218604441600*a^12*b
^3*c^14))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c
^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a
^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (9*x^(1/2)*(167772
16*a^3*b^25*c^4 - 31243722414882816*a^15*b*c^16 + 23890755584*a^4*b^23*c^5

- 1000190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7 - 20918638215168
0*a^7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 7925554690916352*a^9*b^13*
c^10 + 28783015391920128*a^10*b^11*c^11 - 73870688712130560*a^11*b^9*c^12 +
 130973825100677120*a^12*b^7*c^13 - 152242778028376064*a^13*b^5*c^14 + 1038
64266406232064*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a
*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 -
811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 324403
20*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*
a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280
*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*
c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^1
9*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*
a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12
+ 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560
*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 40
09*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2
)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627
776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^
34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28
*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a
^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 +
2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 1040455827456
0*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*
c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 549755
8138880*a^19*b^2*c^24)))^(3/4))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2)
 - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 92
19696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 973
2052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9
 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776
*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14
+ 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 15
7*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3
*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554
432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7
 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158
760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13
- 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^
11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18
+ 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 208091165
49120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^
4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4) - (9*x^(1/2)*(123201*a^4*b^16
 + 483729408*a^12*c^8 - 14619852*a^5*b^14*c + 653342274*a^6*b^12*c^2 - 1310
5503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5 +
 9199443456*a^10*b^4*c^6 + 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c + 167
77216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 +

126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128
*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^1
0*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25
)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c
^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^
6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b
^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 384035
8219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^
5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/
2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*
b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))
/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b
^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^1
0 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^2
4*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 70447529
9840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^1
4*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20
809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*
a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4) + (((3*(3159*a^3*b^14
- 20155392*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340
*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*
a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^
14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 3440
64*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + ((3*(-(81*(b^3
3 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^
29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^
5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^1
7*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 9863540244
48*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^
13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c
^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b
^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(
-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 -
80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c
^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26
*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456
*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^1
7 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293
239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*
b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1
/4)*(703687441776640*a^13*b*c^15 + 671088640*a^3*b^21*c^5 - 26843545600*a^4
*b^19*c^6 + 483183820800*a^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8 + 360777
25286400*a^7*b^13*c^9 - 173173081374720*a^8*b^11*c^10 + 577243604582400*a^9
*b^9*c^11 - 1319413953331200*a^10*b^7*c^12 + 1979120929996800*a^11*b^5*c^13
 - 1759218604441600*a^12*b^3*c^14))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a

*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + (9*x^(1/2)*(16777216*a^3*b^25*c^4 - 31243722414882816*a^15*b*c^16 + 23890755584*a^4*b^23*c^5 - 1000190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7 - 209186382151680*a^7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 7925554690916352*a^9*b^13*c^10 + 28783015391920128*a^10*b^11*c^11 - 73870688712130560*a^11*b^9*c^12 + 130973825100677120*a^12*b^7*c^13 - 152242778028376064*a^13*b^5*c^14 + 103864266406232064*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(3/4))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4) + (9*x^(1/2)*(123201*a^4*b^16 + 483729408*a^12*c^8 - 14619852*a^5*b^14*c + 653

342274*a^6*b^12*c^2 - 13105503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5 + 9199443456*a^10*b^4*c^6 + 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*2i + 2*atan(((((3*(3159*a^3*b^14 - 20155392*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (((-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27 *c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*(703687441776640*a^13*b*c^15 + 671088640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6 + 483183820800*a^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8 + 36077725286400*a^7*b^13*c^9 - 173173081374720*a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11 - 1319413953331200*a^10*b^7*c^12 + 1979120929996800*a^11*b^5*c^13 - 1759218604441600*a^12*b^3*c^14)*3i)/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (9*x^(1/2)*(16777216*a^3*b^25*c^4 - 31243722414882816*a^15*b*c^16 + 23890755584*a^4*b^23*c^5 - 1000190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7 - 209186382151680*a^7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 7925554690916352*a^9*b^13*c^10 + 28783015391920128*a^10*b^11*c^11 - 73870688712130560*a^11*b^9*c^12 + 130973825100677120*a^12*b^7*c^13 - 152242778028376064*a^13*b^5*c^14 + 103864266406232064*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558

274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(3/4)*1i)*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*1i - (9*x^(1/2)*(123201*a^4*b^16 + 483729408*a^12*c^8 - 14619852*a^5*b^14*c + 653342274*a^6*b^12*c^2 - 13105503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5 + 9199443456*a^10*b^4*c^6 + 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4) - (((3*(3159*a^3*b^14 - 20155392*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 +

576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (((-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*(703687441776640*a^13*b*c^15 + 671088640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6 + 483183820800*a^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8 + 36077725286400*a^7*b^13*c^9 - 173173081374720*a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11 - 1319413953331200*a^10*b^7*c^12 + 1979120929996800*a^11*b^5*c^13 - 1759218604441600*a^12*b^3*c^14)*3i)/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + (9*x^(1/2)*(16777216*a^3*b^25*c^4 - 31243722414882816*a^15*b*c^16 + 23890755584*a^4*b^23*c^5 - 1000190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7 - 209186382151680*a^7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 7925554690916352*a^9*b^13*c^10 + 28783015391920128*a^10*b^11*c^11 - 73870688712130560*a^11*b^9*c^12 + 130973825100677120*a^12*b^7*c^13 - 152242778028376064*a^13*b^5*c^14 + 103864266406232064*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876

096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 82
55569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20
*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 520227
9137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^1
5*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 +
 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(3/4)*1i)*(-
(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 1050
9*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5
*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744
*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 98
6354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^1
3*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 12
96*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4
*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a
*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^4
0*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^
4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*
a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 1937
30707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*
b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 -
16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 195850508697
60*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^
24)))^(1/4)*1i + (9*x^(1/2)*(123201*a^4*b^16 + 483729408*a^12*c^8 - 1461985
2*a^5*b^14*c + 653342274*a^6*b^12*c^2 - 13105503216*a^7*b^10*c^3 + 10230607
1520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5 + 9199443456*a^10*b^4*c^6 + 6261
608448*a^11*b^2*c^7))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2
 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^
5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^
8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*
c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c
^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140
233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 4
3376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13
*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531
438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3
*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^
4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/
2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*
c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 +
1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1
270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c
^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 211342589
9520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^
12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19
585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a

^19*b^2*c^24)))^(1/4))/((((3*(3159*a^3*b^14 - 20155392*a^10*c^7 - 367497*a^
4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340*a^6*b^8*c^3 + 1945179360*a^7*b
^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536*(b^18*c -
262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32
256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4
*c^8 + 589824*a^8*b^2*c^9)) - (((-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2)
 - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 92
19696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 973
2052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9
 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776
*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14
+ 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 15
7*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3
*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554
432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7
 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158
760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13
- 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^
11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18
+ 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 208091165
49120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^
4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*(703687441776640*a^13*b*c^15
+ 671088640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6 + 483183820800*a^5*b^17
*c^7 - 5153960755200*a^6*b^15*c^8 + 36077725286400*a^7*b^13*c^9 - 173173081
374720*a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11 - 1319413953331200*a^10
*b^7*c^12 + 1979120929996800*a^11*b^5*c^13 - 1759218604441600*a^12*b^3*c^14
)*3i)/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 -
 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b
^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (9*x^(1/2)*(16777216*a
^3*b^25*c^4 - 31243722414882816*a^15*b*c^16 + 23890755584*a^4*b^23*c^5 - 10
00190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7 - 209186382151680*a^
7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 7925554690916352*a^9*b^13*c^10
 + 28783015391920128*a^10*b^11*c^11 - 73870688712130560*a^11*b^9*c^12 + 130
973825100677120*a^12*b^7*c^13 - 152242778028376064*a^13*b^5*c^14 + 10386426
6406232064*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^2
2*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 8110
08*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a
^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11
*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^1
6*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4
- 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^
7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10
*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 75
62531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^1
5*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a

^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25
)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*
a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c
^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^1
1 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b
^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113
425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^
14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21
 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138
880*a^19*b^2*c^24)))^(3/4)*1i)*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2)
 - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 921
9696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732
052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9
 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*
a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 +
 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157
*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*
(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(335544
32*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7
 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 1587
60960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 -
 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^1
1*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 +
 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 2080911654
9120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4
*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*1i - (9*x^(1/2)*(123201*a^4*b^
16 + 483729408*a^12*c^8 - 14619852*a^5*b^14*c + 653342274*a^6*b^12*c^2 - 13
105503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5
 + 9199443456*a^10*b^4*c^6 + 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c + 1
6777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4
 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 129761
28*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a
^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^
25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27
*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*
c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9
*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840
358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*
b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(
1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^
3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)
))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2
*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c
^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b
^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475

299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*1i + (((3*(3159*a^3*b^14 - 20155392*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (((-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*(703687441776640*a^13*b*c^15 + 671088640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6 + 483183820800*a^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8 + 36077725286400*a^7*b^13*c^9 - 173173081374720*a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11 - 1319413953331200*a^10*b^7*c^12 + 1979120929996800*a^11*b^5*c^13 - 1759218604441600*a^12*b^3*c^14)*3i)/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + (9*x^(1/2)*(16777216*a^3*b^25*c^4 - 31243722414882816*a^15*b*c^16 + 23890755584*a^4*b^23*c^5 - 1000190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7 - 209186382151680*a^7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 7925554690916352*a^9*b^13*c^10 + 28783015391920128*a^10*b^11*c^11 - 73870688712130560*a^11*b^9*c^12 + 130973825100677120*a^12*b^7*c^13 - 152242778028376064*a^13*b^5*c^14 + 103864266406232064*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 10849307

8528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^1
1 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 82122626826
24*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^
2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) -
54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^2
5)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 +
3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^
5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 82555699
20*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15
- 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 520227913728
0*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10
*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056
700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(3/4)*1i)*(-(81*(b
^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*
b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*
c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b
^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 98635402
4448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*
c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4
c^4(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c -
 b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c
*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5
- 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32
*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^
26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 1937307074
56*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c
^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 166472
93239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^1
7*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^
(1/4)*1i + (9*x^(1/2)*(123201*a^4*b^16 + 483729408*a^12*c^8 - 14619852*a^5*
b^14*c + 653342274*a^6*b^12*c^2 - 13105503216*a^7*b^10*c^3 + 102306071520*a
^8*b^8*c^4 - 66486210048*a^9*b^6*c^5 + 9199443456*a^10*b^4*c^6 + 6261608448
*a^11*b^2*c^7))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 105
6*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14
*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9
- 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12))
)*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 +
10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728
*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 4337679
9744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10
+ 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592
*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15
+ 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*
(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 1
07*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 +

b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 124032
0*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087
680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 +
193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a
^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^1
9 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050
869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^
2*c^24)))^(1/4)*1i))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 47110422
5280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b
^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7
*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174
656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c
^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 421376557
0560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c
+ 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c -
b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(109951
1627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^
3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*
b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 440297062
40*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^1
6 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 104045582
74560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*
b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 54
97558138880*a^19*b^2*c^24)))^(1/4) + 2*atan(((((3*(3159*a^3*b^14 - 20155392
*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340*a^6*b^8*c
^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*a^9*b^2*c^
6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5
376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6
*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (((-(81*(b^33 - b^8*(-(4
*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394
248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 14243688
96*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 1084
93078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11
*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262
682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c
- b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2
) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^
2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^
6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 1587609
6*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255
569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c
^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 52022791
37280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*
b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 1
3056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*(7036874

41776640*a^13*b*c^15 + 671088640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6 + 483183820800*a^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8 + 36077725286400*a^7*b^13*c^9 - 173173081374720*a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11 - 1319413953331200*a^10*b^7*c^12 + 1979120929996800*a^11*b^5*c^13 - 1759218604441600*a^12*b^3*c^14)*3i)/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (9*x^(1/2)*(16777216*a^3*b^25*c^4 - 31243722414882816*a^15*b*c^16 + 23890755584*a^4*b^23*c^5 - 1000190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7 - 209186382151680*a^7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 7925554690916352*a^9*b^13*c^10 + 28783015391920128*a^10*b^11*c^11 - 73870688712130560*a^11*b^9*c^12 + 130973825100677120*a^12*b^7*c^13 - 152242778028376064*a^13*b^5*c^14 + 103864266406232064*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(3/4)*1i)*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^

15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 520227913
7280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b
^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13
056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*1i - (9*x
^(1/2)*(123201*a^4*b^16 + 483729408*a^12*c^8 - 14619852*a^5*b^14*c + 653342
274*a^6*b^12*c^2 - 13105503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 - 66
486210048*a^9*b^6*c^5 + 9199443456*a^10*b^4*c^6 + 6261608448*a^11*b^2*c^7))
/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3
- 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*
a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*
b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 -
b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*
c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 +
1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c
^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*
a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13
- 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*
(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)
^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4
*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*
a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9
- 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^
12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^
10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 -
5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239
296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6
*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)
- (((3*(3159*a^3*b^14 - 20155392*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a
^5*b^10*c^2 - 299549340*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a
^8*b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*
a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 12
9024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2
c^9)) - (((-(81(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*
b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 -
140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7
+ 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b
^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562
531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*
b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2
*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^
(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^
20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8
+ 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11
- 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^2
2*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 211342

5899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*(703687441776640*a^13*b*c^15 + 671088640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6 + 483183820800*a^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8 + 36077725286400*a^7*b^13*c^9 - 173173081374720*a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11 - 1319413953331200*a^10*b^7*c^12 + 1979120929996800*a^11*b^5*c^13 - 1759218604441600*a^12*b^3*c^14)*3i)/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) + (9*x^(1/2)*(16777216*a^3*b^25*c^4 - 31243722414882816*a^15*b*c^16 + 23890755584*a^4*b^23*c^5 - 1000190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7 - 209186382151680*a^7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 7925554690916352*a^9*b^13*c^10 + 28783015391920128*a^10*b^11*c^11 - 73870688712130560*a^11*b^9*c^12 + 130973825100677120*a^12*b^7*c^13 - 152242778028376064*a^13*b^5*c^14 + 103864266406232064*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(3/4)*1i)*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^2

0*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*1i + (9*x^(1/2)*(123201*a^4*b^16 + 483729408*a^12*c^8 - 14619852*a^5*b^14*c + 653342274*a^6*b^12*c^2 - 13105503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5 + 9199443456*a^10*b^4*c^6 + 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4))/((((3*(3159*a^3*b^14 - 20155392*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (((-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*

a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^3
0*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^
8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704
475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^1
3*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20
 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 1305670057
9840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*(703687441776640*
a^13*b*c^15 + 671088640*a^3*b^21*c^5 - 26843545600*a^4*b^19*c^6 + 483183820
800*a^5*b^17*c^7 - 5153960755200*a^6*b^15*c^8 + 36077725286400*a^7*b^13*c^9
 - 173173081374720*a^8*b^11*c^10 + 577243604582400*a^9*b^9*c^11 - 131941395
3331200*a^10*b^7*c^12 + 1979120929996800*a^11*b^5*c^13 - 1759218604441600*a
^12*b^3*c^14)*3i)/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^
2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 +
344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (9*x^(1/2)
*(16777216*a^3*b^25*c^4 - 31243722414882816*a^15*b*c^16 + 23890755584*a^4*b
^23*c^5 - 1000190509056*a^5*b^21*c^6 + 18747532247040*a^6*b^19*c^7 - 209186
382151680*a^7*b^17*c^8 + 1544951275978752*a^8*b^15*c^9 - 7925554690916352*a
^9*b^13*c^10 + 28783015391920128*a^10*b^11*c^11 - 73870688712130560*a^11*b^
9*c^12 + 130973825100677120*a^12*b^7*c^13 - 152242778028376064*a^13*b^5*c^1
4 + 103864266406232064*a^14*b^3*c^15))/(4194304*(b^24*c + 16777216*a^12*c^1
3 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^1
6*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8
+ 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 5
0331648*a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 4711
04225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a
^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992
*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 1315
1174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b
^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 42137
65570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^3
1*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a
*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(10
99511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 7296
0*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*
a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029
706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18
*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404
558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a
^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23
 - 5497558138880*a^19*b^2*c^24)))^(3/4)*1i)*(-(81*(b^33 - b^8*(-(4*a*c - b^2
)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^
27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^2
1*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a
^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 38
40358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^1

4*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*1i - (9*x^(1/2)*(123201*a^4*b^16 + 483729408*a^12*c^8 - 14619852*a^5*b^14*c + 653342274*a^6*b^12*c^2 - 13105503216*a^7*b^10*c^3 + 102306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5 + 9199443456*a^10*b^4*c^6 + 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)*1i + (((3*(3159*a^3*b^14 - 20155392*a^10*c^7 - 367497*a^4*b^12*c + 15900219*a^5*b^10*c^2 - 299549340*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 + 2840323968*a^8*b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*b^2*c^9)) - (((-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^

15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c
^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2)
+ 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^2
5 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 124
0320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270
087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14
+ 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 211342589952
0*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*
c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585
050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19
*b^2*c^24)))^(1/4)*(703687441776640*a^13*b*c^15 + 671088640*a^3*b^21*c^5 -
26843545600*a^4*b^19*c^6 + 483183820800*a^5*b^17*c^7 - 5153960755200*a^6*b^
15*c^8 + 36077725286400*a^7*b^13*c^9 - 173173081374720*a^8*b^11*c^10 + 5772
43604582400*a^9*b^9*c^11 - 1319413953331200*a^10*b^7*c^12 + 197912092999680
0*a^11*b^5*c^13 - 1759218604441600*a^12*b^3*c^14)*3i)/(65536*(b^18*c - 2621
44*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*
a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8
+ 589824*a^8*b^2*c^9)) + (9*x^(1/2)*(16777216*a^3*b^25*c^4 - 3124372241488
2816*a^15*b*c^16 + 23890755584*a^4*b^23*c^5 - 1000190509056*a^5*b^21*c^6 +
18747532247040*a^6*b^19*c^7 - 209186382151680*a^7*b^17*c^8 + 15449512759787
52*a^8*b^15*c^9 - 7925554690916352*a^9*b^13*c^10 + 28783015391920128*a^10*b
^11*c^11 - 73870688712130560*a^11*b^9*c^12 + 130973825100677120*a^12*b^7*c^
13 - 152242778028376064*a^13*b^5*c^14 + 103864266406232064*a^14*b^3*c^15))/
(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 -
14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a
^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b
^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 -
b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c
^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 +
1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^
8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a
^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 -
8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(
-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^
25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*
a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a
*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 -
15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^1
2 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^1
0*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 -
5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 166472932392
96*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*
c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(3/4)*
1i)*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16
+ 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 1402337

28*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376
799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^1
0 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 75625314385
92*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^1
5 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^
2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) +
 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25
 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240
320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 12700
87680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14
+ 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520
*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c
^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 195850
50869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*
b^2*c^24)))^(1/4)*1i + (9*x^(1/2)*(123201*a^4*b^16 + 483729408*a^12*c^8 - 1
4619852*a^5*b^14*c + 653342274*a^6*b^12*c^2 - 13105503216*a^7*b^10*c^3 + 10
2306071520*a^8*b^8*c^4 - 66486210048*a^9*b^6*c^5 + 9199443456*a^10*b^4*c^6
+ 6261608448*a^11*b^2*c^7))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^
22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811
008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*
a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^1
1*b^2*c^12)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^
16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4
 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c
^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^1
0*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7
562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^
15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*
a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^2
5)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776
*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*
c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^
11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*
b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 211
3425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a
^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^2
1 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 549755813
8880*a^19*b^2*c^24)))^(1/4)*1i))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2
) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9
219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 97
32052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^
9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 384035821977
6*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14
 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 1
57*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^

```
3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(3355
4432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^
7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 15
8760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13
 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a
^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18
 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116
549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b
^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(1/4)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.1081 $\int \frac{x^{13/2}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=569

$$\frac{x^{7/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^{3/2}\left(x^2\left(28ac+5b^2\right)+24ab\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\left(\sqrt{b^2-4ac}\left(28ac+5b^2\right)+172abc+5b^3\right)\tan^{-1}}{32\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}}$$

```
[Out] 1/4*x^(7/2)*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/16*x^(3/2)*(24*a*b
+(28*a*c+5*b^2)*x^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/64*arctan(2^(1/4)*c^(
1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(5*b^2+28*a*c+(-172*a*b*c-5*b^3
)/(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^2/(-b+(-4*a*c+b^2)^(1/2)
)^(1/4)-1/64*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))
*(5*b^2+28*a*c+(-172*a*b*c-5*b^3)/(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(3/4)/(-4*a
*c+b^2)^2/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/64*arctan(2^(1/4)*c^(1/4)*x^(1/2)
/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(5*b^3+172*a*b*c+(28*a*c+5*b^2)*(-4*a*c+b^2
)^(1/2))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1
/64*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(5*b^3+1
72*a*b*c+(28*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(5
/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A] time = 1.91, antiderivative size = 569, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1115, 1365, 1498, 1510, 298, 205, 208}

$$\frac{\left(\sqrt{b^2-4ac}\left(28ac+5b^2\right)+172abc+5b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(-\frac{172abc+5b^3}{\sqrt{b^2-4ac}}+28ac+5b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right.}{32\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^2\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^(7/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x^(3/2)*
(24*a*b + (5*b^2 + 28*a*c)*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) +
 ((5*b^3 + 172*a*b*c + Sqrt[b^2 - 4*a*c]*(5*b^2 + 28*a*c))*ArcTan[(2^(1/4)*
c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*c^(3/4)*(b^2
- 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + ((5*b^2 + 28*a*c - (5*b^3
+ 172*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt
[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2 -
```

```
 4*a*c])^(1/4)) - ((5*b^3 + 172*a*b*c + Sqrt[b^2 - 4*a*c]*(5*b^2 + 28*a*c))
*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(
3/4)*c^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ((5*b^2
+ 28*a*c - (5*b^3 + 172*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*
Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)
^2*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1498

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Simp[(f^(n - 1)*(f*x)^(m - n + 1)*(a +
 b*x^n + c*x^(2*n))^(p + 1)*(b*d - 2*a*e - (b*e - 2*c*d)*x^n))/(n*(p + 1)*(
```

```
b^2 - 4*a*c)), x] + Dist[f^n/(n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - n)*(
a + b*x^n + c*x^(2*n))^(p + 1)*Simp[(n - m - 1)*(b*d - 2*a*e) + (2*n*p + 2*
n + m + 1)*(b*e - 2*c*d)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m,
n - 1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^{13/2}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^{14}}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{7/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{x^6\left(14a-5bx^4\right)}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}$$

$$= \frac{x^{7/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(24ab+\left(5b^2+28ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(-72ab+\left(5\right.\right.}{a+bx^4}\right.}{16\left(b^2\right.}$$

$$= \frac{x^{7/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(24ab+\left(5b^2+28ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(5b^3+172abc+\sqrt{b}\right.}{}$$

$$= \frac{x^{7/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(24ab+\left(5b^2+28ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(5b^3+172abc+\sqrt{b}\right.}{}$$

$$= \frac{x^{7/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(24ab+\left(5b^2+28ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(5b^3+172abc+\sqrt{b}\right.}{32\,2^{3/4}c^{3/4}}$$

Mathematica [C] time = 0.41, size = 216, normalized size = 0.38

$$\frac{c\left(a+bx^2+cx^4\right)^2\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{28\#1^4ac\log\left(\sqrt{x}-\#1\right)+5\#1^4b^2\log\left(\sqrt{x}-\#1\right)-72ab\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]-16x^{3/2}\left(b^2-\right.}{64c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (4*x^(3/2)*(4*b^3 + 8*a*b*c + 5*b^2*c*x^2 + 28*a*c^2*x^2)*(a + b*x^2 + c*x^
4) - 16*(b^2 - 4*a*c)*x^(3/2)*(b^2*x^2 + a*(b - 2*c*x^2)) + c*(a + b*x^2 +
c*x^4)^2*RootSum[a + b*#1^4 + c*#1^8 & , (-72*a*b*Log[Sqrt[x] - #1] + 5*b^2
*Log[Sqrt[x] - #1]*#1^4 + 28*a*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5)
& ])/(64*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 191.18Unable to convert to
real 1/4 Error: Bad Argument Value
```

maple [C] time = 0.04, size = 242, normalized size = 0.43

$$\frac{\left(\left(28ac+5b^2\right)\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^6-72\,\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^2ab\right)\ln\left(-\text{RootOf}\left(c\_Z^8+b\_Z^4+\right.\right.}{64\left(16a^2c^2-8a\,b^2c+b^4\right)\left(2\,\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7c+\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(3/4/(16*a^2*c^2-8*a*b^2*c+b^4)*a^2*b*x^(3/2)-1/32*a*(4*a*c-37*b^2)/(16*a
^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+9/32*b*(4*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)
*x^(11/2)+1/32*c*(28*a*c+5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(15/2))/(c*x^4
+b*x^2+a)^2+1/64/(16*a^2*c^2-8*a*b^2*c+b^4)*sum(((28*a*c+5*b^2)*_R^6-72*_R^
2*a*b)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(5\,b^{2}c+28\,ac^{2}\right)x^{\frac{15}{2}}+9\left(b^{3}+4\,abc\right)x^{\frac{11}{2}}+24\,a^{2}bx^{\frac{3}{2}}+\left(37\,ab^{2}-4\,a^{2}c\right)x^{\frac{7}{2}}}{16\left(\left(b^{4}c^{2}-8\,ab^{2}c^{3}+16\,a^{2}c^{4}\right)x^{8}+2\left(b^{5}c-8\,ab^{3}c^{2}+16\,a^{2}bc^{3}\right)x^{6}+a^{2}b^{4}-8\,a^{3}b^{2}c+16\,a^{4}c^{2}+\left(b^{6}-6\,ab^{4}c+32\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((5*b^2*c + 28*a*c^2)*x^(15/2) + 9*(b^3 + 4*a*b*c)*x^(11/2) + 24*a^2*b
*x^(3/2) + (37*a*b^2 - 4*a^2*c)*x^(7/2))/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*
c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*
c + 16*a^3*b*c^2)*x^2) + integrate(1/32*((5*b^2 + 28*a*c)*x^(5/2) - 72*a*b*
sqrt(x))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*
c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

mupad [B] time = 8.01, size = 39697, normalized size = 69.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((9*x^(11/2)*(b^3 + 4*a*b*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^(7/2
)*(37*a*b^2 - 4*a^2*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^(15/2)*(
28*a*c + 5*b^2))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a^2*b*x^(3/2))/(2
*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*
b*x^2 + 2*b*c*x^6) - atan(((((386183668047020032*a^16*c^16 + 2097152000*a^3
*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^5 - 515
4027327193088*a^6*b^20*c^6 + 52821290217635840*a^7*b^18*c^7 - 3505726682667
41760*a^8*b^16*c^8 + 1560295235622273024*a^9*b^14*c^9 - 4628236966960300032
*a^10*b^12*c^10 + 8604139182719238144*a^11*b^10*c^11 - 7924026369753743360*
a^12*b^8*c^12 - 1942353261163970560*a^13*b^6*c^13 + 1182321565924274995 2*a^
14*b^4*c^14 - 8419198028392431616*a^15*b^2*c^15)/(268435456*(b^28 + 2684354
56*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4
 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 1
```

96804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 -
 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c
^13 - 56*a*b^26*c)) - (x^(1/2)*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1
/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c
^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*
b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 144629
70429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*
b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 +
150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 2
3125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4
c(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3
 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^3
2*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^2
6*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 19373070745
6*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^
15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 1664729
3239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17
*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(
1/4)*(27584547717644288*a^15*c^16 + 99891544064*a^3*b^24*c^4 - 409256696217
6*a^4*b^22*c^5 + 75824426385408*a^5*b^20*c^6 - 837991069122560*a^6*b^18*c^7
 + 6133342147706880*a^7*b^16*c^8 - 31188471955587072*a^8*b^14*c^9 + 1123431
50323826688*a^9*b^12*c^10 - 286537128244936704*a^10*b^10*c^11 + 50774347459
0679040*a^11*b^8*c^12 - 599365778533253120*a^12*b^6*c^13 + 4363565826456944
64*a^13*b^4*c^14 - 170573835886657536*a^14*b^2*c^15))/(4194304*(b^24 + 1677
7216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c
^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 3
2440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331
648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25
)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^
25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*
a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14
462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a
^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^1
3 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2)
 + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a
*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40
*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4
*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7
*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 1937307
07456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^1
6*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 166
47293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*
a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)
))^(3/4) - (x^(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098125
*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 3874698

62400*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7))/(
4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3
 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976
128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a
^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^
6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c
^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21
*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 66645041479
68*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^1
0 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 26745984
4112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*
a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)
^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627
776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^
34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*
c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9
*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 21
13425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*
a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^
19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 54975581
38880*a^19*b^2*c^22)))^(1/4)*1i - (((386183668047020032*a^16*c^16 + 2097152
000*a^3*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^
5 - 5154027327193088*a^6*b^20*c^6 + 52821290217635840*a^7*b^18*c^7 - 350572
668266741760*a^8*b^16*c^8 + 1560295235622273024*a^9*b^14*c^9 - 462823696696
0300032*a^10*b^12*c^10 + 8604139182719238144*a^11*b^10*c^11 - 7924026369753
743360*a^12*b^8*c^12 - 1942353261163970560*a^13*b^6*c^13 + 1182321565924274
9952*a^14*b^4*c^14 - 8419198028392431616*a^15*b^2*c^15)/(268435456*(b^28 +
268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b
^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*
c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8
*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^1
3*b^2*c^13 - 56*a*b^26*c)) + (x^(1/2)*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)
^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3
*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 2651888332
80*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 +
 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 7045524226048
0*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*
c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1
/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 5437
5*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b
^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*
a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*
a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 1937
30707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*
b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 -

16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 195850508697
60*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^
22)))^(1/4)*(27584547717644288*a^15*c^16 + 99891544064*a^3*b^24*c^4 - 40925
66962176*a^4*b^22*c^5 + 75824426385408*a^5*b^20*c^6 - 837991069122560*a^6*b
^18*c^7 + 6133342147706880*a^7*b^16*c^8 - 31188471955587072*a^8*b^14*c^9 +
112343150323826688*a^9*b^12*c^10 - 286537128244936704*a^10*b^10*c^11 + 5077
43474590679040*a^11*b^8*c^12 - 599365778533253120*a^12*b^6*c^13 + 436356582
645694464*a^13*b^4*c^14 - 170573835886657536*a^14*b^2*c^15))/(4194304*(b^24
 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4
*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*
c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10
- 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c -
b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416
*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188
833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c
^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 704552422
60480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*
b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25
)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) +
54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23
 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240
320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087
680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 +
193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a
^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^1
7 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050
869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^
2*c^22)))^(3/4) + (x^(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c^8 + 28
1098125*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 +
387469862400*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*
c^7))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b
^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6
- 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 692
06016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 +
 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2
*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a
^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664
504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b
^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 2
67459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^
3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c
 - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(109
9511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960
*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^
6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706

240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*1i)/((((38618366804702003 2*a^16*c^16 + 2097152000*a^3*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^5 - 5154027327193088*a^6*b^20*c^6 + 52821290217635840*a^7*b^18*c^7 - 350572668266741760*a^8*b^16*c^8 + 1560295235622273024*a^9*b^14*c^9 - 4628236966960300032*a^10*b^12*c^10 + 8604139182719238144*a^11*b^10*c^11 - 7924026369753743360*a^12*b^8*c^12 - 1942353261163970560*a^13*b^6*c^13 + 11823215659242749952*a^14*b^4*c^14 - 8419198028392431616*a^15*b^2*c^15)/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) - (x^(1/2)*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*(27584547717644288*a^15*c^16 + 99891544064*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c^5 + 75824426385408*a^5*b^20*c^6 - 837991069122560*a^6*b^18*c^7 + 6133342147706880*a^7*b^16*c^8 - 31188471955587072*a^8*b^14*c^9 + 112343150323826688*a^9*b^12*c^10 - 286537128244936704*a^10*b^10*c^11 + 507743474590679040*a^11*b^8*c^12 - 599365778533253120*a^12*b^6*c^13 + 436356582645694464*a^13*b^4*c^14 - 170573835886657536*a^14*b^2*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70

455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 26745984411238
4*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c -
b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(
1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(109951162776*a^
20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6
 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 -
1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*
c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 21134258
99520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b
^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 1
9585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*
a^19*b^2*c^22)))^(3/4) - (x^(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c
^8 + 281098125*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9
*c^4 + 387469862400*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 117420369920*a
^9*b^3*c^7))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 1408
0*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^
12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^
9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625
*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89
000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 254924
09600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7
 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520
*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c
^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416
*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(
-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(335544
32*(109951162776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5
 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 15876
0960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 4
4029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*
b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 1
0404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 208091165491
20*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c
^21 - 5497558138880*a^19*b^2*c^22)))^(1/4) + (((386183668047020032*a^16*c^1
6 + 2097152000*a^3*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 295658569334784*
a^5*b^22*c^5 - 5154027327193088*a^6*b^20*c^6 + 52821290217635840*a^7*b^18*c
^7 - 350572668266741760*a^8*b^16*c^8 + 1560295235622273024*a^9*b^14*c^9 - 4
628236966960300032*a^10*b^12*c^10 + 8604139182719238144*a^11*b^10*c^11 - 79
24026369753743360*a^12*b^8*c^12 - 1942353261163970560*a^13*b^6*c^13 + 11823
215659242749952*a^14*b^4*c^14 - 8419198028392431616*a^15*b^2*c^15)/(2684354
56*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 2
56256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 5622988
8*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 10496245
76*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 93
9524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (x^(1/2)*(-(625*b^31 + 625*b^6*(-(4

*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 2
7186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 -
 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8
*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70
455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 26745984411238
4*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c -
b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(
1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^
20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6
 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 -
1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*
c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 21134258
99520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b
^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 1
9585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*
a^19*b^2*c^22)))^(1/4)*(27584547717644288*a^15*c^16 + 99891544064*a^3*b^24*
c^4 - 4092566962176*a^4*b^22*c^5 + 75824426385408*a^5*b^20*c^6 - 8379910691
22560*a^6*b^18*c^7 + 6133342147706880*a^7*b^16*c^8 - 31188471955587072*a^8*
b^14*c^9 + 112343150323826688*a^9*b^12*c^10 - 286537128244936704*a^10*b^10*
c^11 + 507743474590679040*a^11*b^8*c^12 - 599365778533253120*a^12*b^6*c^13
+ 436356582645694464*a^13*b^4*c^14 - 170573835886657536*a^14*b^2*c^15))/(41
94304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 +
 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 1297612
8*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^1
0*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c))*(-(625*b^31 + 625*b^6*
(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2
 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c
^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968
*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10
- 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 2674598441
12384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*
c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^2
5)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(109951162777
6*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34
*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^
9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b
^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113
425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^
14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19
 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138
880*a^19*b^2*c^22)))^(3/4) + (x^(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10
*b*c^8 + 281098125*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940400*a^6
*b^9*c^4 + 387469862400*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 1174203699
20*a^9*b^3*c^7))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 -
14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^

6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^
6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-
(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15
- 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25
492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17
*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 416332644
3520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b
^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 3
8416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c
^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33
554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*
c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 1
58760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11
 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a
^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16
 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116
549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b
^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4) - (285333125*a^4*b^15*c + 48
189030400*a^11*b*c^8 + 22337507500*a^5*b^13*c^2 + 657473586000*a^6*b^11*c^3
 + 8657411576000*a^7*b^9*c^4 + 43867083462400*a^8*b^7*c^5 + 13299491251200*
a^9*b^5*c^6 + 1381697515520*a^10*b^3*c^7)/(134217728*(b^28 + 268435456*a^14
*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050
048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 19680460
8*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 152672
6656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 5
6*a*b^26*c))))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 1519210463
2320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*
a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 16888
16578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^1
3*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 20666
9464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*
a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c +
1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^
2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4
 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096
*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569
920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13
 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 52022791372
80*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^1
0*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 1305
6700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*2i - 2*atan
(((((386183668047020032*a^16*c^16 + 2097152000*a^3*b^26*c^3 - 7615312560128
*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^5 - 5154027327193088*a^6*b^20*c^
6 + 52821290217635840*a^7*b^18*c^7 - 350572668266741760*a^8*b^16*c^8 + 1560
295235622273024*a^9*b^14*c^9 - 4628236966960300032*a^10*b^12*c^10 + 8604139

182719238144*a^11*b^10*c^11 - 7924026369753743360*a^12*b^8*c^12 - 194235326
1163970560*a^13*b^6*c^13 + 1182321565924274995 2*a^14*b^4*c^14 - 84191980283
92431616*a^15*b^2*c^15)/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b
^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 +
12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 5
24812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11
+ 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) - (x^
(1/2)*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*
b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c
^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a
^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 41
63326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360
*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c
^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^
2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/
2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^
2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*
c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^
24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 7044752
99840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^
14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 2
0809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840
*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*(27584547717644288*a^
15*c^16 + 99891544064*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c^5 + 758244263
85408*a^5*b^20*c^6 - 837991069122560*a^6*b^18*c^7 + 6133342147706880*a^7*b^
16*c^8 - 31188471955587072*a^8*b^14*c^9 + 112343150323826688*a^9*b^12*c^10
- 286537128244936704*a^10*b^10*c^11 + 507743474590679040*a^11*b^8*c^12 - 59
9365778533253120*a^12*b^6*c^13 + 436356582645694464*a^13*b^4*c^14 - 1705738
35886657536*a^14*b^2*c^15)*1i)/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a
^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^
5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 5
7671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*
a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320
*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*
b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 168881657
8560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^
9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464
207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14
*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911
000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^2
5)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3
040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5
*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*
a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 7
04475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a

^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^
18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700
579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(3/4)*1i + (x^(1/2)*(
3705625*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098125*a^4*b^13*c^2 + 78857
79000*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 387469862400*a^7*b^7*c^5 - 4
97953639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7))/(4194304*(b^24 + 16777
216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^
4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32
440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 503316
48*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625
*b^31 + 1519210463232O*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25
*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^
6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 1446
2970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^1
1*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13
- 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) -
23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b
^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c
^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b
^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b
^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707
456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*
c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647
293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^
17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))
^(1/4) - (((386183668047020032*a^16*c^16 + 2097152000*a^3*b^26*c^3 - 761531
2560128*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^5 - 5154027327193088*a^6*
b^20*c^6 + 52821290217635840*a^7*b^18*c^7 - 350572668266741760*a^8*b^16*c^8
+ 1560295235622273024*a^9*b^14*c^9 - 4628236966960300032*a^10*b^12*c^10 +
8604139182719238144*a^11*b^10*c^11 - 7924026369753743360*a^12*b^8*c^12 - 19
42353261163970560*a^13*b^6*c^13 + 11823215659242749952*a^14*b^4*c^14 - 8419
198028392431616*a^15*b^2*c^15)/(268435456*(b^28 + 268435456*a^14*c^14 + 145
6*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^1
8*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*
c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b
^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)
) + (x^(1/2)*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 1519210463232
0*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4
*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 16888165
78560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c
^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 20666946
4207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^1
4*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 191
1000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^
25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 +

3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^
5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920
*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 -
704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*
a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c
^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 1305670
0579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*(2758454771764
4288*a^15*c^16 + 99891544064*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c^5 + 75
824426385408*a^5*b^20*c^6 - 837991069122560*a^6*b^18*c^7 + 6133342147706880
*a^7*b^16*c^8 - 31188471955587072*a^8*b^14*c^9 + 112343150323826688*a^9*b^1
2*c^10 - 286537128244936704*a^10*b^10*c^11 + 507743474590679040*a^11*b^8*c^
12 - 599365778533253120*a^12*b^6*c^13 + 436356582645694464*a^13*b^4*c^14 -
170573835886657536*a^14*b^2*c^15)*1i)/(4194304*(b^24 + 16777216*a^12*c^12 +
 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*
b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*
c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^1
1 - 48*a*b^22*c))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 1519210
4632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 13422976
00*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 16
88816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*
b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 20
6669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 1500091147878
40*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c
 + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c -
 b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*
c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876
096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255
569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c
^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 52022791
37280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*
b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 1
3056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(3/4)*1i - (x^
(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098125*a^4*b^13*c^2
+ 7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 387469862400*a^7*b^7*
c^5 - 497953639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7))/(4194304*(b^24
+ 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*
b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c
^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 -
 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2
) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a
^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 26518883
3280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8
 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260
480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^
5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^

(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54
375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 +
 b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 124032
0*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 127008768
0*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 19
3730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^1
2*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17
- 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 1958505086
9760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*
c^22)))^(1/4))/((((386183668047020032*a^16*c^16 + 2097152000*a^3*b^26*c^3 -
 7615312560128*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^5 - 51540273271930
88*a^6*b^20*c^6 + 52821290217635840*a^7*b^18*c^7 - 350572668266741760*a^8*b
^16*c^8 + 1560295235622273024*a^9*b^14*c^9 - 4628236966960300032*a^10*b^12*
c^10 + 8604139182719238144*a^11*b^10*c^11 - 7924026369753743360*a^12*b^8*c^
12 - 1942353261163970560*a^13*b^6*c^13 + 11823215659242749952*a^14*b^4*c^14
 - 8419198028392431616*a^15*b^2*c^15)/(268435456*(b^28 + 268435456*a^14*c^1
4 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*
a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^
8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656
*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*
b^26*c)) - (x^(1/2)*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 151921
04632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297
600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1
688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9
*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 2
06669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787
840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*
c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c
- b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38
*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 1587
6096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 825
5569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*
c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279
137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15
*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 +
13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*(275845
47717644288*a^15*c^16 + 99891544064*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c
^5 + 75824426385408*a^5*b^20*c^6 - 837991069122560*a^6*b^18*c^7 + 613334214
7706880*a^7*b^16*c^8 - 31188471955587072*a^8*b^14*c^9 + 112343150323826688*
a^9*b^12*c^10 - 286537128244936704*a^10*b^10*c^11 + 507743474590679040*a^11
*b^8*c^12 - 599365778533253120*a^12*b^6*c^13 + 436356582645694464*a^13*b^4*
c^14 - 170573835886657536*a^14*b^2*c^15)*1i)/(4194304*(b^24 + 16777216*a^12
*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 8110
08*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a
^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*

b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(3/4)*1i + (x^(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098125*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 387469862400*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*1i + (((38618366804702003 2*a^16*c^16 + 2097152000*a^3*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^5 - 5154027327193088*a^6*b^20*c^6 + 52821290217635840*a^7*b^18*c^7 - 350572668266741760*a^8*b^16*c^8 + 1560295235622273024*a^9*b^14*c^9 - 4628236966960300032*a^10*b^12*c^10 + 8604139182719238144*a^11*b^10*c^11 - 7924026369753743360*a^12*b^8*c^12 - 1942353261163970560*a^13*b^6*c^13 + 11823215659242749952*a^14*b^4*c^14 - 8419198028392431616*a^15*b^2*c^15)/(268435456*(b^28 + 26843545

6*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (x^(1/2)*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*(27584547717644288*a^15*c^16 + 99891544064*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c^5 + 75824426385408*a^5*b^20*c^6 - 837991069122560*a^6*b^18*c^7 + 6133342147706880*a^7*b^16*c^8 - 31188471955587072*a^8*b^14*c^9 + 112343150323826688*a^9*b^12*c^10 - 286537128244936704*a^10*b^10*c^11 + 507743474590679040*a^11*b^8*c^12 - 599365778533253120*a^12*b^6*c^13 + 436356582645694464*a^13*b^4*c^14 - 170573835886657536*a^14*b^2*c^15)*1i)/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)

))^(3/4)*1i - (x^(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098
125*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 3874
69862400*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7)
)/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*
c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12
976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 6920601
6*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*
c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27
*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^
21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 666450414
7968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c
^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459
844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(
4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^
2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(10995116
27776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*
b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^2
8*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a
^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 +
2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 1040455827456
0*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*
c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 549755
8138880*a^19*b^2*c^22)))^(1/4)*1i + (285333125*a^4*b^15*c + 48189030400*a^1
1*b*c^8 + 22337507500*a^5*b^13*c^2 + 657473586000*a^6*b^11*c^3 + 8657411576
000*a^7*b^9*c^4 + 43867083462400*a^8*b^7*c^5 + 13299491251200*a^9*b^5*c^6 +
1381697515520*a^10*b^3*c^7)/(134217728*(b^28 + 268435456*a^14*c^14 + 1456*
a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*
c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^
8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6
*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)))
)*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^
15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 -
25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b
^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 416332
6443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^1
2*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14
- 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^
2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/
(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^
36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8
+ 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c
^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 70447529984
0*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c
^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809
116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^1

8*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4) - 2*atan(((((386183668047
020032*a^16*c^16 + 2097152000*a^3*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 2
95658569334784*a^5*b^22*c^5 - 5154027327193088*a^6*b^20*c^6 + 5282129021763
5840*a^7*b^18*c^7 - 350572668266741760*a^8*b^16*c^8 + 1560295235622273024*a
^9*b^14*c^9 - 4628236966960300032*a^10*b^12*c^10 + 8604139182719238144*a^11
*b^10*c^11 - 7924026369753743360*a^12*b^8*c^12 - 1942353261163970560*a^13*b
^6*c^13 + 11823215659242749952*a^14*b^4*c^14 - 8419198028392431616*a^15*b^2
c^15)/(268435456(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a
^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^1
6*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10
*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^1
2*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) - (x^(1/2)*(-(625*b^31
 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a
^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600
*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 66
64504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10
*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 -
 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*
c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a
*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1
099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 729
60*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*
a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 440297
06240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*
c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 104045
58274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^
16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 -
 5497558138880*a^19*b^2*c^22)))^(1/4)*(27584547717644288*a^15*c^16 + 998915
44064*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c^5 + 75824426385408*a^5*b^20*c
^6 - 837991069122560*a^6*b^18*c^7 + 6133342147706880*a^7*b^16*c^8 - 3118847
1955587072*a^8*b^14*c^9 + 112343150323826688*a^9*b^12*c^10 - 28653712824493
6704*a^10*b^10*c^11 + 507743474590679040*a^11*b^8*c^12 - 599365778533253120
*a^12*b^6*c^13 + 436356582645694464*a^13*b^4*c^14 - 170573835886657536*a^14
*b^2*c^15)*1i)/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14
080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*
b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*
c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(6
25*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 -
89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 2549
2409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c
^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 41633264435
20*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7
*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 384
16*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2
*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(3355

4432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(3/4)*1i + (x^(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098125*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 387469862400*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4) - (((386183668047020032*a^16*c^16 + 2097152000*a^3*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^5 - 5154027327193088*a^6*b^20*c^6 + 52821290217635840*a^7*b^18*c^7 - 350572668266741760*a^8*b^16*c^8 + 1560295235622273024*a^9*b^14*c^9 - 4628236966960300032*a^10*b^12*c^10 + 8604139182719238144*a^11*b^10*c^11 - 7924026369753743360*a^12*b^8*c^12 - 1942353261163970560*a^13*b^6*c^13 + 11823215659242749952*a^14*b^4*c^14 - 8419198028392431616*a^15*b^2*c^15)/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (x^(1/2)*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 41633264

43520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*
b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 -
38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*
c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(3
3554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36
*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 +
158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^1
1 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*
a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^1
6 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 2080911
6549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*
b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*(27584547717644288*a^15*c^1
6 + 99891544064*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c^5 + 75824426385408*
a^5*b^20*c^6 - 837991069122560*a^6*b^18*c^7 + 6133342147706880*a^7*b^16*c^8
 - 31188471955587072*a^8*b^14*c^9 + 112343150323826688*a^9*b^12*c^10 - 2865
37128244936704*a^10*b^10*c^11 + 507743474590679040*a^11*b^8*c^12 - 59936577
8533253120*a^12*b^6*c^13 + 436356582645694464*a^13*b^4*c^14 - 1705738358866
57536*a^14*b^2*c^15)*1i)/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^2
0*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 37
84704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 5767168
0*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22
c)))(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15
*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*
c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*
a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4
163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 20666946420736
0*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*
c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a
^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1
/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a
^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30
*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b
^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475
299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b
^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 +
20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 1305670057984
0*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(3/4)*1i - (x^(1/2)*(37056
25*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098125*a^4*b^13*c^2 + 7885779000
*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 387469862400*a^7*b^7*c^5 - 497953
639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7))/(4194304*(b^24 + 16777216*a
^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 8
11008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 3244032
0*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^
11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2
) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3

- 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4))/((((386183668047020032*a^16*c^16 + 2097152000*a^3*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^5 - 5154027327193088*a^6*b^20*c^6 + 52821290217635840*a^7*b^18*c^7 - 350572668266741760*a^8*b^16*c^8 + 1560295235622273024*a^9*b^14*c^9 - 4628236966960300032*a^10*b^12*c^10 + 8604139182719238144*a^11*b^10*c^11 - 7924026369753743360*a^12*b^8*c^12 - 1942353261163970560*a^13*b^6*c^13 + 11823215659242749952*a^14*b^4*c^14 - 8419198028392431616*a^15*b^2*c^15)/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) - (x^(1/2)*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*(27584547717644288*a^15*c^16 + 99891544064*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c^5 + 75824426385408*a^5*b^20*c^6 - 837991069122560*a^6*b^18*c^7 + 6133342147706880*a^7*b^16*c^8 - 31188471955587072*a^8*b^14*c^9 + 112343150323826688*a^9*b^12*c^10 - 286537128244936704*a^10*b^10*c^11 + 507743474590679040*a^11*b^8*c^12

- 5993657785332531 20*a^12*b^6*c^13 + 436356582645694464*a^13*b^4*c^14 - 170
573835886657536*a^14*b^2*c^15)*1i)/(4194304*(b^24 + 16777216*a^12*c^12 + 10
56*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^1
4*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8
 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 -
 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 151921046
32320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600
*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688
816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^
13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 2066
69464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840
*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c +
 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b
^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^
4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 1587609
6*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 825556
9920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^1
3 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137
280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^
10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 130
56700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(3/4)*1i + (x^(1
/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098125*a^4*b^13*c^2 +
7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 387469862400*a^7*b^7*c^
5 - 497953639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7))/(4194304*(b^24 +
16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^
16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7
 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 5
0331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2
)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^
3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833
280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8
+ 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 704552422604
80*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5
*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(
1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 543
75*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 +
b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320
*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680
*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193
730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12
*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 -
 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869
760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c
^22)))^(1/4)*1i + (((386183668047020032*a^16*c^16 + 2097152000*a^3*b^26*c^3
 - 7615312560128*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^5 - 515402732719

3088*a^6*b^20*c^6 + 52821290217635840*a^7*b^18*c^7 - 350572668266741760*a^8*b^16*c^8 + 1560295235622273024*a^9*b^14*c^9 - 4628236966960300032*a^10*b^12*c^10 + 8604139182719238144*a^11*b^10*c^11 - 7924026369753743360*a^12*b^8*c^12 - 1942353261163970560*a^13*b^6*c^13 + 11823215659242749952*a^14*b^4*c^14 - 8419198028392431616*a^15*b^2*c^15)/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (x^(1/2)*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*(27584547717644288*a^15*c^16 + 99891544064*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c^5 + 75824426385408*a^5*b^20*c^6 - 837991069122560*a^6*b^18*c^7 + 6133342147706880*a^7*b^16*c^8 - 31188471955587072*a^8*b^14*c^9 + 112343150323826688*a^9*b^12*c^10 - 286537128244936704*a^10*b^10*c^11 + 507743474590679040*a^11*b^8*c^12 - 599365778533253120*a^12*b^6*c^13 + 436356582645694464*a^13*b^4*c^14 - 170573835886657536*a^14*b^2*c^15)*1i)/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26* c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(3/4)*1i - (x^(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098125*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 387469862400*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 1519104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*1i + (285333125*a^4*b^15*c + 48189030400*a^11*b*c^8 + 22337507500*a^5*b^13*c^2 + 657473586000*a^6*b^11*c^3 + 8657411576000*a^7*b^9*c^4 + 43867083462400*a^8*b^7*c^5 + 13299491251200*a^9*b^5*c^6 + 1381697515520*a^10*b^3*c^7)/(134217728*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c))))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 1519104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 15

8760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4) - atan(((((386183668047020032*a^16*c^16 + 2097152000*a^3*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^5 - 5154027327193088*a^6*b^20*c^6 + 52821290217635840*a^7*b^18*c^7 - 350572668266741760*a^8*b^16*c^8 + 1560295235622273024*a^9*b^14*c^9 - 4628236966960300032*a^10*b^12*c^10 + 8604139182719238144*a^11*b^10*c^11 - 7924026369753743360*a^12*b^8*c^12 - 1942353261163970560*a^13*b^6*c^13 + 11823215659242749952*a^14*b^4*c^14 - 8419198028392431616*a^15*b^2*c^15)/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) - (x^(1/2)*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*(27584547717644288*a^15*c^16 + 99891544064*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c^5 + 75824426385408*a^5*b^20*c^6 - 837991069122560*a^6*b^18*c^7 + 6133342147706880*a^7*b^16*c^8 - 31188471955587072*a^8*b^14*c^9 + 112343150323826688*a^9*b^12*c^10 - 286537128244936704*a^10*b^10*c^11 + 507743474590679040*a^11*b^8*c^12 - 599365778533253120*a^12*b^6*c^13 + 436356582645694464*a^13*b^4*c^14 - 170573835886657536*a^14*b^2*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 666450

4147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^1
1*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267
459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*
(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c -
b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(10995
11627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a
^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*
b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 4402970624
0*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14
+ 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 1040455827
4560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b
^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 549
7558138880*a^19*b^2*c^22)))^(3/4) - (x^(1/2)*(3705625*a^3*b^15*c - 64022568
96*a^10*b*c^8 + 281098125*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940
400*a^6*b^9*c^4 + 387469862400*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 117
420369920*a^9*b^3*c^7))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20
*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 378
4704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680
*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*
c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b
*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^
4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^
7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 416
3326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*
a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^
14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2
*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2
))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2
*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c
^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^2
4*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 70447529
9840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^1
4*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20
809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*
a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*1i - (((38618366804702
0032*a^16*c^16 + 2097152000*a^3*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 295
658569334784*a^5*b^22*c^5 - 5154027327193088*a^6*b^20*c^6 + 528212902176358
40*a^7*b^18*c^7 - 350572668266741760*a^8*b^16*c^8 + 1560295235622273024*a^9
*b^14*c^9 - 4628236966960300032*a^10*b^12*c^10 + 8604139182719238144*a^11*b
^10*c^11 - 7924026369753743360*a^12*b^8*c^12 - 1942353261163970560*a^13*b^6
*c^13 + 11823215659242749952*a^14*b^4*c^14 - 8419198028392431616*a^15*b^2*c
^15)/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3
*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*
c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c
^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*

b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (x^(1/2)*((625*b^6*(-(
4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*
b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^
5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 66645
04147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^
11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 26
7459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3
*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c
- b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099
511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*
a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6
*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 440297062
40*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^1
4 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 104045582
74560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*
b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 54
97558138880*a^19*b^2*c^22)))^(1/4)*(27584547717644288*a^15*c^16 + 998915440
64*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c^5 + 75824426385408*a^5*b^20*c^6
- 837991069122560*a^6*b^18*c^7 + 6133342147706880*a^7*b^16*c^8 - 3118847195
5587072*a^8*b^14*c^9 + 112343150323826688*a^9*b^12*c^10 - 28653712824493670
4*a^10*b^10*c^11 + 507743474590679040*a^11*b^8*c^12 - 599365778533253120*a^
12*b^6*c^13 + 436356582645694464*a^13*b^4*c^14 - 170573835886657536*a^14*b^
2*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^
3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c
^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 +
69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*
(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a
^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600
*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 66
64504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10
*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 +
267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*
c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a
*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1
099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 729
60*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*
a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 440297
06240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*
c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 104045
58274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^
16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 -
5497558138880*a^19*b^2*c^22)))^(3/4) + (x^(1/2)*(3705625*a^3*b^15*c - 6402
256896*a^10*b*c^8 + 281098125*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 9552
5940400*a^6*b^9*c^4 + 387469862400*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 -
117420369920*a^9*b^3*c^7))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*

b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 +
 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 5767
1680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b
^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^
15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^2
3*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 168881657856
0*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 +
 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207
360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^
3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000
*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^
(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040
*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^
30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8
*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 7044
75299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13
*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18
+ 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579
840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*1i)/((((3861836680
47020032*a^16*c^16 + 2097152000*a^3*b^26*c^3 - 7615312560128*a^4*b^24*c^4 +
 295658569334784*a^5*b^22*c^5 - 5154027327193088*a^6*b^20*c^6 + 52821290217
635840*a^7*b^18*c^7 - 350572668266741760*a^8*b^16*c^8 + 1560295235622273024
*a^9*b^14*c^9 - 4628236966960300032*a^10*b^12*c^10 + 8604139182719238144*a^
11*b^10*c^11 - 7924026369753743360*a^12*b^8*c^12 - 1942353261163970560*a^13
*b^6*c^13 + 11823215659242749952*a^14*b^4*c^14 - 8419198028392431616*a^15*b
^2*c^15)/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296
*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b
^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^
10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a
^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) - (x^(1/2)*((625*b^6
*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*
a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 2549240960
0*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6
664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^1
0*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12
+ 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3
c^3(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*
a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(
1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72
960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960
*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029
706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18
*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404
558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a
^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21

- 5497558138880*a^19*b^2*c^22)))^(1/4)*(27584547717644288*a^15*c^16 + 99891544064*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c^5 + 75824426385408*a^5*b^20*c^6 - 837991069122560*a^6*b^18*c^7 + 6133342147706880*a^7*b^16*c^8 - 31188471955587072*a^8*b^14*c^9 + 112343150323826688*a^9*b^12*c^10 - 286537128244936704*a^10*b^10*c^11 + 507743474590679040*a^11*b^8*c^12 - 599365778533253120*a^12*b^6*c^13 + 436356582645694464*a^13*b^4*c^14 - 170573835886657536*a^14*b^2*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(3/4) - (x^(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098125*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 387469862400*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280* a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4) + (((386183668047020032*a^16*c^16 + 2097152000*a^3*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^5 - 5154027327193088*a^6*b^20*c^6 + 52821290217635840*a^7*b^18*c^7 - 350572668266741760*a^8*b^16*c^8 + 1560295235622273024*a^9*b^14*c^9 - 4628236966960300032*a^10*b^12*c^10 + 8604139182719238144*a^11*b^10*c^11 - 7924026369753743360*a^12*b^8*c^12 - 1942353261163970560*a^13*b^6*c^13 + 11823215659242749952*a^14*b^4*c^14 - 8419198028392431616*a^15*b^2*c^15)/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (x^(1/2)*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4)*(27584547717644288*a^15*c^16 + 99891544064*a^3*b^24*c^4 - 4092566962176*a^4*b^22*c^5 + 75824426385408*a^5*b^20*c^6 - 837991069122560*a^6*b^18*c^7 + 6133342147706880*a^7*b^16*c^8 - 31188471955587072*a^8*b^14*c^9 + 112343150323826688*a^9*b^12*c^10 - 286537128244936704*a^10*b^10*c^11 + 507743474590679040*a^11*b^8*c^12 - 599365778533253120*a^12*b^6*c^13 + 436356582645694464*a^13*b^4*c^14 - 170573835886657536*a^14*b^2*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416

*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(
-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(335544
32*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5
- 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a^5*b^30*c^8 + 15876
0960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 4
4029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 - 704475299840*a^11*
b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280*a^13*b^14*c^16 + 1
0404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*c^18 + 208091165491
20*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 13056700579840*a^18*b^4*c
^21 - 5497558138880*a^19*b^2*c^22)))^(3/4) + (x^(1/2)*(3705625*a^3*b^15*c -
6402256896*a^10*b*c^8 + 281098125*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 +
95525940400*a^6*b^9*c^4 + 387469862400*a^7*b^7*c^5 - 497953639680*a^8*b^5*
c^6 - 117420369920*a^9*b^3*c^7))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056
*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*
c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 -
57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 4
8*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 151921046323
20*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^
4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816
578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*
c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 2066694
64207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^
14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 19
11000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)
^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 - 80*a*b^38*c^4 +
3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*c^7 - 15876096*a
^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*c^10 + 825556992
0*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*a^10*b^20*c^13 -
704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15 - 5202279137280
*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 16647293239296*a^15*b^10*
c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b^6*c^20 + 130567
00579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/4) - (285333125*
a^4*b^15*c + 48189030400*a^11*b*c^8 + 22337507500*a^5*b^13*c^2 + 6574735860
00*a^6*b^11*c^3 + 8657411576000*a^7*b^9*c^4 + 43867083462400*a^8*b^7*c^5 +
13299491251200*a^9*b^5*c^6 + 1381697515520*a^10*b^3*c^7)/(134217728*(b^28 +
268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*
b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14
*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^
8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^
13*b^2*c^13 - 56*a*b^26*c))))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^3
1 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3
+ 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^
19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970
429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^
9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 15

```
0009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 231
25*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c
*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3 -
 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^32*
c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^26*
c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 193730707456*
a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^15
 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 166472932
39296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17*b
^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(1/
4)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.1082 $\int \frac{x^{11/2}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=569

$$\frac{\sqrt{x}\left(x^2\left(20ac+7b^2\right)+24ab\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\left(\sqrt{b^2-4ac}\left(20ac+7b^2\right)+36abc+7b^3\right)\tan^{-1}}{32\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}\right.-}$$

```
[Out] 1/4*x^(5/2)*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-3/64*arctan(2^(1/4)*
c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(7*b^2+20*a*c+(-36*a*b*c-7*b
^3)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^2/(-b+(-4*a*c+b^2)^(1/
2))^(3/4)-3/64*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4
))*(7*b^2+20*a*c+(-36*a*b*c-7*b^3)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(1/4)/(-4*
a*c+b^2)^2/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-3/64*arctan(2^(1/4)*c^(1/4)*x^(1/2
)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(7*b^3+36*a*b*c+(20*a*c+7*b^2)*(-4*a*c+b^2
)^(1/2))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-3
/64*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(7*b^3+3
6*a*b*c+(20*a*c+7*b^2)*(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(5/
2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/16*(24*a*b+(20*a*c+7*b^2)*x^2)*x^(1/2)/(
-4*a*c+b^2)^2/(c*x^4+b*x^2+a)
```

Rubi [A] time = 1.96, antiderivative size = 569, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1115, 1365, 1498, 1422, 212, 208, 205}

$$\frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\sqrt{x}\left(x^2\left(20ac+7b^2\right)+24ab\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\left(\sqrt{b^2-4ac}\left(20ac+7b^2\right)+36abc+7b^3\right)\tan^{-1}}{32\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}\right.-}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^(5/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[x]*
(24*a*b + (7*b^2 + 20*a*c)*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) -
 (3*(7*b^3 + 36*a*b*c + Sqrt[b^2 - 4*a*c]*(7*b^2 + 20*a*c))*ArcTan[(2^(1/4)
*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(1/4)*(b^2
 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*(7*b^2 + 20*a*c - (7*b
^3 + 36*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sq
rt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2
```

```
 - 4*a*c])^(3/4)) - (3*(7*b^3 + 36*a*b*c + Sqrt[b^2 - 4*a*c]*(7*b^2 + 20*a*
c))*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*
2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*(7
*b^2 + 20*a*c - (7*b^3 + 36*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1
/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(1/4)*(b^2 - 4*
a*c)^2*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
```

```
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1498

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Simp[(f^(n - 1)*(f*x)^(m - n + 1)*(a +
 b*x^n + c*x^(2*n))^(p + 1)*(b*d - 2*a*e - (b*e - 2*c*d)*x^n))/(n*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[f^n/(n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - n)*(
a + b*x^n + c*x^(2*n))^(p + 1)*Simp[(n - m - 1)*(b*d - 2*a*e) + (2*n*p + 2*
n + m + 1)*(b*e - 2*c*d)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m,
n - 1] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^{11/2}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^{12}}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{x^4\left(10a-7bx^4\right)}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}$$

$$= \frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(24ab+\left(7b^2+20ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{-24ab+3\left(7b^2\right.}{a+bx^4+}\right.}{16\left(b^2-\right.}$$

$$= \frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(24ab+\left(7b^2+20ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3\left(7b^3+36abc+\sqrt{\right.\right.}}{}$$

$$= \frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(24ab+\left(7b^2+20ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(3\left(7b^3+36abc+\sqrt{\right.\right.}}{}$$

$$= \frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(24ab+\left(7b^2+20ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{3\left(7b^3+36abc+\sqrt{b\right.}}{32\sqrt[4]{2}\sqrt[4]{c}}$$

Mathematica [C] time = 0.39, size = 219, normalized size = 0.38

$$\frac{3c\left(a+bx^2+cx^4\right)^2\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{20\#1^4ac\log\left(\sqrt{x}-\#1\right)+7\#1^4b^2\log\left(\sqrt{x}-\#1\right)-8ab\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]-16\sqrt{x}\left(b^2\right.}{64c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (4*Sqrt[x]*(4*b^3 + 8*a*b*c + 7*b^2*c*x^2 + 20*a*c^2*x^2)*(a + b*x^2 + c*x^
4) - 16*(b^2 - 4*a*c)*Sqrt[x]*(b^2*x^2 + a*(b - 2*c*x^2)) + 3*c*(a + b*x^2
+ c*x^4)^2*RootSum[a + b*#1^4 + c*#1^8 & , (-8*a*b*Log[Sqrt[x] - #1] + 7*b^
2*Log[Sqrt[x] - #1]*#1^4 + 20*a*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^
7) & ])/(64*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 191.55Unable to convert to
real 1/4 Error: Bad Argument Value
```

maple [C] time = 0.04, size = 241, normalized size = 0.42

$$\frac{3\left(\left(20ac+7b^2\right)\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^4-8ab\right)\ln\left(-\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{64\left(16a^2c^2-8a\,b^2c+b^4\right)\left(2\,\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7c+\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3b\right)}+\frac{\frac{2\left(20ac+7b^2\right)c\,x^{\frac{13}{2}}}{512a^2c^2-256a\,b^2c+32b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(3/4/(16*a^2*c^2-8*a*b^2*c+b^4)*a^2*b*x^(1/2)-3/32*(4*a*c-13*b^2)/(16*a^2
*c^2-8*a*b^2*c+b^4)*a*x^(5/2)+1/32*b*(28*a*c+11*b^2)/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^(9/2)+1/32*c*(20*a*c+7*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(13/2))/(c*
x^4+b*x^2+a)^2+3/64/(16*a^2*c^2-8*a*b^2*c+b^4)*sum(((20*a*c+7*b^2)*_R^4-8*a
*b)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{24\,bc^{2}x^{\frac{17}{2}} + \left(41\,b^{2}c - 20\,ac^{2}\right)x^{\frac{13}{2}} + \left(13\,b^{3} + 20\,abc\right)x^{\frac{9}{2}} + 3\left(3\,ab^{2} + 4\,a^{2}c \cdots\right.}{16\left(\left(b^{4}c^{2} - 8\,ab^{2}c^{3} + 16\,a^{2}c^{4}\right)x^{8} + 2\left(b^{5}c - 8\,ab^{3}c^{2} + 16\,a^{2}bc^{3}\right)x^{6} + a^{2}b^{4} - 8\,a^{3}b^{2}c + 16\,a^{4}c^{2} + \left(b^{6} - 6\,ab^{4}c + 32 \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(24*b*c^2*x^(17/2) + (41*b^2*c - 20*a*c^2)*x^(13/2) + (13*b^3 + 20*a*
b*c)*x^(9/2) + 3*(3*a*b^2 + 4*a^2*c)*x^(5/2))/((b^4*c^2 - 8*a*b^2*c^3 + 16*
a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3
*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2
*b^3*c + 16*a^3*b*c^2)*x^2) + integrate(3/32*(8*b*c*x^(7/2) + 5*(3*b^2 + 4*
a*c)*x^(3/2))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16
*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

mupad [B] time = 8.52, size = 45495, normalized size = 79.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^(9/2)*(11*b^3 + 28*a*b*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*x^(
5/2)*(13*a*b^2 - 4*a^2*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^(13/2
)*(20*a*c + 7*b^2))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a^2*b*x^(1/2))
/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2
*a*b*x^2 + 2*b*c*x^6) - atan((((((3*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2
) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*
a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 398985216
0*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977
994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^1
1 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2
*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^2
5)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 30
40*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*
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b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a^13*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*a^4*b^18*c^6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 46901042872320*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 206158430208000*a^9*b^8*c^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^13 - 615726511554560*a^12*b^2*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c))) - (9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 147907936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*b^19*c^7 + 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 2233932749733888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 1599789418414080*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 16008889300418560*a^12*b^5*c^14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(3/4) + (3*(570240000*a^7*b*c^8 + 2917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 879403392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66

059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4) - (9*x^(1/2)*(43758225*a^2*b^14*c^3 - 10368000000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11404429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 - 22394880000*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*1i - ((((3*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579

840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a
^13*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*
a^4*b^18*c^6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 4
6901042872320*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 20615843020800
0*a^9*b^8*c^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^
13 - 615726511554560*a^12*b^2*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^
2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 +
344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)
) + (9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 14
7907936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*b
^19*c^7 + 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 223
3932749733888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 159978941841
4080*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 16008889300418560*a^1
2*b^5*c^14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a^
12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 81
1008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320
*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^1
1*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401
*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23
*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^
17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a
^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 2307
70606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*
c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)
))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b
^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6
 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c
^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840
*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^
14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 208091
16549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18
*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(3/4) + (3*(570240000*a^7*b*c^8
+ 2917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 879
403392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9
 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b
^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*
a*b^16*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 70464307
2000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040
*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799
680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66
059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^
5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/
2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^
40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*
a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6

*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4) + (9*x^(1/2)*(43758225*a^2*b^14*c^3 - 10368000000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11404429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 - 22394880000*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*1i)/(((((3*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a^13*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*a^4*b^18*c^6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 46901042872320*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 206158430208000*a^9*b^8*c^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^

13 - 615726511554560*a^12*b^2*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c))) - (9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 147907936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*b^19*c^7 + 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 2233932749733888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 1599789418414080*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 16008889300418560*a^12*b^5*c^14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(3/4) + (3*(570240000*a^7*b*c^8 + 2917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 879403392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497

558138880*a^19*b^2*c^20)))^(1/4) - (9*x^(1/2)*(43758225*a^2*b^14*c^3 - 1036
8000000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11404
429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 - 223
94880000*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*
c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784
704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*
a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c
)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^1
4*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^2
1*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*
b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 6605923942
4*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 +
887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 940
0*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1
099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34
*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^
7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^2
2*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 211342
5899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14
*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 -
19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 549755813888
0*a^19*b^2*c^20)))^(1/4) + ((((3*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) -
2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3
*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a
^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994
240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 -
230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-
(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^
(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*
a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^3
0*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b
^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 7044752
99840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^
14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 2
0809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840
*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a^13
*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*a^4
*b^18*c^6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 4690
1042872320*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 206158430208000*a
^9*b^8*c^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^13
- 615726511554560*a^12*b^2*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b
^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344
064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) +
(9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 14790
7936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*b^19

*c^7 + 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 223393
2749733888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 159978941841408
0*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 16008889300418560*a^12*b
^5*c^14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a^12*
c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 81100
8*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^
8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b
^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^
29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^
3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*
c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*
b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 2307706
06080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c -
 b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/
(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36
*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 +
158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9
- 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^
11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14
+ 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 208091165
49120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^
4*c^19 - 5497558138880*a^19*b^2*c^20)))^(3/4) + (3*(570240000*a^7*b*c^8 + 2
917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 879403
392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9 +
576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*
c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b
^16*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 70464307200
0*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^
4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680
*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059
239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c
^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2)
+ 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*
c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3
*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^
28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^
9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2
113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560
*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c
^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558
138880*a^19*b^2*c^20)))^(1/4) + (9*x^(1/2)*(43758225*a^2*b^14*c^3 - 1036800
0000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11404429
344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 - 223948
80000*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2
 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704

*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*2i - atan((((((3*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)

))^(1/4)*(351843720888320*a^13*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a
^3*b^20*c^5 + 167772160000*a^4*b^18*c^6 - 1691143372800*a^5*b^16*c^7 + 1095
2166604800*a^6*b^14*c^8 - 46901042872320*a^7*b^12*c^9 + 129879811031040*a^8
*b^10*c^10 - 206158430208000*a^9*b^8*c^11 + 82463372083200*a^10*b^6*c^12 +
329853488332800*a^11*b^4*c^13 - 615726511554560*a^12*b^2*c^14))/(65536*(b^1
8 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*
c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824
*a^8*b^2*c^8 - 36*a*b^16*c)) - (9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 422212
4650659840*a^14*b*c^16 - 147907936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21
*c^6 - 26607322398720*a^5*b^19*c^7 + 176329882337280*a^6*b^17*c^8 - 7772172
81884160*a^7*b^15*c^9 + 2233932749733888*a^8*b^13*c^10 - 3727344418160640*a
^9*b^11*c^11 + 1599789418414080*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c
^13 - 16008889300418560*a^12*b^5*c^14 + 13792273858822144*a^13*b^3*c^15))/(
4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3
 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976
128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a
^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c))*(-(81*(2401*b^29 + 2
401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*
b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*
b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*
a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 14369
6855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*
c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2
c(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21
 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32
*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26
*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a
^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13
 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 1664729323
9296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^
6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(3/4
) + (3*(570240000*a^7*b*c^8 + 2917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 +
 303385824*a^4*b^7*c^5 + 879403392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(
65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256
*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^
7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*
c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 2824
3200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989
852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 -
19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^
7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^
2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b
^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2
 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096
*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 82555699

20*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 -
704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280
*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*
c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 130567
00579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4) - (9*x^(1/2)*
(43758225*a^2*b^14*c^3 - 10368000000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 41
19250464*a^4*b^10*c^5 + 11404429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 -
8687347200*a^7*b^4*c^8 - 22394880000*a^8*b^2*c^9))/(4194304*(b^24 + 167772
16*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4
- 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 324
40320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 5033164
8*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2
)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^
3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*
a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 1997799
4240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11
+ 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(
-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)
^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040
*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^
30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*
b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475
299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b
^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 +
20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 1305670057984
0*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*1i - ((((3*(-(81*(24
01*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1
323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437
284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 1
3327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c
^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 88785027072
0*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c +
9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776
*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 12403
20*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 12700876
80*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193
730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12
*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 -
16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869
760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c
^20)))^(1/4)*(351843720888320*a^13*c^15 + 251658240*a^2*b^22*c^4 - 97307852
80*a^3*b^20*c^5 + 167772160000*a^4*b^18*c^6 - 1691143372800*a^5*b^16*c^7 +
10952166604800*a^6*b^14*c^8 - 46901042872320*a^7*b^12*c^9 + 129879811031040
*a^8*b^10*c^10 - 206158430208000*a^9*b^8*c^11 + 82463372083200*a^10*b^6*c^1
2 + 329853488332800*a^11*b^4*c^13 - 615726511554560*a^12*b^2*c^14))/(65536*

(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b
^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 58
9824*a^8*b^2*c^8 - 36*a*b^16*c)) + (9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 42
22124650659840*a^14*b*c^16 - 147907936256*a^3*b^23*c^5 + 2590402150400*a^4*
b^21*c^6 - 26607322398720*a^5*b^19*c^7 + 176329882337280*a^6*b^17*c^8 - 777
217281884160*a^7*b^15*c^9 + 2233932749733888*a^8*b^13*c^10 - 37273444181606
40*a^9*b^11*c^11 + 1599789418414080*a^10*b^9*c^12 + 7124835347988480*a^11*b
^7*c^13 - 16008889300418560*a^12*b^5*c^14 + 13792273858822144*a^13*b^3*c^15
))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18
*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 1
2976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 692060
16*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29
 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*
a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*
a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073
280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 1
43696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*
b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a
*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c
^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*
b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*
b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 1937307074
56*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c
^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 166472
93239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^1
7*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^
(3/4) + (3*(570240000*a^7*b*c^8 + 2917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c
^4 + 303385824*a^4*b^7*c^5 + 879403392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7
))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 3
2256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^
4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(
4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 +
28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 -
3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^
8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^1
1*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 1000
0*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c
 - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38
*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 1587
6096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255
569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^
11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 520227913
7280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b
^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13
056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4) + (9*x^(1

/2)*(43758225*a^2*b^14*c^3 - 10368000000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11404429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 - 22394880000*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*1i)/(((((3*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a^13*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*a^4*b^18*c^6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 46901042872320*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 206158430208000*a^9*b^8*c^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^13 - 615726511554560*a^12*b^2*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 147907936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*b^19*c^7 + 176329882337280*a^6*b^17*c^8 -

777217281884160*a^7*b^15*c^9 + 2233932749733888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 1599789418414080*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 16008889300418560*a^12*b^5*c^14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(3/4) + (3*(570240000*a^7*b*c^8 + 2917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 879403392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4) - (9*x^(1/2)*(43758225*a^2*b^14*c^3 - 10368000000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11404429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 - 22394880000*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c

^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4) + ((((3*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a^13*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*a^4*b^18*c^6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 46901042872320*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 206158430208000*a^9*b^8*c^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^13 - 615726511554560*a^12*b^2*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 147907936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*b^19*c^7 + 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 2233932749733888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 1599789418414080*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 16008889300418560*a^12*b^5*c^14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^

6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 6
9206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401
*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 132
3600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 143728
4352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 133
27073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^1
0 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*
a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9
400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a
^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320
*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680
*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 19373
0707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b
^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 1
6647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 1958505086976
0*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^2
0)))^(3/4) + (3*(570240000*a^7*b*c^8 + 2917215*a^2*b^11*c^3 + 49009212*a^3*
b^9*c^4 + 303385824*a^4*b^7*c^5 + 879403392*a^5*b^5*c^6 + 1191801600*a^6*b^
3*c^7))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^
3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a
^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)))*(-(81*(2401*b^29 + 2401*b^
4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c
^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c
^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^
13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 14369685504
0*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 +
 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(
4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a
*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 -
 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 +
 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^
20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202
279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a
^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18
 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4) + (9
x^(1/2)(43758225*a^2*b^14*c^3 - 10368000000*a^9*c^10 + 682628310*a^3*b^12
*c^4 + 4119250464*a^4*b^10*c^5 + 11404429344*a^5*b^8*c^6 + 11263650048*a^6*
b^6*c^7 - 8687347200*a^7*b^4*c^8 - 22394880000*a^8*b^2*c^9))/(4194304*(b^24
 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4
*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*
c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10
 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*
a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28
243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 39
89852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8

- 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*2i - 2*atan((((((((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a^13*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*a^4*b^18*c^6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 46901042872320*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 206158430208000*a^9*b^8*c^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^13 - 615726511554560*a^12*b^2*c^14)*3i)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b

^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344
064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) -
(9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 14790
7936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*b^19
*c^7 + 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 223393
2749733888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 159978941841408
0*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 16008889300418560*a^12*b
^5*c^14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a^12*
c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 81100
8*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^
8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b
^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^
29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^
3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*
c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*
b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 2307706
06080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c -
b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/
(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36
*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 +
158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9
- 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^
11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14
+ 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 208091165
49120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^
4*c^19 - 5497558138880*a^19*b^2*c^20)))^(3/4)*1i - (3*(570240000*a^7*b*c^8
+ 2917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 879
403392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9
+ 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b
^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*
a*b^16*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 70464307
2000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040
*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799
680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66
059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^
5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/
2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^
40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*
a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6
*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240
*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12
+ 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274
560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^
8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497
558138880*a^19*b^2*c^20)))^(1/4)*1i + (9*x^(1/2)*(43758225*a^2*b^14*c^3 - 1

0368000000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11
404429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 -
22394880000*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^
20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3
784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 576716
80*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^2
2*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*
a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*
b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a
^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 6605923
9424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^1
2 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) +
9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c
+ 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b
^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28
*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*
b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 211
3425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a
^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^1
7 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 549755813
8880*a^19*b^2*c^20)))^(1/4) - ((((((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2)
- 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^
3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*
a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 1997799
4240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11
- 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(
-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)
^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040
*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^
30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*
b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475
299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b
^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 +
20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 1305670057984
0*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a^1
3*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*a^
4*b^18*c^6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 469
01042872320*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 206158430208000*
a^9*b^8*c^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^13
 - 615726511554560*a^12*b^2*c^14)*3i)/(65536*(b^18 - 262144*a^9*c^9 + 576*a
^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 +
 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c
)) + (9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 1
47907936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*
b^19*c^7 + 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 22

33932749733888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 15997894184
14080*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 16008889300418560*a^
12*b^5*c^14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a
^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 8
11008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 3244032
0*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^
11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 240
1*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^2
3*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b
^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*
a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230
770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a
*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2
)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*
b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^
6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*
c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 70447529984
0*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c
^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809
116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^1
8*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(3/4)*1i - (3*(570240000*a^7*b*
c^8 + 2917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 +
879403392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9
*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a
^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 -
36*a*b^16*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 7046
43072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 27141
5040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 279
3799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9
+ 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^1
2*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)
^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432
*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72
960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960
*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 4402970
6240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c
^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 1040455
8274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^1
6*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 -
5497558138880*a^19*b^2*c^20)))^(1/4)*1i - (9*x^(1/2)*(43758225*a^2*b^14*c^3
- 10368000000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5
+ 11404429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^
8 - 22394880000*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^
2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5
+ 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57

671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a
*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072
000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*
a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 27937996
80*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 660
59239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5
*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2
) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^4
0*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a
^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*
b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*
a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 +
 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 104045582745
60*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8
*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 54975
58138880*a^19*b^2*c^20)))^(1/4))/(((((((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1
/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 2824320
0*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852
160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 199
77994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c
^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c
^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)
^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 +
3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^
5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*
a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 70
4475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^
13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^1
6 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 130567005
79840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320
*a^13*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 16777216000
0*a^4*b^18*c^6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 -
 46901042872320*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 206158430208
000*a^9*b^8*c^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*
c^13 - 615726511554560*a^12*b^2*c^14)*3i)/(65536*(b^18 - 262144*a^9*c^9 + 5
76*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c
^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^
16*c)) - (9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16
 - 147907936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*
a^5*b^19*c^7 + 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9
+ 2233932749733888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 1599789
418414080*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 1600888930041856
0*a^12*b^5*c^14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 167772
16*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4
 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 324

40320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 5033164
8*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) -
2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3
*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a
^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994
240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 -
230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-
(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^
(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*
a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^3
0*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b
^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 7044752
99840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^
14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 2
0809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840
*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(3/4)*1i - (3*(570240000*a^
7*b*c^8 + 2917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c
^5 + 879403392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144
*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 1290
24*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c
^8 - 36*a*b^16*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 -
704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 2
71415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 -
2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*
c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080
*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)
^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(3355
4432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3
- 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 15876
0960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 440
29706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^
18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 104
04558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120
*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^1
9 - 5497558138880*a^19*b^2*c^20)))^(1/4)*1i + (9*x^(1/2)*(43758225*a^2*b^14
*c^3 - 10368000000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*
c^5 + 11404429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^
4*c^8 - 22394880000*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 105
6*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14
*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8
- 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 -
48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 70464
3072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415
040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793
799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 +

66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12
*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^
(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*
(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 729
60*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*
a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706
240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^
12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558
274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16
*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5
497558138880*a^19*b^2*c^20)))^(1/4)*1i + ((((((81*(2401*b^4*(-(4*a*c - b^2)
^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 -
28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 +
3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^
8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^1
1*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 1000
0*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c
 - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38
*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 1587
6096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255
569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^
11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 520227913
7280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b
^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13
056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(35184372
0888320*a^13*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 1677
72160000*a^4*b^18*c^6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^1
4*c^8 - 46901042872320*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 20615
8430208000*a^9*b^8*c^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^
11*b^4*c^13 - 615726511554560*a^12*b^2*c^14)*3i)/(65536*(b^18 - 262144*a^9*
c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^
5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 -
36*a*b^16*c)) + (9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14
*b*c^16 - 147907936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322
398720*a^5*b^19*c^7 + 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^
15*c^9 + 2233932749733888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 +
1599789418414080*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 160088893
00418560*a^12*b^5*c^14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 +
 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b
^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^
7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 -
50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^
(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243
200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 39898
52160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 1

9977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(3/4)*1i - (3*(570240000*a^7*b*c^8 + 2917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 879403392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*1i - (9*x^(1/2)*(43758225*a^2*b^14*c^3 - 10368000000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11404429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 - 22394880000*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158

760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 4
4029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*
b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 1
0404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 208091165491
20*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c
^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*1i))*((81*(2401*b^4*(-(4*a*c - b
^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2
 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5
 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13
*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*
a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 1
0000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*
a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b
^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 1
5876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8
255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20
*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 520227
9137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^1
5*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 +
 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4) - 2*at
an(((((((-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 70464307200
0*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^
4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680
*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059
239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c
^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2)
- 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*
c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3
*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^
28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^
9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2
113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560
*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c
^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558
138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a^13*c^15 + 251658240*a^2*b^
22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*a^4*b^18*c^6 - 169114337280
0*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 46901042872320*a^7*b^12*c^9
+ 129879811031040*a^8*b^10*c^10 - 206158430208000*a^9*b^8*c^11 + 8246337208
3200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^13 - 615726511554560*a^12*b
^2*c^14)*3i)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^
12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589
824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (9*x^(1/2)*(37748736
00*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 147907936256*a^3*b^23*c^5
+ 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*b^19*c^7 + 17632988233728
0*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 2233932749733888*a^8*b^13*c

^10 - 3727344418160640*a^9*b^11*c^11 + 1599789418414080*a^10*b^9*c^12 + 712
4835347988480*a^11*b^7*c^13 - 16008889300418560*a^12*b^5*c^14 + 13792273858
822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*
c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784
704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*
a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c
)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^
14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^
21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7
*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 660592394
24*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12
- 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 94
00*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c +
1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^3
4*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c
^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^
22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 21134
25899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^1
4*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17
- 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 54975581388
80*a^19*b^2*c^20)))^(3/4)*1i - (3*(570240000*a^7*b*c^8 + 2917215*a^2*b^11*c
^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 879403392*a^5*b^5*c^6 +
1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2
- 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*
b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)))*(-(81*(2
401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 -
1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 143
7284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 +
13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*
c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 8878502707
20*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c
+ 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 109951162777
6*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240
320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087
680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 19
3730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^1
2*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15
- 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 1958505086
9760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*
c^20)))^(1/4)*1i + (9*x^(1/2)*(43758225*a^2*b^14*c^3 - 10368000000*a^9*c^10
+ 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11404429344*a^5*b^8*c
^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 - 22394880000*a^8*b^2
c^9))/(4194304(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*
b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6
- 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69

206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*
b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323
600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284
352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 1332
7073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10
 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a
^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 94
00*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^
20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*
a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*
a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730
707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^
16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16
647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760
*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20
)))^(1/4) - (((((-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704
643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 2714
15040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 27
93799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9
 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^
12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25
)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(3355443
2*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 7
2960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 15876096
0*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 440297
06240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*
c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 104045
58274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^
16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 -
 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a^13*c^15 + 25165824
0*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*a^4*b^18*c^6 - 1691
143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 46901042872320*a^7*b
^12*c^9 + 129879811031040*a^8*b^10*c^10 - 206158430208000*a^9*b^8*c^11 + 82
463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^13 - 61572651155456
0*a^12*b^2*c^14)*3i)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 537
6*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c
^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (9*x^(1/2)*(
3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 147907936256*a^3*b
^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*b^19*c^7 + 176329
882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 2233932749733888*a^
8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 1599789418414080*a^10*b^9*c^
12 + 7124835347988480*a^11*b^7*c^13 - 16008889300418560*a^12*b^5*c^14 + 137
92273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a
^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^
5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 5

7671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*
a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 7046430
72000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 27141504
0*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 279379
9680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 6
6059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b
^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1
/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b
^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960
*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^
6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 4402970624
0*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12
 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 1040455827
4560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b
^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 549
7558138880*a^19*b^2*c^20)))^(3/4)*1i - (3*(570240000*a^7*b*c^8 + 2917215*a^
2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 879403392*a^5*b
^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b
^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344
064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)))*
(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b
*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c
^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^1
5*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a
^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 88
7850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a
*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099
511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^
4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 -
 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c
^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 211342589
9520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^
12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19
585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a
^19*b^2*c^20)))^(1/4)*1i - (9*x^(1/2)*(43758225*a^2*b^14*c^3 - 10368000000*
a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11404429344*a
^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 - 22394880000
*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14
080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*
b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*
c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(8
1*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^1
4 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 +
 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^
7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*

b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850
270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^2
7*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 10995116
27776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 +
1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 127
0087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10
+ 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520
*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c
^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 195850
50869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*
b^2*c^20)))^(1/4))/((((((-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/
2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^
3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*
c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*
b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 2307706
06080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c -
 b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/
(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36
*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 +
158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9
- 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^
11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14
+ 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 208091165
49120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^
4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a^13*c^15 +
251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*a^4*b^18*c^
6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 469010428723
20*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 206158430208000*a^9*b^8*c
^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^13 - 615726
511554560*a^12*b^2*c^14)*3i)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c
^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a
^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (9*x
^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 1479079362
56*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*b^19*c^7
+ 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 22339327497
33888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 1599789418414080*a^1
0*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 16008889300418560*a^12*b^5*c^
14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a^12*c^12
+ 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5
*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8
*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^
11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) +
 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 -
271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6
+ 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11

*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 23077060608
0*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2
)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(335
54432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3
 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 1587
60960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44
029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b
^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10
404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 2080911654912
0*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^
19 - 5497558138880*a^19*b^2*c^20)))^(3/4)*1i - (3*(570240000*a^7*b*c^8 + 29
17215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 8794033
92*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9 + 5
76*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c
^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^
16*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 70464307200
0*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^
4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680
*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059
239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c
^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2)
- 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*
c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3
*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^
28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^
9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2
113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560
*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c
^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558
138880*a^19*b^2*c^20)))^(1/4)*1i + (9*x^(1/2)*(43758225*a^2*b^14*c^3 - 1036
8000000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11404
429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 - 223
94880000*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*
c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784
704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*
a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c
)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^
14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^
21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7
*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 660592394
24*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12
- 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 94
00*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c +
1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^3
4*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c

^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*1i + (((((-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a^13*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*a^4*b^18*c^6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 46901042872320*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 206158430208000*a^9*b^8*c^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^13 - 615726511554560*a^12*b^2*c^14)*3i)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 147907936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*b^19*c^7 + 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 2233932749733888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 1599789418414080*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 16008889300418560*a^12*b^5*c^14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30 *c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(3/4)*1i - (3*(570240000*a^7*b*c^8 + 2917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 879403392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*1i - (9*x^(1/2)*(43758225*a^2*b^14*c^3 - 10368000000*a^9*c^10 + 682628310*a^3*b^12*c^4 + 4119250464*a^4*b^10*c^5 + 11404429344*a^5*b^8*c^6 + 11263650048*a^6*b^6*c^7 - 8687347200*a^7*b^4*c^8 - 22394880000*a^8*b^2*c^9))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18*b^4*c^19 -

```
5497558138880*a^19*b^2*c^20)))^(1/4)*1i))*(-(81*(2401*b^29 + 2401*b^4*(-(4*
a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28
243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 39
89852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8
- 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*
b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*
a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c -
 b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c
^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 158760
96*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 825556
9920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11
 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 52022791372
80*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^1
0*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 1305
6700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2+a)**3,x)
[Out] Timed out
```

3.1083 $\int \frac{x^{9/2}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=533

$$-\frac{3x^{3/2}\left(-4ac+5b^2+8bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\sqrt[4]{c}\left(4b\sqrt{b^2-4ac}+20ac+11b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{}}{\sqrt[4]{-\sqrt{b^2}}}\right.}{16\,2^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

```
[Out] 1/4*x^(3/2)*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-3/16*x^(3/2)*(8*b*c*
x^2-4*a*c+5*b^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3/32*c^(1/4)*arctan(2^(1/4)
*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(11*b^2+20*a*c-4*b*(-4*a*c+
b^2)^(1/2))*2^(1/4)/(-4*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-3/32*c
^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(11*b
^2+20*a*c-4*b*(-4*a*c+b^2)^(1/2))*2^(1/4)/(-4*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^
2)^(1/2))^(1/4)-3/32*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2
)^(1/2))^(1/4))*(11*b^2+20*a*c+4*b*(-4*a*c+b^2)^(1/2))*2^(1/4)/(-4*a*c+b^2)
^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+3/32*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x
^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(11*b^2+20*a*c+4*b*(-4*a*c+b^2)^(1/2)
)*2^(1/4)/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A] time = 1.45, antiderivative size = 533, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1115, 1365, 1500, 1510, 298, 205, 208}

$$-\frac{3x^{3/2}\left(-4ac+5b^2+8bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\sqrt[4]{c}\left(4b\sqrt{b^2-4ac}+20ac+11b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{}}{\sqrt[4]{-\sqrt{b^2}}}\right.}{16\,2^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^(3/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*x^(3/2
)*(5*b^2 - 4*a*c + 8*b*c*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (
3*c^(1/4)*(11*b^2 + 20*a*c + 4*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(3/4)*(b^2 - 4*a*c)^(5/2)*
(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (3*c^(1/4)*(11*b^2 + 20*a*c - 4*b*Sqrt[b^
2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)
])/(16*2^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (3*c^(
1/4)*(11*b^2 + 20*a*c + 4*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqr
```

```
t[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b
- Sqrt[b^2 - 4*a*c])^(1/4)) - (3*c^(1/4)*(11*b^2 + 20*a*c - 4*b*Sqrt[b^2 -
4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/
(16*2^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1500

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^n + c*x^(
2*n))^(p + 1)*(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n))/(a*f*n*(p +
1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a
+ b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m + 2
```

```
*n*(p + 1) + 1)) - a*b*e*(m + 1) + c*(m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2
 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{9/2}}{\left(a+bx^2+cx^4\right)^3}\,dx &= 2\,\text{Subst}\left(\int \frac{x^{10}}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)\\
&= \frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{x^2\left(6a-9bx^4\right)}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}\\
&= \frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3x^{3/2}\left(5b^2-4ac+8bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(3a\left(7b^2+20ac\cdots\right.\right.}{a+bx^4+\cdots}\right.}{16a\left(b^2-\cdots\right.}\\
&= \frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3x^{3/2}\left(5b^2-4ac+8bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3c\left(11b^2+20ac-4b\cdots\right.\right.}{\cdots}\\
&= \frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3x^{3/2}\left(5b^2-4ac+8bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(3\sqrt{c}\left(11b^2+20ac-\cdots\right.\right.}{\cdots}\\
&= \frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3x^{3/2}\left(5b^2-4ac+8bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{3\sqrt[4]{c}\left(11b^2+20ac+4\cdots\right.}{16\,2^{3/4}\left(b^2-\cdots\right.}
\end{aligned}
$$

Mathematica [C] time = 0.38, size = 176, normalized size = 0.33

$$\frac{-3\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{8\#1^4bc\log\left(\sqrt{x}-\#1\right)-20ac\log\left(\sqrt{x}-\#1\right)-7b^2\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]-\frac{12x^{3/2}\left(-4ac+5b^2+8bcx^2\right)}{a+bx^2+cx^4}+\frac{16x^{3/2}(b}{(a+}}{64\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((16*(b^2 - 4*a*c)*x^(3/2)*(2*a + b*x^2))/(a + b*x^2 + c*x^4)^2 - (12*x^(3/
2)*(5*b^2 - 4*a*c + 8*b*c*x^2))/(a + b*x^2 + c*x^4) - 3*RootSum[a + b*#1^4
+ c*#1^8 & , (-7*b^2*Log[Sqrt[x] - #1] - 20*a*c*Log[Sqrt[x] - #1] + 8*b*c*L
og[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(64*(b^2 - 4*a*c)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 191.51Unable to convert to
real 1/4 Error: Bad Argument Value
```

maple [C] time = 0.04, size = 244, normalized size = 0.46

$$-\frac{3\left(8\,\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^6bc+\left(-20ac-7b^2\right)\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^2\right)\ln\left(-\text{RootOf}\left(c\_Z^8+b\_\right.\right.}{64\left(16a^2c^2-8a\,b^2c+b^4\right)\left(2\,\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7c+\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(-1/32*a*(20*a*c+7*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(3/2)-1/32*b*(28*a*c
+11*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+3/32*(4*a*c-13*b^2)*c/(16*a^2*c
^2-8*a*b^2*c+b^4)*x^(11/2)-3/4*b*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(15/2))/(
c*x^4+b*x^2+a)^2-3/64/(16*a^2*c^2-8*a*b^2*c+b^4)*sum((8*_R^6*b*c+(-20*a*c-7
*b^2)*_R^2)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{24\,bc^2x^{\frac{15}{2}} + 3\,(13\,b^2c - 4\,ac^2)x^{\frac{11}{2}} + (11\,b^3 + 28\,abc)x^{\frac{7}{2}} + (7\,ab^2 + 20\,a^2c)}{16\,((b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4)x^8 + 2\,(b^5c - 8\,ab^3c^2 + 16\,a^2bc^3)x^6 + a^2b^4 - 8\,a^3b^2c + 16\,a^4c^2 + (b^6 - 6\,ab^4c + 32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(24*b*c^2*x^(15/2) + 3*(13*b^2*c - 4*a*c^2)*x^(11/2) + (11*b^3 + 28*a
*b*c)*x^(7/2) + (7*a*b^2 + 20*a^2*c)*x^(3/2))/((b^4*c^2 - 8*a*b^2*c^3 + 16*
a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3
*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2
*b^3*c + 16*a^3*b*c^2)*x^2) - integrate(3/32*(8*b*c*x^(5/2) - (7*b^2 + 20*a
*c)*sqrt(x))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*
a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

mupad [B] time = 7.66, size = 37678, normalized size = 70.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] - atan(((((27*(5754585088*a*b^27*c^4 + 309622474381721600*a^14*b*c^17 - 161
128382464*a^2*b^25*c^5 + 1626181992448*a^3*b^23*c^6 - 3983582167040*a^4*b^2
1*c^7 - 56328496087040*a^5*b^19*c^8 + 557813172535296*a^6*b^17*c^9 - 196180
3621859328*a^7*b^15*c^10 + 715782069682176*a^8*b^13*c^11 + 1581647476555776
0*a^9*b^11*c^12 - 39296545576714240*a^10*b^9*c^13 - 32756650414702592*a^11*
b^7*c^14 + 300756012615335936*a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c
^16))/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^
3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16
*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*
c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12
*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) - (9*x^(1/2)*((81*(2401
*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 132
3600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 143728
```

4352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 133
27073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^1
0 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*
a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9
400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a
^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320
*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680
*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 19373
0707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b
^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 1
6647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 1958505086976
0*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^1
9)))^(1/4)*(822083584*a*b^26*c^4 - 14073748835532800*a^14*c^17 - 2795084185
6*a^2*b^24*c^5 + 399431958528*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7 + 1
0329396346880*a^5*b^18*c^8 + 6262062317568*a^6*b^16*c^9 - 202859895324672*a
^7*b^14*c^10 + 658057709223936*a^8*b^12*c^11 + 346346162749440*a^9*b^10*c^1
2 - 8653156510597120*a^10*b^8*c^13 + 28569710136131584*a^11*b^6*c^14 - 4707
6689854857216*a^12*b^4*c^15 + 40250921669623808*a^13*b^2*c^16))/(4194304*(b
^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*
a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^
10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^
10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)
^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 -
28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 +
3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^
8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^1
1*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 1000
0*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c
 - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^
38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 1587
6096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255
569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^
10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 520227913
7280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b
^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13
056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(3/4) - (9*x^(1
/2)*(200930625*a*b^13*c^5 - 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6
 + 7523454960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^
9 - 5453568000*a^6*b^3*c^10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^
2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5
 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57
671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a
*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072
000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*
a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 27937996

80*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 660
59239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5
*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2
) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b
^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a
^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*
b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*
a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 +
 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 104045582745
60*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8
*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 54975
58138880*a^20*b^2*c^19)))^(1/4)*1i - (((27*(5754585088*a*b^27*c^4 + 3096224
74381721600*a^14*b*c^17 - 161128382464*a^2*b^25*c^5 + 1626181992448*a^3*b^2
3*c^6 - 3983582167040*a^4*b^21*c^7 - 56328496087040*a^5*b^19*c^8 + 55781317
2535296*a^6*b^17*c^9 - 1961803621859328*a^7*b^15*c^10 + 715782069682176*a^8
*b^13*c^11 + 15816474765557760*a^9*b^11*c^12 - 39296545576714240*a^10*b^9*c
^13 - 32756650414702592*a^11*b^7*c^14 + 300756012615335936*a^12*b^5*c^15 -
517069532217475072*a^13*b^3*c^16))/(268435456*(b^28 + 268435456*a^14*c^14 +
 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5
*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b
^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^
11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^2
6*c)) + (9*x^(1/2)*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 7
04643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 27
1415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 -
2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c
^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*
a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^
25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554
432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 -
 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760
960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 4402
9706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^1
8*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 1040
4558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*
a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18
 - 5497558138880*a^20*b^2*c^19)))^(1/4)*(822083584*a*b^26*c^4 - 14073748835
532800*a^14*c^17 - 27950841856*a^2*b^24*c^5 + 399431958528*a^3*b^22*c^6 - 2
968896143360*a^4*b^20*c^7 + 10329396346880*a^5*b^18*c^8 + 6262062317568*a^6
*b^16*c^9 - 202859895324672*a^7*b^14*c^10 + 658057709223936*a^8*b^12*c^11 +
 346346162749440*a^9*b^10*c^12 - 8653156510597120*a^10*b^8*c^13 + 285697101
36131584*a^11*b^6*c^14 - 47076689854857216*a^12*b^4*c^15 + 4025092166962380
8*a^13*b^2*c^16))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 -
 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a
^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b

^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(3/4) + (9*x^(1/2)*(200930625*a*b^13*c^5 - 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 7523454960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 5453568000*a^6*b^3*c^10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*1i)/((27*(103680000000*a^8*c^12 + 1406514375*a*b^14*c^5 + 22129159500*a^2*b^12*c^6 + 140297799600*a^3*b^10*c^7 + 460920922560*a^4*b^8*c^8 + 844743271680*a^5*b^6*c^9 + 869387904000*a^6*b^4*c^10 + 469670400000*a^7*b^2*c^11))/(134217728*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (((27*(5754585088*a*b^27*c^4 + 309622474381721600*a^14*b*c^17 - 161128382464*a^2*b^25*c^5 + 1626181992448*a^3*b^23*c^6

- 3983582167040*a^4*b^21*c^7 - 56328496087040*a^5*b^19*c^8 + 557813172535296*a^6*b^17*c^9 - 1961803621859328*a^7*b^15*c^10 + 715782069682176*a^8*b^13*c^11 + 15816474765557760*a^9*b^11*c^12 - 39296545576714240*a^10*b^9*c^13 - 32756650414702592*a^11*b^7*c^14 + 300756012615335936*a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c^16))/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) - (9*x^(1/2)*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*(822083584*a*b^26*c^4 - 14073748835532800*a^14*c^17 - 27950841856*a^2*b^24*c^5 + 399431958528*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7 + 10329396346880*a^5*b^18*c^8 + 6262062317568*a^6*b^16*c^9 - 202859895324672*a^7*b^14*c^10 + 658057709223936*a^8*b^12*c^11 + 346346162749440*a^9*b^10*c^12 - 8653156510597120*a^10*b^8*c^13 + 28569710136131584*a^11*b^6*c^14 - 47076689854857216*a^12*b^4*c^15 + 40250921669623808*a^13*b^2*c^16))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14

- 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 195850508
69760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2
*c^19)))^(3/4) - (9*x^(1/2)*(200930625*a*b^13*c^5 - 3110400000*a^7*b*c^11 +
2093250600*a^2*b^11*c^6 + 7523454960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8
+ 2354261760*a^5*b^5*c^9 - 5453568000*a^6*b^3*c^10))/(4194304*(b^24 + 1677
7216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c
^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 3
2440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331
648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2)
- 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a
^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160
*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 199779
94240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11
- 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*
(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25
)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 304
0*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b
^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9
*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 70447
5299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*
b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 +
20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 130567005798
40*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4) + (((27*(5754585088
*a*b^27*c^4 + 309622474381721600*a^14*b*c^17 - 161128382464*a^2*b^25*c^5 +
1626181992448*a^3*b^23*c^6 - 3983582167040*a^4*b^21*c^7 - 56328496087040*a^
5*b^19*c^8 + 557813172535296*a^6*b^17*c^9 - 1961803621859328*a^7*b^15*c^10
+ 715782069682176*a^8*b^13*c^11 + 15816474765557760*a^9*b^11*c^12 - 3929654
5576714240*a^10*b^9*c^13 - 32756650414702592*a^11*b^7*c^14 + 30075601261533
5936*a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c^16))/(268435456*(b^28 +
268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b
^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*
c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8
*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^1
3*b^2*c^13 - 56*a*b^26*c)) + (9*x^(1/2)*((81*(2401*b^4*(-(4*a*c - b^2)^25)^
(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243
200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 39898
52160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 1
9977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7
*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2
c^2(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^
2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c
+ 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*
a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 825556992
0*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 -
704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*

a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c
^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 1305670
0579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*(822083584*a*b
^26*c^4 - 14073748835532800*a^14*c^17 - 27950841856*a^2*b^24*c^5 + 39943195
8528*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7 + 10329396346880*a^5*b^18*c^
8 + 6262062317568*a^6*b^16*c^9 - 202859895324672*a^7*b^14*c^10 + 6580577092
23936*a^8*b^12*c^11 + 346346162749440*a^9*b^10*c^12 - 8653156510597120*a^10
*b^8*c^13 + 28569710136131584*a^11*b^6*c^14 - 47076689854857216*a^12*b^4*c^
15 + 40250921669623808*a^13*b^2*c^16))/(4194304*(b^24 + 16777216*a^12*c^12
+ 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5
*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8
*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^
11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 -
704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 2
71415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 -
 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*
c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080
*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)
^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(3355
4432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2
- 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 15876
0960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 440
29706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^
18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 104
04558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120
*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^1
8 - 5497558138880*a^20*b^2*c^19)))^(3/4) + (9*x^(1/2)*(200930625*a*b^13*c^5
 - 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 7523454960*a^3*b^9*c^7
 + 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 5453568000*a^6*b^3*c^
10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^
18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 -
 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 6920
6016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4
*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600
*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352
*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 1332707
3280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 -
143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13
*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*
a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*
c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5
*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8
*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707
456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*
c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647

293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*2i - atan(((((27*(5754585088*a*b^27*c^4 + 309622474381721600*a^14*b*c^17 - 161128382464*a^2*b^25*c^5 + 1626181992448*a^3*b^23*c^6 - 3983582167040*a^4*b^21*c^7 - 56328496087040*a^5*b^19*c^8 + 557813172535296*a^6*b^17*c^9 - 1961803621859328*a^7*b^15*c^10 + 715782069682176*a^8*b^13*c^11 + 15816474765557760*a^9*b^11*c^12 - 39296545576714240*a^10*b^9*c^13 - 32756650414702592*a^11*b^7*c^14 + 300756012615335936*a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c^16))/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) - (9*x^(1/2)*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*(822083584*a*b^26*c^4 - 14073748835532800*a^14*c^17 - 27950841856*a^2*b^24*c^5 + 399431958528*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7 + 10329396346880*a^5*b^18*c^8 + 6262062317568*a^6*b^16*c^9 - 202859895324672*a^7*b^14*c^10 + 658057709223936*a^8*b^12*c

^11 + 346346162749440*a^9*b^10*c^12 - 8653156510597120*a^10*b^8*c^13 + 2856
9710136131584*a^11*b^6*c^14 - 47076689854857216*a^12*b^4*c^15 + 40250921669
623808*a^13*b^2*c^16))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*
c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784
704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*
a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c
)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^
14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^
21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7
*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 660592394
24*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12
- 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 94
00*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 +
1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^3
4*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c
^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b
^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 21134
25899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^1
5*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16
- 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 54975581388
80*a^20*b^2*c^19)))^(3/4) - (9*x^(1/2)*(200930625*a*b^13*c^5 - 3110400000*a
^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 7523454960*a^3*b^9*c^7 + 10328580864*
a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 5453568000*a^6*b^3*c^10))/(4194304*(
b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720
*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b
^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c
^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(
-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2
+ 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5
- 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*
c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a
^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10
000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a
c - b^2)^25)^(1/2)))/(33554432(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*
b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15
876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 82
55569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*
c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279
137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16
*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 +
13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*1i - ((
(27*(5754585088*a*b^27*c^4 + 309622474381721600*a^14*b*c^17 - 161128382464*
a^2*b^25*c^5 + 1626181992448*a^3*b^23*c^6 - 3983582167040*a^4*b^21*c^7 - 56
328496087040*a^5*b^19*c^8 + 557813172535296*a^6*b^17*c^9 - 1961803621859328
*a^7*b^15*c^10 + 715782069682176*a^8*b^13*c^11 + 15816474765557760*a^9*b^11

*c^12 - 3929654557671424O*a^10*b^9*c^13 - 32756650414702592*a^11*b^7*c^14 +
 300756012615335936*a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c^16))/(268
435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3
 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 562
29888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049
624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12
 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (9*x^(1/2)*(-(81*(2401*b^29 + 2
401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*
b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*
b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*
a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 14369
6855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*
c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2
c(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20
 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32
*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26
*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a
^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12
 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 1664729323
9296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^
6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4
)*(822083584*a*b^26*c^4 - 14073748835532800*a^14*c^17 - 27950841856*a^2*b^2
4*c^5 + 399431958528*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7 + 1032939634
6880*a^5*b^18*c^8 + 6262062317568*a^6*b^16*c^9 - 202859895324672*a^7*b^14*c
^10 + 658057709223936*a^8*b^12*c^11 + 346346162749440*a^9*b^10*c^12 - 86531
56510597120*a^10*b^8*c^13 + 28569710136131584*a^11*b^6*c^14 - 4707668985485
7216*a^12*b^4*c^15 + 40250921669623808*a^13*b^2*c^16))/(4194304*(b^24 + 167
77216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*
c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 +
32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 5033
1648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c -
b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200
*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 39898521
60*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 1997
7994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^
11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^
2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^
25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3
040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6
*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a
^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704
475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^1
4*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15
 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 1305670057
9840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(3/4) + (9*x^(1/2)*(200

930625*a*b^13*c^5 - 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 75234
54960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 5453
568000*a^6*b^3*c^10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c
^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 37847
04*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a
^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)
))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^1
4*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^2
1*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*
b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 6605923942
4*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 -
887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 940
0*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1
099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34
*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^
6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^
22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 211342
5899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15
*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 -
19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 549755813888
0*a^20*b^2*c^19)))^(1/4)*1i)/((27*(103680000000*a^8*c^12 + 1406514375*a*b^1
4*c^5 + 22129159500*a^2*b^12*c^6 + 140297799600*a^3*b^10*c^7 + 460920922560
*a^4*b^8*c^8 + 844743271680*a^5*b^6*c^9 + 869387904000*a^6*b^4*c^10 + 46967
0400000*a^7*b^2*c^11))/(134217728*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^
24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 +
12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 52
4812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11
+ 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (((2
7*(5754585088*a*b^27*c^4 + 309622474381721600*a^14*b*c^17 - 161128382464*a^
2*b^25*c^5 + 1626181992448*a^3*b^23*c^6 - 3983582167040*a^4*b^21*c^7 - 5632
8496087040*a^5*b^19*c^8 + 557813172535296*a^6*b^17*c^9 - 1961803621859328*a
^7*b^15*c^10 + 715782069682176*a^8*b^13*c^11 + 15816474765557760*a^9*b^11*c
^12 - 39296545576714240*a^10*b^9*c^13 - 32756650414702592*a^11*b^7*c^14 + 3
00756012615335936*a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c^16))/(26843
5456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 +
256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229
888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 104962
4576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 -
939524096*a^13*b^2*c^13 - 56*a*b^26*c)) - (9*x^(1/2)*(-(81*(2401*b^29 + 240
1*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^
25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^
19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^
8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 1436968
55040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^
13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c

*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*(822083584*a*b^26*c^4 - 14073748835532800*a^14*c^17 - 27950841856*a^2*b^24*c^5 + 399431958528*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7 + 10329396346880*a^5*b^18*c^8 + 6262062317568*a^6*b^16*c^9 - 202859895324672*a^7*b^14*c^10 + 658057709223936*a^8*b^12*c^11 + 346346162749440*a^9*b^10*c^12 - 8653156510597120*a^10*b^8*c^13 + 28569710136131584*a^11*b^6*c^14 - 47076689854857216*a^12*b^4*c^15 + 40250921669623808*a^13*b^2*c^16))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(3/4) - (9*x^(1/2)*(200930625*a*b^13*c^5 - 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 7523454960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 5453568000*a^6*b^3*c^10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 109

9511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c
^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6
- 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22
*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 21134258
99520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b
^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 1
9585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*
a^20*b^2*c^19)))^(1/4) + (((27*(5754585088*a*b^27*c^4 + 309622474381721600*
a^14*b*c^17 - 161128382464*a^2*b^25*c^5 + 1626181992448*a^3*b^23*c^6 - 3983
582167040*a^4*b^21*c^7 - 56328496087040*a^5*b^19*c^8 + 557813172535296*a^6*
b^17*c^9 - 1961803621859328*a^7*b^15*c^10 + 715782069682176*a^8*b^13*c^11 +
15816474765557760*a^9*b^11*c^12 - 39296545576714240*a^10*b^9*c^13 - 327566
50414702592*a^11*b^7*c^14 + 300756012615335936*a^12*b^5*c^15 - 517069532217
475072*a^13*b^3*c^16))/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^
24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 +
12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 52
4812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11
+ 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (9*x
^(1/2)*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000
*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4
*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*
a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 660592
39424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^
12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) -
9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40
+ 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*
b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^2
8*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^1
0*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 21
13425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*
a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^
16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 54975581
38880*a^20*b^2*c^19)))^(1/4)*(822083584*a*b^26*c^4 - 14073748835532800*a^14
*c^17 - 27950841856*a^2*b^24*c^5 + 399431958528*a^3*b^22*c^6 - 296889614336
0*a^4*b^20*c^7 + 10329396346880*a^5*b^18*c^8 + 6262062317568*a^6*b^16*c^9 -
202859895324672*a^7*b^14*c^10 + 658057709223936*a^8*b^12*c^11 + 3463461627
49440*a^9*b^10*c^12 - 8653156510597120*a^10*b^8*c^13 + 28569710136131584*a^
11*b^6*c^14 - 47076689854857216*a^12*b^4*c^15 + 40250921669623808*a^13*b^2*
c^16))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*
b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6
- 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69
206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*
b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323
600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284
352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 1332

7073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10
 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a
^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 94
00*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^
21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*
a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*
a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730
707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^
16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16
647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760
*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19
)))^(3/4) + (9*x^(1/2)*(200930625*a*b^13*c^5 - 3110400000*a^7*b*c^11 + 2093
250600*a^2*b^11*c^6 + 7523454960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 23
54261760*a^5*b^5*c^9 - 5453568000*a^6*b^3*c^10))/(4194304*(b^24 + 16777216*
a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 -
811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 324403
20*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a
^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^2
5)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b
^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6
*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 1997799424
0*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 2
30770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4
*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1
/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^
3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*
c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^2
4*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299
840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14
*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 208
09116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a
^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)))*(-(81*(2401*b^29 + 24
01*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b
^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b
^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a
^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696
855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c
^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*
c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 -
 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*
c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*
c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^
11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 -
 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239
296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6

*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)
*2i - 2*atan(((((27*(5754585088*a*b^27*c^4 + 309622474381721600*a^14*b*c^17
 - 161128382464*a^2*b^25*c^5 + 1626181992448*a^3*b^23*c^6 - 3983582167040*a
^4*b^21*c^7 - 56328496087040*a^5*b^19*c^8 + 557813172535296*a^6*b^17*c^9 -
1961803621859328*a^7*b^15*c^10 + 715782069682176*a^8*b^13*c^11 + 1581647476
5557760*a^9*b^11*c^12 - 39296545576714240*a^10*b^9*c^13 - 32756650414702592
*a^11*b^7*c^14 + 300756012615335936*a^12*b^5*c^15 - 517069532217475072*a^13
*b^3*c^16))/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23
296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^
6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9
*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 152672665
6*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) - (x^(1/2)*((81*(
2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 +
 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 14
37284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 -
 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9
*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270
720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c
 + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 10995116277
76*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 124
0320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 127008
7680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 1
93730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^
13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14
 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 195850508
69760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2
c^19)))^(1/4)(822083584*a*b^26*c^4 - 14073748835532800*a^14*c^17 - 279508
41856*a^2*b^24*c^5 + 399431958528*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7
 + 10329396346880*a^5*b^18*c^8 + 6262062317568*a^6*b^16*c^9 - 2028598953246
72*a^7*b^14*c^10 + 658057709223936*a^8*b^12*c^11 + 346346162749440*a^9*b^10
*c^12 - 8653156510597120*a^10*b^8*c^13 + 28569710136131584*a^11*b^6*c^14 -
47076689854857216*a^12*b^4*c^15 + 40250921669623808*a^13*b^2*c^16)*9i)/(419
4304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 +
126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128
*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10
*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c
 - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25
*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19
*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*
b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855
040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13
 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(
-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80
*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4
 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7

+ 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(3/4)*1i + (9*x^(1/2)*(200930625*a*b^13*c^5 - 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 7523454960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 5453568000*a^6*b^3*c^10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4) - (((27*(5754585088*a*b^27*c^4 + 309622474381721600*a^14*b*c^17 - 161128382464*a^2*b^25*c^5 + 1626181992448*a^3*b^23*c^6 - 3983582167040*a^4*b^21*c^7 - 56328496087040*a^5*b^19*c^8 + 557813172535296*a^6*b^17*c^9 - 1961803621859328*a^7*b^15*c^10 + 715782069682176*a^8*b^13*c^11 + 15816474765557760*a^9*b^11*c^12 - 39296545576714240*a^10*b^9*c^13 - 32756650414702592*a^11*b^7*c^14 + 300756012615335936*a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c^16))/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (x^(1/2)*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 1

58760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*(822083584*a*b^26*c^4 - 14073748835532800*a^14*c^17 - 27950841856*a^2*b^24*c^5 + 399431958528*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7 + 10329396346880*a^5*b^18*c^8 + 6262062317568*a^6*b^16*c^9 - 202859895324672*a^7*b^14*c^10 + 658057709223936*a^8*b^12*c^11 + 346346162749440*a^9*b^10*c^12 - 8653156510597120*a^10*b^8*c^13 + 28569710136131584*a^11*b^6*c^14 - 47076689854857216*a^12*b^4*c^15 + 40250921669623808*a^13*b^2*c^16)*9i)/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(3/4)*1i - (9*x^(1/2)*(200930625*a*b^13*c^5 - 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 7523454960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 5453568000*a^6*b^3*c^10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7

+ 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b
^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 520
2279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*
a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^1
7 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4))/((
((27*(5754585088*a*b^27*c^4 + 309622474381721600*a^14*b*c^17 - 161128382464
*a^2*b^25*c^5 + 1626181992448*a^3*b^23*c^6 - 3983582167040*a^4*b^21*c^7 - 5
6328496087040*a^5*b^19*c^8 + 557813172535296*a^6*b^17*c^9 - 196180362185932
8*a^7*b^15*c^10 + 715782069682176*a^8*b^13*c^11 + 15816474765557760*a^9*b^1
1*c^12 - 39296545576714240*a^10*b^9*c^13 - 32756650414702592*a^11*b^7*c^14
+ 300756012615335936*a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c^16))/(26
8435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^
3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56
229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 104
9624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12
 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) - (x^(1/2)*((81*(2401*b^4*(-(4*a
*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^
25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^
19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^
8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 1436968
55040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^
13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c
*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 -
80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c
^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c
^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^1
1*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 -
5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 166472932392
96*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*
c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*
(822083584*a*b^26*c^4 - 140737488355328OO*a^14*c^17 - 27950841856*a^2*b^24*
c^5 + 399431958528*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7 + 103293963468
80*a^5*b^18*c^8 + 6262062317568*a^6*b^16*c^9 - 202859895324672*a^7*b^14*c^1
0 + 658057709223936*a^8*b^12*c^11 + 346346162749440*a^9*b^10*c^12 - 8653156
510597120*a^10*b^8*c^13 + 28569710136131584*a^11*b^6*c^14 - 470766898548572
16*a^12*b^4*c^15 + 40250921669623808*a^13*b^2*c^16)*9i)/(4194304*(b^24 + 16
777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16
*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 +
 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 503
31648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/
2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200
*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 39898521
60*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 1997
7994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^
11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^

2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(3/4)*1i + (9*x^(1/2)*(200930625*a*b^13*c^5 - 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 7523454960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 5453568000*a^6*b^3*c^10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*1i - (27*(103680000000*a^8*c^12 + 1406514375*a*b^14*c^5 + 22129159500*a^2*b^12*c^6 + 140297799600*a^3*b^10*c^7 + 460920922560*a^4*b^8*c^8 + 844743271680*a^5*b^6*c^9 + 869387904000*a^6*b^4*c^10 + 469670400000*a^7*b^2*c^11))/(134217728*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (((27*(5754585088*a*b^27*c^4 + 309622474381721600*a^14*b*c^17 - 161128382464*a^2*b^25*c^5 + 1626181992448*a^3*b^23*c^6 - 3983582167040*a^4*b^21*c^7 - 56328496087040*a^5*b^19*c^8 + 557813172535296*a^6*b^17*c^9 - 1961803621859328*a^7*b^15*c^10 + 715782069682176*a^8*b^13*c^11 + 15816474765557760*a^9*b^11*c^12 - 39296545576714240*a^10*b^9*c^13 - 32756650414702592*a^11*b^7*c^14 + 300756012615335936*a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c^16))/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 562

29888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049
624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12
- 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (x^(1/2)*((81*(2401*b^4*(-(4*a*
c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^2
5*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^1
9*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8
*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 14369685
5040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^1
3 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*
(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 8
0*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^
4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^
7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11
*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5
202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 1664729323929
6*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c
^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*(
822083584*a*b^26*c^4 - 14073748835532800*a^14*c^17 - 27950841856*a^2*b^24*c
^5 + 399431958528*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7 + 1032939634688
0*a^5*b^18*c^8 + 6262062317568*a^6*b^16*c^9 - 202859895324672*a^7*b^14*c^10
 + 658057709223936*a^8*b^12*c^11 + 346346162749440*a^9*b^10*c^12 - 86531565
10597120*a^10*b^8*c^13 + 28569710136131584*a^11*b^6*c^14 - 4707668985485721
6*a^12*b^4*c^15 + 40250921669623808*a^13*b^2*c^16)*9i)/(4194304*(b^24 + 167
77216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*
c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 +
32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 5033
1648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2
) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*
a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 398985216
0*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977
994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^1
1 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2
*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^2
5)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 30
40*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*
b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^
9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 7044
75299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14
*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15
+ 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579
840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(3/4)*1i - (9*x^(1/2)*(2
00930625*a*b^13*c^5 - 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 752
3454960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 54
53568000*a^6*b^3*c^10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20
*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 378

4704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680 *a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22* c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^ 14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^ 21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7 *b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 660592394 24*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 94 00*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^3 4*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c ^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b ^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 21134 25899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^1 5*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 54975581388 80*a^20*b^2*c^19)))^(1/4)*1i))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2 401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b ^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6 *b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 1997799424 0*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 2 30770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4 *a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1 /2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^ 3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30* c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^2 4*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299 840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14 *c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 208 09116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a ^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4) - 2*atan(((((27*(575458 5088*a*b^27*c^4 + 309622474381721600*a^14*b*c^17 - 161128382464*a^2*b^25*c^ 5 + 1626181992448*a^3*b^23*c^6 - 3983582167040*a^4*b^21*c^7 - 5632849608704 0*a^5*b^19*c^8 + 557813172535296*a^6*b^17*c^9 - 1961803621859328*a^7*b^15*c ^10 + 715782069682176*a^8*b^13*c^11 + 15816474765557760*a^9*b^11*c^12 - 392 96545576714240*a^10*b^9*c^13 - 32756650414702592*a^11*b^7*c^14 + 3007560126 15335936*a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c^16))/(268435456*(b^2 8 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a ^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b ^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10 *b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096 *a^13*b^2*c^13 - 56*a*b^26*c)) - (x^(1/2)*(-(81*(2401*b^29 + 2401*b^4*(-(4* a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28 243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 39 89852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8

- 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*
b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*
a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c -
 b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38
*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 158760
96*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 825556
9920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10
 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 52022791372
80*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^1
0*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 1305
6700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*(822083584*
a*b^26*c^4 - 14073748835532800*a^14*c^17 - 27950841856*a^2*b^24*c^5 + 39943
1958528*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7 + 10329396346880*a^5*b^18
*c^8 + 6262062317568*a^6*b^16*c^9 - 202859895324672*a^7*b^14*c^10 + 6580577
09223936*a^8*b^12*c^11 + 346346162749440*a^9*b^10*c^12 - 8653156510597120*a
^10*b^8*c^13 + 28569710136131584*a^11*b^6*c^14 - 47076689854857216*a^12*b^4
*c^15 + 40250921669623808*a^13*b^2*c^16)*9i)/(4194304*(b^24 + 16777216*a^12
*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 8110
08*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a
^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*
b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(
1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*
c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^1
7*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^
9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 23077
0606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c
 - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2))
)/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^
36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5
+ 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^
8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*
a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^1
3 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 2080911
6549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*
b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(3/4)*1i + (9*x^(1/2)*(200930625*
a*b^13*c^5 - 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 7523454960*a
^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 5453568000*
a^6*b^3*c^10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14
080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*
b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*
c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(8
1*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^1
4 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 +
 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^
7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*

b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850
270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^2
7*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 10995116
27776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 +
1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 127
0087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9
+ 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520
*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c
^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 195850
50869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*
b^2*c^19)))^(1/4) - (((27*(5754585088*a*b^27*c^4 + 309622474381721600*a^14*
b*c^17 - 161128382464*a^2*b^25*c^5 + 1626181992448*a^3*b^23*c^6 - 398358216
7040*a^4*b^21*c^7 - 56328496087040*a^5*b^19*c^8 + 557813172535296*a^6*b^17*
c^9 - 1961803621859328*a^7*b^15*c^10 + 715782069682176*a^8*b^13*c^11 + 1581
6474765557760*a^9*b^11*c^12 - 39296545576714240*a^10*b^9*c^13 - 32756650414
702592*a^11*b^7*c^14 + 300756012615335936*a^12*b^5*c^15 - 51706953221747507
2*a^13*b^3*c^16))/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^
2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300
288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 5248122
88*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 152
6726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (x^(1/2)*
(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b
*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c
^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^1
5*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a
^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 88
7850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a
*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099
511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^
3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 -
1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*
c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 211342589
9520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^
12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19
585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a
^20*b^2*c^19)))^(1/4)*(822083584*a*b^26*c^4 - 140737488355328000*a^14*c^17 -
27950841856*a^2*b^24*c^5 + 399431958528*a^3*b^22*c^6 - 2968896143360*a^4*b
^20*c^7 + 10329396346880*a^5*b^18*c^8 + 6262062317568*a^6*b^16*c^9 - 202859
895324672*a^7*b^14*c^10 + 658057709223936*a^8*b^12*c^11 + 346346162749440*a
^9*b^10*c^12 - 8653156510597120*a^10*b^8*c^13 + 28569710136131584*a^11*b^6*
c^14 - 47076689854857216*a^12*b^4*c^15 + 40250921669623808*a^13*b^2*c^16)*9
i)/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18
*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 1
2976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 692060
16*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29

+ 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*
a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*
a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073
280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 1
43696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*
b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a
*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c
^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*
b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*
b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 1937307074
56*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c
^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 166472
93239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^1
8*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^
(3/4)*1i - (9*x^(1/2)*(200930625*a*b^13*c^5 - 3110400000*a^7*b*c^11 + 20932
50600*a^2*b^11*c^6 + 7523454960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 235
4261760*a^5*b^5*c^9 - 5453568000*a^6*b^3*c^10))/(4194304*(b^24 + 16777216*a
^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 8
11008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 3244032
0*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^
11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25
)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^
23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*
b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240
*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 23
0770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*
a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/
2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3
*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c
^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24
*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 7044752998
40*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*
c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 2080
9116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^
19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4))/((((27*(5754585088*a*b^
27*c^4 + 309622474381721600*a^14*b*c^17 - 161128382464*a^2*b^25*c^5 + 16261
81992448*a^3*b^23*c^6 - 3983582167040*a^4*b^21*c^7 - 56328496087040*a^5*b^1
9*c^8 + 557813172535296*a^6*b^17*c^9 - 1961803621859328*a^7*b^15*c^10 + 715
782069682176*a^8*b^13*c^11 + 15816474765557760*a^9*b^11*c^12 - 392965455767
14240*a^10*b^9*c^13 - 32756650414702592*a^11*b^7*c^14 + 300756012615335936*
a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c^16))/(268435456*(b^28 + 26843
5456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c
^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 +
196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10
- 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2

*c^13 - 56*a*b^26*c)) - (x^(1/2)*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*(822083584*a*b^26*c^4 - 14073748835532800*a^14*c^17 - 27950841856*a^2*b^24*c^5 + 399431958528*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7 + 10329396346880*a^5*b^18*c^8 + 6262062317568*a^6*b^16*c^9 - 202859895324672*a^7*b^14*c^10 + 658057709223936*a^8*b^12*c^11 + 346346162749440*a^9*b^10*c^12 - 8653156510597120*a^10*b^8*c^13 + 28569710136131584*a^11*b^6*c^14 - 47076689854857216*a^12*b^4*c^15 + 40250921669623808*a^13*b^2*c^16)*9i)/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(3/4)*1i + (9*x^(1/2)*(200930625*a*b^13*c^5 - 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 7523454960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 5453568000*a^6*b^3*c^10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401*b

^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 13236
00*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 14372843
52*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327
073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10
+ 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*a^
13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 940
0*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^2
1*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a
^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a
^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 1937307
07456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^1
6*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 166
47293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*
a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)
))^(1/4)*1i - (27*(103680000000*a^8*c^12 + 1406514375*a*b^14*c^5 + 22129159
500*a^2*b^12*c^6 + 140297799600*a^3*b^10*c^7 + 460920922560*a^4*b^8*c^8 + 8
44743271680*a^5*b^6*c^9 + 869387904000*a^6*b^4*c^10 + 469670400000*a^7*b^2*
c^11))/(134217728*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a
^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^1
6*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10
*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^1
2*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) + (((27*(5754585088*a*
b^27*c^4 + 309622474381721600*a^14*b*c^17 - 161128382464*a^2*b^25*c^5 + 162
6181992448*a^3*b^23*c^6 - 3983582167040*a^4*b^21*c^7 - 56328496087040*a^5*b
^19*c^8 + 557813172535296*a^6*b^17*c^9 - 1961803621859328*a^7*b^15*c^10 + 7
15782069682176*a^8*b^13*c^11 + 15816474765557760*a^9*b^11*c^12 - 3929654557
6714240*a^10*b^9*c^13 - 32756650414702592*a^11*b^7*c^14 + 30075601261533593
6*a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c^16))/(268435456*(b^28 + 268
435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20
*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7
 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^
10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b
^2*c^13 - 56*a*b^26*c)) + (x^(1/2)*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b
^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*
a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 398985216
0*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977
994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^1
1 + 230770606080*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2
*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^2
5)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 30
40*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*
b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^
9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 7044
75299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14
*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15

+ 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579
840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(1/4)*(822083584*a*b^26*
c^4 - 14073748835532800*a^14*c^17 - 27950841856*a^2*b^24*c^5 + 399431958528
*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7 + 10329396346880*a^5*b^18*c^8 +
6262062317568*a^6*b^16*c^9 - 202859895324672*a^7*b^14*c^10 + 65805770922393
6*a^8*b^12*c^11 + 346346162749440*a^9*b^10*c^12 - 8653156510597120*a^10*b^8
*c^13 + 28569710136131584*a^11*b^6*c^14 - 47076689854857216*a^12*b^4*c^15 +
40250921669623808*a^13*b^2*c^16)*9i)/(4194304*(b^24 + 16777216*a^12*c^12 +
1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*
b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*
c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^1
1 - 48*a*b^22*c)))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) +
704643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 2
71415040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 +
2793799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*
c^9 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080
*a^12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)
^25)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(3355
4432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2
- 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 15876
0960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 440
29706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^
18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 104
04558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120
*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^1
8 - 5497558138880*a^20*b^2*c^19)))^(3/4)*1i - (9*x^(1/2)*(200930625*a*b^13*
c^5 - 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 7523454960*a^3*b^9*
c^7 + 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 5453568000*a^6*b^3
c^10))/(4194304(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3
*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^
6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 6
9206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(81*(2401
*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704643072000*a^14*b*c^14 - 132
3600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 271415040*a^4*b^21*c^4 + 143728
4352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 2793799680*a^7*b^15*c^7 + 133
27073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9 - 66059239424*a^10*b^9*c^1
0 + 143696855040*a^11*b^7*c^11 + 230770606080*a^12*b^5*c^12 - 887850270720*
a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) - 9400*a*b^27*c + 9
400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a
^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320
*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680
*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 19373
0707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b
^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 1
6647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 1958505086976

```
0*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^1
9)))^(1/4)*1i))*(-(81*(2401*b^29 + 2401*b^4*(-(4*a*c - b^2)^25)^(1/2) + 704
643072000*a^14*b*c^14 - 1323600*a^2*b^25*c^2 + 28243200*a^3*b^23*c^3 - 2714
15040*a^4*b^21*c^4 + 1437284352*a^5*b^19*c^5 - 3989852160*a^6*b^17*c^6 + 27
93799680*a^7*b^15*c^7 + 13327073280*a^8*b^13*c^8 - 19977994240*a^9*b^11*c^9
 - 66059239424*a^10*b^9*c^10 + 143696855040*a^11*b^7*c^11 + 230770606080*a^
12*b^5*c^12 - 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25
)^(1/2) - 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(3355443
2*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 7
2960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 15876096
0*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 440297
06240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*
c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 104045
58274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^
17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 -
 5497558138880*a^20*b^2*c^19)))^(1/4) - ((x^(7/2)*(11*b^3 + 28*a*b*c))/(16*
(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^(3/2)*(7*a*b^2 + 20*a^2*c))/(16*(b^4 +
 16*a^2*c^2 - 8*a*b^2*c)) - (3*x^(11/2)*(4*a*c^2 - 13*b^2*c))/(16*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)) + (3*b*c^2*x^(15/2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.1084 $\int \frac{x^{7/2}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=533

$$\frac{c^{3/4}\left(36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(-36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-}}\right.}{16\sqrt[4]{2}\left(b^2-4ac\right)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[Out] -1/32*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))
*(41*b^2+28*a*c-36*b*(-4*a*c+b^2)^(1/2))*2^(3/4)/(-4*a*c+b^2)^(5/2)/(-b+(-4
*a*c+b^2)^(1/2))^(3/4)-1/32*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4
*a*c+b^2)^(1/2))^(1/4))*(41*b^2+28*a*c-36*b*(-4*a*c+b^2)^(1/2))*2^(3/4)/(-4
*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/32*c^(3/4)*arctan(2^(1/4)*c
^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(41*b^2+28*a*c+36*b*(-4*a*c+b
^2)^(1/2))*2^(3/4)/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/32*c^
(3/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(41*b^
2+28*a*c+36*b*(-4*a*c+b^2)^(1/2))*2^(3/4)/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^
2)^(1/2))^(3/4)+1/4*(b*x^2+2*a)*x^(1/2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-1/16
*(24*b*c*x^2-4*a*c+13*b^2)*x^(1/2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)
```

Rubi [A] time = 1.36, antiderivative size = 533, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1115, 1365, 1430, 1422, 212, 208, 205}

$$\frac{c^{3/4}\left(36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(-36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-}}\right.}{16\sqrt[4]{2}\left(b^2-4ac\right)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (Sqrt[x]*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (Sqrt[x]*
(13*b^2 - 4*a*c + 24*b*c*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (
c^(3/4)*(41*b^2 + 28*a*c + 36*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*
Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(1/4)*(b^2 - 4*a*c)^(5/2)*(
-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(41*b^2 + 28*a*c - 36*b*Sqrt[b^2
- 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])
/(16*2^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)
*(41*b^2 + 28*a*c + 36*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x
```

```
])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b - S
qrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(41*b^2 + 28*a*c - 36*b*Sqrt[b^2 - 4*a*
c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(16*
2^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
```

```
*c] ||  !IGtQ[n/2, 0])
```

Rule 1430

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> -Simp[(x*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*(
a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a
*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d*(
2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n + c
*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^8}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{2a-11bx^4}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\sqrt{x}\left(13b^2-4ac+24bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{a\left(5b^2+28ac\right)-72}{a+bx^4+cx}\right.}{16a\left(b^2-4\right.}$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\sqrt{x}\left(13b^2-4ac+24bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(41b^2+28ac-36b\right.\right.}{}$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\sqrt{x}\left(13b^2-4ac+24bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(41b^2+28ac-36b\right.\right.}{1}$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\sqrt{x}\left(13b^2-4ac+24bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{c^{3/4}\left(41b^2+28ac+36\right.}{16\sqrt[4]{2}\left(b^2-4\right.}$$

Mathematica [C] time = 0.44, size = 177, normalized size = 0.33

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{72\#1^4bc\log\left(\sqrt{x}-\#1\right)-28ac\log\left(\sqrt{x}-\#1\right)-5b^2\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]+\frac{4\sqrt{x}\left(28a^2c+a\left(5b^2+36bcx^2-4c^2x^4\right)+b\right.}{\left(a+bx^2+cx^4\right)^2}}{64\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/64*((4*Sqrt[x]*(28*a^2*c + a*(5*b^2 + 36*b*c*x^2 - 4*c^2*x^4) + b*x^2*(9
*b^2 + 37*b*c*x^2 + 24*c^2*x^4)))/(a + b*x^2 + c*x^4)^2 + RootSum[a + b*#1^
4 + c*#1^8 & , (-5*b^2*Log[Sqrt[x] - #1] - 28*a*c*Log[Sqrt[x] - #1] + 72*b*
c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(b^2 - 4*a*c)^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 191.21Unable to convert to
real 1/4 Error: Bad Argument Value
```

maple [C] time = 0.04, size = 237, normalized size = 0.44

$$\frac{\left(-72\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^4bc+28ac+5b^2\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{64\left(16a^2c^2-8a\,b^2c+b^4\right)\left(2\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7c+\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3b\right)}+\frac{-\frac{3b\,c^2x^{\frac{13}{2}}}{2\left(16a^2c^2-8a\,b^2c+b\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(-1/32*a*(28*a*c+5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2)-9/32*b*(4*a*c+
b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)+1/32*c*(4*a*c-37*b^2)/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^(9/2)-3/4*b*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(13/2))/(c*x^4
+b*x^2+a)^2+1/64/(16*a^2*c^2-8*a*b^2*c+b^4)*sum((-72*_R^4*b*c+28*a*c+5*b^2)
/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(5\,b^2c^2 + 28\,ac^3\right)x^{\frac{17}{2}} + 2\left(5\,b^3c + 16\,abc^2\right)x^{\frac{13}{2}} + \left(5\,b^4 + ab^2c + 60\,a^2c^2\right)x^{\frac{9}{2}} + \left(ab^3\right. \cdots}{16\left(\left(ab^4c^2 - 8\,a^2b^2c^3 + 16\,a^3c^4\right)x^8 + a^3b^4 - 8\,a^4b^2c + 16\,a^5c^2 + 2\left(ab^5c - 8\,a^2b^3c^2 + 16\,a^3bc^3\right)x^6 + \left(ab^6 - 6\,a^2b^4c\right.\right. \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((5*b^2*c^2 + 28*a*c^3)*x^(17/2) + 2*(5*b^3*c + 16*a*b*c^2)*x^(13/2) +
 (5*b^4 + a*b^2*c + 60*a^2*c^2)*x^(9/2) + (a*b^3 + 20*a^2*b*c)*x^(5/2))/((a
*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5
*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*
c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2) - integ
rate(1/32*((5*b^2*c + 28*a*c^2)*x^(7/2) + 5*(b^3 + 20*a*b*c)*x^(3/2))/(a^2*
b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*x^4
 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2), x)
```

mupad [B] time = 8.38, size = 47803, normalized size = 89.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] atan(((((171894580*a*b^8*c^7 - 48125*b^10*c^6 - 17210368*a^5*c^11 + 3520856
800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536*(
b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^
10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589
824*a^8*b^2*c^8 - 36*a*b^16*c)) + ((((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 6
25*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^
25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*
a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14
462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a
^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^1
3 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2)
 - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a
```

*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23*c^4 + 1759218604441600*a^12*b*c^15 - 1677721600*a^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 563714457600*a^4*b^17*c^7 - 8375186227200*a^5*b^15*c^8 + 68547678044160*a^6*b^13*c^9 - 360777252864000*a^7*b^11*c^10 + 1278182267289600*a^8*b^9*c^11 - 3051144767078400*a^9*b^7*c^12 + 4727899999436800*a^10*b^5*c^13 - 4310085580881920*a^11*b^3*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (x^(1/2)*(209715200*b^27*c^4 - 629145600*a*b^25*c^5 - 9162010491931852 8*a^13*b*c^17 - 94623498240*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19*c^8 - 197235635650560*a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 15146459867381760*a^7*b^13*c^11 + 63613894492422144*a^8*b^11*c^12 - 180146733873889280*a^9*b^9*c^13 + 342651803680112640*a^10*b^7*c^14 - 419309754368655360*a^11*b^5*c^15 + 296956100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(3/4))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 +

70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 26745984411 2384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 10 99511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34* c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b ^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 21134 25899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^1 7*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 54975581388 80*a^22*b^2*c^19)))^(1/4) - (x^(1/2)*(481890304*a^6*c^13 + 441265825*b^12*c ^7 + 16718255400*a*b^10*c^8 + 151843979760*a^2*b^8*c^9 - 123896495360*a^3*b ^6*c^10 + 12295917312*a^4*b^4*c^11 + 7420127232*a^5*b^2*c^12))/(2097152*(b^ 24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a ^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^1 0*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^1 0 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 2718641 6*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 26518 8833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15* c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242 260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13 *b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^2 5)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 10995116277 76*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 124 0320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 127008 7680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520* a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^ 14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 1958505 0869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b ^2*c^19)))^(1/4)*1i - (((171894580*a*b^8*c^7 - 48125*b^10*c^6 - 17210368*a^ 5*c^11 + 3520856800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167976704*a^4*b^ 2*c^10)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^ 3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a ^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + ((((625*b^6*(-(4*a*c - b^ 2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a ^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112 384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)

^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 109
9511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c
^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6
- 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^
22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 211342
5899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17
*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 -
19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 549755813888
0*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23*c^4 + 1759218604441600*a^12*b*c^1
5 - 1677721600*a^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 563714457600*a^4*b^
17*c^7 - 8375186227200*a^5*b^15*c^8 + 68547678044160*a^6*b^13*c^9 - 3607772
52864000*a^7*b^11*c^10 + 1278182267289600*a^8*b^9*c^11 - 3051144767078400*a
^9*b^7*c^12 + 4727899999436800*a^10*b^5*c^13 - 4310085580881920*a^11*b^3*c^
14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 +
32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*
b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (x^(1/2)*(209715200*b^27*c^4
- 629145600*a*b^25*c^5 - 9162010491931852 8*a^13*b*c^17 - 94623498240*a^2*b
^23*c^6 + 1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19*c^8 - 1972356
35650560*a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 15146459867381760*
a^7*b^13*c^11 + 63613894492422144*a^8*b^11*c^12 - 180146733873889280*a^9*b^
9*c^13 + 342651803680112640*a^10*b^7*c^14 - 419309754368655360*a^11*b^5*c^1
5 + 296956100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777216*a^12*c^12
+ 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5
*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8
*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^
11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 151921
04632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297
600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1
688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9
*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 2
06669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787
840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*
c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c
- b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^
38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 1587
6096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 825
5569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*
c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279
137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18
*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 +
13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(3/4))*((625*
b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 890
00*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 2549240
9600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7
+ 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*

a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4) + (x^(1/2)*(481890304*a^6*c^13 + 441265825*b^12*c^7 + 16718255400*a*b^10*c^8 + 151843979760*a^2*b^8*c^9 - 123896495360*a^3*b^6*c^10 + 12295917312*a^4*b^4*c^11 + 7420127232*a^5*b^2*c^12))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*1i)/((((171894580*a*b^8*c^7 - 48125*b^10*c^6 - 17210368*a^5*c^11 + 3520856800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + ((((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(

4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432
*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 -
72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 1587609
60*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 440
29706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^
18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 104
04558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120
*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^1
8 - 5497558138880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23*c^4 + 17592186044
41600*a^12*b*c^15 - 1677721600*a^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 563
714457600*a^4*b^17*c^7 - 8375186227200*a^5*b^15*c^8 + 68547678044160*a^6*b^
13*c^9 - 360777252864000*a^7*b^11*c^10 + 1278182267289600*a^8*b^9*c^11 - 30
51144767078400*a^9*b^7*c^12 + 4727899999436800*a^10*b^5*c^13 - 431008558088
1920*a^11*b^3*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 537
6*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c
^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (x^(1/2)*(20
9715200*b^27*c^4 - 629145600*a*b^25*c^5 - 91620104919318528*a^13*b*c^17 - 9
4623498240*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b^
19*c^8 - 197235635650560*a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 15
146459867381760*a^7*b^13*c^11 + 63613894492422144*a^8*b^11*c^12 - 180146733
873889280*a^9*b^9*c^13 + 342651803680112640*a^10*b^7*c^14 - 419309754368655
360*a^11*b^5*c^15 + 296956100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 167
77216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*
c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 +
32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 5033
1648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 6
25*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^
25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*
a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14
462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a
^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^1
3 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2)
 - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a
*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*
c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7
*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^1
0*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 1937307
07456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^1
6*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 166
47293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*
a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)
))^(3/4))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a
^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^
23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 16888165785
60*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9

+ 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 20666946420 7360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b ^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 191100 0*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25) ^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 304 0*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b ^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^ 11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704 475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^1 6*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 1305670057 9840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4) - (x^(1/2)*(48189 0304*a^6*c^13 + 441265825*b^12*c^7 + 16718255400*a*b^10*c^8 + 151843979760* a^2*b^8*c^9 - 123896495360*a^3*b^6*c^10 + 12295917312*a^4*b^4*c^11 + 742012 7232*a^5*b^2*c^12))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704 *a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9 *b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c))) *((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^1 5 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^ 17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326 443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12 *b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2 *c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^3 6*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c ^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840 *a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^ 13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 208091 16549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21 *b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4) + (((171894580*a*b^8*c^7 - 48125*b^10*c^6 - 17210368*a^5*c^11 + 3520856800*a^2*b^6*c^8 + 3512738432*a ^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^ 2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + ((((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15* b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c ^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a ^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 41 63326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360 *a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c ^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^

2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23*c^4 + 1759218604441600*a^12*b*c^15 - 1677721600*a^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 563714457600*a^4*b^17*c^7 - 8375186227200*a^5*b^15*c^8 + 68547678044160*a^6*b^13*c^9 - 360777252864000*a^7*b^11*c^10 + 1278182267289600*a^8*b^9*c^11 - 3051144767078400*a^9*b^7*c^12 + 4727899999436800*a^10*b^5*c^13 - 4310085580881920*a^11*b^3*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (x^(1/2)*(209715200*b^27*c^4 - 629145600*a*b^25*c^5 - 9162010491931852 8*a^13*b*c^17 - 94623498240*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19*c^8 - 197235635650560*a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 15146459867381760*a^7*b^13*c^11 + 63613894492422144*a^8*b^11*c^12 - 180146733873889280*a^9*b^9*c^13 + 342651803680112640*a^10*b^7*c^14 - 419309754368655360*a^11*b^5*c^15 + 296956100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(3/4))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*

a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11
- 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114
787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^
29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a
c - b^2)^25)^(1/2))/(33554432(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4
*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 1
5876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 +
8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^
20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202
279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a
^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17
+ 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4) + (x
^(1/2)*(481890304*a^6*c^13 + 441265825*b^12*c^7 + 16718255400*a*b^10*c^8 +
151843979760*a^2*b^8*c^9 - 123896495360*a^3*b^6*c^10 + 12295917312*a^4*b^4*
c^11 + 7420127232*a^5*b^2*c^12))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056
*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*
c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 -
57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 4
8*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 151921046323
20*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^
4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816
578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*
c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 2066694
64207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^
14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 19
11000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)
^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c +
3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a
^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 825556992
0*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 -
704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280
*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*
c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 130567
00579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)))*((625*b^6*(
-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^
2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*
a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 666
4504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*
b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 +
267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c
^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*
c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^
3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 7296
0*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a
^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 4402970

6240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*2i - ((9*x^(5/2)*(b^3 + 4*a*b*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^(1/2)*(5*a*b^2 + 28*a^2*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^(9/2)*(4*a*c^2 - 37*b^2*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*c^2*x^(13/2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + atan(((((171894580*a*b^8*c^7 - 48125*b^10*c^6 - 17210368*a^5*c^11 + 3520856800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (((-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23*c^4 + 1759218604441600*a^12*b*c^15 - 1677721600*a^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 563714457600*a^4*b^17*c^7 - 8375186227200*a^5*b^15*c^8 + 68547678044160*a^6*b^13*c^9 - 360777252864000*a^7*b^11*c^10 + 1278182267289600*a^8*b^9*c^11 - 3051144767078400*a^9*b^7*c^12 + 4727899999436800*a^10*b^5*c^13 - 4310085580881920*a^11*b^3*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (x^(1/2)*(209715200*b^27*c^4 - 629145600*a*b^25*c^5 - 91620104919318528*a^13*b*c^17 - 94623498240*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19*c^8 - 197235635650560*a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 15146459867381760*a^7*b^13*c^11 + 63613894492422144*a^8*b^11*c^12 - 180146733873889280*a^9*b^9*c^13 + 342651803680112640*a^10*b^7*c^14 - 419309754368655360*a^11*b^5*c^15 + 296956100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^

9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625
*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89
000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 254924
09600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7
 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520
*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c
^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416
*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(
-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(335544
32*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2
- 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 15876
0960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 4
4029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*
b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 1
0404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 208091165491
20*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c
^18 - 5497558138880*a^22*b^2*c^19)))^(3/4))*(-(625*b^31 + 625*b^6*(-(4*a*c
- b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 271864
16*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 2651
88833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15
*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 7045524
2260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^1
3*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^
25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2)
+ 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627
776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 12
40320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 12700
87680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9
+ 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520
*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c
^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 195850
50869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*
b^2*c^19)))^(1/4) - (x^(1/2)*(481890304*a^6*c^13 + 441265825*b^12*c^7 + 167
18255400*a*b^10*c^8 + 151843979760*a^2*b^8*c^9 - 123896495360*a^3*b^6*c^10
+ 12295917312*a^4*b^4*c^11 + 7420127232*a^5*b^2*c^12))/(2097152*(b^24 + 167
77216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*
c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 +
32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 5033
1648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^2
5)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b
^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280
*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 1
4462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*
a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^
13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2

) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*
a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23
*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^
7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^
10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730
707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^
16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16
647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760
*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19
)))^(1/4)*1i - (((171894580*a*b^8*c^7 - 48125*b^10*c^6 - 17210368*a^5*c^11
+ 3520856800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167976704*a^4*b^2*c^10)
/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 322
56*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*
c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (((-(625*b^31 + 625*b^6*(-(4*a*c
 - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186
416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265
188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^1
5*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 704552
42260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^
13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)
^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2)
 + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 109951162
7776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1
240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270
087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9
 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 211342589952
0*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*
c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585
050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22
*b^2*c^19)))^(1/4)*(83886080*a*b^23*c^4 + 1759218604441600*a^12*b*c^15 - 16
77721600*a^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 563714457600*a^4*b^17*c^7
 - 8375186227200*a^5*b^15*c^8 + 68547678044160*a^6*b^13*c^9 - 3607772528640
00*a^7*b^11*c^10 + 1278182267289600*a^8*b^9*c^11 - 3051144767078400*a^9*b^7
*c^12 + 4727899999436800*a^10*b^5*c^13 - 4310085580881920*a^11*b^3*c^14))/(
65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256
*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^
7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (x^(1/2)*(209715200*b^27*c^4 - 629
145600*a*b^25*c^5 - 91620104919318528*a^13*b*c^17 - 94623498240*a^2*b^23*c^
6 + 1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19*c^8 - 1972356356505
60*a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 15146459867381760*a^7*b^
13*c^11 + 63613894492422144*a^8*b^11*c^12 - 180146733873889280*a^9*b^9*c^13
 + 342651803680112640*a^10*b^7*c^14 - 419309754368655360*a^11*b^5*c^15 + 29
6956100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056
*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*
c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 -

57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 4
8*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632
320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a
^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 168881
6578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13
*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669
464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a
^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1
911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2
)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c
+ 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*
a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 82555699
20*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10
- 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 520227913728
0*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10
*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056
700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(3/4))*(-(625*b^31
+ 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a
^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600
*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 66
64504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10
*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 -
267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*
c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a
*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a
^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 729
60*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*
a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 440297
06240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*
c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 104045
58274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^
19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 -
5497558138880*a^22*b^2*c^19)))^(1/4) + (x^(1/2)*(481890304*a^6*c^13 + 4412
65825*b^12*c^7 + 16718255400*a*b^10*c^8 + 151843979760*a^2*b^8*c^9 - 123896
495360*a^3*b^6*c^10 + 12295917312*a^4*b^4*c^11 + 7420127232*a^5*b^2*c^12))/
(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^
3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 1297
6128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*
a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b
^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*
c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^2
1*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147
968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^
10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 2674598
44112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4

*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2
)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40
+ 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b
^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28
*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^
12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2
113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560
*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c
^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558
138880*a^22*b^2*c^19)))^(1/4)*1i)/((((171894580*a*b^8*c^7 - 48125*b^10*c^6
- 17210368*a^5*c^11 + 3520856800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167
976704*a^4*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 537
6*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c
^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (((-(625*b^3
1 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*
a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 2549240960
0*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6
664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^1
0*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12
- 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3
c^3(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*
a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(
a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72
960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960
*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029
706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18
*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404
558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a
^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18
- 5497558138880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23*c^4 + 1759218604441
600*a^12*b*c^15 - 1677721600*a^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 56371
4457600*a^4*b^17*c^7 - 8375186227200*a^5*b^15*c^8 + 68547678044160*a^6*b^13
*c^9 - 360777252864000*a^7*b^11*c^10 + 1278182267289600*a^8*b^9*c^11 - 3051
144767078400*a^9*b^7*c^12 + 4727899999436800*a^10*b^5*c^13 - 43100855808819
20*a^11*b^3*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*
a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6
- 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (x^(1/2)*(2097
15200*b^27*c^4 - 629145600*a*b^25*c^5 - 91620104919318528*a^13*b*c^17 - 946
23498240*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19
*c^8 - 197235635650560*a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 1514
6459867381760*a^7*b^13*c^11 + 63613894492422144*a^8*b^11*c^12 - 18014673387
3889280*a^9*b^9*c^13 + 342651803680112640*a^10*b^7*c^14 - 41930975436865536
0*a^11*b^5*c^15 + 296956100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777
216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^
4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32

440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 503316
48*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)
^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^2
5*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a
^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 144
62970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^
11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13
 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2)
+ 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*
b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c
^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*
b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10
*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 19373070
7456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16
*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 1664
7293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a
^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19))
)^(3/4))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a
^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^
23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 16888165785
60*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9
- 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 20666946420
7360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b
^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 191100
0*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)
^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 304
0*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b
^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^
11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704
475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^1
6*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15
 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 1305670057
9840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4) - (x^(1/2)*(48189
0304*a^6*c^13 + 441265825*b^12*c^7 + 16718255400*a*b^10*c^8 + 151843979760*
a^2*b^8*c^9 - 123896495360*a^3*b^6*c^10 + 12295917312*a^4*b^4*c^11 + 742012
7232*a^5*b^2*c^12))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2
 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704
*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9
*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))
*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^
15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 +
 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b
^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 416332
6443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^1
2*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14

- 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4) + (((171894580*a*b^8*c^7 - 48125*b^10*c^6 - 17210368*a^5*c^11 + 3520856800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (((-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23*c^4 + 1759218604441600*a^12*b*c^15 - 1677721600*a^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 563714457600*a^4*b^17*c^7 - 8375186227200*a^5*b^15*c^8 + 68547678044160*a^6*b^13*c^9 - 360777252864000*a^7*b^11*c^10 + 1278182267289600*a^8*b^9*c^11 - 3051144767078400*a^9*b^7*c^12 + 4727899999436800*a^10*b^5*c^13 - 4310085580881920*a^11*b^3*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (x^(1/2)*(209715200*b^27*c^4 - 629145600*a*b^25*c^5 - 91620104919318528*a^13*b*c^17 - 94623498240*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19*c^8 - 197235635650560*a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 15146459867381760*a^7*b^13*c^11 + 63613894492422144*a^8*b^11*c^12 - 180146733873889280*a^9*b^9*c^13 + 342651803680112640*a^10*b^7*c^14 - 419309754368655360*a^11*b^5*c^15 + 296956100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128* a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(3/4))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4) + (x^(1/2)*(481890304*a^6*c^13 + 441265825*b^12*c^7 + 16718255400*a*b^10*c^8 + 151843979760*a^2*b^8*c^9 - 123896495360*a^3*b^6*c^10 + 12295917312*a^4*b^4*c^11 + 7420127232*a^5*b^2*c^12))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 1500091147878

40*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*2i + 2*atan(((((171894580*a*b^8*c^7 - 48125*b^10*c^6 - 17210368*a^5*c^11 + 3520856800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - ((((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 130567005

79840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23
*c^4 + 1759218604441600*a^12*b*c^15 - 1677721600*a^2*b^21*c^5 - 6710886400*
a^3*b^19*c^6 + 563714457600*a^4*b^17*c^7 - 8375186227200*a^5*b^15*c^8 + 685
47678044160*a^6*b^13*c^9 - 360777252864000*a^7*b^11*c^10 + 1278182267289600
*a^8*b^9*c^11 - 3051144767078400*a^9*b^7*c^12 + 4727899999436800*a^10*b^5*c
^13 - 4310085580881920*a^11*b^3*c^14)*1i)/(65536*(b^18 - 262144*a^9*c^9 + 5
76*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c
^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^
16*c)) - (x^(1/2)*(209715200*b^27*c^4 - 629145600*a*b^25*c^5 - 916201049193
18528*a^13*b*c^17 - 94623498240*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^7 +
 1803886264320*a^4*b^19*c^8 - 197235635650560*a^5*b^17*c^9 + 23306210535014
40*a^6*b^15*c^10 - 15146459867381760*a^7*b^13*c^11 + 63613894492422144*a^8*
b^11*c^12 - 180146733873889280*a^9*b^9*c^13 + 342651803680112640*a^10*b^7*c
^14 - 419309754368655360*a^11*b^5*c^15 + 296956100429742080*a^12*b^3*c^16))
/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c
^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 129
76128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016
*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c
 - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*
c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^2
1*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147
968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^
10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 2674598
44112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4
*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2
)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40
+ 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b
^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28
*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^
12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2
113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560
*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c
^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558
138880*a^22*b^2*c^19)))^(3/4)*1i)*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625
*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25
*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^
6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 1446
2970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^1
1*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13
- 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) -
 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b
^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^
20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b
^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*
b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707

456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*1i - (x^(1/2)*(481890304*a^6*c^13 + 441265825*b^12*c^7 + 16718255400*a*b^10*c^8 + 151843979760*a^2*b^8*c^9 - 123896495360*a^3*b^6*c^10 + 12295917312*a^4*b^4*c^11 + 7420127232*a^5*b^2*c^12))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4) - (((171894580*a*b^8*c^7 - 48125*b^10*c^6 - 17210368*a^5*c^11 + 3520856800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - ((((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^2

0*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^
(1/4)*(83886080*a*b^23*c^4 + 1759218604441600*a^12*b*c^15 - 1677721600*a^2*
b^21*c^5 - 6710886400*a^3*b^19*c^6 + 563714457600*a^4*b^17*c^7 - 8375186227
200*a^5*b^15*c^8 + 68547678044160*a^6*b^13*c^9 - 360777252864000*a^7*b^11*c
^10 + 1278182267289600*a^8*b^9*c^11 - 3051144767078400*a^9*b^7*c^12 + 47278
99999436800*a^10*b^5*c^13 - 4310085580881920*a^11*b^3*c^14)*1i)/(65536*(b^1
8 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*
c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824
*a^8*b^2*c^8 - 36*a*b^16*c)) + (x^(1/2)*(209715200*b^27*c^4 - 629145600*a*b
^25*c^5 - 91620104919318528*a^13*b*c^17 - 94623498240*a^2*b^23*c^6 + 129842
2300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19*c^8 - 197235635650560*a^5*b^1
7*c^9 + 2330621053501440*a^6*b^15*c^10 - 15146459867381760*a^7*b^13*c^11 +
63613894492422144*a^8*b^11*c^12 - 180146733873889280*a^9*b^9*c^13 + 3426518
03680112640*a^10*b^7*c^14 - 419309754368655360*a^11*b^5*c^15 + 296956100429
742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*
c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784
704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*
a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c
)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*
c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4
 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7
*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163
326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a
^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^1
4 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*
b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2)
)/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*
b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^
5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^2
4*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299
840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14
*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 208
09116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a
^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(3/4)*1i)*((625*b^6*(-(4*a*c
- b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c
^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21
*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 66645041479
68*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^1
0 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 26745984
4112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*
a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)
^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 +
 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^
34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*
c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^1

2*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 21
13425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*
a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^
16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 54975581
38880*a^22*b^2*c^19)))^(1/4)*1i + (x^(1/2)*(481890304*a^6*c^13 + 441265825*
b^12*c^7 + 16718255400*a*b^10*c^8 + 151843979760*a^2*b^8*c^9 - 123896495360
*a^3*b^6*c^10 + 12295917312*a^4*b^4*c^11 + 7420127232*a^5*b^2*c^12))/(20971
52*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 12
6720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a
^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b
^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)
^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 2
7186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 +
265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8
*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70
455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 26745984411238
4*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c -
b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(
1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 10995
11627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3
+ 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 -
1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22
*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 21134258
99520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b
^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 1
9585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*
a^22*b^2*c^19)))^(1/4))/((((171894580*a*b^8*c^7 - 48125*b^10*c^6 - 17210368
*a^5*c^11 + 3520856800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167976704*a^4
*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12
*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 58982
4*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - ((((625*b^6*(-(4*a*c -
b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^
2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*
c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 666450414796
8*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10
+ 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844
112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a
*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^
25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 +
1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^3
4*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c
^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12
*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 211
3425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a
^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^1

6 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 549755813
8880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23*c^4 + 1759218604441600*a^12*b*
c^15 - 1677721600*a^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 563714457600*a^4
*b^17*c^7 - 8375186227200*a^5*b^15*c^8 + 68547678044160*a^6*b^13*c^9 - 3607
77252864000*a^7*b^11*c^10 + 1278182267289600*a^8*b^9*c^11 - 305114476707840
0*a^9*b^7*c^12 + 4727899999436800*a^10*b^5*c^13 - 4310085580881920*a^11*b^3
*c^14)*1i)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12
*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 58982
4*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (x^(1/2)*(209715200*b^
27*c^4 - 629145600*a*b^25*c^5 - 91620104919318528*a^13*b*c^17 - 94623498240
*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19*c^8 - 1
97235635650560*a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 151464598673
81760*a^7*b^13*c^11 + 63613894492422144*a^8*b^11*c^12 - 180146733873889280*
a^9*b^9*c^13 + 342651803680112640*a^10*b^7*c^14 - 419309754368655360*a^11*b
^5*c^15 + 296956100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777216*a^12
*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 8110
08*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a
^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*
b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 +
15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1
342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c
^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 144629704294
40*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^
11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009
114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a
*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(
4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*
a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4
- 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7
+ 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13
*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5
202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 1664729323929
6*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c
^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(3/4)*1
i)*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c
^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4
- 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*
b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 41633
26443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^
12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14
- 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b
^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))
/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b
^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5
+ 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24

*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 7044752998
40*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*
c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 2080
9116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^
21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*1i - (x^(1/2)*(481890304
*a^6*c^13 + 441265825*b^12*c^7 + 16718255400*a*b^10*c^8 + 151843979760*a^2*
b^8*c^9 - 123896495360*a^3*b^6*c^10 + 12295917312*a^4*b^4*c^11 + 7420127232
*a^5*b^2*c^12))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 1
4080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6
*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6
*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((6
25*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 +
89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 2549
2409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c
^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 41633264435
20*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7
*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 384
16*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2
*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(3355
4432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^
2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158
760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 -
44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^1
4*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 +
10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 2080911654
9120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4
*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*1i + (((171894580*a*b^8*c^7 -
48125*b^10*c^6 - 17210368*a^5*c^11 + 3520856800*a^2*b^6*c^8 + 3512738432*a^
3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2
*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 3
44064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c))
- ((((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b
*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^
4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^
7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 416
3326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*
a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^
14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2
*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2
))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5
*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c
^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^
24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 70447529
9840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^1
4*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20

809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*
a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23*c^4 +
 1759218604441600*a^12*b*c^15 - 1677721600*a^2*b^21*c^5 - 6710886400*a^3*b^
19*c^6 + 563714457600*a^4*b^17*c^7 - 8375186227200*a^5*b^15*c^8 + 685476780
44160*a^6*b^13*c^9 - 360777252864000*a^7*b^11*c^10 + 1278182267289600*a^8*b
^9*c^11 - 3051144767078400*a^9*b^7*c^12 + 4727899999436800*a^10*b^5*c^13 -
4310085580881920*a^11*b^3*c^14)*1i)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2
*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 3
44064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c))
 + (x^(1/2)*(209715200*b^27*c^4 - 629145600*a*b^25*c^5 - 91620104919318528*
a^13*b*c^17 - 94623498240*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^7 + 18038
86264320*a^4*b^19*c^8 - 197235635650560*a^5*b^17*c^9 + 2330621053501440*a^6
*b^15*c^10 - 15146459867381760*a^7*b^13*c^11 + 63613894492422144*a^8*b^11*c
^12 - 180146733873889280*a^9*b^9*c^13 + 342651803680112640*a^10*b^7*c^14 -
419309754368655360*a^11*b^5*c^15 + 296956100429742080*a^12*b^3*c^16))/(2097
152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 1
26720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*
a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*
b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2
)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 -
27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5
+ 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^
8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 7
0455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 2674598441123
84*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c -
 b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^
(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099
511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^
3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 -
 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^2
2*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425
899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*
b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 -
19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880
*a^22*b^2*c^19)))^(3/4)*1i)*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31
+ 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 +
 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19
*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 1446297042
9440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*
c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 1500
09114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125
*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(
-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 8
0*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^
4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c

^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*1i + (x^(1/2)*(481890304*a^6*c^13 + 441265825*b^12*c^7 + 16718255400*a*b^10*c^8 + 151843979760*a^2*b^8*c^9 - 123896495360*a^3*b^6*c^10 + 12295917312*a^4*b^4*c^11 + 7420127232*a^5*b^2*c^12))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*1i))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4) + 2*atan(((((171894580*a*b^8*c^7 - 48125*b^10*c^6 - 17210368*a^5*c^11 + 3520856800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 +

576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8
*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*
b^16*c)) - (((-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 151921046323
20*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^
4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816
578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*
c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 2066694
64207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^
14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 19
11000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)
^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c +
3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a
^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 825556992
0*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 -
704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280
*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*
c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 130567
00579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b
^23*c^4 + 1759218604441600*a^12*b*c^15 - 1677721600*a^2*b^21*c^5 - 67108864
00*a^3*b^19*c^6 + 563714457600*a^4*b^17*c^7 - 8375186227200*a^5*b^15*c^8 +
68547678044160*a^6*b^13*c^9 - 360777252864000*a^7*b^11*c^10 + 1278182267289
600*a^8*b^9*c^11 - 3051144767078400*a^9*b^7*c^12 + 4727899999436800*a^10*b^
5*c^13 - 4310085580881920*a^11*b^3*c^14)*1i)/(65536*(b^18 - 262144*a^9*c^9
+ 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^
8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a
*b^16*c)) - (x^(1/2)*(209715200*b^27*c^4 - 629145600*a*b^25*c^5 - 916201049
19318528*a^13*b*c^17 - 94623498240*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^
7 + 1803886264320*a^4*b^19*c^8 - 197235635650560*a^5*b^17*c^9 + 23306210535
01440*a^6*b^15*c^10 - 15146459867381760*a^7*b^13*c^11 + 63613894492422144*a
^8*b^11*c^12 - 180146733873889280*a^9*b^9*c^13 + 342651803680112640*a^10*b^
7*c^14 - 419309754368655360*a^11*b^5*c^15 + 296956100429742080*a^12*b^3*c^1
6))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^1
8*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 -
12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206
016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 6
25*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b
^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5
*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 666450
4147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^1
1*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267
459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*
(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c -
b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b
^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a
^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*

b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 4402970624
0*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11
 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 1040455827
4560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b
^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 549
7558138880*a^22*b^2*c^19)))^(3/4)*1i)*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)
^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3
*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 2651888332
80*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 +
 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 7045524226048
0*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*
c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1
/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 5437
5*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^
23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*
a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*
a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 1937
30707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*
b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 -
16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 195850508697
60*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^
19)))^(1/4)*1i - (x^(1/2)*(481890304*a^6*c^13 + 441265825*b^12*c^7 + 167182
55400*a*b^10*c^8 + 151843979760*a^2*b^8*c^9 - 123896495360*a^3*b^6*c^10 + 1
2295917312*a^4*b^4*c^11 + 7420127232*a^5*b^2*c^12))/(2097152*(b^24 + 167772
16*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4
 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 324
40320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 5033164
8*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^
(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25
*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^
6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 1446
2970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^1
1*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13
+ 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) +
 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b
^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^
20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b
^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*
b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707
456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*
c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647
293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^
20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))
^(1/4) - (((171894580*a*b^8*c^7 - 48125*b^10*c^6 - 17210368*a^5*c^11 + 3520
856800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(6553

6*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4
*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 +
589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (((-(625*b^31 + 625*b^6*(-(4*a*c - b^2
)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^
3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833
280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8
+ 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 704552422604
80*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5
*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(
1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 543
75*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a
^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320
*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680
*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193
730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15
*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 -
16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869
760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c
^19)))^(1/4)*(83886080*a*b^23*c^4 + 1759218604441600*a^12*b*c^15 - 16777216
00*a^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 563714457600*a^4*b^17*c^7 - 837
5186227200*a^5*b^15*c^8 + 68547678044160*a^6*b^13*c^9 - 360777252864000*a^7
*b^11*c^10 + 1278182267289600*a^8*b^9*c^11 - 3051144767078400*a^9*b^7*c^12
+ 4727899999436800*a^10*b^5*c^13 - 4310085580881920*a^11*b^3*c^14)*1i)/(655
36*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^
4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 +
589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (x^(1/2)*(209715200*b^27*c^4 - 629145
600*a*b^25*c^5 - 91620104919318528*a^13*b*c^17 - 94623498240*a^2*b^23*c^6 +
1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19*c^8 - 197235635650560*
a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 15146459867381760*a^7*b^13*
c^11 + 63613894492422144*a^8*b^11*c^12 - 180146733873889280*a^9*b^9*c^13 +
342651803680112640*a^10*b^7*c^14 - 419309754368655360*a^11*b^5*c^15 + 29695
6100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^
2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5
+ 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57
671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a
*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320
*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*
b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 168881657
8560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^
9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464
207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14
*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911
000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^2
5)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3
040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8

*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*
a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 7
04475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a
^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^
15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700
579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(3/4)*1i)*(-(625*b^31
 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a
^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600
*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 66
64504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10
*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 -
 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*
c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a
*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a
^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 729
60*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*
a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 440297
06240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*
c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 104045
58274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^
19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 -
 5497558138880*a^22*b^2*c^19)))^(1/4)*1i + (x^(1/2)*(481890304*a^6*c^13 + 4
41265825*b^12*c^7 + 16718255400*a*b^10*c^8 + 151843979760*a^2*b^8*c^9 - 123
896495360*a^3*b^6*c^10 + 12295917312*a^4*b^4*c^11 + 7420127232*a^5*b^2*c^12
))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18
*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 1
2976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 692060
16*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 62
5*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^
27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*
b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504
147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11
*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 2674
59844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(
-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c -
b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^
40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^
6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b
^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240
*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11
+ 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274
560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^
8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497
558138880*a^22*b^2*c^19)))^(1/4))/((((171894580*a*b^8*c^7 - 48125*b^10*c^6
- 17210368*a^5*c^11 + 3520856800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167

976704*a^4*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 537
6*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c
^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (((-(625*b^3
1 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*
a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 2549240960
0*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6
664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^1
0*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12
- 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3
c^3(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*
a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(
a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72
960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960
*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029
706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18
*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404
558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a
^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18
- 5497558138880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23*c^4 + 1759218604441
600*a^12*b*c^15 - 1677721600*a^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 56371
4457600*a^4*b^17*c^7 - 8375186227200*a^5*b^15*c^8 + 68547678044160*a^6*b^13
*c^9 - 360777252864000*a^7*b^11*c^10 + 1278182267289600*a^8*b^9*c^11 - 3051
144767078400*a^9*b^7*c^12 + 4727899999436800*a^10*b^5*c^13 - 43100855808819
20*a^11*b^3*c^14)*1i)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 53
76*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*
c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (x^(1/2)*(2
09715200*b^27*c^4 - 629145600*a*b^25*c^5 - 91620104919318528*a^13*b*c^17 -
94623498240*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b
^19*c^8 - 197235635650560*a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 1
5146459867381760*a^7*b^13*c^11 + 63613894492422144*a^8*b^11*c^12 - 18014673
3873889280*a^9*b^9*c^13 + 342651803680112640*a^10*b^7*c^14 - 41930975436865
5360*a^11*b^5*c^15 + 296956100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 16
777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16
*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 +
32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 503
31648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^
25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*
b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 26518883328
0*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 +
14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480
*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c
^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/
2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375
*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^2
3*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a

^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a
^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 19373
0707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b
^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 1
6647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 1958505086976
0*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^1
9)))^(3/4)*1i)*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 1519210463
2320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*
a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 16888
16578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^1
3*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 20666
9464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*
a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c +
1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^
2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c
 + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096
*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569
920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10
 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 52022791372
80*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^1
0*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 1305
6700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*1i - (x^(1/
2)*(481890304*a^6*c^13 + 441265825*b^12*c^7 + 16718255400*a*b^10*c^8 + 1518
43979760*a^2*b^8*c^9 - 123896495360*a^3*b^6*c^10 + 12295917312*a^4*b^4*c^11
 + 7420127232*a^5*b^2*c^12))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2
*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5
+ 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 576
71680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*
b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*
a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b
^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578
560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9
 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 2066694642
07360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*
b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 19110
00*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25
)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 30
40*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*
b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a
^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 70
4475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^
16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^1
5 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 130567005
79840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*1i + (((17189458
0*a*b^8*c^7 - 48125*b^10*c^6 - 17210368*a^5*c^11 + 3520856800*a^2*b^6*c^8 +

3512738432*a^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) - (((-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)*(83886080*a*b^23*c^4 + 1759218604441600*a^12*b*c^15 - 1677721600*a^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 563714457600*a^4*b^17*c^7 - 8375186227200*a^5*b^15*c^8 + 68547678044160*a^6*b^13*c^9 - 360777252864000*a^7*b^11*c^10 + 1278182267289600*a^8*b^9*c^11 - 3051144767078400*a^9*b^7*c^12 + 4727899999436800*a^10*b^5*c^13 - 4310085580881920*a^11*b^3*c^14)*1i)/(65536*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + (x^(1/2)*(209715200*b^27*c^4 - 629145600*a*b^25*c^5 - 91620104919318528*a^13*b*c^17 - 94623498240*a^2*b^23*c^6 + 1298422300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19*c^8 - 197235635650560*a^5*b^17*c^9 + 2330621053501440*a^6*b^15*c^10 - 15146459867381760*a^7*b^13*c^11 + 63613894492422144*a^8*b^11*c^12 - 180146733873889280*a^9*b^9*c^13 + 342651803680112640*a^10*b^7*c^14 - 419309754368655360*a^11*b^5*c^15 + 296956100429742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 7

2960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 15876096
0*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 4402
9706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^1
8*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 1040
4558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*
a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18
 - 5497558138880*a^22*b^2*c^19)))^(3/4)*1i)*(-(625*b^31 + 625*b^6*(-(4*a*c
- b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 271864
16*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 2651
88833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15
*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 7045524
2260480*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^1
3*b^5*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^
25)^(1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2)
+ 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627
776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 12
40320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 12700
87680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9
+ 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520
*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c
^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 195850
50869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*
b^2*c^19)))^(1/4)*1i + (x^(1/2)*(481890304*a^6*c^13 + 441265825*b^12*c^7 +
16718255400*a*b^10*c^8 + 151843979760*a^2*b^8*c^9 - 123896495360*a^3*b^6*c^
10 + 12295917312*a^4*b^4*c^11 + 7420127232*a^5*b^2*c^12))/(2097152*(b^24 +
16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^
16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7
 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 5
0331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2
)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^
3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833
280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8
+ 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 704552422604
80*a^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5
*c^13 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(
1/2) + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 543
75*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a
^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320
*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680
*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193
730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15
*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 -
 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869
760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c
^19)))^(1/4)*1i))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 1519210

```
4632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c^3 - 13422976
00*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*b^19*c^6 + 16
88816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14462970429440*a^9*
b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*b^9*c^11 + 20
6669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 + 1500091147878
40*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 23125*a*b^29*c
 + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c -
 b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^3
8*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876
096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255
569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c
^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 52022791
37280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*
b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 1
3056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.1085 $\int \frac{x^{5/2}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=594

$$\frac{3x^{3/2}\left(cx^2\left(12ac+b^2\right)+b\left(4ac+b^2\right)\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3\sqrt[4]{c}\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+12ac+b^2\right)\tan}{32\, 2^{3/4}a\left(b^2-4ac\right)^2\sqrt[4]{-\sqrt{b^2-}}}$$

```
[Out] -1/4*x^(3/2)*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/16*x^(3/2)*(b*(4*
a*c+b^2)+c*(12*a*c+b^2)*x^2)/a/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3/64*c^(1/4)*
arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b^2+12*a*c-b
^3/(-4*a*c+b^2)^(1/2)+68*a*b*c/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-4*a*c+b^2)^2
/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-3/64*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)
/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b^2+12*a*c-b^3/(-4*a*c+b^2)^(1/2)+68*a*b*c
/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-4*a*c+b^2)^2/(-b-(-4*a*c+b^2)^(1/2))^(1/4)
+3/64*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))
*(b^3-68*a*b*c+(12*a*c+b^2)*(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-4*a*c+b^2)^(5/2
)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-3/64*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2
)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b^3-68*a*b*c+(12*a*c+b^2)*(-4*a*c+b^2)^(1
/2))*2^(1/4)/a/(-4*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A] time = 2.31, antiderivative size = 594, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1115, 1364, 1500, 1510, 298, 205, 208}

$$\frac{3x^{3/2}\left(cx^2\left(12ac+b^2\right)+b\left(4ac+b^2\right)\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3\sqrt[4]{c}\left(-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{68abc}{\sqrt{b^2-4ac}}+12ac+b^2\right)\text{ta}}{32\, 2^{3/4}a\left(b^2-4ac\right)^2\sqrt[4]{-\sqrt{b^2-}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x^(3/2)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*x^(3/
2)*(b*(b^2 + 4*a*c) + c*(b^2 + 12*a*c)*x^2))/(16*a*(b^2 - 4*a*c)^2*(a + b*x
^2 + c*x^4)) + (3*c^(1/4)*(b^2 + 12*a*c - b^3/Sqrt[b^2 - 4*a*c] + (68*a*b*c
)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*
c])^(1/4)])/(32*2^(3/4)*a*(b^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) +
 (3*c^(1/4)*(b^3 - 68*a*b*c + Sqrt[b^2 - 4*a*c]*(b^2 + 12*a*c))*ArcTan[(2^(
1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a*(b^2 -
```

```
 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (3*c^(1/4)*(b^2 + 12*a*c -
b^3/Sqrt[b^2 - 4*a*c] + (68*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1
/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a*(b^2 - 4*a*c)^2
*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (3*c^(1/4)*(b^3 - 68*a*b*c + Sqrt[b^2 -
4*a*c]*(b^2 + 12*a*c))*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4
*a*c])^(1/4)])/(32*2^(3/4)*a*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(
1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1364

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(n - 1)*(d*x)^(m - n + 1)*(b + 2*c*x^n)*(a + b*x^n + c*
x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] - Dist[d^n/(n*(p + 1)*(b^2
- 4*a*c)), Int[(d*x)^(m - n)*(b*(m - n + 1) + 2*c*(m + 2*n*(p + 1) + 1)*x^n
)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] && GtQ[m, n -
1] && LeQ[m, 2*n - 1]
```

Rule 1500

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^n + c*x^(
```

```
2*n))^(p + 1)*(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n))/(a*f*n*(p +
1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a
+ b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m + 2
*n*(p + 1) + 1)) - a*b*e*(m + 1) + c*(m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2
 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2\left(3b-18cx^4\right)}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{3/2}\left(b\left(b^2+4ac\right)+c\left(b^2+12ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{3/2}\left(b\left(b^2+4ac\right)+c\left(b^2+12ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3c\left(b^2+1\ldots\right.\right.}{\ldots}$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{3/2}\left(b\left(b^2+4ac\right)+c\left(b^2+12ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(3\sqrt{c}\left(b^2+\ldots\right.\right.}{\ldots}$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{3/2}\left(b\left(b^2+4ac\right)+c\left(b^2+12ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3\sqrt[4]{c}\left(b^2+\ldots\right.}{3\ldots}$$

Mathematica [C] time = 0.41, size = 222, normalized size = 0.37

$$\frac{3\left(a+bx^2+cx^4\right)^2\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{12\#1^4ac^2\log\left(\sqrt{x}-\#1\right)+\#1^4b^2c\log\left(\sqrt{x}-\#1\right)-28abc\log\left(\sqrt{x}-\#1\right)+b^3\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\& \right.}{64a\left(b^2-4ac\right)^2\left(a+bx^2+cx\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (-16*a*(b^2 - 4*a*c)*x^(3/2)*(b + 2*c*x^2) + 12*x^(3/2)*(b^3 + 4*a*b*c + b^
2*c*x^2 + 12*a*c^2*x^2)*(a + b*x^2 + c*x^4) + 3*(a + b*x^2 + c*x^4)^2*RootS
um[a + b*#1^4 + c*#1^8 & , (b^3*Log[Sqrt[x] - #1] - 28*a*b*c*Log[Sqrt[x] -
#1] + b^2*c*Log[Sqrt[x] - #1]*#1^4 + 12*a*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1
 + 2*c*#1^5) & ])/(64*a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 191.33Unable to convert to
real 1/4 Error: Bad Argument Value
```

maple [C] time = 0.04, size = 277, normalized size = 0.47

$$\frac{3\left(\left(12ac+b^2\right)\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^6c+\left(-28ac+b^2\right)\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^2b\right)\ln\left(-\text{RootOf}\left(c\_Z\right.\right.}{64\left(16a^2c^2-8a\,b^2c+b^4\right)a\left(2\,\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7c+\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(1/32*b*(28*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(3/2)+1/32*(68*a^2*c^2+
7*a*b^2*c+3*b^4)/a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+3/16/a*c*b*(8*a*c+b^2
)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(11/2)+3/32*c^2*(12*a*c+b^2)/a/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^(15/2))/(c*x^4+b*x^2+a)^2+3/64/a/(16*a^2*c^2-8*a*b^2*c+b^4)
*sum((c*(12*a*c+b^2)*_R^6+b*(-28*a*c+b^2)*_R^2)/(2*_R^7*c+_R^3*b)*ln(-_R+x^
(1/2)),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\,(b^2c^2+12\,ac^3)x^{\frac{15}{2}}+6\,(b^3c+8\,abc^2)x^{\frac{11}{2}}+(3\,b^4+7\,ab^2c+68\,a^2c^2)x^{\frac{7}{2}}-(ab}{16\,((ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4)x^8+a^3b^4-8\,a^4b^2c+16\,a^5c^2+2\,(ab^5c-8\,a^2b^3c^2+16\,a^3bc^3)x^6+(ab^6-6\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(b^2*c^2 + 12*a*c^3)*x^(15/2) + 6*(b^3*c + 8*a*b*c^2)*x^(11/2) + (3
*b^4 + 7*a*b^2*c + 68*a^2*c^2)*x^(7/2) - (a*b^3 - 28*a^2*b*c)*x^(3/2))/((a*
b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*
c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c
 + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2) + integr
ate(3/32*((b^2*c + 12*a*c^2)*x^(5/2) + (b^3 - 28*a*b*c)*sqrt(x))/(a^2*b^4 -
 8*a^3*b^2*c + 16*a^4*c^2 + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*x^4 + (a
*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2), x)
```

mupad [B] time = 8.02, size = 42197, normalized size = 71.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((3*x^(11/2)*(b^3*c + 8*a*b*c^2))/(8*(a*b^4 + 16*a^3*c^2 - 8*a^2*b^2*c)) -
(x^(3/2)*(b^3 - 28*a*b*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^(7/2)*(
3*b^4 + 68*a^2*c^2 + 7*a*b^2*c))/(16*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3
*c^2*x^(15/2)*(12*a*c + b^2))/(16*(a*b^4 + 16*a^3*c^2 - 8*a^2*b^2*c)))/(x^4
*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - atan(((((27*(3799
912185593856*a^15*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 14019461
120*a^2*b^26*c^6 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8 -
 81637933056000*a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 356438262153
2160*a^7*b^16*c^11 + 13728399105196032*a^8*b^14*c^12 - 35694820362027008*a^
9*b^12*c^13 + 56529603635707904*a^10*b^10*c^14 - 33767651356442624*a^11*b^8
*c^15 - 51215251621806080*a^12*b^6*c^16 + 114542723335192576*a^13*b^4*c^17
```

- 70615034782285824*a^14*b^2*c^18))/(33554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) - (9*x^(1/2)*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*(5066549580791808*a^15*c^18 + 16777216*a*b^28*c^4 - 1677721600*a^2*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 1491964264448*a^4*b^22*c^7 + 20440823103488*a^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^7*b^16*c^10 - 5727081191178240*a^8*b^14*c^11 + 19380541706993664*a^9*b^12*c^12 - 47173446878101504*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 - 93414507895848960*a^12*b^6*c^15 + 67905838131445760*a^13*b^4*c^16 - 27584547717644288*a^14*b^2*c^17))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a

^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(3/4) + (9*x^(1/2)*(2982998016*a^6*b*c^14 - 173138472*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 166726460160*a^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^13))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*1i - (((27*(3799912185593856*a^15*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b^26*c^6 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8 - 81637933056000*a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 3564382621532160*a^7*b^16*c^11 + 13728399105196032*a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^13 + 56529603635707904*a^10*b^10*c^14 - 33767651356442624*a^11*b^8*c^15 - 51215251621806080*a^12*b^6*c^16 + 114542723335192576*a^13*b^4*c^17 - 70615034782285824*a^14*b^2*c^18))/(33554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) + (9*x^(1/2)*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^

6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^
20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b
^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^1
2*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 1937307
07456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^1
6*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 166
47293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*
a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)
))^(1/4)*(5066549580791808*a^15*c^18 + 16777216*a*b^28*c^4 - 1677721600*a^2
*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 1491964264448*a^4*b^22*c^7 + 2044082
3103488*a^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^7*
b^16*c^10 - 5727081191178240*a^8*b^14*c^11 + 19380541706993664*a^9*b^12*c^1
2 - 47173446878101504*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 - 93
414507895848960*a^12*b^6*c^15 + 67905838131445760*a^13*b^4*c^16 - 275845477
17644288*a^14*b^2*c^17))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b
^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 8110
08*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a
^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^1
3*b^2*c^11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^
16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4
 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c
^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^1
0*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7
562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^
15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*
a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^2
5)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 10
99511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*
c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c
^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14
*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 211
3425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a
^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^1
6 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 549755813
8880*a^24*b^2*c^19)))^(3/4) - (9*x^(1/2)*(2982998016*a^6*b*c^14 - 173138472
*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 166726460160*a^3
*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^13))/(4194304
*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080
*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^1
2*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c
^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*(
-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 -
394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 14243
68896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 1
08493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b

^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*1i)/((((27*(3799912185593856*a^15*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b^26*c^6 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8 - 81637933056000*a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 3564382621532160*a^7*b^16*c^11 + 13728399105196032*a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^13 + 56529603635707904*a^10*b^10*c^14 - 33767651356442624*a^11*b^8*c^15 - 51215251621806080*a^12*b^6*c^16 + 114542723335192576*a^13*b^4*c^17 - 70615034782285824*a^14*b^2*c^18))/(33554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) - (9*x^(1/2)*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*(5066549580791808*a^15*c^18 + 16777216*a*b^28*c^4 - 1677721600*a^2*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 1491964264448*a^4*b^22*c^7 + 20440823103488*a^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^7*b^16*c^10 - 5727081191178240*a^8*b^14*c^11 + 19380541706993664*a^9*b^12*c^12 - 47173446878101504*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 - 93414507895848960*a^12*b^6*c^15 +

67905838131445760*a^13*b^4*c^16 - 27584547717644288*a^14*b^2*c^17))/(419430
4*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 1408
0*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^
12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*
c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*
(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 -
394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424
368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 -
108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*
b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 821
2262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*
a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^
(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c
- b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b
^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 158
76096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 +
8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^
20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202
279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a
^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17
 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19))))^(3/4) + (9
x^(1/2)(2982998016*a^6*b*c^14 - 173138472*a*b^11*c^9 - 123201*b^13*c^8 +
10695194640*a^2*b^9*c^10 - 166726460160*a^3*b^7*c^11 + 147581948160*a^4*b^5
*c^12 + 44937566208*a^5*b^3*c^13))/(4194304*(a^2*b^24 + 16777216*a^14*c^12
- 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*
c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 +
32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50
331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 47110
4225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^
4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*
a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151
174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^
9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 421376
5570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31
*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*
c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5
*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960
*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a
^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 440297
06240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*
c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 104045
58274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^
21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 -
 5497558138880*a^24*b^2*c^19)))^(1/4) - (27*(2114129160*a*b^11*c^10 - 24024
195*b^13*c^9 + 1209323520*a^6*b*c^15 - 61748341200*a^2*b^9*c^11 + 590751532

800*a^3*b^7*c^12 + 227993875200*a^4*b^5*c^13 + 28822210560*a^5*b^3*c^14))/(
16777216*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^
2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300
288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812
288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 1
526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) + (((27*(37999121855938
56*a^15*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b^
26*c^6 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8 - 816379330
56000*a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 3564382621532160*a^7*b
^16*c^11 + 13728399105196032*a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^1
3 + 56529603635707904*a^10*b^10*c^14 - 33767651356442624*a^11*b^8*c^15 - 51
215251621806080*a^12*b^6*c^16 + 114542723335192576*a^13*b^4*c^17 - 70615034
782285824*a^14*b^2*c^18))/(33554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^
3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2
050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 19680
4608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1
526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^1
3)) + (9*x^(1/2)*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280
*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*
c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^1
9*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*
a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12
+ 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560
*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 40
09*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2
)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 +
1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^
34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^2
8*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a
^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 +
2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 1040455827456
0*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*
c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 549755
8138880*a^24*b^2*c^19)))^(1/4)*(5066549580791808*a^15*c^18 + 16777216*a*b^2
8*c^4 - 1677721600*a^2*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 1491964264448*
a^4*b^22*c^7 + 20440823103488*a^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 +
1225740716605440*a^7*b^16*c^10 - 5727081191178240*a^8*b^14*c^11 + 19380541
706993664*a^9*b^12*c^12 - 47173446878101504*a^10*b^10*c^13 + 80798711478747
136*a^11*b^8*c^14 - 93414507895848960*a^12*b^6*c^15 + 67905838131445760*a^1
3*b^4*c^16 - 27584547717644288*a^14*b^2*c^17))/(4194304*(a^2*b^24 + 1677721
6*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 1267
20*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9
*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b
^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(
1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3

+ 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(3/4) - (9*x^(1/2)*(2982998016*a^6*b*c^14 - 173138472*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 166726460160*a^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^13))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b

^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*2i - atan(((((27*(3799912185593856*a^15*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b^26*c^6 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8 - 81637933056000*a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 3564382621532160*a^7*b^16*c^11 + 13728399105196032*a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^13 + 56529603635707904*a^10*b^10*c^14 - 33767651356442624*a^11*b^8*c^15 - 51215251621806080*a^12*b^6*c^16 + 114542723335192576*a^13*b^4*c^17 - 70615034782285824*a^14*b^2*c^18))/(33554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) - (9*x^(1/2)*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*(5066549580791808*a^15*c^18 + 16777216*a*b^28*c^4 - 1677721600*a^2*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 1491964264448*a^4*b^22*c^7 + 20440823103488*a^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^7*b^16*c^10 - 5727081191178240*a^8*b^14*c^11 + 19380541706993664*a^9*b^12*c^12 - 47173446878101504*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 - 93414507895848960*a^12*b^6*c^15 + 67905838131445760*a^13*b^4*c^16 - 27584547717644288*a^14*b^2*c^17))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7

+ 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(3/4) + (9*x^(1/2)*(2982998016*a^6*b*c^14 - 173138472*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 166726460160*a^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^13))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*1i - (((27*(3799912185593856*a^15*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b^26*c^6 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8 - 81637933056000*a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 3564382621532160*a^7*b^16*c^11 + 13728399105196032*a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^13 + 56529603635707904*a^10*b^10*c^14 - 337

67651356442624*a^11*b^8*c^15 - 51215251621806080*a^12*b^6*c^16 + 1145427233
35192576*a^13*b^4*c^17 - 70615034782285824*a^14*b^2*c^18))/(33554432*(a^2*b
^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b
^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^
6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^11*b^10*c
^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726656*a^14*
b^4*c^12 - 939524096*a^15*b^2*c^13)) + (9*x^(1/2)*(-(81*(b^33 - b^8*(-(4*a*
c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248
*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*
a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 1084930
78528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^
11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682
624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b
^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) +
 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^
25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c +
 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a
^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569
920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10
 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 52022791372
80*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^1
0*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 1305
6700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*(5066549580
791808*a^15*c^18 + 16777216*a*b^28*c^4 - 1677721600*a^2*b^26*c^5 + 67947724
800*a^3*b^24*c^6 - 1491964264448*a^4*b^22*c^7 + 20440823103488*a^5*b^20*c^8
 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^7*b^16*c^10 - 57270811
91178240*a^8*b^14*c^11 + 19380541706993664*a^9*b^12*c^12 - 4717344687810150
4*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 - 93414507895848960*a^12
*b^6*c^15 + 67905838131445760*a^13*b^4*c^16 - 27584547717644288*a^14*b^2*c^
17))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^2
0*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 37
84704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 576716
80*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*
(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^
2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^2
3*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8
*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354
024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^
7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a
^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c
 - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6
c(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^2
0 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^
32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12
*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 19373070

7456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16
*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 1664
7293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a
^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19))
)^(3/4) - (9*x^(1/2)*(2982998016*a^6*b*c^14 - 173138472*a*b^11*c^9 - 123201
*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 166726460160*a^3*b^7*c^11 + 14758194
8160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^13))/(4194304*(a^2*b^24 + 1677721
6*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 1267
20*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9
*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b
^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(
1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3
+ 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 -
9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15
*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 384035821
9776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c
^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2)
- 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2
c^3(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(3
3554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36
*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 +
158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*
c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 70447529984
0*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c
^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809
116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^2
3*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*1i)/((((27*(3799912185593
856*a^15*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b
^26*c^6 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8 - 81637933
056000*a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 3564382621532160*a^7*
b^16*c^11 + 13728399105196032*a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^
13 + 56529603635707904*a^10*b^10*c^14 - 33767651356442624*a^11*b^8*c^15 - 5
1215251621806080*a^12*b^6*c^16 + 114542723335192576*a^13*b^4*c^17 - 7061503
4782285824*a^14*b^2*c^18))/(33554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a
^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 -
2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 1968
04608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 -
1526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^
13)) - (9*x^(1/2)*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 47110422528
0*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25
*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^
19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656
*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12
+ 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 421376557056
0*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4

009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*(5066549580791808*a^15*c^18 + 16777216*a*b^28*c^4 - 1677721600*a^2*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 1491964264448*a^4*b^22*c^7 + 20440823103488*a^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^7*b^16*c^10 - 5727081191178240*a^8*b^14*c^11 + 19380541706993664*a^9*b^12*c^12 - 47173446878101504*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 - 93414507895848960*a^12*b^6*c^15 + 67905838131445760*a^13*b^4*c^16 - 27584547717644288*a^14*b^2*c^17))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(3/4) + (9*x^(1/2)*(2982998016*a^6*b*c^14 - 173138472*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 166726460160*a^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^13))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376

799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^1
0 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 75625314385
92*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^1
5 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^
2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) +
107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 109951162777
6*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240
320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 12700
87680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9
+ 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520
*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c
^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 195850
50869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*
b^2*c^19)))^(1/4) - (27*(2114129160*a*b^11*c^10 - 24024195*b^13*c^9 + 12093
23520*a^6*b*c^15 - 61748341200*a^2*b^9*c^11 + 590751532800*a^3*b^7*c^12 + 2
27993875200*a^4*b^5*c^13 + 28822210560*a^5*b^3*c^14))/(16777216*(a^2*b^28 +
268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c
^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 5
6229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 +
1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c
^12 - 939524096*a^15*b^2*c^13)) + (((27*(3799912185593856*a^15*c^19 + 20971
52*b^30*c^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b^26*c^6 - 40259446374
4*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8 - 81637933056000*a^5*b^20*c^9 +
645335479222272*a^6*b^18*c^10 - 3564382621532160*a^7*b^16*c^11 + 137283991
05196032*a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^13 + 5652960363570790
4*a^10*b^10*c^14 - 33767651356442624*a^11*b^8*c^15 - 51215251621806080*a^12
*b^6*c^16 + 114542723335192576*a^13*b^4*c^17 - 70615034782285824*a^14*b^2*c
^18))/(33554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*
b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5
+ 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 -
524812288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c
^11 + 1526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) + (9*x^(1/2)*(-(
81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509
*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*
b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*
a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986
354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13
*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 129
6*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*
a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*
b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*
c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9
*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a
^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 19373
0707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b

^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 1
6647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 1958505086976
0*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^1
9)))^(1/4)*(5066549580791808*a^15*c^18 + 16777216*a*b^28*c^4 - 1677721600*a
^2*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 1491964264448*a^4*b^22*c^7 + 20440
823103488*a^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^
7*b^16*c^10 - 5727081191178240*a^8*b^14*c^11 + 19380541706993664*a^9*b^12*c
^12 - 47173446878101504*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 -
93414507895848960*a^12*b^6*c^15 + 67905838131445760*a^13*b^4*c^16 - 2758454
7717644288*a^14*b^2*c^17))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3
*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 81
1008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320
*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a
^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*
a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c
^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19
*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a
^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 +
7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*
a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 400
9*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)
^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 +
1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^3
4*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28
*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^
14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2
113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560
*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c
^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558
138880*a^24*b^2*c^19)))^(3/4) - (9*x^(1/2)*(2982998016*a^6*b*c^14 - 1731384
72*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 166726460160*a
^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^13))/(41943
04*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 140
80*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b
^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6
*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 - b^8
*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2
- 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 142
4368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 -
108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11
*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 82
12262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4
*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)
^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c
- b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*

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b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15
876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 +
 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b
^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 520
2279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*
a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^1
7 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)))*(
-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 105
09*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^
5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 4337679974
4*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 9
86354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^
13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1
296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(
4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*
a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^2
5*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a
^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680
*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193
730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17
*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 -
 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869
760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c
^19)))^(1/4)*2i - 2*atan(((((27*(3799912185593856*a^15*c^19 + 2097152*b^30*
c^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b^26*c^6 - 402594463744*a^3*b^
24*c^7 + 7074549334016*a^4*b^22*c^8 - 81637933056000*a^5*b^20*c^9 + 6453354
79222272*a^6*b^18*c^10 - 3564382621532160*a^7*b^16*c^11 + 13728399105196032
*a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^13 + 56529603635707904*a^10*b
^10*c^14 - 33767651356442624*a^11*b^8*c^15 - 51215251621806080*a^12*b^6*c^1
6 + 114542723335192576*a^13*b^4*c^17 - 70615034782285824*a^14*b^2*c^18))/(3
3554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2
 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 123002
88*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 5248122
88*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 15
26726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) - (x^(1/2)*(-(81*(b^33 +
 b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*
c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 +
 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c
^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*
a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13
- 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*
(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)
^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4
*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*
a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4
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- 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c
^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^
15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 -
5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239
296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6
*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)
*(5066549580791808*a^15*c^18 + 16777216*a*b^28*c^4 - 1677721600*a^2*b^26*c^
5 + 67947724800*a^3*b^24*c^6 - 1491964264448*a^4*b^22*c^7 + 20440823103488*
a^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^7*b^16*c^1
0 - 5727081191178240*a^8*b^14*c^11 + 19380541706993664*a^9*b^12*c^12 - 4717
3446878101504*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 - 9341450789
5848960*a^12*b^6*c^15 + 67905838131445760*a^13*b^4*c^16 - 27584547717644288
*a^14*b^2*c^17)*9i)/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c
+ 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^
7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b
^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2
c^11)))(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*
c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 14
0233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 +
43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^1
3*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 756253
1438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^
3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b
^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1
/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511
627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 +
1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 -
1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22
*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 21134258
99520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b
^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 1
9585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*
a^24*b^2*c^19)))^(3/4)*1i - (9*x^(1/2)*(2982998016*a^6*b*c^14 - 173138472*a
*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 166726460160*a^3*b
^7*c^11 + 147581948160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^13))/(4194304*(
a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a
^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*
c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9
+ 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*(-(
4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 39
4248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368
896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108
493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^1
1*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 821226
2682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c

- b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4) - (((27*(3799912185593856*a^15*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b^26*c^6 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8 - 81637933056000*a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 3564382621532160*a^7*b^16*c^11 + 13728399105196032*a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^13 + 56529603635707904*a^10*b^10*c^14 - 33767651356442624*a^11*b^8*c^15 - 51215251621806080*a^12*b^6*c^16 + 114542723335192576*a^13*b^4*c^17 - 70615034782285824*a^14*b^2*c^18))/(33554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) + (x^(1/2)*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*(5066549580791808*a^15*c^18 + 16777216*a*b^28*c^4 - 1677721600*a^2*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 1491964264448*a^4*b^22*c^7 + 20440823103488*a^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^7*b^16*c^10 - 5727081191178240*a^8*b^14*c^11 + 19380541706993664*a^9*b^12*c^12 - 47173446878101504*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 - 93414507895848960*a^12*b^6*c^15 + 67905838131445760*a^13*b^4*c^16 - 27584547717644288*a^14*b^2*c^17)*9i)/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^

5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(3/4)*1i + (9*x^(1/2)*(2982998016*a^6*b*c^14 - 173138472*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 166726460160*a^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^13))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4))/((27*(2114129160*a*b^11*c^10 - 24024195*b^13*c^9 + 1209323520*a^6*b*c^15 - 61748341200*a^2*b^9*c^11 + 590751532800*a^3*b^7*c^12 + 227993875200*a^4*b^5*c^13 + 28822210560*a^5*b^3*c^14))/(16777216*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2

- 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 123002
88*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 5248122
88*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 15
26726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) + (((27*(379991218559385
6*a^15*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b^2
6*c^6 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8 - 8163793305
6000*a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 3564382621532160*a^7*b^
16*c^11 + 13728399105196032*a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^13
 + 56529603635707904*a^10*b^10*c^14 - 33767651356442624*a^11*b^8*c^15 - 512
15251621806080*a^12*b^6*c^16 + 114542723335192576*a^13*b^4*c^17 - 706150347
82285824*a^14*b^2*c^18))/(33554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3
*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 20
50048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804
608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 15
26726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13
)) - (x^(1/2)*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^
16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4
 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c
^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^1
0*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7
562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^
15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*
a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^2
5)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 10
99511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*
c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c
^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14
*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 211
3425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a
^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^1
6 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 549755813
8880*a^24*b^2*c^19)))^(1/4)*(5066549580791808*a^15*c^18 + 16777216*a*b^28*c
^4 - 1677721600*a^2*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 1491964264448*a^4
*b^22*c^7 + 20440823103488*a^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 + 12
25740716605440*a^7*b^16*c^10 - 5727081191178240*a^8*b^14*c^11 + 19380541706
993664*a^9*b^12*c^12 - 47173446878101504*a^10*b^10*c^13 + 80798711478747136
*a^11*b^8*c^14 - 93414507895848960*a^12*b^6*c^15 + 67905838131445760*a^13*b
^4*c^16 - 27584547717644288*a^14*b^2*c^17)*9i)/(4194304*(a^2*b^24 + 1677721
6*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 1267
20*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9
*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b
^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(
1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3
+ 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 -
 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15

*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 384035821
9776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c
^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2)
- 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2
c^3(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(3
3554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36
*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 +
158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*
c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 70447529984
0*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c
^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809
116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^2
3*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(3/4)*1i - (9*x^(1/2)*(29829980
16*a^6*b*c^14 - 173138472*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^
9*c^10 - 166726460160*a^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 4493756620
8*a^5*b^3*c^13))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c +
1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b
^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*
c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^
11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^1
6 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 14023
3728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 433
76799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c
^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 756253143
8592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c
^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*
c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2)
- 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627
776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 12
40320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 127
0087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^
9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 21134258995
20*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12
*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 1958
5050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^2
4*b^2*c^19)))^(1/4)*1i + (((27*(3799912185593856*a^15*c^19 + 2097152*b^30*c
^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b^26*c^6 - 402594463744*a^3*b^2
4*c^7 + 7074549334016*a^4*b^22*c^8 - 81637933056000*a^5*b^20*c^9 + 64533547
9222272*a^6*b^18*c^10 - 3564382621532160*a^7*b^16*c^11 + 13728399105196032*
a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^13 + 56529603635707904*a^10*b^
10*c^14 - 33767651356442624*a^11*b^8*c^15 - 51215251621806080*a^12*b^6*c^16
+ 114542723335192576*a^13*b^4*c^17 - 70615034782285824*a^14*b^2*c^18))/(33
554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2
- 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 1230028
8*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 52481228

8*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 152
6726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) + (x^(1/2)*(-(81*(b^33 +
b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c
^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 +
1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^
8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a
^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 -
8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(
-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^
25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*
a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a
^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 -
15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^
7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^1
5*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 -
5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 166472932392
96*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*
c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*
(5066549580791808*a^15*c^18 + 16777216*a*b^28*c^4 - 1677721600*a^2*b^26*c^5
+ 67947724800*a^3*b^24*c^6 - 1491964264448*a^4*b^22*c^7 + 20440823103488*a
^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^7*b^16*c^10
- 5727081191178240*a^8*b^14*c^11 + 19380541706993664*a^9*b^12*c^12 - 47173
446878101504*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 - 93414507895
848960*a^12*b^6*c^15 + 67905838131445760*a^13*b^4*c^16 - 27584547717644288*
a^14*b^2*c^17)*9i)/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c
+ 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7
*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^
8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*
c^11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c
^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140
233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 4
3376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13
*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531
438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3
*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^
4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/
2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 10995116
27776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 +
1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1
270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*
c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 211342589
9520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^
12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19
585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a
^24*b^2*c^19)))^(3/4)*1i + (9*x^(1/2)*(2982998016*a^6*b*c^14 - 173138472*a*

b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 166726460160*a^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^13))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*1i))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4) - 2*atan(((((27*(3799912185593856*a^15*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b^26*c^6 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8 - 81637933056000*a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 3564382621532160*a^7*b^16*c^11 + 13728399105196032*a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^13 + 56529603635707904*a^10*b^10*c^14 - 33767651356442624*a^11*b^8*c^15 - 51215251621806080*a^12*b^6*c^16 + 114542723335192576*a^13*b^4*c^17 - 70615034782285824*a^14*b^2*c^18))/(335544

32*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23
296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^
8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^
11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726
656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) - (x^(1/2)*(-(81*(b^33 - b^8*
(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 -
394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424
368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 -
108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*
b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 821
2262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*
a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^
(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c
- b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b
^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 158
76096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 +
8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^
20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202
279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a
^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17
+ 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*(506
6549580791808*a^15*c^18 + 16777216*a*b^28*c^4 - 1677721600*a^2*b^26*c^5 + 6
7947724800*a^3*b^24*c^6 - 1491964264448*a^4*b^22*c^7 + 20440823103488*a^5*b
^20*c^8 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^7*b^16*c^10 - 5
727081191178240*a^8*b^14*c^11 + 19380541706993664*a^9*b^12*c^12 - 471734468
78101504*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 - 934145078958489
60*a^12*b^6*c^15 + 67905838131445760*a^13*b^4*c^16 - 27584547717644288*a^14
*b^2*c^17)*9i)/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 10
56*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^1
4*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^
8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11
)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16
+ 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 1402337
28*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376
799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^1
0 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 75625314385
92*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^1
5 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^
2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) +
107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 109951162777
6*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240
320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 12700
87680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9
+ 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520
*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c

^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 195850
50869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*
b^2*c^19)))^(3/4)*1i - (9*x^(1/2)*(2982998016*a^6*b*c^14 - 173138472*a*b^11
*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 166726460160*a^3*b^7*c^
11 + 147581948160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^13))/(4194304*(a^2*b
^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^
18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 -
 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69
206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c
 - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*
a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a
^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 10849307
8528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^1
1 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 82122626826
24*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^
2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) +
54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^2
5)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c +
3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^
10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 82555699
20*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10
- 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 520227913728
0*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10
*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056
700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4) - (((27*(379
9912185593856*a^15*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 1401946
1120*a^2*b^26*c^6 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8
- 81637933056000*a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 35643826215
32160*a^7*b^16*c^11 + 13728399105196032*a^8*b^14*c^12 - 35694820362027008*a
^9*b^12*c^13 + 56529603635707904*a^10*b^10*c^14 - 33767651356442624*a^11*b^
8*c^15 - 51215251621806080*a^12*b^6*c^16 + 114542723335192576*a^13*b^4*c^17
 - 70615034782285824*a^14*b^2*c^18))/(33554432*(a^2*b^28 + 268435456*a^16*c
^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b
^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*
c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a^12*b
^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524096*a
^15*b^2*c^13)) + (x^(1/2)*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471
104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*
a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 973205299
2*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 131
51174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*
b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213
765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^
31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*
a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a

^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 729
60*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960
*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 4402
9706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^1
8*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 1040
4558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*
a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18
 - 5497558138880*a^24*b^2*c^19)))^(1/4)*(5066549580791808*a^15*c^18 + 16777
216*a*b^28*c^4 - 1677721600*a^2*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 14919
64264448*a^4*b^22*c^7 + 20440823103488*a^5*b^20*c^8 - 188712273051648*a^6*b
^18*c^9 + 1225740716605440*a^7*b^16*c^10 - 5727081191178240*a^8*b^14*c^11 +
 19380541706993664*a^9*b^12*c^12 - 47173446878101504*a^10*b^10*c^13 + 80798
711478747136*a^11*b^8*c^14 - 93414507895848960*a^12*b^6*c^15 + 679058381314
45760*a^13*b^4*c^16 - 27584547717644288*a^14*b^2*c^17)*9i)/(4194304*(a^2*b^
24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^1
8*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 -
12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 692
06016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c
 - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a
^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^
6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078
528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11
 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 821226268262
4*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2
)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 5
4648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25
)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3
040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^1
0*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 825556992
0*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 -
 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280
*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*
c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 130567
00579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(3/4)*1i + (9*x^(1/
2)*(2982998016*a^6*b*c^14 - 173138472*a*b^11*c^9 - 123201*b^13*c^8 + 106951
94640*a^2*b^9*c^10 - 166726460160*a^3*b^7*c^11 + 147581948160*a^4*b^5*c^12
+ 44937566208*a^5*b^3*c^13))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a
^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 -
811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 324403
20*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648
*a^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 47110422528
0*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25
*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^
19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656
*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12

+ 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 421376557056
0*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4
009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^
2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40
+ 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b
^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^
28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*
a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 +
2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 104045582745
60*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8
*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 54975
58138880*a^24*b^2*c^19)))^(1/4))/((27*(2114129160*a*b^11*c^10 - 24024195*b^
13*c^9 + 1209323520*a^6*b*c^15 - 61748341200*a^2*b^9*c^11 + 590751532800*a^
3*b^7*c^12 + 227993875200*a^4*b^5*c^13 + 28822210560*a^5*b^3*c^14))/(167772
16*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23
296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^
8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^
11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726
656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) + (((27*(3799912185593856*a^1
5*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b^26*c^6
 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8 - 81637933056000*
a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 3564382621532160*a^7*b^16*c^
11 + 13728399105196032*a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^13 + 56
529603635707904*a^10*b^10*c^14 - 33767651356442624*a^11*b^8*c^15 - 51215251
621806080*a^12*b^6*c^16 + 114542723335192576*a^13*b^4*c^17 - 70615034782285
824*a^14*b^2*c^18))/(33554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26
*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048
*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a
^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726
656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) -
(x^(1/2)*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*
c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 14
0233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 +
43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^1
3*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 756253
1438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^
3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b
^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1
/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511
627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 +
1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 -
1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22
*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 21134258
99520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b
^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 1

9585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*(5066549580791808*a^15*c^18 + 16777216*a*b^28*c^4 - 1677721600*a^2*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 1491964264448*a^4*b^22*c^7 + 20440823103488*a^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^7*b^16*c^10 - 5727081191178240*a^8*b^14*c^11 + 19380541706993664*a^9*b^12*c^12 - 47173446878101504*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 - 93414507895848960*a^12*b^6*c^15 + 67905838131445760*a^13*b^4*c^16 - 27584547717644288*a^14*b^2*c^17)*9i)/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(3/4)*1i - (9*x^(1/2)*(2982998016*a^6*b*c^14 - 173138472*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 166726460160*a^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^13))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 12700876

80*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*1i + (((27*(3799912185593856*a^15*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 14019461120*a^2*b^26*c^6 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8 - 81637933056000*a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 3564382621532160*a^7*b^16*c^11 + 13728399105196032*a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^13 + 56529603635707904*a^10*b^10*c^14 - 33767651356442624*a^11*b^8*c^15 - 51215251621806080*a^12*b^6*c^16 + 114542723335192576*a^13*b^4*c^17 - 70615034782285824*a^14*b^2*c^18))/(33554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) + (x^(1/2)*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*(5066549580791808*a^15*c^18 + 16777216*a*b^28*c^4 - 1677721600*a^2*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 1491964264448*a^4*b^22*c^7 + 20440823103488*a^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^7*b^16*c^10 - 5727081191178240*a^8*b^14*c^11 + 19380541706993664*a^9*b^12*c^12 - 47173446878101504*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 - 93414507895848960*a^12*b^6*c^15 + 67905838131445760*a^13*b^4*c^16 - 27584547717644288*a^14*b^2*c^17)*9i)/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 14023372

8*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 433767
99744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10
 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 756253143859
2*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15
 - 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2
*(-(4*a*c - b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) +
107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776
*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 12403
20*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 127008
7680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 +
 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*
a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^
14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 1958505
0869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b
^2*c^19)))^(3/4)*1i + (9*x^(1/2)*(2982998016*a^6*b*c^14 - 173138472*a*b^11*
c^9 - 123201*b^13*c^8 + 10695194640*a^2*b^9*c^10 - 166726460160*a^3*b^7*c^1
1 + 147581948160*a^4*b^5*c^12 + 44937566208*a^5*b^3*c^13))/(4194304*(a^2*b^
24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^1
8*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 -
12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 692
06016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 - b^8*(-(4*a*c
 - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a
^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^
6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078
528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11
 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 821226268262
4*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4*c^4*(-(4*a*c - b^2
)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) + 5
4648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c*(-(4*a*c - b^2)^25
)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3
040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^1
0*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 825556992
0*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 -
 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280
*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*
c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 130567
00579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*1i))*(-(81*(b
^33 - b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*
b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*
c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b
^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 98635402
4448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*
c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 - 1296*a^4
c^4(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c - 4009*a^2*b^4*c^2*(-(4*a*c -
 b^2)^25)^(1/2) + 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) + 107*a*b^6*c

```
*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20
- 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32
*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b
^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 1937307074
56*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c
^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 166472
93239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^2
2*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^
(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.1086 $\int \frac{x^{3/2}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=594

$$-\frac{\sqrt{x}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\sqrt{x}\left(cx^2\left(44ac+b^2\right)+b\left(20ac+b^2\right)\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3c^{3/4}\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+44ac+b^2\right)\tan\ldots}{32\sqrt[4]{2}\,a\left(b^2-4ac\right)^2\left(-\sqrt{b^2-4ac}\ldots\right.}$$

```
[Out] -3/64*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))
*(b^2+44*a*c-b^3/(-4*a*c+b^2)^(1/2)+68*a*b*c/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/
(-4*a*c+b^2)^2/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-3/64*c^(3/4)*arctanh(2^(1/4)*c
^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b^2+44*a*c-b^3/(-4*a*c+b^2)^
(1/2)+68*a*b*c/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-4*a*c+b^2)^2/(-b-(-4*a*c+b^2
)^(1/2))^(3/4)-3/64*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)
^(1/2))^(1/4))*(b^3-68*a*b*c+(44*a*c+b^2)*(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-4
*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-3/64*c^(3/4)*arctanh(2^(1/4)*
c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b^3-68*a*b*c+(44*a*c+b^2)*(
-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-4*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/
4)-1/4*(2*c*x^2+b)*x^(1/2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/16*(b*(20*a*c+b
^2)+c*(44*a*c+b^2)*x^2)*x^(1/2)/a/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)
```

Rubi [A] time = 2.37, antiderivative size = 594, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1115, 1364, 1430, 1422, 212, 208, 205}

$$-\frac{3c^{3/4}\left(-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{68abc}{\sqrt{b^2-4ac}}+44ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,a\left(b^2-4ac\right)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3c^{3/4}\left(\sqrt{b^2-4ac}\left(44ac+b^2\right)-68abc+b^3\right)\tan^{-1}\ldots}{32\sqrt[4]{2}\,a\left(b^2-4ac\right)^{5/2}\left(\sqrt{b^2-4ac}-b\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(Sqrt[x]*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[x]
*(b*(b^2 + 20*a*c) + c*(b^2 + 44*a*c)*x^2))/(16*a*(b^2 - 4*a*c)^2*(a + b*x^
2 + c*x^4)) - (3*c^(3/4)*(b^2 + 44*a*c - b^3/Sqrt[b^2 - 4*a*c] + (68*a*b*c)
/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c
])^(1/4)])/(32*2^(1/4)*a*(b^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) -
(3*c^(3/4)*(b^3 - 68*a*b*c + Sqrt[b^2 - 4*a*c]*(b^2 + 44*a*c))*ArcTan[(2^(1
/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a*(b^2 -
```

```
4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(b^2 + 44*a*c - b
^3/Sqrt[b^2 - 4*a*c] + (68*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/
4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a*(b^2 - 4*a*c)^2*
(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(b^3 - 68*a*b*c + Sqrt[b^2 - 4
*a*c]*(b^2 + 44*a*c))*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*
a*c])^(1/4)])/(32*2^(1/4)*a*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3
/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1364

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(n - 1)*(d*x)^(m - n + 1)*(b + 2*c*x^n)*(a + b*x^n + c*
x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] - Dist[d^n/(n*(p + 1)*(b^2
- 4*a*c)), Int[(d*x)^(m - n)*(b*(m - n + 1) + 2*c*(m + 2*n*(p + 1) + 1)*x^n
)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] && GtQ[m, n -
1] && LeQ[m, 2*n - 1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
```

```
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1430

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> -Simp[(x*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*(
a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a
*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d*(
2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n + c
*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\sqrt{x}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\text{Subst}\left(\int \frac{b-22cx^4}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}$$

$$= -\frac{\sqrt{x}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\sqrt{x}\left(b\left(b^2+20ac\right)+c\left(b^2+44ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\text{Subst}\left(\int \frac{-3b\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{\sqrt{x}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\sqrt{x}\left(b\left(b^2+20ac\right)+c\left(b^2+44ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\left(3c\left(b^2+44\cdots\right.\right.}{\cdots}$$

$$= -\frac{\sqrt{x}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\sqrt{x}\left(b\left(b^2+20ac\right)+c\left(b^2+44ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\left(3c\left(b^2+44\cdots\right.\right.}{\cdots}$$

$$= -\frac{\sqrt{x}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\sqrt{x}\left(b\left(b^2+20ac\right)+c\left(b^2+44ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3c^{3/4}\left(b^2+4\cdots\right.}{32\cdots}$$

Mathematica [C] time = 0.41, size = 224, normalized size = 0.38

$$\frac{3\left(a+bx^2+cx^4\right)^2 \text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{44\#1^4ac^2\log\left(\sqrt{x}-\#1\right)+\#1^4b^2c\log\left(\sqrt{x}-\#1\right)-12abc\log\left(\sqrt{x}-\#1\right)+b^3\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\right.}{64a\left(b^2-4ac\right)^2\left(a+bx^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (-16*a*(b^2 - 4*a*c)*Sqrt[x]*(b + 2*c*x^2) + 4*Sqrt[x]*(b^3 + 20*a*b*c + b^
2*c*x^2 + 44*a*c^2*x^2)*(a + b*x^2 + c*x^4) + 3*(a + b*x^2 + c*x^4)^2*RootS
um[a + b*#1^4 + c*#1^8 & , (b^3*Log[Sqrt[x] - #1] - 12*a*b*c*Log[Sqrt[x] -
#1] + b^2*c*Log[Sqrt[x] - #1]*#1^4 + 44*a*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1
^3 + 2*c*#1^7) & ])/(64*a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 191.63Unable to convert to
real 1/4 Error: Bad Argument Value
```

maple [C] time = 0.04, size = 270, normalized size = 0.45

$$\frac{3\left(\left(44ac+b^2\right)\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^4c-12abc+b^3\right)\ln\left(-\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)+\sqrt{x}\right)}{64\left(16a^2c^2-8a\,b^2c+b^4\right)a\left(2\,\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7c+\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^3b\right)}+\frac{\left(44ac+\right.}{16\left(16a^2c^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(3/32*b*(12*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2)+1/32*(76*a^2*c^2+
13*a*b^2*c+b^4)/a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)+1/16/a*c*b*(32*a*c+b^2
)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(9/2)+1/32*c^2*(44*a*c+b^2)/a/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^(13/2))/(c*x^4+b*x^2+a)^2+3/64/a/(16*a^2*c^2-8*a*b^2*c+b^4)*
sum((c*(44*a*c+b^2)*_R^4-12*a*b*c+b^3)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R
=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\left(b^3c^2-12\,abc^3\right)x^{\frac{17}{2}}+\left(6\,b^4c-71\,ab^2c^2+44\,a^2c^3\right)x^{\frac{13}{2}}+\left(3\,b^5-28\,ab^3c-8\,a^2bc^2\right)x^{\frac{9}{2}}+\left(7\right.}{16\left(\left(a^2b^4c^2-8\,a^3b^2c^3+16\,a^4c^4\right)x^8+a^4b^4-8\,a^5b^2c+16\,a^6c^2+2\left(a^2b^5c-8\,a^3b^3c^2+16\,a^4bc^3\right)x^6+\left(a^2b^6-6\,a^3b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(b^3*c^2 - 12*a*b*c^3)*x^(17/2) + (6*b^4*c - 71*a*b^2*c^2 + 44*a^2*
c^3)*x^(13/2) + (3*b^5 - 28*a*b^3*c - 8*a^2*b*c^2)*x^(9/2) + (7*a*b^4 - 59*
a^2*b^2*c + 76*a^3*c^2)*x^(5/2))/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4
)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 +
 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5
- 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2) + integrate(-3/32*((b^3*c - 12*a*b*c^2)*
x^(7/2) + (b^4 - 13*a*b^2*c - 44*a^2*c^2)*x^(3/2))/(a^3*b^4 - 8*a^4*b^2*c +
 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a
^3*b^3*c + 16*a^4*b*c^2)*x^2), x)
```

mupad [**B**] time = 9.35, size = 54027, normalized size = 90.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] atan(((((3*(230850*a*b^11*c^8 - 4455*b^13*c^7 + 24287662080*a^6*b*c^13 - 36
79344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 - 548653824*a^4*b^5*c^11 + 9760227
840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576
*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5
 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + ((3*
(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 +
 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a
^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^
8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626
713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b
```

^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 299 19144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a *b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4 *a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a* b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27* c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^ 11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680 *a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193 730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19 *b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869 760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c ^19)))^(1/4)*(774056185954304*a^16*c^16 - 16777216*a^4*b^24*c^4 + 889192448 *a^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 + 256355860480*a^7*b^18*c^7 - 2045 478174720*a^8*b^16*c^8 + 10385230921728*a^9*b^14*c^9 - 31026843746304*a^10* b^12*c^10 + 30099130810368*a^11*b^10*c^11 + 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*b^6*c^13 + 1587694790508544*a^14*b^4*c^14 - 170644204 6308352*a^15*b^2*c^15))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^ 8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) - (9*x^(1/2)*(3096224743817216*a^16*b*c^18 - 16777216*a^2*b^29*c^4 + 1157627 904*a^3*b^27*c^5 - 34175188992*a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 5 968393928704*a^6*b^21*c^8 + 40450001993728*a^7*b^19*c^9 - 171227461189632*a ^8*b^17*c^10 + 350881648214016*a^9*b^15*c^11 + 523642412728320*a^10*b^13*c^ 12 - 6226534348095488*a^11*b^11*c^13 + 21186489555615744*a^12*b^9*c^14 - 39 951854506868736*a^13*b^7*c^15 + 42889749576286208*a^14*b^5*c^16 - 225179981 36852480*a^15*b^3*c^17))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b ^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 8110 08*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320 *a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a ^15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 125050657177 60*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27* c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^ 7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^ 15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291 284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^1 5*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^ 25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015* a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^2 5)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 109 9511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34* c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28* c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^1 6*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 21 13425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560* a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(3/4))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4) + (9*x^(1/2)*(245025*b^14*c^9 - 1175522844672*a^7*c^16 - 13142250*a*b^12*c^10 + 966155040*a^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 + 112005110016*a^4*b^6*c^13 + 617614170624*a^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*1i - (((3*(230850*a*b^11*c^8 - 4455*b^13*c^7 + 24287662080*a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^

10 - 548653824*a^4*b^5*c^11 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + ((3*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(774056185954304*a^16*c^16 - 16777216*a^4*b^24*c^4 + 889192448*a^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 + 256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8 + 10385230921728*a^9*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 30099130810368*a^11*b^10*c^11 + 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*b^6*c^13 + 1587694790508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + (9*x^(1/2)*(3096224743817216*a^16*b*c^18 - 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 34175188992*a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21*c^8 + 40450001993728*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 350881648214016*a^9*b^15*c^11 + 523642412728320*a^10*b^13*c^12 - 6226534348095488*a^11*b^11*c^13 + 21186489555615744*a^12*b^9*c^14 - 39951854506868736*a^13*b^7*c^15 + 42889749576286208*a^14*b^5*c^16 - 22517998136852480*a^15*b^3*c^17))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c

^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*
c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2)
- 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^
25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c +
 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096
*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 82555
69920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^
10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 520227913
7280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b
^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13
056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(3/4))*(-(81*(b
^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^
2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*
c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c
^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a
^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13
- 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837
120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c
+ 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b
^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-
(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 8
0*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*
c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^
26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 19373070745
6*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^
12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 1664729
3239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24
*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(
1/4) - (9*x^(1/2)*(245025*b^14*c^9 - 1175522844672*a^7*c^16 - 13142250*a*b^
12*c^10 + 966155040*a^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 + 112005110016
*a^4*b^6*c^13 + 617614170624*a^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/(419
4304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 1
4080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^1
0*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13
*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 +
 b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^3
1*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 +
 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 +
10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b
^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 201
14959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a
^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510
*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^2
5)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*
c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8

*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*1i)/((((3*(230850*a*b^11*c^8 - 4455*b^13*c^7 + 24287662080*a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 - 548653824*a^4*b^5*c^11 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + ((3*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(774056185954304*a^16*c^16 - 16777216*a^4*b^24*c^4 + 889192448*a^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 + 256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8 + 10385230921728*a^9*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 30099130810368*a^11*b^10*c^11 + 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*b^6*c^13 + 1587694790508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) - (9*x^(1/2)*(3096224743817216*a^16*b*c^18 - 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 34175188992*a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21*c^8 + 40450001993728*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 350881648214016*a^9*b^15*c^11 + 523642412728320*a^10*b^13*c^12 - 6226534348095488*a^11*b^11*c^13 + 21186489555615744*a^12*b^9*c^14 - 39951854506868736*a^13*b^7*c^15 + 42889749576286208*a^14*b^5*c^16 - 22517998136852480*a^15*b^3*c^17))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 81100

8*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(3/4))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4) + (9*x^(1/2)*(245025*b^14*c^9 - 1175522844672*a^7*c^16 - 13142250*a*b^12*c^10 + 966155040*a^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 + 112005110016*a^4*b^6*c^13 + 617614170624*a^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a

^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7
*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560
*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^1
2 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 3197447123
7632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c
 - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2)
 + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c
 - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^
40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^
10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^
13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 4402970
6240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c
^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 1040455
8274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^2
3*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 -
5497558138880*a^26*b^2*c^19)))^(1/4) + (((3*(230850*a*b^11*c^8 - 4455*b^13*
c^7 + 24287662080*a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 -
 548653824*a^4*b^5*c^11 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 2621
44*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*
a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c
^7 + 589824*a^12*b^2*c^8)) + ((3*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/
2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 +
1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 1811
90400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83
490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12
*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 3
1974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5
*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^
25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*
(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(3355443
2*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 -
 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158
760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8
- 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^
18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13
+ 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 208091165
49120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^
4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(774056185954304*a^16*c^16 -
16777216*a^4*b^24*c^4 + 889192448*a^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 +
 256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8 + 10385230921728*a^
9*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 30099130810368*a^11*b^10*c^11
+ 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*b^6*c^13 + 158769479
0508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15))/(65536*(a^4*b^18 -
 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 3
2256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*

b^4*c^7 + 589824*a^12*b^2*c^8)) + (9*x^(1/2)*(3096224743817216*a^16*b*c^18
- 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 34175188992*a^4*b^25*c^
6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21*c^8 + 40450001993728
*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 350881648214016*a^9*b^15*c^
11 + 523642412728320*a^10*b^13*c^12 - 6226534348095488*a^11*b^11*c^13 + 211
86489555615744*a^12*b^9*c^14 - 39951854506868736*a^13*b^7*c^15 + 4288974957
6286208*a^14*b^5*c^16 - 22517998136852480*a^15*b^3*c^17))/(4194304*(a^4*b^2
4 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18
*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 -
12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69
206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(4*a*
c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335
*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6
*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a
^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 23
79389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a
^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16
- 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*
(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33
880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^
(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 304
0*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^1
2*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 825556992
0*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 -
704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280
*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*
c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 130567
00579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(3/4))*(-(81*(b^35
+ b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^
31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5
+ 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 +
10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*
b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20
114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*
a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 51
0*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^
25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a
c - b^2)^25)^(1/2)))/(33554432(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^
8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4
- 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c
^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^
17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 -
5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239
296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6
*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)

- (9*x^(1/2)*(245025*b^14*c^9 - 1175522844672*a^7*c^16 - 13142250*a*b^12*c
^10 + 966155040*a^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 + 112005110016*a^4
*b^6*c^13 + 617614170624*a^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/(4194304
*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080
*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^
12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6
*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 + b^1
0*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^
2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 703
16800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 1091
2870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*
c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 2011495
9237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*
b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2
*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(
1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c -
b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^3
8*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 158
76096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 +
8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^
20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202
279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a
^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17
+ 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)))*(-
(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3
910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5
*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*
b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 50262671
3600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9
*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919
144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b
^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a
*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^
8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^
20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11
*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a
^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 19373
0707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b
^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 1
6647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 1958505086976
0*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^1
9)))^(1/4)*2i + atan(((((3*(230850*a*b^11*c^8 - 4455*b^13*c^7 + 24287662080
*a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 - 548653824*a^4*b^
5*c^11 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*
a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 12

9024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*
b^2*c^8)) + ((3*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 125050657177
60*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*
c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^
7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^
15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291
284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^1
5*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^
25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*
a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^2
5)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 109
9511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*
c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*
c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^1
6*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 21
13425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*
a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^
16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 54975581
38880*a^26*b^2*c^19)))^(1/4)*(774056185954304*a^16*c^16 - 16777216*a^4*b^24
*c^4 + 889192448*a^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 + 256355860480*a^7
*b^18*c^7 - 2045478174720*a^8*b^16*c^8 + 10385230921728*a^9*b^14*c^9 - 3102
6843746304*a^10*b^12*c^10 + 30099130810368*a^11*b^10*c^11 + 156680406958080
*a^12*b^8*c^12 - 764160581304320*a^13*b^6*c^13 + 1587694790508544*a^14*b^4*
c^14 - 1706442046308352*a^15*b^2*c^15))/(65536*(a^4*b^18 - 262144*a^13*c^9
- 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4
- 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*
a^12*b^2*c^8)) - (9*x^(1/2)*(3096224743817216*a^16*b*c^18 - 16777216*a^2*b^
29*c^4 + 1157627904*a^3*b^27*c^5 - 34175188992*a^4*b^25*c^6 + 570425344000*
a^5*b^23*c^7 - 5968393928704*a^6*b^21*c^8 + 40450001993728*a^7*b^19*c^9 - 1
71227461189632*a^8*b^17*c^10 + 350881648214016*a^9*b^15*c^11 + 523642412728
320*a^10*b^13*c^12 - 6226534348095488*a^11*b^11*c^13 + 21186489555615744*a^
12*b^9*c^14 - 39951854506868736*a^13*b^7*c^15 + 42889749576286208*a^14*b^5*
c^16 - 22517998136852480*a^15*b^3*c^17))/(4194304*(a^4*b^24 + 16777216*a^16
*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8
*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^1
0*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c
^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2
) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1
329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 18119
0400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 834
90242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*
b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31
974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*
(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^2
5)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(

-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432
*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 -
72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 1587
60960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 -
44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^1
8*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 +
10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 2080911654
9120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4
*c^18 - 5497558138880*a^26*b^2*c^19)))^(3/4))*(-(81*(b^35 - b^10*(-(4*a*c -
b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^
3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^
23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*
b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 23793
89337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14
*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 2
34256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(
4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880
*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/
2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a
^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b
^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a
^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 70
4475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^
20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^1
5 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 130567005
79840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4) + (9*x^(1/2)*(24
5025*b^14*c^9 - 1175522844672*a^7*c^16 - 13142250*a*b^12*c^10 + 966155040*a
^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 + 112005110016*a^4*b^6*c^13 + 61761
4170624*a^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/(4194304*(a^4*b^24 + 1677
7216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 1
26720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128
*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a
^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)
^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^2
9*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^
6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*
c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337
600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*
c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256
*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c
- b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*
b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/
(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^
36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c
^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b

^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 7044752
99840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^
14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 2
0809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840
*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*1i - (((3*(230850*a*b
^11*c^8 - 4455*b^13*c^7 + 24287662080*a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 83
09952*a^3*b^7*c^10 - 548653824*a^4*b^5*c^11 + 9760227840*a^5*b^3*c^12))/(65
536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a
^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6
 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + ((3*(-(81*(b^35 - b^10*(-(
4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 9
1335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800
*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 109128704
00*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11
 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 201149592371
20*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c
^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*
c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2)
 + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^
25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c +
 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096
*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 82555
69920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^
10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 520227913
7280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b
^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13
056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(77405618
5954304*a^16*c^16 - 16777216*a^4*b^24*c^4 + 889192448*a^5*b^22*c^5 - 200655
50336*a^6*b^20*c^6 + 256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8
 + 10385230921728*a^9*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 3009913081
0368*a^11*b^10*c^11 + 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*
b^6*c^13 + 1587694790508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15)
)/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5
376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^
6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + (9*x^(1/2)*(309622474
3817216*a^16*b*c^18 - 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 341
75188992*a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21*
c^8 + 40450001993728*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 3508816
48214016*a^9*b^15*c^11 + 523642412728320*a^10*b^13*c^12 - 6226534348095488*
a^11*b^11*c^13 + 21186489555615744*a^12*b^9*c^14 - 39951854506868736*a^13*b
^7*c^15 + 42889749576286208*a^14*b^5*c^16 - 22517998136852480*a^15*b^3*c^17
))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*
c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784
704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 576716
80*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11))*(-(81*

(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(3/4))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4) - (9*x^(1/2)*(245025*b^14*c^9 - 1175522844672*a^7*c^16 - 13142250*a*b^12*c^10 + 966155040*a^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 + 112005110016*a^4*b^6*c^13 + 617614170624*a^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502

626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^1 3*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 9 5*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45 *a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^ 27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320 *a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087 680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a ^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^1 4 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050 869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^ 2*c^19)))^(1/4)*1i)/((((3*(230850*a*b^11*c^8 - 4455*b^13*c^7 + 24287662080* a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 - 548653824*a^4*b^5 *c^11 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a ^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129 024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b ^2*c^8)) + ((3*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 1250506571776 0*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c ^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^1 5*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 82912 84418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15 *b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^2 5)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a ^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099 511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c ^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c ^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16 *b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 211 3425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a ^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^1 6 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 549755813 8880*a^26*b^2*c^19)))^(1/4)*(774056185954304*a^16*c^16 - 16777216*a^4*b^24* c^4 + 889192448*a^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 + 256355860480*a^7* b^18*c^7 - 2045478174720*a^8*b^16*c^8 + 10385230921728*a^9*b^14*c^9 - 31026 843746304*a^10*b^12*c^10 + 30099130810368*a^11*b^10*c^11 + 156680406958080* a^12*b^8*c^12 - 764160581304320*a^13*b^6*c^13 + 1587694790508544*a^14*b^4*c ^14 - 1706442046308352*a^15*b^2*c^15))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a ^12*b^2*c^8)) - (9*x^(1/2)*(3096224743817216*a^16*b*c^18 - 16777216*a^2*b^2 9*c^4 + 1157627904*a^3*b^27*c^5 - 34175188992*a^4*b^25*c^6 + 570425344000*a

^5*b^23*c^7 - 5968393928704*a^6*b^21*c^8 + 40450001993728*a^7*b^19*c^9 - 17
1227461189632*a^8*b^17*c^10 + 350881648214016*a^9*b^15*c^11 + 5236424127283
20*a^10*b^13*c^12 - 6226534348095488*a^11*b^11*c^13 + 21186489555615744*a^1
2*b^9*c^14 - 39951854506868736*a^13*b^7*c^15 + 42889749576286208*a^14*b^5*c
^16 - 22517998136852480*a^15*b^3*c^17))/(4194304*(a^4*b^24 + 16777216*a^16*
c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*
b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10
*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^
10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2)
 + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 13
29320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190
400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 8349
0242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b
^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 319
74471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(
-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25
)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-
(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*
(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 7
2960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 15876
0960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 -
44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18
*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 +
10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549
120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*
c^18 - 5497558138880*a^26*b^2*c^19)))^(3/4))*(-(81*(b^35 - b^10*(-(4*a*c -
b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3
*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^2
3*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b
^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 237938
9337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*
b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 23
4256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4
*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*
a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2
)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^
9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^
30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^
15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704
475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^2
0*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15
 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 1305670057
9840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4) + (9*x^(1/2)*(245
025*b^14*c^9 - 1175522844672*a^7*c^16 - 13142250*a*b^12*c^10 + 966155040*a^
2*b^10*c^11 - 22497354720*a^3*b^8*c^12 + 112005110016*a^4*b^6*c^13 + 617614

170624*a^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/(4194304*(a^4*b^24 + 16777
216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 12
6720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*
a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^
14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^
25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29
*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6
 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c
^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 23793893376
00*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c
^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*
a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c
 - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b
^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(
33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^3
6*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^
5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^
24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 70447529
9840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^1
4*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20
809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*
a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4) + (((3*(230850*a*b^11*
c^8 - 4455*b^13*c^7 + 24287662080*a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 830995
2*a^3*b^7*c^10 - 548653824*a^4*b^5*c^11 + 9760227840*a^5*b^3*c^12))/(65536*
(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b
^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 5
89824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + ((3*(-(81*(b^35 - b^10*(-(4*a*
c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335
*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6
*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a
^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 23
79389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a
^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16
 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*
(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33
880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^
(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 304
0*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^1
2*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 825556992
0*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 -
 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280
*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*
c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 130567
00579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(774056185954
304*a^16*c^16 - 16777216*a^4*b^24*c^4 + 889192448*a^5*b^22*c^5 - 2006555033

6*a^6*b^20*c^6 + 256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8 + 1
0385230921728*a^9*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 30099130810368
*a^11*b^10*c^11 + 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*b^6*
c^13 + 1587694790508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15))/(6
5536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*
a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^
6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + (9*x^(1/2)*(3096224743817
216*a^16*b*c^18 - 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 3417518
8992*a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21*c^8
+ 40450001993728*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 35088164821
4016*a^9*b^15*c^11 + 523642412728320*a^10*b^13*c^12 - 6226534348095488*a^11
*b^11*c^13 + 21186489555615744*a^12*b^9*c^14 - 39951854506868736*a^13*b^7*c
^15 + 42889749576286208*a^14*b^5*c^16 - 22517998136852480*a^15*b^3*c^17))/(
4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2
- 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*
a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a
^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^3
5 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*
b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^
5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8
+ 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^1
1*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 -
20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 2991914483712
0*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c -
510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2
)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4
*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*
a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^
4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26
*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*
a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12
- 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 166472932
39296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b
^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(3/
4))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^
17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 123568
16*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 66872320
0*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 50
2626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^
13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 -
29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) -
95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*
(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 4
5*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a
^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 124032

0*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 127008
7680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 +
 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*
a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^
14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 1958505
0869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b
^2*c^19)))^(1/4) - (9*x^(1/2)*(245025*b^14*c^9 - 1175522844672*a^7*c^16 - 1
3142250*a*b^12*c^10 + 966155040*a^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 +
112005110016*a^4*b^6*c^13 + 617614170624*a^5*b^4*c^14 + 19430129664*a^6*b^2
c^15))/(4194304(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*
b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 +
 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 5
7671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(
-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 +
3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^
5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8
*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 5026267
13600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^
9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 2991
9144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*
b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*
a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b
^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c
^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^1
1*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*
a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 1937
30707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*
b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 -
16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 195850508697
60*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^
19)))^(1/4)))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760
*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^
4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7
- 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15
*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 829128
4418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*
b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25
)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^
3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)
^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 10995
11627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^
3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^
6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*
b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113
425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^

21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*2i + 2*atan(((((3*(230850*a*b^11*c^8 - 4455*b^13*c^7 + 24287662080*a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 - 548653824*a^4*b^5*c^11 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) - (((-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(774056185954304*a^16*c^16 - 16777216*a^4*b^24*c^4 + 889192448*a^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 + 256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8 + 10385230921728*a^9*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 30099130810368*a^11*b^10*c^11 + 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*b^6*c^13 + 1587694790508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15)*3i)/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) - (9*x^(1/2)*(3096224743817216*a^16*b*c^18 - 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 34175188992*a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21*c^8 + 40450001993728*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 350881648214016*a^9*b^15*c^11 + 523642412728320*a^10*b^13*c^12 - 6226534348095488*a^11*b^11*c^13 + 21186489555615744*a^12*b^9*c^14 - 39951854506868736*a^13*b^7*c^15 + 42889749576286208*a^14*b^5*c^16 - 22517998136852480*a^15*b^3*c^17))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a

^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400
*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 -
2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120
*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^1
6 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^
2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) -
33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25
)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3
040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a
^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569
920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10
 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 52022791372
80*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^1
0*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 1305
6700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(3/4)*1i)*(-(81*(
b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a
^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25
*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*
c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*
a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13
 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 2991914483
7120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c
 + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c -
b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(
-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 -
80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32
*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b
^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 1937307074
56*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c
^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 166472
93239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^2
4*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^
(1/4)*1i + (9*x^(1/2)*(245025*b^14*c^9 - 1175522844672*a^7*c^16 - 13142250*
a*b^12*c^10 + 966155040*a^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 + 11200511
0016*a^4*b^6*c^13 + 617614170624*a^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/
(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2
 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704
*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*
a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^
35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2
*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c
^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^
8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^
11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 -
 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 299191448371

20*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4) - (((3*(230850*a*b^11*c^8 - 4455*b^13*c^7 + 24287662080*a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 - 548653824*a^4*b^5*c^11 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) - (((-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(774056185954304*a^16*c^16 - 16777216*a^4*b^24*c^4 + 889192448*a^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 + 256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8 + 10385230921728*a^9*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 30099130810368*a^11*b^10*c^11 + 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*b^6*c^13 + 1587694790508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15)*3i)/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + (9*x^(1/2)*(3096224743817216*a^16*b*c^18 - 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 34175188992*a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21*c^8 + 40450001993728*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 350881648214016*a^9*b^15*c^11 + 523642412728320*a^10*b^13*c

^12 - 6226534348095488*a^11*b^11*c^13 + 21186489555615744*a^12*b^9*c^14 - 3
9951854506868736*a^13*b^7*c^15 + 42889749576286208*a^14*b^5*c^16 - 22517998
136852480*a^15*b^3*c^17))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*
b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811
008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 3244032
0*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*
a^15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717
760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27
*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c
^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b
^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 829
1284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^
15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)
^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015
*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^
25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 10
99511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34
*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28
*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^
16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2
113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560
*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c
^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558
138880*a^26*b^2*c^19)))^(3/4)*1i)*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1
/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 +
1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181
190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 8
3490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^1
2*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 +
31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^
5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)
^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4
*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(335544
32*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2
- 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 15
8760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8
- 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a
^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13
+ 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116
549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b
^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*1i - (9*x^(1/2)*(245025*b^14
*c^9 - 1175522844672*a^7*c^16 - 13142250*a*b^12*c^10 + 966155040*a^2*b^10*c
^11 - 22497354720*a^3*b^8*c^12 + 112005110016*a^4*b^6*c^13 + 617614170624*a
^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/(4194304*(a^4*b^24 + 16777216*a^16
*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8

*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4))/((((3*(230850*a*b^11*c^8 - 4455*b^13*c^7 + 24287662080*a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 - 548653824*a^4*b^5*c^11 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) - (((-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(774056185954304*a^16*c^16 - 16777216*a^4*b^24*c^4 + 889192448*a^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 + 256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8 + 10385230921728*a^9*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 30099130810368*a^11*b^10

*c^11 + 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*b^6*c^13 + 158
7694790508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15)*3i)/(65536*(a
^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^1
2*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589
824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) - (9*x^(1/2)*(3096224743817216*a^1
6*b*c^18 - 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 34175188992*a^
4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21*c^8 + 40450
001993728*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 350881648214016*a^
9*b^15*c^11 + 523642412728320*a^10*b^13*c^12 - 6226534348095488*a^11*b^11*c
^13 + 21186489555615744*a^12*b^9*c^14 - 39951854506868736*a^13*b^7*c^15 + 4
2889749576286208*a^14*b^5*c^16 - 22517998136852480*a^15*b^3*c^17))/(4194304
*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080
*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^
12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6
*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 + b^1
0*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^
2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 703
16800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 1091
2870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*
c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 2011495
9237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*
b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2
*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(
1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c -
b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^3
8*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 158
76096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 +
8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^
20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202
279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a
^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17
 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(3/4)*1i)*
(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 +
 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a
^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^
8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626
713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b
^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 299
19144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a
*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4
*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*
b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*
c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^
11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680
*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193

730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19
*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 -
 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869
760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c
^19)))^(1/4)*1i + (9*x^(1/2)*(245025*b^14*c^9 - 1175522844672*a^7*c^16 - 13
142250*a*b^12*c^10 + 966155040*a^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 + 1
12005110016*a^4*b^6*c^13 + 617614170624*a^5*b^4*c^14 + 19430129664*a^6*b^2*
c^15))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b
^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 +
3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57
671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-
(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3
910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5
*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*
b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 50262671
3600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9
*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919
144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b
^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a
*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^
8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^
20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11
*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a
^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 19373
0707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b
^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 1
6647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 1958505086976
0*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^1
9)))^(1/4)*1i + (((3*(230850*a*b^11*c^8 - 4455*b^13*c^7 + 24287662080*a^6*b
*c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 - 548653824*a^4*b^5*c^11
 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^
16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a
^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^
8)) - (((-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*
b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12
356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 6687
23200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10
+ 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 829128441856
0*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^
15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2
) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*
c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2)
 - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 10995116277
76*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 12
40320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 12

70087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(774056185954304*a^16*c^16 - 16777216*a^4*b^24*c^4 + 889192448*a^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 + 256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8 + 10385230921728*a^9*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 30099130810368*a^11*b^10*c^11 + 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*b^6*c^13 + 1587694790508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15)*3i)/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + (9*x^(1/2)*(3096224743817216*a^16*b*c^18 - 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 34175188992*a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21*c^8 + 40450001993728*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 350881648214016*a^9*b^15*c^11 + 523642412728320*a^10*b^13*c^12 - 6226534348095488*a^11*b^11*c^13 + 21186489555615744*a^12*b^9*c^14 - 39951854506868736*a^13*b^7*c^15 + 42889749576286208*a^14*b^5*c^16 - 22517998136852480*a^15*b^3*c^17))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(3/4)*1i)*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389

337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b
^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234
256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*
a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a
^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)
))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9
*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^3
0*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^1
5*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 7044
75299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20
*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15
+ 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579
840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*1i - (9*x^(1/2)*(2
45025*b^14*c^9 - 1175522844672*a^7*c^16 - 13142250*a*b^12*c^10 + 966155040*
a^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 + 112005110016*a^4*b^6*c^13 + 6176
14170624*a^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/(4194304*(a^4*b^24 + 167
77216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 +
126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 1297612
8*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*
a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(4*a*c - b^2
)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^
29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c
^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17
*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 237938933
7600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7
*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 23425
6*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*
c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4
*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))
/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b
^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*
c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*
b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475
299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b
^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 +
20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 1305670057984
0*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*1i))*(-(81*(b^35 + b
^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*
c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 7
0316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10
912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^1
3*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114
959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^1
6*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a
^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)

^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4) + 2*atan(((((3*(230850*a*b^11*c^8 - 4455*b^13*c^7 + 24287662080*a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 - 548653824*a^4*b^5*c^11 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) - (((-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(774056185954304*a^16*c^16 - 16777216*a^4*b^24*c^4 + 889192448*a^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 + 256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8 + 10385230921728*a^9*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 30099130810368*a^11*b^10*c^11 + 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*b^6*c^13 + 1587694790508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15)*3i)/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) - (9*x^(1/2)*(3096224743817216*a^16*b*c^18 - 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 34175188992*a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21*c^8 + 40450001993728*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 350881648214016*a^9*b^15*c^11 + 523642412728320*a^10*b^13*c^12 - 6226534348095488*a^11*b^11*c^13 + 21186489555615744*a^12*b^9*c^14 - 39951854506868736*a^13*b^7*c^15 + 42889749576286208*a^14*b^5*c^16 - 225179

98136852480*a^15*b^3*c^17))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(3/4)*1i)*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*1i + (9*x^(1/2)*(245025*b^14*c^9 - 1175522844672*a^7*c^16 - 13142250*a*b^12*c^10 + 966155040*a^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 + 112005110016*a^4*b^6*c^13 + 617614170624*a^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4

*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1
/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 +
 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181
190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 8
3490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^1
2*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 +
31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^
5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)
^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4
*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(335544
32*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2
- 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 15
8760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8
 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a
^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13
 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116
549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b
^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4) - (((3*(230850*a*b^11*c^8 -
4455*b^13*c^7 + 24287662080*a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*
b^7*c^10 - 548653824*a^4*b^5*c^11 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b
^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^
3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*
a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) - (((-(81*(b^35 - b^10*(-(4*a*c - b^2)
^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^2
9*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^
6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*
c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337
600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*
c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256
*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c
 - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*
b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/
(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^
36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c
^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b
^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 7044752
99840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^
14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 2
0809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840
*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(774056185954304*a^16
*c^16 - 16777216*a^4*b^24*c^4 + 889192448*a^5*b^22*c^5 - 20065550336*a^6*b^
20*c^6 + 256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8 + 103852309
21728*a^9*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 30099130810368*a^11*b^
10*c^11 + 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*b^6*c^13 + 1
587694790508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15)*3i)/(65536*

(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b
^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 5
89824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + (9*x^(1/2)*(3096224743817216*a
^16*b*c^18 - 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 34175188992*
a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21*c^8 + 404
50001993728*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 350881648214016*
a^9*b^15*c^11 + 523642412728320*a^10*b^13*c^12 - 6226534348095488*a^11*b^11
*c^13 + 21186489555615744*a^12*b^9*c^14 - 39951854506868736*a^13*b^7*c^15 +
 42889749576286208*a^14*b^5*c^16 - 22517998136852480*a^15*b^3*c^17))/(41943
04*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 140
80*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*
b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b
^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 - b
^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*
c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 7
0316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10
912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^1
3*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114
959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^1
6*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a
^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)
^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c
- b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b
^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 1
5876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7
+ 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*
b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 52
02279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296
*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^
17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(3/4)*1i
)*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17
 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816
*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*
a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 5026
26713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13
*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 2
9919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95
*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-
(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*
a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^2
7*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*
a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 12700876
80*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 1
93730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^
19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14

- 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 195850508 69760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2 *c^19)))^(1/4)*1i - (9*x^(1/2)*(245025*b^14*c^9 - 1175522844672*a^7*c^16 - 13142250*a*b^12*c^10 + 966155040*a^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 + 112005110016*a^4*b^6*c^13 + 617614170624*a^5*b^4*c^14 + 19430129664*a^6*b^ 2*c^15))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6 *b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))* (-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a ^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^ 8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626 713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b ^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 299 19144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a *b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4 *a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a* b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27* c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^ 11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680 *a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193 730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19 *b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869 760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c ^19)))^(1/4))/((((3*(230850*a*b^11*c^8 - 4455*b^13*c^7 + 24287662080*a^6*b* c^13 - 3679344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 - 548653824*a^4*b^5*c^11 + 9760227840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^1 6*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^ 9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) - (((-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b *c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 123 56816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 66872 3200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560 *a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^1 5 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c ^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 109951162777 6*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 124 0320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 127 0087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^ 9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 21134258995

20*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12
*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 1958
5050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^2
6*b^2*c^19)))^(1/4)*(774056185954304*a^16*c^16 - 16777216*a^4*b^24*c^4 + 88
9192448*a^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 + 256355860480*a^7*b^18*c^7
 - 2045478174720*a^8*b^16*c^8 + 10385230921728*a^9*b^14*c^9 - 3102684374630
4*a^10*b^12*c^10 + 30099130810368*a^11*b^10*c^11 + 156680406958080*a^12*b^8
*c^12 - 764160581304320*a^13*b^6*c^13 + 1587694790508544*a^14*b^4*c^14 - 17
06442046308352*a^15*b^2*c^15)*3i)/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a
^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129
024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b
^2*c^8)) - (9*x^(1/2)*(3096224743817216*a^16*b*c^18 - 16777216*a^2*b^29*c^4
 + 1157627904*a^3*b^27*c^5 - 34175188992*a^4*b^25*c^6 + 570425344000*a^5*b^
23*c^7 - 5968393928704*a^6*b^21*c^8 + 40450001993728*a^7*b^19*c^9 - 1712274
61189632*a^8*b^17*c^10 + 350881648214016*a^9*b^15*c^11 + 523642412728320*a^
10*b^13*c^12 - 6226534348095488*a^11*b^11*c^13 + 21186489555615744*a^12*b^9
*c^14 - 39951854506868736*a^13*b^7*c^15 + 42889749576286208*a^14*b^5*c^16 -
 22517998136852480*a^15*b^3*c^17))/(4194304*(a^4*b^24 + 16777216*a^16*c^12
- 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*
c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7
+ 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 -
50331648*a^15*b^2*c^11)))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12
505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320
*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a
^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 834902425
60*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c
^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471
237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a
*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/
2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*
c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*
b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*
a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*
a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029
706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18
*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404
558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a
^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18
- 5497558138880*a^26*b^2*c^19)))^(3/4)*1i)*(-(81*(b^35 - b^10*(-(4*a*c - b^
2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b
^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*
c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^1
7*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 23793893
37600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^
7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 2342

56*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a
*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^
4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2))
)/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*
b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30
*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15
*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 70447
5299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*
b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 +
 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 130567005798
40*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*1i + (9*x^(1/2)*(24
5025*b^14*c^9 - 1175522844672*a^7*c^16 - 13142250*a*b^12*c^10 + 966155040*a
^2*b^10*c^11 - 22497354720*a^3*b^8*c^12 + 112005110016*a^4*b^6*c^13 + 61761
4170624*a^5*b^4*c^14 + 19430129664*a^6*b^2*c^15))/(4194304*(a^4*b^24 + 1677
7216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 1
26720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128
*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a
^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)
^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^2
9*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^
6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*
c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337
600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*
c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256
*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c
 - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*
b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/
(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^
36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c
^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b
^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 7044752
99840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^
14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 2
0809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840
*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*1i + (((3*(230850*a*b
^11*c^8 - 4455*b^13*c^7 + 24287662080*a^6*b*c^13 - 3679344*a^2*b^9*c^9 + 83
09952*a^3*b^7*c^10 - 548653824*a^4*b^5*c^11 + 9760227840*a^5*b^3*c^12))/(65
536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 - 5376*a
^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b^6*c^6
 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) - (((-(81*(b^35 - b^10*(-(4*
a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 913
35*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a
^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400
*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 -
2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120

*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^1
6 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^
2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) +
33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25
)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3
040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a
^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569
920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10
 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16*c^12 - 52022791372
80*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^1
0*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 1305
6700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(1/4)*(7740561859
54304*a^16*c^16 - 16777216*a^4*b^24*c^4 + 889192448*a^5*b^22*c^5 - 20065550
336*a^6*b^20*c^6 + 256355860480*a^7*b^18*c^7 - 2045478174720*a^8*b^16*c^8 +
 10385230921728*a^9*b^14*c^9 - 31026843746304*a^10*b^12*c^10 + 300991308103
68*a^11*b^10*c^11 + 156680406958080*a^12*b^8*c^12 - 764160581304320*a^13*b^
6*c^13 + 1587694790508544*a^14*b^4*c^14 - 1706442046308352*a^15*b^2*c^15)*3
i)/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576*a^6*b^14*c^2 -
5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5 + 344064*a^10*b
^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + (9*x^(1/2)*(30962247
43817216*a^16*b*c^18 - 16777216*a^2*b^29*c^4 + 1157627904*a^3*b^27*c^5 - 34
175188992*a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 5968393928704*a^6*b^21
*c^8 + 40450001993728*a^7*b^19*c^9 - 171227461189632*a^8*b^17*c^10 + 350881
648214016*a^9*b^15*c^11 + 523642412728320*a^10*b^13*c^12 - 6226534348095488
*a^11*b^11*c^13 + 21186489555615744*a^12*b^9*c^14 - 39951854506868736*a^13*
b^7*c^15 + 42889749576286208*a^14*b^5*c^16 - 22517998136852480*a^15*b^3*c^1
7))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20
*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9*b^14*c^5 + 378
4704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12*b^8*c^8 - 57671
680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^2*c^11)))*(-(81
(b^35 - b^10(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910
*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^
25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^1
9*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 50262671360
0*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b^9*c^
13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 29919144
837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a*b^33
*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^4*c^3*(-(4*a*c
- b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c
*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*c^20
- 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^
32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14
*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 19373070
7456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19*b^16
*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 - 1664

7293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a ^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c^19)))^(3/4)*1i)*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a ^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c ^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 82912844 18560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^ 5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^ (1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3* b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511 627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^ 22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 211342 5899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21 *b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 549755813888 0*a^26*b^2*c^19)))^(1/4)*1i - (9*x^(1/2)*(245025*b^14*c^9 - 1175522844672*a ^7*c^16 - 13142250*a*b^12*c^10 + 966155040*a^2*b^10*c^11 - 22497354720*a^3* b^8*c^12 + 112005110016*a^4*b^6*c^13 + 617614170624*a^5*b^4*c^14 + 19430129 664*a^6*b^2*c^15))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 811008*a^9 *b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320*a^12* b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a^15*b^ 2*c^11)))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^1 7*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 66 8723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^1 0 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418 560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5* c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1 /2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) - 2015*a^3*b^ 4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/ 2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 109951162 7776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22 *c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 21134258 99520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b ^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 1 9585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880* a^26*b^2*c^19)))^(1/4)*1i))*(-(81*(b^35 - b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 13293

```
20*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400
*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 8349024
2560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11
*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 319744
71237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 + 234256*a^5*c^5*(-(4
*a*c - b^2)^25)^(1/2) - 95*a*b^33*c - 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(
1/2) - 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) + 33880*a^4*b^2*c^4*(-(4*
a*c - b^2)^25)^(1/2) + 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^
7*b^40 + 1099511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 7296
0*a^10*b^34*c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 15876096
0*a^13*b^28*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 440
29706240*a^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^
18*c^11 + 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 104
04558274560*a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120
*a^23*b^8*c^16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^1
8 - 5497558138880*a^26*b^2*c^19)))^(1/4) + ((x^(9/2)*(b^3*c + 32*a*b*c^2))/
(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (3*x^(1/2)*(b^3 - 12*a*b*c))/(16*(b^
4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^(5/2)*(b^4 + 76*a^2*c^2 + 13*a*b^2*c))/(1
6*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c^2*x^(13/2)*(44*a*c + b^2))/(16*a*(
b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*
x^2 + 2*b*c*x^6)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.1087 $\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=658

$$\frac{x^{3/2}\left(52a^2c^2+bcx^2\left(5b^2-44ac\right)-45ab^2c+5b^4\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\sqrt[4]{c}\left(520a^2c^2-54ab^2c-b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan}{32\,2^{3/4}a^2\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

```
[Out] 1/4*x^(3/2)*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/16*x^(3/
2)*(5*b^4-45*a*b^2*c+52*a^2*c^2+b*c*(-44*a*c+5*b^2)*x^2)/a^2/(-4*a*c+b^2)^2
/(c*x^4+b*x^2+a)-1/64*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^
2)^(1/2))^(1/4))*(5*b^4-54*a*b^2*c+520*a^2*c^2-b*(-44*a*c+5*b^2)*(-4*a*c+b^
2)^(1/2))*2^(1/4)/a^2/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/64
*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(5*
b^4-54*a*b^2*c+520*a^2*c^2-b*(-44*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))*2^(1/4)/a^
2/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/64*c^(1/4)*arctan(2^(1
/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(5*b^4-54*a*b^2*c+520*a^
2*c^2+b*(-44*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))*2^(1/4)/a^2/(-4*a*c+b^2)^(5/2)/
(-b+(-4*a*c+b^2)^(1/2))^(1/4)-1/64*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/
(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(5*b^4-54*a*b^2*c+520*a^2*c^2+b*(-44*a*c+5*b
^2)*(-4*a*c+b^2)^(1/2))*2^(1/4)/a^2/(-4*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^(1/
2))^(1/4)
```

Rubi [A] time = 5.48, antiderivative size = 658, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1115, 1366, 1500, 1510, 298, 205, 208}

$$\frac{x^{3/2}\left(52a^2c^2+bcx^2\left(5b^2-44ac\right)-45ab^2c+5b^4\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\sqrt[4]{c}\left(520a^2c^2-54ab^2c-b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan}{32\,2^{3/4}a^2\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2)
 + (x^(3/2)*(5*b^4 - 45*a*b^2*c + 52*a^2*c^2 + b*c*(5*b^2 - 44*a*c)*x^2))/(
16*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (c^(1/4)*(5*b^4 - 54*a*b^2*c
+ 520*a^2*c^2 - b*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/
4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^
```

```
(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(5*b^4 - 54*a*b^2*c + 520*
a^2*c^2 + b*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqr
t[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^(5/2)*
(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(5*b^4 - 54*a*b^2*c + 520*a^2*c^
2 - b*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])
/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-b -
 Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(5*b^4 - 54*a*b^2*c + 520*a^2*c^2 + b
*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b
+ Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt
[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1366

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^n)*(a + b*x^n + c*x^(
2*n))^(p + 1))/(a*d*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2
 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(m + n*(p
+ 1) + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(m + n*(2*p + 3) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0
] && IGtQ[n, 0] && ILtQ[p, -1]
```

Rule 1500

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^n + c*x^(
2*n))^(p + 1)*(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n))/(a*f*n*(p +
1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a
+ b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m + 2
*n*(p + 1) + 1)) - a*b*e*(m + 1) + c*(m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2
 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{x^2\left(-5b^2+26ac-9bcx^4\right)}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4a\left(b^2-4ac\right)}$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(5b^4-45ab^2c+52a^2c^2+bc\left(5b^2-44ac\right)x^2\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} +$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(5b^4-45ab^2c+52a^2c^2+bc\left(5b^2-44ac\right)x^2\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} -$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(5b^4-45ab^2c+52a^2c^2+bc\left(5b^2-44ac\right)x^2\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} +$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(5b^4-45ab^2c+52a^2c^2+bc\left(5b^2-44ac\right)x^2\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} -$$

<u>Mathematica</u> [C] time = 0.49, size = 254, normalized size = 0.39

$$\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{-44\#1^4abc^2\log\left(\sqrt{x}-\#1\right)+5\#1^4b^3c\log\left(\sqrt{x}-\#1\right)+260a^2c^2\log\left(\sqrt{x}-\#1\right)-49ab^2c\log\left(\sqrt{x}-\#1\right)+5b^4\log\left(\sqrt{x}-\right.}{2\#1^5c+\#1b}\right.}{64a^2\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-16*a*(-b^2 + 4*a*c)*x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(a + b*x^2 + c*x^4)
^2 + (4*x^(3/2)*(5*b^4 - 45*a*b^2*c + 52*a^2*c^2 + 5*b^3*c*x^2 - 44*a*b*c^2
*x^2))/(a + b*x^2 + c*x^4) + RootSum[a + b*#1^4 + c*#1^8 & , (5*b^4*Log[Sqr
t[x] - #1] - 49*a*b^2*c*Log[Sqrt[x] - #1] + 260*a^2*c^2*Log[Sqrt[x] - #1] +
 5*b^3*c*Log[Sqrt[x] - #1]*#1^4 - 44*a*b*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1
+ 2*c*#1^5) & ])/(64*a^2*(b^2 - 4*a*c)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 191.53Unable to convert to
real 1/4 Error: Bad Argument Value
```

maple [C] time = 0.05, size = 321, normalized size = 0.49

$$-\frac{\left(\left(44ac-5b^2\right)\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^6 bc+\left(-260a^2c^2+49a\,b^2c-5b^4\right)\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^2\right)\ln\left(-\right.}{64\left(16a^2c^2-8a\,b^2c+b^4\right)a^2\left(2\,\mathrm{RootOf}\left(c\_Z^8+b\_Z^4+a\right)^7 c+\mathrm{RootOf}\left(c\_Z^8+b\_Z\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(3/32*(28*a^2*c^2-23*a*b^2*c+3*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^(3/2)-
1/32*b*(8*a^2*c^2+36*a*b^2*c-5*b^4)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+
1/32/a^2*c*(52*a^2*c^2-89*a*b^2*c+10*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(11/
2)-1/32*c^2*b*(44*a*c-5*b^2)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(15/2))/(c*x^
4+b*x^2+a)^2-1/64/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*sum((b*c*(44*a*c-5*b^2)*_R
^6+(-260*a^2*c^2+49*a*b^2*c-5*b^4)*_R^2)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),
_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(5\,b^3c^2-44\,abc^3\right)x^{\frac{15}{2}}+\left(10\,b^4c-89\,ab^2c^2+52\,a^2c^3\right)x^{\frac{11}{2}}+\left(5\,b^5-36\,ab^3c-8\,a^2bc^2\right)x^{\frac{7}{2}}+3\left(\right.}{16\left(\left(a^2b^4c^2-8\,a^3b^2c^3+16\,a^4c^4\right)x^8+a^4b^4-8\,a^5b^2c+16\,a^6c^2+2\left(a^2b^5c-8\,a^3b^3c^2+16\,a^4bc^3\right)x^6+\left(a^2b^6-6\,a^3b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")

[Out] 1/16*((5*b^3*c^2 - 44*a*b*c^3)*x^(15/2) + (10*b^4*c - 89*a*b^2*c^2 + 52*a^2*c^3)*x^(11/2) + (5*b^5 - 36*a*b^3*c - 8*a^2*b*c^2)*x^(7/2) + 3*(3*a*b^4 - 23*a^2*b^2*c + 28*a^3*c^2)*x^(3/2))/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2) - integrate(-1/32*((5*b^3*c - 44*a*b*c^2)*x^(5/2) + (5*b^4 - 49*a*b^2*c + 260*a^2*c^2)*sqrt(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), x)

mupad [B] time = 8.75, size = 46948, normalized size = 71.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(a + b*x^2 + c*x^4)^3,x)

[Out] ((x^(11/2)*(10*b^4*c + 52*a^2*c^3 - 89*a*b^2*c^2))/(16*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) - (x^(7/2)*(8*a^2*b*c^2 - 5*b^5 + 36*a*b^3*c))/(16*a*(a*b^4 + 16*a^3*c^2 - 8*a^2*b^2*c)) + (3*x^(3/2)*(3*b^4 + 28*a^2*c^2 - 23*a*b^2*c))/(16*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (b*c^2*x^(15/2)*(44*a*c - 5*b^2))/(16*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + atan(((((2097152000*a*b^33*c^4 + 466178856428188467200*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340020080640*a^3*b^29*c^6 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c^8 - 23398590986584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 1631099300505190400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 45207702606568226816*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 487882094458626375680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 - 1771946621413479153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18 - 1418770116510434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^19*b^2*c^13)) - (x^(1/2)*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 -

285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b
^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^
(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*
c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(
33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b
^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*
c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*
b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475
299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b
^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 +
20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 1305670057984
0*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(2378463553205043200
*a^18*c^19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 - 8147183861
76*a^5*b^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476672*a^7*b^22*c^
8 + 2159815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c^10 + 98862
579421544448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12 + 163543943
3677275136*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 + 9201889778
671288320*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 138416023484
90686464*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18))/(4194304*(a^6*
b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b
^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c
^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9
 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 - 625*b^1
2*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b
^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5
*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 133170
68448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*
b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^
12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 4
8851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189
126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 526
25*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^
6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^
(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-
(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80
*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32
*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b
^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 1937307074
56*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c
^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 166472
93239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^2
6*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^
(3/4) + (x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 + 99803558400
000*a^7*b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^9*c^13 + 19
75414457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483712000*a^6*

b^3*c^16))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a
^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*
c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^
8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11
)))*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^
18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b
^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913
600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^
9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807
000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 3176436974
3282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680
*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a
*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^2
5)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4
c^4(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^
(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 109
9511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^3
4*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^2
8*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a
^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 +
2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 1040455827456
0*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*
c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 549755
8138880*a^28*b^2*c^19)))^(1/4)*1i - (((2097152000*a*b^33*c^4 + 466178856428
188467200*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340020080640*a^3*b^29*
c^6 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c^8 - 233985
90986584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 1631099300505
190400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 4520770260656822
6816*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 48788209445862
6375680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 - 17719466214
13479153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18 - 14187701
16510434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a^20*c^14 -
56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c
^4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^
7 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8
*c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^1
9*b^2*c^13)) + (x^(1/2)*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) +
11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^
3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b
^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 826293
38933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a
^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^
11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15
 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 2856
10000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c

^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2
) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*
(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(3355
4432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*
c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5
+ 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24
*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 7044752998
40*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*
c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 2080
9116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^
27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(2378463553205043200*a^1
8*c^19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 - 814718386176*a
^5*b^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476672*a^7*b^22*c^8 +
2159815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c^10 + 988625794
21544448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12 + 1635439433677
275136*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 + 92018897786712
88320*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 1384160234849068
6464*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18))/(4194304*(a^6*b^24
 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*
c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 -
 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 6
9206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-
(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*
c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^2
7*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 1331706844
8000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17
*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 -
 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851
227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 355771891266
35520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a
*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^
3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2
) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a
c - b^2)^25)^(1/2))/(33554432(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^1
0*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4
 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*
c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a
^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12
- 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 1664729323
9296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^
6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(3/4
) - (x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 + 99803558400000*
a^7*b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^9*c^13 + 197541
4457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483712000*a^6*b^3*
c^16))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b

^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5
+ 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 -
57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*
(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b
*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*
c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*
a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 +
419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 58070005
41921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282
176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^1
6*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c -
 b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(
1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4
*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2
) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511
627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^
3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^
6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*
b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113
425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^
23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16
 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138
880*a^28*b^2*c^19)))^(1/4)*1i)/((((2097152000*a*b^33*c^4 + 4661788564281884
67200*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340020080640*a^3*b^29*c^6
- 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c^8 - 2339859098
6584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 16310993005051904
00*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 45207702606568226816
*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 487882094458626375
680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 - 177194662141347
9153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18 - 141877011651
0434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a^20*c^14 - 56*a
^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^4 -
 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^7 +
196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*c^1
0 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^19*b^
2*c^13)) - (x^(1/2)*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 1127
9020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 +
1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*
c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 8262933893
3248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*
b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c
^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 5
2725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 28561000
0*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(
-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) -

28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(2378463553205043200*a^18*c^19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 - 814718386176*a^5*b^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476672*a^7*b^22*c^8 + 2159815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c^10 + 98862579421544448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12 + 1635439433677275136*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 + 9201889778671288320*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 13841602348490686464*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(3/4) + (x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 + 99803558400000*a^7*b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^9*c^13 + 1975414457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483712000*a^6*b^3*c^16))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 37

84704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 5767 1680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(6 25*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^1 8 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7* b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 4197 01532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 580700054192 1280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176* a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^ 5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 10995116277 76*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22 *c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 21134258 99520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b ^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 1 9585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880* a^28*b^2*c^19)))^(1/4) + (((2097152000*a*b^33*c^4 + 466178856428188467200*a ^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340020080640*a^3*b^29*c^6 - 12030 0087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c^8 - 23398590986584064 *a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 1631099300505190400*a^8* b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 45207702606568226816*a^10*b ^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 487882094458626375680*a^1 2*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 - 1771946621413479153664 *a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18 - 1418770116510434197 504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a^20*c^14 - 56*a^7*b^26 *c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^4 - 205004 8*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^7 + 1968046 08*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*c^10 - 152 6726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^19*b^2*c^13)) + (x^(1/2)*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326 912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954 201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1 756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^ 9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^ 11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 3 1764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360 025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*c ^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 2854520 1*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c -

b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b
^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960
*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960
*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 4402
9706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^1
8*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 1040
4558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*
a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18
 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(2378463553205043200*a^18*c^19 - 41
9430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 - 814718386176*a^5*b^26*c^6
 + 15745652097024*a^6*b^24*c^7 - 214134184476672*a^7*b^22*c^8 + 21598155728
48640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c^10 + 98862579421544448*a^
10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12 + 1635439433677275136*a^12
*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 + 9201889778671288320*a^14*
b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 13841602348490686464*a^16*b
^4*c^17 - 8502514621498785792*a^17*b^2*c^18))/(4194304*(a^6*b^24 + 16777216
*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 12672
0*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a
^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^1
6*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-(4*a*c - b^
2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758
230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188
531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^
21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 173
7502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 1542259399
1966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 4885122788622336
0*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*
b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 3
80775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c
 - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 1215786
00*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^2
5)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c +
3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096
*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 82555
69920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^
10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 520227913
7280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b
^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13
056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(3/4) - (x^(1/2
)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 + 99803558400000*a^7*b*c^17
 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^9*c^13 + 1975414457856*a^4
*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483712000*a^6*b^3*c^16))/(419
4304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 1
4080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a
^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^

15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4) + (80318101760000000*a^7*c^19 - 6746163125*b^14*c^12 + 572489781500*a*b^12*c^13 - 15194313373200*a^2*b^10*c^14 + 226647361174720*a^3*b^8*c^15 - 2095830057168640*a^4*b^6*c^16 + 12493373163648000*a^5*b^4*c^17 - 44688231411200000*a^6*b^2*c^18)/(134217728*(a^6*b^28 + 268435456*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^19*b^2*c^13))))*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 +

10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549 120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4* c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*2i + atan(((((2097152000*a*b^33 *c^4 + 4661788564281884672OO*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340 020080640*a^3*b^29*c^6 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^ 5*b^25*c^8 - 23398590986584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c ^10 - 1631099300505190400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 45207702606568226816*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^ 14 - 487882094458626375680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9 *c^16 - 1771946621413479153664*a^14*b^7*c^17 + 2014068018680264916992*a^15* b^5*c^18 - 1418770116510434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268 435456*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56 229888*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c ^12 - 939524096*a^19*b^2*c^13)) - (x^(1/2)*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 5 7758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^ 8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 154225 93991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 488512278862 23360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a ^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4* a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121 578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^ 2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38* c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 1587 6096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8 255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^2 0*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 52022 79137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^ 24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(2378 463553205043200*a^18*c^19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c ^5 - 814718386176*a^5*b^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476 672*a^7*b^22*c^8 + 2159815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^ 18*c^10 + 98862579421544448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c ^12 + 1635439433677275136*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^ 14 + 9201889778671288320*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 13841602348490686464*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18)) /(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^ 2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784 704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 576716

80*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625
*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18
+ 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 -
16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^
23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701
532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 58070005419212
80*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^
14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*
c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^
25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) -
4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*
a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21
375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776
*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 12
40320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 12
70087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c
^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899
520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^1
2*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 195
85050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^
28*b^2*c^19)))^(3/4) + (x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^1
1 + 99803558400000*a^7*b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^
3*b^9*c^13 + 1975414457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 1099
0483712000*a^6*b^3*c^16))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*
b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 81
1008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440
320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 5033164
8*a^17*b^2*c^11)))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279
020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1
109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c
^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933
248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b
^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^
13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52
725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000
*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-
(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 2
8545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*
a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*
(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 -
72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158
760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8
- 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^
20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13
+ 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 208091165

49120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^
4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*1i - (((2097152000*a*b^33*c^4
+ 4661788564281884672000*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 53400200
80640*a^3*b^29*c^6 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^
25*c^8 - 23398590986584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10
- 1631099300505190400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 4
5207702606568226816*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 -
487882094458626375680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^1
6 - 1771946621413479153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*
c^18 - 1418770116510434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 2684354
56*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256
256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 562298
88*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049
624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12
- 939524096*a^19*b^2*c^13)) + (x^(1/2)*(-(625*b^37 + 625*b^12*(-(4*a*c - b^
2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758
230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188
531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^
21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 173
7502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 1542259399
1966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 4885122788622336
0*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*
b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 3
80775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c
- b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 1215786
00*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^2
5)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c +
3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096
*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 82555
69920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^
10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 520227913
7280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b
^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13
056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(23784635
53205043200*a^18*c^19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 -
814718386176*a^5*b^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476672*
a^7*b^22*c^8 + 2159815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c
^10 + 98862579421544448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12
+ 1635439433677275136*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 +
9201889778671288320*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 1
3841602348490686464*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18))/(41
94304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 -
14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*
a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a
^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^3

7 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 21
68275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 1628
5749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c
^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 4197015327
33440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a
^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b
^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16
 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^
(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 407
5730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c
 - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*
a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^2
9*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 124032
0*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 127008
7680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 +
 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*
a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^
14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 1958505
0869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b
^2*c^19)))^(3/4) - (x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 +
99803558400000*a^7*b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^
9*c^13 + 1975414457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483
712000*a^6*b^3*c^16))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22
*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008
*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*
a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^
17*b^2*c^11)))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 112790203
26912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 11099
54201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 -
 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*
a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*
c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 +
 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 527253
60025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6
c^6(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a
*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545
201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c
 - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9
*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 729
60*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 1587609
60*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44
029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b
^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10
404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 2080911654912
0*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^

18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*1i)/((((2097152000*a*b^33*c^4 + 4
66178856428188467200*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 534002008064
0*a^3*b^29*c^6 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c
^8 - 23398590986584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 16
31099300505190400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 45207
702606568226816*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 487
882094458626375680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 -
1771946621413479153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18
 - 1418770116510434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a
^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*
a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a
^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 10496245
76*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 93
9524096*a^19*b^2*c^13)) - (x^(1/2)*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^2
5)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*
a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 1885317
80400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c
^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502
295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966
720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^
15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*
c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 38077
5*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^
2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a
^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(
1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040
*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^1
4*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 825556992
0*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 -
 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280
*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*
c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 130567
00579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(237846355320
5043200*a^18*c^19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 - 814
718386176*a^5*b^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476672*a^7*
b^22*c^8 + 2159815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c^10
+ 98862579421544448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12 + 16
35439433677275136*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 + 920
1889778671288320*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 13841
602348490686464*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18))/(419430
4*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 1408
0*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12
*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*
b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 +
625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 216827

5*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749
400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 +
 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 41970153273344
0*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*
b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c
^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 3
5577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2
) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730
*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^
2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^
10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^
20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^
13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680
*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193
730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21
*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 -
 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869
760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c
^19)))^(3/4) + (x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 + 9980
3558400000*a^7*b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^9*c^
13 + 1975414457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 109904837120
00*a^6*b^3*c^16))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c +
 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^1
1*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14
*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b
^2*c^11)))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 1127902032691
2000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 110995420
1*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 175
6313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*
b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11
 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 317
64369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 5272536002
5927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6
*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c -
 b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*
a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^
2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^4
0 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a
^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a
^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 440297
06240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*
c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 104045
58274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^
25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 -
 5497558138880*a^28*b^2*c^19)))^(1/4) + (((2097152000*a*b^33*c^4 + 46617885

6428188467200*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340020080640*a^3*b^29*c^6 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c^8 - 23398590986584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 1631099300505190400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 45207702606568226816*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 487882094458626375680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 - 1771946621413479153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18 - 1418770116510434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^19*b^2*c^13)) + (x^(1/2)*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(2378463553205043200*a^18*c^19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 - 814718386176*a^5*b^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476672*a^7*b^22*c^8 + 2159815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c^10 + 98862579421544448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12 + 1635439433677275136*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 + 9201889778671288320*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 13841602348490686464*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5

*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 133170
68448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*
b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^
12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 4
8851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189
126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 526
25*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^
6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^
(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-
(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80
*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32
*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b
^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 1937307074
56*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c
^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 166472
93239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^2
6*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^
(3/4) - (x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 + 99803558400
000*a^7*b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^9*c^13 + 19
75414457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483712000*a^6*
b^3*c^16))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a
^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*
c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^
8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11
)))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^
18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b
^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913
600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^
9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807
000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 3176436974
3282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680
*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a
*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^2
5)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4
c^4(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^
(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 109
9511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^3
4*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^2
8*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a
^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 +
2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 1040455827456
0*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*
c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 549755
8138880*a^28*b^2*c^19)))^(1/4) + (80318101760000000*a^7*c^19 - 6746163125*b
^14*c^12 + 572489781500*a*b^12*c^13 - 15194313373200*a^2*b^10*c^14 + 226647

361174720*a^3*b^8*c^15 - 2095830057168640*a^4*b^6*c^16 + 12493373163648000*a^5*b^4*c^17 - 44688231411200000*a^6*b^2*c^18)/(134217728*(a^6*b^28 + 268435456*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^19*b^2*c^13))))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*2i + 2*atan(((((2097152000*a*b^33*c^4 + 4661788564281884672000*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340020080640*a^3*b^29*c^6 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c^8 - 23398590986584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 1631099300505190400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 45207702606568226816*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 487882094458626375680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 - 1771946621413479153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18 - 1418770116510434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^19*b^2*c^13)) - (x^(1/2)*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^

14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35
577189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2)
 - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*
a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2
)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^1
0*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^2
0 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^1
3*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*
a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 1937
30707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*
b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 -
16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 195850508697
60*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^
19)))^(1/4)*(2378463553205043200*a^18*c^19 - 419430400*a^3*b^30*c^4 + 26675
773440*a^4*b^28*c^5 - 814718386176*a^5*b^26*c^6 + 15745652097024*a^6*b^24*c
^7 - 214134184476672*a^7*b^22*c^8 + 2159815572848640*a^8*b^20*c^9 - 1661536
0157450240*a^9*b^18*c^10 + 98862579421544448*a^10*b^16*c^11 - 4569839705385
86112*a^11*b^14*c^12 + 1635439433677275136*a^12*b^12*c^13 - 448054836609417
2160*a^13*b^10*c^14 + 9201889778671288320*a^14*b^8*c^15 - 13675039531022155
776*a^15*b^6*c^16 + 13841602348490686464*a^16*b^4*c^17 - 850251462149878579
2*a^17*b^2*c^18)*1i)/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*
c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*
a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a
^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^1
7*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 1127902032
6912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 110995
4201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 -
1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a
^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c
^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 +
31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 5272536
0025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*
c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*
c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 285452
01*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c -
 b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*
b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 7296
0*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 15876096
0*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 440
29706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^
18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 104
04558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120
*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^1
8 - 5497558138880*a^28*b^2*c^19)))^(3/4)*1i - (x^(1/2)*(30525625*b^15*c^10
 - 1297573875*a*b^13*c^11 + 99803558400000*a^7*b*c^17 + 27786809400*a^2*b^11

*c^12 - 311511417680*a^3*b^9*c^13 + 1975414457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483712000*a^6*b^3*c^16))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4) - (((2097152000*a*b^33*c^4 + 466178856428188467200*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340020080640*a^3*b^29*c^6 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c^8 - 23398590986584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 1631099300505190400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 45207702606568226816*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 487882094458626375680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 - 1771946621413479153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18 - 1418770116510434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^19*b^2*c^13)) + (x^(1/2)*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 355

77189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2)
- 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a
^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)
^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10
c(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20
- 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13
*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a
^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 19373
0707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b
^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 1
6647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 1958505086976
0*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^1
9)))^(1/4)*(2378463553205043200*a^18*c^19 - 419430400*a^3*b^30*c^4 + 266757
73440*a^4*b^28*c^5 - 814718386176*a^5*b^26*c^6 + 15745652097024*a^6*b^24*c^
7 - 214134184476672*a^7*b^22*c^8 + 2159815572848640*a^8*b^20*c^9 - 16615360
157450240*a^9*b^18*c^10 + 98862579421544448*a^10*b^16*c^11 - 45698397053858
6112*a^11*b^14*c^12 + 1635439433677275136*a^12*b^12*c^13 - 4480548366094172
160*a^13*b^10*c^14 + 9201889778671288320*a^14*b^8*c^15 - 136750395310221557
76*a^15*b^6*c^16 + 13841602348490686464*a^16*b^4*c^17 - 8502514621498785792
*a^17*b^2*c^18)*1i)/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c
+ 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a
^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^
14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17
*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326
912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954
201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1
756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^
9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^
11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 3
1764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360
025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*c
^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c
- b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 2854520
1*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c -
b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b
^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960
*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960
*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 4402
9706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^1
8*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 1040
4558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*
a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18
- 5497558138880*a^28*b^2*c^19)))^(3/4)*1i + (x^(1/2)*(30525625*b^15*c^10 -
1297573875*a*b^13*c^11 + 99803558400000*a^7*b*c^17 + 27786809400*a^2*b^11*
c^12 - 311511417680*a^3*b^9*c^13 + 1975414457856*a^4*b^7*c^14 - 47539805913

60*a^5*b^5*c^15 - 10990483712000*a^6*b^3*c^16))/(4194304*(a^6*b^24 + 167772
16*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126
720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128
*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a
^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-(4*a*c -
b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 577
58230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 1
88531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*
b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1
737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593
991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223
360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^1
7*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c -
380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*
c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 12157
8600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)
^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c
+ 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 158760
96*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 825
5569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*
c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279
137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24
*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 +
13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4))/((((20
97152000*a*b^33*c^4 + 4661788856428188467200*a^17*b*c^20 - 151833804800*a^2*
b^31*c^5 + 5340020080640*a^3*b^29*c^6 - 120300087803904*a^4*b^27*c^7 + 1933
149881761792*a^5*b^25*c^8 - 23398590986584064*a^6*b^23*c^9 + 21987825226350
5920*a^7*b^21*c^10 - 1631099300505190400*a^8*b^19*c^11 + 962501480402858803
2*a^9*b^17*c^12 - 45207702606568226816*a^10*b^15*c^13 + 1680270722876120760
32*a^11*b^13*c^14 - 487882094458626375680*a^12*b^11*c^15 + 1082673222923122
114560*a^13*b^9*c^16 - 1771946621413479153664*a^14*b^7*c^17 + 2014068018680
264916992*a^15*b^5*c^18 - 1418770116510434197504*a^16*b^3*c^19)/(268435456*
(a^6*b^28 + 268435456*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296
*a^9*b^22*c^3 + 256256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^1
2*b^16*c^6 - 56229888*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288*a
^15*b^10*c^9 + 1049624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 152672
6656*a^18*b^4*c^12 - 939524096*a^19*b^2*c^13)) - (x^(1/2)*(-(625*b^37 - 625
b^12(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a
^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400
*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13
317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a
^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^1
3*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14
- 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 3557
7189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) -

52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(2378463553205043200*a^18*c^19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 - 814718386176*a^5*b^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476672*a^7*b^22*c^8 + 2159815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c^10 + 98862579421544448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12 + 1635439433677275136*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 + 9201889778671288320*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 13841602348490686464*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18)*1i)/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(3/4)*1i - (x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 + 99803558400000*a^7*b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^9*c^13 + 1975414457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483712000*a^6*b^3*c^16))/(4194304*(a^6*b^24 + 1677721

6*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 1267
20*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*
a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^
16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-(4*a*c - b
^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 5775
8230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 18
8531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b
^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 17
37502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 154225939
91966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 488512278862233
60*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17
*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c -
380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c
 - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578
600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^
25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c +
 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 1587609
6*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255
569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c
^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 52022791
37280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*
b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 1
3056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*1i + (((
2097152000*a*b^33*c^4 + 466178856428188467200*a^17*b*c^20 - 151833804800*a^
2*b^31*c^5 + 5340020080640*a^3*b^29*c^6 - 120300087803904*a^4*b^27*c^7 + 19
33149881761792*a^5*b^25*c^8 - 23398590986584064*a^6*b^23*c^9 + 219878252263
505920*a^7*b^21*c^10 - 1631099300505190400*a^8*b^19*c^11 + 9625014804028588
032*a^9*b^17*c^12 - 45207702606568226816*a^10*b^15*c^13 + 16802707228761207
6032*a^11*b^13*c^14 - 487882094458626375680*a^12*b^11*c^15 + 10826732229231
22114560*a^13*b^9*c^16 - 1771946621413479153664*a^14*b^7*c^17 + 20140680186
80264916992*a^15*b^5*c^18 - 1418770116510434197504*a^16*b^3*c^19)/(26843545
6*(a^6*b^28 + 268435456*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 232
96*a^9*b^22*c^3 + 256256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a
^12*b^16*c^6 - 56229888*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288
*a^15*b^10*c^9 + 1049624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526
726656*a^18*b^4*c^12 - 939524096*a^19*b^2*c^13)) + (x^(1/2)*(-(625*b^37 - 6
25*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275
*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 162857494
00*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 +
13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440
*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b
^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^
14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35
577189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2)
 - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*

a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2
)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^1
0*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^2
0 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^1
3*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*
a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 1937
30707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*
b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 -
16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 195850508697
60*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^
19)))^(1/4)*(2378463553205043200*a^18*c^19 - 419430400*a^3*b^30*c^4 + 26675
773440*a^4*b^28*c^5 - 814718386176*a^5*b^26*c^6 + 15745652097024*a^6*b^24*c
^7 - 214134184476672*a^7*b^22*c^8 + 2159815572848640*a^8*b^20*c^9 - 1661536
0157450240*a^9*b^18*c^10 + 98862579421544448*a^10*b^16*c^11 - 4569839705385
86112*a^11*b^14*c^12 + 1635439433677275136*a^12*b^12*c^13 - 448054836609417
2160*a^13*b^10*c^14 + 9201889778671288320*a^14*b^8*c^15 - 13675039531022155
776*a^15*b^6*c^16 + 13841602348490686464*a^16*b^4*c^17 - 850251462149878579
2*a^17*b^2*c^18)*1i)/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*
c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*
a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a
^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^1
7*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 1127902032
6912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 110995
4201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 -
1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a
^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c
^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 +
31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 5272536
0025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*
c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*
c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 285452
01*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c -
b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*
b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 7296
0*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 15876096
0*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 440
29706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^
18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 104
04558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120
*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^1
8 - 5497558138880*a^28*b^2*c^19)))^(3/4)*1i + (x^(1/2)*(30525625*b^15*c^10
- 1297573875*a*b^13*c^11 + 99803558400000*a^7*b*c^17 + 27786809400*a^2*b^11
*c^12 - 311511417680*a^3*b^9*c^13 + 1975414457856*a^4*b^7*c^14 - 4753980591
360*a^5*b^5*c^15 - 10990483712000*a^6*b^3*c^16))/(4194304*(a^6*b^24 + 16777
216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 12

6720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 1297612
8*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*
a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 - 625*b^12*(-(4*a*c -
 b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57
758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 +
188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8
*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 -
1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 1542259
3991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 4885122788622
3360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^
17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c
- 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a
*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 1215
78600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2
)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c
 + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876
096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 82
55569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20
*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 520227
9137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^2
4*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 +
 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*1i - (
80318101760000000*a^7*c^19 - 6746163125*b^14*c^12 + 572489781500*a*b^12*c^1
3 - 15194313373200*a^2*b^10*c^14 + 226647361174720*a^3*b^8*c^15 - 209583005
7168640*a^4*b^6*c^16 + 12493373163648000*a^5*b^4*c^17 - 44688231411200000*a
^6*b^2*c^18)/(134217728*(a^6*b^28 + 268435456*a^20*c^14 - 56*a^7*b^26*c + 1
456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^4 - 2050048*a^11
*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^7 + 196804608*a^1
4*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*c^10 - 152672665
6*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^19*b^2*c^13))))*(-
(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c
^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^
4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^
7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 41
9701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541
921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 3176436974328217
6*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*
b^5*c^16 - 35577189126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b
^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/
2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(
-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2)
+ 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 109951162
7776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3
+ 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6
- 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^

22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 211342
5899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23
*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 -
19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 549755813888
0*a^28*b^2*c^19)))^(1/4) + 2*atan(((((2097152000*a*b^33*c^4 + 4661788564281
88467200*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340020080640*a^3*b^29*c
^6 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c^8 - 2339859
0986584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 16310993005051
90400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 4520770260656822
6816*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 487882094458626
375680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 - 177194662141
3479153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18 - 141877011
6510434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a^20*c^14 - 5
6*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^
4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^7
+ 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*
c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^19
*b^2*c^13)) - (x^(1/2)*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 1
1279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3
+ 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^
25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 8262933
8933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^
11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^1
1*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15
+ 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 28561
0000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^
2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2)
+ 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(
-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554
432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c
^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 +
158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*
c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 70447529984
0*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c
^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809
116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^2
7*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(2378463553205043200*a^18
*c^19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 - 814718386176*a^
5*b^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476672*a^7*b^22*c^8 + 2
159815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c^10 + 9886257942
1544448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12 + 16354394336772
75136*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 + 920188977867128
8320*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 13841602348490686
464*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18)*1i)/(4194304*(a^6*b^
24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^1

8*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(3/4)*1i - (x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 + 99803558400000*a^7*b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^9*c^13 + 1975414457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483712000*a^6*b^3*c^16))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 +

2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 104045582745
60*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8
*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 54975
58138880*a^28*b^2*c^19)))^(1/4) - (((2097152000*a*b^33*c^4 + 46617885642818
8467200*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340020080640*a^3*b^29*c^
6 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c^8 - 23398590
986584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 163109930050519
0400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 452077026065682268
16*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 4878820944586263
75680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 - 1771946621413
479153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18 - 1418770116
510434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a^20*c^14 - 56
*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^4
 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^7
+ 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*c
^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^19*
b^2*c^13)) + (x^(1/2)*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11
279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3
+ 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^2
5*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338
933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^1
1*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11
*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 +
 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610
000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2
*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2)
+ 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-
(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(335544
32*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^
2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 +
158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c
^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840
*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^
13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 208091
16549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27
*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(2378463553205043200*a^18*
c^19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 - 814718386176*a^5
*b^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476672*a^7*b^22*c^8 + 21
59815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c^10 + 98862579421
544448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12 + 163543943367727
5136*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 + 9201889778671288
320*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 13841602348490686
464*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18)*1i)/(4194304*(a^6*b^2
4 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18
*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6

- 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(3/4)*1i + (x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 + 99803558400000*a^7*b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^9*c^13 + 1975414457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483712000*a^6*b^3*c^16))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 1040455827456

0*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*
c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 549755
8138880*a^28*b^2*c^19)))^(1/4))/((((2097152000*a*b^33*c^4 + 466178856428188
467200*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340020080640*a^3*b^29*c^6
 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c^8 - 233985909
86584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 1631099300505190
400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 4520770260656822681
6*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 48788209445862637
5680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 - 17719466214134
79153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18 - 14187701165
10434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a^20*c^14 - 56*
a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^4
 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^7 +
 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*c^
10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^19*b
^2*c^13)) - (x^(1/2)*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 112
79020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 +
 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25
*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 826293389
33248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11
*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*
c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 +
52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 2856100
00*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*
(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) +
 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(
4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(3355443
2*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2
 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 1
58760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^
8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*
a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^1
3 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 2080911
6549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*
b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(2378463553205043200*a^18*c
^19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 - 814718386176*a^5*
b^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476672*a^7*b^22*c^8 + 215
9815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c^10 + 988625794215
44448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12 + 1635439433677275
136*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 + 92018897786712883
20*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 1384160234849068646
4*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18)*1i)/(4194304*(a^6*b^24
 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*
c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 -
 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 6

9206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(3/4)*1i - (x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 + 99803558400000*a^7*b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^9*c^13 + 1975414457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483712000*a^6*b^3*c^16))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c

^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558
138880*a^28*b^2*c^19)))^(1/4)*1i + (((2097152000*a*b^33*c^4 + 4661788564281
88467200*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340020080640*a^3*b^29*c
^6 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c^8 - 2339859
0986584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 16310993005051
90400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 4520770260656822б
816*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 487882094458626
375680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 - 177194662141
3479153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18 - 141877011
6510434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a^20*c^14 - 5
6*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^20*c^
4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^14*c^7
 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16*b^8*
c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096*a^19
*b^2*c^13)) + (x^(1/2)*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 1
1279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3
 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^
25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 8262933
8933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^
11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^1
1*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15
 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 28561
0000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^
2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2)
 + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(
-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554
432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c
^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 +
 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*
c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 70447529984
0*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c
^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809
116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^2
7*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(237846355320504320O*a^18
*c^19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 - 814718386176*a^
5*b^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476672*a^7*b^22*c^8 + 2
159815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c^10 + 9886257942
1544448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12 + 16354394336772
75136*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 + 920188977867128
8320*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 13841602348490686
464*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18)*1i)/(4194304*(a^6*b^
24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^1
8*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6
 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 +
 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 + 625*b^12*

(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(3/4)*1i + (x^(1/2)*(30525625*b^15*c^10 - 1297573875*a*b^13*c^11 + 99803558400000*a^7*b*c^17 + 27786809400*a^2*b^11*c^12 - 311511417680*a^3*b^9*c^13 + 1975414457856*a^4*b^7*c^14 - 4753980591360*a^5*b^5*c^15 - 10990483712000*a^6*b^3*c^16))/(4194304*(a^6*b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056700579840*a^27*b^4*c^18 - 54975

```
58138880*a^28*b^2*c^19)))^(1/4)*1i - (80318101760000000*a^7*c^19 - 67461631
25*b^14*c^12 + 572489781500*a*b^12*c^13 - 15194313373200*a^2*b^10*c^14 + 22
6647361174720*a^3*b^8*c^15 - 2095830057168640*a^4*b^6*c^16 + 12493373163648
000*a^5*b^4*c^17 - 44688231411200000*a^6*b^2*c^18)/(134217728*(a^6*b^28 + 2
68435456*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3
 + 256256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 -
56229888*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9
+ 1049624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4
*c^12 - 939524096*a^19*b^2*c^13))))*(-(625*b^37 + 625*b^12*(-(4*a*c - b^2)^
25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230
*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531
780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*
c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 173750
2295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 1542259399196
6720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a
^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3
*c^17 + 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c + 3807
75*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) - 4075730*a^3*b^6*c^3*(-(4*a*c - b
^2)^25)^(1/2) + 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 121578600*
a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) - 21375*a*b^10*c*(-(4*a*c - b^2)^25)^
(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 304
0*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^
14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 82555699
20*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10
- 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 520227913728
0*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10
*c^15 + 20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 13056
700579840*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.1088 $\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=658

$$\frac{\sqrt{x}\left(60a^2c^2+bcx^2\left(7b^2-52ac\right)-55ab^2c+7b^4\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3c^{3/4}\left(280a^2c^2-66ab^2c-b\left(7b^2-52ac\right)\sqrt{b^2-4ac}+7b^4\right)\tan}{32\sqrt[4]{2}\,a^2\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[Out] 3/64*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*
(7*b^4-66*a*b^2*c+280*a^2*c^2-b*(-52*a*c+7*b^2)*(-4*a*c+b^2)^(1/2))*2^(3/4)
/a^2/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+3/64*c^(3/4)*arctanh(
2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(7*b^4-66*a*b^2*c+28
0*a^2*c^2-b*(-52*a*c+7*b^2)*(-4*a*c+b^2)^(1/2))*2^(3/4)/a^2/(-4*a*c+b^2)^(5
/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-3/64*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/
2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(7*b^4-66*a*b^2*c+280*a^2*c^2+b*(-52*a*c+
7*b^2)*(-4*a*c+b^2)^(1/2))*2^(3/4)/a^2/(-4*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^
(1/2))^(3/4)-3/64*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^
(1/2))^(1/4))*(7*b^4-66*a*b^2*c+280*a^2*c^2+b*(-52*a*c+7*b^2)*(-4*a*c+b^2)^
(1/2))*2^(3/4)/a^2/(-4*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*(b*
c*x^2-2*a*c+b^2)*x^(1/2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/16*(7*b^4-55*a*
b^2*c+60*a^2*c^2+b*c*(-52*a*c+7*b^2)*x^2)*x^(1/2)/a^2/(-4*a*c+b^2)^2/(c*x^4
+b*x^2+a)
```

Rubi [A] time = 5.79, antiderivative size = 658, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1115, 1345, 1430, 1422, 212, 208, 205}

$$\frac{\sqrt{x}\left(60a^2c^2+bcx^2\left(7b^2-52ac\right)-55ab^2c+7b^4\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3c^{3/4}\left(280a^2c^2-66ab^2c-b\left(7b^2-52ac\right)\sqrt{b^2-4ac}+7b^4\right)\tan}{32\sqrt[4]{2}\,a^2\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2)
 + (Sqrt[x]*(7*b^4 - 55*a*b^2*c + 60*a^2*c^2 + b*c*(7*b^2 - 52*a*c)*x^2))/(
16*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*c^(3/4)*(7*b^4 - 66*a*b^2*
c + 280*a^2*c^2 - b*(7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(
1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c
```

```
)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(7*b^4 - 66*a*b^2*c +
280*a^2*c^2 + b*(7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c)^(5
/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (3*c^(3/4)*(7*b^4 - 66*a*b^2*c + 280*
a^2*c^2 - b*(7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sq
rt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c)^(5/2)
*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(7*b^4 - 66*a*b^2*c + 280*a^2
*c^2 + b*(7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[
x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-
b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1345

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(
x*(b^2 - 2*a*c + b*c*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^
2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + n*
(p + 1)*(b^2 - 4*a*c) + b*c*(n*(2*p + 3) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^
(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*
c, 0] && ILtQ[p, -1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1430

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> -Simp[(x*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*(
a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a
*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d*(
2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n + c
*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\mathrm{Subst}\left(\int \frac{1}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\mathrm{Subst}\left(\int \frac{b^2-2ac-8\left(b^2-4ac\right)-11bcx^4}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4a\left(b^2-4ac\right)}$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(7b^4-55ab^2c+60a^2c^2+bc\left(7b^2-52ac\right)x^2\right.}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(7b^4-55ab^2c+60a^2c^2+bc\left(7b^2-52ac\right)x^2\right.}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(7b^4-55ab^2c+60a^2c^2+bc\left(7b^2-52ac\right)x^2\right.}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(7b^4-55ab^2c+60a^2c^2+bc\left(7b^2-52ac\right)x^2\right.}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Mathematica [C] time = 0.46, size = 258, normalized size = 0.39

$$\frac{3\mathrm{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{-52\#1^4abc^2\log\left(\sqrt{x}-\#1\right)+7\#1^4b^3c\log\left(\sqrt{x}-\#1\right)+140a^2c^2\log\left(\sqrt{x}-\#1\right)-59ab^2c\log\left(\sqrt{x}-\#1\right)+7b^4\log\left(\sqrt{x}\right.}{2\#1^7c+\#1^3b}\right.}{64a^2\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-16*a*(-b^2 + 4*a*c)*Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(a + b*x^2 + c*x^4)
^2 + (4*Sqrt[x]*(7*b^4 - 55*a*b^2*c + 60*a^2*c^2 + 7*b^3*c*x^2 - 52*a*b*c^2
*x^2))/(a + b*x^2 + c*x^4) + 3*RootSum[a + b*#1^4 + c*#1^8 & , (7*b^4*Log[S
qrt[x] - #1] - 59*a*b^2*c*Log[Sqrt[x] - #1] + 140*a^2*c^2*Log[Sqrt[x] - #1]
```

```
 + 7*b^3*c*Log[Sqrt[x] - #1]*#1^4 - 52*a*b*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#
1^3 + 2*c*#1^7) & ])/(64*a^2*(b^2 - 4*a*c)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 191.11Unable to convert to
real 1/4 Error: Bad Argument Value
```

maple [C] time = 0.04, size = 316, normalized size = 0.48

$$\frac{3\left(\left(-52ac+7b^{2}\right)\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{4}bc+140a^{2}c^{2}-59a\,b^{2}c+7b^{4}\right)\ln\left(-\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)+\right.}{64\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)a^{2}\left(2\,\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{7}c+\mathrm{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)^{3}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(1/32*(92*a^2*c^2-79*a*b^2*c+11*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^(1/2)
-1/32*b*(8*a^2*c^2+44*a*b^2*c-7*b^4)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)
+1/32/a^2*c*(60*a^2*c^2-107*a*b^2*c+14*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(9
/2)-1/32*c^2*b*(52*a*c-7*b^2)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(13/2))/(c*x
^4+b*x^2+a)^2+3/64/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*sum((b*c*(-52*a*c+7*b^2)*
_R^4+140*a^2*c^2-59*a*b^2*c+7*b^4)/(2*_R^7*c+_R^3*b)*ln(-_R+x^(1/2)),_R=Roo
tOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\left(7\,b^{4}c^{2}-59\,ab^{2}c^{3}+140\,a^{2}c^{4}\right)x^{\frac{17}{2}}+\left(42\,b^{5}c-347\,ab^{3}c^{2}+788\,a^{2}bc^{3}\right)x^{\frac{13}{2}}+\left(21\,b^{6}-121\,ab^{4}c-41\,a^{2}b^{2}c^{2}+900\,a\right.}{16\left(a^{5}b^{4}-8\,a^{6}b^{2}c+16\,a^{7}c^{2}+\left(a^{3}b^{4}c^{2}-8\,a^{4}b^{2}c^{3}+16\,a^{5}c^{4}\right)x^{8}+2\left(a^{3}b^{5}c-8\,a^{4}b^{3}c^{2}+16\,a^{5}bc^{3}\right)x^{6}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")

[Out] 1/16*(3*(7*b^4*c^2 - 59*a*b^2*c^3 + 140*a^2*c^4)*x^(17/2) + (42*b^5*c - 347*a*b^3*c^2 + 788*a^2*b*c^3)*x^(13/2) + (21*b^6 - 121*a*b^4*c - 41*a^2*b^2*c^2 + 900*a^3*c^3)*x^(9/2) + (49*a*b^5 - 398*a^2*b^3*c + 832*a^3*b*c^2)*x^(5/2) + 32*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*sqrt(x))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + (a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^2) - integrate(3/32*((7*b^4*c - 59*a*b^2*c^2 + 140*a^2*c^3)*x^(7/2) + (7*b^5 - 66*a*b^3*c + 192*a^2*b*c^2)*x^(3/2))/(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^4 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2), x)

mupad [B] time = 9.85, size = 60099, normalized size = 91.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(1/2)*(a + b*x^2 + c*x^4)^3),x)

[Out] ((x^(9/2)*(14*b^4*c + 60*a^2*c^3 - 107*a*b^2*c^2))/(16*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^(1/2)*(11*b^4 + 92*a^2*c^2 - 79*a*b^2*c))/(16*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^(5/2)*(8*a^2*b*c^2 - 7*b^5 + 44*a*b^3*c))/(16*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (b*c^2*x^(13/2)*(52*a*c - 7*b^2))/(16*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - atan((((((9*x^(1/2)*(1546704997025054720*a^19*b*c^19 - 822083584*a^4*b^31*c^4 + 50851741696*a^5*b^29*c^5 - 1473677099008*a^6*b^27*c^6 + 26523687976960*a^7*b^25*c^7 - 331351626612736*a^8*b^23*c^8 + 3041476258824192*a^9*b^21*c^9 - 21176692735213568*a^10*b^19*c^10 + 113812892427485184*a^11*b^17*c^11 - 475720885626470400*a^12*b^15*c^12 + 1545406748670558208*a^13*b^13*c^13 - 3867206695260258304*a^14*b^11*c^14 + 7315227880965799936*a^15*b^9*c^15 - 10117494892562219008*a^16*b^7*c^16 + 9650897342106173440*a^17*b^5*c^17 - 5672002255696429056*a^18*b^3*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) - (3*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^1

2*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*(3377699720527872*a^19*b*c^16 + 117440512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5 + 132070244352*a^9*b^21*c^6 - 1828045455360*a^10*b^19*c^7 + 17136919511040*a^11*b^17*c^8 - 114572547588096*a^12*b^15*c^9 + 559926296444928*a^13*b^13*c^10 - 2014580179992576*a^14*b^11*c^11 + 5294148487741440*a^15*b^9*c^12 - 9906599766261760*a^16*b^7*c^13 + 12525636463624192*a^17*b^5*c^14 - 9605333580251136*a^18*b^3*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(3/4) + (3*(4356374400000*a^8*c^16 + 18475695*b^16*c^8 - 685712223*a*b^14* c^9 + 11424393414*a^2*b^12*c^10 - 110892005343*a^3*b^10*c^11 + 681741235260 *a^4*b^8*c^12 - 2694857597280*a^5*b^6*c^13 + 6582295198080*a^6*b^4*c^14 - 8 763424992000*a^7*b^2*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^1 6*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024 *a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2 *c^8)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566 784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 311254 4495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600* a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17* c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836 636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 112249500440 98560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a *b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c ^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/ 2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6 *(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33 554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^ 36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c ^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b ^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 7044752 99840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^ 14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 2 0809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840 *a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4) - (9*x^(1/2)*(1219784 832000000*a^8*c^19 + 1755191025*b^16*c^11 - 67599928620*a*b^14*c^12 + 11724 33971394*a^2*b^12*c^13 - 11911732472304*a^3*b^10*c^14 + 77626373024736*a^4* b^8*c^15 - 333603251301888*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17 - 15 56843742720000*a^7*b^2*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48* a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^ 4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 5 0331648*a^19*b^2*c^11)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3* b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 40693634220 0*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551 027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200 *a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15 *b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^ 17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25) ^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a *c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 1080

25400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2
)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38
*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 158
76096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 +
8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^
20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202
279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a
^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17
 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*1i +
 ((((9*x^(1/2)*(1546704997025054720*a^19*b*c^19 - 822083584*a^4*b^31*c^4 +
50851741696*a^5*b^29*c^5 - 1473677099008*a^6*b^27*c^6 + 26523687976960*a^7*
b^25*c^7 - 331351626612736*a^8*b^23*c^8 + 3041476258824192*a^9*b^21*c^9 - 2
1176692735213568*a^10*b^19*c^10 + 113812892427485184*a^11*b^17*c^11 - 47572
0885626470400*a^12*b^15*c^12 + 1545406748670558208*a^13*b^13*c^13 - 3867206
695260258304*a^14*b^11*c^14 + 7315227880965799936*a^15*b^9*c^15 - 101174948
92562219008*a^16*b^7*c^16 + 9650897342106173440*a^17*b^5*c^17 - 56720022556
96429056*a^18*b^3*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b
^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 8
11008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 3244
0320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 503316
48*a^19*b^2*c^11)) + (3*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1
/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b
^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200
*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 -
113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 17489235510
27200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*
a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*
b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^1
7 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^
(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7
556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*
c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 10802
5400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)
^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*
c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 1587
6096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8
255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^2
0*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 52022
79137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^
26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17
+ 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*(3377
699720527872*a^19*b*c^16 + 117440512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5
 + 132070244352*a^9*b^21*c^6 - 1828045455360*a^10*b^19*c^7 + 17136919511040
*a^11*b^17*c^8 - 114572547588096*a^12*b^15*c^9 + 559926296444928*a^13*b^13*
c^10 - 2014580179992576*a^14*b^11*c^11 + 5294148487741440*a^15*b^9*c^12 - 9

906599766261760*a^16*b^7*c^13 + 12525636463624192*a^17*b^5*c^14 - 960533358
0251136*a^18*b^3*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c
+ 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^1
3*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8
)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 24054165667840
00*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495
*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276
813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*
b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11
 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 216
83350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 328366360
93972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 1122495004409856
0*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^3
7*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(
-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) +
 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(
4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(335544
32*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c
^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 +
 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*
c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 70447529984
0*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c
^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809
116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^2
9*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(3/4) - (3*(4356374400000*a^8*c
^16 + 18475695*b^16*c^8 - 685712223*a*b^14*c^9 + 11424393414*a^2*b^12*c^10
- 110892005343*a^3*b^10*c^11 + 681741235260*a^4*b^8*c^12 - 2694857597280*a^
5*b^6*c^13 + 6582295198080*a^6*b^4*c^14 - 8763424992000*a^7*b^2*c^15))/(655
36*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a
^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*
c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 - 2401*
b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2
*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*
a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 213
41140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a
^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^1
5*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^1
4 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 243
59874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a
^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-
(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 3
4052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a
*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 737
45*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 109951162777
6*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1

240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1
270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*
c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 211342589
9520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^
12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19
585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a
^30*b^2*c^19)))^(1/4) - (9*x^(1/2)*(1219784832000000*a^8*c^19 + 1755191025*
b^16*c^11 - 67599928620*a*b^14*c^12 + 1172433971394*a^2*b^12*c^13 - 1191173
2472304*a^3*b^10*c^14 + 77626373024736*a^4*b^8*c^15 - 333603251301888*a^5*b
^6*c^16 + 930302051212800*a^6*b^4*c^17 - 1556843742720000*a^7*b^2*c^18))/(4
194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2
- 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 37847
04*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 5767168
0*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)))*(-(81*(
2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c
^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c
^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a
^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 +
492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 50526441
61945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470
080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^
16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*
c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 99666
0*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b
^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a
^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2
)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^
40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*
a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*
a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029
706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18
*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404
558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a
^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18
- 5497558138880*a^30*b^2*c^19)))^(1/4)*1i)/(((((9*x^(1/2)*(1546704997025054
720*a^19*b*c^19 - 822083584*a^4*b^31*c^4 + 50851741696*a^5*b^29*c^5 - 14736
77099008*a^6*b^27*c^6 + 26523687976960*a^7*b^25*c^7 - 331351626612736*a^8*b
^23*c^8 + 3041476258824192*a^9*b^21*c^9 - 21176692735213568*a^10*b^19*c^10
+ 113812892427485184*a^11*b^17*c^11 - 475720885626470400*a^12*b^15*c^12 + 1
545406748670558208*a^13*b^13*c^13 - 3867206695260258304*a^14*b^11*c^14 + 73
15227880965799936*a^15*b^9*c^15 - 10117494892562219008*a^16*b^7*c^16 + 9650
897342106173440*a^17*b^5*c^17 - 5672002255696429056*a^18*b^3*c^18))/(419430
4*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 140
80*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^
14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^1

7*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) - (3*(-(81*(2
401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^
19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^
4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^
7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 4
92398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 505264416
1945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 216833504234700
80*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^1
6*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c
^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660
*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^
2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^
5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)
^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^4
0 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a
^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a
^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 440297
06240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*
c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 104045
58274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^
27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 -
 5497558138880*a^30*b^2*c^19)))^(1/4)*(3377699720527872*a^19*b*c^16 + 11744
0512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5 + 132070244352*a^9*b^21*c^6 - 1
828045455360*a^10*b^19*c^7 + 17136919511040*a^11*b^17*c^8 - 114572547588096
*a^12*b^15*c^9 + 559926296444928*a^13*b^13*c^10 - 2014580179992576*a^14*b^1
1*c^11 + 5294148487741440*a^15*b^9*c^12 - 9906599766261760*a^16*b^7*c^13 +
12525636463624192*a^17*b^5*c^14 - 9605333580251136*a^18*b^3*c^15))/(65536*(
a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*
b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6
 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 - 2401*b^14
*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^3
5*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*
b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 2134114
0889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*
b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^
12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 -
30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 2435987
4477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c
^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a
*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052
295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c -
 b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a
*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^
31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 12403
20*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 12700

87680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(3/4) + (3*(4356374400000*a^8*c^16 + 18475695*b^16*c^8 - 685712223*a*b^14*c^9 + 11424393414*a^2*b^12*c^10 - 110892005343*a^3*b^10*c^11 + 681741235260*a^4*b^8*c^12 - 2694857597280*a^5*b^6*c^13 + 6582295198080*a^6*b^4*c^14 - 8763424992000*a^7*b^2*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4) - (9*x^(1/2)*(1219784832000000*a^8*c^19 + 1755191025*b^16*c^11 - 67599928620*a*b^14*c^12 + 1172433971394*a^2*b^12*c^13 - 11911732472304*a^3*b^10*c^14 + 77626373024736*a^4*b^8*c^15 - 333603251301888*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17 - 1556843742720000*a^7*b^2*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 117565

81147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701
511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*
a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c
 - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^2
5)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6
c^4(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(
1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(
4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 8
0*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^3
2*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*
b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707
456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*
c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647
293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^
28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))
^(1/4) - ((((9*x^(1/2)*(1546704997025054720*a^19*b*c^19 - 822083584*a^4*b^3
1*c^4 + 50851741696*a^5*b^29*c^5 - 1473677099008*a^6*b^27*c^6 + 26523687976
960*a^7*b^25*c^7 - 331351626612736*a^8*b^23*c^8 + 3041476258824192*a^9*b^21
*c^9 - 21176692735213568*a^10*b^19*c^10 + 113812892427485184*a^11*b^17*c^11
 - 475720885626470400*a^12*b^15*c^12 + 1545406748670558208*a^13*b^13*c^13 -
 3867206695260258304*a^14*b^11*c^14 + 7315227880965799936*a^15*b^9*c^15 - 1
0117494892562219008*a^16*b^7*c^16 + 9650897342106173440*a^17*b^5*c^17 - 567
2002255696429056*a^18*b^3*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 -
48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16
*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^
7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10
 - 50331648*a^19*b^2*c^11)) + (3*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2
)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 1808519
65*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 4069
36342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^2
3*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 174
8923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 1175658114
7443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 309290257015111
68*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17
*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b
^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(
1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4
*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2)
 - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*
c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^
12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^
4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26
*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*
a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12
 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 166472932

39296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*(3377699720527872*a^19*b*c^16 + 117440512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5 + 132070244352*a^9*b^21*c^6 - 1828045455360*a^10*b^19*c^7 + 17136919511040*a^11*b^17*c^8 - 114572547588096*a^12*b^15*c^9 + 559926296444928*a^13*b^13*c^10 - 2014580179992576*a^14*b^11*c^11 + 5294148487741440*a^15*b^9*c^12 - 9906599766261760*a^16*b^7*c^13 + 12525636463624192*a^17*b^5*c^14 - 9605333580251136*a^18*b^3*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(3/4) - (3*(4356374400000*a^8*c^16 + 18475695*b^16*c^8 - 685712223*a*b^14*c^9 + 11424393414*a^2*b^12*c^10 - 110892005343*a^3*b^10*c^11 + 681741235260*a^4*b^8*c^12 - 2694857597280*a^5*b^6*c^13 + 6582295198080*a^6*b^4*c^14 - 8763424992000*a^7*b^2*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24

010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^1
0*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(
1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^
5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/
2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099
511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34
*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28
*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^
20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2
113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560
*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c
^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558
138880*a^30*b^2*c^19)))^(1/4) - (9*x^(1/2)*(1219784832000000*a^8*c^19 + 175
5191025*b^16*c^11 - 67599928620*a*b^14*c^12 + 1172433971394*a^2*b^12*c^13 -
 11911732472304*a^3*b^10*c^14 + 77626373024736*a^4*b^8*c^15 - 3336032513018
88*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17 - 1556843742720000*a^7*b^2*c
^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b
^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5
 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 -
 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)))
(-(81(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*
a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^
4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813
600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^2
1*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 +
5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 216833
50423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 328366360939
72480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a
^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c
 - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4
*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87
808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a
*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*
(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2
- 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 15
8760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8
 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a
^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13
 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116
549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b
^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)))*(-(81*(2401*b^39 - 2401*b^
14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b
^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^
5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341
140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^1

0*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*2i - atan((((((9*x^(1/2)*(154670499702505472 0*a^19*b*c^19 - 822083584*a^4*b^31*c^4 + 50851741696*a^5*b^29*c^5 - 1473677099008*a^6*b^27*c^6 + 26523687976960*a^7*b^25*c^7 - 331351626612736*a^8*b^23*c^8 + 3041476258824192*a^9*b^21*c^9 - 21176692735213568*a^10*b^19*c^10 + 113812892427485184*a^11*b^17*c^11 - 475720885626470400*a^12*b^15*c^12 + 1545406748670558208*a^13*b^13*c^13 - 3867206695260258304*a^14*b^11*c^14 + 7315227880965799936*a^15*b^9*c^15 - 10117494892562219008*a^16*b^7*c^16 + 9650897342106173440*a^17*b^5*c^17 - 5672002255696429056*a^18*b^3*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) - (3*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 211342

5899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*(3377699720527872*a^19*b*c^16 + 117440512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5 + 132070244352*a^9*b^21*c^6 - 1828045455360*a^10*b^19*c^7 + 17136919511040*a^11*b^17*c^8 - 114572547588096*a^12*b^15*c^9 + 559926296444928*a^13*b^13*c^10 - 2014580179992576*a^14*b^11*c^11 + 5294148487741440*a^15*b^9*c^12 - 9906599766261760*a^16*b^7*c^13 + 12525636463624192*a^17*b^5*c^14 - 9605333580251136*a^18*b^3*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(3/4) + (3*(4356374400000*a^8*c^16 + 18475695*b^16*c^8 - 685712223*a*b^14*c^9 + 11424393414*a^2*b^12*c^10 - 110892005343*a^3*b^10*c^11 + 681741235260*a^4*b^8*c^12 - 2694857597280*a^5*b^6*c^13 + 6582295198080*a^6*b^4*c^14 - 8763424992000*a^7*b^2*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 3283663

6093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098
560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b
^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3
*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2)
 - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(
-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(3355
4432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36
*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5
 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^2
4*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299
840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14
*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 208
09116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a
^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4) - (9*x^(1/2)*(121978483
2000000*a^8*c^19 + 1755191025*b^16*c^11 - 67599928620*a*b^14*c^12 + 1172433
971394*a^2*b^12*c^13 - 11911732472304*a^3*b^10*c^14 + 77626373024736*a^4*b^
8*c^15 - 333603251301888*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17 - 1556
843742720000*a^7*b^2*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^
9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4
 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 3
2440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 503
31648*a^19*b^2*c^11)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/
2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^
33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*
a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 1
13330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 174892355102
7200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a
^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b
^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17
 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(
1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 75
56115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c
 - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025
400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^
25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c
 + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876
096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 82
55569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20
*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 520227
9137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^2
6*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 +
 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*1i + (
(((9*x^(1/2)*(15467049970250547 20*a^19*b*c^19 - 822083584*a^4*b^31*c^4 + 50
851741696*a^5*b^29*c^5 - 1473677099008*a^6*b^27*c^6 + 26523687976960*a^7*b^
25*c^7 - 331351626612736*a^8*b^23*c^8 + 3041476258824192*a^9*b^21*c^9 - 211

76692735213568*a^10*b^19*c^10 + 113812892427485184*a^11*b^17*c^11 - 4757208
85626470400*a^12*b^15*c^12 + 1545406748670558208*a^13*b^13*c^13 - 386720669
5260258304*a^14*b^11*c^14 + 7315227880965799936*a^15*b^9*c^15 - 10117494892
562219008*a^16*b^7*c^16 + 9650897342106173440*a^17*b^5*c^17 - 5672002255696
429056*a^18*b^3*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^2
2*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811
008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 324403
20*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648
*a^19*b^2*c^11)) + (3*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2
) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^3
3*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a
^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 11
3330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027
200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^
13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^
9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17
+ 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1
/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 755
6115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c
- b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 1080254
00*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^2
5)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c
+ 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 158760
96*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 825
5569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*
c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279
137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26
*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 +
13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*(337769
9720527872*a^19*b*c^16 + 117440512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5 +
 132070244352*a^9*b^21*c^6 - 1828045455360*a^10*b^19*c^7 + 17136919511040*a
^11*b^17*c^8 - 114572547588096*a^12*b^15*c^9 + 559926296444928*a^13*b^13*c^
10 - 2014580179992576*a^14*b^11*c^11 + 5294148487741440*a^15*b^9*c^12 - 990
6599766261760*a^16*b^7*c^13 + 12525636463624192*a^17*b^5*c^14 - 96053335802
51136*a^18*b^3*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c +
576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*
b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8))
)*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000
*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a
^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 327681
3600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^
21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 +
 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683
350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093
972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*

a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*
c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(
4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 8
7808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*
a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432
*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2
 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 1
58760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^
8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*
a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^1
3 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 2080911
6549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*
b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(3/4) - (3*(4356374400000*a^8*c^1
6 + 18475695*b^16*c^8 - 685712223*a*b^14*c^9 + 11424393414*a^2*b^12*c^10 -
110892005343*a^3*b^10*c^11 + 681741235260*a^4*b^8*c^12 - 2694857597280*a^5*
b^6*c^13 + 6582295198080*a^6*b^4*c^14 - 8763424992000*a^7*b^2*c^15))/(65536
*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^1
1*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^
6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 + 2401*b^
14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b
^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^
5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341
140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^1
0*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*
c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14
 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359
874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7
c^7(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4
*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 340
52295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c
 - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745
*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*
a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 124
0320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 127
0087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^
9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 21134258995
20*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12
*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 1958
5050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^3
0*b^2*c^19)))^(1/4) - (9*x^(1/2)*(1219784832000000*a^8*c^19 + 1755191025*b^
16*c^11 - 67599928620*a*b^14*c^12 + 1172433971394*a^2*b^12*c^13 - 119117324
72304*a^3*b^10*c^14 + 77626373024736*a^4*b^8*c^15 - 333603251301888*a^5*b^6
*c^16 + 930302051212800*a^6*b^4*c^17 - 1556843742720000*a^7*b^2*c^18))/(419
4304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 -
14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704

*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*
a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)))*(-(81*(24
01*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^1
9 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4
 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7
*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 49
2398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161
945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 216833504234700
80*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16
*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^
18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*
a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2
)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5
*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^
25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40
 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^
14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^
17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 4402970
6240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c
^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 1040455
8274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^2
7*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 -
5497558138880*a^30*b^2*c^19)))^(1/4)*1i)/(((((9*x^(1/2)*(154670499702505472
0*a^19*b*c^19 - 822083584*a^4*b^31*c^4 + 50851741696*a^5*b^29*c^5 - 1473677
099008*a^6*b^27*c^6 + 26523687976960*a^7*b^25*c^7 - 331351626612736*a^8*b^2
3*c^8 + 3041476258824192*a^9*b^21*c^9 - 21176692735213568*a^10*b^19*c^10 +
113812892427485184*a^11*b^17*c^11 - 475720885626470400*a^12*b^15*c^12 + 154
5406748670558208*a^13*b^13*c^13 - 3867206695260258304*a^14*b^11*c^14 + 7315
227880965799936*a^15*b^9*c^15 - 10117494892562219008*a^16*b^7*c^16 + 965089
7342106173440*a^17*b^5*c^17 - 5672002255696429056*a^18*b^3*c^18))/(4194304*
(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080
*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14
*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*
b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) - (3*(-(81*(240
1*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19
 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4
- 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*
b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492
398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 50526441619
45600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080
*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*
b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^1
8 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a
^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)
^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*

b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^2
5)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40
+ 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^1
4*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^1
7*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706
240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^
11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558
274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27
*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5
497558138880*a^30*b^2*c^19)))^(1/4)*(3377699720527872*a^19*b*c^16 + 1174405
12*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5 + 132070244352*a^9*b^21*c^6 - 182
8045455360*a^10*b^19*c^7 + 17136919511040*a^11*b^17*c^8 - 114572547588096*a
^12*b^15*c^9 + 559926296444928*a^13*b^13*c^10 - 2014580179992576*a^14*b^11*
c^11 + 5294148487741440*a^15*b^9*c^12 - 9906599766261760*a^16*b^7*c^13 + 12
525636463624192*a^17*b^5*c^14 - 9605333580251136*a^18*b^3*c^15))/(65536*(a^
8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^
12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 -
589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 + 2401*b^14*(
-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*
c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^
29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 213411408
89600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^
19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12
 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30
929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 243598744
77424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7
*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c
 - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 3405229
5*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b
^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b
^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31
*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320
*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087
680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 +
193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a
^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^1
4 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050
869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^
2*c^19)))^(3/4) + (3*(4356374400000*a^8*c^16 + 18475695*b^16*c^8 - 68571222
3*a*b^14*c^9 + 11424393414*a^2*b^12*c^10 - 110892005343*a^3*b^10*c^11 + 681
741235260*a^4*b^8*c^12 - 2694857597280*a^5*b^6*c^13 + 6582295198080*a^6*b^4
*c^14 - 8763424992000*a^7*b^2*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 3
6*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4
 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824
*a^16*b^2*c^8)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2

405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4) - (9*x^(1/2)*(1219784832000000*a^8*c^19 + 1755191025*b^16*c^11 - 67599928620*a*b^14*c^12 + 1172433971394*a^2*b^12*c^13 - 11911732472304*a^3*b^10*c^14 + 77626373024736*a^4*b^8*c^15 - 333603251301888*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17 - 1556843742720000*a^7*b^2*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^

12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 1664729
3239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28
*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(
1/4) - ((((9*x^(1/2)*(1546704997025054720*a^19*b*c^19 - 822083584*a^4*b^31*
c^4 + 50851741696*a^5*b^29*c^5 - 1473677099008*a^6*b^27*c^6 + 2652368797696
0*a^7*b^25*c^7 - 331351626612736*a^8*b^23*c^8 + 3041476258824192*a^9*b^21*c
^9 - 21176692735213568*a^10*b^19*c^10 + 113812892427485184*a^11*b^17*c^11 -
 475720885626470400*a^12*b^15*c^12 + 1545406748670558208*a^13*b^13*c^13 - 3
867206695260258304*a^14*b^11*c^14 + 7315227880965799936*a^15*b^9*c^15 - 101
17494892562219008*a^16*b^7*c^16 + 9650897342106173440*a^17*b^5*c^17 - 56720
02255696429056*a^18*b^3*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48
*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c
^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7
 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 -
50331648*a^19*b^2*c^11)) + (3*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^
25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965
*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936
342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*
c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 17489
23551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 117565811474
43200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168
*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b
^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2
)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/
2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(
-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) +
 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c
- b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12
*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4
- 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c
^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^
21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 -
 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239
296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6
*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)
*(3377699720527872*a^19*b*c^16 + 117440512*a^7*b^25*c^4 - 5804916736*a^8*b^
23*c^5 + 132070244352*a^9*b^21*c^6 - 1828045455360*a^10*b^19*c^7 + 17136919
511040*a^11*b^17*c^8 - 114572547588096*a^12*b^15*c^9 + 559926296444928*a^13
*b^13*c^10 - 2014580179992576*a^14*b^11*c^11 + 5294148487741440*a^15*b^9*c^
12 - 9906599766261760*a^16*b^7*c^13 + 12525636463624192*a^17*b^5*c^14 - 960
5333580251136*a^18*b^3*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b
^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 1290
24*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b
^2*c^8)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 24054165
66784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112

544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6
- 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 11333074802560
0*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^1
7*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13
+ 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 328
36636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 1122495004
4098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795
*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8
c^3(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(
1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c
^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(
33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*
b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30
*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19
*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 70447
5299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*
b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 +
20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 130567005798
40*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(3/4) - (3*(4356374400000
*a^8*c^16 + 18475695*b^16*c^8 - 685712223*a*b^14*c^9 + 11424393414*a^2*b^12
*c^10 - 110892005343*a^3*b^10*c^11 + 681741235260*a^4*b^8*c^12 - 2694857597
280*a^5*b^6*c^13 + 6582295198080*a^6*b^4*c^14 - 8763424992000*a^7*b^2*c^15)
)/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 -
5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^1
4*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 +
2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 74450
60*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 403026
63491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7
+ 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 49239818937
3440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^
12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^
11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16
- 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 2401
0000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*
c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/
2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*
(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2)
- 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 109951
1627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c
^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c
^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20
*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 211
3425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a
^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^1
6 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 549755813

8880*a^30*b^2*c^19)))^(1/4) - (9*x^(1/2)*(1219784832000000*a^8*c^19 + 17551
91025*b^16*c^11 - 67599928620*a*b^14*c^12 + 1172433971394*a^2*b^12*c^13 - 1
1911732472304*a^3*b^10*c^14 + 77626373024736*a^4*b^8*c^15 - 333603251301888
*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17 - 1556843742720000*a^7*b^2*c^1
8))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^2
0*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 +
 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 5
7671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)))*(
-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^
19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*
b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 327681360
0400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*
c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 50
52644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350
423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972
480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^1
8*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c +
 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a
*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 8780
8681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c
 - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a
^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 -
72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 1587
60960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 -
 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^2
2*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 +
 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 2080911654
9120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4
*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)))*(-(81*(2401*b^39 + 2401*b^14
*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^3
5*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*
b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 2134114
0889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*
b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^
12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 -
30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 2435987
4477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c
^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a
*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052
295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c -
 b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a
*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^
31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 12403
20*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 12700
87680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9

+ 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520
*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c
^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 195850
50869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*
b^2*c^19)))^(1/4)*2i - 2*atan((((((9*x^(1/2)*(1546704997025054720*a^19*b*c^
19 - 822083584*a^4*b^31*c^4 + 50851741696*a^5*b^29*c^5 - 1473677099008*a^6*
b^27*c^6 + 26523687976960*a^7*b^25*c^7 - 331351626612736*a^8*b^23*c^8 + 304
1476258824192*a^9*b^21*c^9 - 21176692735213568*a^10*b^19*c^10 + 11381289242
7485184*a^11*b^17*c^11 - 475720885626470400*a^12*b^15*c^12 + 15454067486705
58208*a^13*b^13*c^13 - 3867206695260258304*a^14*b^11*c^14 + 731522788096579
9936*a^15*b^9*c^15 - 10117494892562219008*a^16*b^7*c^16 + 96508973421061734
40*a^17*b^5*c^17 - 5672002255696429056*a^18*b^3*c^18))/(4194304*(a^8*b^24 +
16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*
c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 -
12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 6
9206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) - ((-(81*(2401*b^39 - 2401
b^14(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^
2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491
*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21
341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*
a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^
15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^
14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24
359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*
a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(
-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) -
34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*
a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73
745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 10995116277
76*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 +
1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 -
1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22
*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 21134258
99520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b
^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 1
9585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*
a^30*b^2*c^19)))^(1/4)*(3377699720527872*a^19*b*c^16 + 117440512*a^7*b^25*c
^4 - 5804916736*a^8*b^23*c^5 + 132070244352*a^9*b^21*c^6 - 1828045455360*a^
10*b^19*c^7 + 17136919511040*a^11*b^17*c^8 - 114572547588096*a^12*b^15*c^9
+ 559926296444928*a^13*b^13*c^10 - 2014580179992576*a^14*b^11*c^11 + 529414
8487741440*a^15*b^9*c^12 - 9906599766261760*a^16*b^7*c^13 + 125256364636241
92*a^17*b^5*c^14 - 9605333580251136*a^18*b^3*c^15)*3i)/(65536*(a^8*b^18 - 2
62144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 3
2256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^1
5*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c -

b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 1808 51965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 4 06936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8* b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 1175658 1147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 309290257015 11168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a ^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6* c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1 /2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4 *a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80 *a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32 *c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b ^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 1937307074 56*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c ^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 166472 93239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^2 8*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^ (3/4)*1i - (3*(4356374400000*a^8*c^16 + 18475695*b^16*c^8 - 685712223*a*b^1 4*c^9 + 11424393414*a^2*b^12*c^10 - 110892005343*a^3*b^10*c^11 + 6817412352 60*a^4*b^8*c^12 - 2694857597280*a^5*b^6*c^13 + 6582295198080*a^6*b^4*c^14 - 8763424992000*a^7*b^2*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b ^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 1290 24*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b ^2*c^8)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 24054165 66784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112 544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 11333074802560 0*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^1 7*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 328 36636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 1122495004 4098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795 *a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8 *c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c ^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13* b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30 *c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19 *b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 70447 5299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24* b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 +

20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 130567005798
40*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*1i + (9*x^(1/2)*(12
19784832000000*a^8*c^19 + 1755191025*b^16*c^11 - 67599928620*a*b^14*c^12 +
1172433971394*a^2*b^12*c^13 - 11911732472304*a^3*b^10*c^14 + 77626373024736
*a^4*b^8*c^15 - 333603251301888*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17
- 1556843742720000*a^7*b^2*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12
- 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^
16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*
c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^1
0 - 50331648*a^19*b^2*c^11)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^
25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965
*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936
342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*
c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 17489
23551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 117565811474
43200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168
*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b
^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2
)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/
2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(
-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) -
108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c
- b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12
*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4
- 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c
^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^
21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 -
5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239
296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6
*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)
+ ((((9*x^(1/2)*(1546704997025054720*a^19*b*c^19 - 822083584*a^4*b^31*c^4
+ 50851741696*a^5*b^29*c^5 - 1473677099008*a^6*b^27*c^6 + 26523687976960*a^
7*b^25*c^7 - 331351626612736*a^8*b^23*c^8 + 3041476258824192*a^9*b^21*c^9 -
21176692735213568*a^10*b^19*c^10 + 113812892427485184*a^11*b^17*c^11 - 475
720885626470400*a^12*b^15*c^12 + 1545406748670558208*a^13*b^13*c^13 - 38672
06695260258304*a^14*b^11*c^14 + 7315227880965799936*a^15*b^9*c^15 - 1011749
4892562219008*a^16*b^7*c^16 + 9650897342106173440*a^17*b^5*c^17 - 567200225
5696429056*a^18*b^3*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9
*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 -
811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32
440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 5033
1648*a^19*b^2*c^11)) + ((-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1
/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b
^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200
*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 -

113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 17489235510
27200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*
a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*
b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^1
7 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^
(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7
556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*
c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 10802
5400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)
^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*
c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 1587
6096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8
255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^2
0*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 52022
79137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^
26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17
+ 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*(3377
699720527872*a^19*b*c^16 + 117440512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5
 + 132070244352*a^9*b^21*c^6 - 1828045455360*a^10*b^19*c^7 + 17136919511040
*a^11*b^17*c^8 - 114572547588096*a^12*b^15*c^9 + 559926296444928*a^13*b^13*
c^10 - 2014580179992576*a^14*b^11*c^11 + 5294148487741440*a^15*b^9*c^12 - 9
906599766261760*a^16*b^7*c^13 + 12525636463624192*a^17*b^5*c^14 - 960533358
0251136*a^18*b^3*c^15)*3i)/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16
*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*
a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*
c^8)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 24054165667
84000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544
495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3
276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a
^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c
^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 +
21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 328366
36093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 1122495004409
8560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*
b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^
3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2
) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*
(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(335
54432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^3
6*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^
5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^
24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 70447529
9840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^1
4*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20
809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*

a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(3/4)*1i + (3*(4356374400000*a^8*c^16 + 18475695*b^16*c^8 - 685712223*a*b^14*c^9 + 11424393414*a^2*b^12*c^10 - 110892005343*a^3*b^10*c^11 + 681741235260*a^4*b^8*c^12 - 2694857597280*a^5*b^6*c^13 + 6582295198080*a^6*b^4*c^14 - 8763424992000*a^7*b^2*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*1i + (9*x^(1/2)*(1219784832000000*a^8*c^19 + 1755191025*b^16*c^11 - 67599928620*a*b^14*c^12 + 1172433971394*a^2*b^12*c^13 - 11911732472304*a^3*b^10*c^14 + 77626373024736*a^4*b^8*c^15 - 333603251301888*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17 - 1556843742720000*a^7*b^2*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(

4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 8
7808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*
a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432
*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2
 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 1
58760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^
8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*
a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^1
3 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 2080911
6549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*
b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4))/(((((9*x^(1/2)*(1546704997
025054720*a^19*b*c^19 - 822083584*a^4*b^31*c^4 + 50851741696*a^5*b^29*c^5 -
 1473677099008*a^6*b^27*c^6 + 26523687976960*a^7*b^25*c^7 - 331351626612736
*a^8*b^23*c^8 + 3041476258824192*a^9*b^21*c^9 - 21176692735213568*a^10*b^19
*c^10 + 113812892427485184*a^11*b^17*c^11 - 475720885626470400*a^12*b^15*c^
12 + 1545406748670558208*a^13*b^13*c^13 - 3867206695260258304*a^14*b^11*c^1
4 + 7315227880965799936*a^15*b^9*c^15 - 10117494892562219008*a^16*b^7*c^16
+ 9650897342106173440*a^17*b^5*c^17 - 5672002255696429056*a^18*b^3*c^18))/(
4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2
 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784
704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 576716
80*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) - ((-(8
1*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*
b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^3
1*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 327681360040
0*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9
 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 50526
44161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423
470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480
*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b
^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 99
6660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c
 - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 8780868
1*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c -
b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11
*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 729
60*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 1587609
60*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44
029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b
^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10
404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 2080911654912
0*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^
18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*(3377699720527872*a^19*b*c^16 + 1
17440512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5 + 132070244352*a^9*b^21*c^6
 - 1828045455360*a^10*b^19*c^7 + 17136919511040*a^11*b^17*c^8 - 11457254758

8096*a^12*b^15*c^9 + 559926296444928*a^13*b^13*c^10 - 2014580179992576*a^14 *b^11*c^11 + 5294148487741440*a^15*b^9*c^12 - 9906599766261760*a^16*b^7*c^1 3 + 12525636463624192*a^17*b^5*c^14 - 9605333580251136*a^18*b^3*c^15)*3i)/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 537 6*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b ^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 - 24 01*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060* a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 403026634 91*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 49239818937344 0*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12* b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11* c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 2401000 0*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2 *(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 109951162 7776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^ 22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 211342 5899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25 *b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 549755813888 0*a^30*b^2*c^19)))^(3/4)*1i - (3*(4356374400000*a^8*c^16 + 18475695*b^16*c^ 8 - 685712223*a*b^14*c^9 + 11424393414*a^2*b^12*c^10 - 110892005343*a^3*b^1 0*c^11 + 681741235260*a^4*b^8*c^12 - 2694857597280*a^5*b^6*c^13 + 658229519 8080*a^6*b^4*c^14 - 8763424992000*a^7*b^2*c^15))/(65536*(a^8*b^18 - 262144* a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a ^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4* c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^2 5)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965* a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 4069363 42200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c ^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 174892 3551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 1175658114744 3200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168* a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^ 5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2) ^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(- (4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c -

b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*
b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 -
15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^
7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^2
1*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 -
5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 166472932392
96*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*
c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*
1i + (9*x^(1/2)*(1219784832000000*a^8*c^19 + 1755191025*b^16*c^11 - 6759992
8620*a*b^14*c^12 + 1172433971394*a^2*b^12*c^13 - 11911732472304*a^3*b^10*c^
14 + 77626373024736*a^4*b^8*c^15 - 333603251301888*a^5*b^6*c^16 + 930302051
212800*a^6*b^4*c^17 - 1556843742720000*a^7*b^2*c^18))/(4194304*(a^8*b^24 +
16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c
^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 -
12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69
206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)))*(-(81*(2401*b^39 - 2401*b^
14*(-(4*a*c - b^2)^25)^(1/2) - 240541656784000*a^19*b*c^19 + 7445060*a^2*b
^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^
5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341
140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^1
0*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*
c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14
- 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359
874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7
c^7(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4
*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 340
52295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c
 - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745
*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*
a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 124
0320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 127
0087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^
9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 21134258995
20*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12
*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 1958
5050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^3
0*b^2*c^19)))^(1/4)*1i - ((((9*x^(1/2)*(1546704997025054720*a^19*b*c^19 - 8
22083584*a^4*b^31*c^4 + 50851741696*a^5*b^29*c^5 - 1473677099008*a^6*b^27*c
^6 + 26523687976960*a^7*b^25*c^7 - 331351626612736*a^8*b^23*c^8 + 304147625
8824192*a^9*b^21*c^9 - 21176692735213568*a^10*b^19*c^10 + 11381289242748518
4*a^11*b^17*c^11 - 475720885626470400*a^12*b^15*c^12 + 1545406748670558208*
a^13*b^13*c^13 - 3867206695260258304*a^14*b^11*c^14 + 7315227880965799936*a
^15*b^9*c^15 - 10117494892562219008*a^16*b^7*c^16 + 9650897342106173440*a^1
7*b^5*c^17 - 5672002255696429056*a^18*b^3*c^18))/(4194304*(a^8*b^24 + 16777
216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 +

126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976
128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 6920601
6*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) + ((-(81*(2401*b^39 - 2401*b^14*
(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35
*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b
^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140
889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b
^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^1
2 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 3
0929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874
477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^
7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*
c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 340522
95*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c -
b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*
b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^3
1*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 124032
0*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 127008
7680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 +
193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*
a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^
14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 1958505
0869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b
^2*c^19)))^(1/4)*(3377699720527872*a^19*b*c^16 + 117440512*a^7*b^25*c^4 - 5
804916736*a^8*b^23*c^5 + 132070244352*a^9*b^21*c^6 - 1828045455360*a^10*b^1
9*c^7 + 17136919511040*a^11*b^17*c^8 - 114572547588096*a^12*b^15*c^9 + 5599
26296444928*a^13*b^13*c^10 - 2014580179992576*a^14*b^11*c^11 + 529414848774
1440*a^15*b^9*c^12 - 9906599766261760*a^16*b^7*c^13 + 12525636463624192*a^1
7*b^5*c^14 - 9605333580251136*a^18*b^3*c^15)*3i)/(65536*(a^8*b^18 - 262144*
a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a
^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*
c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^2
5)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*
a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 4069363
42200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c
^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 174892
3551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 1175658114744
3200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*
a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^
5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)
^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2
) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-
(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) -
108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c -
b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*

b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 -
 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^
7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^2
1*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 -
5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 166472932392
96*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*
c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(3/4)*
1i + (3*(4356374400000*a^8*c^16 + 18475695*b^16*c^8 - 685712223*a*b^14*c^9
+ 11424393414*a^2*b^12*c^10 - 110892005343*a^3*b^10*c^11 + 681741235260*a^4
*b^8*c^12 - 2694857597280*a^5*b^6*c^13 + 6582295198080*a^6*b^4*c^14 - 87634
24992000*a^7*b^2*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c
+ 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^1
3*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8
)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 24054165667840
00*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495
*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276
813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*
b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11
 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 216
83350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 328366360
93972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 1122495004409856
0*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^3
7*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(
-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) +
 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(
4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(335544
32*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c
^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 +
 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*
c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 70447529984
0*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c
^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809
116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^2
9*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*1i + (9*x^(1/2)*(12197848
32000000*a^8*c^19 + 1755191025*b^16*c^11 - 67599928620*a*b^14*c^12 + 117243
3971394*a^2*b^12*c^13 - 11911732472304*a^3*b^10*c^14 + 77626373024736*a^4*b
^8*c^15 - 333603251301888*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17 - 155
6843742720000*a^7*b^2*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a
^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4
 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 +
32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50
331648*a^19*b^2*c^11)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1
/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b
^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200
*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 -

113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 17489235510 27200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200* a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15* b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^1 7 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^ (1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7 556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a* c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 10802 5400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2) ^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38* c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 1587 6096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8 255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^2 0*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 52022 79137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^ 26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*1i))* (-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a ^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4 *b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 32768136 00400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21 *c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5 052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 2168335 0423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 3283663609397 2480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^ 18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4* a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 878 08681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a* c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158 760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^ 22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 208091165 49120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^ 4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4) - 2*atan((((((9*x^(1/2)*(1546 704997025054720*a^19*b*c^19 - 822083584*a^4*b^31*c^4 + 50851741696*a^5*b^29 *c^5 - 1473677099008*a^6*b^27*c^6 + 26523687976960*a^7*b^25*c^7 - 331351626 612736*a^8*b^23*c^8 + 3041476258824192*a^9*b^21*c^9 - 21176692735213568*a^1 0*b^19*c^10 + 113812892427485184*a^11*b^17*c^11 - 475720885626470400*a^12*b ^15*c^12 + 1545406748670558208*a^13*b^13*c^13 - 3867206695260258304*a^14*b^ 11*c^14 + 7315227880965799936*a^15*b^9*c^15 - 10117494892562219008*a^16*b^7 *c^16 + 9650897342106173440*a^17*b^5*c^17 - 5672002255696429056*a^18*b^3*c^

18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) - ((-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*(3377699720527872*a^19*b*c^16 + 117440512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5 + 132070244352*a^9*b^21*c^6 - 1828045455360*a^10*b^19*c^7 + 17136919511040*a^11*b^17*c^8 - 114572547588096*a^12*b^15*c^9 + 559926296444928*a^13*b^13*c^10 - 2014580179992576*a^14*b^11*c^11 + 5294148487741440*a^15*b^9*c^12 - 9906599766261760*a^16*b^7*c^13 + 12525636463624192*a^17*b^5*c^14 - 9605333580251136*a^18*b^3*c^15)*3i)/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1

/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 109
9511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^3
4*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^2
8*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a
^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 +
2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 1040455827456
0*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*
c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 549755
8138880*a^30*b^2*c^19)))^(3/4)*1i - (3*(4356374400000*a^8*c^16 + 18475695*b
^16*c^8 - 685712223*a*b^14*c^9 + 11424393414*a^2*b^12*c^10 - 110892005343*a
^3*b^10*c^11 + 681741235260*a^4*b^8*c^12 - 2694857597280*a^5*b^6*c^13 + 658
2295198080*a^6*b^4*c^14 - 8763424992000*a^7*b^2*c^15))/(65536*(a^8*b^18 - 2
62144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 3
2256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^1
5*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c -
b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 1808
51965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 4
06936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*
b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 -
1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 1175658
1147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 309290257015
11168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a
^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c
- b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25
)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*
c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1
/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4
*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80
*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32
*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b
^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 1937307074
56*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c
^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 166472
93239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^2
8*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^
(1/4)*1i + (9*x^(1/2)*(1219784832000000*a^8*c^19 + 1755191025*b^16*c^11 - 6
7599928620*a*b^14*c^12 + 1172433971394*a^2*b^12*c^13 - 11911732472304*a^3*b
^10*c^14 + 77626373024736*a^4*b^8*c^15 - 333603251301888*a^5*b^6*c^16 + 930
302051212800*a^6*b^4*c^17 - 1556843742720000*a^7*b^2*c^18))/(4194304*(a^8*b
^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*
b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*
c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^
9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)))*(-(81*(2401*b^39 + 2
401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060
*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663

491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4) + ((((9*x^(1/2)*(1546704997025054720*a^19*b*c^19 - 822083584*a^4*b^31*c^4 + 50851741696*a^5*b^29*c^5 - 1473677099008*a^6*b^27*c^6 + 26523687976960*a^7*b^25*c^7 - 331351626612736*a^8*b^23*c^8 + 3041476258824192*a^9*b^21*c^9 - 21176692735213568*a^10*b^19*c^10 + 113812892427485184*a^11*b^17*c^11 - 475720885626470400*a^12*b^15*c^12 + 1545406748670558208*a^13*b^13*c^13 - 3867206695260258304*a^14*b^11*c^14 + 7315227880965799936*a^15*b^9*c^15 - 10117494892562219008*a^16*b^7*c^16 + 9650897342106173440*a^17*b^5*c^17 - 5672002255696429056*a^18*b^3*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) + ((-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 127

0087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*(3377699720527872*a^19*b*c^16 + 117440512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5 + 132070244352*a^9*b^21*c^6 - 1828045455360*a^10*b^19*c^7 + 17136919511040*a^11*b^17*c^8 - 114572547588096*a^12*b^15*c^9 + 559926296444928*a^13*b^13*c^10 - 2014580179992576*a^14*b^11*c^11 + 5294148487741440*a^15*b^9*c^12 - 9906599766261760*a^16*b^7*c^13 + 12525636463624192*a^17*b^5*c^14 - 9605333580251136*a^18*b^3*c^15)*3i)/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(3/4)*1i + (3*(4356374400000*a^8*c^16 + 18475695*b^16*c^8 - 685712223*a*b^14*c^9 + 11424393414*a^2*b^12*c^10 - 110892005343*a^3*b^10*c^11 + 681741235260*a^4*b^8*c^12 - 2694857597280*a^5*b^6*c^13 + 6582295198080*a^6*b^4*c^14 - 8763424992000*a^7*b^2*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c

^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*1i + (9*x^(1/2)*(1219784832000000*a^8*c^19 + 1755191025*b^16*c^11 - 67599928620*a*b^14*c^12 + 1172433971394*a^2*b^12*c^13 - 11911732472304*a^3*b^10*c^14 + 77626373024736*a^4*b^8*c^15 - 333603251301888*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17 - 1556843742720000*a^7*b^2*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4))/(((((9*x^(1/2)*(1546704997025054720*a^19*b*c^19 - 822083584*a^4*b^31*c^4 + 5

0851741696*a^5*b^29*c^5 - 1473677099008*a^6*b^27*c^6 + 26523687976960*a^7*b^25*c^7 - 331351626612736*a^8*b^23*c^8 + 3041476258824192*a^9*b^21*c^9 - 21176692735213568*a^10*b^19*c^10 + 113812892427485184*a^11*b^17*c^11 - 475720885626470400*a^12*b^15*c^12 + 1545406748670558208*a^13*b^13*c^13 - 3867206695260258304*a^14*b^11*c^14 + 7315227880965799936*a^15*b^9*c^15 - 10117494892562219008*a^16*b^7*c^16 + 9650897342106173440*a^17*b^5*c^17 - 5672002255696429056*a^18*b^3*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) - ((-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*(3377699720527872*a^19*b*c^16 + 117440512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5 + 132070244352*a^9*b^21*c^6 - 1828045455360*a^10*b^19*c^7 + 17136919511040*a^11*b^17*c^8 - 114572547588096*a^12*b^15*c^9 + 559926296444928*a^13*b^13*c^10 - 2014580179992576*a^14*b^11*c^11 + 5294148487741440*a^15*b^9*c^12 - 9906599766261760*a^16*b^7*c^13 + 12525636463624192*a^17*b^5*c^14 - 9605333580251136*a^18*b^3*c^15)*3i)/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 216

83350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 328366360
93972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 1122495004409856
0*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^3
7*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(
-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) -
87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(
4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(335544
32*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c
^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 +
158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*
c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 70447529984
0*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c
^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809
116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^2
9*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(3/4)*1i - (3*(4356374400000*a^
8*c^16 + 18475695*b^16*c^8 - 685712223*a*b^14*c^9 + 11424393414*a^2*b^12*c^
10 - 110892005343*a^3*b^10*c^11 + 681741235260*a^4*b^8*c^12 - 2694857597280
*a^5*b^6*c^13 + 6582295198080*a^6*b^4*c^14 - 8763424992000*a^7*b^2*c^15))/(
65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 537
6*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b
^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 + 24
01*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*
a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 403026634
91*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 +
21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 49239818937344
0*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*
b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*
c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 -
24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 2401000
0*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2
*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2)
+ 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(
4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) -
73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 109951162
7776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3
+ 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6
- 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^
22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 211342
5899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25
*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 -
19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 549755813888
0*a^30*b^2*c^19)))^(1/4)*1i + (9*x^(1/2)*(1219784832000000*a^8*c^19 + 17551
91025*b^16*c^11 - 67599928620*a*b^14*c^12 + 1172433971394*a^2*b^12*c^13 - 1
1911732472304*a^3*b^10*c^14 + 77626373024736*a^4*b^8*c^15 - 333603251301888
*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17 - 1556843742720000*a^7*b^2*c^1

8))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^2
0*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 +
 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 5
7671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)))*(
-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^
19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*
b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 327681360
0400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*
c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 50
52644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350
423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972
480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^1
8*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c +
 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a
*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 8780
8681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c
 - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a
^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 -
72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 1587
60960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 -
 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^2
2*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 +
 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 2080911654
9120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4
*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*1i - ((((9*x^(1/2)*(1546704997
025054720*a^19*b*c^19 - 822083584*a^4*b^31*c^4 + 50851741696*a^5*b^29*c^5 -
 1473677099008*a^6*b^27*c^6 + 26523687976960*a^7*b^25*c^7 - 331351626612736
*a^8*b^23*c^8 + 3041476258824192*a^9*b^21*c^9 - 21176692735213568*a^10*b^19
*c^10 + 113812892427485184*a^11*b^17*c^11 - 475720885626470400*a^12*b^15*c^
12 + 1545406748670558208*a^13*b^13*c^13 - 3867206695260258304*a^14*b^11*c^1
4 + 7315227880965799936*a^15*b^9*c^15 - 10117494892562219008*a^16*b^7*c^16
+ 9650897342106173440*a^17*b^5*c^17 - 5672002255696429056*a^18*b^3*c^18))/(
4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2
 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784
704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 576716
80*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) + ((-(8
1*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*
b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^3
1*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 327681360040
0*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9
 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 50526
44161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423
470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480
*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b
^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 99

6660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c
- b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 8780868
1*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c -
b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11
*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 729
60*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 1587609
60*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44
029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b
^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10
404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 2080911654912
0*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^
18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*(3377699720527872*a^19*b*c^16 + 1
17440512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5 + 132070244352*a^9*b^21*c^6
 - 1828045455360*a^10*b^19*c^7 + 17136919511040*a^11*b^17*c^8 - 11457254758
8096*a^12*b^15*c^9 + 559926296444928*a^13*b^13*c^10 - 2014580179992576*a^14
*b^11*c^11 + 5294148487741440*a^15*b^9*c^12 - 9906599766261760*a^16*b^7*c^1
3 + 12525636463624192*a^17*b^5*c^14 - 9605333580251136*a^18*b^3*c^15)*3i)/(
65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 537
6*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b
^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 + 24
01*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*
a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 403026634
91*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 +
21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 49239818937344
0*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*
b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*
c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 -
24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 2401000
0*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2
*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2)
+ 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(
4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) -
73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 109951162
7776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3
+ 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6
- 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^
22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 211342
5899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25
*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 -
 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 549755813888
0*a^30*b^2*c^19)))^(3/4)*1i + (3*(4356374400000*a^8*c^16 + 18475695*b^16*c^
8 - 685712223*a*b^14*c^9 + 11424393414*a^2*b^12*c^10 - 110892005343*a^3*b^1
0*c^11 + 681741235260*a^4*b^8*c^12 - 2694857597280*a^5*b^6*c^13 + 658229519
8080*a^6*b^4*c^14 - 8763424992000*a^7*b^2*c^15))/(65536*(a^8*b^18 - 262144*
a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a

^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*1i + (9*x^(1/2)*(1219784832000000*a^8*c^19 + 1755191025*b^16*c^11 - 67599928620*a*b^14*c^12 + 1172433971394*a^2*b^12*c^13 - 11911732472304*a^3*b^10*c^14 + 77626373024736*a^4*b^8*c^15 - 333603251301888*a^5*b^6*c^16 + 930302051212800*a^6*b^4*c^17 - 1556843742720000*a^7*b^2*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) - 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 127

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0087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^
9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 21134258995
20*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12
*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 1958
5050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^3
0*b^2*c^19)))^(1/4)*1i))*(-(81*(2401*b^39 + 2401*b^14*(-(4*a*c - b^2)^25)^(
1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*
b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 40693634220
0*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 -
 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551
027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200
*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15
*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^
17 + 11224950044098560*a^18*b^3*c^18 - 24010000*a^7*c^7*(-(4*a*c - b^2)^25)
^(1/2) - 193795*a*b^37*c + 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) -
7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) + 34052295*a^4*b^6*c^4*(-(4*a
*c - b^2)^25)^(1/2) - 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) + 1080
25400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) - 73745*a*b^12*c*(-(4*a*c - b^2
)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38
*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 158
76096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 +
8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^
20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202
279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a
^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17
 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.1089 $\int (dx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=147

$$\frac{2(dx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] 2/5*(d*x)^(5/2)*AppellF1(5/4,-1/2,-1/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/d/(1+2*c*x^2/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2(dx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*(d*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -1/2, -1/2, 9/4, (-2*c
*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*d*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
```

```
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int (dx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx = \frac{\sqrt{a+bx^2+cx^4}\int (dx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{2(dx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.58, size = 365, normalized size = 2.48

$$\frac{2d\sqrt{dx}\left(2x^2\left(10ac-3b^2\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-10ab\sqrt{\frac{-\sqrt{b^2-\cdots}}{b-\cdots}}\cdots}{225c\sqrt{a+bx^2+c\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*d*Sqrt[d*x]*(5*(2*b + 5*c*x^2)*(a + b*x^2 + c*x^4) - 10*a*b*Sqrt[(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 2*(-3*
b^2 + 10*a*c)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*
a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Appe
llF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
+ Sqrt[b^2 - 4*a*c])]))/(225*c*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{cx^4+bx^2+a}\,\sqrt{dx}\,dx, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)*d*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4 + bx^2 + a}\,(dx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}}\sqrt{c\,x^4 + b\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x)
[Out] int((d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4 + bx^2 + a}\,(dx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^{3/2}\,\sqrt{c\,x^4 + b\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}}\sqrt{a + bx^2 + cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((d*x)**(3/2)*sqrt(a + b*x**2 + c*x**4), x)
```

3.1090 $\int \sqrt{dx}\,\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=147

$$\frac{2(dx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] 2/3*(d*x)^(3/2)*AppellF1(3/4,-1/2,-1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/d/(1+2*c*x^2/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2(dx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -1/2, -1/2, 7/4, (-2*c
*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*d*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
```

```
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \sqrt{dx}\,\sqrt{a+bx^2+cx^4}\,dx = \frac{\sqrt{a+bx^2+cx^4}\int \sqrt{dx}\,\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{2(dx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.43, size = 342, normalized size = 2.33

$$\frac{2x\sqrt{dx}\left(6bx^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\,F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+28a\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\right.}{147\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*x*Sqrt[d*x]*(21*(a + b*x^2 + c*x^4) + 28*a*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 6*b*x^2*Sqrt[(b - Sqrt[
b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(147*Sqr
t[a + b*x^2 + c*x^4])
```

fricas [F] time = 1.17, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\sqrt{cx^4+bx^2+a}\,\sqrt{dx}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4 + bx^2 + a}\,\sqrt{dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\,\sqrt{c\,x^4 + b\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
[Out] int((d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4 + bx^2 + a}\,\sqrt{dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d\,x}\,\sqrt{c\,x^4 + b\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\,\sqrt{a + bx^2 + cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(sqrt(d*x)*sqrt(a + b*x**2 + c*x**4), x)
```

3.1091 $\int \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{dx}}\,dx$

Optimal. Leaf size=145

$$\frac{2\sqrt{dx}\,\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] 2*AppellF1(1/4,-1/2,-1/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-
4*a*c+b^2)^(1/2)))*(d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/d/(1+2*c*x^2/(b-(-4*a*
c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2\sqrt{dx}\,\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/Sqrt[d*x],x]
```

```
[Out] (2*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -1/2, -1/2, 5/4, (-2*c*x
^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[1
 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
```

```
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{dx}}\,dx = \frac{\sqrt{a+bx^2+cx^4}\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}{\sqrt{dx}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{2\sqrt{dx}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.38, size = 342, normalized size = 2.36

$$\frac{2x\left(2bx^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+20a\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-\cdots}}{\sqrt{b\cdots}}}\right.}{25\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/Sqrt[d*x],x]
```

```
[Out] (2*x*(5*(a + b*x^2 + c*x^4) + 20*a*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(
b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 2*b*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(25*Sqrt[d*x]*Sqrt[a
 + b*x^2 + c*x^4])
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\,\sqrt{dx}}{dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/sqrt(d*x), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^4 + b\,x^2 + a}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/(d*x)^(1/2),x)
[Out] int((c*x^4+b*x^2+a)^(1/2)/(d*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/sqrt(d*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4 + b\,x^2 + a}}{\sqrt{d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/(d*x)^(1/2),x)
[Out] int((a + b*x^2 + c*x^4)^(1/2)/(d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/(d*x)**(1/2),x)
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/sqrt(d*x), x)
```

3.1092 $\int \frac{\sqrt{a+bx^2+cx^4}}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{2\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -2*AppellF1(-1/4,-1/2,-1/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+
(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/d/(d*x)^(1/2)/(1+2*c*x^2/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$-\frac{2\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/(d*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -1/2, -1/2, 3/4, (-2*c*x^2)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[d*x]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
```

```
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(dx)^{3/2}}\,dx = \frac{\sqrt{a+bx^2+cx^4}\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}{(dx)^{3/2}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= -\frac{2\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.36, size = 345, normalized size = 2.38

$$\frac{x\left(28bx^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+24cx^4\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2}}{}}\right.}{21(dx)^{3/2}\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/(d*x)^(3/2),x]
```

```
[Out] (x*(-42*(a + b*x^2 + c*x^4) + 28*b*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^
2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 24*c*x^4*Sqrt[(b - Sqrt[b^2 -
4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(21*(d*x)^(3/2
)*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\,\sqrt{dx}}{d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(d^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/(d*x)^(3/2), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^4 + b\,x^2 + a}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/(d*x)^(3/2),x)
[Out] int((c*x^4+b*x^2+a)^(1/2)/(d*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/(d*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4 + b\,x^2 + a}}{(d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/(d*x)^(3/2),x)
[Out] int((a + b*x^2 + c*x^4)^(1/2)/(d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/(d*x)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/(d*x)**(3/2), x)
```

3.1093 $\int (dx)^{3/2} \left(a+bx^2+cx^4\right)^{3/2} \, dx$

Optimal. Leaf size=148

$$\frac{2a(dx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] 2/5*a*(d*x)^(5/2)*AppellF1(5/4,-3/2,-3/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/d/(1+2*c*x^2/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2a(dx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*a*(d*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -3/2, -3/2, 9/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*d*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
```

```
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int (dx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}\,dx=\frac{\left(a\sqrt{a+bx^2+cx^4}\right)\int (dx)^{3/2}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$=\frac{2a(dx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.88, size = 459, normalized size = 3.10

$$\frac{2d\sqrt{dx}\left(4x^2\left(260a^2c^2-157ab^2c+21b^4\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*d*Sqrt[d*x]*(5*(-28*b^4*x^2 - 8*b^3*c*x^4 + 305*b^2*c^2*x^6 + 480*b*c^3*
x^8 + 195*c^4*x^10 + a^2*c*(176*b + 455*c*x^2) + a*(-28*b^3 + 196*b^2*c*x^2
 + 916*b*c^2*x^4 + 650*c^3*x^6)) + 20*a*b*(7*b^2 - 44*a*c)*Sqrt[(b - Sqrt[b
^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 4*(21*b^4
- 157*a*b^2*c + 260*a^2*c^2)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(16575*c^2*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cdx^5+bdx^3+adx\right)\sqrt{cx^4+bx^2+a}\,\sqrt{dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*d*x^5 + b*d*x^3 + a*d*x)*sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} (dx)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}} \left(c\,x^4 + b\,x^2 + a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(c*x^4+b*x^2+a)^(3/2),x)
[Out] int((d*x)^(3/2)*(c*x^4+b*x^2+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} (dx)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {(d\,x)}^{3/2}\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}} \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((d*x)**(3/2)*(a + b*x**2 + c*x**4)**(3/2), x)
```

3.1094 $\int \sqrt{dx} \left(a+bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=148

$$\frac{2a(dx)^{3/2}\sqrt{a+bx^2+cx^4}\, F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] 2/3*a*(d*x)^(3/2)*AppellF1(3/4,-3/2,-3/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/d/(1+2*c*x^2/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2a(dx)^{3/2}\sqrt{a+bx^2+cx^4}\, F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*a*(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -3/2, -3/2, 7/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*d*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
```

```
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \sqrt{dx}\left(a+bx^2+cx^4\right)^{3/2}dx = \frac{\left(a\sqrt{a+bx^2+cx^4}\right)\int \sqrt{dx}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{2a(dx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.71, size = 417, normalized size = 2.82

$$\frac{2x\sqrt{dx}\Big(7\left(209a^2c+12ab^2+328abcx^2+286ac^2x^4+12b^3x^2+131b^2cx^4+196bc^2x^6+77c^3x^8\right)+12bx^2\left(36ac-5\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*x]*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*x*Sqrt[d*x]*(7*(12*a*b^2 + 209*a^2*c + 12*b^3*x^2 + 328*a*b*c*x^2 + 131*
b^2*c*x^4 + 286*a*c^2*x^4 + 196*b*c^2*x^6 + 77*c^3*x^8) - 28*a*(3*b^2 - 44*
a*c)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2
, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
4*a*c])] + 12*b*(-5*b^2 + 36*a*c)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2
)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(8085*c*Sqrt[a + b*x^2 + c*x^4
])
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\sqrt{dx}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} \sqrt{dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\, \left(c\,x^4 + b\,x^2 + a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x)
[Out] int((d*x)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} \sqrt{dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d\,x}\, \left(c\,x^4 + b\,x^2 + a\right)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\, \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(sqrt(d*x)*(a + b*x**2 + c*x**4)**(3/2), x)
```

3.1095 $\int \frac{(a+bx^2+cx^4)^{3/2}}{\sqrt{dx}}\,dx$

Optimal. Leaf size=146

$$\frac{2a\sqrt{dx}\,\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] 2*a*AppellF1(1/4,-3/2,-3/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+
(-4*a*c+b^2)^(1/2)))*(d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/d/(1+2*c*x^2/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2a\sqrt{dx}\,\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/Sqrt[d*x],x]
```

```
[Out] (2*a*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -3/2, -3/2, 5/4, (-2*c
*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]])/(d*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
```

```
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{\sqrt{dx}}\,dx = \frac{\left(a\sqrt{a+bx^2+cx^4}\right)\int \frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{\sqrt{dx}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$
$$= \frac{2a\sqrt{dx}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.71, size = 415, normalized size = 2.84

$$\frac{2x\left(5\left(51a^2c+4ab^2+76abcx^2+66ac^2x^4+4b^3x^2+29b^2cx^4+40bc^2x^6+15c^3x^8\right)-4bx^2\left(3b^2-28ac\right)\sqrt{\frac{-\sqrt{b^2-4ac}}{b-\sqrt{b^2}}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/Sqrt[d*x],x]
```

```
[Out] (2*x*(5*(4*a*b^2 + 51*a^2*c + 4*b^3*x^2 + 76*a*b*c*x^2 + 29*b^2*c*x^4 + 66*
a*c^2*x^4 + 40*b*c^2*x^6 + 15*c^3*x^8) - 20*a*(b^2 - 36*a*c)*Sqrt[(b - Sqrt
[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - 4*b*(3*b
^2 - 28*a*c)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
 Sqrt[b^2 - 4*a*c])]))/(975*c*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\sqrt{dx}}{dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(1/2),x, algorithm="fricas")
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x)/(d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(1/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/sqrt(d*x), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^4 + b\,x^2 + a\right)^{\frac{3}{2}}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/(d*x)^(1/2),x)
[Out] int((c*x^4+b*x^2+a)^(3/2)/(d*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(1/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/sqrt(d*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4 + b\,x^2 + a\right)^{3/2}}{\sqrt{d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/(d*x)^(1/2),x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/(d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{\sqrt{dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/(d*x)**(1/2),x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/sqrt(d*x), x)
```

3.1096 $\int \frac{(a+bx^2+cx^4)^{3/2}}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{2a\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] -2*a*AppellF1(-1/4,-3/2,-3/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(
b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/d/(d*x)^(1/2)/(1+2*c*x^2/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$-\frac{2a\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/(d*x)^(3/2),x]
```

```
[Out] (-2*a*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -3/2, -3/2, 3/4, (-2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[d*x]*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
```

```
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{(dx)^{3/2}}\,dx = \frac{\left(a\sqrt{a+bx^2+cx^4}\right)\int \frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{(dx)^{3/2}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= -\frac{2a\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.73, size = 384, normalized size = 2.63

$$\frac{x\left(14\left(-77a^2-64abx^2-70acx^4+13b^2x^4+20bcx^6+7c^2x^8\right)+896abx^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{3}{4}\right.\right.}{539(dx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/(d*x)^(3/2),x]
```

```
[Out] (x*(14*(-77*a^2 - 64*a*b*x^2 + 13*b^2*x^4 - 70*a*c*x^4 + 20*b*c*x^6 + 7*c^2
*x^8) + 896*a*b*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 -
4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Ap
pellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])] + 24*(b^2 + 28*a*c)*x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(539*(d*x)^(3/2)*Sqr
t[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\sqrt{dx}}{d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(3/2),x, algorithm="fricas")
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x)/(d^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/(d*x)^(3/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/(d*x)^(3/2),x)
[Out] int((c*x^4+b*x^2+a)^(3/2)/(d*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/(d*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2+a\right)^{3/2}}{(d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/(d*x)^(3/2),x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/(d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/(d*x)**(3/2),x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/(d*x)**(3/2), x)
```

3.1097 $\int \frac{(dx)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=147

$$\frac{2(dx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{a+bx^2+cx^4}}$$

```
[Out] 2/5*(d*x)^(5/2)*AppellF1(5/4,1/2,1/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+
2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/d/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2(dx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*(d*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*d*Sqrt[a + b*x^2
+ c*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^{3/2}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^2+cx^4}}$$
$$= \frac{2(dx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] time = 0.09, size = 173, normalized size = 1.18

$$\frac{2x(dx)^{3/2}\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{5\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^(3/2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*x*(d*x)^(3/2)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
 Sqrt[b^2 - 4*a*c])])/(5*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx}\,dx}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*d*x/sqrt(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((d*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
[Out] int((d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((d*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^{3/2}}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((d*x)^(3/2)/(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((d*x)**(3/2)/sqrt(a + b*x**2 + c*x**4), x)
```

3.1098 $\int \frac{\sqrt{dx}}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=147

$$\frac{2(dx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{a+bx^2+cx^4}}$$

```
[Out] 2/3*(d*x)^(3/2)*AppellF1(3/4,1/2,1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+
2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/d/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2(dx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*(d*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*d*Sqrt[a + b*x^2
+ c*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{\sqrt{dx}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^2+cx^4}}$$
$$= \frac{2(dx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] time = 0.10, size = 173, normalized size = 1.18

$$\frac{2x\sqrt{dx}\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{3\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*x]/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*x*Sqrt[d*x]*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c
])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF
1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])])/(3*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx}}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{\sqrt{c\,x^{4}+b\,x^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
[Out] int((d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{\sqrt{cx^{4}+bx^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x}}{\sqrt{c\,x^{4}+b\,x^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((d*x)^(1/2)/(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{\sqrt{a+bx^{2}+cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(sqrt(d*x)/sqrt(a + b*x**2 + c*x**4), x)
```

3.1099 $\int \frac{1}{\sqrt{dx}\,\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=145

$$\frac{2\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{a+bx^2+cx^4}}$$

```
[Out] 2*AppellF1(1/4,1/2,1/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*
a*c+b^2)^(1/2)))*(d*x)^(1/2)*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*
c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/d/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*Sqrt[d*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[a + b*x^2 + c*
x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\sqrt{a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\sqrt{dx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^2+cx^4}}$$
$$= \frac{2\sqrt{dx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] time = 0.08, size = 171, normalized size = 1.18

$$\frac{2x\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*x*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2
, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
4*a*c])])/(Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\sqrt{dx}}{cdx^5+bdx^3+adx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(c*d*x^5 + b*d*x^3 + a*d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
[Out] int(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}\,\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{d\,x}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int(1/((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\,\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(1/2)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(1/(sqrt(d*x)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.1100 $\int \frac{1}{(dx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=145

$$-\frac{2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

```
[Out] -2*AppellF1(-1/4,1/2,1/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-
4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(
-4*a*c+b^2)^(1/2)))^(1/2)/d/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$-\frac{2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[-1/4, 1/2, 1/2, 3/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[d*x]*Sqrt[a + b*x^2 +
c*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{(dx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^2+cx^4}}$$

$$= -\frac{2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

Mathematica [B] time = 0.48, size = 348, normalized size = 2.40

$$\frac{x\left(14bx^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+18cx^4\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\cdots}\right.}{21a(dx)^{3/2}\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (x*(-42*(a + b*x^2 + c*x^4) + 14*b*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^
2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 18*c*x^4*Sqrt[(b - Sqrt[b^2 -
4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(21*a*(d*x)^(3
/2)*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\,\sqrt{dx}}{cd^2x^6+bd^2x^4+ad^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(c*d^2*x^6 + b*d^2*x^4 + a*d^2*x
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}\,(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}}\sqrt{c\,x^4 + b\,x^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
[Out] int(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}\,(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(d\,x)^{3/2}\,\sqrt{c\,x^4 + b\,x^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int(1/((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}}\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/((d*x)**(3/2)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.1101 $\int \frac{(dx)^{3/2}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=150

$$\frac{2(dx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5ad\sqrt{a+bx^2+cx^4}}$$

```
[Out] 2/5*(d*x)^(5/2)*AppellF1(5/4,3/2,3/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+
2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/d/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2(dx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5ad\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*(d*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 3/2, 3/2, 9/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*a*d*Sqrt[a + b*x^
2 + c*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
```

```
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^{3/2}}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}}$$

$$= \frac{2(dx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5ad\sqrt{a+bx^2+cx^4}}$$

Mathematica [B] time = 0.34, size = 348, normalized size = 2.32

$$\frac{d\sqrt{dx}\left(2cx^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\,F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+5b\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\cdots}\right.}{5\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^(3/2)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (d*Sqrt[d*x]*(-5*(b + 2*c*x^2) + 5*b*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 2*c*x^2*Sqrt[(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(5*(b^2 - 4*a*c)*S
qrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\,\sqrt{dx}\,dx}{c^2x^8+2\,bcx^6+\left(b^2+2\,ac\right)x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)*d*x/(c^2*x^8 + 2*b*c*x^6 + (b^2
+ 2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((d*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{\left(c\,x^4 + b\,x^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
[Out] int((d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((d*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^{3/2}}{\left(c\,x^4 + b\,x^2 + a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((d*x)^(3/2)/(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((d*x)**(3/2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

3.1102 $\int \frac{\sqrt{dx}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=150

$$\frac{2(dx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3ad\sqrt{a+bx^2+cx^4}}$$

```
[Out] 2/3*(d*x)^(3/2)*AppellF1(3/4,3/2,3/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+
2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/d/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2(dx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3ad\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*(d*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 3/2, 3/2, 7/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*a*d*Sqrt[a + b*x^
2 + c*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
```

```
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{\sqrt{dx}}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}}$$

$$= \frac{2(dx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3ad\sqrt{a+bx^2+cx^4}}$$

Mathematica [B] time = 0.51, size = 367, normalized size = 2.45

$$\frac{x\sqrt{dx}\left(9bcx^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+7\left(2ac+b^2\right)\sqrt{\frac{-\sqrt{b^2-4a}}{b-\sqrt{b}}}\cdots\right.}{21a\left(4ac-b^2\right)\sqrt{a+bx^2+\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*x]/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*Sqrt[d*x]*(-21*(b^2 - 2*a*c + b*c*x^2) + 7*(b^2 + 2*a*c)*Sqrt[(b - Sqrt[
b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 9*b*c*x^2
*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/
2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a
*c])]))/(21*a*(-b^2 + 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\,\sqrt{dx}}{c^2x^8+2\,bcx^6+\left(b^2+2\,ac\right)x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*
a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{\left(c\,x^4 + b\,x^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x}}{{\left(c\,x^4 + b\,x^2 + a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((d*x)^(1/2)/(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(sqrt(d*x)/(a + b*x**2 + c*x**4)**(3/2), x)
```

3.1103 $\int \frac{1}{\sqrt{dx}\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=148

$$\frac{2\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad\sqrt{a+bx^2+cx^4}}$$

```
[Out] 2*AppellF1(1/4,3/2,3/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*
a*c+b^2)^(1/2)))*(d*x)^(1/2)*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*
c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/d/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (2*Sqrt[d*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 3/2, 3/2, 5/4, (-2*c*x^2)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*d*Sqrt[a + b*x^2 +
c*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
```

```
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\sqrt{dx}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}}$$

$$=\frac{2\sqrt{dx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad\sqrt{a+bx^2+cx^4}}$$

Mathematica [B] time = 0.47, size = 366, normalized size = 2.47

$$\frac{x\left(bcx^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\,F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-5\left(b^2-6ac\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\cdots\right.}{5a\sqrt{dx}\left(4ac-b^2\right)\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[d*x]*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (x*(-5*(b^2 - 2*a*c + b*c*x^2) - 5*(b^2 - 6*a*c)*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + b*c*x^2*Sqrt[(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(5*a*(
-b^2 + 4*a*c)*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\,\sqrt{dx}}{c^2dx^9+2\,bcdx^7+\left(b^2+2\,ac\right)dx^5+2\,abdx^3+a^2dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(c^2*d*x^9 + 2*b*c*d*x^7 + (b^2
+ 2*a*c)*d*x^5 + 2*a*b*d*x^3 + a^2*d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x)), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\,\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{d\,x}\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(1/((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\,\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(1/2)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral(1/(sqrt(d*x)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.1104 $\int \frac{1}{(dx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=148

$$-\frac{2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad\sqrt{dx}\,\sqrt{a+bx^2+cx^4}}$$

```
[Out] -2*AppellF1(-1/4,3/2,3/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-
4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(
-4*a*c+b^2)^(1/2)))^(1/2)/a/d/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$-\frac{2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad\sqrt{dx}\,\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[-1/4, 3/2, 3/2, 3/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*d*Sqrt[d*x]*Sqrt[a + b*x^2
+ c*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
```

```
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{(dx)^{3/2}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}}$$

$$= -\frac{2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

Mathematica [B] time = 0.69, size = 409, normalized size = 2.76

$$-\frac{x\left(-7\left(8a^2c+a\left(-2b^2+11bcx^2+10c^2x^4\right)-3b^2x^2\left(b+cx^2\right)\right)-7bx^2\left(b^2-3ac\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+}{\sqrt{b^2-4ac}-}}\right.}{7a^2(d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -1/7*(x*(-7*(8*a^2*c - 3*b^2*x^2*(b + c*x^2) + a*(-2*b^2 + 11*b*c*x^2 + 10*
c^2*x^4)) - 7*b*(b^2 - 3*a*c)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 3*c*(-3*b^2 + 10*a*c)*x^4*Sqrt[(b -
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (
-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(a
^2*(b^2 - 4*a*c)*(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\,\sqrt{dx}}{c^2d^2x^{10}+2\,bcd^2x^8+\left(b^2+2\,ac\right)d^2x^6+2\,abd^2x^4+a^2d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(c^2*d^2*x^10 + 2*b*c*d^2*x^8 +
(b^2 + 2*a*c)*d^2*x^6 + 2*a*b*d^2*x^4 + a^2*d^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} (dx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2)), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}} \left(c\,x^4 + b\,x^2 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} (dx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(d\,x)^{3/2} \left(c\,x^4 + b\,x^2 + a\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(1/((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral(1/((d*x)**(3/2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.1105 $\int (dx)^m \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=156

$$\frac{a^3(dx)^{m+1}}{d(m+1)}+\frac{3a^2b(dx)^{m+3}}{d^3(m+3)}+\frac{3c\left(ac+b^2\right)(dx)^{m+9}}{d^9(m+9)}+\frac{b\left(6ac+b^2\right)(dx)^{m+7}}{d^7(m+7)}+\frac{3a\left(ac+b^2\right)(dx)^{m+5}}{d^5(m+5)}+\frac{3bc^2(dx)^{m+11}}{d^{11}(m+11)}+\frac{c^3(\ldots}{d^{13}(\ldots}$$

```
[Out] a^3*(d*x)^(1+m)/d/(1+m)+3*a^2*b*(d*x)^(3+m)/d^3/(3+m)+3*a*(a*c+b^2)*(d*x)^(
5+m)/d^5/(5+m)+b*(6*a*c+b^2)*(d*x)^(7+m)/d^7/(7+m)+3*c*(a*c+b^2)*(d*x)^(9+m
)/d^9/(9+m)+3*b*c^2*(d*x)^(11+m)/d^11/(11+m)+c^3*(d*x)^(13+m)/d^13/(13+m)
```

Rubi [A] time = 0.10, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$\frac{3a^2b(dx)^{m+3}}{d^3(m+3)}+\frac{a^3(dx)^{m+1}}{d(m+1)}+\frac{3a\left(ac+b^2\right)(dx)^{m+5}}{d^5(m+5)}+\frac{b\left(6ac+b^2\right)(dx)^{m+7}}{d^7(m+7)}+\frac{3c\left(ac+b^2\right)(dx)^{m+9}}{d^9(m+9)}+\frac{3bc^2(dx)^{m+11}}{d^{11}(m+11)}+\frac{c^3(\ldots}{d^{13}(\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*(d*x)^(1 + m))/(d*(1 + m)) + (3*a^2*b*(d*x)^(3 + m))/(d^3*(3 + m)) + (
3*a*(b^2 + a*c)*(d*x)^(5 + m))/(d^5*(5 + m)) + (b*(b^2 + 6*a*c)*(d*x)^(7 +
m))/(d^7*(7 + m)) + (3*c*(b^2 + a*c)*(d*x)^(9 + m))/(d^9*(9 + m)) + (3*b*c^
2*(d*x)^(11 + m))/(d^11*(11 + m)) + (c^3*(d*x)^(13 + m))/(d^13*(13 + m))
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int (dx)^m \left(a+bx^2+cx^4\right)^3 \, dx = \int \left(a^3(dx)^m+\frac{3a^2b(dx)^{2+m}}{d^2}+\frac{3a\left(b^2+ac\right)(dx)^{4+m}}{d^4}+\frac{b\left(b^2+6ac\right)(dx)^{6+m}}{d^6}+\frac{3c\,(\ldots}{\ldots}\right.$$

$$=\frac{a^3(dx)^{1+m}}{d(1+m)}+\frac{3a^2b(dx)^{3+m}}{d^3(3+m)}+\frac{3a\left(b^2+ac\right)(dx)^{5+m}}{d^5(5+m)}+\frac{b\left(b^2+6ac\right)(dx)^{7+m}}{d^7(7+m)}+\frac{3c\,(b^2\ldots}{\ldots}$$

Mathematica [A] time = 0.13, size = 111, normalized size = 0.71

$$x(dx)^m\left(\frac{a^3}{m+1}+\frac{3a^2bx^2}{m+3}+\frac{3cx^8\left(ac+b^2\right)}{m+9}+\frac{bx^6\left(6ac+b^2\right)}{m+7}+\frac{3ax^4\left(ac+b^2\right)}{m+5}+\frac{3bc^2x^{10}}{m+11}+\frac{c^3x^{12}}{m+13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] x*(d*x)^m*(a^3/(1 + m) + (3*a^2*b*x^2)/(3 + m) + (3*a*(b^2 + a*c)*x^4)/(5 +
 m) + (b*(b^2 + 6*a*c)*x^6)/(7 + m) + (3*c*(b^2 + a*c)*x^8)/(9 + m) + (3*b*
c^2*x^10)/(11 + m) + (c^3*x^12)/(13 + m))
```

fricas [B] time = 0.69, size = 594, normalized size = 3.81

$$\frac{\left(\left(c^3m^6+36\,c^3m^5+505\,c^3m^4+3480\,c^3m^3+12139\,c^3m^2+19524\,c^3m+10395\,c^3\right)x^{13}+3\left(bc^2m^6+38\,bc^2m^5+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] ((c^3*m^6 + 36*c^3*m^5 + 505*c^3*m^4 + 3480*c^3*m^3 + 12139*c^3*m^2 + 19524
*c^3*m + 10395*c^3)*x^13 + 3*(b*c^2*m^6 + 38*b*c^2*m^5 + 555*b*c^2*m^4 + 39
40*b*c^2*m^3 + 14039*b*c^2*m^2 + 22902*b*c^2*m + 12285*b*c^2)*x^11 + 3*((b^
2*c + a*c^2)*m^6 + 40*(b^2*c + a*c^2)*m^5 + 613*(b^2*c + a*c^2)*m^4 + 4528*
(b^2*c + a*c^2)*m^3 + 15015*b^2*c + 15015*a*c^2 + 16627*(b^2*c + a*c^2)*m^2
 + 27688*(b^2*c + a*c^2)*m)*x^9 + ((b^3 + 6*a*b*c)*m^6 + 42*(b^3 + 6*a*b*c)
*m^5 + 679*(b^3 + 6*a*b*c)*m^4 + 5292*(b^3 + 6*a*b*c)*m^3 + 19305*b^3 + 115
830*a*b*c + 20335*(b^3 + 6*a*b*c)*m^2 + 34986*(b^3 + 6*a*b*c)*m)*x^7 + 3*((
a*b^2 + a^2*c)*m^6 + 44*(a*b^2 + a^2*c)*m^5 + 753*(a*b^2 + a^2*c)*m^4 + 628
0*(a*b^2 + a^2*c)*m^3 + 27027*a*b^2 + 27027*a^2*c + 25979*(a*b^2 + a^2*c)*m
^2 + 47436*(a*b^2 + a^2*c)*m)*x^5 + 3*(a^2*b*m^6 + 46*a^2*b*m^5 + 835*a^2*b
*m^4 + 7540*a^2*b*m^3 + 34759*a^2*b*m^2 + 73054*a^2*b*m + 45045*a^2*b)*x^3
+ (a^3*m^6 + 48*a^3*m^5 + 925*a^3*m^4 + 9120*a^3*m^3 + 48259*a^3*m^2 + 1290
72*a^3*m + 135135*a^3)*x)*(d*x)^m/(m^7 + 49*m^6 + 973*m^5 + 10045*m^4 + 573
79*m^3 + 177331*m^2 + 264207*m + 135135)
```

giac [B] time = 0.23, size = 1132, normalized size = 7.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] ((d*x)^m*c^3*m^6*x^13 + 36*(d*x)^m*c^3*m^5*x^13 + 3*(d*x)^m*b*c^2*m^6*x^11
+ 505*(d*x)^m*c^3*m^4*x^13 + 114*(d*x)^m*b*c^2*m^5*x^11 + 3480*(d*x)^m*c^3*
m^3*x^13 + 3*(d*x)^m*b^2*c*m^6*x^9 + 3*(d*x)^m*a*c^2*m^6*x^9 + 1665*(d*x)^m
*b*c^2*m^4*x^11 + 12139*(d*x)^m*c^3*m^2*x^13 + 120*(d*x)^m*b^2*c*m^5*x^9 +
120*(d*x)^m*a*c^2*m^5*x^9 + 11820*(d*x)^m*b*c^2*m^3*x^11 + 19524*(d*x)^m*c^
3*m*x^13 + (d*x)^m*b^3*m^6*x^7 + 6*(d*x)^m*a*b*c*m^6*x^7 + 1839*(d*x)^m*b^2
*c*m^4*x^9 + 1839*(d*x)^m*a*c^2*m^4*x^9 + 42117*(d*x)^m*b*c^2*m^2*x^11 + 10
395*(d*x)^m*c^3*x^13 + 42*(d*x)^m*b^3*m^5*x^7 + 252*(d*x)^m*a*b*c*m^5*x^7 +
 13584*(d*x)^m*b^2*c*m^3*x^9 + 13584*(d*x)^m*a*c^2*m^3*x^9 + 68706*(d*x)^m*
b*c^2*m*x^11 + 3*(d*x)^m*a*b^2*m^6*x^5 + 3*(d*x)^m*a^2*c*m^6*x^5 + 679*(d*x
)^m*b^3*m^4*x^7 + 4074*(d*x)^m*a*b*c*m^4*x^7 + 49881*(d*x)^m*b^2*c*m^2*x^9
+ 49881*(d*x)^m*a*c^2*m^2*x^9 + 36855*(d*x)^m*b*c^2*x^11 + 132*(d*x)^m*a*b^
2*m^5*x^5 + 132*(d*x)^m*a^2*c*m^5*x^5 + 5292*(d*x)^m*b^3*m^3*x^7 + 31752*(d
*x)^m*a*b*c*m^3*x^7 + 83064*(d*x)^m*b^2*c*m*x^9 + 83064*(d*x)^m*a*c^2*m*x^9
 + 3*(d*x)^m*a^2*b*m^6*x^3 + 2259*(d*x)^m*a*b^2*m^4*x^5 + 2259*(d*x)^m*a^2*
c*m^4*x^5 + 20335*(d*x)^m*b^3*m^2*x^7 + 122010*(d*x)^m*a*b*c*m^2*x^7 + 4504
5*(d*x)^m*b^2*c*x^9 + 45045*(d*x)^m*a*c^2*x^9 + 138*(d*x)^m*a^2*b*m^5*x^3 +
 18840*(d*x)^m*a*b^2*m^3*x^5 + 18840*(d*x)^m*a^2*c*m^3*x^5 + 34986*(d*x)^m*
b^3*m*x^7 + 209916*(d*x)^m*a*b*c*m*x^7 + (d*x)^m*a^3*m^6*x + 2505*(d*x)^m*a
^2*b*m^4*x^3 + 77937*(d*x)^m*a*b^2*m^2*x^5 + 77937*(d*x)^m*a^2*c*m^2*x^5 +
19305*(d*x)^m*b^3*x^7 + 115830*(d*x)^m*a*b*c*x^7 + 48*(d*x)^m*a^3*m^5*x + 2
2620*(d*x)^m*a^2*b*m^3*x^3 + 142308*(d*x)^m*a*b^2*m*x^5 + 142308*(d*x)^m*a^
2*c*m*x^5 + 925*(d*x)^m*a^3*m^4*x + 104277*(d*x)^m*a^2*b*m^2*x^3 + 81081*(d
*x)^m*a*b^2*x^5 + 81081*(d*x)^m*a^2*c*x^5 + 9120*(d*x)^m*a^3*m^3*x + 219162
*(d*x)^m*a^2*b*m*x^3 + 48259*(d*x)^m*a^3*m^2*x + 135135*(d*x)^m*a^2*b*x^3 +
 129072*(d*x)^m*a^3*m*x + 135135*(d*x)^m*a^3*x)/(m^7 + 49*m^6 + 973*m^5 + 1
0045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135)
```

maple [B] time = 0.01, size = 782, normalized size = 5.01

$$\frac{\left(c^3m^6x^{12}+36c^3m^5x^{12}+3b\,c^2m^6x^{10}+505c^3m^4x^{12}+114b\,c^2m^5x^{10}+3480c^3m^3x^{12}+3a\,c^2m^6x^8+3b^2c\,m^6x^8+1\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] x*(c^3*m^6*x^12+36*c^3*m^5*x^12+3*b*c^2*m^6*x^10+505*c^3*m^4*x^12+114*b*c^2
*m^5*x^10+3480*c^3*m^3*x^12+3*a*c^2*m^6*x^8+3*b^2*c*m^6*x^8+1665*b*c^2*m^4*
x^10+12139*c^3*m^2*x^12+120*a*c^2*m^5*x^8+120*b^2*c*m^5*x^8+11820*b*c^2*m^3
*x^10+19524*c^3*m*x^12+6*a*b*c*m^6*x^6+1839*a*c^2*m^4*x^8+b^3*m^6*x^6+1839*
b^2*c*m^4*x^8+42117*b*c^2*m^2*x^10+10395*c^3*x^12+252*a*b*c*m^5*x^6+13584*a
*c^2*m^3*x^8+42*b^3*m^5*x^6+13584*b^2*c*m^3*x^8+68706*b*c^2*m*x^10+3*a^2*c*
m^6*x^4+3*a*b^2*m^6*x^4+4074*a*b*c*m^4*x^6+49881*a*c^2*m^2*x^8+679*b^3*m^4*
x^6+49881*b^2*c*m^2*x^8+36855*b*c^2*x^10+132*a^2*c*m^5*x^4+132*a*b^2*m^5*x^
```

```
4+31752*a*b*c*m^3*x^6+83064*a*c^2*m*x^8+5292*b^3*m^3*x^6+83064*b^2*c*m*x^8+
3*a^2*b*m^6*x^2+2259*a^2*c*m^4*x^4+2259*a*b^2*m^4*x^4+122010*a*b*c*m^2*x^6+
45045*a*c^2*x^8+20335*b^3*m^2*x^6+45045*b^2*c*x^8+138*a^2*b*m^5*x^2+18840*a
^2*c*m^3*x^4+18840*a*b^2*m^3*x^4+209916*a*b*c*m*x^6+34986*b^3*m*x^6+a^3*m^6
+2505*a^2*b*m^4*x^2+77937*a^2*c*m^2*x^4+77937*a*b^2*m^2*x^4+115830*a*b*c*x^
6+19305*b^3*x^6+48*a^3*m^5+22620*a^2*b*m^3*x^2+142308*a^2*c*m*x^4+142308*a*
b^2*m*x^4+925*a^3*m^4+104277*a^2*b*m^2*x^2+81081*a^2*c*x^4+81081*a*b^2*x^4+
9120*a^3*m^3+219162*a^2*b*m*x^2+48259*a^3*m^2+135135*a^2*b*x^2+129072*a^3*m
+135135*a^3)*(d*x)^m/(m+13)/(m+11)/(m+9)/(m+7)/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.31, size = 195, normalized size = 1.25

$$\frac{c^3d^mx^{13}x^m}{m+13}+\frac{3\,bc^2d^mx^{11}x^m}{m+11}+\frac{3\,b^2cd^mx^9x^m}{m+9}+\frac{3\,ac^2d^mx^9x^m}{m+9}+\frac{b^3d^mx^7x^m}{m+7}+\frac{6\,abcd^mx^7x^m}{m+7}+\frac{3\,ab^2d^mx^5x^m}{m+5}+\frac{3\,a^2cd^mx^5x^m}{m+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] c^3*d^m*x^13*x^m/(m + 13) + 3*b*c^2*d^m*x^11*x^m/(m + 11) + 3*b^2*c*d^m*x^9
*x^m/(m + 9) + 3*a*c^2*d^m*x^9*x^m/(m + 9) + b^3*d^m*x^7*x^m/(m + 7) + 6*a*
b*c*d^m*x^7*x^m/(m + 7) + 3*a*b^2*d^m*x^5*x^m/(m + 5) + 3*a^2*c*d^m*x^5*x^m
/(m + 5) + 3*a^2*b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*a^3/(d*(m + 1))
```

mupad [B] time = 4.83, size = 546, normalized size = 3.50

$$\frac{a^3\,x\,{\left(d\,x\right)}^m\,\left(m^6+48\,m^5+925\,m^4+9120\,m^3+48259\,m^2+129072\,m+135135\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{c^3\,x^{13}\,{\left(d\,x\right)}^m\,\left(m^6+36\,m^5\right.}{m^7+49\,m^6+973\,m^5+10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (a^3*x*(d*x)^m*(129072*m + 48259*m^2 + 9120*m^3 + 925*m^4 + 48*m^5 + m^6 +
135135))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6
+ m^7 + 135135) + (c^3*x^13*(d*x)^m*(19524*m + 12139*m^2 + 3480*m^3 + 505*m
^4 + 36*m^5 + m^6 + 10395))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4
+ 973*m^5 + 49*m^6 + m^7 + 135135) + (3*a^2*b*x^3*(d*x)^m*(73054*m + 34759*
m^2 + 7540*m^3 + 835*m^4 + 46*m^5 + m^6 + 45045))/(264207*m + 177331*m^2 +
57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (3*b*c^2*x^11*(d
*x)^m*(22902*m + 14039*m^2 + 3940*m^3 + 555*m^4 + 38*m^5 + m^6 + 12285))/(2
64207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135
135) + (3*a*x^5*(d*x)^m*(a*c + b^2)*(47436*m + 25979*m^2 + 6280*m^3 + 753*m
^4 + 44*m^5 + m^6 + 27027))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4
+ 973*m^5 + 49*m^6 + m^7 + 135135) + (b*x^7*(d*x)^m*(6*a*c + b^2)*(34986*m
+ 20335*m^2 + 5292*m^3 + 679*m^4 + 42*m^5 + m^6 + 19305))/(264207*m + 17733
```

```
1*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (3*c*x^9
*(d*x)^m*(a*c + b^2)*(27688*m + 16627*m^2 + 4528*m^3 + 613*m^4 + 40*m^5 + m
^6 + 15015))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*
m^6 + m^7 + 135135)
```

sympy [A] time = 7.27, size = 4451, normalized size = 28.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Piecewise(((-a**3/(12*x**12) - 3*a**2*b/(10*x**10) - 3*a**2*c/(8*x**8) - 3*
a*b**2/(8*x**8) - a*b*c/x**6 - 3*a*c**2/(4*x**4) - b**3/(6*x**6) - 3*b**2*c
/(4*x**4) - 3*b*c**2/(2*x**2) + c**3*log(x))/d**13, Eq(m, -13)), ((-a**3/(1
0*x**10) - 3*a**2*b/(8*x**8) - a**2*c/(2*x**6) - a*b**2/(2*x**6) - 3*a*b*c/
(2*x**4) - 3*a*c**2/(2*x**2) - b**3/(4*x**4) - 3*b**2*c/(2*x**2) + 3*b*c**2
*log(x) + c**3*x**2/2)/d**11, Eq(m, -11)), ((-a**3/(8*x**8) - a**2*b/(2*x**
6) - 3*a**2*c/(4*x**4) - 3*a*b**2/(4*x**4) - 3*a*b*c/x**2 + 3*a*c**2*log(x)
 - b**3/(2*x**2) + 3*b**2*c*log(x) + 3*b*c**2*x**2/2 + c**3*x**4/4)/d**9, E
q(m, -9)), ((-a**3/(6*x**6) - 3*a**2*b/(4*x**4) - 3*a**2*c/(2*x**2) - 3*a*b
**2/(2*x**2) + 6*a*b*c*log(x) + 3*a*c**2*x**2/2 + b**3*log(x) + 3*b**2*c*x*
*2/2 + 3*b*c**2*x**4/4 + c**3*x**6/6)/d**7, Eq(m, -7)), ((-a**3/(4*x**4) -
3*a**2*b/(2*x**2) + 3*a**2*c*log(x) + 3*a*b**2*log(x) + 3*a*b*c*x**2 + 3*a*
c**2*x**4/4 + b**3*x**2/2 + 3*b**2*c*x**4/4 + b*c**2*x**6/2 + c**3*x**8/8)/
d**5, Eq(m, -5)), ((-a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a**2*c*x**2/2 + 3*
a*b**2*x**2/2 + 3*a*b*c*x**4/2 + a*c**2*x**6/2 + b**3*x**4/4 + b**2*c*x**6/
2 + 3*b*c**2*x**8/8 + c**3*x**10/10)/d**3, Eq(m, -3)), ((a**3*log(x) + 3*a*
*2*b*x**2/2 + 3*a**2*c*x**4/4 + 3*a*b**2*x**4/4 + a*b*c*x**6 + 3*a*c**2*x**
8/8 + b**3*x**6/6 + 3*b**2*c*x**8/8 + 3*b*c**2*x**10/10 + c**3*x**12/12)/d,
 Eq(m, -1)), (a**3*d**m*m**6*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4
 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 48*a**3*d**m*m**5*x*x**m
/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 26420
7*m + 135135) + 925*a**3*d**m*m**4*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 1004
5*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 9120*a**3*d**m*m**
3*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2
 + 264207*m + 135135) + 48259*a**3*d**m*m**2*x*x**m/(m**7 + 49*m**6 + 973*m
**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 129072*a
**3*d**m*m*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 17
7331*m**2 + 264207*m + 135135) + 135135*a**3*d**m*x*x**m/(m**7 + 49*m**6 +
973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 3*a
**2*b*d**m*m**6*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m
**3 + 177331*m**2 + 264207*m + 135135) + 138*a**2*b*d**m*m**5*x**3*x**m/(m*
*7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m
+ 135135) + 2505*a**2*b*d**m*m**4*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10
```

045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 22620*a**2*b*d**
m*m**3*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177
331*m**2 + 264207*m + 135135) + 104277*a**2*b*d**m*m**2*x**3*x**m/(m**7 + 4
9*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 1351
35) + 219162*a**2*b*d**m*m*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**
4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 135135*a**2*b*d**m*x**3
*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 +
264207*m + 135135) + 3*a**2*c*d**m*m**6*x**5*x**m/(m**7 + 49*m**6 + 973*m**
5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 132*a**2*c
*d**m*m**5*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 +
 177331*m**2 + 264207*m + 135135) + 2259*a**2*c*d**m*m**4*x**5*x**m/(m**7 +
 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 13
5135) + 18840*a**2*c*d**m*m**3*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045
*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 77937*a**2*c*d**m*m
2*x5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331
*m**2 + 264207*m + 135135) + 142308*a**2*c*d**m*m*x**5*x**m/(m**7 + 49*m**6
 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) +
81081*a**2*c*d**m*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379
*m**3 + 177331*m**2 + 264207*m + 135135) + 3*a*b**2*d**m*m**6*x**5*x**m/(m*
*7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m
+ 135135) + 132*a*b**2*d**m*m**5*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 100
45*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 2259*a*b**2*d**m*
m**4*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 17733
1*m**2 + 264207*m + 135135) + 18840*a*b**2*d**m*m**3*x**5*x**m/(m**7 + 49*m
6 + 973*m5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
 + 77937*a*b**2*d**m*m**2*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4
 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 142308*a*b**2*d**m*m*x**
5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 +
 264207*m + 135135) + 81081*a*b**2*d**m*x**5*x**m/(m**7 + 49*m**6 + 973*m**
5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 6*a*b*c*d*
*m*m**6*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 17
7331*m**2 + 264207*m + 135135) + 252*a*b*c*d**m*m**5*x**7*x**m/(m**7 + 49*m
6 + 973*m5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
 + 4074*a*b*c*d**m*m**4*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 +
 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 31752*a*b*c*d**m*m**3*x**7
*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 +
264207*m + 135135) + 122010*a*b*c*d**m*m**2*x**7*x**m/(m**7 + 49*m**6 + 973
*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 209916
*a*b*c*d**m*m*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**
3 + 177331*m**2 + 264207*m + 135135) + 115830*a*b*c*d**m*x**7*x**m/(m**7 +
49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135
135) + 3*a*c**2*d**m*m**6*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4
 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 120*a*c**2*d**m*m**5*x**
9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 +
 264207*m + 135135) + 1839*a*c**2*d**m*m**4*x**9*x**m/(m**7 + 49*m**6 + 973

*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 13584*
a*c**2*d**m*m**3*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*
m**3 + 177331*m**2 + 264207*m + 135135) + 49881*a*c**2*d**m*m**2*x**9*x**m/
(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207
*m + 135135) + 83064*a*c**2*d**m*m*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 1
0045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 45045*a*c**2*d*
*m*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*
m**2 + 264207*m + 135135) + b**3*d**m*m**6*x**7*x**m/(m**7 + 49*m**6 + 973*
m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 42*b**3
*d**m*m**5*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 +
177331*m**2 + 264207*m + 135135) + 679*b**3*d**m*m**4*x**7*x**m/(m**7 + 49
*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 13513
5) + 5292*b**3*d**m*m**3*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4
+ 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 20335*b**3*d**m*m**2*x**7
*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 +
264207*m + 135135) + 34986*b**3*d**m*m*x**7*x**m/(m**7 + 49*m**6 + 973*m**5
+ 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 19305*b**3*
d**m*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 17733
1*m**2 + 264207*m + 135135) + 3*b**2*c*d**m*m**6*x**9*x**m/(m**7 + 49*m**6
+ 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 1
20*b**2*c*d**m*m**5*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 573
79*m**3 + 177331*m**2 + 264207*m + 135135) + 1839*b**2*c*d**m*m**4*x**9*x**
m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 2642
07*m + 135135) + 13584*b**2*c*d**m*m**3*x**9*x**m/(m**7 + 49*m**6 + 973*m**
5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 49881*b**2
*c*d**m*m**2*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3
+ 177331*m**2 + 264207*m + 135135) + 83064*b**2*c*d**m*m*x**9*x**m/(m**7 +
49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 13
5135) + 45045*b**2*c*d**m*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4
+ 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 3*b*c**2*d**m*m**6*x**11
*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 +
264207*m + 135135) + 114*b*c**2*d**m*m**5*x**11*x**m/(m**7 + 49*m**6 + 973*
m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 1665*b*
c**2*d**m*m**4*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m
3 + 177331*m2 + 264207*m + 135135) + 11820*b*c**2*d**m*m**3*x**11*x**m/
(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207
*m + 135135) + 42117*b*c**2*d**m*m**2*x**11*x**m/(m**7 + 49*m**6 + 973*m**5
+ 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 68706*b*c**
2*d**m*m*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 +
177331*m**2 + 264207*m + 135135) + 36855*b*c**2*d**m*x**11*x**m/(m**7 + 49*
m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135
) + c**3*d**m*m**6*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 573
79*m**3 + 177331*m**2 + 264207*m + 135135) + 36*c**3*d**m*m**5*x**13*x**m/(
m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*
m + 135135) + 505*c**3*d**m*m**4*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 + 10

```
045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 3480*c**3*d**m*m
**3*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 17733
1*m**2 + 264207*m + 135135) + 12139*c**3*d**m*m**2*x**13*x**m/(m**7 + 49*m*
*6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
+ 19524*c**3*d**m*m*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57
379*m**3 + 177331*m**2 + 264207*m + 135135) + 10395*c**3*d**m*x**13*x**m/(m
**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m
 + 135135), True))
```

3.1106 $\int (dx)^m \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=101

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{\left(2ac+b^2\right)(dx)^{m+5}}{d^5(m+5)}+\frac{2ab(dx)^{m+3}}{d^3(m+3)}+\frac{2bc(dx)^{m+7}}{d^7(m+7)}+\frac{c^2(dx)^{m+9}}{d^9(m+9)}$$

```
[Out] a^2*(d*x)^(1+m)/d/(1+m)+2*a*b*(d*x)^(3+m)/d^3/(3+m)+(2*a*c+b^2)*(d*x)^(5+m)
/d^5/(5+m)+2*b*c*(d*x)^(7+m)/d^7/(7+m)+c^2*(d*x)^(9+m)/d^9/(9+m)
```

Rubi [A] time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1108}

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{\left(2ac+b^2\right)(dx)^{m+5}}{d^5(m+5)}+\frac{2ab(dx)^{m+3}}{d^3(m+3)}+\frac{2bc(dx)^{m+7}}{d^7(m+7)}+\frac{c^2(dx)^{m+9}}{d^9(m+9)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(3 + m))/(d^3*(3 + m)) + ((b
^2 + 2*a*c)*(d*x)^(5 + m))/(d^5*(5 + m)) + (2*b*c*(d*x)^(7 + m))/(d^7*(7 +
m)) + (c^2*(d*x)^(9 + m))/(d^9*(9 + m))
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a+bx^2+cx^4\right)^2 \, dx &= \int \left(a^2(dx)^m+\frac{2ab(dx)^{2+m}}{d^2}+\frac{\left(b^2+2ac\right)(dx)^{4+m}}{d^4}+\frac{2bc(dx)^{6+m}}{d^6}+\frac{c^2(dx)^{8+m}}{d^8}\right) dx\\ &= \frac{a^2(dx)^{1+m}}{d(1+m)}+\frac{2ab(dx)^{3+m}}{d^3(3+m)}+\frac{\left(b^2+2ac\right)(dx)^{5+m}}{d^5(5+m)}+\frac{2bc(dx)^{7+m}}{d^7(7+m)}+\frac{c^2(dx)^{9+m}}{d^9(9+m)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 70, normalized size = 0.69

$$x(dx)^m\left(\frac{a^2}{m+1}+\frac{x^4\left(2ac+b^2\right)}{m+5}+\frac{2abx^2}{m+3}+\frac{2bcx^6}{m+7}+\frac{c^2x^8}{m+9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^2)/(3 + m) + ((b^2 + 2*a*c)*x^4)/(5 + m)
+ (2*b*c*x^6)/(7 + m) + (c^2*x^8)/(9 + m))
```

fricas [B] time = 0.66, size = 241, normalized size = 2.39

$$\frac{\left(\left(c^2m^4 + 16\,c^2m^3 + 86\,c^2m^2 + 176\,c^2m + 105\,c^2\right)x^9 + 2\left(bcm^4 + 18\,bcm^3 + 104\,bcm^2 + 222\,bcm + 135\,bc\right)x^7 + \left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] ((c^2*m^4 + 16*c^2*m^3 + 86*c^2*m^2 + 176*c^2*m + 105*c^2)*x^9 + 2*(b*c*m^4
 + 18*b*c*m^3 + 104*b*c*m^2 + 222*b*c*m + 135*b*c)*x^7 + ((b^2 + 2*a*c)*m^4
 + 20*(b^2 + 2*a*c)*m^3 + 130*(b^2 + 2*a*c)*m^2 + 189*b^2 + 378*a*c + 300*(
b^2 + 2*a*c)*m)*x^5 + 2*(a*b*m^4 + 22*a*b*m^3 + 164*a*b*m^2 + 458*a*b*m + 3
15*a*b)*x^3 + (a^2*m^4 + 24*a^2*m^3 + 206*a^2*m^2 + 744*a^2*m + 945*a^2)*x)
*(d*x)^m/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

giac [B] time = 0.18, size = 449, normalized size = 4.45

$$\frac{(dx)^m\,c^2m^4x^9 + 16\,(dx)^m\,c^2m^3x^9 + 2\,(dx)^m\,bcm^4x^7 + 86\,(dx)^m\,c^2m^2x^9 + 36\,(dx)^m\,bcm^3x^7 + 176\,(dx)^m\,c^2mx^9 +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] ((d*x)^m*c^2*m^4*x^9 + 16*(d*x)^m*c^2*m^3*x^9 + 2*(d*x)^m*b*c*m^4*x^7 + 86*
(d*x)^m*c^2*m^2*x^9 + 36*(d*x)^m*b*c*m^3*x^7 + 176*(d*x)^m*c^2*m*x^9 + (d*x
)^m*b^2*m^4*x^5 + 2*(d*x)^m*a*c*m^4*x^5 + 208*(d*x)^m*b*c*m^2*x^7 + 105*(d*
x)^m*c^2*x^9 + 20*(d*x)^m*b^2*m^3*x^5 + 40*(d*x)^m*a*c*m^3*x^5 + 444*(d*x)^
m*b*c*m*x^7 + 2*(d*x)^m*a*b*m^4*x^3 + 130*(d*x)^m*b^2*m^2*x^5 + 260*(d*x)^m
*a*c*m^2*x^5 + 270*(d*x)^m*b*c*x^7 + 44*(d*x)^m*a*b*m^3*x^3 + 300*(d*x)^m*b
^2*m*x^5 + 600*(d*x)^m*a*c*m*x^5 + (d*x)^m*a^2*m^4*x + 328*(d*x)^m*a*b*m^2*
x^3 + 189*(d*x)^m*b^2*x^5 + 378*(d*x)^m*a*c*x^5 + 24*(d*x)^m*a^2*m^3*x + 91
6*(d*x)^m*a*b*m*x^3 + 206*(d*x)^m*a^2*m^2*x + 630*(d*x)^m*a*b*x^3 + 744*(d*
x)^m*a^2*m*x + 945*(d*x)^m*a^2*x)/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*
m + 945)
```

maple [B] time = 0.01, size = 301, normalized size = 2.98

$$\frac{\left(c^2m^4x^8 + 16c^2m^3x^8 + 2bc\,m^4x^6 + 86c^2m^2x^8 + 36bc\,m^3x^6 + 176c^2m\,x^8 + 2ac\,m^4x^4 + b^2m^4x^4 + 208bc\,m^2x^6 + 1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] x*(c^2*m^4*x^8+16*c^2*m^3*x^8+2*b*c*m^4*x^6+86*c^2*m^2*x^8+36*b*c*m^3*x^6+1
76*c^2*m*x^8+2*a*c*m^4*x^4+b^2*m^4*x^4+208*b*c*m^2*x^6+105*c^2*x^8+40*a*c*m
^3*x^4+20*b^2*m^3*x^4+444*b*c*m*x^6+2*a*b*m^4*x^2+260*a*c*m^2*x^4+130*b^2*m
^2*x^4+270*b*c*x^6+44*a*b*m^3*x^2+600*a*c*m*x^4+300*b^2*m*x^4+a^2*m^4+328*a
*b*m^2*x^2+378*a*c*x^4+189*b^2*x^4+24*a^2*m^3+916*a*b*m*x^2+206*a^2*m^2+630
*a*b*x^2+744*a^2*m+945*a^2)*(d*x)^m/(m+9)/(m+7)/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.11, size = 110, normalized size = 1.09

$$\frac{c^2 d^m x^9 x^m}{m+9} + \frac{2\,bcd^m x^7 x^m}{m+7} + \frac{b^2 d^m x^5 x^m}{m+5} + \frac{2\,acd^m x^5 x^m}{m+5} + \frac{2\,abd^m x^3 x^m}{m+3} + \frac{(dx)^{m+1}\,a^2}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] c^2*d^m*x^9*x^m/(m + 9) + 2*b*c*d^m*x^7*x^m/(m + 7) + b^2*d^m*x^5*x^m/(m +
5) + 2*a*c*d^m*x^5*x^m/(m + 5) + 2*a*b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*
a^2/(d*(m + 1))
```

mupad [B] time = 4.58, size = 260, normalized size = 2.57

$$(d\,x)^m\,\left(\frac{c^2\,x^9\,\left(m^4+16\,m^3+86\,m^2+176\,m+105\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{x^5\,\left(b^2+2\,a\,c\right)\,\left(m^4+20\,m^3+130\,m^2+300\,m+189\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] (d*x)^m*((c^2*x^9*(176*m + 86*m^2 + 16*m^3 + m^4 + 105))/(1689*m + 950*m^2
+ 230*m^3 + 25*m^4 + m^5 + 945) + (x^5*(2*a*c + b^2)*(300*m + 130*m^2 + 20*
m^3 + m^4 + 189))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (a^2*
x*(744*m + 206*m^2 + 24*m^3 + m^4 + 945))/(1689*m + 950*m^2 + 230*m^3 + 25*
m^4 + m^5 + 945) + (2*a*b*x^3*(458*m + 164*m^2 + 22*m^3 + m^4 + 315))/(1689
*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (2*b*c*x^7*(222*m + 104*m^2
+ 18*m^3 + m^4 + 135))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945))
```

sympy [A] time = 2.87, size = 1486, normalized size = 14.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Piecewise(((-a**2/(8*x**8) - a*b/(3*x**6) - a*c/(2*x**4) - b**2/(4*x**4) -
b*c/x**2 + c**2*log(x))/d**9, Eq(m, -9)), ((-a**2/(6*x**6) - a*b/(2*x**4) -
 a*c/x**2 - b**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2/2)/d**7, Eq(m, -7)), (
(-a**2/(4*x**4) - a*b/x**2 + 2*a*c*log(x) + b**2*log(x) + b*c*x**2 + c**2*x
**4/4)/d**5, Eq(m, -5)), ((-a**2/(2*x**2) + 2*a*b*log(x) + a*c*x**2 + b**2*
x**2/2 + b*c*x**4/2 + c**2*x**6/6)/d**3, Eq(m, -3)), ((a**2*log(x) + a*b*x*
*2 + a*c*x**4/2 + b**2*x**4/4 + b*c*x**6/3 + c**2*x**8/8)/d, Eq(m, -1)), (a
**2*d**m*m**4*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945)
+ 24*a**2*d**m*m**3*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m +
 945) + 206*a**2*d**m*m**2*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1
689*m + 945) + 744*a**2*d**m*m*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2
 + 1689*m + 945) + 945*a**2*d**m*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m*
*2 + 1689*m + 945) + 2*a*b*d**m*m**4*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 +
 950*m**2 + 1689*m + 945) + 44*a*b*d**m*m**3*x**3*x**m/(m**5 + 25*m**4 + 23
0*m**3 + 950*m**2 + 1689*m + 945) + 328*a*b*d**m*m**2*x**3*x**m/(m**5 + 25*
m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 916*a*b*d**m*m*x**3*x**m/(m**5
 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 630*a*b*d**m*x**3*x**m/(
m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 2*a*c*d**m*m**4*x**5
*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 40*a*c*d**m*m
**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 260*a
*c*d**m*m**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945
) + 600*a*c*d**m*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
 + 945) + 378*a*c*d**m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 16
89*m + 945) + b**2*d**m*m**4*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**
2 + 1689*m + 945) + 20*b**2*d**m*m**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3
+ 950*m**2 + 1689*m + 945) + 130*b**2*d**m*m**2*x**5*x**m/(m**5 + 25*m**4 +
 230*m**3 + 950*m**2 + 1689*m + 945) + 300*b**2*d**m*m*x**5*x**m/(m**5 + 25
*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 189*b**2*d**m*x**5*x**m/(m**5
 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 2*b*c*d**m*m**4*x**7*x**
m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 36*b*c*d**m*m**3*
x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 208*b*c*d
**m*m**2*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) +
444*b*c*d**m*m*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 9
45) + 270*b*c*d**m*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
 + 945) + c**2*d**m*m**4*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
1689*m + 945) + 16*c**2*d**m*m**3*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 95
0*m**2 + 1689*m + 945) + 86*c**2*d**m*m**2*x**9*x**m/(m**5 + 25*m**4 + 230*
m**3 + 950*m**2 + 1689*m + 945) + 176*c**2*d**m*m*x**9*x**m/(m**5 + 25*m**4
 + 230*m**3 + 950*m**2 + 1689*m + 945) + 105*c**2*d**m*x**9*x**m/(m**5 + 25
*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945), True))
```

3.1107 $\int (dx)^m \left(a+bx^2+cx^4\right) dx$

Optimal. Leaf size=52

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+3}}{d^3(m+3)}+\frac{c(dx)^{m+5}}{d^5(m+5)}$$

```
[Out] a*(d*x)^(1+m)/d/(1+m)+b*(d*x)^(3+m)/d^3/(3+m)+c*(d*x)^(5+m)/d^5/(5+m)
```

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+3}}{d^3(m+3)}+\frac{c(dx)^{m+5}}{d^5(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*(d*x)^(1 + m))/(d*(1 + m)) + (b*(d*x)^(3 + m))/(d^3*(3 + m)) + (c*(d*x)^
(5 + m))/(d^5*(5 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a+bx^2+cx^4\right) dx &= \int \left(a(dx)^m+\frac{b(dx)^{2+m}}{d^2}+\frac{c(dx)^{4+m}}{d^4}\right) dx\\ &= \frac{a(dx)^{1+m}}{d(1+m)}+\frac{b(dx)^{3+m}}{d^3(3+m)}+\frac{c(dx)^{5+m}}{d^5(5+m)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 0.67

$$x(dx)^m\left(\frac{a}{m+1}+\frac{bx^2}{m+3}+\frac{cx^4}{m+5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^2 + c*x^4),x]
```

```
[Out] x*(d*x)^m*(a/(1 + m) + (b*x^2)/(3 + m) + (c*x^4)/(5 + m))
```

fricas [A] time = 0.51, size = 71, normalized size = 1.37

$$\frac{\left(\left(cm^{2}+4\,cm+3\,c\right)x^{5}+\left(bm^{2}+6\,bm+5\,b\right)x^{3}+\left(am^{2}+8\,am+15\,a\right)x\right)(dx)^{m}}{m^{3}+9\,m^{2}+23\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] ((c*m^2 + 4*c*m + 3*c)*x^5 + (b*m^2 + 6*b*m + 5*b)*x^3 + (a*m^2 + 8*a*m + 1
5*a)*x)*(d*x)^m/(m^3 + 9*m^2 + 23*m + 15)
```

giac [B] time = 0.16, size = 119, normalized size = 2.29

$$\frac{(dx)^{m}\,cm^{2}x^{5}+4\,(dx)^{m}\,cmx^{5}+(dx)^{m}\,bm^{2}x^{3}+3\,(dx)^{m}\,cx^{5}+6\,(dx)^{m}\,bmx^{3}+(dx)^{m}\,am^{2}x+5\,(dx)^{m}\,bx^{3}+8\,(dx}{m^{3}+9\,m^{2}+23\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] ((d*x)^m*c*m^2*x^5 + 4*(d*x)^m*c*m*x^5 + (d*x)^m*b*m^2*x^3 + 3*(d*x)^m*c*x^
5 + 6*(d*x)^m*b*m*x^3 + (d*x)^m*a*m^2*x + 5*(d*x)^m*b*x^3 + 8*(d*x)^m*a*m*x
 + 15*(d*x)^m*a*x)/(m^3 + 9*m^2 + 23*m + 15)
```

maple [A] time = 0.00, size = 78, normalized size = 1.50

$$\frac{\left(c\,m^{2}x^{4}+4cm\,x^{4}+b\,m^{2}x^{2}+3c\,x^{4}+6bm\,x^{2}+a\,m^{2}+5b\,x^{2}+8am+15a\right)x\,(dx)^{m}}{(m+5)\,(m+3)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^4+b*x^2+a),x)
```

```
[Out] x*(c*m^2*x^4+4*c*m*x^4+b*m^2*x^2+3*c*x^4+6*b*m*x^2+a*m^2+5*b*x^2+8*a*m+15*a
)*(d*x)^m/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.09, size = 50, normalized size = 0.96

$$\frac{cd^{m}x^{5}x^{m}}{m+5}+\frac{bd^{m}x^{3}x^{m}}{m+3}+\frac{(dx)^{m+1}\,a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] c*d^m*x^5*x^m/(m + 5) + b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*a/(d*(m + 1))
```

mupad [B] time = 4.40, size = 89, normalized size = 1.71

$$(d\,x)^m\,\left(\frac{b\,x^3\,\left(m^2+6\,m+5\right)}{m^3+9\,m^2+23\,m+15}+\frac{c\,x^5\,\left(m^2+4\,m+3\right)}{m^3+9\,m^2+23\,m+15}+\frac{a\,x\,\left(m^2+8\,m+15\right)}{m^3+9\,m^2+23\,m+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^2 + c*x^4),x)
```

```
[Out] (d*x)^m*((b*x^3*(6*m + m^2 + 5))/(23*m + 9*m^2 + m^3 + 15) + (c*x^5*(4*m +
m^2 + 3))/(23*m + 9*m^2 + m^3 + 15) + (a*x*(8*m + m^2 + 15))/(23*m + 9*m^2
+ m^3 + 15))
```

sympy [A] time = 0.99, size = 314, normalized size = 6.04

$$\begin{cases} \frac{-\frac{a}{4x^4}-\frac{b}{2x^2}+c\log(x)}{d^5} \\ \frac{-\frac{a}{2x^2}+b\log(x)+\frac{cx^2}{2}}{d^3} \\ \frac{a\log(x)+\frac{bx^2}{2}+\frac{cx^4}{4}}{d} \\ \frac{ad^m m^2 x x^m}{m^3+9m^2+23m+15}+\frac{8ad^m m x x^m}{m^3+9m^2+23m+15}+\frac{15ad^m x x^m}{m^3+9m^2+23m+15}+\frac{bd^m m^2 x^3 x^m}{m^3+9m^2+23m+15}+\frac{6bd^m m x^3 x^m}{m^3+9m^2+23m+15}+\frac{5bd^m x^3 x^m}{m^3+9m^2+23m+15}+\frac{cd^m m}{m^3+9m^2\cdots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**4+b*x**2+a),x)
```

```
[Out] Piecewise(((-a/(4*x**4) - b/(2*x**2) + c*log(x))/d**5, Eq(m, -5)), ((-a/(2*
x**2) + b*log(x) + c*x**2/2)/d**3, Eq(m, -3)), ((a*log(x) + b*x**2/2 + c*x*
*4/4)/d, Eq(m, -1)), (a*d**m*m**2*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 8*a*
d**m*m*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 15*a*d**m*x*x**m/(m**3 + 9*m**2
 + 23*m + 15) + b*d**m*m**2*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 6*b*d**
m*m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 5*b*d**m*x**3*x**m/(m**3 + 9*m*
*2 + 23*m + 15) + c*d**m*m**2*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 4*c*d
**m*m*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 3*c*d**m*x**5*x**m/(m**3 + 9*
m**2 + 23*m + 15), True))
```

3.1108 $\int \frac{(dx)^m}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=173

$$\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] 2*c*(d*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^
2)^(1/2)))/d/(1+m)/(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)-2*c*(d*x)^(1+m
)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/d/(
1+m)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.25, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1131, 364}

$$\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m
)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(
1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1131

```
Int[((d_.)*(x_))^(m_.)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*x^2), x]
, x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c
, d, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(dx)^m}{a+bx^2+cx^4}\,dx = \frac{c\int \frac{(dx)^m}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{(dx)^m}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}}$$
$$= \frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d(1+m)} - \frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}$$

Mathematica [C] time = 0.06, size = 82, normalized size = 0.47

$$\frac{(dx)^m \text{RootSum}\left[\#1^4c+\#1^2b+a\&,\frac{\left(\frac{x}{x-\#1}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)}{2\#1^3c+\#1b}\&\right]}{2m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m/(a + b*x^2 + c*x^4),x]
[Out] ((d*x)^m*RootSum[a + b*#1^2 + c*#1^4 & , Hypergeometric2F1[-m, -m, 1 - m, -
(#1/(x - #1))]/((x/(x - #1))^m*(b*#1 + 2*c*#1^3)) & ])/(2*m)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{cx^4+bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((d*x)^m/(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((d*x)^m/(c*x^4 + b*x^2 + a), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{c\,x^4 + b\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^4+b*x^2+a),x)
[Out] int((d*x)^m/(c*x^4+b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((d*x)^m/(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{c\,x^4 + b\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^2 + c*x^4),x)
[Out] int((d*x)^m/(a + b*x^2 + c*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + bx^2 + cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**4+b*x**2+a),x)
[Out] Integral((d*x)**m/(a + b*x**2 + c*x**4), x)
```

3.1109 $\int \frac{(dx)^m}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=315

$$\frac{c(dx)^{m+1}\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2ad(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(-b(1-m)\sqrt{b^2\right.}{2}$$

```
[Out] 1/2*(d*x)^(1+m)*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/d/(c*x^4+b*x^2+a)-1/2*c*
(d*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b+(-4*a*c+b^2)^(
1/2)))*(b^2*(1-m)-4*a*c*(3-m)-b*(1-m)*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3
/2)/d/(1+m)/(b+(-4*a*c+b^2)^(1/2))+1/2*c*(d*x)^(1+m)*hypergeom([1, 1/2+1/2*
m],[3/2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))*(b^2*(1-m)-4*a*c*(3-m)+b*(1
-m)*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)/d/(1+m)/(b-(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.70, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1121, 1285, 364}

$$\frac{c(dx)^{m+1}\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2ad(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(-b(1-m)\sqrt{b^2\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*d*(a + b*x^2 + c
*x^4)) + (c*(b^2*(1 - m) + b*Sqrt[b^2 - 4*a*c]*(1 - m) - 4*a*c*(3 - m))*(d*
x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m))
 - (c*(b^2*(1 - m) - b*Sqrt[b^2 - 4*a*c]*(1 - m) - 4*a*c*(3 - m))*(d*x)^(1
+ m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1285

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b
*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*
e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{(dx)^{1+m}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)d\left(a+bx^2+cx^4\right)} - \frac{\int \frac{(dx)^m\left(-b^2(1-m)+2ac(3-m)-bc(1-m)x^2\right)}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)d\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(b^2(1-m)-b\sqrt{b^2-4ac}\,(1-m)-4ac(3-m)\right)\right)\int}{4a\left(b^2-4ac\right)^{3/2}}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)d\left(a+bx^2+cx^4\right)} + \frac{c\left(b^2(1-m)+b\sqrt{b^2-4ac}\,(1-m)-4ac(3-m)\right)(dx}{2a\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4a\right.}}$$

Mathematica [C] time = 0.08, size = 78, normalized size = 0.25

$$\frac{x(dx)^m F_1\left(\frac{m+1}{2};2,2;\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{a^2(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(d*x)^m*AppellF1[(1 + m)/2, 2, 2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(a^2*(1 + m))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{c^2x^8 + 2\,bcx^6 + (b^2 + 2\,ac)x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x^4 + 2*a*b*x^2 + a^2
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(cx^4 + bx^2 + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(c\,x^4 + b\,x^2 + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] int((d*x)^m/(c*x^4+b*x^2+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(cx^4 + bx^2 + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^m}{{\left(c\,x^4+b\,x^2+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^2 + c*x^4)^2,x)
[Out] int((d*x)^m/(a + b*x^2 + c*x^4)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

3.1110 $\int (dx)^m \left(a+bx^2+cx^4\right)^{3/2} \, dx$

Optimal. Leaf size=158

$$\frac{a(dx)^{m+1}\sqrt{a+bx^2+cx^4}\, F_1\left(\frac{m+1}{2};-\frac{3}{2},-\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] a*(d*x)^(1+m)*AppellF1(1/2+1/2*m,-3/2,-3/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b^
2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/d/(1+m)/(1
+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(
1/2)
```

Rubi [A] time = 0.15, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1141, 510}

$$\frac{a(dx)^{m+1}\sqrt{a+bx^2+cx^4}\, F_1\left(\frac{m+1}{2};-\frac{3}{2},-\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -3/2, -3/2, (3
 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
```

```
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int (dx)^m \left(a+bx^2+cx^4\right)^{3/2}\,dx = \frac{\left(a\sqrt{a+bx^2+cx^4}\right)\int (dx)^m \left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$
$$= \frac{a(dx)^{1+m}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1+m}{2};-\frac{3}{2},-\frac{3}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.28, size = 357, normalized size = 2.26

$$\frac{x(dx)^m\sqrt{a+bx^2+cx^4}\left(a\left(m^2+8m+15\right)F_1\left(\frac{m+1}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+(m+1)x^2\left(c(m+3)x\right.\right.}{(m+1)(m+3)(m+5)\sqrt{\frac{-\sqrt{b^2-4a}}{b-\sqrt{b}}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + b*x^2 + c*x^4]*(a*(15 + 8*m + m^2)*AppellF1[(1 + m)/2,
-1/2, -1/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
Sqrt[b^2 - 4*a*c])] + (1 + m)*x^2*(b*(5 + m)*AppellF1[(3 + m)/2, -1/2, -1/2
, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
 4*a*c])] + c*(3 + m)*x^2*AppellF1[(5 + m)/2, -1/2, -1/2, (7 + m)/2, (-2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))/((1 +
m)*(3 + m)*(5 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^4+bx^2+a\right)^{\frac{3}{2}}(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^m, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \left(c\,x^4 + b\,x^2 + a\right)^{\frac{3}{2}} (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^4+b*x^2+a)^(3/2),x)
[Out] int((d*x)^m*(c*x^4+b*x^2+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m \left(c\,x^4 + b\,x^2 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int((d*x)^m*(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + bx^2 + cx^4\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((d*x)**m*(a + b*x**2 + c*x**4)**(3/2), x)
```

3.1111 $\int (dx)^m \sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=157

$$\frac{(dx)^{m+1}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{m+1}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (d*x)^(1+m)*AppellF1(1/2+1/2*m,-1/2,-1/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b^2)
^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/d/(1+m)/(1+2
*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/
2)
```

Rubi [A] time = 0.14, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1141, 510}

$$\frac{(dx)^{m+1}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{m+1}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -1/2, -1/2, (3 +
 m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])])/(d*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
```

```
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int (dx)^m\sqrt{a+bx^2+cx^4}\,dx = \frac{\sqrt{a+bx^2+cx^4}\int (dx)^m\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{(dx)^{1+m}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1+m}{2};-\frac{1}{2},-\frac{1}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [A] time = 0.10, size = 181, normalized size = 1.15

$$\frac{x(dx)^m\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{m+1}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{(m+1)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -1/2, -1/2, (3 + m)/
2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])
/((1 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^4+bx^2+a}\,(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(d*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4+bx^2+a}\,(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^m, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{c\,x^{4}+b\,x^{2}+a}\;\left(d x\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^4+b*x^2+a)^(1/2),x)
[Out] int((d*x)^m*(c*x^4+b*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^{4}+bx^{2}+a}\,\left(dx\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,x\right)}^m\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((d*x)^m*(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(dx\right)^{m} \sqrt{a + bx^{2} + cx^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((d*x)**m*sqrt(a + b*x**2 + c*x**4), x)
```

3.1112 $\int \frac{(dx)^m}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=157

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{m+1}{2};\frac{1}{2},\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (d*x)^(1+m)*AppellF1(1/2+1/2*m,1/2,1/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(
1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/d/(1+m)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1141, 510}

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{m+1}{2};\frac{1}{2},\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 1/2, 1/2, (3 + m)/2, (-2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*(1 +
m)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^m}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^2+cx^4}}$$

$$= \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{2};\frac{1}{2},\frac{1}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] time = 0.11, size = 181, normalized size = 1.15

$$\frac{x(dx)^m\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\,F_1\left(\frac{m+1}{2};\frac{1}{2},\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{(m+1)\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1
 + m)/2, 1/2, 1/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)
/(-b + Sqrt[b^2 - 4*a*c])])/((1 + m)*Sqrt[a + b*x^2 + c*x^4])
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((d*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{m}}{\sqrt{c\,x^{4}+b\,x^{2}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^4+b*x^2+a)^(1/2),x)
[Out] int((d*x)^m/(c*x^4+b*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{m}}{\sqrt{cx^{4}+bx^{2}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((d*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^{m}}{\sqrt{c\,x^{4}+b\,x^{2}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((d*x)^m/(a + b*x^2 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{m}}{\sqrt{a+bx^{2}+cx^{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((d*x)**m/sqrt(a + b*x**2 + c*x**4), x)
```

3.1113 $\int \frac{(dx)^m}{(a+bx^2+cx^4)^{3/2}} dx$

Optimal. Leaf size=160

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{2};\frac{3}{2},\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (d*x)^(1+m)*AppellF1(1/2+1/2*m,3/2,3/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(
1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/d/(1+m)/(c*x^4+b*x^2+a)^(1/
2)
```

Rubi [A] time = 0.14, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1141, 510}

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{2};\frac{3}{2},\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 3/2, 3/2, (3 + m)/2, (-2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*d*(1
+ m)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
```

```
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^m}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}}$$

$$= \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{2};\frac{3}{2},\frac{3}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad(1+m)\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] time = 0.17, size = 221, normalized size = 1.38

$$\frac{x(dx)^m\left(\sqrt{b^2-4ac}-b-2cx^2\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}\right)^{3/2}F_1\left(\frac{m+1}{2};\frac{3}{2},\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{(m+1)\left(\sqrt{b^2-4ac}-b\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(d*x)^m*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]))^(3/2)*AppellF1[(1 + m)/2, 3/2, 3/2, (3 + m)/2, (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/((-b + Sqr
t[b^2 - 4*a*c])*(1 + m)*(a + b*x^2 + c*x^4)^(3/2))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\,(dx)^m}{c^2x^8+2\,bcx^6+\left(b^2+2\,ac\right)x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(d*x)^m/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*
c)*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((d*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^4+b*x^2+a)^(3/2),x)
[Out] int((d*x)^m/(c*x^4+b*x^2+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((d*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int((d*x)^m/(a + b*x^2 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((d*x)**m/(a + b*x**2 + c*x**4)**(3/2), x)
```

3.1114 $\int (dx)^m \left(a+bx^2+cx^4\right)^p\,dx$

Optimal. Leaf size=155

$$\frac{(dx)^{m+1}\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{m+1}{2};-p,-p;\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)}$$

```
[Out] (d*x)^(1+m)*(c*x^4+b*x^2+a)^p*AppellF1(1/2+1/2*m,-p,-p,3/2+1/2*m,-2*c*x^2/(
b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/((1+2*c*x^2/
(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] time = 0.10, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1141, 510}

$$\frac{(dx)^{m+1}\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{m+1}{2};-p,-p;\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^2 + c*x^4)^p*AppellF1[(1 + m)/2, -p, -p, (3 + m)/2,
 (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(
d*(1 + m)*(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]))^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p\,dx = \left(\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p\right)\int (dx)^m \left(1+\right.$$

$$= \frac{(dx)^{1+m}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{1+m}{2};-p,-p;\frac{3+}{}\right.}{d(1+m)}$$

Mathematica [A] time = 0.24, size = 179, normalized size = 1.15

$$\frac{x(dx)^m\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{m+1}{2};-p,-p;\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x*(d*x)^m*(a + b*x^2 + c*x^4)^p*AppellF1[(1 + m)/2, -p, -p, (3 + m)/2, (-2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/((1 +
 m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqr
t[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^4+bx^2+a\right)^p (dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p*(d*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4+bx^2+a\right)^p (dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*(d*x)^m, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (dx)^{m} \left(c\,x^{4}+b\,x^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^4+b*x^2+a)^p,x)
[Out] int((d*x)^m*(c*x^4+b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{4}+bx^{2}+a\right)^{p} (dx)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^p*(d*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^{m}\,\left(c\,x^{4}+b\,x^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^2 + c*x^4)^p,x)
[Out] int((d*x)^m*(a + b*x^2 + c*x^4)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**4+b*x**2+a)**p,x)
[Out] Timed out
```

3.1115 $\int x^7 \left(a+bx^2+cx^4\right)^p \, dx$

Optimal. Leaf size=257

$$-\frac{b2^{p-2}\left(6ac-b^2(p+3)\right)\left(a+bx^2+cx^4\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c^3(p+1)(2p+3)\sqrt{b^2-4ac}}+\frac{(-2a}{}$$

```
[Out] 1/4*x^4*(c*x^4+b*x^2+a)^(1+p)/c/(2+p)+1/8*(b^2*(2+p)*(3+p)-2*a*c*(3+2*p)-2*
b*c*(1+p)*(3+p)*x^2)*(c*x^4+b*x^2+a)^(1+p)/c^3/(2+p)/(2*p^2+5*p+3)-2^(-2+p)
*b*(6*a*c-b^2*(3+p))*(c*x^4+b*x^2+a)^(1+p)*hypergeom([-p, 1+p],[2+p],1/2*(2
*c*x^2+(-4*a*c+b^2)^(1/2)+b)/(-4*a*c+b^2)^(1/2))*((-2*c*x^2+(-4*a*c+b^2)^(1
/2)-b)/(-4*a*c+b^2)^(1/2))^(-1-p)/c^3/(1+p)/(3+2*p)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1114, 742, 779, 624}

$$-\frac{b2^{p-2}\left(6ac-b^2(p+3)\right)\left(a+bx^2+cx^4\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c^3(p+1)(2p+3)\sqrt{b^2-4ac}}+\frac{(-2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^4*(a + b*x^2 + c*x^4)^(1 + p))/(4*c*(2 + p)) + ((b^2*(2 + p)*(3 + p) - 2
*a*c*(3 + 2*p) - 2*b*c*(1 + p)*(3 + p)*x^2)*(a + b*x^2 + c*x^4)^(1 + p))/(8
*c^3*(1 + p)*(2 + p)*(3 + 2*p)) - (2^(-2 + p)*b*(6*a*c - b^2*(3 + p))*(-((b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^2 + c
*x^4)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)/(2*Sqrt[b^2 - 4*a*c])])/(c^3*Sqrt[b^2 - 4*a*c]*(1 + p)*(3 + 2*p))
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
```

```
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int x^7\left(a+bx^2+cx^4\right)^p\,dx=\frac{1}{2}\text{Subst}\left(\int x^3\left(a+bx+cx^2\right)^p\,dx,x,x^2\right)$$

$$=\frac{x^4\left(a+bx^2+cx^4\right)^{1+p}}{4c(2+p)}+\frac{\text{Subst}\left(\int x(-2a-b(3+p)x)\left(a+bx+cx^2\right)^p\,dx,x,x^2\right)}{4c(2+p)}$$

$$=\frac{x^4\left(a+bx^2+cx^4\right)^{1+p}}{4c(2+p)}+\frac{\left(b^2(2+p)(3+p)-2ac(3+2p)-2bc(1+p)(3+p)x^2\right)\left(a+\right.}{8c^3(1+p)(2+p)(3+2p)}$$

$$=\frac{x^4\left(a+bx^2+cx^4\right)^{1+p}}{4c(2+p)}+\frac{\left(b^2(2+p)(3+p)-2ac(3+2p)-2bc(1+p)(3+p)x^2\right)\left(a+\right.}{8c^3(1+p)(2+p)(3+2p)}$$

Mathematica [C] time = 0.23, size = 162, normalized size = 0.63

$$\frac{1}{8}x^8\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^2+cx^4\right)^pF_1\left(4;-p,-p;5;-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{}{\sqrt{}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^8*(a + b*x^2 + c*x^4)^p*AppellF1[4, -p, -p, 5, (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(8*((b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^4 + bx^2 + a\right)^p x^7, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p*x^7, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2 + a\right)^p x^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^7, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x^7 \left(c\,x^4 + b\,x^2 + a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(c*x^4+b*x^2+a)^p,x)
```

```
[Out] int(x^7*(c*x^4+b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2 + a\right)^p x^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^7 \left(c\,x^4+b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2 + c*x^4)^p,x)
[Out] int(x^7*(a + b*x^2 + c*x^4)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{7} \left(a + bx^{2} + cx^{4}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(c*x**4+b*x**2+a)**p,x)
[Out] Integral(x**7*(a + b*x**2 + c*x**4)**p, x)
```

3.1116 $\int x^5 \left(a + bx^2 + cx^4\right)^p \, dx$

Optimal. Leaf size=223

$$\frac{2^{p-1}\left(2ac - b^2(p+2)\right)\left(a + bx^2 + cx^4\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1} {}_2F_1\left(-p, p+1; p+2; \frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c^2(p+1)(2p+3)\sqrt{b^2-4ac}} - \frac{b(p+2}{4c^2}$$

```
[Out] -1/4*b*(2+p)*(c*x^4+b*x^2+a)^(1+p)/c^2/(2*p^2+5*p+3)+1/2*x^2*(c*x^4+b*x^2+a
)^(1+p)/c/(3+2*p)+2^(-1+p)*(2*a*c-b^2*(2+p))*(c*x^4+b*x^2+a)^(1+p)*hypergeo
m([-p, 1+p],[2+p],1/2*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/(-4*a*c+b^2)^(1/2))*((
-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2))^(-1-p)/c^2/(1+p)/(3+2*p)
/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1114, 742, 640, 624}

$$\frac{2^{p-1}\left(2ac - b^2(p+2)\right)\left(a + bx^2 + cx^4\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1} {}_2F_1\left(-p, p+1; p+2; \frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c^2(p+1)(2p+3)\sqrt{b^2-4ac}} - \frac{b(p+2}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] -(b*(2 + p)*(a + b*x^2 + c*x^4)^(1 + p))/(4*c^2*(1 + p)*(3 + 2*p)) + (x^2*(
a + b*x^2 + c*x^4)^(1 + p))/(2*c*(3 + 2*p)) + (2^(-1 + p)*(2*a*c - b^2*(2 +
 p))*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a +
 b*x^2 + c*x^4)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)/(2*Sqrt[b^2 - 4*a*c])])/(c^2*Sqrt[b^2 - 4*a*c]*(1 + p)*(
3 + 2*p))
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int x^5\left(a+bx^2+cx^4\right)^p\,dx=\frac{1}{2}\operatorname{Subst}\left(\int x^2\left(a+bx+cx^2\right)^p\,dx,x,x^2\right)$$

$$=\frac{x^2\left(a+bx^2+cx^4\right)^{1+p}}{2c(3+2p)}+\frac{\operatorname{Subst}\left(\int(-a-b(2+p)x)\left(a+bx+cx^2\right)^p\,dx,x,x^2\right)}{2c(3+2p)}$$

$$=-\frac{b(2+p)\left(a+bx^2+cx^4\right)^{1+p}}{4c^2(1+p)(3+2p)}+\frac{x^2\left(a+bx^2+cx^4\right)^{1+p}}{2c(3+2p)}-\frac{\left(2ac-b^2(2+p)\right)\operatorname{Subst}\left(\int\left(\right.\right.}{4c^2(3+}$$

$$=-\frac{b(2+p)\left(a+bx^2+cx^4\right)^{1+p}}{4c^2(1+p)(3+2p)}+\frac{x^2\left(a+bx^2+cx^4\right)^{1+p}}{2c(3+2p)}+\frac{2^{-1+p}\left(2ac-b^2(2+p)\right)\left(-\frac{b-}{}\right.}{}$$

Mathematica [C] time = 0.22, size = 162, normalized size = 0.73

$$\frac{1}{6}x^6\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^2+cx^4\right)^pF_1\left(3;-p,-p;4;-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{}{\sqrt{b}}\right.$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^5*(a + b*x^2 + c*x^4)^p,x]

[Out] (x^6*(a + b*x^2 + c*x^4)^p*AppellF1[3, -p, -p, 4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(6*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^4 + bx^2 + a\right)^p x^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")

[Out] integral((c*x^4 + b*x^2 + a)^p*x^5, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2 + a\right)^p x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(c*x^4+b*x^2+a)^p,x, algorithm="giac")

[Out] integrate((c*x^4 + b*x^2 + a)^p*x^5, x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int x^5 \left(c\,x^4 + b\,x^2 + a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(c*x^4+b*x^2+a)^p,x)

[Out] int(x^5*(c*x^4+b*x^2+a)^p,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2 + a\right)^p x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^2 + a)^p*x^5, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5 \left(c\,x^4+b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2 + c*x^4)^p,x)
```

```
[Out] int(x^5*(a + b*x^2 + c*x^4)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \left(a+bx^2+cx^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] Integral(x**5*(a + b*x**2 + c*x**4)**p, x)
```

3.1117 $\int x^3 \left(a+bx^2+cx^4\right)^p \, dx$

Optimal. Leaf size=160

$$\frac{b2^{p-1}\left(a+bx^2+cx^4\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c(p+1)\sqrt{b^2-4ac}}+\frac{\left(a+bx^2+cx^4\right)^{p+1}}{4c(p+1)}$$

```
[Out] 1/4*(c*x^4+b*x^2+a)^(1+p)/c/(1+p)+2^(-1+p)*b*(c*x^4+b*x^2+a)^(1+p)*hypergeo
m([-p, 1+p],[2+p],1/2*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/(-4*a*c+b^2)^(1/2))*((
-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2))^(-1-p)/c/(1+p)/(-4*a*c+b
^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1114, 640, 624}

$$\frac{b2^{p-1}\left(a+bx^2+cx^4\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c(p+1)\sqrt{b^2-4ac}}+\frac{\left(a+bx^2+cx^4\right)^{p+1}}{4c(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (a + b*x^2 + c*x^4)^(1 + p)/(4*c*(1 + p)) + (2^(-1 + p)*b*(-((b - Sqrt[b^2
- 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^2 + c*x^4)^(1 + p
)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(2*
Sqrt[b^2 - 4*a*c])])/(c*Sqrt[b^2 - 4*a*c]*(1 + p))
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int x^3 \left(a+bx^2+cx^4\right)^p \, dx = \frac{1}{2}\text{Subst}\left(\int x\left(a+bx+cx^2\right)^p \, dx, x, x^2\right)$$
$$= \frac{\left(a+bx^2+cx^4\right)^{1+p}}{4c(1+p)} - \frac{b\,\text{Subst}\left(\int \left(a+bx+cx^2\right)^p \, dx, x, x^2\right)}{4c}$$
$$= \frac{\left(a+bx^2+cx^4\right)^{1+p}}{4c(1+p)} + \frac{2^{-1+p} b \left(-\frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{b^2-4ac}}\right)^{-1-p} \left(a+bx^2+cx^4\right)^{1+p} {}_2F_1\left(-p, 1+p\right.}{c\sqrt{b^2-4ac}\,(1+p)}$$

Mathematica [C] time = 0.20, size = 162, normalized size = 1.01

$$\frac{1}{4}x^4\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(2;-p,-p;3;-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^4*(a + b*x^2 + c*x^4)^p*AppellF1[2, -p, -p, 3, (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(4*((b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^4+bx^2+a\right)^p x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2 + a\right)^p x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^3, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^3 \left(c\,x^4 + b\,x^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2+a)^p,x)
[Out] int(x^3*(c*x^4+b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2 + a\right)^p x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left(c\,x^4 + b\,x^2 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2 + c*x^4)^p,x)
[Out] int(x^3*(a + b*x^2 + c*x^4)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a + bx^2 + cx^4\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)**p,x)
[Out] Integral(x**3*(a + b*x**2 + c*x**4)**p, x)
```

3.1118 $\int x \left(a + bx^2 + cx^4\right)^p \, dx$

Optimal. Leaf size=126

$$-\frac{2^p \left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1} \left(a + bx^2 + cx^4\right)^{p+1} {}_2F_1\left(-p, p+1; p+2; \frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{(p+1)\sqrt{b^2-4ac}}$$

```
[Out] -2^p*(c*x^4+b*x^2+a)^(1+p)*hypergeom([-p, 1+p],[2+p],1/2*(2*c*x^2+(-4*a*c+b
^2)^(1/2)+b)/(-4*a*c+b^2)^(1/2))*((-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b
^2)^(1/2))^(-1-p)/(1+p)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1107, 624}

$$-\frac{2^p \left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1} \left(a + bx^2 + cx^4\right)^{p+1} {}_2F_1\left(-p, p+1; p+2; \frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{(p+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] -((2^p*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a
 + b*x^2 + c*x^4)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(2*Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(1 + p)))
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int x\left(a+bx^2+cx^4\right)^p\,dx=\frac{1}{2}\operatorname{Subst}\left(\int\left(a+bx+cx^2\right)^p\,dx,x,x^2\right)$$

$$=-\frac{2^p\left(-\frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx^2+cx^4\right)^{1+p}\,{}_2F_1\left(-p,1+p;2+p;\frac{b+\sqrt{b^2-4ac}+2cx^2}{2\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(1+p)}$$

Mathematica [A] time = 0.10, size = 135, normalized size = 1.07

$$\frac{2^{p-2}\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p\,{}_2F_1\left(-p,p+1;p+2;\frac{-2cx^2-b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (2^(-2 + p)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(a + b*x^2 + c*x^4)^p*Hyperge
ometric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)/(2*Sqrt[b^2
 - 4*a*c])])/(c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c
])^p)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(cx^4+bx^2+a\right)^p x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^4+bx^2+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x \left(c\, x^{4}+b\, x^{2}+a \right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^p,x)
[Out] int(x*(c*x^4+b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{4}+bx^{2}+a\right)^{p} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^p*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(c\,x^4+b\,x^2+a\right)^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2 + c*x^4)^p,x)
[Out] int(x*(a + b*x^2 + c*x^4)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(a + bx^{2} + cx^{4}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**p,x)
[Out] Integral(x*(a + b*x**2 + c*x**4)**p, x)
```

3.1119 $\int \frac{(a+bx^2+cx^4)^p}{x}\,dx$

Optimal. Leaf size=152

$$\frac{4^{p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^pF_1\left(-2p;-p,-p;1-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^2},-\frac{b+\sqrt{b^2-4ac}}{2cx^2}\right)}{p}$$

```
[Out] 4^(-1+p)*(c*x^4+b*x^2+a)^p*AppellF1(-2*p,-p,-p,1-2*p,1/2*(-b-(-4*a*c+b^2)^(
1/2))/c/x^2,1/2*(-b+(-4*a*c+b^2)^(1/2))/c/x^2)/p/(((2*c*x^2-(-4*a*c+b^2)^(1
/2)+b)/c/x^2)^p)/(((2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/c/x^2)^p)
```

Rubi [A] time = 0.15, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1114, 758, 133}

$$\frac{4^{p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^pF_1\left(-2p;-p,-p;1-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^2},-\frac{b+\sqrt{b^2-4ac}}{2cx^2}\right)}{p}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^p/x,x]
```

```
[Out] (4^(-1 + p)*(a + b*x^2 + c*x^4)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -(b - Sqr
t[b^2 - 4*a*c])/(2*c*x^2), -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)])/(p*((b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c
*x^2))^p)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Dist[((1/(d + e*x))^(2*p)*(a + b*
x + c*x^2)^p)/(e*((e*(b - q + 2*c*x))/(2*c*(d + e*x)))^p*((e*(b + q + 2*c*x
))/(2*c*(d + e*x)))^p), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - (e*(b -
q))/(2*c))*x, x]^p*Simp[1 - (d - (e*(b + q))/(2*c))*x, x]^p, x], x, 1/(d +
e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
```

```
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m,
0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^p}{x}\,dx=\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^p}{x}\,dx,x,x^2\right)$$

$$=-\left(\left(2^{-1+2p}\left(\frac{1}{x^2}\right)^{2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)\right.\right.$$

$$=\frac{4^{-1+p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(-2p;-p,-p;1-2p;-\frac{b}{}\right.}{p}$$

Mathematica [A] time = 0.20, size = 152, normalized size = 1.00

$$\frac{4^{p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(-2p;-p,-p;1-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^2},\frac{\sqrt{b^2-4ac}-b}{2cx^2}\right)}{p}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2 + c*x^4)^p/x,x]
```

```
[Out] (4^(-1 + p)*(a + b*x^2 + c*x^4)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(b +
 Sqrt[b^2 - 4*a*c])/(c*x^2), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)])/(p*((b -
Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/
(c*x^2))^p)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(cx^4+bx^2+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2 + a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^4 + b\,x^2 + a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^p/x,x)
```

```
[Out] int((c*x^4+b*x^2+a)^p/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2 + a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4 + b\,x^2 + a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^p/x,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^p/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2 + cx^4\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**p/x,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**p/x, x)
```

3.1120 $\int \frac{(a+bx^2+cx^4)^p}{x^3} dx$

Optimal. Leaf size=166

$$-\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(1-2p;-p,-p;2(1-p);-\frac{b-\sqrt{b^2-4ac}}{2cx^2},-\frac{b+\sqrt{b^2-4ac}}{2cx^2}\right)}{(1-2p)x^2}$$

```
[Out] -2^(-1+2*p)*(c*x^4+b*x^2+a)^p*AppellF1(1-2*p,-p,-p,2-2*p,1/2*(-b-(-4*a*c+b^
2)^(1/2))/c/x^2,1/2*(-b+(-4*a*c+b^2)^(1/2))/c/x^2)/(1-2*p)/x^2/(((2*c*x^2-(
-4*a*c+b^2)^(1/2)+b)/c/x^2)^p)/(((2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/c/x^2)^p)
```

Rubi [A] time = 0.13, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1114, 758, 133}

$$-\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(1-2p;-p,-p;2(1-p);-\frac{b-\sqrt{b^2-4ac}}{2cx^2},-\frac{b+\sqrt{b^2-4ac}}{2cx^2}\right)}{(1-2p)x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^p/x^3,x]
```

```
[Out] -((2^(-1 + 2*p)*(a + b*x^2 + c*x^4)^p*AppellF1[1 - 2*p, -p, -p, 2*(1 - p),
-(b - Sqrt[b^2 - 4*a*c])/(2*c*x^2), -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)])/((
1 - 2*p)*x^2*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)/(c*x^2))^p))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Dist[((1/(d + e*x))^(2*p)*(a + b*
x + c*x^2)^p)/(e*((e*(b - q + 2*c*x))/(2*c*(d + e*x)))^p*((e*(b + q + 2*c*x
))/(2*c*(d + e*x)))^p), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - (e*(b -
q))/(2*c))*x, x]^p*Simp[1 - (d - (e*(b + q))/(2*c))*x, x]^p, x], x, 1/(d +
e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
```

```
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m,
0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^p}{x^3}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^p}{x^2}\,dx, x, x^2\right)$$

$$= -\left(\left(2^{-1+2p}\left(\frac{1}{x^2}\right)^{2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)\right.\right.$$

$$= -\frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(1-2p;-p,-p;2(1-\right.}{(1-2p)x^2}$$

Mathematica [A] time = 0.20, size = 163, normalized size = 0.98

$$\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(1-2p;-p,-p;2-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^2},\frac{\sqrt{b^2-4ac}-b}{2cx^2}\right)}{(2p-1)x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2 + c*x^4)^p/x^3,x]
```

```
[Out] (2^(-1 + 2*p)*(a + b*x^2 + c*x^4)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -1/2
*(b + Sqrt[b^2 - 4*a*c])/(c*x^2), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)])/((-1
 + 2*p)*x^2*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2 -
4*a*c] + 2*c*x^2)/(c*x^2))^p)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(cx^4+bx^2+a\right)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2 + a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^3, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^4 + b\,x^2 + a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^p/x^3,x)
```

```
[Out] int((c*x^4+b*x^2+a)^p/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2 + a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4 + b\,x^2 + a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^p/x^3,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^p/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**p/x**3,x)
```

```
[Out] Timed out
```

3.1121 $\int \frac{(a+bx^2+cx^4)^p}{x^5} \, dx$

Optimal. Leaf size=164

$$-\frac{4^{p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^pF_1\left(2(1-p);-p,-p;3-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^2},-\frac{b+\sqrt{b^2-4}}{2cx^2}\right.}{(1-p)x^4}$$

```
[Out] -4^(-1+p)*(c*x^4+b*x^2+a)^p*AppellF1(2-2*p,-p,-p,3-2*p,1/2*(-b-(-4*a*c+b^2)
^(1/2))/c/x^2,1/2*(-b+(-4*a*c+b^2)^(1/2))/c/x^2)/(1-p)/x^4/(((2*c*x^2-(-4*a
*c+b^2)^(1/2)+b)/c/x^2)^p)/(((2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/c/x^2)^p)
```

Rubi [A] time = 0.13, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1114, 758, 133}

$$-\frac{4^{p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^pF_1\left(2(1-p);-p,-p;3-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^2},-\frac{b+\sqrt{b^2-4}}{2cx^2}\right.}{(1-p)x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^p/x^5,x]
```

```
[Out] -((4^(-1 + p)*(a + b*x^2 + c*x^4)^p*AppellF1[2*(1 - p), -p, -p, 3 - 2*p, -(
b - Sqrt[b^2 - 4*a*c])/(2*c*x^2), -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)])/((1
- p)*x^4*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(c*x^2))^p))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Dist[((1/(d + e*x))^(2*p)*(a + b*
x + c*x^2)^p)/(e*((e*(b - q + 2*c*x))/(2*c*(d + e*x)))^p*((e*(b + q + 2*c*x
))/(2*c*(d + e*x)))^p), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - (e*(b -
q))/(2*c))*x, x]^p*Simp[1 - (d - (e*(b + q))/(2*c))*x, x]^p, x], x, 1/(d +
e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
```

```
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m,
0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^p}{x^5}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^p}{x^3}\,dx, x, x^2\right)$$

$$= -\left(\left(2^{-1+2p}\left(\frac{1}{x^2}\right)^{2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)\right.\right.$$

$$= -\frac{4^{-1+p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(2(1-p);-p,-p;3-\right.}{(1-p)x^4}$$

Mathematica [A] time = 0.22, size = 159, normalized size = 0.97

$$\frac{4^{p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(2-2p;-p,-p;3-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^2},\frac{\sqrt{b^2-4ac}-b}{2cx^2}\right)}{(p-1)x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2 + c*x^4)^p/x^5,x]
```

```
[Out] (4^(-1 + p)*(a + b*x^2 + c*x^4)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, -1/2*(
b + Sqrt[b^2 - 4*a*c])/(c*x^2), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)])/((-1 +
 p)*x^4*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(c*x^2))^p)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^4+bx^2+a\right)^p}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^5,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2 + a\right)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^5,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^5, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^4 + b\,x^2 + a\right)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^p/x^5,x)
```

```
[Out] int((c*x^4+b*x^2+a)^p/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^2 + a\right)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^5,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4 + b\,x^2 + a\right)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^p/x^5,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^p/x^5, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**p/x**5,x)
```

```
[Out] Timed out
```

3.1122 $\int x^4 \left(a+bx^2+cx^4\right)^p \, dx$

Optimal. Leaf size=138

$$\frac{1}{5}x^5\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{5}{2};-p,-p;\frac{7}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2c}{b+\sqrt{b}}\right.$$

```
[Out] 1/5*x^5*(c*x^4+b*x^2+a)^p*AppellF1(5/2,-p,-p,7/2,-2*c*x^2/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^
p)/((1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] time = 0.10, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1141, 510}

$$\frac{1}{5}x^5\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{5}{2};-p,-p;\frac{7}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2c}{b+\sqrt{b}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^5*(a + b*x^2 + c*x^4)^p*AppellF1[5/2, -p, -p, 7/2, (-2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*(1 + (2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int x^4 \left(a+bx^2+cx^4\right)^p \, dx = \left(\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p\right)\int x^4\left(1+\frac{2c}{b-\sqrt{b^2}}\right.$$
$$= \frac{1}{5}x^5\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{5}{2};-p,-p;\frac{7}{2}\right.$$

Mathematica [A] time = 0.17, size = 166, normalized size = 1.20

$$\frac{1}{5}x^5\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{5}{2};-p,-p;\frac{7}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^5*(a + b*x^2 + c*x^4)^p*AppellF1[5/2, -p, -p, 7/2, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(5*((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^4+bx^2+a\right)^p x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4+bx^2+a\right)^p x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^4, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^4 \left(c\,x^4+b\,x^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2+a)^p,x)
```

```
[Out] int(x^4*(c*x^4+b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2 + a\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\left(c\,x^4 + b\,x^2 + a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2 + c*x^4)^p,x)
```

```
[Out] int(x^4*(a + b*x^2 + c*x^4)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\left(a + bx^2 + cx^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] Integral(x**4*(a + b*x**2 + c*x**4)**p, x)
```

3.1123 $\int x^2 \left(a+bx^2+cx^4\right)^p \, dx$

Optimal. Leaf size=138

$$\frac{1}{3}x^3\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2}}\right.$$

```
[Out] 1/3*x^3*(c*x^4+b*x^2+a)^p*AppellF1(3/2,-p,-p,5/2,-2*c*x^2/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^
p)/((1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1141, 510}

$$\frac{1}{3}x^3\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^3*(a + b*x^2 + c*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*(1 + (2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int x^2 \left(a+bx^2+cx^4\right)^p \, dx = \left(\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p\right)\int x^2\left(1+\frac{2}{b-\sqrt{b}}\right.$$
$$= \frac{1}{3}x^3\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{3}{2};-p,-p;\right.$$

Mathematica [A] time = 0.15, size = 166, normalized size = 1.20

$$\frac{1}{3}x^3\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^3*(a + b*x^2 + c*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(3*((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^4+bx^2+a\right)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4+bx^2+a\right)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^2, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x^2 \left(c\,x^4+b\,x^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)^p,x)
```

```
[Out] int(x^2*(c*x^4+b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^4 + bx^2 + a)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(c\,x^4 + b\,x^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2 + c*x^4)^p,x)
```

```
[Out] int(x^2*(a + b*x^2 + c*x^4)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + bx^2 + cx^4\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] Integral(x**2*(a + b*x**2 + c*x**4)**p, x)
```

3.1124 $\int \left(a+bx^2+cx^4\right)^p\,dx$

Optimal. Leaf size=133

$$x\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-}}\right.$$

```
[Out] x*(c*x^4+b*x^2+a)^p*AppellF1(1/2,-p,-p,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^p)/((1
+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] time = 0.06, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1105, 429}

$$x\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x*(a + b*x^2 + c*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*c*x^2)/(b - Sqrt[b^
2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/((1 + (2*c*x^2)/(b - Sqrt
[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (
2*c*x^2)/(b + q))^FracPart[p]*(1 + (2*c*x^2)/(b - q))^FracPart[p]), Int[(1
+ (2*c*x^2)/(b + q))^p*(1 + (2*c*x^2)/(b - q))^p, x], x]] /; FreeQ[{a, b, c
, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(a+bx^2+cx^4\right)^p\,dx = \left(\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p\right)\int\left(1+\frac{2cx^2}{b-\sqrt{b^2-4a}}\right.$$
$$= x\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{}{b-}\right.$$

Mathematica [A] time = 0.16, size = 161, normalized size = 1.21

$$x\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{}{\sqrt{b^2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x*(a + b*x^2 + c*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(((b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^4+bx^2+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4+bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \left(c\,x^4+b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^p,x)
```

```
[Out] int((c*x^4+b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^2 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c\,x^4 + b\,x^2 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^p,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + bx^2 + cx^4\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**p,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**p, x)
```

3.1125 $\int \frac{(a+bx^2+cx^4)^p}{x^2}\,dx$

Optimal. Leaf size=136

$$-\frac{\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^pF_1\left(-\frac{1}{2};-p,-p;\frac{1}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{x}$$

```
[Out] -(c*x^4+b*x^2+a)^p*AppellF1(-1/2,-p,-p,1/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/x/((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^p)/(
(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] time = 0.09, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1141, 510}

$$-\frac{\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^pF_1\left(-\frac{1}{2};-p,-p;\frac{1}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^p/x^2,x]
```

```
[Out] -(((a + b*x^2 + c*x^4)^p*AppellF1[-1/2, -p, -p, 1/2, (-2*c*x^2)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(x*(1 + (2*c*x^2)/(b - S
qrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^p}{x^2}\,dx = \left(\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p\right)\int \frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^p}{x}$$

$$= -\frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(-\frac{1}{2};-p,-p;\frac{1}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\right.}{x}$$

Mathematica [A] time = 0.16, size = 164, normalized size = 1.21

$$-\frac{\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(-\frac{1}{2};-p,-p;\frac{1}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2 + c*x^4)^p/x^2,x]
```

```
[Out] -(((a + b*x^2 + c*x^4)^p*AppellF1[-1/2, -p, -p, 1/2, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(x*((b - Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c]))^p))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx^4+bx^2+a)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^4+bx^2+a)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^2, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^p/x^2,x)
[Out] int((c*x^4+b*x^2+a)^p/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{4}+bx^{2}+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^2,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^p/x^2,x)
[Out] int((a + b*x^2 + c*x^4)^p/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**p/x**2,x)
[Out] Timed out
```

3.1126 $\int \frac{(a+bx^2+cx^4)^p}{x^4}\,dx$

Optimal. Leaf size=138

$$-\frac{\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(-\frac{3}{2};-p,-p;-\frac{1}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3x^3}$$

```
[Out] -1/3*(c*x^4+b*x^2+a)^p*AppellF1(-3/2,-p,-p,-1/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1
/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/x^3/((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
))^p)/((1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1141, 510}

$$-\frac{\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(-\frac{3}{2};-p,-p;-\frac{1}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^p/x^4,x]
```

```
[Out] -((a + b*x^2 + c*x^4)^p*AppellF1[-3/2, -p, -p, -1/2, (-2*c*x^2)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*x^3*(1 + (2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^p}{x^4}\,dx = \left(\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p\right)\int \frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^p \left(\right.}{x^4}$$

$$= -\frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(-\frac{3}{2};-p,-p;-\frac{1}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\right.}{3x^3}$$

Mathematica [A] time = 0.17, size = 166, normalized size = 1.20

$$-\frac{\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(-\frac{3}{2};-p,-p;-\frac{1}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{3x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2 + c*x^4)^p/x^4,x]
```

```
[Out] -1/3*((a + b*x^2 + c*x^4)^p*AppellF1[-3/2, -p, -p, -1/2, (-2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(x^3*((b - Sqrt[b^2
- 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(cx^4+bx^2+a\right)^p}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^4,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^2+a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^4, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{p}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^p/x^4,x)
[Out] int((c*x^4+b*x^2+a)^p/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{4}+bx^{2}+a\right)^{p}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^4,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{p}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^p/x^4,x)
[Out] int((a + b*x^2 + c*x^4)^p/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**p/x**4,x)
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
    func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```